Computer Algebra Independent Integration Tests

Summer 2023 edition

3-Logarithms/57-3.1.4-f-x-^m-d+e-x^r-^q-a+b-log-c-x^n-^p

Nasser M. Abbasi

September 5, 2023 Compiled on September 5, 2023 at 11:35am

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [456]. This is test number [57].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (456)	0.00 (0)
Mathematica	98.46 (449)	1.54 (7)
Maple	73.03 (333)	26.97 (123)
Sympy	62.06 (283)	37.94 (173)
Fricas	61.40 (280)	38.60 (176)
Maxima	49.34 (225)	50.66 (231)
Giac	41.67 (190)	58.33 (266)
Mupad	32.02 (146)	67.98 (310)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

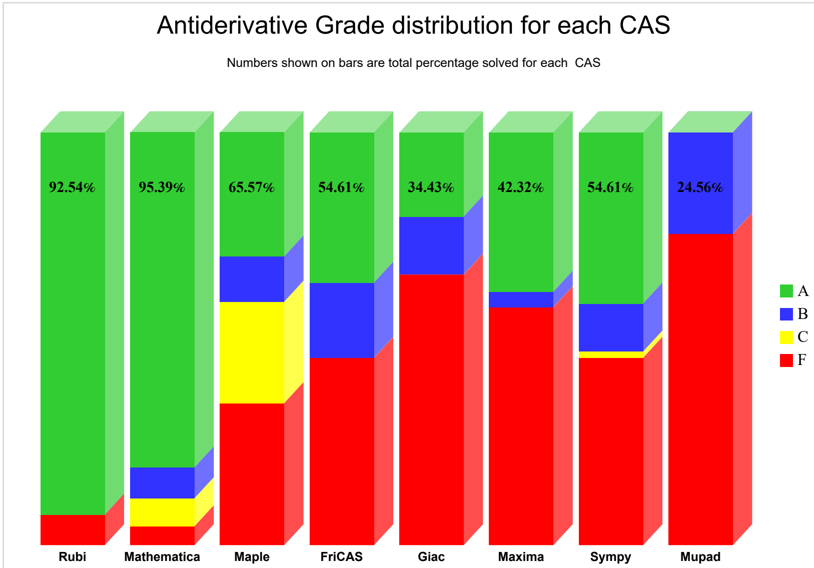
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

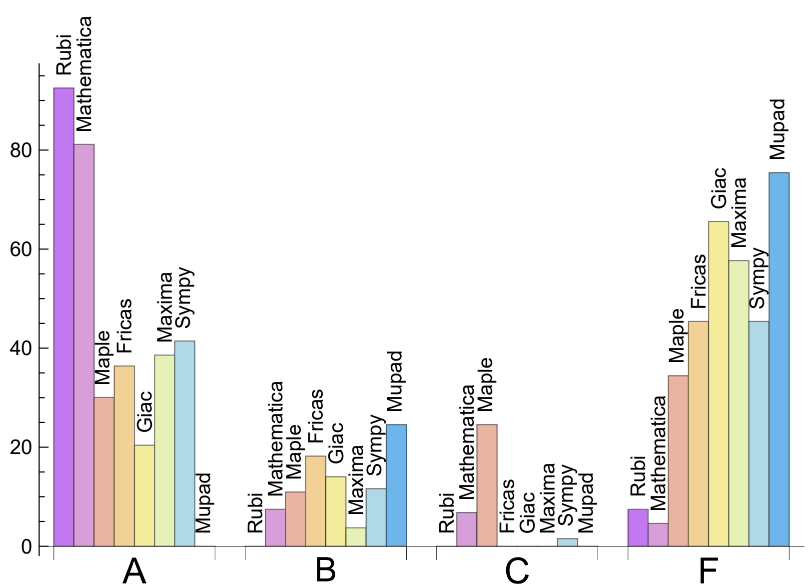
System	% A grade	% B grade	% C grade	% F grade
Rubi	92.544	0.000	0.000	7.456
Mathematica	81.140	7.456	6.798	4.605
Sympy	41.447	11.623	1.535	45.395
Maxima	38.596	3.728	0.000	57.675
Fricas	36.404	18.202	0.000	45.395
Maple	30.044	10.965	24.561	34.430
Giac	20.395	14.035	0.000	65.570
Mupad	0.000	24.561	0.000	75.439

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	7	100.00	0.00	0.00
Rubi	0	0.00	0.00	0.00
Maple	123	100.00	0.00	0.00
Fricas	176	92.61	0.00	7.39
Sympy	173	72.25	23.70	4.05
Maxima	231	56.28	0.43	43.29
Giac	266	99.62	0.00	0.38
Mupad	310	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.15
Maxima	0.22
Fricas	0.31
Mathematica	0.34
Giac	0.35
Mupad	0.44
Maple	2.47
Sympy	25.72

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	85.30	1.11	82.00	1.09
Maxima	123.27	1.47	109.00	1.19
Rubi	150.20	1.00	126.00	1.00
Mathematica	167.43	1.31	125.00	1.03
Giac	222.13	2.09	132.00	1.44
Fricas	266.51	2.18	159.00	1.74
Maple	344.15	2.50	177.00	1.71
Sympy	402.59	3.29	207.00	1.87

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

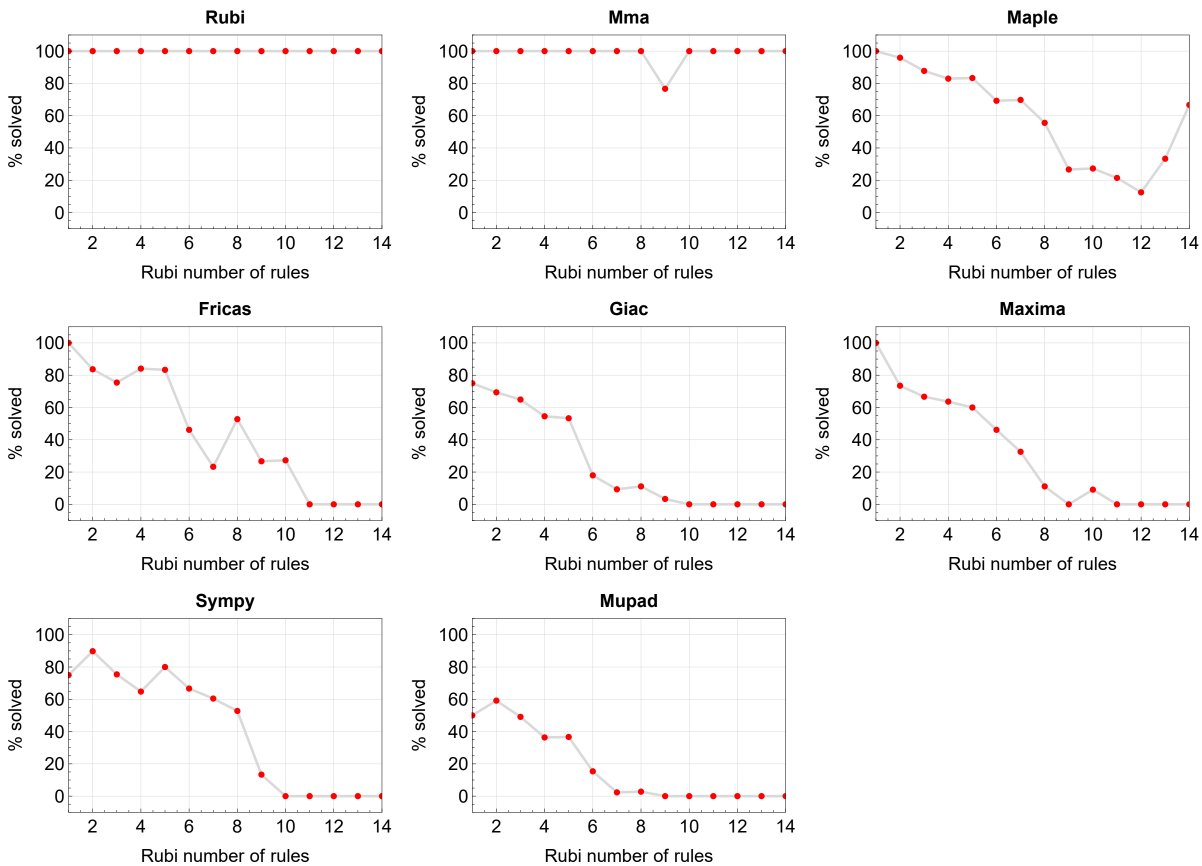


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

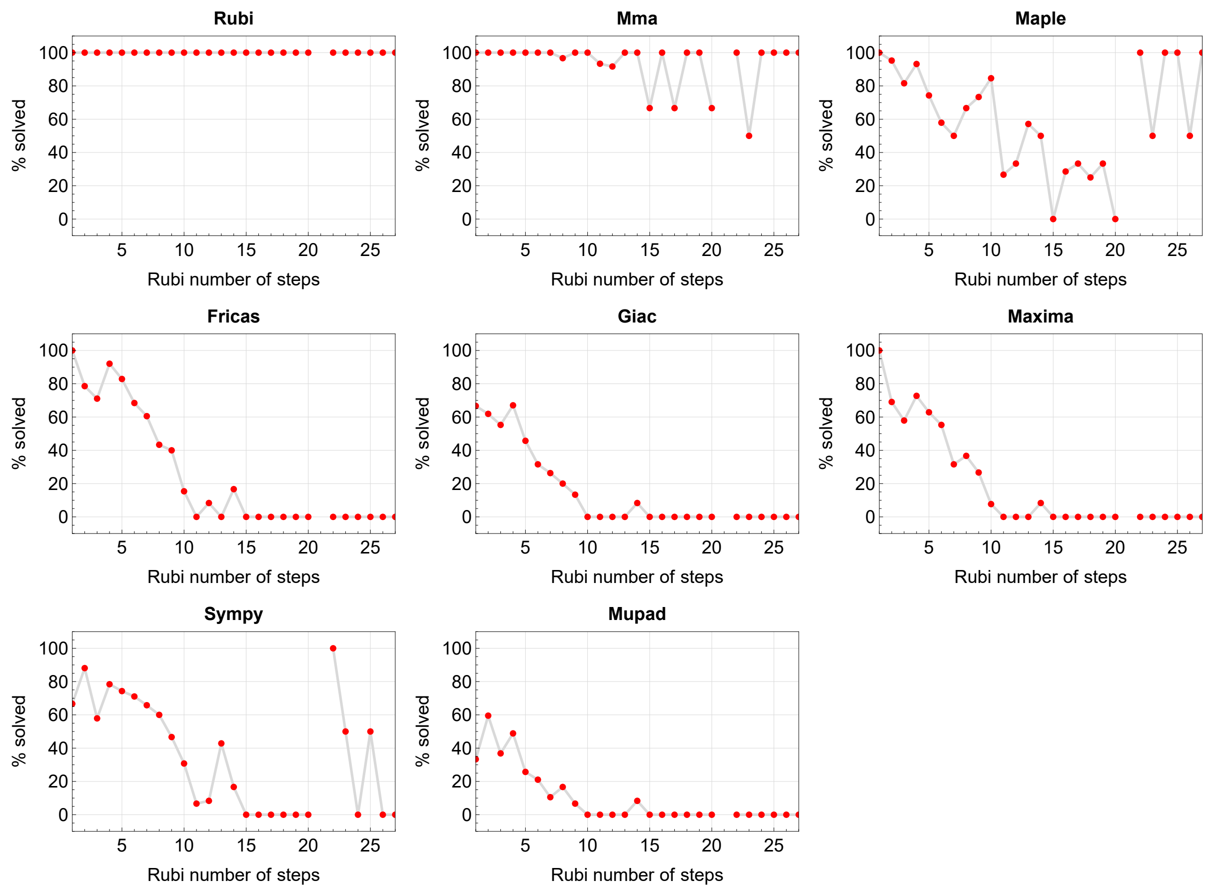


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

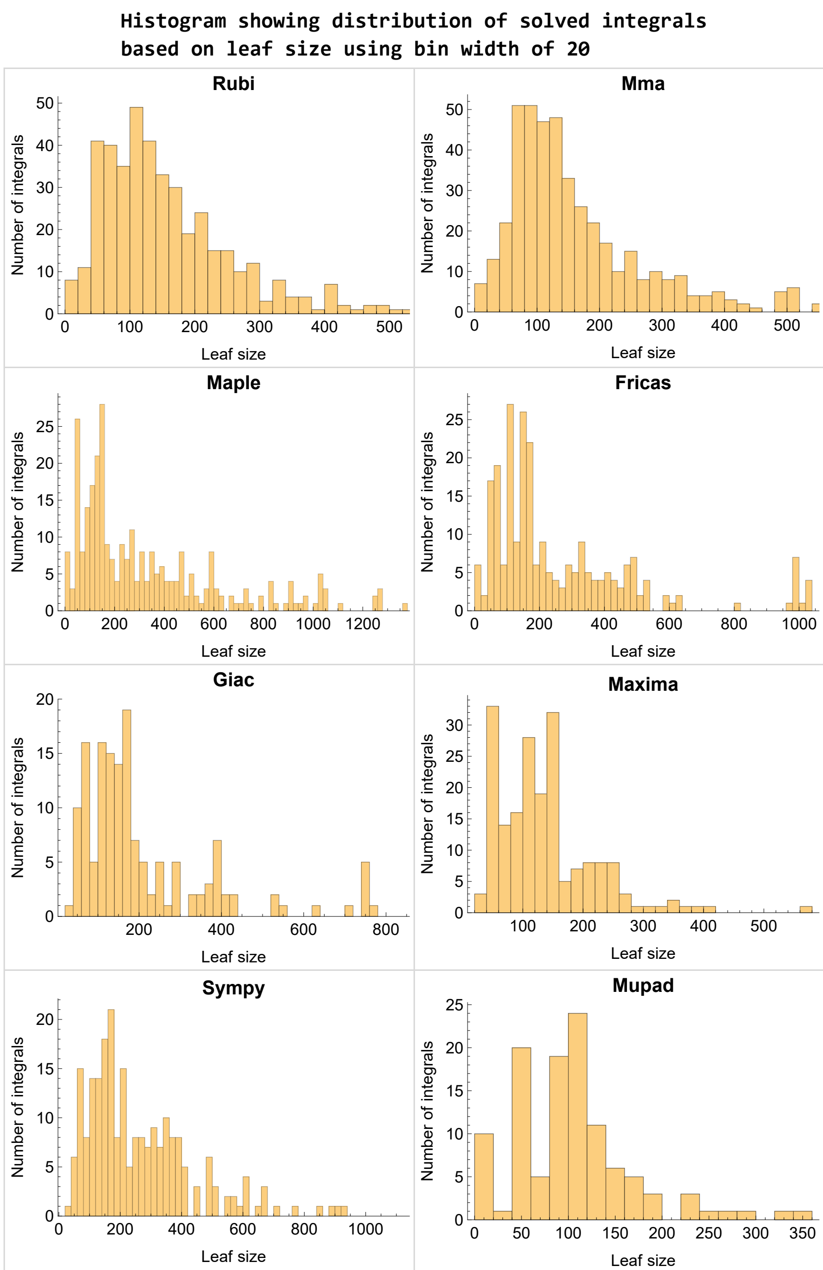


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

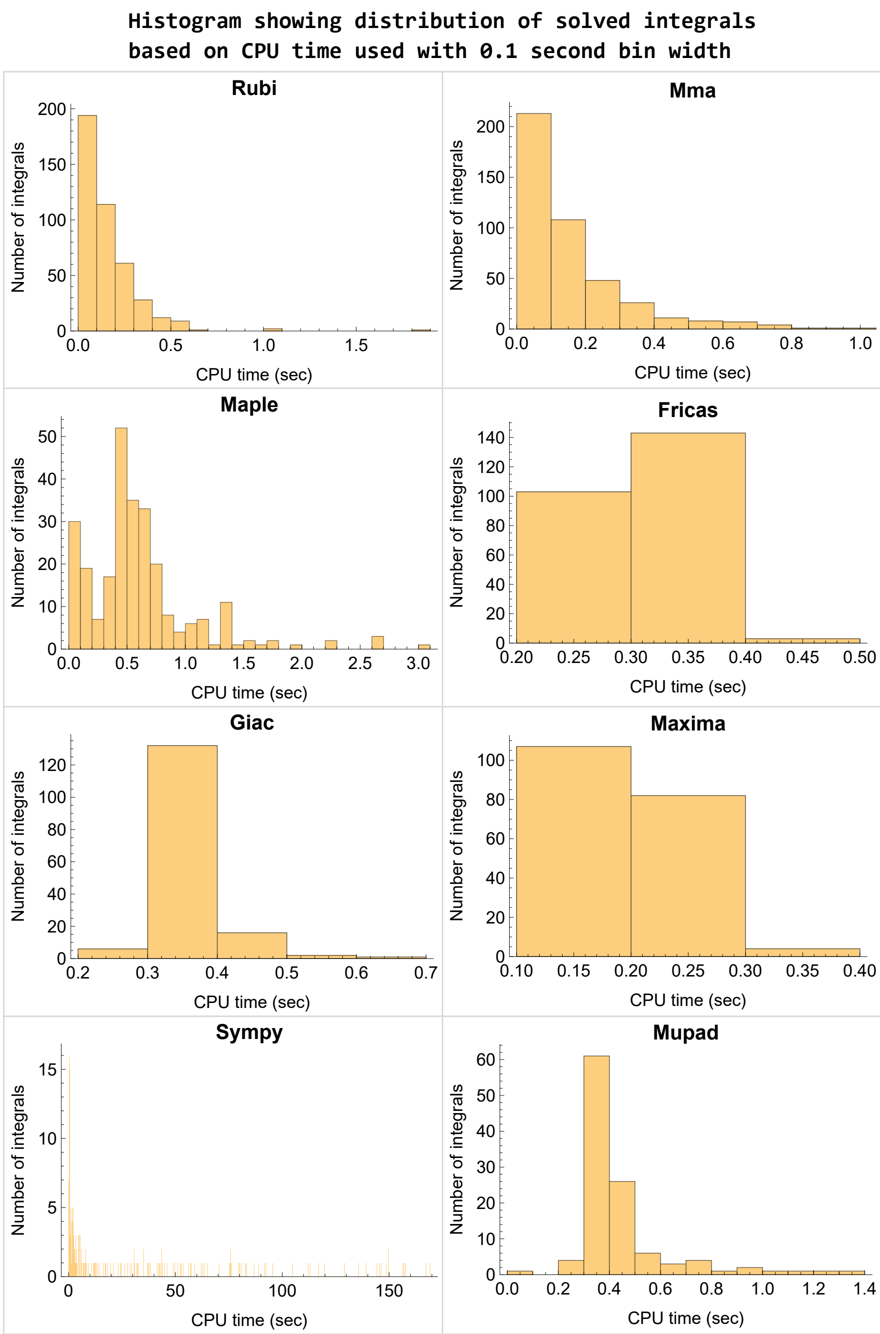


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

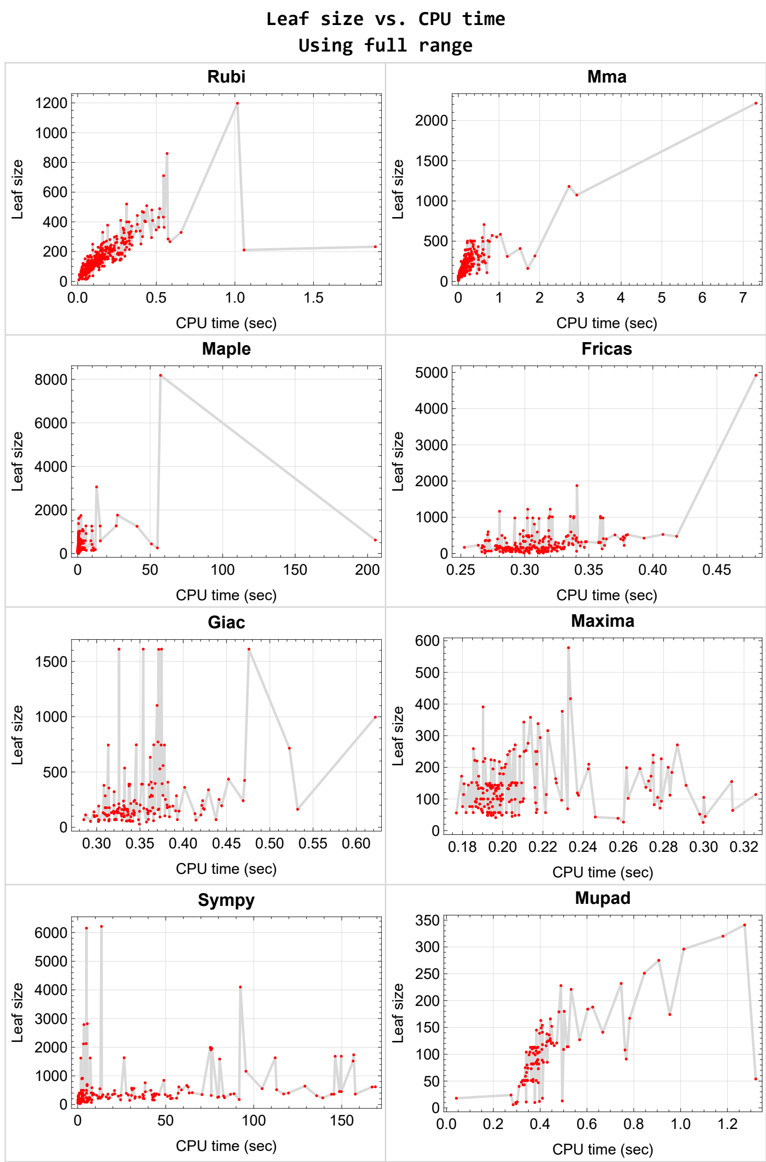


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{127, 128, 129, 157, 158, 159, 160, 161, 166, 167, 168, 170, 249, 250, 322, 323, 327, 328, 406, 407, 409, 410, 411, 412, 413, 414, 416, 417, 418, 419, 444, 445, 452, 453}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {166, 167, 168, 170, 322, 323, 406, 407, 409, 410, 411, 412, 413, 414, 416, 417, 418, 419, 444, 445}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {355, 363, 364, 365, 366, 408, 415, 424, 425, 426, 430, 431, 432}

Maple {31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 71, 72, 73, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 240, 317, 326, 329, 330, 331, 332, 333, 334, 335, 336, 408, 415, 424, 425, 426, 430, 455, 456}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

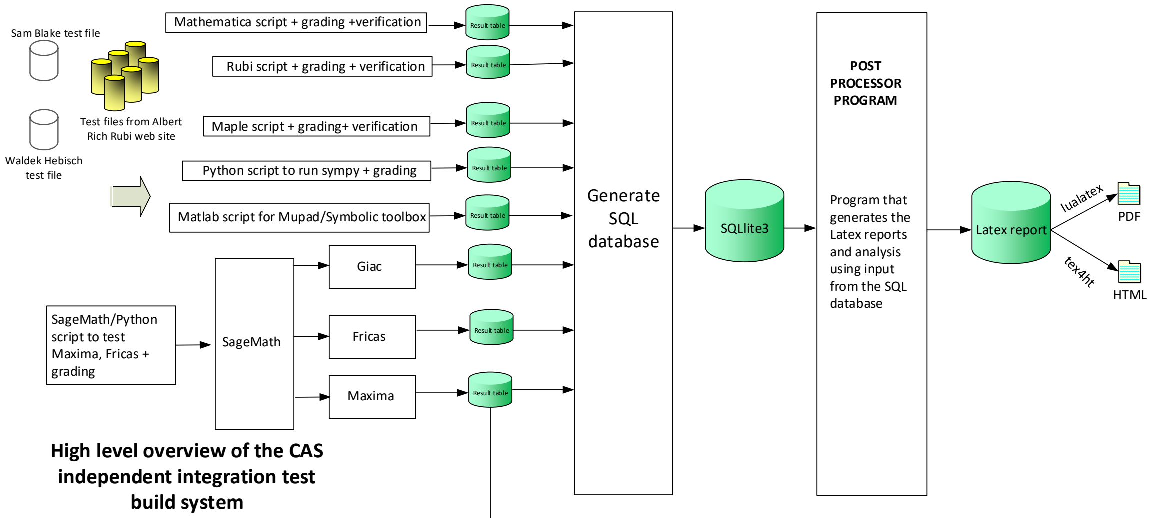
```
integrand = evalin(symengine,'cos(x)*sin(x)')
```

```
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 162, 163, 164, 165, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 408, 415, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 446, 447, 448, 449, 450, 451, 454, 455, 456 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 162, 163, 164, 165, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 220, 223, 226, 227, 228, 229, 230, 232, 233, 241, 242, 243, 245, 246, 247, 251, 252, 253, 260, 261, 262, 263, 264, 265, 272, 273, 274, 275, 276, 277, 278, 282, 283, 284, 285, 286, 287, 288, 289, 293, 294, 295, 296, 297, 298, 299, 300, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 408, 415, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 432, 440, 441, 442, 443, 446, 447, 448, 449, 450, 451, 454, 455, 456 }

B grade { 56, 65, 115, 121, 166, 167, 168, 170, 213, 236, 237, 238, 239, 240, 244, 322, 323, 363, 406, 407, 409, 410, 411, 412, 413, 414, 416, 417, 418, 419, 430, 431, 444, 445 }

C grade { 221, 222, 224, 225, 231, 234, 235, 248, 254, 255, 256, 257, 258, 259, 266, 267, 268, 269, 270, 271, 279, 280, 281, 290, 291, 292, 301, 302, 303, 304, 312 }

F normal fail { 433, 434, 435, 436, 437, 438, 439 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 42, 49, 57, 67, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 120, 147, 165, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 223, 233, 241, 243, 275, 321, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 359, 360, 361, 362, 370, 371, 372, 376, 377, 378, 382, 395, 420, 421, 422, 423, 427, 428, 429, 443 }

B grade { 48, 56, 58, 65, 66, 68, 69, 70, 162, 163, 164, 232, 318, 319, 320, 367, 368, 369, 373, 374, 375, 379, 380, 381, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 440, 441, 442, 454 }

C grade { 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 71, 72, 73, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 240, 242, 244, 245, 246, 317, 326, 329, 330, 331, 332, 333, 334, 335, 336, 408, 415, 424, 425, 426, 430, 455, 456 }

F normal fail { 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 169, 247, 248, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 324, 325, 350, 355, 356, 357, 358, 363, 364, 365, 366, 431, 432, 433, 434, 435, 436, 437, 438, 439, 446, 447, 448, 449, 450, 451 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 42, 49, 57, 58, 67, 68, 74, 75, 81, 82, 83, 130, 131, 132, 133, 137, 138, 139, 140, 144, 145, 146, 147, 151, 152, 153, 154, 165, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 223, 233, 241, 242, 251, 252, 253, 260, 261, 262, 263, 264, 265, 272, 273, 274, 275, 276, 277, 278, 283, 284, 285, 286, 287, 288, 289, 293, 294, 295, 296, 297, 298, 299, 300, 305, 306, 307, 308, 309, 310, 315, 316, 317, 321, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 370, 382, 395, 408, 420, 421, 422, 423, 424, 443 }

B grade { 22, 48, 56, 65, 66, 69, 70, 76, 77, 78, 79, 80, 84, 85, 86, 87, 88, 89, 90, 91, 162, 163, 164, 198, 232, 318, 319, 320, 345, 346, 347, 350, 365, 366, 367, 368, 369, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 415, 425, 426, 427, 428, 429, 430, 431, 432, 440, 441, 442, 454 }

C grade { }

F normal fail { 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 71, 72, 73, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 134, 135, 136, 141, 142, 143, 148, 149, 150, 155, 156, 169, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 240, 243, 244, 245, 246, 247, 248, 254, 255, 256, 257, 258, 259, 266, 267, 268, 269, 270, 271, 279, 280, 281, 282, 290, 291, 292, 301, 302, 303, 304, 311, 312, 313, 314, 324, 325, 326, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 446, 447, 448, 449, 450, 451, 455, 456 }

F(-1) timedout fail { }

F(-2) exception fail { 124, 125, 126, 127, 128, 129, 433, 434, 435, 436, 437, 438, 439 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 42, 49, 57, 58, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 120, 130, 131, 132, 133, 137, 138, 139, 140, 144, 145, 146, 147, 151, 152, 153, 154, 162, 163, 164, 165, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 223, 233, 244, 275, 299, 309, 310, 315, 317, 318, 319, 320, 321, 337, 338, 339, 340, 341, 342, 343, 344, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 367, 368, 369, 370, 373, 374, 375, 379, 380, 381, 382, 385, 386, 387, 392, 393, 394, 395, 398, 399, 400, 421, 422, 423, 427, 428, 429, 440, 441, 442, 443 }

B grade { 48, 56, 65, 66, 70, 74, 75, 232, 241, 242, 243, 345, 346, 347, 348, 349, 454 }

C grade { }

F normal fail { 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 59, 60, 61, 62, 63, 71, 72, 73, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 134, 135, 136, 141, 142, 143, 148, 149, 150, 155, 156, 169, 210, 211, 212, 213, 214, 215, 221, 222, 224, 225, 231, 234, 235, 305, 306, 311, 312, 313, 314, 316, 329, 330, 331, 332, 333, 334, 335, 336, 350, 355, 363, 364, 365, 366, 408, 415, 420, 424, 425, 426, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 446, 447, 455, 456 }

F(-1) timedout fail { 64 }

F(-2) exception fail { 216, 217, 218, 219, 220, 226, 227, 228, 229, 230, 236, 237, 238, 239, 240, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 300, 301, 302, 303, 304, 307, 308, 324, 325, 326, 371, 372, 376, 377, 378, 383, 384, 388, 389, 390, 391, 396, 397, 401, 402, 403, 404, 405, 448, 449, 450, 451, 452, 453 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 28, 29, 30, 42, 49, 57, 58, 67, 68, 69, 70, 81, 144, 145, 146, 147, 154, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 223, 233, 251, 252, 253, 275, 317, 352, 353, 354, 361, 375, 454 }

B grade { 22, 27, 48, 56, 65, 66, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 162, 163, 164, 165, 198, 232, 318, 319, 320, 321, 351, 356, 357, 358, 359, 360, 362, 367, 368, 369, 371, 372, 373, 374, 376, 377, 378, 379, 380, 381, 385, 386, 387, 392, 393, 394, 398, 399, 400, 440, 441, 442, 443 }

C grade { }

F normal fail { 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 71, 72, 73, 74, 75, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 155, 156, 169, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 324, 325, 326, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 355, 363, 364, 365, 366, 370, 382, 383, 384, 388, 389, 390, 391, 395, 396, 397, 401, 402, 403, 404, 405, 408, 415, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 446, 447, 448, 449, 450, 451, 455, 456 }

F(-1) timedout fail { }

F(-2) exception fail { 170 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 42, 48, 49, 56, 57, 58, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 223, 232, 233, 241, 242, 243, 244, 345, 346, 347, 348, 349, 454 }

C grade { }

F normal fail { }

F(-1) timedout fail { 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 71, 72, 73, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 162, 163, 164, 165, 169, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 240, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 408, 415, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 446, 447, 448, 449, 450, 451, 455, 456 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 120, 130, 131, 132, 133, 138, 139, 140, 144, 145, 146, 147, 151, 152, 153, 154, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 221, 225, 231, 234, 242, 251, 252, 253, 263, 264, 265, 275, 276, 277, 278, 286, 287, 288, 289, 297, 298, 299, 300, 317, 329, 330, 331, 332, 335, 336, 337, 338, 339, 340, 343, 344, 352, 353, 360, 361, 386, 387, 388, 389, 390, 394, 395, 396, 400, 401, 402, 408, 415, 421, 424, 425 }

B grade { 42, 48, 49, 56, 57, 58, 65, 66, 67, 68, 69, 70, 162, 163, 164, 165, 223, 232, 233, 318, 319, 320, 321, 351, 354, 359, 362, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 392, 422, 423, 427, 428, 429, 443, 454 }

C grade { 35, 241, 243, 334, 342, 348, 349 }

F normal fail { 34, 41, 43, 74, 75, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 134, 135, 136, 142, 143, 148, 149, 150, 155, 156, 216, 217, 218, 219, 220, 222, 226, 227, 228, 229, 230, 236, 237, 238, 239, 244, 245, 246, 247, 248, 254, 255, 256, 257, 258, 259, 260, 261, 262, 266, 267, 270, 271, 272, 273, 279, 280, 281, 282, 283, 284, 285, 290, 291, 292, 293, 294, 295, 304, 305, 306, 309, 310, 311, 314, 315, 316, 326, 333, 341, 350, 355, 356, 357, 363, 364, 430, 431, 434, 435, 436, 437, 450, 451, 455, 456 }

F(-1) timedout fail { 137, 141, 214, 215, 224, 235, 240, 268, 269, 274, 296, 301, 302, 303, 307, 308, 312, 313, 324, 325, 358, 365, 366, 385, 391, 393, 397, 398, 399, 403, 404, 405, 426, 432, 433, 438, 439, 446, 447, 448, 449 }

F(-2) exception fail { 345, 346, 347, 420, 440, 441, 442 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	58	57	69	66	69	51
N.S.	1	1.00	1.00	1.21	1.19	1.44	1.38	1.44	1.06
time (sec)	N/A	0.034	0.020	0.126	0.198	0.286	0.341	0.321	0.337

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	45	58	57	69	66	69	51
N.S.	1	1.00	0.94	1.21	1.19	1.44	1.38	1.44	1.06
time (sec)	N/A	0.033	0.019	0.070	0.196	0.314	0.243	0.322	0.331

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	58	57	69	66	69	51
N.S.	1	1.00	1.00	1.21	1.19	1.44	1.38	1.44	1.06
time (sec)	N/A	0.027	0.015	0.072	0.194	0.272	0.178	0.393	0.344

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	55	50	49	61	56	58	43
N.S.	1	1.00	1.15	1.04	1.02	1.27	1.17	1.21	0.90
time (sec)	N/A	0.017	0.004	0.086	0.199	0.312	0.141	0.314	0.332

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	43	46	41	45	65	47	40
N.S.	1	1.00	0.98	1.05	0.93	1.02	1.48	1.07	0.91
time (sec)	N/A	0.036	0.004	0.078	0.197	0.290	0.169	0.358	0.310

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	50	48	53	49	50	53	55	59
N.S.	1	1.04	1.00	1.10	1.02	1.04	1.10	1.15	1.23
time (sec)	N/A	0.041	0.020	0.079	0.195	0.293	1.640	0.342	0.337

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	41	47	57	53	58	57	47
N.S.	1	1.00	0.68	0.78	0.95	0.88	0.97	0.95	0.78
time (sec)	N/A	0.032	0.018	0.056	0.193	0.281	0.245	0.314	0.392

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	47	48	57	57	68	59	49
N.S.	1	1.00	0.82	0.84	1.00	1.00	1.19	1.04	0.86
time (sec)	N/A	0.033	0.018	0.054	0.194	0.290	0.317	0.366	0.369

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	81	101	100	118	121	123	82
N.S.	1	1.00	1.09	1.36	1.35	1.59	1.64	1.66	1.11
time (sec)	N/A	0.059	0.037	1.543	0.195	0.284	0.476	0.341	0.394

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	81	101	100	118	116	123	82
N.S.	1	1.00	1.09	1.36	1.35	1.59	1.57	1.66	1.11
time (sec)	N/A	0.051	0.030	0.611	0.190	0.286	0.347	0.414	0.369

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	81	101	100	118	121	123	82
N.S.	1	1.00	1.09	1.36	1.35	1.59	1.64	1.66	1.11
time (sec)	N/A	0.041	0.027	0.865	0.187	0.293	0.268	0.331	0.373

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	77	91	90	110	102	109	73
N.S.	1	1.00	1.10	1.30	1.29	1.57	1.46	1.56	1.04
time (sec)	N/A	0.025	0.029	0.399	0.194	0.293	0.188	0.335	0.362

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	83	97	84	98	131	96	75
N.S.	1	1.00	1.04	1.21	1.05	1.22	1.64	1.20	0.94
time (sec)	N/A	0.049	0.036	0.619	0.189	0.296	0.254	0.339	0.337

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	76	96	83	98	112	93	99
N.S.	1	1.00	0.97	1.23	1.06	1.26	1.44	1.19	1.27
time (sec)	N/A	0.053	0.038	0.452	0.193	0.301	0.329	0.373	0.353

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	97	90	101	99	102	99
N.S.	1	1.00	1.00	1.15	1.07	1.20	1.18	1.21	1.18
time (sec)	N/A	0.055	0.037	0.612	0.187	0.295	2.069	0.312	0.379

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	76	91	100	103	104	110	82
N.S.	1	1.00	1.01	1.21	1.33	1.37	1.39	1.47	1.09
time (sec)	N/A	0.047	0.029	0.588	0.189	0.294	0.321	0.343	0.392

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	80	91	100	106	122	111	85
N.S.	1	1.00	0.84	0.96	1.05	1.12	1.28	1.17	0.89
time (sec)	N/A	0.053	0.031	0.577	0.193	0.277	0.413	0.311	0.430

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	80	91	100	106	117	111	85
N.S.	1	1.00	0.84	0.96	1.05	1.12	1.23	1.17	0.89
time (sec)	N/A	0.071	0.033	0.566	0.195	0.278	0.539	0.314	0.414

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	133	144	143	167	170	177	113
N.S.	1	1.00	1.33	1.44	1.43	1.67	1.70	1.77	1.13
time (sec)	N/A	0.077	0.049	11.004	0.198	0.285	0.659	0.365	0.371

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	133	144	143	167	175	177	113
N.S.	1	1.00	1.33	1.44	1.43	1.67	1.75	1.77	1.13
time (sec)	N/A	0.069	0.040	5.903	0.200	0.288	0.502	0.365	0.371

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	130	142	141	167	167	174	112
N.S.	1	1.00	1.07	1.16	1.16	1.37	1.37	1.43	0.92
time (sec)	N/A	0.062	0.087	0.670	0.194	0.297	0.352	0.332	0.377

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	110	134	133	159	156	163	104
N.S.	1	1.00	1.29	1.58	1.56	1.87	1.84	1.92	1.22
time (sec)	N/A	0.030	0.033	0.456	0.198	0.302	0.265	0.532	0.356

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	123	144	127	149	199	145	106
N.S.	1	1.00	1.01	1.18	1.04	1.22	1.63	1.19	0.87
time (sec)	N/A	0.061	0.043	0.533	0.197	0.290	0.357	0.319	0.383

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	118	145	127	149	182	146	154
N.S.	1	1.00	0.99	1.22	1.07	1.25	1.53	1.23	1.29
time (sec)	N/A	0.060	0.052	0.849	0.192	0.298	0.446	0.362	0.409

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	115	143	125	150	182	141	139
N.S.	1	1.00	0.97	1.21	1.06	1.27	1.54	1.19	1.18
time (sec)	N/A	0.061	0.054	0.456	0.196	0.323	0.438	0.322	0.397

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	122	144	133	151	144	149	136
N.S.	1	1.00	0.97	1.14	1.06	1.20	1.14	1.18	1.08
time (sec)	N/A	0.073	0.054	0.497	0.202	0.308	2.457	0.391	0.439

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	109	134	143	152	158	162	118
N.S.	1	1.00	1.21	1.49	1.59	1.69	1.76	1.80	1.31
time (sec)	N/A	0.053	0.037	0.424	0.198	0.266	0.441	0.358	0.410

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	113	134	143	155	168	163	120
N.S.	1	1.00	0.80	0.94	1.01	1.09	1.18	1.15	0.85
time (sec)	N/A	0.067	0.038	0.424	0.197	0.302	0.556	0.424	0.447

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	113	134	143	155	177	163	121
N.S.	1	1.00	0.85	1.01	1.08	1.17	1.33	1.23	0.91
time (sec)	N/A	0.067	0.040	0.817	0.204	0.294	0.739	0.336	0.471

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	113	134	143	155	172	163	121
N.S.	1	1.00	0.85	1.01	1.08	1.17	1.29	1.23	0.91
time (sec)	N/A	0.063	0.037	0.447	0.207	0.281	0.954	0.334	0.453

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	148	148	142	272	0	0	267	0	0
N.S.	1	1.00	0.96	1.84	0.00	0.00	1.80	0.00	0.00
time (sec)	N/A	0.113	0.051	0.671	0.000	0.000	16.237	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	107	107	105	233	0	0	218	0	0
N.S.	1	1.00	0.98	2.18	0.00	0.00	2.04	0.00	0.00
time (sec)	N/A	0.091	0.036	0.395	0.000	0.000	12.897	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	69	69	66	188	0	0	163	0	0
N.S.	1	1.00	0.96	2.72	0.00	0.00	2.36	0.00	0.00
time (sec)	N/A	0.061	0.023	0.393	0.000	0.000	8.061	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	39	39	37	151	0	0	0	0	0
N.S.	1	1.00	0.95	3.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.018	0.006	0.398	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	44	44	63	181	0	0	175	0	0
N.S.	1	1.00	1.43	4.11	0.00	0.00	3.98	0.00	0.00
time (sec)	N/A	0.037	0.024	0.751	0.000	0.000	5.526	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	74	74	88	221	0	0	216	0	0
N.S.	1	1.00	1.19	2.99	0.00	0.00	2.92	0.00	0.00
time (sec)	N/A	0.085	0.056	0.763	0.000	0.000	32.453	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	110	110	124	264	0	0	265	0	0
N.S.	1	1.00	1.13	2.40	0.00	0.00	2.41	0.00	0.00
time (sec)	N/A	0.127	0.127	0.448	0.000	0.000	35.163	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	150	150	159	309	0	0	314	0	0
N.S.	1	1.00	1.06	2.06	0.00	0.00	2.09	0.00	0.00
time (sec)	N/A	0.188	0.114	0.474	0.000	0.000	52.480	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	152	152	141	298	0	0	323	0	0
N.S.	1	1.00	0.93	1.96	0.00	0.00	2.12	0.00	0.00
time (sec)	N/A	0.160	0.079	0.411	0.000	0.000	24.564	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	98	107	98	250	0	0	269	0	0
N.S.	1	1.09	1.00	2.55	0.00	0.00	2.74	0.00	0.00
time (sec)	N/A	0.113	0.060	0.408	0.000	0.000	12.077	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	65	65	71	205	0	0	0	0	0
N.S.	1	1.00	1.09	3.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.044	0.040	0.785	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	41	54	63	51	153	64	54
N.S.	1	1.00	1.05	1.38	1.62	1.31	3.92	1.64	1.38
time (sec)	N/A	0.013	0.019	0.395	0.190	0.317	0.621	0.329	1.321

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	80	80	96	229	0	0	0	0	0
N.S.	1	1.00	1.20	2.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.087	0.053	0.436	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	114	114	120	276	0	0	318	0	0
N.S.	1	1.00	1.05	2.42	0.00	0.00	2.79	0.00	0.00
time (sec)	N/A	0.123	0.087	0.444	0.000	0.000	30.604	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	154	154	165	331	0	0	376	0	0
N.S.	1	1.00	1.07	2.15	0.00	0.00	2.44	0.00	0.00
time (sec)	N/A	0.152	0.137	0.497	0.000	0.000	43.486	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	149	149	150	303	0	0	391	0	0
N.S.	1	1.00	1.01	2.03	0.00	0.00	2.62	0.00	0.00
time (sec)	N/A	0.174	0.091	0.443	0.000	0.000	27.599	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	107	107	122	258	0	0	347	0	0
N.S.	1	1.00	1.14	2.41	0.00	0.00	3.24	0.00	0.00
time (sec)	N/A	0.090	0.076	0.433	0.000	0.000	19.769	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	75	137	114	115	398	124	108
N.S.	1	1.00	1.21	2.21	1.84	1.85	6.42	2.00	1.74
time (sec)	N/A	0.032	0.082	0.412	0.200	0.289	1.792	0.320	0.763

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	53	132	99	107	415	104	91
N.S.	1	1.00	0.70	1.74	1.30	1.41	5.46	1.37	1.20
time (sec)	N/A	0.023	0.036	0.616	0.206	0.290	1.881	0.300	0.768

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	134	134	141	273	0	0	352	0	0
N.S.	1	1.00	1.05	2.04	0.00	0.00	2.63	0.00	0.00
time (sec)	N/A	0.152	0.078	0.900	0.000	0.000	36.997	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	171	171	173	324	0	0	444	0	0
N.S.	1	1.00	1.01	1.89	0.00	0.00	2.60	0.00	0.00
time (sec)	N/A	0.166	0.105	0.555	0.000	0.000	38.432	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	217	217	227	386	0	0	496	0	0
N.S.	1	1.00	1.05	1.78	0.00	0.00	2.29	0.00	0.00
time (sec)	N/A	0.206	0.226	0.681	0.000	0.000	41.528	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	229	249	405	0	0	617	0	0
N.S.	1	1.00	1.09	1.77	0.00	0.00	2.69	0.00	0.00
time (sec)	N/A	0.311	0.182	0.559	0.000	0.000	62.664	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	183	183	207	355	0	0	563	0	0
N.S.	1	1.00	1.13	1.94	0.00	0.00	3.08	0.00	0.00
time (sec)	N/A	0.253	0.140	0.527	0.000	0.000	31.903	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	141	141	179	310	0	0	518	0	0
N.S.	1	1.00	1.27	2.20	0.00	0.00	3.67	0.00	0.00
time (sec)	N/A	0.138	0.148	0.542	0.000	0.000	30.940	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	172	165	179	178	677	202	167
N.S.	1	1.00	2.18	2.09	2.27	2.25	8.57	2.56	2.11
time (sec)	N/A	0.049	0.080	0.507	0.200	0.297	5.379	0.381	0.782

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	135	205	150	162	661	163	141
N.S.	1	1.00	1.15	1.75	1.28	1.38	5.65	1.39	1.21
time (sec)	N/A	0.059	0.064	0.507	0.193	0.299	5.317	0.335	0.667

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	66	188	144	160	700	147	127
N.S.	1	1.00	0.69	1.98	1.52	1.68	7.37	1.55	1.34
time (sec)	N/A	0.028	0.040	0.490	0.196	0.319	5.353	0.360	0.569

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	174	174	222	316	0	0	510	0	0
N.S.	1	1.00	1.28	1.82	0.00	0.00	2.93	0.00	0.00
time (sec)	N/A	0.207	0.115	0.710	0.000	0.000	58.961	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	211	211	231	370	0	0	614	0	0
N.S.	1	1.00	1.09	1.75	0.00	0.00	2.91	0.00	0.00
time (sec)	N/A	0.209	0.166	0.754	0.000	0.000	57.442	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	263	263	276	438	0	0	668	0	0
N.S.	1	1.00	1.05	1.67	0.00	0.00	2.54	0.00	0.00
time (sec)	N/A	0.249	0.197	0.924	0.000	0.000	61.944	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	329	329	403	561	0	0	1686	0	0
N.S.	1	1.00	1.22	1.71	0.00	0.00	5.12	0.00	0.00
time (sec)	N/A	0.658	0.300	1.670	0.000	0.000	149.758	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	285	285	356	511	0	0	1632	0	0
N.S.	1	1.00	1.25	1.79	0.00	0.00	5.73	0.00	0.00
time (sec)	N/A	0.575	0.326	1.335	0.000	0.000	112.194	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-1)	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	243	243	333	466	0	0	1588	0	0
N.S.	1	1.00	1.37	1.92	0.00	0.00	6.53	0.00	0.00
time (sec)	N/A	0.345	0.297	1.302	0.000	0.000	80.640	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	335	395	377	361	1911	424	341
N.S.	1	1.00	2.46	2.90	2.77	2.65	14.05	3.12	2.51
time (sec)	N/A	0.080	0.192	1.500	0.230	0.305	75.709	0.471	1.274

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	316	435	358	356	1972	394	320
N.S.	1	1.00	1.94	2.67	2.20	2.18	12.10	2.42	1.96
time (sec)	N/A	0.088	0.182	1.352	0.214	0.312	75.511	0.376	1.181

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	281	413	338	343	1979	361	296
N.S.	1	1.00	1.24	1.83	1.50	1.52	8.76	1.60	1.31
time (sec)	N/A	0.142	0.175	1.335	0.218	0.299	75.704	0.402	1.013

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	192	383	316	333	1986	322	275
N.S.	1	1.00	0.96	1.92	1.59	1.67	9.98	1.62	1.38
time (sec)	N/A	0.114	0.129	1.336	0.222	0.300	75.270	0.320	0.907

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	160	404	294	323	1992	283	251
N.S.	1	1.00	0.92	2.32	1.69	1.86	11.45	1.63	1.44
time (sec)	N/A	0.080	0.099	1.270	0.219	0.308	75.495	0.364	0.845

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	99	386	276	310	1955	252	232
N.S.	1	1.00	0.65	2.54	1.82	2.04	12.86	1.66	1.53
time (sec)	N/A	0.047	0.103	1.316	0.213	0.324	76.214	0.441	0.746

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	294	294	349	445	0	0	1518	0	0
N.S.	1	1.00	1.19	1.51	0.00	0.00	5.16	0.00	0.00
time (sec)	N/A	0.470	0.249	2.211	0.000	0.000	156.444	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	339	339	401	508	0	0	1685	0	0
N.S.	1	1.00	1.18	1.50	0.00	0.00	4.97	0.00	0.00
time (sec)	N/A	0.397	0.391	2.681	0.000	0.000	146.271	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	401	401	486	594	0	0	1737	0	0
N.S.	1	1.00	1.21	1.48	0.00	0.00	4.33	0.00	0.00
time (sec)	N/A	0.432	0.327	3.621	0.000	0.000	156.804	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	48	11	0	0	8
N.S.	1	1.00	1.00	0.75	4.00	0.92	0.00	0.00	0.67
time (sec)	N/A	0.008	0.004	0.371	0.206	0.269	0.000	0.000	0.298

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	11	7	45	9	0	0	6
N.S.	1	1.00	1.10	0.70	4.50	0.90	0.00	0.00	0.60
time (sec)	N/A	0.007	0.003	0.383	0.205	0.304	0.000	0.000	0.285

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	82	156	151	219	185	241	116
N.S.	1	1.00	0.75	1.43	1.39	2.01	1.70	2.21	1.06
time (sec)	N/A	0.094	0.041	0.148	0.209	0.307	0.347	0.469	0.460

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	82	155	150	219	184	238	116
N.S.	1	1.00	0.75	1.42	1.38	2.01	1.69	2.18	1.06
time (sec)	N/A	0.073	0.041	0.131	0.204	0.267	0.260	0.331	0.425

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	77	141	136	200	163	215	104
N.S.	1	1.00	0.76	1.40	1.35	1.98	1.61	2.13	1.03
time (sec)	N/A	0.048	0.035	0.181	0.216	0.281	0.189	0.339	0.397

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	59	114	101	144	138	144	85
N.S.	1	1.00	0.84	1.63	1.44	2.06	1.97	2.06	1.21
time (sec)	N/A	0.055	0.017	0.164	0.210	0.278	0.245	0.350	0.383

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	63	114	114	149	141	162	138
N.S.	1	1.00	0.88	1.58	1.58	2.07	1.96	2.25	1.92
time (sec)	N/A	0.076	0.030	0.170	0.222	0.284	1.820	0.329	0.429

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	90	130	150	179	165	192	109
N.S.	1	1.00	0.87	1.26	1.46	1.74	1.60	1.86	1.06
time (sec)	N/A	0.093	0.037	0.116	0.199	0.287	0.252	0.302	0.500

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	82	131	151	187	185	195	114
N.S.	1	1.00	0.75	1.20	1.39	1.72	1.70	1.79	1.05
time (sec)	N/A	0.093	0.040	0.123	0.206	0.268	0.323	0.331	0.515

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	82	131	151	188	187	195	114
N.S.	1	1.00	0.75	1.20	1.39	1.72	1.72	1.79	1.05
time (sec)	N/A	0.097	0.041	0.138	0.207	0.280	0.402	0.444	0.520

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	149	262	250	364	311	408	180
N.S.	1	1.00	0.84	1.47	1.40	2.04	1.75	2.29	1.01
time (sec)	N/A	0.151	0.062	4.697	0.217	0.272	0.475	0.382	0.503

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	134	262	250	363	308	408	179
N.S.	1	1.00	0.75	1.47	1.40	2.04	1.73	2.29	1.01
time (sec)	N/A	0.120	0.065	2.641	0.216	0.270	0.350	0.363	0.480

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	135	247	235	347	286	385	166
N.S.	1	1.00	0.78	1.43	1.36	2.01	1.65	2.23	0.96
time (sec)	N/A	0.079	0.052	0.601	0.208	0.272	0.260	0.360	0.444

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	114	223	198	293	269	285	152
N.S.	1	1.00	0.83	1.63	1.45	2.14	1.96	2.08	1.11
time (sec)	N/A	0.147	0.029	0.748	0.199	0.286	0.367	0.392	0.449

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	107	226	200	291	255	274	228
N.S.	1	1.00	0.80	1.70	1.50	2.19	1.92	2.06	1.71
time (sec)	N/A	0.115	0.031	0.751	0.201	0.293	0.432	0.361	0.489

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	117	217	210	291	258	290	221
N.S.	1	1.00	0.85	1.58	1.53	2.12	1.88	2.12	1.61
time (sec)	N/A	0.129	0.058	0.707	0.201	0.311	2.468	0.363	0.532

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	131	238	250	326	287	354	184
N.S.	1	1.00	0.78	1.42	1.49	1.94	1.71	2.11	1.10
time (sec)	N/A	0.143	0.066	0.719	0.211	0.314	0.335	0.364	0.603

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	134	238	251	332	309	355	188
N.S.	1	1.00	0.75	1.34	1.41	1.87	1.74	1.99	1.06
time (sec)	N/A	0.140	0.065	0.503	0.203	0.299	0.432	0.314	0.624

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	271	271	211	732	0	0	0	0	0
N.S.	1	1.00	0.78	2.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.183	0.118	0.569	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	200	200	158	637	0	0	0	0	0
N.S.	1	1.00	0.79	3.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.156	0.067	0.487	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	130	130	103	528	0	0	0	0	0
N.S.	1	1.00	0.79	4.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.108	0.054	0.429	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	72	72	68	445	0	0	0	0	0
N.S.	1	1.00	0.94	6.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.046	0.022	0.371	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	79	79	94	528	0	0	0	0	0
N.S.	1	1.00	1.19	6.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.078	0.040	0.415	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	135	135	130	615	0	0	0	0	0
N.S.	1	1.00	0.96	4.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.146	0.078	0.414	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	204	204	185	731	0	0	0	0	0
N.S.	1	1.00	0.91	3.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.218	0.101	0.458	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	273	273	237	828	0	0	0	0	0
N.S.	1	1.00	0.87	3.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.328	0.096	0.533	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	281	281	240	824	0	0	0	0	0
N.S.	1	1.00	0.85	2.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.205	0.142	0.552	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	203	203	186	700	0	0	0	0	0
N.S.	1	1.00	0.92	3.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.178	0.112	0.511	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	143	143	142	609	0	0	0	0	0
N.S.	1	1.00	0.99	4.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.138	0.097	0.415	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	77	77	81	369	0	0	0	0	0
N.S.	1	1.00	1.05	4.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	0.037	0.397	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	151	151	166	683	0	0	0	0	0
N.S.	1	1.00	1.10	4.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.166	0.120	0.431	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	211	211	223	790	0	0	0	0	0
N.S.	1	1.00	1.06	3.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	0.223	0.445	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	285	285	268	924	0	0	0	0	0
N.S.	1	1.00	0.94	3.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.250	0.142	0.485	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	296	327	258	827	0	0	0	0	0
N.S.	1	1.10	0.87	2.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.294	0.189	0.519	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	232	262	212	738	0	0	0	0	0
N.S.	1	1.13	0.91	3.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.302	0.165	0.451	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	112	112	155	484	0	0	0	0	0
N.S.	1	1.00	1.38	4.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	0.152	0.515	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	126	126	146	435	0	0	0	0	0
N.S.	1	1.00	1.16	3.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.101	0.069	0.478	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	257	268	232	793	0	0	0	0	0
N.S.	1	1.04	0.90	3.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	0.177	0.519	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	322	335	290	908	0	0	0	0	0
N.S.	1	1.04	0.90	2.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.339	0.294	0.497	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	398	430	344	943	0	0	0	0	0
N.S.	1	1.08	0.86	2.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.515	0.444	0.562	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	333	364	298	854	0	0	0	0	0
N.S.	1	1.09	0.89	2.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.515	0.316	0.603	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	161	161	371	593	0	0	0	0	0
N.S.	1	1.00	2.30	3.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.148	0.325	0.520	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	210	222	281	508	0	0	0	0	0
N.S.	1	1.06	1.34	2.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.156	0.162	0.559	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	203	203	211	495	0	0	0	0	0
N.S.	1	1.00	1.04	2.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.179	0.115	0.513	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	351	363	318	894	0	0	0	0	0
N.S.	1	1.03	0.91	2.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.550	0.276	0.684	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	420	432	378	1015	0	0	0	0	0
N.S.	1	1.03	0.90	2.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.545	0.453	0.655	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	157	96	180	132	0	374	0	0
N.S.	1	1.47	0.90	1.68	1.23	0.00	3.50	0.00	0.00
time (sec)	N/A	0.130	0.088	0.319	0.203	0.000	20.968	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	113	113	243	967	0	0	0	0	0
N.S.	1	1.00	2.15	8.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.103	0.142	0.552	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	217	217	432	1373	0	0	0	0	0
N.S.	1	1.00	1.99	6.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	0.363	0.553	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	361	409	706	1607	0	0	0	0	0
N.S.	1	1.13	1.96	4.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.476	0.632	0.629	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	189	189	169	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	0.202	0.000	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	298	298	287	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.348	0.256	0.000	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	402	402	366	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.429	0.416	0.000	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	19	22	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	0.86	1.00	1.00
time (sec)	N/A	0.038	7.966	0.026	0.264	0.000	0.372	0.426	0.340

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	20	22	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	0.91	1.00	1.00
time (sec)	N/A	0.069	5.644	0.025	0.284	0.000	0.838	0.316	0.393

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	20	22	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	0.91	1.00	1.00
time (sec)	N/A	0.128	13.808	0.024	0.263	0.000	1.730	0.376	0.388

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	242	183	0	227	495	347	0	0
N.S.	1	1.00	0.76	0.00	0.94	2.05	1.43	0.00	0.00
time (sec)	N/A	0.150	0.166	0.000	0.279	0.334	70.597	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	192	192	151	0	184	396	284	0	0
N.S.	1	1.00	0.79	0.00	0.96	2.06	1.48	0.00	0.00
time (sec)	N/A	0.122	0.134	0.000	0.284	0.314	51.038	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	116	0	143	291	221	0	0
N.S.	1	1.00	0.82	0.00	1.01	2.05	1.56	0.00	0.00
time (sec)	N/A	0.067	0.084	0.000	0.291	0.334	56.221	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	77	0	93	184	144	0	0
N.S.	1	1.00	0.82	0.00	0.99	1.96	1.53	0.00	0.00
time (sec)	N/A	0.027	0.060	0.000	0.279	0.326	29.543	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	211	332	0	0	0	0	0	0
N.S.	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.163	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	392	0	0	0	0	0	0
N.S.	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	0.203	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	298	298	500	0	0	0	0	0	0
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.233	0.337	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	263	263	187	0	239	595	0	0	0
N.S.	1	1.00	0.71	0.00	0.91	2.26	0.00	0.00	0.00
time (sec)	N/A	0.168	0.192	0.000	0.275	0.336	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	213	213	153	0	196	496	643	0	0
N.S.	1	1.00	0.72	0.00	0.92	2.33	3.02	0.00	0.00
time (sec)	N/A	0.161	0.150	0.000	0.268	0.339	129.067	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	120	0	155	391	517	0	0
N.S.	1	1.00	0.74	0.00	0.95	2.40	3.17	0.00	0.00
time (sec)	N/A	0.078	0.115	0.000	0.314	0.340	113.002	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	87	0	105	288	377	0	0
N.S.	1	1.00	0.76	0.00	0.91	2.50	3.28	0.00	0.00
time (sec)	N/A	0.037	0.062	0.000	0.277	0.326	88.800	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	255	255	391	0	0	0	0	0	0
N.S.	1	1.00	1.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.304	0.188	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	259	259	480	0	0	0	0	0	0
N.S.	1	1.00	1.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.241	0.220	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	293	293	501	0	0	0	0	0	0
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	0.352	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	150	0	215	395	396	252	0
N.S.	1	1.00	0.69	0.00	0.99	1.82	1.82	1.16	0.00
time (sec)	N/A	0.149	0.147	0.000	0.275	0.320	57.096	0.379	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	169	118	0	172	296	337	192	0
N.S.	1	1.00	0.70	0.00	1.02	1.75	1.99	1.14	0.00
time (sec)	N/A	0.114	0.124	0.000	0.274	0.358	42.283	0.388	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	80	0	127	189	272	131	0
N.S.	1	1.00	0.67	0.00	1.07	1.59	2.29	1.10	0.00
time (sec)	N/A	0.063	0.074	0.000	0.273	0.306	32.670	0.359	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	55	62	82	116	131	74	0
N.S.	1	1.00	0.80	0.90	1.19	1.68	1.90	1.07	0.00
time (sec)	N/A	0.024	0.032	0.400	0.275	0.304	2.295	0.322	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	249	0	0	0	0	0	0
N.S.	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.143	0.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	226	226	392	0	0	0	0	0	0
N.S.	1	1.00	1.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.182	0.168	0.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	304	304	501	0	0	0	0	0	0
N.S.	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	0.226	0.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	159	0	200	435	369	0	0
N.S.	1	1.00	0.82	0.00	1.03	2.24	1.90	0.00	0.00
time (sec)	N/A	0.128	0.086	0.000	0.282	0.340	116.990	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	146	146	124	0	157	330	308	0	0
N.S.	1	1.00	0.85	0.00	1.08	2.26	2.11	0.00	0.00
time (sec)	N/A	0.109	0.083	0.000	0.273	0.319	135.748	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	83	0	112	223	177	0	0
N.S.	1	1.00	0.88	0.00	1.19	2.37	1.88	0.00	0.00
time (sec)	N/A	0.066	0.049	0.000	0.283	0.378	91.742	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	71	155	88	71	0
N.S.	1	1.00	1.00	0.00	1.34	2.92	1.66	1.34	0.00
time (sec)	N/A	0.021	0.033	0.000	0.278	0.317	3.852	0.360	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	201	201	295	0	0	0	0	0	0
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.208	0.188	0.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	253	506	0	0	0	0	0	0
N.S.	1	1.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.331	0.299	0.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	31	19	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.35	0.83	1.09	1.09
time (sec)	N/A	0.054	1.756	0.003	0.220	0.279	0.792	0.301	0.267

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	29	17	23	23
N.S.	1	1.00	1.10	1.00	1.10	1.38	0.81	1.10	1.10
time (sec)	N/A	0.036	1.210	0.020	0.237	0.286	0.849	0.396	0.273

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	27	17	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.35	0.85	1.10	1.10
time (sec)	N/A	0.020	0.023	0.022	0.239	0.292	0.992	0.307	0.256

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	33	19	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.43	0.83	1.09	1.09
time (sec)	N/A	0.056	0.887	0.029	0.242	0.272	1.441	0.304	0.275

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	37	20	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.61	0.87	1.09	1.09
time (sec)	N/A	0.056	1.019	0.003	0.319	0.293	1.125	0.293	0.276

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	152	1746	271	1222	6156	536	0
N.S.	1	1.00	0.72	8.27	1.28	5.79	29.18	2.54	0.00
time (sec)	N/A	0.178	0.168	2.220	0.287	0.320	4.987	0.332	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	108	911	195	633	2791	374	0
N.S.	1	1.00	0.71	5.95	1.27	4.14	18.24	2.44	0.00
time (sec)	N/A	0.124	0.104	1.054	0.243	0.319	3.510	0.337	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	64	353	119	235	899	212	0
N.S.	1	1.00	0.67	3.72	1.25	2.47	9.46	2.23	0.00
time (sec)	N/A	0.057	0.054	0.217	0.237	0.334	2.389	0.326	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	32	68	57	52	141	95	0
N.S.	1	1.00	0.70	1.48	1.24	1.13	3.07	2.07	0.00
time (sec)	N/A	0.012	0.012	0.082	0.202	0.313	2.069	0.384	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	72	23	25	31	20	25	25
N.S.	1	1.00	3.13	1.00	1.09	1.35	0.87	1.09	1.09
time (sec)	N/A	0.040	0.129	0.035	0.252	0.319	2.083	0.341	0.473

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	72	23	25	42	22	25	25
N.S.	1	1.00	3.13	1.00	1.09	1.83	0.96	1.09	1.09
time (sec)	N/A	0.034	0.118	0.036	0.273	0.343	4.029	0.337	0.510

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	173	15	112	17	15	17	17
N.S.	1	1.00	11.53	1.00	7.47	1.13	1.00	1.13	1.13
time (sec)	N/A	0.012	0.161	0.033	0.296	0.332	7.503	0.417	0.284

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	61	0	0	0	238	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	3.50	0.00	0.00
time (sec)	N/A	0.020	0.017	0.000	0.000	0.000	6.235	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	17	17	89	17	19	19	15	0	19
N.S.	1	1.00	5.24	1.00	1.12	1.12	0.88	0.00	1.12
time (sec)	N/A	0.020	0.042	0.027	0.241	0.303	4.980	0.000	0.300

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	69	58	57	69	66	69	51
N.S.	1	1.00	1.44	1.21	1.19	1.44	1.38	1.44	1.06
time (sec)	N/A	0.028	0.005	0.379	0.205	0.293	0.873	0.322	0.339

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	69	58	57	69	66	69	51
N.S.	1	1.00	1.44	1.21	1.19	1.44	1.38	1.44	1.06
time (sec)	N/A	0.028	0.005	0.204	0.190	0.325	0.458	0.348	0.332

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	69	58	57	67	66	69	51
N.S.	1	1.00	1.47	1.23	1.21	1.43	1.40	1.47	1.09
time (sec)	N/A	0.023	0.005	0.168	0.221	0.325	0.237	0.438	0.325

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	57	58	49	55	78	52	48
N.S.	1	1.00	1.10	1.12	0.94	1.06	1.50	1.00	0.92
time (sec)	N/A	0.046	0.001	0.076	0.208	0.289	0.224	0.302	0.320

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	54	57	58	49	59	63	56	66
N.S.	1	1.04	1.10	1.12	0.94	1.13	1.21	1.08	1.27
time (sec)	N/A	0.034	0.005	0.080	0.200	0.303	1.293	0.334	0.342

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	69	53	57	60	68	65	51
N.S.	1	1.00	1.21	0.93	1.00	1.05	1.19	1.14	0.89
time (sec)	N/A	0.032	0.004	0.078	0.196	0.316	0.403	0.303	0.383

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	69	58	57	69	66	69	51
N.S.	1	1.00	1.44	1.21	1.19	1.44	1.38	1.44	1.06
time (sec)	N/A	0.028	0.005	0.245	0.193	0.303	0.676	0.285	0.345

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	69	58	57	69	66	69	51
N.S.	1	1.00	1.44	1.21	1.19	1.44	1.38	1.44	1.06
time (sec)	N/A	0.027	0.004	0.157	0.198	0.274	0.330	0.302	0.344

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	55	50	49	61	56	58	43
N.S.	1	1.00	1.15	1.04	1.02	1.27	1.17	1.21	0.90
time (sec)	N/A	0.013	0.002	0.074	0.194	0.311	0.181	0.336	0.327

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	49	50	49	58	46	54	51
N.S.	1	1.00	1.11	1.14	1.11	1.32	1.05	1.23	1.16
time (sec)	N/A	0.027	0.003	0.061	0.192	0.289	0.174	0.293	0.346

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	63	53	57	59	58	65	51
N.S.	1	1.00	1.19	1.00	1.08	1.11	1.09	1.23	0.96
time (sec)	N/A	0.031	0.005	0.066	0.199	0.285	0.317	0.415	0.362

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	69	54	57	63	68	67	53
N.S.	1	1.00	1.21	0.95	1.00	1.11	1.19	1.18	0.93
time (sec)	N/A	0.033	0.005	0.078	0.217	0.321	0.517	0.309	0.370

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	84	101	100	118	116	123	82
N.S.	1	1.00	1.14	1.36	1.35	1.59	1.57	1.66	1.11
time (sec)	N/A	0.059	0.029	1.366	0.199	0.294	1.688	0.366	0.386

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	87	101	100	118	116	123	82
N.S.	1	1.00	1.18	1.36	1.35	1.59	1.57	1.66	1.11
time (sec)	N/A	0.058	0.029	0.787	0.198	0.284	0.873	0.328	0.369

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	85	101	100	116	116	123	82
N.S.	1	1.00	1.12	1.33	1.32	1.53	1.53	1.62	1.08
time (sec)	N/A	0.043	0.027	0.721	0.200	0.305	0.477	0.367	0.367

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	82	103	88	104	133	100	80
N.S.	1	1.00	0.92	1.16	0.99	1.17	1.49	1.12	0.90
time (sec)	N/A	0.055	0.034	0.544	0.216	0.290	0.469	0.332	0.344

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	83	104	91	108	139	106	110
N.S.	1	1.00	0.91	1.14	1.00	1.19	1.53	1.16	1.21
time (sec)	N/A	0.070	0.043	0.512	0.203	0.296	0.555	0.297	0.366

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	82	103	90	108	105	103	102
N.S.	1	1.00	0.91	1.14	1.00	1.20	1.17	1.14	1.13
time (sec)	N/A	0.066	0.040	0.478	0.209	0.294	1.711	0.379	0.389

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	95	101	100	118	121	123	82
N.S.	1	1.00	1.28	1.36	1.35	1.59	1.64	1.66	1.11
time (sec)	N/A	0.049	0.026	0.951	0.195	0.284	1.232	0.309	0.371

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	95	101	100	118	121	123	82
N.S.	1	1.00	1.28	1.36	1.35	1.59	1.64	1.66	1.11
time (sec)	N/A	0.051	0.027	0.544	0.198	0.311	0.663	0.359	0.362

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	89	93	92	112	110	112	74
N.S.	1	1.00	1.03	1.08	1.07	1.30	1.28	1.30	0.86
time (sec)	N/A	0.027	0.025	0.448	0.210	0.291	0.344	0.420	0.347

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	86	96	94	109	100	103	102
N.S.	1	1.00	1.04	1.16	1.13	1.31	1.20	1.24	1.23
time (sec)	N/A	0.052	0.026	0.782	0.199	0.293	0.338	0.329	0.371

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	80	96	92	110	100	112	90
N.S.	1	1.00	0.98	1.17	1.12	1.34	1.22	1.37	1.10
time (sec)	N/A	0.059	0.030	0.473	0.186	0.323	0.406	0.287	0.390

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	86	97	100	111	112	119	88
N.S.	1	1.00	0.95	1.07	1.10	1.22	1.23	1.31	0.97
time (sec)	N/A	0.064	0.031	0.481	0.197	0.288	0.567	0.310	0.387

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	97	100	112	122	119	89
N.S.	1	1.00	1.00	1.02	1.05	1.18	1.28	1.25	0.94
time (sec)	N/A	0.057	0.035	0.474	0.190	0.313	0.954	0.343	0.409

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	120	144	143	167	175	177	113
N.S.	1	1.00	1.20	1.44	1.43	1.67	1.75	1.77	1.13
time (sec)	N/A	0.079	0.039	9.546	0.193	0.286	2.960	0.314	0.376

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	120	144	143	167	170	177	113
N.S.	1	1.00	0.92	1.11	1.10	1.28	1.31	1.36	0.87
time (sec)	N/A	0.115	0.043	1.320	0.198	0.328	1.726	0.311	0.368

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	118	144	143	165	170	177	113
N.S.	1	1.00	1.30	1.58	1.57	1.81	1.87	1.95	1.24
time (sec)	N/A	0.055	0.040	0.731	0.192	0.325	0.909	0.381	0.359

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	116	150	133	155	212	149	112
N.S.	1	1.00	0.89	1.15	1.02	1.19	1.63	1.15	0.86
time (sec)	N/A	0.075	0.047	0.725	0.188	0.287	0.939	0.336	0.385

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	115	149	133	155	209	156	163
N.S.	1	1.00	0.88	1.14	1.02	1.18	1.60	1.19	1.24
time (sec)	N/A	0.089	0.060	0.747	0.194	0.321	1.005	0.331	0.403

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	115	149	133	157	209	154	149
N.S.	1	1.00	0.88	1.14	1.02	1.20	1.60	1.18	1.14
time (sec)	N/A	0.087	0.056	0.898	0.190	0.293	1.040	0.324	0.405

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	133	144	143	167	170	177	113
N.S.	1	1.00	1.33	1.44	1.43	1.67	1.70	1.77	1.13
time (sec)	N/A	0.057	0.035	1.369	0.194	0.330	2.277	0.310	0.376

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	133	144	143	167	175	177	113
N.S.	1	1.00	1.33	1.44	1.43	1.67	1.75	1.77	1.13
time (sec)	N/A	0.058	0.036	0.809	0.203	0.308	1.248	0.326	0.390

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	124	134	133	161	156	163	104
N.S.	1	1.00	1.02	1.11	1.10	1.33	1.29	1.35	0.86
time (sec)	N/A	0.033	0.035	0.631	0.197	0.287	0.678	0.324	0.341

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	123	140	135	159	146	151	145
N.S.	1	1.00	1.04	1.19	1.14	1.35	1.24	1.28	1.23
time (sec)	N/A	0.061	0.041	0.675	0.189	0.298	0.633	0.343	0.384

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	112	138	137	156	155	159	141
N.S.	1	1.00	0.93	1.14	1.13	1.29	1.28	1.31	1.17
time (sec)	N/A	0.068	0.041	0.651	0.194	0.282	0.710	0.371	0.410

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	115	140	135	160	146	165	125
N.S.	1	1.00	0.97	1.19	1.14	1.36	1.24	1.40	1.06
time (sec)	N/A	0.062	0.040	0.673	0.197	0.309	0.690	0.372	0.434

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	140	143	160	158	171	123
N.S.	1	1.00	1.00	1.10	1.13	1.26	1.24	1.35	0.97
time (sec)	N/A	0.078	0.048	0.684	0.188	0.287	0.962	0.354	0.429

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	133	140	143	161	177	171	125
N.S.	1	1.00	1.00	1.05	1.08	1.21	1.33	1.29	0.94
time (sec)	N/A	0.072	0.045	0.674	0.195	0.331	1.750	0.423	0.444

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	121	121	174	343	0	0	257	0	0
N.S.	1	1.00	1.44	2.83	0.00	0.00	2.12	0.00	0.00
time (sec)	N/A	0.124	0.081	0.618	0.000	0.000	35.173	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	83	83	135	284	0	0	202	0	0
N.S.	1	1.00	1.63	3.42	0.00	0.00	2.43	0.00	0.00
time (sec)	N/A	0.099	0.051	0.439	0.000	0.000	17.419	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	49	49	94	244	0	0	141	0	0
N.S.	1	1.00	1.92	4.98	0.00	0.00	2.88	0.00	0.00
time (sec)	N/A	0.035	0.025	0.396	0.000	0.000	3.269	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	49	49	126	274	0	0	144	0	0
N.S.	1	1.00	2.57	5.59	0.00	0.00	2.94	0.00	0.00
time (sec)	N/A	0.044	0.059	0.416	0.000	0.000	5.995	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	83	83	157	317	0	0	0	0	0
N.S.	1	1.00	1.89	3.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	0.078	0.487	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	121	121	196	369	0	0	0	0	0
N.S.	1	1.00	1.62	3.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.138	0.128	0.636	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	167	167	208	365	0	0	0	0	0
N.S.	1	1.00	1.25	2.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.132	0.102	0.493	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	132	132	170	317	0	0	0	0	0
N.S.	1	1.00	1.29	2.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.108	0.072	0.414	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	105	105	107	263	0	0	0	0	0
N.S.	1	1.00	1.02	2.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.044	0.032	0.403	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	134	134	173	325	0	0	0	0	0
N.S.	1	1.00	1.29	2.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	0.098	0.457	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	165	165	211	369	0	0	0	0	0
N.S.	1	1.00	1.28	2.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.158	0.134	0.555	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	129	129	287	351	0	0	316	0	0
N.S.	1	1.00	2.22	2.72	0.00	0.00	2.45	0.00	0.00
time (sec)	N/A	0.152	0.351	0.542	0.000	0.000	44.054	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	95	95	321	305	0	0	0	0	0
N.S.	1	1.00	3.38	3.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.137	0.147	0.471	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	74	74	71	61	292	71	73
N.S.	1	1.00	1.48	1.48	1.42	1.22	5.84	1.42	1.46
time (sec)	N/A	0.028	0.056	0.427	0.193	0.340	23.119	0.314	0.392

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	82	82	279	338	0	0	0	0	0
N.S.	1	1.00	3.40	4.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.093	0.259	0.589	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	126	126	334	380	0	0	364	0	0
N.S.	1	1.00	2.65	3.02	0.00	0.00	2.89	0.00	0.00
time (sec)	N/A	0.158	0.346	0.687	0.000	0.000	157.726	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	191	191	296	559	0	0	0	0	0
N.S.	1	1.00	1.55	2.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.195	0.361	0.524	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	164	164	258	516	0	0	0	0	0
N.S.	1	1.00	1.57	3.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.183	0.328	0.487	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	164	164	289	449	0	0	0	0	0
N.S.	1	1.00	1.76	2.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	0.332	0.523	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	183	183	328	568	0	0	0	0	0
N.S.	1	1.00	1.79	3.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.158	0.481	0.601	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	224	224	361	622	0	0	0	0	0
N.S.	1	1.00	1.61	2.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.231	0.445	0.859	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	152	152	498	359	0	0	403	0	0
N.S.	1	1.00	3.28	2.36	0.00	0.00	2.65	0.00	0.00
time (sec)	N/A	0.194	0.346	0.704	0.000	0.000	63.335	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	129	130	128	126	612	140	129
N.S.	1	1.00	1.90	1.91	1.88	1.85	9.00	2.06	1.90
time (sec)	N/A	0.052	0.082	0.624	0.197	0.329	167.209	0.297	0.443

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	111	143	109	118	619	116	109
N.S.	1	1.00	1.35	1.74	1.33	1.44	7.55	1.41	1.33
time (sec)	N/A	0.042	0.058	0.563	0.199	0.290	169.103	0.337	0.407

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	115	115	396	390	0	0	403	0	0
N.S.	1	1.00	3.44	3.39	0.00	0.00	3.50	0.00	0.00
time (sec)	N/A	0.142	0.610	1.071	0.000	0.000	119.755	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	162	162	507	440	0	0	0	0	0
N.S.	1	1.00	3.13	2.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	0.726	1.178	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	211	211	495	900	0	0	0	0	0
N.S.	1	1.00	2.35	4.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.302	0.766	0.694	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	187	187	497	826	0	0	0	0	0
N.S.	1	1.00	2.66	4.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.605	0.651	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	210	210	544	664	0	0	0	0	0
N.S.	1	1.00	2.59	3.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.096	0.590	0.627	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	219	219	552	964	0	0	0	0	0
N.S.	1	1.00	2.52	4.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.943	1.005	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	260	260	584	1029	0	0	0	0	0
N.S.	1	1.00	2.25	3.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.297	1.035	1.386	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	76	14	94	0	11
N.S.	1	1.00	1.00	0.71	4.47	0.82	5.53	0.00	0.65
time (sec)	N/A	0.028	0.008	0.438	0.180	0.299	4.078	0.000	0.302

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	17	53	58	13	117	0	10
N.S.	1	1.00	1.06	3.31	3.62	0.81	7.31	0.00	0.62
time (sec)	N/A	0.028	0.005	0.400	0.192	0.311	2.838	0.000	0.296

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	31	20	48	0	85	0	18
N.S.	1	1.00	1.41	0.91	2.18	0.00	3.86	0.00	0.82
time (sec)	N/A	0.016	0.006	0.361	0.194	0.000	5.443	0.000	0.042

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	65	26	26	0	0	0	24
N.S.	1	1.00	2.03	0.81	0.81	0.00	0.00	0.00	0.75
time (sec)	N/A	0.029	0.008	0.615	0.300	0.000	0.000	0.000	0.276

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	68	53	0	0	0	0	0
N.S.	1	1.00	1.10	0.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.023	0.625	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	72	55	0	0	0	0	0
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	0.022	0.681	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	509	509	432	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.439	0.490	0.000	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	711	711	1073	0	0	0	0	0	0
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.546	2.912	0.000	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	55	20	24	24
N.S.	1	1.00	1.09	1.00	1.09	2.50	0.91	1.09	1.09
time (sec)	N/A	0.021	2.379	0.004	0.227	0.298	77.409	0.322	0.312

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	261	102	1	24	24
N.S.	1	1.00	1.09	1.00	11.86	4.64	0.05	1.09	1.09
time (sec)	N/A	0.020	10.338	0.018	0.229	0.278	0.000	0.312	0.350

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	208	208	251	0	0	414	490	288	0
N.S.	1	1.00	1.21	0.00	0.00	1.99	2.36	1.38	0.00
time (sec)	N/A	0.173	0.132	0.000	0.000	0.337	20.885	0.377	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	204	0	0	309	343	215	0
N.S.	1	1.00	1.32	0.00	0.00	2.01	2.23	1.40	0.00
time (sec)	N/A	0.130	0.102	0.000	0.000	0.342	14.268	0.346	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	136	0	0	202	218	141	0
N.S.	1	1.00	1.33	0.00	0.00	1.98	2.14	1.38	0.00
time (sec)	N/A	0.064	0.071	0.000	0.000	0.325	11.605	0.336	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	220	220	203	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	0.215	0.000	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	252	252	303	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.299	0.302	0.000	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	469	469	276	0	0	0	0	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.411	0.381	0.000	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	409	410	250	0	0	0	0	0	0
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.273	0.271	0.000	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	330	330	237	0	0	0	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.159	0.230	0.000	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	345	345	183	0	0	0	0	0	0
N.S.	1	1.00	0.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	0.331	0.000	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	99	0	0	212	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	1.89	0.00	0.00	0.00
time (sec)	N/A	0.070	0.105	0.000	0.000	0.343	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	170	170	145	0	0	323	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	1.90	0.00	0.00	0.00
time (sec)	N/A	0.107	0.131	0.000	0.000	0.348	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	230	230	180	0	0	426	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	1.85	0.00	0.00	0.00
time (sec)	N/A	0.130	0.162	0.000	0.000	0.378	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	231	256	0	0	514	1161	0	0
N.S.	1	1.00	1.11	0.00	0.00	2.23	5.03	0.00	0.00
time (sec)	N/A	0.188	0.224	0.000	0.000	0.370	95.554	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	177	227	0	0	409	845	0	0
N.S.	1	1.00	1.28	0.00	0.00	2.31	4.77	0.00	0.00
time (sec)	N/A	0.134	0.132	0.000	0.000	0.361	48.939	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	181	0	0	304	573	0	0
N.S.	1	1.00	1.45	0.00	0.00	2.43	4.58	0.00	0.00
time (sec)	N/A	0.078	0.096	0.000	0.000	0.359	30.638	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	260	260	301	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.292	0.496	0.000	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	349	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.328	0.588	0.000	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	464	464	331	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.419	0.630	0.000	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	378	378	314	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.190	0.547	0.000	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	400	400	329	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.326	0.628	0.000	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	400	400	269	0	0	0	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.312	0.497	0.000	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	114	0	0	314	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	2.28	0.00	0.00	0.00
time (sec)	N/A	0.080	0.153	0.000	0.000	0.358	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	196	196	145	0	0	423	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	2.16	0.00	0.00	0.00
time (sec)	N/A	0.124	0.193	0.000	0.000	0.393	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	256	256	178	0	0	526	0	0	0
N.S.	1	1.00	0.70	0.00	0.00	2.05	0.00	0.00	0.00
time (sec)	N/A	0.158	0.224	0.000	0.000	0.408	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	53	75	39	54	71	54	0
N.S.	1	1.00	0.88	1.25	0.65	0.90	1.18	0.90	0.00
time (sec)	N/A	0.030	0.032	0.440	0.257	0.318	7.761	0.339	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	182	182	204	0	0	314	359	0	0
N.S.	1	1.00	1.12	0.00	0.00	1.73	1.97	0.00	0.00
time (sec)	N/A	0.174	0.150	0.000	0.000	0.344	17.139	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	145	0	0	207	236	0	0
N.S.	1	1.00	1.12	0.00	0.00	1.60	1.83	0.00	0.00
time (sec)	N/A	0.113	0.119	0.000	0.000	0.320	12.282	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	91	0	0	124	129	0	0
N.S.	1	1.00	1.25	0.00	0.00	1.70	1.77	0.00	0.00
time (sec)	N/A	0.059	0.074	0.000	0.000	0.322	2.166	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	162	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.190	0.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	258	258	229	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.279	0.667	0.000	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	359	359	205	0	0	0	0	0	0
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	0.506	0.000	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	250	250	186	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	0.360	0.000	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	127	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	1.57	0.00	0.00	0.00
time (sec)	N/A	0.062	0.073	0.000	0.000	0.360	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	110	0	0	223	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	1.55	0.00	0.00	0.00
time (sec)	N/A	0.090	0.102	0.000	0.000	0.362	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	204	204	147	0	0	326	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	0.144	0.166	0.000	0.000	0.347	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	209	209	195	0	0	461	374	0	0
N.S.	1	1.00	0.93	0.00	0.00	2.21	1.79	0.00	0.00
time (sec)	N/A	0.201	0.165	0.000	0.000	0.377	44.806	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	160	0	0	356	308	0	0
N.S.	1	1.00	1.01	0.00	0.00	2.25	1.95	0.00	0.00
time (sec)	N/A	0.159	0.140	0.000	0.000	0.340	37.361	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	118	0	0	245	167	0	0
N.S.	1	1.00	1.18	0.00	0.00	2.45	1.67	0.00	0.00
time (sec)	N/A	0.128	0.120	0.000	0.000	0.304	24.360	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	77	0	0	169	94	0	0
N.S.	1	1.00	1.35	0.00	0.00	2.96	1.65	0.00	0.00
time (sec)	N/A	0.053	0.110	0.000	0.000	0.325	4.625	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	209	209	241	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.235	0.258	0.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	287	218	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.301	0.214	0.000	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	328	328	217	0	0	0	0	0	0
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.324	0.324	0.000	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	70	0	0	172	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	2.97	0.00	0.00	0.00
time (sec)	N/A	0.023	0.070	0.000	0.000	0.331	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	103	0	0	241	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	0.092	0.095	0.000	0.000	0.338	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	176	144	0	0	370	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	2.10	0.00	0.00	0.00
time (sec)	N/A	0.123	0.112	0.000	0.000	0.377	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	236	236	180	0	0	473	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	2.00	0.00	0.00	0.00
time (sec)	N/A	0.208	0.137	0.000	0.000	0.419	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	212	212	240	0	0	504	556	0	0
N.S.	1	1.00	1.13	0.00	0.00	2.38	2.62	0.00	0.00
time (sec)	N/A	0.223	0.196	0.000	0.000	0.379	104.749	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	205	0	0	401	415	0	0
N.S.	1	1.00	1.32	0.00	0.00	2.59	2.68	0.00	0.00
time (sec)	N/A	0.161	0.161	0.000	0.000	0.364	65.068	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	137	0	137	325	337	0	0
N.S.	1	1.00	1.27	0.00	1.27	3.01	3.12	0.00	0.00
time (sec)	N/A	0.114	0.194	0.000	0.271	0.348	28.707	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	97	0	0	267	272	0	0
N.S.	1	1.00	1.15	0.00	0.00	3.18	3.24	0.00	0.00
time (sec)	N/A	0.068	0.168	0.000	0.000	0.337	13.693	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	251	251	273	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.297	0.308	0.000	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	337	337	227	0	0	0	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.371	0.185	0.000	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	443	443	199	0	0	0	0	0	0
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.379	0.212	0.000	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	383	383	244	0	0	0	0	0	0
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.380	0.593	0.000	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	101	0	0	277	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	3.11	0.00	0.00	0.00
time (sec)	N/A	0.067	0.130	0.000	0.000	0.330	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	113	116	0	0	337	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	2.98	0.00	0.00	0.00
time (sec)	N/A	0.043	0.102	0.000	0.000	0.330	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	144	0	0	399	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.111	0.127	0.000	0.000	0.375	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	230	230	182	0	0	520	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	2.26	0.00	0.00	0.00
time (sec)	N/A	0.178	0.147	0.000	0.000	0.380	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	251	251	163	0	199	125	0	0	0
N.S.	1	1.00	0.65	0.00	0.79	0.50	0.00	0.00	0.00
time (sec)	N/A	0.402	0.298	0.000	0.262	0.330	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	113	0	105	66	0	0	0
N.S.	1	1.00	0.76	0.00	0.71	0.45	0.00	0.00	0.00
time (sec)	N/A	0.223	0.147	0.000	0.300	0.300	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	301	301	310	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.412	1.204	0.000	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	489	489	255	0	0	0	0	0	0
N.S.	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.521	0.636	0.000	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	406	406	316	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.427	1.882	0.000	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	248	248	217	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.150	0.381	0.000	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	70	0	102	73	0	0	0
N.S.	1	1.00	0.49	0.00	0.72	0.51	0.00	0.00	0.00
time (sec)	N/A	0.268	0.160	0.000	0.262	0.300	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	252	252	116	0	0	135	0	0	0
N.S.	1	1.00	0.46	0.00	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.331	0.210	0.000	0.000	0.316	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	34	34	27	119	27	27	29	28	0
N.S.	1	1.00	0.79	3.50	0.79	0.79	0.85	0.82	0.00
time (sec)	N/A	0.027	0.018	0.425	0.260	0.283	1.214	0.349	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	156	1761	271	1222	6217	558	0
N.S.	1	1.00	0.74	8.35	1.28	5.79	29.46	2.64	0.00
time (sec)	N/A	1.058	0.167	27.451	0.206	0.302	13.507	0.377	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	112	916	195	633	2820	396	0
N.S.	1	1.00	0.73	5.99	1.27	4.14	18.43	2.59	0.00
time (sec)	N/A	0.132	0.100	5.717	0.195	0.299	5.452	0.368	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	68	343	119	235	920	234	0
N.S.	1	1.00	0.72	3.61	1.25	2.47	9.68	2.46	0.00
time (sec)	N/A	0.076	0.054	0.901	0.190	0.292	2.709	0.343	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	32	68	57	52	141	95	0
N.S.	1	1.00	0.70	1.48	1.24	1.13	3.07	2.07	0.00
time (sec)	N/A	0.010	0.008	0.004	0.185	0.289	2.176	0.331	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	108	25	27	33	22	27	27
N.S.	1	1.00	4.32	1.00	1.08	1.32	0.88	1.08	1.08
time (sec)	N/A	0.044	0.704	0.030	0.217	0.285	7.106	0.379	0.412

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	108	25	27	44	24	27	27
N.S.	1	1.00	4.32	1.00	1.08	1.76	0.96	1.08	1.08
time (sec)	N/A	0.042	0.137	0.044	0.221	0.283	162.221	0.419	0.451

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1198	1198	2215	0	0	0	0	0	0
N.S.	1	1.00	1.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.016	7.313	0.000	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	860	860	1180	0	0	0	0	0	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.569	2.720	0.000	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	520	520	571	598	0	0	0	0	0
N.S.	1	1.00	1.10	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.311	0.831	1.131	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	55	1	24	24
N.S.	1	1.00	1.09	1.00	1.09	2.50	0.05	1.09	1.09
time (sec)	N/A	0.021	3.846	0.006	0.216	0.261	0.000	0.310	0.349

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	261	102	1	24	24
N.S.	1	1.00	1.09	1.00	11.86	4.64	0.05	1.09	1.09
time (sec)	N/A	0.019	17.202	0.035	0.228	0.276	0.000	0.327	0.391

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	185	185	171	313	0	0	316	0	0
N.S.	1	1.00	0.92	1.69	0.00	0.00	1.71	0.00	0.00
time (sec)	N/A	0.133	0.070	0.338	0.000	0.000	75.860	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	148	148	142	272	0	0	267	0	0
N.S.	1	1.00	0.96	1.84	0.00	0.00	1.80	0.00	0.00
time (sec)	N/A	0.128	0.049	0.180	0.000	0.000	80.120	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	107	107	105	233	0	0	218	0	0
N.S.	1	1.00	0.98	2.18	0.00	0.00	2.04	0.00	0.00
time (sec)	N/A	0.083	0.036	0.127	0.000	0.000	53.551	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	69	69	66	188	0	0	163	0	0
N.S.	1	1.00	0.96	2.72	0.00	0.00	2.36	0.00	0.00
time (sec)	N/A	0.053	0.023	0.109	0.000	0.000	42.526	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	39	39	37	151	0	0	0	0	0
N.S.	1	1.00	0.95	3.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.006	0.076	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	44	44	63	181	0	0	173	0	0
N.S.	1	1.00	1.43	4.11	0.00	0.00	3.93	0.00	0.00
time (sec)	N/A	0.044	0.025	0.095	0.000	0.000	6.043	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	95	95	88	221	0	0	216	0	0
N.S.	1	1.00	0.93	2.33	0.00	0.00	2.27	0.00	0.00
time (sec)	N/A	0.101	0.057	0.128	0.000	0.000	37.151	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	135	135	124	264	0	0	265	0	0
N.S.	1	1.00	0.92	1.96	0.00	0.00	1.96	0.00	0.00
time (sec)	N/A	0.120	0.133	0.184	0.000	0.000	42.968	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	156	204	210	0	299	0	0
N.S.	1	1.00	0.92	1.20	1.24	0.00	1.76	0.00	0.00
time (sec)	N/A	0.128	0.056	0.127	0.243	0.000	82.828	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	125	168	164	0	253	0	0
N.S.	1	1.00	0.92	1.24	1.21	0.00	1.86	0.00	0.00
time (sec)	N/A	0.110	0.044	0.049	0.226	0.000	79.519	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	99	126	112	0	207	0	0
N.S.	1	1.00	1.01	1.29	1.14	0.00	2.11	0.00	0.00
time (sec)	N/A	0.081	0.028	0.036	0.238	0.000	52.730	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	64	88	69	0	156	0	0
N.S.	1	1.00	1.02	1.40	1.10	0.00	2.48	0.00	0.00
time (sec)	N/A	0.055	0.022	0.033	0.232	0.000	43.429	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	34	59	43	0	0	0	0
N.S.	1	1.00	0.94	1.64	1.19	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	0.006	0.027	0.246	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	54	81	67	0	170	0	0
N.S.	1	1.00	1.32	1.98	1.63	0.00	4.15	0.00	0.00
time (sec)	N/A	0.053	0.018	0.042	0.217	0.000	5.953	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	77	115	96	0	206	0	0
N.S.	1	1.00	0.92	1.37	1.14	0.00	2.45	0.00	0.00
time (sec)	N/A	0.093	0.061	0.046	0.229	0.000	37.730	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	110	157	151	0	252	0	0
N.S.	1	1.00	0.91	1.30	1.25	0.00	2.08	0.00	0.00
time (sec)	N/A	0.111	0.096	0.057	0.227	0.000	42.654	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	52	39	0	0	13
N.S.	1	1.00	1.00	0.82	3.06	2.29	0.00	0.00	0.76
time (sec)	N/A	0.047	0.011	0.794	0.298	0.297	0.000	0.000	0.495

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	17	13	45	50	0	0	12
N.S.	1	1.00	1.06	0.81	2.81	3.12	0.00	0.00	0.75
time (sec)	N/A	0.044	0.010	0.480	0.301	0.303	0.000	0.000	0.397

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	21	19	64	55	0	0	18
N.S.	1	1.00	1.05	0.95	3.20	2.75	0.00	0.00	0.90
time (sec)	N/A	0.051	0.009	0.508	0.314	0.303	0.000	0.000	0.410

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	16	11	72	13	82	0	10
N.S.	1	1.00	1.14	0.79	5.14	0.93	5.86	0.00	0.71
time (sec)	N/A	0.053	0.004	0.777	0.188	0.314	4.883	0.000	0.377

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	21	12	81	14	92	0	11
N.S.	1	1.00	1.24	0.71	4.76	0.82	5.41	0.00	0.65
time (sec)	N/A	0.061	0.005	0.667	0.199	0.310	5.017	0.000	0.340

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	23	0	0	89	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	3.42	0.00	0.00	0.00
time (sec)	N/A	0.066	0.009	0.000	0.000	0.283	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	140	256	253	193	333	339	0
N.S.	1	1.00	0.82	1.50	1.48	1.13	1.95	1.98	0.00
time (sec)	N/A	0.154	0.110	54.990	0.212	0.294	8.482	0.429	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	101	181	180	135	241	241	0
N.S.	1	1.00	0.71	1.27	1.27	0.95	1.70	1.70	0.00
time (sec)	N/A	0.139	0.079	12.181	0.199	0.286	4.897	0.422	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	113	61	105	109	76	156	146	0
N.S.	1	1.26	0.68	1.17	1.21	0.84	1.73	1.62	0.00
time (sec)	N/A	0.084	0.046	1.743	0.197	0.296	3.007	0.395	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	29	48	48	42	82	64	0
N.S.	1	1.00	0.76	1.26	1.26	1.11	2.16	1.68	0.00
time (sec)	N/A	0.012	0.008	0.114	0.193	0.296	2.072	0.338	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	141	0	0	77	0	0	0
N.S.	1	1.00	1.83	0.00	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.136	0.304	0.000	0.000	0.288	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	89	0	97	89	0	202	0
N.S.	1	1.00	1.29	0.00	1.41	1.29	0.00	2.93	0.00
time (sec)	N/A	0.077	0.280	0.000	0.204	0.318	0.000	0.350	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	150	137	0	152	167	0	633	0
N.S.	1	1.00	0.91	0.00	1.01	1.11	0.00	4.22	0.00
time (sec)	N/A	0.155	0.275	0.000	0.205	0.314	0.000	0.362	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	188	188	178	0	210	242	0	1102	0
N.S.	1	1.00	0.95	0.00	1.12	1.29	0.00	5.86	0.00
time (sec)	N/A	0.159	0.292	0.000	0.217	0.319	0.000	0.370	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	285	617	578	592	760	995	0
N.S.	1	1.00	0.77	1.66	1.55	1.59	2.04	2.67	0.00
time (sec)	N/A	0.343	0.185	205.349	0.233	0.310	38.319	0.622	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	207	439	417	419	552	715	0
N.S.	1	1.00	0.69	1.47	1.40	1.41	1.85	2.40	0.00
time (sec)	N/A	0.343	0.147	50.855	0.234	0.315	12.604	0.523	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	125	261	257	244	352	435	0
N.S.	1	1.00	0.55	1.15	1.14	1.08	1.56	1.92	0.00
time (sec)	N/A	0.209	0.092	10.749	0.205	0.324	9.277	0.452	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	67	119	117	124	184	198	0
N.S.	1	1.00	0.97	1.72	1.70	1.80	2.67	2.87	0.00
time (sec)	N/A	0.036	0.016	0.387	0.196	0.329	6.767	0.421	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	502	0	0	178	0	0	0
N.S.	1	1.00	3.89	0.00	0.00	1.38	0.00	0.00	0.00
time (sec)	N/A	0.223	0.400	0.000	0.000	0.314	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	157	0	0	266	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	1.93	0.00	0.00	0.00
time (sec)	N/A	0.235	0.510	0.000	0.000	0.296	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	214	214	207	0	0	535	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	2.50	0.00	0.00	0.00
time (sec)	N/A	0.329	0.488	0.000	0.000	0.296	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	346	346	240	0	0	810	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	2.34	0.00	0.00	0.00
time (sec)	N/A	0.499	0.598	0.000	0.000	0.307	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	73	174	76	159	398	132	0
N.S.	1	1.00	1.24	2.95	1.29	2.69	6.75	2.24	0.00
time (sec)	N/A	0.055	0.062	4.804	0.184	0.304	8.569	0.325	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	73	169	76	159	398	132	0
N.S.	1	1.00	1.24	2.86	1.29	2.69	6.75	2.24	0.00
time (sec)	N/A	0.072	0.059	1.324	0.195	0.313	2.856	0.324	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	73	174	76	159	398	132	0
N.S.	1	1.00	1.24	2.95	1.29	2.69	6.75	2.24	0.00
time (sec)	N/A	0.053	0.063	0.362	0.185	0.316	0.858	0.317	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	68	56	64	131	0	0
N.S.	1	1.00	1.02	1.28	1.06	1.21	2.47	0.00	0.00
time (sec)	N/A	0.061	0.061	0.217	0.193	0.310	2.282	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	72	135	0	140	495	389	0
N.S.	1	1.00	1.01	1.90	0.00	1.97	6.97	5.48	0.00
time (sec)	N/A	0.051	0.076	0.224	0.000	0.314	2.037	0.338	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	72	135	0	140	495	390	0
N.S.	1	1.00	1.01	1.90	0.00	1.97	6.97	5.49	0.00
time (sec)	N/A	0.053	0.070	0.377	0.000	0.301	3.419	0.337	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	73	169	76	159	398	132	0
N.S.	1	1.00	1.24	2.86	1.29	2.69	6.75	2.24	0.00
time (sec)	N/A	0.057	0.061	2.615	0.191	0.299	5.027	0.307	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	73	169	76	159	398	132	0
N.S.	1	1.00	1.24	2.86	1.29	2.69	6.75	2.24	0.00
time (sec)	N/A	0.064	0.059	0.697	0.187	0.305	1.509	0.344	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	53	144	68	138	323	110	0
N.S.	1	1.00	0.93	2.53	1.19	2.42	5.67	1.93	0.00
time (sec)	N/A	0.025	0.083	0.194	0.189	0.280	0.474	0.314	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	127	0	130	348	186	0
N.S.	1	1.00	1.00	1.90	0.00	1.94	5.19	2.78	0.00
time (sec)	N/A	0.058	0.070	0.208	0.000	0.290	2.181	0.384	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	72	135	0	140	495	390	0
N.S.	1	1.00	1.01	1.90	0.00	1.97	6.97	5.49	0.00
time (sec)	N/A	0.055	0.072	0.223	0.000	0.289	2.571	0.352	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	72	135	0	140	495	390	0
N.S.	1	1.00	1.01	1.90	0.00	1.97	6.97	5.49	0.00
time (sec)	N/A	0.056	0.066	0.747	0.000	0.295	5.035	0.353	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	118	583	148	489	1634	744	0
N.S.	1	1.00	1.15	5.66	1.44	4.75	15.86	7.22	0.00
time (sec)	N/A	0.109	0.177	15.649	0.192	0.301	26.357	0.374	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	118	588	148	488	1625	744	0
N.S.	1	1.00	1.15	5.71	1.44	4.74	15.78	7.22	0.00
time (sec)	N/A	0.130	0.170	5.228	0.192	0.305	7.097	0.378	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	116	577	148	488	1622	744	0
N.S.	1	1.00	1.14	5.66	1.45	4.78	15.90	7.29	0.00
time (sec)	N/A	0.105	0.163	1.789	0.181	0.301	1.726	0.313	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	90	122	114	115	216	0	0
N.S.	1	1.00	0.87	1.17	1.10	1.11	2.08	0.00	0.00
time (sec)	N/A	0.105	0.136	1.444	0.326	0.293	2.439	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	120	474	0	457	2118	0	0
N.S.	1	1.00	0.89	3.51	0.00	3.39	15.69	0.00	0.00
time (sec)	N/A	0.121	0.215	1.149	0.000	0.303	3.299	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	121	475	0	457	2127	0	0
N.S.	1	1.00	0.90	3.52	0.00	3.39	15.76	0.00	0.00
time (sec)	N/A	0.116	0.205	1.099	0.000	0.303	4.816	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	124	586	152	497	0	746	0
N.S.	1	1.00	1.18	5.58	1.45	4.73	0.00	7.10	0.00
time (sec)	N/A	0.111	0.174	9.098	0.187	0.303	0.000	0.346	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	124	581	152	497	240	746	0
N.S.	1	1.00	1.18	5.53	1.45	4.73	2.29	7.10	0.00
time (sec)	N/A	0.109	0.173	3.028	0.185	0.282	83.393	0.366	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	107	509	144	466	211	244	0
N.S.	1	1.00	0.95	4.50	1.27	4.12	1.87	2.16	0.00
time (sec)	N/A	0.049	0.115	1.010	0.183	0.316	2.283	0.344	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	121	471	0	455	209	0	0
N.S.	1	1.00	0.98	3.83	0.00	3.70	1.70	0.00	0.00
time (sec)	N/A	0.113	0.198	1.108	0.000	0.319	3.285	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	478	0	466	233	0	0
N.S.	1	1.00	1.00	3.76	0.00	3.67	1.83	0.00	0.00
time (sec)	N/A	0.112	0.204	1.119	0.000	0.299	18.042	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	478	0	466	233	0	0
N.S.	1	1.00	1.00	3.76	0.00	3.67	1.83	0.00	0.00
time (sec)	N/A	0.115	0.198	1.096	0.000	0.332	139.218	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	473	0	466	0	0	0
N.S.	1	1.00	1.00	3.72	0.00	3.67	0.00	0.00	0.00
time (sec)	N/A	0.115	0.206	3.262	0.000	0.319	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	172	1249	218	1011	4100	1609	0
N.S.	1	1.00	1.17	8.50	1.48	6.88	27.89	10.95	0.00
time (sec)	N/A	0.244	0.241	40.921	0.190	0.321	92.425	0.372	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	178	1262	222	1022	0	1611	0
N.S.	1	1.00	1.19	8.47	1.49	6.86	0.00	10.81	0.00
time (sec)	N/A	0.277	0.240	15.464	0.186	0.338	0.000	0.476	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	178	1267	222	1024	357	1611	0
N.S.	1	1.00	1.19	8.50	1.49	6.87	2.40	10.81	0.00
time (sec)	N/A	0.231	0.230	5.648	0.196	0.336	86.831	0.375	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	132	177	172	169	299	0	0
N.S.	1	1.00	0.87	1.16	1.13	1.11	1.97	0.00	0.00
time (sec)	N/A	0.111	0.240	3.322	0.188	0.346	2.913	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	181	1039	0	981	347	0	0
N.S.	1	1.00	0.95	5.44	0.00	5.14	1.82	0.00	0.00
time (sec)	N/A	0.283	0.272	3.593	0.000	0.361	46.140	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	181	1039	0	980	0	0	0
N.S.	1	1.00	0.95	5.44	0.00	5.13	0.00	0.00	0.00
time (sec)	N/A	0.268	0.282	3.445	0.000	0.319	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	184	1269	228	1023	0	1611	0
N.S.	1	1.00	1.22	8.40	1.51	6.77	0.00	10.67	0.00
time (sec)	N/A	0.275	0.230	26.564	0.191	0.359	0.000	0.354	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	176	1257	224	1022	0	1611	0
N.S.	1	1.00	1.19	8.49	1.51	6.91	0.00	10.89	0.00
time (sec)	N/A	0.272	0.229	9.671	0.194	0.319	0.000	0.326	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	159	1108	220	983	325	379	0
N.S.	1	1.00	0.94	6.56	1.30	5.82	1.92	2.24	0.00
time (sec)	N/A	0.062	0.162	3.408	0.187	0.306	3.895	0.308	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	181	1035	0	967	323	0	0
N.S.	1	1.00	1.01	5.78	0.00	5.40	1.80	0.00	0.00
time (sec)	N/A	0.262	0.254	3.769	0.000	0.311	10.764	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	180	1039	0	980	347	0	0
N.S.	1	1.00	0.94	5.44	0.00	5.13	1.82	0.00	0.00
time (sec)	N/A	0.256	0.262	3.471	0.000	0.338	49.681	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	187	1041	0	981	0	0	0
N.S.	1	1.00	1.02	5.69	0.00	5.36	0.00	0.00	0.00
time (sec)	N/A	0.260	0.265	3.648	0.000	0.359	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	188	1041	0	981	0	0	0
N.S.	1	1.00	1.03	5.69	0.00	5.36	0.00	0.00	0.00
time (sec)	N/A	0.273	0.263	3.286	0.000	0.302	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	182	1044	0	981	0	0	0
N.S.	1	1.00	0.95	5.47	0.00	5.14	0.00	0.00	0.00
time (sec)	N/A	0.273	0.271	9.999	0.000	0.292	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	87	23	25	29	20	25	25
N.S.	1	1.00	3.78	1.00	1.09	1.26	0.87	1.09	1.09
time (sec)	N/A	0.045	0.092	0.056	0.230	0.257	6.413	0.308	0.484

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	87	21	23	25	19	23	23
N.S.	1	1.00	4.14	1.00	1.10	1.19	0.90	1.10	1.10
time (sec)	N/A	0.029	0.076	0.053	0.227	0.266	2.492	0.304	0.426

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	A	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	54	54	108	246	0	93	452	0	0
N.S.	1	1.00	2.00	4.56	0.00	1.72	8.37	0.00	0.00
time (sec)	N/A	0.052	0.113	0.635	0.000	0.269	149.775	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	86	23	25	29	20	25	25
N.S.	1	1.00	3.74	1.00	1.09	1.26	0.87	1.09	1.09
time (sec)	N/A	0.044	0.089	0.053	0.240	0.287	5.708	0.298	0.433

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	87	23	25	29	20	25	25
N.S.	1	1.00	3.78	1.00	1.09	1.26	0.87	1.09	1.09
time (sec)	N/A	0.049	0.084	0.060	0.243	0.296	4.023	0.345	0.477

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	69	20	22	22	17	22	22
N.S.	1	1.00	3.45	1.00	1.10	1.10	0.85	1.10	1.10
time (sec)	N/A	0.013	0.065	0.047	0.228	0.258	1.586	0.334	0.651

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	83	23	25	29	20	25	25
N.S.	1	1.00	3.61	1.00	1.09	1.26	0.87	1.09	1.09
time (sec)	N/A	0.045	0.085	0.056	0.243	0.255	3.808	0.406	0.425

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	140	23	25	42	22	25	25
N.S.	1	1.00	6.09	1.00	1.09	1.83	0.96	1.09	1.09
time (sec)	N/A	0.045	0.165	0.060	0.231	0.243	56.206	0.319	0.574

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	140	21	23	38	20	23	23
N.S.	1	1.00	6.67	1.00	1.10	1.81	0.95	1.10	1.10
time (sec)	N/A	0.029	0.162	0.059	0.229	0.260	10.048	0.311	0.546

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	102	102	132	342	0	214	360	0	0
N.S.	1	1.00	1.29	3.35	0.00	2.10	3.53	0.00	0.00
time (sec)	N/A	0.156	0.210	1.197	0.000	0.278	144.219	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	139	23	25	45	22	25	25
N.S.	1	1.00	6.04	1.00	1.09	1.96	0.96	1.09	1.09
time (sec)	N/A	0.046	0.164	0.058	0.234	0.266	117.217	0.324	0.464

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	140	23	25	42	22	25	25
N.S.	1	1.00	6.09	1.00	1.09	1.83	0.96	1.09	1.09
time (sec)	N/A	0.042	0.156	0.068	0.230	0.264	23.097	0.301	0.534

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	161	20	22	35	19	22	22
N.S.	1	1.00	8.05	1.00	1.10	1.75	0.95	1.10	1.10
time (sec)	N/A	0.012	1.707	0.051	0.224	0.262	9.127	0.398	0.556

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	135	23	25	45	22	25	25
N.S.	1	1.00	5.87	1.00	1.09	1.96	0.96	1.09	1.09
time (sec)	N/A	0.041	0.139	0.066	0.239	0.279	54.173	0.302	0.473

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	0	45	0	0	0
N.S.	1	1.00	1.00	0.84	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.097	0.016	0.510	0.000	0.266	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	132	177	172	169	299	0	0
N.S.	1	1.00	0.87	1.16	1.13	1.11	1.97	0.00	0.00
time (sec)	N/A	0.109	0.177	0.000	0.180	0.253	2.789	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	90	122	114	115	216	0	0
N.S.	1	1.00	0.87	1.17	1.10	1.11	2.08	0.00	0.00
time (sec)	N/A	0.088	0.094	0.000	0.180	0.316	2.348	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	68	56	64	131	0	0
N.S.	1	1.00	1.02	1.28	1.06	1.21	2.47	0.00	0.00
time (sec)	N/A	0.058	0.034	0.014	0.177	0.270	2.271	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	A	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	54	54	108	246	0	93	452	0	0
N.S.	1	1.00	2.00	4.56	0.00	1.72	8.37	0.00	0.00
time (sec)	N/A	0.053	0.032	0.000	0.000	0.285	148.700	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	102	102	132	342	0	214	360	0	0
N.S.	1	1.00	1.29	3.35	0.00	2.10	3.53	0.00	0.00
time (sec)	N/A	0.169	0.092	0.431	0.000	0.277	145.563	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	169	169	170	473	0	401	0	0	0
N.S.	1	1.00	1.01	2.80	0.00	2.37	0.00	0.00	0.00
time (sec)	N/A	0.300	0.158	4.003	0.000	0.279	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	262	418	391	521	588	0	0
N.S.	1	1.00	1.07	1.71	1.60	2.13	2.40	0.00	0.00
time (sec)	N/A	0.214	0.254	9.882	0.190	0.271	8.035	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	179	281	259	353	408	0	0
N.S.	1	1.00	1.11	1.75	1.61	2.19	2.53	0.00	0.00
time (sec)	N/A	0.173	0.174	3.382	0.185	0.269	7.698	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	109	149	131	193	245	0	0
N.S.	1	1.00	1.36	1.86	1.64	2.41	3.06	0.00	0.00
time (sec)	N/A	0.093	0.094	0.831	0.197	0.306	6.867	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	94	94	270	580	0	228	0	0	0
N.S.	1	1.00	2.87	6.17	0.00	2.43	0.00	0.00	0.00
time (sec)	N/A	0.088	0.189	0.801	0.000	0.264	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	182	182	397	0	0	600	0	0	0
N.S.	1	1.00	2.18	0.00	0.00	3.30	0.00	0.00	0.00
time (sec)	N/A	0.282	0.246	0.000	0.000	0.271	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	267	267	459	0	0	1165	0	0	0
N.S.	1	1.00	1.72	0.00	0.00	4.36	0.00	0.00	0.00
time (sec)	N/A	0.587	0.368	0.000	0.000	0.280	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	327	327	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	284	284	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.291	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	240	240	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.233	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	174	174	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	225	225	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	271	271	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.287	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	314	314	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	178	8183	343	4918	0	772	0
N.S.	1	1.00	0.76	35.12	1.47	21.11	0.00	3.31	0.00
time (sec)	N/A	1.894	0.302	57.146	0.210	0.480	0.000	0.371	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	124	3059	240	1875	0	528	0
N.S.	1	1.00	0.75	18.54	1.45	11.36	0.00	3.20	0.00
time (sec)	N/A	0.147	0.166	13.002	0.202	0.341	0.000	0.373	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	70	660	137	431	0	285	0
N.S.	1	1.00	0.72	6.80	1.41	4.44	0.00	2.94	0.00
time (sec)	N/A	0.068	0.081	1.915	0.195	0.285	0.000	0.310	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	32	68	57	52	141	95	0
N.S.	1	1.00	0.70	1.48	1.24	1.13	3.07	2.07	0.00
time (sec)	N/A	0.013	0.009	0.085	0.182	0.313	2.116	0.297	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	111	25	27	33	22	27	27
N.S.	1	1.00	4.44	1.00	1.08	1.32	0.88	1.08	1.08
time (sec)	N/A	0.045	0.158	0.076	0.258	0.265	6.047	0.327	0.447

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	177	25	27	46	24	27	27
N.S.	1	1.00	7.08	1.00	1.08	1.84	0.96	1.08	1.08
time (sec)	N/A	0.046	0.296	0.063	0.259	0.272	23.527	0.340	0.518

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	143	0	0	0	0	0	0
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.513	0.000	0.000	0.000	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	98	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	0.459	0.000	0.000	0.000	0.000	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	480	480	408	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.471	1.518	0.000	0.000	0.000	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	350	350	304	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	0.753	0.000	0.000	0.000	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	220	220	200	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.190	0.354	0.000	0.000	0.000	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	107	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.046	0.056	0.000	0.000	0.000	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	0	29	24	29	29
N.S.	1	1.00	1.07	1.00	0.00	1.07	0.89	1.07	1.07
time (sec)	N/A	0.064	1.132	0.129	0.000	0.270	92.380	0.353	0.385

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	0	42	1	29	29
N.S.	1	1.00	1.07	1.00	0.00	1.56	0.04	1.07	1.07
time (sec)	N/A	0.066	1.337	0.145	0.000	0.278	0.000	0.379	0.433

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	108	284	218	215	908	199	174
N.S.	1	1.00	0.94	2.47	1.90	1.87	7.90	1.73	1.51
time (sec)	N/A	0.069	0.109	0.685	0.198	0.277	2.372	0.320	0.954

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	202	202	244	740	0	0	0	0	0
N.S.	1	1.00	1.21	3.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	0.155	0.774	0.000	0.000	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	295	295	339	1652	0	0	0	0	0
N.S.	1	1.00	1.15	5.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.340	0.234	1.197	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [113] had the largest ratio of [.6087000000000000019]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	19	0.158
2	A	4	3	1.00	19	0.158
3	A	4	3	1.00	17	0.176
4	A	2	1	1.00	16	0.062
5	A	4	3	1.00	19	0.158
6	A	4	4	1.04	19	0.210
7	A	4	4	1.00	19	0.210
8	A	4	3	1.00	19	0.158
9	A	4	4	1.00	21	0.190
10	A	4	4	1.00	21	0.190
11	A	4	4	1.00	19	0.210
12	A	4	4	1.00	18	0.222
13	A	3	3	1.00	21	0.143
14	A	3	3	1.00	21	0.143
15	A	4	4	1.00	21	0.190
16	A	4	4	1.00	21	0.190
17	A	4	4	1.00	21	0.190
18	A	4	4	1.00	21	0.190
19	A	4	4	1.00	21	0.190
20	A	4	4	1.00	21	0.190
21	A	5	4	1.00	19	0.210
22	A	4	4	1.00	18	0.222
23	A	4	3	1.00	21	0.143
24	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	3	3	1.00	21	0.143
26	A	5	4	1.00	21	0.190
27	A	4	4	1.00	21	0.190
28	A	5	6	1.00	21	0.286
29	A	4	4	1.00	21	0.190
30	A	4	4	1.00	21	0.190
31	A	8	6	1.00	21	0.286
32	A	7	6	1.00	21	0.286
33	A	6	5	1.00	19	0.263
34	A	2	2	1.00	18	0.111
35	A	2	2	1.00	21	0.095
36	A	4	4	1.00	21	0.190
37	A	6	4	1.00	21	0.190
38	A	8	4	1.00	21	0.190
39	A	8	7	1.00	21	0.333
40	A	7	6	1.09	21	0.286
41	A	3	3	1.00	19	0.158
42	A	2	2	1.00	18	0.111
43	A	5	5	1.00	21	0.238
44	A	7	7	1.00	21	0.333
45	A	8	7	1.00	21	0.333
46	A	8	6	1.00	21	0.286
47	A	4	3	1.00	21	0.143
48	A	3	2	1.00	19	0.105
49	A	3	2	1.00	18	0.111
50	A	9	7	1.00	21	0.333
51	A	10	8	1.00	21	0.381
52	A	11	8	1.00	21	0.381
53	A	10	7	1.00	21	0.333
54	A	9	6	1.00	21	0.286
55	A	5	3	1.00	21	0.143
56	A	3	2	1.00	21	0.095
57	A	4	4	1.00	19	0.210
58	A	3	2	1.00	18	0.111
59	A	13	7	1.00	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	13	8	1.00	21	0.381
61	A	14	8	1.00	21	0.381
62	A	13	7	1.00	21	0.333
63	A	12	6	1.00	21	0.286
64	A	8	3	1.00	21	0.143
65	A	3	2	1.00	21	0.095
66	A	5	6	1.00	21	0.286
67	A	4	4	1.00	21	0.190
68	A	4	4	1.00	21	0.190
69	A	4	4	1.00	19	0.210
70	A	3	2	1.00	18	0.111
71	A	25	7	1.00	21	0.333
72	A	22	8	1.00	21	0.381
73	A	23	8	1.00	21	0.381
74	A	1	1	1.00	13	0.077
75	A	1	1	1.00	14	0.071
76	A	6	3	1.00	21	0.143
77	A	6	3	1.00	19	0.158
78	A	7	5	1.00	18	0.278
79	A	6	5	1.00	21	0.238
80	A	6	5	1.00	21	0.238
81	A	6	3	1.00	21	0.143
82	A	6	3	1.00	21	0.143
83	A	6	3	1.00	21	0.143
84	A	8	3	1.00	23	0.130
85	A	8	3	1.00	21	0.143
86	A	5	4	1.00	20	0.200
87	A	14	8	1.00	23	0.348
88	A	9	7	1.00	23	0.304
89	A	8	5	1.00	23	0.217
90	A	8	3	1.00	23	0.130
91	A	8	3	1.00	23	0.130
92	A	12	8	1.00	23	0.348
93	A	10	8	1.00	23	0.348
94	A	8	6	1.00	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
95	A	3	3	1.00	20	0.150
96	A	3	3	1.00	23	0.130
97	A	6	6	1.00	23	0.261
98	A	9	6	1.00	23	0.261
99	A	12	6	1.00	23	0.261
100	A	13	10	1.00	23	0.435
101	A	11	8	1.00	23	0.348
102	A	8	6	1.00	21	0.286
103	A	3	3	1.00	20	0.150
104	A	7	7	1.00	23	0.304
105	A	10	9	1.00	23	0.391
106	A	12	9	1.00	23	0.391
107	A	17	13	1.10	23	0.565
108	A	14	11	1.13	23	0.478
109	A	4	4	1.00	21	0.190
110	A	6	6	1.00	20	0.300
111	A	14	10	1.04	23	0.435
112	A	16	13	1.04	23	0.565
113	A	27	14	1.08	23	0.609
114	A	24	12	1.09	23	0.522
115	A	5	4	1.00	23	0.174
116	A	8	7	1.06	21	0.333
117	A	10	7	1.00	20	0.350
118	A	25	11	1.03	23	0.478
119	A	26	14	1.03	23	0.609
120	A	8	7	1.47	13	0.538
121	A	4	4	1.00	23	0.174
122	A	9	7	1.00	23	0.304
123	A	18	9	1.13	23	0.391
124	A	10	7	1.00	20	0.350
125	A	14	7	1.00	22	0.318
126	A	18	7	1.00	22	0.318
127	N/A	0	0	1.00	22	0.000
128	N/A	0	0	1.00	22	0.000
129	N/A	0	0	1.00	22	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	8	7	1.00	23	0.304
131	A	6	6	1.00	23	0.261
132	A	7	7	1.00	21	0.333
133	A	5	4	1.00	20	0.200
134	A	12	11	1.00	23	0.478
135	A	11	9	1.00	23	0.391
136	A	16	11	1.00	23	0.478
137	A	9	7	1.00	23	0.304
138	A	6	6	1.00	23	0.261
139	A	8	7	1.00	21	0.333
140	A	6	4	1.00	20	0.200
141	A	18	11	1.00	23	0.478
142	A	14	10	1.00	23	0.435
143	A	16	11	1.00	23	0.478
144	A	7	7	1.00	23	0.304
145	A	6	6	1.00	23	0.261
146	A	6	7	1.00	21	0.333
147	A	4	4	1.00	20	0.200
148	A	7	8	1.00	23	0.348
149	A	11	10	1.00	23	0.435
150	A	16	11	1.00	23	0.478
151	A	6	6	1.00	23	0.261
152	A	6	6	1.00	23	0.261
153	A	5	6	1.00	21	0.286
154	A	3	3	1.00	20	0.150
155	A	11	10	1.00	23	0.435
156	A	15	13	1.00	23	0.565
157	N/A	0	0	1.00	23	0.000
158	N/A	0	0	1.00	21	0.000
159	N/A	0	0	1.00	20	0.000
160	N/A	0	0	1.00	23	0.000
161	N/A	0	0	1.00	23	0.000
162	A	3	3	1.00	23	0.130
163	A	4	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
164	A	3	2	1.00	21	0.095
165	A	1	1	1.00	16	0.062
166	N/A	0	0	1.00	23	0.000
167	N/A	0	0	1.00	23	0.000
168	N/A	0	0	1.00	15	0.000
169	A	2	2	1.00	14	0.143
170	N/A	0	0	1.00	17	0.000
171	A	2	2	1.00	21	0.095
172	A	2	2	1.00	21	0.095
173	A	4	3	1.00	19	0.158
174	A	4	4	1.00	21	0.190
175	A	3	3	1.04	21	0.143
176	A	4	3	1.00	21	0.143
177	A	2	2	1.00	21	0.095
178	A	2	2	1.00	21	0.095
179	A	2	1	1.00	18	0.056
180	A	2	2	1.00	21	0.095
181	A	4	3	1.00	21	0.143
182	A	4	3	1.00	21	0.143
183	A	4	5	1.00	23	0.217
184	A	4	5	1.00	23	0.217
185	A	5	5	1.00	21	0.238
186	A	3	4	1.00	23	0.174
187	A	7	6	1.00	23	0.261
188	A	5	5	1.00	23	0.217
189	A	2	2	1.00	23	0.087
190	A	2	2	1.00	23	0.087
191	A	2	2	1.00	20	0.100
192	A	2	2	1.00	23	0.087
193	A	2	2	1.00	23	0.087
194	A	4	4	1.00	23	0.174
195	A	4	4	1.00	23	0.174
196	A	4	5	1.00	23	0.217
197	A	6	6	1.00	23	0.261

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
198	A	5	5	1.00	21	0.238
199	A	5	5	1.00	23	0.217
200	A	7	6	1.00	23	0.261
201	A	7	6	1.00	23	0.261
202	A	2	2	1.00	23	0.087
203	A	2	2	1.00	23	0.087
204	A	2	2	1.00	20	0.100
205	A	2	2	1.00	23	0.087
206	A	3	3	1.00	23	0.130
207	A	2	2	1.00	23	0.087
208	A	4	4	1.00	23	0.174
209	A	4	4	1.00	23	0.174
210	A	6	6	1.00	23	0.261
211	A	5	6	1.00	23	0.261
212	A	2	2	1.00	21	0.095
213	A	2	2	1.00	23	0.087
214	A	4	4	1.00	23	0.174
215	A	6	4	1.00	23	0.174
216	A	10	9	1.00	23	0.391
217	A	9	8	1.00	23	0.348
218	A	5	5	1.00	20	0.250
219	A	7	7	1.00	23	0.304
220	A	9	7	1.00	23	0.304
221	A	7	8	1.00	23	0.348
222	A	6	7	1.00	23	0.304
223	A	2	2	1.00	21	0.095
224	A	3	3	1.00	23	0.130
225	A	5	5	1.00	23	0.217
226	A	16	10	1.00	23	0.435
227	A	14	8	1.00	23	0.348
228	A	7	6	1.00	20	0.300
229	A	8	8	1.00	23	0.348
230	A	10	8	1.00	23	0.348
231	A	10	9	1.00	23	0.391
232	A	4	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
233	A	4	3	1.00	21	0.143
234	A	4	3	1.00	23	0.130
235	A	6	5	1.00	23	0.217
236	A	24	9	1.00	23	0.391
237	A	19	9	1.00	23	0.391
238	A	10	7	1.00	20	0.350
239	A	9	8	1.00	23	0.348
240	A	11	8	1.00	23	0.348
241	A	2	2	1.00	18	0.111
242	A	2	2	1.00	19	0.105
243	A	2	3	1.00	12	0.250
244	A	4	4	1.00	10	0.400
245	A	3	4	1.00	19	0.210
246	A	3	4	1.00	21	0.190
247	A	16	6	1.00	22	0.273
248	A	20	6	1.00	22	0.273
249	N/A	0	0	1.00	22	0.000
250	N/A	0	0	1.00	22	0.000
251	A	7	8	1.00	25	0.320
252	A	8	9	1.00	25	0.360
253	A	6	5	1.00	23	0.217
254	A	12	9	1.00	25	0.360
255	A	14	11	1.00	25	0.440
256	A	19	13	1.00	25	0.520
257	A	11	12	1.00	25	0.480
258	A	11	11	1.00	22	0.500
259	A	11	10	1.00	25	0.400
260	A	5	4	1.00	25	0.160
261	A	7	8	1.00	25	0.320
262	A	8	9	1.00	25	0.360
263	A	7	8	1.00	25	0.320
264	A	9	9	1.00	25	0.360
265	A	7	5	1.00	23	0.217
266	A	17	9	1.00	25	0.360
267	A	18	12	1.00	25	0.480

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
268	A	19	13	1.00	25	0.520
269	A	16	11	1.00	22	0.500
270	A	14	12	1.00	25	0.480
271	A	11	10	1.00	25	0.400
272	A	6	4	1.00	25	0.160
273	A	8	8	1.00	25	0.320
274	A	9	9	1.00	25	0.360
275	A	6	5	1.00	13	0.385
276	A	7	8	1.00	25	0.320
277	A	7	9	1.00	25	0.360
278	A	5	5	1.00	23	0.217
279	A	8	9	1.00	25	0.360
280	A	14	12	1.00	25	0.480
281	A	12	12	1.00	25	0.480
282	A	7	7	1.00	22	0.318
283	A	4	4	1.00	25	0.160
284	A	6	8	1.00	25	0.320
285	A	7	9	1.00	25	0.360
286	A	7	8	1.00	25	0.320
287	A	7	8	1.00	25	0.320
288	A	6	8	1.00	25	0.320
289	A	4	4	1.00	23	0.174
290	A	11	9	1.00	25	0.360
291	A	12	12	1.00	25	0.480
292	A	11	11	1.00	25	0.440
293	A	3	3	1.00	22	0.136
294	A	5	7	1.00	25	0.280
295	A	6	8	1.00	25	0.320
296	A	8	10	1.00	25	0.400
297	A	9	8	1.00	25	0.320
298	A	7	8	1.00	25	0.320
299	A	6	8	1.00	25	0.320
300	A	5	5	1.00	23	0.217
301	A	15	9	1.00	25	0.360
302	A	13	14	1.00	25	0.560

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
303	A	12	13	1.00	25	0.520
304	A	11	11	1.00	25	0.440
305	A	4	4	1.00	25	0.160
306	A	5	5	1.00	22	0.227
307	A	6	9	1.00	25	0.360
308	A	7	10	1.00	25	0.400
309	A	8	10	1.00	33	0.303
310	A	6	6	1.00	31	0.194
311	A	8	9	1.00	33	0.273
312	A	13	11	1.00	33	0.333
313	A	12	12	1.00	33	0.364
314	A	7	7	1.00	30	0.233
315	A	4	4	1.00	33	0.121
316	A	6	8	1.00	33	0.242
317	A	5	5	1.00	13	0.385
318	A	3	3	1.00	25	0.120
319	A	4	4	1.00	25	0.160
320	A	3	2	1.00	23	0.087
321	A	1	1	1.00	16	0.062
322	N/A	0	0	1.00	25	0.000
323	N/A	0	0	1.00	25	0.000
324	A	26	6	1.00	22	0.273
325	A	20	6	1.00	22	0.273
326	A	14	11	1.00	20	0.550
327	N/A	0	0	1.00	22	0.000
328	N/A	0	0	1.00	22	0.000
329	A	9	7	1.00	23	0.304
330	A	8	7	1.00	23	0.304
331	A	7	7	1.00	21	0.333
332	A	6	6	1.00	20	0.300
333	A	3	3	1.00	23	0.130
334	A	2	2	1.00	23	0.087
335	A	6	7	1.00	23	0.304
336	A	7	7	1.00	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
337	A	9	7	1.00	21	0.333
338	A	8	7	1.00	21	0.333
339	A	7	7	1.00	19	0.368
340	A	6	6	1.00	18	0.333
341	A	3	3	1.00	21	0.143
342	A	2	2	1.00	21	0.095
343	A	6	7	1.00	21	0.333
344	A	7	7	1.00	21	0.333
345	A	2	2	1.00	22	0.091
346	A	2	2	1.00	23	0.087
347	A	2	2	1.00	25	0.080
348	A	4	4	1.00	18	0.222
349	A	4	4	1.00	18	0.222
350	A	3	3	1.00	22	0.136
351	A	5	4	1.00	27	0.148
352	A	5	4	1.00	27	0.148
353	A	5	4	1.26	25	0.160
354	A	1	1	1.00	18	0.056
355	A	3	3	1.00	27	0.111
356	A	3	3	1.00	27	0.111
357	A	5	4	1.00	27	0.148
358	A	5	4	1.00	27	0.148
359	A	7	7	1.00	29	0.241
360	A	7	8	1.00	29	0.276
361	A	7	8	1.00	27	0.296
362	A	2	2	1.00	20	0.100
363	A	4	4	1.00	29	0.138
364	A	4	4	1.00	29	0.138
365	A	7	7	1.00	29	0.241
366	A	12	9	1.00	29	0.310
367	A	4	3	1.00	21	0.143
368	A	4	3	1.00	21	0.143
369	A	4	3	1.00	19	0.158
370	A	4	4	1.00	21	0.190
371	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
372	A	2	2	1.00	21	0.095
373	A	4	3	1.00	21	0.143
374	A	4	3	1.00	21	0.143
375	A	3	2	1.00	18	0.111
376	A	4	3	1.00	21	0.143
377	A	2	2	1.00	21	0.095
378	A	2	2	1.00	21	0.095
379	A	4	4	1.00	23	0.174
380	A	4	4	1.00	23	0.174
381	A	4	4	1.00	21	0.190
382	A	5	6	1.00	23	0.261
383	A	4	4	1.00	23	0.174
384	A	4	4	1.00	23	0.174
385	A	4	4	1.00	23	0.174
386	A	4	4	1.00	23	0.174
387	A	2	2	1.00	20	0.100
388	A	3	3	1.00	23	0.130
389	A	4	4	1.00	23	0.174
390	A	4	4	1.00	23	0.174
391	A	4	4	1.00	23	0.174
392	A	4	4	1.00	23	0.174
393	A	4	4	1.00	23	0.174
394	A	4	4	1.00	21	0.190
395	A	5	6	1.00	23	0.261
396	A	4	4	1.00	23	0.174
397	A	4	4	1.00	23	0.174
398	A	4	4	1.00	23	0.174
399	A	4	4	1.00	23	0.174
400	A	2	2	1.00	20	0.100
401	A	3	3	1.00	23	0.130
402	A	4	4	1.00	23	0.174
403	A	4	4	1.00	23	0.174
404	A	4	4	1.00	23	0.174
405	A	4	4	1.00	23	0.174
406	N/A	0	0	1.00	23	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
407	N/A	0	0	1.00	21	0.000
408	A	2	2	1.00	23	0.087
409	N/A	0	0	1.00	23	0.000
410	N/A	0	0	1.00	23	0.000
411	N/A	0	0	1.00	20	0.000
412	N/A	0	0	1.00	23	0.000
413	N/A	0	0	1.00	23	0.000
414	N/A	0	0	1.00	21	0.000
415	A	5	5	1.00	23	0.217
416	N/A	0	0	1.00	23	0.000
417	N/A	0	0	1.00	23	0.000
418	N/A	0	0	1.00	20	0.000
419	N/A	0	0	1.00	23	0.000
420	A	4	4	1.00	25	0.160
421	A	5	6	1.00	23	0.261
422	A	5	6	1.00	23	0.261
423	A	4	4	1.00	21	0.190
424	A	2	2	1.00	23	0.087
425	A	5	5	1.00	23	0.217
426	A	10	8	1.00	23	0.348
427	A	10	5	1.00	25	0.200
428	A	8	5	1.00	25	0.200
429	A	6	5	1.00	23	0.217
430	A	3	3	1.00	25	0.120
431	A	7	6	1.00	25	0.240
432	A	14	8	1.00	25	0.320
433	A	23	9	1.00	25	0.360
434	A	17	9	1.00	25	0.360
435	A	12	9	1.00	25	0.360
436	A	8	9	1.00	25	0.360
437	A	11	9	1.00	25	0.360
438	A	15	9	1.00	25	0.360
439	A	20	9	1.00	25	0.360

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
440	A	9	5	1.00	25	0.200
441	A	7	5	1.00	25	0.200
442	A	5	4	1.00	23	0.174
443	A	1	1	1.00	16	0.062
444	N/A	0	0	1.00	25	0.000
445	N/A	0	0	1.00	25	0.000
446	A	3	3	1.00	26	0.115
447	A	3	3	1.00	32	0.094
448	A	13	4	1.00	27	0.148
449	A	10	4	1.00	27	0.148
450	A	7	4	1.00	25	0.160
451	A	2	2	1.00	18	0.111
452	N/A	0	0	1.00	27	0.000
453	N/A	0	0	1.00	27	0.000
454	A	3	2	1.00	23	0.087
455	A	8	7	1.00	25	0.280
456	A	11	9	1.00	25	0.360

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^3(d+ex)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int x^3(d+ex)(a+b\log(cx^n))\,dx=-\frac{1}{16}bdnx^4-\frac{1}{25}benx^5+\frac{1}{20}\left(5dx^4+4ex^5\right)(a+b\log(cx^n))$$

```
[Out] -1/16*b*d*n*x^4-1/25*b*e*n*x^5+1/20*(4*e*x^5+5*d*x^4)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {45, 2371, 12}

$$\int x^3(d+ex)(a+b\log(cx^n))\,dx=\frac{1}{20}\left(5dx^4+4ex^5\right)(a+b\log(cx^n))-\frac{1}{16}bdnx^4-\frac{1}{25}benx^5$$

```
[In] Int[x^3*(d + e*x)*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/16*(b*d*n*x^4) - (b*e*n*x^5)/25 + ((5*d*x^4 + 4*e*x^5)*(a + b*Log[c*x^n]
))/20
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{20}\left(5dx^4+4ex^5\right)(a+b\log(cx^n))-(bn)\int\frac{1}{20}x^3(5d+4ex)\,dx\\
&= \frac{1}{20}\left(5dx^4+4ex^5\right)(a+b\log(cx^n))-\frac{1}{20}(bn)\int x^3(5d+4ex)\,dx\\
&= \frac{1}{20}\left(5dx^4+4ex^5\right)(a+b\log(cx^n))-\frac{1}{20}(bn)\int\left(5dx^3+4ex^4\right)\,dx\\
&= -\frac{1}{16}bdnx^4-\frac{1}{25}benx^5+\frac{1}{20}\left(5dx^4+4ex^5\right)(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int x^3(d+ex)(a+b\log(cx^n))\,dx=\frac{1}{400}x^4(20a(5d+4ex)-bn(25d+16ex)+20b(5d+4ex)\log(cx^n))$$

```
[In] Integrate[x^3*(d + e*x)*(a + b*Log[c*x^n]),x]
[Out] (x^4*(20*a*(5*d + 4*e*x) - b*n*(25*d + 16*e*x) + 20*b*(5*d + 4*e*x)*Log[c*x
^n]))/400
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

method	result
parallelrisch	$\frac{x^5\ln(c\,x^n)be}{5}-\frac{ben\,x^5}{25}+\frac{ae\,x^5}{5}+\frac{x^4\ln(c\,x^n)bd}{4}-\frac{bdn\,x^4}{16}+\frac{ad\,x^4}{4}$
risch	$\frac{b\,x^4(4ex+5d)\ln(x^n)}{20}-\frac{i\pi be\,x^5\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)}{10}+\frac{i\pi be\,x^5\operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^n)^2}{10}+\frac{i\pi be\,x^5\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x}{10}$

```
[In] int(x^3*(e*x+d)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*ln(c*x^n)*b*e-1/25*b*e*n*x^5+1/5*a*e*x^5+1/4*x^4*ln(c*x^n)*b*d-1/16
*b*d*n*x^4+1/4*a*d*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int x^3(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{25}\,(ben-5\,ae)x^5-\frac{1}{16}\,(bdn-4\,ad)x^4+\frac{1}{20}\,\left(4\,bex^5+5\,bdx^4\right)\log\left(c\right)+\frac{1}{20}\,\left(4\,benx^5+5\,bdnx^4\right)\log\left(x\right)$$

```
[In] integrate(x^3*(e*x+d)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/25*(b*e*n - 5*a*e)*x^5 - 1/16*(b*d*n - 4*a*d)*x^4 + 1/20*(4*b*e*x^5 + 5*
b*d*x^4)*log(c) + 1/20*(4*b*e*n*x^5 + 5*b*d*n*x^4)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int x^3(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{adx^4}{4}+\frac{aex^5}{5}-\frac{bdnx^4}{16}+\frac{bdx^4\log\left(cx^n\right)}{4}-\frac{benx^5}{25}+\frac{bex^5\log\left(cx^n\right)}{5}$$

```
[In] integrate(x**3*(e*x+d)*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d*x**4/4 + a*e*x**5/5 - b*d*n*x**4/16 + b*d*x**4*log(c*x**n)/4 - b*e*n*x*
*5/25 + b*e*x**5*log(c*x**n)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int x^3(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{25}\,benx^5+\frac{1}{5}\,bex^5\log\left(cx^n\right)-\frac{1}{16}\,bdnx^4+\frac{1}{5}\,aex^5+\frac{1}{4}\,bdx^4\log\left(cx^n\right)+\frac{1}{4}\,adx^4$$

```
[In] integrate(x^3*(e*x+d)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/25*b*e*n*x^5 + 1/5*b*e*x^5*log(c*x^n) - 1/16*b*d*n*x^4 + 1/5*a*e*x^5 + 1
/4*b*d*x^4*log(c*x^n) + 1/4*a*d*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int x^3(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{1}{5}\,benx^5\log\left(x\right) - \frac{1}{25}\,benx^5 + \frac{1}{5}\,bex^5\log\left(c\right) + \frac{1}{4}\,bdnx^4\log\left(x\right) - \frac{1}{16}\,bdnx^4 + \frac{1}{5}\,aex^5 + \frac{1}{4}\,bdx^4\log\left(c\right) + \frac{1}{4}\,adx^4$$

```
[In] integrate(x^3*(e*x+d)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/5*b*e*n*x^5*log(x) - 1/25*b*e*n*x^5 + 1/5*b*e*x^5*log(c) + 1/4*b*d*n*x^4*
log(x) - 1/16*b*d*n*x^4 + 1/5*a*e*x^5 + 1/4*b*d*x^4*log(c) + 1/4*a*d*x^4
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^3(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx = \ln\left(c\,x^n\right)\,\left(\frac{b\,e\,x^5}{5}+\frac{b\,d\,x^4}{4}\right) + \frac{d\,x^4\,\left(4\,a-b\,n\right)}{16} + \frac{e\,x^5\,\left(5\,a-b\,n\right)}{25}$$

```
[In] int(x^3*(a + b*log(c*x^n))*(d + e*x),x)
```

```
[Out] log(c*x^n)*((b*d*x^4)/4 + (b*e*x^5)/5) + (d*x^4*(4*a - b*n))/16 + (e*x^5*(5
*a - b*n))/25
```

3.2 $\int x^2(d+ex)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int x^2(d+ex)(a+b\log(cx^n))\,dx=-\frac{1}{9}bdnx^3-\frac{1}{16}benx^4+\frac{1}{12}\left(4dx^3+3ex^4\right)(a+b\log(cx^n))$$

```
[Out] -1/9*b*d*n*x^3-1/16*b*e*n*x^4+1/12*(3*e*x^4+4*d*x^3)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {45, 2371, 12}

$$\int x^2(d+ex)(a+b\log(cx^n))\,dx=\frac{1}{12}\left(4dx^3+3ex^4\right)(a+b\log(cx^n))-\frac{1}{9}bdnx^3-\frac{1}{16}benx^4$$

```
[In] Int[x^2*(d + e*x)*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/9*(b*d*n*x^3) - (b*e*n*x^4)/16 + ((4*d*x^3 + 3*e*x^4)*(a + b*Log[c*x^n])
)/12
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{12}\left(4dx^3+3ex^4\right)\left(a+b\log\left(cx^n\right)\right)-(bn)\int\frac{1}{12}x^2(4d+3ex)\,dx\\
&= \frac{1}{12}\left(4dx^3+3ex^4\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{12}(bn)\int x^2(4d+3ex)\,dx\\
&= \frac{1}{12}\left(4dx^3+3ex^4\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{12}(bn)\int\left(4dx^2+3ex^3\right)\,dx\\
&= -\frac{1}{9}bdnx^3-\frac{1}{16}benx^4+\frac{1}{12}\left(4dx^3+3ex^4\right)\left(a+b\log\left(cx^n\right)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{1}{144}x^3(48ad-16bdn+36aex-9benx+12b(4d+3ex)\log\left(cx^n\right))$$

```
[In] Integrate[x^2*(d + e*x)*(a + b*Log[c*x^n]),x]
[Out] (x^3*(48*a*d - 16*b*d*n + 36*a*e*x - 9*b*e*n*x + 12*b*(4*d + 3*e*x)*Log[c*x
^n]))/144
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

method	result
parallelrisch	$\frac{x^4be\ln(cx^n)}{4}-\frac{benx^4}{16}+\frac{x^4ae}{4}+\frac{x^3\ln(cx^n)bd}{3}-\frac{bdnx^3}{9}+\frac{x^3ad}{3}$
risch	$\frac{bx^3(3ex+4d)\ln(x^n)}{12}-\frac{i\pi bex^4\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)}{8}+\frac{i\pi bex^4\operatorname{csgn}(ic)\operatorname{csgn}(icx^n)^2}{8}+\frac{i\pi bex^4\operatorname{csgn}(ix^n)\operatorname{csgn}(icx}{8}$

```
[In] int(x^2*(e*x+d)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*b*e*ln(c*x^n)-1/16*b*e*n*x^4+1/4*x^4*a*e+1/3*x^3*ln(c*x^n)*b*d-1/9*
b*d*n*x^3+1/3*x^3*a*d
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{16}\,(ben-4\,ae)x^4-\frac{1}{9}\,(bdn-3\,ad)x^3+\frac{1}{12}\,\left(3\,bex^4+4\,bdx^3\right)\log\left(c\right)+\frac{1}{12}\,\left(3\,benx^4+4\,bdnx^3\right)\log\left(x\right)$$

```
[In] integrate(x^2*(e*x+d)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/16*(b*e*n - 4*a*e)*x^4 - 1/9*(b*d*n - 3*a*d)*x^3 + 1/12*(3*b*e*x^4 + 4*b
*d*x^3)*log(c) + 1/12*(3*b*e*n*x^4 + 4*b*d*n*x^3)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{adx^3}{3}+\frac{aex^4}{4}-\frac{bdnx^3}{9}+\frac{bdx^3\log\left(cx^n\right)}{3}-\frac{benx^4}{16}+\frac{bex^4\log\left(cx^n\right)}{4}$$

```
[In] integrate(x**2*(e*x+d)*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d*x**3/3 + a*e*x**4/4 - b*d*n*x**3/9 + b*d*x**3*log(c*x**n)/3 - b*e*n*x**
4/16 + b*e*x**4*log(c*x**n)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{16}\,benx^4+\frac{1}{4}\,bex^4\log\left(cx^n\right)-\frac{1}{9}\,bdnx^3+\frac{1}{4}\,aex^4+\frac{1}{3}\,bdx^3\log\left(cx^n\right)+\frac{1}{3}\,adx^3$$

```
[In] integrate(x^2*(e*x+d)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/16*b*e*n*x^4 + 1/4*b*e*x^4*log(c*x^n) - 1/9*b*d*n*x^3 + 1/4*a*e*x^4 + 1/
3*b*d*x^3*log(c*x^n) + 1/3*a*d*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{1}{4}\,benx^4\log\left(x\right) - \frac{1}{16}\,benx^4 + \frac{1}{4}\,bex^4\log\left(c\right) + \frac{1}{3}\,bdnx^3\log\left(x\right) - \frac{1}{9}\,bdnx^3 + \frac{1}{4}\,aex^4 + \frac{1}{3}\,bdx^3\log\left(c\right) + \frac{1}{3}\,adx^3$$

```
[In] integrate(x^2*(e*x+d)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*e*n*x^4*log(x) - 1/16*b*e*n*x^4 + 1/4*b*e*x^4*log(c) + 1/3*b*d*n*x^3*
log(x) - 1/9*b*d*n*x^3 + 1/4*a*e*x^4 + 1/3*b*d*x^3*log(c) + 1/3*a*d*x^3
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx = \ln\left(c\,x^n\right)\,\left(\frac{b\,e\,x^4}{4}+\frac{b\,d\,x^3}{3}\right) + \frac{d\,x^3\,\left(3\,a-b\,n\right)}{9} + \frac{e\,x^4\,\left(4\,a-b\,n\right)}{16}$$

```
[In] int(x^2*(a + b*log(c*x^n))*(d + e*x),x)
```

```
[Out] log(c*x^n)*((b*d*x^3)/3 + (b*e*x^4)/4) + (d*x^3*(3*a - b*n))/9 + (e*x^4*(4*
a - b*n))/16
```

3.3 $\int x(d+ex)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int x(d+ex)(a+b\log(cx^n))\,dx=-\frac{1}{4}bdnx^2-\frac{1}{9}benx^3+\frac{1}{6}\left(3dx^2+2ex^3\right)(a+b\log(cx^n))$$

```
[Out] -1/4*b*d*n*x^2-1/9*b*e*n*x^3+1/6*(2*e*x^3+3*d*x^2)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {45, 2371, 12}

$$\int x(d+ex)(a+b\log(cx^n))\,dx=\frac{1}{6}\left(3dx^2+2ex^3\right)(a+b\log(cx^n))-\frac{1}{4}bdnx^2-\frac{1}{9}benx^3$$

```
[In] Int[x*(d + e*x)*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/4*(b*d*n*x^2) - (b*e*n*x^3)/9 + ((3*d*x^2 + 2*e*x^3)*(a + b*Log[c*x^n]))
/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{6}\left(3dx^2+2ex^3\right)(a+b\log(cx^n))-(bn)\int\frac{1}{6}x(3d+2ex)\,dx \\ &= \frac{1}{6}\left(3dx^2+2ex^3\right)(a+b\log(cx^n))-\frac{1}{6}(bn)\int x(3d+2ex)\,dx \\ &= \frac{1}{6}\left(3dx^2+2ex^3\right)(a+b\log(cx^n))-\frac{1}{6}(bn)\int\left(3dx+2ex^2\right)\,dx \\ &= -\frac{1}{4}bdnx^2-\frac{1}{9}benx^3+\frac{1}{6}\left(3dx^2+2ex^3\right)(a+b\log(cx^n)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int x(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{1}{36}x^2(6a(3d+2ex)-bn(9d+4ex)+6b(3d+2ex)\log\left(cx^n\right))$$

```
[In] Integrate[x*(d + e*x)*(a + b*Log[c*x^n]),x]
[Out] (x^2*(6*a*(3*d + 2*e*x) - b*n*(9*d + 4*e*x) + 6*b*(3*d + 2*e*x)*Log[c*x^n])
)/36
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

method	result
parallelrisch	$\frac{x^3 be\ln(c\,x^n)}{3}-\frac{ben\,x^3}{9}+\frac{x^3 ae}{3}+\frac{x^2\ln(c\,x^n)bd}{2}-\frac{bdn\,x^2}{4}+\frac{ad\,x^2}{2}$
risch	$\frac{b\,x^2(2ex+3d)\ln(x^n)}{6}-\frac{i\pi be\,x^3\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)}{6}+\frac{i\pi be\,x^3\operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^n)^2}{6}+\frac{i\pi be\,x^3\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x$

```
[In] int(x*(e*x+d)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*b*e*ln(c*x^n)-1/9*b*e*n*x^3+1/3*x^3*a*e+1/2*x^2*ln(c*x^n)*b*d-1/4*b
*d*n*x^2+1/2*a*d*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int x(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{1}{9}\,(ben-3\,ae)x^3 - \frac{1}{4}\,(bdn-2\,ad)x^2 + \frac{1}{6}\left(2\,bex^3+3\,bdx^2\right)\log\left(c\right) + \frac{1}{6}\left(2\,benx^3+3\,bdnx^2\right)\log\left(x\right)$$

```
[In] integrate(x*(e*x+d)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/9*(b*e*n - 3*a*e)*x^3 - 1/4*(b*d*n - 2*a*d)*x^2 + 1/6*(2*b*e*x^3 + 3*b*d
*x^2)*log(c) + 1/6*(2*b*e*n*x^3 + 3*b*d*n*x^2)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int x(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{adx^2}{2} + \frac{aex^3}{3} - \frac{bdnx^2}{4} + \frac{bdx^2\log\left(cx^n\right)}{2} - \frac{benx^3}{9} + \frac{bex^3\log\left(cx^n\right)}{3}$$

```
[In] integrate(x*(e*x+d)*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d*x**2/2 + a*e*x**3/3 - b*d*n*x**2/4 + b*d*x**2*log(c*x**n)/2 - b*e*n*x**
3/9 + b*e*x**3*log(c*x**n)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int x(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{1}{9}\,benx^3 + \frac{1}{3}\,bex^3\log\left(cx^n\right) - \frac{1}{4}\,bdnx^2 + \frac{1}{3}\,aex^3 + \frac{1}{2}\,bdx^2\log\left(cx^n\right) + \frac{1}{2}\,adx^2$$

```
[In] integrate(x*(e*x+d)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/9*b*e*n*x^3 + 1/3*b*e*x^3*log(c*x^n) - 1/4*b*d*n*x^2 + 1/3*a*e*x^3 + 1/2
*b*d*x^2*log(c*x^n) + 1/2*a*d*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int x(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{1}{3}\,benx^3\log\left(x\right) - \frac{1}{9}\,benx^3 + \frac{1}{3}\,bex^3\log\left(c\right) + \frac{1}{2}\,bdnx^2\log\left(x\right) - \frac{1}{4}\,bdnx^2 + \frac{1}{3}\,aex^3 + \frac{1}{2}\,bdx^2\log\left(c\right) + \frac{1}{2}\,adx^2$$

```
[In] integrate(x*(e*x+d)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*e*n*x^3*log(x) - 1/9*b*e*n*x^3 + 1/3*b*e*x^3*log(c) + 1/2*b*d*n*x^2*l
og(x) - 1/4*b*d*n*x^2 + 1/3*a*e*x^3 + 1/2*b*d*x^2*log(c) + 1/2*a*d*x^2
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx = \ln\left(c\,x^n\right)\,\left(\frac{b\,e\,x^3}{3}+\frac{b\,d\,x^2}{2}\right) + \frac{d\,x^2\,\left(2\,a-b\,n\right)}{4} + \frac{e\,x^3\,\left(3\,a-b\,n\right)}{9}$$

```
[In] int(x*(a + b*log(c*x^n))*(d + e*x),x)
```

```
[Out] log(c*x^n)*((b*d*x^2)/2 + (b*e*x^3)/3) + (d*x^2*(2*a - b*n))/4 + (e*x^3*(3*
a - b*n))/9
```

3.4 $\int (d+ex)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int (d+ex)(a+b\log(cx^n))\,dx=-bdnx-\frac{1}{4}benx^2+dx(a+b\log(cx^n))+\frac{1}{2}ex^2(a+b\log(cx^n))$$

```
[Out] -b*d*n*x-1/4*b*e*n*x^2+d*x*(a+b*ln(c*x^n))+1/2*e*x^2*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2350}

$$\int (d+ex)(a+b\log(cx^n))\,dx=dx(a+b\log(cx^n))+\frac{1}{2}ex^2(a+b\log(cx^n))-bdnx-\frac{1}{4}benx^2$$

```
[In] Int[(d + e*x)*(a + b*Log[c*x^n]),x]
[Out] -(b*d*n*x) - (b*e*n*x^2)/4 + d*x*(a + b*Log[c*x^n]) + (e*x^2*(a + b*Log[c*x
^n]))/2
```

Rule 2350

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Dist[a + b*Log[c*x^n], u
, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=dx(a+b\log(cx^n))+\frac{1}{2}ex^2(a+b\log(cx^n))-(bn)\int\left(d+\frac{ex}{2}\right)dx\\&=-bdnx-\frac{1}{4}benx^2+dx(a+b\log(cx^n))+\frac{1}{2}ex^2(a+b\log(cx^n))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int (d+ex) (a+b\log (cx^n)) \, dx = adx-bdnx+\frac{1}{2}aex^2-\frac{1}{4}benx^2+bdx\log (cx^n)+\frac{1}{2}bex^2\log (cx^n)$$

```
[In] Integrate[(d + e*x)*(a + b*Log[c*x^n]),x]
```

```
[Out] a*d*x - b*d*n*x + (a*e*x^2)/2 - (b*e*n*x^2)/4 + b*d*x*Log[c*x^n] + (b*e*x^2
*Log[c*x^n])/2
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{be\,x^2\ln(c\,x^n)}{2}-\frac{ben\,x^2}{4}+\frac{ae\,x^2}{2}+x\ln(c\,x^n)\,bd-bdnx+xad$
default	$xad+\frac{ae\,x^2}{2}+bd(x\ln(c\,x^n)-nx)+\frac{be\,x^2\ln(c\,\mathrm{e}^{n\ln(x)})}{2}-\frac{ben\,x^2}{4}$
parts	$a\left(\frac{1}{2}e\,x^2+dx\right)+b\left(x\ln(c\,x^n)\,d-xdn+\frac{e\,x^2\ln(c\,\mathrm{e}^{n\ln(x)})}{2}-\frac{en\,x^2}{4}\right)$
norman	$\left(-\frac{1}{4}ben+\frac{1}{2}ae\right)x^2+(-bdn+ad)\,x+bdx\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)+\frac{be\,x^2\ln(c\,\mathrm{e}^{n\ln(x)})}{2}$
risch	$\frac{bx(ex+2d)\ln(x^n)}{2}-\frac{i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)be\,x^2}{4}+\frac{i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2be\,x^2}{4}+\frac{i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2be\,x}{4}$

```
[In] int((e*x+d)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*e*x^2*ln(c*x^n)-1/4*b*e*n*x^2+1/2*a*e*x^2+x*ln(c*x^n)*b*d-b*d*n*x+x*a
*d
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int (d+ex) (a+b\log (cx^n)) \, dx = -\frac{1}{4}(ben-2\,ae)x^2-(bdn-ad)x + \frac{1}{2}\left(bex^2+2\,bdx\right)\log (c)+\frac{1}{2}\left(benx^2+2\,bdnx\right)\log (x)$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/4*(b*e*n - 2*a*e)*x^2 - (b*d*n - a*d)*x + 1/2*(b*e*x^2 + 2*b*d*x)*log(c)
 + 1/2*(b*e*n*x^2 + 2*b*d*n*x)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

$$\int (d+ex)\,(a+b\log{(cx^n)})\;dx = adx + \frac{aex^2}{2} - bdnx + bdx\log{(cx^n)} - \frac{benx^2}{4} + \frac{bex^2\log{(cx^n)}}{2}$$

```
[In] integrate((e*x+d)*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d*x + a*e*x**2/2 - b*d*n*x + b*d*x*log(c*x**n) - b*e*n*x**2/4 + b*e*x**2*
log(c*x**n)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int (d+ex)\,(a+b\log{(cx^n)})\;dx = -\frac{1}{4}\,benx^2 + \frac{1}{2}\,bex^2\log{(cx^n)} - bdnx + \frac{1}{2}\,aex^2 + bdx\log{(cx^n)} + adx$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/4*b*e*n*x^2 + 1/2*b*e*x^2*log(c*x^n) - b*d*n*x + 1/2*a*e*x^2 + b*d*x*log
(c*x^n) + a*d*x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int (d+ex)\,(a+b\log{(cx^n)})\;dx = \frac{1}{2}\,benx^2\log{(x)} - \frac{1}{4}\,benx^2 + \frac{1}{2}\,bex^2\log{(c)} + bdnx\log{(x)} - bdnx + \frac{1}{2}\,aex^2 + bdx\log{(c)} + adx$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*e*n*x^2*log(x) - 1/4*b*e*n*x^2 + 1/2*b*e*x^2*log(c) + b*d*n*x*log(x)
- b*d*n*x + 1/2*a*e*x^2 + b*d*x*log(c) + a*d*x
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int (d+ex)\,(a+b\log{(cx^n)})\;dx=\ln{(c\,x^n)}\,\left(\frac{b\,e\,x^2}{2}+b\,d\,x\right)+d\,x\,(a-b\,n)+\frac{e\,x^2\,(2\,a-b\,n)}{4}$$

```
[In] int((a + b*log(c*x^n))*(d + e*x),x)
[Out] log(c*x^n)*(b*d*x + (b*e*x^2)/2) + d*x*(a - b*n) + (e*x^2*(2*a - b*n))/4
```

3.5 $\int \frac{(d+ex)(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = aex - benx + bex\log\left(cx^n\right) + \frac{d\left(a+b\log\left(cx^n\right)\right)^2}{2bn}$$

```
[Out] a*e*x-b*e*n*x+b*e*x*ln(c*x^n)+1/2*d*(a+b*ln(c*x^n))^2/b/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2388, 2338, 2332}

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{d\left(a+b\log\left(cx^n\right)\right)^2}{2bn} + aex + bex\log\left(cx^n\right) - benx$$

```
[In] Int[((d + e*x)*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] a*e*x - b*e*n*x + b*e*x*Log[c*x^n] + (d*(a + b*Log[c*x^n])^2)/(2*b*n)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rubi steps

$$\begin{aligned}\text{integral} &= d\int \frac{a+b\log\left(cx^n\right)}{x}\,dx + e\int \left(a+b\log\left(cx^n\right)\right)\,dx\\ &= aex + \frac{d\left(a+b\log\left(cx^n\right)\right)^2}{2bn} + (be)\int \log\left(cx^n\right)\,dx\\ &= aex - benx + bex\log\left(cx^n\right) + \frac{d\left(a+b\log\left(cx^n\right)\right)^2}{2bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = aex - benx + ad\log(x) + bex\log\left(cx^n\right) + \frac{bd\log^2\left(cx^n\right)}{2n}$$

```
[In] Integrate[((d + e*x)*(a + b*Log[c*x^n]))/x,x]
[Out] a*e*x - b*e*n*x + a*d*Log[x] + b*e*x*Log[c*x^n] + (b*d*Log[c*x^n]^2)/(2*n)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

method	result
default	$\ln(x)\,ad + aex + bex\ln\left(c\,\mathrm{e}^{n\ln(x)}\right) + \frac{bd\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^2}{2n} - benx$
parts	$\ln(x)\,ad + aex + bex\ln\left(c\,\mathrm{e}^{n\ln(x)}\right) + \frac{bd\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^2}{2n} - benx$
parallelrisch	$\frac{2x\ln(c\,x^n)ben - 2xbe\,n^2 + 2\ln(x)adn + 2xaen + bd\ln(c\,x^n)^2}{2n}$
norman	$(-ben + ae)\,x + \frac{ad\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)}{n} + bex\ln\left(c\,\mathrm{e}^{n\ln(x)}\right) + \frac{bd\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^2}{2n}$
risch	$(bex + bd\ln(x))\ln(x^n) - \frac{bdn\ln(x)^2}{2} - \frac{i\pi bex\,\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)}{2} + \frac{i\pi bex\,\operatorname{csgn}(ic)\operatorname{csgn}(icx^n)^2}{2} + i\pi b$

```
[In] int((e*x+d)*(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*a*d+a*e*x+b*e*x*ln(c*exp(n*ln(x)))+1/2*b*d/n*ln(c*exp(n*ln(x)))^2-b*e
*n*x
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)(a+b\log(cx^n))}{x}\,dx = \frac{1}{2}\,bdn\log(x)^2 + bex\log(c) - (ben-ae)x + (benx + bd\log(c) + ad)\log(x)$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] 1/2*b*d*n*log(x)^2 + b*e*x*log(c) - (b*e*n - a*e)*x + (b*e*n*x + b*d*log(c)
 + a*d)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.48

$$\int \frac{(d+ex)(a+b\log(cx^n))}{x}\,dx = \begin{cases} \frac{ad\log(cx^n)}{n} + aex + \frac{bd\log(cx^n)^2}{2n} - benx + bex\log(cx^n) & \text{for } n \neq 0 \\ (a+b\log(c))(d\log(x)+ex) & \text{otherwise} \end{cases}$$

```
[In] integrate((e*x+d)*(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise((a*d*log(c*x**n)/n + a*e*x + b*d*log(c*x**n)**2/(2*n) - b*e*n*x +
 b*e*x*log(c*x**n), Ne(n, 0)), ((a + b*log(c))*(d*log(x) + e*x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)(a+b\log(cx^n))}{x}\,dx = -benx + bex\log(cx^n) + aex + \frac{bd\log(cx^n)^2}{2\,n} + ad\log(x)$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] -b*e*n*x + b*e*x*log(c*x^n) + a*e*x + 1/2*b*d*log(c*x^n)^2/n + a*d*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)(a+b\log(cx^n))}{x}\,dx = benx\log(x) + \frac{1}{2}bdn\log(x)^2 - (ben - be\log(c) - ae)x + (bd\log(c) + ad)\log(x)$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))/x,x, algorithm="giac")
[Out] b*e*n*x*log(x) + 1/2*b*d*n*log(x)^2 - (b*e*n - b*e*log(c) - a*e)*x + (b*d*l
og(c) + a*d)*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)(a+b\log(cx^n))}{x}\,dx = a\,d\,\ln(x) + e\,x\,(a-b\,n) + b\,e\,x\,\ln(c\,x^n) + \frac{b\,d\ln(c\,x^n)^2}{2\,n}$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x))/x,x)
[Out] a*d*log(x) + e*x*(a - b*n) + b*e*x*log(c*x^n) + (b*d*log(c*x^n)^2)/(2*n)
```

3.6 $\int \frac{(d+ex)(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \frac{(d+ex)(a+b\log(cx^n))}{x^2}\,dx = -\frac{bdn}{x}-\frac{d(a+b\log(cx^n))}{x}+\frac{e(a+b\log(cx^n))^2}{2bn}$$

```
[Out] -b*d*n/x-d*(a+b*ln(c*x^n))/x+1/2*e*(a+b*ln(c*x^n))^2/b/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {45, 2372, 14, 2338}

$$\int \frac{(d+ex)(a+b\log(cx^n))}{x^2}\,dx = -\frac{d(a+b\log(cx^n))}{x}+e\log(x)(a+b\log(cx^n))-\frac{bdn}{x}-\frac{1}{2}ben\log^2(x)$$

```
[In] Int[((d + e*x)*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -((b*d*n)/x) - (b*e*n*Log[x]^2)/2 - (d*(a + b*Log[c*x^n]))/x + e*Log[x]*(a
+ b*Log[c*x^n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d(a+b\log(cx^n))}{x}+e\log(x)(a+b\log(cx^n))-(bn)\int\frac{-d+ex\log(x)}{x^2}\,dx\\
&= -\frac{d(a+b\log(cx^n))}{x}+e\log(x)(a+b\log(cx^n))-(bn)\int\left(-\frac{d}{x^2}+\frac{e\log(x)}{x}\right)dx\\
&= -\frac{bdn}{x}-\frac{d(a+b\log(cx^n))}{x}+e\log(x)(a+b\log(cx^n))-(ben)\int\frac{\log(x)}{x}\,dx\\
&= -\frac{bdn}{x}-\frac{1}{2}ben\log^2(x)-\frac{d(a+b\log(cx^n))}{x}+e\log(x)(a+b\log(cx^n))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$
\int\frac{(d+ex)(a+b\log(cx^n))}{x^2}\,dx=-\frac{bdn}{x}-\frac{d(a+b\log(cx^n))}{x}+\frac{e(a+b\log(cx^n))^2}{2bn}
$$

```
[In] Integrate[((d + e*x)*(a + b*Log[c*x^n]))/x^2,x]
[Out] -((b*d*n)/x) - (d*(a + b*Log[c*x^n]))/x + (e*(a + b*Log[c*x^n])^2)/(2*b*n)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

method	result
parallelrisch	$\frac{2\ln(x)xaen+be\ln(c\,x^{n})^{2}x-2\ln(c\,x^{n})bdn-2bd\,n^{2}-2adn}{2xn}$
risch	$-\frac{b(-ex\ln(x)+d)\ln(x^{n})}{x}-\frac{i\ln(x)\pi be\ \operatorname{csgn}(ic)\operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})x-i\ln(x)\pi be\ \operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^{n})^{2}x-i\ln(x)\pi be\ \operatorname{csgn}(i}{}$

```
[In] int((e*x+d)*(a+b*ln(c*x^n))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/x*(2*ln(x)*x*a*e*n+b*e*ln(c*x^n)^2*x-2*ln(c*x^n)*b*d*n-2*b*d*n^2-2*a*d*
n)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)\,(a+b\log{(cx^n)})}{x^2}\,dx = \frac{benx\log{(x)}^2-2\,bdn-2\,bd\log{(c)}-2\,ad+2\,(bex\log{(c)}-bdn+aex)\log{(x)}}{2\,x}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*e*n*x*log(x)^2 - 2*b*d*n - 2*b*d*log(c) - 2*a*d + 2*(b*e*x*log(c) -
b*d*n + a*e*x)*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)\,(a+b\log{(cx^n)})}{x^2}\,dx = -\frac{ad}{x}+ae\log{(x)}+bd\left(-\frac{n}{x}-\frac{\log{(cx^n)}}{x}\right) - be\left(\begin{cases}-\log{(c)}\log{(x)} & \text{for}\ n=0\\ -\frac{\log{(cx^n)}^2}{2n} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((e*x+d)*(a+b*ln(c*x**n))/x**2,x)
```

```
[Out] -a*d/x + a*e*log(x) + b*d*(-n/x - log(c*x**n)/x) - b*e*Piecewise((-log(c)*l
og(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)\,(a+b\log{(cx^n)})}{x^2}\,dx = \frac{be\log{(cx^n)}^2}{2\,n} + ae\log{(x)} - \frac{bdn}{x} - \frac{bd\log{(cx^n)}}{x} - \frac{ad}{x}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*e*log(c*x^n)^2/n + a*e*log(x) - b*d*n/x - b*d*log(c*x^n)/x - a*d/x
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)\,(a+b\log{(cx^n)})}{x^2}\,dx = \frac{1}{2}\,ben\log{(x)}^2 - bdn\left(\frac{\log{(x)}}{x} + \frac{1}{x}\right) + be\log{(c)}\log{(|x|)} + ae\log{(|x|)} - \frac{bd\log{(c)}}{x} - \frac{ad}{x}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*e*n*log(x)^2 - b*d*n*(log(x)/x + 1/x) + b*e*log(c)*log(abs(x)) + a*e*
log(abs(x)) - b*d*log(c)/x - a*d/x
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)\,(a+b\log{(cx^n)})}{x^2}\,dx = \ln{(x)}\,\left(a\,e+b\,e\,n\right) - \frac{a\,d+b\,d\,n}{x} - \frac{\ln{(c\,x^n)}\,\left(b\,d+b\,e\,x\right)}{x} + \frac{b\,e\ln{(c\,x^n)}^2}{2\,n}$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x))/x^2,x)
```

```
[Out] log(x)*(a*e + b*e*n) - (a*d + b*d*n)/x - (log(c*x^n)*(b*d + b*e*x))/x + (b*
e*log(c*x^n)^2)/(2*n)
```

3.7 $\int \frac{(d+ex)(a+b\log(cx^n))}{x^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 60

$$\int \frac{(d+ex)(a+b\log(cx^n))}{x^3} dx = -\frac{bdn}{4x^2} - \frac{ben}{x} + \frac{be^2n\log(x)}{2d} - \frac{(d+ex)^2(a+b\log(cx^n))}{2dx^2}$$

```
[Out] -1/4*b*d*n/x^2-b*e*n/x+1/2*b*e^2*n*ln(x)/d-1/2*(e*x+d)^2*(a+b*ln(c*x^n))/d/
x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {37, 2372, 12, 45}

$$\int \frac{(d+ex)(a+b\log(cx^n))}{x^3} dx = -\frac{(d+ex)^2(a+b\log(cx^n))}{2dx^2} + \frac{be^2n\log(x)}{2d} - \frac{bdn}{4x^2} - \frac{ben}{x}$$

```
[In] Int[((d + e*x)*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/4*(b*d*n)/x^2 - (b*e*n)/x + (b*e^2*n*Log[x])/(2*d) - ((d + e*x)^2*(a + b
*Log[c*x^n]))/(2*d*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d+ex)^2\,(a+b\log(cx^n))}{2dx^2} - (bn)\int -\frac{(d+ex)^2}{2dx^3}\,dx \\
&= -\frac{(d+ex)^2\,(a+b\log(cx^n))}{2dx^2} + \frac{(bn)\int \frac{(d+ex)^2}{x^3}\,dx}{2d} \\
&= -\frac{(d+ex)^2\,(a+b\log(cx^n))}{2dx^2} + \frac{(bn)\int \left(\frac{d^2}{x^3}+\frac{2de}{x^2}+\frac{e^2}{x}\right)dx}{2d} \\
&= -\frac{bdn}{4x^2} - \frac{ben}{x} + \frac{be^2n\log(x)}{2d} - \frac{(d+ex)^2\,(a+b\log(cx^n))}{2dx^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)\,(a+b\log(cx^n))}{x^3}\,dx = -\frac{2a(d+2ex)+bn(d+4ex)+2b(d+2ex)\log(cx^n)}{4x^2}$$

```
[In] Integrate[((d + e*x)*(a + b*Log[c*x^n]))/x^3,x]
[Out] -1/4*(2*a*(d + 2*e*x) + b*n*(d + 4*e*x) + 2*b*(d + 2*e*x)*Log[c*x^n])/x^2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

method	result
parallelrisch	$-\frac{4bex\ln(c\,x^n)+4benx+4aex+2b\ln(c\,x^n)d+bdn+2ad}{4x^2}$
risch	$-\frac{b(2ex+d)\ln(x^n)}{2x^2}-\frac{-2i\pi bex\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+2i\pi bex\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+2i\pi bex\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{}$

```
[In] int((e*x+d)*(a+b*ln(c*x^n))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^2*(4*b*e*x*ln(c*x^n)+4*b*e*n*x+4*a*e*x+2*b*ln(c*x^n)*d+b*d*n+2*a*d)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)(a+b\log(cx^n))}{x^3}\,dx = -\frac{bdn+2\,ad+4\,(ben+ae)x+2\,(2\,bex+bd)\log(c)+2\,(2\,benx+bdn)\log(x)}{4\,x^2}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*d*n + 2*a*d + 4*(b*e*n + a*e)*x + 2*(2*b*e*x + b*d)*log(c) + 2*(2*b
*e*n*x + b*d*n)*log(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)(a+b\log(cx^n))}{x^3}\,dx = -\frac{ad}{2x^2}-\frac{ae}{x}-\frac{bdn}{4x^2}-\frac{bd\log(cx^n)}{2x^2}-\frac{ben}{x}-\frac{be\log(cx^n)}{x}$$

```
[In] integrate((e*x+d)*(a+b*ln(c*x**n))/x**3,x)
```

```
[Out] -a*d/(2*x**2) - a*e/x - b*d*n/(4*x**2) - b*d*log(c*x**n)/(2*x**2) - b*e*n/x
 - b*e*log(c*x**n)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)(a+b\log(cx^n))}{x^3}\,dx = -\frac{ben}{x} - \frac{be\log(cx^n)}{x} - \frac{bdn}{4x^2} - \frac{ae}{x} - \frac{bd\log(cx^n)}{2x^2} - \frac{ad}{2x^2}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))/x^3,x, algorithm="maxima")
```

```
[Out] -b*e*n/x - b*e*log(c*x^n)/x - 1/4*b*d*n/x^2 - a*e/x - 1/2*b*d*log(c*x^n)/x^
2 - 1/2*a*d/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)(a+b\log(cx^n))}{x^3}\,dx = -\frac{(2benx+bdn)\log(x)}{2x^2} - \frac{4benx+4bex\log(c)+bdn+4aex+2bd\log(c)+2ad}{4x^2}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*e*n*x + b*d*n)*log(x)/x^2 - 1/4*(4*b*e*n*x + 4*b*e*x*log(c) + b*d
*n + 4*a*e*x + 2*b*d*log(c) + 2*a*d)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)(a+b\log(cx^n))}{x^3}\,dx = -\frac{a\,d+x\,(2\,a\,e+2\,b\,e\,n)+\frac{b\,d\,n}{2}}{2\,x^2} - \frac{\ln(c\,x^n)\,\left(\frac{b\,d}{2}+b\,e\,x\right)}{x^2}$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x))/x^3,x)
```

```
[Out] - (a*d + x*(2*a*e + 2*b*e*n) + (b*d*n)/2)/(2*x^2) - (log(c*x^n)*((b*d)/2 +
b*e*x))/x^2
```

3.8 $\int \frac{(d+ex)(a+b\log(cx^n))}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(d+ex)(a+b\log(cx^n))}{x^4}\,dx = -\frac{bdn}{9x^3}-\frac{ben}{4x^2}-\frac{d(a+b\log(cx^n))}{3x^3}-\frac{e(a+b\log(cx^n))}{2x^2}$$

```
[Out] -1/9*b*d*n/x^3-1/4*b*e*n/x^2-1/3*d*(a+b*ln(c*x^n))/x^3-1/2*e*(a+b*ln(c*x^n)
)/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {45, 2372, 12}

$$\int \frac{(d+ex)(a+b\log(cx^n))}{x^4}\,dx = -\frac{d(a+b\log(cx^n))}{3x^3}-\frac{e(a+b\log(cx^n))}{2x^2}-\frac{bdn}{9x^3}-\frac{ben}{4x^2}$$

```
[In] Int[((d + e*x)*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] -1/9*(b*d*n)/x^3 - (b*e*n)/(4*x^2) - (d*(a + b*Log[c*x^n]))/(3*x^3) - (e*(a
 + b*Log[c*x^n]))/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d(a+b\log(cx^n))}{3x^3} - \frac{e(a+b\log(cx^n))}{2x^2} - (bn)\int \frac{-2d-3ex}{6x^4}\,dx \\
&= -\frac{d(a+b\log(cx^n))}{3x^3} - \frac{e(a+b\log(cx^n))}{2x^2} - \frac{1}{6}(bn)\int \frac{-2d-3ex}{x^4}\,dx \\
&= -\frac{d(a+b\log(cx^n))}{3x^3} - \frac{e(a+b\log(cx^n))}{2x^2} - \frac{1}{6}(bn)\int \left(-\frac{2d}{x^4}-\frac{3e}{x^3}\right)dx \\
&= -\frac{bdn}{9x^3} - \frac{ben}{4x^2} - \frac{d(a+b\log(cx^n))}{3x^3} - \frac{e(a+b\log(cx^n))}{2x^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$
\int \frac{(d+ex)(a+b\log(cx^n))}{x^4}\,dx = -\frac{6a(2d+3ex)+bn(4d+9ex)+6b(2d+3ex)\log(cx^n)}{36x^3}
$$

```
[In] Integrate[((d + e*x)*(a + b*Log[c*x^n]))/x^4,x]
[Out] -1/36*(6*a*(2*d + 3*e*x) + b*n*(4*d + 9*e*x) + 6*b*(2*d + 3*e*x)*Log[c*x^n]
)/x^3
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

method	result
parallelrisch	$-\frac{18bex\ln(cx^n)+9benx+18aex+12b\ln(cx^n)d+4bdn+12ad}{36x^3}$
risch	$-\frac{b(3ex+2d)\ln(x^n)}{6x^3} - \frac{-9i\pi bex\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)+9i\pi bex\ \mathrm{csgn}(ic)\,\mathrm{csgn}(icx^n)^2+9i\pi bex\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2-}{}$

```
[In] int((e*x+d)*(a+b*ln(c*x^n))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/36/x^3*(18*b*e*x*ln(c*x^n)+9*b*e*n*x+18*a*e*x+12*b*ln(c*x^n)*d+4*b*d*n+1
2*a*d)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)}{x^4}\,dx = -\frac{4\,bdn+12\,ad+9\,(ben+2\,ae)x+6\,(3\,bex+2\,bd)\log\left(c\right)+6\,(3\,benx+2\,bdn)\log\left(x\right)}{36\,x^3}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))/x^4,x, algorithm="fricas")
```

```
[Out] -1/36*(4*b*d*n + 12*a*d + 9*(b*e*n + 2*a*e)*x + 6*(3*b*e*x + 2*b*d)*log(c)
+ 6*(3*b*e*n*x + 2*b*d*n)*log(x))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)}{x^4}\,dx = -\frac{ad}{3x^3}-\frac{ae}{2x^2}-\frac{bdn}{9x^3}-\frac{bd\log\left(cx^n\right)}{3x^3}-\frac{ben}{4x^2}-\frac{be\log\left(cx^n\right)}{2x^2}$$

```
[In] integrate((e*x+d)*(a+b*ln(c*x**n))/x**4,x)
```

```
[Out] -a*d/(3*x**3) - a*e/(2*x**2) - b*d*n/(9*x**3) - b*d*log(c*x**n)/(3*x**3) -
b*e*n/(4*x**2) - b*e*log(c*x**n)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)}{x^4}\,dx = -\frac{ben}{4\,x^2}-\frac{be\log\left(cx^n\right)}{2\,x^2}-\frac{bdn}{9\,x^3}-\frac{ae}{2\,x^2}-\frac{bd\log\left(cx^n\right)}{3\,x^3}-\frac{ad}{3\,x^3}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))/x^4,x, algorithm="maxima")
```

```
[Out] -1/4*b*e*n/x^2 - 1/2*b*e*log(c*x^n)/x^2 - 1/9*b*d*n/x^3 - 1/2*a*e/x^2 - 1/3
*b*d*log(c*x^n)/x^3 - 1/3*a*d/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)}{x^4}\,dx$$
$$= -\frac{(3\,benx+2\,bdn)\log\left(x\right)}{6\,x^3} - \frac{9\,benx+18\,bex\log\left(c\right)+4\,bdn+18\,aex+12\,bd\log\left(c\right)+12\,ad}{36\,x^3}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*e*n*x + 2*b*d*n)*log(x)/x^3 - 1/36*(9*b*e*n*x + 18*b*e*x*log(c) +
 4*b*d*n + 18*a*e*x + 12*b*d*log(c) + 12*a*d)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)}{x^4}\,dx = -\frac{2\,a\,d+x\,\left(3\,a\,e+\frac{3\,b\,e\,n}{2}\right)+\frac{2\,b\,d\,n}{3}}{6\,x^3} - \frac{\ln\left(c\,x^n\right)\,\left(\frac{b\,d}{3}+\frac{b\,e\,x}{2}\right)}{x^3}$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x))/x^4,x)
```

```
[Out] - (2*a*d + x*(3*a*e + (3*b*e*n)/2) + (2*b*d*n)/3)/(6*x^3) - (log(c*x^n)*((b
*d)/3 + (b*e*x)/2))/x^3
```

3.9 $\int x^3(d+ex)^2 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 74

$$\int x^3(d+ex)^2 (a+b\log(cx^n))\,dx = -\frac{1}{16}bd^2nx^4-\frac{2}{25}bdenx^5-\frac{1}{36}be^2nx^6 + \frac{1}{60}\left(15d^2x^4+24dex^5+10e^2x^6\right)(a+b\log(cx^n))$$

```
[Out] -1/16*b*d^2*n*x^4-2/25*b*d*e*n*x^5-1/36*b*e^2*n*x^6+1/60*(10*e^2*x^6+24*d*e
*x^5+15*d^2*x^4)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {45, 2371, 12, 14}

$$\int x^3(d+ex)^2 (a+b\log(cx^n))\,dx = \frac{1}{60}\left(15d^2x^4+24dex^5+10e^2x^6\right)(a+b\log(cx^n)) - \frac{1}{16}bd^2nx^4-\frac{2}{25}bdenx^5-\frac{1}{36}be^2nx^6$$

```
[In] Int[x^3*(d + e*x)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/16*(b*d^2*n*x^4) - (2*b*d*e*n*x^5)/25 - (b*e^2*n*x^6)/36 + ((15*d^2*x^4
+ 24*d*e*x^5 + 10*e^2*x^6)*(a + b*Log[c*x^n]))/60
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{60}\left(15d^2x^4+24dex^5+10e^2x^6\right)\left(a+b\log\left(cx^n\right)\right) \\
&\quad -(bn)\int \frac{1}{60}x^3\left(15d^2+24dex+10e^2x^2\right)\,dx \\
&= \frac{1}{60}\left(15d^2x^4+24dex^5+10e^2x^6\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{60}(bn)\int x^3\left(15d^2+24dex+10e^2x^2\right)\,dx \\
&= \frac{1}{60}\left(15d^2x^4+24dex^5+10e^2x^6\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{60}(bn)\int \left(15d^2x^3+24dex^4+10e^2x^5\right)\,dx \\
&= -\frac{1}{16}bd^2nx^4-\frac{2}{25}bdenx^5-\frac{1}{36}be^2nx^6+\frac{1}{60}\left(15d^2x^4+24dex^5+10e^2x^6\right)\left(a+b\log\left(cx^n\right)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09

$$\int x^3(d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{x^4(60a(15d^2+24dex+10e^2x^2)-bn(225d^2+288dex+100e^2x^2)+60b(15d^2+24dex+10e^2x^2)\log\left(cx^n\right))}{3600}$$

```
[In] Integrate[x^3*(d + e*x)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^4*(60*a*(15*d^2 + 24*d*e*x + 10*e^2*x^2) - b*n*(225*d^2 + 288*d*e*x + 10
0*e^2*x^2) + 60*b*(15*d^2 + 24*d*e*x + 10*e^2*x^2)*Log[c*x^n]))/3600
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36

method	result
parallelrisch	$\frac{x^{6} \ln(c\,x^{n}) b\,e^{2}}{6}-\frac{b\,e^{2} n\,x^{6}}{36}+\frac{a\,e^{2} x^{6}}{6}+\frac{2 x^{5} \ln(c\,x^{n}) b d e}{5}-\frac{2 b d e n\,x^{5}}{25}+\frac{2 a d e\,x^{5}}{5}+\frac{x^{4} \ln(c\,x^{n}) b\,d^{2}}{4}-\frac{b\,d^{2} n\,x^{4}}{16}+\frac{a\,d^{2} x}{4}$
risch	$\frac{b\,x^{4}\left(10 e^{2} x^{2}+24 d e x+15 d^{2}\right) \ln(x^{n})}{60}-\frac{i \pi b d e\,x^{5} \operatorname{csgn}(i c\,x^{n})^{3}}{5}+\frac{i \pi b\,e^{2} x^{6} \operatorname{csgn}(i c) \operatorname{csgn}(i c\,x^{n})^{2}}{12}+\frac{i \pi b\,d^{2} x^{4} \operatorname{csgn}(i x^{n}) \operatorname{csgn}(ic}{8}$

```
[In] int(x^3*(e*x+d)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^6*ln(c*x^n)*b*e^2-1/36*b*e^2*n*x^6+1/6*a*e^2*x^6+2/5*x^5*ln(c*x^n)*b*
d*e-2/25*b*d*e*n*x^5+2/5*a*d*e*x^5+1/4*x^4*ln(c*x^n)*b*d^2-1/16*b*d^2*n*x^4
+1/4*a*d^2*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.59

$$\int x^3(d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{1}{36}\left(be^2n-6\,ae^2\right)x^6-\frac{2}{25}\left(bden-5\,ade\right)x^5 -\frac{1}{16}\left(bd^2n-4\,ad^2\right)x^4 +\frac{1}{60}\left(10\,be^2x^6+24\,bdex^5+15\,bd^2x^4\right)\log\left(c\right) +\frac{1}{60}\left(10\,be^2nx^6+24\,bdenx^5+15\,bd^2nx^4\right)\log\left(x\right)$$

```
[In] integrate(x^3*(e*x+d)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/36*(b*e^2*n - 6*a*e^2)*x^6 - 2/25*(b*d*e*n - 5*a*d*e)*x^5 - 1/16*(b*d^2*
n - 4*a*d^2)*x^4 + 1/60*(10*b*e^2*x^6 + 24*b*d*e*x^5 + 15*b*d^2*x^4)*log(c)
 + 1/60*(10*b*e^2*n*x^6 + 24*b*d*e*n*x^5 + 15*b*d^2*n*x^4)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.64

$$\int x^3(d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{ad^2x^4}{4}+\frac{2adex^5}{5}+\frac{ae^2x^6}{6}-\frac{bd^2nx^4}{16}+\frac{bd^2x^4\log\left(cx^n\right)}{4} -\frac{2bdenx^5}{25}+\frac{2bdex^5\log\left(cx^n\right)}{5}-\frac{be^2nx^6}{36}+\frac{be^2x^6\log\left(cx^n\right)}{6}$$

```
[In] integrate(x**3*(e*x+d)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**2*x**4/4 + 2*a*d*e*x**5/5 + a*e**2*x**6/6 - b*d**2*n*x**4/16 + b*d**2*
x**4*log(c*x**n)/4 - 2*b*d*e*n*x**5/25 + 2*b*d*e*x**5*log(c*x**n)/5 - b*e**
2*n*x**6/36 + b*e**2*x**6*log(c*x**n)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int x^3(d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{1}{36}\,be^2nx^6+\frac{1}{6}\,be^2x^6\log\left(cx^n\right)-\frac{2}{25}\,bdenx^5+\frac{1}{6}\,ae^2x^6+\frac{2}{5}\,bdex^5\log\left(cx^n\right)-\frac{1}{16}\,bd^2nx^4+\frac{2}{5}\,adex^5+\frac{1}{4}\,bd^2x^4\log\left(cx^n\right)+\frac{1}{4}\,ad^2x^4$$

```
[In] integrate(x^3*(e*x+d)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/36*b*e^2*n*x^6 + 1/6*b*e^2*x^6*log(c*x^n) - 2/25*b*d*e*n*x^5 + 1/6*a*e^2
*x^6 + 2/5*b*d*e*x^5*log(c*x^n) - 1/16*b*d^2*n*x^4 + 2/5*a*d*e*x^5 + 1/4*b*
d^2*x^4*log(c*x^n) + 1/4*a*d^2*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.66

$$\int x^3(d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{1}{6}\,be^2nx^6\log\left(x\right)-\frac{1}{36}\,be^2nx^6+\frac{1}{6}\,be^2x^6\log\left(c\right)+\frac{2}{5}\,bdenx^5\log\left(x\right)-\frac{2}{25}\,bdenx^5+\frac{1}{6}\,ae^2x^6+\frac{2}{5}\,bdex^5\log\left(c\right)+\frac{1}{4}\,bd^2nx^4\log\left(x\right)-\frac{1}{16}\,bd^2nx^4+\frac{2}{5}\,adex^5+\frac{1}{4}\,bd^2x^4\log\left(c\right)+\frac{1}{4}\,ad^2x^4$$

```
[In] integrate(x^3*(e*x+d)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/6*b*e^2*n*x^6*log(x) - 1/36*b*e^2*n*x^6 + 1/6*b*e^2*x^6*log(c) + 2/5*b*d*
e*n*x^5*log(x) - 2/25*b*d*e*n*x^5 + 1/6*a*e^2*x^6 + 2/5*b*d*e*x^5*log(c) +
1/4*b*d^2*n*x^4*log(x) - 1/16*b*d^2*n*x^4 + 2/5*a*d*e*x^5 + 1/4*b*d^2*x^4*l
og(c) + 1/4*a*d^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int x^3(d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx=\ln\left(c\,x^n\right)\,\left(\frac{b\,d^2\,x^4}{4}+\frac{2\,b\,d\,e\,x^5}{5}+\frac{b\,e^2\,x^6}{6}\right)+\frac{d^2\,x^4\,\left(4\,a-b\,n\right)}{16}+\frac{e^2\,x^6\,\left(6\,a-b\,n\right)}{36}+\frac{2\,d\,e\,x^5\,\left(5\,a-b\,n\right)}{25}$$

```
[In] int(x^3*(a + b*log(c*x^n))*(d + e*x)^2,x)
```

```
[Out] log(c*x^n)*((b*d^2*x^4)/4 + (b*e^2*x^6)/6 + (2*b*d*e*x^5)/5) + (d^2*x^4*(4*
a - b*n))/16 + (e^2*x^6*(6*a - b*n))/36 + (2*d*e*x^5*(5*a - b*n))/25
```

3.10 $\int x^2(d+ex)^2 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 74

$$\int x^2(d+ex)^2 (a+b\log(cx^n))\,dx = -\frac{1}{9}bd^2nx^3 - \frac{1}{8}bdenx^4 - \frac{1}{25}be^2nx^5 + \frac{1}{30}\left(10d^2x^3+15dex^4+6e^2x^5\right)(a+b\log(cx^n))$$

```
[Out] -1/9*b*d^2*n*x^3-1/8*b*d*e*n*x^4-1/25*b*e^2*n*x^5+1/30*(6*e^2*x^5+15*d*e*x^
4+10*d^2*x^3)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {45, 2371, 12, 14}

$$\int x^2(d+ex)^2 (a+b\log(cx^n))\,dx = \frac{1}{30}\left(10d^2x^3+15dex^4+6e^2x^5\right)(a+b\log(cx^n)) - \frac{1}{9}bd^2nx^3 - \frac{1}{8}bdenx^4 - \frac{1}{25}be^2nx^5$$

```
[In] Int[x^2*(d + e*x)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/9*(b*d^2*n*x^3) - (b*d*e*n*x^4)/8 - (b*e^2*n*x^5)/25 + ((10*d^2*x^3 + 15
*d*e*x^4 + 6*e^2*x^5)*(a + b*Log[c*x^n]))/30
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{30}\left(10d^2x^3+15dex^4+6e^2x^5\right)\left(a+b\log\left(cx^n\right)\right) \\
&\quad -(bn)\int \frac{1}{30}x^2\left(10d^2+15dex+6e^2x^2\right)\,dx \\
&= \frac{1}{30}\left(10d^2x^3+15dex^4+6e^2x^5\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{30}(bn)\int x^2\left(10d^2+15dex+6e^2x^2\right)\,dx \\
&= \frac{1}{30}\left(10d^2x^3+15dex^4+6e^2x^5\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{30}(bn)\int \left(10d^2x^2+15dex^3+6e^2x^4\right)\,dx \\
&= -\frac{1}{9}bd^2nx^3-\frac{1}{8}bdenx^4-\frac{1}{25}be^2nx^5+\frac{1}{30}\left(10d^2x^3+15dex^4+6e^2x^5\right)\left(a+b\log\left(cx^n\right)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09

$$
\int x^2(d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx
= \frac{x^3\left(60a\left(10d^2+15dex+6e^2x^2\right)-bn\left(200d^2+225dex+72e^2x^2\right)+60b\left(10d^2+15dex+6e^2x^2\right)\log\left(cx^n\right)\right)}{1800}
$$

```
[In] Integrate[x^2*(d + e*x)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^3*(60*a*(10*d^2 + 15*d*e*x + 6*e^2*x^2) - b*n*(200*d^2 + 225*d*e*x + 72*
e^2*x^2) + 60*b*(10*d^2 + 15*d*e*x + 6*e^2*x^2)*Log[c*x^n]))/1800
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36

method	result
parallelrisch	$\frac{x^{5} b \ln\left(c\,x^{n}\right) e^{2}}{5}-\frac{b\,e^{2} n\,x^{5}}{25}+\frac{x^{5} a\,e^{2}}{5}+\frac{x^{4} b \ln\left(c\,x^{n}\right) d e}{2}-\frac{b d e n\,x^{4}}{8}+\frac{x^{4} a d e}{2}+\frac{x^{3} b \ln\left(c\,x^{n}\right) d^{2}}{3}-\frac{b\,d^{2} n\,x^{3}}{9}+\frac{a\,d^{2} x^{3}}{3}$
risch	$\frac{b\,x^{3} \left(6 e^{2} x^{2}+15 d e x+10 d^{2}\right) \ln\left(x^{n}\right)}{30}+\frac{i \pi b\,d^{2} x^{3} \operatorname{csgn}\left(i x^{n}\right) \operatorname{csgn}\left(i c\,x^{n}\right)^{2}}{6}-\frac{i \pi b\,d^{2} x^{3} \operatorname{csgn}\left(i c\,x^{n}\right)^{3}}{6}+\frac{i \pi b\,e^{2} x^{5} \operatorname{csgn}\left(i c\right) \operatorname{csgn}\left(i c\,x^{n}\right.}{10}$

```
[In] int(x^2*(e*x+d)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*b*ln(c*x^n)*e^2-1/25*b*e^2*n*x^5+1/5*x^5*a*e^2+1/2*x^4*b*ln(c*x^n)*
d*e-1/8*b*d*e*n*x^4+1/2*x^4*a*d*e+1/3*x^3*b*ln(c*x^n)*d^2-1/9*b*d^2*n*x^3+1
/3*a*d^2*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.59

$$\int x^2(d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{1}{25}\left(be^2n-5\,ae^2\right)x^5 - \frac{1}{8}\left(bden-4\,ade\right)x^4 - \frac{1}{9}\left(bd^2n-3\,ad^2\right)x^3 + \frac{1}{30}\left(6\,be^2x^5+15\,bdex^4+10\,bd^2x^3\right)\log\left(c\right) + \frac{1}{30}\left(6\,be^2nx^5+15\,bdenx^4+10\,bd^2nx^3\right)\log\left(x\right)$$

```
[In] integrate(x^2*(e*x+d)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/25*(b*e^2*n - 5*a*e^2)*x^5 - 1/8*(b*d*e*n - 4*a*d*e)*x^4 - 1/9*(b*d^2*n
- 3*a*d^2)*x^3 + 1/30*(6*b*e^2*x^5 + 15*b*d*e*x^4 + 10*b*d^2*x^3)*log(c) +
1/30*(6*b*e^2*n*x^5 + 15*b*d*e*n*x^4 + 10*b*d^2*n*x^3)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.57

$$\int x^2(d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{ad^2x^3}{3}+\frac{adex^4}{2}+\frac{ae^2x^5}{5}-\frac{bd^2nx^3}{9}+\frac{bd^2x^3\log\left(cx^n\right)}{3} - \frac{bdenx^4}{8}+\frac{bdex^4\log\left(cx^n\right)}{2}-\frac{be^2nx^5}{25}+\frac{be^2x^5\log\left(cx^n\right)}{5}$$

```
[In] integrate(x**2*(e*x+d)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**2*x**3/3 + a*d*e*x**4/2 + a*e**2*x**5/5 - b*d**2*n*x**3/9 + b*d**2*x**
3*log(c*x**n)/3 - b*d*e*n*x**4/8 + b*d*e*x**4*log(c*x**n)/2 - b*e**2*n*x**5
/25 + b*e**2*x**5*log(c*x**n)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int x^2(d+ex)^2(a+b\log(cx^n))\,dx = -\frac{1}{25}\,be^2nx^5 + \frac{1}{5}\,be^2x^5\log(cx^n) - \frac{1}{8}\,bdenx^4 + \frac{1}{5}\,ae^2x^5 + \frac{1}{2}\,bdex^4\log(cx^n) - \frac{1}{9}\,bd^2nx^3 + \frac{1}{2}\,adex^4 + \frac{1}{3}\,bd^2x^3\log(cx^n) + \frac{1}{3}\,ad^2x^3$$

```
[In] integrate(x^2*(e*x+d)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/25*b*e^2*n*x^5 + 1/5*b*e^2*x^5*log(c*x^n) - 1/8*b*d*e*n*x^4 + 1/5*a*e^2*
x^5 + 1/2*b*d*e*x^4*log(c*x^n) - 1/9*b*d^2*n*x^3 + 1/2*a*d*e*x^4 + 1/3*b*d^
2*x^3*log(c*x^n) + 1/3*a*d^2*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.66

$$\int x^2(d+ex)^2(a+b\log(cx^n))\,dx = \frac{1}{5}\,be^2nx^5\log(x) - \frac{1}{25}\,be^2nx^5 + \frac{1}{5}\,be^2x^5\log(c) + \frac{1}{2}\,bdenx^4\log(x) - \frac{1}{8}\,bdenx^4 + \frac{1}{5}\,ae^2x^5 + \frac{1}{2}\,bdex^4\log(c) + \frac{1}{3}\,bd^2nx^3\log(x) - \frac{1}{9}\,bd^2nx^3 + \frac{1}{2}\,adex^4 + \frac{1}{3}\,bd^2x^3\log(c) + \frac{1}{3}\,ad^2x^3$$

```
[In] integrate(x^2*(e*x+d)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/5*b*e^2*n*x^5*log(x) - 1/25*b*e^2*n*x^5 + 1/5*b*e^2*x^5*log(c) + 1/2*b*d*
e*n*x^4*log(x) - 1/8*b*d*e*n*x^4 + 1/5*a*e^2*x^5 + 1/2*b*d*e*x^4*log(c) + 1
/3*b*d^2*n*x^3*log(x) - 1/9*b*d^2*n*x^3 + 1/2*a*d*e*x^4 + 1/3*b*d^2*x^3*log
(c) + 1/3*a*d^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int x^2(d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx=\ln\left(c\,x^n\right)\,\left(\frac{b\,d^2\,x^3}{3}+\frac{b\,d\,e\,x^4}{2}+\frac{b\,e^2\,x^5}{5}\right)+\frac{d^2\,x^3\,\left(3\,a-b\,n\right)}{9}+\frac{e^2\,x^5\,\left(5\,a-b\,n\right)}{25}+\frac{d\,e\,x^4\,\left(4\,a-b\,n\right)}{8}$$

```
[In] int(x^2*(a + b*log(c*x^n))*(d + e*x)^2,x)
```

```
[Out] log(c*x^n)*((b*d^2*x^3)/3 + (b*e^2*x^5)/5 + (b*d*e*x^4)/2) + (d^2*x^3*(3*a
- b*n))/9 + (e^2*x^5*(5*a - b*n))/25 + (d*e*x^4*(4*a - b*n))/8
```

3.11 $\int x(d+ex)^2 (a+b\log(cx^n)) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int x(d+ex)^2 (a+b\log(cx^n)) \, dx = -\frac{1}{4}bd^2nx^2 - \frac{2}{9}bdenx^3 - \frac{1}{16}be^2nx^4 + \frac{1}{12}\left(6d^2x^2+8dex^3+3e^2x^4\right)(a+b\log(cx^n))$$

```
[Out] -1/4*b*d^2*n*x^2-2/9*b*d*e*n*x^3-1/16*b*e^2*n*x^4+1/12*(3*e^2*x^4+8*d*e*x^3
+6*d^2*x^2)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {45, 2371, 12, 14}

$$\int x(d+ex)^2 (a+b\log(cx^n)) \, dx = \frac{1}{12}\left(6d^2x^2+8dex^3+3e^2x^4\right)(a+b\log(cx^n)) - \frac{1}{4}bd^2nx^2 - \frac{2}{9}bdenx^3 - \frac{1}{16}be^2nx^4$$

```
[In] Int[x*(d + e*x)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/4*(b*d^2*n*x^2) - (2*b*d*e*n*x^3)/9 - (b*e^2*n*x^4)/16 + ((6*d^2*x^2 + 8
*d*e*x^3 + 3*e^2*x^4)*(a + b*Log[c*x^n]))/12
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{12}\left(6d^2x^2+8dex^3+3e^2x^4\right)\left(a+b\log\left(cx^n\right)\right)-(bn)\int\frac{1}{12}x\left(6d^2+8dex+3e^2x^2\right)\,dx\\
&= \frac{1}{12}\left(6d^2x^2+8dex^3+3e^2x^4\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{12}(bn)\int x\left(6d^2+8dex+3e^2x^2\right)\,dx\\
&= \frac{1}{12}\left(6d^2x^2+8dex^3+3e^2x^4\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{12}(bn)\int\left(6d^2x+8dex^2+3e^2x^3\right)\,dx\\
&= -\frac{1}{4}bd^2nx^2-\frac{2}{9}bdenx^3-\frac{1}{16}be^2nx^4+\frac{1}{12}\left(6d^2x^2+8dex^3+3e^2x^4\right)\left(a+b\log\left(cx^n\right)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09

$$
\int x(d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{1}{144}x^2\left(12a\left(6d^2+8dex+3e^2x^2\right)-bn\left(36d^2+32dex+9e^2x^2\right)+12b\left(6d^2+8dex+3e^2x^2\right)\log\left(cx^n\right)\right)
$$

```
[In] Integrate[x*(d + e*x)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^2*(12*a*(6*d^2 + 8*d*e*x + 3*e^2*x^2) - b*n*(36*d^2 + 32*d*e*x + 9*e^2*x
^2) + 12*b*(6*d^2 + 8*d*e*x + 3*e^2*x^2)*Log[c*x^n]))/144
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36

method	result
parallelrisch	$\frac{x^4 b \ln(c\,x^n) e^2}{4} - \frac{b\,e^2 n\,x^4}{16} + \frac{x^4 a\,e^2}{4} + \frac{2x^3 b \ln(c\,x^n) de}{3} - \frac{2bden\,x^3}{9} + \frac{2x^3 ade}{3} + \frac{x^2 b \ln(c\,x^n) d^2}{2} - \frac{b\,d^2 n\,x^2}{4} + \frac{a\,d^2 x^2}{2}$
risch	$\frac{b\,x^2 (3e^2x^2+8dex+6d^2) \ln(x^n)}{12} - \frac{i\pi b\,d^2 x^2 \operatorname{csgn}(ic\,x^n)^3}{4} + \frac{i\pi bde\,x^3 \operatorname{csgn}(ic) \operatorname{csgn}(ic\,x^n)^2}{3} + \frac{i\pi bde\,x^3 \operatorname{csgn}(ix^n) \operatorname{csgn}(ic\,x^n)}{3}$

```
[In] int(x*(e*x+d)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*b*ln(c*x^n)*e^2-1/16*b*e^2*n*x^4+1/4*x^4*a*e^2+2/3*x^3*b*ln(c*x^n)*
d*e-2/9*b*d*e*n*x^3+2/3*x^3*a*d*e+1/2*x^2*b*ln(c*x^n)*d^2-1/4*b*d^2*n*x^2+1
/2*a*d^2*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.59

$$\begin{aligned}\int x(d+ex)^2\,(a+b\log{(cx^n)})\;dx =& -\frac{1}{16}\left(be^2n-4\,ae^2\right)x^4\\ &-\frac{2}{9}\,(bden-3\,ade)x^3-\frac{1}{4}\left(bd^2n-2\,ad^2\right)x^2\\ &+\frac{1}{12}\left(3\,be^2x^4+8\,bdex^3+6\,bd^2x^2\right)\log{(c)}\\ &+\frac{1}{12}\left(3\,be^2nx^4+8\,bdenx^3+6\,bd^2nx^2\right)\log{(x)}\end{aligned}$$

```
[In] integrate(x*(e*x+d)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/16*(b*e^2*n - 4*a*e^2)*x^4 - 2/9*(b*d*e*n - 3*a*d*e)*x^3 - 1/4*(b*d^2*n
- 2*a*d^2)*x^2 + 1/12*(3*b*e^2*x^4 + 8*b*d*e*x^3 + 6*b*d^2*x^2)*log(c) + 1/
12*(3*b*e^2*n*x^4 + 8*b*d*e*n*x^3 + 6*b*d^2*n*x^2)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.64

$$\begin{aligned}\int x(d+ex)^2\,(a+b\log{(cx^n)})\;dx =& \frac{ad^2x^2}{2}+\frac{2adex^3}{3}+\frac{ae^2x^4}{4}-\frac{bd^2nx^2}{4}+\frac{bd^2x^2\log{(cx^n)}}{2}\\ &-\frac{2bdenx^3}{9}+\frac{2bdex^3\log{(cx^n)}}{3}-\frac{be^2nx^4}{16}+\frac{be^2x^4\log{(cx^n)}}{4}\end{aligned}$$

```
[In] integrate(x*(e*x+d)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**2*x**2/2 + 2*a*d*e*x**3/3 + a*e**2*x**4/4 - b*d**2*n*x**2/4 + b*d**2*x
**2*log(c*x**n)/2 - 2*b*d*e*n*x**3/9 + 2*b*d*e*x**3*log(c*x**n)/3 - b*e**2*
n*x**4/16 + b*e**2*x**4*log(c*x**n)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int x(d+ex)^2(a+b\log(cx^n))\,dx = -\frac{1}{16}be^2nx^4 + \frac{1}{4}be^2x^4\log(cx^n) - \frac{2}{9}bdenx^3 + \frac{1}{4}ae^2x^4 + \frac{2}{3}bdex^3\log(cx^n) - \frac{1}{4}bd^2nx^2 + \frac{2}{3}adex^3 + \frac{1}{2}bd^2x^2\log(cx^n) + \frac{1}{2}ad^2x^2$$

```
[In] integrate(x*(e*x+d)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/16*b*e^2*n*x^4 + 1/4*b*e^2*x^4*log(c*x^n) - 2/9*b*d*e*n*x^3 + 1/4*a*e^2*
x^4 + 2/3*b*d*e*x^3*log(c*x^n) - 1/4*b*d^2*n*x^2 + 2/3*a*d*e*x^3 + 1/2*b*d^
2*x^2*log(c*x^n) + 1/2*a*d^2*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.66

$$\int x(d+ex)^2(a+b\log(cx^n))\,dx = \frac{1}{4}be^2nx^4\log(x) - \frac{1}{16}be^2nx^4 + \frac{1}{4}be^2x^4\log(c) + \frac{2}{3}bdenx^3\log(x) - \frac{2}{9}bdenx^3 + \frac{1}{4}ae^2x^4 + \frac{2}{3}bdex^3\log(c) + \frac{1}{2}bd^2nx^2\log(x) - \frac{1}{4}bd^2nx^2 + \frac{2}{3}adex^3 + \frac{1}{2}bd^2x^2\log(c) + \frac{1}{2}ad^2x^2$$

```
[In] integrate(x*(e*x+d)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*e^2*n*x^4*log(x) - 1/16*b*e^2*n*x^4 + 1/4*b*e^2*x^4*log(c) + 2/3*b*d*
e*n*x^3*log(x) - 2/9*b*d*e*n*x^3 + 1/4*a*e^2*x^4 + 2/3*b*d*e*x^3*log(c) + 1
/2*b*d^2*n*x^2*log(x) - 1/4*b*d^2*n*x^2 + 2/3*a*d*e*x^3 + 1/2*b*d^2*x^2*log
(c) + 1/2*a*d^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int x(d+ex)^2\,(a+b\log{(cx^n)})\ dx = \ln{(c\,x^n)}\,\left(\frac{b\,d^2\,x^2}{2}+\frac{2\,b\,d\,e\,x^3}{3}+\frac{b\,e^2\,x^4}{4}\right)+\frac{d^2\,x^2\,(2\,a-b\,n)}{4}+\frac{e^2\,x^4\,(4\,a-b\,n)}{16}+\frac{2\,d\,e\,x^3\,(3\,a-b\,n)}{9}$$

```
[In] int(x*(a + b*log(c*x^n))*(d + e*x)^2,x)
```

```
[Out] log(c*x^n)*((b*d^2*x^2)/2 + (b*e^2*x^4)/4 + (2*b*d*e*x^3)/3) + (d^2*x^2*(2*
a - b*n))/4 + (e^2*x^4*(4*a - b*n))/16 + (2*d*e*x^3*(3*a - b*n))/9
```

3.12 $\int (d+ex)^2 (a+b\log(cx^n)) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 70

$$\int (d+ex)^2 (a+b\log(cx^n)) \, dx = -bd^2nx - \frac{1}{2}bdenx^2 - \frac{1}{9}be^2nx^3 - \frac{bd^3n\log(x)}{3e} + \frac{(d+ex)^3(a+b\log(cx^n))}{3e}$$

```
[Out] -b*d^2*n*x-1/2*b*d*e*n*x^2-1/9*b*e^2*n*x^3-1/3*b*d^3*n*ln(x)/e+1/3*(e*x+d)^
3*(a+b*ln(c*x^n))/e
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {32, 2350, 12, 45}

$$\int (d+ex)^2 (a+b\log(cx^n)) \, dx = \frac{(d+ex)^3(a+b\log(cx^n))}{3e} - \frac{bd^3n\log(x)}{3e} - bd^2nx - \frac{1}{2}bdenx^2 - \frac{1}{9}be^2nx^3$$

```
[In] Int[(d + e*x)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -(b*d^2*n*x) - (b*d*e*n*x^2)/2 - (b*e^2*n*x^3)/9 - (b*d^3*n*Log[x])/(3*e) +
 ((d + e*x)^3*(a + b*Log[c*x^n]))/(3*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2350

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Dist[a + b*Log[c*x^n], u
, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^3\,(a+b\log(cx^n))}{3e} - (bn)\int \frac{(d+ex)^3}{3ex}\,dx \\
&= \frac{(d+ex)^3\,(a+b\log(cx^n))}{3e} - \frac{(bn)\int \frac{(d+ex)^3}{x}\,dx}{3e} \\
&= \frac{(d+ex)^3\,(a+b\log(cx^n))}{3e} - \frac{(bn)\int \left(3d^2e+\frac{d^3}{x}+3de^2x+e^3x^2\right)\,dx}{3e} \\
&= -bd^2nx-\frac{1}{2}bdenx^2-\frac{1}{9}be^2nx^3-\frac{bd^3n\log(x)}{3e}+\frac{(d+ex)^3\,(a+b\log(cx^n))}{3e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int (d+ex)^2\,(a+b\log(cx^n))\,dx = \frac{1}{18}x\big(6a\big(3d^2+3dex+e^2x^2\big)-bn\big(18d^2+9dex+2e^2x^2\big)+6b\big(3d^2+3dex+e^2x^2\big)\log(cx^n)\big)$$

```
[In] Integrate[(d + e*x)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] (x*(6*a*(3*d^2 + 3*d*e*x + e^2*x^2) - b*n*(18*d^2 + 9*d*e*x + 2*e^2*x^2) +
6*b*(3*d^2 + 3*d*e*x + e^2*x^2)*Log[c*x^n]))/18
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.30

method	result
parallelrisch	$\frac{b\ln(c\,x^n)e^2x^3}{3}-\frac{b\,e^2n\,x^3}{9}+\frac{a\,e^2x^3}{3}+b\ln\left(c\,x^n\right)de\,x^2-\frac{bden\,x^2}{2}+ade\,x^2+xb\ln\left(c\,x^n\right)d^2-b\,d^2nx+$
risch	$\frac{(ex+d)^3b\ln(x^n)}{3e}+\frac{i\pi b\,d^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2x}{2}-\frac{i\pi b\,d^2\,\mathrm{csgn}(ic\,x^n)^3x}{2}-\frac{ie\pi bd\,x^2\,\mathrm{csgn}(ic\,x^n)^3}{2}-\frac{i\pi b\,d^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix}{2}$

```
[In] int((e*x+d)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*ln(c*x^n)*e^2*x^3-1/9*b*e^2*n*x^3+1/3*a*e^2*x^3+b*ln(c*x^n)*d*e*x^2-1
/2*b*d*e*n*x^2+a*d*e*x^2+x*b*ln(c*x^n)*d^2-b*d^2*n*x+a*d^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.57

$$\int (d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{1}{9}\left(be^2n-3\,ae^2\right)x^3-\frac{1}{2}\left(bden-2\,ade\right)x^2 - \left(bd^2n-ad^2\right)x+\frac{1}{3}\left(be^2x^3+3\,bdex^2+3\,bd^2x\right)\log\left(c\right) + \frac{1}{3}\left(be^2nx^3+3\,bdenx^2+3\,bd^2nx\right)\log\left(x\right)$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/9*(b*e^2*n - 3*a*e^2)*x^3 - 1/2*(b*d*e*n - 2*a*d*e)*x^2 - (b*d^2*n - a*d
^2)*x + 1/3*(b*e^2*x^3 + 3*b*d*e*x^2 + 3*b*d^2*x)*log(c) + 1/3*(b*e^2*n*x^3
 + 3*b*d*e*n*x^2 + 3*b*d^2*n*x)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.46

$$\int (d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx = ad^2x+adex^2+\frac{ae^2x^3}{3}-bd^2nx+bd^2x\log\left(cx^n\right) - \frac{bdenx^2}{2}+bdex^2\log\left(cx^n\right)-\frac{be^2nx^3}{9}+\frac{be^2x^3\log\left(cx^n\right)}{3}$$

```
[In] integrate((e*x+d)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 - b*d**2*n*x + b*d**2*x*log(c*x**n) -
 b*d*e*n*x**2/2 + b*d*e*x**2*log(c*x**n) - b*e**2*n*x**3/9 + b*e**2*x**3*lo
g(c*x**n)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

$$\int (d+ex)^2 (a+b\log(cx^n))\,dx = -\frac{1}{9}be^2nx^3 + \frac{1}{3}be^2x^3\log(cx^n) - \frac{1}{2}bdenx^2 + \frac{1}{3}ae^2x^3 + bdex^2\log(cx^n) - bd^2nx + adex^2 + bd^2x\log(cx^n) + ad^2x$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/9*b*e^2*n*x^3 + 1/3*b*e^2*x^3*log(c*x^n) - 1/2*b*d*e*n*x^2 + 1/3*a*e^2*x
^3 + b*d*e*x^2*log(c*x^n) - b*d^2*n*x + a*d*e*x^2 + b*d^2*x*log(c*x^n) + a*
d^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.56

$$\int (d+ex)^2 (a+b\log(cx^n))\,dx = \frac{1}{3}be^2nx^3\log(x) - \frac{1}{9}be^2nx^3 + \frac{1}{3}be^2x^3\log(c) + bdenx^2\log(x) - \frac{1}{2}bdenx^2 + \frac{1}{3}ae^2x^3 + bdex^2\log(c) + bd^2nx\log(x) - bd^2nx + adex^2 + bd^2x\log(c) + ad^2x$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*e^2*n*x^3*log(x) - 1/9*b*e^2*n*x^3 + 1/3*b*e^2*x^3*log(c) + b*d*e*n*x
^2*log(x) - 1/2*b*d*e*n*x^2 + 1/3*a*e^2*x^3 + b*d*e*x^2*log(c) + b*d^2*n*x*
log(x) - b*d^2*n*x + a*d*e*x^2 + b*d^2*x*log(c) + a*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int (d+ex)^2 (a+b\log(cx^n))\,dx = \ln(c\,x^n)\left(b\,d^2\,x + b\,d\,e\,x^2 + \frac{b\,e^2\,x^3}{3}\right) + \frac{e^2\,x^3\,(3\,a-b\,n)}{9} + d^2\,x\,(a-b\,n) + \frac{d\,e\,x^2\,(2\,a-b\,n)}{2}$$

```
[In] int((a + b*log(c*x^n))*(d + e*x)^2,x)
```

```
[Out] log(c*x^n)*((b*e^2*x^3)/3 + b*d^2*x + b*d*e*x^2) + (e^2*x^3*(3*a - b*n))/9
+ d^2*x*(a - b*n) + (d*e*x^2*(2*a - b*n))/2
```

3.13 $\int \frac{(d+ex)^2(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{(d+ex)^2(a+b\log(cx^n))}{x}\,dx = -\frac{1}{4}bn(4d+ex)^2-\frac{1}{2}bd^2n\log^2(x)+2dex(a+b\log(cx^n)) +\frac{1}{2}e^2x^2(a+b\log(cx^n))+d^2\log(x)\,(a+b\log(cx^n))$$

```
[Out] -1/4*b*n*(e*x+4*d)^2-1/2*b*d^2*n*ln(x)^2+2*d*e*x*(a+b*ln(c*x^n))+1/2*e^2*x^
2*(a+b*ln(c*x^n))+d^2*ln(x)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {45, 2372, 2338}

$$\int \frac{(d+ex)^2(a+b\log(cx^n))}{x}\,dx = d^2\log(x)\,(a+b\log(cx^n))+2dex(a+b\log(cx^n)) +\frac{1}{2}e^2x^2(a+b\log(cx^n))-\frac{1}{2}bd^2n\log^2(x)-\frac{1}{4}bn(4d+ex)^2$$

```
[In] Int[((d + e*x)^2*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] -1/4*(b*n*(4*d + e*x)^2) - (b*d^2*n*Log[x]^2)/2 + 2*d*e*x*(a + b*Log[c*x^n]
) + (e^2*x^2*(a + b*Log[c*x^n]))/2 + d^2*Log[x]*(a + b*Log[c*x^n])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2dex(a+b\log(cx^n))+\frac{1}{2}e^2x^2(a+b\log(cx^n)) \\
&\quad +d^2\log(x)(a+b\log(cx^n))-(bn)\int\left(\frac{1}{2}e(4d+ex)+\frac{d^2\log(x)}{x}\right)dx \\
&= -\frac{1}{4}bn(4d+ex)^2+2dex(a+b\log(cx^n))+\frac{1}{2}e^2x^2(a+b\log(cx^n)) \\
&\quad +d^2\log(x)(a+b\log(cx^n))-\left(bd^2n\right)\int\frac{\log(x)}{x}\,dx \\
&= -\frac{1}{4}bn(4d+ex)^2-\frac{1}{2}bd^2n\log^2(x)+2dex(a+b\log(cx^n)) \\
&\quad +\frac{1}{2}e^2x^2(a+b\log(cx^n))+d^2\log(x)(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int\frac{(d+ex)^2(a+b\log(cx^n))}{x}\,dx = 2adex-2bdenx-\frac{1}{4}be^2nx^2+2bdex\log(cx^n)+\frac{1}{2}e^2x^2(a+b\log(cx^n))+\frac{d^2(a+b\log(cx^n))^2}{2bn}$$

```
[In] Integrate[((d + e*x)^2*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] 2*a*d*e*x - 2*b*d*e*n*x - (b*e^2*n*x^2)/4 + 2*b*d*e*x*Log[c*x^n] + (e^2*x^2
*(a + b*Log[c*x^n]))/2 + (d^2*(a + b*Log[c*x^n])^2)/(2*b*n)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.21

method	result
parallelrisch	$\frac{2x^{2} \ln(c\,x^{n}) b\,e^{2} n - x^{2} b\,e^{2} n^{2} + 2x^{2} a\,e^{2} n + 8x \ln(c\,x^{n}) bden - 8xbde\,n^{2} + 4 \ln(x) a\,d^{2} n + 8xaden + 2b\,d^{2} \ln(c\,x^{n})^{2}}{4n}$
risch	$\left(\frac{x^{2} b\,e^{2}}{2} + 2bdex + b\,d^{2} \ln(x)\right) \ln(x^{n}) - \frac{b\,d^{2} n \ln(x)^{2}}{2} + \frac{i \ln(x) \pi b\,d^{2} \operatorname{csgn}(ic) \operatorname{csgn}(ic\,x^{n})^{2}}{2} + \frac{i\pi b\,e^{2} x^{2} \operatorname{csgn}(ic) \operatorname{cs}}{4}$

```
[In] int((e*x+d)^2*(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*x^2*ln(c*x^n)*b*e^2*n-x^2*b*e^2*n^2+2*x^2*a*e^2*n+8*x*ln(c*x^n)*b*d*
e*n-8*x*b*d*e*n^2+4*ln(x)*a*d^2*n+8*x*a*d*e*n+2*b*d^2*ln(c*x^n)^2)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x}\,dx = \frac{1}{2}\,bd^2n\log(x)^2 - \frac{1}{4}\left(be^2n - 2\,ae^2\right)x^2 - 2\,(bden - ade)x + \frac{1}{2}\left(be^2x^2 + 4\,bdex\right)\log(c) + \frac{1}{2}\left(be^2nx^2 + 4\,bdenx + 2\,bd^2\log(c) + 2\,ad^2\right)\log(x)$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] 1/2*b*d^2*n*log(x)^2 - 1/4*(b*e^2*n - 2*a*e^2)*x^2 - 2*(b*d*e*n - a*d*e)*x
+ 1/2*(b*e^2*x^2 + 4*b*d*e*x)*log(c) + 1/2*(b*e^2*n*x^2 + 4*b*d*e*n*x + 2*b
*d^2*log(c) + 2*a*d^2)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x}\,dx = \begin{cases} \frac{ad^2\log(cx^n)}{n} + 2adex + \frac{ae^2x^2}{2} + \frac{bd^2\log(cx^n)^2}{2n} - 2bdenx + 2bdex\log(cx^n) - \frac{be^2nx^2}{4} + \frac{be^2x^2\log(cx^n)}{2} & \text{for}\ n \neq 0 \\ (a+b\log(c))\left(d^2\log(x) + 2dex + \frac{e^2x^2}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((e*x+d)**2*(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise((a*d**2*log(c*x**n)/n + 2*a*d*e*x + a*e**2*x**2/2 + b*d**2*log(c*
x**n)**2/(2*n) - 2*b*d*e*n*x + 2*b*d*e*x*log(c*x**n) - b*e**2*n*x**2/4 + b*
e**2*x**2*log(c*x**n)/2, Ne(n, 0)), ((a + b*log(c))*(d**2*log(x) + 2*d*e*x
+ e**2*x**2/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^2(a+b\log(cx^n))}{x}\,dx = -\frac{1}{4}\,be^2nx^2+\frac{1}{2}\,be^2x^2\log(cx^n)-2\,bdenx+\frac{1}{2}\,ae^2x^2+2\,bdex\log(cx^n)+2\,adex+\frac{bd^2\log(cx^n)^2}{2\,n}+ad^2\log(x)$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] -1/4*b*e^2*n*x^2 + 1/2*b*e^2*x^2*log(c*x^n) - 2*b*d*e*n*x + 1/2*a*e^2*x^2 +
 2*b*d*e*x*log(c*x^n) + 2*a*d*e*x + 1/2*b*d^2*log(c*x^n)^2/n + a*d^2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^2(a+b\log(cx^n))}{x}\,dx = \frac{1}{2}\,bd^2n\log(x)^2-\frac{1}{4}\left(be^2n-2\,be^2\log(c)-2\,ae^2\right)x^2-2\left(bden-bde\log(c)-ade\right)x+\frac{1}{2}\left(be^2nx^2+4\,bdenx\right)\log(x)+\left(bd^2\log(c)+ad^2\right)\log(x)$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] 1/2*b*d^2*n*log(x)^2 - 1/4*(b*e^2*n - 2*b*e^2*log(c) - 2*a*e^2)*x^2 - 2*(b*
d*e*n - b*d*e*log(c) - a*d*e)*x + 1/2*(b*e^2*n*x^2 + 4*b*d*e*n*x)*log(x) +
(b*d^2*log(c) + a*d^2)*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^2\,(a+b\log(cx^n))}{x}\,dx = \ln(c\,x^n)\,\left(\frac{b\,e^2\,x^2}{2}+2\,b\,d\,e\,x\right)+\frac{e^2\,x^2\,(2\,a-b\,n)}{4} + a\,d^2\,\ln(x)+\frac{b\,d^2\,\ln(c\,x^n)^2}{2\,n}+2\,d\,e\,x\,(a-b\,n)$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^2)/x,x)
```

```
[Out] log(c*x^n)*((b*e^2*x^2)/2 + 2*b*d*e*x) + (e^2*x^2*(2*a - b*n))/4 + a*d^2*lo
g(x) + (b*d^2*log(c*x^n)^2)/(2*n) + 2*d*e*x*(a - b*n)
```

3.14 $\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^2} dx = -\frac{bd^2n}{x} - be^2nx - bden\log^2(x) - \frac{d^2(a+b\log(cx^n))}{x} + e^2x(a+b\log(cx^n)) + 2de\log(x)(a+b\log(cx^n))$$

```
[Out] -b*d^2*n/x-b*e^2*n*x-b*d*e*n*ln(x)^2-d^2*(a+b*ln(c*x^n))/x+e^2*x*(a+b*ln(c*
x^n))+2*d*e*ln(x)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {45, 2372, 2338}

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^2} dx = -\frac{d^2(a+b\log(cx^n))}{x} + 2de\log(x)(a+b\log(cx^n)) + e^2x(a+b\log(cx^n)) - \frac{bd^2n}{x} - bden\log^2(x) - be^2nx$$

```
[In] Int[((d + e*x)^2*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -((b*d^2*n)/x) - b*e^2*n*x - b*d*e*n*Log[x]^2 - (d^2*(a + b*Log[c*x^n]))/x
+ e^2*x*(a + b*Log[c*x^n]) + 2*d*e*Log[x]*(a + b*Log[c*x^n])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{x}+e^2x(a+b\log(cx^n)) \\
&\quad +2de\log(x)\,(a+b\log(cx^n))-(bn)\int\left(e^2-\frac{d^2}{x^2}+\frac{2de\log(x)}{x}\right)dx \\
&= -\frac{bd^2n}{x}-be^2nx-\frac{d^2(a+b\log(cx^n))}{x}+e^2x(a+b\log(cx^n)) \\
&\quad +2de\log(x)\,(a+b\log(cx^n))-(2bden)\int\frac{\log(x)}{x}\,dx \\
&= -\frac{bd^2n}{x}-be^2nx-bden\log^2(x)-\frac{d^2(a+b\log(cx^n))}{x} \\
&\quad +e^2x(a+b\log(cx^n))+2de\log(x)\,(a+b\log(cx^n))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$
\begin{aligned}
\int\frac{(d+ex)^2\,(a+b\log(cx^n))}{x^2}\,dx &= -\frac{bd^2n}{x}+ae^2x-be^2nx+be^2x\log(cx^n) \\
&\quad -\frac{d^2(a+b\log(cx^n))}{x}+\frac{de(a+b\log(cx^n))^2}{bn}
\end{aligned}
$$

```
[In] Integrate[((d + e*x)^2*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -((b*d^2*n)/x) + a*e^2*x - b*e^2*n*x + b*e^2*x*Log[c*x^n] - (d^2*(a + b*Log
[c*x^n]))/x + (d*e*(a + b*Log[c*x^n])^2)/(b*n)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23

method	result
parallelrisch	$\frac{x^2 \ln(c x^n) b e^2 n - x^2 b e^2 n^2 + 2 \ln(x) x a d e n + x^2 a e^2 n + b d e \ln(c x^n)^2 x - \ln(c x^n) b d^2 n - b d^2 n^2 - a d^2 n}{x n}$
risch	$-\frac{b(-2dex\ln(x) - e^2x^2 + d^2)\ln(x^n)}{x} - \frac{-2i\ln(x)\pi bde\ \mathrm{csgn}(ix^n)\ \mathrm{csgn}(icx^n)^2 x + i\pi b d^2\ \mathrm{csgn}(ic)\ \mathrm{csgn}(icx^n)^2 + i\pi b e^2 x^2\ \mathrm{csgn}(i$

[In] int((e*x+d)^2*(a+b*ln(c*x^n))/x^2,x,method=_RETURNVERBOSE)

[Out] 1/x*(x^2*ln(c*x^n)*b*e^2*n-x^2*b*e^2*n^2+2*ln(x)*x*a*d*e*n+x^2*a*e^2*n+b*d*e*ln(c*x^n)^2*x-ln(c*x^n)*b*d^2*n-b*d^2*n^2-a*d^2*n)/n

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^2}\,dx = \frac{bdenx\log(x)^2 - bd^2n - ad^2 - (be^2n - ae^2)x^2 + (be^2x^2 - bd^2)\log(c) + (be^2nx^2 + 2\,bdex\log(c) - bd^2n +}{x}$$

[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x^2,x, algorithm="fricas")

[Out] (b*d*e*n*x*log(x)^2 - b*d^2*n - a*d^2 - (b*e^2*n - a*e^2)*x^2 + (b*e^2*x^2 - b*d^2)*log(c) + (b*e^2*n*x^2 + 2*b*d*e*x*log(c) - b*d^2*n + 2*a*d*e*x)*log(x))/x

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^2}\,dx = \begin{cases} -\frac{ad^2}{x} + \frac{2ade\log(cx^n)}{n} + ae^2x - \frac{bd^2n}{x} - \frac{bd^2\log(cx^n)}{x} + \frac{bde\log(cx^n)^2}{n} - be^2nx + be^2x\log(cx^n) & \text{for } n \neq 0 \\ (a+b\log(c))\left(-\frac{d^2}{x} + 2de\log(x) + e^2x\right) & \text{otherwise} \end{cases}$$

[In] integrate((e*x+d)**2*(a+b*ln(c*x**n))/x**2,x)

[Out] Piecewise((-a*d**2/x + 2*a*d*e*log(c*x**n)/n + a*e**2*x - b*d**2*n/x - b*d**2*log(c*x**n)/x + b*d*e*log(c*x**n)**2/n - b*e**2*n*x + b*e**2*x*log(c*x**n), Ne(n, 0)), ((a + b*log(c))*(-d**2/x + 2*d*e*log(x) + e**2*x), True))

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^2}\,dx = -be^2nx + be^2x\log(cx^n) + ae^2x + \frac{bde\log(cx^n)^2}{n} + 2\,ade\log(x) - \frac{bd^2n}{x} - \frac{bd^2\log(cx^n)}{x} - \frac{ad^2}{x}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
```

```
[Out] -b*e^2*n*x + b*e^2*x*log(c*x^n) + a*e^2*x + b*d*e*log(c*x^n)^2/n + 2*a*d*e*
log(x) - b*d^2*n/x - b*d^2*log(c*x^n)/x - a*d^2/x
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^2}\,dx = bden\log(x)^2 + (x\log(x)-x)be^2n - bd^2n\left(\frac{\log(x)}{x}+\frac{1}{x}\right) + be^2x\log(c) + 2\,bde\log(c)\log(|x|) + ae^2x + 2\,ade\log(|x|) - \frac{bd^2\log(c)}{x} - \frac{ad^2}{x}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

```
[Out] b*d*e*n*log(x)^2 + (x*log(x) - x)*b*e^2*n - b*d^2*n*(log(x)/x + 1/x) + b*e^
2*x*log(c) + 2*b*d*e*log(c)*log(abs(x)) + a*e^2*x + 2*a*d*e*log(abs(x)) - b
*d^2*log(c)/x - a*d^2/x
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^2}\,dx = \ln(x)\,\left(2\,a\,d\,e+2\,b\,d\,e\,n\right) - \frac{a\,d^2+b\,d^2\,n}{x} - \ln(c\,x^n)\,\left(\frac{b\,d^2+2\,b\,d\,e\,x+b\,e^2\,x^2}{x} - 2\,b\,e^2\,x\right) + e^2\,x\,(a-b\,n) + \frac{b\,d\,e\ln(c\,x^n)^2}{n}$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^2)/x^2,x)
```

```
[Out] log(x)*(2*a*d*e + 2*b*d*e*n) - (a*d^2 + b*d^2*n)/x - log(c*x^n)*((b*d^2 + b
*e^2*x^2 + 2*b*d*e*x)/x - 2*b*e^2*x) + e^2*x*(a - b*n) + (b*d*e*log(c*x^n)^
2)/n
```

3.15 $\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{(d+ex)^2\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx = -\frac{bn(d+4ex)^2}{4x^2}-\frac{1}{2}be^2n\log^2(x)-\frac{d^2\left(a+b\log\left(cx^n\right)\right)}{2x^2} -\frac{2de\left(a+b\log\left(cx^n\right)\right)}{x}+e^2\log(x)\left(a+b\log\left(cx^n\right)\right)$$

```
[Out] -1/4*b*n*(4*e*x+d)^2/x^2-1/2*b*e^2*n*ln(x)^2-1/2*d^2*(a+b*ln(c*x^n))/x^2-2*
d*e*(a+b*ln(c*x^n))/x+e^2*ln(x)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {45, 2372, 37, 2338}

$$\int \frac{(d+ex)^2\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx = -\frac{d^2\left(a+b\log\left(cx^n\right)\right)}{2x^2}-\frac{2de\left(a+b\log\left(cx^n\right)\right)}{x} +e^2\log(x)\left(a+b\log\left(cx^n\right)\right) -\frac{bn(d+4ex)^2}{4x^2}-\frac{1}{2}be^2n\log^2(x)$$

```
[In] Int[((d + e*x)^2*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/4*(b*n*(d + 4*e*x)^2)/x^2 - (b*e^2*n*Log[x]^2)/2 - (d^2*(a + b*Log[c*x^n
]))/(2*x^2) - (2*d*e*(a + b*Log[c*x^n]))/x + e^2*Log[x]*(a + b*Log[c*x^n])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{2x^2} - \frac{2de(a+b\log(cx^n))}{x} \\
&\quad + e^2\log(x)\,(a+b\log(cx^n)) - (bn)\int\left(-\frac{d(d+4ex)}{2x^3} + \frac{e^2\log(x)}{x}\right)dx \\
&= -\frac{d^2(a+b\log(cx^n))}{2x^2} - \frac{2de(a+b\log(cx^n))}{x} + e^2\log(x)\,(a+b\log(cx^n)) \\
&\quad + \frac{1}{2}(bdn)\int\frac{d+4ex}{x^3}\,dx - \left(be^2n\right)\int\frac{\log(x)}{x}\,dx \\
&= -\frac{bn(d+4ex)^2}{4x^2} - \frac{1}{2}be^2n\log^2(x) - \frac{d^2(a+b\log(cx^n))}{2x^2} \\
&\quad - \frac{2de(a+b\log(cx^n))}{x} + e^2\log(x)\,(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^3}\,dx = -\frac{bd^2n}{4x^2}-\frac{2bden}{x}-\frac{d^2(a+b\log(cx^n))}{2x^2}-\frac{2de(a+b\log(cx^n))}{x}+\frac{e^2(a+b\log(cx^n))^2}{2bn}$$

```
[In] Integrate[((d + e*x)^2*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/4*(b*d^2*n)/x^2 - (2*b*d*e*n)/x - (d^2*(a + b*Log[c*x^n]))/(2*x^2) - (2*
d*e*(a + b*Log[c*x^n]))/x + (e^2*(a + b*Log[c*x^n])^2)/(2*b*n)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

method	result
parallelrisch	$\frac{4\ln(x)x^2 a e^2 n+2e^2 b\ln(c x^n)^2 x^2-8x\ln(c x^n)bden-8xbde n^2-8xaden-2\ln(c x^n)b d^2 n-b d^2 n^2-2a d^2 n}{4x^2 n}$
risch	$-\frac{b(-2e^2\ln(x)x^2+4dex+d^2)\ln(x^n)}{2x^2}-\frac{-2i\ln(x)\pi b e^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic x^n)^2 x^2+4i\pi bdex\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic x^n)^2+4i\pi bdex\,\mathrm{cs}$

```
[In] int((e*x+d)^2*(a+b*ln(c*x^n))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/x^2*(4*ln(x)*x^2*a*e^2*n+2*e^2*b*ln(c*x^n)^2*x^2-8*x*ln(c*x^n)*b*d*e*n-
8*x*b*d*e*n^2-8*x*a*d*e*n-2*ln(c*x^n)*b*d^2*n-b*d^2*n^2-2*a*d^2*n)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^3}\,dx = \frac{2\,be^2nx^2\log(x)^2-bd^2n-2\,ad^2-8\,(bden+ade)x-2\,(4\,bdex+bd^2)\log(c)+2\,(2\,be^2x^2\log(c)-4\,bdenx}{4\,x^2}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*e^2*n*x^2*log(x)^2 - b*d^2*n - 2*a*d^2 - 8*(b*d*e*n + a*d*e)*x - 2
*(4*b*d*e*x + b*d^2)*log(c) + 2*(2*b*e^2*x^2*log(c) - 4*b*d*e*n*x + 2*a*e^2
*x^2 - b*d^2*n)*log(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^3}\,dx = -\frac{ad^2}{2x^2} - \frac{2ade}{x} + ae^2\log(x) + bd^2\left(-\frac{n}{4x^2} - \frac{\log(cx^n)}{2x^2}\right) + 2bde\left(-\frac{n}{x} - \frac{\log(cx^n)}{x}\right) - be^2\left(\begin{cases} -\log(c)\log(x) & \text{for } n = 0 \\ -\frac{\log(cx^n)^2}{2n} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x+d)**2*(a+b*ln(c*x**n))/x**3,x)
```

```
[Out] -a*d**2/(2*x**2) - 2*a*d*e/x + a*e**2*log(x) + b*d**2*(-n/(4*x**2) - log(c*
x**n)/(2*x**2)) + 2*b*d*e*(-n/x - log(c*x**n)/x) - b*e**2*Piecewise((-log(c
)*log(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^3}\,dx = \frac{be^2\log(cx^n)^2}{2\,n} + ae^2\log(x) - \frac{2\,bden}{x} - \frac{2\,bde\log(cx^n)}{x} - \frac{bd^2n}{4\,x^2} - \frac{2\,ade}{x} - \frac{bd^2\log(cx^n)}{2\,x^2} - \frac{ad^2}{2\,x^2}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*e^2*log(c*x^n)^2/n + a*e^2*log(x) - 2*b*d*e*n/x - 2*b*d*e*log(c*x^n)/
x - 1/4*b*d^2*n/x^2 - 2*a*d*e/x - 1/2*b*d^2*log(c*x^n)/x^2 - 1/2*a*d^2/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^3}\,dx = \frac{1}{2}\,be^2n\log(x)^2 - 2\,bden\left(\frac{\log(x)}{x} + \frac{1}{x}\right) - \frac{1}{4}\,bd^2n\left(\frac{2\,\log(x)}{x^2} + \frac{1}{x^2}\right) + be^2\log(c)\log(|x|) + ae^2\log(|x|) - \frac{2\,bde\log(c)}{x} - \frac{2\,ade}{x} - \frac{bd^2\log(c)}{2\,x^2} - \frac{ad^2}{2\,x^2}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b*e^2*n*log(x)^2 - 2*b*d*e*n*(log(x)/x + 1/x) - 1/4*b*d^2*n*(2*log(x)/x
^2 + 1/x^2) + b*e^2*log(c)*log(abs(x)) + a*e^2*log(abs(x)) - 2*b*d*e*log(c)
/x - 2*a*d*e/x - 1/2*b*d^2*log(c)/x^2 - 1/2*a*d^2/x^2
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^3} \, dx = \ln(x) \left(a\,e^2+\frac{3\,b\,e^2\,n}{2}\right) - \frac{a\,d^2+x\,(4\,a\,d\,e+4\,b\,d\,e\,n)+\frac{b\,d^2\,n}{2}}{2\,x^2} - \frac{\ln(c\,x^n)\,\left(\frac{b\,d^2}{2}+2\,b\,d\,e\,x+\frac{3\,b\,e^2\,x^2}{2}\right)}{x^2} + \frac{b\,e^2\,\ln(c\,x^n)^2}{2\,n}$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^2)/x^3,x)
```

```
[Out] log(x)*(a*e^2 + (3*b*e^2*n)/2) - (a*d^2 + x*(4*a*d*e + 4*b*d*e*n) + (b*d^2*
n)/2)/(2*x^2) - (log(c*x^n)*((b*d^2)/2 + (3*b*e^2*x^2)/2 + 2*b*d*e*x))/x^2
+ (b*e^2*log(c*x^n)^2)/(2*n)
```

3.16 $\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^4}\,dx = -\frac{bd^2n}{9x^3}-\frac{bden}{2x^2}-\frac{be^2n}{x}+\frac{be^3n\log(x)}{3d}-\frac{(d+ex)^3(a+b\log(cx^n))}{3dx^3}$$

```
[Out] -1/9*b*d^2*n/x^3-1/2*b*d*e*n/x^2-b*e^2*n/x+1/3*b*e^3*n*ln(x)/d-1/3*(e*x+d)^
3*(a+b*ln(c*x^n))/d/x^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {37, 2372, 12, 45}

$$\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^4}\,dx = -\frac{(d+ex)^3(a+b\log(cx^n))}{3dx^3}-\frac{bd^2n}{9x^3}+\frac{be^3n\log(x)}{3d}-\frac{bden}{2x^2}-\frac{be^2n}{x}$$

```
[In] Int[((d + e*x)^2*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] -1/9*(b*d^2*n)/x^3 - (b*d*e*n)/(2*x^2) - (b*e^2*n)/x + (b*e^3*n*Log[x])/(3*
d) - ((d + e*x)^3*(a + b*Log[c*x^n]))/(3*d*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(d+ex)^3(a+b\log(cx^n))}{3dx^3}-(bn)\int -\frac{(d+ex)^3}{3dx^4}\,dx\\
&= -\frac{(d+ex)^3(a+b\log(cx^n))}{3dx^3}+\frac{(bn)\int \frac{(d+ex)^3}{x^4}\,dx}{3d}\\
&= -\frac{(d+ex)^3(a+b\log(cx^n))}{3dx^3}+\frac{(bn)\int \left(\frac{d^3}{x^4}+\frac{3d^2e}{x^3}+\frac{3de^2}{x^2}+\frac{e^3}{x}\right)dx}{3d}\\
&= -\frac{bd^2n}{9x^3}-\frac{bden}{2x^2}-\frac{be^2n}{x}+\frac{be^3n\log(x)}{3d}-\frac{(d+ex)^3(a+b\log(cx^n))}{3dx^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$
\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^4}\,dx = -\frac{6a(d^2+3dex+3e^2x^2)+bn(2d^2+9dex+18e^2x^2)+6b(d^2+3dex+3e^2x^2)\log(cx^n)}{18x^3}
$$

```
[In] Integrate[((d + e*x)^2*(a + b*Log[c*x^n]))/x^4,x]
[Out] -1/18*(6*a*(d^2 + 3*d*e*x + 3*e^2*x^2) + b*n*(2*d^2 + 9*d*e*x + 18*e^2*x^2)
 + 6*b*(d^2 + 3*d*e*x + 3*e^2*x^2)*Log[c*x^n])/x^3
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.21

method	result
parallelrisch	$-\frac{18b\ln(c\,x^{n})e^{2}x^{2}+18b\,e^{2}n\,x^{2}+18a\,e^{2}x^{2}+18b\ln(c\,x^{n})dex+9bdenx+18adex+6b\ln(c\,x^{n})d^{2}+2b\,d^{2}n+6a\,d^{2}}{18x^{3}}$
risch	$-\frac{b(3e^{2}x^{2}+3dex+d^{2})\ln(x^{n})}{3x^{3}}-\frac{9i\pi b\,e^{2}x^{2}\operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^{n})^{2}+9i\pi b\,e^{2}x^{2}\operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})^{2}-9i\pi b\,e^{2}x^{2}\operatorname{csgn}(ic\,x^{n})^{3}-9i}{}$

```
[In] int((e*x+d)^2*(a+b*ln(c*x^n))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18/x^3*(18*b*ln(c*x^n)*e^2*x^2+18*b*e^2*n*x^2+18*a*e^2*x^2+18*b*ln(c*x^n
)*d*e*x+9*b*d*e*n*x+18*a*d*e*x+6*b*ln(c*x^n)*d^2+2*b*d^2*n+6*a*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.37

$$\int \frac{(d+ex)^2\,(a+b\log{(cx^n)})}{x^4}\,dx = -\frac{2\,bd^2n+6\,ad^2+18\,(be^2n+ae^2)x^2+9\,(bden+2\,ade)x+6\,(3\,be^2x^2+3\,bdex+bd^2)\log{(c)}+6\,(3\,be^2nx^2}{18\,x^3}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x^4,x, algorithm="fricas")
```

```
[Out] -1/18*(2*b*d^2*n + 6*a*d^2 + 18*(b*e^2*n + a*e^2)*x^2 + 9*(b*d*e*n + 2*a*d*
e)*x + 6*(3*b*e^2*x^2 + 3*b*d*e*x + b*d^2)*log(c) + 6*(3*b*e^2*n*x^2 + 3*b*
d*e*n*x + b*d^2*n)*log(x))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)^2\,(a+b\log{(cx^n)})}{x^4}\,dx = -\frac{ad^2}{3x^3}-\frac{ade}{x^2}-\frac{ae^2}{x}-\frac{bd^2n}{9x^3}-\frac{bd^2\log{(cx^n)}}{3x^3} - \frac{bden}{2x^2}-\frac{bde\log{(cx^n)}}{x^2}-\frac{be^2n}{x}-\frac{be^2\log{(cx^n)}}{x}$$

```
[In] integrate((e*x+d)**2*(a+b*ln(c*x**n))/x**4,x)
```

```
[Out] -a*d**2/(3*x**3) - a*d*e/x**2 - a*e**2/x - b*d**2*n/(9*x**3) - b*d**2*log(c
*x**n)/(3*x**3) - b*d*e*n/(2*x**2) - b*d*e*log(c*x**n)/x**2 - b*e**2*n/x -
b*e**2*log(c*x**n)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^4}\,dx = -\frac{be^2n}{x} - \frac{be^2\log(cx^n)}{x} - \frac{bden}{2\,x^2} - \frac{ae^2}{x} - \frac{bde\log(cx^n)}{x^2} - \frac{bd^2n}{9\,x^3} - \frac{ade}{x^2} - \frac{bd^2\log(cx^n)}{3\,x^3} - \frac{ad^2}{3\,x^3}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x^4,x, algorithm="maxima")
```

```
[Out] -b*e^2*n/x - b*e^2*log(c*x^n)/x - 1/2*b*d*e*n/x^2 - a*e^2/x - b*d*e*log(c*x
^n)/x^2 - 1/9*b*d^2*n/x^3 - a*d*e/x^2 - 1/3*b*d^2*log(c*x^n)/x^3 - 1/3*a*d^
2/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^4}\,dx = -\frac{(3\,be^2nx^2+3\,bdenx+bd^2n)\log(x)}{3\,x^3} - \frac{18\,be^2nx^2+18\,be^2x^2\log(c)+9\,bdenx+18\,ae^2x^2+18\,bdex\log(c)+2\,bd^2n+18\,adex+6\,bd^2\log(c)+}{18\,x^3}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b*e^2*n*x^2 + 3*b*d*e*n*x + b*d^2*n)*log(x)/x^3 - 1/18*(18*b*e^2*n*
x^2 + 18*b*e^2*x^2*log(c) + 9*b*d*e*n*x + 18*a*e^2*x^2 + 18*b*d*e*x*log(c)
+ 2*b*d^2*n + 18*a*d*e*x + 6*b*d^2*log(c) + 6*a*d^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^4}\,dx = -\frac{x^2\,(3\,a\,e^2+3\,b\,e^2\,n)+a\,d^2+x\,\left(3\,a\,d\,e+\frac{3\,b\,d\,e\,n}{2}\right)+\frac{b\,d^2\,n}{3}}{3\,x^3} - \frac{\ln(c\,x^n)\,\left(\frac{b\,d^2}{3}+b\,d\,e\,x+b\,e^2\,x^2\right)}{x^3}$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^2)/x^4,x)
```

```
[Out] - (x^2*(3*a*e^2 + 3*b*e^2*n) + a*d^2 + x*(3*a*d*e + (3*b*d*e*n)/2) + (b*d^2
*n)/3)/(3*x^3) - (log(c*x^n)*((b*d^2)/3 + b*e^2*x^2 + b*d*e*x))/x^3
```

3.17 $\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^5}\,dx = -\frac{bd^2n}{16x^4} - \frac{2bden}{9x^3} - \frac{be^2n}{4x^2} - \frac{d^2(a+b\log(cx^n))}{4x^4} - \frac{2de(a+b\log(cx^n))}{3x^3} - \frac{e^2(a+b\log(cx^n))}{2x^2}$$

```
[Out] -1/16*b*d^2*n/x^4-2/9*b*d*e*n/x^3-1/4*b*e^2*n/x^2-1/4*d^2*(a+b*ln(c*x^n))/x
^4-2/3*d*e*(a+b*ln(c*x^n))/x^3-1/2*e^2*(a+b*ln(c*x^n))/x^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {45, 2372, 12, 14}

$$\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^5}\,dx = -\frac{d^2(a+b\log(cx^n))}{4x^4} - \frac{2de(a+b\log(cx^n))}{3x^3} - \frac{e^2(a+b\log(cx^n))}{2x^2} - \frac{bd^2n}{16x^4} - \frac{2bden}{9x^3} - \frac{be^2n}{4x^2}$$

```
[In] Int[((d + e*x)^2*(a + b*Log[c*x^n]))/x^5,x]
```

```
[Out] -1/16*(b*d^2*n)/x^4 - (2*b*d*e*n)/(9*x^3) - (b*e^2*n)/(4*x^2) - (d^2*(a + b
*Log[c*x^n]))/(4*x^4) - (2*d*e*(a + b*Log[c*x^n]))/(3*x^3) - (e^2*(a + b*Lo
g[c*x^n]))/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{4x^4} - \frac{2de(a+b\log(cx^n))}{3x^3} - \frac{e^2(a+b\log(cx^n))}{2x^2} - (bn)\int \frac{-3d^2-8dex-6e^2x^2}{12x^5}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{4x^4} - \frac{2de(a+b\log(cx^n))}{3x^3} - \frac{e^2(a+b\log(cx^n))}{2x^2} - \frac{1}{12}(bn)\int \frac{-3d^2-8dex-6e^2x^2}{x^5}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{4x^4} - \frac{2de(a+b\log(cx^n))}{3x^3} - \frac{e^2(a+b\log(cx^n))}{2x^2} - \frac{1}{12}(bn)\int \left(-\frac{3d^2}{x^5} - \frac{8de}{x^4} - \frac{6e^2}{x^3}\right)dx \\
&= -\frac{bd^2n}{16x^4} - \frac{2bden}{9x^3} - \frac{be^2n}{4x^2} - \frac{d^2(a+b\log(cx^n))}{4x^4} - \frac{2de(a+b\log(cx^n))}{3x^3} - \frac{e^2(a+b\log(cx^n))}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^2 (a+b\log (cx^n))}{x^5} \, dx = -\frac{12a(3d^2+8dex+6e^2x^2)+bn(9d^2+32dex+36e^2x^2)+12b(3d^2+8dex+6e^2x^2)\log (cx^n)}{144x^4}$$

```
[In] Integrate[((d + e*x)^2*(a + b*Log[c*x^n]))/x^5,x]
```

```
[Out] -1/144*(12*a*(3*d^2 + 8*d*e*x + 6*e^2*x^2) + b*n*(9*d^2 + 32*d*e*x + 36*e^2
*x^2) + 12*b*(3*d^2 + 8*d*e*x + 6*e^2*x^2)*Log[c*x^n])/x^4
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

method	result
parallelrisch	$-\frac{72b\ln(c\,x^n)e^2x^2+36b\,e^2n\,x^2+72a\,e^2x^2+96b\ln(c\,x^n)dex+32bdenx+96adex+36b\ln(c\,x^n)d^2+9b\,d^2n+36a\,d^2}{144x^4}$
risch	$-\frac{b(6e^2x^2+8dex+3d^2)\ln(x^n)}{12x^4} - \frac{18i\pi b\,d^2\operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^n)^2-36i\pi b\,e^2x^2\operatorname{csgn}(ic\,x^n)^3-48i\pi bdex\operatorname{csgn}(ic\,x^n)^3+48i\pi bdex\ \text{cs}}{}$

```
[In] int((e*x+d)^2*(a+b*ln(c*x^n))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/144/x^4*(72*b*ln(c*x^n)*e^2*x^2+36*b*e^2*n*x^2+72*a*e^2*x^2+96*b*ln(c*x^
n)*d*e*x+32*b*d*e*n*x+96*a*d*e*x+36*b*ln(c*x^n)*d^2+9*b*d^2*n+36*a*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^2 (a+b\log (cx^n))}{x^5} \, dx = -\frac{9\,bd^2n+36\,ad^2+36\,(be^2n+2\,ae^2)x^2+32\,(bden+3\,ade)x+12\,(6\,be^2x^2+8\,bdex+3\,bd^2)\log (c)+12\,(6}{144\,x^4}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x^5,x, algorithm="fricas")
```

```
[Out] -1/144*(9*b*d^2*n + 36*a*d^2 + 36*(b*e^2*n + 2*a*e^2)*x^2 + 32*(b*d*e*n + 3
*a*d*e)*x + 12*(6*b*e^2*x^2 + 8*b*d*e*x + 3*b*d^2)*log(c) + 12*(6*b*e^2*n*x
^2 + 8*b*d*e*n*x + 3*b*d^2*n)*log(x))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^5}\,dx = -\frac{ad^2}{4x^4} - \frac{2ade}{3x^3} - \frac{ae^2}{2x^2} - \frac{bd^2n}{16x^4} - \frac{bd^2\log(cx^n)}{4x^4} - \frac{2bden}{9x^3} - \frac{2bde\log(cx^n)}{3x^3} - \frac{be^2n}{4x^2} - \frac{be^2\log(cx^n)}{2x^2}$$

```
[In] integrate((e*x+d)**2*(a+b*ln(c*x**n))/x**5,x)
```

```
[Out] -a*d**2/(4*x**4) - 2*a*d*e/(3*x**3) - a*e**2/(2*x**2) - b*d**2*n/(16*x**4)
- b*d**2*log(c*x**n)/(4*x**4) - 2*b*d*e*n/(9*x**3) - 2*b*d*e*log(c*x**n)/(3
*x**3) - b*e**2*n/(4*x**2) - b*e**2*log(c*x**n)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^5}\,dx = -\frac{be^2n}{4\,x^2} - \frac{be^2\log(cx^n)}{2\,x^2} - \frac{2\,bden}{9\,x^3} - \frac{ae^2}{2\,x^2} - \frac{2\,bde\log(cx^n)}{3\,x^3} - \frac{bd^2n}{16\,x^4} - \frac{2\,ade}{3\,x^3} - \frac{bd^2\log(cx^n)}{4\,x^4} - \frac{ad^2}{4\,x^4}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*b*e^2*n/x^2 - 1/2*b*e^2*log(c*x^n)/x^2 - 2/9*b*d*e*n/x^3 - 1/2*a*e^2/x
^2 - 2/3*b*d*e*log(c*x^n)/x^3 - 1/16*b*d^2*n/x^4 - 2/3*a*d*e/x^3 - 1/4*b*d^
2*log(c*x^n)/x^4 - 1/4*a*d^2/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^5}\,dx = -\frac{(6\,be^2nx^2 + 8\,bdenx + 3\,bd^2n)\log(x)}{12\,x^4} - \frac{36\,be^2nx^2 + 72\,be^2x^2\log(c) + 32\,bdenx + 72\,ae^2x^2 + 96\,bdex\log(c) + 9\,bd^2n + 96\,adex + 36\,bd^2\log(c)}{144\,x^4}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x^5,x, algorithm="giac")
```

```
[Out] -1/12*(6*b*e^2*n*x^2 + 8*b*d*e*n*x + 3*b*d^2*n)*log(x)/x^4 - 1/144*(36*b*e^
2*n*x^2 + 72*b*e^2*x^2*log(c) + 32*b*d*e*n*x + 72*a*e^2*x^2 + 96*b*d*e*x*lo
g(c) + 9*b*d^2*n + 96*a*d*e*x + 36*b*d^2*log(c) + 36*a*d^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)^2\left(a+b\log\left(cx^n\right)\right)}{x^5}\,dx$$
$$= -\frac{x^2\left(6\,a\,e^2+3\,b\,e^2\,n\right)+3\,a\,d^2+x\left(8\,a\,d\,e+\frac{8\,b\,d\,e\,n}{3}\right)+\frac{3\,b\,d^2\,n}{4}}{12\,x^4}$$
$$-\frac{\ln\left(c\,x^n\right)\left(\frac{b\,d^2}{4}+\frac{2\,b\,d\,e\,x}{3}+\frac{b\,e^2\,x^2}{2}\right)}{x^4}$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^2)/x^5,x)
```

```
[Out] - (x^2*(6*a*e^2 + 3*b*e^2*n) + 3*a*d^2 + x*(8*a*d*e + (8*b*d*e*n)/3) + (3*b
*d^2*n)/4)/(12*x^4) - (log(c*x^n)*((b*d^2)/4 + (b*e^2*x^2)/2 + (2*b*d*e*x)/
3))/x^4
```

3.18 $\int \frac{(d+ex)^2(a+b\log(cx^n))}{x^6} dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^6} dx = -\frac{bd^2n}{25x^5} - \frac{bden}{8x^4} - \frac{be^2n}{9x^3} - \frac{d^2(a+b\log(cx^n))}{5x^5} - \frac{de(a+b\log(cx^n))}{2x^4} - \frac{e^2(a+b\log(cx^n))}{3x^3}$$

```
[Out] -1/25*b*d^2*n/x^5-1/8*b*d*e*n/x^4-1/9*b*e^2*n/x^3-1/5*d^2*(a+b*ln(c*x^n))/x
^5-1/2*d*e*(a+b*ln(c*x^n))/x^4-1/3*e^2*(a+b*ln(c*x^n))/x^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {45, 2372, 12, 14}

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^6} dx = -\frac{d^2(a+b\log(cx^n))}{5x^5} - \frac{de(a+b\log(cx^n))}{2x^4} - \frac{e^2(a+b\log(cx^n))}{3x^3} - \frac{bd^2n}{25x^5} - \frac{bden}{8x^4} - \frac{be^2n}{9x^3}$$

```
[In] Int[((d + e*x)^2*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] -1/25*(b*d^2*n)/x^5 - (b*d*e*n)/(8*x^4) - (b*e^2*n)/(9*x^3) - (d^2*(a + b*L
og[c*x^n]))/(5*x^5) - (d*e*(a + b*Log[c*x^n]))/(2*x^4) - (e^2*(a + b*Log[c*
x^n]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{5x^5}-\frac{de(a+b\log(cx^n))}{2x^4} \\
&\quad -\frac{e^2(a+b\log(cx^n))}{3x^3}-(bn)\int\frac{-6d^2-15dex-10e^2x^2}{30x^6}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{5x^5}-\frac{de(a+b\log(cx^n))}{2x^4} \\
&\quad -\frac{e^2(a+b\log(cx^n))}{3x^3}-\frac{1}{30}(bn)\int\frac{-6d^2-15dex-10e^2x^2}{x^6}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{5x^5}-\frac{de(a+b\log(cx^n))}{2x^4} \\
&\quad -\frac{e^2(a+b\log(cx^n))}{3x^3}-\frac{1}{30}(bn)\int\left(-\frac{6d^2}{x^6}-\frac{15de}{x^5}-\frac{10e^2}{x^4}\right)dx \\
&= -\frac{bd^2n}{25x^5}-\frac{bden}{8x^4}-\frac{be^2n}{9x^3}-\frac{d^2(a+b\log(cx^n))}{5x^5}-\frac{de(a+b\log(cx^n))}{2x^4}-\frac{e^2(a+b\log(cx^n))}{3x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^6}\,dx = -\frac{60a(6d^2+15dex+10e^2x^2)+bn(72d^2+225dex+200e^2x^2)+60b(6d^2+15dex+10e^2x^2)\log(cx^n)}{1800x^5}$$

```
[In] Integrate[((d + e*x)^2*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] -1/1800*(60*a*(6*d^2 + 15*d*e*x + 10*e^2*x^2) + b*n*(72*d^2 + 225*d*e*x + 2
00*e^2*x^2) + 60*b*(6*d^2 + 15*d*e*x + 10*e^2*x^2)*Log[c*x^n])/x^5
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

method	result
parallelrisch	$-\frac{600b\ln(cx^n)e^2x^2+200b\,e^2n\,x^2+600a\,e^2x^2+900b\ln(cx^n)dex+225bdenx+900adex+360b\ln(cx^n)d^2+72b\,d^2n+360a\,d^2}{1800x^5}$
risch	$-\frac{b(10e^2x^2+15dex+6d^2)\ln(x^n)}{30x^5}-\frac{-180i\pi b\,d^2\operatorname{csgn}(icx^n)^3-450i\pi bdex\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)+180i\pi b\,d^2\operatorname{csgn}(ic)\,c$

```
[In] int((e*x+d)^2*(a+b*ln(c*x^n))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/1800/x^5*(600*b*ln(c*x^n)*e^2*x^2+200*b*e^2*n*x^2+600*a*e^2*x^2+900*b*ln
(c*x^n)*d*e*x+225*b*d*e*n*x+900*a*d*e*x+360*b*ln(c*x^n)*d^2+72*b*d^2*n+360*
a*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^6}\,dx = -\frac{72\,bd^2n+360\,ad^2+200\,(be^2n+3\,ae^2)x^2+225\,(bden+4\,ade)x+60\,(10\,be^2x^2+15\,bdex+6\,bd^2)\log(c}{1800\,x^5}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x^6,x, algorithm="fricas")
```

```
[Out] -1/1800*(72*b*d^2*n + 360*a*d^2 + 200*(b*e^2*n + 3*a*e^2)*x^2 + 225*(b*d*e*
n + 4*a*d*e)*x + 60*(10*b*e^2*x^2 + 15*b*d*e*x + 6*b*d^2)*log(c) + 60*(10*b
*e^2*n*x^2 + 15*b*d*e*n*x + 6*b*d^2*n)*log(x))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^6}\,dx = -\frac{ad^2}{5x^5} - \frac{ade}{2x^4} - \frac{ae^2}{3x^3} - \frac{bd^2n}{25x^5} - \frac{bd^2\log(cx^n)}{5x^5} - \frac{bden}{8x^4} - \frac{bde\log(cx^n)}{2x^4} - \frac{be^2n}{9x^3} - \frac{be^2\log(cx^n)}{3x^3}$$

```
[In] integrate((e*x+d)**2*(a+b*ln(c*x**n))/x**6,x)
```

```
[Out] -a*d**2/(5*x**5) - a*d*e/(2*x**4) - a*e**2/(3*x**3) - b*d**2*n/(25*x**5) -
b*d**2*log(c*x**n)/(5*x**5) - b*d*e*n/(8*x**4) - b*d*e*log(c*x**n)/(2*x**4)
 - b*e**2*n/(9*x**3) - b*e**2*log(c*x**n)/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^6}\,dx = -\frac{be^2n}{9\,x^3} - \frac{be^2\log(cx^n)}{3\,x^3} - \frac{bden}{8\,x^4} - \frac{ae^2}{3\,x^3} - \frac{bde\log(cx^n)}{2\,x^4} - \frac{bd^2n}{25\,x^5} - \frac{ade}{2\,x^4} - \frac{bd^2\log(cx^n)}{5\,x^5} - \frac{ad^2}{5\,x^5}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x^6,x, algorithm="maxima")
```

```
[Out] -1/9*b*e^2*n/x^3 - 1/3*b*e^2*log(c*x^n)/x^3 - 1/8*b*d*e*n/x^4 - 1/3*a*e^2/x
^3 - 1/2*b*d*e*log(c*x^n)/x^4 - 1/25*b*d^2*n/x^5 - 1/2*a*d*e/x^4 - 1/5*b*d^
2*log(c*x^n)/x^5 - 1/5*a*d^2/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))}{x^6}\,dx = -\frac{(10\,be^2nx^2 + 15\,bdenx + 6\,bd^2n)\log(x)}{30\,x^5} - \frac{200\,be^2nx^2 + 600\,be^2x^2\log(c) + 225\,bdenx + 600\,ae^2x^2 + 900\,bdex\log(c) + 72\,bd^2n + 900\,adex + 360\,bd}{1800\,x^5}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))/x^6,x, algorithm="giac")
```

```
[Out] -1/30*(10*b*e^2*n*x^2 + 15*b*d*e*n*x + 6*b*d^2*n)*log(x)/x^5 - 1/1800*(200*
b*e^2*n*x^2 + 600*b*e^2*x^2*log(c) + 225*b*d*e*n*x + 600*a*e^2*x^2 + 900*b*
d*e*x*log(c) + 72*b*d^2*n + 900*a*d*e*x + 360*b*d^2*log(c) + 360*a*d^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)^2\,(a+b\log(cx^n))}{x^6}\,dx$$
$$=-\frac{x^2\left(10\,a\,e^2+\frac{10\,b\,e^2\,n}{3}\right)+6\,a\,d^2+x\left(15\,a\,d\,e+\frac{15\,b\,d\,e\,n}{4}\right)+\frac{6\,b\,d^2\,n}{5}}{30\,x^5}-\frac{\ln(c\,x^n)\,\left(\frac{b\,d^2}{5}+\frac{b\,d\,e\,x}{2}+\frac{b\,e^2\,x^2}{3}\right)}{x^5}$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^2)/x^6,x)
```

```
[Out] - (x^2*(10*a*e^2 + (10*b*e^2*n)/3) + 6*a*d^2 + x*(15*a*d*e + (15*b*d*e*n)/4
) + (6*b*d^2*n)/5)/(30*x^5) - (log(c*x^n)*((b*d^2)/5 + (b*e^2*x^2)/3 + (b*d
*e*x)/2))/x^5
```

3.19 $\int x^3(d+ex)^3 (a+b\log(cx^n))\, dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int x^3(d+ex)^3 (a+b\log(cx^n))\, dx = -\frac{1}{16}bd^3nx^4-\frac{3}{25}bd^2enx^5-\frac{1}{12}bde^2nx^6-\frac{1}{49}be^3nx^7+\frac{1}{140}\left(35d^3x^4+84d^2ex^5+70de^2x^6+20e^3x^7\right)(a+b\log(cx^n))$$

```
[Out] -1/16*b*d^3*n*x^4-3/25*b*d^2*e*n*x^5-1/12*b*d*e^2*n*x^6-1/49*b*e^3*n*x^7+1/
140*(20*e^3*x^7+70*d*e^2*x^6+84*d^2*e*x^5+35*d^3*x^4)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {45, 2371, 12, 14}

$$\int x^3(d+ex)^3 (a+b\log(cx^n))\, dx = \frac{1}{140}\left(35d^3x^4+84d^2ex^5+70de^2x^6+20e^3x^7\right)(a+b\log(cx^n))-\frac{1}{16}bd^3nx^4-\frac{3}{25}bd^2enx^5-\frac{1}{12}bde^2nx^6-\frac{1}{49}be^3nx^7$$

```
[In] Int[x^3*(d + e*x)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/16*(b*d^3*n*x^4) - (3*b*d^2*e*n*x^5)/25 - (b*d*e^2*n*x^6)/12 - (b*e^3*n*
x^7)/49 + ((35*d^3*x^4 + 84*d^2*e*x^5 + 70*d*e^2*x^6 + 20*e^3*x^7)*(a + b*L
og[c*x^n]))/140
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{140}\left(35d^3x^4+84d^2ex^5+70de^2x^6+20e^3x^7\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int \frac{1}{140}x^3\left(35d^3+84d^2ex+70de^2x^2+20e^3x^3\right)\,dx \\
&= \frac{1}{140}\left(35d^3x^4+84d^2ex^5+70de^2x^6+20e^3x^7\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{140}(bn)\int x^3\left(35d^3+84d^2ex+70de^2x^2+20e^3x^3\right)\,dx \\
&= \frac{1}{140}\left(35d^3x^4+84d^2ex^5+70de^2x^6+20e^3x^7\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{140}(bn)\int \left(35d^3x^3+84d^2ex^4+70de^2x^5+20e^3x^6\right)\,dx \\
&= -\frac{1}{16}bd^3nx^4-\frac{3}{25}bd^2enx^5-\frac{1}{12}bde^2nx^6-\frac{1}{49}be^3nx^7 \\
&\quad +\frac{1}{140}\left(35d^3x^4+84d^2ex^5+70de^2x^6+20e^3x^7\right)(a+b\log(cx^n))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.33

$$\int x^3(d+ex)^3\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{16}bd^3nx^4-\frac{3}{25}bd^2enx^5-\frac{1}{12}bde^2nx^6-\frac{1}{49}be^3nx^7+\frac{1}{4}d^3x^4(a+b\log\left(cx^n\right))+\frac{3}{5}d^2ex^5(a+b\log\left(cx^n\right))+\frac{1}{2}de^2x^6(a+b\log\left(cx^n\right))+\frac{1}{7}e^3x^7(a+b\log\left(cx^n\right))$$

```
[In] Integrate[x^3*(d + e*x)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/16*(b*d^3*n*x^4) - (3*b*d^2*e*n*x^5)/25 - (b*d*e^2*n*x^6)/12 - (b*e^3*n*
x^7)/49 + (d^3*x^4*(a + b*Log[c*x^n]))/4 + (3*d^2*e*x^5*(a + b*Log[c*x^n]))
/5 + (d*e^2*x^6*(a + b*Log[c*x^n]))/2 + (e^3*x^7*(a + b*Log[c*x^n]))/7
```

Maple [A] (verified)

Time = 11.00 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.44

method	result
parallelrisch	$\frac{x^7\ln(cx^n)be^3}{7}-\frac{be^3nx^7}{49}+\frac{ae^3x^7}{7}+\frac{x^6\ln(cx^n)bde^2}{2}-\frac{bde^2nx^6}{12}+\frac{ade^2x^6}{2}+\frac{3x^5\ln(cx^n)bd^2e}{5}-\frac{3bd^2enx^5}{25}+$
risch	$\frac{ae^3x^7}{7}+\frac{ad^3x^4}{4}+\frac{3ad^2ex^5}{5}+\frac{ade^2x^6}{2}+\frac{i\pi be^3x^7\,\mathrm{csgn}(ic)\,\mathrm{csgn}(icx^n)^2}{14}+\frac{i\pi be^3x^7\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2}{14}-$

```
[In] int(x^3*(e*x+d)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*x^7*ln(c*x^n)*b*e^3-1/49*b*e^3*n*x^7+1/7*a*e^3*x^7+1/2*x^6*ln(c*x^n)*b*
d*e^2-1/12*b*d*e^2*n*x^6+1/2*a*d*e^2*x^6+3/5*x^5*ln(c*x^n)*b*d^2*e-3/25*b*d
^2*e*n*x^5+3/5*a*d^2*e*x^5+1/4*x^4*ln(c*x^n)*b*d^3-1/16*b*d^3*n*x^4+1/4*a*d
^3*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.67

$$\int x^3(d+ex)^3\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{49}\left(be^3n-7\,ae^3\right)x^7-\frac{1}{12}\left(bde^2n-6\,ade^2\right)x^6-\frac{3}{25}\left(bd^2en-5\,ad^2e\right)x^5-\frac{1}{16}\left(bd^3n-4\,ad^3\right)x^4+\frac{1}{140}\left(20\,be^3x^7+70\,bde^2x^6+84\,bd^2ex^5+35\,bd^3x^4\right)\log\left(c\right)+\frac{1}{140}\left(20\,be^3nx^7+70\,bde^2nx^6+84\,bd^2enx^5+35\,bd^3nx^4\right)\log\left(x\right)$$

```
[In] integrate(x^3*(e*x+d)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/49*(b*e^3*n - 7*a*e^3)*x^7 - 1/12*(b*d*e^2*n - 6*a*d*e^2)*x^6 - 3/25*(b*
d^2*e*n - 5*a*d^2*e)*x^5 - 1/16*(b*d^3*n - 4*a*d^3)*x^4 + 1/140*(20*b*e^3*x
^7 + 70*b*d*e^2*x^6 + 84*b*d^2*e*x^5 + 35*b*d^3*x^4)*log(c) + 1/140*(20*b*e
^3*n*x^7 + 70*b*d*e^2*n*x^6 + 84*b*d^2*e*n*x^5 + 35*b*d^3*n*x^4)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.70

$$\int x^3(d+ex)^3\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{ad^3x^4}{4}+\frac{3ad^2ex^5}{5}+\frac{ade^2x^6}{2}+\frac{ae^3x^7}{7}-\frac{bd^3nx^4}{16}+\frac{bd^3x^4\log\left(cx^n\right)}{4}-\frac{3bd^2enx^5}{25}+\frac{3bd^2ex^5\log\left(cx^n\right)}{5}-\frac{bde^2nx^6}{12}+\frac{bde^2x^6\log\left(cx^n\right)}{2}-\frac{be^3nx^7}{49}+\frac{be^3x^7\log\left(cx^n\right)}{7}$$

```
[In] integrate(x**3*(e*x+d)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**3*x**4/4 + 3*a*d**2*e*x**5/5 + a*d*e**2*x**6/2 + a*e**3*x**7/7 - b*d**
3*n*x**4/16 + b*d**3*x**4*log(c*x**n)/4 - 3*b*d**2*e*n*x**5/25 + 3*b*d**2*e
*x**5*log(c*x**n)/5 - b*d*e**2*n*x**6/12 + b*d*e**2*x**6*log(c*x**n)/2 - b*
e**3*n*x**7/49 + b*e**3*x**7*log(c*x**n)/7
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.43

$$\int x^3(d+ex)^3\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{49}\,be^3nx^7+\frac{1}{7}\,be^3x^7\log\left(cx^n\right)-\frac{1}{12}\,bde^2nx^6+\frac{1}{7}\,ae^3x^7+\frac{1}{2}\,bde^2x^6\log\left(cx^n\right)-\frac{3}{25}\,bd^2enx^5+\frac{1}{2}\,ade^2x^6+\frac{3}{5}\,bd^2ex^5\log\left(cx^n\right)-\frac{1}{16}\,bd^3nx^4+\frac{3}{5}\,ad^2ex^5+\frac{1}{4}\,bd^3x^4\log\left(cx^n\right)+\frac{1}{4}\,ad^3x^4$$

```
[In] integrate(x^3*(e*x+d)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/49*b*e^3*n*x^7 + 1/7*b*e^3*x^7*log(c*x^n) - 1/12*b*d*e^2*n*x^6 + 1/7*a*e
^3*x^7 + 1/2*b*d*e^2*x^6*log(c*x^n) - 3/25*b*d^2*e*n*x^5 + 1/2*a*d*e^2*x^6
+ 3/5*b*d^2*e*x^5*log(c*x^n) - 1/16*b*d^3*n*x^4 + 3/5*a*d^2*e*x^5 + 1/4*b*d
^3*x^4*log(c*x^n) + 1/4*a*d^3*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.77

$$\int x^3(d+ex)^3(a+b\log(cx^n))\,dx = \frac{1}{7}be^3nx^7\log(x) - \frac{1}{49}be^3nx^7 + \frac{1}{7}be^3x^7\log(c) + \frac{1}{2}bde^2nx^6\log(x) - \frac{1}{12}bde^2nx^6 + \frac{1}{7}ae^3x^7 + \frac{1}{2}bde^2x^6\log(c) + \frac{3}{5}bd^2enx^5\log(x) - \frac{3}{25}bd^2enx^5 + \frac{1}{2}ade^2x^6 + \frac{3}{5}bd^2ex^5\log(c) + \frac{1}{4}bd^3nx^4\log(x) - \frac{1}{16}bd^3nx^4 + \frac{3}{5}ad^2ex^5 + \frac{1}{4}bd^3x^4\log(c) + \frac{1}{4}ad^3x^4$$

```
[In] integrate(x^3*(e*x+d)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/7*b*e^3*n*x^7*log(x) - 1/49*b*e^3*n*x^7 + 1/7*b*e^3*x^7*log(c) + 1/2*b*d*
e^2*n*x^6*log(x) - 1/12*b*d*e^2*n*x^6 + 1/7*a*e^3*x^7 + 1/2*b*d*e^2*x^6*log
(c) + 3/5*b*d^2*e*n*x^5*log(x) - 3/25*b*d^2*e*n*x^5 + 1/2*a*d*e^2*x^6 + 3/5
*b*d^2*e*x^5*log(c) + 1/4*b*d^3*n*x^4*log(x) - 1/16*b*d^3*n*x^4 + 3/5*a*d^2
*e*x^5 + 1/4*b*d^3*x^4*log(c) + 1/4*a*d^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13

$$\int x^3(d+ex)^3(a+b\log(cx^n))\,dx = \ln(c\,x^n)\left(\frac{b\,d^3\,x^4}{4} + \frac{3\,b\,d^2\,e\,x^5}{5} + \frac{b\,d\,e^2\,x^6}{2} + \frac{b\,e^3\,x^7}{7}\right) + \frac{d^3\,x^4\,(4\,a-b\,n)}{16} + \frac{e^3\,x^7\,(7\,a-b\,n)}{49} + \frac{3\,d^2\,e\,x^5\,(5\,a-b\,n)}{25} + \frac{d\,e^2\,x^6\,(6\,a-b\,n)}{12}$$

```
[In] int(x^3*(a + b*log(c*x^n))*(d + e*x)^3,x)
```

```
[Out] log(c*x^n)*((b*d^3*x^4)/4 + (b*e^3*x^7)/7 + (3*b*d^2*e*x^5)/5 + (b*d*e^2*x^
6)/2) + (d^3*x^4*(4*a - b*n))/16 + (e^3*x^7*(7*a - b*n))/49 + (3*d^2*e*x^5*
(5*a - b*n))/25 + (d*e^2*x^6*(6*a - b*n))/12
```

3.20 $\int x^2(d+ex)^3 (a+b\log(cx^n)) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int x^2(d+ex)^3 (a+b\log(cx^n)) \, dx = -\frac{1}{9}bd^3nx^3 - \frac{3}{16}bd^2enx^4 - \frac{3}{25}bde^2nx^5 - \frac{1}{36}be^3nx^6 + \frac{1}{60}\left(20d^3x^3+45d^2ex^4+36de^2x^5+10e^3x^6\right)(a + b\log(cx^n))$$

```
[Out] -1/9*b*d^3*n*x^3-3/16*b*d^2*e*n*x^4-3/25*b*d*e^2*n*x^5-1/36*b*e^3*n*x^6+1/6
0*(10*e^3*x^6+36*d*e^2*x^5+45*d^2*e*x^4+20*d^3*x^3)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {45, 2371, 12, 14}

$$\int x^2(d+ex)^3 (a+b\log(cx^n)) \, dx = \frac{1}{60}\left(20d^3x^3+45d^2ex^4+36de^2x^5+10e^3x^6\right)(a+b\log(cx^n)) - \frac{1}{9}bd^3nx^3 - \frac{3}{16}bd^2enx^4 - \frac{3}{25}bde^2nx^5 - \frac{1}{36}be^3nx^6$$

```
[In] Int[x^2*(d + e*x)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/9*(b*d^3*n*x^3) - (3*b*d^2*e*n*x^4)/16 - (3*b*d*e^2*n*x^5)/25 - (b*e^3*n
*x^6)/36 + ((20*d^3*x^3 + 45*d^2*e*x^4 + 36*d*e^2*x^5 + 10*e^3*x^6)*(a + b*
Log[c*x^n]))/60
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{60}\left(20d^3x^3+45d^2ex^4+36de^2x^5+10e^3x^6\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int \frac{1}{60}x^2\left(20d^3+45d^2ex+36de^2x^2+10e^3x^3\right)\,dx \\
&= \frac{1}{60}\left(20d^3x^3+45d^2ex^4+36de^2x^5+10e^3x^6\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{60}(bn)\int x^2\left(20d^3+45d^2ex+36de^2x^2+10e^3x^3\right)\,dx \\
&= \frac{1}{60}\left(20d^3x^3+45d^2ex^4+36de^2x^5+10e^3x^6\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{60}(bn)\int \left(20d^3x^2+45d^2ex^3+36de^2x^4+10e^3x^5\right)\,dx \\
&= -\frac{1}{9}bd^3nx^3-\frac{3}{16}bd^2enx^4-\frac{3}{25}bde^2nx^5-\frac{1}{36}be^3nx^6 \\
&\quad +\frac{1}{60}\left(20d^3x^3+45d^2ex^4+36de^2x^5+10e^3x^6\right)(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.33

$$\int x^2(d+ex)^3(a+b\log(cx^n))\,dx=-\frac{1}{9}bd^3nx^3-\frac{3}{16}bd^2enx^4-\frac{3}{25}bde^2nx^5-\frac{1}{36}be^3nx^6+\frac{1}{3}d^3x^3(a+b\log(cx^n))+\frac{3}{4}d^2ex^4(a+b\log(cx^n))+\frac{3}{5}de^2x^5(a+b\log(cx^n))+\frac{1}{6}e^3x^6(a+b\log(cx^n))$$

```
[In] Integrate[x^2*(d + e*x)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/9*(b*d^3*n*x^3) - (3*b*d^2*e*n*x^4)/16 - (3*b*d*e^2*n*x^5)/25 - (b*e^3*n
*x^6)/36 + (d^3*x^3*(a + b*Log[c*x^n]))/3 + (3*d^2*e*x^4*(a + b*Log[c*x^n])
)/4 + (3*d*e^2*x^5*(a + b*Log[c*x^n]))/5 + (e^3*x^6*(a + b*Log[c*x^n]))/6
```

Maple [A] (verified)

Time = 5.90 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.44

method	result
parallelrisch	$\frac{x^6 b\ln(cx^n)e^3}{6}-\frac{be^3nx^6}{36}+\frac{x^6ae^3}{6}+\frac{3x^5b\ln(cx^n)de^2}{5}-\frac{3bde^2nx^5}{25}+\frac{3x^5ade^2}{5}+\frac{3x^4b\ln(cx^n)d^2e}{4}-\frac{3bd^2enx^4}{16}$
risch	$\frac{ad^3x^3}{3}+\frac{x^6ae^3}{6}-\frac{i\pi bd^3x^3\operatorname{csgn}(icx^n)^3}{6}-\frac{i\pi be^3x^6\operatorname{csgn}(icx^n)^3}{12}+\frac{3x^5ade^2}{5}+\frac{3x^4ad^2e}{4}-\frac{3i\pi bde^2x^5\operatorname{csgn}(ic)\operatorname{csgn}}{10}$

```
[In] int(x^2*(e*x+d)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^6*b*ln(c*x^n)*e^3-1/36*b*e^3*n*x^6+1/6*x^6*a*e^3+3/5*x^5*b*ln(c*x^n)*
d*e^2-3/25*b*d*e^2*n*x^5+3/5*x^5*a*d*e^2+3/4*x^4*b*ln(c*x^n)*d^2*e-3/16*b*d
^2*e*n*x^4+3/4*x^4*a*d^2*e+1/3*x^3*b*ln(c*x^n)*d^3-1/9*b*d^3*n*x^3+1/3*a*d^
3*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.67

$$\int x^2(d+ex)^3(a+b\log(cx^n))\,dx=-\frac{1}{36}\left(be^3n-6\,ae^3\right)x^6-\frac{3}{25}\left(bde^2n-5\,ade^2\right)x^5-\frac{3}{16}\left(bd^2en-4\,ad^2e\right)x^4-\frac{1}{9}\left(bd^3n-3\,ad^3\right)x^3+\frac{1}{60}\left(10\,be^3x^6+36\,bde^2x^5+45\,bd^2ex^4+20\,bd^3x^3\right)\log(c)+\frac{1}{60}\left(10\,be^3nx^6+36\,bde^2nx^5+45\,bd^2enx^4+20\,bd^3nx^3\right)\log(x)$$

```
[In] integrate(x^2*(e*x+d)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/36*(b*e^3*n - 6*a*e^3)*x^6 - 3/25*(b*d*e^2*n - 5*a*d*e^2)*x^5 - 3/16*(b*
d^2*e*n - 4*a*d^2*e)*x^4 - 1/9*(b*d^3*n - 3*a*d^3)*x^3 + 1/60*(10*b*e^3*x^6
 + 36*b*d*e^2*x^5 + 45*b*d^2*e*x^4 + 20*b*d^3*x^3)*log(c) + 1/60*(10*b*e^3*
n*x^6 + 36*b*d*e^2*n*x^5 + 45*b*d^2*e*n*x^4 + 20*b*d^3*n*x^3)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.75

$$\int x^2(d+ex)^3\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{ad^3x^3}{3}+\frac{3ad^2ex^4}{4}+\frac{3ade^2x^5}{5}+\frac{ae^3x^6}{6}-\frac{bd^3nx^3}{9}+\frac{bd^3x^3\log\left(cx^n\right)}{3}-\frac{3bd^2enx^4}{16}+\frac{3bd^2ex^4\log\left(cx^n\right)}{4}-\frac{3bde^2nx^5}{25}+\frac{3bde^2x^5\log\left(cx^n\right)}{5}-\frac{be^3nx^6}{36}+\frac{be^3x^6\log\left(cx^n\right)}{6}$$

```
[In] integrate(x**2*(e*x+d)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**3*x**3/3 + 3*a*d**2*e*x**4/4 + 3*a*d*e**2*x**5/5 + a*e**3*x**6/6 - b*d
**3*n*x**3/9 + b*d**3*x**3*log(c*x**n)/3 - 3*b*d**2*e*n*x**4/16 + 3*b*d**2*
e*x**4*log(c*x**n)/4 - 3*b*d*e**2*n*x**5/25 + 3*b*d*e**2*x**5*log(c*x**n)/5
 - b*e**3*n*x**6/36 + b*e**3*x**6*log(c*x**n)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.43

$$\int x^2(d+ex)^3\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{36}\,be^3nx^6+\frac{1}{6}\,be^3x^6\log\left(cx^n\right)-\frac{3}{25}\,bde^2nx^5+\frac{1}{6}\,ae^3x^6+\frac{3}{5}\,bde^2x^5\log\left(cx^n\right)-\frac{3}{16}\,bd^2enx^4+\frac{3}{5}\,ade^2x^5+\frac{3}{4}\,bd^2ex^4\log\left(cx^n\right)-\frac{1}{9}\,bd^3nx^3+\frac{3}{4}\,ad^2ex^4+\frac{1}{3}\,bd^3x^3\log\left(cx^n\right)+\frac{1}{3}\,ad^3x^3$$

```
[In] integrate(x^2*(e*x+d)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/36*b*e^3*n*x^6 + 1/6*b*e^3*x^6*log(c*x^n) - 3/25*b*d*e^2*n*x^5 + 1/6*a*e
^3*x^6 + 3/5*b*d*e^2*x^5*log(c*x^n) - 3/16*b*d^2*e*n*x^4 + 3/5*a*d*e^2*x^5
+ 3/4*b*d^2*e*x^4*log(c*x^n) - 1/9*b*d^3*n*x^3 + 3/4*a*d^2*e*x^4 + 1/3*b*d^
3*x^3*log(c*x^n) + 1/3*a*d^3*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.77

$$\int x^2(d+ex)^3\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{1}{6}\,be^3nx^6\log\left(x\right)-\frac{1}{36}\,be^3nx^6+\frac{1}{6}\,be^3x^6\log\left(c\right)+\frac{3}{5}\,bde^2nx^5\log\left(x\right)-\frac{3}{25}\,bde^2nx^5+\frac{1}{6}\,ae^3x^6+\frac{3}{5}\,bde^2x^5\log\left(c\right)+\frac{3}{4}\,bd^2enx^4\log\left(x\right)-\frac{3}{16}\,bd^2enx^4+\frac{3}{5}\,ade^2x^5+\frac{3}{4}\,bd^2ex^4\log\left(c\right)+\frac{1}{3}\,bd^3nx^3\log\left(x\right)-\frac{1}{9}\,bd^3nx^3+\frac{3}{4}\,ad^2ex^4+\frac{1}{3}\,bd^3x^3\log\left(c\right)+\frac{1}{3}\,ad^3x^3$$

```
[In] integrate(x^2*(e*x+d)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/6*b*e^3*n*x^6*log(x) - 1/36*b*e^3*n*x^6 + 1/6*b*e^3*x^6*log(c) + 3/5*b*d*
e^2*n*x^5*log(x) - 3/25*b*d*e^2*n*x^5 + 1/6*a*e^3*x^6 + 3/5*b*d*e^2*x^5*log
(c) + 3/4*b*d^2*e*n*x^4*log(x) - 3/16*b*d^2*e*n*x^4 + 3/5*a*d*e^2*x^5 + 3/4
*b*d^2*e*x^4*log(c) + 1/3*b*d^3*n*x^3*log(x) - 1/9*b*d^3*n*x^3 + 3/4*a*d^2*
e*x^4 + 1/3*b*d^3*x^3*log(c) + 1/3*a*d^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13

$$\int x^2(d+ex)^3\left(a+b\log\left(cx^n\right)\right)\,dx=\ln\left(c\,x^n\right)\,\left(\frac{b\,d^3\,x^3}{3}+\frac{3\,b\,d^2\,e\,x^4}{4}+\frac{3\,b\,d\,e^2\,x^5}{5}+\frac{b\,e^3\,x^6}{6}\right)+\frac{d^3\,x^3\,\left(3\,a-b\,n\right)}{9}+\frac{e^3\,x^6\,\left(6\,a-b\,n\right)}{36}+\frac{3\,d^2\,e\,x^4\,\left(4\,a-b\,n\right)}{16}+\frac{3\,d\,e^2\,x^5\,\left(5\,a-b\,n\right)}{25}$$

```
[In] int(x^2*(a + b*log(c*x^n))*(d + e*x)^3,x)
```

```
[Out] log(c*x^n)*((b*d^3*x^3)/3 + (b*e^3*x^6)/6 + (3*b*d^2*e*x^4)/4 + (3*b*d*e^2*
x^5)/5) + (d^3*x^3*(3*a - b*n))/9 + (e^3*x^6*(6*a - b*n))/36 + (3*d^2*e*x^4
*(4*a - b*n))/16 + (3*d*e^2*x^5*(5*a - b*n))/25
```

3.21 $\int x(d+ex)^3 (a+b\log(cx^n)) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 122

$$\int x(d+ex)^3 (a+b\log(cx^n)) \, dx = \frac{bd^4nx}{5e} + \frac{3}{20}bd^3nx^2 + \frac{1}{15}bd^2enx^3 + \frac{1}{80}bde^2nx^4 - \frac{bn(d+ex)^5}{25e^2} + \frac{bd^5n\log(x)}{20e^2} - \frac{1}{20}\left(\frac{5d(d+ex)^4}{e^2} - \frac{4(d+ex)^5}{e^2}\right)(a+b\log(cx^n))$$

```
[Out] 1/5*b*d^4*n*x/e+3/20*b*d^3*n*x^2+1/15*b*d^2*e*n*x^3+1/80*b*d*e^2*n*x^4-1/25
*b*n*(e*x+d)^5/e^2+1/20*b*d^5*n*ln(x)/e^2-1/20*(5*d*(e*x+d)^4/e^2-4*(e*x+d)
^5/e^2)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {45, 2371, 12, 81}

$$\int x(d+ex)^3 (a+b\log(cx^n)) \, dx = -\frac{1}{20}\left(\frac{5d(d+ex)^4}{e^2} - \frac{4(d+ex)^5}{e^2}\right)(a+b\log(cx^n)) + \frac{bd^5n\log(x)}{20e^2} + \frac{bd^4nx}{5e} + \frac{3}{20}bd^3nx^2 + \frac{1}{15}bd^2enx^3 + \frac{1}{80}bde^2nx^4 - \frac{bn(d+ex)^5}{25e^2}$$

```
[In] Int[x*(d + e*x)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] (b*d^4*n*x)/(5*e) + (3*b*d^3*n*x^2)/20 + (b*d^2*e*n*x^3)/15 + (b*d*e^2*n*x^
4)/80 - (b*n*(d + e*x)^5)/(25*e^2) + (b*d^5*n*Log[x])/(20*e^2) - (((5*d*(d
+ e*x)^4)/e^2 - (4*(d + e*x)^5)/e^2)*(a + b*Log[c*x^n]))/20
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{20}\left(\frac{5d(d+ex)^4}{e^2}-\frac{4(d+ex)^5}{e^2}\right)(a+b\log(cx^n))-(bn)\int\frac{(d+ex)^4(-d+4ex)}{20e^2x}\,dx\\
&= -\frac{1}{20}\left(\frac{5d(d+ex)^4}{e^2}-\frac{4(d+ex)^5}{e^2}\right)(a+b\log(cx^n))-\frac{(bn)\int\frac{(d+ex)^4(-d+4ex)}{x}\,dx}{20e^2}\\
&= -\frac{bn(d+ex)^5}{25e^2}-\frac{1}{20}\left(\frac{5d(d+ex)^4}{e^2}-\frac{4(d+ex)^5}{e^2}\right)(a+b\log(cx^n))+\frac{(bdn)\int\frac{(d+ex)^4}{x}\,dx}{20e^2}\\
&= -\frac{bn(d+ex)^5}{25e^2}-\frac{1}{20}\left(\frac{5d(d+ex)^4}{e^2}-\frac{4(d+ex)^5}{e^2}\right)(a+b\log(cx^n))\\
&\quad+\frac{(bdn)\int\left(4d^3e+\frac{d^4}{x}+6d^2e^2x+4de^3x^2+e^4x^3\right)dx}{20e^2}
\end{aligned}
$$

$$= \frac{bd^4nx}{5e} + \frac{3}{20}bd^3nx^2 + \frac{1}{15}bd^2enx^3 + \frac{1}{80}bde^2nx^4 - \frac{bn(d+ex)^5}{25e^2} + \frac{bd^5n\log(x)}{20e^2} - \frac{1}{20}\left(\frac{5d(d+ex)^4}{e^2} - \frac{4(d+ex)^5}{e^2}\right)(a+b\log(cx^n))$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int x(d+ex)^3(a+b\log(cx^n))\,dx = -\frac{1}{4}bd^3nx^2 - \frac{1}{3}bd^2enx^3 - \frac{3}{16}bde^2nx^4 - \frac{1}{25}be^3nx^5 + \frac{1}{2}d^3x^2(a+b\log(cx^n)) + d^2ex^3(a+b\log(cx^n)) + \frac{3}{4}de^2x^4(a+b\log(cx^n)) + \frac{1}{5}e^3x^5(a+b\log(cx^n))$$

```
[In] Integrate[x*(d + e*x)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/4*(b*d^3*n*x^2) - (b*d^2*e*n*x^3)/3 - (3*b*d*e^2*n*x^4)/16 - (b*e^3*n*x^
5)/25 + (d^3*x^2*(a + b*Log[c*x^n]))/2 + d^2*e*x^3*(a + b*Log[c*x^n]) + (3*
d*e^2*x^4*(a + b*Log[c*x^n]))/4 + (e^3*x^5*(a + b*Log[c*x^n]))/5
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.16

method	result
parallelrisch	$\frac{x^5 b\ln(cx^n)e^3}{5} - \frac{be^3nx^5}{25} + \frac{x^5ae^3}{5} + \frac{3x^4b\ln(cx^n)de^2}{4} - \frac{3bde^2nx^4}{16} + \frac{3x^4ade^2}{4} + x^3b\ln(cx^n)d^2e - \frac{bd^2enx^3}{3}$
risch	$\frac{x^5ae^3}{5} + \ln(c)bd^2ex^3 + \frac{3\ln(c)bde^2x^4}{4} - \frac{3i\pi bde^2x^4\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)}{8} - \frac{i\pi bd^2ex^3\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\,\mathrm{c}}{2}$

```
[In] int(x*(e*x+d)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*b*ln(c*x^n)*e^3-1/25*b*e^3*n*x^5+1/5*x^5*a*e^3+3/4*x^4*b*ln(c*x^n)*
d*e^2-3/16*b*d*e^2*n*x^4+3/4*x^4*a*d*e^2+x^3*b*ln(c*x^n)*d^2*e-1/3*b*d^2*e*
n*x^3+x^3*a*d^2*e+1/2*x^2*b*ln(c*x^n)*d^3-1/4*b*d^3*n*x^2+1/2*a*d^3*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.37

$$\begin{aligned}
&\int x(d+ex)^3\,(a+b\log{(cx^n)})\;dx \\
&= -\frac{1}{25}\left(be^3n-5\,ae^3\right)x^5-\frac{3}{16}\left(bde^2n-4\,ade^2\right)x^4-\frac{1}{3}\left(bd^2en-3\,ad^2e\right)x^3 \\
&\quad -\frac{1}{4}\left(bd^3n-2\,ad^3\right)x^2+\frac{1}{20}\left(4\,be^3x^5+15\,bde^2x^4+20\,bd^2ex^3+10\,bd^3x^2\right)\log{(c)} \\
&\quad +\frac{1}{20}\left(4\,be^3nx^5+15\,bde^2nx^4+20\,bd^2enx^3+10\,bd^3nx^2\right)\log{(x)}
\end{aligned}$$

```
[In] integrate(x*(e*x+d)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/25*(b*e^3*n - 5*a*e^3)*x^5 - 3/16*(b*d*e^2*n - 4*a*d*e^2)*x^4 - 1/3*(b*d
^2*e*n - 3*a*d^2*e)*x^3 - 1/4*(b*d^3*n - 2*a*d^3)*x^2 + 1/20*(4*b*e^3*x^5 +
 15*b*d*e^2*x^4 + 20*b*d^2*e*x^3 + 10*b*d^3*x^2)*log(c) + 1/20*(4*b*e^3*n*x
^5 + 15*b*d*e^2*n*x^4 + 20*b*d^2*e*n*x^3 + 10*b*d^3*n*x^2)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.37

$$\begin{aligned}
\int x(d+ex)^3\,(a+b\log{(cx^n)})\;dx &= \frac{ad^3x^2}{2}+ad^2ex^3+\frac{3ade^2x^4}{4}+\frac{ae^3x^5}{5} \\
&\quad -\frac{bd^3nx^2}{4}+\frac{bd^3x^2\log{(cx^n)}}{2}-\frac{bd^2enx^3}{3} \\
&\quad +bd^2ex^3\log{(cx^n)}-\frac{3bde^2nx^4}{16} \\
&\quad +\frac{3bde^2x^4\log{(cx^n)}}{4}-\frac{be^3nx^5}{25}+\frac{be^3x^5\log{(cx^n)}}{5}
\end{aligned}$$

```
[In] integrate(x*(e*x+d)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**3*x**2/2 + a*d**2*e*x**3 + 3*a*d*e**2*x**4/4 + a*e**3*x**5/5 - b*d**3*
n*x**2/4 + b*d**3*x**2*log(c*x**n)/2 - b*d**2*e*n*x**3/3 + b*d**2*e*x**3*lo
g(c*x**n) - 3*b*d*e**2*n*x**4/16 + 3*b*d*e**2*x**4*log(c*x**n)/4 - b*e**3*n
*x**5/25 + b*e**3*x**5*log(c*x**n)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.16

$$\int x(d+ex)^3(a+b\log(cx^n))\,dx = -\frac{1}{25}\,be^3nx^5 + \frac{1}{5}\,be^3x^5\log(cx^n) - \frac{3}{16}\,bde^2nx^4 + \frac{1}{5}\,ae^3x^5 + \frac{3}{4}\,bde^2x^4\log(cx^n) - \frac{1}{3}\,bd^2enx^3 + \frac{3}{4}\,ade^2x^4 + bd^2ex^3\log(cx^n) - \frac{1}{4}\,bd^3nx^2 + ad^2ex^3 + \frac{1}{2}\,bd^3x^2\log(cx^n) + \frac{1}{2}\,ad^3x^2$$

```
[In] integrate(x*(e*x+d)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/25*b*e^3*n*x^5 + 1/5*b*e^3*x^5*log(c*x^n) - 3/16*b*d*e^2*n*x^4 + 1/5*a*e
^3*x^5 + 3/4*b*d*e^2*x^4*log(c*x^n) - 1/3*b*d^2*e*n*x^3 + 3/4*a*d*e^2*x^4 +
 b*d^2*e*x^3*log(c*x^n) - 1/4*b*d^3*n*x^2 + a*d^2*e*x^3 + 1/2*b*d^3*x^2*log
(c*x^n) + 1/2*a*d^3*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.43

$$\int x(d+ex)^3(a+b\log(cx^n))\,dx = \frac{1}{5}\,be^3nx^5\log(x) - \frac{1}{25}\,be^3nx^5 + \frac{1}{5}\,be^3x^5\log(c) + \frac{3}{4}\,bde^2nx^4\log(x) - \frac{3}{16}\,bde^2nx^4 + \frac{1}{5}\,ae^3x^5 + \frac{3}{4}\,bde^2x^4\log(c) + bd^2enx^3\log(x) - \frac{1}{3}\,bd^2enx^3 + \frac{3}{4}\,ade^2x^4 + bd^2ex^3\log(c) + \frac{1}{2}\,bd^3nx^2\log(x) - \frac{1}{4}\,bd^3nx^2 + ad^2ex^3 + \frac{1}{2}\,bd^3x^2\log(c) + \frac{1}{2}\,ad^3x^2$$

```
[In] integrate(x*(e*x+d)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/5*b*e^3*n*x^5*log(x) - 1/25*b*e^3*n*x^5 + 1/5*b*e^3*x^5*log(c) + 3/4*b*d*
e^2*n*x^4*log(x) - 3/16*b*d*e^2*n*x^4 + 1/5*a*e^3*x^5 + 3/4*b*d*e^2*x^4*log
(c) + b*d^2*e*n*x^3*log(x) - 1/3*b*d^2*e*n*x^3 + 3/4*a*d*e^2*x^4 + b*d^2*e*
x^3*log(c) + 1/2*b*d^3*n*x^2*log(x) - 1/4*b*d^3*n*x^2 + a*d^2*e*x^3 + 1/2*b
*d^3*x^2*log(c) + 1/2*a*d^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int x(d+ex)^3\,(a+b\log{(cx^n)})\,dx=\ln{(c\,x^n)}\,\left(\frac{b\,d^3\,x^2}{2}+b\,d^2\,e\,x^3+\frac{3\,b\,d\,e^2\,x^4}{4}+\frac{b\,e^3\,x^5}{5}\right)+\frac{d^3\,x^2\,(2\,a-b\,n)}{4}+\frac{e^3\,x^5\,(5\,a-b\,n)}{25}+\frac{d^2\,e\,x^3\,(3\,a-b\,n)}{3}+\frac{3\,d\,e^2\,x^4\,(4\,a-b\,n)}{16}$$

```
[In] int(x*(a + b*log(c*x^n))*(d + e*x)^3,x)
```

```
[Out] log(c*x^n)*((b*d^3*x^2)/2 + (b*e^3*x^5)/5 + b*d^2*e*x^3 + (3*b*d*e^2*x^4)/4
) + (d^3*x^2*(2*a - b*n))/4 + (e^3*x^5*(5*a - b*n))/25 + (d^2*e*x^3*(3*a -
b*n))/3 + (3*d*e^2*x^4*(4*a - b*n))/16
```

3.22 $\int (d+ex)^3 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int (d+ex)^3 (a+b\log(cx^n))\,dx = -bd^3nx - \frac{3}{4}bd^2enx^2 - \frac{1}{3}bde^2nx^3 - \frac{1}{16}be^3nx^4 - \frac{bd^4n\log(x)}{4e} + \frac{(d+ex)^4(a+b\log(cx^n))}{4e}$$

```
[Out] -b*d^3*n*x-3/4*b*d^2*e*n*x^2-1/3*b*d*e^2*n*x^3-1/16*b*e^3*n*x^4-1/4*b*d^4*n
*ln(x)/e+1/4*(e*x+d)^4*(a+b*ln(c*x^n))/e
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {32, 2350, 12, 45}

$$\int (d+ex)^3 (a+b\log(cx^n))\,dx = \frac{(d+ex)^4(a+b\log(cx^n))}{4e} - \frac{bd^4n\log(x)}{4e} - bd^3nx - \frac{3}{4}bd^2enx^2 - \frac{1}{3}bde^2nx^3 - \frac{1}{16}be^3nx^4$$

```
[In] Int[(d + e*x)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -(b*d^3*n*x) - (3*b*d^2*e*n*x^2)/4 - (b*d*e^2*n*x^3)/3 - (b*e^3*n*x^4)/16 -
 (b*d^4*n*Log[x])/(4*e) + ((d + e*x)^4*(a + b*Log[c*x^n]))/(4*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2350

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Dist[a + b*Log[c*x^n], u
, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^4\,(a+b\log{(cx^n)})}{4e} - (bn)\int \frac{(d+ex)^4}{4ex}\,dx \\
&= \frac{(d+ex)^4\,(a+b\log{(cx^n)})}{4e} - \frac{(bn)\int \frac{(d+ex)^4}{x}\,dx}{4e} \\
&= \frac{(d+ex)^4\,(a+b\log{(cx^n)})}{4e} - \frac{(bn)\int \left(4d^3e+\frac{d^4}{x}+6d^2e^2x+4de^3x^2+e^4x^3\right)\,dx}{4e} \\
&= -bd^3nx-\frac{3}{4}bd^2enx^2-\frac{1}{3}bde^2nx^3-\frac{1}{16}be^3nx^4-\frac{bd^4n\log(x)}{4e}+\frac{(d+ex)^4\,(a+b\log{(cx^n)})}{4e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

$$\int (d+ex)^3\,(a+b\log{(cx^n)})\;dx = \frac{1}{48}x\big(12a\big(4d^3+6d^2ex+4de^2x^2+e^3x^3\big) - bn\big(48d^3+36d^2ex+16de^2x^2+3e^3x^3\big) + 12b\big(4d^3+6d^2ex+4de^2x^2+e^3x^3\big)\log{(cx^n)}\big)$$

```
[In] Integrate[(d + e*x)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] (x*(12*a*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - b*n*(48*d^3 + 36*d^2
*e*x + 16*d*e^2*x^2 + 3*e^3*x^3) + 12*b*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 +
e^3*x^3)*Log[c*x^n]))/48
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.58

method	result
parallelrisch	$\frac{b\ln(c\,x^{n})e^{3}x^{4}}{4}-\frac{b\,e^{3}n\,x^{4}}{16}+\frac{a\,e^{3}x^{4}}{4}+b\ln\left(c\,x^{n}\right)d\,e^{2}x^{3}-\frac{bd\,e^{2}n\,x^{3}}{3}+ad\,e^{2}x^{3}+\frac{3b\ln(c\,x^{n})d^{2}e\,x^{2}}{2}-\frac{3b\,d^{2}en\,x^{2}}{4}$
risch	$\frac{i\pi b\,d^{3}\operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})^{2}x}{2}+\frac{ie^{3}\pi b\,x^{4}\operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^{n})^{2}}{8}+\frac{a\,e^{3}x^{4}}{4}-\frac{ie^{2}\pi bd\,x^{3}\operatorname{csgn}(ic)\operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})}{2}-$

```
[In] int((e*x+d)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b*ln(c*x^n)*e^3*x^4-1/16*b*e^3*n*x^4+1/4*a*e^3*x^4+b*ln(c*x^n)*d*e^2*x^
3-1/3*b*d*e^2*n*x^3+a*d*e^2*x^3+3/2*b*ln(c*x^n)*d^2*e*x^2-3/4*b*d^2*e*n*x^2
+3/2*a*d^2*e*x^2+x*b*ln(c*x^n)*d^3-b*d^3*n*x+a*d^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(75) = 150.

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.87

$$\int (d+ex)^3\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{16}\left(be^3n-4\,ae^3\right)x^4-\frac{1}{3}\left(bde^2n-3\,ade^2\right)x^3-\frac{3}{4}\left(bd^2en-2\,ad^2e\right)x^2-\left(bd^3n-ad^3\right)x+\frac{1}{4}\left(be^3x^4+4\,bde^2x^3+6\,bd^2ex^2+4\,bd^3x\right)\log\left(c\right)+\frac{1}{4}\left(be^3nx^4+4\,bde^2nx^3+6\,bd^2enx^2+4\,bd^3nx\right)\log\left(x\right)$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/16*(b*e^3*n - 4*a*e^3)*x^4 - 1/3*(b*d*e^2*n - 3*a*d*e^2)*x^3 - 3/4*(b*d^
2*e*n - 2*a*d^2*e)*x^2 - (b*d^3*n - a*d^3)*x + 1/4*(b*e^3*x^4 + 4*b*d*e^2*x
^3 + 6*b*d^2*e*x^2 + 4*b*d^3*x)*log(c) + 1/4*(b*e^3*n*x^4 + 4*b*d*e^2*n*x^3
 + 6*b*d^2*e*n*x^2 + 4*b*d^3*n*x)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.84

$$\int (d+ex)^3\left(a+b\log\left(cx^n\right)\right)\,dx=ad^3x+\frac{3ad^2ex^2}{2}+ade^2x^3+\frac{ae^3x^4}{4}-bd^3nx+bd^3x\log\left(cx^n\right)-\frac{3bd^2enx^2}{4}+\frac{3bd^2ex^2\log\left(cx^n\right)}{2}-\frac{bde^2nx^3}{3}+bde^2x^3\log\left(cx^n\right)-\frac{be^3nx^4}{16}+\frac{be^3x^4\log\left(cx^n\right)}{4}$$

```
[In] integrate((e*x+d)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 - b*d**3*n*x +
 b*d**3*x*log(c*x**n) - 3*b*d**2*e*n*x**2/4 + 3*b*d**2*e*x**2*log(c*x**n)/2
 - b*d*e**2*n*x**3/3 + b*d*e**2*x**3*log(c*x**n) - b*e**3*n*x**4/16 + b*e**
3*x**4*log(c*x**n)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

$$\int (d+ex)^3 (a+b\log(cx^n))\,dx = -\frac{1}{16}\,be^3nx^4 + \frac{1}{4}\,be^3x^4\log(cx^n) - \frac{1}{3}\,bde^2nx^3 + \frac{1}{4}\,ae^3x^4 + bde^2x^3\log(cx^n) - \frac{3}{4}\,bd^2enx^2 + ade^2x^3 + \frac{3}{2}\,bd^2ex^2\log(cx^n) - bd^3nx + \frac{3}{2}\,ad^2ex^2 + bd^3x\log(cx^n) + ad^3x$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/16*b*e^3*n*x^4 + 1/4*b*e^3*x^4*log(c*x^n) - 1/3*b*d*e^2*n*x^3 + 1/4*a*e^
3*x^4 + b*d*e^2*x^3*log(c*x^n) - 3/4*b*d^2*e*n*x^2 + a*d*e^2*x^3 + 3/2*b*d^
2*e*x^2*log(c*x^n) - b*d^3*n*x + 3/2*a*d^2*e*x^2 + b*d^3*x*log(c*x^n) + a*d
^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(75) = 150$.

Time = 0.53 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.92

$$\int (d+ex)^3 (a+b\log(cx^n))\,dx = \frac{1}{4}\,be^3nx^4\log(x) - \frac{1}{16}\,be^3nx^4 + \frac{1}{4}\,be^3x^4\log(c) + bde^2nx^3\log(x) - \frac{1}{3}\,bde^2nx^3 + \frac{1}{4}\,ae^3x^4 + bde^2x^3\log(c) + \frac{3}{2}\,bd^2enx^2\log(x) - \frac{3}{4}\,bd^2enx^2 + ade^2x^3 + \frac{3}{2}\,bd^2ex^2\log(c) + bd^3nx\log(x) - bd^3nx + \frac{3}{2}\,ad^2ex^2 + bd^3x\log(c) + ad^3x$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*e^3*n*x^4*log(x) - 1/16*b*e^3*n*x^4 + 1/4*b*e^3*x^4*log(c) + b*d*e^2*
n*x^3*log(x) - 1/3*b*d*e^2*n*x^3 + 1/4*a*e^3*x^4 + b*d*e^2*x^3*log(c) + 3/2
```

```
*b*d^2*e*n*x^2*log(x) - 3/4*b*d^2*e*n*x^2 + a*d*e^2*x^3 + 3/2*b*d^2*e*x^2*l
og(c) + b*d^3*n*x*log(x) - b*d^3*n*x + 3/2*a*d^2*e*x^2 + b*d^3*x*log(c) + a
*d^3*x
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

$$\int (d+ex)^3 \left(a+b\log\left(cx^n\right)\right)\,dx = \ln\left(c\,x^n\right)\,\left(b\,d^3\,x+\frac{3\,b\,d^2\,e\,x^2}{2}+b\,d\,e^2\,x^3+\frac{b\,e^3\,x^4}{4}\right) + \frac{e^3\,x^4\,\left(4\,a-b\,n\right)}{16}+d^3\,x\,\left(a-b\,n\right) + \frac{3\,d^2\,e\,x^2\,\left(2\,a-b\,n\right)}{4}+\frac{d\,e^2\,x^3\,\left(3\,a-b\,n\right)}{3}$$

```
[In] int((a + b*log(c*x^n))*(d + e*x)^3,x)

[Out] log(c*x^n)*((b*e^3*x^4)/4 + b*d^3*x + (3*b*d^2*e*x^2)/2 + b*d*e^2*x^3) + (e
^3*x^4*(4*a - b*n))/16 + d^3*x*(a - b*n) + (3*d^2*e*x^2*(2*a - b*n))/4 + (d
*e^2*x^3*(3*a - b*n))/3
```

3.23 $\int \frac{(d+ex)^3(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \frac{(d+ex)^3\,(a+b\log(cx^n))}{x}\,dx = -3bd^2enx-\frac{3}{4}bde^2nx^2-\frac{1}{9}be^3nx^3-\frac{1}{2}bd^3n\log^2(x)+3d^2ex(a+b\log(cx^n))+\frac{3}{2}de^2x^2(a+b\log(cx^n))+\frac{1}{3}e^3x^3(a+b\log(cx^n))+d^3\log(x)\,(a+b\log(cx^n))$$

```
[Out] -3*b*d^2*e*n*x-3/4*b*d*e^2*n*x^2-1/9*b*e^3*n*x^3-1/2*b*d^3*n*ln(x)^2+3*d^2*
e*x*(a+b*ln(c*x^n))+3/2*d*e^2*x^2*(a+b*ln(c*x^n))+1/3*e^3*x^3*(a+b*ln(c*x^n
))+d^3*ln(x)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {45, 2372, 2338}

$$\int \frac{(d+ex)^3\,(a+b\log(cx^n))}{x}\,dx = d^3\log(x)\,(a+b\log(cx^n))+3d^2ex(a+b\log(cx^n))+\frac{3}{2}de^2x^2(a+b\log(cx^n))+\frac{1}{3}e^3x^3(a+b\log(cx^n))-\frac{1}{2}bd^3n\log^2(x)-3bd^2enx-\frac{3}{4}bde^2nx^2-\frac{1}{9}be^3nx^3$$

```
[In] Int[((d + e*x)^3*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] -3*b*d^2*e*n*x - (3*b*d*e^2*n*x^2)/4 - (b*e^3*n*x^3)/9 - (b*d^3*n*Log[x]^2)
/2 + 3*d^2*e*x*(a + b*Log[c*x^n]) + (3*d*e^2*x^2*(a + b*Log[c*x^n]))/2 + (e
^3*x^3*(a + b*Log[c*x^n]))/3 + d^3*Log[x]*(a + b*Log[c*x^n])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3d^2ex(a+b\log(cx^n))+\frac{3}{2}de^2x^2(a+b\log(cx^n))+\frac{1}{3}e^3x^3(a+b\log(cx^n))\\
&\quad+d^3\log(x)(a+b\log(cx^n))-(bn)\int\left(\frac{1}{6}e\left(18d^2+9dex+2e^2x^2\right)+\frac{d^3\log(x)}{x}\right)dx\\
&=3d^2ex(a+b\log(cx^n))+\frac{3}{2}de^2x^2(a+b\log(cx^n))+\frac{1}{3}e^3x^3(a+b\log(cx^n))+d^3\log(x)(a\\
&\quad+b\log(cx^n))-\left(bd^3n\right)\int\frac{\log(x)}{x}\,dx-\frac{1}{6}(ben)\int\left(18d^2+9dex+2e^2x^2\right)dx\\
&=-3bd^2enx-\frac{3}{4}bde^2nx^2-\frac{1}{9}be^3nx^3-\frac{1}{2}bd^3n\log^2(x)+3d^2ex(a+b\log(cx^n))\\
&\quad+\frac{3}{2}de^2x^2(a+b\log(cx^n))+\frac{1}{3}e^3x^3(a+b\log(cx^n))+d^3\log(x)(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01

$$\begin{aligned}
\int\frac{(d+ex)^3(a+b\log(cx^n))}{x}\,dx &= 3ad^2ex-3bd^2enx-\frac{3}{4}bde^2nx^2-\frac{1}{9}be^3nx^3\\
&\quad+3bd^2ex\log(cx^n)+\frac{3}{2}de^2x^2(a+b\log(cx^n))\\
&\quad+\frac{1}{3}e^3x^3(a+b\log(cx^n))+\frac{d^3(a+b\log(cx^n))^2}{2bn}
\end{aligned}$$

```
[In] Integrate[((d + e*x)^3*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] 3*a*d^2*e*x - 3*b*d^2*e*n*x - (3*b*d*e^2*n*x^2)/4 - (b*e^3*n*x^3)/9 + 3*b*d
^2*e*x*Log[c*x^n] + (3*d*e^2*x^2*(a + b*Log[c*x^n]))/2 + (e^3*x^3*(a + b*Lo
g[c*x^n]))/3 + (d^3*(a + b*Log[c*x^n])^2)/(2*b*n)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.18

method	result
parallelrisch	$\frac{12x^3\ln(cx^n)be^3n-4x^3be^3n^2+12x^3ae^3n+54x^2\ln(cx^n)bde^2n-27x^2bde^2n^2+54x^2ade^2n+108x\ln(cx^n)bd^2en-108xbd^2en^{}}{36n}$
risch	$\frac{\ln(c)be^3x^3}{3}-\frac{3i\pi bde^2x^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)}{4}-\frac{3i\pi bd^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)ex}{2}-\frac{i\ln(x)\pi bd^3\,\mathrm{csgn}(ic}{}$

```
[In] int((e*x+d)^3*(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*(12*x^3*ln(c*x^n)*b*e^3*n-4*x^3*b*e^3*n^2+12*x^3*a*e^3*n+54*x^2*ln(c*x
^n)*b*d*e^2*n-27*x^2*b*d*e^2*n^2+54*x^2*a*d*e^2*n+108*x*ln(c*x^n)*b*d^2*e*n
-108*x*b*d^2*e*n^2+36*ln(x)*a*d^3*n+108*x*a*d^2*e*n+18*b*d^3*ln(c*x^n)^2)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$
$$=\frac{1}{2}\,bd^3n\log\left(x\right)^2-\frac{1}{9}\left(be^3n-3\,ae^3\right)x^3-\frac{3}{4}\left(bde^2n-2\,ade^2\right)x^2$$
$$-3\left(bd^2en-ad^2e\right)x+\frac{1}{6}\left(2\,be^3x^3+9\,bde^2x^2+18\,bd^2ex\right)\log\left(c\right)$$
$$+\frac{1}{6}\left(2\,be^3nx^3+9\,bde^2nx^2+18\,bd^2enx+6\,bd^3\log\left(c\right)+6\,ad^3\right)\log\left(x\right)$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] 1/2*b*d^3*n*log(x)^2 - 1/9*(b*e^3*n - 3*a*e^3)*x^3 - 3/4*(b*d*e^2*n - 2*a*d
*e^2)*x^2 - 3*(b*d^2*e*n - a*d^2*e)*x + 1/6*(2*b*e^3*x^3 + 9*b*d*e^2*x^2 +
18*b*d^2*e*x)*log(c) + 1/6*(2*b*e^3*n*x^3 + 9*b*d*e^2*n*x^2 + 18*b*d^2*e*n*
x + 6*b*d^3*log(c) + 6*a*d^3)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.63

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x}\,dx$$
$$= \begin{cases} \frac{ad^3\log(cx^n)}{n} + 3ad^2ex + \frac{3ade^2x^2}{2} + \frac{ae^3x^3}{3} + \frac{bd^3\log(cx^n)^2}{2n} - 3bd^2enx + 3bd^2ex\log(cx^n) - \frac{3bde^2nx^2}{4} + \frac{3bde^2x^2\log(c \\ (a+b\log(c))\left(d^3\log(x) + 3d^2ex + \frac{3de^2x^2}{2} + \frac{e^3x^3}{3}\right) \end{cases}$$

```
[In] integrate((e*x+d)**3*(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise((a*d**3*log(c*x**n)/n + 3*a*d**2*e*x + 3*a*d*e**2*x**2/2 + a*e**3
*x**3/3 + b*d**3*log(c*x**n)**2/(2*n) - 3*b*d**2*e*n*x + 3*b*d**2*e*x*log(c
*x**n) - 3*b*d*e**2*n*x**2/4 + 3*b*d*e**2*x**2*log(c*x**n)/2 - b*e**3*n*x**
3/9 + b*e**3*x**3*log(c*x**n)/3, Ne(n, 0)), ((a + b*log(c))*(d**3*log(x) +
3*d**2*e*x + 3*d*e**2*x**2/2 + e**3*x**3/3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x}\,dx = -\frac{1}{9}\,be^3nx^3 + \frac{1}{3}\,be^3x^3\log(cx^n) - \frac{3}{4}\,bde^2nx^2 + \frac{1}{3}\,ae^3x^3 + \frac{3}{2}\,bde^2x^2\log(cx^n) - 3\,bd^2enx + \frac{3}{2}\,ade^2x^2 + 3\,bd^2ex\log(cx^n) + 3\,ad^2ex + \frac{bd^3\log(cx^n)^2}{2\,n} + ad^3\log(x)$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] -1/9*b*e^3*n*x^3 + 1/3*b*e^3*x^3*log(c*x^n) - 3/4*b*d*e^2*n*x^2 + 1/3*a*e^3
*x^3 + 3/2*b*d*e^2*x^2*log(c*x^n) - 3*b*d^2*e*n*x + 3/2*a*d*e^2*x^2 + 3*b*d
^2*e*x*log(c*x^n) + 3*a*d^2*e*x + 1/2*b*d^3*log(c*x^n)^2/n + a*d^3*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)^3\,(a+b\log(cx^n))}{x}\,dx = \frac{1}{2}\,bd^3n\log(x)^2 - \frac{1}{9}\left(be^3n - 3\,be^3\log(c) - 3\,ae^3\right)x^3 - \frac{3}{4}\left(bde^2n - 2\,bde^2\log(c) - 2\,ade^2\right)x^2 - 3\left(bd^2en - bd^2e\log(c) - ad^2e\right)x + \frac{1}{6}\left(2\,be^3nx^3 + 9\,bde^2nx^2 + 18\,bd^2enx\right)\log(x) + \left(bd^3\log(c) + ad^3\right)\log(x)$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] 1/2*b*d^3*n*log(x)^2 - 1/9*(b*e^3*n - 3*b*e^3*log(c) - 3*a*e^3)*x^3 - 3/4*(
b*d*e^2*n - 2*b*d*e^2*log(c) - 2*a*d*e^2)*x^2 - 3*(b*d^2*e*n - b*d^2*e*log(
c) - a*d^2*e)*x + 1/6*(2*b*e^3*n*x^3 + 9*b*d*e^2*n*x^2 + 18*b*d^2*e*n*x)*lo
g(x) + (b*d^3*log(c) + a*d^3)*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^3\,(a+b\log(cx^n))}{x}\,dx = \ln(c\,x^n)\,\left(3\,b\,d^2\,e\,x + \frac{3\,b\,d\,e^2\,x^2}{2} + \frac{b\,e^3\,x^3}{3}\right) + \frac{e^3\,x^3\,(3\,a-b\,n)}{9} + a\,d^3\,\ln(x) + \frac{b\,d^3\ln(c\,x^n)^2}{2\,n} + \frac{3\,d\,e^2\,x^2\,(2\,a-b\,n)}{4} + 3\,d^2\,e\,x\,(a-b\,n)$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^3)/x,x)
```

```
[Out] log(c*x^n)*((b*e^3*x^3)/3 + 3*b*d^2*e*x + (3*b*d*e^2*x^2)/2) + (e^3*x^3*(3*
a - b*n))/9 + a*d^3*log(x) + (b*d^3*log(c*x^n)^2)/(2*n) + (3*d*e^2*x^2*(2*a
 - b*n))/4 + 3*d^2*e*x*(a - b*n)
```

3.24 $\int \frac{(d+ex)^3(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{(d+ex)^3\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx = -\frac{bd^3n}{x}-3bde^2nx-\frac{1}{4}be^3nx^2-\frac{3}{2}bd^2en\log^2(x) - \frac{d^3(a+b\log\left(cx^n\right))}{x}+3de^2x(a+b\log\left(cx^n\right)) + \frac{1}{2}e^3x^2(a+b\log\left(cx^n\right))+3d^2e\log(x)\left(a+b\log\left(cx^n\right)\right)$$

```
[Out] -b*d^3*n/x-3*b*d*e^2*n*x-1/4*b*e^3*n*x^2-3/2*b*d^2*e*n*ln(x)^2-d^3*(a+b*ln(
c*x^n))/x+3*d*e^2*x*(a+b*ln(c*x^n))+1/2*e^3*x^2*(a+b*ln(c*x^n))+3*d^2*e*ln(
x)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {45, 2372, 2338}

$$\int \frac{(d+ex)^3\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx = -\frac{d^3(a+b\log\left(cx^n\right))}{x}+3d^2e\log(x)\left(a+b\log\left(cx^n\right)\right) + 3de^2x(a+b\log\left(cx^n\right))+\frac{1}{2}e^3x^2(a+b\log\left(cx^n\right)) - \frac{bd^3n}{x}-\frac{3}{2}bd^2en\log^2(x)-3bde^2nx-\frac{1}{4}be^3nx^2$$

```
[In] Int[((d + e*x)^3*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -((b*d^3*n)/x) - 3*b*d*e^2*n*x - (b*e^3*n*x^2)/4 - (3*b*d^2*e*n*Log[x]^2)/2
 - (d^3*(a + b*Log[c*x^n]))/x + 3*d*e^2*x*(a + b*Log[c*x^n]) + (e^3*x^2*(a
+ b*Log[c*x^n]))/2 + 3*d^2*e*Log[x]*(a + b*Log[c*x^n])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{x} + 3de^2x(a+b\log(cx^n)) + \frac{1}{2}e^3x^2(a+b\log(cx^n)) \\
&\quad + 3d^2e\log(x)\,(a+b\log(cx^n)) - (bn)\int\left(3de^2 - \frac{d^3}{x^2} + \frac{e^3x}{2} + \frac{3d^2e\log(x)}{x}\right)dx \\
&= -\frac{bd^3n}{x} - 3bde^2nx - \frac{1}{4}be^3nx^2 - \frac{d^3(a+b\log(cx^n))}{x} + 3de^2x(a+b\log(cx^n)) \\
&\quad + \frac{1}{2}e^3x^2(a+b\log(cx^n)) + 3d^2e\log(x)\,(a+b\log(cx^n)) - \left(3bd^2en\right)\int\frac{\log(x)}{x}\,dx \\
&= -\frac{bd^3n}{x} - 3bde^2nx - \frac{1}{4}be^3nx^2 - \frac{3}{2}bd^2en\log^2(x) - \frac{d^3(a+b\log(cx^n))}{x} \\
&\quad + 3de^2x(a+b\log(cx^n)) + \frac{1}{2}e^3x^2(a+b\log(cx^n)) + 3d^2e\log(x)\,(a+b\log(cx^n))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$
\begin{aligned}
\int \frac{(d+ex)^3\,(a+b\log(cx^n))}{x^2}\,dx &= -\frac{bd^3n}{x} + 3ade^2x - 3bde^2nx - \frac{1}{4}be^3nx^2 \\
&\quad + 3bde^2x\log(cx^n) - \frac{d^3(a+b\log(cx^n))}{x} \\
&\quad + \frac{1}{2}e^3x^2(a+b\log(cx^n)) + \frac{3d^2e(a+b\log(cx^n))^2}{2bn}
\end{aligned}
$$

```
[In] Integrate[((d + e*x)^3*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -((b*d^3*n)/x) + 3*a*d*e^2*x - 3*b*d*e^2*n*x - (b*e^3*n*x^2)/4 + 3*b*d*e^2*
x*Log[c*x^n] - (d^3*(a + b*Log[c*x^n]))/x + (e^3*x^2*(a + b*Log[c*x^n]))/2
+ (3*d^2*e*(a + b*Log[c*x^n])^2)/(2*b*n)
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.22

method	result
parallelrisch	$\frac{2x^3\ln(c\,x^n)b\,e^3n-x^3b\,e^3n^2+2x^3a\,e^3n+12x^2\ln(c\,x^n)bd\,e^2n-12x^2bd\,e^2n^2+12\ln(x)xa\,d^2en+12x^2ad\,e^2n+6e\,d^2b\ln(c\,x^n)^2x-4\,\mathrm{l}}{4xn}$
risch	$-\frac{b\left(-e^3x^3-6e\,d^2\ln(x)x-6d\,e^2x^2+2d^3\right)\ln(x^n)}{2x}-\frac{-2\ln(c)b\,e^3x^3+6i\ln(x)\pi b\,d^2e\,\mathrm{csgn}(ic\,x^n)^3x+6i\pi bd\,e^2x^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)}{}$

```
[In] int((e*x+d)^3*(a+b*ln(c*x^n))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/x*(2*x^3*ln(c*x^n)*b*e^3*n-x^3*b*e^3*n^2+2*x^3*a*e^3*n+12*x^2*ln(c*x^n)
*b*d*e^2*n-12*x^2*b*d*e^2*n^2+12*ln(x)*x*a*d^2*e*n+12*x^2*a*d*e^2*n+6*e*d^2
*b*ln(c*x^n)^2*x-4*ln(c*x^n)*b*d^3*n-4*b*d^3*n^2-4*a*d^3*n)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^3\,(a+b\log{(cx^n)})}{x^2}\,dx$$
$$=\frac{6\,bd^2enx\log{(x)}^2-4\,bd^3n-4\,ad^3-(be^3n-2\,ae^3)x^3-12\,(bde^2n-ade^2)x^2+2\,(be^3x^3+6\,bde^2x^2-2\,bd^3)}{4\,x}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b*d^2*e*n*x*log(x)^2 - 4*b*d^3*n - 4*a*d^3 - (b*e^3*n - 2*a*e^3)*x^3
 - 12*(b*d*e^2*n - a*d*e^2)*x^2 + 2*(b*e^3*x^3 + 6*b*d*e^2*x^2 - 2*b*d^3)*l
og(c) + 2*(b*e^3*n*x^3 + 6*b*d*e^2*n*x^2 + 6*b*d^2*e*x*log(c) - 2*b*d^3*n +
 6*a*d^2*e*x)*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^2}\,dx = \begin{cases} -\frac{ad^3}{x} + \frac{3ad^2e\log(cx^n)}{n} + 3ade^2x + \frac{ae^3x^2}{2} - \frac{bd^3n}{x} - \frac{bd^3\log(cx^n)}{x} + \frac{3bd^2e\log(cx^n)^2}{2n} - 3bde^2nx + 3bde^2x\log(cx^n) - \\ (a+b\log(c))\left(-\frac{d^3}{x} + 3d^2e\log(x) + 3de^2x + \frac{e^3x^2}{2}\right) \end{cases}$$

```
[In] integrate((e*x+d)**3*(a+b*ln(c*x**n))/x**2,x)
```

```
[Out] Piecewise((-a*d**3/x + 3*a*d**2*e*log(c*x**n)/n + 3*a*d*e**2*x + a*e**3*x**
2/2 - b*d**3*n/x - b*d**3*log(c*x**n)/x + 3*b*d**2*e*log(c*x**n)**2/(2*n) -
 3*b*d*e**2*n*x + 3*b*d*e**2*x*log(c*x**n) - b*e**3*n*x**2/4 + b*e**3*x**2*
log(c*x**n)/2, Ne(n, 0)), ((a + b*log(c))*(-d**3/x + 3*d**2*e*log(x) + 3*d*
e**2*x + e**3*x**2/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^2}\,dx = -\frac{1}{4}\,be^3nx^2 + \frac{1}{2}\,be^3x^2\log(cx^n) - 3\,bde^2nx + \frac{1}{2}\,ae^3x^2 + 3\,bde^2x\log(cx^n) + 3\,ade^2x + \frac{3\,bd^2e\log(cx^n)^2}{2\,n} + 3\,ad^2e\log(x) - \frac{bd^3n}{x} - \frac{bd^3\log(cx^n)}{x} - \frac{ad^3}{x}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*b*e^3*n*x^2 + 1/2*b*e^3*x^2*log(c*x^n) - 3*b*d*e^2*n*x + 1/2*a*e^3*x^2
 + 3*b*d*e^2*x*log(c*x^n) + 3*a*d*e^2*x + 3/2*b*d^2*e*log(c*x^n)^2/n + 3*a*
d^2*e*log(x) - b*d^3*n/x - b*d^3*log(c*x^n)/x - a*d^3/x
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^2} dx = \frac{1}{2} be^3x^2\log(c) + \frac{3}{2} bd^2en\log(x)^2 + 3(x\log(x) - x)bde^2n + \frac{1}{4}(2x^2\log(x) - x^2)be^3n + \frac{1}{2}ae^3x^2 - bd^3n\left(\frac{\log(x)}{x} + \frac{1}{x}\right) + 3bde^2x\log(c) + 3bd^2e\log(c)\log(|x|) + 3ade^2x + 3ad^2e\log(|x|) - \frac{bd^3\log(c)}{x} - \frac{ad^3}{x}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*e^3*x^2*log(c) + 3/2*b*d^2*e*n*log(x)^2 + 3*(x*log(x) - x)*b*d*e^2*n
+ 1/4*(2*x^2*log(x) - x^2)*b*e^3*n + 1/2*a*e^3*x^2 - b*d^3*n*(log(x)/x + 1/
x) + 3*b*d*e^2*x*log(c) + 3*b*d^2*e*log(c)*log(abs(x)) + 3*a*d*e^2*x + 3*a*
d^2*e*log(abs(x)) - b*d^3*log(c)/x - a*d^3/x
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^2} dx = \ln(x)\ (3\,a\,d^2\,e + 3\,b\,d^2\,e\,n) - \ln(c\,x^n)\ \left(\frac{b\,d^3 + 3\,b\,d^2\,e\,x + 3\,b\,d\,e^2\,x^2 + b\,e^3\,x^3}{x} - \frac{\frac{3\,b\,e^3\,x^3}{2} + 6\,b\,d\,e^2\,x^2}{x}\right) - \frac{a\,d^3 + b\,d^3\,n}{x} + \frac{e^3\,x^2\,(2\,a - b\,n)}{4} + 3\,d\,e^2\,x\,(a - b\,n) + \frac{3\,b\,d^2\,e\ln(c\,x^n)^2}{2\,n}$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^3)/x^2,x)
```

```
[Out] log(x)*(3*a*d^2*e + 3*b*d^2*e*n) - log(c*x^n)*((b*d^3 + b*e^3*x^3 + 3*b*d^2
*e*x + 3*b*d*e^2*x^2)/x - ((3*b*e^3*x^3)/2 + 6*b*d*e^2*x^2)/x) - (a*d^3 + b
*d^3*n)/x + (e^3*x^2*(2*a - b*n))/4 + 3*d*e^2*x*(a - b*n) + (3*b*d^2*e*log(
c*x^n)^2)/(2*n)
```

3.25 $\int \frac{(d+ex)^3(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 118

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^3}\,dx = -\frac{bd^3n}{4x^2}-\frac{3bd^2en}{x}-be^3nx-\frac{3}{2}bde^2n\log^2(x) -\frac{d^3(a+b\log(cx^n))}{2x^2}-\frac{3d^2e(a+b\log(cx^n))}{x} +e^3x(a+b\log(cx^n))+3de^2\log(x)(a+b\log(cx^n))$$

```
[Out] -1/4*b*d^3*n/x^2-3*b*d^2*e*n/x-b*e^3*n*x-3/2*b*d*e^2*n*ln(x)^2-1/2*d^3*(a+b
*ln(c*x^n))/x^2-3*d^2*e*(a+b*ln(c*x^n))/x+e^3*x*(a+b*ln(c*x^n))+3*d*e^2*ln(
x)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {45, 2372, 2338}

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^3}\,dx = -\frac{d^3(a+b\log(cx^n))}{2x^2}-\frac{3d^2e(a+b\log(cx^n))}{x} +3de^2\log(x)(a+b\log(cx^n))+e^3x(a+b\log(cx^n)) -\frac{bd^3n}{4x^2}-\frac{3bd^2en}{x}-\frac{3}{2}bde^2n\log^2(x)-be^3nx$$

```
[In] Int[((d + e*x)^3*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/4*(b*d^3*n)/x^2 - (3*b*d^2*e*n)/x - b*e^3*n*x - (3*b*d*e^2*n*Log[x]^2)/2
 - (d^3*(a + b*Log[c*x^n]))/(2*x^2) - (3*d^2*e*(a + b*Log[c*x^n]))/x + e^3*
x*(a + b*Log[c*x^n]) + 3*d*e^2*Log[x]*(a + b*Log[c*x^n])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{2x^2} - \frac{3d^2e(a+b\log(cx^n))}{x} + e^3x(a+b\log(cx^n)) \\
&\quad + 3de^2\log(x)(a+b\log(cx^n)) - (bn)\int\left(e^3 - \frac{d^3}{2x^3} - \frac{3d^2e}{x^2} + \frac{3de^2\log(x)}{x}\right)dx \\
&= -\frac{bd^3n}{4x^2} - \frac{3bd^2en}{x} - be^3nx - \frac{d^3(a+b\log(cx^n))}{2x^2} - \frac{3d^2e(a+b\log(cx^n))}{x} \\
&\quad + e^3x(a+b\log(cx^n)) + 3de^2\log(x)(a+b\log(cx^n)) - \left(3bde^2n\right)\int\frac{\log(x)}{x}\,dx \\
&= -\frac{bd^3n}{4x^2} - \frac{3bd^2en}{x} - be^3nx - \frac{3}{2}bde^2n\log^2(x) - \frac{d^3(a+b\log(cx^n))}{2x^2} \\
&\quad - \frac{3d^2e(a+b\log(cx^n))}{x} + e^3x(a+b\log(cx^n)) + 3de^2\log(x)(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\begin{aligned}
\int\frac{(d+ex)^3(a+b\log(cx^n))}{x^3}\,dx &= -\frac{bd^3n}{4x^2} - \frac{3bd^2en}{x} + ae^3x - be^3nx \\
&\quad + be^3x\log(cx^n) - \frac{d^3(a+b\log(cx^n))}{2x^2} \\
&\quad - \frac{3d^2e(a+b\log(cx^n))}{x} + \frac{3de^2(a+b\log(cx^n))^2}{2bn}
\end{aligned}$$

```
[In] Integrate[((d + e*x)^3*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/4*(b*d^3*n)/x^2 - (3*b*d^2*e*n)/x + a*e^3*x - b*e^3*n*x + b*e^3*x*Log[c*
x^n] - (d^3*(a + b*Log[c*x^n]))/(2*x^2) - (3*d^2*e*(a + b*Log[c*x^n]))/x +
(3*d*e^2*(a + b*Log[c*x^n])^2)/(2*b*n)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.21

method	result
parallelrisch	$\frac{4x^3\ln(cx^n)be^3n-4x^3be^3n^2+12\ln(x)x^2ade^2n+4x^3ae^3n+6e^2db\ln(cx^n)^2x^2-12x\ln(cx^n)bd^2en-12xbd^2en^2-12xad^2en-}{4x^2n}$
risch	$-\frac{b(-6e^2d\ln(x)x^2-2e^3x^3+6d^2ex+d^3)\ln(x^n)}{2x^2}-\frac{-4\ln(c)be^3x^3+6i\ln(x)\pi bde^2\,\mathrm{csgn}(icx^n)^3x^2-6i\pi bd^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)}{}$

```
[In] int((e*x+d)^3*(a+b*ln(c*x^n))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/x^2*(4*x^3*ln(c*x^n)*b*e^3*n-4*x^3*b*e^3*n^2+12*ln(x)*x^2*a*d*e^2*n+4*x
^3*a*e^3*n+6*e^2*d*b*ln(c*x^n)^2*x^2-12*x*ln(c*x^n)*b*d^2*e*n-12*x*b*d^2*e*
n^2-12*x*a*d^2*e*n-2*ln(c*x^n)*b*d^3*n-b*d^3*n^2-2*a*d^3*n)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^3(a+b\log(cx^n))}{x^3}\,dx$$
$$=\frac{6\,bde^2nx^2\log(x)^2-bd^3n-2\,ad^3-4\,(be^3n-ae^3)x^3-12\,(bd^2en+ad^2e)x+2\,(2\,be^3x^3-6\,bd^2ex-bd^3)\,\mathrm{l}}{4\,x^2}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b*d*e^2*n*x^2*log(x)^2 - b*d^3*n - 2*a*d^3 - 4*(b*e^3*n - a*e^3)*x^3
 - 12*(b*d^2*e*n + a*d^2*e)*x + 2*(2*b*e^3*x^3 - 6*b*d^2*e*x - b*d^3)*log(c
) + 2*(2*b*e^3*n*x^3 + 6*b*d*e^2*x^2*log(c) - 6*b*d^2*e*n*x + 6*a*d*e^2*x^2
 - b*d^3*n)*log(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex)^3\,(a+b\log(cx^n))}{x^3}\,dx$$
$$=\begin{cases} -\frac{ad^3}{2x^2}-\frac{3ad^2e}{x}+\frac{3ade^2\log(cx^n)}{n}+ae^3x-\frac{bd^3n}{4x^2}-\frac{bd^3\log(cx^n)}{2x^2}-\frac{3bd^2en}{x}-\frac{3bd^2e\log(cx^n)}{x}+\frac{3bde^2\log(cx^n)^2}{2n}-be^3nx+b \\ (a+b\log(c))\left(-\frac{d^3}{2x^2}-\frac{3d^2e}{x}+3de^2\log(x)+e^3x\right) \end{cases}$$

```
[In] integrate((e*x+d)**3*(a+b*ln(c*x**n))/x**3,x)
```

```
[Out] Piecewise((-a*d**3/(2*x**2) - 3*a*d**2*e/x + 3*a*d*e**2*log(c*x**n)/n + a*e
**3*x - b*d**3*n/(4*x**2) - b*d**3*log(c*x**n)/(2*x**2) - 3*b*d**2*e*n/x -
3*b*d**2*e*log(c*x**n)/x + 3*b*d*e**2*log(c*x**n)**2/(2*n) - b*e**3*n*x + b
*e**3*x*log(c*x**n), Ne(n, 0)), ((a + b*log(c))*(-d**3/(2*x**2) - 3*d**2*e/
x + 3*d*e**2*log(x) + e**3*x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^3\,(a+b\log(cx^n))}{x^3}\,dx = -be^3nx+be^3x\log(cx^n)+ae^3x+\frac{3\,bde^2\log(cx^n)^2}{2\,n}+3\,ade^2\log(x)-\frac{3\,bd^2en}{x}-\frac{3\,bd^2e\log(cx^n)}{x}-\frac{bd^3n}{4\,x^2}-\frac{3\,ad^2e}{x}-\frac{bd^3\log(cx^n)}{2\,x^2}-\frac{ad^3}{2\,x^2}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^3,x, algorithm="maxima")
```

```
[Out] -b*e^3*n*x + b*e^3*x*log(c*x^n) + a*e^3*x + 3/2*b*d*e^2*log(c*x^n)^2/n + 3*
a*d*e^2*log(x) - 3*b*d^2*e*n/x - 3*b*d^2*e*log(c*x^n)/x - 1/4*b*d^3*n/x^2 -
 3*a*d^2*e/x - 1/2*b*d^3*log(c*x^n)/x^2 - 1/2*a*d^3/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^3}\,dx = \frac{3}{2}bde^2n\log(x)^2 + (x\log(x)-x)be^3n - 3\,bd^2en\left(\frac{\log(x)}{x}+\frac{1}{x}\right) - \frac{1}{4}\,bd^3n\left(\frac{2\,\log(x)}{x^2}+\frac{1}{x^2}\right) + be^3x\log(c) + 3\,bde^2\log(c)\log(|x|) + ae^3x + 3\,ade^2\log(|x|) - \frac{3\,bd^2e\log(c)}{x} - \frac{3\,ad^2e}{x} - \frac{bd^3\log(c)}{2\,x^2} - \frac{ad^3}{2\,x^2}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

```
[Out] 3/2*b*d*e^2*n*log(x)^2 + (x*log(x) - x)*b*e^3*n - 3*b*d^2*e*n*(log(x)/x + 1
/x) - 1/4*b*d^3*n*(2*log(x)/x^2 + 1/x^2) + b*e^3*x*log(c) + 3*b*d*e^2*log(c
)*log(abs(x)) + a*e^3*x + 3*a*d*e^2*log(abs(x)) - 3*b*d^2*e*log(c)/x - 3*a*
d^2*e/x - 1/2*b*d^3*log(c)/x^2 - 1/2*a*d^3/x^2
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^3}\,dx = \ln(x)\,\left(3\,a\,d\,e^2+\frac{9\,b\,d\,e^2\,n}{2}\right) - \ln(c\,x^n)\,\left(\frac{\frac{b\,d^3}{2}+3\,b\,d^2\,e\,x+\frac{9\,b\,d\,e^2\,x^2}{2}+2\,b\,e^3\,x^3}{x^2} - 3\,b\,e^3\,x\right) - \frac{x\,(6\,a\,d^2\,e+6\,b\,d^2\,e\,n)+a\,d^3+\frac{b\,d^3\,n}{2}}{2\,x^2} + e^3\,x\,(a-b\,n) + \frac{3\,b\,d\,e^2\ln(c\,x^n)^2}{2\,n}$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^3)/x^3,x)
```

```
[Out] log(x)*(3*a*d*e^2 + (9*b*d*e^2*n)/2) - log(c*x^n)*(((b*d^3)/2 + 2*b*e^3*x^3
 + 3*b*d^2*e*x + (9*b*d*e^2*x^2)/2)/x^2 - 3*b*e^3*x) - (x*(6*a*d^2*e + 6*b*
d^2*e*n) + a*d^3 + (b*d^3*n)/2)/(2*x^2) + e^3*x*(a - b*n) + (3*b*d*e^2*log(
c*x^n)^2)/(2*n)
```

3.26 $\int \frac{(d+ex)^3(a+b\log(cx^n))}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{(d+ex)^3(a+b\log(cx^n))}{x^4}\,dx = -\frac{bd^3n}{9x^3}-\frac{3bd^2en}{4x^2}-\frac{3bde^2n}{x}-\frac{1}{2}be^3n\log^2(x)-\frac{d^3(a+b\log(cx^n))}{3x^3}-\frac{3d^2e(a+b\log(cx^n))}{2x^2}-\frac{3de^2(a+b\log(cx^n))}{x}+e^3\log(x)(a+b\log(cx^n))$$

```
[Out] -1/9*b*d^3*n/x^3-3/4*b*d^2*e*n/x^2-3*b*d*e^2*n/x-1/2*b*e^3*n*ln(x)^2-1/3*d^
3*(a+b*ln(c*x^n))/x^3-3/2*d^2*e*(a+b*ln(c*x^n))/x^2-3*d*e^2*(a+b*ln(c*x^n))
/x+e^3*ln(x)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {45, 2372, 14, 2338}

$$\int \frac{(d+ex)^3(a+b\log(cx^n))}{x^4}\,dx = -\frac{d^3(a+b\log(cx^n))}{3x^3}-\frac{3d^2e(a+b\log(cx^n))}{2x^2}-\frac{3de^2(a+b\log(cx^n))}{x}+e^3\log(x)(a+b\log(cx^n))-\frac{bd^3n}{9x^3}-\frac{3bd^2en}{4x^2}-\frac{3bde^2n}{x}-\frac{1}{2}be^3n\log^2(x)$$

```
[In] Int[((d + e*x)^3*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] -1/9*(b*d^3*n)/x^3 - (3*b*d^2*e*n)/(4*x^2) - (3*b*d*e^2*n)/x - (b*e^3*n*Log
[x]^2)/2 - (d^3*(a + b*Log[c*x^n]))/(3*x^3) - (3*d^2*e*(a + b*Log[c*x^n]))/
(2*x^2) - (3*d*e^2*(a + b*Log[c*x^n]))/x + e^3*Log[x]*(a + b*Log[c*x^n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{3x^3}-\frac{3d^2e(a+b\log(cx^n))}{2x^2}-\frac{3de^2(a+b\log(cx^n))}{x} \\
&\quad +e^3\log(x)\,(a+b\log(cx^n))-(bn)\int\left(-\frac{d(2d^2+9dex+18e^2x^2)}{6x^4}+\frac{e^3\log(x)}{x}\right)dx \\
&= -\frac{d^3(a+b\log(cx^n))}{3x^3}-\frac{3d^2e(a+b\log(cx^n))}{2x^2} \\
&\quad -\frac{3de^2(a+b\log(cx^n))}{x}+e^3\log(x)\,(a+b\log(cx^n)) \\
&\quad +\frac{1}{6}(bdn)\int\frac{2d^2+9dex+18e^2x^2}{x^4}\,dx-(be^3n)\int\frac{\log(x)}{x}\,dx \\
&= -\frac{1}{2}be^3n\log^2(x)-\frac{d^3(a+b\log(cx^n))}{3x^3}-\frac{3d^2e(a+b\log(cx^n))}{2x^2}-\frac{3de^2(a+b\log(cx^n))}{x} \\
&\quad +e^3\log(x)\,(a+b\log(cx^n))+\frac{1}{6}(bdn)\int\left(\frac{2d^2}{x^4}+\frac{9de}{x^3}+\frac{18e^2}{x^2}\right)dx
\end{aligned}$$

$$
= -\frac{bd^3n}{9x^3} - \frac{3bd^2en}{4x^2} - \frac{3bde^2n}{x} - \frac{1}{2}be^3n\log^2(x) - \frac{d^3(a+b\log(cx^n))}{3x^3} - \frac{3d^2e(a+b\log(cx^n))}{2x^2} - \frac{3de^2(a+b\log(cx^n))}{x} + e^3\log(x)\left(a+b\log(cx^n)\right)
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.97

$$
\int \frac{(d+ex)^3\left(a+b\log(cx^n)\right)}{x^4}\,dx = -\frac{bd^3n}{9x^3} - \frac{3bd^2en}{4x^2} - \frac{3bde^2n}{x} - \frac{d^3(a+b\log(cx^n))}{3x^3} - \frac{3d^2e(a+b\log(cx^n))}{2x^2} - \frac{3de^2(a+b\log(cx^n))}{x} + \frac{e^3(a+b\log(cx^n))^2}{2bn}
$$

```
[In] Integrate[((d + e*x)^3*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] -1/9*(b*d^3*n)/x^3 - (3*b*d^2*e*n)/(4*x^2) - (3*b*d*e^2*n)/x - (d^3*(a + b*
Log[c*x^n]))/(3*x^3) - (3*d^2*e*(a + b*Log[c*x^n]))/(2*x^2) - (3*d*e^2*(a +
 b*Log[c*x^n]))/x + (e^3*(a + b*Log[c*x^n])^2)/(2*b*n)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{36\ln(x)x^3a\,e^3n+18e^3b\ln(c\,x^n)^2x^3-108x^2\ln(c\,x^n)bd\,e^2n-108x^2bd\,e^2n^2-108x^2ad\,e^2n-54x\ln(c\,x^n)b\,d^2en-27xb\,d^2e\,n^2-54xa}{36x^3n}$
risch	$-\frac{b(-6e^3\ln(x)x^3+18d\,e^2x^2+9d^2ex+2d^3)\ln(x^n)}{6x^3}-\frac{-54i\pi bd\,e^2x^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)-27i\pi b\,d^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)}{}$

```
[In] int((e*x+d)^3*(a+b*ln(c*x^n))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/36/x^3*(36*ln(x)*x^3*a*e^3*n+18*e^3*b*ln(c*x^n)^2*x^3-108*x^2*ln(c*x^n)*b
*d*e^2*n-108*x^2*b*d*e^2*n^2-108*x^2*a*d*e^2*n-54*x*ln(c*x^n)*b*d^2*e*n-27*
x*b*d^2*e*n^2-54*x*a*d^2*e*n-12*ln(c*x^n)*b*d^3*n-4*b*d^3*n^2-12*a*d^3*n)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^4}\,dx = \frac{18\,be^3nx^3\log(x)^2 - 4\,bd^3n - 12\,ad^3 - 108\,(bde^2n + ade^2)x^2 - 27\,(bd^2en + 2\,ad^2e)x - 6\,(18\,bde^2x^2 + 9\,bd}{36\,x^3}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^4,x, algorithm="fricas")
```

```
[Out] 1/36*(18*b*e^3*n*x^3*log(x)^2 - 4*b*d^3*n - 12*a*d^3 - 108*(b*d*e^2*n + a*d
*e^2)*x^2 - 27*(b*d^2*e*n + 2*a*d^2*e)*x - 6*(18*b*d*e^2*x^2 + 9*b*d^2*e*x
+ 2*b*d^3)*log(c) + 6*(6*b*e^3*x^3*log(c) - 18*b*d*e^2*n*x^2 + 6*a*e^3*x^3
- 9*b*d^2*e*n*x - 2*b*d^3*n)*log(x))/x^3
```

Sympy [A] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^4}\,dx = & -\frac{ad^3}{3x^3} - \frac{3ad^2e}{2x^2} - \frac{3ade^2}{x} + ae^3\log(x) \\ & + bd^3\left(-\frac{n}{9x^3} - \frac{\log(cx^n)}{3x^3}\right) + 3bd^2e\left(-\frac{n}{4x^2} - \frac{\log(cx^n)}{2x^2}\right) \\ & + 3bde^2\left(-\frac{n}{x} - \frac{\log(cx^n)}{x}\right) \\ & - be^3\left(\begin{cases} -\log(c)\log(x) & \text{for } n = 0 \\ -\frac{\log(cx^n)^2}{2n} & \text{otherwise} \end{cases}\right)\end{aligned}$$

```
[In] integrate((e*x+d)**3*(a+b*ln(c*x**n))/x**4,x)
```

```
[Out] -a*d**3/(3*x**3) - 3*a*d**2*e/(2*x**2) - 3*a*d*e**2/x + a*e**3*log(x) + b*d
**3*(-n/(9*x**3) - log(c*x**n)/(3*x**3)) + 3*b*d**2*e*(-n/(4*x**2) - log(c*
x**n)/(2*x**2)) + 3*b*d*e**2*(-n/x - log(c*x**n)/x) - b*e**3*Piecewise((-lo
g(c)*log(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^4}\,dx = \frac{be^3\log(cx^n)^2}{2n} + ae^3\log(x) - \frac{3\,bde^2n}{x} - \frac{3\,bde^2\log(cx^n)}{x} - \frac{3\,bd^2en}{4\,x^2} - \frac{3\,ade^2}{x} - \frac{3\,bd^2e\log(cx^n)}{2\,x^2} - \frac{bd^3n}{9\,x^3} - \frac{3\,ad^2e}{2\,x^2} - \frac{bd^3\log(cx^n)}{3\,x^3} - \frac{ad^3}{3\,x^3}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*b*e^3*log(c*x^n)^2/n + a*e^3*log(x) - 3*b*d*e^2*n/x - 3*b*d*e^2*log(c*x
^n)/x - 3/4*b*d^2*e*n/x^2 - 3*a*d*e^2/x - 3/2*b*d^2*e*log(c*x^n)/x^2 - 1/9*
b*d^3*n/x^3 - 3/2*a*d^2*e/x^2 - 1/3*b*d^3*log(c*x^n)/x^3 - 1/3*a*d^3/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^4}\,dx = \frac{1}{2}\,be^3n\log(x)^2 - 3\,bde^2n\left(\frac{\log(x)}{x} + \frac{1}{x}\right) - \frac{3}{4}\,bd^2en\left(\frac{2\,\log(x)}{x^2} + \frac{1}{x^2}\right) - \frac{1}{9}\,bd^3n\left(\frac{3\,\log(x)}{x^3} + \frac{1}{x^3}\right) + be^3\log(c)\log(|x|) + ae^3\log(|x|) - \frac{3\,bde^2\log(c)}{x} - \frac{3\,ade^2}{x} - \frac{3\,bd^2e\log(c)}{2\,x^2} - \frac{3\,ad^2e}{2\,x^2} - \frac{bd^3\log(c)}{3\,x^3} - \frac{ad^3}{3\,x^3}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^4,x, algorithm="giac")
```

```
[Out] 1/2*b*e^3*n*log(x)^2 - 3*b*d*e^2*n*(log(x)/x + 1/x) - 3/4*b*d^2*e*n*(2*log(
x)/x^2 + 1/x^2) - 1/9*b*d^3*n*(3*log(x)/x^3 + 1/x^3) + b*e^3*log(c)*log(abs
(x)) + a*e^3*log(abs(x)) - 3*b*d*e^2*log(c)/x - 3*a*d*e^2/x - 3/2*b*d^2*e*l
og(c)/x^2 - 3/2*a*d^2*e/x^2 - 1/3*b*d^3*log(c)/x^3 - 1/3*a*d^3/x^3
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^4}\,dx$$
$$= \ln(x)\left(a\,e^3+\frac{11\,b\,e^3\,n}{6}\right) - \frac{x\left(9\,a\,d^2\,e+\frac{9\,b\,d^2\,e\,n}{2}\right)+2\,a\,d^3+x^2\,(18\,a\,d\,e^2+18\,b\,d\,e^2\,n)+\frac{2\,b\,d^3\,n}{3}}{6\,x^3} - \frac{\ln(c\,x^n)\,\left(\frac{b\,d^3}{3}+\frac{3\,b\,d^2\,e\,x}{2}+3\,b\,d\,e^2\,x^2+\frac{11\,b\,e^3\,x^3}{6}\right)}{x^3}+\frac{b\,e^3\ln(c\,x^n)^2}{2\,n}$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^3)/x^4,x)
```

```
[Out] log(x)*(a*e^3 + (11*b*e^3*n)/6) - (x*(9*a*d^2*e + (9*b*d^2*e*n)/2) + 2*a*d^
3 + x^2*(18*a*d*e^2 + 18*b*d*e^2*n) + (2*b*d^3*n)/3)/(6*x^3) - (log(c*x^n)*
((b*d^3)/3 + (11*b*e^3*x^3)/6 + (3*b*d^2*e*x)/2 + 3*b*d*e^2*x^2))/x^3 + (b*
e^3*log(c*x^n)^2)/(2*n)
```

3.27 $\int \frac{(d+ex)^3(a+b\log(cx^n))}{x^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{(d+ex)^3(a+b\log(cx^n))}{x^5}\,dx = -\frac{bd^3n}{16x^4}-\frac{bd^2en}{3x^3}-\frac{3bde^2n}{4x^2}-\frac{be^3n}{x}+\frac{be^4n\log(x)}{4d}-\frac{(d+ex)^4(a+b\log(cx^n))}{4dx^4}$$

```
[Out] -1/16*b*d^3*n/x^4-1/3*b*d^2*e*n/x^3-3/4*b*d*e^2*n/x^2-b*e^3*n/x+1/4*b*e^4*n
*ln(x)/d-1/4*(e*x+d)^4*(a+b*ln(c*x^n))/d/x^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {37, 2372, 12, 45}

$$\int \frac{(d+ex)^3(a+b\log(cx^n))}{x^5}\,dx = -\frac{(d+ex)^4(a+b\log(cx^n))}{4dx^4}-\frac{bd^3n}{16x^4}-\frac{bd^2en}{3x^3}+\frac{be^4n\log(x)}{4d}-\frac{3bde^2n}{4x^2}-\frac{be^3n}{x}$$

```
[In] Int[((d + e*x)^3*(a + b*Log[c*x^n]))/x^5,x]
```

```
[Out] -1/16*(b*d^3*n)/x^4 - (b*d^2*e*n)/(3*x^3) - (3*b*d*e^2*n)/(4*x^2) - (b*e^3*
n)/x + (b*e^4*n*Log[x])/(4*d) - ((d + e*x)^4*(a + b*Log[c*x^n]))/(4*d*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d+ex)^4\,(a+b\log(cx^n))}{4dx^4}-(bn)\int -\frac{(d+ex)^4}{4dx^5}\,dx\\
&= -\frac{(d+ex)^4\,(a+b\log(cx^n))}{4dx^4}+\frac{(bn)\int\frac{(d+ex)^4}{x^5}\,dx}{4d}\\
&= -\frac{(d+ex)^4\,(a+b\log(cx^n))}{4dx^4}+\frac{(bn)\int\left(\frac{d^4}{x^5}+\frac{4d^3e}{x^4}+\frac{6d^2e^2}{x^3}+\frac{4de^3}{x^2}+\frac{e^4}{x}\right)dx}{4d}\\
&= -\frac{bd^3n}{16x^4}-\frac{bd^2en}{3x^3}-\frac{3bde^2n}{4x^2}-\frac{be^3n}{x}+\frac{be^4n\log(x)}{4d}-\frac{(d+ex)^4\,(a+b\log(cx^n))}{4dx^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.21

$$\int\frac{(d+ex)^3\,(a+b\log(cx^n))}{x^5}\,dx=$$

$$-\frac{12a(d^3+4d^2ex+6de^2x^2+4e^3x^3)+bn(3d^3+16d^2ex+36de^2x^2+48e^3x^3)+12b(d^3+4d^2ex+6de^2x^2}{48x^4}$$

```
[In] Integrate[((d + e*x)^3*(a + b*Log[c*x^n]))/x^5,x]
```

```
[Out] -1/48*(12*a*(d^3 + 4*d^2*e*x + 6*d*e^2*x^2 + 4*e^3*x^3) + b*n*(3*d^3 + 16*d
^2*e*x + 36*d*e^2*x^2 + 48*e^3*x^3) + 12*b*(d^3 + 4*d^2*e*x + 6*d*e^2*x^2 +
 4*e^3*x^3)*Log[c*x^n])/x^4
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.49

method	result
parallelrisch	$-\frac{48b\ln(c\,x^n)e^3x^3+48b\,e^3n\,x^3+48a\,e^3x^3+72b\ln(c\,x^n)d\,e^2x^2+36bd\,e^2n\,x^2+72ad\,e^2x^2+48b\ln(c\,x^n)d^2ex+16b\,d^2enx+48a\,d^2ex}{48x^4}$
risch	$-\frac{b(4e^3x^3+6d\,e^2x^2+4d^2ex+d^3)\ln(x^n)}{4x^4}-\frac{48\ln(c)b\,e^3x^3-36i\pi bd\,e^2x^2\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)-24i\pi b\,d^2\operatorname{csgn}(ic)\operatorname{csgn}(i}{}$

[In] int((e*x+d)^3*(a+b*ln(c*x^n))/x^5,x,method=_RETURNVERBOSE)

[Out] -1/48/x^4*(48*b*ln(c*x^n)*e^3*x^3+48*b*e^3*n*x^3+48*a*e^3*x^3+72*b*ln(c*x^n
)*d*e^2*x^2+36*b*d*e^2*n*x^2+72*a*d*e^2*x^2+48*b*ln(c*x^n)*d^2*e*x+16*b*d^2
*e*n*x+48*a*d^2*e*x+12*b*ln(c*x^n)*d^3+3*b*d^3*n+12*a*d^3)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.69

$$\int \frac{(d+ex)^3\,(a+b\log(cx^n))}{x^5}\,dx = -\frac{3\,bd^3n+12\,ad^3+48\,(be^3n+ae^3)x^3+36\,(bde^2n+2\,ade^2)x^2+16\,(bd^2en+3\,ad^2e)x+12\,(4\,be^3x^3+6\,bd}{48\,x^4}$$

[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^5,x, algorithm="fricas")

[Out] -1/48*(3*b*d^3*n + 12*a*d^3 + 48*(b*e^3*n + a*e^3)*x^3 + 36*(b*d*e^2*n + 2*
a*d*e^2)*x^2 + 16*(b*d^2*e*n + 3*a*d^2*e)*x + 12*(4*b*e^3*x^3 + 6*b*d*e^2*x
^2 + 4*b*d^2*e*x + b*d^3)*log(c) + 12*(4*b*e^3*n*x^3 + 6*b*d*e^2*n*x^2 + 4*
b*d^2*e*n*x + b*d^3*n)*log(x))/x^4

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex)^3\,(a+b\log(cx^n))}{x^5}\,dx = -\frac{ad^3}{4x^4}-\frac{ad^2e}{x^3}-\frac{3ade^2}{2x^2}-\frac{ae^3}{x}-\frac{bd^3n}{16x^4} -\frac{bd^3\log(cx^n)}{4x^4}-\frac{bd^2en}{3x^3}-\frac{bd^2e\log(cx^n)}{x^3} -\frac{3bde^2n}{4x^2}-\frac{3bde^2\log(cx^n)}{2x^2}-\frac{be^3n}{x}-\frac{be^3\log(cx^n)}{x}$$

[In] integrate((e*x+d)**3*(a+b*ln(c*x**n))/x**5,x)

```
[Out] -a*d**3/(4*x**4) - a*d**2*e/x**3 - 3*a*d*e**2/(2*x**2) - a*e**3/x - b*d**3*
n/(16*x**4) - b*d**3*log(c*x**n)/(4*x**4) - b*d**2*e*n/(3*x**3) - b*d**2*e*
log(c*x**n)/x**3 - 3*b*d*e**2*n/(4*x**2) - 3*b*d*e**2*log(c*x**n)/(2*x**2)
- b*e**3*n/x - b*e**3*log(c*x**n)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.59

$$\int \frac{(d+ex)^3\,(a+b\log(cx^n))}{x^5}\,dx = -\frac{be^3n}{x} - \frac{be^3\log(cx^n)}{x} - \frac{3\,bde^2n}{4\,x^2} - \frac{ae^3}{x} - \frac{3\,bde^2\log(cx^n)}{2\,x^2} - \frac{bd^2en}{3\,x^3} - \frac{3\,ade^2}{2\,x^2} - \frac{bd^2e\log(cx^n)}{x^3} - \frac{bd^3n}{16\,x^4} - \frac{ad^2e}{x^3} - \frac{bd^3\log(cx^n)}{4\,x^4} - \frac{ad^3}{4\,x^4}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^5,x, algorithm="maxima")
```

```
[Out] -b*e^3*n/x - b*e^3*log(c*x^n)/x - 3/4*b*d*e^2*n/x^2 - a*e^3/x - 3/2*b*d*e^2
*log(c*x^n)/x^2 - 1/3*b*d^2*e*n/x^3 - 3/2*a*d*e^2/x^2 - b*d^2*e*log(c*x^n)/
x^3 - 1/16*b*d^3*n/x^4 - a*d^2*e/x^3 - 1/4*b*d^3*log(c*x^n)/x^4 - 1/4*a*d^3
/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(80) = 160$.

Time = 0.36 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.80

$$\int \frac{(d+ex)^3\,(a+b\log(cx^n))}{x^5}\,dx = -\frac{(4\,be^3nx^3 + 6\,bde^2nx^2 + 4\,bd^2enx + bd^3n)\log(x)}{4\,x^4} - \frac{48\,be^3nx^3 + 48\,be^3x^3\log(c) + 36\,bde^2nx^2 + 48\,ae^3x^3 + 72\,bde^2x^2\log(c) + 16\,bd^2enx + 72\,ade^2x^2 + 48\,b}{48\,x^4}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(4*b*e^3*n*x^3 + 6*b*d*e^2*n*x^2 + 4*b*d^2*e*n*x + b*d^3*n)*log(x)/x^4
 - 1/48*(48*b*e^3*n*x^3 + 48*b*e^3*x^3*log(c) + 36*b*d*e^2*n*x^2 + 48*a*e^3
*x^3 + 72*b*d*e^2*x^2*log(c) + 16*b*d^2*e*n*x + 72*a*d*e^2*x^2 + 48*b*d^2*e
*x*log(c) + 3*b*d^3*n + 48*a*d^2*e*x + 12*b*d^3*log(c) + 12*a*d^3)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^3 (a+b\log (cx^n))}{x^5} \, dx$$
$$= -\frac{x^3 (4 a e^3 + 4 b e^3 n) + x \left(4 a d^2 e + \frac{4 b d^2 e n}{3}\right) + a d^3 + x^2 (6 a d e^2 + 3 b d e^2 n) + \frac{b d^3 n}{4}}{4 x^4}$$
$$- \frac{\ln (c x^n) \left(\frac{b d^3}{4} + b d^2 e x + \frac{3 b d e^2 x^2}{2} + b e^3 x^3\right)}{x^4}$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^3)/x^5,x)
```

```
[Out] - (x^3*(4*a*e^3 + 4*b*e^3*n) + x*(4*a*d^2*e + (4*b*d^2*e*n)/3) + a*d^3 + x^
2*(6*a*d*e^2 + 3*b*d*e^2*n) + (b*d^3*n)/4)/(4*x^4) - (log(c*x^n)*((b*d^3)/4
 + b*e^3*x^3 + b*d^2*e*x + (3*b*d*e^2*x^2)/2))/x^4
```

3.28 $\int \frac{(d+ex)^3(a+b\log(cx^n))}{x^6}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 142

$$\int \frac{(d+ex)^3\,(a+b\log(cx^n))}{x^6}\,dx = \frac{bd^2en}{80x^4}+\frac{bde^2n}{15x^3}+\frac{3be^3n}{20x^2}+\frac{be^4n}{5dx}-\frac{bn(d+ex)^5}{25d^2x^5}-\frac{be^5n\log(x)}{20d^2} -\frac{(d+ex)^4\,(a+b\log(cx^n))}{5dx^5}+\frac{e(d+ex)^4\,(a+b\log(cx^n))}{20d^2x^4}$$

```
[Out] 1/80*b*d^2*e*n/x^4+1/15*b*d*e^2*n/x^3+3/20*b*e^3*n/x^2+1/5*b*e^4*n/d/x-1/25
*b*n*(e*x+d)^5/d^2/x^5-1/20*b*e^5*n*ln(x)/d^2-1/5*(e*x+d)^4*(a+b*ln(c*x^n))
/d/x^5+1/20*e*(e*x+d)^4*(a+b*ln(c*x^n))/d^2/x^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {47, 37, 2372, 12, 79, 45}

$$\int \frac{(d+ex)^3\,(a+b\log(cx^n))}{x^6}\,dx = \frac{e(d+ex)^4\,(a+b\log(cx^n))}{20d^2x^4} -\frac{(d+ex)^4\,(a+b\log(cx^n))}{5dx^5}-\frac{be^5n\log(x)}{20d^2} -\frac{bn(d+ex)^5}{25d^2x^5}+\frac{bd^2en}{80x^4}+\frac{be^4n}{5dx}+\frac{bde^2n}{15x^3}+\frac{3be^3n}{20x^2}$$

```
[In] Int[((d + e*x)^3*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] (b*d^2*e*n)/(80*x^4) + (b*d*e^2*n)/(15*x^3) + (3*b*e^3*n)/(20*x^2) + (b*e^4
*n)/(5*d*x) - (b*n*(d + e*x)^5)/(25*d^2*x^5) - (b*e^5*n*Log[x])/(20*d^2) -
((d + e*x)^4*(a + b*Log[c*x^n]))/(5*d*x^5) + (e*(d + e*x)^4*(a + b*Log[c*x^
n]))/(20*d^2*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(d+ex)^4\left(a+b\log\left(cx^n\right)\right)}{5dx^5}+\frac{e(d+ex)^4\left(a+b\log\left(cx^n\right)\right)}{20d^2x^4}\\&\quad-(bn)\int\frac{(-4d+ex)(d+ex)^4}{20d^2x^6}\,dx\\&=-\frac{(d+ex)^4\left(a+b\log\left(cx^n\right)\right)}{5dx^5}+\frac{e(d+ex)^4\left(a+b\log\left(cx^n\right)\right)}{20d^2x^4}-\frac{(bn)\int\frac{(-4d+ex)(d+ex)^4}{x^6}\,dx}{20d^2}\\&=-\frac{bn(d+ex)^5}{25d^2x^5}-\frac{(d+ex)^4\left(a+b\log\left(cx^n\right)\right)}{5dx^5}+\frac{e(d+ex)^4\left(a+b\log\left(cx^n\right)\right)}{20d^2x^4}-\frac{(ben)\int\frac{(d+ex)^4}{x^5}\,dx}{20d^2}\\&=-\frac{bn(d+ex)^5}{25d^2x^5}-\frac{(d+ex)^4\left(a+b\log\left(cx^n\right)\right)}{5dx^5}+\frac{e(d+ex)^4\left(a+b\log\left(cx^n\right)\right)}{20d^2x^4}\\&\quad-\frac{(ben)\int\left(\frac{d^4}{x^5}+\frac{4d^3e}{x^4}+\frac{6d^2e^2}{x^3}+\frac{4de^3}{x^2}+\frac{e^4}{x}\right)dx}{20d^2}\\&=\frac{bd^2en}{80x^4}+\frac{bde^2n}{15x^3}+\frac{3be^3n}{20x^2}+\frac{be^4n}{5dx}-\frac{bn(d+ex)^5}{25d^2x^5}-\frac{be^5n\log(x)}{20d^2}\\&\quad-\frac{(d+ex)^4\left(a+b\log\left(cx^n\right)\right)}{5dx^5}+\frac{e(d+ex)^4\left(a+b\log\left(cx^n\right)\right)}{20d^2x^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.80

$$\int\frac{(d+ex)^3\left(a+b\log\left(cx^n\right)\right)}{x^6}\,dx=\\-\frac{60a(4d^3+15d^2ex+20de^2x^2+10e^3x^3)+bn(48d^3+225d^2ex+400de^2x^2+300e^3x^3)+60b(4d^3+15d^2}{1200x^5}$$

```
[In] Integrate[((d + e*x)^3*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] -1/1200*(60*a*(4*d^3 + 15*d^2*e*x + 20*d*e^2*x^2 + 10*e^3*x^3) + b*n*(48*d^
3 + 225*d^2*e*x + 400*d*e^2*x^2 + 300*e^3*x^3) + 60*b*(4*d^3 + 15*d^2*e*x +
 20*d*e^2*x^2 + 10*e^3*x^3)*Log[c*x^n])/x^5
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

method	result
parallelrisch	$-\frac{600b\ln(c\,x^n)e^3x^3+300b\,e^3n\,x^3+600a\,e^3x^3+1200b\ln(c\,x^n)d\,e^2x^2+400bd\,e^2n\,x^2+1200ad\,e^2x^2+900b\ln(c\,x^n)d^2ex+225b\,d^2en}{1200x^5}$
risch	$-\frac{b(10e^3x^3+20d\,e^2x^2+15d^2ex+4d^3)\ln(x^n)}{20x^5}-\frac{600\ln(c)b\,e^3x^3-600i\pi bd\,e^2x^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)-450i\pi b\,d^2\,\mathrm{csgn}(i}{}$

[In] int((e*x+d)^3*(a+b*ln(c*x^n))/x^6,x,method=_RETURNVERBOSE)

[Out] -1/1200/x^5*(600*b*ln(c*x^n)*e^3*x^3+300*b*e^3*n*x^3+600*a*e^3*x^3+1200*b*l n(c*x^n)*d*e^2*x^2+400*b*d*e^2*n*x^2+1200*a*d*e^2*x^2+900*b*ln(c*x^n)*d^2*e *x+225*b*d^2*e*n*x+900*a*d^2*e*x+240*b*ln(c*x^n)*d^3+48*b*d^3*n+240*a*d^3)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^3\,(a+b\log(cx^n))}{x^6}\,dx = -\frac{48\,bd^3n+240\,ad^3+300\,(be^3n+2\,ae^3)x^3+400\,(bde^2n+3\,ade^2)x^2+225\,(bd^2en+4\,ad^2e)x+60\,(10\,be^3}{1200}$$

[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^6,x, algorithm="fricas")

[Out] -1/1200*(48*b*d^3*n + 240*a*d^3 + 300*(b*e^3*n + 2*a*e^3)*x^3 + 400*(b*d*e^ 2*n + 3*a*d*e^2)*x^2 + 225*(b*d^2*e*n + 4*a*d^2*e)*x + 60*(10*b*e^3*x^3 + 2 0*b*d*e^2*x^2 + 15*b*d^2*e*x + 4*b*d^3)*log(c) + 60*(10*b*e^3*n*x^3 + 20*b* d*e^2*n*x^2 + 15*b*d^2*e*n*x + 4*b*d^3*n)*log(x))/x^5

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^3\,(a+b\log(cx^n))}{x^6}\,dx = -\frac{ad^3}{5x^5}-\frac{3ad^2e}{4x^4}-\frac{ade^2}{x^3}-\frac{ae^3}{2x^2}-\frac{bd^3n}{25x^5}-\frac{bd^3\log(cx^n)}{5x^5}-\frac{3bd^2en}{16x^4}-\frac{3bd^2e\log(cx^n)}{4x^4}-\frac{bde^2n}{3x^3}-\frac{bde^2\log(cx^n)}{x^3}-\frac{be^3n}{4x^2}-\frac{be^3\log(cx^n)}{2x^2}$$

[In] integrate((e*x+d)**3*(a+b*ln(c*x**n))/x**6,x)

```
[Out] -a*d**3/(5*x**5) - 3*a*d**2*e/(4*x**4) - a*d*e**2/x**3 - a*e**3/(2*x**2) -
b*d**3*n/(25*x**5) - b*d**3*log(c*x**n)/(5*x**5) - 3*b*d**2*e*n/(16*x**4) -
 3*b*d**2*e*log(c*x**n)/(4*x**4) - b*d*e**2*n/(3*x**3) - b*d*e**2*log(c*x**
n)/x**3 - b*e**3*n/(4*x**2) - b*e**3*log(c*x**n)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^6}\,dx = -\frac{be^3n}{4\,x^2} - \frac{be^3\log(cx^n)}{2\,x^2} - \frac{bde^2n}{3\,x^3} - \frac{ae^3}{2\,x^2} - \frac{bde^2\log(cx^n)}{x^3} - \frac{3\,bd^2en}{16\,x^4} - \frac{ade^2}{x^3} - \frac{3\,bd^2e\log(cx^n)}{4\,x^4} - \frac{bd^3n}{25\,x^5} - \frac{3\,ad^2e}{4\,x^4} - \frac{bd^3\log(cx^n)}{5\,x^5} - \frac{ad^3}{5\,x^5}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^6,x, algorithm="maxima")
```

```
[Out] -1/4*b*e^3*n/x^2 - 1/2*b*e^3*log(c*x^n)/x^2 - 1/3*b*d*e^2*n/x^3 - 1/2*a*e^3
/x^2 - b*d*e^2*log(c*x^n)/x^3 - 3/16*b*d^2*e*n/x^4 - a*d*e^2/x^3 - 3/4*b*d^
2*e*log(c*x^n)/x^4 - 1/25*b*d^3*n/x^5 - 3/4*a*d^2*e/x^4 - 1/5*b*d^3*log(c*x
^n)/x^5 - 1/5*a*d^3/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^6}\,dx = -\frac{(10\,be^3nx^3 + 20\,bde^2nx^2 + 15\,bd^2enx + 4\,bd^3n)\log(x)}{20\,x^5} - \frac{300\,be^3nx^3 + 600\,be^3x^3\log(c) + 400\,bde^2nx^2 + 600\,ae^3x^3 + 1200\,bde^2x^2\log(c) + 225\,bd^2enx + 1200\,ad}{1200\,x^5}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^6,x, algorithm="giac")
```

```
[Out] -1/20*(10*b*e^3*n*x^3 + 20*b*d*e^2*n*x^2 + 15*b*d^2*e*n*x + 4*b*d^3*n)*log(
x)/x^5 - 1/1200*(300*b*e^3*n*x^3 + 600*b*e^3*x^3*log(c) + 400*b*d*e^2*n*x^2
 + 600*a*e^3*x^3 + 1200*b*d*e^2*x^2*log(c) + 225*b*d^2*e*n*x + 1200*a*d*e^2
*x^2 + 900*b*d^2*e*x*log(c) + 48*b*d^3*n + 900*a*d^2*e*x + 240*b*d^3*log(c)
 + 240*a*d^3)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^6}\,dx = -\frac{x^3 (10\,a\,e^3+5\,b\,e^3\,n)+x \left(15\,a\,d^2\,e+\frac{15\,b\,d^2\,e\,n}{4}\right)+4\,a\,d^3+x^2 \left(20\,a\,d\,e^2+\frac{20\,b\,d\,e^2\,n}{3}\right)+\frac{4\,b\,d^3\,n}{5}}{20\,x^5} - \frac{\ln(c\,x^n) \left(\frac{b\,d^3}{5}+\frac{3\,b\,d^2\,e\,x}{4}+b\,d\,e^2\,x^2+\frac{b\,e^3\,x^3}{2}\right)}{x^5}$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^3)/x^6,x)
```

```
[Out] - (x^3*(10*a*e^3 + 5*b*e^3*n) + x*(15*a*d^2*e + (15*b*d^2*e*n)/4) + 4*a*d^3
 + x^2*(20*a*d*e^2 + (20*b*d*e^2*n)/3) + (4*b*d^3*n)/5)/(20*x^5) - (log(c*x
^n)*((b*d^3)/5 + (b*e^3*x^3)/2 + (3*b*d^2*e*x)/4 + b*d*e^2*x^2))/x^5
```

3.29 $\int \frac{(d+ex)^3(a+b\log(cx^n))}{x^7}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 133

$$\int \frac{(d+ex)^3\,(a+b\log{(cx^n)})}{x^7}\,dx = -\frac{bd^3n}{36x^6}-\frac{3bd^2en}{25x^5}-\frac{3bde^2n}{16x^4}-\frac{be^3n}{9x^3}-\frac{d^3(a+b\log{(cx^n)})}{6x^6}-\frac{3d^2e(a+b\log{(cx^n)})}{5x^5}-\frac{3de^2(a+b\log{(cx^n)})}{4x^4}-\frac{e^3(a+b\log{(cx^n)})}{3x^3}$$

```
[Out] -1/36*b*d^3*n/x^6-3/25*b*d^2*e*n/x^5-3/16*b*d*e^2*n/x^4-1/9*b*e^3*n/x^3-1/6
*d^3*(a+b*ln(c*x^n))/x^6-3/5*d^2*e*(a+b*ln(c*x^n))/x^5-3/4*d*e^2*(a+b*ln(c*
x^n))/x^4-1/3*e^3*(a+b*ln(c*x^n))/x^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {45, 2372, 12, 14}

$$\int \frac{(d+ex)^3\,(a+b\log{(cx^n)})}{x^7}\,dx = -\frac{d^3(a+b\log{(cx^n)})}{6x^6}-\frac{3d^2e(a+b\log{(cx^n)})}{5x^5}-\frac{3de^2(a+b\log{(cx^n)})}{4x^4}-\frac{e^3(a+b\log{(cx^n)})}{3x^3}-\frac{bd^3n}{36x^6}-\frac{3bd^2en}{25x^5}-\frac{3bde^2n}{16x^4}-\frac{be^3n}{9x^3}$$

```
[In] Int[((d + e*x)^3*(a + b*Log[c*x^n]))/x^7,x]
```

```
[Out] -1/36*(b*d^3*n)/x^6 - (3*b*d^2*e*n)/(25*x^5) - (3*b*d*e^2*n)/(16*x^4) - (b*
e^3*n)/(9*x^3) - (d^3*(a + b*Log[c*x^n]))/(6*x^6) - (3*d^2*e*(a + b*Log[c*x
```

```
^n]))/(5*x^5) - (3*d*e^2*(a + b*Log[c*x^n]))/(4*x^4) - (e^3*(a + b*Log[c*x^
n]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{6x^6}-\frac{3d^2e(a+b\log(cx^n))}{5x^5}-\frac{3de^2(a+b\log(cx^n))}{4x^4}\\
&\quad-\frac{e^3(a+b\log(cx^n))}{3x^3}-(bn)\int\frac{-10d^3-36d^2ex-45de^2x^2-20e^3x^3}{60x^7}\,dx\\
&=-\frac{d^3(a+b\log(cx^n))}{6x^6}-\frac{3d^2e(a+b\log(cx^n))}{5x^5}-\frac{3de^2(a+b\log(cx^n))}{4x^4}\\
&\quad-\frac{e^3(a+b\log(cx^n))}{3x^3}-\frac{1}{60}(bn)\int\frac{-10d^3-36d^2ex-45de^2x^2-20e^3x^3}{x^7}\,dx\\
&=-\frac{d^3(a+b\log(cx^n))}{6x^6}-\frac{3d^2e(a+b\log(cx^n))}{5x^5}-\frac{3de^2(a+b\log(cx^n))}{4x^4}\\
&\quad-\frac{e^3(a+b\log(cx^n))}{3x^3}-\frac{1}{60}(bn)\int\left(-\frac{10d^3}{x^7}-\frac{36d^2e}{x^6}-\frac{45de^2}{x^5}-\frac{20e^3}{x^4}\right)dx
\end{aligned}$$

$$= -\frac{bd^3n}{36x^6} - \frac{3bd^2en}{25x^5} - \frac{3bde^2n}{16x^4} - \frac{be^3n}{9x^3} - \frac{d^3(a + b\log(cx^n))}{6x^6} - \frac{3d^2e(a + b\log(cx^n))}{5x^5} - \frac{3de^2(a + b\log(cx^n))}{4x^4} - \frac{e^3(a + b\log(cx^n))}{3x^3}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85

$$\int \frac{(d + ex)^3 (a + b\log(cx^n))}{x^7} dx = -\frac{60a(10d^3 + 36d^2ex + 45de^2x^2 + 20e^3x^3) + bn(100d^3 + 432d^2ex + 675de^2x^2 + 400e^3x^3) + 60b(10d^3 + 3}{3600x^6}$$

```
[In] Integrate[((d + e*x)^3*(a + b*Log[c*x^n]))/x^7,x]
```

```
[Out] -1/3600*(60*a*(10*d^3 + 36*d^2*e*x + 45*d*e^2*x^2 + 20*e^3*x^3) + b*n*(100*
d^3 + 432*d^2*e*x + 675*d*e^2*x^2 + 400*e^3*x^3) + 60*b*(10*d^3 + 36*d^2*e*
x + 45*d*e^2*x^2 + 20*e^3*x^3)*Log[c*x^n])/x^6
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.01

method	result
parallelrisch	$-\frac{1200b\ln(cx^n)e^3x^3+400b e^3n x^3+1200a e^3x^3+2700b\ln(cx^n)d e^2x^2+675bd e^2n x^2+2700ad e^2x^2+2160b\ln(cx^n)d^2ex+432b}{3600x^6}$
risch	$-\frac{b(20e^3x^3+45d e^2x^2+36d^2ex+10d^3)\ln(x^n)}{60x^6} - \frac{1200\ln(c)b e^3x^3-1350i\pi bd e^2x^2 \operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)-1080i\pi b d^2}{}$

```
[In] int((e*x+d)^3*(a+b*ln(c*x^n))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3600/x^6*(1200*b*ln(c*x^n)*e^3*x^3+400*b*e^3*n*x^3+1200*a*e^3*x^3+2700*b
*ln(c*x^n)*d*e^2*x^2+675*b*d*e^2*n*x^2+2700*a*d*e^2*x^2+2160*b*ln(c*x^n)*d^
2*e*x+432*b*d^2*e*n*x+2160*a*d^2*e*x+600*b*ln(c*x^n)*d^3+100*b*d^3*n+600*a*
d^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^7}\,dx = -\frac{100\,bd^3n + 600\,ad^3 + 400\,(be^3n + 3\,ae^3)x^3 + 675\,(bde^2n + 4\,ade^2)x^2 + 432\,(bd^2en + 5\,ad^2e)x + 60\,(20\,be}{3600}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^7,x, algorithm="fricas")
```

```
[Out] -1/3600*(100*b*d^3*n + 600*a*d^3 + 400*(b*e^3*n + 3*a*e^3)*x^3 + 675*(b*d*e
^2*n + 4*a*d*e^2)*x^2 + 432*(b*d^2*e*n + 5*a*d^2*e)*x + 60*(20*b*e^3*x^3 +
45*b*d*e^2*x^2 + 36*b*d^2*e*x + 10*b*d^3)*log(c) + 60*(20*b*e^3*n*x^3 + 45*
b*d*e^2*n*x^2 + 36*b*d^2*e*n*x + 10*b*d^3*n)*log(x))/x^6
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^7}\,dx = -\frac{ad^3}{6x^6} - \frac{3ad^2e}{5x^5} - \frac{3ade^2}{4x^4} - \frac{ae^3}{3x^3} - \frac{bd^3n}{36x^6} - \frac{bd^3\log(cx^n)}{6x^6} - \frac{3bd^2en}{25x^5} - \frac{3bd^2e\log(cx^n)}{5x^5} - \frac{3bde^2n}{16x^4} - \frac{3bde^2\log(cx^n)}{4x^4} - \frac{be^3n}{9x^3} - \frac{be^3\log(cx^n)}{3x^3}$$

```
[In] integrate((e*x+d)**3*(a+b*ln(c*x**n))/x**7,x)
```

```
[Out] -a*d**3/(6*x**6) - 3*a*d**2*e/(5*x**5) - 3*a*d*e**2/(4*x**4) - a*e**3/(3*x*
*3) - b*d**3*n/(36*x**6) - b*d**3*log(c*x**n)/(6*x**6) - 3*b*d**2*e*n/(25*x
**5) - 3*b*d**2*e*log(c*x**n)/(5*x**5) - 3*b*d*e**2*n/(16*x**4) - 3*b*d*e**
2*log(c*x**n)/(4*x**4) - b*e**3*n/(9*x**3) - b*e**3*log(c*x**n)/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^7}\,dx = -\frac{be^3n}{9\,x^3} - \frac{be^3\log(cx^n)}{3\,x^3} - \frac{3\,bde^2n}{16\,x^4} - \frac{ae^3}{3\,x^3} - \frac{3\,bde^2\log(cx^n)}{4\,x^4} - \frac{3\,bd^2en}{25\,x^5} - \frac{3\,ade^2}{4\,x^4} - \frac{3\,bd^2e\log(cx^n)}{5\,x^5} - \frac{bd^3n}{36\,x^6} - \frac{3\,ad^2e}{5\,x^5} - \frac{bd^3\log(cx^n)}{6\,x^6} - \frac{ad^3}{6\,x^6}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^7,x, algorithm="maxima")
```

```
[Out] -1/9*b*e^3*n/x^3 - 1/3*b*e^3*log(c*x^n)/x^3 - 3/16*b*d*e^2*n/x^4 - 1/3*a*e^
3/x^3 - 3/4*b*d*e^2*log(c*x^n)/x^4 - 3/25*b*d^2*e*n/x^5 - 3/4*a*d*e^2/x^4 -
 3/5*b*d^2*e*log(c*x^n)/x^5 - 1/36*b*d^3*n/x^6 - 3/5*a*d^2*e/x^5 - 1/6*b*d^
3*log(c*x^n)/x^6 - 1/6*a*d^3/x^6
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^3\,(a+b\log{(cx^n)})}{x^7}\,dx = -\frac{(20\,be^3nx^3+45\,bde^2nx^2+36\,bd^2enx+10\,bd^3n)\log{(x)}}{60\,x^6} - \frac{400\,be^3nx^3+1200\,be^3x^3\log{(c)}+675\,bde^2nx^2+1200\,ae^3x^3+2700\,bde^2x^2\log{(c)}+432\,bd^2enx+2700}{3600\,x^6}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^7,x, algorithm="giac")
```

```
[Out] -1/60*(20*b*e^3*n*x^3 + 45*b*d*e^2*n*x^2 + 36*b*d^2*e*n*x + 10*b*d^3*n)*log
(x)/x^6 - 1/3600*(400*b*e^3*n*x^3 + 1200*b*e^3*x^3*log(c) + 675*b*d*e^2*n*x
^2 + 1200*a*e^3*x^3 + 2700*b*d*e^2*x^2*log(c) + 432*b*d^2*e*n*x + 2700*a*d*
e^2*x^2 + 2160*b*d^2*e*x*log(c) + 100*b*d^3*n + 2160*a*d^2*e*x + 600*b*d^3*
log(c) + 600*a*d^3)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^3\,(a+b\log{(cx^n)})}{x^7}\,dx = -\frac{x^3\left(20\,a\,e^3+\frac{20\,b\,e^3\,n}{3}\right)+x\left(36\,a\,d^2\,e+\frac{36\,b\,d^2\,e\,n}{5}\right)+10\,a\,d^3+x^2\left(45\,a\,d\,e^2+\frac{45\,b\,d\,e^2\,n}{4}\right)+\frac{5\,b\,d^3\,n}{3}}{60\,x^6} - \frac{\ln{(c\,x^n)}\,\left(\frac{b\,d^3}{6}+\frac{3\,b\,d^2\,e\,x}{5}+\frac{3\,b\,d\,e^2\,x^2}{4}+\frac{b\,e^3\,x^3}{3}\right)}{x^6}$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^3)/x^7,x)
```

```
[Out] - (x^3*(20*a*e^3 + (20*b*e^3*n)/3) + x*(36*a*d^2*e + (36*b*d^2*e*n)/5) + 10
*a*d^3 + x^2*(45*a*d*e^2 + (45*b*d*e^2*n)/4) + (5*b*d^3*n)/3)/(60*x^6) - (l
og(c*x^n)*((b*d^3)/6 + (b*e^3*x^3)/3 + (3*b*d^2*e*x)/5 + (3*b*d*e^2*x^2)/4)
)/x^6
```

3.30 $\int \frac{(d+ex)^3(a+b\log(cx^n))}{x^8}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 133

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^8}\,dx = -\frac{bd^3n}{49x^7}-\frac{bd^2en}{12x^6}-\frac{3bde^2n}{25x^5}-\frac{be^3n}{16x^4}-\frac{d^3(a+b\log(cx^n))}{7x^7}-\frac{d^2e(a+b\log(cx^n))}{2x^6}-\frac{3de^2(a+b\log(cx^n))}{5x^5}-\frac{e^3(a+b\log(cx^n))}{4x^4}$$

```
[Out] -1/49*b*d^3*n/x^7-1/12*b*d^2*e*n/x^6-3/25*b*d*e^2*n/x^5-1/16*b*e^3*n/x^4-1/
7*d^3*(a+b*ln(c*x^n))/x^7-1/2*d^2*e*(a+b*ln(c*x^n))/x^6-3/5*d*e^2*(a+b*ln(c
*x^n))/x^5-1/4*e^3*(a+b*ln(c*x^n))/x^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {45, 2372, 12, 14}

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^8}\,dx = -\frac{d^3(a+b\log(cx^n))}{7x^7}-\frac{d^2e(a+b\log(cx^n))}{2x^6}-\frac{3de^2(a+b\log(cx^n))}{5x^5}-\frac{e^3(a+b\log(cx^n))}{4x^4}-\frac{bd^3n}{49x^7}-\frac{bd^2en}{12x^6}-\frac{3bde^2n}{25x^5}-\frac{be^3n}{16x^4}$$

```
[In] Int[((d + e*x)^3*(a + b*Log[c*x^n]))/x^8,x]
```

```
[Out] -1/49*(b*d^3*n)/x^7 - (b*d^2*e*n)/(12*x^6) - (3*b*d*e^2*n)/(25*x^5) - (b*e^
3*n)/(16*x^4) - (d^3*(a + b*Log[c*x^n]))/(7*x^7) - (d^2*e*(a + b*Log[c*x^n]
```

```
))/(2*x^6) - (3*d*e^2*(a + b*Log[c*x^n]))/(5*x^5) - (e^3*(a + b*Log[c*x^n])
)/(4*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{7x^7}-\frac{d^2e(a+b\log(cx^n))}{2x^6}-\frac{3de^2(a+b\log(cx^n))}{5x^5} \\
&\quad -\frac{e^3(a+b\log(cx^n))}{4x^4}-(bn)\int\frac{-20d^3-70d^2ex-84de^2x^2-35e^3x^3}{140x^8}\,dx \\
&= -\frac{d^3(a+b\log(cx^n))}{7x^7}-\frac{d^2e(a+b\log(cx^n))}{2x^6}-\frac{3de^2(a+b\log(cx^n))}{5x^5} \\
&\quad -\frac{e^3(a+b\log(cx^n))}{4x^4}-\frac{1}{140}(bn)\int\frac{-20d^3-70d^2ex-84de^2x^2-35e^3x^3}{x^8}\,dx \\
&= -\frac{d^3(a+b\log(cx^n))}{7x^7}-\frac{d^2e(a+b\log(cx^n))}{2x^6}-\frac{3de^2(a+b\log(cx^n))}{5x^5} \\
&\quad -\frac{e^3(a+b\log(cx^n))}{4x^4}-\frac{1}{140}(bn)\int\left(-\frac{20d^3}{x^8}-\frac{70d^2e}{x^7}-\frac{84de^2}{x^6}-\frac{35e^3}{x^5}\right)dx
\end{aligned}$$

$$= -\frac{bd^3n}{49x^7} - \frac{bd^2en}{12x^6} - \frac{3bde^2n}{25x^5} - \frac{be^3n}{16x^4} - \frac{d^3(a+b\log(cx^n))}{7x^7} - \frac{d^2e(a+b\log(cx^n))}{2x^6} - \frac{3de^2(a+b\log(cx^n))}{5x^5} - \frac{e^3(a+b\log(cx^n))}{4x^4}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^8} dx = -\frac{420a(20d^3+70d^2ex+84de^2x^2+35e^3x^3)+bn(1200d^3+4900d^2ex+7056de^2x^2+3675e^3x^3)+420b(20d}{58800x^7}$$

```
[In] Integrate[((d + e*x)^3*(a + b*Log[c*x^n]))/x^8,x]
```

```
[Out] -1/58800*(420*a*(20*d^3 + 70*d^2*e*x + 84*d*e^2*x^2 + 35*e^3*x^3) + b*n*(12
00*d^3 + 4900*d^2*e*x + 7056*d*e^2*x^2 + 3675*e^3*x^3) + 420*b*(20*d^3 + 70
*d^2*e*x + 84*d*e^2*x^2 + 35*e^3*x^3)*Log[c*x^n])/x^7
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.01

method	result
parallelrisch	$-\frac{14700b\ln(cx^n)e^3x^3+3675be^3nx^3+14700ae^3x^3+35280b\ln(cx^n)de^2x^2+7056bde^2nx^2+35280ade^2x^2+29400b\ln(cx^n)d^2ex}{58800x^7}$
risch	$-\frac{b(35e^3x^3+84de^2x^2+70d^2ex+20d^3)\ln(x^n)}{140x^7} - \frac{14700\ln(c)be^3x^3-17640i\pi bde^2x^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)-14700i\pi b\,d}{}$

```
[In] int((e*x+d)^3*(a+b*ln(c*x^n))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/58800/x^7*(14700*b*ln(c*x^n)*e^3*x^3+3675*b*e^3*n*x^3+14700*a*e^3*x^3+35
280*b*ln(c*x^n)*d*e^2*x^2+7056*b*d*e^2*n*x^2+35280*a*d*e^2*x^2+29400*b*ln(c
*x^n)*d^2*e*x+4900*b*d^2*e*n*x+29400*a*d^2*e*x+8400*b*ln(c*x^n)*d^3+1200*b*
d^3*n+8400*a*d^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^8}\,dx = -\frac{1200\,bd^3n + 8400\,ad^3 + 3675\,(be^3n + 4\,ae^3)x^3 + 7056\,(bde^2n + 5\,ade^2)x^2 + 4900\,(bd^2en + 6\,ad^2e)x + 4}{}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^8,x, algorithm="fricas")
```

```
[Out] -1/58800*(1200*b*d^3*n + 8400*a*d^3 + 3675*(b*e^3*n + 4*a*e^3)*x^3 + 7056*(
b*d*e^2*n + 5*a*d*e^2)*x^2 + 4900*(b*d^2*e*n + 6*a*d^2*e)*x + 420*(35*b*e^3
*x^3 + 84*b*d*e^2*x^2 + 70*b*d^2*e*x + 20*b*d^3)*log(c) + 420*(35*b*e^3*n*x
^3 + 84*b*d*e^2*n*x^2 + 70*b*d^2*e*n*x + 20*b*d^3*n)*log(x))/x^7
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^8}\,dx = -\frac{ad^3}{7x^7} - \frac{ad^2e}{2x^6} - \frac{3ade^2}{5x^5} - \frac{ae^3}{4x^4} - \frac{bd^3n}{49x^7} - \frac{bd^3\log(cx^n)}{7x^7} - \frac{bd^2en}{12x^6} - \frac{bd^2e\log(cx^n)}{2x^6} - \frac{3bde^2n}{25x^5} - \frac{3bde^2\log(cx^n)}{5x^5} - \frac{be^3n}{16x^4} - \frac{be^3\log(cx^n)}{4x^4}$$

```
[In] integrate((e*x+d)**3*(a+b*ln(c*x**n))/x**8,x)
```

```
[Out] -a*d**3/(7*x**7) - a*d**2*e/(2*x**6) - 3*a*d*e**2/(5*x**5) - a*e**3/(4*x**4
) - b*d**3*n/(49*x**7) - b*d**3*log(c*x**n)/(7*x**7) - b*d**2*e*n/(12*x**6)
 - b*d**2*e*log(c*x**n)/(2*x**6) - 3*b*d*e**2*n/(25*x**5) - 3*b*d*e**2*log(
c*x**n)/(5*x**5) - b*e**3*n/(16*x**4) - b*e**3*log(c*x**n)/(4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^3 (a+b\log(cx^n))}{x^8}\,dx = -\frac{be^3n}{16\,x^4} - \frac{be^3\log(cx^n)}{4\,x^4} - \frac{3\,bde^2n}{25\,x^5} - \frac{ae^3}{4\,x^4} - \frac{3\,bde^2\log(cx^n)}{5\,x^5} - \frac{bd^2en}{12\,x^6} - \frac{3\,ade^2}{5\,x^5} - \frac{bd^2e\log(cx^n)}{2\,x^6} - \frac{bd^3n}{49\,x^7} - \frac{ad^2e}{2\,x^6} - \frac{bd^3\log(cx^n)}{7\,x^7} - \frac{ad^3}{7\,x^7}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^8,x, algorithm="maxima")
```

```
[Out] -1/16*b*e^3*n/x^4 - 1/4*b*e^3*log(c*x^n)/x^4 - 3/25*b*d*e^2*n/x^5 - 1/4*a*e
^3/x^4 - 3/5*b*d*e^2*log(c*x^n)/x^5 - 1/12*b*d^2*e*n/x^6 - 3/5*a*d*e^2/x^5
- 1/2*b*d^2*e*log(c*x^n)/x^6 - 1/49*b*d^3*n/x^7 - 1/2*a*d^2*e/x^6 - 1/7*b*d
^3*log(c*x^n)/x^7 - 1/7*a*d^3/x^7
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^3\,(a+b\log{(cx^n)})}{x^8}\,dx = -\frac{(35\,be^3nx^3+84\,bde^2nx^2+70\,bd^2enx+20\,bd^3n)\log{(x)}}{140\,x^7} - \frac{3675\,be^3nx^3+14700\,be^3x^3\log{(c)}+7056\,bde^2nx^2+14700\,ae^3x^3+35280\,bde^2x^2\log{(c)}+4900\,bd^2enx+}{58800\,x^7}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))/x^8,x, algorithm="giac")
```

```
[Out] -1/140*(35*b*e^3*n*x^3 + 84*b*d*e^2*n*x^2 + 70*b*d^2*e*n*x + 20*b*d^3*n)*lo
g(x)/x^7 - 1/58800*(3675*b*e^3*n*x^3 + 14700*b*e^3*x^3*log(c) + 7056*b*d*e^
2*n*x^2 + 14700*a*e^3*x^3 + 35280*b*d*e^2*x^2*log(c) + 4900*b*d^2*e*n*x + 3
5280*a*d*e^2*x^2 + 29400*b*d^2*e*x*log(c) + 1200*b*d^3*n + 29400*a*d^2*e*x
+ 8400*b*d^3*log(c) + 8400*a*d^3)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^3\,(a+b\log{(cx^n)})}{x^8}\,dx = -\frac{x^3\left(35\,a\,e^3+\frac{35\,b\,e^3\,n}{4}\right)+x\left(70\,a\,d^2\,e+\frac{35\,b\,d^2\,e\,n}{3}\right)+20\,a\,d^3+x^2\left(84\,a\,d\,e^2+\frac{84\,b\,d\,e^2\,n}{5}\right)+\frac{20\,b\,d^3\,n}{7}}{140\,x^7} - \frac{\ln{(c\,x^n)}\,\left(\frac{b\,d^3}{7}+\frac{b\,d^2\,e\,x}{2}+\frac{3\,b\,d\,e^2\,x^2}{5}+\frac{b\,e^3\,x^3}{4}\right)}{x^7}$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^3)/x^8,x)
```

```
[Out] - (x^3*(35*a*e^3 + (35*b*e^3*n)/4) + x*(70*a*d^2*e + (35*b*d^2*e*n)/3) + 20
*a*d^3 + x^2*(84*a*d*e^2 + (84*b*d*e^2*n)/5) + (20*b*d^3*n)/7)/(140*x^7) -
(log(c*x^n)*((b*d^3)/7 + (b*e^3*x^3)/4 + (b*d^2*e*x)/2 + (3*b*d*e^2*x^2)/5)
)/x^7
```

3.31 $\int \frac{x^3(a+b\log(cx^n))}{d+ex}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex}\,dx = \frac{ad^2x}{e^3}-\frac{bd^2nx}{e^3}+\frac{bdnx^2}{4e^2}-\frac{bnx^3}{9e}+\frac{bd^2x\log(cx^n)}{e^3}-\frac{dx^2(a+b\log(cx^n))}{2e^2}+\frac{x^3(a+b\log(cx^n))}{3e}-\frac{d^3(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^4}-\frac{bd^3n\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4}$$

```
[Out] a*d^2*x/e^3-b*d^2*n*x/e^3+1/4*b*d*n*x^2/e^2-1/9*b*n*x^3/e+b*d^2*x*ln(c*x^n)
/e^3-1/2*d*x^2*(a+b*ln(c*x^n))/e^2+1/3*x^3*(a+b*ln(c*x^n))/e-d^3*(a+b*ln(c*
x^n))*ln(1+e*x/d)/e^4-b*d^3*n*polylog(2,-e*x/d)/e^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {45, 2393, 2332, 2341, 2354, 2438}

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex}\,dx = -\frac{d^3\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{e^4}-\frac{dx^2(a+b\log(cx^n))}{2e^2}+\frac{x^3(a+b\log(cx^n))}{3e}+\frac{ad^2x}{e^3}+\frac{bd^2x\log(cx^n)}{e^3}-\frac{bd^3n\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4}-\frac{bd^2nx}{e^3}+\frac{bdnx^2}{4e^2}-\frac{bnx^3}{9e}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n]))/(d + e*x),x]
```

```
[Out] (a*d^2*x)/e^3 - (b*d^2*n*x)/e^3 + (b*d*n*x^2)/(4*e^2) - (b*n*x^3)/(9*e) + (
b*d^2*x*Log[c*x^n])/e^3 - (d*x^2*(a + b*Log[c*x^n]))/(2*e^2) + (x^3*(a + b*
```

```
Log[c*x^n]))/(3*e) - (d^3*(a + b*Log[c*x^n])*Log[1 + (e*x)/d])/e^4 - (b*d^3
*n*PolyLog[2, -((e*x)/d)])/e^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2(a+b\log(cx^n))}{e^3} - \frac{dx(a+b\log(cx^n))}{e^2} + \frac{x^2(a+b\log(cx^n))}{e} - \frac{d^3(a+b\log(cx^n))}{e^3(d+ex)}\right) dx$$

$$
\begin{aligned}
&= \frac{d^2 \int (a + b\log(cx^n))\, dx}{e^3} - \frac{d^3 \int \frac{a+b\log(cx^n)}{d+ex}\, dx}{e^3} \\
&\quad - \frac{d \int x(a + b\log(cx^n))\, dx}{e^2} + \frac{\int x^2(a + b\log(cx^n))\, dx}{e} \\
&= \frac{ad^2x}{e^3} + \frac{bdnx^2}{4e^2} - \frac{bnx^3}{9e} - \frac{dx^2(a + b\log(cx^n))}{2e^2} + \frac{x^3(a + b\log(cx^n))}{3e} \\
&\quad - \frac{d^3(a + b\log(cx^n))\log\left(1 + \frac{ex}{d}\right)}{e^4} + \frac{(bd^2)\int \log(cx^n)\, dx}{e^3} + \frac{(bd^3n)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}\, dx}{e^4} \\
&= \frac{ad^2x}{e^3} - \frac{bd^2nx}{e^3} + \frac{bdnx^2}{4e^2} - \frac{bnx^3}{9e} + \frac{bd^2x\log(cx^n)}{e^3} - \frac{dx^2(a + b\log(cx^n))}{2e^2} \\
&\quad + \frac{x^3(a + b\log(cx^n))}{3e} - \frac{d^3(a + b\log(cx^n))\log\left(1 + \frac{ex}{d}\right)}{e^4} - \frac{bd^3n\mathrm{Li}_2\left(-\frac{ex}{d}\right)}{e^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.96

$$
\int \frac{x^3(a + b\log(cx^n))}{d + ex}\, dx = \frac{36ad^2ex - 36bd^2enx - 18ade^2x^2 + 9bde^2nx^2 + 12ae^3x^3 - 4be^3nx^3 - 36ad^3\log\left(1 + \frac{ex}{d}\right) + 6b\log(cx^n)\,(e\ldots}{36e^4}
$$

[In] Integrate[(x^3*(a + b*Log[c*x^n]))/(d + e*x),x]

[Out] (36*a*d^2*e*x - 36*b*d^2*e*n*x - 18*a*d*e^2*x^2 + 9*b*d*e^2*n*x^2 + 12*a*e^
3*x^3 - 4*b*e^3*n*x^3 - 36*a*d^3*Log[1 + (e*x)/d] + 6*b*Log[c*x^n]*(e*x*(6*
d^2 - 3*d*e*x + 2*e^2*x^2) - 6*d^3*Log[1 + (e*x)/d]) - 36*b*d^3*n*PolyLog[2
, -((e*x)/d)])/(36*e^4)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.67 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.84

method	result
risch	$\frac{b\ln(x^n)x^3}{3e} - \frac{b\ln(x^n)d\,x^2}{2e^2} + \frac{b\ln(x^n)x\,d^2}{e^3} - \frac{b\ln(x^n)d^3\ln(ex+d)}{e^4} - \frac{bn\,x^3}{9e} + \frac{bdn\,x^2}{4e^2} - \frac{b\,d^2nx}{e^3} - \frac{49bn\,d^3}{36e^4} + \frac{bn\,d^3\ln(ex+d\ldots}{e^4}$

[In] int(x^3*(a+b*ln(c*x^n))/(e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/3*b*ln(x^n)/e*x^3-1/2*b*ln(x^n)/e^2*d*x^2+b*ln(x^n)/e^3*x*d^2-b*ln(x^n)*d
^3/e^4*ln(e*x+d)-1/9*b*n*x^3/e+1/4*b*d*n*x^2/e^2-b*d^2*n*x/e^3-49/36*b*n*d^
3/e^4+b*n*d^3/e^4*ln(e*x+d)*ln(-e*x/d)+b*n*d^3/e^4*dilog(-e*x/d)+(-1/2*I*b*

```
Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2
+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+
a)*(1/e^3*(1/3*e^2*x^3-1/2*d*e*x^2+d^2*x)-d^3/e^4*ln(e*x+d))
```

Fricas [F]

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{ex+d}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d),x, algorithm="fricas")
[Out] integral((b*x^3*log(c*x^n) + a*x^3)/(e*x + d), x)
```

Sympy [A] (verification not implemented)

Time = 16.24 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.80

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex}\,dx = -\frac{ad^3\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^3} + \frac{ad^2x}{e^3} - \frac{adx^2}{2e^2} + \frac{ax^3}{3e} + \frac{bd^3n\left(\begin{cases}\frac{x}{d} & \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(d)\log(x)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(d)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{otherwise}\end{cases}}{e} & \end{cases}\right)}{e^3} - \frac{bd^3\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{e^3} - \frac{bd^2nx}{e^3} + \frac{bd^2x\log(cx^n)}{e^3} + \frac{bdnx^2}{4e^2} - \frac{bdx^2\log(cx^n)}{2e^2} - \frac{bnx^3}{9e} + \frac{bx^3\log(cx^n)}{3e}$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))/(e*x+d),x)
[Out] -a*d**3*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e**3 + a*d**2*x/
e**3 - a*d*x**2/(2*e**2) + a*x**3/(3*e) + b*d**3*n*Piecewise((x/d, Eq(e, 0)
```

```
), (Piecewise((-polylog(2, e*x*exp_polar(I*pi)/d), (Abs(x) < 1) & (1/Abs(x)
 < 1)), (log(d)*log(x) - polylog(2, e*x*exp_polar(I*pi)/d), Abs(x) < 1), (-
log(d)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/d), 1/Abs(x) < 1), (-meije
rg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0
)), x)*log(d) - polylog(2, e*x*exp_polar(I*pi)/d), True))/e, True))/e**3 -
b*d**3*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))*log(c*x**n)/e**3
- b*d**2*n*x/e**3 + b*d**2*x*log(c*x**n)/e**3 + b*d*n*x**2/(4*e**2) - b*d*x
**2*log(c*x**n)/(2*e**2) - b*n*x**3/(9*e) + b*x**3*log(c*x**n)/(3*e)
```

Maxima [F]

$$\int \frac{x^3(a + b\log(cx^n))}{d + ex}\,dx = \int \frac{(b\log(cx^n) + a)x^3}{ex + d}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d),x, algorithm="maxima")
[Out] -1/6*a*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) + b
*integrate((x^3*log(c) + x^3*log(x^n))/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^3(a + b\log(cx^n))}{d + ex}\,dx = \int \frac{(b\log(cx^n) + a)x^3}{ex + d}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^3/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\log(cx^n))}{d + ex}\,dx = \int \frac{x^3\,(a + b\,\ln(c\,x^n))}{d + e\,x}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n)))/(d + e*x),x)
[Out] int((x^3*(a + b*log(c*x^n)))/(d + e*x), x)
```

3.32 $\int \frac{x^2(a+b\log(cx^n))}{d+ex}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex}\,dx = -\frac{adx}{e^2}+\frac{bdnx}{e^2}-\frac{bnx^2}{4e}-\frac{bdx\log(cx^n)}{e^2}+\frac{x^2(a+b\log(cx^n))}{2e} + \frac{d^2(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^3}+\frac{bd^2n\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^3}$$

```
[Out] -a*d*x/e^2+b*d*n*x/e^2-1/4*b*n*x^2/e-b*d*x*ln(c*x^n)/e^2+1/2*x^2*(a+b*ln(c*
x^n))/e+d^2*(a+b*ln(c*x^n))*ln(1+e*x/d)/e^3+b*d^2*n*polylog(2,-e*x/d)/e^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {45, 2393, 2332, 2341, 2354, 2438}

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex}\,dx = \frac{d^2\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{e^3}+\frac{x^2(a+b\log(cx^n))}{2e}-\frac{adx}{e^2} - \frac{bdx\log(cx^n)}{e^2}+\frac{bd^2n\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^3}+\frac{bdnx}{e^2}-\frac{bnx^2}{4e}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/(d + e*x),x]
```

```
[Out] -((a*d*x)/e^2) + (b*d*n*x)/e^2 - (b*n*x^2)/(4*e) - (b*d*x*Log[c*x^n])/e^2 +
 (x^2*(a + b*Log[c*x^n]))/(2*e) + (d^2*(a + b*Log[c*x^n])*Log[1 + (e*x)/d])
/e^3 + (b*d^2*n*PolyLog[2, -((e*x)/d)])/e^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{d(a+b\log(cx^n))}{e^2}+\frac{x(a+b\log(cx^n))}{e}+\frac{d^2(a+b\log(cx^n))}{e^2(d+ex)}\right)dx \\
&= -\frac{d\int(a+b\log(cx^n))\,dx}{e^2}+\frac{d^2\int\frac{a+b\log(cx^n)}{d+ex}\,dx}{e^2}+\frac{\int x(a+b\log(cx^n))\,dx}{e} \\
&= -\frac{adx}{e^2}-\frac{bnx^2}{4e}+\frac{x^2(a+b\log(cx^n))}{2e}+\frac{d^2(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^3} \\
&\quad -\frac{(bd)\int\log(cx^n)\,dx}{e^2}-\frac{(bd^2n)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^3}
\end{aligned}$$

$$= -\frac{adx}{e^2} + \frac{bdnx}{e^2} - \frac{bnx^2}{4e} - \frac{bdx\log(cx^n)}{e^2} + \frac{x^2(a+b\log(cx^n))}{2e} + \frac{d^2(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^3} + \frac{bd^2n\text{Li}_2\left(-\frac{ex}{d}\right)}{e^3}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex}\,dx = \frac{-4adex + 4bdenx + 2ae^2x^2 - be^2nx^2 + 4ad^2\log\left(1+\frac{ex}{d}\right) + 2b\log(cx^n)\left(ex(-2d+ex) + 2d^2\log\left(1+\frac{ex}{d}\right)\right)}{4e^3}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*x),x]
```

```
[Out] (-4*a*d*e*x + 4*b*d*e*n*x + 2*a*e^2*x^2 - b*e^2*n*x^2 + 4*a*d^2*Log[1 + (e*
x)/d] + 2*b*Log[c*x^n]*(e*x*(-2*d + e*x) + 2*d^2*Log[1 + (e*x)/d]) + 4*b*d^
2*n*PolyLog[2, -((e*x)/d)])/(4*e^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.18

method	result
risch	$\frac{b\ln(x^n)x^2}{2e} - \frac{b\ln(x^n)dx}{e^2} + \frac{b\ln(x^n)d^2\ln(ex+d)}{e^3} - \frac{bn\,d^2\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{e^3} - \frac{bn\,d^2\,\text{dilog}\left(-\frac{ex}{d}\right)}{e^3} - \frac{bn\,x^2}{4e} + \frac{bdnx}{e^2} + \frac{5bn\,d^2}{4e^3} -$

```
[In] int(x^2*(a+b*ln(c*x^n))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*ln(x^n)/e*x^2-b*ln(x^n)/e^2*d*x+b*ln(x^n)*d^2/e^3*ln(e*x+d)-b*n*d^2/e
^3*ln(e*x+d)*ln(-e*x/d)-b*n*d^2/e^3*dilog(-e*x/d)-1/4*b*n*x^2/e+b*d*n*x/e^2
+5/4*b*n*d^2/e^3+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*P
i*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*
Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/e^2*(1/2*e*x^2-d*x)+d^2/e^3*ln(e*x+d))
```

Fricas [F]

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{ex+d}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d),x, algorithm="fricas")
[Out] integral((b*x^2*log(c*x^n) + a*x^2)/(e*x + d), x)
```

Sympy [A] (verification not implemented)

Time = 12.90 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.04

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex}\,dx = \frac{ad^2\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^2} - \frac{adx}{e^2} + \frac{ax^2}{2e} - \frac{bd^2n\left(\begin{cases}\frac{x}{d} & \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(d)\log(x)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(d)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{otherwise}\end{cases}}{e} & \end{cases}\right)}{e^2} + \frac{bd^2\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{e^2} + \frac{bdnx}{e^2} - \frac{bdx\log(cx^n)}{e^2} - \frac{bnx^2}{4e} + \frac{bx^2\log(cx^n)}{2e}$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(e*x+d),x)
[Out] a*d**2*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e**2 - a*d*x/e**2
 + a*x**2/(2*e) - b*d**2*n*Piecewise((x/d, Eq(e, 0)), (Piecewise((-polylog(
2, e*x*exp_polar(I*pi)/d), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) -
 polylog(2, e*x*exp_polar(I*pi)/d), Abs(x) < 1), (-log(d)*log(1/x) - polylo
g(2, e*x*exp_polar(I*pi)/d), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0)
, ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(
2, e*x*exp_polar(I*pi)/d), True))/e, True))/e**2 + b*d**2*Piecewise((x/d, E
q(e, 0)), (log(d + e*x)/e, True))*log(c*x**n)/e**2 + b*d*n*x/e**2 - b*d*x*l
og(c*x**n)/e**2 - b*n*x**2/(4*e) + b*x**2*log(c*x**n)/(2*e)
```

Maxima [F]

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{ex+d}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + b*integrate((x^2*log
(c) + x^2*log(x^n))/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{ex+d}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,x^n))}{d+e\,x}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e*x),x)
```

```
[Out] int((x^2*(a + b*log(c*x^n)))/(d + e*x), x)
```

3.33 $\int \frac{x(a+b\log(cx^n))}{d+ex}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{x(a+b\log(cx^n))}{d+ex}\,dx = \frac{ax}{e}-\frac{bnx}{e}+\frac{bx\log(cx^n)}{e} - \frac{d(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^2}-\frac{bdn\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^2}$$

```
[Out] a*x/e-b*n*x/e+b*x*ln(c*x^n)/e-d*(a+b*ln(c*x^n))*ln(1+e*x/d)/e^2-b*d*n*polyl
og(2,-e*x/d)/e^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {45, 2393, 2332, 2354, 2438}

$$\int \frac{x(a+b\log(cx^n))}{d+ex}\,dx = -\frac{d\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{e^2}+\frac{ax}{e} + \frac{bx\log(cx^n)}{e}-\frac{bdn\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^2}-\frac{bnx}{e}$$

```
[In] Int[(x*(a + b*Log[c*x^n]))/(d + e*x),x]
```

```
[Out] (a*x)/e - (b*n*x)/e + (b*x*Log[c*x^n])/e - (d*(a + b*Log[c*x^n])*Log[1 + (e
*x)/d])/e^2 - (b*d*n*PolyLog[2, -((e*x)/d)])/e^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a+b\log\left(cx^n\right)}{e}-\frac{d(a+b\log\left(cx^n\right))}{e(d+ex)}\right)\,dx\\
&= \frac{\int \left(a+b\log\left(cx^n\right)\right)\,dx}{e}-\frac{d\int \frac{a+b\log\left(cx^n\right)}{d+ex}\,dx}{e}\\
&= \frac{ax}{e}-\frac{d(a+b\log\left(cx^n\right))\log\left(1+\frac{ex}{d}\right)}{e^2}+\frac{b\int \log\left(cx^n\right)\,dx}{e}+\frac{(bdn)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^2}\\
&= \frac{ax}{e}-\frac{bnx}{e}+\frac{bx\log\left(cx^n\right)}{e}-\frac{d(a+b\log\left(cx^n\right))\log\left(1+\frac{ex}{d}\right)}{e^2}-\frac{bdn\text{Li}_2\left(-\frac{ex}{d}\right)}{e^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{x(a+b\log(cx^n))}{d+ex}\,dx = \frac{aex-benx-ad\log\left(1+\frac{ex}{d}\right)+b\log(cx^n)\left(ex-d\log\left(1+\frac{ex}{d}\right)\right)-bdn\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^2}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e*x),x]
```

```
[Out] (a*e*x - b*e*n*x - a*d*Log[1 + (e*x)/d] + b*Log[c*x^n]*(e*x - d*Log[1 + (e*
x)/d]) - b*d*n*PolyLog[2, -((e*x)/d)])/e^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.72

method	result
risch	$\frac{b\ln(x^n)x}{e} - \frac{b\ln(x^n)d\ln(ex+d)}{e^2} - \frac{bnx}{e} - \frac{bnd}{e^2} + \frac{bnd\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{e^2} + \frac{bnd\,\mathrm{dilog}\left(-\frac{ex}{d}\right)}{e^2} + \Big(-\frac{ib\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csg}}{2}$

```
[In] int(x*(a+b*ln(c*x^n))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] b*ln(x^n)/e*x-b*ln(x^n)*d/e^2*ln(e*x+d)-b*n*x/e-b*n*d/e^2+b*n*d/e^2*ln(e*x+
d)*ln(-e*x/d)+b*n*d/e^2*dilog(-e*x/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*cs
gn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csg
n(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(x/e-d/e^2*ln(e*x+d))
```

Fricas [F]

$$\int \frac{x(a+b\log(cx^n))}{d+ex}\,dx = \int \frac{(b\log(cx^n)+a)x}{ex+d}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*log(c*x^n) + a*x)/(e*x + d), x)
```

Sympy [A] (verification not implemented)

Time = 8.06 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.36

$$\int \frac{x(a+b\log(cx^n))}{d+ex}\,dx = -\frac{ad\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e} + \frac{ax}{e} + \frac{bdn\left(\begin{cases}\frac{x}{d} \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(d)\log(x)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(d)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{otherwise}\end{cases}}{e}\end{cases}\right)}{e} - \frac{bd\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{e} - \frac{bnx}{e} + \frac{bx\log(cx^n)}{e}$$

[In] integrate(x*(a+b*ln(c*x**n))/(e*x+d),x)

[Out] -a*d*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e + a*x/e + b*d*n*Piecewise((x/d, Eq(e, 0)), (Piecewise((-polylog(2, e*x*exp_polar(I*pi)/d), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x*exp_polar(I*pi)/d), Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/d), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x*exp_polar(I*pi)/d), True))/e, True))/e - b*d*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))*log(c*x**n)/e - b*n*x/e + b*x*log(c*x**n)/e

Maxima [F]

$$\int \frac{x(a+b\log(cx^n))}{d+ex}\,dx = \int \frac{(b\log(cx^n)+a)x}{ex+d}\,dx$$

[In] integrate(x*(a+b*log(c*x^n))/(e*x+d),x, algorithm="maxima")

[Out] a*(x/e - d*log(e*x + d)/e^2) + b*integrate((x*log(c) + x*log(x^n))/(e*x + d), x)

Giac [F]

$$\int \frac{x(a+b\log(cx^n))}{d+ex}\,dx = \int \frac{(b\log(cx^n)+a)x}{ex+d}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(cx^n))}{d+ex}\,dx = \int \frac{x\,(a+b\,\ln(c\,x^n))}{d+e\,x}\,dx$$

```
[In] int((x*(a + b*log(c*x^n)))/(d + e*x),x)
[Out] int((x*(a + b*log(c*x^n)))/(d + e*x), x)
```

3.34 $\int \frac{a+b\log(cx^n)}{d+ex}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 39

$$\int \frac{a+b\log(cx^n)}{d+ex}\,dx=\frac{(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e}+\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e}$$

```
[Out] (a+b*ln(c*x^n))*ln(1+e*x/d)/e+b*n*polylog(2,-e*x/d)/e
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2354, 2438}

$$\int \frac{a+b\log(cx^n)}{d+ex}\,dx=\frac{\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{e}+\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e}$$

```
[In] Int[(a + b*Log[c*x^n])/(d + e*x),x]
[Out] ((a + b*Log[c*x^n])*Log[1 + (e*x)/d])/e + (b*n*PolyLog[2, -((e*x)/d)])/e
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e} - \frac{(bn)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e}\\ &= \frac{(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e} + \frac{bn\text{Li}_2\left(-\frac{ex}{d}\right)}{e}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int\frac{a+b\log(cx^n)}{d+ex}\,dx = \frac{(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)+bn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*x),x]
[Out] ((a + b*Log[c*x^n])*Log[1 + (e*x)/d] + b*n*PolyLog[2, -((e*x)/d)])/e
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.87

method	result
risch	$\frac{b\ln(x^n)\ln(ex+d)}{e} - \frac{bn\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{e} - \frac{bn\,\text{dilog}\left(-\frac{ex}{d}\right)}{e} + \frac{\left(-\frac{ib\pi\,\text{csgn}(ic)\,\text{csgn}(ix^n)\,\text{csgn}(icx^n)}{2} + \frac{ib\pi\,\text{csgn}(ic)\,\text{csgn}(icx^n)^2}{2} + ib\right.}{e}$

```
[In] int((a+b*ln(c*x^n))/(e*x+d),x,method=_RETURNVERBOSE)
[Out] b*ln(x^n)*ln(e*x+d)/e-b/e*n*ln(e*x+d)*ln(-e*x/d)-b/e*n*dilog(-e*x/d)+(-1/2*
I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^
n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln
(c)+a)*ln(e*x+d)/e
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^n\right)}{d+ex}\,dx=\int \frac{b\log\left(cx^n\right)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d),x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{a+b\log\left(cx^n\right)}{d+ex}\,dx=\int \frac{a+b\log\left(cx^n\right)}{d+ex}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/(e*x+d),x)
[Out] Integral((a + b*log(c*x**n))/(d + e*x), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(cx^n\right)}{d+ex}\,dx=\int \frac{b\log\left(cx^n\right)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d),x, algorithm="maxima")
[Out] b*integrate((log(c) + log(x^n))/(e*x + d), x) + a*log(e*x + d)/e
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{d+ex}\,dx=\int \frac{b\log\left(cx^n\right)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{d + ex}\,dx = \int \frac{a + b\,\ln(c\,x^n)}{d + e\,x}\,dx$$

```
[In] int((a + b*log(c*x^n))/(d + e*x),x)
[Out] int((a + b*log(c*x^n))/(d + e*x), x)
```

3.35 $\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 44

$$\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx = -\frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d}+\frac{bn\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d}$$

```
[Out] -ln(1+d/e/x)*(a+b*ln(c*x^n))/d+b*n*polylog(2,-d/e/x)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx = \frac{bn\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d}-\frac{\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x)),x]
[Out] -((Log[1 + d/(e*x)]*(a + b*Log[c*x^n]))/d) + (b*n*PolyLog[2, -(d/(e*x))])/d
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = -\frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d} + \frac{(bn)\int\frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d}$$
$$= -\frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d} + \frac{bn\mathrm{Li}_2\left(-\frac{d}{ex}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.43

$$\int\frac{a+b\log(cx^n)}{x(d+ex)}\,dx = \frac{(a+b\log(cx^n))\left(a+b\log(cx^n)-2bn\log\left(1+\frac{ex}{d}\right)\right)}{2bdn} - \frac{bn\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{d}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x)),x]
[Out] ((a + b*Log[c*x^n])*(a + b*Log[c*x^n] - 2*b*n*Log[1 + (e*x)/d]))/(2*b*d*n)
- (b*n*PolyLog[2, -((e*x)/d)])/d
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.75 (sec) , antiderivative size = 181, normalized size of antiderivative = 4.11

method	result
risch	$-\frac{b\ln(x^n)\ln(ex+d)}{d}+\frac{b\ln(x^n)\ln(x)}{d}-\frac{bn\ln(x)^2}{2d}+\frac{bn\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{d}+\frac{bn\,\mathrm{dilog}\left(-\frac{ex}{d}\right)}{d}+\left(-\frac{ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csg}}{2}\right.$

```
[In] int((a+b*ln(c*x^n))/x/(e*x+d),x,method=_RETURNVERBOSE)
[Out] -b*ln(x^n)/d*ln(e*x+d)+b*ln(x^n)/d*ln(x)-1/2*b*n/d*ln(x)^2+b*n/d*ln(e*x+d)*
ln(-e*x/d)+b*n/d*dilog(-e*x/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*
x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x
^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-1/d*ln(e*x+d)+1/d*ln(x))
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx=\int \frac{b\log(cx^n)+a}{(ex+d)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d),x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(e*x^2 + d*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.53 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.98

$$\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx=-\frac{2ae\left(\begin{cases}\frac{1}{2e}+\frac{x}{d} & \text{for } e=0\\ -\frac{\log(-2ex)}{2e} & \text{otherwise}\end{cases}\right)}{d}-\frac{2ae\left(\begin{cases}\frac{1}{2e}+\frac{x}{d} & \text{for } e=0\\ \frac{\log(2d+2ex)}{2e} & \text{otherwise}\end{cases}\right)}{d}+bn\left(\begin{cases}-\frac{1}{ex} & \text{fo}\\ \frac{\begin{cases}\mathrm{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{for } \frac{1}{|x|}<1\wedge|x|<1\\ \log(e)\log(x)+\mathrm{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{for } |x|<1\\ -\log(e)\log\left(\frac{1}{x}\right)+\mathrm{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log(e)+\mathrm{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{otherwise}\end{cases}}{d} & \text{ot}\end{cases}\right)-b\left(\begin{cases}\frac{1}{ex} & \text{for } d=0\\ \frac{\log\left(\frac{d}{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate((a+b*ln(c*x**n))/x/(e*x+d),x)
[Out] -2*a*e*Piecewise((1/(2*e) + x/d, Eq(e, 0)), (-log(-2*e*x)/(2*e), True))/d -
 2*a*e*Piecewise((1/(2*e) + x/d, Eq(e, 0)), (log(2*d + 2*e*x)/(2*e), True))
/d + b*n*Piecewise((-1/(e*x), Eq(d, 0)), (Piecewise((polylog(2, d*exp_polar
(I*pi)/(e*x)), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(e)*log(x) + polylog(2,
d*exp_polar(I*pi)/(e*x)), Abs(x) < 1), (-log(e)*log(1/x) + polylog(2, d*exp
_polar(I*pi)/(e*x)), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x
)*log(e) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(e) + polylog(2, d*exp
_polar(I*pi)/(e*x)), True))/d, True)) - b*Piecewise((1/(e*x), Eq(d, 0)), (l
og(d/x + e)/d, True))*log(c*x**n)
```

Maxima [F]

$$\int \frac{a + b \log(cx^n)}{x(d + ex)} \, dx = \int \frac{b \log(cx^n) + a}{(ex + d)x} \, dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -a*(log(e*x + d)/d - log(x)/d) + b*integrate((log(c) + log(x^n))/(e*x^2 + d
*x), x)
```

Giac [F]

$$\int \frac{a + b \log(cx^n)}{x(d + ex)} \, dx = \int \frac{b \log(cx^n) + a}{(ex + d)x} \, dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \log(cx^n)}{x(d + ex)} \, dx = \int \frac{a + b \ln(c\,x^n)}{x\,(d + e\,x)} \, dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x*(d + e*x)), x)
```

3.36 $\int \frac{a+b\log(cx^n)}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)}\,dx = -\frac{bn}{dx}-\frac{a+b\log(cx^n)}{dx}+\frac{e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^2} - \frac{ben\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^2}$$

```
[Out] -b*n/d/x+(-a-b*ln(c*x^n))/d/x+e*ln(1+d/e/x)*(a+b*ln(c*x^n))/d^2-b*e*n*polyl
og(2,-d/e/x)/d^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2380, 2341, 2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)}\,dx = \frac{e\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^2} - \frac{a+b\log(cx^n)}{dx}-\frac{ben\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^2}-\frac{bn}{dx}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^2*(d + e*x)),x]
```

```
[Out] -((b*n)/(d*x)) - (a + b*Log[c*x^n])/(d*x) + (e*Log[1 + d/(e*x)]*(a + b*Log[
c*x^n]))/d^2 - (b*e*n*PolyLog[2, -(d/(e*x))])/d^2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d} - \frac{e\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d}\\
&= -\frac{bn}{dx} - \frac{a+b\log(cx^n)}{dx} + \frac{e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^2} - \frac{(ben)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^2}\\
&= -\frac{bn}{dx} - \frac{a+b\log(cx^n)}{dx} + \frac{e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^2} - \frac{ben\mathrm{Li}_2\left(-\frac{d}{ex}\right)}{d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)}\,dx = -\frac{\frac{2bdn}{x} + \frac{2d(a+b\log(cx^n))}{x} + \frac{e(a+b\log(cx^n))^2}{bn} - 2e(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right) - 2ben\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{2d^2}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*x)),x]
```

```
[Out] -1/2*((2*b*d*n)/x + (2*d*(a + b*Log[c*x^n]))/x + (e*(a + b*Log[c*x^n])^2)/(
b*n) - 2*e*(a + b*Log[c*x^n])*Log[1 + (e*x)/d] - 2*b*e*n*PolyLog[2, -((e*x)
/d)])/d^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.99

method	result
risch	$\frac{b\ln(x^n)e\ln(ex+d)}{d^2} - \frac{b\ln(x^n)}{dx} - \frac{b\ln(x^n)e\ln(x)}{d^2} - \frac{bne\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{d^2} - \frac{bne\,\mathrm{dilog}\left(-\frac{ex}{d}\right)}{d^2} - \frac{bn}{dx} + \frac{bne\ln(x)^2}{2d^2} + \Big(-\frac{ib\pi}{}$

[In] int((a+b*ln(c*x^n))/x^2/(e*x+d),x,method=_RETURNVERBOSE)

[Out] b*ln(x^n)*e/d^2*ln(e*x+d)-b*ln(x^n)/d/x-b*ln(x^n)*e/d^2*ln(x)-b*n*e/d^2*ln(e*x+d)*ln(-e*x/d)-b*n*e/d^2*dilog(-e*x/d)-b*n/d/x+1/2*b*n*e/d^2*ln(x)^2+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(e/d^2*ln(e*x+d)-1/d/x-e/d^2*ln(x))

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)x^2}\,dx$$

[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d),x, algorithm="fricas")

[Out] integral((b*log(c*x^n) + a)/(e*x^3 + d*x^2), x)

Sympy [A] (verification not implemented)

Time = 32.45 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.92

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)}\,dx = -\frac{a}{dx}+\frac{ae^2\left(\begin{cases}\frac{x}{d} & \text{for}\ e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{d^2}-\frac{ae\log(x)}{d^2}-\frac{bn}{dx}-\frac{b\log(cx^n)}{dx}$$
$$-\frac{be^2n\left(\begin{cases}\frac{x}{d} \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for}\ \frac{1}{|x|}<1\wedge|x|<1\\ \log(d)\log(x)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for}\ |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for}\ \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log(d)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{otherwise}\end{cases}}{e}\end{cases}\right)}{d^2}$$
$$+\frac{be^2\left(\begin{cases}\frac{x}{d} & \text{for}\ e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^2}+\frac{ben\log(x)^2}{2d^2}-\frac{be\log(x)\log(cx^n)}{d^2}$$

```
[In] integrate((a+b*ln(c*x**n))/x**2/(e*x+d),x)
```

```
[Out] -a/(d*x) + a*e**2*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/d**2 -
 a*e*log(x)/d**2 - b*n/(d*x) - b*log(c*x**n)/(d*x) - b*e**2*n*Piecewise((x/
d, Eq(e, 0)), (Piecewise((-polylog(2, e*x*exp_polar(I*pi)/d), (Abs(x) < 1)
& (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x*exp_polar(I*pi)/d), Abs(
x) < 1), (-log(d)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/d), 1/Abs(x) <
1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1), ()),
 ((), (0, 0)), x)*log(d) - polylog(2, e*x*exp_polar(I*pi)/d), True))/e, Tru
e))/d**2 + b*e**2*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))*log(c*
x**n)/d**2 + b*e*n*log(x)**2/(2*d**2) - b*e*log(x)*log(c*x**n)/d**2
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) + b*integrate((log(c) + log
(x^n))/(e*x^3 + d*x^2), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^2\,(d+e\,x)}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*x)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*x)), x)
```

3.37 $\int \frac{a+b\log(cx^n)}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)}\,dx = -\frac{bn}{4dx^2}+\frac{ben}{d^2x}-\frac{a+b\log(cx^n)}{2dx^2}+\frac{e(a+b\log(cx^n))}{d^2x} - \frac{e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^3}+\frac{be^2n\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^3}$$

```
[Out] -1/4*b*n/d/x^2+b*e*n/d^2/x+1/2*(-a-b*ln(c*x^n))/d/x^2+e*(a+b*ln(c*x^n))/d^2
/x-e^2*ln(1+d/e/x)*(a+b*ln(c*x^n))/d^3+b*e^2*n*polylog(2,-d/e/x)/d^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2380, 2341, 2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)}\,dx = -\frac{e^2\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^3}+\frac{e(a+b\log(cx^n))}{d^2x} - \frac{a+b\log(cx^n)}{2dx^2}+\frac{be^2n\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^3}+\frac{ben}{d^2x}-\frac{bn}{4dx^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^3*(d + e*x)),x]
```

```
[Out] -1/4*(b*n)/(d*x^2) + (b*e*n)/(d^2*x) - (a + b*Log[c*x^n])/(2*d*x^2) + (e*(a
 + b*Log[c*x^n]))/(d^2*x) - (e^2*Log[1 + d/(e*x)]*(a + b*Log[c*x^n]))/d^3 +
 (b*e^2*n*PolyLog[2, -(d/(e*x))])/d^3
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{a+b\log(cx^n)}{x^3}\,dx}{d} - \frac{e\int \frac{a+b\log(cx^n)}{x^2(d+ex)}\,dx}{d}\\
&= -\frac{bn}{4dx^2} - \frac{a+b\log(cx^n)}{2dx^2} - \frac{e\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d^2}\\
&= -\frac{bn}{4dx^2} + \frac{ben}{d^2x} - \frac{a+b\log(cx^n)}{2dx^2} + \frac{e(a+b\log(cx^n))}{d^2x}\\
&\quad - \frac{e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^3} + \frac{(be^2n)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^3}\\
&= -\frac{bn}{4dx^2} + \frac{ben}{d^2x} - \frac{a+b\log(cx^n)}{2dx^2} + \frac{e(a+b\log(cx^n))}{d^2x}\\
&\quad - \frac{e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^3} + \frac{be^2n\text{Li}_2\left(-\frac{d}{ex}\right)}{d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.13

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)}\,dx = -\frac{\frac{bd^2n}{x^2}-\frac{4bden}{x}+\frac{2d^2(a+b\log(cx^n))}{x^2}-\frac{4de(a+b\log(cx^n))}{x}-\frac{2e^2(a+b\log(cx^n))^2}{bn}+4e^2(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)+4b}{4d^3}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*(d + e*x)),x]
```

```
[Out] -1/4*((b*d^2*n)/x^2 - (4*b*d*e*n)/x + (2*d^2*(a + b*Log[c*x^n]))/x^2 - (4*d
*e*(a + b*Log[c*x^n]))/x - (2*e^2*(a + b*Log[c*x^n])^2)/(b*n) + 4*e^2*(a +
b*Log[c*x^n])*Log[1 + (e*x)/d] + 4*b*e^2*n*PolyLog[2, -((e*x)/d)])/d^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.40

method	result
risch	$-\frac{b\ln(x^n)e^2\ln(ex+d)}{d^3}-\frac{b\ln(x^n)}{2d\,x^2}+\frac{b\ln(x^n)e^2\ln(x)}{d^3}+\frac{b\ln(x^n)e}{d^2x}+\frac{ben}{d^2x}-\frac{bn}{4d\,x^2}-\frac{bn\,e^2\ln(x)^2}{2d^3}+\frac{bn\,e^2\ln(ex+d)\ln(-\frac{ex}{d}}{d^3}$

```
[In] int((a+b*ln(c*x^n))/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -b*ln(x^n)*e^2/d^3*ln(e*x+d)-1/2*b*ln(x^n)/d/x^2+b*ln(x^n)*e^2/d^3*ln(x)+b*
ln(x^n)*e/d^2/x+b*e*n/d^2/x-1/4*b*n/d/x^2-1/2*b*n*e^2/d^3*ln(x)^2+b*n*e^2/d
^3*ln(e*x+d)*ln(-e*x/d)+b*n*e^2/d^3*dilog(-e*x/d)+(-1/2*I*b*Pi*csgn(I*c)*cs
gn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csg
n(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-e^2/d^3*ln
(e*x+d)-1/2/d/x^2+e^2/d^3*ln(x)+e/d^2/x)
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e*x^4 + d*x^3), x)
```

Sympy [A] (verification not implemented)

Time = 35.16 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.41

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)}\,dx = -\frac{a}{2dx^2}+\frac{ae}{d^2x}-\frac{ae^3\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{d^3}+\frac{ae^2\log(x)}{d^3}-\frac{bn}{4dx^2}-\frac{b\log(cx^n)}{2dx^2}+\frac{ben}{d^2x}+\frac{be\log(cx^n)}{d^2x}+\frac{be^3n\left(\begin{cases}\frac{x}{d}\\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1\wedge|x|<1\\ \log(d)\log(x)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log(d)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{otherwise}\end{cases}}{e}\end{cases}\right)}{d^3}-\frac{be^3\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^3}-\frac{be^2n\log(x)^2}{2d^3}+\frac{be^2\log(x)\log(cx^n)}{d^3}$$

```
[In] integrate((a+b*ln(c*x**n))/x**3/(e*x+d),x)
```

```
[Out] -a/(2*d*x**2) + a*e/(d**2*x) - a*e**3*Piecewise((x/d, Eq(e, 0)), (log(d + e
*x)/e, True))/d**3 + a*e**2*log(x)/d**3 - b*n/(4*d*x**2) - b*log(c*x**n)/(2
*d*x**2) + b*e*n/(d**2*x) + b*e*log(c*x**n)/(d**2*x) + b*e**3*n*Piecewise((
x/d, Eq(e, 0)), (Piecewise((-polylog(2, e*x*exp_polar(I*pi)/d), (Abs(x) < 1
) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x*exp_polar(I*pi)/d), Ab
s(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/d), 1/Abs(x)
< 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1), ()
), ((), (0, 0)), x)*log(d) - polylog(2, e*x*exp_polar(I*pi)/d), True))/e, T
rue))/d**3 - b*e**3*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))*log(
c*x**n)/d**3 - b*e**2*n*log(x)**2/(2*d**3) + b*e**2*log(x)*log(c*x**n)/d**3
```

Maxima [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^3(d+ex)}\,dx = \int \frac{b\log\left(cx^n\right)+a}{(ex+d)x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d),x, algorithm="maxima")
[Out] -1/2*a*(2*e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2))
+ b*integrate((log(c) + log(x^n))/(e*x^4 + d*x^3), x)
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^3(d+ex)}\,dx = \int \frac{b\log\left(cx^n\right)+a}{(ex+d)x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x^3(d+ex)}\,dx = \int \frac{a+b\,\ln\left(c\,x^n\right)}{x^3\,\left(d+e\,x\right)}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e*x)),x)
[Out] int((a + b*log(c*x^n))/(x^3*(d + e*x)), x)
```

3.38 $\int \frac{a+b\log(cx^n)}{x^4(d+ex)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 150

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex)}\,dx=-\frac{bn}{9dx^3}+\frac{ben}{4d^2x^2}-\frac{be^2n}{d^3x}-\frac{a+b\log(cx^n)}{3dx^3}+\frac{e(a+b\log(cx^n))}{2d^2x^2}-\frac{e^2(a+b\log(cx^n))}{d^3x}+\frac{e^3\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4}-\frac{be^3n\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^4}$$

```
[Out] -1/9*b*n/d/x^3+1/4*b*e*n/d^2/x^2-b*e^2*n/d^3/x+1/3*(-a-b*ln(c*x^n))/d/x^3+1
/2*e*(a+b*ln(c*x^n))/d^2/x^2-e^2*(a+b*ln(c*x^n))/d^3/x+e^3*ln(1+d/e/x)*(a+b
*ln(c*x^n))/d^4-b*e^3*n*polylog(2,-d/e/x)/d^4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2380, 2341, 2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex)}\,dx=\frac{e^3\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^4}-\frac{e^2(a+b\log(cx^n))}{d^3x}+\frac{e(a+b\log(cx^n))}{2d^2x^2}-\frac{a+b\log(cx^n)}{3dx^3}-\frac{be^3n\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^4}-\frac{be^2n}{d^3x}+\frac{ben}{4d^2x^2}-\frac{bn}{9dx^3}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^4*(d + e*x)),x]
```

```
[Out] -1/9*(b*n)/(d*x^3) + (b*e*n)/(4*d^2*x^2) - (b*e^2*n)/(d^3*x) - (a + b*Log[c
*x^n])/(3*d*x^3) + (e*(a + b*Log[c*x^n]))/(2*d^2*x^2) - (e^2*(a + b*Log[c*x
^n]))/(d^3*x) + (e^3*Log[1 + d/(e*x)]*(a + b*Log[c*x^n]))/d^4 - (b*e^3*n*Po
lyLog[2, -(d/(e*x))])/d^4
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{a+b\log(cx^n)}{x^4}\,dx}{d} - \frac{e\int \frac{a+b\log(cx^n)}{x^3(d+ex)}\,dx}{d}\\
&= -\frac{bn}{9dx^3} - \frac{a+b\log(cx^n)}{3dx^3} - \frac{e\int \frac{a+b\log(cx^n)}{x^3}\,dx}{d^2} + \frac{e^2\int \frac{a+b\log(cx^n)}{x^2(d+ex)}\,dx}{d^2}\\
&= -\frac{bn}{9dx^3} + \frac{ben}{4d^2x^2} - \frac{a+b\log(cx^n)}{3dx^3} + \frac{e(a+b\log(cx^n))}{2d^2x^2}\\
&\quad + \frac{e^2\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d^3} - \frac{e^3\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d^3}\\
&= -\frac{bn}{9dx^3} + \frac{ben}{4d^2x^2} - \frac{be^2n}{d^3x} - \frac{a+b\log(cx^n)}{3dx^3} + \frac{e(a+b\log(cx^n))}{2d^2x^2}\\
&\quad - \frac{e^2(a+b\log(cx^n))}{d^3x} + \frac{e^3\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4} - \frac{(be^3n)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^4}
\end{aligned}
$$

$$= -\frac{bn}{9dx^3} + \frac{ben}{4d^2x^2} - \frac{be^2n}{d^3x} - \frac{a+b\log(cx^n)}{3dx^3} + \frac{e(a+b\log(cx^n))}{2d^2x^2} - \frac{e^2(a+b\log(cx^n))}{d^3x} + \frac{e^3\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4} - \frac{be^3n\text{Li}_2\left(-\frac{d}{ex}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.06

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex)}\,dx = \frac{-\frac{4bd^3n}{x^3} + \frac{9bd^2en}{x^2} - \frac{36bde^2n}{x} - \frac{12d^3(a+b\log(cx^n))}{x^3} + \frac{18d^2e(a+b\log(cx^n))}{x^2} - \frac{36de^2(a+b\log(cx^n))}{x} - \frac{18e^3(a+b\log(cx^n))^2}{bn} + 36e^3(a}{36d^4}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^4*(d + e*x)),x]
```

```
[Out] ((-4*b*d^3*n)/x^3 + (9*b*d^2*e*n)/x^2 - (36*b*d*e^2*n)/x - (12*d^3*(a + b*L
og[c*x^n]))/x^3 + (18*d^2*e*(a + b*Log[c*x^n]))/x^2 - (36*d*e^2*(a + b*Log[
c*x^n]))/x - (18*e^3*(a + b*Log[c*x^n])^2)/(b*n) + 36*e^3*(a + b*Log[c*x^n]
)*Log[1 + (e*x)/d] + 36*b*e^3*n*PolyLog[2, -((e*x)/d)])/(36*d^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.47 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.06

method	result
risch	$\frac{b\ln(x^n)e^3\ln(ex+d)}{d^4} - \frac{b\ln(x^n)}{3d\,x^3} - \frac{b\ln(x^n)e^2}{d^3x} + \frac{b\ln(x^n)e}{2d^2x^2} - \frac{b\ln(x^n)e^3\ln(x)}{d^4} - \frac{b\,e^2n}{d^3x} + \frac{ben}{4d^2x^2} - \frac{bn}{9d\,x^3} + \frac{bn\,e^3\ln(x)^2}{2d^4} - \frac{b}{}$

```
[In] int((a+b*ln(c*x^n))/x^4/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] b*ln(x^n)*e^3/d^4*ln(e*x+d)-1/3*b*ln(x^n)/d/x^3-b*ln(x^n)*e^2/d^3/x+1/2*b*l
n(x^n)*e/d^2/x^2-b*ln(x^n)*e^3/d^4*ln(x)-b*e^2*n/d^3/x+1/4*b*e*n/d^2/x^2-1/
9*b*n/d/x^3+1/2*b*n*e^3/d^4*ln(x)^2-b*n*e^3/d^4*ln(e*x+d)*ln(-e*x/d)-b*n*e^
3/d^4*dilog(-e*x/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*
b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I
*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(e^3/d^4*ln(e*x+d)-1/3/d/x^3-e^2/d^3/x+1/2
*e/d^2/x^2-e^3/d^4*ln(x))
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex)}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x+d),x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(e*x^5 + d*x^4), x)
```

Sympy [A] (verification not implemented)

Time = 52.48 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.09

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex)}\,dx = -\frac{a}{3dx^3} + \frac{ae}{2d^2x^2} - \frac{ae^2}{d^3x} + \frac{ae^4\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{d^4} - \frac{ae^3\log(x)}{d^4} - \frac{bn}{9dx^3} - \frac{b\log(cx^n)}{3dx^3} + \frac{ben}{4d^2x^2} + \frac{be\log(cx^n)}{2d^2x^2} - \frac{be^2n}{d^3x} - \frac{be^2\log(cx^n)}{d^3x} - \frac{be^4n\left(\begin{cases}\frac{x}{d} \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(d)\log(x)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(d)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{otherwise}\end{cases}}{e}\end{cases}\right)}{d^4} + \frac{be^4\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^4} + \frac{be^3n\log(x)^2}{2d^4} - \frac{be^3\log(x)\log(cx^n)}{d^4}$$

```
[In] integrate((a+b*ln(c*x**n))/x**4/(e*x+d),x)
[Out] -a/(3*d*x**3) + a*e/(2*d**2*x**2) - a*e**2/(d**3*x) + a*e**4*Piecewise((x/d
, Eq(e, 0)), (log(d + e*x)/e, True))/d**4 - a*e**3*log(x)/d**4 - b*n/(9*d*x
**3) - b*log(c*x**n)/(3*d*x**3) + b*e*n/(4*d**2*x**2) + b*e*log(c*x**n)/(2*
d**2*x**2) - b*e**2*n/(d**3*x) - b*e**2*log(c*x**n)/(d**3*x) - b*e**4*n*Pie
cewise((x/d, Eq(e, 0)), (Piecewise((-polylog(2, e*x*exp_polar(I*pi)/d), (Ab
s(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x*exp_polar(I*pi
```

```
)/d), Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/d), 1
/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1
, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x*exp_polar(I*pi)/d), Tru
e))/e, True))/d**4 + b*e**4*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, Tru
e))*log(c*x**n)/d**4 + b*e**3*n*log(x)**2/(2*d**4) - b*e**3*log(x)*log(c*x*
*n)/d**4
```

Maxima [F]

$$\int \frac{a + b\log\left(cx^n\right)}{x^4(d + ex)}\,dx = \int \frac{b\log\left(cx^n\right) + a}{(ex + d)x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/6*a*(6*e^3*log(e*x + d)/d^4 - 6*e^3*log(x)/d^4 - (6*e^2*x^2 - 3*d*e*x + 2
*d^2)/(d^3*x^3)) + b*integrate((log(c) + log(x^n))/(e*x^5 + d*x^4), x)
```

Giac [F]

$$\int \frac{a + b\log\left(cx^n\right)}{x^4(d + ex)}\,dx = \int \frac{b\log\left(cx^n\right) + a}{(ex + d)x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(cx^n\right)}{x^4(d + ex)}\,dx = \int \frac{a + b\,\ln\left(c\,x^n\right)}{x^4\,\left(d + e\,x\right)}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^4*(d + e*x)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^4*(d + e*x)), x)
```

3.39 $\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^2}\,dx = \frac{3bdnx}{e^3}-\frac{d(3a+bn)x}{e^3}-\frac{3bnx^2}{4e^2}-\frac{3bdx\log(cx^n)}{e^3}-\frac{x^3(a+b\log(cx^n))}{e(d+ex)}+\frac{x^2(3a+bn+3b\log(cx^n))}{2e^2}+\frac{d^2(3a+bn+3b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^4}+\frac{3bd^2n\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4}$$

```
[Out] 3*b*d*n*x/e^3-d*(b*n+3*a)*x/e^3-3/4*b*n*x^2/e^2-3*b*d*x*ln(c*x^n)/e^3-x^3*(
a+b*ln(c*x^n))/e/(e*x+d)+1/2*x^2*(3*a+b*n+3*b*ln(c*x^n))/e^2+d^2*(3*a+b*n+3
*b*ln(c*x^n))*ln(1+e*x/d)/e^4+3*b*d^2*n*polylog(2,-e*x/d)/e^4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2384, 45, 2393, 2332, 2341, 2354, 2438}

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^2}\,dx = \frac{d^2\log\left(\frac{ex}{d}+1\right)(3a+3b\log(cx^n)+bn)}{e^4}-\frac{x^3(a+b\log(cx^n))}{e(d+ex)}+\frac{x^2(3a+3b\log(cx^n)+bn)}{2e^2}-\frac{dx(3a+bn)}{e^3}-\frac{3bdx\log(cx^n)}{e^3}+\frac{3bd^2n\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4}+\frac{3bdnx}{e^3}-\frac{3bnx^2}{4e^2}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n]))/(d + e*x)^2,x]
```

```
[Out] (3*b*d*n*x)/e^3 - (d*(3*a + b*n)*x)/e^3 - (3*b*n*x^2)/(4*e^2) - (3*b*d*x*Lo
g[c*x^n])/e^3 - (x^3*(a + b*Log[c*x^n]))/(e*(d + e*x)) + (x^2*(3*a + b*n +
3*b*Log[c*x^n]))/(2*e^2) + (d^2*(3*a + b*n + 3*b*Log[c*x^n])*Log[1 + (e*x)/
d])/e^4 + (3*b*d^2*n*PolyLog[2, -((e*x)/d)])/e^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3(a+b\log(cx^n))}{e(d+ex)}+\frac{\int\frac{x^2(3a+bn+3b\log(cx^n))}{d+ex}\,dx}{e}\\
&= -\frac{x^3(a+b\log(cx^n))}{e(d+ex)}+\frac{\int\left(-\frac{d(3a+bn+3b\log(cx^n))}{e^2}+\frac{x(3a+bn+3b\log(cx^n))}{e}+\frac{d^2(3a+bn+3b\log(cx^n))}{e^2(d+ex)}\right)dx}{e}\\
&= -\frac{x^3(a+b\log(cx^n))}{e(d+ex)}-\frac{d\int(3a+bn+3b\log(cx^n))\,dx}{e^3}\\
&\quad+\frac{d^2\int\frac{3a+bn+3b\log(cx^n)}{d+ex}\,dx}{e^3}+\frac{\int x(3a+bn+3b\log(cx^n))\,dx}{e^2}\\
&= -\frac{d(3a+bn)x}{e^3}-\frac{3bnx^2}{4e^2}-\frac{x^3(a+b\log(cx^n))}{e(d+ex)}+\frac{x^2(3a+bn+3b\log(cx^n))}{2e^2}\\
&\quad+\frac{d^2(3a+bn+3b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^4}-\frac{(3bd)\int\log(cx^n)\,dx}{e^3}\\
&\quad-\frac{(3bd^2n)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^4}\\
&= \frac{3bdnx}{e^3}-\frac{d(3a+bn)x}{e^3}-\frac{3bnx^2}{4e^2}-\frac{3bdx\log(cx^n)}{e^3}\\
&\quad-\frac{x^3(a+b\log(cx^n))}{e(d+ex)}+\frac{x^2(3a+bn+3b\log(cx^n))}{2e^2}\\
&\quad+\frac{d^2(3a+bn+3b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^4}+\frac{3bd^2n\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.93

$$\int\frac{x^3(a+b\log(cx^n))}{(d+ex)^2}\,dx$$
$$=\frac{-8adex+8bdenx-be^2nx^2-8bdex\log(cx^n)+2e^2x^2(a+b\log(cx^n))+\frac{4d^3(a+b\log(cx^n))}{d+ex}-4bd^2n(\log(x)-}{4e^4}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/(d + e*x)^2,x]
[Out] (-8*a*d*e*x + 8*b*d*e*n*x - b*e^2*n*x^2 - 8*b*d*e*x*Log[c*x^n] + 2*e^2*x^2*
(a + b*Log[c*x^n]) + (4*d^3*(a + b*Log[c*x^n]))/(d + e*x) - 4*b*d^2*n*(Log[
x] - Log[d + e*x]) + 12*d^2*(a + b*Log[c*x^n])*Log[1 + (e*x)/d] + 12*b*d^2*
n*PolyLog[2, -((e*x)/d)])/(4*e^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.41 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.96

method	result
risch	$\frac{b\ln(x^n)x^2}{2e^2} - \frac{2b\ln(x^n)dx}{e^3} + \frac{3b\ln(x^n)d^2\ln(ex+d)}{e^4} + \frac{b\ln(x^n)d^3}{e^4(ex+d)} - \frac{3bn\,d^2\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{e^4} - \frac{3bn\,d^2\,\mathrm{dilog}\left(-\frac{ex}{d}\right)}{e^4} - \frac{bn\,x^2}{4e^2} +$

```
[In] int(x^3*(a+b*ln(c*x^n))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*ln(x^n)/e^2*x^2-2*b*ln(x^n)/e^3*d*x+3*b*ln(x^n)/e^4*d^2*ln(e*x+d)+b*l
n(x^n)*d^3/e^4/(e*x+d)-3*b*n/e^4*d^2*ln(e*x+d)*ln(-e*x/d)-3*b*n/e^4*d^2*dil
og(-e*x/d)-1/4*b*n*x^2/e^2+2*b*d*n*x/e^3+9/4*b*n/e^4*d^2+b*n/e^4*d^2*ln(e*x
+d)-b*n/e^4*d^2*ln(e*x)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/
2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1
/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/e^3*(1/2*e*x^2-2*d*x)+3/e^4*d^2*ln(
e*x+d)+d^3/e^4/(e*x+d))
```

Fricas [F]

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{(ex+d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*log(c*x^n) + a*x^3)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [A] (verification not implemented)

Time = 24.56 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.12

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^2}\,dx$$
$$= -\frac{ad^3\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x} & \text{otherwise}\end{cases}\right)}{e^3} + \frac{3ad^2\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^3} - \frac{2adx}{e^3} + \frac{ax^2}{2e^2}$$
$$+ \frac{bd^3n\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{\log(x)}{de} + \frac{\log\left(\frac{d}{e}+x\right)}{de} & \text{otherwise}\end{cases}\right)}{e^3} - \frac{bd^3\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x} & \text{otherwise}\end{cases}\right)\log(cx^n)}{e^3}$$
$$- \frac{3bd^2n\left(\begin{cases}\frac{x}{d} \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(d)\log(x)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(d)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{otherwise}\end{cases}}{e}\end{cases}\right)}{e^3}$$
$$+ \frac{3bd^2\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{e^3}$$
$$+ \frac{2bdnx}{e^3} - \frac{2bdx\log(cx^n)}{e^3} - \frac{bnx^2}{4e^2} + \frac{bx^2\log(cx^n)}{2e^2}$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))/(e*x+d)**2,x)
```

```
[Out] -a*d**3*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))/e**3 + 3*a
*d**2*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e**3 - 2*a*d*x/e**
3 + a*x**2/(2*e**2) + b*d**3*n*Piecewise((x/d**2, Eq(e, 0)), (-log(x)/(d*e)
 + log(d/e + x)/(d*e), True))/e**3 - b*d**3*Piecewise((x/d**2, Eq(e, 0)), (
-1/(d*e + e**2*x), True))*log(c*x**n)/e**3 - 3*b*d**2*n*Piecewise((x/d, Eq(
e, 0)), (Piecewise((-polylog(2, e*x*exp_polar(I*pi)/d), (Abs(x) < 1) & (1/A
bs(x) < 1)), (log(d)*log(x) - polylog(2, e*x*exp_polar(I*pi)/d), Abs(x) < 1
), (-log(d)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/d), 1/Abs(x) < 1), (-
meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), ((),
(0, 0)), x)*log(d) - polylog(2, e*x*exp_polar(I*pi)/d), True))/e, True))/e*
```

```
*3 + 3*b*d**2*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))*log(c*x**n
)/e**3 + 2*b*d*n*x/e**3 - 2*b*d*x*log(c*x**n)/e**3 - b*n*x**2/(4*e**2) + b*
x**2*log(c*x**n)/(2*e**2)
```

Maxima [F]

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{(ex+d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^2,x, algorithm="maxima")
[Out] 1/2*(2*d^3/(e^5*x + d*e^4) + 6*d^2*log(e*x + d)/e^4 + (e*x^2 - 4*d*x)/e^3)*
a + b*integrate((x^3*log(c) + x^3*log(x^n))/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Giac [F]

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{(ex+d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^3/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^2}\,dx = \int \frac{x^3\,(a+b\,\ln(c\,x^n))}{(d+e\,x)^2}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n)))/(d + e*x)^2,x)
[Out] int((x^3*(a + b*log(c*x^n)))/(d + e*x)^2, x)
```

3.40 $\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^2}\,dx = -\frac{bnx}{e^2}+\frac{2x(a+b\log(cx^n))}{e^2}-\frac{x^2(a+b\log(cx^n))}{e(d+ex)} - \frac{d(2a+bn+2b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^3} - \frac{2bdn\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^3}$$

```
[Out] -b*n*x/e^2+2*x*(a+b*ln(c*x^n))/e^2-x^2*(a+b*ln(c*x^n))/e/(e*x+d)-d*(2*a+b*n
+2*b*ln(c*x^n))*ln(1+e*x/d)/e^3-2*b*d*n*polylog(2,-e*x/d)/e^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2384, 45, 2393, 2332, 2354, 2438}

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^2}\,dx = -\frac{d\log\left(\frac{ex}{d}+1\right)(2a+2b\log(cx^n)+bn)}{e^3}-\frac{x^2(a+b\log(cx^n))}{e(d+ex)} + \frac{x(2a+bn)}{e^2}+\frac{2bx\log(cx^n)}{e^2}-\frac{2bdn\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^3}-\frac{2bnx}{e^2}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/(d + e*x)^2,x]
```

```
[Out] (-2*b*n*x)/e^2 + ((2*a + b*n)*x)/e^2 + (2*b*x*Log[c*x^n])/e^2 - (x^2*(a + b
*Log[c*x^n]))/(e*(d + e*x)) - (d*(2*a + b*n + 2*b*Log[c*x^n])*Log[1 + (e*x)
/d])/e^3 - (2*b*d*n*PolyLog[2, -((e*x)/d)])/e^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^2(a+b\log(cx^n))}{e(d+ex)}+\frac{\int \frac{x(2a+bn+2b\log(cx^n))}{d+ex}\,dx}{e}\\ &= -\frac{x^2(a+b\log(cx^n))}{e(d+ex)}+\frac{\int\left(\frac{2a+bn+2b\log(cx^n)}{e}-\frac{d(2a+bn+2b\log(cx^n))}{e(d+ex)}\right)dx}{e}\end{aligned}$$

$$= -\frac{x^2(a+b\log(cx^n))}{e(d+ex)} + \frac{\int (2a+bn+2b\log(cx^n))\,dx}{e^2} - \frac{d\int \frac{2a+bn+2b\log(cx^n)}{d+ex}\,dx}{e^2}$$

$$= \frac{(2a+bn)x}{e^2} - \frac{x^2(a+b\log(cx^n))}{e(d+ex)} - \frac{d(2a+bn+2b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^3} + \frac{(2b)\int \log(cx^n)\,dx}{e^2} + \frac{(2bdn)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^3}$$

$$= -\frac{2bnx}{e^2} + \frac{(2a+bn)x}{e^2} + \frac{2bx\log(cx^n)}{e^2} - \frac{x^2(a+b\log(cx^n))}{e(d+ex)} - \frac{d(2a+bn+2b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^3} - \frac{2bdn\text{Li}_2\left(-\frac{ex}{d}\right)}{e^3}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^2}\,dx = \frac{aex - benx + bex\log(cx^n) - \frac{d^2(a+b\log(cx^n))}{d+ex} + bdn(\log(x)-\log(d+ex)) - 2d(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^3}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*x)^2,x]
```

```
[Out] (a*e*x - b*e*n*x + b*e*x*Log[c*x^n] - (d^2*(a + b*Log[c*x^n]))/(d + e*x) +
b*d*n*(Log[x] - Log[d + e*x]) - 2*d*(a + b*Log[c*x^n])*Log[1 + (e*x)/d] - 2
*b*d*n*PolyLog[2, -((e*x)/d)])/e^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.41 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.55

method	result
risch	$\frac{b\ln(x^n)x}{e^2} - \frac{2b\ln(x^n)d\ln(ex+d)}{e^3} - \frac{b\ln(x^n)d^2}{e^3(ex+d)} - \frac{bnd\ln(ex+d)}{e^3} + \frac{bnd\ln(ex)}{e^3} - \frac{bnx}{e^2} - \frac{bnd}{e^3} + \frac{2bnd\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{e^3} +$

```
[In] int(x^2*(a+b*ln(c*x^n))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*ln(x^n)/e^2*x-2*b*ln(x^n)/e^3*d*ln(e*x+d)-b*ln(x^n)/e^3*d^2/(e*x+d)-b*n/e
^3*d*ln(e*x+d)+b*n/e^3*d*ln(e*x)-b*n*x/e^2-b*n/e^3*d+2*b*n/e^3*d*ln(e*x+d)*
ln(-e*x/d)+2*b*n/e^3*d*dilog(-e*x/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csg
n(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn
(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(x/e^2-2/e^3*d*ln(e*x+d)-
1/e^3*d^2/(e*x+d))
```

Fricas [F]

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{(ex+d)^2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((b*x^2*log(c*x^n) + a*x^2)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [A] (verification not implemented)

Time = 12.08 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.74

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^2}\,dx = \frac{ad^2\left(\begin{cases} \frac{x}{d^2} & \text{for } e=0 \\ -\frac{1}{de+e^2x} & \text{otherwise} \end{cases}\right)}{e^2} - \frac{2ad\left(\begin{cases} \frac{x}{d} & \text{for } e=0 \\ \frac{\log(d+ex)}{e} & \text{otherwise} \end{cases}\right)}{e^2} + \frac{ax}{e^2} - \frac{bd^2n\left(\begin{cases} \frac{x}{d^2} & \text{for } e=0 \\ -\frac{\log(x)}{de} + \frac{\log\left(\frac{d}{e}+x\right)}{de} & \text{otherwise} \end{cases}\right)}{e^2} + \frac{bd^2\left(\begin{cases} \frac{x}{d^2} & \text{for } e=0 \\ -\frac{1}{de+e^2x} & \text{otherwise} \end{cases}\right)\log(cx^n)}{e^2} + \frac{2bdn\left(\begin{cases} \frac{x}{d} \\ \frac{\begin{cases} -\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1 \\ \log(d)\log(x) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } |x|<1 \\ -\log(d)\log\left(\frac{1}{x}\right) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(d) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(d) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{otherwise} \end{cases}}{e} \end{cases}\right)}{e^2} - \frac{2bd\left(\begin{cases} \frac{x}{d} & \text{for } e=0 \\ \frac{\log(d+ex)}{e} & \text{otherwise} \end{cases}\right)\log(cx^n)}{e^2} - \frac{bnx}{e^2} + \frac{bx\log(cx^n)}{e^2}$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(e*x+d)**2,x)
```

```
[Out] a*d**2*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))/e**2 - 2*a*
d*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e**2 + a*x/e**2 - b*d*
*2*n*Piecewise((x/d**2, Eq(e, 0)), (-log(x)/(d*e) + log(d/e + x)/(d*e), Tru
e))/e**2 + b*d**2*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))*
log(c*x**n)/e**2 + 2*b*d*n*Piecewise((x/d, Eq(e, 0)), (Piecewise((-polylog(
2, e*x*exp_polar(I*pi)/d), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) -
 polylog(2, e*x*exp_polar(I*pi)/d), Abs(x) < 1), (-log(d)*log(1/x) - polylo
g(2, e*x*exp_polar(I*pi)/d), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0)
, ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(
2, e*x*exp_polar(I*pi)/d), True))/e, True))/e**2 - 2*b*d*Piecewise((x/d, Eq
(e, 0)), (log(d + e*x)/e, True))*log(c*x**n)/e**2 - b*n*x/e**2 + b*x*log(c*
x**n)/e**2
```

Maxima [F]

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^2}\,dx=\int \frac{(b\log(cx^n)+a)x^2}{(ex+d)^2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -a*(d^2/(e^4*x + d*e^3) - x/e^2 + 2*d*log(e*x + d)/e^3) + b*integrate((x^2*
log(c) + x^2*log(x^n))/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Giac [F]

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^2}\,dx=\int \frac{(b\log(cx^n)+a)x^2}{(ex+d)^2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^2}\,dx=\int \frac{x^2\,(a+b\,\ln(c\,x^n))}{(d+e\,x)^2}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e*x)^2,x)
```

```
[Out] int((x^2*(a + b*log(c*x^n)))/(d + e*x)^2, x)
```

3.41 $\int \frac{x(a+b\log(cx^n))}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^2}\,dx = -\frac{x(a+b\log(cx^n))}{e(d+ex)}+\frac{(a+bn+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^2}+\frac{bn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^2}$$

```
[Out] -x*(a+b*ln(c*x^n))/e/(e*x+d)+(a+b*n+b*ln(c*x^n))*ln(1+e*x/d)/e^2+b*n*polylo
g(2,-e*x/d)/e^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2384, 2354, 2438}

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^2}\,dx = \frac{\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n)+bn)}{e^2}-\frac{x(a+b\log(cx^n))}{e(d+ex)}+\frac{bn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^2}$$

```
[In] Int[(x*(a + b*Log[c*x^n]))/(d + e*x)^2,x]
```

```
[Out] -((x*(a + b*Log[c*x^n]))/(e*(d + e*x))) + ((a + b*n + b*Log[c*x^n])*Log[1 +
 (e*x)/d])/e^2 + (b*n*PolyLog[2, -((e*x)/d)])/e^2
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
```

```
Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(a+b\log(cx^n))}{e(d+ex)} + \frac{\int \frac{a+bn+b\log(cx^n)}{d+ex}\,dx}{e} \\
&= -\frac{x(a+b\log(cx^n))}{e(d+ex)} + \frac{(a+bn+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^2} - \frac{(bn)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^2} \\
&= -\frac{x(a+b\log(cx^n))}{e(d+ex)} + \frac{(a+bn+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^2} + \frac{bn\text{Li}_2\left(-\frac{ex}{d}\right)}{e^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^2}\,dx = \frac{\frac{d(a+b\log(cx^n))}{d+ex} - bn(\log(x)-\log(d+ex)) + (a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right) + bn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^2}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e*x)^2,x]
```

```
[Out] ((d*(a + b*Log[c*x^n]))/(d + e*x) - b*n*(Log[x] - Log[d + e*x]) + (a + b*Lo
g[c*x^n])*Log[1 + (e*x)/d] + b*n*PolyLog[2, -((e*x)/d)])/e^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.78 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.15

method	result
risch	$\frac{b\ln(x^n)\ln(ex+d)}{e^2}+\frac{b\ln(x^n)d}{e^2(ex+d)}-\frac{bn\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{e^2}-\frac{bn\,\mathrm{dilog}\left(-\frac{ex}{d}\right)}{e^2}+\frac{bn\ln(ex+d)}{e^2}-\frac{bn\ln(ex)}{e^2}+\left(-\frac{ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}\right.$

[In] int(x*(a+b*ln(c*x^n))/(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] b*ln(x^n)/e^2*ln(e*x+d)+b*ln(x^n)/e^2*d/(e*x+d)-b*n/e^2*ln(e*x+d)*ln(-e*x/d)-b*n/e^2*dilog(-e*x/d)+b*n/e^2*ln(e*x+d)-b*n/e^2*ln(e*x)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/e^2*ln(e*x+d)+1/e^2*d/(e*x+d))

Fricas [F]

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^2}\,dx=\int \frac{(b\log(cx^n)+a)x}{(ex+d)^2}\,dx$$

[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^2,x, algorithm="fricas")

[Out] integral((b*x*log(c*x^n) + a*x)/(e^2*x^2 + 2*d*e*x + d^2), x)

Sympy [F]

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^2}\,dx=\int \frac{x(a+b\log(cx^n))}{(d+ex)^2}\,dx$$

[In] integrate(x*(a+b*ln(c*x**n))/(e*x+d)**2,x)

[Out] Integral(x*(a + b*log(c*x**n))/(d + e*x)**2, x)

Maxima [F]

$$\int \frac{x(a + b\log(cx^n))}{(d + ex)^2}\,dx = \int \frac{(b\log(cx^n) + a)x}{(ex + d)^2}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] a*(d/(e^3*x + d*e^2) + log(e*x + d)/e^2) + b*integrate((x*log(c) + x*log(x^
n))/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Giac [F]

$$\int \frac{x(a + b\log(cx^n))}{(d + ex)^2}\,dx = \int \frac{(b\log(cx^n) + a)x}{(ex + d)^2}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\log(cx^n))}{(d + ex)^2}\,dx = \int \frac{x\,(a + b\,\ln(c\,x^n))}{(d + e\,x)^2}\,dx$$

```
[In] int((x*(a + b*log(c*x^n)))/(d + e*x)^2,x)
```

```
[Out] int((x*(a + b*log(c*x^n)))/(d + e*x)^2, x)
```

3.42 $\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 39

$$\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx=\frac{x(a+b\log(cx^n))}{d(d+ex)}-\frac{bn\log(d+ex)}{de}$$

```
[Out] x*(a+b*ln(c*x^n))/d/(e*x+d)-b*n*ln(e*x+d)/d/e
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2351, 31}

$$\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx=\frac{x(a+b\log(cx^n))}{d(d+ex)}-\frac{bn\log(d+ex)}{de}$$

```
[In] Int[(a + b*Log[c*x^n])/(d + e*x)^2,x]
```

```
[Out] (x*(a + b*Log[c*x^n]))/(d*(d + e*x)) - (b*n*Log[d + e*x])/(d*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x(a+b\log(cx^n))}{d(d+ex)} - \frac{(bn)\int \frac{1}{d+ex}\,dx}{d}\\ &= \frac{x(a+b\log(cx^n))}{d(d+ex)} - \frac{bn\log(d+ex)}{de}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx = \frac{-\frac{a+b\log(cx^n)}{d+ex} + \frac{bn(\log(x)-\log(d+ex))}{d}}{e}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*x)^2,x]
[Out] (-((a + b*Log[c*x^n])/(d + e*x)) + (b*n*(Log[x] - Log[d + e*x]))/d)/e
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

method	result
parallelrisch	$-\frac{\ln(ex+d)benx+\ln(ex+d)bdn-bex\ln(c\,x^n)-aex}{d(ex+d)e}$
risch	$-\frac{b\ln(x^n)}{e(ex+d)} - \frac{-i\pi bd\ \operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)+i\pi bd\ \operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^n)^2+i\pi bd\ \operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2-i\pi bd\operatorname{csgn}(ic\,x}{2(ex+d)ed}$

```
[In] int((a+b*ln(c*x^n))/(e*x+d)^2,x,method=_RETURNVERBOSE)
[Out] -(ln(e*x+d)*b*e*n*x+ln(e*x+d)*b*d*n-b*e*x*ln(c*x^n)-a*e*x)/d/(e*x+d)/e
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx = \frac{benx\log(x) - bd\log(c) - ad - (benx+bdn)\log(ex+d)}{de^2x+d^2e}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d)^2,x, algorithm="fricas")
[Out] (b*e*n*x*log(x) - b*d*log(c) - a*d - (b*e*n*x + b*d*n)*log(e*x + d))/(d*e^2
*x + d^2*e)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(31) = 62$.

Time = 0.62 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.92

$$\int \frac{a + b\log(cx^n)}{(d + ex)^2}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{a}{x} - \frac{bn}{x} - \frac{b\log(cx^n)}{x}\right) & \text{for } d = 0 \wedge e = 0 \\ \frac{ax - bnx + bx\log(cx^n)}{d^2} & \text{for } e = 0 \\ \frac{-\frac{a}{x} - \frac{bn}{x} - \frac{b\log(cx^n)}{x}}{e^2} & \text{for } d = 0 \\ -\frac{ad}{d^2e + de^2x} - \frac{bdn\log\left(\frac{d}{e} + x\right)}{d^2e + de^2x} - \frac{benx\log\left(\frac{d}{e} + x\right)}{d^2e + de^2x} + \frac{bex\log(cx^n)}{d^2e + de^2x} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*ln(c*x**n))/(e*x+d)**2,x)
```

```
[Out] Piecewise((zoo*(-a/x - b*n/x - b*log(c*x**n)/x), Eq(d, 0) & Eq(e, 0)), ((a*
x - b*n*x + b*x*log(c*x**n))/d**2, Eq(e, 0)), ((-a/x - b*n/x - b*log(c*x**n
)/x)/e**2, Eq(d, 0)), (-a*d/(d**2*e + d*e**2*x) - b*d*n*log(d/e + x)/(d**2*
e + d*e**2*x) - b*e*n*x*log(d/e + x)/(d**2*e + d*e**2*x) + b*e*x*log(c*x**n
)/(d**2*e + d*e**2*x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int \frac{a + b\log(cx^n)}{(d + ex)^2}\,dx = -bn\left(\frac{\log(ex + d)}{de} - \frac{\log(x)}{de}\right) - \frac{b\log(cx^n)}{e^2x + de} - \frac{a}{e^2x + de}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -b*n*(log(e*x + d)/(d*e) - log(x)/(d*e)) - b*log(c*x^n)/(e^2*x + d*e) - a/(
e^2*x + d*e)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.64

$$\int \frac{a+b\log\left(cx^n\right)}{(d+ex)^2}\,dx = -\frac{bn\log\left(x\right)}{e^2x+de}-\frac{bn\log\left(ex+d\right)}{de}+\frac{bn\log\left(x\right)}{de}-\frac{b\log\left(c\right)+a}{e^2x+de}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -b*n*log(x)/(e^2*x + d*e) - b*n*log(e*x + d)/(d*e) + b*n*log(x)/(d*e) - (b*
log(c) + a)/(e^2*x + d*e)
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int \frac{a+b\log\left(cx^n\right)}{(d+ex)^2}\,dx = -\frac{a}{x\,e^2+d\,e}-\frac{b\,\ln\left(c\,x^n\right)}{e\,\left(d+e\,x\right)}-\frac{2\,b\,n\,\mathrm{atanh}\left(\frac{2\,e\,x}{d}+1\right)}{d\,e}$$

```
[In] int((a + b*log(c*x^n))/(d + e*x)^2,x)
```

```
[Out] - a/(d*e + e^2*x) - (b*log(c*x^n))/(e*(d + e*x)) - (2*b*n*atanh((2*e*x)/d +
 1))/(d*e)
```

3.43 $\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx = -\frac{ex(a+b\log(cx^n))}{d^2(d+ex)}-\frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^2}+\frac{bn\log(d+ex)}{d^2}+\frac{bn\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^2}$$

```
[Out] -e*x*(a+b*ln(c*x^n))/d^2/(e*x+d)-ln(1+d/e/x)*(a+b*ln(c*x^n))/d^2+b*n*ln(e*x
+d)/d^2+b*n*polylog(2,-d/e/x)/d^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2389, 2379, 2438, 2351, 31}

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx = -\frac{\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^2}-\frac{ex(a+b\log(cx^n))}{d^2(d+ex)}+\frac{bn\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^2}+\frac{bn\log(d+ex)}{d^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x)^2),x]
```

```
[Out] -((e*x*(a + b*Log[c*x^n]))/(d^2*(d + e*x))) - (Log[1 + d/(e*x)]*(a + b*Log[
c*x^n]))/d^2 + (b*n*Log[d + e*x])/d^2 + (b*n*PolyLog[2, -(d/(e*x))])/d^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d} - \frac{e\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{d}$$

$$= -\frac{ex(a+b\log(cx^n))}{d^2(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^2} + \frac{(bn)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^2} + \frac{(ben)\int \frac{1}{d+ex}\,dx}{d^2}$$

$$= -\frac{ex(a+b\log(cx^n))}{d^2(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^2} + \frac{bn\log(d+ex)}{d^2} + \frac{bn\text{Li}_2\left(-\frac{d}{ex}\right)}{d^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx$$
$$=\frac{\frac{2d(a+b\log(cx^n))}{d+ex}+\frac{(a+b\log(cx^n))^2}{bn}-2bn(\log(x)-\log(d+ex))-2(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)-2bn\,\mathrm{PolyLog}\left(\right.}{2d^2}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x)^2),x]
```

```
[Out] ((2*d*(a + b*Log[c*x^n]))/(d + e*x) + (a + b*Log[c*x^n])^2/(b*n) - 2*b*n*(L
og[x] - Log[d + e*x]) - 2*(a + b*Log[c*x^n])*Log[1 + (e*x)/d] - 2*b*n*PolyL
og[2, -((e*x)/d)])/(2*d^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.86

method	result
risch	$-\frac{b\ln(x^n)\ln(ex+d)}{d^2}+\frac{b\ln(x^n)}{d(ex+d)}+\frac{b\ln(x^n)\ln(x)}{d^2}-\frac{bn\ln(x)^2}{2d^2}+\frac{bn\ln(ex+d)}{d^2}-\frac{bn\ln(x)}{d^2}+\frac{bn\ln(ex+d)\ln(-\frac{ex}{d})}{d^2}+\frac{bn\,\mathrm{dilog}(}{d^2}$

```
[In] int((a+b*ln(c*x^n))/x/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b*ln(x^n)/d^2*ln(e*x+d)+b*ln(x^n)/d/(e*x+d)+b*ln(x^n)/d^2*ln(x)-1/2*b*n/d^
2*ln(x)^2+b*n*ln(e*x+d)/d^2-b*n/d^2*ln(x)+b*n/d^2*ln(e*x+d)*ln(-e*x/d)+b*n/
d^2*dilog(-e*x/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*
Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b
*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-1/d^2*ln(e*x+d)+1/d/(e*x+d)+1/d^2*ln(x))
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx=\int \frac{b\log(cx^n)+a}{(ex+d)^2x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e^2*x^3 + 2*d*e*x^2 + d^2*x), x)
```

Sympy [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx = \int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x/(e*x+d)**2,x)
[Out] Integral((a + b*log(c*x**n))/(x*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)^2x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^2,x, algorithm="maxima")
[Out] a*(1/(d*e*x + d^2) - log(e*x + d)/d^2 + log(x)/d^2) + b*integrate((log(c) +
 log(x^n))/(e^2*x^3 + 2*d*e*x^2 + d^2*x), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)^2x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x + d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x\,(d+e\,x)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x)^2),x)
[Out] int((a + b*log(c*x^n))/(x*(d + e*x)^2), x)
```

3.44 $\int \frac{a+b\log(cx^n)}{x^2(d+ex)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^2}\,dx = -\frac{bn}{d^2x} - \frac{a+b\log(cx^n)}{d^2x} + \frac{e^2x(a+b\log(cx^n))}{d^3(d+ex)} + \frac{2e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^3} - \frac{ben\log(d+ex)}{d^3} - \frac{2ben\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^3}$$

```
[Out] -b*n/d^2/x+(-a-b*ln(c*x^n))/d^2/x+e^2*x*(a+b*ln(c*x^n))/d^3/(e*x+d)+2*e*ln(
1+d/e/x)*(a+b*ln(c*x^n))/d^3-b*e*n*ln(e*x+d)/d^3-2*b*e*n*polylog(2,-d/e/x)/
d^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {46, 2393, 2341, 2351, 31, 2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^2}\,dx = \frac{e^2x(a+b\log(cx^n))}{d^3(d+ex)} + \frac{2e\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^3} - \frac{a+b\log(cx^n)}{d^2x} - \frac{2ben\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^3} - \frac{ben\log(d+ex)}{d^3} - \frac{bn}{d^2x}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^2*(d + e*x)^2),x]
```

```
[Out] -((b*n)/(d^2*x)) - (a + b*Log[c*x^n])/(d^2*x) + (e^2*x*(a + b*Log[c*x^n]))/
(d^3*(d + e*x)) + (2*e*Log[1 + d/(e*x)]*(a + b*Log[c*x^n]))/d^3 - (b*e*n*Lo
g[d + e*x])/d^3 - (2*b*e*n*PolyLog[2, -(d/(e*x))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a+b\log(cx^n)}{d^2x^2}+\frac{e^2(a+b\log(cx^n))}{d^2(d+ex)^2}-\frac{2e(a+b\log(cx^n))}{d^2x(d+ex)}\right)dx\\
&= \frac{\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d^2}-\frac{(2e)\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d^2}+\frac{e^2\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{d^2}\\
&= -\frac{bn}{d^2x}-\frac{a+b\log(cx^n)}{d^2x}+\frac{e^2x(a+b\log(cx^n))}{d^3(d+ex)}\\
&\quad+\frac{2e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^3}-\frac{(2ben)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^3}-\frac{(be^2n)\int \frac{1}{d+ex}\,dx}{d^3}\\
&= -\frac{bn}{d^2x}-\frac{a+b\log(cx^n)}{d^2x}+\frac{e^2x(a+b\log(cx^n))}{d^3(d+ex)}\\
&\quad+\frac{2e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^3}-\frac{ben\log(d+ex)}{d^3}-\frac{2ben\text{Li}_2\left(-\frac{d}{ex}\right)}{d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.05

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^2}\,dx = -\frac{\frac{bdn}{x}+\frac{d(a+b\log(cx^n))}{x}+\frac{de(a+b\log(cx^n))}{d+ex}+\frac{e(a+b\log(cx^n))^2}{bn}-ben(\log(x)-\log(d+ex))-2e(a+b\log(cx^n))\log}{d^3}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*x)^2),x]
```

```
[Out] -(((b*d*n)/x + (d*(a + b*Log[c*x^n]))/x + (d*e*(a + b*Log[c*x^n]))/(d + e*x
) + (e*(a + b*Log[c*x^n])^2)/(b*n) - b*e*n*(Log[x] - Log[d + e*x]) - 2*e*(a
 + b*Log[c*x^n])*Log[1 + (e*x)/d] - 2*b*e*n*PolyLog[2, -((e*x)/d)])/d^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.42

method	result
risch	$-\frac{b\ln(x^n)e}{d^2(ex+d)}+\frac{2b\ln(x^n)e\ln(ex+d)}{d^3}-\frac{b\ln(x^n)}{d^2x}-\frac{2b\ln(x^n)e\ln(x)}{d^3}+\frac{bne\ln(x)^2}{d^3}-\frac{2bne\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{d^3}-\frac{2bne\,\text{dilog}\left(-\frac{ex}{d}\right)}{d^3}$

```
[In] int((a+b*ln(c*x^n))/x^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b*ln(x^n)/d^2*e/(e*x+d)+2*b*ln(x^n)/d^3*e*ln(e*x+d)-b*ln(x^n)/d^2/x-2*b*ln
(x^n)/d^3*e*ln(x)+b*n/d^3*e*ln(x)^2-2*b*n/d^3*e*ln(e*x+d)*ln(-e*x/d)-2*b*n/
d^3*e*dilog(-e*x/d)-b*e*n*ln(e*x+d)/d^3-b*n/d^2/x+b*n/d^3*e*ln(x)+(-1/2*I*b
*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^
2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)
+a)*(-1/d^2*e/(e*x+d)+2/d^3*e*ln(e*x+d)-1/d^2/x-2/d^3*e*ln(x))
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{2}(d+ex)^{2}}\,dx=\int \frac{b\log\left(cx^{n}\right)+a}{\left(ex+d\right)^{2}x^{2}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2), x)
```

Sympy [A] (verification not implemented)

Time = 30.60 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.79

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^2}\,dx = \frac{ae^2\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x} & \text{otherwise}\end{cases}\right)}{d^2} - \frac{a}{d^2x} + \frac{2ae^2\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{d^3} - \frac{2ae\log(x)}{d^3} - \frac{be^2n\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{\log(x)}{de}+\frac{\log\left(\frac{d}{e}+x\right)}{de} & \text{otherwise}\end{cases}\right)}{d^2} + \frac{be^2\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^2} - \frac{bn}{d^2x} - \frac{b\log(cx^n)}{d^2x} - \frac{2be^2n\left(\begin{cases}\frac{x}{d} & \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1\wedge|x|<1\\ \log(d)\log(x)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log(d)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{otherwise}\end{cases}}{e} & \end{cases}\right)}{d^3} + \frac{2be^2\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^3} + \frac{ben\log(x)^2}{d^3} - \frac{2be\log(x)\log(cx^n)}{d^3}$$

```
[In] integrate((a+b*ln(c*x**n))/x**2/(e*x+d)**2,x)
```

```
[Out] a*e**2*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))/d**2 - a/(d
**2*x) + 2*a*e**2*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/d**3 -
 2*a*e*log(x)/d**3 - b*e**2*n*Piecewise((x/d**2, Eq(e, 0)), (-log(x)/(d*e)
+ log(d/e + x)/(d*e), True))/d**2 + b*e**2*Piecewise((x/d**2, Eq(e, 0)), (-
1/(d*e + e**2*x), True))*log(c*x**n)/d**2 - b*n/(d**2*x) - b*log(c*x**n)/(d
```

```
**2*x) - 2*b*e**2*n*Piecewise((x/d, Eq(e, 0)), (Piecewise((-polylog(2, e*x*
exp_polar(I*pi)/d), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylo
g(2, e*x*exp_polar(I*pi)/d), Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*
x*exp_polar(I*pi)/d), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()),
x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x*
exp_polar(I*pi)/d), True))/e, True))/d**3 + 2*b*e**2*Piecewise((x/d, Eq(e,
0)), (log(d + e*x)/e, True))*log(c*x**n)/d**3 + b*e*n*log(x)**2/d**3 - 2*b*
e*log(x)*log(c*x**n)/d**3
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^2}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)^2x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -a*((2*e*x + d)/(d^2*e*x^2 + d^3*x) - 2*e*log(e*x + d)/d^3 + 2*e*log(x)/d^3
) + b*integrate((log(c) + log(x^n))/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^2}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)^2x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x + d)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^2}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^2\,(d+e\,x)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*x)^2),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*x)^2), x)
```

3.45 $\int \frac{a+b\log(cx^n)}{x^3(d+ex)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 154

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^2}\,dx = -\frac{bn}{4d^2x^2}+\frac{2ben}{d^3x}-\frac{a+b\log(cx^n)}{2d^2x^2}+\frac{2e(a+b\log(cx^n))}{d^3x} -\frac{e^3x(a+b\log(cx^n))}{d^4(d+ex)}-\frac{3e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4} +\frac{be^2n\log(d+ex)}{d^4}+\frac{3be^2n\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^4}$$

```
[Out] -1/4*b*n/d^2/x^2+2*b*e*n/d^3/x+1/2*(-a-b*ln(c*x^n))/d^2/x^2+2*e*(a+b*ln(c*x
^n))/d^3/x-e^3*x*(a+b*ln(c*x^n))/d^4/(e*x+d)-3*e^2*ln(1+d/e/x)*(a+b*ln(c*x^
n))/d^4+b*e^2*n*ln(e*x+d)/d^4+3*b*e^2*n*polylog(2,-d/e/x)/d^4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {46, 2393, 2341, 2351, 31, 2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^2}\,dx = -\frac{e^3x(a+b\log(cx^n))}{d^4(d+ex)}-\frac{3e^2\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^4} +\frac{2e(a+b\log(cx^n))}{d^3x}-\frac{a+b\log(cx^n)}{2d^2x^2} +\frac{3be^2n\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^4}+\frac{be^2n\log(d+ex)}{d^4}+\frac{2ben}{d^3x}-\frac{bn}{4d^2x^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^3*(d + e*x)^2),x]
```

```
[Out] -1/4*(b*n)/(d^2*x^2) + (2*b*e*n)/(d^3*x) - (a + b*Log[c*x^n])/(2*d^2*x^2) +
 (2*e*(a + b*Log[c*x^n]))/(d^3*x) - (e^3*x*(a + b*Log[c*x^n]))/(d^4*(d + e*
x)) - (3*e^2*Log[1 + d/(e*x)]*(a + b*Log[c*x^n]))/d^4 + (b*e^2*n*Log[d + e*
x])/d^4 + (3*b*e^2*n*PolyLog[2, -(d/(e*x))])/d^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log(cx^n)}{d^2x^3} - \frac{2e(a+b\log(cx^n))}{d^3x^2} - \frac{e^3(a+b\log(cx^n))}{d^3(d+ex)^2} + \frac{3e^2(a+b\log(cx^n))}{d^3x(d+ex)}\right) dx$$

$$= \frac{\int \frac{a+b\log(cx^n)}{x^3}\,dx}{d^2} - \frac{(2e)\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d^3} + \frac{(3e^2)\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d^3} - \frac{e^3\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{d^3}$$

$$= -\frac{bn}{4d^2x^2} + \frac{2ben}{d^3x} - \frac{a+b\log(cx^n)}{2d^2x^2} + \frac{2e(a+b\log(cx^n))}{d^3x} - \frac{e^3x(a+b\log(cx^n))}{d^4(d+ex)} - \frac{3e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4} + \frac{(3be^2n)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^4} + \frac{(be^3n)\int \frac{1}{d+ex}\,dx}{d^4}$$

$$= -\frac{bn}{4d^2x^2} + \frac{2ben}{d^3x} - \frac{a+b\log(cx^n)}{2d^2x^2} + \frac{2e(a+b\log(cx^n))}{d^3x} - \frac{e^3x(a+b\log(cx^n))}{d^4(d+ex)} - \frac{3e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4} + \frac{be^2n\log(d+ex)}{d^4} + \frac{3be^2n\text{Li}_2\left(-\frac{d}{ex}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^2}\,dx = -\frac{\frac{bd^2n}{x^2} - \frac{8bden}{x} + \frac{2d^2(a+b\log(cx^n))}{x^2} - \frac{8de(a+b\log(cx^n))}{x} - \frac{4de^2(a+b\log(cx^n))}{d+ex} - \frac{6e^2(a+b\log(cx^n))^2}{bn} + 4be^2n(\log(x)-\log(d}{4d^4}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*(d + e*x)^2),x]
```

```
[Out] -1/4*((b*d^2*n)/x^2 - (8*b*d*e*n)/x + (2*d^2*(a + b*Log[c*x^n]))/x^2 - (8*d
*e*(a + b*Log[c*x^n]))/x - (4*d*e^2*(a + b*Log[c*x^n]))/(d + e*x) - (6*e^2*
(a + b*Log[c*x^n])^2)/(b*n) + 4*b*e^2*n*(Log[x] - Log[d + e*x]) + 12*e^2*(a
 + b*Log[c*x^n])*Log[1 + (e*x)/d] + 12*b*e^2*n*PolyLog[2, -((e*x)/d)])/d^4
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.15

method	result
risch	$-\frac{3b\ln(x^n)e^2\ln(ex+d)}{d^4}+\frac{b\ln(x^n)e^2}{d^3(ex+d)}-\frac{b\ln(x^n)}{2d^2x^2}+\frac{3b\ln(x^n)e^2\ln(x)}{d^4}+\frac{2b\ln(x^n)e}{d^3x}+\frac{b\,e^2n\ln(ex+d)}{d^4}-\frac{bn}{4d^2x^2}+\frac{2ben}{d^3x}-$

[In] int((a+b*ln(c*x^n))/x^3/(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] -3*b*ln(x^n)/d^4*e^2*ln(e*x+d)+b*ln(x^n)/d^3*e^2/(e*x+d)-1/2*b*ln(x^n)/d^2/x^2+3*b*ln(x^n)/d^4*e^2*ln(x)+2*b*ln(x^n)/d^3*e/x+b*e^2*n*ln(e*x+d)/d^4-1/4*b*n/d^2/x^2+2*b*e*n/d^3/x-b*n/d^4*e^2*ln(x)-3/2*b*n/d^4*e^2*ln(x)^2+3*b*n/d^4*e^2*ln(e*x+d)*ln(-e*x/d)+3*b*n/d^4*e^2*dilog(-e*x/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-3/d^4*e^2*ln(e*x+d)+1/d^3*e^2/(e*x+d)-1/2/d^2/x^2+3/d^4*e^2*ln(x)+2/d^3*e/x)

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^2}\,dx=\int \frac{b\log(cx^n)+a}{(ex+d)^2x^3}\,dx$$

[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d)^2,x, algorithm="fricas")

[Out] integral((b*log(c*x^n) + a)/(e^2*x^5 + 2*d*e*x^4 + d^2*x^3), x)

Sympy [A] (verification not implemented)

Time = 43.49 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.44

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^2}\,dx$$

$$= -\frac{a}{2d^2x^2} - \frac{ae^3\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x} & \text{otherwise}\end{cases}\right)}{d^3} + \frac{2ae}{d^3x} - \frac{3ae^3\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{d^4} + \frac{3ae^2\log(x)}{d^4} - \frac{bn}{4d^2x^2} - \frac{b\log(cx^n)}{2d^2x^2} + \frac{be^3n\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{\log(x)}{de}+\frac{\log\left(\frac{d}{e}+x\right)}{de} & \text{otherwise}\end{cases}\right)}{d^3} - \frac{be^3\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^3} + \frac{2ben}{d^3x} + \frac{2be\log(cx^n)}{d^3x} + \frac{3be^3n\left(\begin{cases}\frac{x}{d} \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(d)\log(x)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(d)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{otherwise}\end{cases}}{e}\end{cases}\right)}{d^4} - \frac{3be^3\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^4} - \frac{3be^2n\log(x)^2}{2d^4} + \frac{3be^2\log(x)\log(cx^n)}{d^4}$$

```
[In] integrate((a+b*ln(c*x**n))/x**3/(e*x+d)**2,x)
```

```
[Out] -a/(2*d**2*x**2) - a*e**3*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x),
 True))/d**3 + 2*a*e/(d**3*x) - 3*a*e**3*Piecewise((x/d, Eq(e, 0)), (log(d
+ e*x)/e, True))/d**4 + 3*a*e**2*log(x)/d**4 - b*n/(4*d**2*x**2) - b*log(c*
x**n)/(2*d**2*x**2) + b*e**3*n*Piecewise((x/d**2, Eq(e, 0)), (-log(x)/(d*e)
 + log(d/e + x)/(d*e), True))/d**3 - b*e**3*Piecewise((x/d**2, Eq(e, 0)), (
-1/(d*e + e**2*x), True))*log(c*x**n)/d**3 + 2*b*e*n/(d**3*x) + 2*b*e*log(c
*x**n)/(d**3*x) + 3*b*e**3*n*Piecewise((x/d, Eq(e, 0)), (Piecewise((-polylo
g(2, e*x*exp_polar(I*pi)/d), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x)
```

```
 - polylog(2, e*x*exp_polar(I*pi)/d), Abs(x) < 1), (-log(d)*log(1/x) - poly
log(2, e*x*exp_polar(I*pi)/d), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0,
0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylo
g(2, e*x*exp_polar(I*pi)/d), True))/e, True))/d**4 - 3*b*e**3*Piecewise((x/
d, Eq(e, 0)), (log(d + e*x)/e, True))*log(c*x**n)/d**4 - 3*b*e**2*n*log(x)*
*2/(2*d**4) + 3*b*e**2*log(x)*log(c*x**n)/d**4
```

Maxima [F]

$$\int \frac{a + b\log(cx^n)}{x^3(d + ex)^2}\,dx = \int \frac{b\log(cx^n) + a}{(ex + d)^2 x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*((6*e^2*x^2 + 3*d*e*x - d^2)/(d^3*e*x^3 + d^4*x^2) - 6*e^2*log(e*x +
d)/d^4 + 6*e^2*log(x)/d^4) + b*integrate((log(c) + log(x^n))/(e^2*x^5 + 2*d
*e*x^4 + d^2*x^3), x)
```

Giac [F]

$$\int \frac{a + b\log(cx^n)}{x^3(d + ex)^2}\,dx = \int \frac{b\log(cx^n) + a}{(ex + d)^2 x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x + d)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{x^3(d + ex)^2}\,dx = \int \frac{a + b\,\ln(c\,x^n)}{x^3\,(d + e\,x)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e*x)^2),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^3*(d + e*x)^2), x)
```

3.46 $\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 149

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^3}\,dx = -\frac{3bnx}{e^3}+\frac{(6a+5bn)x}{2e^3}+\frac{3bx\log(cx^n)}{e^3} - \frac{x^3(a+b\log(cx^n))}{2e(d+ex)^2}-\frac{x^2(3a+bn+3b\log(cx^n))}{2e^2(d+ex)} - \frac{d(6a+5bn+6b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{2e^4} - \frac{3bdn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4}$$

```
[Out] -3*b*n*x/e^3+1/2*(5*b*n+6*a)*x/e^3+3*b*x*ln(c*x^n)/e^3-1/2*x^3*(a+b*ln(c*x^
n))/e/(e*x+d)^2-1/2*x^2*(3*a+b*n+3*b*ln(c*x^n))/e^2/(e*x+d)-1/2*d*(6*a+5*b*
n+6*b*ln(c*x^n))*ln(1+e*x/d)/e^4-3*b*d*n*polylog(2,-e*x/d)/e^4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2384, 45, 2393, 2332, 2354, 2438}

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^3}\,dx = -\frac{d\log\left(\frac{ex}{d}+1\right)(6a+6b\log(cx^n)+5bn)}{2e^4} - \frac{x^2(3a+3b\log(cx^n)+bn)}{2e^2(d+ex)}-\frac{x^3(a+b\log(cx^n))}{2e(d+ex)^2} + \frac{x(6a+5bn)}{2e^3}+\frac{3bx\log(cx^n)}{e^3}-\frac{3bdn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4}-\frac{3bnx}{e^3}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n]))/(d + e*x)^3,x]
```

```
[Out] (-3*b*n*x)/e^3 + ((6*a + 5*b*n)*x)/(2*e^3) + (3*b*x*Log[c*x^n])/e^3 - (x^3*
(a + b*Log[c*x^n]))/(2*e*(d + e*x)^2) - (x^2*(3*a + b*n + 3*b*Log[c*x^n]))/
(2*e^2*(d + e*x)) - (d*(6*a + 5*b*n + 6*b*Log[c*x^n])*Log[1 + (e*x)/d])/(2*
e^4) - (3*b*d*n*PolyLog[2, -((e*x)/d)])/e^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^3(a+b\log(cx^n))}{2e(d+ex)^2}+\frac{\int \frac{x^2(3a+bn+3b\log(cx^n))}{(d+ex)^2}\,dx}{2e}\\
&= -\frac{x^3(a+b\log(cx^n))}{2e(d+ex)^2}-\frac{x^2(3a+bn+3b\log(cx^n))}{2e^2(d+ex)}+\frac{\int \frac{x(3bn+2(3a+bn)+6b\log(cx^n))}{d+ex}\,dx}{2e^2}\\
&= -\frac{x^3(a+b\log(cx^n))}{2e(d+ex)^2}-\frac{x^2(3a+bn+3b\log(cx^n))}{2e^2(d+ex)}\\
&\quad+\frac{\int\left(\frac{3bn+2(3a+bn)+6b\log(cx^n)}{e}-\frac{d(3bn+2(3a+bn)+6b\log(cx^n))}{e(d+ex)}\right)dx}{2e^2}\\
&= -\frac{x^3(a+b\log(cx^n))}{2e(d+ex)^2}-\frac{x^2(3a+bn+3b\log(cx^n))}{2e^2(d+ex)}\\
&\quad+\frac{\int(3bn+2(3a+bn)+6b\log(cx^n))\,dx}{2e^3}-\frac{d\int \frac{3bn+2(3a+bn)+6b\log(cx^n)}{d+ex}\,dx}{2e^3}\\
&= \frac{(6a+5bn)x}{2e^3}-\frac{x^3(a+b\log(cx^n))}{2e(d+ex)^2}-\frac{x^2(3a+bn+3b\log(cx^n))}{2e^2(d+ex)}\\
&\quad-\frac{d(6a+5bn+6b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{2e^4}\\
&\quad+\frac{(3b)\int\log(cx^n)\,dx}{e^3}+\frac{(3bdn)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^4}\\
&= -\frac{3bnx}{e^3}+\frac{(6a+5bn)x}{2e^3}+\frac{3bx\log(cx^n)}{e^3}\\
&\quad-\frac{x^3(a+b\log(cx^n))}{2e(d+ex)^2}-\frac{x^2(3a+bn+3b\log(cx^n))}{2e^2(d+ex)}\\
&\quad-\frac{d(6a+5bn+6b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{2e^4}-\frac{3bdn\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.01

$$
\int\frac{x^3(a+b\log(cx^n))}{(d+ex)^3}\,dx
$$
$$
=\frac{2aex-2benx+2bex\log(cx^n)+\frac{d^3(a+b\log(cx^n))}{(d+ex)^2}-\frac{6d^2(a+b\log(cx^n))}{d+ex}+6bdn(\log(x)-\log(d+ex))-bdn\left(\frac{d}{d+ex}\right.}{2e^4}
$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/(d + e*x)^3,x]
```

```
[Out] (2*a*e*x - 2*b*e*n*x + 2*b*e*x*Log[c*x^n] + (d^3*(a + b*Log[c*x^n]))/(d + e
*x)^2 - (6*d^2*(a + b*Log[c*x^n]))/(d + e*x) + 6*b*d*n*(Log[x] - Log[d + e*
x]) - b*d*n*(d/(d + e*x) + Log[x] - Log[d + e*x]) - 6*d*(a + b*Log[c*x^n])*
Log[1 + (e*x)/d] - 6*b*d*n*PolyLog[2, -((e*x)/d)])/(2*e^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.03

method	result
risch	$\frac{b\ln(x^n)x}{e^3} - \frac{3b\ln(x^n)d\ln(ex+d)}{e^4} - \frac{3b\ln(x^n)d^2}{e^4(ex+d)} + \frac{b\ln(x^n)d^3}{2e^4(ex+d)^2} - \frac{bnx}{e^3} - \frac{bnd}{e^4} - \frac{5bnd\ln(ex+d)}{2e^4} - \frac{bn\,d^2}{2e^4(ex+d)} + \frac{5bnd\ln(e}{2e^4}$

[In] int(x^3*(a+b*ln(c*x^n))/(e*x+d)^3,x,method=_RETURNVERBOSE)

[Out] b*ln(x^n)*x/e^3-3*b*ln(x^n)/e^4*d*ln(e*x+d)-3*b*ln(x^n)/e^4*d^2/(e*x+d)+1/2*b*ln(x^n)*d^3/e^4/(e*x+d)^2-b*n*x/e^3-b*n/e^4*d-5/2*b*n/e^4*d*ln(e*x+d)-1/2*b*n/e^4*d^2/(e*x+d)+5/2*b*n/e^4*d*ln(e*x)+3*b*n/e^4*d*ln(e*x+d)*ln(-e*x/d)+3*b*n/e^4*d*dilog(-e*x/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(x/e^3-3/e^4*d*ln(e*x+d)-3/e^4*d^2/(e*x+d)+1/2*d^3/e^4/(e*x+d)^2)

Fricas [F]

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{(ex+d)^3}\,dx$$

[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^3,x, algorithm="fricas")

[Out] integral((b*x^3*log(c*x^n) + a*x^3)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)

Sympy [A] (verification not implemented)

Time = 27.60 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.62

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^3}\,dx = -\frac{ad^3\left(\begin{cases}\frac{x}{d^3} & \text{for } e=0\\ -\frac{1}{2e(d+ex)^2} & \text{otherwise}\end{cases}\right)}{e^3} + \frac{3ad^2\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x} & \text{otherwise}\end{cases}\right)}{e^3} - \frac{3ad\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^3} + \frac{ax}{e^3} + \frac{bd^3n\left(\begin{cases}\frac{x}{d^3} & \text{for } e=0\\ -\frac{1}{2d^2e+2de^2x} - \frac{\log(x)}{2d^2e} + \frac{\log\left(\frac{d}{e}+x\right)}{2d^2e} & \text{otherwise}\end{cases}\right)}{e^3} - \frac{bd^3\left(\begin{cases}\frac{x}{d^3} & \text{for } e=0\\ -\frac{1}{2e(d+ex)^2} & \text{otherwise}\end{cases}\right)\log(cx^n)}{e^3} - \frac{3bd^2n\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{\log(x)}{de} + \frac{\log\left(\frac{d}{e}+x\right)}{de} & \text{otherwise}\end{cases}\right)}{e^3} + \frac{3bd^2\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x} & \text{otherwise}\end{cases}\right)\log(cx^n)}{e^3} + \frac{3bdn\left(\begin{cases}\frac{x}{d} & \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(d)\log(x) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(d) + G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(d) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{otherwise}\end{cases}}{e} & \end{cases}\right)}{e^3} - \frac{3bd\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{e^3} - \frac{bnx}{e^3} + \frac{bx\log(cx^n)}{e^3}$$

[In] integrate(x**3*(a+b*ln(c*x**n))/(e*x+d)**3,x)

[Out] -a*d**3*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))/e**3 + 3*a*d**2*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))/e**3 - 3

```
*a*d*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e**3 + a*x/e**3 + b
*d**3*n*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*d**2*e + 2*d*e**2*x) - log(x)/
(2*d**2*e) + log(d/e + x)/(2*d**2*e), True))/e**3 - b*d**3*Piecewise((x/d**
3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))*log(c*x**n)/e**3 - 3*b*d**2*n*
Piecewise((x/d**2, Eq(e, 0)), (-log(x)/(d*e) + log(d/e + x)/(d*e), True))/e
**3 + 3*b*d**2*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))*log
(c*x**n)/e**3 + 3*b*d*n*Piecewise((x/d, Eq(e, 0)), (Piecewise((-polylog(2,
e*x*exp_polar(I*pi)/d), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - po
lylog(2, e*x*exp_polar(I*pi)/d), Abs(x) < 1), (-log(d)*log(1/x) - polylog(2
, e*x*exp_polar(I*pi)/d), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), (
)), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2,
e*x*exp_polar(I*pi)/d), True))/e, True))/e**3 - 3*b*d*Piecewise((x/d, Eq(e,
 0)), (log(d + e*x)/e, True))*log(c*x**n)/e**3 - b*n*x/e**3 + b*x*log(c*x**
n)/e**3
```

Maxima [F]

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{(ex+d)^3}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a*((6*d^2*e*x + 5*d^3)/(e^6*x^2 + 2*d*e^5*x + d^2*e^4) - 2*x/e^3 + 6*d
*log(e*x + d)/e^4) + b*integrate((x^3*log(c) + x^3*log(x^n))/(e^3*x^3 + 3*d
*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Giac [F]

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{(ex+d)^3}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^3/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^3}\,dx = \int \frac{x^3\,(a+b\,\ln(c\,x^n))}{(d+e\,x)^3}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n)))/(d + e*x)^3,x)
[Out] int((x^3*(a + b*log(c*x^n)))/(d + e*x)^3, x)
```

3.47 $\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^3} dx = -\frac{x^2(a+b\log(cx^n))}{2e(d+ex)^2} - \frac{x(2a+bn+2b\log(cx^n))}{2e^2(d+ex)} + \frac{(2a+3bn+2b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{2e^3} + \frac{bn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^3}$$

```
[Out] -1/2*x^2*(a+b*ln(c*x^n))/e/(e*x+d)^2-1/2*x*(2*a+b*n+2*b*ln(c*x^n))/e^2/(e*x
+d)+1/2*(2*a+3*b*n+2*b*ln(c*x^n))*ln(1+e*x/d)/e^3+b*n*polylog(2,-e*x/d)/e^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2384, 2354, 2438}

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^3} dx = \frac{\log\left(\frac{ex}{d}+1\right)(2a+2b\log(cx^n)+3bn)}{2e^3} - \frac{x(2a+2b\log(cx^n)+bn)}{2e^2(d+ex)} - \frac{x^2(a+b\log(cx^n))}{2e(d+ex)^2} + \frac{bn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^3}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/(d + e*x)^3,x]
```

```
[Out] -1/2*(x^2*(a + b*Log[c*x^n]))/(e*(d + e*x)^2) - (x*(2*a + b*n + 2*b*Log[c*x
^n]))/(2*e^2*(d + e*x)) + ((2*a + 3*b*n + 2*b*Log[c*x^n])*Log[1 + (e*x)/d])
/(2*e^3) + (b*n*PolyLog[2, -((e*x)/d)])/e^3
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2(a+b\log(cx^n))}{2e(d+ex)^2}+\frac{\int \frac{x(2a+bn+2b\log(cx^n))}{(d+ex)^2}\,dx}{2e}\\
&= -\frac{x^2(a+b\log(cx^n))}{2e(d+ex)^2}-\frac{x(2a+bn+2b\log(cx^n))}{2e^2(d+ex)}+\frac{\int \frac{2a+3bn+2b\log(cx^n)}{d+ex}\,dx}{2e^2}\\
&= -\frac{x^2(a+b\log(cx^n))}{2e(d+ex)^2}-\frac{x(2a+bn+2b\log(cx^n))}{2e^2(d+ex)}\\
&\quad+\frac{(2a+3bn+2b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{2e^3}-\frac{(bn)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^3}\\
&= -\frac{x^2(a+b\log(cx^n))}{2e(d+ex)^2}-\frac{x(2a+bn+2b\log(cx^n))}{2e^2(d+ex)}\\
&\quad+\frac{(2a+3bn+2b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{2e^3}+\frac{bn\text{Li}_2\left(-\frac{ex}{d}\right)}{e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.14

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^3}\,dx = \frac{-\frac{d^2(a+b\log(cx^n))}{(d+ex)^2}+\frac{4d(a+b\log(cx^n))}{d+ex}-4bn(\log(x)-\log(d+ex))+bn\left(\frac{d}{d+ex}+\log(x)-\log(d+ex)\right)+2(a+b\dots}{2e^3}$$

[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*x)^3,x]

[Out] (-((d^2*(a + b*Log[c*x^n]))/(d + e*x)^2) + (4*d*(a + b*Log[c*x^n]))/(d + e*x) - 4*b*n*(Log[x] - Log[d + e*x]) + b*n*(d/(d + e*x) + Log[x] - Log[d + e*x]) + 2*(a + b*Log[c*x^n])*Log[1 + (e*x)/d] + 2*b*n*PolyLog[2, -((e*x)/d)])/(2*e^3)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.41

method	result
risch	$\frac{b\ln(x^n)\ln(ex+d)}{e^3}+\frac{2b\ln(x^n)d}{e^3(ex+d)}-\frac{b\ln(x^n)d^2}{2e^3(ex+d)^2}+\frac{bnd}{2e^3(ex+d)}+\frac{3bn\ln(ex+d)}{2e^3}-\frac{3bn\ln(ex)}{2e^3}-\frac{bn\ln(ex+d)\ln(-\frac{ex}{d})}{e^3}-\frac{bn\,d\dots}{}$

[In] int(x^2*(a+b*ln(c*x^n))/(e*x+d)^3,x,method=_RETURNVERBOSE)

[Out] b*ln(x^n)/e^3*ln(e*x+d)+2*b*ln(x^n)/e^3*d/(e*x+d)-1/2*b*ln(x^n)/e^3*d^2/(e*x+d)^2+1/2*b*n/e^3*d/(e*x+d)+3/2*b*n/e^3*ln(e*x+d)-3/2*b*n/e^3*ln(e*x)-b*n/e^3*ln(e*x+d)*ln(-e*x/d)-b*n/e^3*dilog(-e*x/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/e^3*ln(e*x+d)+2/e^3*d/(e*x+d)-1/2/e^3*d^2/(e*x+d)^2)

Fricas [F]

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{(ex+d)^3}\,dx$$

[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^3,x, algorithm="fricas")

[Out] integral((b*x^2*log(c*x^n) + a*x^2)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)

Sympy [A] (verification not implemented)

Time = 19.77 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.24

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^3}\,dx$$
$$= \frac{ad^2\left(\begin{cases}\frac{x}{d^3} & \text{for } e=0\\ -\frac{1}{2e(d+ex)^2} & \text{otherwise}\end{cases}\right)}{e^2} - \frac{2ad\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x} & \text{otherwise}\end{cases}\right)}{e^2}$$
$$+ \frac{a\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^2} - \frac{bd^2n\left(\begin{cases}\frac{x}{d^3} & \text{for } e=0\\ -\frac{1}{2d^2e+2de^2x} - \frac{\log(x)}{2d^2e} + \frac{\log\left(\frac{d}{e}+x\right)}{2d^2e} & \text{otherwise}\end{cases}\right)}{e^2}$$
$$+ \frac{bd^2\left(\begin{cases}\frac{x}{d^3} & \text{for } e=0\\ -\frac{1}{2e(d+ex)^2} & \text{otherwise}\end{cases}\right)\log(cx^n)}{e^2}$$
$$+ \frac{2bdn\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{\log(x)}{de} + \frac{\log\left(\frac{d}{e}+x\right)}{de} & \text{otherwise}\end{cases}\right)}{e^2} - \frac{2bd\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x} & \text{otherwise}\end{cases}\right)\log(cx^n)}{e^2}$$
$$- \frac{bn\left(\begin{cases}\frac{x}{d} \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(d)\log(x) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(d) + G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(d) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{otherwise}\end{cases}}{e}\end{cases}\right)}{e^2}$$
$$+ \frac{b\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{e^2}$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(e*x+d)**3,x)
```

```
[Out] a*d**2*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))/e**2 -
2*a*d*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))/e**2 + a*Pie
cewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e**2 - b*d**2*n*Piecewise((
```

```
x/d**3, Eq(e, 0)), (-1/(2*d**2*e + 2*d*e**2*x) - log(x)/(2*d**2*e) + log(d/
e + x)/(2*d**2*e), True))/e**2 + b*d**2*Piecewise((x/d**3, Eq(e, 0)), (-1/(
2*e*(d + e*x)**2), True))*log(c*x**n)/e**2 + 2*b*d*n*Piecewise((x/d**2, Eq(
e, 0)), (-log(x)/(d*e) + log(d/e + x)/(d*e), True))/e**2 - 2*b*d*Piecewise(
(x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))*log(c*x**n)/e**2 - b*n*Piece
wise((x/d, Eq(e, 0)), (Piecewise((-polylog(2, e*x*exp_polar(I*pi)/d), (Abs(
x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x*exp_polar(I*pi)/
d), Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/d), 1/A
bs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1,
1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x*exp_polar(I*pi)/d), True)
)/e, True))/e**2 + b*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))*log
(c*x**n)/e**2
```

Maxima [F]

$$\int \frac{x^2(a+b\log{(cx^n)})}{(d+ex)^3}\,dx = \int \frac{(b\log{(cx^n)}+a)x^2}{(ex+d)^3}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^3,x, algorithm="maxima")
[Out] 1/2*a*((4*d*e*x + 3*d^2)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3) + 2*log(e*x + d)/e
^3) + b*integrate((x^2*log(c) + x^2*log(x^n))/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^
2*e*x + d^3), x)
```

Giac [F]

$$\int \frac{x^2(a+b\log{(cx^n)})}{(d+ex)^3}\,dx = \int \frac{(b\log{(cx^n)}+a)x^2}{(ex+d)^3}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^2/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log{(cx^n)})}{(d+ex)^3}\,dx = \int \frac{x^2\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{{\left(d+e\,x\right)}^3}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e*x)^3,x)
[Out] int((x^2*(a + b*log(c*x^n)))/(d + e*x)^3, x)
```

3.48 $\int \frac{x(a+b\log(cx^n))}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 62

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^3}\,dx = -\frac{bn}{2e^2(d+ex)} + \frac{x^2(a+b\log(cx^n))}{2d(d+ex)^2} - \frac{bn\log(d+ex)}{2de^2}$$

```
[Out] -1/2*b*n/e^2/(e*x+d)+1/2*x^2*(a+b*ln(c*x^n))/d/(e*x+d)^2-1/2*b*n*ln(e*x+d)/
d/e^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2373, 45}

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^3}\,dx = \frac{x^2(a+b\log(cx^n))}{2d(d+ex)^2} - \frac{bn}{2e^2(d+ex)} - \frac{bn\log(d+ex)}{2de^2}$$

```
[In] Int[(x*(a + b*Log[c*x^n]))/(d + e*x)^3,x]
```

```
[Out] -1/2*(b*n)/(e^2*(d + e*x)) + (x^2*(a + b*Log[c*x^n]))/(2*d*(d + e*x)^2) - (
b*n*Log[d + e*x])/(2*d*e^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^2(a+b\log(cx^n))}{2d(d+ex)^2} - \frac{(bn)\int \frac{x}{(d+ex)^2}\,dx}{2d}\\ &= \frac{x^2(a+b\log(cx^n))}{2d(d+ex)^2} - \frac{(bn)\int \left(-\frac{d}{e(d+ex)^2}+\frac{1}{e(d+ex)}\right)dx}{2d}\\ &= -\frac{bn}{2e^2(d+ex)} + \frac{x^2(a+b\log(cx^n))}{2d(d+ex)^2} - \frac{bn\log(d+ex)}{2de^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.21

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^3}\,dx = \frac{bn\log(x) - \frac{bdn(d+ex)+ad(d+2ex)+bd(d+2ex)\log(cx^n)+bn(d+ex)^2\log(d+ex)}{(d+ex)^2}}{2de^2}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e*x)^3,x]
```

```
[Out] (b*n*Log[x] - (b*d*n*(d + e*x) + a*d*(d + 2*e*x) + b*d*(d + 2*e*x)*Log[c*x^
n] + b*n*(d + e*x)^2*Log[d + e*x])/(d + e*x)^2)/(2*d*e^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(56) = 112.

Time = 0.41 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.21

method	result
parallelrisch	$\frac{\ln(x)b e^2 n x^2-\ln(ex+d)b e^2 n x^2+2\ln(x)bdenx-2\ln(ex+d)bdenx+\ln(x)b d^2 n-\ln(ex+d)b d^2 n-2b\ln(c x^n)dex-bdenx-2adex}{2e^2(ex+d)^2 d}$
risch	$-\frac{b(2ex+d)\ln(x^n)}{2(ex+d)^2 e^2} - \frac{2i\pi bdex\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2+2i\pi bdex\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2-i\pi b\,d^2\,\mathrm{csgn}(ic\,x^n)^3+i\pi b\,d^2\,\mathrm{csgn}(ix^n)}{}$

```
[In] int(x*(a+b*ln(c*x^n))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(ln(x)*b*e^2*n*x^2-ln(e*x+d)*b*e^2*n*x^2+2*ln(x)*b*d*e*n*x-2*ln(e*x+d)*
b*d*e*n*x+ln(x)*b*d^2*n-ln(e*x+d)*b*d^2*n-2*b*ln(c*x^n)*d*e*x-b*d*e*n*x-2*a
*d*e*x-b*ln(c*x^n)*d^2-b*d^2*n-a*d^2)/e^2/(e*x+d)^2/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(56) = 112.

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.85

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^3}\,dx = \frac{be^2nx^2\log(x) - bd^2n - ad^2 - (bden + 2\,ade)x - (be^2nx^2 + 2\,bdenx + bd^2n)\log(ex+d) - (2\,bdex + bd^2)\,\mathrm{l}}{2\,(de^4x^2 + 2\,d^2e^3x + d^3e^2)}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*e^2*n*x^2*log(x) - b*d^2*n - a*d^2 - (b*d*e*n + 2*a*d*e)*x - (b*e^2*
n*x^2 + 2*b*d*e*n*x + b*d^2*n)*log(e*x + d) - (2*b*d*e*x + b*d^2)*log(c))/(
d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(53) = 106.

Time = 1.79 (sec) , antiderivative size = 398, normalized size of antiderivative = 6.42

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^3}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{a}{x} - \frac{bn}{x} - \frac{b\log(cx^n)}{x}\right) \\ \frac{\frac{ax^2}{2} - \frac{bnx^2}{4} + \frac{bx^2\log(cx^n)}{2}}{d^3} \\ \frac{-\frac{a}{x} - \frac{bn}{x} - \frac{b\log(cx^n)}{x}}{e^3} \\ -\frac{ad^2}{2d^3e^2+4d^2e^3x+2de^4x^2} - \frac{2adex}{2d^3e^2+4d^2e^3x+2de^4x^2} - \frac{bd^2n\log\left(\frac{d}{e}+x\right)}{2d^3e^2+4d^2e^3x+2de^4x^2} - \frac{bd^2n}{2d^3e^2+4d^2e^3x+2de^4x^2} - \frac{2bdenx\log\left(\frac{d}{e}+x\right)}{2d^3e^2+4d^2e^3x+2de^4x^2} - \end{cases}$$

```
[In] integrate(x*(a+b*ln(c*x**n))/(e*x+d)**3,x)
```

```
[Out] Piecewise((zoo*(-a/x - b*n/x - b*log(c*x**n)/x), Eq(d, 0) & Eq(e, 0)), ((a*
x**2/2 - b*n*x**2/4 + b*x**2*log(c*x**n)/2)/d**3, Eq(e, 0)), ((-a/x - b*n/x
 - b*log(c*x**n)/x)/e**3, Eq(d, 0)), (-a*d**2/(2*d**3*e**2 + 4*d**2*e**3*x
+ 2*d*e**4*x**2) - 2*a*d*e*x/(2*d**3*e**2 + 4*d**2*e**3*x + 2*d*e**4*x**2)
- b*d**2*n*log(d/e + x)/(2*d**3*e**2 + 4*d**2*e**3*x + 2*d*e**4*x**2) - b*d
**2*n/(2*d**3*e**2 + 4*d**2*e**3*x + 2*d*e**4*x**2) - 2*b*d*e*n*x*log(d/e +
```

```
 x)/(2*d**3*e**2 + 4*d**2*e**3*x + 2*d*e**4*x**2) - b*d*e*n*x/(2*d**3*e**2
+ 4*d**2*e**3*x + 2*d*e**4*x**2) - b*e**2*n*x**2*log(d/e + x)/(2*d**3*e**2
+ 4*d**2*e**3*x + 2*d*e**4*x**2) + b*e**2*x**2*log(c*x**n)/(2*d**3*e**2 + 4
*d**2*e**3*x + 2*d*e**4*x**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(56) = 112$.

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.84

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^3}\,dx = -\frac{1}{2}bn\left(\frac{1}{e^3x+de^2}+\frac{\log(ex+d)}{de^2}-\frac{\log(x)}{de^2}\right) - \frac{(2\,ex+d)b\log(cx^n)}{2\,(e^4x^2+2\,de^3x+d^2e^2)} - \frac{(2\,ex+d)a}{2\,(e^4x^2+2\,de^3x+d^2e^2)}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*n*(1/(e^3*x + d*e^2) + log(e*x + d)/(d*e^2) - log(x)/(d*e^2)) - 1/2*
(2*e*x + d)*b*log(c*x^n)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) - 1/2*(2*e*x + d)*
a/(e^4*x^2 + 2*d*e^3*x + d^2*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(56) = 112$.

Time = 0.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.00

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^3}\,dx = -\frac{(2\,benx+bdn)\log(x)}{2\,(e^4x^2+2\,de^3x+d^2e^2)} - \frac{benx+2\,bex\log(c)+bdn+2\,aex+bd\log(c)+ad}{2\,(e^4x^2+2\,de^3x+d^2e^2)} - \frac{bn\log(ex+d)}{2\,de^2} + \frac{bn\log(x)}{2\,de^2}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*e*n*x + b*d*n)*log(x)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) - 1/2*(b*e*
n*x + 2*b*e*x*log(c) + b*d*n + 2*a*e*x + b*d*log(c) + a*d)/(e^4*x^2 + 2*d*e
^3*x + d^2*e^2) - 1/2*b*n*log(e*x + d)/(d*e^2) + 1/2*b*n*log(x)/(d*e^2)
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.74

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^3}\,dx = -\frac{a\,d+x\,(2\,a\,e+b\,e\,n)+b\,d\,n}{2\,d^2\,e^2+4\,d\,e^3\,x+2\,e^4\,x^2} - \frac{\ln\left(c\,x^n\right)\,\left(\frac{b\,d}{2\,e^2}+\frac{b\,x}{e}\right)}{d^2+2\,d\,e\,x+e^2\,x^2} - \frac{b\,n\,\mathrm{atanh}\left(\frac{2\,e\,x}{d}+1\right)}{d\,e^2}$$

```
[In] int((x*(a + b*log(c*x^n)))/(d + e*x)^3,x)
```

```
[Out] - (a*d + x*(2*a*e + b*e*n) + b*d*n)/(2*d^2*e^2 + 2*e^4*x^2 + 4*d*e^3*x) - (
log(c*x^n)*((b*d)/(2*e^2) + (b*x)/e))/(d^2 + e^2*x^2 + 2*d*e*x) - (b*n*atan
h((2*e*x)/d + 1))/(d*e^2)
```

3.49 $\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 76

$$\int \frac{a+b\log\left(cx^n\right)}{(d+ex)^3}\,dx=\frac{bn}{2de(d+ex)}+\frac{bn\log(x)}{2d^2e}-\frac{a+b\log\left(cx^n\right)}{2e(d+ex)^2}-\frac{bn\log(d+ex)}{2d^2e}$$

```
[Out] 1/2*b*n/d/e/(e*x+d)+1/2*b*n*ln(x)/d^2/e+1/2*(-a-b*ln(c*x^n))/e/(e*x+d)^2-1/
2*b*n*ln(e*x+d)/d^2/e
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2356, 46}

$$\int \frac{a+b\log\left(cx^n\right)}{(d+ex)^3}\,dx=-\frac{a+b\log\left(cx^n\right)}{2e(d+ex)^2}+\frac{bn\log(x)}{2d^2e}-\frac{bn\log(d+ex)}{2d^2e}+\frac{bn}{2de(d+ex)}$$

```
[In] Int[(a + b*Log[c*x^n])/(d + e*x)^3,x]
```

```
[Out] (b*n)/(2*d*e*(d + e*x)) + (b*n*Log[x])/(2*d^2*e) - (a + b*Log[c*x^n])/(2*e*
(d + e*x)^2) - (b*n*Log[d + e*x])/(2*d^2*e)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\log\left(cx^n\right)}{2e(d+ex)^2}+\frac{(bn)\int\frac{1}{x(d+ex)^2}\,dx}{2e}\\
&= -\frac{a+b\log\left(cx^n\right)}{2e(d+ex)^2}+\frac{(bn)\int\left(\frac{1}{d^2x}-\frac{e}{d(d+ex)^2}-\frac{e}{d^2(d+ex)}\right)dx}{2e}\\
&= \frac{bn}{2de(d+ex)}+\frac{bn\log(x)}{2d^2e}-\frac{a+b\log\left(cx^n\right)}{2e(d+ex)^2}-\frac{bn\log(d+ex)}{2d^2e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int\frac{a+b\log\left(cx^n\right)}{(d+ex)^3}\,dx=\frac{-\frac{a+b\log(cx^n)}{(d+ex)^2}+\frac{bn\left(\frac{d}{d+ex}+\log(x)-\log(d+ex)\right)}{d^2}}{2e}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*x)^3,x]
[Out] (-((a + b*Log[c*x^n])/(d + e*x)^2) + (b*n*(d/(d + e*x) + Log[x] - Log[d + e
*x]))/d^2)/(2*e)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.74

method	result
parallelrisch	$\frac{2\ln(x)x^2b\,e^3n-2\ln(ex+d)x^2b\,e^3n+4\ln(x)xbd\,e^2n-4\ln(ex+d)xbd\,e^2n-b\,e^3n\,x^2+2\ln(x)b\,d^2en-2\ln(ex+d)b\,d^2en-2b\ln(c\,x^n)}{4e^2d^2(ex+d)^2}$
risch	$-\frac{b\ln(x^n)}{2e(ex+d)^2}-\frac{-i\pi b\,d^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+i\pi b\,d^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+i\pi b\,d^2\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-i\pi b\,d^2\,\mathrm{csg}}{}$

```
[In] int((a+b*ln(c*x^n))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*ln(x)*x^2*b*e^3*n-2*ln(e*x+d)*x^2*b*e^3*n+4*ln(x)*x*b*d*e^2*n-4*ln(e
*x+d)*x*b*d*e^2*n-b*e^3*n*x^2+2*ln(x)*b*d^2*e*n-2*ln(e*x+d)*b*d^2*e*n-2*b*l
n(c*x^n)*d^2*e+e*d^2*b*n-2*e*d^2*a)/e^2/d^2/(e*x+d)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.41

$$\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx = \frac{bdenx + bd^2n - bd^2\log(c) - ad^2 - (be^2nx^2 + 2\,bdenx + bd^2n)\log(ex+d) + (be^2nx^2 + 2\,bdenx)\log(x)}{2\,(d^2e^3x^2 + 2\,d^3e^2x + d^4e)}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*e*n*x + b*d^2*n - b*d^2*log(c) - a*d^2 - (b*e^2*n*x^2 + 2*b*d*e*n*
x + b*d^2*n)*log(e*x + d) + (b*e^2*n*x^2 + 2*b*d*e*n*x)*log(x))/(d^2*e^3*x^
2 + 2*d^3*e^2*x + d^4*e)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(66) = 132$.

Time = 1.88 (sec) , antiderivative size = 415, normalized size of antiderivative = 5.46

$$\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{a}{2x^2} - \frac{bn}{4x^2} - \frac{b\log(cx^n)}{2x^2}\right) \\ \frac{ax-bnx+bx\log(cx^n)}{d^3} \\ \frac{-\frac{a}{2x^2} - \frac{bn}{4x^2} - \frac{b\log(cx^n)}{2x^2}}{e^3} \\ -\frac{ad^2}{2d^4e+4d^3e^2x+2d^2e^3x^2} - \frac{bd^2n\log\left(\frac{d}{e}+x\right)}{2d^4e+4d^3e^2x+2d^2e^3x^2} + \frac{bd^2n}{2d^4e+4d^3e^2x+2d^2e^3x^2} - \frac{2bdenx\log\left(\frac{d}{e}+x\right)}{2d^4e+4d^3e^2x+2d^2e^3x^2} + \frac{bdenx}{2d^4e+4d^3e^2x+2d^2e^3x^2} - \end{cases}$$

```
[In] integrate((a+b*ln(c*x**n))/(e*x+d)**3,x)
```

```
[Out] Piecewise((zoo*(-a/(2*x**2) - b*n/(4*x**2) - b*log(c*x**n)/(2*x**2)), Eq(d,
 0) & Eq(e, 0)), ((a*x - b*n*x + b*x*log(c*x**n))/d**3, Eq(e, 0)), ((-a/(2*
x**2) - b*n/(4*x**2) - b*log(c*x**n)/(2*x**2))/e**3, Eq(d, 0)), (-a*d**2/(2
*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2) - b*d**2*n*log(d/e + x)/(2*d**4
*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2) + b*d**2*n/(2*d**4*e + 4*d**3*e**2*x
 + 2*d**2*e**3*x**2) - 2*b*d*e*n*x*log(d/e + x)/(2*d**4*e + 4*d**3*e**2*x +
```

```
2*d**2*e**3*x**2) + b*d*e*n*x/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2
) + 2*b*d*e*x*log(c*x**n)/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2) - b
*e**2*n*x**2*log(d/e + x)/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2) + b
*e**2*x**2*log(c*x**n)/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30

$$\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx = \frac{1}{2}bn\left(\frac{1}{de^2x+d^2e}-\frac{\log(ex+d)}{d^2e}+\frac{\log(x)}{d^2e}\right)-\frac{b\log(cx^n)}{2\,(e^3x^2+2\,de^2x+d^2e)}-\frac{a}{2\,(e^3x^2+2\,de^2x+d^2e)}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*n*(1/(d*e^2*x + d^2*e) - log(e*x + d)/(d^2*e) + log(x)/(d^2*e)) - 1/2
*b*log(c*x^n)/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 1/2*a/(e^3*x^2 + 2*d*e^2*x +
d^2*e)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.37

$$\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx = -\frac{bn\log(x)}{2\,(e^3x^2+2\,de^2x+d^2e)}+\frac{benx+bdn-bd\log(c)-ad}{2\,(de^3x^2+2\,d^2e^2x+d^3e)}-\frac{bn\log(ex+d)}{2\,d^2e}+\frac{bn\log(x)}{2\,d^2e}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/2*b*n*log(x)/(e^3*x^2 + 2*d*e^2*x + d^2*e) + 1/2*(b*e*n*x + b*d*n - b*d*
log(c) - a*d)/(d*e^3*x^2 + 2*d^2*e^2*x + d^3*e) - 1/2*b*n*log(e*x + d)/(d^2
*e) + 1/2*b*n*log(x)/(d^2*e)
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20

$$\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx = \frac{b\,n-a+\frac{b\,e\,n\,x}{d}}{2\,d^2\,e+4\,d\,e^2\,x+2\,e^3\,x^2} - \frac{b\,\ln\left(c\,x^n\right)}{2\,e\,\left(d^2+2\,d\,e\,x+e^2\,x^2\right)} - \frac{b\,n\,\mathrm{atanh}\left(\frac{2\,e\,x}{d}+1\right)}{d^2\,e}$$

```
[In] int((a + b*log(c*x^n))/(d + e*x)^3,x)
```

```
[Out] (b*n - a + (b*e*n*x)/d)/(2*d^2*e + 2*e^3*x^2 + 4*d*e^2*x) - (b*log(c*x^n))/
(2*e*(d^2 + e^2*x^2 + 2*d*e*x)) - (b*n*atanh((2*e*x)/d + 1))/(d^2*e)
```

3.50 $\int \frac{a+b\log(cx^n)}{x(d+ex)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 134

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^3}\,dx=-\frac{bn}{2d^2(d+ex)}-\frac{bn\log(x)}{2d^3}+\frac{a+b\log(cx^n)}{2d(d+ex)^2}-\frac{ex(a+b\log(cx^n))}{d^3(d+ex)}-\frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^3}+\frac{3bn\log(d+ex)}{2d^3}+\frac{bn\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^3}$$

```
[Out] -1/2*b*n/d^2/(e*x+d)-1/2*b*n*ln(x)/d^3+1/2*(a+b*ln(c*x^n))/d/(e*x+d)^2-e*x*
(a+b*ln(c*x^n))/d^3/(e*x+d)-ln(1+d/e/x)*(a+b*ln(c*x^n))/d^3+3/2*b*n*ln(e*x+
d)/d^3+b*n*polylog(2,-d/e/x)/d^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^3}\,dx=-\frac{\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^3}-\frac{ex(a+b\log(cx^n))}{d^3(d+ex)}+\frac{a+b\log(cx^n)}{2d(d+ex)^2}+\frac{bn\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^3}+\frac{3bn\log(d+ex)}{2d^3}-\frac{bn\log(x)}{2d^3}-\frac{bn}{2d^2(d+ex)}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x)^3),x]
```

```
[Out] -1/2*(b*n)/(d^2*(d + e*x)) - (b*n*Log[x])/(2*d^3) + (a + b*Log[c*x^n])/(2*d
*(d + e*x)^2) - (e*x*(a + b*Log[c*x^n]))/(d^3*(d + e*x)) - (Log[1 + d/(e*x)
```

```
]*(a + b*Log[c*x^n]))/d^3 + (3*b*n*Log[d + e*x])/(2*d^3) + (b*n*PolyLog[2,
-(d/(e*x))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx}{d} - \frac{e\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx}{d}\\
&= \frac{a+b\log(cx^n)}{2d(d+ex)^2} + \frac{\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d^2} - \frac{e\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{d^2} - \frac{(bn)\int \frac{1}{x(d+ex)^2}\,dx}{2d}\\
&= \frac{a+b\log(cx^n)}{2d(d+ex)^2} - \frac{ex(a+b\log(cx^n))}{d^3(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^3}\\
&\quad + \frac{(bn)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^3} - \frac{(bn)\int \left(\frac{1}{d^2x} - \frac{e}{d(d+ex)^2} - \frac{e}{d^2(d+ex)}\right)dx}{2d} + \frac{(ben)\int \frac{1}{d+ex}\,dx}{d^3}\\
&= -\frac{bn}{2d^2(d+ex)} - \frac{bn\log(x)}{2d^3} + \frac{a+b\log(cx^n)}{2d(d+ex)^2} - \frac{ex(a+b\log(cx^n))}{d^3(d+ex)}\\
&\quad - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^3} + \frac{3bn\log(d+ex)}{2d^3} + \frac{bn\mathrm{Li}_2\left(-\frac{d}{ex}\right)}{d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.05

$$
\int \frac{a+b\log(cx^n)}{x(d+ex)^3}\,dx = \frac{\frac{d^2(a+b\log(cx^n))}{(d+ex)^2} + \frac{2d(a+b\log(cx^n))}{d+ex} + \frac{(a+b\log(cx^n))^2}{bn} - 2bn(\log(x)-\log(d+ex)) + bn\left(-\frac{d}{d+ex} - \log(x) + \log(d+e\right.}{2d^3}
$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x)^3),x]
```

```
[Out] ((d^2*(a + b*Log[c*x^n]))/(d + e*x)^2 + (2*d*(a + b*Log[c*x^n]))/(d + e*x)
+ (a + b*Log[c*x^n])^2/(b*n) - 2*b*n*(Log[x] - Log[d + e*x]) + b*n*(-(d/(d
+ e*x)) - Log[x] + Log[d + e*x]) - 2*(a + b*Log[c*x^n])*Log[1 + (e*x)/d] -
2*b*n*PolyLog[2, -((e*x)/d)])/(2*d^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.90 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.04

method	result
risch	$-\frac{b\ln(x^n)\ln(ex+d)}{d^3}+\frac{b\ln(x^n)}{d^2(ex+d)}+\frac{b\ln(x^n)}{2d(ex+d)^2}+\frac{b\ln(x^n)\ln(x)}{d^3}-\frac{bn}{2d^2(ex+d)}+\frac{3bn\ln(ex+d)}{2d^3}-\frac{3bn\ln(x)}{2d^3}-\frac{bn\ln(x)^2}{2d^3}+$

```
[In] int((a+b*ln(c*x^n))/x/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b*ln(x^n)/d^3*ln(e*x+d)+b*ln(x^n)/d^2/(e*x+d)+1/2*b*ln(x^n)/d/(e*x+d)^2+b*
ln(x^n)/d^3*ln(x)-1/2*b*n/d^2/(e*x+d)+3/2*b*n*ln(e*x+d)/d^3-3/2*b*n*ln(x)/d
^3-1/2*b*n/d^3*ln(x)^2+b*n/d^3*ln(e*x+d)*ln(-e*x/d)+b*n/d^3*dilog(-e*x/d)+(
-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I
*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3
+b*ln(c)+a)*(-1/d^3*ln(e*x+d)+1/d^2/(e*x+d)+1/2/d/(e*x+d)^2+1/d^3*ln(x))
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^3}\,dx=\int \frac{b\log(cx^n)+a}{(ex+d)^3x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x),
x)
```

Sympy [A] (verification not implemented)

Time = 37.00 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.63

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^3}\,dx = -\frac{ae\left(\begin{cases}\frac{x}{d^3} & \text{for } e=0\\ -\frac{1}{2e(d+ex)^2} & \text{otherwise}\end{cases}\right)}{d} - \frac{ae\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x} & \text{otherwise}\end{cases}\right)}{d^2} - \frac{ae\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{d^3} + \frac{a\log(x)}{d^3} + \frac{be^2n\left(\begin{cases}-\frac{1}{e^3x} & \text{for } d=0\\ -\frac{1}{2de^2+2e^3x}-\frac{\log(d+ex)}{2de^2} & \text{otherwise}\end{cases}\right)}{d^2} - \frac{be^2\left(\begin{cases}\frac{1}{e^3x} & \text{for } d=0\\ -\frac{1}{2d\left(\frac{d}{x}+e\right)^2} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^2} - \frac{2ben\left(\begin{cases}-\frac{1}{e^2x} & \text{for } d=0\\ -\frac{\log(d^2+dex)}{de} & \text{otherwise}\end{cases}\right)}{d^2} + \frac{2be\left(\begin{cases}\frac{1}{e^2x} & \text{for } d=0\\ -\frac{1}{\frac{d^2}{x}+de} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^2} + \frac{bn\left(\begin{cases}-\frac{1}{ex} & \text{fo}\\ \frac{\begin{cases}\operatorname{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{for } \frac{1}{|x|}<1\wedge|x|<1\\ \log(e)\log(x)+\operatorname{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{for } |x|<1\\ -\log(e)\log\left(\frac{1}{x}\right)+\operatorname{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log(e)+\operatorname{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{otherwise}\end{cases}}{d} & \text{ot}\end{cases}\right)}{d^2} - \frac{b\left(\begin{cases}\frac{1}{ex} & \text{for } d=0\\ \frac{\log\left(\frac{d}{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^2}$$

```
[In] integrate((a+b*ln(c*x**n))/x/(e*x+d)**3,x)
```

```
[Out] -a*e*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))/d - a*e*P
iecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))/d**2 - a*e*Piecewis
e((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/d**3 + a*log(x)/d**3 + b*e**2*n*
Piecewise((-1/(e**3*x), Eq(d, 0)), (-1/(2*d*e**2 + 2*e**3*x) - log(d + e*x)
/(2*d*e**2), True))/d**2 - b*e**2*Piecewise((1/(e**3*x), Eq(d, 0)), (-1/(2*
```

```
d*(d/x + e)**2), True))*log(c*x**n)/d**2 - 2*b*e*n*Piecewise((-1/(e**2*x),
Eq(d, 0)), (-log(d**2 + d*e*x)/(d*e), True))/d**2 + 2*b*e*Piecewise((1/(e**
2*x), Eq(d, 0)), (-1/(d**2/x + d*e), True))*log(c*x**n)/d**2 + b*n*Piecewis
e((-1/(e*x), Eq(d, 0)), (Piecewise((polylog(2, d*exp_polar(I*pi)/(e*x)), (A
bs(x) < 1) & (1/Abs(x) < 1)), (log(e)*log(x) + polylog(2, d*exp_polar(I*pi)
/(e*x)), Abs(x) < 1), (-log(e)*log(1/x) + polylog(2, d*exp_polar(I*pi)/(e*x
)), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(e) + meijer
g(((1, 1), ()), ((), (0, 0)), x)*log(e) + polylog(2, d*exp_polar(I*pi)/(e*x
)), True))/d, True))/d**2 - b*Piecewise((1/(e*x), Eq(d, 0)), (log(d/x + e)/
d, True))*log(c*x**n)/d**2
```

Maxima [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x(d+ex)^3}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex+d\right)^3x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^3,x, algorithm="maxima")
[Out] 1/2*a*((2*e*x + 3*d)/(d^2*e^2*x^2 + 2*d^3*e*x + d^4) - 2*log(e*x + d)/d^3 +
 2*log(x)/d^3) + b*integrate((log(c) + log(x^n))/(e^3*x^4 + 3*d*e^2*x^3 + 3
*d^2*e*x^2 + d^3*x), x)
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x(d+ex)^3}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex+d\right)^3x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x + d)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x(d+ex)^3}\,dx=\int \frac{a+b\,\ln\left(c\,x^n\right)}{x\,{\left(d+e\,x\right)}^3}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x)^3),x)
[Out] int((a + b*log(c*x^n))/(x*(d + e*x)^3), x)
```

3.51 $\int \frac{a+b\log(cx^n)}{x^2(d+ex)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 171

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^3}\,dx = -\frac{bn}{d^3x}+\frac{ben}{2d^3(d+ex)}+\frac{ben\log(x)}{2d^4}-\frac{a+b\log(cx^n)}{d^3x} -\frac{e(a+b\log(cx^n))}{2d^2(d+ex)^2}+\frac{2e^2x(a+b\log(cx^n))}{d^4(d+ex)} +\frac{3e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4} -\frac{5ben\log(d+ex)}{2d^4}-\frac{3ben\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^4}$$

```
[Out] -b*n/d^3/x+1/2*b*e*n/d^3/(e*x+d)+1/2*b*e*n*ln(x)/d^4+(-a-b*ln(c*x^n))/d^3/x
-1/2*e*(a+b*ln(c*x^n))/d^2/(e*x+d)^2+2*e^2*x*(a+b*ln(c*x^n))/d^4/(e*x+d)+3*
e*ln(1+d/e/x)*(a+b*ln(c*x^n))/d^4-5/2*b*e*n*ln(e*x+d)/d^4-3*b*e*n*polylog(2
,-d/e/x)/d^4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {46, 2393, 2341, 2356, 2351, 31, 2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^3}\,dx = \frac{2e^2x(a+b\log(cx^n))}{d^4(d+ex)}+\frac{3e\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^4} -\frac{a+b\log(cx^n)}{d^3x}-\frac{e(a+b\log(cx^n))}{2d^2(d+ex)^2}-\frac{3ben\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^4} +\frac{ben\log(x)}{2d^4}-\frac{5ben\log(d+ex)}{2d^4}+\frac{ben}{2d^3(d+ex)}-\frac{bn}{d^3x}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^2*(d + e*x)^3),x]
```

```
[Out] -((b*n)/(d^3*x)) + (b*e*n)/(2*d^3*(d + e*x)) + (b*e*n*Log[x])/(2*d^4) - (a
+ b*Log[c*x^n])/(d^3*x) - (e*(a + b*Log[c*x^n]))/(2*d^2*(d + e*x)^2) + (2*e
^2*x*(a + b*Log[c*x^n]))/(d^4*(d + e*x)) + (3*e*Log[1 + d/(e*x)]*(a + b*Log
[c*x^n]))/d^4 - (5*b*e*n*Log[d + e*x])/(2*d^4) - (3*b*e*n*PolyLog[2, -(d/(e
*x))])/d^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
```

```
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log(cx^n)}{d^3x^2} + \frac{e^2(a+b\log(cx^n))}{d^2(d+ex)^3} + \frac{2e^2(a+b\log(cx^n))}{d^3(d+ex)^2} - \frac{3e(a+b\log(cx^n))}{d^3x(d+ex)}\right) dx$$

$$= \frac{\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d^3} - \frac{(3e)\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d^3} + \frac{(2e^2)\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{d^3} + \frac{e^2\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx}{d^2}$$

$$= -\frac{bn}{d^3x} - \frac{a+b\log(cx^n)}{d^3x} - \frac{e(a+b\log(cx^n))}{2d^2(d+ex)^2} + \frac{2e^2x(a+b\log(cx^n))}{d^4(d+ex)} + \frac{3e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4} - \frac{(3ben)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^4} + \frac{(ben)\int \frac{1}{x(d+ex)^2}\,dx}{2d^2} - \frac{(2be^2n)\int \frac{1}{d+ex}\,dx}{d^4}$$

$$= -\frac{bn}{d^3x} - \frac{a+b\log(cx^n)}{d^3x} - \frac{e(a+b\log(cx^n))}{2d^2(d+ex)^2} + \frac{2e^2x(a+b\log(cx^n))}{d^4(d+ex)} + \frac{3e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4} - \frac{2ben\log(d+ex)}{d^4} - \frac{3ben\text{Li}_2\left(-\frac{d}{ex}\right)}{d^4} + \frac{(ben)\int \left(\frac{1}{d^2x} - \frac{e}{d(d+ex)^2} - \frac{e}{d^2(d+ex)}\right) dx}{2d^2}$$

$$= -\frac{bn}{d^3x} + \frac{ben}{2d^3(d+ex)} + \frac{ben\log(x)}{2d^4} - \frac{a+b\log(cx^n)}{d^3x} - \frac{e(a+b\log(cx^n))}{2d^2(d+ex)^2} + \frac{2e^2x(a+b\log(cx^n))}{d^4(d+ex)} + \frac{3e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4} - \frac{5ben\log(d+ex)}{2d^4} - \frac{3ben\text{Li}_2\left(-\frac{d}{ex}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^3}\,dx$$
$$= \frac{-\frac{2bdn}{x}-\frac{2d(a+b\log(cx^n))}{x}-\frac{d^2e(a+b\log(cx^n))}{(d+ex)^2}-\frac{4de(a+b\log(cx^n))}{d+ex}-\frac{3e(a+b\log(cx^n))^2}{bn}+4ben(\log(x)-\log(d+ex))+}{2d^4}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*x)^3),x]
```

```
[Out] ((-2*b*d*n)/x - (2*d*(a + b*Log[c*x^n]))/x - (d^2*e*(a + b*Log[c*x^n]))/(d
+ e*x)^2 - (4*d*e*(a + b*Log[c*x^n]))/(d + e*x) - (3*e*(a + b*Log[c*x^n])^2
)/(b*n) + 4*b*e*n*(Log[x] - Log[d + e*x]) + b*e*n*(d/(d + e*x) + Log[x] - L
og[d + e*x]) + 6*e*(a + b*Log[c*x^n])*Log[1 + (e*x)/d] + 6*b*e*n*PolyLog[2,
 -((e*x)/d)])/(2*d^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.89

method	result
risch	$-\frac{b\ln(x^n)e}{2d^2(ex+d)^2}+\frac{3b\ln(x^n)e\ln(ex+d)}{d^4}-\frac{2b\ln(x^n)e}{d^3(ex+d)}-\frac{b\ln(x^n)}{d^3x}-\frac{3b\ln(x^n)e\ln(x)}{d^4}+\frac{3bne\ln(x)^2}{2d^4}-\frac{3bne\ln(ex+d)\ln(-\frac{ex}{d})}{d^4}$

```
[In] int((a+b*ln(c*x^n))/x^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*ln(x^n)/d^2/(e*x+d)^2*e+3*b*ln(x^n)/d^4*e*ln(e*x+d)-2*b*ln(x^n)/d^3*
e/(e*x+d)-b*ln(x^n)/d^3/x-3*b*ln(x^n)/d^4*e*ln(x)+3/2*b*n/d^4*e*ln(x)^2-3*b
*n/d^4*e*ln(e*x+d)*ln(-e*x/d)-3*b*n/d^4*e*dilog(-e*x/d)+1/2*b*e*n/d^3/(e*x+
d)-5/2*b*e*n*ln(e*x+d)/d^4-b*n/d^3/x+5/2*b*e*n*ln(x)/d^4+(-1/2*I*b*Pi*csgn(
I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b
*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-1/2
/d^2/(e*x+d)^2*e+3/d^4*e*ln(e*x+d)-2/d^3*e/(e*x+d)-1/d^3/x-3/d^4*e*ln(x))
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^3}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)^3x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2)
, x)
```

Sympy [A] (verification not implemented)

Time = 38.43 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.60

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^3}\,dx = \frac{ae^2\left(\begin{cases}\frac{x}{d^3} & \text{for } e=0\\ -\frac{1}{2e(d+ex)^2} & \text{otherwise}\end{cases}\right)}{d^2} + \frac{2ae^2\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x} & \text{otherwise}\end{cases}\right)}{d^3} - \frac{a}{d^3x} + \frac{3ae^2\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{d^4} - \frac{3ae\log(x)}{d^4} - \frac{be^2n\left(\begin{cases}\frac{x}{d^3} & \text{for } e=0\\ -\frac{1}{2d^2e+2de^2x} - \frac{\log(x)}{2d^2e} + \frac{\log\left(\frac{d}{e}+x\right)}{2d^2e} & \text{otherwise}\end{cases}\right)}{d^2} + \frac{be^2\left(\begin{cases}\frac{x}{d^3} & \text{for } e=0\\ -\frac{1}{2e(d+ex)^2} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^2} - \frac{2be^2n\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{\log(x)}{de} + \frac{\log\left(\frac{d}{e}+x\right)}{de} & \text{otherwise}\end{cases}\right)}{d^3} + \frac{2be^2\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^3} - \frac{bn}{d^3x} - \frac{b\log(cx^n)}{d^3x} - \frac{3be^2n\left(\begin{cases}\frac{x}{d} & \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|< \\ \log(d)\log(x) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } |x|<1 \\ -\log(d)\log\left(\frac{1}{x}\right) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(d) + G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(d) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{otherwise}\end{cases}}{e} & \end{cases}\right)}{d^4} + \frac{3be^2\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^4} + \frac{3ben\log(x)^2}{2d^4} - \frac{3be\log(x)\log(cx^n)}{d^4}$$

```
[In] integrate((a+b*ln(c*x**n))/x**2/(e*x+d)**3,x)
```

```
[Out] a*e**2*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))/d**2 +
2*a*e**2*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))/d**3 - a/
```

```
(d**3*x) + 3*a*e**2*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/d**4
 - 3*a*e*log(x)/d**4 - b*e**2*n*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*d**2*e
 + 2*d*e**2*x) - log(x)/(2*d**2*e) + log(d/e + x)/(2*d**2*e), True))/d**2 +
 b*e**2*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))*log(c*
x**n)/d**2 - 2*b*e**2*n*Piecewise((x/d**2, Eq(e, 0)), (-log(x)/(d*e) + log(
d/e + x)/(d*e), True))/d**3 + 2*b*e**2*Piecewise((x/d**2, Eq(e, 0)), (-1/(d
*e + e**2*x), True))*log(c*x**n)/d**3 - b*n/(d**3*x) - b*log(c*x**n)/(d**3*
x) - 3*b*e**2*n*Piecewise((x/d, Eq(e, 0)), (Piecewise((-polylog(2, e*x*exp_
polar(I*pi)/d), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2,
 e*x*exp_polar(I*pi)/d), Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x*ex
p_polar(I*pi)/d), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*l
og(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x*exp_
polar(I*pi)/d), True))/e, True))/d**4 + 3*b*e**2*Piecewise((x/d, Eq(e, 0)),
 (log(d + e*x)/e, True))*log(c*x**n)/d**4 + 3*b*e*n*log(x)**2/(2*d**4) - 3*
b*e*log(x)*log(c*x**n)/d**4
```

Maxima [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^2(d+ex)^3}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex+d\right)^3x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a*((6*e^2*x^2 + 9*d*e*x + 2*d^2)/(d^3*e^2*x^3 + 2*d^4*e*x^2 + d^5*x) -
 6*e*log(e*x + d)/d^4 + 6*e*log(x)/d^4) + b*integrate((log(c) + log(x^n))/(
e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2), x)
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^2(d+ex)^3}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex+d\right)^3x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x + d)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^3}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^2\,{(d+e\,x)}^3}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*x)^3),x)
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*x)^3), x)
```

3.52 $\int \frac{a+b\log(cx^n)}{x^3(d+ex)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 217

$$\begin{aligned}\int \frac{a+b\log(cx^n)}{x^3(d+ex)^3}\,dx = &-\frac{bn}{4d^3x^2}+\frac{3ben}{d^4x}-\frac{be^2n}{2d^4(d+ex)}-\frac{be^2n\log(x)}{2d^5}\\ &-\frac{a+b\log(cx^n)}{2d^3x^2}+\frac{3e(a+b\log(cx^n))}{d^4x}+\frac{e^2(a+b\log(cx^n))}{2d^3(d+ex)^2}\\ &-\frac{3e^3x(a+b\log(cx^n))}{d^5(d+ex)}-\frac{6e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^5}\\ &+\frac{7be^2n\log(d+ex)}{2d^5}+\frac{6be^2n\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^5}\end{aligned}$$

```
[Out] -1/4*b*n/d^3/x^2+3*b*e*n/d^4/x-1/2*b*e^2*n/d^4/(e*x+d)-1/2*b*e^2*n*ln(x)/d^
5+1/2*(-a-b*ln(c*x^n))/d^3/x^2+3*e*(a+b*ln(c*x^n))/d^4/x+1/2*e^2*(a+b*ln(c*
x^n))/d^3/(e*x+d)^2-3*e^3*x*(a+b*ln(c*x^n))/d^5/(e*x+d)-6*e^2*ln(1+d/e/x)*(
a+b*ln(c*x^n))/d^5+7/2*b*e^2*n*ln(e*x+d)/d^5+6*b*e^2*n*polylog(2,-d/e/x)/d^
5
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used

= {46, 2393, 2341, 2356, 2351, 31, 2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^3}\,dx = -\frac{3e^3x(a+b\log(cx^n))}{d^5(d+ex)} - \frac{6e^2\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^5} + \frac{3e(a+b\log(cx^n))}{d^4x} + \frac{e^2(a+b\log(cx^n))}{2d^3(d+ex)^2} - \frac{a+b\log(cx^n)}{2d^3x^2} + \frac{6be^2n\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^5} - \frac{be^2n\log(x)}{2d^5} + \frac{7be^2n\log(d+ex)}{2d^5} - \frac{be^2n}{2d^4(d+ex)} + \frac{3ben}{d^4x} - \frac{bn}{4d^3x^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^3*(d + e*x)^3),x]
```

```
[Out] -1/4*(b*n)/(d^3*x^2) + (3*b*e*n)/(d^4*x) - (b*e^2*n)/(2*d^4*(d + e*x)) - (b
*e^2*n*Log[x])/(2*d^5) - (a + b*Log[c*x^n])/(2*d^3*x^2) + (3*e*(a + b*Log[c
*x^n]))/(d^4*x) + (e^2*(a + b*Log[c*x^n]))/(2*d^3*(d + e*x)^2) - (3*e^3*x*(
a + b*Log[c*x^n]))/(d^5*(d + e*x)) - (6*e^2*Log[1 + d/(e*x)]*(a + b*Log[c*x
^n]))/d^5 + (7*b*e^2*n*Log[d + e*x])/(2*d^5) + (6*b*e^2*n*PolyLog[2, -(d/(e
*x))])/d^5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log(cx^n)}{d^3x^3} - \frac{3e(a+b\log(cx^n))}{d^4x^2} - \frac{e^3(a+b\log(cx^n))}{d^3(d+ex)^3} - \frac{3e^3(a+b\log(cx^n))}{d^4(d+ex)^2} + \frac{6e^2(a+b\log(cx^n))}{d^4x(d+ex)}\right) dx$$

$$= \frac{\int \frac{a+b\log(cx^n)}{x^3}\,dx}{d^3} - \frac{(3e)\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d^4} + \frac{(6e^2)\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d^4} - \frac{(3e^3)\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{d^4} - \frac{e^3\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx}{d^3}$$

$$= -\frac{bn}{4d^3x^2} + \frac{3ben}{d^4x} - \frac{a+b\log(cx^n)}{2d^3x^2} + \frac{3e(a+b\log(cx^n))}{d^4x} + \frac{e^2(a+b\log(cx^n))}{2d^3(d+ex)^2} - \frac{3e^3x(a+b\log(cx^n))}{d^5(d+ex)} - \frac{6e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^5} + \frac{(6be^2n)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^5} - \frac{(be^2n)\int \frac{1}{x(d+ex)^2}\,dx}{2d^3} + \frac{(3be^3n)\int \frac{1}{d+ex}\,dx}{d^5}$$

$$= -\frac{bn}{4d^3x^2} + \frac{3ben}{d^4x} - \frac{a+b\log(cx^n)}{2d^3x^2} + \frac{3e(a+b\log(cx^n))}{d^4x} + \frac{e^2(a+b\log(cx^n))}{2d^3(d+ex)^2} - \frac{3e^3x(a+b\log(cx^n))}{d^5(d+ex)} - \frac{6e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^5} + \frac{3be^2n\log(d+ex)}{d^5} + \frac{6be^2n\text{Li}_2\left(-\frac{d}{ex}\right)}{d^5} - \frac{(be^2n)\int\left(\frac{1}{d^2x} - \frac{e}{d(d+ex)^2} - \frac{e}{d^2(d+ex)}\right)dx}{2d^3}$$

$$= -\frac{bn}{4d^3x^2} + \frac{3ben}{d^4x} - \frac{be^2n}{2d^4(d+ex)} - \frac{be^2n\log(x)}{2d^5} - \frac{a+b\log(cx^n)}{2d^3x^2} + \frac{3e(a+b\log(cx^n))}{d^4x} + \frac{e^2(a+b\log(cx^n))}{2d^3(d+ex)^2} - \frac{3e^3x(a+b\log(cx^n))}{d^5(d+ex)} - \frac{6e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^5} + \frac{7be^2n\log(d+ex)}{2d^5} + \frac{6be^2n\text{Li}_2\left(-\frac{d}{ex}\right)}{d^5}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.05

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^3}\,dx = -\frac{\frac{bd^2n}{x^2} - \frac{12bden}{x} + \frac{2d^2(a+b\log(cx^n))}{x^2} - \frac{12de(a+b\log(cx^n))}{x} - \frac{2d^2e^2(a+b\log(cx^n))}{(d+ex)^2} - \frac{12de^2(a+b\log(cx^n))}{d+ex} - \frac{12e^2(a+b\log(cx^n))^2}{bn}}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*(d + e*x)^3),x]
```

```
[Out] -1/4*((b*d^2*n)/x^2 - (12*b*d*e*n)/x + (2*d^2*(a + b*Log[c*x^n]))/x^2 - (12
*d*e*(a + b*Log[c*x^n]))/x - (2*d^2*e^2*(a + b*Log[c*x^n]))/(d + e*x)^2 - (
12*d*e^2*(a + b*Log[c*x^n]))/(d + e*x) - (12*e^2*(a + b*Log[c*x^n])^2)/(b*n
) + 12*b*e^2*n*(Log[x] - Log[d + e*x]) + (2*b*e^2*n*(d + (d + e*x)*Log[x] -
 (d + e*x)*Log[d + e*x]))/(d + e*x) + 24*e^2*(a + b*Log[c*x^n])*Log[1 + (e*
x)/d] + 24*b*e^2*n*PolyLog[2, -((e*x)/d)])/d^5
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.68 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.78

method	result
risch	$-\frac{6b\ln(x^n)e^2\ln(ex+d)}{d^5} + \frac{3b\ln(x^n)e^2}{d^4(ex+d)} + \frac{b\ln(x^n)e^2}{2d^3(ex+d)^2} - \frac{b\ln(x^n)}{2d^3x^2} + \frac{6b\ln(x^n)e^2\ln(x)}{d^5} + \frac{3b\ln(x^n)e}{d^4x} - \frac{be^2n}{2d^4(ex+d)} + \frac{7be^2n}{}$

```
[In] int((a+b*ln(c*x^n))/x^3/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -6*b*ln(x^n)/d^5*e^2*ln(e*x+d)+3*b*ln(x^n)/d^4*e^2/(e*x+d)+1/2*b*ln(x^n)/d^
3*e^2/(e*x+d)^2-1/2*b*ln(x^n)/d^3/x^2+6*b*ln(x^n)/d^5*e^2*ln(x)+3*b*ln(x^n)
/d^4*e/x-1/2*b*e^2*n/d^4/(e*x+d)+7/2*b*e^2*n*ln(e*x+d)/d^5-1/4*b*n/d^3/x^2+
3*b*e*n/d^4/x-7/2*b*e^2*n*ln(x)/d^5-3*b*n/d^5*e^2*ln(x)^2+6*b*n/d^5*e^2*ln(
e*x+d)*ln(-e*x/d)+6*b*n/d^5*e^2*dilog(-e*x/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I
*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*
x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-6/d^5*e^2*ln(e
*x+d)+3/d^4*e^2/(e*x+d)+1/2/d^3*e^2/(e*x+d)^2-1/2/d^3/x^2+6/d^5*e^2*ln(x)+3
/d^4*e/x)
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^3(d+ex)^3}\,dx = \int \frac{b\log\left(cx^n\right)+a}{\left(ex+d\right)^3x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4 + d^3*x^3)
, x)
```

Sympy [A] (verification not implemented)

Time = 41.53 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.29

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^3}\,dx = -\frac{ae^3\left(\begin{cases}\frac{x}{d^3} & \text{for } e=0\\ -\frac{1}{2e(d+ex)^2} & \text{otherwise}\end{cases}\right)}{d^3} - \frac{a}{2d^3x^2} - \frac{3ae^3\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x} & \text{otherwise}\end{cases}\right)}{d^4} + \frac{3ae}{d^4x} - \frac{6ae^3\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{d^5}$$
$$+ \frac{6ae^2\log(x)}{d^5} + \frac{be^3n\left(\begin{cases}\frac{x}{d^3} & \text{for } e=0\\ -\frac{1}{2d^2e+2de^2x} - \frac{\log(x)}{2d^2e} + \frac{\log\left(\frac{d}{e}+x\right)}{2d^2e} & \text{otherwise}\end{cases}\right)}{d^3}$$
$$- \frac{be^3\left(\begin{cases}\frac{x}{d^3} & \text{for } e=0\\ -\frac{1}{2e(d+ex)^2} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^3} - \frac{bn}{4d^3x^2}$$
$$- \frac{b\log(cx^n)}{2d^3x^2} + \frac{3be^3n\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{\log(x)}{de} + \frac{\log\left(\frac{d}{e}+x\right)}{de} & \text{otherwise}\end{cases}\right)}{d^4}$$
$$- \frac{3be^3\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^4} + \frac{3ben}{d^4x} + \frac{3be\log(cx^n)}{d^4x}$$
$$+ \frac{6be^3n\left(\begin{cases}\frac{x}{d} \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|< \\ \log(d)\log(x)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } |x|<1 \\ -\log(d)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(d) + G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(d) - \operatorname{Li}_2\left(\frac{exe^{i\pi}}{d}\right) & \text{otherwise}\end{cases}}{e}\end{cases}\right)}{d^5}$$
$$- \frac{6be^3\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^5} - \frac{3be^2n\log(x)^2}{d^5} + \frac{6be^2\log(x)\log(cx^n)}{d^5}$$

```
[In] integrate((a+b*ln(c*x**n))/x**3/(e*x+d)**3,x)
```

```
[Out] -a*e**3*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))/d**3 -
 a/(2*d**3*x**2) - 3*a*e**3*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x
), True))/d**4 + 3*a*e/(d**4*x) - 6*a*e**3*Piecewise((x/d, Eq(e, 0)), (log(
d + e*x)/e, True))/d**5 + 6*a*e**2*log(x)/d**5 + b*e**3*n*Piecewise((x/d**3
, Eq(e, 0)), (-1/(2*d**2*e + 2*d*e**2*x) - log(x)/(2*d**2*e) + log(d/e + x)
/(2*d**2*e), True))/d**3 - b*e**3*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d
 + e*x)**2), True))*log(c*x**n)/d**3 - b*n/(4*d**3*x**2) - b*log(c*x**n)/(2
*d**3*x**2) + 3*b*e**3*n*Piecewise((x/d**2, Eq(e, 0)), (-log(x)/(d*e) + log
(d/e + x)/(d*e), True))/d**4 - 3*b*e**3*Piecewise((x/d**2, Eq(e, 0)), (-1/(
d*e + e**2*x), True))*log(c*x**n)/d**4 + 3*b*e*n/(d**4*x) + 3*b*e*log(c*x**
n)/(d**4*x) + 6*b*e**3*n*Piecewise((x/d, Eq(e, 0)), (Piecewise((-polylog(2,
 e*x*exp_polar(I*pi)/d), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - p
olylog(2, e*x*exp_polar(I*pi)/d), Abs(x) < 1), (-log(d)*log(1/x) - polylog(
2, e*x*exp_polar(I*pi)/d), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0),
()), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2,
 e*x*exp_polar(I*pi)/d), True))/e, True))/d**5 - 6*b*e**3*Piecewise((x/d, E
q(e, 0)), (log(d + e*x)/e, True))*log(c*x**n)/d**5 - 3*b*e**2*n*log(x)**2/d
**5 + 6*b*e**2*log(x)*log(c*x**n)/d**5
```

Maxima [F]

$$\int \frac{a + b\log\left(cx^{n}\right)}{x^{3}(d + ex)^{3}}\, dx = \int \frac{b\log\left(cx^{n}\right) + a}{\left(ex + d\right)^{3}x^{3}}\, dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*((12*e^3*x^3 + 18*d*e^2*x^2 + 4*d^2*e*x - d^3)/(d^4*e^2*x^4 + 2*d^5*e
*x^3 + d^6*x^2) - 12*e^2*log(e*x + d)/d^5 + 12*e^2*log(x)/d^5) + b*integrat
e((log(c) + log(x^n))/(e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4 + d^3*x^3), x)
```

Giac [F]

$$\int \frac{a + b\log\left(cx^{n}\right)}{x^{3}(d + ex)^{3}}\, dx = \int \frac{b\log\left(cx^{n}\right) + a}{\left(ex + d\right)^{3}x^{3}}\, dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x + d)^3*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^3}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^3\,(d+e\,x)^3}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e*x)^3),x)
[Out] int((a + b*log(c*x^n))/(x^3*(d + e*x)^3), x)
```

3.53 $\int \frac{x^5(a+b\log(cx^n))}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 229

$$\begin{aligned}\int \frac{x^5(a+b\log(cx^n))}{(d+ex)^4}\,dx =& \frac{10bdnx}{e^5}-\frac{d(60a+47bn)x}{6e^5}-\frac{5bnx^2}{2e^4}\\ &-\frac{10bdx\log(cx^n)}{e^5}-\frac{x^5(a+b\log(cx^n))}{3e(d+ex)^3}\\ &-\frac{x^4(5a+bn+5b\log(cx^n))}{6e^2(d+ex)^2}-\frac{x^3(20a+9bn+20b\log(cx^n))}{6e^3(d+ex)}\\ &+\frac{x^2(60a+47bn+60b\log(cx^n))}{12e^4}\\ &+\frac{d^2(60a+47bn+60b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{6e^6}\\ &+\frac{10bd^2n\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^6}\end{aligned}$$

```
[Out] 10*b*d*n*x/e^5-1/6*d*(47*b*n+60*a)*x/e^5-5/2*b*n*x^2/e^4-10*b*d*x*ln(c*x^n)
/e^5-1/3*x^5*(a+b*ln(c*x^n))/e/(e*x+d)^3-1/6*x^4*(5*a+b*n+5*b*ln(c*x^n))/e^
2/(e*x+d)^2-1/6*x^3*(20*a+9*b*n+20*b*ln(c*x^n))/e^3/(e*x+d)+1/12*x^2*(60*a+
47*b*n+60*b*ln(c*x^n))/e^4+1/6*d^2*(60*a+47*b*n+60*b*ln(c*x^n))*ln(1+e*x/d)
/e^6+10*b*d^2*n*polylog(2,-e*x/d)/e^6
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2384, 45, 2393, 2332, 2341, 2354, 2438}

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex)^4}\,dx = \frac{d^2\log\left(\frac{ex}{d}+1\right)(60a+60b\log(cx^n)+47bn)}{6e^6} - \frac{x^3(20a+20b\log(cx^n)+9bn)}{6e^3(d+ex)} - \frac{x^4(5a+5b\log(cx^n)+bn)}{6e^2(d+ex)^2} - \frac{x^5(a+b\log(cx^n))}{3e(d+ex)^3} + \frac{x^2(60a+60b\log(cx^n)+47bn)}{12e^4} - \frac{dx(60a+47bn)}{6e^5} - \frac{10bdx\log(cx^n)}{e^5} + \frac{10bd^2n\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^6} + \frac{10bdnx}{e^5} - \frac{5bnx^2}{2e^4}$$

```
[In] Int[(x^5*(a + b*Log[c*x^n]))/(d + e*x)^4,x]
```

```
[Out] (10*b*d*n*x)/e^5 - (d*(60*a + 47*b*n)*x)/(6*e^5) - (5*b*n*x^2)/(2*e^4) - (1
0*b*d*x*Log[c*x^n])/e^5 - (x^5*(a + b*Log[c*x^n]))/(3*e*(d + e*x)^3) - (x^4
*(5*a + b*n + 5*b*Log[c*x^n]))/(6*e^2*(d + e*x)^2) - (x^3*(20*a + 9*b*n + 2
0*b*Log[c*x^n]))/(6*e^3*(d + e*x)) + (x^2*(60*a + 47*b*n + 60*b*Log[c*x^n])
)/(12*e^4) + (d^2*(60*a + 47*b*n + 60*b*Log[c*x^n])*Log[1 + (e*x)/d])/(6*e^
6) + (10*b*d^2*n*PolyLog[2, -((e*x)/d)])/e^6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = -\frac{x^5(a+b\log(cx^n))}{3e(d+ex)^3} + \frac{\int \frac{x^4(5a+bn+5b\log(cx^n))}{(d+ex)^3}\,dx}{3e}$$

$$= -\frac{x^5(a+b\log(cx^n))}{3e(d+ex)^3} - \frac{x^4(5a+bn+5b\log(cx^n))}{6e^2(d+ex)^2} + \frac{\int \frac{x^3(5bn+4(5a+bn)+20b\log(cx^n))}{(d+ex)^2}\,dx}{6e^2}$$

$$= -\frac{x^5(a+b\log(cx^n))}{3e(d+ex)^3} - \frac{x^4(5a+bn+5b\log(cx^n))}{6e^2(d+ex)^2} - \frac{x^3(20a+9bn+20b\log(cx^n))}{6e^3(d+ex)} + \frac{\int \frac{x^2(20bn+3(5bn+4(5a+bn))+60b\log(cx^n))}{d+ex}\,dx}{6e^3}$$

$$= -\frac{x^5(a+b\log(cx^n))}{3e(d+ex)^3} - \frac{x^4(5a+bn+5b\log(cx^n))}{6e^2(d+ex)^2} - \frac{x^3(20a+9bn+20b\log(cx^n))}{6e^3(d+ex)} + \frac{\int \left(-\frac{d(20bn+3(5bn+4(5a+bn))+60b\log(cx^n))}{e^2} + \frac{x(20bn+3(5bn+4(5a+bn))+60b\log(cx^n))}{e} + \frac{d^2(20bn+3(5bn+4(5a+bn))+60}{e^2(d+ex)}\right.}{6e^3}$$

$$= -\frac{x^5(a+b\log(cx^n))}{3e(d+ex)^3} - \frac{x^4(5a+bn+5b\log(cx^n))}{6e^2(d+ex)^2} - \frac{x^3(20a+9bn+20b\log(cx^n))}{6e^3(d+ex)} - \frac{d\int(20bn+3(5bn+4(5a+bn))+60b\log(cx^n))\,dx}{6e^5} + \frac{d^2\int\frac{20bn+3(5bn+4(5a+bn))+60b\log(cx^n)}{d+ex}\,dx}{6e^5} + \frac{\int x(20bn+3(5bn+4(5a+bn))+60b\log(cx^n))\,dx}{6e^4}$$

$$= -\frac{d(60a+47bn)x}{6e^5} - \frac{5bnx^2}{2e^4} - \frac{x^5(a+b\log(cx^n))}{3e(d+ex)^3} - \frac{x^4(5a+bn+5b\log(cx^n))}{6e^2(d+ex)^2} - \frac{x^3(20a+9bn+20b\log(cx^n))}{6e^3(d+ex)} + \frac{x^2(60a+47bn+60b\log(cx^n))}{12e^4} + \frac{d^2(60a+47bn+60b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{6e^6} - \frac{(10bd)\int\log(cx^n)\,dx}{e^5} - \frac{(10bd^2n)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^6}$$

$$= \frac{10bdnx}{e^5} - \frac{d(60a+47bn)x}{6e^5} - \frac{5bnx^2}{2e^4} - \frac{10bdx\log(cx^n)}{e^5} - \frac{x^5(a+b\log(cx^n))}{3e(d+ex)^3} - \frac{x^4(5a+bn+5b\log(cx^n))}{6e^2(d+ex)^2} - \frac{x^3(20a+9bn+20b\log(cx^n))}{6e^3(d+ex)} + \frac{x^2(60a+47bn+60b\log(cx^n))}{12e^4} + \frac{d^2(60a+47bn+60b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{6e^6} + \frac{10bd^2n\text{Li}_2\left(-\frac{ex}{d}\right)}{e^6}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.09

$$\int\frac{x^5(a+b\log(cx^n))}{(d+ex)^4}\,dx = \frac{-48adex+48bdenx-3be^2nx^2-48bdex\log(cx^n)+6e^2x^2(a+b\log(cx^n))+\frac{4d^5(a+b\log(cx^n))}{(d+ex)^3}-\frac{30d^4(a+b\log(}{(d+ex)^2}}{}$$

```
[In] Integrate[(x^5*(a + b*Log[c*x^n]))/(d + e*x)^4,x]
```

```
[Out] (-48*a*d*e*x + 48*b*d*e*n*x - 3*b*e^2*n*x^2 - 48*b*d*e*x*Log[c*x^n] + 6*e^2
*x^2*(a + b*Log[c*x^n]) + (4*d^5*(a + b*Log[c*x^n]))/(d + e*x)^3 - (30*d^4*
(a + b*Log[c*x^n]))/(d + e*x)^2 + (120*d^3*(a + b*Log[c*x^n]))/(d + e*x) -
2*b*d^2*n*((d*(3*d + 2*e*x))/(d + e*x)^2 + 2*Log[x] - 2*Log[d + e*x]) - 120
*b*d^2*n*(Log[x] - Log[d + e*x]) + 30*b*d^2*n*(d/(d + e*x) + Log[x] - Log[d
 + e*x]) + 120*d^2*(a + b*Log[c*x^n])*Log[1 + (e*x)/d] + 120*b*d^2*n*PolyLo
g[2, -((e*x)/d)])/(12*e^6)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.77

method	result
risch	$\frac{b\ln(x^n)x^2}{2e^4} - \frac{4b\ln(x^n)dx}{e^5} + \frac{b\ln(x^n)d^5}{3e^6(ex+d)^3} + \frac{10b\ln(x^n)d^2\ln(ex+d)}{e^6} + \frac{10b\ln(x^n)d^3}{e^6(ex+d)} - \frac{5b\ln(x^n)d^4}{2e^6(ex+d)^2} - \frac{bn\,x^2}{4e^4} + \frac{4bdnx}{e^5} + \frac{17bn}{4e}$

[In] int(x^5*(a+b*ln(c*x^n))/(e*x+d)^4,x,method=_RETURNVERBOSE)

[Out] 1/2*b*ln(x^n)/e^4*x^2-4*b*ln(x^n)/e^5*d*x+1/3*b*ln(x^n)*d^5/e^6/(e*x+d)^3+10*b*ln(x^n)/e^6*d^2*ln(e*x+d)+10*b*ln(x^n)/e^6*d^3/(e*x+d)-5/2*b*ln(x^n)/e^6*d^4/(e*x+d)^2-1/4*b*n*x^2/e^4+4*b*d*n*x/e^5+17/4*b*n/e^6*d^2+47/6*b*n/e^6*d^2*ln(e*x+d)+13/6*b*n/e^6*d^3/(e*x+d)-1/6*b*n/e^6*d^4/(e*x+d)^2-47/6*b*n/e^6*d^2*ln(e*x)-10*b*n/e^6*d^2*ln(e*x+d)*ln(-e*x/d)-10*b*n/e^6*d^2*dilog(-e*x/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/e^5*(1/2*e*x^2-4*d*x)+1/3*d^5/e^6/(e*x+d)^3+10/e^6*d^2*ln(e*x+d)+10/e^6*d^3/(e*x+d)-5/2/e^6*d^4/(e*x+d)^2)

Fricas [F]

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)x^5}{(ex+d)^4}\,dx$$

[In] integrate(x^5*(a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="fricas")

[Out] integral((b*x^5*log(c*x^n) + a*x^5)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)

Sympy [A] (verification not implemented)

Time = 62.66 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.69

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex)^4}\,dx = \text{Too large to display}$$

[In] integrate(x**5*(a+b*ln(c*x**n))/(e*x+d)**4,x)

[Out] -a*d**5*Piecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), True))/e**5 + 5*a*d**4*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))/e**5 - 10*a*d**3*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))/e**5 + 10*a*d**2*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e**5 - 4*a*d

```
*x/e**5 + a*x**2/(2*e**4) + b*d**5*n*Piecewise((x/d**4, Eq(e, 0)), (-3*d/(6
*d**4*e + 12*d**3*e**2*x + 6*d**2*e**3*x**2) - 2*e*x/(6*d**4*e + 12*d**3*e*
*2*x + 6*d**2*e**3*x**2) - log(x)/(3*d**3*e) + log(d/e + x)/(3*d**3*e), Tru
e))/e**5 - b*d**5*Piecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), Tru
e))*log(c*x**n)/e**5 - 5*b*d**4*n*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*d**2
*e + 2*d*e**2*x) - log(x)/(2*d**2*e) + log(d/e + x)/(2*d**2*e), True))/e**5
 + 5*b*d**4*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))*lo
g(c*x**n)/e**5 + 10*b*d**3*n*Piecewise((x/d**2, Eq(e, 0)), (-log(x)/(d*e) +
 log(d/e + x)/(d*e), True))/e**5 - 10*b*d**3*Piecewise((x/d**2, Eq(e, 0)),
(-1/(d*e + e**2*x), True))*log(c*x**n)/e**5 - 10*b*d**2*n*Piecewise((x/d, E
q(e, 0)), (Piecewise((-polylog(2, e*x*exp_polar(I*pi)/d), (Abs(x) < 1) & (1
/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x*exp_polar(I*pi)/d), Abs(x) <
 1), (-log(d)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/d), 1/Abs(x) < 1),
(-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), (()
, (0, 0)), x)*log(d) - polylog(2, e*x*exp_polar(I*pi)/d), True))/e, True))/
e**5 + 10*b*d**2*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))*log(c*x
**n)/e**5 + 4*b*d*n*x/e**5 - 4*b*d*x*log(c*x**n)/e**5 - b*n*x**2/(4*e**4) +
 b*x**2*log(c*x**n)/(2*e**4)
```

Maxima [F]

$$\int \frac{x^5(a + b\log(cx^n))}{(d + ex)^4} \, dx = \int \frac{(b\log(cx^n) + a)x^5}{(ex + d)^4} \, dx$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*a*((60*d^3*e^2*x^2 + 105*d^4*e*x + 47*d^5)/(e^9*x^3 + 3*d*e^8*x^2 + 3*d
^2*e^7*x + d^3*e^6) + 60*d^2*log(e*x + d)/e^6 + 3*(e*x^2 - 8*d*x)/e^5) + b*
integrate((x^5*log(c) + x^5*log(x^n))/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^
2 + 4*d^3*e*x + d^4), x)
```

Giac [F]

$$\int \frac{x^5(a + b\log(cx^n))}{(d + ex)^4} \, dx = \int \frac{(b\log(cx^n) + a)x^5}{(ex + d)^4} \, dx$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^5/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex)^4}\,dx=\int \frac{x^5\,(a+b\,\ln(c\,x^n))}{(d+e\,x)^4}\,dx$$

```
[In] int((x^5*(a + b*log(c*x^n)))/(d + e*x)^4,x)
[Out] int((x^5*(a + b*log(c*x^n)))/(d + e*x)^4, x)
```

3.54 $\int \frac{x^4(a+b\log(cx^n))}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 183

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex)^4}\,dx = -\frac{4bnx}{e^4}+\frac{(12a+13bn)x}{3e^4}+\frac{4bx\log(cx^n)}{e^4}-\frac{x^4(a+b\log(cx^n))}{3e(d+ex)^3} - \frac{x^3(4a+bn+4b\log(cx^n))}{6e^2(d+ex)^2}-\frac{x^2(12a+7bn+12b\log(cx^n))}{6e^3(d+ex)} - \frac{d(12a+13bn+12b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{3e^5} - \frac{4bdn\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^5}$$

```
[Out] -4*b*n*x/e^4+1/3*(13*b*n+12*a)*x/e^4+4*b*x*ln(c*x^n)/e^4-1/3*x^4*(a+b*ln(c*
x^n))/e/(e*x+d)^3-1/6*x^3*(4*a+b*n+4*b*ln(c*x^n))/e^2/(e*x+d)^2-1/6*x^2*(12
*a+7*b*n+12*b*ln(c*x^n))/e^3/(e*x+d)-1/3*d*(12*a+13*b*n+12*b*ln(c*x^n))*ln(
1+e*x/d)/e^5-4*b*d*n*polylog(2,-e*x/d)/e^5
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

= {2384, 45, 2393, 2332, 2354, 2438}

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex)^4}\,dx = -\frac{d\log\left(\frac{ex}{d}+1\right)(12a+12b\log(cx^n)+13bn)}{3e^5} - \frac{x^2(12a+12b\log(cx^n)+7bn)}{6e^3(d+ex)} - \frac{x^3(4a+4b\log(cx^n)+bn)}{6e^2(d+ex)^2} - \frac{x^4(a+b\log(cx^n))}{3e(d+ex)^3} + \frac{x(12a+13bn)}{3e^4} + \frac{4bx\log(cx^n)}{e^4} - \frac{4bdn\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^5} - \frac{4bnx}{e^4}$$

```
[In] Int[(x^4*(a + b*Log[c*x^n]))/(d + e*x)^4,x]
```

```
[Out] (-4*b*n*x)/e^4 + ((12*a + 13*b*n)*x)/(3*e^4) + (4*b*x*Log[c*x^n])/e^4 - (x^
4*(a + b*Log[c*x^n]))/(3*e*(d + e*x)^3) - (x^3*(4*a + b*n + 4*b*Log[c*x^n])
)/(6*e^2*(d + e*x)^2) - (x^2*(12*a + 7*b*n + 12*b*Log[c*x^n]))/(6*e^3*(d +
e*x)) - (d*(12*a + 13*b*n + 12*b*Log[c*x^n])*Log[1 + (e*x)/d])/(3*e^5) - (4
*b*d*n*PolyLog[2, -((e*x)/d)])/e^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^4(a+b\log(cx^n))}{3e(d+ex)^3}+\frac{\int\frac{x^3(4a+bn+4b\log(cx^n))}{(d+ex)^3}\,dx}{3e}\\
&= -\frac{x^4(a+b\log(cx^n))}{3e(d+ex)^3}-\frac{x^3(4a+bn+4b\log(cx^n))}{6e^2(d+ex)^2}+\frac{\int\frac{x^2(4bn+3(4a+bn)+12b\log(cx^n))}{(d+ex)^2}\,dx}{6e^2}\\
&= -\frac{x^4(a+b\log(cx^n))}{3e(d+ex)^3}-\frac{x^3(4a+bn+4b\log(cx^n))}{6e^2(d+ex)^2}\\
&\quad -\frac{x^2(12a+7bn+12b\log(cx^n))}{6e^3(d+ex)}+\frac{\int\frac{x(12bn+2(4bn+3(4a+bn))+24b\log(cx^n))}{d+ex}\,dx}{6e^3}\\
&= -\frac{x^4(a+b\log(cx^n))}{3e(d+ex)^3}-\frac{x^3(4a+bn+4b\log(cx^n))}{6e^2(d+ex)^2}-\frac{x^2(12a+7bn+12b\log(cx^n))}{6e^3(d+ex)}\\
&\quad +\frac{\int\left(\frac{12bn+2(4bn+3(4a+bn))+24b\log(cx^n)}{e}-\frac{d(12bn+2(4bn+3(4a+bn))+24b\log(cx^n))}{e(d+ex)}\right)dx}{6e^3}\\
&= -\frac{x^4(a+b\log(cx^n))}{3e(d+ex)^3}-\frac{x^3(4a+bn+4b\log(cx^n))}{6e^2(d+ex)^2}-\frac{x^2(12a+7bn+12b\log(cx^n))}{6e^3(d+ex)}\\
&\quad +\frac{\int(12bn+2(4bn+3(4a+bn))+24b\log(cx^n))\,dx}{6e^4}\\
&\quad -\frac{d\int\frac{12bn+2(4bn+3(4a+bn))+24b\log(cx^n)}{d+ex}\,dx}{6e^4}\\
&= \frac{(12a+13bn)x}{3e^4}-\frac{x^4(a+b\log(cx^n))}{3e(d+ex)^3}-\frac{x^3(4a+bn+4b\log(cx^n))}{6e^2(d+ex)^2}\\
&\quad -\frac{x^2(12a+7bn+12b\log(cx^n))}{6e^3(d+ex)}-\frac{d(12a+13bn+12b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{3e^5}\\
&\quad +\frac{(4b)\int\log(cx^n)\,dx}{e^4}+\frac{(4bdn)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^5}
\end{aligned}$$

$$= -\frac{4bnx}{e^4} + \frac{(12a+13bn)x}{3e^4} + \frac{4bx\log(cx^n)}{e^4} - \frac{x^4(a+b\log(cx^n))}{3e(d+ex)^3} - \frac{x^3(4a+bn+4b\log(cx^n))}{6e^2(d+ex)^2} - \frac{x^2(12a+7bn+12b\log(cx^n))}{6e^3(d+ex)} - \frac{d(12a+13bn+12b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{3e^5} - \frac{4bdn\text{Li}_2\left(-\frac{ex}{d}\right)}{e^5}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.13

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex)^4}\,dx$$

$$= \frac{6aex - 6benx + 6bex\log(cx^n) - \frac{2d^4(a+b\log(cx^n))}{(d+ex)^3} + \frac{12d^3(a+b\log(cx^n))}{(d+ex)^2} - \frac{36d^2(a+b\log(cx^n))}{d+ex} + bdn\left(\frac{d(3d+2ex)}{(d+ex)^2} + 2\log\right.}{}$$

```
[In] Integrate[(x^4*(a + b*Log[c*x^n]))/(d + e*x)^4,x]
```

```
[Out] (6*a*e*x - 6*b*e*n*x + 6*b*e*x*Log[c*x^n] - (2*d^4*(a + b*Log[c*x^n]))/(d +
 e*x)^3 + (12*d^3*(a + b*Log[c*x^n]))/(d + e*x)^2 - (36*d^2*(a + b*Log[c*x^
n]))/(d + e*x) + b*d*n*((d*(3*d + 2*e*x))/(d + e*x)^2 + 2*Log[x] - 2*Log[d
+ e*x]) + 36*b*d*n*(Log[x] - Log[d + e*x]) - 12*b*d*n*(d/(d + e*x) + Log[x]
 - Log[d + e*x]) - 24*d*(a + b*Log[c*x^n])*Log[1 + (e*x)/d] - 24*b*d*n*Poly
Log[2, -((e*x)/d)])/(6*e^5)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.53 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.94

method	result
risch	$\frac{b\ln(x^n)x}{e^4} - \frac{b\ln(x^n)d^4}{3e^5(ex+d)^3} - \frac{4b\ln(x^n)d\ln(ex+d)}{e^5} - \frac{6b\ln(x^n)d^2}{e^5(ex+d)} + \frac{2b\ln(x^n)d^3}{e^5(ex+d)^2} - \frac{bnx}{e^4} - \frac{bnd}{e^5} + \frac{bn\,d^3}{6e^5(ex+d)^2} - \frac{13bnd\ln(ex+}{3e^5}$

```
[In] int(x^4*(a+b*ln(c*x^n))/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] b*ln(x^n)/e^4*x-1/3*b*ln(x^n)/e^5*d^4/(e*x+d)^3-4*b*ln(x^n)/e^5*d*ln(e*x+d)
-6*b*ln(x^n)/e^5*d^2/(e*x+d)+2*b*ln(x^n)/e^5*d^3/(e*x+d)^2-b*n*x/e^4-b*n/e^
5*d+1/6*b*n/e^5*d^3/(e*x+d)^2-13/3*b*n/e^5*d*ln(e*x+d)-5/3*b*n/e^5*d^2/(e*x
+d)+13/3*b*n/e^5*d*ln(e*x)+4*b*n/e^5*d*ln(e*x+d)*ln(-e*x/d)+4*b*n/e^5*d*dil
og(-e*x/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn
(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csg
n(I*c*x^n)^3+b*ln(c)+a)*(x/e^4-1/3/e^5*d^4/(e*x+d)^3-4/e^5*d*ln(e*x+d)-6/e^
5*d^2/(e*x+d)+2/e^5*d^3/(e*x+d)^2)
```

Fricas [F]

$$\int \frac{x^4(a+b\log{(cx^n)})}{(d+ex)^4}\,dx=\int \frac{(b\log{(cx^n)}+a)x^4}{(ex+d)^4}\,dx$$

[In] integrate(x^4*(a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="fricas")

[Out] integral((b*x^4*log(c*x^n) + a*x^4)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)

Sympy [A] (verification not implemented)

Time = 31.90 (sec) , antiderivative size = 563, normalized size of antiderivative = 3.08

$$\int \frac{x^4(a+b\log{(cx^n)})}{(d+ex)^4}\,dx=\text{Too large to display}$$

[In] integrate(x**4*(a+b*ln(c*x**n))/(e*x+d)**4,x)

[Out] a*d**4*Piecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), True))/e**4 - 4*a*d**3*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))/e**4 + 6*a*d**2*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))/e**4 - 4*a*d*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e**4 + a*x/e**4 - b*d**4*n*Piecewise((x/d**4, Eq(e, 0)), (-3*d/(6*d**4*e + 12*d**3*e**2*x + 6*d**2*e**3*x**2) - 2*e*x/(6*d**4*e + 12*d**3*e**2*x + 6*d**2*e**3*x**2) - log(x)/(3*d**3*e) + log(d/e + x)/(3*d**3*e), True))/e**4 + b*d**4*Piecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), True))*log(c*x**n)/e**4 + 4*b*d**3*n*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*d**2*e + 2*d*e**2*x) - log(x)/(2*d**2*e) + log(d/e + x)/(2*d**2*e), True))/e**4 - 4*b*d**3*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))*log(c*x**n)/e**4 - 6*b*d**2*n*Piecewise((x/d**2, Eq(e, 0)), (-log(x)/(d*e) + log(d/e + x)/(d*e), True))/e**4 + 6*b*d**2*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))*log(c*x**n)/e**4 + 4*b*d*n*Piecewise((x/d, Eq(e, 0)), (Piecewise((-polylog(2, e*x*exp_polar(I*pi)/d), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x*exp_polar(I*pi)/d), Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/d), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x*exp_polar(I*pi)/d), True))/e, True))/e**4 - 4*b*d*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))*log(c*x**n)/e**4 - b*n*x/e**4 + b*x*log(c*x**n)/e**4

Maxima [F]

$$\int \frac{x^4(a+b\log{(cx^n)})}{(d+ex)^4}\,dx = \int \frac{(b\log{(cx^n)}+a)x^4}{(ex+d)^4}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/3*a*((18*d^2*e^2*x^2 + 30*d^3*e*x + 13*d^4)/(e^8*x^3 + 3*d*e^7*x^2 + 3*d
^2*e^6*x + d^3*e^5) - 3*x/e^4 + 12*d*log(e*x + d)/e^5) + b*integrate((x^4*l
og(c) + x^4*log(x^n))/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x +
d^4), x)
```

Giac [F]

$$\int \frac{x^4(a+b\log{(cx^n)})}{(d+ex)^4}\,dx = \int \frac{(b\log{(cx^n)}+a)x^4}{(ex+d)^4}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^4/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log{(cx^n)})}{(d+ex)^4}\,dx = \int \frac{x^4\,(a+b\,\ln{(c\,x^n)})}{(d+e\,x)^4}\,dx$$

```
[In] int((x^4*(a + b*log(c*x^n)))/(d + e*x)^4,x)
```

```
[Out] int((x^4*(a + b*log(c*x^n)))/(d + e*x)^4, x)
```

3.55 $\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 141

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^4}\,dx = -\frac{x^3(a+b\log(cx^n))}{3e(d+ex)^3}-\frac{x^2(3a+bn+3b\log(cx^n))}{6e^2(d+ex)^2}-\frac{x(6a+5bn+6b\log(cx^n))}{6e^3(d+ex)}+\frac{(6a+11bn+6b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{6e^4}+\frac{bn\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4}$$

```
[Out] -1/3*x^3*(a+b*ln(c*x^n))/e/(e*x+d)^3-1/6*x^2*(3*a+b*n+3*b*ln(c*x^n))/e^2/(e
*x+d)^2-1/6*x*(6*a+5*b*n+6*b*ln(c*x^n))/e^3/(e*x+d)+1/6*(6*a+11*b*n+6*b*ln(
c*x^n))*ln(1+e*x/d)/e^4+b*n*polylog(2,-e*x/d)/e^4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2384, 2354, 2438}

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^4}\,dx = \frac{\log\left(\frac{ex}{d}+1\right)(6a+6b\log(cx^n)+11bn)}{6e^4}-\frac{x(6a+6b\log(cx^n)+5bn)}{6e^3(d+ex)}-\frac{x^2(3a+3b\log(cx^n)+bn)}{6e^2(d+ex)^2}-\frac{x^3(a+b\log(cx^n))}{3e(d+ex)^3}+\frac{bn\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n]))/(d + e*x)^4,x]
```

```
[Out] -1/3*(x^3*(a + b*Log[c*x^n]))/(e*(d + e*x)^3) - (x^2*(3*a + b*n + 3*b*Log[c
*x^n]))/(6*e^2*(d + e*x)^2) - (x*(6*a + 5*b*n + 6*b*Log[c*x^n]))/(6*e^3*(d
+ e*x)) + ((6*a + 11*b*n + 6*b*Log[c*x^n])*Log[1 + (e*x)/d])/(6*e^4) + (b*n
*PolyLog[2, -((e*x)/d)])/e^4
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^3(a+b\log(cx^n))}{3e(d+ex)^3}+\frac{\int \frac{x^2(3a+bn+3b\log(cx^n))}{(d+ex)^3}\,dx}{3e}\\
&= -\frac{x^3(a+b\log(cx^n))}{3e(d+ex)^3}-\frac{x^2(3a+bn+3b\log(cx^n))}{6e^2(d+ex)^2}+\frac{\int \frac{x(3bn+2(3a+bn)+6b\log(cx^n))}{(d+ex)^2}\,dx}{6e^2}\\
&= -\frac{x^3(a+b\log(cx^n))}{3e(d+ex)^3}-\frac{x^2(3a+bn+3b\log(cx^n))}{6e^2(d+ex)^2}\\
&\quad -\frac{x(6a+5bn+6b\log(cx^n))}{6e^3(d+ex)}+\frac{\int \frac{9bn+2(3a+bn)+6b\log(cx^n)}{d+ex}\,dx}{6e^3}\\
&= -\frac{x^3(a+b\log(cx^n))}{3e(d+ex)^3}-\frac{x^2(3a+bn+3b\log(cx^n))}{6e^2(d+ex)^2}-\frac{x(6a+5bn+6b\log(cx^n))}{6e^3(d+ex)}\\
&\quad +\frac{(6a+11bn+6b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{6e^4}-\frac{(bn)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^4}\\
&= -\frac{x^3(a+b\log(cx^n))}{3e(d+ex)^3}-\frac{x^2(3a+bn+3b\log(cx^n))}{6e^2(d+ex)^2}-\frac{x(6a+5bn+6b\log(cx^n))}{6e^3(d+ex)}\\
&\quad +\frac{(6a+11bn+6b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{6e^4}+\frac{bn\,\text{Li}_2\left(-\frac{ex}{d}\right)}{e^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.27

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^4}\,dx$$
$$= \frac{\frac{2d^3(a+b\log(cx^n))}{(d+ex)^3}-\frac{9d^2(a+b\log(cx^n))}{(d+ex)^2}+\frac{18d(a+b\log(cx^n))}{d+ex}-bn\left(\frac{d(3d+2ex)}{(d+ex)^2}+2\log(x)-2\log(d+ex)\right)-18bn(\log(x$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/(d + e*x)^4,x]
```

```
[Out] ((2*d^3*(a + b*Log[c*x^n]))/(d + e*x)^3 - (9*d^2*(a + b*Log[c*x^n]))/(d + e
*x)^2 + (18*d*(a + b*Log[c*x^n]))/(d + e*x) - b*n*((d*(3*d + 2*e*x))/(d + e
*x)^2 + 2*Log[x] - 2*Log[d + e*x]) - 18*b*n*(Log[x] - Log[d + e*x]) + 9*b*n
*(d/(d + e*x) + Log[x] - Log[d + e*x]) + 6*(a + b*Log[c*x^n])*Log[1 + (e*x)
/d] + 6*b*n*PolyLog[2, -((e*x)/d)])/(6*e^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.54 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.20

method	result
risch	$\frac{b\ln(x^n)d^3}{3e^4(ex+d)^3}+\frac{b\ln(x^n)\ln(ex+d)}{e^4}+\frac{3b\ln(x^n)d}{e^4(ex+d)}-\frac{3b\ln(x^n)d^2}{2e^4(ex+d)^2}+\frac{7bnd}{6e^4(ex+d)}+\frac{11bn\ln(ex+d)}{6e^4}-\frac{bn\,d^2}{6e^4(ex+d)^2}-\frac{11bn\ln(ex}{6e^4}$

```
[In] int(x^3*(a+b*ln(c*x^n))/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*ln(x^n)/e^4*d^3/(e*x+d)^3+b*ln(x^n)/e^4*ln(e*x+d)+3*b*ln(x^n)/e^4*d/(
e*x+d)-3/2*b*ln(x^n)/e^4*d^2/(e*x+d)^2+7/6*b*n/e^4*d/(e*x+d)+11/6*b*n/e^4*l
n(e*x+d)-1/6*b*n/e^4*d^2/(e*x+d)^2-11/6*b*n/e^4*ln(e*x)-b*n/e^4*ln(e*x+d)*l
n(-e*x/d)-b*n/e^4*dilog(-e*x/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c
*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*
x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/3/e^4*d^3/(e*x+d)^3+1/e^4*l
n(e*x+d)+3/e^4*d/(e*x+d)-3/2/e^4*d^2/(e*x+d)^2)
```

Fricas [F]

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{(ex+d)^4}\,dx$$

[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="fricas")

[Out] integral((b*x^3*log(c*x^n) + a*x^3)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)

Sympy [A] (verification not implemented)

Time = 30.94 (sec) , antiderivative size = 518, normalized size of antiderivative = 3.67

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^4}\,dx = \text{Too large to display}$$

[In] integrate(x**3*(a+b*ln(c*x**n))/(e*x+d)**4,x)

[Out] -a*d**3*Piecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), True))/e**3 + 3*a*d**2*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))/e**3 - 3*a*d*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))/e**3 + a*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e**3 + b*d**3*n*Piecewise((x/d**4, Eq(e, 0)), (-3*d/(6*d**4*e + 12*d**3*e**2*x + 6*d**2*e**3*x**2) - 2*e*x/(6*d**4*e + 12*d**3*e**2*x + 6*d**2*e**3*x**2) - log(x)/(3*d**3*e) + log(d/e + x)/(3*d**3*e), True))/e**3 - b*d**3*Piecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), True))*log(c*x**n)/e**3 - 3*b*d**2*n*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*d**2*e + 2*d*e**2*x) - log(x)/(2*d**2*e) + log(d/e + x)/(2*d**2*e), True))/e**3 + 3*b*d**2*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))*log(c*x**n)/e**3 + 3*b*d*n*Piecewise((x/d**2, Eq(e, 0)), (-log(x)/(d*e) + log(d/e + x)/(d*e), True))/e**3 - 3*b*d*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))*log(c*x**n)/e**3 - b*n*Piecewise((x/d, Eq(e, 0)), (Piecewise((-polylog(2, e*x*exp_polar(I*pi)/d), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x*exp_polar(I*pi)/d), Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/d), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x*exp_polar(I*pi)/d), True))/e, True))/e**3 + b*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))*log(c*x**n)/e**3

Maxima [F]

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{(ex+d)^4}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*a*((18*d*e^2*x^2 + 27*d^2*e*x + 11*d^3)/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*
e^5*x + d^3*e^4) + 6*log(e*x + d)/e^4) + b*integrate((x^3*log(c) + x^3*log(
x^n))/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Giac [F]

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{(ex+d)^4}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^3/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^4}\,dx = \int \frac{x^3\,(a+b\,\ln(c\,x^n))}{(d+e\,x)^4}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n)))/(d + e*x)^4,x)
```

```
[Out] int((x^3*(a + b*log(c*x^n)))/(d + e*x)^4, x)
```

3.56 $\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^4}\,dx=\frac{bdn}{6e^3(d+ex)^2}-\frac{2bn}{3e^3(d+ex)}+\frac{x^3(a+b\log(cx^n))}{3d(d+ex)^3}-\frac{bn\log(d+ex)}{3de^3}$$

```
[Out] 1/6*b*d*n/e^3/(e*x+d)^2-2/3*b*n/e^3/(e*x+d)+1/3*x^3*(a+b*ln(c*x^n))/d/(e*x+
d)^3-1/3*b*n*ln(e*x+d)/d/e^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2373, 45}

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^4}\,dx=\frac{x^3(a+b\log(cx^n))}{3d(d+ex)^3}-\frac{2bn}{3e^3(d+ex)}+\frac{bdn}{6e^3(d+ex)^2}-\frac{bn\log(d+ex)}{3de^3}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/(d + e*x)^4,x]
```

```
[Out] (b*d*n)/(6*e^3*(d + e*x)^2) - (2*b*n)/(3*e^3*(d + e*x)) + (x^3*(a + b*Log[c
*x^n]))/(3*d*(d + e*x)^3) - (b*n*Log[d + e*x])/(3*d*e^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^3(a+b\log(cx^n))}{3d(d+ex)^3} - \frac{(bn)\int\frac{x^2}{(d+ex)^3}\,dx}{3d}\\ &= \frac{x^3(a+b\log(cx^n))}{3d(d+ex)^3} - \frac{(bn)\int\left(\frac{d^2}{e^2(d+ex)^3} - \frac{2d}{e^2(d+ex)^2} + \frac{1}{e^2(d+ex)}\right)dx}{3d}\\ &= \frac{bdn}{6e^3(d+ex)^2} - \frac{2bn}{3e^3(d+ex)} + \frac{x^3(a+b\log(cx^n))}{3d(d+ex)^3} - \frac{bn\log(d+ex)}{3de^3}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. $2(79) = 158$.

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.18

$$\int\frac{x^2(a+b\log(cx^n))}{(d+ex)^4}\,dx = -\frac{ad^2}{3e^3(d+ex)^3} + \frac{ad}{e^3(d+ex)^2} + \frac{bdn}{6e^3(d+ex)^2} - \frac{a}{e^3(d+ex)} - \frac{2bn}{3e^3(d+ex)} + \frac{bn\log(x)}{3de^3} - \frac{bd^2\log(cx^n)}{3e^3(d+ex)^3} + \frac{bd\log(cx^n)}{e^3(d+ex)^2} - \frac{b\log(cx^n)}{e^3(d+ex)} - \frac{bn\log(d+ex)}{3de^3}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*x)^4,x]
```

```
[Out] -1/3*(a*d^2)/(e^3*(d + e*x)^3) + (a*d)/(e^3*(d + e*x)^2) + (b*d*n)/(6*e^3*(
d + e*x)^2) - a/(e^3*(d + e*x)) - (2*b*n)/(3*e^3*(d + e*x)) + (b*n*Log[x])/
(3*d*e^3) - (b*d^2*Log[c*x^n])/(3*e^3*(d + e*x)^3) + (b*d*Log[c*x^n])/(e^3*
(d + e*x)^2) - (b*Log[c*x^n])/(e^3*(d + e*x)) - (b*n*Log[d + e*x])/(3*d*e^3
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(71) = 142$.

Time = 0.51 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.09

method	result
parallelrisch	$\frac{-2\ln(ex+d)x^3 b\, e^3 n^2 - 6\ln(ex+d)x^2 bd\, e^2 n^2 + 2x^3 \ln(c\, x^n) b\, e^3 n - 6\ln(ex+d)xb\, d^2 e\, n^2 - 4x^2 bd\, e^2 n^2 - 2\ln(ex+d)b\, d^3 n^2 - 6x^2 ad\, e^2}{6nd\, e^3 (ex+d)^3}$
risch	$-\frac{b(3e^2x^2+3dex+d^2)\ln(x^n)}{3(ex+d)^3 e^3} - \frac{-6\ln(-x)bd\, e^2 n\, x^2 - 6\ln(-x)b\, d^2 enx - 3i\pi bd\, e^2 x^2 \operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\, x^n) - 3i\pi b\, d^2 \operatorname{csg}}{}$

```
[In] int(x^2*(a+b*ln(c*x^n))/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-2*ln(e*x+d)*x^3*b*e^3*n^2-6*ln(e*x+d)*x^2*b*d*e^2*n^2+2*x^3*ln(c*x^n)
*b*e^3*n-6*ln(e*x+d)*x*b*d^2*e*n^2-4*x^2*b*d*e^2*n^2-2*ln(e*x+d)*b*d^3*n^2-
6*x^2*a*d*e^2*n-7*x*b*d^2*e*n^2-6*x*a*d^2*e*n-3*b*d^3*n^2-2*a*d^3*n)/n/d/e^
3/(e*x+d)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(71) = 142$.

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.25

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^4}\,dx = \frac{2\,be^3nx^3\log(x) - 3\,bd^3n - 2\,ad^3 - 2\,(2\,bde^2n + 3\,ade^2)x^2 - (7\,bd^2en + 6\,ad^2e)x - 2\,(be^3nx^3 + 3\,bde^2nx^2 +}{6\,(de^6x^3 + 3\,d^2e^5x^2 + 3\,d^3e^4x + d^4e^3)}$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*e^3*n*x^3*log(x) - 3*b*d^3*n - 2*a*d^3 - 2*(2*b*d*e^2*n + 3*a*d*e^
2)*x^2 - (7*b*d^2*e*n + 6*a*d^2*e)*x - 2*(b*e^3*n*x^3 + 3*b*d*e^2*n*x^2 + 3
*b*d^2*e*n*x + b*d^3*n)*log(e*x + d) - 2*(3*b*d*e^2*x^2 + 3*b*d^2*e*x + b*d
^3)*log(c))/(d*e^6*x^3 + 3*d^2*e^5*x^2 + 3*d^3*e^4*x + d^4*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(71) = 142$.

Time = 5.38 (sec) , antiderivative size = 677, normalized size of antiderivative = 8.57

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^4}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{a}{x}-\frac{bn}{x}-\frac{b\log(cx^n)}{x}\right) \\ \frac{\frac{ax^3}{3}-\frac{bnx^3}{9}+\frac{bx^3\log(cx^n)}{3}}{d^4} \\ \frac{-\frac{a}{x}-\frac{bn}{x}-\frac{b\log(cx^n)}{x}}{e^4} \\ -\frac{2ad^3}{6d^4e^3+18d^3e^4x+18d^2e^5x^2+6de^6x^3}-\frac{6ad^2ex}{6d^4e^3+18d^3e^4x+18d^2e^5x^2+6de^6x^3}-\frac{6ade^2x^2}{6d^4e^3+18d^3e^4x+18d^2e^5x^2+6de^6x^3}-\frac{2bd^3n\,\mathrm{l}}{6d^4e^3+18d^3e^4x\ldots} \end{cases}$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(e*x+d)**4,x)
```

```
[Out] Piecewise((zoo*(-a/x - b*n/x - b*log(c*x**n)/x), Eq(d, 0) & Eq(e, 0)), ((a*
x**3/3 - b*n*x**3/9 + b*x**3*log(c*x**n)/3)/d**4, Eq(e, 0)), ((-a/x - b*n/x
 - b*log(c*x**n)/x)/e**4, Eq(d, 0)), (-2*a*d**3/(6*d**4*e**3 + 18*d**3*e**4
*x + 18*d**2*e**5*x**2 + 6*d*e**6*x**3) - 6*a*d**2*e*x/(6*d**4*e**3 + 18*d*
*3*e**4*x + 18*d**2*e**5*x**2 + 6*d*e**6*x**3) - 6*a*d*e**2*x**2/(6*d**4*e*
*3 + 18*d**3*e**4*x + 18*d**2*e**5*x**2 + 6*d*e**6*x**3) - 2*b*d**3*n*log(d
/e + x)/(6*d**4*e**3 + 18*d**3*e**4*x + 18*d**2*e**5*x**2 + 6*d*e**6*x**3)
- 3*b*d**3*n/(6*d**4*e**3 + 18*d**3*e**4*x + 18*d**2*e**5*x**2 + 6*d*e**6*x
**3) - 6*b*d**2*e*n*x*log(d/e + x)/(6*d**4*e**3 + 18*d**3*e**4*x + 18*d**2*
e**5*x**2 + 6*d*e**6*x**3) - 7*b*d**2*e*n*x/(6*d**4*e**3 + 18*d**3*e**4*x +
 18*d**2*e**5*x**2 + 6*d*e**6*x**3) - 6*b*d*e**2*n*x**2*log(d/e + x)/(6*d**
4*e**3 + 18*d**3*e**4*x + 18*d**2*e**5*x**2 + 6*d*e**6*x**3) - 4*b*d*e**2*n
*x**2/(6*d**4*e**3 + 18*d**3*e**4*x + 18*d**2*e**5*x**2 + 6*d*e**6*x**3) -
2*b*e**3*n*x**3*log(d/e + x)/(6*d**4*e**3 + 18*d**3*e**4*x + 18*d**2*e**5*x
**2 + 6*d*e**6*x**3) + 2*b*e**3*x**3*log(c*x**n)/(6*d**4*e**3 + 18*d**3*e**
4*x + 18*d**2*e**5*x**2 + 6*d*e**6*x**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(71) = 142.

Time = 0.20 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.27

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^4}\,dx = -\frac{1}{6}\,bn\left(\frac{4\,ex+3\,d}{e^5x^2+2\,de^4x+d^2e^3}+\frac{2\,\log(ex+d)}{de^3}-\frac{2\,\log(x)}{de^3}\right) - \frac{(3\,e^2x^2+3\,dex+d^2)b\log(cx^n)}{3\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)} - \frac{(3\,e^2x^2+3\,dex+d^2)a}{3\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)}$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/6*b*n*((4*e*x + 3*d)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3) + 2*log(e*x + d)/(d
*e^3) - 2*log(x)/(d*e^3)) - 1/3*(3*e^2*x^2 + 3*d*e*x + d^2)*b*log(c*x^n)/(e
^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3) - 1/3*(3*e^2*x^2 + 3*d*e*x +
d^2)*a/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(71) = 142.

Time = 0.38 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.56

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^4}\,dx = -\frac{(3\,be^2nx^2+3\,bdenx+bd^2n)\log(x)}{3\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)} - \frac{4\,be^2nx^2+6\,be^2x^2\log(c)+7\,bdenx+6\,ae^2x^2+6\,bdex\log(c)+3\,bd^2n+6\,adex+2\,bd^2\log(c)+2\,ad^2}{6\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)} - \frac{bn\log(ex+d)}{3\,de^3} + \frac{bn\log(x)}{3\,de^3}$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b*e^2*n*x^2 + 3*b*d*e*n*x + b*d^2*n)*log(x)/(e^6*x^3 + 3*d*e^5*x^2
+ 3*d^2*e^4*x + d^3*e^3) - 1/6*(4*b*e^2*n*x^2 + 6*b*e^2*x^2*log(c) + 7*b*d*
e*n*x + 6*a*e^2*x^2 + 6*b*d*e*x*log(c) + 3*b*d^2*n + 6*a*d*e*x + 2*b*d^2*lo
g(c) + 2*a*d^2)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3) - 1/3*b*n*l
og(e*x + d)/(d*e^3) + 1/3*b*n*log(x)/(d*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.11

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^4}\,dx = -\frac{x^2\,(3\,a\,e^2+2\,b\,e^2\,n)+a\,d^2+x\,\left(3\,a\,d\,e+\frac{7\,b\,d\,e\,n}{2}\right)+\frac{3\,b\,d^2\,n}{2}}{3\,d^3\,e^3+9\,d^2\,e^4\,x+9\,d\,e^5\,x^2+3\,e^6\,x^3} - \frac{\ln(c\,x^n)\,\left(\frac{b\,d^2}{3\,e^3}+\frac{b\,x^2}{e}+\frac{b\,d\,x}{e^2}\right)}{d^3+3\,d^2\,e\,x+3\,d\,e^2\,x^2+e^3\,x^3} - \frac{2\,b\,n\,\mathrm{atanh}\left(\frac{2\,e\,x}{d}+1\right)}{3\,d\,e^3}$$

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e*x)^4,x)
```

```
[Out] - (x^2*(3*a*e^2 + 2*b*e^2*n) + a*d^2 + x*(3*a*d*e + (7*b*d*e*n)/2) + (3*b*d
^2*n)/2)/(3*d^3*e^3 + 3*e^6*x^3 + 9*d^2*e^4*x + 9*d*e^5*x^2) - (log(c*x^n)*
((b*d^2)/(3*e^3) + (b*x^2)/e + (b*d*x)/e^2))/(d^3 + e^3*x^3 + 3*d*e^2*x^2 +
 3*d^2*e*x) - (2*b*n*atanh((2*e*x)/d + 1))/(3*d*e^3)
```

3.57 $\int \frac{x(a+b\log(cx^n))}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 117

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^4}\,dx = -\frac{bn}{6e^2(d+ex)^2}+\frac{bn}{6de^2(d+ex)}+\frac{bn\log(x)}{6d^2e^2}+\frac{d(a+b\log(cx^n))}{3e^2(d+ex)^3}-\frac{a+b\log(cx^n)}{2e^2(d+ex)^2}-\frac{bn\log(d+ex)}{6d^2e^2}$$

```
[Out] -1/6*b*n/e^2/(e*x+d)^2+1/6*b*n/d/e^2/(e*x+d)+1/6*b*n*ln(x)/d^2/e^2+1/3*d*(a
+b*ln(c*x^n))/e^2/(e*x+d)^3+1/2*(-a-b*ln(c*x^n))/e^2/(e*x+d)^2-1/6*b*n*ln(e
*x+d)/d^2/e^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {45, 2382, 12, 78}

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^4}\,dx = -\frac{a+b\log(cx^n)}{2e^2(d+ex)^2}+\frac{d(a+b\log(cx^n))}{3e^2(d+ex)^3}+\frac{bn\log(x)}{6d^2e^2}-\frac{bn\log(d+ex)}{6d^2e^2}+\frac{bn}{6de^2(d+ex)}-\frac{bn}{6e^2(d+ex)^2}$$

```
[In] Int[(x*(a + b*Log[c*x^n]))/(d + e*x)^4,x]
```

```
[Out] -1/6*(b*n)/(e^2*(d + e*x)^2) + (b*n)/(6*d*e^2*(d + e*x)) + (b*n*Log[x])/(6*
d^2*e^2) + (d*(a + b*Log[c*x^n]))/(3*e^2*(d + e*x)^3) - (a + b*Log[c*x^n])/
(2*e^2*(d + e*x)^2) - (b*n*Log[d + e*x])/(6*d^2*e^2)
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{d(a+b\log(cx^n))}{3e^2(d+ex)^3}-\frac{a+b\log(cx^n)}{2e^2(d+ex)^2}-(bn)\int\frac{-d-3ex}{6e^2x(d+ex)^3}\,dx\\
&= \frac{d(a+b\log(cx^n))}{3e^2(d+ex)^3}-\frac{a+b\log(cx^n)}{2e^2(d+ex)^2}-\frac{(bn)\int\frac{-d-3ex}{x(d+ex)^3}\,dx}{6e^2}\\
&= \frac{d(a+b\log(cx^n))}{3e^2(d+ex)^3}-\frac{a+b\log(cx^n)}{2e^2(d+ex)^2}-\frac{(bn)\int\left(-\frac{1}{d^2x}-\frac{2e}{(d+ex)^3}+\frac{e}{d(d+ex)^2}+\frac{e}{d^2(d+ex)}\right)dx}{6e^2}\\
&= -\frac{bn}{6e^2(d+ex)^2}+\frac{bn}{6de^2(d+ex)}+\frac{bn\log(x)}{6d^2e^2}\\
&\quad+\frac{d(a+b\log(cx^n))}{3e^2(d+ex)^3}-\frac{a+b\log(cx^n)}{2e^2(d+ex)^2}-\frac{bn\log(d+ex)}{6d^2e^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.15

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^4}\,dx = \frac{d(a+b\log(cx^n))}{3e^2(d+ex)^3} - \frac{a+b\log(cx^n)}{2e^2(d+ex)^2} - \frac{bn\left(\frac{1}{(d+ex)^2}+\frac{2}{d(d+ex)}+\frac{2\log(x)}{d^2}-\frac{2\log(d+ex)}{d^2}\right)}{6e^2} + \frac{bn\left(\frac{1}{d(d+ex)}+\frac{\log(x)}{d^2}-\frac{\log(d+ex)}{d^2}\right)}{2e^2}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e*x)^4,x]
```

```
[Out] (d*(a + b*Log[c*x^n]))/(3*e^2*(d + e*x)^3) - (a + b*Log[c*x^n])/(2*e^2*(d +
 e*x)^2) - (b*n*((d + e*x)^(-2) + 2/(d*(d + e*x)) + (2*Log[x])/d^2 - (2*Log
[d + e*x])/d^2))/(6*e^2) + (b*n*(1/(d*(d + e*x)) + Log[x]/d^2 - Log[d + e*x
]/d^2))/(2*e^2)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.75

method	result
parallelrisch	$\frac{-3x\ln(c\,x^n)b\,d^3e^2+xb\,d^3e^2n+x^2b\,d^2e^3n+\ln(x)b\,d^4en-\ln(ex+d)b\,d^4en-\ln(c\,x^n)b\,d^4e+3x^2a\,d^2e^3+x^3ad\,e^4+3\ln(x)xb\,d^3e^2n-}{6d^3e^3(ex+d)^3}$
risch	$-\frac{b(3ex+d)\ln(x^n)}{6(ex+d)^3e^2}-\frac{3i\pi b\,d^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2ex+i\pi b\,d^3\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2-i\pi b\,d^3\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)-3i}{}$

```
[In] int(x*(a+b*ln(c*x^n))/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-3*x*ln(c*x^n)*b*d^3*e^2+x*b*d^3*e^2*n+x^2*b*d^2*e^3*n+ln(x)*b*d^4*e*n
-ln(e*x+d)*b*d^4*e*n-ln(c*x^n)*b*d^4*e+3*x^2*a*d^2*e^3+x^3*a*d*e^4+3*ln(x)*
x*b*d^3*e^2*n-3*ln(e*x+d)*x*b*d^3*e^2*n+ln(x)*x^3*b*d*e^4*n-ln(e*x+d)*x^3*b
*d*e^4*n+3*ln(x)*x^2*b*d^2*e^3*n-3*ln(e*x+d)*x^2*b*d^2*e^3*n)/d^3/e^3/(e*x+
d)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.38

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^4}\,dx = \frac{bde^2nx^2 - ad^3 + (bd^2en - 3\,ad^2e)x - (be^3nx^3 + 3\,bde^2nx^2 + 3\,bd^2enx + bd^3n)\log(ex+d) - (3\,bd^2ex + bd}{6\,(d^2e^5x^3 + 3\,d^3e^4x^2 + 3\,d^4e^3x + d^5e^2)}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(b*d*e^2*n*x^2 - a*d^3 + (b*d^2*e*n - 3*a*d^2*e)*x - (b*e^3*n*x^3 + 3*b
*d*e^2*n*x^2 + 3*b*d^2*e*n*x + b*d^3*n)*log(e*x + d) - (3*b*d^2*e*x + b*d^3
)*log(c) + (b*e^3*n*x^3 + 3*b*d*e^2*n*x^2)*log(x))/(d^2*e^5*x^3 + 3*d^3*e^4
*x^2 + 3*d^4*e^3*x + d^5*e^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. 2(112) = 224.

Time = 5.32 (sec) , antiderivative size = 661, normalized size of antiderivative = 5.65

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^4}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{a}{2x^2} - \frac{bn}{4x^2} - \frac{b\log(cx^n)}{2x^2}\right) \\ \frac{\frac{ax^2}{2} - \frac{bnx^2}{4} + \frac{bx^2\log(cx^n)}{2}}{d^4} \\ \frac{-\frac{a}{2x^2} - \frac{bn}{4x^2} - \frac{b\log(cx^n)}{2x^2}}{e^4} \\ -\frac{ad^3}{6d^5e^2+18d^4e^3x+18d^3e^4x^2+6d^2e^5x^3} - \frac{3ad^2ex}{6d^5e^2+18d^4e^3x+18d^3e^4x^2+6d^2e^5x^3} - \frac{bd^3n\log\left(\frac{d}{e}+x\right)}{6d^5e^2+18d^4e^3x+18d^3e^4x^2+6d^2e^5x^3} - \frac{3bd^2en}{6d^5e^2+18d^4e^3x} \end{cases}$$

```
[In] integrate(x*(a+b*ln(c*x**n))/(e*x+d)**4,x)
```

```
[Out] Piecewise((zoo*(-a/(2*x**2) - b*n/(4*x**2) - b*log(c*x**n)/(2*x**2)), Eq(d,
 0) & Eq(e, 0)), ((a*x**2/2 - b*n*x**2/4 + b*x**2*log(c*x**n)/2)/d**4, Eq(e
, 0)), ((-a/(2*x**2) - b*n/(4*x**2) - b*log(c*x**n)/(2*x**2))/e**4, Eq(d, 0
)), (-a*d**3/(6*d**5*e**2 + 18*d**4*e**3*x + 18*d**3*e**4*x**2 + 6*d**2*e**
5*x**3) - 3*a*d**2*e*x/(6*d**5*e**2 + 18*d**4*e**3*x + 18*d**3*e**4*x**2 +
6*d**2*e**5*x**3) - b*d**3*n*log(d/e + x)/(6*d**5*e**2 + 18*d**4*e**3*x + 1
8*d**3*e**4*x**2 + 6*d**2*e**5*x**3) - 3*b*d**2*e*n*x*log(d/e + x)/(6*d**5*
e**2 + 18*d**4*e**3*x + 18*d**3*e**4*x**2 + 6*d**2*e**5*x**3) + b*d**2*e*n*
x/(6*d**5*e**2 + 18*d**4*e**3*x + 18*d**3*e**4*x**2 + 6*d**2*e**5*x**3) - 3
```

```
*b*d*e**2*n*x**2*log(d/e + x)/(6*d**5*e**2 + 18*d**4*e**3*x + 18*d**3*e**4*
x**2 + 6*d**2*e**5*x**3) + b*d*e**2*n*x**2/(6*d**5*e**2 + 18*d**4*e**3*x +
18*d**3*e**4*x**2 + 6*d**2*e**5*x**3) + 3*b*d*e**2*x**2*log(c*x**n)/(6*d**5
*e**2 + 18*d**4*e**3*x + 18*d**3*e**4*x**2 + 6*d**2*e**5*x**3) - b*e**3*n*x
**3*log(d/e + x)/(6*d**5*e**2 + 18*d**4*e**3*x + 18*d**3*e**4*x**2 + 6*d**2
*e**5*x**3) + b*e**3*x**3*log(c*x**n)/(6*d**5*e**2 + 18*d**4*e**3*x + 18*d*
*3*e**4*x**2 + 6*d**2*e**5*x**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.28

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^4}\,dx = \frac{1}{6}\,bn\left(\frac{x}{de^3x^2+2\,d^2e^2x+d^3e}-\frac{\log(ex+d)}{d^2e^2}+\frac{\log(x)}{d^2e^2}\right) - \frac{(3\,ex+d)b\log(cx^n)}{6\,(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2)} - \frac{(3\,ex+d)a}{6\,(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2)}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*b*n*(x/(d*e^3*x^2 + 2*d^2*e^2*x + d^3*e) - log(e*x + d)/(d^2*e^2) + log
(x)/(d^2*e^2)) - 1/6*(3*e*x + d)*b*log(c*x^n)/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^
2*e^3*x + d^3*e^2) - 1/6*(3*e*x + d)*a/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x
 + d^3*e^2)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.39

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^4}\,dx = -\frac{(3\,benx+bdn)\log(x)}{6\,(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2)} + \frac{be^2nx^2+bdenx-3\,bdex\log(c)-3\,adex-bd^2\log(c)-ad^2}{6\,(de^5x^3+3\,d^2e^4x^2+3\,d^3e^3x+d^4e^2)} - \frac{bn\log(ex+d)}{6\,d^2e^2} + \frac{bn\log(x)}{6\,d^2e^2}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*e*n*x + b*d*n)*log(x)/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*
e^2) + 1/6*(b*e^2*n*x^2 + b*d*e*n*x - 3*b*d*e*x*log(c) - 3*a*d*e*x - b*d^2*
log(c) - a*d^2)/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4*e^2) - 1/6*b
*n*log(e*x + d)/(d^2*e^2) + 1/6*b*n*log(x)/(d^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.21

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^4}\,dx = -\frac{a\,d+x\,(3\,a\,e-b\,e\,n)-\frac{b\,e^2\,n\,x^2}{d}}{6\,d^3\,e^2+18\,d^2\,e^3\,x+18\,d\,e^4\,x^2+6\,e^5\,x^3} - \frac{\ln\left(c\,x^n\right)\,\left(\frac{b\,d}{6\,e^2}+\frac{b\,x}{2\,e}\right)}{d^3+3\,d^2\,e\,x+3\,d\,e^2\,x^2+e^3\,x^3} - \frac{b\,n\,\mathrm{atanh}\left(\frac{2\,e\,x}{d}+1\right)}{3\,d^2\,e^2}$$

```
[In] int((x*(a + b*log(c*x^n)))/(d + e*x)^4,x)
```

```
[Out] - (a*d + x*(3*a*e - b*e*n) - (b*e^2*n*x^2)/d)/(6*d^3*e^2 + 6*e^5*x^3 + 18*d
^2*e^3*x + 18*d*e^4*x^2) - (log(c*x^n)*((b*d)/(6*e^2) + (b*x)/(2*e)))/(d^3
+ e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x) - (b*n*atanh((2*e*x)/d + 1))/(3*d^2*e^
2)
```

3.58 $\int \frac{a+b\log(cx^n)}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 95

$$\int \frac{a+b\log(cx^n)}{(d+ex)^4}\,dx = \frac{bn}{6de(d+ex)^2}+\frac{bn}{3d^2e(d+ex)}+\frac{bn\log(x)}{3d^3e}-\frac{a+b\log(cx^n)}{3e(d+ex)^3}-\frac{bn\log(d+ex)}{3d^3e}$$

```
[Out] 1/6*b*n/d/e/(e*x+d)^2+1/3*b*n/d^2/e/(e*x+d)+1/3*b*n*ln(x)/d^3/e+1/3*(-a-b*l
n(c*x^n))/e/(e*x+d)^3-1/3*b*n*ln(e*x+d)/d^3/e
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2356, 46}

$$\int \frac{a+b\log(cx^n)}{(d+ex)^4}\,dx = -\frac{a+b\log(cx^n)}{3e(d+ex)^3}+\frac{bn\log(x)}{3d^3e}-\frac{bn\log(d+ex)}{3d^3e}+\frac{bn}{3d^2e(d+ex)}+\frac{bn}{6de(d+ex)^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(d + e*x)^4,x]
```

```
[Out] (b*n)/(6*d*e*(d + e*x)^2) + (b*n)/(3*d^2*e*(d + e*x)) + (b*n*Log[x])/(3*d^3
*e) - (a + b*Log[c*x^n])/(3*e*(d + e*x)^3) - (b*n*Log[d + e*x])/(3*d^3*e)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\log\left(cx^n\right)}{3e(d+ex)^3}+\frac{(bn)\int\frac{1}{x(d+ex)^3}\,dx}{3e}\\
&= -\frac{a+b\log\left(cx^n\right)}{3e(d+ex)^3}+\frac{(bn)\int\left(\frac{1}{d^3x}-\frac{e}{d(d+ex)^3}-\frac{e}{d^2(d+ex)^2}-\frac{e}{d^3(d+ex)}\right)dx}{3e}\\
&= \frac{bn}{6de(d+ex)^2}+\frac{bn}{3d^2e(d+ex)}+\frac{bn\log(x)}{3d^3e}-\frac{a+b\log\left(cx^n\right)}{3e(d+ex)^3}-\frac{bn\log(d+ex)}{3d^3e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int\frac{a+b\log\left(cx^n\right)}{(d+ex)^4}\,dx=\frac{-\frac{a+b\log(cx^n)}{(d+ex)^3}+\frac{bn\left(\frac{d(3d+2ex)}{(d+ex)^2}+2\log(x)-2\log(d+ex)\right)}{2d^3}}{3e}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*x)^4,x]
[Out] (-((a + b*Log[c*x^n])/(d + e*x)^3) + (b*n*((d*(3*d + 2*e*x))/(d + e*x)^2 +
2*Log[x] - 2*Log[d + e*x]))/(2*d^3))/(3*e)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(88) = 176.

Time = 0.49 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.98

method	result
parallelrisch	$\frac{-5x^3b\,e^5n-6\ln(c\,x^n)b\,d^3e^2+4b\,d^3e^2n-6a\,d^3e^2+18\ln(x)x^2bd\,e^4n-18\ln(ex+d)x^2bd\,e^4n+18\ln(x)xb\,d^2e^3n-18\ln(ex+d)xb\,d^2}{18e^3d^3(ex+d)^3}$
risch	$-\frac{b\ln(x^n)}{3e(ex+d)^3}-\frac{2\ln(ex+d)b\,e^3n\,x^3-2\ln(-x)b\,e^3n\,x^3-i\pi b\,d^3\,\mathrm{csgn}(ic\,x^n)^3+i\pi b\,d^3\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2+i\pi b\,d^3\,\mathrm{csgn}(ic)\,\mathrm{cs}}{}$

```
[In] int((a+b*ln(c*x^n))/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*(-5*x^3*b*e^5*n-6*ln(c*x^n)*b*d^3*e^2+4*b*d^3*e^2*n-6*a*d^3*e^2+18*ln(
x)*x^2*b*d*e^4*n-18*ln(e*x+d)*x^2*b*d*e^4*n+18*ln(x)*x*b*d^2*e^3*n-18*ln(e*
x+d)*x*b*d^2*e^3*n+6*ln(x)*b*d^3*e^2*n-6*ln(e*x+d)*b*d^3*e^2*n-9*x^2*b*d*e^
4*n+6*ln(x)*x^3*b*e^5*n-6*ln(e*x+d)*x^3*b*e^5*n)/e^3/d^3/(e*x+d)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.68

$$\int \frac{a+b\log(cx^n)}{(d+ex)^4}\,dx = \frac{2\,bde^2nx^2+5\,bd^2enx+3\,bd^3n-2\,bd^3\log(c)-2\,ad^3-2\,(be^3nx^3+3\,bde^2nx^2+3\,bd^2enx+bd^3n)\log(ex}{6\,(d^3e^4x^3+3\,d^4e^3x^2+3\,d^5e^2x+d^6e)}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*d*e^2*n*x^2 + 5*b*d^2*e*n*x + 3*b*d^3*n - 2*b*d^3*log(c) - 2*a*d^3
 - 2*(b*e^3*n*x^3 + 3*b*d*e^2*n*x^2 + 3*b*d^2*e*n*x + b*d^3*n)*log(e*x + d)
 + 2*(b*e^3*n*x^3 + 3*b*d*e^2*n*x^2 + 3*b*d^2*e*n*x)*log(x))/(d^3*e^4*x^3 +
 3*d^4*e^3*x^2 + 3*d^5*e^2*x + d^6*e)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(83) = 166$.

Time = 5.35 (sec) , antiderivative size = 700, normalized size of antiderivative = 7.37

$$\int \frac{a+b\log(cx^n)}{(d+ex)^4}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{a}{3x^3}-\frac{bn}{9x^3}-\frac{b\log(cx^n)}{3x^3}\right) \\ \frac{ax-bnx+bx\log(cx^n)}{d^4} \\ \frac{-\frac{a}{3x^3}-\frac{bn}{9x^3}-\frac{b\log(cx^n)}{3x^3}}{e^4} \\ -\frac{2ad^3}{6d^6e+18d^5e^2x+18d^4e^3x^2+6d^3e^4x^3}-\frac{2bd^3n\log\left(\frac{d}{e}+x\right)}{6d^6e+18d^5e^2x+18d^4e^3x^2+6d^3e^4x^3}+\frac{3bd^3n}{6d^6e+18d^5e^2x+18d^4e^3x^2+6d^3e^4x^3}-\frac{6bd^2enx}{6d^6e+18d^5e^2x+} \end{cases}$$

```
[In] integrate((a+b*ln(c*x**n))/(e*x+d)**4,x)
```

```
[Out] Piecewise((zoo*(-a/(3*x**3) - b*n/(9*x**3) - b*log(c*x**n)/(3*x**3)), Eq(d,
 0) & Eq(e, 0)), ((a*x - b*n*x + b*x*log(c*x**n))/d**4, Eq(e, 0)), ((-a/(3*
x**3) - b*n/(9*x**3) - b*log(c*x**n)/(3*x**3))/e**4, Eq(d, 0)), (-2*a*d**3/
(6*d**6*e + 18*d**5*e**2*x + 18*d**4*e**3*x**2 + 6*d**3*e**4*x**3) - 2*b*d*
*3*n*log(d/e + x)/(6*d**6*e + 18*d**5*e**2*x + 18*d**4*e**3*x**2 + 6*d**3*e
**4*x**3) + 3*b*d**3*n/(6*d**6*e + 18*d**5*e**2*x + 18*d**4*e**3*x**2 + 6*d
**3*e**4*x**3) - 6*b*d**2*e*n*x*log(d/e + x)/(6*d**6*e + 18*d**5*e**2*x + 1
8*d**4*e**3*x**2 + 6*d**3*e**4*x**3) + 5*b*d**2*e*n*x/(6*d**6*e + 18*d**5*e
**2*x + 18*d**4*e**3*x**2 + 6*d**3*e**4*x**3) + 6*b*d**2*e*x*log(c*x**n)/(6
*d**6*e + 18*d**5*e**2*x + 18*d**4*e**3*x**2 + 6*d**3*e**4*x**3) - 6*b*d*e*
*2*n*x**2*log(d/e + x)/(6*d**6*e + 18*d**5*e**2*x + 18*d**4*e**3*x**2 + 6*d
**3*e**4*x**3) + 2*b*d*e**2*n*x**2/(6*d**6*e + 18*d**5*e**2*x + 18*d**4*e**
3*x**2 + 6*d**3*e**4*x**3) + 6*b*d*e**2*x**2*log(c*x**n)/(6*d**6*e + 18*d**
5*e**2*x + 18*d**4*e**3*x**2 + 6*d**3*e**4*x**3) - 2*b*e**3*n*x**3*log(d/e
+ x)/(6*d**6*e + 18*d**5*e**2*x + 18*d**4*e**3*x**2 + 6*d**3*e**4*x**3) + 2
*b*e**3*x**3*log(c*x**n)/(6*d**6*e + 18*d**5*e**2*x + 18*d**4*e**3*x**2 + 6
*d**3*e**4*x**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.52

$$\int \frac{a+b\log(cx^n)}{(d+ex)^4}\,dx = \frac{1}{6}\,bn\left(\frac{2\,ex+3\,d}{d^2e^3x^2+2\,d^3e^2x+d^4e}-\frac{2\,\log(ex+d)}{d^3e}+\frac{2\,\log(x)}{d^3e}\right) - \frac{b\log(cx^n)}{3\,(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e)} - \frac{a}{3\,(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e)}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*b*n*((2*e*x + 3*d)/(d^2*e^3*x^2 + 2*d^3*e^2*x + d^4*e) - 2*log(e*x + d)
/(d^3*e) + 2*log(x)/(d^3*e)) - 1/3*b*log(c*x^n)/(e^4*x^3 + 3*d*e^3*x^2 + 3*
d^2*e^2*x + d^3*e) - 1/3*a/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.55

$$\int \frac{a+b\log(cx^n)}{(d+ex)^4}\,dx = -\frac{bn\log(x)}{3\,(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e)} + \frac{2\,be^2nx^2+5\,bdenx+3\,bd^2n-2\,bd^2\log(c)-2\,ad^2}{6\,(d^2e^4x^3+3\,d^3e^3x^2+3\,d^4e^2x+d^5e)} - \frac{bn\log(ex+d)}{3\,d^3e} + \frac{bn\log(x)}{3\,d^3e}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -1/3*b*n*log(x)/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e) + 1/6*(2*b*e^
2*n*x^2 + 5*b*d*e*n*x + 3*b*d^2*n - 2*b*d^2*log(c) - 2*a*d^2)/(d^2*e^4*x^3
+ 3*d^3*e^3*x^2 + 3*d^4*e^2*x + d^5*e) - 1/3*b*n*log(e*x + d)/(d^3*e) + 1/3
*b*n*log(x)/(d^3*e)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.34

$$\int \frac{a+b\log(cx^n)}{(d+ex)^4}\,dx = \frac{\frac{3\,b\,n}{2}-a+\frac{b\,e^2\,n\,x^2}{d^2}+\frac{5\,b\,e\,n\,x}{2\,d}}{3\,d^3\,e+9\,d^2\,e^2\,x+9\,d\,e^3\,x^2+3\,e^4\,x^3} - \frac{b\,\ln(c\,x^n)}{3\,e\,\left(d^3+3\,d^2\,e\,x+3\,d\,e^2\,x^2+e^3\,x^3\right)} - \frac{2\,b\,n\,\mathrm{atanh}\left(\frac{2\,e\,x}{d}+1\right)}{3\,d^3\,e}$$

```
[In] int((a + b*log(c*x^n))/(d + e*x)^4,x)
```

```
[Out] ((3*b*n)/2 - a + (b*e^2*n*x^2)/d^2 + (5*b*e*n*x)/(2*d))/(3*d^3*e + 3*e^4*x^
3 + 9*d^2*e^2*x + 9*d*e^3*x^2) - (b*log(c*x^n))/(3*e*(d^3 + e^3*x^3 + 3*d*e
^2*x^2 + 3*d^2*e*x)) - (2*b*n*atanh((2*e*x)/d + 1))/(3*d^3*e)
```

3.59 $\int \frac{a+b\log(cx^n)}{x(d+ex)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^4}\,dx = -\frac{bn}{6d^2(d+ex)^2}-\frac{5bn}{6d^3(d+ex)}-\frac{5bn\log(x)}{6d^4}+\frac{a+b\log(cx^n)}{3d(d+ex)^3}+\frac{a+b\log(cx^n)}{2d^2(d+ex)^2}-\frac{ex(a+b\log(cx^n))}{d^4(d+ex)}-\frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4}+\frac{11bn\log(d+ex)}{6d^4}+\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^4}$$

```
[Out] -1/6*b*n/d^2/(e*x+d)^2-5/6*b*n/d^3/(e*x+d)-5/6*b*n*ln(x)/d^4+1/3*(a+b*ln(c*
x^n))/d/(e*x+d)^3+1/2*(a+b*ln(c*x^n))/d^2/(e*x+d)^2-e*x*(a+b*ln(c*x^n))/d^4
/(e*x+d)-ln(1+d/e/x)*(a+b*ln(c*x^n))/d^4+11/6*b*n*ln(e*x+d)/d^4+b*n*polylog
(2,-d/e/x)/d^4
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^4}\,dx = -\frac{\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^4}-\frac{ex(a+b\log(cx^n))}{d^4(d+ex)}+\frac{a+b\log(cx^n)}{2d^2(d+ex)^2}+\frac{a+b\log(cx^n)}{3d(d+ex)^3}+\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^4}+\frac{11bn\log(d+ex)}{6d^4}-\frac{5bn\log(x)}{6d^4}-\frac{5bn}{6d^3(d+ex)}-\frac{bn}{6d^2(d+ex)^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x)^4),x]
```

```
[Out] -1/6*(b*n)/(d^2*(d + e*x)^2) - (5*b*n)/(6*d^3*(d + e*x)) - (5*b*n*Log[x])/(
6*d^4) + (a + b*Log[c*x^n])/(3*d*(d + e*x)^3) + (a + b*Log[c*x^n])/(2*d^2*(
d + e*x)^2) - (e*x*(a + b*Log[c*x^n]))/(d^4*(d + e*x)) - (Log[1 + d/(e*x)]*
(a + b*Log[c*x^n]))/d^4 + (11*b*n*Log[d + e*x])/(6*d^4) + (b*n*PolyLog[2, -
(d/(e*x))])/d^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
```

```
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{a+b\log(cx^n)}{x(d+ex)^3}\,dx}{d} - \frac{e\int \frac{a+b\log(cx^n)}{(d+ex)^4}\,dx}{d}\\
&= \frac{a+b\log(cx^n)}{3d(d+ex)^3} + \frac{\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx}{d^2} - \frac{e\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx}{d^2} - \frac{(bn)\int \frac{1}{x(d+ex)^3}\,dx}{3d}\\
&= \frac{a+b\log(cx^n)}{3d(d+ex)^3} + \frac{a+b\log(cx^n)}{2d^2(d+ex)^2} + \frac{\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d^3} - \frac{e\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{d^3}\\
&\quad - \frac{(bn)\int \frac{1}{x(d+ex)^2}\,dx}{2d^2} - \frac{(bn)\int \left(\frac{1}{d^3x} - \frac{e}{d(d+ex)^3} - \frac{e}{d^2(d+ex)^2} - \frac{e}{d^3(d+ex)}\right)dx}{3d}\\
&= -\frac{bn}{6d^2(d+ex)^2} - \frac{bn}{3d^3(d+ex)} - \frac{bn\log(x)}{3d^4} + \frac{a+b\log(cx^n)}{3d(d+ex)^3} + \frac{a+b\log(cx^n)}{2d^2(d+ex)^2}\\
&\quad - \frac{ex(a+b\log(cx^n))}{d^4(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4} + \frac{bn\log(d+ex)}{3d^4}\\
&\quad + \frac{(bn)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^4} - \frac{(bn)\int \left(\frac{1}{d^2x} - \frac{e}{d(d+ex)^2} - \frac{e}{d^2(d+ex)}\right)dx}{2d^2} + \frac{(ben)\int \frac{1}{d+ex}\,dx}{d^4}\\
&= -\frac{bn}{6d^2(d+ex)^2} - \frac{5bn}{6d^3(d+ex)} - \frac{5bn\log(x)}{6d^4} + \frac{a+b\log(cx^n)}{3d(d+ex)^3} + \frac{a+b\log(cx^n)}{2d^2(d+ex)^2}\\
&\quad - \frac{ex(a+b\log(cx^n))}{d^4(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4} + \frac{11bn\log(d+ex)}{6d^4}\\
&\quad + \frac{bn\text{Li}_2\left(-\frac{d}{ex}\right)}{d^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.28

$$\int \frac{a+b\log\left(cx^n\right)}{x(d+ex)^4}\,dx$$
$$= \frac{\frac{3a^2}{bn}+\frac{2ad^3}{(d+ex)^3}+\frac{3ad^2}{(d+ex)^2}-\frac{bd^2n}{(d+ex)^2}+\frac{6ad}{d+ex}-\frac{5bdn}{d+ex}-11bn\log(x)+\frac{6a\log(cx^n)}{n}+\frac{2bd^3\log(cx^n)}{(d+ex)^3}+\frac{3bd^2\log(cx^n)}{(d+ex)^2}+\frac{6bd}{}}{6}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x)^4),x]
```

```
[Out] ((3*a^2)/(b*n) + (2*a*d^3)/(d + e*x)^3 + (3*a*d^2)/(d + e*x)^2 - (b*d^2*n)/
(d + e*x)^2 + (6*a*d)/(d + e*x) - (5*b*d*n)/(d + e*x) - 11*b*n*Log[x] + (6*
a*Log[c*x^n])/n + (2*b*d^3*Log[c*x^n])/(d + e*x)^3 + (3*b*d^2*Log[c*x^n])/(
d + e*x)^2 + (6*b*d*Log[c*x^n])/(d + e*x) + (3*b*Log[c*x^n]^2)/n + 11*b*n*L
og[d + e*x] - 6*a*Log[1 + (e*x)/d] - 6*b*Log[c*x^n]*Log[1 + (e*x)/d] - 6*b*
n*PolyLog[2, -((e*x)/d)])/(6*d^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.71 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{b\ln(x^n)\ln(ex+d)}{d^4}+\frac{b\ln(x^n)}{d^3(ex+d)}+\frac{b\ln(x^n)}{2d^2(ex+d)^2}+\frac{b\ln(x^n)}{3d(ex+d)^3}+\frac{b\ln(x^n)\ln(x)}{d^4}-\frac{5bn}{6d^3(ex+d)}-\frac{bn}{6d^2(ex+d)^2}+\frac{11bn\ln(ex}{6d^4}$

```
[In] int((a+b*ln(c*x^n))/x/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -b*ln(x^n)/d^4*ln(e*x+d)+b*ln(x^n)/d^3/(e*x+d)+1/2*b*ln(x^n)/d^2/(e*x+d)^2+
1/3*b*ln(x^n)/d/(e*x+d)^3+b*ln(x^n)/d^4*ln(x)-5/6*b*n/d^3/(e*x+d)-1/6*b*n/d
^2/(e*x+d)^2+11/6*b*n*ln(e*x+d)/d^4-11/6*b*n*ln(x)/d^4-1/2*b*n/d^4*ln(x)^2+
b*n/d^4*ln(e*x+d)*ln(-e*x/d)+b*n/d^4*dilog(-e*x/d)+(-1/2*I*b*Pi*csgn(I*c)*c
sgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*cs
gn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-1/d^4*ln(
e*x+d)+1/d^3/(e*x+d)+1/2/d^2/(e*x+d)^2+1/3/d/(e*x+d)^3+1/d^4*ln(x))
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^4}\,dx=\int \frac{b\log(cx^n)+a}{(ex+d)^4x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e^4*x^5 + 4*d*e^3*x^4 + 6*d^2*e^2*x^3 + 4*d^3*
e*x^2 + d^4*x), x)
```

Sympy [A] (verification not implemented)

Time = 58.96 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.93

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^4}\,dx = -\frac{ae\left(\begin{cases}\frac{x}{d^4} & \text{for } e=0\\ -\frac{1}{3e(d+ex)^3} & \text{otherwise}\end{cases}\right)}{d} - \frac{ae\left(\begin{cases}\frac{x}{d^3} & \text{for } e=0\\ -\frac{1}{2e(d+ex)^2} & \text{otherwise}\end{cases}\right)}{d^2}$$

$$-\frac{ae\left(\begin{cases}\frac{x}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x} & \text{otherwise}\end{cases}\right)}{d^3} - \frac{ae\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{d^4} + \frac{a\log(x)}{d^4}$$

$$-\frac{be^3n\left(\begin{cases}-\frac{1}{e^4x} & \text{for } d=0\\ -\frac{3d}{6d^2e^3+12de^4x+6e^5x^2} - \frac{4ex}{6d^2e^3+12de^4x+6e^5x^2} - \frac{\log(d+ex)}{3de^3} & \text{otherwise}\end{cases}\right)}{d^3}$$

$$+\frac{be^3\left(\begin{cases}\frac{1}{e^4x} & \text{for } d=0\\ -\frac{1}{3d\left(\frac{d}{x}+e\right)^3} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^3}$$

$$+\frac{3be^2n\left(\begin{cases}-\frac{1}{e^3x} & \text{for } d=0\\ -\frac{1}{2de^2+2e^3x} - \frac{\log(d+ex)}{2de^2} & \text{otherwise}\end{cases}\right)}{d^3}$$

$$-\frac{3be^2\left(\begin{cases}\frac{1}{e^3x} & \text{for } d=0\\ -\frac{1}{2d\left(\frac{d}{x}+e\right)^2} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^3}$$

$$-\frac{3ben\left(\begin{cases}-\frac{1}{e^2x} & \text{for } d=0\\ -\frac{\log(d^2+dex)}{de} & \text{otherwise}\end{cases}\right)}{d^3} + \frac{3be\left(\begin{cases}\frac{1}{e^2x} & \text{for } d=0\\ -\frac{1}{\frac{d^2}{x}+de} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^3}$$

$$+\frac{bn\left(\begin{cases}-\frac{1}{ex} \\ \frac{\begin{cases}\operatorname{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(e)\log(x)+\operatorname{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{for } |x|<1\\ -\log(e)\log\left(\frac{1}{x}\right)+\operatorname{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(e)+\operatorname{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{otherwise}\end{cases}}{d}\end{cases}\right)}{d^3}$$

$$-\frac{b\left(\begin{cases}\frac{1}{ex} & \text{for } d=0\\ \frac{\log\left(\frac{d}{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^3}$$

```
[In] integrate((a+b*ln(c*x**n))/x/(e*x+d)**4,x)
```

```
[Out] -a*e*Piecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), True))/d - a*e*P
iecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))/d**2 - a*e*Piec
ewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))/d**3 - a*e*Piecewise((
x/d, Eq(e, 0)), (log(d + e*x)/e, True))/d**4 + a*log(x)/d**4 - b*e**3*n*Pie
cewise((-1/(e**4*x), Eq(d, 0)), (-3*d/(6*d**2*e**3 + 12*d*e**4*x + 6*e**5*x
**2) - 4*e*x/(6*d**2*e**3 + 12*d*e**4*x + 6*e**5*x**2) - log(d + e*x)/(3*d*
e**3), True))/d**3 + b*e**3*Piecewise((1/(e**4*x), Eq(d, 0)), (-1/(3*d*(d/x
 + e)**3), True))*log(c*x**n)/d**3 + 3*b*e**2*n*Piecewise((-1/(e**3*x), Eq(
d, 0)), (-1/(2*d*e**2 + 2*e**3*x) - log(d + e*x)/(2*d*e**2), True))/d**3 -
3*b*e**2*Piecewise((1/(e**3*x), Eq(d, 0)), (-1/(2*d*(d/x + e)**2), True))*l
og(c*x**n)/d**3 - 3*b*e*n*Piecewise((-1/(e**2*x), Eq(d, 0)), (-log(d**2 + d
*e*x)/(d*e), True))/d**3 + 3*b*e*Piecewise((1/(e**2*x), Eq(d, 0)), (-1/(d**
2/x + d*e), True))*log(c*x**n)/d**3 + b*n*Piecewise((-1/(e*x), Eq(d, 0)), (
Piecewise((polylog(2, d*exp_polar(I*pi)/(e*x)), (Abs(x) < 1) & (1/Abs(x) <
1)), (log(e)*log(x) + polylog(2, d*exp_polar(I*pi)/(e*x)), Abs(x) < 1), (-l
og(e)*log(1/x) + polylog(2, d*exp_polar(I*pi)/(e*x)), 1/Abs(x) < 1), (-meij
erg(((), (1, 1)), ((0, 0), ()), x)*log(e) + meijerg(((1, 1), ()), ((), (0,
0)), x)*log(e) + polylog(2, d*exp_polar(I*pi)/(e*x)), True))/d, True))/d**3
 - b*Piecewise((1/(e*x), Eq(d, 0)), (log(d/x + e)/d, True))*log(c*x**n)/d**
3
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^4}\,dx=\int \frac{b\log(cx^n)+a}{(ex+d)^4x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*a*((6*e^2*x^2 + 15*d*e*x + 11*d^2)/(d^3*e^3*x^3 + 3*d^4*e^2*x^2 + 3*d^5
*e*x + d^6) - 6*log(e*x + d)/d^4 + 6*log(x)/d^4) + b*integrate((log(c) + lo
g(x^n))/(e^4*x^5 + 4*d*e^3*x^4 + 6*d^2*e^2*x^3 + 4*d^3*e*x^2 + d^4*x), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^4}\,dx=\int \frac{b\log(cx^n)+a}{(ex+d)^4x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x + d)^4*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^4}\,dx=\int \frac{a+b\,\ln(c\,x^n)}{x\,(d+e\,x)^4}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x)^4),x)
[Out] int((a + b*log(c*x^n))/(x*(d + e*x)^4), x)
```

3.60 $\int \frac{a+b\log(cx^n)}{x^2(d+ex)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 211

$$\begin{aligned}\int \frac{a+b\log(cx^n)}{x^2(d+ex)^4}\,dx = &-\frac{bn}{d^4x}+\frac{ben}{6d^3(d+ex)^2}+\frac{4ben}{3d^4(d+ex)}+\frac{4ben\log(x)}{3d^5}\\&-\frac{a+b\log(cx^n)}{d^4x}-\frac{e(a+b\log(cx^n))}{3d^2(d+ex)^3}-\frac{e(a+b\log(cx^n))}{d^3(d+ex)^2}\\&+\frac{3e^2x(a+b\log(cx^n))}{d^5(d+ex)}+\frac{4e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^5}\\&-\frac{13ben\log(d+ex)}{3d^5}-\frac{4ben\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^5}\end{aligned}$$

```
[Out] -b*n/d^4/x+1/6*b*e*n/d^3/(e*x+d)^2+4/3*b*e*n/d^4/(e*x+d)+4/3*b*e*n*ln(x)/d^
5+(-a-b*ln(c*x^n))/d^4/x-1/3*e*(a+b*ln(c*x^n))/d^2/(e*x+d)^3-e*(a+b*ln(c*x^
n))/d^3/(e*x+d)^2+3*e^2*x*(a+b*ln(c*x^n))/d^5/(e*x+d)+4*e*ln(1+d/e/x)*(a+b*
ln(c*x^n))/d^5-13/3*b*e*n*ln(e*x+d)/d^5-4*b*e*n*polylog(2,-d/e/x)/d^5
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used

$= \{46, 2393, 2341, 2356, 2351, 31, 2379, 2438\}$

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^4}\,dx = \frac{3e^2x(a+b\log(cx^n))}{d^5(d+ex)} + \frac{4e\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^5} - \frac{a+b\log(cx^n)}{d^4x} - \frac{e(a+b\log(cx^n))}{d^3(d+ex)^2} - \frac{e(a+b\log(cx^n))}{3d^2(d+ex)^3} - \frac{4ben\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^5} + \frac{4ben\log(x)}{3d^5} - \frac{13ben\log(d+ex)}{3d^5} + \frac{4ben}{3d^4(d+ex)} - \frac{bn}{d^4x} + \frac{ben}{6d^3(d+ex)^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^2*(d + e*x)^4),x]
```

```
[Out] -((b*n)/(d^4*x)) + (b*e*n)/(6*d^3*(d + e*x)^2) + (4*b*e*n)/(3*d^4*(d + e*x)
) + (4*b*e*n*Log[x])/(3*d^5) - (a + b*Log[c*x^n])/(d^4*x) - (e*(a + b*Log[c
*x^n]))/(3*d^2*(d + e*x)^3) - (e*(a + b*Log[c*x^n]))/(d^3*(d + e*x)^2) + (3
*e^2*x*(a + b*Log[c*x^n]))/(d^5*(d + e*x)) + (4*e*Log[1 + d/(e*x)]*(a + b*L
og[c*x^n]))/d^5 - (13*b*e*n*Log[d + e*x])/(3*d^5) - (4*b*e*n*PolyLog[2, -(d
/(e*x))])/d^5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a+b\log(cx^n)}{d^4x^2}+\frac{e^2(a+b\log(cx^n))}{d^2(d+ex)^4}+\frac{2e^2(a+b\log(cx^n))}{d^3(d+ex)^3}\right. \\
&\qquad \left.+\frac{3e^2(a+b\log(cx^n))}{d^4(d+ex)^2}-\frac{4e(a+b\log(cx^n))}{d^4x(d+ex)}\right)dx \\
&= \frac{\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d^4}-\frac{(4e)\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d^4}+\frac{(3e^2)\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{d^4} \\
&\quad +\frac{(2e^2)\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx}{d^3}+\frac{e^2\int \frac{a+b\log(cx^n)}{(d+ex)^4}\,dx}{d^2} \\
&= -\frac{bn}{d^4x}-\frac{a+b\log(cx^n)}{d^4x}-\frac{e(a+b\log(cx^n))}{3d^2(d+ex)^3}-\frac{e(a+b\log(cx^n))}{d^3(d+ex)^2} \\
&\quad +\frac{3e^2x(a+b\log(cx^n))}{d^5(d+ex)}+\frac{4e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^5}-\frac{(4ben)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^5} \\
&\quad +\frac{(ben)\int \frac{1}{x(d+ex)^2}\,dx}{d^3}+\frac{(ben)\int \frac{1}{x(d+ex)^3}\,dx}{3d^2}-\frac{(3be^2n)\int \frac{1}{d+ex}\,dx}{d^5}
\end{aligned}
$$

$$= -\frac{bn}{d^4x} - \frac{a+b\log(cx^n)}{d^4x} - \frac{e(a+b\log(cx^n))}{3d^2(d+ex)^3} - \frac{e(a+b\log(cx^n))}{d^3(d+ex)^2} + \frac{3e^2x(a+b\log(cx^n))}{d^5(d+ex)} + \frac{4e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^5} - \frac{3ben\log(d+ex)}{d^5} - \frac{4ben\text{Li}_2\left(-\frac{d}{ex}\right)}{d^5} + \frac{(ben)\int\left(\frac{1}{d^2x} - \frac{e}{d(d+ex)^2} - \frac{e}{d^2(d+ex)}\right)dx}{d^3} + \frac{(ben)\int\left(\frac{1}{d^3x} - \frac{e}{d(d+ex)^3} - \frac{e}{d^2(d+ex)^2} - \frac{e}{d^3(d+ex)}\right)dx}{3d^2}$$

$$= -\frac{bn}{d^4x} + \frac{ben}{6d^3(d+ex)^2} + \frac{4ben}{3d^4(d+ex)} + \frac{4ben\log(x)}{3d^5} - \frac{a+b\log(cx^n)}{d^4x} - \frac{e(a+b\log(cx^n))}{3d^2(d+ex)^3} - \frac{e(a+b\log(cx^n))}{d^3(d+ex)^2} + \frac{3e^2x(a+b\log(cx^n))}{d^5(d+ex)} + \frac{4e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^5} - \frac{13ben\log(d+ex)}{3d^5} - \frac{4ben\text{Li}_2\left(-\frac{d}{ex}\right)}{d^5}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.09

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^4}\,dx = \frac{-\frac{6bdn}{x} - \frac{6d(a+b\log(cx^n))}{x} - \frac{2d^3e(a+b\log(cx^n))}{(d+ex)^3} - \frac{6d^2e(a+b\log(cx^n))}{(d+ex)^2} - \frac{18de(a+b\log(cx^n))}{d+ex} - \frac{12e(a+b\log(cx^n))^2}{bn} + ben\left(\frac{d(3d+}{(d+e}\right.}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*x)^4),x]
```

```
[Out] ((-6*b*d*n)/x - (6*d*(a + b*Log[c*x^n]))/x - (2*d^3*e*(a + b*Log[c*x^n]))/(
d + e*x)^3 - (6*d^2*e*(a + b*Log[c*x^n]))/(d + e*x)^2 - (18*d*e*(a + b*Log[
c*x^n]))/(d + e*x) - (12*e*(a + b*Log[c*x^n])^2)/(b*n) + b*e*n*((d*(3*d + 2
*e*x))/(d + e*x)^2 + 2*Log[x] - 2*Log[d + e*x]) + 18*b*e*n*(Log[x] - Log[d
+ e*x]) + 6*b*e*n*(d/(d + e*x) + Log[x] - Log[d + e*x]) + 24*e*(a + b*Log[c
*x^n])*Log[1 + (e*x)/d] + 24*b*e*n*PolyLog[2, -((e*x)/d)])/(6*d^5)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.75 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.75

method	result
risch	$-\frac{b\ln(x^n)e}{3d^2(ex+d)^3}+\frac{4b\ln(x^n)e\ln(ex+d)}{d^5}-\frac{3b\ln(x^n)e}{d^4(ex+d)}-\frac{b\ln(x^n)e}{d^3(ex+d)^2}-\frac{b\ln(x^n)}{d^4x}-\frac{4b\ln(x^n)e\ln(x)}{d^5}+\frac{2bne\ln(x)^2}{d^5}-\frac{4bne\ln(ex}{}$

```
[In] int((a+b*ln(c*x^n))/x^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b*ln(x^n)/d^2/(e*x+d)^3*e+4*b*ln(x^n)/d^5*e*ln(e*x+d)-3*b*ln(x^n)/d^4*
e/(e*x+d)-b*ln(x^n)/d^3/(e*x+d)^2*e-b*ln(x^n)/d^4/x-4*b*ln(x^n)/d^5*e*ln(x)
+2*b*n/d^5*e*ln(x)^2-4*b*n/d^5*e*ln(e*x+d)*ln(-e*x/d)-4*b*n/d^5*e*dilog(-e*
x/d)+4/3*b*e*n/d^4/(e*x+d)-13/3*b*e*n*ln(e*x+d)/d^5+1/6*b*e*n/d^3/(e*x+d)^2
-b*n/d^4/x+13/3*b*e*n*ln(x)/d^5+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c
*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*
x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-1/3/d^2/(e*x+d)^3*e+4/d^5*e*
ln(e*x+d)-3/d^4*e/(e*x+d)-1/d^3/(e*x+d)^2*e-1/d^4/x-4/d^5*e*ln(x))
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^4}\,dx=\int \frac{b\log(cx^n)+a}{(ex+d)^4x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e^4*x^6 + 4*d*e^3*x^5 + 6*d^2*e^2*x^4 + 4*d^3*
e*x^3 + d^4*x^2), x)
```

Sympy [A] (verification not implemented)

Time = 57.44 (sec) , antiderivative size = 614, normalized size of antiderivative = 2.91

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^4}\,dx=\text{Too large to display}$$

```
[In] integrate((a+b*ln(c*x**n))/x**2/(e*x+d)**4,x)
```

```
[Out] a*e**2*Piecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), True))/d**2 +
2*a*e**2*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))/d**3
+ 3*a*e**2*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))/d**4 -
a/(d**4*x) + 4*a*e**2*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/d*
*5 - 4*a*e*log(x)/d**5 - b*e**2*n*Piecewise((x/d**4, Eq(e, 0)), (-3*d/(6*d*
```

```
*4*e + 12*d**3*e**2*x + 6*d**2*e**3*x**2) - 2*e*x/(6*d**4*e + 12*d**3*e**2*
x + 6*d**2*e**3*x**2) - log(x)/(3*d**3*e) + log(d/e + x)/(3*d**3*e), True))
/d**2 + b*e**2*Piecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), True))
*log(c*x**n)/d**2 - 2*b*e**2*n*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*d**2*e
+ 2*d*e**2*x) - log(x)/(2*d**2*e) + log(d/e + x)/(2*d**2*e), True))/d**3 +
2*b*e**2*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))*log(c
*x**n)/d**3 - 3*b*e**2*n*Piecewise((x/d**2, Eq(e, 0)), (-log(x)/(d*e) + log
(d/e + x)/(d*e), True))/d**4 + 3*b*e**2*Piecewise((x/d**2, Eq(e, 0)), (-1/(
d*e + e**2*x), True))*log(c*x**n)/d**4 - b*n/(d**4*x) - b*log(c*x**n)/(d**4
*x) - 4*b*e**2*n*Piecewise((x/d, Eq(e, 0)), (Piecewise((-polylog(2, e*x*exp
_polar(I*pi)/d), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2
, e*x*exp_polar(I*pi)/d), Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x*e
xp_polar(I*pi)/d), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*
log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x*exp
_polar(I*pi)/d), True))/e, True))/d**5 + 4*b*e**2*Piecewise((x/d, Eq(e, 0))
, (log(d + e*x)/e, True))*log(c*x**n)/d**5 + 2*b*e*n*log(x)**2/d**5 - 4*b*e
*log(x)*log(c*x**n)/d**5
```

Maxima [F]

$$\int \frac{a + b\log(cx^n)}{x^2(d + ex)^4}\,dx = \int \frac{b\log(cx^n) + a}{(ex + d)^4x^2}\,dx$$

[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d)^4,x, algorithm="maxima")

```
[Out] -1/3*a*((12*e^3*x^3 + 30*d*e^2*x^2 + 22*d^2*e*x + 3*d^3)/(d^4*e^3*x^4 + 3*d
^5*e^2*x^3 + 3*d^6*e*x^2 + d^7*x) - 12*e*log(e*x + d)/d^5 + 12*e*log(x)/d^5
) + b*integrate((log(c) + log(x^n))/(e^4*x^6 + 4*d*e^3*x^5 + 6*d^2*e^2*x^4
+ 4*d^3*e*x^3 + d^4*x^2), x)
```

Giac [F]

$$\int \frac{a + b\log(cx^n)}{x^2(d + ex)^4}\,dx = \int \frac{b\log(cx^n) + a}{(ex + d)^4x^2}\,dx$$

[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d)^4,x, algorithm="giac")

[Out] integrate((b*log(c*x^n) + a)/((e*x + d)^4*x^2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^4}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^2\,(d+e\,x)^4}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*x)^4),x)
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*x)^4), x)
```

3.61 $\int \frac{a+b\log(cx^n)}{x^3(d+ex)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 263

$$\begin{aligned}\int \frac{a+b\log(cx^n)}{x^3(d+ex)^4}\,dx =& -\frac{bn}{4d^4x^2}+\frac{4ben}{d^5x}-\frac{be^2n}{6d^4(d+ex)^2}-\frac{11be^2n}{6d^5(d+ex)}\\&-\frac{11be^2n\log(x)}{6d^6}-\frac{a+b\log(cx^n)}{2d^4x^2}+\frac{4e(a+b\log(cx^n))}{d^5x}\\&+\frac{e^2(a+b\log(cx^n))}{3d^3(d+ex)^3}+\frac{3e^2(a+b\log(cx^n))}{2d^4(d+ex)^2}\\&-\frac{6e^3x(a+b\log(cx^n))}{d^6(d+ex)}-\frac{10e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^6}\\&+\frac{47be^2n\log(d+ex)}{6d^6}+\frac{10be^2n\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^6}\end{aligned}$$

```
[Out] -1/4*b*n/d^4/x^2+4*b*e*n/d^5/x-1/6*b*e^2*n/d^4/(e*x+d)^2-11/6*b*e^2*n/d^5/(
e*x+d)-11/6*b*e^2*n*ln(x)/d^6+1/2*(-a-b*ln(c*x^n))/d^4/x^2+4*e*(a+b*ln(c*x^
n))/d^5/x+1/3*e^2*(a+b*ln(c*x^n))/d^3/(e*x+d)^3+3/2*e^2*(a+b*ln(c*x^n))/d^4
/(e*x+d)^2-6*e^3*x*(a+b*ln(c*x^n))/d^6/(e*x+d)-10*e^2*ln(1+d/e/x)*(a+b*ln(c
*x^n))/d^6+47/6*b*e^2*n*ln(e*x+d)/d^6+10*b*e^2*n*polylog(2,-d/e/x)/d^6
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {46, 2393, 2341, 2356, 2351, 31, 2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^4}\,dx = -\frac{6e^3x(a+b\log(cx^n))}{d^6(d+ex)} - \frac{10e^2\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^6} + \frac{4e(a+b\log(cx^n))}{d^5x} + \frac{3e^2(a+b\log(cx^n))}{2d^4(d+ex)^2} - \frac{a+b\log(cx^n)}{2d^4x^2} + \frac{e^2(a+b\log(cx^n))}{3d^3(d+ex)^3} + \frac{10be^2n\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^6} - \frac{11be^2n\log(x)}{6d^6} + \frac{47be^2n\log(d+ex)}{6d^6} - \frac{11be^2n}{6d^5(d+ex)} + \frac{4ben}{d^5x} - \frac{be^2n}{6d^4(d+ex)^2} - \frac{bn}{4d^4x^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^3*(d + e*x)^4),x]
```

```
[Out] -1/4*(b*n)/(d^4*x^2) + (4*b*e*n)/(d^5*x) - (b*e^2*n)/(6*d^4*(d + e*x)^2) -
(11*b*e^2*n)/(6*d^5*(d + e*x)) - (11*b*e^2*n*Log[x])/(6*d^6) - (a + b*Log[c
*x^n])/(2*d^4*x^2) + (4*e*(a + b*Log[c*x^n]))/(d^5*x) + (e^2*(a + b*Log[c*x
^n]))/(3*d^3*(d + e*x)^3) + (3*e^2*(a + b*Log[c*x^n]))/(2*d^4*(d + e*x)^2)
- (6*e^3*x*(a + b*Log[c*x^n]))/(d^6*(d + e*x)) - (10*e^2*Log[1 + d/(e*x)]*(
a + b*Log[c*x^n]))/d^6 + (47*b*e^2*n*Log[d + e*x])/(6*d^6) + (10*b*e^2*n*Po
lyLog[2, -(d/(e*x))])/d^6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
```

```
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log(cx^n)}{d^4x^3} - \frac{4e(a+b\log(cx^n))}{d^5x^2} - \frac{e^3(a+b\log(cx^n))}{d^3(d+ex)^4} - \frac{3e^3(a+b\log(cx^n))}{d^4(d+ex)^3} - \frac{6e^3(a+b\log(cx^n))}{d^5(d+ex)^2} + \frac{10e^2(a+b\log(cx^n))}{d^5x(d+ex)}\right) dx$$

$$= \frac{\int \frac{a+b\log(cx^n)}{x^3}\,dx}{d^4} - \frac{(4e)\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d^5} + \frac{(10e^2)\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d^5} - \frac{(6e^3)\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{d^5} - \frac{(3e^3)\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx}{d^4} - \frac{e^3\int \frac{a+b\log(cx^n)}{(d+ex)^4}\,dx}{d^3}$$

$$= -\frac{bn}{4d^4x^2} + \frac{4ben}{d^5x} - \frac{a+b\log(cx^n)}{2d^4x^2} + \frac{4e(a+b\log(cx^n))}{d^5x} + \frac{e^2(a+b\log(cx^n))}{3d^3(d+ex)^3} + \frac{3e^2(a+b\log(cx^n))}{2d^4(d+ex)^2} - \frac{6e^3x(a+b\log(cx^n))}{d^6(d+ex)} - \frac{10e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^6} + \frac{(10be^2n)\int\frac{\log\left(1+\frac{d}{ex}\right)}{x}dx}{d^6} - \frac{(3be^2n)\int\frac{1}{x(d+ex)^2}dx}{2d^4} - \frac{(be^2n)\int\frac{1}{x(d+ex)^3}dx}{3d^3} + \frac{(6be^3n)\int\frac{1}{d+ex}dx}{d^6}$$

$$= -\frac{bn}{4d^4x^2} + \frac{4ben}{d^5x} - \frac{a+b\log(cx^n)}{2d^4x^2} + \frac{4e(a+b\log(cx^n))}{d^5x} + \frac{e^2(a+b\log(cx^n))}{3d^3(d+ex)^3} + \frac{3e^2(a+b\log(cx^n))}{2d^4(d+ex)^2} - \frac{6e^3x(a+b\log(cx^n))}{d^6(d+ex)} - \frac{10e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^6} + \frac{6be^2n\log(d+ex)}{d^6} + \frac{10be^2n\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^6} - \frac{(3be^2n)\int\left(\frac{1}{d^2x}-\frac{e}{d(d+ex)^2}-\frac{e}{d^2(d+ex)}\right)dx}{2d^4} - \frac{(be^2n)\int\left(\frac{1}{d^3x}-\frac{e}{d(d+ex)^3}-\frac{e}{d^2(d+ex)^2}-\frac{e}{d^3(d+ex)}\right)dx}{3d^3}$$

$$= -\frac{bn}{4d^4x^2} + \frac{4ben}{d^5x} - \frac{be^2n}{6d^4(d+ex)^2} - \frac{11be^2n}{6d^5(d+ex)} - \frac{11be^2n\log(x)}{6d^6} - \frac{a+b\log(cx^n)}{2d^4x^2} + \frac{4e(a+b\log(cx^n))}{d^5x} + \frac{e^2(a+b\log(cx^n))}{3d^3(d+ex)^3} + \frac{3e^2(a+b\log(cx^n))}{2d^4(d+ex)^2} - \frac{6e^3x(a+b\log(cx^n))}{d^6(d+ex)} - \frac{10e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^6} + \frac{47be^2n\log(d+ex)}{6d^6} + \frac{10be^2n\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^6}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.05

$$\int\frac{a+b\log(cx^n)}{x^3(d+ex)^4}\,dx$$

$$= \frac{-\frac{3bd^2n}{x^2} + \frac{48bden}{x} - \frac{18bde^2n}{d+ex} - \frac{2bde^2n(3d+2ex)}{(d+ex)^2} - 22be^2n\log(x) - \frac{6d^2(a+b\log(cx^n))}{x^2} + \frac{48de(a+b\log(cx^n))}{x} + \frac{4d^3e^2(a+b\log(cx^n))}{(d+ex)^3}}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*(d + e*x)^4),x]
```

```
[Out] ((-3*b*d^2*n)/x^2 + (48*b*d*e*n)/x - (18*b*d*e^2*n)/(d + e*x) - (2*b*d*e^2*
n*(3*d + 2*e*x))/(d + e*x)^2 - 22*b*e^2*n*Log[x] - (6*d^2*(a + b*Log[c*x^n]
))/x^2 + (48*d*e*(a + b*Log[c*x^n]))/x + (4*d^3*e^2*(a + b*Log[c*x^n]))/(d
+ e*x)^3 + (18*d^2*e^2*(a + b*Log[c*x^n]))/(d + e*x)^2 + (72*d*e^2*(a + b*L
```

```
og[c*x^n]))/(d + e*x) + (60*e^2*(a + b*Log[c*x^n])^2)/(b*n) - 72*b*e^2*n*(L
og[x] - Log[d + e*x]) + 22*b*e^2*n*Log[d + e*x] - 120*e^2*(a + b*Log[c*x^n]
)*Log[1 + (e*x)/d] - 120*b*e^2*n*PolyLog[2, -((e*x)/d)])/(12*d^6)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.92 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{10b\ln(x^n)e^2\ln(ex+d)}{d^6}+\frac{6b\ln(x^n)e^2}{d^5(ex+d)}+\frac{3b\ln(x^n)e^2}{2d^4(ex+d)^2}+\frac{b\ln(x^n)e^2}{3d^3(ex+d)^3}-\frac{b\ln(x^n)}{2d^4x^2}+\frac{10b\ln(x^n)e^2\ln(x)}{d^6}+\frac{4b\ln(x^n)e}{d^5x}-\frac{1}{6d}$

```
[In] int((a+b*ln(c*x^n))/x^3/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -10*b*ln(x^n)/d^6*e^2*ln(e*x+d)+6*b*ln(x^n)/d^5*e^2/(e*x+d)+3/2*b*ln(x^n)/d
^4*e^2/(e*x+d)^2+1/3*b*ln(x^n)/d^3/(e*x+d)^3*e^2-1/2*b*ln(x^n)/d^4/x^2+10*b
*ln(x^n)/d^6*e^2*ln(x)+4*b*ln(x^n)/d^5*e/x-11/6*b*e^2*n/d^5/(e*x+d)+47/6*b*
e^2*n*ln(e*x+d)/d^6-1/6*b*e^2*n/d^4/(e*x+d)^2-1/4*b*n/d^4/x^2+4*b*e*n/d^5/x
-47/6*b*e^2*n*ln(x)/d^6-5*b*n/d^6*e^2*ln(x)^2+10*b*n/d^6*e^2*ln(e*x+d)*ln(-
e*x/d)+10*b*n/d^6*e^2*dilog(-e*x/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn
(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(
I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-10/d^6*e^2*ln(e*x+d)+6/d
^5*e^2/(e*x+d)+3/2/d^4*e^2/(e*x+d)^2+1/3/d^3/(e*x+d)^3*e^2-1/2/d^4/x^2+10/d
^6*e^2*ln(x)+4/d^5*e/x)
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^4}\,dx=\int \frac{b\log(cx^n)+a}{(ex+d)^4x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e^4*x^7 + 4*d*e^3*x^6 + 6*d^2*e^2*x^5 + 4*d^3*
e*x^4 + d^4*x^3), x)
```

Sympy [A] (verification not implemented)

Time = 61.94 (sec) , antiderivative size = 668, normalized size of antiderivative = 2.54

$$\int \frac{a + b \log (cx^n)}{x^3(d + ex)^4}\, dx = \text{Too large to display}$$

```
[In] integrate((a+b*ln(c*x**n))/x**3/(e*x+d)**4,x)
```

```
[Out] -a*e**3*Piecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), True))/d**3 -
 3*a*e**3*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))/d**4
 - a/(2*d**4*x**2) - 6*a*e**3*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2
*x), True))/d**5 + 4*a*e/(d**5*x) - 10*a*e**3*Piecewise((x/d, Eq(e, 0)), (l
og(d + e*x)/e, True))/d**6 + 10*a*e**2*log(x)/d**6 + b*e**3*n*Piecewise((x/
d**4, Eq(e, 0)), (-3*d/(6*d**4*e + 12*d**3*e**2*x + 6*d**2*e**3*x**2) - 2*e
*x/(6*d**4*e + 12*d**3*e**2*x + 6*d**2*e**3*x**2) - log(x)/(3*d**3*e) + log
(d/e + x)/(3*d**3*e), True))/d**3 - b*e**3*Piecewise((x/d**4, Eq(e, 0)), (-
1/(3*e*(d + e*x)**3), True))*log(c*x**n)/d**3 + 3*b*e**3*n*Piecewise((x/d**
3, Eq(e, 0)), (-1/(2*d**2*e + 2*d*e**2*x) - log(x)/(2*d**2*e) + log(d/e + x
)/(2*d**2*e), True))/d**4 - 3*b*e**3*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e
*(d + e*x)**2), True))*log(c*x**n)/d**4 - b*n/(4*d**4*x**2) - b*log(c*x**n)
/(2*d**4*x**2) + 6*b*e**3*n*Piecewise((x/d**2, Eq(e, 0)), (-log(x)/(d*e) +
log(d/e + x)/(d*e), True))/d**5 - 6*b*e**3*Piecewise((x/d**2, Eq(e, 0)), (-
1/(d*e + e**2*x), True))*log(c*x**n)/d**5 + 4*b*e*n/(d**5*x) + 4*b*e*log(c*
x**n)/(d**5*x) + 10*b*e**3*n*Piecewise((x/d, Eq(e, 0)), (Piecewise((-polylo
g(2, e*x*exp_polar(I*pi)/d), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x)
 - polylog(2, e*x*exp_polar(I*pi)/d), Abs(x) < 1), (-log(d)*log(1/x) - poly
log(2, e*x*exp_polar(I*pi)/d), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0,
0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylo
g(2, e*x*exp_polar(I*pi)/d), True))/e, True))/d**6 - 10*b*e**3*Piecewise((x
/d, Eq(e, 0)), (log(d + e*x)/e, True))*log(c*x**n)/d**6 - 5*b*e**2*n*log(x)
**2/d**6 + 10*b*e**2*log(x)*log(c*x**n)/d**6
```

Maxima [F]

$$\int \frac{a + b \log (cx^n)}{x^3(d + ex)^4}\, dx = \int \frac{b \log (cx^n) + a}{(ex + d)^4 x^3}\, dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*a*((60*e^4*x^4 + 150*d*e^3*x^3 + 110*d^2*e^2*x^2 + 15*d^3*e*x - 3*d^4)/
(d^5*e^3*x^5 + 3*d^6*e^2*x^4 + 3*d^7*e*x^3 + d^8*x^2) - 60*e^2*log(e*x + d)
/d^6 + 60*e^2*log(x)/d^6) + b*integrate((log(c) + log(x^n))/(e^4*x^7 + 4*d*
e^3*x^6 + 6*d^2*e^2*x^5 + 4*d^3*e*x^4 + d^4*x^3), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^4}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)^4x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x + d)^4*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^4}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^3\,(d+e\,x)^4}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e*x)^4),x)
[Out] int((a + b*log(c*x^n))/(x^3*(d + e*x)^4), x)
```

3.62 $\int \frac{x^8(a+b\log(cx^n))}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 329

$$\begin{aligned}\int \frac{x^8(a+b\log(cx^n))}{(d+ex)^7}\,dx =& \frac{28bdnx}{e^8}-\frac{d(280a+341bn)x}{10e^8}-\frac{7bnx^2}{e^7}\\ &-\frac{28bdx\log(cx^n)}{e^8}-\frac{x^8(a+b\log(cx^n))}{6e(d+ex)^6}\\ &-\frac{x^7(8a+bn+8b\log(cx^n))}{30e^2(d+ex)^5}-\frac{x^6(56a+15bn+56b\log(cx^n))}{120e^3(d+ex)^4}\\ &-\frac{x^5(168a+73bn+168b\log(cx^n))}{180e^4(d+ex)^3}\\ &+\frac{x^2(280a+341bn+280b\log(cx^n))}{20e^7}\\ &-\frac{x^4(840a+533bn+840b\log(cx^n))}{360e^5(d+ex)^2}\\ &-\frac{x^3(840a+743bn+840b\log(cx^n))}{90e^6(d+ex)}\\ &+\frac{d^2(280a+341bn+280b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{10e^9}\\ &+\frac{28bd^2n\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^9}\end{aligned}$$

```
[Out] 28*b*d*n*x/e^8-1/10*d*(341*b*n+280*a)*x/e^8-7*b*n*x^2/e^7-28*b*d*x*ln(c*x^n
)/e^8-1/6*x^8*(a+b*ln(c*x^n))/e/(e*x+d)^6-1/30*x^7*(8*a+b*n+8*b*ln(c*x^n))/
e^2/(e*x+d)^5-1/120*x^6*(56*a+15*b*n+56*b*ln(c*x^n))/e^3/(e*x+d)^4-1/180*x^
5*(168*a+73*b*n+168*b*ln(c*x^n))/e^4/(e*x+d)^3+1/20*x^2*(280*a+341*b*n+280*
b*ln(c*x^n))/e^7-1/360*x^4*(840*a+533*b*n+840*b*ln(c*x^n))/e^5/(e*x+d)^2-1/
90*x^3*(840*a+743*b*n+840*b*ln(c*x^n))/e^6/(e*x+d)+1/10*d^2*(280*a+341*b*n+
280*b*ln(c*x^n))*ln(1+e*x/d)/e^9+28*b*d^2*n*polylog(2,-e*x/d)/e^9
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2384, 45, 2393, 2332, 2341, 2354, 2438}

$$\int \frac{x^8(a+b\log(cx^n))}{(d+ex)^7}\,dx = \frac{d^2\log\left(\frac{ex}{d}+1\right)(280a+280b\log(cx^n)+341bn)}{10e^9} - \frac{x^3(840a+840b\log(cx^n)+743bn)}{90e^6(d+ex)} - \frac{x^4(840a+840b\log(cx^n)+533bn)}{360e^5(d+ex)^2} - \frac{x^5(168a+168b\log(cx^n)+73bn)}{180e^4(d+ex)^3} - \frac{x^6(56a+56b\log(cx^n)+15bn)}{120e^3(d+ex)^4} - \frac{x^7(8a+8b\log(cx^n)+bn)}{30e^2(d+ex)^5} - \frac{x^8(a+b\log(cx^n))}{6e(d+ex)^6} + \frac{x^2(280a+280b\log(cx^n)+341bn)}{20e^7} - \frac{dx(280a+341bn)}{10e^8} - \frac{28bdx\log(cx^n)}{e^8} + \frac{28bd^2n\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^9} + \frac{28bdnx}{e^8} - \frac{7bnx^2}{e^7}$$

```
[In] Int[(x^8*(a + b*Log[c*x^n]))/(d + e*x)^7,x]
```

```
[Out] (28*b*d*n*x)/e^8 - (d*(280*a + 341*b*n)*x)/(10*e^8) - (7*b*n*x^2)/e^7 - (28
*b*d*x*Log[c*x^n])/e^8 - (x^8*(a + b*Log[c*x^n]))/(6*e*(d + e*x)^6) - (x^7*
(8*a + b*n + 8*b*Log[c*x^n]))/(30*e^2*(d + e*x)^5) - (x^6*(56*a + 15*b*n +
56*b*Log[c*x^n]))/(120*e^3*(d + e*x)^4) - (x^5*(168*a + 73*b*n + 168*b*Log[
c*x^n]))/(180*e^4*(d + e*x)^3) + (x^2*(280*a + 341*b*n + 280*b*Log[c*x^n]))
/(20*e^7) - (x^4*(840*a + 533*b*n + 840*b*Log[c*x^n]))/(360*e^5*(d + e*x)^2
) - (x^3*(840*a + 743*b*n + 840*b*Log[c*x^n]))/(90*e^6*(d + e*x)) + (d^2*(2
80*a + 341*b*n + 280*b*Log[c*x^n])*Log[1 + (e*x)/d])/(10*e^9) + (28*b*d^2*n
*PolyLog[2, -((e*x)/d)])/e^9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^8(a+b\log(cx^n))}{6e(d+ex)^6}+\frac{\int\frac{x^7(8a+bn+8b\log(cx^n))}{(d+ex)^6}\,dx}{6e}\\
&= -\frac{x^8(a+b\log(cx^n))}{6e(d+ex)^6}-\frac{x^7(8a+bn+8b\log(cx^n))}{30e^2(d+ex)^5}+\frac{\int\frac{x^6(8bn+7(8a+bn)+56b\log(cx^n))}{(d+ex)^5}\,dx}{30e^2}\\
&= -\frac{x^8(a+b\log(cx^n))}{6e(d+ex)^6}-\frac{x^7(8a+bn+8b\log(cx^n))}{30e^2(d+ex)^5}\\
&\quad -\frac{x^6(56a+15bn+56b\log(cx^n))}{120e^3(d+ex)^4}+\frac{\int\frac{x^5(56bn+6(8bn+7(8a+bn))+336b\log(cx^n))}{(d+ex)^4}\,dx}{120e^3}
\end{aligned}
$$

$$=-\frac{x^8(a+b\log(cx^n))}{6e(d+ex)^6}-\frac{x^7(8a+bn+8b\log(cx^n))}{30e^2(d+ex)^5}-\frac{x^6(56a+15bn+56b\log(cx^n))}{120e^3(d+ex)^4}-\frac{x^5(168a+73bn+168b\log(cx^n))}{180e^4(d+ex)^3}+\frac{\int \frac{x^4(336bn+5(56bn+6(8bn+7(8a+bn)))+1680b\log(cx^n))}{(d+ex)^3}\,dx}{360e^4}$$

$$=-\frac{x^8(a+b\log(cx^n))}{6e(d+ex)^6}-\frac{x^7(8a+bn+8b\log(cx^n))}{30e^2(d+ex)^5}-\frac{x^6(56a+15bn+56b\log(cx^n))}{120e^3(d+ex)^4}-\frac{x^5(168a+73bn+168b\log(cx^n))}{180e^4(d+ex)^3}-\frac{x^4(840a+533bn+840b\log(cx^n))}{360e^5(d+ex)^2}+\frac{\int \frac{x^3(1680bn+4(336bn+5(56bn+6(8bn+7(8a+bn))))+6720b\log(cx^n))}{(d+ex)^2}\,dx}{720e^5}$$

$$=-\frac{x^8(a+b\log(cx^n))}{6e(d+ex)^6}-\frac{x^7(8a+bn+8b\log(cx^n))}{30e^2(d+ex)^5}-\frac{x^6(56a+15bn+56b\log(cx^n))}{120e^3(d+ex)^4}-\frac{x^5(168a+73bn+168b\log(cx^n))}{180e^4(d+ex)^3}-\frac{x^4(840a+533bn+840b\log(cx^n))}{360e^5(d+ex)^2}-\frac{x^3(840a+743bn+840b\log(cx^n))}{90e^6(d+ex)}+\frac{\int \frac{x^2(6720bn+3(1680bn+4(336bn+5(56bn+6(8bn+7(8a+bn)))))+20160b\log(cx^n))}{d+ex}\,dx}{720e^6}$$

$$=-\frac{x^8(a+b\log(cx^n))}{6e(d+ex)^6}-\frac{x^7(8a+bn+8b\log(cx^n))}{30e^2(d+ex)^5}-\frac{x^6(56a+15bn+56b\log(cx^n))}{120e^3(d+ex)^4}-\frac{x^5(168a+73bn+168b\log(cx^n))}{180e^4(d+ex)^3}-\frac{x^4(840a+533bn+840b\log(cx^n))}{360e^5(d+ex)^2}-\frac{x^3(840a+743bn+840b\log(cx^n))}{90e^6(d+ex)}+\frac{\int \left(-\frac{d(6720bn+3(1680bn+4(336bn+5(56bn+6(8bn+7(8a+bn)))))+20160b\log(cx^n))}{e^2}+\frac{x(6720bn+3(1680bn+4(336bn+5(56bn}{}\right.}{7}$$

$$=-\frac{x^8(a+b\log(cx^n))}{6e(d+ex)^6}-\frac{x^7(8a+bn+8b\log(cx^n))}{30e^2(d+ex)^5}-\frac{x^6(56a+15bn+56b\log(cx^n))}{120e^3(d+ex)^4}-\frac{x^5(168a+73bn+168b\log(cx^n))}{180e^4(d+ex)^3}-\frac{x^4(840a+533bn+840b\log(cx^n))}{360e^5(d+ex)^2}-\frac{x^3(840a+743bn+840b\log(cx^n))}{90e^6(d+ex)}-\frac{d\int (6720bn+3(1680bn+4(336bn+5(56bn+6(8bn+7(8a+bn)))))+20160b\log(cx^n))\,dx}{720e^8}+\frac{d^2\int \frac{6720bn+3(1680bn+4(336bn+5(56bn+6(8bn+7(8a+bn)))))+20160b\log(cx^n)}{d+ex}\,dx}{720e^8}+\frac{\int x(6720bn+3(1680bn+4(336bn+5(56bn+6(8bn+7(8a+bn)))))+20160b\log(cx^n))\,dx}{720e^7}$$

$$= -\frac{d(280a+341bn)x}{10e^8} - \frac{7bnx^2}{e^7} - \frac{x^8(a+b\log(cx^n))}{6e(d+ex)^6} - \frac{x^7(8a+bn+8b\log(cx^n))}{30e^2(d+ex)^5} - \frac{x^6(56a+15bn+56b\log(cx^n))}{120e^3(d+ex)^4} - \frac{x^5(168a+73bn+168b\log(cx^n))}{180e^4(d+ex)^3} + \frac{x^2(280a+341bn+280b\log(cx^n))}{20e^7} - \frac{x^4(840a+533bn+840b\log(cx^n))}{360e^5(d+ex)^2} - \frac{x^3(840a+743bn+840b\log(cx^n))}{90e^6(d+ex)} + \frac{d^2(280a+341bn+280b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{10e^9} - \frac{(28bd)\int\log(cx^n)\,dx}{e^8} - \frac{(28bd^2n)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^9}$$

$$= \frac{28bdnx}{e^8} - \frac{d(280a+341bn)x}{10e^8} - \frac{7bnx^2}{e^7} - \frac{28bdx\log(cx^n)}{e^8} - \frac{x^8(a+b\log(cx^n))}{6e(d+ex)^6} - \frac{x^7(8a+bn+8b\log(cx^n))}{30e^2(d+ex)^5} - \frac{x^6(56a+15bn+56b\log(cx^n))}{120e^3(d+ex)^4} - \frac{x^5(168a+73bn+168b\log(cx^n))}{180e^4(d+ex)^3} + \frac{x^2(280a+341bn+280b\log(cx^n))}{20e^7} - \frac{x^4(840a+533bn+840b\log(cx^n))}{360e^5(d+ex)^2} - \frac{x^3(840a+743bn+840b\log(cx^n))}{90e^6(d+ex)} + \frac{d^2(280a+341bn+280b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{10e^9} + \frac{28bd^2n\text{Li}_2\left(-\frac{ex}{d}\right)}{e^9}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.22

$$\int\frac{x^8(a+b\log(cx^n))}{(d+ex)^7}\,dx = \frac{-2520adex+2520bdenx+180ae^2x^2-90be^2nx^2-\frac{60ad^8}{(d+ex)^6}+\frac{576ad^7}{(d+ex)^5}+\frac{12bd^7n}{(d+ex)^5}-\frac{2520ad^6}{(d+ex)^4}-\frac{129bd^6n}{(d+ex)^4}+\frac{6720ad^5}{(d+ex)^3}}{}$$

```
[In] Integrate[(x^8*(a + b*Log[c*x^n]))/(d + e*x)^7,x]
```

```
[Out] (-2520*a*d*e*x + 2520*b*d*e*n*x + 180*a*e^2*x^2 - 90*b*e^2*n*x^2 - (60*a*d^
8)/(d + e*x)^6 + (576*a*d^7)/(d + e*x)^5 + (12*b*d^7*n)/(d + e*x)^5 - (2520
*a*d^6)/(d + e*x)^4 - (129*b*d^6*n)/(d + e*x)^4 + (6720*a*d^5)/(d + e*x)^3
+ (668*b*d^5*n)/(d + e*x)^3 - (12600*a*d^4)/(d + e*x)^2 - (2358*b*d^4*n)/(d
 + e*x)^2 + (20160*a*d^3)/(d + e*x) + (7884*b*d^3*n)/(d + e*x) - 12276*b*d^
2*n*Log[x] - 2520*b*d*e*x*Log[c*x^n] + 180*b*e^2*x^2*Log[c*x^n] - (60*b*d^8
*Log[c*x^n])/(d + e*x)^6 + (576*b*d^7*Log[c*x^n])/(d + e*x)^5 - (2520*b*d^6
*Log[c*x^n])/(d + e*x)^4 + (6720*b*d^5*Log[c*x^n])/(d + e*x)^3 - (12600*b*d
```

```
^4*Log[c*x^n])/(d + e*x)^2 + (20160*b*d^3*Log[c*x^n])/(d + e*x) + 12276*b*d
^2*n*Log[d + e*x] + 10080*a*d^2*Log[1 + (e*x)/d] + 10080*b*d^2*Log[c*x^n]*L
og[1 + (e*x)/d] + 10080*b*d^2*n*PolyLog[2, -((e*x)/d)])/(360*e^9)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.67 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.71

method	result
risch	$\frac{b\ln(x^n)x^2}{2e^7} - \frac{7b\ln(x^n)dx}{e^8} + \frac{8b\ln(x^n)d^7}{5e^9(ex+d)^5} - \frac{b\ln(x^n)d^8}{6e^9(ex+d)^6} + \frac{56b\ln(x^n)d^5}{3e^9(ex+d)^3} + \frac{28b\ln(x^n)d^2\ln(ex+d)}{e^9} + \frac{56b\ln(x^n)d^3}{e^9(ex+d)} - \frac{35b\ln}{e^9(e}$

```
[In] int(x^8*(a+b*ln(c*x^n))/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*ln(x^n)/e^7*x^2-7*b*ln(x^n)/e^8*d*x+8/5*b*ln(x^n)/e^9*d^7/(e*x+d)^5-1
/6*b*ln(x^n)*d^8/e^9/(e*x+d)^6+56/3*b*ln(x^n)/e^9*d^5/(e*x+d)^3+28*b*ln(x^n
)/e^9*d^2*ln(e*x+d)+56*b*ln(x^n)/e^9*d^3/(e*x+d)-35*b*ln(x^n)/e^9*d^4/(e*x+
d)^2-7*b*ln(x^n)/e^9*d^6/(e*x+d)^4-1/4*b*n*x^2/e^7+7*b*d*n*x/e^8+29/4*b*n/e
^9*d^2+341/10*b*n/e^9*d^2*ln(e*x+d)+219/10*b*n/e^9*d^3/(e*x+d)-131/20*b*n/e
^9*d^4/(e*x+d)^2+167/90*b*n/e^9*d^5/(e*x+d)^3-43/120*b*n/e^9*d^6/(e*x+d)^4+
1/30*b*n/e^9*d^7/(e*x+d)^5-341/10*b*n/e^9*d^2*ln(e*x)-28*b*n/e^9*d^2*ln(e*x
+d)*ln(-e*x/d)-28*b*n/e^9*d^2*dilog(-e*x/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x
^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^
n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/e^8*(1/2*e*x^2-
7*d*x)+8/5/e^9*d^7/(e*x+d)^5-1/6*d^8/e^9/(e*x+d)^6+56/3/e^9*d^5/(e*x+d)^3+2
8/e^9*d^2*ln(e*x+d)+56/e^9*d^3/(e*x+d)-35/e^9*d^4/(e*x+d)^2-7/e^9*d^6/(e*x+
d)^4)
```

Fricas [F]

$$\int \frac{x^8(a+b\log{(cx^n)})}{(d+ex)^7}\,dx = \int \frac{(b\log{(cx^n)}+a)x^8}{(ex+d)^7}\,dx$$

```
[In] integrate(x^8*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] integral((b*x^8*log(c*x^n) + a*x^8)/(e^7*x^7 + 7*d*e^6*x^6 + 21*d^2*e^5*x^5
 + 35*d^3*e^4*x^4 + 35*d^4*e^3*x^3 + 21*d^5*e^2*x^2 + 7*d^6*e*x + d^7), x)
```

Sympy [A] (verification not implemented)

Time = 149.76 (sec) , antiderivative size = 1686, normalized size of antiderivative = 5.12

$$\int \frac{x^8(a+b\log(cx^n))}{(d+ex)^7}\,dx = \text{Too large to display}$$

```
[In] integrate(x**8*(a+b*ln(c*x**n))/(e*x+d)**7,x)
```

```
[Out] a*d**8*Piecewise((x/d**7, Eq(e, 0)), (-1/(6*e*(d + e*x)**6), True))/e**8 -
8*a*d**7*Piecewise((x/d**6, Eq(e, 0)), (-1/(5*e*(d + e*x)**5), True))/e**8
+ 28*a*d**6*Piecewise((x/d**5, Eq(e, 0)), (-1/(4*e*(d + e*x)**4), True))/e*
*8 - 56*a*d**5*Piecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), True))
/e**8 + 70*a*d**4*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), Tru
e))/e**8 - 56*a*d**3*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True
))/e**8 + 28*a*d**2*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e**8
 - 7*a*d*x/e**8 + a*x**2/(2*e**7) - b*d**8*n*Piecewise((x/d**7, Eq(e, 0)),
(-137*d**4/(360*d**10*e + 1800*d**9*e**2*x + 3600*d**8*e**3*x**2 + 3600*d**
7*e**4*x**3 + 1800*d**6*e**5*x**4 + 360*d**5*e**6*x**5) - 385*d**3*e*x/(360
*d**10*e + 1800*d**9*e**2*x + 3600*d**8*e**3*x**2 + 3600*d**7*e**4*x**3 + 1
800*d**6*e**5*x**4 + 360*d**5*e**6*x**5) - 470*d**2*e**2*x**2/(360*d**10*e
+ 1800*d**9*e**2*x + 3600*d**8*e**3*x**2 + 3600*d**7*e**4*x**3 + 1800*d**6*
e**5*x**4 + 360*d**5*e**6*x**5) - 270*d*e**3*x**3/(360*d**10*e + 1800*d**9*
e**2*x + 3600*d**8*e**3*x**2 + 3600*d**7*e**4*x**3 + 1800*d**6*e**5*x**4 +
360*d**5*e**6*x**5) - 60*e**4*x**4/(360*d**10*e + 1800*d**9*e**2*x + 3600*d
**8*e**3*x**2 + 3600*d**7*e**4*x**3 + 1800*d**6*e**5*x**4 + 360*d**5*e**6*x
**5) - log(x)/(6*d**6*e) + log(d/e + x)/(6*d**6*e), True))/e**8 + b*d**8*Pi
ecewise((x/d**7, Eq(e, 0)), (-1/(6*e*(d + e*x)**6), True))*log(c*x**n)/e**8
 + 8*b*d**7*n*Piecewise((x/d**6, Eq(e, 0)), (-25*d**3/(60*d**8*e + 240*d**7
*e**2*x + 360*d**6*e**3*x**2 + 240*d**5*e**4*x**3 + 60*d**4*e**5*x**4) - 52
*d**2*e*x/(60*d**8*e + 240*d**7*e**2*x + 360*d**6*e**3*x**2 + 240*d**5*e**4
*x**3 + 60*d**4*e**5*x**4) - 42*d*e**2*x**2/(60*d**8*e + 240*d**7*e**2*x +
360*d**6*e**3*x**2 + 240*d**5*e**4*x**3 + 60*d**4*e**5*x**4) - 12*e**3*x**3
/(60*d**8*e + 240*d**7*e**2*x + 360*d**6*e**3*x**2 + 240*d**5*e**4*x**3 + 6
0*d**4*e**5*x**4) - log(x)/(5*d**5*e) + log(d/e + x)/(5*d**5*e), True))/e**
8 - 8*b*d**7*Piecewise((x/d**6, Eq(e, 0)), (-1/(5*e*(d + e*x)**5), True))*l
og(c*x**n)/e**8 - 28*b*d**6*n*Piecewise((x/d**5, Eq(e, 0)), (-11*d**2/(24*d
**6*e + 72*d**5*e**2*x + 72*d**4*e**3*x**2 + 24*d**3*e**4*x**3) - 15*d*e*x/
(24*d**6*e + 72*d**5*e**2*x + 72*d**4*e**3*x**2 + 24*d**3*e**4*x**3) - 6*e*
*2*x**2/(24*d**6*e + 72*d**5*e**2*x + 72*d**4*e**3*x**2 + 24*d**3*e**4*x**3
) - log(x)/(4*d**4*e) + log(d/e + x)/(4*d**4*e), True))/e**8 + 28*b*d**6*Pi
ecewise((x/d**5, Eq(e, 0)), (-1/(4*e*(d + e*x)**4), True))*log(c*x**n)/e**8
 + 56*b*d**5*n*Piecewise((x/d**4, Eq(e, 0)), (-3*d/(6*d**4*e + 12*d**3*e**2
*x + 6*d**2*e**3*x**2) - 2*e*x/(6*d**4*e + 12*d**3*e**2*x + 6*d**2*e**3*x**
2) - log(x)/(3*d**3*e) + log(d/e + x)/(3*d**3*e), True))/e**8 - 56*b*d**5*P
```

```
iecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), True))*log(c*x**n)/e**
8 - 70*b*d**4*n*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*d**2*e + 2*d*e**2*x) -
 log(x)/(2*d**2*e) + log(d/e + x)/(2*d**2*e), True))/e**8 + 70*b*d**4*Piece
wise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))*log(c*x**n)/e**8 +
56*b*d**3*n*Piecewise((x/d**2, Eq(e, 0)), (-log(x)/(d*e) + log(d/e + x)/(d*
e), True))/e**8 - 56*b*d**3*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x
), True))*log(c*x**n)/e**8 - 28*b*d**2*n*Piecewise((x/d, Eq(e, 0)), (Piecew
ise((-polylog(2, e*x*exp_polar(I*pi)/d), (Abs(x) < 1) & (1/Abs(x) < 1)), (l
og(d)*log(x) - polylog(2, e*x*exp_polar(I*pi)/d), Abs(x) < 1), (-log(d)*log
(1/x) - polylog(2, e*x*exp_polar(I*pi)/d), 1/Abs(x) < 1), (-meijerg(((), (1
, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log
(d) - polylog(2, e*x*exp_polar(I*pi)/d), True))/e, True))/e**8 + 28*b*d**2*
Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))*log(c*x**n)/e**8 + 7*b*d
*n*x/e**8 - 7*b*d*x*log(c*x**n)/e**8 - b*n*x**2/(4*e**7) + b*x**2*log(c*x**
n)/(2*e**7)
```

Maxima [F]

$$\int \frac{x^8(a+b\log(cx^n))}{(d+ex)^7}\,dx = \int \frac{(b\log(cx^n)+a)x^8}{(ex+d)^7}\,dx$$

```
[In] integrate(x^8*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] 1/30*a*((1680*d^3*e^5*x^5 + 7350*d^4*e^4*x^4 + 13160*d^5*e^3*x^3 + 11970*d^
6*e^2*x^2 + 5508*d^7*e*x + 1023*d^8)/(e^15*x^6 + 6*d*e^14*x^5 + 15*d^2*e^13
*x^4 + 20*d^3*e^12*x^3 + 15*d^4*e^11*x^2 + 6*d^5*e^10*x + d^6*e^9) + 840*d^
2*log(e*x + d)/e^9 + 15*(e*x^2 - 14*d*x)/e^8) + b*integrate((x^8*log(c) + x
^8*log(x^n))/(e^7*x^7 + 7*d*e^6*x^6 + 21*d^2*e^5*x^5 + 35*d^3*e^4*x^4 + 35*
d^4*e^3*x^3 + 21*d^5*e^2*x^2 + 7*d^6*e*x + d^7), x)
```

Giac [F]

$$\int \frac{x^8(a+b\log(cx^n))}{(d+ex)^7}\,dx = \int \frac{(b\log(cx^n)+a)x^8}{(ex+d)^7}\,dx$$

```
[In] integrate(x^8*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^8/(e*x + d)^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(a+b\log(cx^n))}{(d+ex)^7}\,dx = \int \frac{x^8\,(a+b\,\ln(c\,x^n))}{(d+e\,x)^7}\,dx$$

```
[In] int((x^8*(a + b*log(c*x^n)))/(d + e*x)^7,x)
[Out] int((x^8*(a + b*log(c*x^n)))/(d + e*x)^7, x)
```

3.63 $\int \frac{x^7(a+b\log(cx^n))}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 285

$$\begin{aligned}\int \frac{x^7(a+b\log(cx^n))}{(d+ex)^7}\,dx &= -\frac{7bnx}{e^7}+\frac{(140a+223bn)x}{20e^7}+\frac{7bx\log(cx^n)}{e^7}-\frac{x^7(a+b\log(cx^n))}{6e(d+ex)^6}\\ &\quad-\frac{x^6(7a+bn+7b\log(cx^n))}{30e^2(d+ex)^5}-\frac{x^5(42a+13bn+42b\log(cx^n))}{120e^3(d+ex)^4}\\ &\quad-\frac{x^2(140a+153bn+140b\log(cx^n))}{40e^6(d+ex)}\\ &\quad-\frac{x^4(210a+107bn+210b\log(cx^n))}{360e^4(d+ex)^3}\\ &\quad-\frac{x^3(420a+319bn+420b\log(cx^n))}{360e^5(d+ex)^2}\\ &\quad-\frac{d(140a+223bn+140b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{20e^8}\\ &\quad-\frac{7bdn\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^8}\end{aligned}$$

```
[Out] -7*b*n*x/e^7+1/20*(223*b*n+140*a)*x/e^7+7*b*x*ln(c*x^n)/e^7-1/6*x^7*(a+b*ln
(c*x^n))/e/(e*x+d)^6-1/30*x^6*(7*a+b*n+7*b*ln(c*x^n))/e^2/(e*x+d)^5-1/120*x
^5*(42*a+13*b*n+42*b*ln(c*x^n))/e^3/(e*x+d)^4-1/40*x^2*(140*a+153*b*n+140*b
*ln(c*x^n))/e^6/(e*x+d)-1/360*x^4*(210*a+107*b*n+210*b*ln(c*x^n))/e^4/(e*x+
d)^3-1/360*x^3*(420*a+319*b*n+420*b*ln(c*x^n))/e^5/(e*x+d)^2-1/20*d*(140*a+
223*b*n+140*b*ln(c*x^n))*ln(1+e*x/d)/e^8-7*b*d*n*polylog(2,-e*x/d)/e^8
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2384, 45, 2393, 2332, 2354, 2438}

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex)^7}\,dx = -\frac{d\log\left(\frac{ex}{d}+1\right)(140a+140b\log(cx^n)+223bn)}{20e^8} - \frac{x^2(140a+140b\log(cx^n)+153bn)}{40e^6(d+ex)} - \frac{x^3(420a+420b\log(cx^n)+319bn)}{360e^5(d+ex)^2} - \frac{x^4(210a+210b\log(cx^n)+107bn)}{360e^4(d+ex)^3} - \frac{x^5(42a+42b\log(cx^n)+13bn)}{120e^3(d+ex)^4} - \frac{x^6(7a+7b\log(cx^n)+bn)}{30e^2(d+ex)^5} - \frac{x^7(a+b\log(cx^n))}{6e(d+ex)^6} + \frac{x(140a+223bn)}{20e^7} + \frac{7bx\log(cx^n)}{e^7} - \frac{7bdn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^8} - \frac{7bnx}{e^7}$$

```
[In] Int[(x^7*(a + b*Log[c*x^n]))/(d + e*x)^7,x]
```

```
[Out] (-7*b*n*x)/e^7 + ((140*a + 223*b*n)*x)/(20*e^7) + (7*b*x*Log[c*x^n])/e^7 -
(x^7*(a + b*Log[c*x^n]))/(6*e*(d + e*x)^6) - (x^6*(7*a + b*n + 7*b*Log[c*x^
n]))/(30*e^2*(d + e*x)^5) - (x^5*(42*a + 13*b*n + 42*b*Log[c*x^n]))/(120*e^
3*(d + e*x)^4) - (x^2*(140*a + 153*b*n + 140*b*Log[c*x^n]))/(40*e^6*(d + e*
x)) - (x^4*(210*a + 107*b*n + 210*b*Log[c*x^n]))/(360*e^4*(d + e*x)^3) - (x
^3*(420*a + 319*b*n + 420*b*Log[c*x^n]))/(360*e^5*(d + e*x)^2) - (d*(140*a
+ 223*b*n + 140*b*Log[c*x^n])*Log[1 + (e*x)/d])/(20*e^8) - (7*b*d*n*PolyLog
[2, -((e*x)/d)])/e^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^7(a+b\log(cx^n))}{6e(d+ex)^6}+\frac{\int \frac{x^6(7a+bn+7b\log(cx^n))}{(d+ex)^6}\,dx}{6e}\\
&= -\frac{x^7(a+b\log(cx^n))}{6e(d+ex)^6}-\frac{x^6(7a+bn+7b\log(cx^n))}{30e^2(d+ex)^5}+\frac{\int \frac{x^5(7bn+6(7a+bn)+42b\log(cx^n))}{(d+ex)^5}\,dx}{30e^2}\\
&= -\frac{x^7(a+b\log(cx^n))}{6e(d+ex)^6}-\frac{x^6(7a+bn+7b\log(cx^n))}{30e^2(d+ex)^5}\\
&\quad -\frac{x^5(42a+13bn+42b\log(cx^n))}{120e^3(d+ex)^4}+\frac{\int \frac{x^4(42bn+5(7bn+6(7a+bn))+210b\log(cx^n))}{(d+ex)^4}\,dx}{120e^3}\\
&= -\frac{x^7(a+b\log(cx^n))}{6e(d+ex)^6}-\frac{x^6(7a+bn+7b\log(cx^n))}{30e^2(d+ex)^5}-\frac{x^5(42a+13bn+42b\log(cx^n))}{120e^3(d+ex)^4}\\
&\quad -\frac{x^4(210a+107bn+210b\log(cx^n))}{360e^4(d+ex)^3}+\frac{\int \frac{x^3(210bn+4(42bn+5(7bn+6(7a+bn)))+840b\log(cx^n))}{(d+ex)^3}\,dx}{360e^4}
\end{aligned}$$

$$= -\frac{x^7(a+b\log(cx^n))}{6e(d+ex)^6} - \frac{x^6(7a+bn+7b\log(cx^n))}{30e^2(d+ex)^5} - \frac{x^5(42a+13bn+42b\log(cx^n))}{120e^3(d+ex)^4} - \frac{x^4(210a+107bn+210b\log(cx^n))}{360e^4(d+ex)^3} - \frac{x^3(420a+319bn+420b\log(cx^n))}{360e^5(d+ex)^2} + \frac{\int \frac{x^2(840bn+3(210bn+4(42bn+5(7bn+6(7a+bn))))+2520b\log(cx^n))}{(d+ex)^2}dx}{720e^5}$$

$$= -\frac{x^7(a+b\log(cx^n))}{6e(d+ex)^6} - \frac{x^6(7a+bn+7b\log(cx^n))}{30e^2(d+ex)^5} - \frac{x^5(42a+13bn+42b\log(cx^n))}{120e^3(d+ex)^4} - \frac{x^2(140a+153bn+140b\log(cx^n))}{40e^6(d+ex)} - \frac{x^4(210a+107bn+210b\log(cx^n))}{360e^4(d+ex)^3} - \frac{x^3(420a+319bn+420b\log(cx^n))}{360e^5(d+ex)^2} + \frac{\int \frac{x(2520bn+2(840bn+3(210bn+4(42bn+5(7bn+6(7a+bn)))))+5040b\log(cx^n))}{d+ex}dx}{720e^6}$$

$$= -\frac{x^7(a+b\log(cx^n))}{6e(d+ex)^6} - \frac{x^6(7a+bn+7b\log(cx^n))}{30e^2(d+ex)^5} - \frac{x^5(42a+13bn+42b\log(cx^n))}{120e^3(d+ex)^4} - \frac{x^2(140a+153bn+140b\log(cx^n))}{40e^6(d+ex)} - \frac{x^4(210a+107bn+210b\log(cx^n))}{360e^4(d+ex)^3} - \frac{x^3(420a+319bn+420b\log(cx^n))}{360e^5(d+ex)^2} + \frac{\int \left(\frac{2520bn+2(840bn+3(210bn+4(42bn+5(7bn+6(7a+bn)))))+5040b\log(cx^n)}{e} - \frac{d(2520bn+2(840bn+3(210bn+4(42bn+5(7bn+6 \cdots}{e(d+ex)} \cdots}{720e^6}$$

$$= -\frac{x^7(a+b\log(cx^n))}{6e(d+ex)^6} - \frac{x^6(7a+bn+7b\log(cx^n))}{30e^2(d+ex)^5} - \frac{x^5(42a+13bn+42b\log(cx^n))}{120e^3(d+ex)^4} - \frac{x^2(140a+153bn+140b\log(cx^n))}{40e^6(d+ex)} - \frac{x^4(210a+107bn+210b\log(cx^n))}{360e^4(d+ex)^3} - \frac{x^3(420a+319bn+420b\log(cx^n))}{360e^5(d+ex)^2} + \frac{\int (2520bn+2(840bn+3(210bn+4(42bn+5(7bn+6(7a+bn)))))+5040b\log(cx^n))\,dx}{720e^7} - \frac{d\int \frac{2520bn+2(840bn+3(210bn+4(42bn+5(7bn+6(7a+bn)))))+5040b\log(cx^n)}{d+ex}dx}{720e^7}$$

$$
\begin{aligned}
&= \frac{(140a+223bn)x}{20e^7} - \frac{x^7(a+b\log(cx^n))}{6e(d+ex)^6} - \frac{x^6(7a+bn+7b\log(cx^n))}{30e^2(d+ex)^5} \\
&\quad - \frac{x^5(42a+13bn+42b\log(cx^n))}{120e^3(d+ex)^4} - \frac{x^2(140a+153bn+140b\log(cx^n))}{40e^6(d+ex)} \\
&\quad - \frac{x^4(210a+107bn+210b\log(cx^n))}{360e^4(d+ex)^3} - \frac{x^3(420a+319bn+420b\log(cx^n))}{360e^5(d+ex)^2} \\
&\quad - \frac{d(140a+223bn+140b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{20e^8} \\
&\quad + \frac{(7b)\int\log(cx^n)\,dx}{e^7} + \frac{(7bdn)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^8} \\
&= -\frac{7bnx}{e^7} + \frac{(140a+223bn)x}{20e^7} + \frac{7bx\log(cx^n)}{e^7} \\
&\quad - \frac{x^7(a+b\log(cx^n))}{6e(d+ex)^6} - \frac{x^6(7a+bn+7b\log(cx^n))}{30e^2(d+ex)^5} \\
&\quad - \frac{x^5(42a+13bn+42b\log(cx^n))}{120e^3(d+ex)^4} - \frac{x^2(140a+153bn+140b\log(cx^n))}{40e^6(d+ex)} \\
&\quad - \frac{x^4(210a+107bn+210b\log(cx^n))}{360e^4(d+ex)^3} - \frac{x^3(420a+319bn+420b\log(cx^n))}{360e^5(d+ex)^2} \\
&\quad - \frac{d(140a+223bn+140b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{20e^8} - \frac{7bdn\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^8}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.25

$$
\int \frac{x^7(a+b\log(cx^n))}{(d+ex)^7}\,dx =
$$

$$
-\frac{-360aex + 360benx - \frac{60ad^7}{(d+ex)^6} + \frac{504ad^6}{(d+ex)^5} + \frac{12bd^6n}{(d+ex)^5} - \frac{1890ad^5}{(d+ex)^4} - \frac{111bd^5n}{(d+ex)^4} + \frac{4200ad^4}{(d+ex)^3} + \frac{482bd^4n}{(d+ex)^3} - \frac{6300ad^3}{(d+ex)^2} - \frac{137}{(d+}
$$

```
[In] Integrate[(x^7*(a + b*Log[c*x^n]))/(d + e*x)^7,x]
```

```
[Out] -1/360*(-360*a*e*x + 360*b*e*n*x - (60*a*d^7)/(d + e*x)^6 + (504*a*d^6)/(d
+ e*x)^5 + (12*b*d^6*n)/(d + e*x)^5 - (1890*a*d^5)/(d + e*x)^4 - (111*b*d^5
*n)/(d + e*x)^4 + (4200*a*d^4)/(d + e*x)^3 + (482*b*d^4*n)/(d + e*x)^3 - (6
300*a*d^3)/(d + e*x)^2 - (1377*b*d^3*n)/(d + e*x)^2 + (7560*a*d^2)/(d + e*x
) + (3546*b*d^2*n)/(d + e*x) - 4014*b*d*n*Log[x] - 360*b*e*x*Log[c*x^n] - (
60*b*d^7*Log[c*x^n])/(d + e*x)^6 + (504*b*d^6*Log[c*x^n])/(d + e*x)^5 - (18
90*b*d^5*Log[c*x^n])/(d + e*x)^4 + (4200*b*d^4*Log[c*x^n])/(d + e*x)^3 - (6
300*b*d^3*Log[c*x^n])/(d + e*x)^2 + (7560*b*d^2*Log[c*x^n])/(d + e*x) + 401
4*b*d*n*Log[d + e*x] + 2520*a*d*Log[1 + (e*x)/d] + 2520*b*d*Log[c*x^n]*Log[
1 + (e*x)/d] + 2520*b*d*n*PolyLog[2, -((e*x)/d)])/e^8
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.34 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.79

method	result
risch	$\frac{b\ln(x^n)x}{e^7}+\frac{b\ln(x^n)d^7}{6e^8(ex+d)^6}-\frac{35b\ln(x^n)d^4}{3e^8(ex+d)^3}-\frac{7b\ln(x^n)d\ln(ex+d)}{e^8}-\frac{21b\ln(x^n)d^2}{e^8(ex+d)}+\frac{35b\ln(x^n)d^3}{2e^8(ex+d)^2}+\frac{21b\ln(x^n)d^5}{4e^8(ex+d)^4}-\frac{7b\ln(x^n)}{5e^8(ex+}$

```
[In] int(x^7*(a+b*ln(c*x^n))/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

```
[Out] b*ln(x^n)/e^7*x+1/6*b*ln(x^n)/e^8*d^7/(e*x+d)^6-35/3*b*ln(x^n)/e^8*d^4/(e*x
+d)^3-7*b*ln(x^n)/e^8*d*ln(e*x+d)-21*b*ln(x^n)/e^8*d^2/(e*x+d)+35/2*b*ln(x^
n)/e^8*d^3/(e*x+d)^2+21/4*b*ln(x^n)/e^8*d^5/(e*x+d)^4-7/5*b*ln(x^n)/e^8*d^6
/(e*x+d)^5-b*n*x/e^7-b*n/e^8*d-223/20*b*n/e^8*d*ln(e*x+d)-197/20*b*n/e^8*d^
2/(e*x+d)+153/40*b*n/e^8*d^3/(e*x+d)^2-241/180*b*n/e^8*d^4/(e*x+d)^3+37/120
*b*n/e^8*d^5/(e*x+d)^4-1/30*b*n/e^8*d^6/(e*x+d)^5+223/20*b*n/e^8*d*ln(e*x)+
7*b*n/e^8*d*ln(e*x+d)*ln(-e*x/d)+7*b*n/e^8*d*dilog(-e*x/d)+(-1/2*I*b*Pi*csg
n(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I
*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(x/
e^7+1/6/e^8*d^7/(e*x+d)^6-35/3/e^8*d^4/(e*x+d)^3-7/e^8*d*ln(e*x+d)-21/e^8*d
^2/(e*x+d)+35/2/e^8*d^3/(e*x+d)^2+21/4/e^8*d^5/(e*x+d)^4-7/5/e^8*d^6/(e*x+d
)^5)
```

Fricas [F]

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex)^7}\,dx=\int \frac{(b\log(cx^n)+a)x^7}{(ex+d)^7}\,dx$$

```
[In] integrate(x^7*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] integral((b*x^7*log(c*x^n) + a*x^7)/(e^7*x^7 + 7*d*e^6*x^6 + 21*d^2*e^5*x^5
 + 35*d^3*e^4*x^4 + 35*d^4*e^3*x^3 + 21*d^5*e^2*x^2 + 7*d^6*e*x + d^7), x)
```

Sympy [A] (verification not implemented)

Time = 112.19 (sec) , antiderivative size = 1632, normalized size of antiderivative = 5.73

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex)^7}\,dx=\text{Too large to display}$$

```
[In] integrate(x**7*(a+b*ln(c*x**n))/(e*x+d)**7,x)
```

```
[Out] -a*d**7*Piecewise((x/d**7, Eq(e, 0)), (-1/(6*e*(d + e*x)**6), True))/e**7 +
 7*a*d**6*Piecewise((x/d**6, Eq(e, 0)), (-1/(5*e*(d + e*x)**5), True))/e**7
```

```
 - 21*a*d**5*Piecewise((x/d**5, Eq(e, 0)), (-1/(4*e*(d + e*x)**4), True))/e
**7 + 35*a*d**4*Piecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), True)
)/e**7 - 35*a*d**3*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), Tr
ue))/e**7 + 21*a*d**2*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), Tru
e))/e**7 - 7*a*d*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e**7 +
a*x/e**7 + b*d**7*n*Piecewise((x/d**7, Eq(e, 0)), (-137*d**4/(360*d**10*e +
 1800*d**9*e**2*x + 3600*d**8*e**3*x**2 + 3600*d**7*e**4*x**3 + 1800*d**6*e
**5*x**4 + 360*d**5*e**6*x**5) - 385*d**3*e*x/(360*d**10*e + 1800*d**9*e**2
*x + 3600*d**8*e**3*x**2 + 3600*d**7*e**4*x**3 + 1800*d**6*e**5*x**4 + 360*
d**5*e**6*x**5) - 470*d**2*e**2*x**2/(360*d**10*e + 1800*d**9*e**2*x + 3600
*d**8*e**3*x**2 + 3600*d**7*e**4*x**3 + 1800*d**6*e**5*x**4 + 360*d**5*e**6
*x**5) - 270*d*e**3*x**3/(360*d**10*e + 1800*d**9*e**2*x + 3600*d**8*e**3*x
**2 + 3600*d**7*e**4*x**3 + 1800*d**6*e**5*x**4 + 360*d**5*e**6*x**5) - 60*
e**4*x**4/(360*d**10*e + 1800*d**9*e**2*x + 3600*d**8*e**3*x**2 + 3600*d**7
*e**4*x**3 + 1800*d**6*e**5*x**4 + 360*d**5*e**6*x**5) - log(x)/(6*d**6*e)
+ log(d/e + x)/(6*d**6*e), True))/e**7 - b*d**7*Piecewise((x/d**7, Eq(e, 0)
), (-1/(6*e*(d + e*x)**6), True))*log(c*x**n)/e**7 - 7*b*d**6*n*Piecewise((
x/d**6, Eq(e, 0)), (-25*d**3/(60*d**8*e + 240*d**7*e**2*x + 360*d**6*e**3*x
**2 + 240*d**5*e**4*x**3 + 60*d**4*e**5*x**4) - 52*d**2*e*x/(60*d**8*e + 24
0*d**7*e**2*x + 360*d**6*e**3*x**2 + 240*d**5*e**4*x**3 + 60*d**4*e**5*x**4
) - 42*d*e**2*x**2/(60*d**8*e + 240*d**7*e**2*x + 360*d**6*e**3*x**2 + 240*
d**5*e**4*x**3 + 60*d**4*e**5*x**4) - 12*e**3*x**3/(60*d**8*e + 240*d**7*e*
*2*x + 360*d**6*e**3*x**2 + 240*d**5*e**4*x**3 + 60*d**4*e**5*x**4) - log(x
)/(5*d**5*e) + log(d/e + x)/(5*d**5*e), True))/e**7 + 7*b*d**6*Piecewise((x
/d**6, Eq(e, 0)), (-1/(5*e*(d + e*x)**5), True))*log(c*x**n)/e**7 + 21*b*d*
*5*n*Piecewise((x/d**5, Eq(e, 0)), (-11*d**2/(24*d**6*e + 72*d**5*e**2*x +
72*d**4*e**3*x**2 + 24*d**3*e**4*x**3) - 15*d*e*x/(24*d**6*e + 72*d**5*e**2
*x + 72*d**4*e**3*x**2 + 24*d**3*e**4*x**3) - 6*e**2*x**2/(24*d**6*e + 72*d
**5*e**2*x + 72*d**4*e**3*x**2 + 24*d**3*e**4*x**3) - log(x)/(4*d**4*e) + l
og(d/e + x)/(4*d**4*e), True))/e**7 - 21*b*d**5*Piecewise((x/d**5, Eq(e, 0)
), (-1/(4*e*(d + e*x)**4), True))*log(c*x**n)/e**7 - 35*b*d**4*n*Piecewise(
(x/d**4, Eq(e, 0)), (-3*d/(6*d**4*e + 12*d**3*e**2*x + 6*d**2*e**3*x**2) -
2*e*x/(6*d**4*e + 12*d**3*e**2*x + 6*d**2*e**3*x**2) - log(x)/(3*d**3*e) +
log(d/e + x)/(3*d**3*e), True))/e**7 + 35*b*d**4*Piecewise((x/d**4, Eq(e, 0
)), (-1/(3*e*(d + e*x)**3), True))*log(c*x**n)/e**7 + 35*b*d**3*n*Piecewise
((x/d**3, Eq(e, 0)), (-1/(2*d**2*e + 2*d*e**2*x) - log(x)/(2*d**2*e) + log(
d/e + x)/(2*d**2*e), True))/e**7 - 35*b*d**3*Piecewise((x/d**3, Eq(e, 0)),
(-1/(2*e*(d + e*x)**2), True))*log(c*x**n)/e**7 - 21*b*d**2*n*Piecewise((x/
d**2, Eq(e, 0)), (-log(x)/(d*e) + log(d/e + x)/(d*e), True))/e**7 + 21*b*d*
*2*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))*log(c*x**n)/e**
7 + 7*b*d*n*Piecewise((x/d, Eq(e, 0)), (Piecewise((-polylog(2, e*x*exp_pola
r(I*pi)/d), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x
*exp_polar(I*pi)/d), Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x*exp_po
lar(I*pi)/d), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d
) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x*exp_pola
```

```
r(I*pi)/d), True))/e, True))/e**7 - 7*b*d*Piecewise((x/d, Eq(e, 0)), (log(d
 + e*x)/e, True))*log(c*x**n)/e**7 - b*n*x/e**7 + b*x*log(c*x**n)/e**7
```

Maxima [F]

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex)^7}\,dx = \int \frac{(b\log(cx^n)+a)x^7}{(ex+d)^7}\,dx$$

```
[In] integrate(x^7*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] -1/60*a*((1260*d^2*e^5*x^5 + 5250*d^3*e^4*x^4 + 9100*d^4*e^3*x^3 + 8085*d^5
*e^2*x^2 + 3654*d^6*e*x + 669*d^7)/(e^14*x^6 + 6*d*e^13*x^5 + 15*d^2*e^12*x
^4 + 20*d^3*e^11*x^3 + 15*d^4*e^10*x^2 + 6*d^5*e^9*x + d^6*e^8) - 60*x/e^7
+ 420*d*log(e*x + d)/e^8) + b*integrate((x^7*log(c) + x^7*log(x^n))/(e^7*x^
7 + 7*d*e^6*x^6 + 21*d^2*e^5*x^5 + 35*d^3*e^4*x^4 + 35*d^4*e^3*x^3 + 21*d^5
*e^2*x^2 + 7*d^6*e*x + d^7), x)
```

Giac [F]

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex)^7}\,dx = \int \frac{(b\log(cx^n)+a)x^7}{(ex+d)^7}\,dx$$

```
[In] integrate(x^7*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^7/(e*x + d)^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex)^7}\,dx = \int \frac{x^7\,(a+b\,\ln(c\,x^n))}{(d+e\,x)^7}\,dx$$

```
[In] int((x^7*(a + b*log(c*x^n)))/(d + e*x)^7,x)
```

```
[Out] int((x^7*(a + b*log(c*x^n)))/(d + e*x)^7, x)
```

3.64 $\int \frac{x^6(a+b\log(cx^n))}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 243

$$\begin{aligned}\int \frac{x^6(a+b\log(cx^n))}{(d+ex)^7}\,dx &= -\frac{x^6(a+b\log(cx^n))}{6e(d+ex)^6}-\frac{x^5(6a+bn+6b\log(cx^n))}{30e^2(d+ex)^5}\\ &\quad-\frac{x^2(20a+19bn+20b\log(cx^n))}{40e^5(d+ex)^2}\\ &\quad-\frac{x(20a+29bn+20b\log(cx^n))}{20e^6(d+ex)}\\ &\quad-\frac{x^4(30a+11bn+30b\log(cx^n))}{120e^3(d+ex)^4}\\ &\quad-\frac{x^3(60a+37bn+60b\log(cx^n))}{180e^4(d+ex)^3}\\ &\quad+\frac{(20a+49bn+20b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{20e^7}\\ &\quad+\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^7}\end{aligned}$$

```
[Out] -1/6*x^6*(a+b*ln(c*x^n))/e/(e*x+d)^6-1/30*x^5*(6*a+b*n+6*b*ln(c*x^n))/e^2/(
e*x+d)^5-1/40*x^2*(20*a+19*b*n+20*b*ln(c*x^n))/e^5/(e*x+d)^2-1/20*x*(20*a+2
9*b*n+20*b*ln(c*x^n))/e^6/(e*x+d)-1/120*x^4*(30*a+11*b*n+30*b*ln(c*x^n))/e^
3/(e*x+d)^4-1/180*x^3*(60*a+37*b*n+60*b*ln(c*x^n))/e^4/(e*x+d)^3+1/20*(20*a
+49*b*n+20*b*ln(c*x^n))*ln(1+e*x/d)/e^7+b*n*polylog(2,-e*x/d)/e^7
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2384, 2354, 2438}

$$\int \frac{x^6(a+b\log(cx^n))}{(d+ex)^7}\,dx = \frac{\log\left(\frac{ex}{d}+1\right)(20a+20b\log(cx^n)+49bn)}{20e^7} - \frac{x(20a+20b\log(cx^n)+29bn)}{20e^6(d+ex)} - \frac{x^2(20a+20b\log(cx^n)+19bn)}{40e^5(d+ex)^2} - \frac{x^3(60a+60b\log(cx^n)+37bn)}{180e^4(d+ex)^3} - \frac{x^4(30a+30b\log(cx^n)+11bn)}{120e^3(d+ex)^4} - \frac{x^5(6a+6b\log(cx^n)+bn)}{30e^2(d+ex)^5} - \frac{x^6(a+b\log(cx^n))}{6e(d+ex)^6} + \frac{bn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^7}$$

```
[In] Int[(x^6*(a + b*Log[c*x^n]))/(d + e*x)^7,x]
```

```
[Out] -1/6*(x^6*(a + b*Log[c*x^n]))/(e*(d + e*x)^6) - (x^5*(6*a + b*n + 6*b*Log[c
*x^n]))/(30*e^2*(d + e*x)^5) - (x^2*(20*a + 19*b*n + 20*b*Log[c*x^n]))/(40*
e^5*(d + e*x)^2) - (x*(20*a + 29*b*n + 20*b*Log[c*x^n]))/(20*e^6*(d + e*x))
 - (x^4*(30*a + 11*b*n + 30*b*Log[c*x^n]))/(120*e^3*(d + e*x)^4) - (x^3*(60
*a + 37*b*n + 60*b*Log[c*x^n]))/(180*e^4*(d + e*x)^3) + ((20*a + 49*b*n + 2
0*b*Log[c*x^n])*Log[1 + (e*x)/d])/(20*e^7) + (b*n*PolyLog[2, -((e*x)/d)])/e
^7
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = -\frac{x^6(a+b\log(cx^n))}{6e(d+ex)^6}+\frac{\int \frac{x^5(6a+bn+6b\log(cx^n))}{(d+ex)^6}\,dx}{6e}$$

$$= -\frac{x^6(a+b\log(cx^n))}{6e(d+ex)^6}-\frac{x^5(6a+bn+6b\log(cx^n))}{30e^2(d+ex)^5}+\frac{\int \frac{x^4(6bn+5(6a+bn)+30b\log(cx^n))}{(d+ex)^5}\,dx}{30e^2}$$

$$= -\frac{x^6(a+b\log(cx^n))}{6e(d+ex)^6}-\frac{x^5(6a+bn+6b\log(cx^n))}{30e^2(d+ex)^5} -\frac{x^4(30a+11bn+30b\log(cx^n))}{120e^3(d+ex)^4}+\frac{\int \frac{x^3(30bn+4(6bn+5(6a+bn))+120b\log(cx^n))}{(d+ex)^4}\,dx}{120e^3}$$

$$= -\frac{x^6(a+b\log(cx^n))}{6e(d+ex)^6}-\frac{x^5(6a+bn+6b\log(cx^n))}{30e^2(d+ex)^5}-\frac{x^4(30a+11bn+30b\log(cx^n))}{120e^3(d+ex)^4} -\frac{x^3(60a+37bn+60b\log(cx^n))}{180e^4(d+ex)^3}+\frac{\int \frac{x^2(120bn+3(30bn+4(6bn+5(6a+bn)))+360b\log(cx^n))}{(d+ex)^3}\,dx}{360e^4}$$

$$= -\frac{x^6(a+b\log(cx^n))}{6e(d+ex)^6}-\frac{x^5(6a+bn+6b\log(cx^n))}{30e^2(d+ex)^5}-\frac{x^2(20a+19bn+20b\log(cx^n))}{40e^5(d+ex)^2} -\frac{x^4(30a+11bn+30b\log(cx^n))}{120e^3(d+ex)^4}-\frac{x^3(60a+37bn+60b\log(cx^n))}{180e^4(d+ex)^3} +\frac{\int \frac{x(360bn+2(120bn+3(30bn+4(6bn+5(6a+bn))))+720b\log(cx^n))}{(d+ex)^2}\,dx}{720e^5}$$

$$= -\frac{x^6(a+b\log(cx^n))}{6e(d+ex)^6}-\frac{x^5(6a+bn+6b\log(cx^n))}{30e^2(d+ex)^5} -\frac{x^2(20a+19bn+20b\log(cx^n))}{40e^5(d+ex)^2}-\frac{x(20a+29bn+20b\log(cx^n))}{20e^6(d+ex)} -\frac{x^4(30a+11bn+30b\log(cx^n))}{120e^3(d+ex)^4}-\frac{x^3(60a+37bn+60b\log(cx^n))}{180e^4(d+ex)^3} +\frac{\int \frac{1080bn+2(120bn+3(30bn+4(6bn+5(6a+bn))))+720b\log(cx^n)}{d+ex}\,dx}{720e^6}$$

$$= -\frac{x^6(a+b\log(cx^n))}{6e(d+ex)^6} - \frac{x^5(6a+bn+6b\log(cx^n))}{30e^2(d+ex)^5} - \frac{x^2(20a+19bn+20b\log(cx^n))}{40e^5(d+ex)^2} - \frac{x(20a+29bn+20b\log(cx^n))}{20e^6(d+ex)} - \frac{x^4(30a+11bn+30b\log(cx^n))}{120e^3(d+ex)^4} - \frac{x^3(60a+37bn+60b\log(cx^n))}{180e^4(d+ex)^3} + \frac{(20a+49bn+20b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{20e^7} - \frac{(bn)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^7}$$

$$= -\frac{x^6(a+b\log(cx^n))}{6e(d+ex)^6} - \frac{x^5(6a+bn+6b\log(cx^n))}{30e^2(d+ex)^5} - \frac{x^2(20a+19bn+20b\log(cx^n))}{40e^5(d+ex)^2} - \frac{x(20a+29bn+20b\log(cx^n))}{20e^6(d+ex)} - \frac{x^4(30a+11bn+30b\log(cx^n))}{120e^3(d+ex)^4} - \frac{x^3(60a+37bn+60b\log(cx^n))}{180e^4(d+ex)^3} + \frac{(20a+49bn+20b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{20e^7} + \frac{bn\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^7}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.37

$$\int \frac{x^6(a+b\log(cx^n))}{(d+ex)^7}\,dx = \frac{-882bn\log(x) + \frac{-60ad^6+432ad^5(d+ex)+12bd^5n(d+ex)-1350ad^4(d+ex)^2-93bd^4n(d+ex)^2+2400ad^3(d+ex)^3+326bd^3n(d+ex)^3-2700ad}{}}{}$$

```
[In] Integrate[(x^6*(a + b*Log[c*x^n]))/(d + e*x)^7,x]
```

```
[Out] (-882*b*n*Log[x] + (-60*a*d^6 + 432*a*d^5*(d + e*x) + 12*b*d^5*n*(d + e*x)
- 1350*a*d^4*(d + e*x)^2 - 93*b*d^4*n*(d + e*x)^2 + 2400*a*d^3*(d + e*x)^3
+ 326*b*d^3*n*(d + e*x)^3 - 2700*a*d^2*(d + e*x)^4 - 711*b*d^2*n*(d + e*x)^
4 + 2160*a*d*(d + e*x)^5 + 1278*b*d*n*(d + e*x)^5 - 60*b*d^6*Log[c*x^n] + 4
32*b*d^5*(d + e*x)*Log[c*x^n] - 1350*b*d^4*(d + e*x)^2*Log[c*x^n] + 2400*b*
d^3*(d + e*x)^3*Log[c*x^n] - 2700*b*d^2*(d + e*x)^4*Log[c*x^n] + 2160*b*d*(
d + e*x)^5*Log[c*x^n] + 882*b*n*(d + e*x)^6*Log[d + e*x] + 360*a*(d + e*x)^
6*Log[1 + (e*x)/d] + 360*b*(d + e*x)^6*Log[c*x^n]*Log[1 + (e*x)/d])/(d + e*
x)^6 + 360*b*n*PolyLog[2, -((e*x)/d)])/(360*e^7)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.30 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.92

method	result
risch	$-\frac{b\ln(x^n)d^6}{6e^7(ex+d)^6}+\frac{20b\ln(x^n)d^3}{3e^7(ex+d)^3}+\frac{b\ln(x^n)\ln(ex+d)}{e^7}+\frac{6b\ln(x^n)d}{e^7(ex+d)}-\frac{15b\ln(x^n)d^2}{2e^7(ex+d)^2}-\frac{15b\ln(x^n)d^4}{4e^7(ex+d)^4}+\frac{6b\ln(x^n)d^5}{5e^7(ex+d)^5}+\frac{71b}{20e^7(e}$

```
[In] int(x^6*(a+b*ln(c*x^n))/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*b*ln(x^n)/e^7*d^6/(e*x+d)^6+20/3*b*ln(x^n)/e^7*d^3/(e*x+d)^3+b*ln(x^n)
/e^7*ln(e*x+d)+6*b*ln(x^n)/e^7*d/(e*x+d)-15/2*b*ln(x^n)/e^7*d^2/(e*x+d)^2-1
5/4*b*ln(x^n)/e^7*d^4/(e*x+d)^4+6/5*b*ln(x^n)/e^7*d^5/(e*x+d)^5+71/20*b*n/e
^7*d/(e*x+d)+49/20*b*n/e^7*ln(e*x+d)-79/40*b*n/e^7*d^2/(e*x+d)^2+163/180*b*
n/e^7*d^3/(e*x+d)^3-31/120*b*n/e^7*d^4/(e*x+d)^4+1/30*b*n/e^7*d^5/(e*x+d)^5
-49/20*b*n/e^7*ln(e*x)-b*n/e^7*ln(e*x+d)*ln(-e*x/d)-b*n/e^7*dilog(-e*x/d)+(
-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I
*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3
+b*ln(c)+a)*(-1/6/e^7*d^6/(e*x+d)^6+20/3/e^7*d^3/(e*x+d)^3+1/e^7*ln(e*x+d)+
6/e^7*d/(e*x+d)-15/2/e^7*d^2/(e*x+d)^2-15/4/e^7*d^4/(e*x+d)^4+6/5/e^7*d^5/(
e*x+d)^5)
```

Fricas [F]

$$\int \frac{x^6(a+b\log{(cx^n)})}{(d+ex)^7}\,dx = \int \frac{(b\log{(cx^n)}+a)x^6}{(ex+d)^7}\,dx$$

```
[In] integrate(x^6*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] integral((b*x^6*log(c*x^n) + a*x^6)/(e^7*x^7 + 7*d*e^6*x^6 + 21*d^2*e^5*x^5
 + 35*d^3*e^4*x^4 + 35*d^4*e^3*x^3 + 21*d^5*e^2*x^2 + 7*d^6*e*x + d^7), x)
```

Sympy [A] (verification not implemented)

Time = 80.64 (sec) , antiderivative size = 1588, normalized size of antiderivative = 6.53

$$\int \frac{x^6(a+b\log{(cx^n)})}{(d+ex)^7}\,dx = \text{Too large to display}$$

```
[In] integrate(x**6*(a+b*ln(c*x**n))/(e*x+d)**7,x)
```

```
[Out] a*d**6*Piecewise((x/d**7, Eq(e, 0)), (-1/(6*e*(d + e*x)**6), True))/e**6 -
6*a*d**5*Piecewise((x/d**6, Eq(e, 0)), (-1/(5*e*(d + e*x)**5), True))/e**6
```

+ 15*a*d**4*Piecewise((x/d**5, Eq(e, 0)), (-1/(4*e*(d + e*x)**4), True))/e*
*6 - 20*a*d**3*Piecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), True))
/e**6 + 15*a*d**2*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), Tru
e))/e**6 - 6*a*d*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))/e
6 + a*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e6 - b*d**6*n*
Piecewise((x/d**7, Eq(e, 0)), (-137*d**4/(360*d**10*e + 1800*d**9*e**2*x +
3600*d**8*e**3*x**2 + 3600*d**7*e**4*x**3 + 1800*d**6*e**5*x**4 + 360*d**5*
e**6*x**5) - 385*d**3*e*x/(360*d**10*e + 1800*d**9*e**2*x + 3600*d**8*e**3*
x**2 + 3600*d**7*e**4*x**3 + 1800*d**6*e**5*x**4 + 360*d**5*e**6*x**5) - 47
0*d**2*e**2*x**2/(360*d**10*e + 1800*d**9*e**2*x + 3600*d**8*e**3*x**2 + 36
00*d**7*e**4*x**3 + 1800*d**6*e**5*x**4 + 360*d**5*e**6*x**5) - 270*d*e**3*
x**3/(360*d**10*e + 1800*d**9*e**2*x + 3600*d**8*e**3*x**2 + 3600*d**7*e**4
*x**3 + 1800*d**6*e**5*x**4 + 360*d**5*e**6*x**5) - 60*e**4*x**4/(360*d**10
*e + 1800*d**9*e**2*x + 3600*d**8*e**3*x**2 + 3600*d**7*e**4*x**3 + 1800*d*
*6*e**5*x**4 + 360*d**5*e**6*x**5) - log(x)/(6*d**6*e) + log(d/e + x)/(6*d*
*6*e), True))/e**6 + b*d**6*Piecewise((x/d**7, Eq(e, 0)), (-1/(6*e*(d + e*x
)**6), True))*log(c*x**n)/e**6 + 6*b*d**5*n*Piecewise((x/d**6, Eq(e, 0)), (
-25*d**3/(60*d**8*e + 240*d**7*e**2*x + 360*d**6*e**3*x**2 + 240*d**5*e**4*
x**3 + 60*d**4*e**5*x**4) - 52*d**2*e*x/(60*d**8*e + 240*d**7*e**2*x + 360*
d**6*e**3*x**2 + 240*d**5*e**4*x**3 + 60*d**4*e**5*x**4) - 42*d*e**2*x**2/(
60*d**8*e + 240*d**7*e**2*x + 360*d**6*e**3*x**2 + 240*d**5*e**4*x**3 + 60*
d**4*e**5*x**4) - 12*e**3*x**3/(60*d**8*e + 240*d**7*e**2*x + 360*d**6*e**3
*x**2 + 240*d**5*e**4*x**3 + 60*d**4*e**5*x**4) - log(x)/(5*d**5*e) + log(d
/e + x)/(5*d**5*e), True))/e**6 - 6*b*d**5*Piecewise((x/d**6, Eq(e, 0)), (-
1/(5*e*(d + e*x)**5), True))*log(c*x**n)/e**6 - 15*b*d**4*n*Piecewise((x/d*
*5, Eq(e, 0)), (-11*d**2/(24*d**6*e + 72*d**5*e**2*x + 72*d**4*e**3*x**2 +
24*d**3*e**4*x**3) - 15*d*e*x/(24*d**6*e + 72*d**5*e**2*x + 72*d**4*e**3*x*
*2 + 24*d**3*e**4*x**3) - 6*e**2*x**2/(24*d**6*e + 72*d**5*e**2*x + 72*d**4
*e**3*x**2 + 24*d**3*e**4*x**3) - log(x)/(4*d**4*e) + log(d/e + x)/(4*d**4*
e), True))/e**6 + 15*b*d**4*Piecewise((x/d**5, Eq(e, 0)), (-1/(4*e*(d + e*x
)**4), True))*log(c*x**n)/e**6 + 20*b*d**3*n*Piecewise((x/d**4, Eq(e, 0)),
(-3*d/(6*d**4*e + 12*d**3*e**2*x + 6*d**2*e**3*x**2) - 2*e*x/(6*d**4*e + 12
*d**3*e**2*x + 6*d**2*e**3*x**2) - log(x)/(3*d**3*e) + log(d/e + x)/(3*d**3
*e), True))/e**6 - 20*b*d**3*Piecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*
x)**3), True))*log(c*x**n)/e**6 - 15*b*d**2*n*Piecewise((x/d**3, Eq(e, 0)),
(-1/(2*d**2*e + 2*d*e**2*x) - log(x)/(2*d**2*e) + log(d/e + x)/(2*d**2*e),
True))/e**6 + 15*b*d**2*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**
2), True))*log(c*x**n)/e**6 + 6*b*d*n*Piecewise((x/d**2, Eq(e, 0)), (-log(x
)/(d*e) + log(d/e + x)/(d*e), True))/e**6 - 6*b*d*Piecewise((x/d**2, Eq(e,
0)), (-1/(d*e + e**2*x), True))*log(c*x**n)/e**6 - b*n*Piecewise((x/d, Eq(e
, 0)), (Piecewise((-polylog(2, e*x*exp_polar(I*pi)/d), (Abs(x) < 1) & (1/Ab
s(x) < 1)), (log(d)*log(x) - polylog(2, e*x*exp_polar(I*pi)/d), Abs(x) < 1)
, (-log(d)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/d), 1/Abs(x) < 1), (-m
eijerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (
0, 0)), x)*log(d) - polylog(2, e*x*exp_polar(I*pi)/d), True))/e, True))/e**

```
6 + b*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))*log(c*x**n)/e**6
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x^6(a+b\log(cx^n))}{(d+ex)^7}\,dx = \text{Timed out}$$

```
[In] integrate(x^6*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{x^6(a+b\log(cx^n))}{(d+ex)^7}\,dx = \int \frac{(b\log(cx^n)+a)x^6}{(ex+d)^7}\,dx$$

```
[In] integrate(x^6*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^6/(e*x + d)^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a+b\log(cx^n))}{(d+ex)^7}\,dx = \int \frac{x^6\,(a+b\,\ln(c\,x^n))}{(d+e\,x)^7}\,dx$$

```
[In] int((x^6*(a + b*log(c*x^n)))/(d + e*x)^7,x)
[Out] int((x^6*(a + b*log(c*x^n)))/(d + e*x)^7, x)
```

3.65 $\int \frac{x^5(a+b\log(cx^n))}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex)^7}\,dx = -\frac{bd^4n}{30e^6(d+ex)^5}+\frac{5bd^3n}{24e^6(d+ex)^4}-\frac{5bd^2n}{9e^6(d+ex)^3}+\frac{5bdn}{6e^6(d+ex)^2}-\frac{5bn}{6e^6(d+ex)}+\frac{x^6(a+b\log(cx^n))}{6d(d+ex)^6}-\frac{bn\log(d+ex)}{6de^6}$$

```
[Out] -1/30*b*d^4*n/e^6/(e*x+d)^5+5/24*b*d^3*n/e^6/(e*x+d)^4-5/9*b*d^2*n/e^6/(e*x
+d)^3+5/6*b*d*n/e^6/(e*x+d)^2-5/6*b*n/e^6/(e*x+d)+1/6*x^6*(a+b*ln(c*x^n))/d
/(e*x+d)^6-1/6*b*n*ln(e*x+d)/d/e^6
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2373, 45}

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex)^7}\,dx = \frac{x^6(a+b\log(cx^n))}{6d(d+ex)^6}-\frac{bd^4n}{30e^6(d+ex)^5}+\frac{5bd^3n}{24e^6(d+ex)^4}-\frac{5bd^2n}{9e^6(d+ex)^3}-\frac{5bn}{6e^6(d+ex)}+\frac{5bdn}{6e^6(d+ex)^2}-\frac{bn\log(d+ex)}{6de^6}$$

```
[In] Int[(x^5*(a + b*Log[c*x^n]))/(d + e*x)^7,x]
```

```
[Out] -1/30*(b*d^4*n)/(e^6*(d + e*x)^5) + (5*b*d^3*n)/(24*e^6*(d + e*x)^4) - (5*b
*d^2*n)/(9*e^6*(d + e*x)^3) + (5*b*d*n)/(6*e^6*(d + e*x)^2) - (5*b*n)/(6*e^
6*(d + e*x)) + (x^6*(a + b*Log[c*x^n]))/(6*d*(d + e*x)^6) - (b*n*Log[d + e*
x])/(6*d*e^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^6(a+b\log(cx^n))}{6d(d+ex)^6} - \frac{(bn)\int \frac{x^5}{(d+ex)^6}\,dx}{6d} \\
&= \frac{x^6(a+b\log(cx^n))}{6d(d+ex)^6} \\
&\quad - \frac{(bn)\int \left(-\frac{d^5}{e^5(d+ex)^6} + \frac{5d^4}{e^5(d+ex)^5} - \frac{10d^3}{e^5(d+ex)^4} + \frac{10d^2}{e^5(d+ex)^3} - \frac{5d}{e^5(d+ex)^2} + \frac{1}{e^5(d+ex)}\right)dx}{6d} \\
&= -\frac{bd^4n}{30e^6(d+ex)^5} + \frac{5bd^3n}{24e^6(d+ex)^4} - \frac{5bd^2n}{9e^6(d+ex)^3} + \frac{5bdn}{6e^6(d+ex)^2} \\
&\quad - \frac{5bn}{6e^6(d+ex)} + \frac{x^6(a+b\log(cx^n))}{6d(d+ex)^6} - \frac{bn\log(d+ex)}{6de^6}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 335 vs. 2(136) = 272.

Time = 0.19 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.46

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex)^7}\,dx =$$

$$-\frac{60ad^6 + 137bd^6n + 360ad^5ex + 762bd^5enx + 900ad^4e^2x^2 + 1725bd^4e^2nx^2 + 1200ad^3e^3x^3 + 2000bd^3e^3n}{}$$

```
[In] Integrate[(x^5*(a + b*Log[c*x^n]))/(d + e*x)^7,x]
```

```
[Out] -1/360*(60*a*d^6 + 137*b*d^6*n + 360*a*d^5*e*x + 762*b*d^5*e*n*x + 900*a*d^
4*e^2*x^2 + 1725*b*d^4*e^2*n*x^2 + 1200*a*d^3*e^3*x^3 + 2000*b*d^3*e^3*n*x^
```

```
3 + 900*a*d^2*e^4*x^4 + 1200*b*d^2*e^4*n*x^4 + 360*a*d*e^5*x^5 + 300*b*d*e^
5*n*x^5 - 60*b*n*(d + e*x)^6*Log[x] + 60*b*d*(d^5 + 6*d^4*e*x + 15*d^3*e^2*
x^2 + 20*d^2*e^3*x^3 + 15*d*e^4*x^4 + 6*e^5*x^5)*Log[c*x^n] + 60*b*d^6*n*Lo
g[d + e*x] + 360*b*d^5*e*n*x*Log[d + e*x] + 900*b*d^4*e^2*n*x^2*Log[d + e*x
] + 1200*b*d^3*e^3*n*x^3*Log[d + e*x] + 900*b*d^2*e^4*n*x^4*Log[d + e*x] +
360*b*d*e^5*n*x^5*Log[d + e*x] + 60*b*e^6*n*x^6*Log[d + e*x])/(d*e^6*(d + e
*x)^6)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(122) = 244.

Time = 1.50 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.90

method	result
parallelrisch	$\frac{-900\ln(ex+d)b\,d^2e^4n\,x^4+50b\,e^6n\,x^6-900\ln(ex+d)b\,d^4e^2n\,x^2-87b\,d^6n-360x\ln(c\,x^n)b\,d^5e-900x^2\ln(c\,x^n)b\,d^4e^2-1200x^3\ln(c}{}$
risch	Expression too large to display

```
[In] int(x^5*(a+b*ln(c*x^n))/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/360*(-900*ln(e*x+d)*b*d^2*e^4*n*x^4+50*b*e^6*n*x^6-900*ln(e*x+d)*b*d^4*e^
2*n*x^2-87*b*d^6*n-360*x*ln(c*x^n)*b*d^5*e-900*x^2*ln(c*x^n)*b*d^4*e^2-1200
*x^3*ln(c*x^n)*b*d^3*e^3-900*x^4*ln(c*x^n)*b*d^2*e^4-360*x^5*ln(c*x^n)*b*d*
e^5+60*ln(x)*x^6*b*e^6*n-1200*ln(e*x+d)*b*d^3*e^3*n*x^3-360*ln(e*x+d)*b*d*e
^5*n*x^5-975*b*d^4*e^2*n*x^2-450*b*d^2*e^4*n*x^4-1000*b*d^3*e^3*n*x^3-462*b
*d^5*e*n*x-360*ln(e*x+d)*b*d^5*e*n*x-60*ln(e*x+d)*b*e^6*n*x^6-60*ln(c*x^n)*
b*d^6-60*ln(e*x+d)*b*d^6*n+60*a*e^6*x^6+1200*ln(x)*x^3*b*d^3*e^3*n+900*ln(x
)*x^2*b*d^4*e^2*n+360*ln(x)*x*b*d^5*e*n+360*ln(x)*x^5*b*d*e^5*n+900*ln(x)*x
^4*b*d^2*e^4*n+60*ln(x)*b*d^6*n)/d/e^6/(e*x+d)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(122) = 244.

Time = 0.31 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.65

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex)^7}\,dx$$
$$= \frac{60\,be^6nx^6\log(x) - 137\,bd^6n - 60\,ad^6 - 60\,(5\,bde^5n + 6\,ade^5)x^5 - 300\,(4\,bd^2e^4n + 3\,ad^2e^4)x^4 - 400\,(5\,bd^3e^3}{}$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] 1/360*(60*b*e^6*n*x^6*log(x) - 137*b*d^6*n - 60*a*d^6 - 60*(5*b*d*e^5*n + 6
*a*d*e^5)*x^5 - 300*(4*b*d^2*e^4*n + 3*a*d^2*e^4)*x^4 - 400*(5*b*d^3*e^3*n
```

```
+ 3*a*d^3*e^3)*x^3 - 75*(23*b*d^4*e^2*n + 12*a*d^4*e^2)*x^2 - 6*(127*b*d^5*
e*n + 60*a*d^5*e)*x - 60*(b*e^6*n*x^6 + 6*b*d*e^5*n*x^5 + 15*b*d^2*e^4*n*x^
4 + 20*b*d^3*e^3*n*x^3 + 15*b*d^4*e^2*n*x^2 + 6*b*d^5*e*n*x + b*d^6*n)*log(
e*x + d) - 60*(6*b*d*e^5*x^5 + 15*b*d^2*e^4*x^4 + 20*b*d^3*e^3*x^3 + 15*b*d
^4*e^2*x^2 + 6*b*d^5*e*x + b*d^6)*log(c))/(d*e^12*x^6 + 6*d^2*e^11*x^5 + 15
*d^3*e^10*x^4 + 20*d^4*e^9*x^3 + 15*d^5*e^8*x^2 + 6*d^6*e^7*x + d^7*e^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1911 vs. $2(133) = 266$.

Time = 75.71 (sec) , antiderivative size = 1911, normalized size of antiderivative = 14.05

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex)^7}\,dx = \text{Too large to display}$$

```
[In] integrate(x**5*(a+b*ln(c*x**n))/(e*x+d)**7,x)
```

```
[Out] Piecewise((zoo*(-a/x - b*n/x - b*log(c*x**n)/x), Eq(d, 0) & Eq(e, 0)), ((a*
x**6/6 - b*n*x**6/36 + b*x**6*log(c*x**n)/6)/d**7, Eq(e, 0)), ((-a/x - b*n/
x - b*log(c*x**n)/x)/e**7, Eq(d, 0)), (-60*a*d**6/(360*d**7*e**6 + 2160*d**
6*e**7*x + 5400*d**5*e**8*x**2 + 7200*d**4*e**9*x**3 + 5400*d**3*e**10*x**4
 + 2160*d**2*e**11*x**5 + 360*d*e**12*x**6) - 360*a*d**5*e*x/(360*d**7*e**6
 + 2160*d**6*e**7*x + 5400*d**5*e**8*x**2 + 7200*d**4*e**9*x**3 + 5400*d**3
*e**10*x**4 + 2160*d**2*e**11*x**5 + 360*d*e**12*x**6) - 900*a*d**4*e**2*x*
*2/(360*d**7*e**6 + 2160*d**6*e**7*x + 5400*d**5*e**8*x**2 + 7200*d**4*e**9
*x**3 + 5400*d**3*e**10*x**4 + 2160*d**2*e**11*x**5 + 360*d*e**12*x**6) - 1
200*a*d**3*e**3*x**3/(360*d**7*e**6 + 2160*d**6*e**7*x + 5400*d**5*e**8*x**
2 + 7200*d**4*e**9*x**3 + 5400*d**3*e**10*x**4 + 2160*d**2*e**11*x**5 + 360
*d*e**12*x**6) - 900*a*d**2*e**4*x**4/(360*d**7*e**6 + 2160*d**6*e**7*x + 5
400*d**5*e**8*x**2 + 7200*d**4*e**9*x**3 + 5400*d**3*e**10*x**4 + 2160*d**2
*e**11*x**5 + 360*d*e**12*x**6) - 360*a*d*e**5*x**5/(360*d**7*e**6 + 2160*d
**6*e**7*x + 5400*d**5*e**8*x**2 + 7200*d**4*e**9*x**3 + 5400*d**3*e**10*x*
*4 + 2160*d**2*e**11*x**5 + 360*d*e**12*x**6) - 60*b*d**6*n*log(d/e + x)/(3
60*d**7*e**6 + 2160*d**6*e**7*x + 5400*d**5*e**8*x**2 + 7200*d**4*e**9*x**3
 + 5400*d**3*e**10*x**4 + 2160*d**2*e**11*x**5 + 360*d*e**12*x**6) - 137*b*
d**6*n/(360*d**7*e**6 + 2160*d**6*e**7*x + 5400*d**5*e**8*x**2 + 7200*d**4*
e**9*x**3 + 5400*d**3*e**10*x**4 + 2160*d**2*e**11*x**5 + 360*d*e**12*x**6)
 - 360*b*d**5*e*n*x*log(d/e + x)/(360*d**7*e**6 + 2160*d**6*e**7*x + 5400*d
**5*e**8*x**2 + 7200*d**4*e**9*x**3 + 5400*d**3*e**10*x**4 + 2160*d**2*e**1
1*x**5 + 360*d*e**12*x**6) - 762*b*d**5*e*n*x/(360*d**7*e**6 + 2160*d**6*e*
*7*x + 5400*d**5*e**8*x**2 + 7200*d**4*e**9*x**3 + 5400*d**3*e**10*x**4 + 2
160*d**2*e**11*x**5 + 360*d*e**12*x**6) - 900*b*d**4*e**2*n*x**2*log(d/e +
x)/(360*d**7*e**6 + 2160*d**6*e**7*x + 5400*d**5*e**8*x**2 + 7200*d**4*e**9
*x**3 + 5400*d**3*e**10*x**4 + 2160*d**2*e**11*x**5 + 360*d*e**12*x**6) - 1
725*b*d**4*e**2*n*x**2/(360*d**7*e**6 + 2160*d**6*e**7*x + 5400*d**5*e**8*x
```

```
**2 + 7200*d**4*e**9*x**3 + 5400*d**3*e**10*x**4 + 2160*d**2*e**11*x**5 + 3
60*d*e**12*x**6) - 1200*b*d**3*e**3*n*x**3*log(d/e + x)/(360*d**7*e**6 + 21
60*d**6*e**7*x + 5400*d**5*e**8*x**2 + 7200*d**4*e**9*x**3 + 5400*d**3*e**1
0*x**4 + 2160*d**2*e**11*x**5 + 360*d*e**12*x**6) - 2000*b*d**3*e**3*n*x**3
/(360*d**7*e**6 + 2160*d**6*e**7*x + 5400*d**5*e**8*x**2 + 7200*d**4*e**9*x
**3 + 5400*d**3*e**10*x**4 + 2160*d**2*e**11*x**5 + 360*d*e**12*x**6) - 900
*b*d**2*e**4*n*x**4*log(d/e + x)/(360*d**7*e**6 + 2160*d**6*e**7*x + 5400*d
**5*e**8*x**2 + 7200*d**4*e**9*x**3 + 5400*d**3*e**10*x**4 + 2160*d**2*e**1
1*x**5 + 360*d*e**12*x**6) - 1200*b*d**2*e**4*n*x**4/(360*d**7*e**6 + 2160*
d**6*e**7*x + 5400*d**5*e**8*x**2 + 7200*d**4*e**9*x**3 + 5400*d**3*e**10*x
**4 + 2160*d**2*e**11*x**5 + 360*d*e**12*x**6) - 360*b*d*e**5*n*x**5*log(d/
e + x)/(360*d**7*e**6 + 2160*d**6*e**7*x + 5400*d**5*e**8*x**2 + 7200*d**4*
e**9*x**3 + 5400*d**3*e**10*x**4 + 2160*d**2*e**11*x**5 + 360*d*e**12*x**6)
 - 300*b*d*e**5*n*x**5/(360*d**7*e**6 + 2160*d**6*e**7*x + 5400*d**5*e**8*x
**2 + 7200*d**4*e**9*x**3 + 5400*d**3*e**10*x**4 + 2160*d**2*e**11*x**5 + 3
60*d*e**12*x**6) - 60*b*e**6*n*x**6*log(d/e + x)/(360*d**7*e**6 + 2160*d**6
*e**7*x + 5400*d**5*e**8*x**2 + 7200*d**4*e**9*x**3 + 5400*d**3*e**10*x**4
+ 2160*d**2*e**11*x**5 + 360*d*e**12*x**6) + 60*b*e**6*x**6*log(c*x**n)/(36
0*d**7*e**6 + 2160*d**6*e**7*x + 5400*d**5*e**8*x**2 + 7200*d**4*e**9*x**3
+ 5400*d**3*e**10*x**4 + 2160*d**2*e**11*x**5 + 360*d*e**12*x**6), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(122) = 244$.

Time = 0.23 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.77

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex)^7}\,dx = -\frac{1}{360}\,bn\left(\frac{300\,e^4x^4+900\,de^3x^3+1100\,d^2e^2x^2+625\,d^3ex+137\,d^4}{e^{11}x^5+5\,de^{10}x^4+10\,d^2e^9x^3+10\,d^3e^8x^2+5\,d^4e^7x+d^5e^6}+\frac{60\,\log(ex+d)}{de^6}-\frac{60\,\log(x)}{de^6}\right) - \frac{(6\,e^5x^5+15\,de^4x^4+20\,d^2e^3x^3+15\,d^3e^2x^2+6\,d^4ex+d^5)b\log(cx^n)}{6\,(e^{12}x^6+6\,de^{11}x^5+15\,d^2e^{10}x^4+20\,d^3e^9x^3+15\,d^4e^8x^2+6\,d^5e^7x+d^6e^6)} - \frac{(6\,e^5x^5+15\,de^4x^4+20\,d^2e^3x^3+15\,d^3e^2x^2+6\,d^4ex+d^5)a}{6\,(e^{12}x^6+6\,de^{11}x^5+15\,d^2e^{10}x^4+20\,d^3e^9x^3+15\,d^4e^8x^2+6\,d^5e^7x+d^6e^6)}$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] -1/360*b*n*((300*e^4*x^4 + 900*d*e^3*x^3 + 1100*d^2*e^2*x^2 + 625*d^3*e*x +
 137*d^4)/(e^11*x^5 + 5*d*e^10*x^4 + 10*d^2*e^9*x^3 + 10*d^3*e^8*x^2 + 5*d^
4*e^7*x + d^5*e^6) + 60*log(e*x + d)/(d*e^6) - 60*log(x)/(d*e^6)) - 1/6*(6*
e^5*x^5 + 15*d*e^4*x^4 + 20*d^2*e^3*x^3 + 15*d^3*e^2*x^2 + 6*d^4*e*x + d^5)
*b*log(c*x^n)/(e^12*x^6 + 6*d*e^11*x^5 + 15*d^2*e^10*x^4 + 20*d^3*e^9*x^3 +
 15*d^4*e^8*x^2 + 6*d^5*e^7*x + d^6*e^6) - 1/6*(6*e^5*x^5 + 15*d*e^4*x^4 +
20*d^2*e^3*x^3 + 15*d^3*e^2*x^2 + 6*d^4*e*x + d^5)*a/(e^12*x^6 + 6*d*e^11*x
```

```
^5 + 15*d^2*e^10*x^4 + 20*d^3*e^9*x^3 + 15*d^4*e^8*x^2 + 6*d^5*e^7*x + d^6*
e^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(122) = 244$.

Time = 0.47 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.12

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex)^7}\,dx$$
$$= -\frac{(6\,be^5nx^5 + 15\,bde^4nx^4 + 20\,bd^2e^3nx^3 + 15\,bd^3e^2nx^2 + 6\,bd^4enx + bd^5n)\log(x)}{6\,(e^{12}x^6 + 6\,de^{11}x^5 + 15\,d^2e^{10}x^4 + 20\,d^3e^9x^3 + 15\,d^4e^8x^2 + 6\,d^5e^7x + d^6e^6)}$$
$$- \frac{300\,be^5nx^5 + 360\,be^5x^5\log(c) + 1200\,bde^4nx^4 + 360\,ae^5x^5 + 900\,bde^4x^4\log(c) + 2000\,bd^2e^3nx^3 + 900\,d}{3}$$
$$- \frac{bn\log(ex+d)}{6\,de^6} + \frac{bn\log(x)}{6\,de^6}$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] -1/6*(6*b*e^5*n*x^5 + 15*b*d*e^4*n*x^4 + 20*b*d^2*e^3*n*x^3 + 15*b*d^3*e^2*
n*x^2 + 6*b*d^4*e*n*x + b*d^5*n)*log(x)/(e^12*x^6 + 6*d*e^11*x^5 + 15*d^2*e
^10*x^4 + 20*d^3*e^9*x^3 + 15*d^4*e^8*x^2 + 6*d^5*e^7*x + d^6*e^6) - 1/360*
(300*b*e^5*n*x^5 + 360*b*e^5*x^5*log(c) + 1200*b*d*e^4*n*x^4 + 360*a*e^5*x^
5 + 900*b*d*e^4*x^4*log(c) + 2000*b*d^2*e^3*n*x^3 + 900*a*d*e^4*x^4 + 1200*
b*d^2*e^3*x^3*log(c) + 1725*b*d^3*e^2*n*x^2 + 1200*a*d^2*e^3*x^3 + 900*b*d^
3*e^2*x^2*log(c) + 762*b*d^4*e*n*x + 900*a*d^3*e^2*x^2 + 360*b*d^4*e*x*log(
c) + 137*b*d^5*n + 360*a*d^4*e*x + 60*b*d^5*log(c) + 60*a*d^5)/(e^12*x^6 +
6*d*e^11*x^5 + 15*d^2*e^10*x^4 + 20*d^3*e^9*x^3 + 15*d^4*e^8*x^2 + 6*d^5*e^
7*x + d^6*e^6) - 1/6*b*n*log(e*x + d)/(d*e^6) + 1/6*b*n*log(x)/(d*e^6)
```

Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.51

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex)^7}\,dx =$$
$$-\frac{x^5\,(6\,a\,e^5+5\,b\,e^5\,n)+x\,\left(6\,a\,d^4\,e+\frac{127\,b\,d^4\,e\,n}{10}\right)+a\,d^5+x^3\,\left(20\,a\,d^2\,e^3+\frac{100\,b\,d^2\,e^3\,n}{3}\right)+x^2\,\left(15\,a\,d^3\,e^2+\frac{11}{}\right.}{6\,d^6\,e^6+36\,d^5\,e^7\,x+90\,d^4\,e^8\,x^2+120\,d^3\,e^9\,x^3+90\,d^2\,e^{10}\,x^4+36\,d\,e^{1}}$$
$$-\frac{\ln(c\,x^n)\,\left(\frac{b\,d^5}{6\,e^6}+\frac{b\,x^5}{e}+\frac{10\,b\,d^2\,x^3}{3\,e^3}+\frac{5\,b\,d^3\,x^2}{2\,e^4}+\frac{5\,b\,d\,x^4}{2\,e^2}+\frac{b\,d^4\,x}{e^5}\right)}{d^6+6\,d^5\,e\,x+15\,d^4\,e^2\,x^2+20\,d^3\,e^3\,x^3+15\,d^2\,e^4\,x^4+6\,d\,e^5\,x^5+e^6\,x^6}$$
$$-\frac{b\,n\,\mathrm{atanh}\left(\frac{2\,e\,x}{d}+1\right)}{3\,d\,e^6}$$

```
[In] int((x^5*(a + b*log(c*x^n)))/(d + e*x)^7,x)
```

```
[Out] - (x^5*(6*a*e^5 + 5*b*e^5*n) + x*(6*a*d^4*e + (127*b*d^4*e*n)/10) + a*d^5 +
 x^3*(20*a*d^2*e^3 + (100*b*d^2*e^3*n)/3) + x^2*(15*a*d^3*e^2 + (115*b*d^3*
e^2*n)/4) + x^4*(15*a*d*e^4 + 20*b*d*e^4*n) + (137*b*d^5*n)/60)/(6*d^6*e^6
+ 6*e^12*x^6 + 36*d^5*e^7*x + 36*d*e^11*x^5 + 90*d^4*e^8*x^2 + 120*d^3*e^9*
x^3 + 90*d^2*e^10*x^4) - (log(c*x^n)*((b*d^5)/(6*e^6) + (b*x^5)/e + (10*b*d
^2*x^3)/(3*e^3) + (5*b*d^3*x^2)/(2*e^4) + (5*b*d*x^4)/(2*e^2) + (b*d^4*x)/e
^5))/(d^6 + e^6*x^6 + 6*d*e^5*x^5 + 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 + 15*d^
2*e^4*x^4 + 6*d^5*e*x) - (b*n*atanh((2*e*x)/d + 1))/(3*d*e^6)
```

3.66 $\int \frac{x^4(a+b\log(cx^n))}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 163

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex)^7}\,dx = -\frac{bnx^5}{30d^2(d+ex)^5}+\frac{bd^2n}{120e^5(d+ex)^4}-\frac{2bdn}{45e^5(d+ex)^3}+\frac{bn}{10e^5(d+ex)^2}-\frac{2bn}{15de^5(d+ex)}+\frac{x^5(a+b\log(cx^n))}{6d(d+ex)^6}+\frac{x^5(a+b\log(cx^n))}{30d^2(d+ex)^5}-\frac{bn\log(d+ex)}{30d^2e^5}$$

```
[Out] -1/30*b*n*x^5/d^2/(e*x+d)^5+1/120*b*d^2*n/e^5/(e*x+d)^4-2/45*b*d*n/e^5/(e*x
+d)^3+1/10*b*n/e^5/(e*x+d)^2-2/15*b*n/d/e^5/(e*x+d)+1/6*x^5*(a+b*ln(c*x^n))
/d/(e*x+d)^6+1/30*x^5*(a+b*ln(c*x^n))/d^2/(e*x+d)^5-1/30*b*n*ln(e*x+d)/d^2/
e^5
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {47, 37, 2382, 12, 79, 45}

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex)^7}\,dx = \frac{x^5(a+b\log(cx^n))}{30d^2(d+ex)^5}+\frac{x^5(a+b\log(cx^n))}{6d(d+ex)^6}+\frac{bd^2n}{120e^5(d+ex)^4}-\frac{bn\log(d+ex)}{30d^2e^5}-\frac{bnx^5}{30d^2(d+ex)^5}-\frac{2bn}{15de^5(d+ex)}+\frac{bn}{10e^5(d+ex)^2}-\frac{2bdn}{45e^5(d+ex)^3}$$

```
[In] Int[(x^4*(a + b*Log[c*x^n]))/(d + e*x)^7,x]
```

```
[Out] -1/30*(b*n*x^5)/(d^2*(d + e*x)^5) + (b*d^2*n)/(120*e^5*(d + e*x)^4) - (2*b*
d*n)/(45*e^5*(d + e*x)^3) + (b*n)/(10*e^5*(d + e*x)^2) - (2*b*n)/(15*d*e^5*
(d + e*x)) + (x^5*(a + b*Log[c*x^n]))/(6*d*(d + e*x)^6) + (x^5*(a + b*Log[c
*x^n]))/(30*d^2*(d + e*x)^5) - (b*n*Log[d + e*x])/(30*d^2*e^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] || !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
```

```
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^5(a+b\log(cx^n))}{6d(d+ex)^6}+\frac{x^5(a+b\log(cx^n))}{30d^2(d+ex)^5}-(bn)\int\frac{x^4(6d+ex)}{30d^2(d+ex)^6}\,dx\\
&= \frac{x^5(a+b\log(cx^n))}{6d(d+ex)^6}+\frac{x^5(a+b\log(cx^n))}{30d^2(d+ex)^5}-\frac{(bn)\int\frac{x^4(6d+ex)}{(d+ex)^6}\,dx}{30d^2}\\
&= -\frac{bnx^5}{30d^2(d+ex)^5}+\frac{x^5(a+b\log(cx^n))}{6d(d+ex)^6}+\frac{x^5(a+b\log(cx^n))}{30d^2(d+ex)^5}-\frac{(bn)\int\frac{x^4}{(d+ex)^5}\,dx}{30d^2}\\
&= -\frac{bnx^5}{30d^2(d+ex)^5}+\frac{x^5(a+b\log(cx^n))}{6d(d+ex)^6}+\frac{x^5(a+b\log(cx^n))}{30d^2(d+ex)^5}\\
&\quad-\frac{(bn)\int\left(\frac{d^4}{e^4(d+ex)^5}-\frac{4d^3}{e^4(d+ex)^4}+\frac{6d^2}{e^4(d+ex)^3}-\frac{4d}{e^4(d+ex)^2}+\frac{1}{e^4(d+ex)}\right)dx}{30d^2}\\
&= -\frac{bnx^5}{30d^2(d+ex)^5}+\frac{bd^2n}{120e^5(d+ex)^4}-\frac{2bdn}{45e^5(d+ex)^3}+\frac{bn}{10e^5(d+ex)^2}\\
&\quad-\frac{2bn}{15de^5(d+ex)}+\frac{x^5(a+b\log(cx^n))}{6d(d+ex)^6}+\frac{x^5(a+b\log(cx^n))}{30d^2(d+ex)^5}-\frac{bn\log(d+ex)}{30d^2e^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.94

$$
\int\frac{x^4(a+b\log(cx^n))}{(d+ex)^7}\,dx = -\frac{12ad^6+13bd^6n+72ad^5ex+66bd^5enx+180ad^4e^2x^2+129bd^4e^2nx^2+240ad^3e^3x^3+112bd^3e^3nx^3+1\ldots}{}
$$

```
[In] Integrate[(x^4*(a + b*Log[c*x^n]))/(d + e*x)^7,x]
```

```
[Out] -1/360*(12*a*d^6 + 13*b*d^6*n + 72*a*d^5*e*x + 66*b*d^5*e*n*x + 180*a*d^4*e
^2*x^2 + 129*b*d^4*e^2*n*x^2 + 240*a*d^3*e^3*x^3 + 112*b*d^3*e^3*n*x^3 + 18
0*a*d^2*e^4*x^4 + 24*b*d^2*e^4*n*x^4 - 12*b*d*e^5*n*x^5 - 12*b*n*(d + e*x)^
6*Log[x] + 12*b*d^2*(d^4 + 6*d^3*e*x + 15*d^2*e^2*x^2 + 20*d*e^3*x^3 + 15*e
^4*x^4)*Log[c*x^n] + 12*b*d^6*n*Log[d + e*x] + 72*b*d^5*e*n*x*Log[d + e*x]
+ 180*b*d^4*e^2*n*x^2*Log[d + e*x] + 240*b*d^3*e^3*n*x^3*Log[d + e*x] + 180
*b*d^2*e^4*n*x^4*Log[d + e*x] + 72*b*d*e^5*n*x^5*Log[d + e*x] + 12*b*e^6*n*
x^6*Log[d + e*x])/(d^2*e^5*(d + e*x)^6)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(147) = 294$.

Time = 1.35 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.67

method	result
parallelrisch	$\frac{-72x\ln(cx^n)b\,d^5e^2-180x^2\ln(cx^n)b\,d^4e^3-240x^3\ln(cx^n)b\,d^3e^4-180x^4\ln(cx^n)b\,d^2e^5-78xb\,d^5e^2n-159x^2b\,d^4e^3n-152x^3b\,d^3e}{}$
risch	Expression too large to display

```
[In] int(x^4*(a+b*ln(c*x^n))/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/360*(-72*x*ln(c*x^n)*b*d^5*e^2-180*x^2*ln(c*x^n)*b*d^4*e^3-240*x^3*ln(c*x
^n)*b*d^3*e^4-180*x^4*ln(c*x^n)*b*d^2*e^5-78*x*b*d^5*e^2*n-159*x^2*b*d^4*e^
3*n-152*x^3*b*d^3*e^4*n-54*x^4*b*d^2*e^5*n+12*ln(x)*x^6*b*e^7*n-12*ln(e*x+d
)*x^6*b*e^7*n+12*ln(x)*b*d^6*e*n-12*ln(e*x+d)*b*d^6*e*n-12*a*d^6*e-240*ln(e
*x+d)*x^3*b*d^3*e^4*n+180*ln(x)*x^2*b*d^4*e^3*n-180*ln(e*x+d)*x^2*b*d^4*e^3
*n+72*ln(x)*x*b*d^5*e^2*n-72*ln(e*x+d)*x*b*d^5*e^2*n+72*ln(x)*x^5*b*d*e^6*n
-72*ln(e*x+d)*x^5*b*d*e^6*n+180*ln(x)*x^4*b*d^2*e^5*n-180*ln(e*x+d)*x^4*b*d
^2*e^5*n+240*ln(x)*x^3*b*d^3*e^4*n-15*b*d^6*e*n-12*ln(c*x^n)*b*d^6*e-2*x^6*
b*e^7*n-72*x*a*d^5*e^2-180*x^2*a*d^4*e^3-240*x^3*a*d^3*e^4-180*x^4*a*d^2*e^
5)/d^2/e^6/(e*x+d)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(147) = 294$.

Time = 0.31 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.18

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex)^7}\,dx = \frac{12\,bde^5nx^5-13\,bd^6n-12\,ad^6-12\,(2\,bd^2e^4n+15\,ad^2e^4)x^4-16\,(7\,bd^3e^3n+15\,ad^3e^3)x^3-3\,(43\,bd^4e^2n+}{}$$

```
[In] integrate(x^4*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] 1/360*(12*b*d*e^5*n*x^5 - 13*b*d^6*n - 12*a*d^6 - 12*(2*b*d^2*e^4*n + 15*a*
d^2*e^4)*x^4 - 16*(7*b*d^3*e^3*n + 15*a*d^3*e^3)*x^3 - 3*(43*b*d^4*e^2*n +
60*a*d^4*e^2)*x^2 - 6*(11*b*d^5*e*n + 12*a*d^5*e)*x - 12*(b*e^6*n*x^6 + 6*b
*d*e^5*n*x^5 + 15*b*d^2*e^4*n*x^4 + 20*b*d^3*e^3*n*x^3 + 15*b*d^4*e^2*n*x^2
 + 6*b*d^5*e*n*x + b*d^6*n)*log(e*x + d) - 12*(15*b*d^2*e^4*x^4 + 20*b*d^3*
e^3*x^3 + 15*b*d^4*e^2*x^2 + 6*b*d^5*e*x + b*d^6)*log(c) + 12*(b*e^6*n*x^6
+ 6*b*d*e^5*n*x^5)*log(x))/(d^2*e^11*x^6 + 6*d^3*e^10*x^5 + 15*d^4*e^9*x^4
+ 20*d^5*e^8*x^3 + 15*d^6*e^7*x^2 + 6*d^7*e^6*x + d^8*e^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1972 vs. 2(155) = 310.

Time = 75.51 (sec) , antiderivative size = 1972, normalized size of antiderivative = 12.10

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex)^7}\,dx = \text{Too large to display}$$

```
[In] integrate(x**4*(a+b*ln(c*x**n))/(e*x+d)**7,x)
```

```
[Out] Piecewise((zoo*(-a/(2*x**2) - b*n/(4*x**2) - b*log(c*x**n)/(2*x**2)), Eq(d,
 0) & Eq(e, 0)), ((a*x**5/5 - b*n*x**5/25 + b*x**5*log(c*x**n)/5)/d**7, Eq(
e, 0)), ((-a/(2*x**2) - b*n/(4*x**2) - b*log(c*x**n)/(2*x**2))/e**7, Eq(d,
0)), (-12*a*d**6/(360*d**8*e**5 + 2160*d**7*e**6*x + 5400*d**6*e**7*x**2 +
7200*d**5*e**8*x**3 + 5400*d**4*e**9*x**4 + 2160*d**3*e**10*x**5 + 360*d**2
*e**11*x**6) - 72*a*d**5*e*x/(360*d**8*e**5 + 2160*d**7*e**6*x + 5400*d**6*
e**7*x**2 + 7200*d**5*e**8*x**3 + 5400*d**4*e**9*x**4 + 2160*d**3*e**10*x**
5 + 360*d**2*e**11*x**6) - 180*a*d**4*e**2*x**2/(360*d**8*e**5 + 2160*d**7*
e**6*x + 5400*d**6*e**7*x**2 + 7200*d**5*e**8*x**3 + 5400*d**4*e**9*x**4 +
2160*d**3*e**10*x**5 + 360*d**2*e**11*x**6) - 240*a*d**3*e**3*x**3/(360*d**
8*e**5 + 2160*d**7*e**6*x + 5400*d**6*e**7*x**2 + 7200*d**5*e**8*x**3 + 540
0*d**4*e**9*x**4 + 2160*d**3*e**10*x**5 + 360*d**2*e**11*x**6) - 180*a*d**2
*e**4*x**4/(360*d**8*e**5 + 2160*d**7*e**6*x + 5400*d**6*e**7*x**2 + 7200*d
**5*e**8*x**3 + 5400*d**4*e**9*x**4 + 2160*d**3*e**10*x**5 + 360*d**2*e**11
*x**6) - 12*b*d**6*n*log(d/e + x)/(360*d**8*e**5 + 2160*d**7*e**6*x + 5400*
d**6*e**7*x**2 + 7200*d**5*e**8*x**3 + 5400*d**4*e**9*x**4 + 2160*d**3*e**1
0*x**5 + 360*d**2*e**11*x**6) - 13*b*d**6*n/(360*d**8*e**5 + 2160*d**7*e**6
*x + 5400*d**6*e**7*x**2 + 7200*d**5*e**8*x**3 + 5400*d**4*e**9*x**4 + 2160
*d**3*e**10*x**5 + 360*d**2*e**11*x**6) - 72*b*d**5*e*n*x*log(d/e + x)/(360
*d**8*e**5 + 2160*d**7*e**6*x + 5400*d**6*e**7*x**2 + 7200*d**5*e**8*x**3 +
 5400*d**4*e**9*x**4 + 2160*d**3*e**10*x**5 + 360*d**2*e**11*x**6) - 66*b*d
**5*e*n*x/(360*d**8*e**5 + 2160*d**7*e**6*x + 5400*d**6*e**7*x**2 + 7200*d*
*5*e**8*x**3 + 5400*d**4*e**9*x**4 + 2160*d**3*e**10*x**5 + 360*d**2*e**11*
x**6) - 180*b*d**4*e**2*n*x**2*log(d/e + x)/(360*d**8*e**5 + 2160*d**7*e**6
*x + 5400*d**6*e**7*x**2 + 7200*d**5*e**8*x**3 + 5400*d**4*e**9*x**4 + 2160
*d**3*e**10*x**5 + 360*d**2*e**11*x**6) - 129*b*d**4*e**2*n*x**2/(360*d**8*
e**5 + 2160*d**7*e**6*x + 5400*d**6*e**7*x**2 + 7200*d**5*e**8*x**3 + 5400*
d**4*e**9*x**4 + 2160*d**3*e**10*x**5 + 360*d**2*e**11*x**6) - 240*b*d**3*e
**3*n*x**3*log(d/e + x)/(360*d**8*e**5 + 2160*d**7*e**6*x + 5400*d**6*e**7*
x**2 + 7200*d**5*e**8*x**3 + 5400*d**4*e**9*x**4 + 2160*d**3*e**10*x**5 + 3
60*d**2*e**11*x**6) - 112*b*d**3*e**3*n*x**3/(360*d**8*e**5 + 2160*d**7*e**
6*x + 5400*d**6*e**7*x**2 + 7200*d**5*e**8*x**3 + 5400*d**4*e**9*x**4 + 216
0*d**3*e**10*x**5 + 360*d**2*e**11*x**6) - 180*b*d**2*e**4*n*x**4*log(d/e +
 x)/(360*d**8*e**5 + 2160*d**7*e**6*x + 5400*d**6*e**7*x**2 + 7200*d**5*e**
8*x**3 + 5400*d**4*e**9*x**4 + 2160*d**3*e**10*x**5 + 360*d**2*e**11*x**6)
```

```
- 24*b*d**2*e**4*n*x**4/(360*d**8*e**5 + 2160*d**7*e**6*x + 5400*d**6*e**7*
x**2 + 7200*d**5*e**8*x**3 + 5400*d**4*e**9*x**4 + 2160*d**3*e**10*x**5 + 3
60*d**2*e**11*x**6) - 72*b*d*e**5*n*x**5*log(d/e + x)/(360*d**8*e**5 + 2160
*d**7*e**6*x + 5400*d**6*e**7*x**2 + 7200*d**5*e**8*x**3 + 5400*d**4*e**9*x
**4 + 2160*d**3*e**10*x**5 + 360*d**2*e**11*x**6) + 12*b*d*e**5*n*x**5/(360
*d**8*e**5 + 2160*d**7*e**6*x + 5400*d**6*e**7*x**2 + 7200*d**5*e**8*x**3 +
 5400*d**4*e**9*x**4 + 2160*d**3*e**10*x**5 + 360*d**2*e**11*x**6) + 72*b*d
*e**5*x**5*log(c*x**n)/(360*d**8*e**5 + 2160*d**7*e**6*x + 5400*d**6*e**7*x
**2 + 7200*d**5*e**8*x**3 + 5400*d**4*e**9*x**4 + 2160*d**3*e**10*x**5 + 36
0*d**2*e**11*x**6) - 12*b*e**6*n*x**6*log(d/e + x)/(360*d**8*e**5 + 2160*d*
*7*e**6*x + 5400*d**6*e**7*x**2 + 7200*d**5*e**8*x**3 + 5400*d**4*e**9*x**4
 + 2160*d**3*e**10*x**5 + 360*d**2*e**11*x**6) + 12*b*e**6*x**6*log(c*x**n)
/(360*d**8*e**5 + 2160*d**7*e**6*x + 5400*d**6*e**7*x**2 + 7200*d**5*e**8*x
**3 + 5400*d**4*e**9*x**4 + 2160*d**3*e**10*x**5 + 360*d**2*e**11*x**6), Tr
ue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(147) = 294$.

Time = 0.21 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.20

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex)^7}\,dx$$
$$= \frac{1}{360}\,bn\left(\frac{12\,e^4x^4-36\,de^3x^3-76\,d^2e^2x^2-53\,d^3ex-13\,d^4}{de^{10}x^5+5\,d^2e^9x^4+10\,d^3e^8x^3+10\,d^4e^7x^2+5\,d^5e^6x+d^6e^5}-\frac{12\,\log(ex+d)}{d^2e^5}+\frac{12\,\log(x)}{d^2e^5}\right)$$
$$-\frac{(15\,e^4x^4+20\,de^3x^3+15\,d^2e^2x^2+6\,d^3ex+d^4)b\log(cx^n)}{30\,(e^{11}x^6+6\,de^{10}x^5+15\,d^2e^9x^4+20\,d^3e^8x^3+15\,d^4e^7x^2+6\,d^5e^6x+d^6e^5)}$$
$$-\frac{(15\,e^4x^4+20\,de^3x^3+15\,d^2e^2x^2+6\,d^3ex+d^4)a}{30\,(e^{11}x^6+6\,de^{10}x^5+15\,d^2e^9x^4+20\,d^3e^8x^3+15\,d^4e^7x^2+6\,d^5e^6x+d^6e^5)}$$

```
[In] integrate(x^4*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] 1/360*b*n*((12*e^4*x^4 - 36*d*e^3*x^3 - 76*d^2*e^2*x^2 - 53*d^3*e*x - 13*d^
4)/(d*e^10*x^5 + 5*d^2*e^9*x^4 + 10*d^3*e^8*x^3 + 10*d^4*e^7*x^2 + 5*d^5*e^
6*x + d^6*e^5) - 12*log(e*x + d)/(d^2*e^5) + 12*log(x)/(d^2*e^5)) - 1/30*(1
5*e^4*x^4 + 20*d*e^3*x^3 + 15*d^2*e^2*x^2 + 6*d^3*e*x + d^4)*b*log(c*x^n)/(
e^11*x^6 + 6*d*e^10*x^5 + 15*d^2*e^9*x^4 + 20*d^3*e^8*x^3 + 15*d^4*e^7*x^2
+ 6*d^5*e^6*x + d^6*e^5) - 1/30*(15*e^4*x^4 + 20*d*e^3*x^3 + 15*d^2*e^2*x^2
 + 6*d^3*e*x + d^4)*a/(e^11*x^6 + 6*d*e^10*x^5 + 15*d^2*e^9*x^4 + 20*d^3*e^
8*x^3 + 15*d^4*e^7*x^2 + 6*d^5*e^6*x + d^6*e^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(147) = 294$.

Time = 0.38 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.42

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex)^7}\,dx$$
$$= -\frac{(15\,be^4nx^4 + 20\,bde^3nx^3 + 15\,bd^2e^2nx^2 + 6\,bd^3enx + bd^4n)\log(x)}{30\,(e^{11}x^6 + 6\,de^{10}x^5 + 15\,d^2e^9x^4 + 20\,d^3e^8x^3 + 15\,d^4e^7x^2 + 6\,d^5e^6x + d^6e^5)}$$
$$+ \frac{12\,be^5nx^5 - 24\,bde^4nx^4 - 180\,bde^4x^4\log(c) - 112\,bd^2e^3nx^3 - 180\,ade^4x^4 - 240\,bd^2e^3x^3\log(c) - 129\,bd}{360\,(de^{11}x^6 + 6\,d^2e^{10}x^5}$$
$$- \frac{bn\log(ex+d)}{30\,d^2e^5} + \frac{bn\log(x)}{30\,d^2e^5}$$

```
[In] integrate(x^4*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] -1/30*(15*b*e^4*n*x^4 + 20*b*d*e^3*n*x^3 + 15*b*d^2*e^2*n*x^2 + 6*b*d^3*e*n
*x + b*d^4*n)*log(x)/(e^11*x^6 + 6*d*e^10*x^5 + 15*d^2*e^9*x^4 + 20*d^3*e^8
*x^3 + 15*d^4*e^7*x^2 + 6*d^5*e^6*x + d^6*e^5) + 1/360*(12*b*e^5*n*x^5 - 24
*b*d*e^4*n*x^4 - 180*b*d*e^4*x^4*log(c) - 112*b*d^2*e^3*n*x^3 - 180*a*d*e^4
*x^4 - 240*b*d^2*e^3*x^3*log(c) - 129*b*d^3*e^2*n*x^2 - 240*a*d^2*e^3*x^3 -
 180*b*d^3*e^2*x^2*log(c) - 66*b*d^4*e*n*x - 180*a*d^3*e^2*x^2 - 72*b*d^4*e
*x*log(c) - 13*b*d^5*n - 72*a*d^4*e*x - 12*b*d^5*log(c) - 12*a*d^5)/(d*e^11
*x^6 + 6*d^2*e^10*x^5 + 15*d^3*e^9*x^4 + 20*d^4*e^8*x^3 + 15*d^5*e^7*x^2 +
6*d^6*e^6*x + d^7*e^5) - 1/30*b*n*log(e*x + d)/(d^2*e^5) + 1/30*b*n*log(x)/
(d^2*e^5)
```

Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.96

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex)^7}\,dx =$$
$$-\frac{x^4\,(15\,a\,e^4 + 2\,b\,e^4\,n) + x\left(6\,a\,d^3\,e + \frac{11\,b\,d^3\,e\,n}{2}\right) + a\,d^4 + x^2\left(15\,a\,d^2\,e^2 + \frac{43\,b\,d^2\,e^2\,n}{4}\right) + x^3\left(20\,a\,d\,e^3 + \frac{28\,b}{}\right.}{30\,d^6\,e^5 + 180\,d^5\,e^6\,x + 450\,d^4\,e^7\,x^2 + 600\,d^3\,e^8\,x^3 + 450\,d^2\,e^9\,x^4 + 180\,d\,e^{10}\,x^5 + 30}$$
$$-\frac{\ln(c\,x^n)\left(\frac{b\,d^4}{30\,e^5} + \frac{b\,x^4}{2\,e} + \frac{b\,d^2\,x^2}{2\,e^3} + \frac{2\,b\,d\,x^3}{3\,e^2} + \frac{b\,d^3\,x}{5\,e^4}\right)}{d^6 + 6\,d^5\,e\,x + 15\,d^4\,e^2\,x^2 + 20\,d^3\,e^3\,x^3 + 15\,d^2\,e^4\,x^4 + 6\,d\,e^5\,x^5 + e^6\,x^6}$$
$$-\frac{b\,n\,\mathrm{atanh}\left(\frac{2\,e\,x}{d} + 1\right)}{15\,d^2\,e^5}$$

```
[In] int((x^4*(a + b*log(c*x^n)))/(d + e*x)^7,x)
```

```
[Out] - (x^4*(15*a*e^4 + 2*b*e^4*n) + x*(6*a*d^3*e + (11*b*d^3*e*n)/2) + a*d^4 +
x^2*(15*a*d^2*e^2 + (43*b*d^2*e^2*n)/4) + x^3*(20*a*d*e^3 + (28*b*d*e^3*n)/
```

```
3) + (13*b*d^4*n)/12 - (b*e^5*n*x^5)/d)/(30*d^6*e^5 + 30*e^11*x^6 + 180*d^5
*e^6*x + 180*d*e^10*x^5 + 450*d^4*e^7*x^2 + 600*d^3*e^8*x^3 + 450*d^2*e^9*x
^4) - (log(c*x^n)*((b*d^4)/(30*e^5) + (b*x^4)/(2*e) + (b*d^2*x^2)/(2*e^3) +
 (2*b*d*x^3)/(3*e^2) + (b*d^3*x)/(5*e^4)))/(d^6 + e^6*x^6 + 6*d*e^5*x^5 + 1
5*d^4*e^2*x^2 + 20*d^3*e^3*x^3 + 15*d^2*e^4*x^4 + 6*d^5*e*x) - (b*n*atanh((
2*e*x)/d + 1))/(15*d^2*e^5)
```

3.67 $\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 226

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^7}\,dx = -\frac{bd^2n}{30e^4(d+ex)^5}+\frac{13bdn}{120e^4(d+ex)^4}-\frac{19bn}{180e^4(d+ex)^3}+\frac{bn}{120de^4(d+ex)^2}+\frac{bn}{60d^2e^4(d+ex)}+\frac{bn\log(x)}{60d^3e^4}+\frac{d^3(a+b\log(cx^n))}{6e^4(d+ex)^6}-\frac{3d^2(a+b\log(cx^n))}{5e^4(d+ex)^5}+\frac{3d(a+b\log(cx^n))}{4e^4(d+ex)^4}-\frac{a+b\log(cx^n)}{3e^4(d+ex)^3}-\frac{bn\log(d+ex)}{60d^3e^4}$$

```
[Out] -1/30*b*d^2*n/e^4/(e*x+d)^5+13/120*b*d*n/e^4/(e*x+d)^4-19/180*b*n/e^4/(e*x+
d)^3+1/120*b*n/d/e^4/(e*x+d)^2+1/60*b*n/d^2/e^4/(e*x+d)+1/60*b*n*ln(x)/d^3/
e^4+1/6*d^3*(a+b*ln(c*x^n))/e^4/(e*x+d)^6-3/5*d^2*(a+b*ln(c*x^n))/e^4/(e*x+
d)^5+3/4*d*(a+b*ln(c*x^n))/e^4/(e*x+d)^4+1/3*(-a-b*ln(c*x^n))/e^4/(e*x+d)^3
-1/60*b*n*ln(e*x+d)/d^3/e^4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used

= {45, 2382, 12, 1634}

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^7}\,dx = \frac{d^3(a+b\log(cx^n))}{6e^4(d+ex)^6} - \frac{3d^2(a+b\log(cx^n))}{5e^4(d+ex)^5} + \frac{3d(a+b\log(cx^n))}{4e^4(d+ex)^4} - \frac{a+b\log(cx^n)}{3e^4(d+ex)^3} + \frac{bn\log(x)}{60d^3e^4} - \frac{bn\log(d+ex)}{60d^3e^4} - \frac{bd^2n}{30e^4(d+ex)^5} + \frac{bn}{60d^2e^4(d+ex)} + \frac{13bdn}{120e^4(d+ex)^4} - \frac{19bn}{180e^4(d+ex)^3} + \frac{bn}{120de^4(d+ex)^2}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n]))/(d + e*x)^7,x]
```

```
[Out] -1/30*(b*d^2*n)/(e^4*(d + e*x)^5) + (13*b*d*n)/(120*e^4*(d + e*x)^4) - (19*
b*n)/(180*e^4*(d + e*x)^3) + (b*n)/(120*d*e^4*(d + e*x)^2) + (b*n)/(60*d^2*
e^4*(d + e*x)) + (b*n*Log[x])/(60*d^3*e^4) + (d^3*(a + b*Log[c*x^n]))/(6*e^
4*(d + e*x)^6) - (3*d^2*(a + b*Log[c*x^n]))/(5*e^4*(d + e*x)^5) + (3*d*(a +
 b*Log[c*x^n]))/(4*e^4*(d + e*x)^4) - (a + b*Log[c*x^n])/(3*e^4*(d + e*x)^3
) - (b*n*Log[d + e*x])/(60*d^3*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{d^3(a+b\log(cx^n))}{6e^4(d+ex)^6} - \frac{3d^2(a+b\log(cx^n))}{5e^4(d+ex)^5} + \frac{3d(a+b\log(cx^n))}{4e^4(d+ex)^4} - \frac{a+b\log(cx^n)}{3e^4(d+ex)^3} - (bn)\int \frac{-d^3-6d^2ex-15de^2x^2-20e^3x^3}{60e^4x(d+ex)^6}\,dx$$

$$= \frac{d^3(a+b\log(cx^n))}{6e^4(d+ex)^6} - \frac{3d^2(a+b\log(cx^n))}{5e^4(d+ex)^5} + \frac{3d(a+b\log(cx^n))}{4e^4(d+ex)^4} - \frac{a+b\log(cx^n)}{3e^4(d+ex)^3} - \frac{(bn)\int \frac{-d^3-6d^2ex-15de^2x^2-20e^3x^3}{x(d+ex)^6}\,dx}{60e^4}$$

$$= \frac{d^3(a+b\log(cx^n))}{6e^4(d+ex)^6} - \frac{3d^2(a+b\log(cx^n))}{5e^4(d+ex)^5} + \frac{3d(a+b\log(cx^n))}{4e^4(d+ex)^4} - \frac{a+b\log(cx^n)}{3e^4(d+ex)^3} - \frac{(bn)\int \left(-\frac{1}{d^3x} - \frac{10d^2e}{(d+ex)^6} + \frac{26de}{(d+ex)^5} - \frac{19e}{(d+ex)^4} + \frac{e}{d(d+ex)^3} + \frac{e}{d^2(d+ex)^2} + \frac{e}{d^3(d+ex)}\right)dx}{60e^4}$$

$$= -\frac{bd^2n}{30e^4(d+ex)^5} + \frac{13bdn}{120e^4(d+ex)^4} - \frac{19bn}{180e^4(d+ex)^3} + \frac{bn}{120de^4(d+ex)^2} + \frac{bn}{60d^2e^4(d+ex)} + \frac{bn\log(x)}{60d^3e^4} + \frac{d^3(a+b\log(cx^n))}{6e^4(d+ex)^6} - \frac{3d^2(a+b\log(cx^n))}{5e^4(d+ex)^5} + \frac{3d(a+b\log(cx^n))}{4e^4(d+ex)^4} - \frac{a+b\log(cx^n)}{3e^4(d+ex)^3} - \frac{bn\log(d+ex)}{60d^3e^4}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.24

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^7}\,dx = \frac{ad^3}{6e^4(d+ex)^6} - \frac{3ad^2}{5e^4(d+ex)^5} - \frac{bd^2n}{30e^4(d+ex)^5} + \frac{3ad}{4e^4(d+ex)^4} + \frac{13bdn}{120e^4(d+ex)^4} - \frac{a}{3e^4(d+ex)^3} - \frac{19bn}{180e^4(d+ex)^3} + \frac{bn}{120de^4(d+ex)^2} + \frac{bn}{60d^2e^4(d+ex)} + \frac{bn\log(x)}{60d^3e^4} + \frac{bd^3\log(cx^n)}{6e^4(d+ex)^6} - \frac{3bd^2\log(cx^n)}{5e^4(d+ex)^5} + \frac{3bd\log(cx^n)}{4e^4(d+ex)^4} - \frac{b\log(cx^n)}{3e^4(d+ex)^3} - \frac{bn\log(d+ex)}{60d^3e^4}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/(d + e*x)^7,x]
```

```
[Out] (a*d^3)/(6*e^4*(d + e*x)^6) - (3*a*d^2)/(5*e^4*(d + e*x)^5) - (b*d^2*n)/(30
*e^4*(d + e*x)^5) + (3*a*d)/(4*e^4*(d + e*x)^4) + (13*b*d*n)/(120*e^4*(d +
e*x)^4) - a/(3*e^4*(d + e*x)^3) - (19*b*n)/(180*e^4*(d + e*x)^3) + (b*n)/(1
20*d*e^4*(d + e*x)^2) + (b*n)/(60*d^2*e^4*(d + e*x)) + (b*n*Log[x])/(60*d^3
*e^4) + (b*d^3*Log[c*x^n])/(6*e^4*(d + e*x)^6) - (3*b*d^2*Log[c*x^n])/(5*e^
4*(d + e*x)^5) + (3*b*d*Log[c*x^n])/(4*e^4*(d + e*x)^4) - (b*Log[c*x^n])/(3
*e^4*(d + e*x)^3) - (b*n*Log[d + e*x])/(60*d^3*e^4)
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.83

method	result
parallelrisch	$\frac{-450x^2\ln(c\,x^n)b\,d^4e^4-600x^3\ln(c\,x^n)b\,d^3e^5-36x^5bd\,e^7n-96xb\,d^5e^3n-150x^2b\,d^4e^4n-50x^3b\,d^3e^5n+30\ln(x)x^6b\,e^8n-30\ln(ex-}{}$
risch	$-\frac{b(20e^3x^3+15d\,e^2x^2+6d^2ex+d^3)\ln(x^n)}{60(ex+d)^6e^4}+\frac{-90\ln(ex+d)b\,d^2e^4n\,x^4-120\ln(c)b\,d^3e^3x^3-90\ln(c)b\,d^4e^2x^2-36\ln(c)b\,d^5ex+18}{}$

```
[In] int(x^3*(a+b*ln(c*x^n))/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1800*(-450*x^2*ln(c*x^n)*b*d^4*e^4-600*x^3*ln(c*x^n)*b*d^3*e^5-36*x^5*b*d
*e^7*n-96*x*b*d^5*e^3*n-150*x^2*b*d^4*e^4*n-50*x^3*b*d^3*e^5*n+30*ln(x)*x^6
*b*e^8*n-30*ln(e*x+d)*x^6*b*e^8*n+30*ln(x)*b*d^6*e^2*n-30*ln(e*x+d)*b*d^6*e
^2*n-180*x*ln(c*x^n)*b*d^5*e^3-30*a*d^6*e^2+180*ln(x)*x^5*b*d*e^7*n-180*ln(
e*x+d)*x^5*b*d*e^7*n+450*ln(x)*x^4*b*d^2*e^6*n-450*ln(e*x+d)*x^4*b*d^2*e^6*
n+600*ln(x)*x^3*b*d^3*e^5*n-600*ln(e*x+d)*x^3*b*d^3*e^5*n+450*ln(x)*x^2*b*d
^4*e^4*n-450*ln(e*x+d)*x^2*b*d^4*e^4*n+180*ln(x)*x*b*d^5*e^3*n-180*ln(e*x+d
)*x*b*d^5*e^3*n-21*b*d^6*e^2*n-30*ln(c*x^n)*b*d^6*e^2-11*x^6*b*e^8*n-180*x*
a*d^5*e^3-450*x^2*a*d^4*e^4-600*x^3*a*d^3*e^5)/d^3/e^6/(e*x+d)^6
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.52

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^7}\,dx = \frac{6\,bde^5nx^5+33\,bd^2e^4nx^4-2\,bd^6n-6\,ad^6+2\,(17\,bd^3e^3n-60\,ad^3e^3)x^3+3\,(bd^4e^2n-30\,ad^4e^2)x^2-6\,(bd^5e}{}$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] 1/360*(6*b*d*e^5*n*x^5 + 33*b*d^2*e^4*n*x^4 - 2*b*d^6*n - 6*a*d^6 + 2*(17*b
*d^3*e^3*n - 60*a*d^3*e^3)*x^3 + 3*(b*d^4*e^2*n - 30*a*d^4*e^2)*x^2 - 6*(b*
d^5*e*n + 6*a*d^5*e)*x - 6*(b*e^6*n*x^6 + 6*b*d*e^5*n*x^5 + 15*b*d^2*e^4*n*
x^4 + 20*b*d^3*e^3*n*x^3 + 15*b*d^4*e^2*n*x^2 + 6*b*d^5*e*n*x + b*d^6*n)*lo
g(e*x + d) - 6*(20*b*d^3*e^3*x^3 + 15*b*d^4*e^2*x^2 + 6*b*d^5*e*x + b*d^6)*
log(c) + 6*(b*e^6*n*x^6 + 6*b*d*e^5*n*x^5 + 15*b*d^2*e^4*n*x^4)*log(x))/(d^
3*e^10*x^6 + 6*d^4*e^9*x^5 + 15*d^5*e^8*x^4 + 20*d^6*e^7*x^3 + 15*d^7*e^6*x
^2 + 6*d^8*e^5*x + d^9*e^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1979 vs. $2(224) = 448$.

Time = 75.70 (sec) , antiderivative size = 1979, normalized size of antiderivative = 8.76

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^7}\,dx = \text{Too large to display}$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))/(e*x+d)**7,x)
```

```
[Out] Piecewise((zoo*(-a/(3*x**3) - b*n/(9*x**3) - b*log(c*x**n)/(3*x**3)), Eq(d,
 0) & Eq(e, 0)), ((a*x**4/4 - b*n*x**4/16 + b*x**4*log(c*x**n)/4)/d**7, Eq(
e, 0)), ((-a/(3*x**3) - b*n/(9*x**3) - b*log(c*x**n)/(3*x**3))/e**7, Eq(d,
0)), (-6*a*d**6/(360*d**9*e**4 + 2160*d**8*e**5*x + 5400*d**7*e**6*x**2 + 7
200*d**6*e**7*x**3 + 5400*d**5*e**8*x**4 + 2160*d**4*e**9*x**5 + 360*d**3*e
**10*x**6) - 36*a*d**5*e*x/(360*d**9*e**4 + 2160*d**8*e**5*x + 5400*d**7*e*
*6*x**2 + 7200*d**6*e**7*x**3 + 5400*d**5*e**8*x**4 + 2160*d**4*e**9*x**5 +
 360*d**3*e**10*x**6) - 90*a*d**4*e**2*x**2/(360*d**9*e**4 + 2160*d**8*e**5
*x + 5400*d**7*e**6*x**2 + 7200*d**6*e**7*x**3 + 5400*d**5*e**8*x**4 + 2160
*d**4*e**9*x**5 + 360*d**3*e**10*x**6) - 120*a*d**3*e**3*x**3/(360*d**9*e**
4 + 2160*d**8*e**5*x + 5400*d**7*e**6*x**2 + 7200*d**6*e**7*x**3 + 5400*d**
5*e**8*x**4 + 2160*d**4*e**9*x**5 + 360*d**3*e**10*x**6) - 6*b*d**6*n*log(d
/e + x)/(360*d**9*e**4 + 2160*d**8*e**5*x + 5400*d**7*e**6*x**2 + 7200*d**6
*e**7*x**3 + 5400*d**5*e**8*x**4 + 2160*d**4*e**9*x**5 + 360*d**3*e**10*x**
6) - 2*b*d**6*n/(360*d**9*e**4 + 2160*d**8*e**5*x + 5400*d**7*e**6*x**2 + 7
200*d**6*e**7*x**3 + 5400*d**5*e**8*x**4 + 2160*d**4*e**9*x**5 + 360*d**3*e
**10*x**6) - 36*b*d**5*e*n*x*log(d/e + x)/(360*d**9*e**4 + 2160*d**8*e**5*x
 + 5400*d**7*e**6*x**2 + 7200*d**6*e**7*x**3 + 5400*d**5*e**8*x**4 + 2160*d
**4*e**9*x**5 + 360*d**3*e**10*x**6) - 6*b*d**5*e*n*x/(360*d**9*e**4 + 2160
*d**8*e**5*x + 5400*d**7*e**6*x**2 + 7200*d**6*e**7*x**3 + 5400*d**5*e**8*x
**4 + 2160*d**4*e**9*x**5 + 360*d**3*e**10*x**6) - 90*b*d**4*e**2*n*x**2*lo
g(d/e + x)/(360*d**9*e**4 + 2160*d**8*e**5*x + 5400*d**7*e**6*x**2 + 7200*d
**6*e**7*x**3 + 5400*d**5*e**8*x**4 + 2160*d**4*e**9*x**5 + 360*d**3*e**10*
x**6) + 3*b*d**4*e**2*n*x**2/(360*d**9*e**4 + 2160*d**8*e**5*x + 5400*d**7*
e**6*x**2 + 7200*d**6*e**7*x**3 + 5400*d**5*e**8*x**4 + 2160*d**4*e**9*x**5
 + 360*d**3*e**10*x**6) - 120*b*d**3*e**3*n*x**3*log(d/e + x)/(360*d**9*e**
4 + 2160*d**8*e**5*x + 5400*d**7*e**6*x**2 + 7200*d**6*e**7*x**3 + 5400*d**
5*e**8*x**4 + 2160*d**4*e**9*x**5 + 360*d**3*e**10*x**6) + 34*b*d**3*e**3*n
*x**3/(360*d**9*e**4 + 2160*d**8*e**5*x + 5400*d**7*e**6*x**2 + 7200*d**6*e
**7*x**3 + 5400*d**5*e**8*x**4 + 2160*d**4*e**9*x**5 + 360*d**3*e**10*x**6)
 - 90*b*d**2*e**4*n*x**4*log(d/e + x)/(360*d**9*e**4 + 2160*d**8*e**5*x + 5
400*d**7*e**6*x**2 + 7200*d**6*e**7*x**3 + 5400*d**5*e**8*x**4 + 2160*d**4*
e**9*x**5 + 360*d**3*e**10*x**6) + 33*b*d**2*e**4*n*x**4/(360*d**9*e**4 + 2
160*d**8*e**5*x + 5400*d**7*e**6*x**2 + 7200*d**6*e**7*x**3 + 5400*d**5*e**
8*x**4 + 2160*d**4*e**9*x**5 + 360*d**3*e**10*x**6) + 90*b*d**2*e**4*x**4*l
```

```
og(c*x**n)/(360*d**9*e**4 + 2160*d**8*e**5*x + 5400*d**7*e**6*x**2 + 7200*d
**6*e**7*x**3 + 5400*d**5*e**8*x**4 + 2160*d**4*e**9*x**5 + 360*d**3*e**10*
x**6) - 36*b*d*e**5*n*x**5*log(d/e + x)/(360*d**9*e**4 + 2160*d**8*e**5*x +
 5400*d**7*e**6*x**2 + 7200*d**6*e**7*x**3 + 5400*d**5*e**8*x**4 + 2160*d**
4*e**9*x**5 + 360*d**3*e**10*x**6) + 6*b*d*e**5*n*x**5/(360*d**9*e**4 + 216
0*d**8*e**5*x + 5400*d**7*e**6*x**2 + 7200*d**6*e**7*x**3 + 5400*d**5*e**8*
x**4 + 2160*d**4*e**9*x**5 + 360*d**3*e**10*x**6) + 36*b*d*e**5*x**5*log(c*
x**n)/(360*d**9*e**4 + 2160*d**8*e**5*x + 5400*d**7*e**6*x**2 + 7200*d**6*e
**7*x**3 + 5400*d**5*e**8*x**4 + 2160*d**4*e**9*x**5 + 360*d**3*e**10*x**6)
 - 6*b*e**6*n*x**6*log(d/e + x)/(360*d**9*e**4 + 2160*d**8*e**5*x + 5400*d*
*7*e**6*x**2 + 7200*d**6*e**7*x**3 + 5400*d**5*e**8*x**4 + 2160*d**4*e**9*x
**5 + 360*d**3*e**10*x**6) + 6*b*e**6*x**6*log(c*x**n)/(360*d**9*e**4 + 216
0*d**8*e**5*x + 5400*d**7*e**6*x**2 + 7200*d**6*e**7*x**3 + 5400*d**5*e**8*
x**4 + 2160*d**4*e**9*x**5 + 360*d**3*e**10*x**6), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.50

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^7}\,dx$$
$$= \frac{1}{360}\,bn\left(\frac{6\,e^4x^4+27\,de^3x^3+7\,d^2e^2x^2-4\,d^3ex-2\,d^4}{d^2e^9x^5+5\,d^3e^8x^4+10\,d^4e^7x^3+10\,d^5e^6x^2+5\,d^6e^5x+d^7e^4}-\frac{6\,\log(ex+d)}{d^3e^4}+\frac{6\,\log(x)}{d^3e^4}\right)$$
$$-\frac{(20\,e^3x^3+15\,de^2x^2+6\,d^2ex+d^3)b\log(cx^n)}{60\,(e^{10}x^6+6\,de^9x^5+15\,d^2e^8x^4+20\,d^3e^7x^3+15\,d^4e^6x^2+6\,d^5e^5x+d^6e^4)}$$
$$-\frac{(20\,e^3x^3+15\,de^2x^2+6\,d^2ex+d^3)a}{60\,(e^{10}x^6+6\,de^9x^5+15\,d^2e^8x^4+20\,d^3e^7x^3+15\,d^4e^6x^2+6\,d^5e^5x+d^6e^4)}$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] 1/360*b*n*((6*e^4*x^4 + 27*d*e^3*x^3 + 7*d^2*e^2*x^2 - 4*d^3*e*x - 2*d^4)/(
d^2*e^9*x^5 + 5*d^3*e^8*x^4 + 10*d^4*e^7*x^3 + 10*d^5*e^6*x^2 + 5*d^6*e^5*x
 + d^7*e^4) - 6*log(e*x + d)/(d^3*e^4) + 6*log(x)/(d^3*e^4)) - 1/60*(20*e^3
*x^3 + 15*d*e^2*x^2 + 6*d^2*e*x + d^3)*b*log(c*x^n)/(e^10*x^6 + 6*d*e^9*x^5
 + 15*d^2*e^8*x^4 + 20*d^3*e^7*x^3 + 15*d^4*e^6*x^2 + 6*d^5*e^5*x + d^6*e^4
) - 1/60*(20*e^3*x^3 + 15*d*e^2*x^2 + 6*d^2*e*x + d^3)*a/(e^10*x^6 + 6*d*e^
9*x^5 + 15*d^2*e^8*x^4 + 20*d^3*e^7*x^3 + 15*d^4*e^6*x^2 + 6*d^5*e^5*x + d^
6*e^4)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.60

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^7}\,dx$$
$$= -\frac{(20\,be^3nx^3+15\,bde^2nx^2+6\,bd^2enx+bd^3n)\log(x)}{60\,(e^{10}x^6+6\,de^9x^5+15\,d^2e^8x^4+20\,d^3e^7x^3+15\,d^4e^6x^2+6\,d^5e^5x+d^6e^4)}$$
$$+\frac{6\,be^5nx^5+33\,bde^4nx^4+34\,bd^2e^3nx^3-120\,bd^2e^3x^3\log(c)+3\,bd^3e^2nx^2-120\,ad^2e^3x^3-90\,bd^3e^2x^2\log}{360\,(d^2e^{10}x^6+6\,d^3e^9x^5+15\,d^4e^8x^4+20\,d^5e^7}$$
$$-\frac{bn\log(ex+d)}{60\,d^3e^4}+\frac{bn\log(x)}{60\,d^3e^4}$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] -1/60*(20*b*e^3*n*x^3 + 15*b*d*e^2*n*x^2 + 6*b*d^2*e*n*x + b*d^3*n)*log(x)/
(e^10*x^6 + 6*d*e^9*x^5 + 15*d^2*e^8*x^4 + 20*d^3*e^7*x^3 + 15*d^4*e^6*x^2
+ 6*d^5*e^5*x + d^6*e^4) + 1/360*(6*b*e^5*n*x^5 + 33*b*d*e^4*n*x^4 + 34*b*d
^2*e^3*n*x^3 - 120*b*d^2*e^3*x^3*log(c) + 3*b*d^3*e^2*n*x^2 - 120*a*d^2*e^3
*x^3 - 90*b*d^3*e^2*x^2*log(c) - 6*b*d^4*e*n*x - 90*a*d^3*e^2*x^2 - 36*b*d^
4*e*x*log(c) - 2*b*d^5*n - 36*a*d^4*e*x - 6*b*d^5*log(c) - 6*a*d^5)/(d^2*e^
10*x^6 + 6*d^3*e^9*x^5 + 15*d^4*e^8*x^4 + 20*d^5*e^7*x^3 + 15*d^6*e^6*x^2 +
 6*d^7*e^5*x + d^8*e^4) - 1/60*b*n*log(e*x + d)/(d^3*e^4) + 1/60*b*n*log(x)
/(d^3*e^4)
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.31

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^7}\,dx =$$
$$-\frac{x^3\left(20\,a\,e^3-\frac{17\,b\,e^3\,n}{3}\right)+x\,(6\,a\,d^2\,e+b\,d^2\,e\,n)+a\,d^3+x^2\left(15\,a\,d\,e^2-\frac{b\,d\,e^2\,n}{2}\right)+\frac{b\,d^3\,n}{3}-\frac{11\,b\,e^4\,n\,x^4}{2\,d}-\frac{b\,e^5\,n}{d^2}}{60\,d^6\,e^4+360\,d^5\,e^5\,x+900\,d^4\,e^6\,x^2+1200\,d^3\,e^7\,x^3+900\,d^2\,e^8\,x^4+360\,d\,e^9\,x^5+60\,e^{10}\,x^6}$$
$$-\frac{\ln(c\,x^n)\left(\frac{b\,d^3}{60\,e^4}+\frac{b\,x^3}{3\,e}+\frac{b\,d\,x^2}{4\,e^2}+\frac{b\,d^2\,x}{10\,e^3}\right)}{d^6+6\,d^5\,e\,x+15\,d^4\,e^2\,x^2+20\,d^3\,e^3\,x^3+15\,d^2\,e^4\,x^4+6\,d\,e^5\,x^5+e^6\,x^6}$$
$$-\frac{b\,n\,\mathrm{atanh}\left(\frac{2\,e\,x}{d}+1\right)}{30\,d^3\,e^4}$$

```
[In] int((x^3*(a + b*log(c*x^n)))/(d + e*x)^7,x)
```

```
[Out] - (x^3*(20*a*e^3 - (17*b*e^3*n)/3) + x*(6*a*d^2*e + b*d^2*e*n) + a*d^3 + x^
2*(15*a*d*e^2 - (b*d*e^2*n)/2) + (b*d^3*n)/3 - (11*b*e^4*n*x^4)/(2*d) - (b*
```

```
e^5*n*x^5)/d^2)/(60*d^6*e^4 + 60*e^10*x^6 + 360*d^5*e^5*x + 360*d*e^9*x^5 +
 900*d^4*e^6*x^2 + 1200*d^3*e^7*x^3 + 900*d^2*e^8*x^4) - (log(c*x^n)*((b*d^
3)/(60*e^4) + (b*x^3)/(3*e) + (b*d*x^2)/(4*e^2) + (b*d^2*x)/(10*e^3)))/(d^6
 + e^6*x^6 + 6*d*e^5*x^5 + 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 + 15*d^2*e^4*x^4
 + 6*d^5*e*x) - (b*n*atanh((2*e*x)/d + 1))/(30*d^3*e^4)
```

3.68 $\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^7} dx$

Optimal result

Integrand size = 21, antiderivative size = 199

$$\begin{aligned}\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^7} dx = {} & \frac{bdn}{30e^3(d+ex)^5} - \frac{7bn}{120e^3(d+ex)^4} + \frac{bn}{180de^3(d+ex)^3} \\ & + \frac{bn}{120d^2e^3(d+ex)^2} + \frac{bn}{60d^3e^3(d+ex)} + \frac{bn\log(x)}{60d^4e^3} \\ & - \frac{d^2(a+b\log(cx^n))}{6e^3(d+ex)^6} + \frac{2d(a+b\log(cx^n))}{5e^3(d+ex)^5} \\ & - \frac{a+b\log(cx^n)}{4e^3(d+ex)^4} - \frac{bn\log(d+ex)}{60d^4e^3}\end{aligned}$$

```
[Out] 1/30*b*d*n/e^3/(e*x+d)^5-7/120*b*n/e^3/(e*x+d)^4+1/180*b*n/d/e^3/(e*x+d)^3+
1/120*b*n/d^2/e^3/(e*x+d)^2+1/60*b*n/d^3/e^3/(e*x+d)+1/60*b*n*ln(x)/d^4/e^3
-1/6*d^2*(a+b*ln(c*x^n))/e^3/(e*x+d)^6+2/5*d*(a+b*ln(c*x^n))/e^3/(e*x+d)^5+
1/4*(-a-b*ln(c*x^n))/e^3/(e*x+d)^4-1/60*b*n*ln(e*x+d)/d^4/e^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used

$= \{45, 2382, 12, 907\}$

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^7}\,dx = -\frac{d^2(a+b\log(cx^n))}{6e^3(d+ex)^6} + \frac{2d(a+b\log(cx^n))}{5e^3(d+ex)^5} - \frac{a+b\log(cx^n)}{4e^3(d+ex)^4} + \frac{bn\log(x)}{60d^4e^3} - \frac{bn\log(d+ex)}{60d^4e^3} + \frac{bn}{60d^3e^3(d+ex)} + \frac{bn}{120d^2e^3(d+ex)^2} + \frac{bdn}{30e^3(d+ex)^5} - \frac{7bn}{120e^3(d+ex)^4} + \frac{bn}{180de^3(d+ex)^3}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/(d + e*x)^7,x]
```

```
[Out] (b*d*n)/(30*e^3*(d + e*x)^5) - (7*b*n)/(120*e^3*(d + e*x)^4) + (b*n)/(180*d
*e^3*(d + e*x)^3) + (b*n)/(120*d^2*e^3*(d + e*x)^2) + (b*n)/(60*d^3*e^3*(d
+ e*x)) + (b*n*Log[x])/(60*d^4*e^3) - (d^2*(a + b*Log[c*x^n]))/(6*e^3*(d +
e*x)^6) + (2*d*(a + b*Log[c*x^n]))/(5*e^3*(d + e*x)^5) - (a + b*Log[c*x^n])
/(4*e^3*(d + e*x)^4) - (b*n*Log[d + e*x])/(60*d^4*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{6e^3(d+ex)^6}+\frac{2d(a+b\log(cx^n))}{5e^3(d+ex)^5} \\
&\quad -\frac{a+b\log(cx^n)}{4e^3(d+ex)^4}-(bn)\int\frac{-d^2-6dex-15e^2x^2}{60e^3x(d+ex)^6}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{6e^3(d+ex)^6}+\frac{2d(a+b\log(cx^n))}{5e^3(d+ex)^5}-\frac{a+b\log(cx^n)}{4e^3(d+ex)^4}-\frac{(bn)\int\frac{-d^2-6dex-15e^2x^2}{x(d+ex)^6}\,dx}{60e^3} \\
&= -\frac{d^2(a+b\log(cx^n))}{6e^3(d+ex)^6}+\frac{2d(a+b\log(cx^n))}{5e^3(d+ex)^5}-\frac{a+b\log(cx^n)}{4e^3(d+ex)^4} \\
&\quad -\frac{(bn)\int\left(-\frac{1}{d^4x}+\frac{10de}{(d+ex)^6}-\frac{14e}{(d+ex)^5}+\frac{e}{d(d+ex)^4}+\frac{e}{d^2(d+ex)^3}+\frac{e}{d^3(d+ex)^2}+\frac{e}{d^4(d+ex)}\right)dx}{60e^3} \\
&= \frac{bdn}{30e^3(d+ex)^5}-\frac{7bn}{120e^3(d+ex)^4}+\frac{bn}{180de^3(d+ex)^3}+\frac{bn}{120d^2e^3(d+ex)^2}+\frac{bn}{60d^3e^3(d+ex)} \\
&\quad +\frac{bn\log(x)}{60d^4e^3}-\frac{d^2(a+b\log(cx^n))}{6e^3(d+ex)^6}+\frac{2d(a+b\log(cx^n))}{5e^3(d+ex)^5}-\frac{a+b\log(cx^n)}{4e^3(d+ex)^4}-\frac{bn\log(d+ex)}{60d^4e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.96

$$\int\frac{x^2(a+b\log(cx^n))}{(d+ex)^7}\,dx$$
$$=\frac{-60ad^6+144ad^5(d+ex)+12bd^5n(d+ex)-90ad^4(d+ex)^2-21bd^4n(d+ex)^2+2bd^3n(d+ex)^3+3b\ldots}{}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*x)^7,x]
```

```
[Out] (-60*a*d^6 + 144*a*d^5*(d + e*x) + 12*b*d^5*n*(d + e*x) - 90*a*d^4*(d + e*x
)^2 - 21*b*d^4*n*(d + e*x)^2 + 2*b*d^3*n*(d + e*x)^3 + 3*b*d^2*n*(d + e*x)^
4 + 6*b*d*n*(d + e*x)^5 + 6*b*n*(d + e*x)^6*Log[x] - 60*b*d^6*Log[c*x^n] +
144*b*d^5*(d + e*x)*Log[c*x^n] - 90*b*d^4*(d + e*x)^2*Log[c*x^n] - 6*b*n*(d
 + e*x)^6*Log[d + e*x])/(360*d^4*e^3*(d + e*x)^6)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(182) = 364.

Time = 1.34 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.92

method	result
parallelrisch	$\frac{360 \ln(x) x^5 b d e^8 n - 360 \ln(ex+d) x^5 b d e^8 n + 900 \ln(x) x^4 b d^2 e^7 n - 900 \ln(ex+d) x^4 b d^2 e^7 n + 1200 \ln(x) x^3 b d^3 e^6 n - 1200 \ln(ex+d) x^3}{}$
risch	$-\frac{b(15e^2x^2+6dex+d^2)\ln(x^n)}{60(ex+d)^6 e^3} + \frac{-90 \ln(ex+d) b d^2 e^4 n x^4 - 90 \ln(c) b d^4 e^2 x^2 - 36 \ln(c) b d^5 ex + 18 i\pi b d^5 ex \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csg}}{}$

```
[In] int(x^2*(a+b*ln(c*x^n))/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3600*(360*ln(x)*x^5*b*d*e^8*n-360*ln(e*x+d)*x^5*b*d*e^8*n+900*ln(x)*x^4*b
*d^2*e^7*n-900*ln(e*x+d)*x^4*b*d^2*e^7*n+1200*ln(x)*x^3*b*d^3*e^6*n-1200*ln
(e*x+d)*x^3*b*d^3*e^6*n+900*ln(x)*x^2*b*d^4*e^5*n-900*ln(e*x+d)*x^2*b*d^4*e
^5*n+360*ln(x)*x*b*d^5*e^4*n-360*ln(e*x+d)*x*b*d^5*e^4*n-360*x*ln(c*x^n)*b*
d^5*e^4-900*x^2*ln(c*x^n)*b*d^4*e^5-162*x^5*b*d*e^8*n-42*x*b*d^5*e^4*n+75*x
^2*b*d^4*e^5*n-225*x^4*b*d^2*e^7*n+60*ln(x)*x^6*b*e^9*n-60*ln(e*x+d)*x^6*b*
e^9*n+60*ln(x)*b*d^6*e^3*n-60*ln(e*x+d)*b*d^6*e^3*n-60*a*d^6*e^3-17*b*d^6*e
^3*n-60*ln(c*x^n)*b*d^6*e^3-37*x^6*b*e^9*n-360*x*a*d^5*e^4-900*x^2*a*d^4*e^
5)/d^4/e^6/(e*x+d)^6
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.67

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^7}\,dx = \frac{6\,bde^5nx^5 + 33\,bd^2e^4nx^4 + 74\,bd^3e^3nx^3 + 2\,bd^6n - 6\,ad^6 + 9\,(7\,bd^4e^2n - 10\,ad^4e^2)x^2 + 18\,(bd^5en - 2\,ad^5e)}{}$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] 1/360*(6*b*d*e^5*n*x^5 + 33*b*d^2*e^4*n*x^4 + 74*b*d^3*e^3*n*x^3 + 2*b*d^6*
n - 6*a*d^6 + 9*(7*b*d^4*e^2*n - 10*a*d^4*e^2)*x^2 + 18*(b*d^5*e*n - 2*a*d^
5*e)*x - 6*(b*e^6*n*x^6 + 6*b*d*e^5*n*x^5 + 15*b*d^2*e^4*n*x^4 + 20*b*d^3*e
^3*n*x^3 + 15*b*d^4*e^2*n*x^2 + 6*b*d^5*e*n*x + b*d^6*n)*log(e*x + d) - 6*(
15*b*d^4*e^2*x^2 + 6*b*d^5*e*x + b*d^6)*log(c) + 6*(b*e^6*n*x^6 + 6*b*d*e^5
*n*x^5 + 15*b*d^2*e^4*n*x^4 + 20*b*d^3*e^3*n*x^3)*log(x))/(d^4*e^9*x^6 + 6*
d^5*e^8*x^5 + 15*d^6*e^7*x^4 + 20*d^7*e^6*x^3 + 15*d^8*e^5*x^2 + 6*d^9*e^4*
x + d^10*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1986 vs. 2(196) = 392.

Time = 75.27 (sec) , antiderivative size = 1986, normalized size of antiderivative = 9.98

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^7}\,dx = \text{Too large to display}$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(e*x+d)**7,x)
```

```
[Out] Piecewise((zoo*(-a/(4*x**4) - b*n/(16*x**4) - b*log(c*x**n)/(4*x**4)), Eq(d
, 0) & Eq(e, 0)), ((a*x**3/3 - b*n*x**3/9 + b*x**3*log(c*x**n)/3)/d**7, Eq(
e, 0)), ((-a/(4*x**4) - b*n/(16*x**4) - b*log(c*x**n)/(4*x**4))/e**7, Eq(d,
 0)), (-6*a*d**6/(360*d**10*e**3 + 2160*d**9*e**4*x + 5400*d**8*e**5*x**2 +
 7200*d**7*e**6*x**3 + 5400*d**6*e**7*x**4 + 2160*d**5*e**8*x**5 + 360*d**4
*e**9*x**6) - 36*a*d**5*e*x/(360*d**10*e**3 + 2160*d**9*e**4*x + 5400*d**8*
e**5*x**2 + 7200*d**7*e**6*x**3 + 5400*d**6*e**7*x**4 + 2160*d**5*e**8*x**5
 + 360*d**4*e**9*x**6) - 90*a*d**4*e**2*x**2/(360*d**10*e**3 + 2160*d**9*e*
*4*x + 5400*d**8*e**5*x**2 + 7200*d**7*e**6*x**3 + 5400*d**6*e**7*x**4 + 21
60*d**5*e**8*x**5 + 360*d**4*e**9*x**6) - 6*b*d**6*n*log(d/e + x)/(360*d**1
0*e**3 + 2160*d**9*e**4*x + 5400*d**8*e**5*x**2 + 7200*d**7*e**6*x**3 + 540
0*d**6*e**7*x**4 + 2160*d**5*e**8*x**5 + 360*d**4*e**9*x**6) + 2*b*d**6*n/(
360*d**10*e**3 + 2160*d**9*e**4*x + 5400*d**8*e**5*x**2 + 7200*d**7*e**6*x*
*3 + 5400*d**6*e**7*x**4 + 2160*d**5*e**8*x**5 + 360*d**4*e**9*x**6) - 36*b
*d**5*e*n*x*log(d/e + x)/(360*d**10*e**3 + 2160*d**9*e**4*x + 5400*d**8*e**
5*x**2 + 7200*d**7*e**6*x**3 + 5400*d**6*e**7*x**4 + 2160*d**5*e**8*x**5 +
360*d**4*e**9*x**6) + 18*b*d**5*e*n*x/(360*d**10*e**3 + 2160*d**9*e**4*x +
5400*d**8*e**5*x**2 + 7200*d**7*e**6*x**3 + 5400*d**6*e**7*x**4 + 2160*d**5
*e**8*x**5 + 360*d**4*e**9*x**6) - 90*b*d**4*e**2*n*x**2*log(d/e + x)/(360*
d**10*e**3 + 2160*d**9*e**4*x + 5400*d**8*e**5*x**2 + 7200*d**7*e**6*x**3 +
 5400*d**6*e**7*x**4 + 2160*d**5*e**8*x**5 + 360*d**4*e**9*x**6) + 63*b*d**
4*e**2*n*x**2/(360*d**10*e**3 + 2160*d**9*e**4*x + 5400*d**8*e**5*x**2 + 72
00*d**7*e**6*x**3 + 5400*d**6*e**7*x**4 + 2160*d**5*e**8*x**5 + 360*d**4*e*
*9*x**6) - 120*b*d**3*e**3*n*x**3*log(d/e + x)/(360*d**10*e**3 + 2160*d**9*
e**4*x + 5400*d**8*e**5*x**2 + 7200*d**7*e**6*x**3 + 5400*d**6*e**7*x**4 +
2160*d**5*e**8*x**5 + 360*d**4*e**9*x**6) + 74*b*d**3*e**3*n*x**3/(360*d**1
0*e**3 + 2160*d**9*e**4*x + 5400*d**8*e**5*x**2 + 7200*d**7*e**6*x**3 + 540
0*d**6*e**7*x**4 + 2160*d**5*e**8*x**5 + 360*d**4*e**9*x**6) + 120*b*d**3*e
**3*x**3*log(c*x**n)/(360*d**10*e**3 + 2160*d**9*e**4*x + 5400*d**8*e**5*x*
*2 + 7200*d**7*e**6*x**3 + 5400*d**6*e**7*x**4 + 2160*d**5*e**8*x**5 + 360*
d**4*e**9*x**6) - 90*b*d**2*e**4*n*x**4*log(d/e + x)/(360*d**10*e**3 + 2160
*d**9*e**4*x + 5400*d**8*e**5*x**2 + 7200*d**7*e**6*x**3 + 5400*d**6*e**7*x
**4 + 2160*d**5*e**8*x**5 + 360*d**4*e**9*x**6) + 33*b*d**2*e**4*n*x**4/(36
0*d**10*e**3 + 2160*d**9*e**4*x + 5400*d**8*e**5*x**2 + 7200*d**7*e**6*x**3
 + 5400*d**6*e**7*x**4 + 2160*d**5*e**8*x**5 + 360*d**4*e**9*x**6) + 90*b*d
```

```
**2*e**4*x**4*log(c*x**n)/(360*d**10*e**3 + 2160*d**9*e**4*x + 5400*d**8*e*
*5*x**2 + 7200*d**7*e**6*x**3 + 5400*d**6*e**7*x**4 + 2160*d**5*e**8*x**5 +
 360*d**4*e**9*x**6) - 36*b*d*e**5*n*x**5*log(d/e + x)/(360*d**10*e**3 + 21
60*d**9*e**4*x + 5400*d**8*e**5*x**2 + 7200*d**7*e**6*x**3 + 5400*d**6*e**7
*x**4 + 2160*d**5*e**8*x**5 + 360*d**4*e**9*x**6) + 6*b*d*e**5*n*x**5/(360*
d**10*e**3 + 2160*d**9*e**4*x + 5400*d**8*e**5*x**2 + 7200*d**7*e**6*x**3 +
 5400*d**6*e**7*x**4 + 2160*d**5*e**8*x**5 + 360*d**4*e**9*x**6) + 36*b*d*e
**5*x**5*log(c*x**n)/(360*d**10*e**3 + 2160*d**9*e**4*x + 5400*d**8*e**5*x*
*2 + 7200*d**7*e**6*x**3 + 5400*d**6*e**7*x**4 + 2160*d**5*e**8*x**5 + 360*
d**4*e**9*x**6) - 6*b*e**6*n*x**6*log(d/e + x)/(360*d**10*e**3 + 2160*d**9*
e**4*x + 5400*d**8*e**5*x**2 + 7200*d**7*e**6*x**3 + 5400*d**6*e**7*x**4 +
2160*d**5*e**8*x**5 + 360*d**4*e**9*x**6) + 6*b*e**6*x**6*log(c*x**n)/(360*
d**10*e**3 + 2160*d**9*e**4*x + 5400*d**8*e**5*x**2 + 7200*d**7*e**6*x**3 +
 5400*d**6*e**7*x**4 + 2160*d**5*e**8*x**5 + 360*d**4*e**9*x**6), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.59

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^7}\,dx = \frac{1}{360}\,bn\left(\frac{6\,e^4x^4+27\,de^3x^3+47\,d^2e^2x^2+16\,d^3ex+2\,d^4}{d^3e^8x^5+5\,d^4e^7x^4+10\,d^5e^6x^3+10\,d^6e^5x^2+5\,d^7e^4x+d^8e^3}-\frac{6\,\log(ex+d)}{d^4e^3}+\frac{6\,\log(x)}{d^4e^3}\right) - \frac{(15\,e^2x^2+6\,dex+d^2)b\log(cx^n)}{60\,(e^9x^6+6\,de^8x^5+15\,d^2e^7x^4+20\,d^3e^6x^3+15\,d^4e^5x^2+6\,d^5e^4x+d^6e^3)} - \frac{(15\,e^2x^2+6\,dex+d^2)a}{60\,(e^9x^6+6\,de^8x^5+15\,d^2e^7x^4+20\,d^3e^6x^3+15\,d^4e^5x^2+6\,d^5e^4x+d^6e^3)}$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] 1/360*b*n*((6*e^4*x^4 + 27*d*e^3*x^3 + 47*d^2*e^2*x^2 + 16*d^3*e*x + 2*d^4)
/(d^3*e^8*x^5 + 5*d^4*e^7*x^4 + 10*d^5*e^6*x^3 + 10*d^6*e^5*x^2 + 5*d^7*e^4
*x + d^8*e^3) - 6*log(e*x + d)/(d^4*e^3) + 6*log(x)/(d^4*e^3)) - 1/60*(15*e
^2*x^2 + 6*d*e*x + d^2)*b*log(c*x^n)/(e^9*x^6 + 6*d*e^8*x^5 + 15*d^2*e^7*x^
4 + 20*d^3*e^6*x^3 + 15*d^4*e^5*x^2 + 6*d^5*e^4*x + d^6*e^3) - 1/60*(15*e^2
*x^2 + 6*d*e*x + d^2)*a/(e^9*x^6 + 6*d*e^8*x^5 + 15*d^2*e^7*x^4 + 20*d^3*e^
6*x^3 + 15*d^4*e^5*x^2 + 6*d^5*e^4*x + d^6*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.62

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^7}\,dx = -\frac{(15\,be^2nx^2+6\,bdenx+bd^2n)\log(x)}{60\,(e^9x^6+6\,de^8x^5+15\,d^2e^7x^4+20\,d^3e^6x^3+15\,d^4e^5x^2+6\,d^5e^4x+d^6e^3)} + \frac{6\,be^5nx^5+33\,bde^4nx^4+74\,bd^2e^3nx^3+63\,bd^3e^2nx^2-90\,bd^3e^2x^2\log(c)+18\,bd^4enx-90\,ad^3e^2x^2-36}{360\,(d^3e^9x^6+6\,d^4e^8x^5+15\,d^5e^7x^4+20\,d^6e^6x^3+15\,d^7e^5x^2+}$$
$$-\frac{bn\log(ex+d)}{60\,d^4e^3}+\frac{bn\log(x)}{60\,d^4e^3}$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] -1/60*(15*b*e^2*n*x^2 + 6*b*d*e*n*x + b*d^2*n)*log(x)/(e^9*x^6 + 6*d*e^8*x^
5 + 15*d^2*e^7*x^4 + 20*d^3*e^6*x^3 + 15*d^4*e^5*x^2 + 6*d^5*e^4*x + d^6*e^
3) + 1/360*(6*b*e^5*n*x^5 + 33*b*d*e^4*n*x^4 + 74*b*d^2*e^3*n*x^3 + 63*b*d^
3*e^2*n*x^2 - 90*b*d^3*e^2*x^2*log(c) + 18*b*d^4*e*n*x - 90*a*d^3*e^2*x^2 -
 36*b*d^4*e*x*log(c) + 2*b*d^5*n - 36*a*d^4*e*x - 6*b*d^5*log(c) - 6*a*d^5)
/(d^3*e^9*x^6 + 6*d^4*e^8*x^5 + 15*d^5*e^7*x^4 + 20*d^6*e^6*x^3 + 15*d^7*e^
5*x^2 + 6*d^8*e^4*x + d^9*e^3) - 1/60*b*n*log(e*x + d)/(d^4*e^3) + 1/60*b*n
*log(x)/(d^4*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.38

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^7}\,dx = \frac{\frac{b\,d^2\,n}{3}-a\,d^2-x\,(6\,a\,d\,e-3\,b\,d\,e\,n)-x^2\left(15\,a\,e^2-\frac{21\,b\,e^2\,n}{2}\right)+\frac{37\,b\,e^3\,n\,x^3}{3\,d}+\frac{11\,b\,e^4\,n\,x^4}{2\,d^2}+\frac{b\,e^5\,n\,x^5}{d^3}}{60\,d^6\,e^3+360\,d^5\,e^4\,x+900\,d^4\,e^5\,x^2+1200\,d^3\,e^6\,x^3+900\,d^2\,e^7\,x^4+360\,d\,e^8\,x^5+60\,e^9\,x^6} - \frac{\ln(c\,x^n)\left(\frac{b\,d^2}{60\,e^3}+\frac{b\,x^2}{4\,e}+\frac{b\,d\,x}{10\,e^2}\right)}{d^6+6\,d^5\,e\,x+15\,d^4\,e^2\,x^2+20\,d^3\,e^3\,x^3+15\,d^2\,e^4\,x^4+6\,d\,e^5\,x^5+e^6\,x^6} - \frac{b\,n\,\mathrm{atanh}\left(\frac{2\,e\,x}{d}+1\right)}{30\,d^4\,e^3}$$

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e*x)^7,x)
```

```
[Out] ((b*d^2*n)/3 - a*d^2 - x*(6*a*d*e - 3*b*d*e*n) - x^2*(15*a*e^2 - (21*b*e^2*
n)/2) + (37*b*e^3*n*x^3)/(3*d) + (11*b*e^4*n*x^4)/(2*d^2) + (b*e^5*n*x^5)/d
^3)/(60*d^6*e^3 + 60*e^9*x^6 + 360*d^5*e^4*x + 360*d*e^8*x^5 + 900*d^4*e^5*
```

```
x^2 + 1200*d^3*e^6*x^3 + 900*d^2*e^7*x^4) - (log(c*x^n)*((b*d^2)/(60*e^3) +
 (b*x^2)/(4*e) + (b*d*x)/(10*e^2)))/(d^6 + e^6*x^6 + 6*d*e^5*x^5 + 15*d^4*e
^2*x^2 + 20*d^3*e^3*x^3 + 15*d^2*e^4*x^4 + 6*d^5*e*x) - (b*n*atanh((2*e*x)/
d + 1))/(30*d^4*e^3)
```

3.69 $\int \frac{x(a+b\log(cx^n))}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 174

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^7}\,dx = -\frac{bn}{30e^2(d+ex)^5} + \frac{bn}{120de^2(d+ex)^4} + \frac{bn}{90d^2e^2(d+ex)^3} + \frac{bn}{60d^3e^2(d+ex)^2} + \frac{bn}{30d^4e^2(d+ex)} + \frac{bn\log(x)}{30d^5e^2} + \frac{d(a+b\log(cx^n))}{6e^2(d+ex)^6} - \frac{a+b\log(cx^n)}{5e^2(d+ex)^5} - \frac{bn\log(d+ex)}{30d^5e^2}$$

```
[Out] -1/30*b*n/e^2/(e*x+d)^5+1/120*b*n/d/e^2/(e*x+d)^4+1/90*b*n/d^2/e^2/(e*x+d)^
3+1/60*b*n/d^3/e^2/(e*x+d)^2+1/30*b*n/d^4/e^2/(e*x+d)+1/30*b*n*ln(x)/d^5/e^
2+1/6*d*(a+b*ln(c*x^n))/e^2/(e*x+d)^6+1/5*(-a-b*ln(c*x^n))/e^2/(e*x+d)^5-1/
30*b*n*ln(e*x+d)/d^5/e^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {45, 2382, 12, 78}

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^7}\,dx = -\frac{a+b\log(cx^n)}{5e^2(d+ex)^5} + \frac{d(a+b\log(cx^n))}{6e^2(d+ex)^6} + \frac{bn\log(x)}{30d^5e^2} - \frac{bn\log(d+ex)}{30d^5e^2} + \frac{bn}{30d^4e^2(d+ex)} + \frac{bn}{60d^3e^2(d+ex)^2} + \frac{bn}{90d^2e^2(d+ex)^3} + \frac{bn}{120de^2(d+ex)^4} - \frac{bn}{30e^2(d+ex)^5}$$

```
[In] Int[(x*(a + b*Log[c*x^n]))/(d + e*x)^7,x]
```

```
[Out] -1/30*(b*n)/(e^2*(d + e*x)^5) + (b*n)/(120*d*e^2*(d + e*x)^4) + (b*n)/(90*d
^2*e^2*(d + e*x)^3) + (b*n)/(60*d^3*e^2*(d + e*x)^2) + (b*n)/(30*d^4*e^2*(d
 + e*x)) + (b*n*Log[x])/(30*d^5*e^2) + (d*(a + b*Log[c*x^n]))/(6*e^2*(d + e
*x)^6) - (a + b*Log[c*x^n])/(5*e^2*(d + e*x)^5) - (b*n*Log[d + e*x])/(30*d^
5*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(a+b\log(cx^n))}{6e^2(d+ex)^6} - \frac{a+b\log(cx^n)}{5e^2(d+ex)^5} - (bn)\int \frac{-d-6ex}{30e^2x(d+ex)^6}\,dx \\
&= \frac{d(a+b\log(cx^n))}{6e^2(d+ex)^6} - \frac{a+b\log(cx^n)}{5e^2(d+ex)^5} - \frac{(bn)\int \frac{-d-6ex}{x(d+ex)^6}\,dx}{30e^2} \\
&= \frac{d(a+b\log(cx^n))}{6e^2(d+ex)^6} - \frac{a+b\log(cx^n)}{5e^2(d+ex)^5} \\
&\quad - \frac{(bn)\int \left(-\frac{1}{d^5x} - \frac{5e}{(d+ex)^6} + \frac{e}{d(d+ex)^5} + \frac{e}{d^2(d+ex)^4} + \frac{e}{d^3(d+ex)^3} + \frac{e}{d^4(d+ex)^2} + \frac{e}{d^5(d+ex)}\right)dx}{30e^2}
\end{aligned}$$

$$= -\frac{bn}{30e^2(d+ex)^5} + \frac{bn}{120de^2(d+ex)^4} + \frac{bn}{90d^2e^2(d+ex)^3} + \frac{bn}{60d^3e^2(d+ex)^2} + \frac{bn}{30d^4e^2(d+ex)} + \frac{bn\log(x)}{30d^5e^2} + \frac{d(a+b\log(cx^n))}{6e^2(d+ex)^6} - \frac{a+b\log(cx^n)}{5e^2(d+ex)^5} - \frac{bn\log(d+ex)}{30d^5e^2}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.92

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^7}\,dx = \frac{60ad^6 - 72ad^5(d+ex) - 12bd^5n(d+ex) + 3bd^4n(d+ex)^2 + 4bd^3n(d+ex)^3 + 6bd^2n(d+ex)^4 + 12bdn(}{360d^5e^2(}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e*x)^7,x]
```

```
[Out] (60*a*d^6 - 72*a*d^5*(d + e*x) - 12*b*d^5*n*(d + e*x) + 3*b*d^4*n*(d + e*x)
^2 + 4*b*d^3*n*(d + e*x)^3 + 6*b*d^2*n*(d + e*x)^4 + 12*b*d*n*(d + e*x)^5 +
 12*b*n*(d + e*x)^6*Log[x] + 60*b*d^6*Log[c*x^n] - 72*b*d^5*(d + e*x)*Log[c
*x^n] - 12*b*n*(d + e*x)^6*Log[d + e*x])/(360*d^5*e^2*(d + e*x)^6)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(159) = 318$.

Time = 1.27 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.32

method	result
parallelrisch	$\frac{12\ln(x)x^6bd\,e^{10}n-12\ln(ex+d)x^6bd\,e^{10}n-66x^5b\,d^2e^9n-13x^6bd\,e^{10}n+12xb\,d^6e^5n-24x^2b\,d^5e^6n-112x^3b\,d^4e^7n-129x^4b\,d^3e^8}{}$
risch	$-\frac{b(6ex+d)\ln(x^n)}{30(ex+d)^6e^2} - \frac{180\ln(ex+d)b\,d^2e^4n\,x^4+72\ln(c)b\,d^5ex-36i\pi b\,d^5ex\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)-72\ln(-x)bd\,e^5n\,x}{}$

```
[In] int(x*(a+b*ln(c*x^n))/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/360*(12*ln(x)*x^6*b*d*e^10*n-12*ln(e*x+d)*x^6*b*d*e^10*n-66*x^5*b*d^2*e^9
*n-13*x^6*b*d*e^10*n+12*x*b*d^6*e^5*n-24*x^2*b*d^5*e^6*n-112*x^3*b*d^4*e^7*
n-129*x^4*b*d^3*e^8*n+12*ln(x)*b*d^7*e^4*n-12*ln(e*x+d)*b*d^7*e^4*n+72*ln(x
)*x^5*b*d^2*e^9*n-72*ln(e*x+d)*x^5*b*d^2*e^9*n+180*ln(x)*x^4*b*d^3*e^8*n-18
0*ln(e*x+d)*x^4*b*d^3*e^8*n+240*ln(x)*x^3*b*d^4*e^7*n-240*ln(e*x+d)*x^3*b*d
^4*e^7*n+180*ln(x)*x^2*b*d^5*e^6*n-180*ln(e*x+d)*x^2*b*d^5*e^6*n+72*ln(x)*x
*b*d^6*e^5*n-72*ln(e*x+d)*x*b*d^6*e^5*n-72*x*ln(c*x^n)*b*d^6*e^5-12*ln(c*x^
n)*b*d^7*e^4+72*x^5*a*d^2*e^9+12*x^6*a*d*e^10+180*x^2*a*d^5*e^6+240*x^3*a*d
^4*e^7+180*x^4*a*d^3*e^8)/e^6/d^6/(e*x+d)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(156) = 312.

Time = 0.31 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.86

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^7}\,dx$$
$$= \frac{12\,bde^5nx^5+66\,bd^2e^4nx^4+148\,bd^3e^3nx^3+171\,bd^4e^2nx^2+13\,bd^6n-12\,ad^6+18\,(5\,bd^5en-4\,ad^5e)x-12}{}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] 1/360*(12*b*d*e^5*n*x^5 + 66*b*d^2*e^4*n*x^4 + 148*b*d^3*e^3*n*x^3 + 171*b*
d^4*e^2*n*x^2 + 13*b*d^6*n - 12*a*d^6 + 18*(5*b*d^5*e*n - 4*a*d^5*e)*x - 12
*(b*e^6*n*x^6 + 6*b*d*e^5*n*x^5 + 15*b*d^2*e^4*n*x^4 + 20*b*d^3*e^3*n*x^3 +
 15*b*d^4*e^2*n*x^2 + 6*b*d^5*e*n*x + b*d^6*n)*log(e*x + d) - 12*(6*b*d^5*e
*x + b*d^6)*log(c) + 12*(b*e^6*n*x^6 + 6*b*d*e^5*n*x^5 + 15*b*d^2*e^4*n*x^4
 + 20*b*d^3*e^3*n*x^3 + 15*b*d^4*e^2*n*x^2)*log(x))/(d^5*e^8*x^6 + 6*d^6*e^
7*x^5 + 15*d^7*e^6*x^4 + 20*d^8*e^5*x^3 + 15*d^9*e^4*x^2 + 6*d^10*e^3*x + d
^11*e^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1992 vs. 2(168) = 336.

Time = 75.49 (sec) , antiderivative size = 1992, normalized size of antiderivative = 11.45

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^7}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(a+b*ln(c*x**n))/(e*x+d)**7,x)
```

```
[Out] Piecewise((zoo*(-a/(5*x**5) - b*n/(25*x**5) - b*log(c*x**n)/(5*x**5)), Eq(d
, 0) & Eq(e, 0)), ((a*x**2/2 - b*n*x**2/4 + b*x**2*log(c*x**n)/2)/d**7, Eq(
e, 0)), ((-a/(5*x**5) - b*n/(25*x**5) - b*log(c*x**n)/(5*x**5))/e**7, Eq(d,
 0)), (-12*a*d**6/(360*d**11*e**2 + 2160*d**10*e**3*x + 5400*d**9*e**4*x**2
 + 7200*d**8*e**5*x**3 + 5400*d**7*e**6*x**4 + 2160*d**6*e**7*x**5 + 360*d*
*5*e**8*x**6) - 72*a*d**5*e*x/(360*d**11*e**2 + 2160*d**10*e**3*x + 5400*d*
*9*e**4*x**2 + 7200*d**8*e**5*x**3 + 5400*d**7*e**6*x**4 + 2160*d**6*e**7*x
**5 + 360*d**5*e**8*x**6) - 12*b*d**6*n*log(d/e + x)/(360*d**11*e**2 + 2160
*d**10*e**3*x + 5400*d**9*e**4*x**2 + 7200*d**8*e**5*x**3 + 5400*d**7*e**6*
x**4 + 2160*d**6*e**7*x**5 + 360*d**5*e**8*x**6) + 13*b*d**6*n/(360*d**11*e
**2 + 2160*d**10*e**3*x + 5400*d**9*e**4*x**2 + 7200*d**8*e**5*x**3 + 5400*
d**7*e**6*x**4 + 2160*d**6*e**7*x**5 + 360*d**5*e**8*x**6) - 72*b*d**5*e*n*
x*log(d/e + x)/(360*d**11*e**2 + 2160*d**10*e**3*x + 5400*d**9*e**4*x**2 +
```

```
7200*d**8*e**5*x**3 + 5400*d**7*e**6*x**4 + 2160*d**6*e**7*x**5 + 360*d**5*
e**8*x**6) + 90*b*d**5*e*n*x/(360*d**11*e**2 + 2160*d**10*e**3*x + 5400*d**
9*e**4*x**2 + 7200*d**8*e**5*x**3 + 5400*d**7*e**6*x**4 + 2160*d**6*e**7*x*
*5 + 360*d**5*e**8*x**6) - 180*b*d**4*e**2*n*x**2*log(d/e + x)/(360*d**11*e
**2 + 2160*d**10*e**3*x + 5400*d**9*e**4*x**2 + 7200*d**8*e**5*x**3 + 5400*
d**7*e**6*x**4 + 2160*d**6*e**7*x**5 + 360*d**5*e**8*x**6) + 171*b*d**4*e**
2*n*x**2/(360*d**11*e**2 + 2160*d**10*e**3*x + 5400*d**9*e**4*x**2 + 7200*d
**8*e**5*x**3 + 5400*d**7*e**6*x**4 + 2160*d**6*e**7*x**5 + 360*d**5*e**8*x
**6) + 180*b*d**4*e**2*x**2*log(c*x**n)/(360*d**11*e**2 + 2160*d**10*e**3*x
 + 5400*d**9*e**4*x**2 + 7200*d**8*e**5*x**3 + 5400*d**7*e**6*x**4 + 2160*d
**6*e**7*x**5 + 360*d**5*e**8*x**6) - 240*b*d**3*e**3*n*x**3*log(d/e + x)/(
360*d**11*e**2 + 2160*d**10*e**3*x + 5400*d**9*e**4*x**2 + 7200*d**8*e**5*x
**3 + 5400*d**7*e**6*x**4 + 2160*d**6*e**7*x**5 + 360*d**5*e**8*x**6) + 148
*b*d**3*e**3*n*x**3/(360*d**11*e**2 + 2160*d**10*e**3*x + 5400*d**9*e**4*x*
*2 + 7200*d**8*e**5*x**3 + 5400*d**7*e**6*x**4 + 2160*d**6*e**7*x**5 + 360*
d**5*e**8*x**6) + 240*b*d**3*e**3*x**3*log(c*x**n)/(360*d**11*e**2 + 2160*d
**10*e**3*x + 5400*d**9*e**4*x**2 + 7200*d**8*e**5*x**3 + 5400*d**7*e**6*x*
*4 + 2160*d**6*e**7*x**5 + 360*d**5*e**8*x**6) - 180*b*d**2*e**4*n*x**4*log
(d/e + x)/(360*d**11*e**2 + 2160*d**10*e**3*x + 5400*d**9*e**4*x**2 + 7200*
d**8*e**5*x**3 + 5400*d**7*e**6*x**4 + 2160*d**6*e**7*x**5 + 360*d**5*e**8*
x**6) + 66*b*d**2*e**4*n*x**4/(360*d**11*e**2 + 2160*d**10*e**3*x + 5400*d*
*9*e**4*x**2 + 7200*d**8*e**5*x**3 + 5400*d**7*e**6*x**4 + 2160*d**6*e**7*x
**5 + 360*d**5*e**8*x**6) + 180*b*d**2*e**4*x**4*log(c*x**n)/(360*d**11*e**
2 + 2160*d**10*e**3*x + 5400*d**9*e**4*x**2 + 7200*d**8*e**5*x**3 + 5400*d*
*7*e**6*x**4 + 2160*d**6*e**7*x**5 + 360*d**5*e**8*x**6) - 72*b*d*e**5*n*x*
*5*log(d/e + x)/(360*d**11*e**2 + 2160*d**10*e**3*x + 5400*d**9*e**4*x**2 +
 7200*d**8*e**5*x**3 + 5400*d**7*e**6*x**4 + 2160*d**6*e**7*x**5 + 360*d**5
*e**8*x**6) + 12*b*d*e**5*n*x**5/(360*d**11*e**2 + 2160*d**10*e**3*x + 5400
*d**9*e**4*x**2 + 7200*d**8*e**5*x**3 + 5400*d**7*e**6*x**4 + 2160*d**6*e**
7*x**5 + 360*d**5*e**8*x**6) + 72*b*d*e**5*x**5*log(c*x**n)/(360*d**11*e**2
 + 2160*d**10*e**3*x + 5400*d**9*e**4*x**2 + 7200*d**8*e**5*x**3 + 5400*d**
7*e**6*x**4 + 2160*d**6*e**7*x**5 + 360*d**5*e**8*x**6) - 12*b*e**6*n*x**6*
log(d/e + x)/(360*d**11*e**2 + 2160*d**10*e**3*x + 5400*d**9*e**4*x**2 + 72
00*d**8*e**5*x**3 + 5400*d**7*e**6*x**4 + 2160*d**6*e**7*x**5 + 360*d**5*e*
*8*x**6) + 12*b*e**6*x**6*log(c*x**n)/(360*d**11*e**2 + 2160*d**10*e**3*x +
 5400*d**9*e**4*x**2 + 7200*d**8*e**5*x**3 + 5400*d**7*e**6*x**4 + 2160*d**
6*e**7*x**5 + 360*d**5*e**8*x**6), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.69

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^7}\,dx$$
$$=\frac{1}{360}\,bn\left(\frac{12\,e^4x^4+54\,de^3x^3+94\,d^2e^2x^2+77\,d^3ex+13\,d^4}{d^4e^7x^5+5\,d^5e^6x^4+10\,d^6e^5x^3+10\,d^7e^4x^2+5\,d^8e^3x+d^9e^2}-\frac{12\,\log(ex+d)}{d^5e^2}+\frac{12\,\log(x)}{d^5e^2}\right)$$
$$-\frac{(6\,ex+d)b\log(cx^n)}{30\,(e^8x^6+6\,de^7x^5+15\,d^2e^6x^4+20\,d^3e^5x^3+15\,d^4e^4x^2+6\,d^5e^3x+d^6e^2)}$$
$$-\frac{(6\,ex+d)a}{30\,(e^8x^6+6\,de^7x^5+15\,d^2e^6x^4+20\,d^3e^5x^3+15\,d^4e^4x^2+6\,d^5e^3x+d^6e^2)}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] 1/360*b*n*((12*e^4*x^4 + 54*d*e^3*x^3 + 94*d^2*e^2*x^2 + 77*d^3*e*x + 13*d^
4)/(d^4*e^7*x^5 + 5*d^5*e^6*x^4 + 10*d^6*e^5*x^3 + 10*d^7*e^4*x^2 + 5*d^8*e
^3*x + d^9*e^2) - 12*log(e*x + d)/(d^5*e^2) + 12*log(x)/(d^5*e^2)) - 1/30*(
6*e*x + d)*b*log(c*x^n)/(e^8*x^6 + 6*d*e^7*x^5 + 15*d^2*e^6*x^4 + 20*d^3*e^
5*x^3 + 15*d^4*e^4*x^2 + 6*d^5*e^3*x + d^6*e^2) - 1/30*(6*e*x + d)*a/(e^8*x
^6 + 6*d*e^7*x^5 + 15*d^2*e^6*x^4 + 20*d^3*e^5*x^3 + 15*d^4*e^4*x^2 + 6*d^5
*e^3*x + d^6*e^2)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.63

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^7}\,dx$$
$$=-\frac{(6\,benx+bdn)\log(x)}{30\,(e^8x^6+6\,de^7x^5+15\,d^2e^6x^4+20\,d^3e^5x^3+15\,d^4e^4x^2+6\,d^5e^3x+d^6e^2)}$$
$$+\frac{12\,be^5nx^5+66\,bde^4nx^4+148\,bd^2e^3nx^3+171\,bd^3e^2nx^2+90\,bd^4enx-72\,bd^4ex\log(c)+13\,bd^5n-72\,ad}{360\,(d^4e^8x^6+6\,d^5e^7x^5+15\,d^6e^6x^4+20\,d^7e^5x^3+15\,d^8e^4x^2+6\,d^9e^3x+d^{10}e^2}$$
$$-\frac{bn\log(ex+d)}{30\,d^5e^2}+\frac{bn\log(x)}{30\,d^5e^2}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] -1/30*(6*b*e*n*x + b*d*n)*log(x)/(e^8*x^6 + 6*d*e^7*x^5 + 15*d^2*e^6*x^4 +
20*d^3*e^5*x^3 + 15*d^4*e^4*x^2 + 6*d^5*e^3*x + d^6*e^2) + 1/360*(12*b*e^5*
n*x^5 + 66*b*d*e^4*n*x^4 + 148*b*d^2*e^3*n*x^3 + 171*b*d^3*e^2*n*x^2 + 90*b
*d^4*e*n*x - 72*b*d^4*e*x*log(c) + 13*b*d^5*n - 72*a*d^4*e*x - 12*b*d^5*log
```

```
(c) - 12*a*d^5)/(d^4*e^8*x^6 + 6*d^5*e^7*x^5 + 15*d^6*e^6*x^4 + 20*d^7*e^5*
x^3 + 15*d^8*e^4*x^2 + 6*d^9*e^3*x + d^10*e^2) - 1/30*b*n*log(e*x + d)/(d^5
*e^2) + 1/30*b*n*log(x)/(d^5*e^2)
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.44

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^7}\,dx$$
$$= \frac{\frac{13\,b\,d\,n}{12}-x\left(6\,a\,e-\frac{15\,b\,e\,n}{2}\right)-a\,d+\frac{57\,b\,e^2\,n\,x^2}{4\,d}+\frac{37\,b\,e^3\,n\,x^3}{3\,d^2}+\frac{11\,b\,e^4\,n\,x^4}{2\,d^3}+\frac{b\,e^5\,n\,x^5}{d^4}}{30\,d^6\,e^2+180\,d^5\,e^3\,x+450\,d^4\,e^4\,x^2+600\,d^3\,e^5\,x^3+450\,d^2\,e^6\,x^4+180\,d\,e^7\,x^5+30\,e^8\,x^6}$$
$$-\frac{\ln\left(c\,x^n\right)\,\left(\frac{b\,d}{30\,e^2}+\frac{b\,x}{5\,e}\right)}{d^6+6\,d^5\,e\,x+15\,d^4\,e^2\,x^2+20\,d^3\,e^3\,x^3+15\,d^2\,e^4\,x^4+6\,d\,e^5\,x^5+e^6\,x^6}$$
$$-\frac{b\,n\,\mathrm{atanh}\left(\frac{2\,e\,x}{d}+1\right)}{15\,d^5\,e^2}$$

```
[In] int((x*(a + b*log(c*x^n)))/(d + e*x)^7,x)
```

```
[Out] ((13*b*d*n)/12 - x*(6*a*e - (15*b*e*n)/2) - a*d + (57*b*e^2*n*x^2)/(4*d) +
(37*b*e^3*n*x^3)/(3*d^2) + (11*b*e^4*n*x^4)/(2*d^3) + (b*e^5*n*x^5)/d^4)/(3
0*d^6*e^2 + 30*e^8*x^6 + 180*d^5*e^3*x + 180*d*e^7*x^5 + 450*d^4*e^4*x^2 +
600*d^3*e^5*x^3 + 450*d^2*e^6*x^4) - (log(c*x^n)*((b*d)/(30*e^2) + (b*x)/(5
*e)))/(d^6 + e^6*x^6 + 6*d*e^5*x^5 + 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 + 15*d
^2*e^4*x^4 + 6*d^5*e*x) - (b*n*atanh((2*e*x)/d + 1))/(15*d^5*e^2)
```

3.70 $\int \frac{a+b\log(cx^n)}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 152

$$\int \frac{a+b\log(cx^n)}{(d+ex)^7}\,dx = \frac{bn}{30de(d+ex)^5}+\frac{bn}{24d^2e(d+ex)^4}+\frac{bn}{18d^3e(d+ex)^3}+\frac{bn}{12d^4e(d+ex)^2}+\frac{bn}{6d^5e(d+ex)}+\frac{bn\log(x)}{6d^6e}-\frac{a+b\log(cx^n)}{6e(d+ex)^6}-\frac{bn\log(d+ex)}{6d^6e}$$

```
[Out] 1/30*b*n/d/e/(e*x+d)^5+1/24*b*n/d^2/e/(e*x+d)^4+1/18*b*n/d^3/e/(e*x+d)^3+1/
12*b*n/d^4/e/(e*x+d)^2+1/6*b*n/d^5/e/(e*x+d)+1/6*b*n*ln(x)/d^6/e+1/6*(-a-b*
ln(c*x^n))/e/(e*x+d)^6-1/6*b*n*ln(e*x+d)/d^6/e
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2356, 46}

$$\int \frac{a+b\log(cx^n)}{(d+ex)^7}\,dx = -\frac{a+b\log(cx^n)}{6e(d+ex)^6}+\frac{bn\log(x)}{6d^6e}-\frac{bn\log(d+ex)}{6d^6e}+\frac{bn}{6d^5e(d+ex)}+\frac{bn}{12d^4e(d+ex)^2}+\frac{bn}{18d^3e(d+ex)^3}+\frac{bn}{24d^2e(d+ex)^4}+\frac{bn}{30de(d+ex)^5}$$

```
[In] Int[(a + b*Log[c*x^n])/(d + e*x)^7,x]
```

```
[Out] (b*n)/(30*d*e*(d + e*x)^5) + (b*n)/(24*d^2*e*(d + e*x)^4) + (b*n)/(18*d^3*e
*(d + e*x)^3) + (b*n)/(12*d^4*e*(d + e*x)^2) + (b*n)/(6*d^5*e*(d + e*x)) +
```

```
(b*n*Log[x])/(6*d^6*e) - (a + b*Log[c*x^n])/(6*e*(d + e*x)^6) - (b*n*Log[d
+ e*x])/(6*d^6*e)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\log\left(cx^n\right)}{6e(d+ex)^6}+\frac{(bn)\int\frac{1}{x(d+ex)^6}\,dx}{6e}\\
&= -\frac{a+b\log\left(cx^n\right)}{6e(d+ex)^6}\\
&\quad+\frac{(bn)\int\left(\frac{1}{d^6x}-\frac{e}{d(d+ex)^6}-\frac{e}{d^2(d+ex)^5}-\frac{e}{d^3(d+ex)^4}-\frac{e}{d^4(d+ex)^3}-\frac{e}{d^5(d+ex)^2}-\frac{e}{d^6(d+ex)}\right)dx}{6e}\\
&= \frac{bn}{30de(d+ex)^5}+\frac{bn}{24d^2e(d+ex)^4}+\frac{bn}{18d^3e(d+ex)^3}+\frac{bn}{12d^4e(d+ex)^2}\\
&\quad+\frac{bn}{6d^5e(d+ex)}+\frac{bn\log(x)}{6d^6e}-\frac{a+b\log\left(cx^n\right)}{6e(d+ex)^6}-\frac{bn\log(d+ex)}{6d^6e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.65

$$\int\frac{a+b\log\left(cx^n\right)}{(d+ex)^7}\,dx=\frac{-\frac{a+b\log\left(cx^n\right)}{(d+ex)^6}+\frac{bn\left(\frac{d\left(137d^4+385d^3ex+470d^2e^2x^2+270de^3x^3+60e^4x^4\right)}{(d+ex)^5}+60\log(x)-60\log(d+ex)\right)}{60d^6}}{6e}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*x)^7,x]
```

```
[Out] (-((a + b*Log[c*x^n])/(d + e*x)^6) + (b*n*((d*(137*d^4 + 385*d^3*e*x + 470*
d^2*e^2*x^2 + 270*d*e^3*x^3 + 60*e^4*x^4))/(d + e*x)^5 + 60*Log[x] - 60*Log
[d + e*x]))/(60*d^6))/(6*e)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(139) = 278.

Time = 1.32 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.54

method	result
parallelrisch	$\frac{-360\ln(ex+d)x^5bd\,e^{10}n+900\ln(x)x^4b\,d^2e^9n-900\ln(ex+d)x^4b\,d^2e^9n+1200\ln(x)x^3b\,d^3e^8n-1200\ln(ex+d)x^3b\,d^3e^8n+360\ln(}{}$
risch	$-\frac{b\ln(x^n)}{6e(ex+d)^6}-\frac{900\ln(ex+d)b\,d^2e^4n\,x^4-360\ln(-x)bd\,e^5n\,x^5-30i\pi b\,d^6\operatorname{csgn}(ic\,x^n)^3+900\ln(ex+d)b\,d^4e^2n\,x^2-137b\,d^6n-900}{}$

```
[In] int((a+b*ln(c*x^n))/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/360*(-360*ln(e*x+d)*x^5*b*d*e^10*n+900*ln(x)*x^4*b*d^2*e^9*n-900*ln(e*x+d
)*x^4*b*d^2*e^9*n+1200*ln(x)*x^3*b*d^3*e^8*n-1200*ln(e*x+d)*x^3*b*d^3*e^8*n
+360*ln(x)*x^5*b*d*e^10*n+60*ln(x)*x^6*b*e^11*n-60*ln(e*x+d)*x^6*b*e^11*n+9
00*x^4*a*d^2*e^9+360*x^5*a*d*e^10+360*x*a*d^5*e^6+900*x^2*a*d^4*e^7+1200*x^
3*a*d^3*e^8-2000*x^3*b*d^3*e^8*n-1200*x^2*b*d^4*e^7*n-300*x*b*d^5*e^6*n+60*
ln(x)*b*d^6*e^5*n-60*ln(e*x+d)*b*d^6*e^5*n+900*ln(x)*x^2*b*d^4*e^7*n-900*ln
(e*x+d)*x^2*b*d^4*e^7*n+360*ln(x)*x*b*d^5*e^6*n-360*ln(e*x+d)*x*b*d^5*e^6*n
-762*x^5*b*d*e^10*n-1725*x^4*b*d^2*e^9*n-137*x^6*b*e^11*n-60*ln(c*x^n)*b*d^
6*e^5+60*x^6*a*e^11)/e^6/d^6/(e*x+d)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(136) = 272.

Time = 0.32 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.04

$$\int \frac{a+b\log(cx^n)}{(d+ex)^7}\,dx = \frac{60\,bde^5nx^5+330\,bd^2e^4nx^4+740\,bd^3e^3nx^3+855\,bd^4e^2nx^2+522\,bd^5enx+137\,bd^6n-60\,bd^6\log(c)-60\,a}{}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] 1/360*(60*b*d*e^5*n*x^5 + 330*b*d^2*e^4*n*x^4 + 740*b*d^3*e^3*n*x^3 + 855*b
*d^4*e^2*n*x^2 + 522*b*d^5*e*n*x + 137*b*d^6*n - 60*b*d^6*log(c) - 60*a*d^6
 - 60*(b*e^6*n*x^6 + 6*b*d*e^5*n*x^5 + 15*b*d^2*e^4*n*x^4 + 20*b*d^3*e^3*n*
x^3 + 15*b*d^4*e^2*n*x^2 + 6*b*d^5*e*n*x + b*d^6*n)*log(e*x + d) + 60*(b*e^
6*n*x^6 + 6*b*d*e^5*n*x^5 + 15*b*d^2*e^4*n*x^4 + 20*b*d^3*e^3*n*x^3 + 15*b*
d^4*e^2*n*x^2 + 6*b*d^5*e*n*x)*log(x))/(d^6*e^7*x^6 + 6*d^7*e^6*x^5 + 15*d^
8*e^5*x^4 + 20*d^9*e^4*x^3 + 15*d^10*e^3*x^2 + 6*d^11*e^2*x + d^12*e)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1955 vs. 2(134) = 268.

Time = 76.21 (sec) , antiderivative size = 1955, normalized size of antiderivative = 12.86

$$\int \frac{a+b\log (cx^n)}{(d+ex)^7}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*ln(c*x**n))/(e*x+d)**7,x)
```

```
[Out] Piecewise((zoo*(-a/(6*x**6) - b*n/(36*x**6) - b*log(c*x**n)/(6*x**6)), Eq(d
, 0) & Eq(e, 0)), ((a*x - b*n*x + b*x*log(c*x**n))/d**7, Eq(e, 0)), ((-a/(6
*x**6) - b*n/(36*x**6) - b*log(c*x**n)/(6*x**6))/e**7, Eq(d, 0)), (-60*a*d*
*6/(360*d**12*e + 2160*d**11*e**2*x + 5400*d**10*e**3*x**2 + 7200*d**9*e**4
*x**3 + 5400*d**8*e**5*x**4 + 2160*d**7*e**6*x**5 + 360*d**6*e**7*x**6) - 6
0*b*d**6*n*log(d/e + x)/(360*d**12*e + 2160*d**11*e**2*x + 5400*d**10*e**3*
x**2 + 7200*d**9*e**4*x**3 + 5400*d**8*e**5*x**4 + 2160*d**7*e**6*x**5 + 36
0*d**6*e**7*x**6) + 137*b*d**6*n/(360*d**12*e + 2160*d**11*e**2*x + 5400*d*
*10*e**3*x**2 + 7200*d**9*e**4*x**3 + 5400*d**8*e**5*x**4 + 2160*d**7*e**6*
x**5 + 360*d**6*e**7*x**6) - 360*b*d**5*e*n*x*log(d/e + x)/(360*d**12*e + 2
160*d**11*e**2*x + 5400*d**10*e**3*x**2 + 7200*d**9*e**4*x**3 + 5400*d**8*e
**5*x**4 + 2160*d**7*e**6*x**5 + 360*d**6*e**7*x**6) + 522*b*d**5*e*n*x/(36
0*d**12*e + 2160*d**11*e**2*x + 5400*d**10*e**3*x**2 + 7200*d**9*e**4*x**3
+ 5400*d**8*e**5*x**4 + 2160*d**7*e**6*x**5 + 360*d**6*e**7*x**6) + 360*b*d
**5*e*x*log(c*x**n)/(360*d**12*e + 2160*d**11*e**2*x + 5400*d**10*e**3*x**2
 + 7200*d**9*e**4*x**3 + 5400*d**8*e**5*x**4 + 2160*d**7*e**6*x**5 + 360*d*
*6*e**7*x**6) - 900*b*d**4*e**2*n*x**2*log(d/e + x)/(360*d**12*e + 2160*d**
11*e**2*x + 5400*d**10*e**3*x**2 + 7200*d**9*e**4*x**3 + 5400*d**8*e**5*x**
4 + 2160*d**7*e**6*x**5 + 360*d**6*e**7*x**6) + 855*b*d**4*e**2*n*x**2/(360
*d**12*e + 2160*d**11*e**2*x + 5400*d**10*e**3*x**2 + 7200*d**9*e**4*x**3 +
 5400*d**8*e**5*x**4 + 2160*d**7*e**6*x**5 + 360*d**6*e**7*x**6) + 900*b*d*
*4*e**2*x**2*log(c*x**n)/(360*d**12*e + 2160*d**11*e**2*x + 5400*d**10*e**3
*x**2 + 7200*d**9*e**4*x**3 + 5400*d**8*e**5*x**4 + 2160*d**7*e**6*x**5 + 3
60*d**6*e**7*x**6) - 1200*b*d**3*e**3*n*x**3*log(d/e + x)/(360*d**12*e + 21
60*d**11*e**2*x + 5400*d**10*e**3*x**2 + 7200*d**9*e**4*x**3 + 5400*d**8*e*
*5*x**4 + 2160*d**7*e**6*x**5 + 360*d**6*e**7*x**6) + 740*b*d**3*e**3*n*x**
3/(360*d**12*e + 2160*d**11*e**2*x + 5400*d**10*e**3*x**2 + 7200*d**9*e**4*
x**3 + 5400*d**8*e**5*x**4 + 2160*d**7*e**6*x**5 + 360*d**6*e**7*x**6) + 12
00*b*d**3*e**3*x**3*log(c*x**n)/(360*d**12*e + 2160*d**11*e**2*x + 5400*d**
10*e**3*x**2 + 7200*d**9*e**4*x**3 + 5400*d**8*e**5*x**4 + 2160*d**7*e**6*x
**5 + 360*d**6*e**7*x**6) - 900*b*d**2*e**4*n*x**4*log(d/e + x)/(360*d**12*
e + 2160*d**11*e**2*x + 5400*d**10*e**3*x**2 + 7200*d**9*e**4*x**3 + 5400*d
**8*e**5*x**4 + 2160*d**7*e**6*x**5 + 360*d**6*e**7*x**6) + 330*b*d**2*e**4
*n*x**4/(360*d**12*e + 2160*d**11*e**2*x + 5400*d**10*e**3*x**2 + 7200*d**9
*e**4*x**3 + 5400*d**8*e**5*x**4 + 2160*d**7*e**6*x**5 + 360*d**6*e**7*x**6
```

```
) + 900*b*d**2*e**4*x**4*log(c*x**n)/(360*d**12*e + 2160*d**11*e**2*x + 540
0*d**10*e**3*x**2 + 7200*d**9*e**4*x**3 + 5400*d**8*e**5*x**4 + 2160*d**7*e
**6*x**5 + 360*d**6*e**7*x**6) - 360*b*d*e**5*n*x**5*log(d/e + x)/(360*d**1
2*e + 2160*d**11*e**2*x + 5400*d**10*e**3*x**2 + 7200*d**9*e**4*x**3 + 5400
*d**8*e**5*x**4 + 2160*d**7*e**6*x**5 + 360*d**6*e**7*x**6) + 60*b*d*e**5*n
*x**5/(360*d**12*e + 2160*d**11*e**2*x + 5400*d**10*e**3*x**2 + 7200*d**9*e
**4*x**3 + 5400*d**8*e**5*x**4 + 2160*d**7*e**6*x**5 + 360*d**6*e**7*x**6)
+ 360*b*d*e**5*x**5*log(c*x**n)/(360*d**12*e + 2160*d**11*e**2*x + 5400*d**
10*e**3*x**2 + 7200*d**9*e**4*x**3 + 5400*d**8*e**5*x**4 + 2160*d**7*e**6*x
**5 + 360*d**6*e**7*x**6) - 60*b*e**6*n*x**6*log(d/e + x)/(360*d**12*e + 21
60*d**11*e**2*x + 5400*d**10*e**3*x**2 + 7200*d**9*e**4*x**3 + 5400*d**8*e*
*5*x**4 + 2160*d**7*e**6*x**5 + 360*d**6*e**7*x**6) + 60*b*e**6*x**6*log(c*
x**n)/(360*d**12*e + 2160*d**11*e**2*x + 5400*d**10*e**3*x**2 + 7200*d**9*e
**4*x**3 + 5400*d**8*e**5*x**4 + 2160*d**7*e**6*x**5 + 360*d**6*e**7*x**6),
 True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(136) = 272$.

Time = 0.21 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.82

$$\int \frac{a + b\log(cx^n)}{(d + ex)^7}\,dx$$
$$= \frac{1}{360}\,bn\left(\frac{60\,e^4x^4 + 270\,de^3x^3 + 470\,d^2e^2x^2 + 385\,d^3ex + 137\,d^4}{d^5e^6x^5 + 5\,d^6e^5x^4 + 10\,d^7e^4x^3 + 10\,d^8e^3x^2 + 5\,d^9e^2x + d^{10}e} - \frac{60\,\log(ex + d)}{d^6e} + \frac{60\,\log(x)}{d^6e}\right)$$
$$- \frac{b\log(cx^n)}{6\,(e^7x^6 + 6\,de^6x^5 + 15\,d^2e^5x^4 + 20\,d^3e^4x^3 + 15\,d^4e^3x^2 + 6\,d^5e^2x + d^6e)}$$
$$- \frac{a}{6\,(e^7x^6 + 6\,de^6x^5 + 15\,d^2e^5x^4 + 20\,d^3e^4x^3 + 15\,d^4e^3x^2 + 6\,d^5e^2x + d^6e)}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] 1/360*b*n*((60*e^4*x^4 + 270*d*e^3*x^3 + 470*d^2*e^2*x^2 + 385*d^3*e*x + 13
7*d^4)/(d^5*e^6*x^5 + 5*d^6*e^5*x^4 + 10*d^7*e^4*x^3 + 10*d^8*e^3*x^2 + 5*d
^9*e^2*x + d^10*e) - 60*log(e*x + d)/(d^6*e) + 60*log(x)/(d^6*e)) - 1/6*b*l
og(c*x^n)/(e^7*x^6 + 6*d*e^6*x^5 + 15*d^2*e^5*x^4 + 20*d^3*e^4*x^3 + 15*d^4
*e^3*x^2 + 6*d^5*e^2*x + d^6*e) - 1/6*a/(e^7*x^6 + 6*d*e^6*x^5 + 15*d^2*e^5
*x^4 + 20*d^3*e^4*x^3 + 15*d^4*e^3*x^2 + 6*d^5*e^2*x + d^6*e)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.66

$$\int \frac{a+b\log(cx^n)}{(d+ex)^7}\,dx$$
$$= -\frac{bn\log(x)}{6\,(e^7x^6+6\,de^6x^5+15\,d^2e^5x^4+20\,d^3e^4x^3+15\,d^4e^3x^2+6\,d^5e^2x+d^6e)} + \frac{60\,be^5nx^5+330\,bde^4nx^4+740\,bd^2e^3nx^3+855\,bd^3e^2nx^2+522\,bd^4enx+137\,bd^5n-60\,bd^5\log(c)-60\,}{360\,(d^5e^7x^6+6\,d^6e^6x^5+15\,d^7e^5x^4+20\,d^8e^4x^3+15\,d^9e^3x^2+6\,d^{10}e^2x+d^{11}e)} - \frac{bn\log(ex+d)}{6\,d^6e} + \frac{bn\log(x)}{6\,d^6e}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] -1/6*b*n*log(x)/(e^7*x^6 + 6*d*e^6*x^5 + 15*d^2*e^5*x^4 + 20*d^3*e^4*x^3 +
15*d^4*e^3*x^2 + 6*d^5*e^2*x + d^6*e) + 1/360*(60*b*e^5*n*x^5 + 330*b*d*e^4
*n*x^4 + 740*b*d^2*e^3*n*x^3 + 855*b*d^3*e^2*n*x^2 + 522*b*d^4*e*n*x + 137*
b*d^5*n - 60*b*d^5*log(c) - 60*a*d^5)/(d^5*e^7*x^6 + 6*d^6*e^6*x^5 + 15*d^7
*e^5*x^4 + 20*d^8*e^4*x^3 + 15*d^9*e^3*x^2 + 6*d^10*e^2*x + d^11*e) - 1/6*b
*n*log(e*x + d)/(d^6*e) + 1/6*b*n*log(x)/(d^6*e)
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.53

$$\int \frac{a+b\log(cx^n)}{(d+ex)^7}\,dx$$
$$= \frac{\frac{137\,b\,n}{60}-a+\frac{57\,b\,e^2\,n\,x^2}{4\,d^2}+\frac{37\,b\,e^3\,n\,x^3}{3\,d^3}+\frac{11\,b\,e^4\,n\,x^4}{2\,d^4}+\frac{b\,e^5\,n\,x^5}{d^5}+\frac{87\,b\,e\,n\,x}{10\,d}}{6\,d^6\,e+36\,d^5\,e^2\,x+90\,d^4\,e^3\,x^2+120\,d^3\,e^4\,x^3+90\,d^2\,e^5\,x^4+36\,d\,e^6\,x^5+6\,e^7\,x^6} - \frac{b\,\ln(c\,x^n)}{6\,e\,(d^6+6\,d^5\,e\,x+15\,d^4\,e^2\,x^2+20\,d^3\,e^3\,x^3+15\,d^2\,e^4\,x^4+6\,d\,e^5\,x^5+e^6\,x^6)} - \frac{b\,n\,\mathrm{atanh}\left(\frac{2\,e\,x}{d}+1\right)}{3\,d^6\,e}$$

```
[In] int((a + b*log(c*x^n))/(d + e*x)^7,x)
```

```
[Out] ((137*b*n)/60 - a + (57*b*e^2*n*x^2)/(4*d^2) + (37*b*e^3*n*x^3)/(3*d^3) + (
11*b*e^4*n*x^4)/(2*d^4) + (b*e^5*n*x^5)/d^5 + (87*b*e*n*x)/(10*d))/(6*d^6*e
 + 6*e^7*x^6 + 36*d^5*e^2*x + 36*d*e^6*x^5 + 90*d^4*e^3*x^2 + 120*d^3*e^4*x
^3 + 90*d^2*e^5*x^4) - (b*log(c*x^n))/(6*e*(d^6 + e^6*x^6 + 6*d*e^5*x^5 + 1
5*d^4*e^2*x^2 + 20*d^3*e^3*x^3 + 15*d^2*e^4*x^4 + 6*d^5*e*x)) - (b*n*atanh(
(2*e*x)/d + 1))/(3*d^6*e)
```

3.71 $\int \frac{a+b\log(cx^n)}{x(d+ex)^7}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 294

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^7}\,dx = -\frac{bn}{30d^2(d+ex)^5} - \frac{11bn}{120d^3(d+ex)^4} - \frac{37bn}{180d^4(d+ex)^3} - \frac{19bn}{40d^5(d+ex)^2} - \frac{29bn}{20d^6(d+ex)} - \frac{29bn\log(x)}{20d^7} + \frac{a+b\log(cx^n)}{6d(d+ex)^6} + \frac{a+b\log(cx^n)}{5d^2(d+ex)^5} + \frac{a+b\log(cx^n)}{4d^3(d+ex)^4} + \frac{a+b\log(cx^n)}{3d^4(d+ex)^3} + \frac{a+b\log(cx^n)}{2d^5(d+ex)^2} - \frac{ex(a+b\log(cx^n))}{d^7(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^7} + \frac{49bn\log(d+ex)}{20d^7} + \frac{bn\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^7}$$

```
[Out] -1/30*b*n/d^2/(e*x+d)^5-11/120*b*n/d^3/(e*x+d)^4-37/180*b*n/d^4/(e*x+d)^3-1
9/40*b*n/d^5/(e*x+d)^2-29/20*b*n/d^6/(e*x+d)-29/20*b*n*ln(x)/d^7+1/6*(a+b*l
n(c*x^n))/d/(e*x+d)^6+1/5*(a+b*ln(c*x^n))/d^2/(e*x+d)^5+1/4*(a+b*ln(c*x^n))
/d^3/(e*x+d)^4+1/3*(a+b*ln(c*x^n))/d^4/(e*x+d)^3+1/2*(a+b*ln(c*x^n))/d^5/(e
*x+d)^2-e*x*(a+b*ln(c*x^n))/d^7/(e*x+d)-ln(1+d/e/x)*(a+b*ln(c*x^n))/d^7+49/
20*b*n*ln(e*x+d)/d^7+b*n*polylog(2,-d/e/x)/d^7
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^7}\,dx = -\frac{\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^7}-\frac{ex(a+b\log(cx^n))}{d^7(d+ex)} +\frac{a+b\log(cx^n)}{2d^5(d+ex)^2}+\frac{a+b\log(cx^n)}{3d^4(d+ex)^3}+\frac{a+b\log(cx^n)}{4d^3(d+ex)^4} +\frac{a+b\log(cx^n)}{5d^2(d+ex)^5}+\frac{a+b\log(cx^n)}{6d(d+ex)^6}+\frac{bn\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^7} +\frac{49bn\log(d+ex)}{20d^7}-\frac{29bn\log(x)}{20d^7}-\frac{29bn}{20d^6(d+ex)}-\frac{19bn}{40d^5(d+ex)^2} -\frac{37bn}{180d^4(d+ex)^3}-\frac{11bn}{120d^3(d+ex)^4}-\frac{bn}{30d^2(d+ex)^5}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x)^7),x]
```

```
[Out] -1/30*(b*n)/(d^2*(d + e*x)^5) - (11*b*n)/(120*d^3*(d + e*x)^4) - (37*b*n)/(
180*d^4*(d + e*x)^3) - (19*b*n)/(40*d^5*(d + e*x)^2) - (29*b*n)/(20*d^6*(d
+ e*x)) - (29*b*n*Log[x])/(20*d^7) + (a + b*Log[c*x^n])/(6*d*(d + e*x)^6) +
 (a + b*Log[c*x^n])/(5*d^2*(d + e*x)^5) + (a + b*Log[c*x^n])/(4*d^3*(d + e*
x)^4) + (a + b*Log[c*x^n])/(3*d^4*(d + e*x)^3) + (a + b*Log[c*x^n])/(2*d^5*
(d + e*x)^2) - (e*x*(a + b*Log[c*x^n]))/(d^7*(d + e*x)) - (Log[1 + d/(e*x)]
*(a + b*Log[c*x^n]))/d^7 + (49*b*n*Log[d + e*x])/(20*d^7) + (b*n*PolyLog[2,
 -(d/(e*x))])/d^7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
```

```
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{a+b\log(cx^n)}{x(d+ex)^6}\,dx}{d}-\frac{e\int \frac{a+b\log(cx^n)}{(d+ex)^7}\,dx}{d}\\
&= \frac{a+b\log(cx^n)}{6d(d+ex)^6}+\frac{\int \frac{a+b\log(cx^n)}{x(d+ex)^5}\,dx}{d^2}-\frac{e\int \frac{a+b\log(cx^n)}{(d+ex)^6}\,dx}{d^2}-\frac{(bn)\int \frac{1}{x(d+ex)^6}\,dx}{6d}\\
&= \frac{a+b\log(cx^n)}{6d(d+ex)^6}+\frac{a+b\log(cx^n)}{5d^2(d+ex)^5}+\frac{\int \frac{a+b\log(cx^n)}{x(d+ex)^4}\,dx}{d^3}-\frac{e\int \frac{a+b\log(cx^n)}{(d+ex)^5}\,dx}{d^3}-\frac{(bn)\int \frac{1}{x(d+ex)^5}\,dx}{5d^2}\\
&\quad -\frac{(bn)\int \left(\frac{1}{d^6x}-\frac{e}{d(d+ex)^6}-\frac{e}{d^2(d+ex)^5}-\frac{e}{d^3(d+ex)^4}-\frac{e}{d^4(d+ex)^3}-\frac{e}{d^5(d+ex)^2}-\frac{e}{d^6(d+ex)}\right)dx}{6d}\end{aligned}$$

$$= -\frac{bn}{30d^2(d+ex)^5} - \frac{bn}{24d^3(d+ex)^4} - \frac{bn}{18d^4(d+ex)^3} - \frac{bn}{12d^5(d+ex)^2} - \frac{bn}{6d^6(d+ex)} - \frac{bn\log(x)}{6d^7} + \frac{a+b\log(cx^n)}{6d(d+ex)^6} + \frac{a+b\log(cx^n)}{5d^2(d+ex)^5} + \frac{a+b\log(cx^n)}{4d^3(d+ex)^4} + \frac{bn\log(d+ex)}{6d^7} + \frac{\int \frac{a+b\log(cx^n)}{x(d+ex)^3}\,dx}{d^4} - \frac{e\int \frac{a+b\log(cx^n)}{(d+ex)^4}\,dx}{d^4} - \frac{(bn)\int \frac{1}{x(d+ex)^4}\,dx}{4d^3} - \frac{(bn)\int \left(\frac{1}{d^5x} - \frac{e}{d(d+ex)^5} - \frac{e}{d^2(d+ex)^4} - \frac{e}{d^3(d+ex)^3} - \frac{e}{d^4(d+ex)^2} - \frac{e}{d^5(d+ex)}\right)dx}{5d^2}$$

$$= -\frac{bn}{30d^2(d+ex)^5} - \frac{11bn}{120d^3(d+ex)^4} - \frac{11bn}{90d^4(d+ex)^3} - \frac{11bn}{60d^5(d+ex)^2} - \frac{11bn}{30d^6(d+ex)} - \frac{11bn\log(x)}{30d^7} + \frac{a+b\log(cx^n)}{6d(d+ex)^6} + \frac{a+b\log(cx^n)}{5d^2(d+ex)^5} + \frac{a+b\log(cx^n)}{4d^3(d+ex)^4} + \frac{a+b\log(cx^n)}{3d^4(d+ex)^3} + \frac{11bn\log(d+ex)}{30d^7} + \frac{\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx}{d^5} - \frac{e\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx}{d^5} - \frac{(bn)\int \frac{1}{x(d+ex)^3}\,dx}{3d^4} - \frac{(bn)\int \left(\frac{1}{d^4x} - \frac{e}{d(d+ex)^4} - \frac{e}{d^2(d+ex)^3} - \frac{e}{d^3(d+ex)^2} - \frac{e}{d^4(d+ex)}\right)dx}{4d^3}$$

$$= -\frac{bn}{30d^2(d+ex)^5} - \frac{11bn}{120d^3(d+ex)^4} - \frac{37bn}{180d^4(d+ex)^3} - \frac{37bn}{120d^5(d+ex)^2} - \frac{37bn}{60d^6(d+ex)} - \frac{37bn\log(x)}{60d^7} + \frac{a+b\log(cx^n)}{6d(d+ex)^6} + \frac{a+b\log(cx^n)}{5d^2(d+ex)^5} + \frac{a+b\log(cx^n)}{4d^3(d+ex)^4} + \frac{a+b\log(cx^n)}{3d^4(d+ex)^3} + \frac{a+b\log(cx^n)}{2d^5(d+ex)^2} + \frac{37bn\log(d+ex)}{60d^7} + \frac{\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d^6} - \frac{e\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{d^6} - \frac{(bn)\int \frac{1}{x(d+ex)^2}\,dx}{2d^5} - \frac{(bn)\int \left(\frac{1}{d^3x} - \frac{e}{d(d+ex)^3} - \frac{e}{d^2(d+ex)^2} - \frac{e}{d^3(d+ex)}\right)dx}{3d^4}$$

$$= -\frac{bn}{30d^2(d+ex)^5} - \frac{11bn}{120d^3(d+ex)^4} - \frac{37bn}{180d^4(d+ex)^3} - \frac{19bn}{40d^5(d+ex)^2} - \frac{19bn}{20d^6(d+ex)} - \frac{19bn\log(x)}{20d^7} + \frac{a+b\log(cx^n)}{6d(d+ex)^6} + \frac{a+b\log(cx^n)}{5d^2(d+ex)^5} + \frac{a+b\log(cx^n)}{4d^3(d+ex)^4} + \frac{a+b\log(cx^n)}{3d^4(d+ex)^3} + \frac{a+b\log(cx^n)}{2d^5(d+ex)^2} - \frac{ex(a+b\log(cx^n))}{d^7(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^7} + \frac{19bn\log(d+ex)}{20d^7} + \frac{(bn)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^7} - \frac{(bn)\int \left(\frac{1}{d^2x} - \frac{e}{d(d+ex)^2} - \frac{e}{d^2(d+ex)}\right)dx}{2d^5} + \frac{(ben)\int \frac{1}{d+ex}\,dx}{d^7}$$

$$= -\frac{bn}{30d^2(d+ex)^5} - \frac{11bn}{120d^3(d+ex)^4} - \frac{37bn}{180d^4(d+ex)^3} - \frac{19bn}{40d^5(d+ex)^2} - \frac{29bn}{20d^6(d+ex)} - \frac{29bn\log(x)}{20d^7} + \frac{a+b\log(cx^n)}{6d(d+ex)^6} + \frac{a+b\log(cx^n)}{5d^2(d+ex)^5} + \frac{a+b\log(cx^n)}{4d^3(d+ex)^4} + \frac{a+b\log(cx^n)}{3d^4(d+ex)^3} + \frac{a+b\log(cx^n)}{2d^5(d+ex)^2} - \frac{ex(a+b\log(cx^n))}{d^7(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^7} + \frac{49bn\log(d+ex)}{20d^7} + \frac{bn\text{Li}_2\left(-\frac{d}{ex}\right)}{d^7}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.19

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^7}\,dx = \frac{\frac{60ad^6}{(d+ex)^6} + \frac{72ad^5}{(d+ex)^5} - \frac{12bd^5n}{(d+ex)^5} + \frac{90ad^4}{(d+ex)^4} - \frac{33bd^4n}{(d+ex)^4} + \frac{120ad^3}{(d+ex)^3} - \frac{74bd^3n}{(d+ex)^3} + \frac{180ad^2}{(d+ex)^2} - \frac{171bd^2n}{(d+ex)^2} + \frac{360ad}{d+ex} - \frac{522bdn}{d+ex} - 882b}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x)^7),x]
```

```
[Out] ((60*a*d^6)/(d + e*x)^6 + (72*a*d^5)/(d + e*x)^5 - (12*b*d^5*n)/(d + e*x)^5
 + (90*a*d^4)/(d + e*x)^4 - (33*b*d^4*n)/(d + e*x)^4 + (120*a*d^3)/(d + e*x
)^3 - (74*b*d^3*n)/(d + e*x)^3 + (180*a*d^2)/(d + e*x)^2 - (171*b*d^2*n)/(d
 + e*x)^2 + (360*a*d)/(d + e*x) - (522*b*d*n)/(d + e*x) - 882*b*n*Log[x] +
(360*a*Log[c*x^n])/n + (60*b*d^6*Log[c*x^n])/(d + e*x)^6 + (72*b*d^5*Log[c*
x^n])/(d + e*x)^5 + (90*b*d^4*Log[c*x^n])/(d + e*x)^4 + (120*b*d^3*Log[c*x^
n])/(d + e*x)^3 + (180*b*d^2*Log[c*x^n])/(d + e*x)^2 + (360*b*d*Log[c*x^n])
/(d + e*x) + (180*b*Log[c*x^n]^2)/n + 882*b*n*Log[d + e*x] - 360*a*Log[1 +
(e*x)/d] - 360*b*Log[c*x^n]*Log[1 + (e*x)/d] - 360*b*n*PolyLog[2, -((e*x)/d
)])/(360*d^7)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.21 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.51

method	result
risch	$-\frac{b\ln(x^n)\ln(ex+d)}{d^7} + \frac{b\ln(x^n)}{d^6(ex+d)} + \frac{b\ln(x^n)}{2d^5(ex+d)^2} + \frac{b\ln(x^n)}{3d^4(ex+d)^3} + \frac{b\ln(x^n)}{4d^3(ex+d)^4} + \frac{b\ln(x^n)}{5d^2(ex+d)^5} + \frac{b\ln(x^n)}{6d(ex+d)^6} + \frac{b\ln(x^n)\ln(x}{d^7}$

```
[In] int((a+b*ln(c*x^n))/x/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -b*ln(x^n)/d^7*ln(e*x+d)+b*ln(x^n)/d^6/(e*x+d)+1/2*b*ln(x^n)/d^5/(e*x+d)^2+
1/3*b*ln(x^n)/d^4/(e*x+d)^3+1/4*b*ln(x^n)/d^3/(e*x+d)^4+1/5*b*ln(x^n)/d^2/(
```

```
e*x+d)^5+1/6*b*ln(x^n)/d/(e*x+d)^6+b*ln(x^n)/d^7*ln(x)-29/20*b*n/d^6/(e*x+d
)-19/40*b*n/d^5/(e*x+d)^2-37/180*b*n/d^4/(e*x+d)^3-11/120*b*n/d^3/(e*x+d)^4
-1/30*b*n/d^2/(e*x+d)^5+49/20*b*n*ln(e*x+d)/d^7-49/20*b*n*ln(x)/d^7-1/2*b*n
/d^7*ln(x)^2+b*n/d^7*ln(e*x+d)*ln(-e*x/d)+b*n/d^7*dilog(-e*x/d)+(-1/2*I*b*P
i*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+
1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a
)*(-1/d^7*ln(e*x+d)+1/d^6/(e*x+d)+1/2/d^5/(e*x+d)^2+1/3/d^4/(e*x+d)^3+1/4/d
^3/(e*x+d)^4+1/5/d^2/(e*x+d)^5+1/6/d/(e*x+d)^6+1/d^7*ln(x))
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^7}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)^7x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e^7*x^8 + 7*d*e^6*x^7 + 21*d^2*e^5*x^6 + 35*d^
3*e^4*x^5 + 35*d^4*e^3*x^4 + 21*d^5*e^2*x^3 + 7*d^6*e*x^2 + d^7*x), x)
```

Sympy [A] (verification not implemented)

Time = 156.44 (sec) , antiderivative size = 1518, normalized size of antiderivative = 5.16

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^7}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*ln(c*x**n))/x/(e*x+d)**7,x)
```

```
[Out] -a*e*Piecewise((x/d**7, Eq(e, 0)), (-1/(6*e*(d + e*x)**6), True))/d - a*e*P
iecewise((x/d**6, Eq(e, 0)), (-1/(5*e*(d + e*x)**5), True))/d**2 - a*e*Piec
ewise((x/d**5, Eq(e, 0)), (-1/(4*e*(d + e*x)**4), True))/d**3 - a*e*Piecewi
se((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), True))/d**4 - a*e*Piecewise(
(x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))/d**5 - a*e*Piecewise((x/
d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))/d**6 - a*e*Piecewise((x/d, Eq(e
, 0)), (log(d + e*x)/e, True))/d**7 + a*log(x)/d**7 + b*e**6*n*Piecewise((-
1/(e**7*x), Eq(d, 0)), (-137*d**4/(360*d**5*e**6 + 1800*d**4*e**7*x + 3600*
d**3*e**8*x**2 + 3600*d**2*e**9*x**3 + 1800*d*e**10*x**4 + 360*e**11*x**5)
- 625*d**3*e*x/(360*d**5*e**6 + 1800*d**4*e**7*x + 3600*d**3*e**8*x**2 + 36
00*d**2*e**9*x**3 + 1800*d*e**10*x**4 + 360*e**11*x**5) - 1100*d**2*e**2*x*
*2/(360*d**5*e**6 + 1800*d**4*e**7*x + 3600*d**3*e**8*x**2 + 3600*d**2*e**9
*x**3 + 1800*d*e**10*x**4 + 360*e**11*x**5) - 900*d*e**3*x**3/(360*d**5*e**
6 + 1800*d**4*e**7*x + 3600*d**3*e**8*x**2 + 3600*d**2*e**9*x**3 + 1800*d*e
**10*x**4 + 360*e**11*x**5) - 300*e**4*x**4/(360*d**5*e**6 + 1800*d**4*e**7
*x + 3600*d**3*e**8*x**2 + 3600*d**2*e**9*x**3 + 1800*d*e**10*x**4 + 360*e*
```

```
*11*x**5) - log(d + e*x)/(6*d*e**6), True))/d**6 - b*e**6*Piecewise((1/(e**
7*x), Eq(d, 0)), (-1/(6*d*(d/x + e)**6), True))*log(c*x**n)/d**6 - 6*b*e**5
*n*Piecewise((-1/(e**6*x), Eq(d, 0)), (-25*d**3/(60*d**4*e**5 + 240*d**3*e*
*6*x + 360*d**2*e**7*x**2 + 240*d*e**8*x**3 + 60*e**9*x**4) - 88*d**2*e*x/(
60*d**4*e**5 + 240*d**3*e**6*x + 360*d**2*e**7*x**2 + 240*d*e**8*x**3 + 60*
e**9*x**4) - 108*d*e**2*x**2/(60*d**4*e**5 + 240*d**3*e**6*x + 360*d**2*e**
7*x**2 + 240*d*e**8*x**3 + 60*e**9*x**4) - 48*e**3*x**3/(60*d**4*e**5 + 240
*d**3*e**6*x + 360*d**2*e**7*x**2 + 240*d*e**8*x**3 + 60*e**9*x**4) - log(d
 + e*x)/(5*d*e**5), True))/d**6 + 6*b*e**5*Piecewise((1/(e**6*x), Eq(d, 0))
, (-1/(5*d*(d/x + e)**5), True))*log(c*x**n)/d**6 + 15*b*e**4*n*Piecewise((
-1/(e**5*x), Eq(d, 0)), (-11*d**2/(24*d**3*e**4 + 72*d**2*e**5*x + 72*d*e**
6*x**2 + 24*e**7*x**3) - 27*d*e*x/(24*d**3*e**4 + 72*d**2*e**5*x + 72*d*e**
6*x**2 + 24*e**7*x**3) - 18*e**2*x**2/(24*d**3*e**4 + 72*d**2*e**5*x + 72*d
*e**6*x**2 + 24*e**7*x**3) - log(d + e*x)/(4*d*e**4), True))/d**6 - 15*b*e*
*4*Piecewise((1/(e**5*x), Eq(d, 0)), (-1/(4*d*(d/x + e)**4), True))*log(c*x
**n)/d**6 - 20*b*e**3*n*Piecewise((-1/(e**4*x), Eq(d, 0)), (-3*d/(6*d**2*e*
*3 + 12*d*e**4*x + 6*e**5*x**2) - 4*e*x/(6*d**2*e**3 + 12*d*e**4*x + 6*e**5
*x**2) - log(d + e*x)/(3*d*e**3), True))/d**6 + 20*b*e**3*Piecewise((1/(e**
4*x), Eq(d, 0)), (-1/(3*d*(d/x + e)**3), True))*log(c*x**n)/d**6 + 15*b*e**
2*n*Piecewise((-1/(e**3*x), Eq(d, 0)), (-1/(2*d*e**2 + 2*e**3*x) - log(d +
e*x)/(2*d*e**2), True))/d**6 - 15*b*e**2*Piecewise((1/(e**3*x), Eq(d, 0)),
(-1/(2*d*(d/x + e)**2), True))*log(c*x**n)/d**6 - 6*b*e*n*Piecewise((-1/(e*
*2*x), Eq(d, 0)), (-log(d**2 + d*e*x)/(d*e), True))/d**6 + 6*b*e*Piecewise(
(1/(e**2*x), Eq(d, 0)), (-1/(d**2/x + d*e), True))*log(c*x**n)/d**6 + b*n*P
iecewise((-1/(e*x), Eq(d, 0)), (Piecewise((polylog(2, d*exp_polar(I*pi)/(e*
x)), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(e)*log(x) + polylog(2, d*exp_pola
r(I*pi)/(e*x)), Abs(x) < 1), (-log(e)*log(1/x) + polylog(2, d*exp_polar(I*p
i)/(e*x)), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(e) +
 meijerg(((1, 1), ()), ((), (0, 0)), x)*log(e) + polylog(2, d*exp_polar(I*p
i)/(e*x)), True))/d, True))/d**6 - b*Piecewise((1/(e*x), Eq(d, 0)), (log(d/
x + e)/d, True))*log(c*x**n)/d**6
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^7}\,dx=\int \frac{b\log(cx^n)+a}{(ex+d)^7x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] 1/60*a*((60*e^5*x^5 + 330*d*e^4*x^4 + 740*d^2*e^3*x^3 + 855*d^3*e^2*x^2 + 5
22*d^4*e*x + 147*d^5)/(d^6*e^6*x^6 + 6*d^7*e^5*x^5 + 15*d^8*e^4*x^4 + 20*d^
9*e^3*x^3 + 15*d^10*e^2*x^2 + 6*d^11*e*x + d^12) - 60*log(e*x + d)/d^7 + 60
*log(x)/d^7) + b*integrate((log(c) + log(x^n))/(e^7*x^8 + 7*d*e^6*x^7 + 21*
d^2*e^5*x^6 + 35*d^3*e^4*x^5 + 35*d^4*e^3*x^4 + 21*d^5*e^2*x^3 + 7*d^6*e*x^
2 + d^7*x), x)
```

Giac [F]

$$\int \frac{a + b\log(cx^n)}{x(d + ex)^7} \, dx = \int \frac{b\log(cx^n) + a}{(ex + d)^7 x} \, dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^7,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x + d)^7*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{x(d + ex)^7} \, dx = \int \frac{a + b\ln(c\,x^n)}{x\,(d + e\,x)^7} \, dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x)^7),x)
[Out] int((a + b*log(c*x^n))/(x*(d + e*x)^7), x)
```

3.72 $\int \frac{a+b\log(cx^n)}{x^2(d+ex)^7}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 339

$$\begin{aligned}\int \frac{a+b\log(cx^n)}{x^2(d+ex)^7}\,dx=&-\frac{bn}{d^7x}+\frac{ben}{30d^3(d+ex)^5}+\frac{17ben}{120d^4(d+ex)^4}+\frac{79ben}{180d^5(d+ex)^3}\\&+\frac{53ben}{40d^6(d+ex)^2}+\frac{103ben}{20d^7(d+ex)}+\frac{103ben\log(x)}{20d^8}\\&-\frac{a+b\log(cx^n)}{d^7x}-\frac{e(a+b\log(cx^n))}{6d^2(d+ex)^6}-\frac{2e(a+b\log(cx^n))}{5d^3(d+ex)^5}\\&-\frac{3e(a+b\log(cx^n))}{4d^4(d+ex)^4}-\frac{4e(a+b\log(cx^n))}{3d^5(d+ex)^3}-\frac{5e(a+b\log(cx^n))}{2d^6(d+ex)^2}\\&+\frac{6e^2x(a+b\log(cx^n))}{d^8(d+ex)}+\frac{7e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^8}\\&-\frac{223ben\log(d+ex)}{20d^8}-\frac{7ben\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^8}\end{aligned}$$

```
[Out] -b*n/d^7/x+1/30*b*e*n/d^3/(e*x+d)^5+17/120*b*e*n/d^4/(e*x+d)^4+79/180*b*e*n
/d^5/(e*x+d)^3+53/40*b*e*n/d^6/(e*x+d)^2+103/20*b*e*n/d^7/(e*x+d)+103/20*b*
e*n*ln(x)/d^8+(-a-b*ln(c*x^n))/d^7/x-1/6*e*(a+b*ln(c*x^n))/d^2/(e*x+d)^6-2/
5*e*(a+b*ln(c*x^n))/d^3/(e*x+d)^5-3/4*e*(a+b*ln(c*x^n))/d^4/(e*x+d)^4-4/3*e
*(a+b*ln(c*x^n))/d^5/(e*x+d)^3-5/2*e*(a+b*ln(c*x^n))/d^6/(e*x+d)^2+6*e^2*x*
(a+b*ln(c*x^n))/d^8/(e*x+d)+7*e*ln(1+d/e/x)*(a+b*ln(c*x^n))/d^8-223/20*b*e*
n*ln(e*x+d)/d^8-7*b*e*n*polylog(2,-d/e/x)/d^8
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {46, 2393, 2341, 2356, 2351, 31, 2379, 2438}

$$\begin{aligned}\int \frac{a+b\log(cx^n)}{x^2(d+ex)^7}\,dx &= \frac{6e^2x(a+b\log(cx^n))}{d^8(d+ex)}+\frac{7e\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^8}\\ &-\frac{a+b\log(cx^n)}{d^7x}-\frac{5e(a+b\log(cx^n))}{2d^6(d+ex)^2}-\frac{4e(a+b\log(cx^n))}{3d^5(d+ex)^3}\\ &-\frac{3e(a+b\log(cx^n))}{4d^4(d+ex)^4}-\frac{2e(a+b\log(cx^n))}{5d^3(d+ex)^5}\\ &-\frac{e(a+b\log(cx^n))}{6d^2(d+ex)^6}-\frac{7ben\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^8}+\frac{103ben\log(x)}{20d^8}\\ &-\frac{223ben\log(d+ex)}{20d^8}+\frac{103ben}{20d^7(d+ex)}-\frac{bn}{d^7x}+\frac{53ben}{40d^6(d+ex)^2}\\ &+\frac{79ben}{180d^5(d+ex)^3}+\frac{17ben}{120d^4(d+ex)^4}+\frac{ben}{30d^3(d+ex)^5}\end{aligned}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^2*(d + e*x)^7),x]
```

```
[Out] -((b*n)/(d^7*x)) + (b*e*n)/(30*d^3*(d + e*x)^5) + (17*b*e*n)/(120*d^4*(d +
e*x)^4) + (79*b*e*n)/(180*d^5*(d + e*x)^3) + (53*b*e*n)/(40*d^6*(d + e*x)^2
) + (103*b*e*n)/(20*d^7*(d + e*x)) + (103*b*e*n*Log[x])/(20*d^8) - (a + b*L
og[c*x^n])/(d^7*x) - (e*(a + b*Log[c*x^n]))/(6*d^2*(d + e*x)^6) - (2*e*(a +
 b*Log[c*x^n]))/(5*d^3*(d + e*x)^5) - (3*e*(a + b*Log[c*x^n]))/(4*d^4*(d +
e*x)^4) - (4*e*(a + b*Log[c*x^n]))/(3*d^5*(d + e*x)^3) - (5*e*(a + b*Log[c*
x^n]))/(2*d^6*(d + e*x)^2) + (6*e^2*x*(a + b*Log[c*x^n]))/(d^8*(d + e*x)) +
 (7*e*Log[1 + d/(e*x)]*(a + b*Log[c*x^n]))/d^8 - (223*b*e*n*Log[d + e*x])/(
20*d^8) - (7*b*e*n*PolyLog[2, -(d/(e*x))])/d^8
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log(cx^n)}{d^7x^2} + \frac{e^2(a+b\log(cx^n))}{d^2(d+ex)^7} + \frac{2e^2(a+b\log(cx^n))}{d^3(d+ex)^6} + \frac{3e^2(a+b\log(cx^n))}{d^4(d+ex)^5} + \frac{4e^2(a+b\log(cx^n))}{d^5(d+ex)^4} + \frac{5e^2(a+b\log(cx^n))}{d^6(d+ex)^3} + \frac{6e^2(a+b\log(cx^n))}{d^7(d+ex)^2} - \frac{7e(a+b\log(cx^n))}{d^7x(d+ex)}\right) dx$$

$$= \frac{\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d^7} - \frac{(7e)\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d^7} + \frac{(6e^2)\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{d^7} + \frac{(5e^2)\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx}{d^6} + \frac{(4e^2)\int \frac{a+b\log(cx^n)}{(d+ex)^4}\,dx}{d^5} + \frac{(3e^2)\int \frac{a+b\log(cx^n)}{(d+ex)^5}\,dx}{d^4} + \frac{(2e^2)\int \frac{a+b\log(cx^n)}{(d+ex)^6}\,dx}{d^3} + \frac{e^2\int \frac{a+b\log(cx^n)}{(d+ex)^7}\,dx}{d^2}$$

$$= -\frac{bn}{d^7x} - \frac{a+b\log(cx^n)}{d^7x} - \frac{e(a+b\log(cx^n))}{6d^2(d+ex)^6} - \frac{2e(a+b\log(cx^n))}{5d^3(d+ex)^5} - \frac{3e(a+b\log(cx^n))}{4d^4(d+ex)^4} - \frac{4e(a+b\log(cx^n))}{3d^5(d+ex)^3} - \frac{5e(a+b\log(cx^n))}{2d^6(d+ex)^2} + \frac{6e^2x(a+b\log(cx^n))}{d^8(d+ex)} + \frac{7e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^8} - \frac{(7ben)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^8} + \frac{(5ben)\int \frac{1}{x(d+ex)^2}\,dx}{2d^6} + \frac{(4ben)\int \frac{1}{x(d+ex)^3}\,dx}{3d^5} + \frac{(3ben)\int \frac{1}{x(d+ex)^4}\,dx}{4d^4} + \frac{(2ben)\int \frac{1}{x(d+ex)^5}\,dx}{5d^3} + \frac{(ben)\int \frac{1}{x(d+ex)^6}\,dx}{6d^2} - \frac{(6be^2n)\int \frac{1}{d+ex}\,dx}{d^8}$$

$$= -\frac{bn}{d^7x} - \frac{a+b\log(cx^n)}{d^7x} - \frac{e(a+b\log(cx^n))}{6d^2(d+ex)^6} - \frac{2e(a+b\log(cx^n))}{5d^3(d+ex)^5} - \frac{3e(a+b\log(cx^n))}{4d^4(d+ex)^4} - \frac{4e(a+b\log(cx^n))}{3d^5(d+ex)^3} - \frac{5e(a+b\log(cx^n))}{2d^6(d+ex)^2} + \frac{6e^2x(a+b\log(cx^n))}{d^8(d+ex)} + \frac{7e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^8} - \frac{6ben\log(d+ex)}{d^8} - \frac{7ben\text{Li}_2\left(-\frac{d}{ex}\right)}{d^8} + \frac{(5ben)\int \left(\frac{1}{d^2x} - \frac{e}{d(d+ex)^2} - \frac{e}{d^2(d+ex)}\right) dx}{2d^6} + \frac{(4ben)\int \left(\frac{1}{d^3x} - \frac{e}{d(d+ex)^3} - \frac{e}{d^2(d+ex)^2} - \frac{e}{d^3(d+ex)}\right) dx}{3d^5} + \frac{(3ben)\int \left(\frac{1}{d^4x} - \frac{e}{d(d+ex)^4} - \frac{e}{d^2(d+ex)^3} - \frac{e}{d^3(d+ex)^2} - \frac{e}{d^4(d+ex)}\right) dx}{4d^4} + \frac{(2ben)\int \left(\frac{1}{d^5x} - \frac{e}{d(d+ex)^5} - \frac{e}{d^2(d+ex)^4} - \frac{e}{d^3(d+ex)^3} - \frac{e}{d^4(d+ex)^2} - \frac{e}{d^5(d+ex)}\right) dx}{5d^3} + \frac{(ben)\int \left(\frac{1}{d^6x} - \frac{e}{d(d+ex)^6} - \frac{e}{d^2(d+ex)^5} - \frac{e}{d^3(d+ex)^4} - \frac{e}{d^4(d+ex)^3} - \frac{e}{d^5(d+ex)^2} - \frac{e}{d^6(d+ex)}\right) dx}{6d^2}$$

$$= -\frac{bn}{d^7x} + \frac{ben}{30d^3(d+ex)^5} + \frac{17ben}{120d^4(d+ex)^4} + \frac{79ben}{180d^5(d+ex)^3} + \frac{53ben}{40d^6(d+ex)^2} + \frac{103ben}{20d^7(d+ex)} + \frac{103ben\log(x)}{20d^8} - \frac{a+b\log(cx^n)}{d^7x} - \frac{e(a+b\log(cx^n))}{6d^2(d+ex)^6} - \frac{2e(a+b\log(cx^n))}{5d^3(d+ex)^5} - \frac{3e(a+b\log(cx^n))}{4d^4(d+ex)^4} - \frac{4e(a+b\log(cx^n))}{3d^5(d+ex)^3} - \frac{5e(a+b\log(cx^n))}{2d^6(d+ex)^2} + \frac{6e^2x(a+b\log(cx^n))}{d^8(d+ex)} + \frac{7e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^8} - \frac{223ben\log(d+ex)}{20d^8} - \frac{7ben\text{Li}_2\left(-\frac{d}{ex}\right)}{d^8}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.18

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^7}\,dx = -\frac{\frac{360ad}{x} + \frac{360bdn}{x} + \frac{60ad^6e}{(d+ex)^6} + \frac{144ad^5e}{(d+ex)^5} - \frac{12bd^5en}{(d+ex)^5} + \frac{270ad^4e}{(d+ex)^4} - \frac{51bd^4en}{(d+ex)^4} + \frac{480ad^3e}{(d+ex)^3} - \frac{158bd^3en}{(d+ex)^3} + \frac{900ad^2e}{(d+ex)^2} - \frac{477bd^2en}{(d+ex)^2} + \cdots}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*x)^7),x]
```

```
[Out] -1/360*((360*a*d)/x + (360*b*d*n)/x + (60*a*d^6*e)/(d + e*x)^6 + (144*a*d^5
*e)/(d + e*x)^5 - (12*b*d^5*e*n)/(d + e*x)^5 + (270*a*d^4*e)/(d + e*x)^4 -
(51*b*d^4*e*n)/(d + e*x)^4 + (480*a*d^3*e)/(d + e*x)^3 - (158*b*d^3*e*n)/(d
 + e*x)^3 + (900*a*d^2*e)/(d + e*x)^2 - (477*b*d^2*e*n)/(d + e*x)^2 + (2160
*a*d*e)/(d + e*x) - (1854*b*d*e*n)/(d + e*x) - 4014*b*e*n*Log[x] + (2520*a*
e*Log[c*x^n])/n + (360*b*d*Log[c*x^n])/x + (60*b*d^6*e*Log[c*x^n])/(d + e*x
)^6 + (144*b*d^5*e*Log[c*x^n])/(d + e*x)^5 + (270*b*d^4*e*Log[c*x^n])/(d +
e*x)^4 + (480*b*d^3*e*Log[c*x^n])/(d + e*x)^3 + (900*b*d^2*e*Log[c*x^n])/(d
 + e*x)^2 + (2160*b*d*e*Log[c*x^n])/(d + e*x) + (1260*b*e*Log[c*x^n]^2)/n +
 4014*b*e*n*Log[d + e*x] - 2520*a*e*Log[1 + (e*x)/d] - 2520*b*e*Log[c*x^n]*
Log[1 + (e*x)/d] - 2520*b*e*n*PolyLog[2, -((e*x)/d)])/d^8
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.68 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.50

method	result
risch	$-\frac{b\ln(x^n)e}{6d^2(ex+d)^6} + \frac{7b\ln(x^n)e\ln(ex+d)}{d^8} - \frac{6b\ln(x^n)e}{d^7(ex+d)} - \frac{5b\ln(x^n)e}{2d^6(ex+d)^2} - \frac{4b\ln(x^n)e}{3d^5(ex+d)^3} - \frac{3b\ln(x^n)e}{4d^4(ex+d)^4} - \frac{2b\ln(x^n)e}{5d^3(ex+d)^5} - \frac{b\ln(x^n)}{d^7x}$

```
[In] int((a+b*ln(c*x^n))/x^2/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*b*ln(x^n)/d^2/(e*x+d)^6*e+7*b*ln(x^n)/d^8*e*ln(e*x+d)-6*b*ln(x^n)/d^7*
e/(e*x+d)-5/2*b*ln(x^n)/d^6/(e*x+d)^2*e-4/3*b*ln(x^n)/d^5/(e*x+d)^3*e-3/4*b
*ln(x^n)/d^4/(e*x+d)^4*e-2/5*b*ln(x^n)/d^3/(e*x+d)^5*e-b*ln(x^n)/d^7/x-7*b*
ln(x^n)/d^8*e*ln(x)+7/2*b*n/d^8*e*ln(x)^2-7*b*n/d^8*e*ln(e*x+d)*ln(-e*x/d)-
7*b*n/d^8*e*dilog(-e*x/d)+103/20*b*e*n/d^7/(e*x+d)-223/20*b*e*n*ln(e*x+d)/d
^8+53/40*b*e*n/d^6/(e*x+d)^2+79/180*b*e*n/d^5/(e*x+d)^3+17/120*b*e*n/d^4/(e
*x+d)^4+1/30*b*e*n/d^3/(e*x+d)^5-b*n/d^7/x+223/20*b*e*n*ln(x)/d^8+(-1/2*I*b
*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^
2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)
+a)*(-1/6/d^2/(e*x+d)^6*e+7/d^8*e*ln(e*x+d)-6/d^7*e/(e*x+d)-5/2/d^6/(e*x+d)
^2*e-4/3/d^5/(e*x+d)^3*e-3/4/d^4/(e*x+d)^4*e-2/5/d^3/(e*x+d)^5*e-1/d^7/x-7/
d^8*e*ln(x))
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^2(d+ex)^7}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex+d\right)^7x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d)^7,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e^7*x^9 + 7*d*e^6*x^8 + 21*d^2*e^5*x^7 + 35*d^
3*e^4*x^6 + 35*d^4*e^3*x^5 + 21*d^5*e^2*x^4 + 7*d^6*e*x^3 + d^7*x^2), x)
```

Sympy [A] (verification not implemented)

Time = 146.27 (sec) , antiderivative size = 1685, normalized size of antiderivative = 4.97

$$\int \frac{a+b\log\left(cx^n\right)}{x^2(d+ex)^7}\,dx=\text{Too large to display}$$

```
[In] integrate((a+b*ln(c*x**n))/x**2/(e*x+d)**7,x)
```

```
[Out] a*e**2*Piecewise((x/d**7, Eq(e, 0)), (-1/(6*e*(d + e*x)**6), True))/d**2 +
2*a*e**2*Piecewise((x/d**6, Eq(e, 0)), (-1/(5*e*(d + e*x)**5), True))/d**3
+ 3*a*e**2*Piecewise((x/d**5, Eq(e, 0)), (-1/(4*e*(d + e*x)**4), True))/d**
4 + 4*a*e**2*Piecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), True))/d
**5 + 5*a*e**2*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))
/d**6 + 6*a*e**2*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))/d
**7 - a/(d**7*x) + 7*a*e**2*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, Tru
e))/d**8 - 7*a*e*log(x)/d**8 - b*e**2*n*Piecewise((x/d**7, Eq(e, 0)), (-137
*d**4/(360*d**10*e + 1800*d**9*e**2*x + 3600*d**8*e**3*x**2 + 3600*d**7*e**
4*x**3 + 1800*d**6*e**5*x**4 + 360*d**5*e**6*x**5) - 385*d**3*e*x/(360*d**1
```

```
0*e + 1800*d**9*e**2*x + 3600*d**8*e**3*x**2 + 3600*d**7*e**4*x**3 + 1800*d
**6*e**5*x**4 + 360*d**5*e**6*x**5) - 470*d**2*e**2*x**2/(360*d**10*e + 180
0*d**9*e**2*x + 3600*d**8*e**3*x**2 + 3600*d**7*e**4*x**3 + 1800*d**6*e**5*
x**4 + 360*d**5*e**6*x**5) - 270*d*e**3*x**3/(360*d**10*e + 1800*d**9*e**2*
x + 3600*d**8*e**3*x**2 + 3600*d**7*e**4*x**3 + 1800*d**6*e**5*x**4 + 360*d
**5*e**6*x**5) - 60*e**4*x**4/(360*d**10*e + 1800*d**9*e**2*x + 3600*d**8*e
**3*x**2 + 3600*d**7*e**4*x**3 + 1800*d**6*e**5*x**4 + 360*d**5*e**6*x**5)
- log(x)/(6*d**6*e) + log(d/e + x)/(6*d**6*e), True))/d**2 + b*e**2*Piecewi
se((x/d**7, Eq(e, 0)), (-1/(6*e*(d + e*x)**6), True))*log(c*x**n)/d**2 - 2*
b*e**2*n*Piecewise((x/d**6, Eq(e, 0)), (-25*d**3/(60*d**8*e + 240*d**7*e**2
*x + 360*d**6*e**3*x**2 + 240*d**5*e**4*x**3 + 60*d**4*e**5*x**4) - 52*d**2
*e*x/(60*d**8*e + 240*d**7*e**2*x + 360*d**6*e**3*x**2 + 240*d**5*e**4*x**3
 + 60*d**4*e**5*x**4) - 42*d*e**2*x**2/(60*d**8*e + 240*d**7*e**2*x + 360*d
**6*e**3*x**2 + 240*d**5*e**4*x**3 + 60*d**4*e**5*x**4) - 12*e**3*x**3/(60*
d**8*e + 240*d**7*e**2*x + 360*d**6*e**3*x**2 + 240*d**5*e**4*x**3 + 60*d**
4*e**5*x**4) - log(x)/(5*d**5*e) + log(d/e + x)/(5*d**5*e), True))/d**3 + 2
*b*e**2*Piecewise((x/d**6, Eq(e, 0)), (-1/(5*e*(d + e*x)**5), True))*log(c*
x**n)/d**3 - 3*b*e**2*n*Piecewise((x/d**5, Eq(e, 0)), (-11*d**2/(24*d**6*e
+ 72*d**5*e**2*x + 72*d**4*e**3*x**2 + 24*d**3*e**4*x**3) - 15*d*e*x/(24*d*
*6*e + 72*d**5*e**2*x + 72*d**4*e**3*x**2 + 24*d**3*e**4*x**3) - 6*e**2*x**
2/(24*d**6*e + 72*d**5*e**2*x + 72*d**4*e**3*x**2 + 24*d**3*e**4*x**3) - lo
g(x)/(4*d**4*e) + log(d/e + x)/(4*d**4*e), True))/d**4 + 3*b*e**2*Piecewise
((x/d**5, Eq(e, 0)), (-1/(4*e*(d + e*x)**4), True))*log(c*x**n)/d**4 - 4*b*
e**2*n*Piecewise((x/d**4, Eq(e, 0)), (-3*d/(6*d**4*e + 12*d**3*e**2*x + 6*d
**2*e**3*x**2) - 2*e*x/(6*d**4*e + 12*d**3*e**2*x + 6*d**2*e**3*x**2) - log
(x)/(3*d**3*e) + log(d/e + x)/(3*d**3*e), True))/d**5 + 4*b*e**2*Piecewise(
(x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), True))*log(c*x**n)/d**5 - 5*b*e
**2*n*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*d**2*e + 2*d*e**2*x) - log(x)/(2
*d**2*e) + log(d/e + x)/(2*d**2*e), True))/d**6 + 5*b*e**2*Piecewise((x/d**
3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))*log(c*x**n)/d**6 - 6*b*e**2*n*
Piecewise((x/d**2, Eq(e, 0)), (-log(x)/(d*e) + log(d/e + x)/(d*e), True))/d
**7 + 6*b*e**2*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))*log
(c*x**n)/d**7 - b*n/(d**7*x) - b*log(c*x**n)/(d**7*x) - 7*b*e**2*n*Piecewis
e((x/d, Eq(e, 0)), (Piecewise((-polylog(2, e*x*exp_polar(I*pi)/d), (Abs(x)
< 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x*exp_polar(I*pi)/d),
 Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x*exp_polar(I*pi)/d), 1/Abs(
x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1),
 ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x*exp_polar(I*pi)/d), True))/e
, True))/d**8 + 7*b*e**2*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))
*log(c*x**n)/d**8 + 7*b*e*n*log(x)**2/(2*d**8) - 7*b*e*log(x)*log(c*x**n)/d
**8
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^7}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)^7x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] -1/60*a*((420*e^6*x^6 + 2310*d*e^5*x^5 + 5180*d^2*e^4*x^4 + 5985*d^3*e^3*x^
3 + 3654*d^4*e^2*x^2 + 1029*d^5*e*x + 60*d^6)/(d^7*e^6*x^7 + 6*d^8*e^5*x^6
+ 15*d^9*e^4*x^5 + 20*d^10*e^3*x^4 + 15*d^11*e^2*x^3 + 6*d^12*e*x^2 + d^13*
x) - 420*e*log(e*x + d)/d^8 + 420*e*log(x)/d^8) + b*integrate((log(c) + log
(x^n))/(e^7*x^9 + 7*d*e^6*x^8 + 21*d^2*e^5*x^7 + 35*d^3*e^4*x^6 + 35*d^4*e^
3*x^5 + 21*d^5*e^2*x^4 + 7*d^6*e*x^3 + d^7*x^2), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^7}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)^7x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d)^7,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x + d)^7*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^7}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^2\,(d+e\,x)^7}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*x)^7),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*x)^7), x)
```

3.73 $\int \frac{a+b\log(cx^n)}{x^3(d+ex)^7}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 401

$$\begin{aligned}\int \frac{a+b\log(cx^n)}{x^3(d+ex)^7}\,dx = &-\frac{bn}{4d^7x^2}+\frac{7ben}{d^8x}-\frac{be^2n}{30d^4(d+ex)^5}-\frac{23be^2n}{120d^5(d+ex)^4}-\frac{34be^2n}{45d^6(d+ex)^3}\\ &-\frac{14be^2n}{5d^7(d+ex)^2}-\frac{131be^2n}{10d^8(d+ex)}-\frac{131be^2n\log(x)}{10d^9}-\frac{a+b\log(cx^n)}{2d^7x^2}\\ &+\frac{7e(a+b\log(cx^n))}{d^8x}+\frac{e^2(a+b\log(cx^n))}{6d^3(d+ex)^6}+\frac{3e^2(a+b\log(cx^n))}{5d^4(d+ex)^5}\\ &+\frac{3e^2(a+b\log(cx^n))}{2d^5(d+ex)^4}+\frac{10e^2(a+b\log(cx^n))}{3d^6(d+ex)^3}+\frac{15e^2(a+b\log(cx^n))}{2d^7(d+ex)^2}\\ &-\frac{21e^3x(a+b\log(cx^n))}{d^9(d+ex)}-\frac{28e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^9}\\ &+\frac{341be^2n\log(d+ex)}{10d^9}+\frac{28be^2n\,\mathrm{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^9}\end{aligned}$$

```
[Out] -1/4*b*n/d^7/x^2+7*b*e*n/d^8/x-1/30*b*e^2*n/d^4/(e*x+d)^5-23/120*b*e^2*n/d^
5/(e*x+d)^4-34/45*b*e^2*n/d^6/(e*x+d)^3-14/5*b*e^2*n/d^7/(e*x+d)^2-131/10*b
*e^2*n/d^8/(e*x+d)-131/10*b*e^2*n*ln(x)/d^9+1/2*(-a-b*ln(c*x^n))/d^7/x^2+7*
e*(a+b*ln(c*x^n))/d^8/x+1/6*e^2*(a+b*ln(c*x^n))/d^3/(e*x+d)^6+3/5*e^2*(a+b*
ln(c*x^n))/d^4/(e*x+d)^5+3/2*e^2*(a+b*ln(c*x^n))/d^5/(e*x+d)^4+10/3*e^2*(a+
b*ln(c*x^n))/d^6/(e*x+d)^3+15/2*e^2*(a+b*ln(c*x^n))/d^7/(e*x+d)^2-21*e^3*x*
(a+b*ln(c*x^n))/d^9/(e*x+d)-28*e^2*ln(1+d/e/x)*(a+b*ln(c*x^n))/d^9+341/10*b
*e^2*n*ln(e*x+d)/d^9+28*b*e^2*n*polylog(2,-d/e/x)/d^9
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {46, 2393, 2341, 2356, 2351, 31, 2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^7}\,dx = -\frac{21e^3x(a+b\log(cx^n))}{d^9(d+ex)} - \frac{28e^2\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^9} + \frac{7e(a+b\log(cx^n))}{d^8x} + \frac{15e^2(a+b\log(cx^n))}{2d^7(d+ex)^2} - \frac{a+b\log(cx^n)}{2d^7x^2} + \frac{10e^2(a+b\log(cx^n))}{3d^6(d+ex)^3} + \frac{3e^2(a+b\log(cx^n))}{2d^5(d+ex)^4} + \frac{3e^2(a+b\log(cx^n))}{5d^4(d+ex)^5} + \frac{e^2(a+b\log(cx^n))}{6d^3(d+ex)^6} + \frac{28be^2n\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^9} - \frac{131be^2n\log(x)}{10d^9} + \frac{341be^2n\log(d+ex)}{10d^9} - \frac{131be^2n}{10d^8(d+ex)} + \frac{7ben}{d^8x} - \frac{14be^2n}{5d^7(d+ex)^2} - \frac{bn}{4d^7x^2} - \frac{34be^2n}{45d^6(d+ex)^3} - \frac{23be^2n}{120d^5(d+ex)^4} - \frac{be^2n}{30d^4(d+ex)^5}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^3*(d + e*x)^7),x]
```

```
[Out] -1/4*(b*n)/(d^7*x^2) + (7*b*e*n)/(d^8*x) - (b*e^2*n)/(30*d^4*(d + e*x)^5) - (23*b*e^2*n)/(120*d^5*(d + e*x)^4) - (34*b*e^2*n)/(45*d^6*(d + e*x)^3) - (14*b*e^2*n)/(5*d^7*(d + e*x)^2) - (131*b*e^2*n)/(10*d^8*(d + e*x)) - (131*b*e^2*n*Log[x])/(10*d^9) - (a + b*Log[c*x^n])/(2*d^7*x^2) + (7*e*(a + b*Log[c*x^n]))/(d^8*x) + (e^2*(a + b*Log[c*x^n]))/(6*d^3*(d + e*x)^6) + (3*e^2*(a + b*Log[c*x^n]))/(5*d^4*(d + e*x)^5) + (3*e^2*(a + b*Log[c*x^n]))/(2*d^5*(d + e*x)^4) + (10*e^2*(a + b*Log[c*x^n]))/(3*d^6*(d + e*x)^3) + (15*e^2*(a + b*Log[c*x^n]))/(2*d^7*(d + e*x)^2) - (21*e^3*x*(a + b*Log[c*x^n]))/(d^9*(d + e*x)) - (28*e^2*Log[1 + d/(e*x)]*(a + b*Log[c*x^n]))/d^9 + (341*b*e^2*n*Log[d + e*x])/(10*d^9) + (28*b*e^2*n*PolyLog[2, -(d/(e*x))])/d^9
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral}=\int \left(\frac{a+b\log(cx^n)}{d^7x^3}-\frac{7e(a+b\log(cx^n))}{d^8x^2}-\frac{e^3(a+b\log(cx^n))}{d^3(d+ex)^7}-\frac{3e^3(a+b\log(cx^n))}{d^4(d+ex)^6}-\frac{6e^3(a+b\log(cx^n))}{d^5(d+ex)^5}-\frac{10e^3(a+b\log(cx^n))}{d^6(d+ex)^4}-\frac{15e^3(a+b\log(cx^n))}{d^7(d+ex)^3}-\frac{21e^3(a+b\log(cx^n))}{d^8(d+ex)^2}+\frac{28e^2(a+b\log(cx^n))}{d^8x(d+ex)}\right)dx$$

$$=\frac{\int \frac{a+b\log(cx^n)}{x^3}\,dx}{d^7}-\frac{(7e)\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d^8}+\frac{(28e^2)\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d^8}-\frac{(21e^3)\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{d^8}-\frac{(15e^3)\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx}{d^7}-\frac{(10e^3)\int \frac{a+b\log(cx^n)}{(d+ex)^4}\,dx}{d^6}-\frac{(6e^3)\int \frac{a+b\log(cx^n)}{(d+ex)^5}\,dx}{d^5}-\frac{(3e^3)\int \frac{a+b\log(cx^n)}{(d+ex)^6}\,dx}{d^4}-\frac{e^3\int \frac{a+b\log(cx^n)}{(d+ex)^7}\,dx}{d^3}$$

$$=-\frac{bn}{4d^7x^2}+\frac{7ben}{d^8x}-\frac{a+b\log(cx^n)}{2d^7x^2}+\frac{7e(a+b\log(cx^n))}{d^8x}+\frac{e^2(a+b\log(cx^n))}{6d^3(d+ex)^6}+\frac{3e^2(a+b\log(cx^n))}{5d^4(d+ex)^5}+\frac{3e^2(a+b\log(cx^n))}{2d^5(d+ex)^4}+\frac{10e^2(a+b\log(cx^n))}{3d^6(d+ex)^3}+\frac{15e^2(a+b\log(cx^n))}{2d^7(d+ex)^2}-\frac{21e^3x(a+b\log(cx^n))}{d^9(d+ex)}-\frac{28e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^9}+\frac{(28be^2n)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^9}-\frac{(15be^2n)\int \frac{1}{x(d+ex)^2}\,dx}{2d^7}-\frac{(10be^2n)\int \frac{1}{x(d+ex)^3}\,dx}{3d^6}-\frac{(3be^2n)\int \frac{1}{x(d+ex)^4}\,dx}{2d^5}-\frac{(3be^2n)\int \frac{1}{x(d+ex)^5}\,dx}{5d^4}-\frac{(be^2n)\int \frac{1}{x(d+ex)^6}\,dx}{6d^3}+\frac{(21be^3n)\int \frac{1}{d+ex}\,dx}{d^9}$$

$$= -\frac{bn}{4d^7x^2} + \frac{7ben}{d^8x} - \frac{a+b\log(cx^n)}{2d^7x^2} + \frac{7e(a+b\log(cx^n))}{d^8x} + \frac{e^2(a+b\log(cx^n))}{6d^3(d+ex)^6} + \frac{3e^2(a+b\log(cx^n))}{5d^4(d+ex)^5} + \frac{3e^2(a+b\log(cx^n))}{2d^5(d+ex)^4} + \frac{10e^2(a+b\log(cx^n))}{3d^6(d+ex)^3} + \frac{15e^2(a+b\log(cx^n))}{2d^7(d+ex)^2} - \frac{21e^3x(a+b\log(cx^n))}{d^9(d+ex)} - \frac{28e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^9} + \frac{21be^2n\log(d+ex)}{d^9} + \frac{28be^2n\text{Li}_2\left(-\frac{d}{ex}\right)}{d^9} - \frac{(15be^2n)\int\left(\frac{1}{d^2x} - \frac{e}{d(d+ex)^2} - \frac{e}{d^2(d+ex)}\right)dx}{2d^7} - \frac{(10be^2n)\int\left(\frac{1}{d^3x} - \frac{e}{d(d+ex)^3} - \frac{e}{d^2(d+ex)^2} - \frac{e}{d^3(d+ex)}\right)dx}{3d^6} - \frac{(3be^2n)\int\left(\frac{1}{d^4x} - \frac{e}{d(d+ex)^4} - \frac{e}{d^2(d+ex)^3} - \frac{e}{d^3(d+ex)^2} - \frac{e}{d^4(d+ex)}\right)dx}{2d^5} - \frac{(3be^2n)\int\left(\frac{1}{d^5x} - \frac{e}{d(d+ex)^5} - \frac{e}{d^2(d+ex)^4} - \frac{e}{d^3(d+ex)^3} - \frac{e}{d^4(d+ex)^2} - \frac{e}{d^5(d+ex)}\right)dx}{5d^4} - \frac{(be^2n)\int\left(\frac{1}{d^6x} - \frac{e}{d(d+ex)^6} - \frac{e}{d^2(d+ex)^5} - \frac{e}{d^3(d+ex)^4} - \frac{e}{d^4(d+ex)^3} - \frac{e}{d^5(d+ex)^2} - \frac{e}{d^6(d+ex)}\right)dx}{6d^3}$$

$$= -\frac{bn}{4d^7x^2} + \frac{7ben}{d^8x} - \frac{be^2n}{30d^4(d+ex)^5} - \frac{23be^2n}{120d^5(d+ex)^4} - \frac{34be^2n}{45d^6(d+ex)^3} - \frac{14be^2n}{5d^7(d+ex)^2} - \frac{131be^2n}{10d^8(d+ex)} - \frac{131be^2n\log(x)}{10d^9} - \frac{a+b\log(cx^n)}{2d^7x^2} + \frac{7e(a+b\log(cx^n))}{d^8x} + \frac{e^2(a+b\log(cx^n))}{6d^3(d+ex)^6} + \frac{3e^2(a+b\log(cx^n))}{5d^4(d+ex)^5} + \frac{3e^2(a+b\log(cx^n))}{2d^5(d+ex)^4} + \frac{10e^2(a+b\log(cx^n))}{3d^6(d+ex)^3} + \frac{15e^2(a+b\log(cx^n))}{2d^7(d+ex)^2} - \frac{21e^3x(a+b\log(cx^n))}{d^9(d+ex)} - \frac{28e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^9} + \frac{341be^2n\log(d+ex)}{10d^9} + \frac{28be^2n\text{Li}_2\left(-\frac{d}{ex}\right)}{d^9}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.21

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^7}\,dx = \frac{-\frac{180ad^2}{x^2} - \frac{90bd^2n}{x^2} + \frac{2520ade}{x} + \frac{2520bden}{x} + \frac{60ad^6e^2}{(d+ex)^6} + \frac{216ad^5e^2}{(d+ex)^5} - \frac{12bd^5e^2n}{(d+ex)^5} + \frac{540ad^4e^2}{(d+ex)^4} - \frac{69bd^4e^2n}{(d+ex)^4} + \frac{1200ad^3e^2}{(d+ex)^3} - \frac{272bd^3e^2}{(d+ex)}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*(d + e*x)^7),x]
```

[Out] ((-180*a*d^2)/x^2 - (90*b*d^2*n)/x^2 + (2520*a*d*e)/x + (2520*b*d*e*n)/x + (60*a*d^6*e^2)/(d + e*x)^6 + (216*a*d^5*e^2)/(d + e*x)^5 - (12*b*d^5*e^2*n) /(d + e*x)^5 + (540*a*d^4*e^2)/(d + e*x)^4 - (69*b*d^4*e^2*n)/(d + e*x)^4 + (1200*a*d^3*e^2)/(d + e*x)^3 - (272*b*d^3*e^2*n)/(d + e*x)^3 + (2700*a*d^2 *e^2)/(d + e*x)^2 - (1008*b*d^2*e^2*n)/(d + e*x)^2 + (7560*a*d*e^2)/(d + e* x) - (4716*b*d*e^2*n)/(d + e*x) - 12276*b*e^2*n*Log[x] + (10080*a*e^2*Log[c *x^n])/n - (180*b*d^2*Log[c*x^n])/x^2 + (2520*b*d*e*Log[c*x^n])/x + (60*b*d ^6*e^2*Log[c*x^n])/(d + e*x)^6 + (216*b*d^5*e^2*Log[c*x^n])/(d + e*x)^5 + (540*b*d^4*e^2*Log[c*x^n])/(d + e*x)^4 + (1200*b*d^3*e^2*Log[c*x^n])/(d + e* x)^3 + (2700*b*d^2*e^2*Log[c*x^n])/(d + e*x)^2 + (7560*b*d*e^2*Log[c*x^n])/ (d + e*x) + (5040*b*e^2*Log[c*x^n]^2)/n + 12276*b*e^2*n*Log[d + e*x] - 1008 0*a*e^2*Log[1 + (e*x)/d] - 10080*b*e^2*Log[c*x^n]*Log[1 + (e*x)/d] - 10080* b*e^2*n*PolyLog[2, -((e*x)/d)])/(360*d^9)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.62 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.48

method	result
risch	$-\frac{28b\ln(x^n)e^2\ln(ex+d)}{d^9}+\frac{21b\ln(x^n)e^2}{d^8(ex+d)}+\frac{15b\ln(x^n)e^2}{2d^7(ex+d)^2}+\frac{10b\ln(x^n)e^2}{3d^6(ex+d)^3}+\frac{3b\ln(x^n)e^2}{2d^5(ex+d)^4}+\frac{3b\ln(x^n)e^2}{5d^4(ex+d)^5}+\frac{b\ln(x^n)e^2}{6d^3(ex+d)^6}-$

[In] int((a+b*ln(c*x^n))/x^3/(e*x+d)^7,x,method=_RETURNVERBOSE)

[Out] -28*b*ln(x^n)/d^9*e^2*ln(e*x+d)+21*b*ln(x^n)/d^8*e^2/(e*x+d)+15/2*b*ln(x^n) /d^7*e^2/(e*x+d)^2+10/3*b*ln(x^n)/d^6/(e*x+d)^3*e^2+3/2*b*ln(x^n)/d^5/(e*x+ d)^4*e^2+3/5*b*ln(x^n)/d^4/(e*x+d)^5*e^2+1/6*b*ln(x^n)/d^3/(e*x+d)^6*e^2-1/ 2*b*ln(x^n)/d^7/x^2+28*b*ln(x^n)/d^9*e^2*ln(x)+7*b*ln(x^n)/d^8*e/x-131/10*b *e^2*n/d^8/(e*x+d)+341/10*b*e^2*n*ln(e*x+d)/d^9-14/5*b*e^2*n/d^7/(e*x+d)^2- 34/45*b*e^2*n/d^6/(e*x+d)^3-23/120*b*e^2*n/d^5/(e*x+d)^4-1/30*b*e^2*n/d^4/(e*x+d)^5-1/4*b*n/d^7/x^2+7*b*e*n/d^8/x-341/10*b*e^2*n*ln(x)/d^9-14*b*n/d^9* e^2*ln(x)^2+28*b*n/d^9*e^2*ln(e*x+d)*ln(-e*x/d)+28*b*n/d^9*e^2*dilog(-e*x/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csg n(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-28/d^9*e^2*ln(e*x+d)+21/d^8*e^2/(e*x+d)+15/2/d^7*e^2/(e*x+ d)^2+10/3/d^6/(e*x+d)^3*e^2+3/2/d^5/(e*x+d)^4*e^2+3/5/d^4/(e*x+d)^5*e^2+1/6 /d^3/(e*x+d)^6*e^2-1/2/d^7/x^2+28/d^9*e^2*ln(x)+7/d^8*e/x)

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^7}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)^7x^3}\,dx$$

[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d)^7,x, algorithm="fricas")

[Out] integral((b*log(c*x^n) + a)/(e^7*x^10 + 7*d*e^6*x^9 + 21*d^2*e^5*x^8 + 35*d^3*e^4*x^7 + 35*d^4*e^3*x^6 + 21*d^5*e^2*x^5 + 7*d^6*e*x^4 + d^7*x^3), x)

Sympy [A] (verification not implemented)

Time = 156.80 (sec) , antiderivative size = 1737, normalized size of antiderivative = 4.33

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^7}\,dx = \text{Too large to display}$$

[In] integrate((a+b*ln(c*x**n))/x**3/(e*x+d)**7,x)

[Out] -a*e**3*Piecewise((x/d**7, Eq(e, 0)), (-1/(6*e*(d + e*x)**6), True))/d**3 - 3*a*e**3*Piecewise((x/d**6, Eq(e, 0)), (-1/(5*e*(d + e*x)**5), True))/d**4 - 6*a*e**3*Piecewise((x/d**5, Eq(e, 0)), (-1/(4*e*(d + e*x)**4), True))/d**5 - 10*a*e**3*Piecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3), True))/d**6 - 15*a*e**3*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))/d**7 - a/(2*d**7*x**2) - 21*a*e**3*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x), True))/d**8 + 7*a*e/(d**8*x) - 28*a*e**3*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/d**9 + 28*a*e**2*log(x)/d**9 + b*e**3*n*Piecewise((x/d**7, Eq(e, 0)), (-137*d**4/(360*d**10*e + 1800*d**9*e**2*x + 3600*d**8*e**3*x**2 + 3600*d**7*e**4*x**3 + 1800*d**6*e**5*x**4 + 360*d**5*e**6*x**5) - 385*d**3*e*x/(360*d**10*e + 1800*d**9*e**2*x + 3600*d**8*e**3*x**2 + 3600*d**7*e**4*x**3 + 1800*d**6*e**5*x**4 + 360*d**5*e**6*x**5) - 470*d**2*e**2*x**2/(360*d**10*e + 1800*d**9*e**2*x + 3600*d**8*e**3*x**2 + 3600*d**7*e**4*x**3 + 1800*d**6*e**5*x**4 + 360*d**5*e**6*x**5) - 270*d*e**3*x**3/(360*d**10*e + 1800*d**9*e**2*x + 3600*d**8*e**3*x**2 + 3600*d**7*e**4*x**3 + 1800*d**6*e**5*x**4 + 360*d**5*e**6*x**5) - 60*e**4*x**4/(360*d**10*e + 1800*d**9*e**2*x + 3600*d**8*e**3*x**2 + 3600*d**7*e**4*x**3 + 1800*d**6*e**5*x**4 + 360*d**5*e**6*x**5) - log(x)/(6*d**6*e) + log(d/e + x)/(6*d**6*e), True))/d**3 - b*e**3*Piecewise((x/d**7, Eq(e, 0)), (-1/(6*e*(d + e*x)**6), True))*log(c*x**n)/d**3 + 3*b*e**3*n*Piecewise((x/d**6, Eq(e, 0)), (-25*d**3/(60*d**8*e + 240*d**7*e**2*x + 360*d**6*e**3*x**2 + 240*d**5*e**4*x**3 + 60*d**4*e**5*x**4) - 52*d**2*e*x/(60*d**8*e + 240*d**7*e**2*x + 360*d**6*e**3*x**2 + 240*d**5*e**4*x**3 + 60*d**4*e**5*x**4) - 42*d*e**2*x**2/(60*d**8*e + 240*d**7*e**2*x + 360*d**6*e**3*x**2 + 240*d**5*e**4*x**3 + 60*d**4*e**5*x**4) - 12*e**3*x**3/(60*d**8*e + 240*d**7*e**2*x + 360*d**6*e**3*x**

```
2 + 240*d**5*e**4*x**3 + 60*d**4*e**5*x**4) - log(x)/(5*d**5*e) + log(d/e +
 x)/(5*d**5*e), True))/d**4 - 3*b*e**3*Piecewise((x/d**6, Eq(e, 0)), (-1/(5
*e*(d + e*x)**5), True))*log(c*x**n)/d**4 + 6*b*e**3*n*Piecewise((x/d**5, E
q(e, 0)), (-11*d**2/(24*d**6*e + 72*d**5*e**2*x + 72*d**4*e**3*x**2 + 24*d*
*3*e**4*x**3) - 15*d*e*x/(24*d**6*e + 72*d**5*e**2*x + 72*d**4*e**3*x**2 +
24*d**3*e**4*x**3) - 6*e**2*x**2/(24*d**6*e + 72*d**5*e**2*x + 72*d**4*e**3
*x**2 + 24*d**3*e**4*x**3) - log(x)/(4*d**4*e) + log(d/e + x)/(4*d**4*e), T
rue))/d**5 - 6*b*e**3*Piecewise((x/d**5, Eq(e, 0)), (-1/(4*e*(d + e*x)**4),
 True))*log(c*x**n)/d**5 + 10*b*e**3*n*Piecewise((x/d**4, Eq(e, 0)), (-3*d/
(6*d**4*e + 12*d**3*e**2*x + 6*d**2*e**3*x**2) - 2*e*x/(6*d**4*e + 12*d**3*
e**2*x + 6*d**2*e**3*x**2) - log(x)/(3*d**3*e) + log(d/e + x)/(3*d**3*e), T
rue))/d**6 - 10*b*e**3*Piecewise((x/d**4, Eq(e, 0)), (-1/(3*e*(d + e*x)**3)
, True))*log(c*x**n)/d**6 + 15*b*e**3*n*Piecewise((x/d**3, Eq(e, 0)), (-1/(
2*d**2*e + 2*d*e**2*x) - log(x)/(2*d**2*e) + log(d/e + x)/(2*d**2*e), True)
)/d**7 - 15*b*e**3*Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), Tr
ue))*log(c*x**n)/d**7 - b*n/(4*d**7*x**2) - b*log(c*x**n)/(2*d**7*x**2) + 2
1*b*e**3*n*Piecewise((x/d**2, Eq(e, 0)), (-log(x)/(d*e) + log(d/e + x)/(d*e
), True))/d**8 - 21*b*e**3*Piecewise((x/d**2, Eq(e, 0)), (-1/(d*e + e**2*x)
, True))*log(c*x**n)/d**8 + 7*b*e*n/(d**8*x) + 7*b*e*log(c*x**n)/(d**8*x) +
 28*b*e**3*n*Piecewise((x/d, Eq(e, 0)), (Piecewise((-polylog(2, e*x*exp_pol
ar(I*pi)/d), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*
x*exp_polar(I*pi)/d), Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x*exp_p
olar(I*pi)/d), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(
d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x*exp_pol
ar(I*pi)/d), True))/e, True))/d**9 - 28*b*e**3*Piecewise((x/d, Eq(e, 0)), (
log(d + e*x)/e, True))*log(c*x**n)/d**9 - 14*b*e**2*n*log(x)**2/d**9 + 28*b
*e**2*log(x)*log(c*x**n)/d**9
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^7}\,dx=\int \frac{b\log(cx^n)+a}{(ex+d)^7x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d)^7,x, algorithm="maxima")
```

```
[Out] 1/30*a*((840*e^7*x^7 + 4620*d*e^6*x^6 + 10360*d^2*e^5*x^5 + 11970*d^3*e^4*x
^4 + 7308*d^4*e^3*x^3 + 2058*d^5*e^2*x^2 + 120*d^6*e*x - 15*d^7)/(d^8*e^6*x
^8 + 6*d^9*e^5*x^7 + 15*d^10*e^4*x^6 + 20*d^11*e^3*x^5 + 15*d^12*e^2*x^4 +
6*d^13*e*x^3 + d^14*x^2) - 840*e^2*log(e*x + d)/d^9 + 840*e^2*log(x)/d^9) +
 b*integrate((log(c) + log(x^n))/(e^7*x^10 + 7*d*e^6*x^9 + 21*d^2*e^5*x^8 +
 35*d^3*e^4*x^7 + 35*d^4*e^3*x^6 + 21*d^5*e^2*x^5 + 7*d^6*e*x^4 + d^7*x^3),
 x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^7}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)^7x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d)^7,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x + d)^7*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex)^7}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^3\,(d+e\,x)^7}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e*x)^7),x)
[Out] int((a + b*log(c*x^n))/(x^3*(d + e*x)^7), x)
```

3.74 $\int \frac{\log(cx)}{1-cx}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{\log(cx)}{1-cx}\,dx = \frac{\text{PolyLog}(2,1-cx)}{c}$$

```
[Out] polylog(2,-c*x+1)/c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2352}

$$\int \frac{\log(cx)}{1-cx}\,dx = \frac{\text{PolyLog}(2,1-cx)}{c}$$

```
[In] Int[Log[c*x]/(1 - c*x),x]
```

```
[Out] PolyLog[2, 1 - c*x]/c
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Li}_2(1-cx)}{c}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log(cx)}{1-cx}\,dx = \frac{\text{PolyLog}(2,1-cx)}{c}$$

```
[In] Integrate[Log[c*x]/(1 - c*x),x]
[Out] PolyLog[2, 1 - c*x]/c
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\text{dilog}(xc)}{c}$	9
default	$\frac{\text{dilog}(xc)}{c}$	9
risch	$\frac{\text{dilog}(xc)}{c}$	9
parts	$-\frac{\ln(xc)\ln(xc-1)}{c}+\frac{\text{dilog}(xc)+\ln(xc-1)\ln(xc)}{c}$	37

```
[In] int(ln(x*c)/(-c*x+1),x,method=_RETURNVERBOSE)
[Out] 1/c*dilog(x*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{\log(cx)}{1-cx}\,dx = \frac{\text{Li}_2(-cx+1)}{c}$$

```
[In] integrate(log(c*x)/(-c*x+1),x, algorithm="fricas")
[Out] dilog(-c*x + 1)/c
```

Sympy [F]

$$\int \frac{\log(cx)}{1-cx}\,dx = -\int \frac{\log{(cx)}}{cx-1}\,dx$$

```
[In] integrate(ln(c*x)/(-c*x+1),x)
```

```
[Out] -Integral(log(c*x)/(c*x - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(11) = 22$.

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 4.00

$$\int \frac{\log(cx)}{1-cx}\,dx = -\frac{\log{(cx-1)}\log{(cx)}}{c} + \frac{\log{(cx-1)}\log{(x)}}{c} - \frac{\log{(-cx+1)}\log{(x)} + \mathrm{Li}_2(cx)}{c}$$

```
[In] integrate(log(c*x)/(-c*x+1),x, algorithm="maxima")
```

```
[Out] -log(c*x - 1)*log(c*x)/c + log(c*x - 1)*log(x)/c - (log(-c*x + 1)*log(x) +
dilog(c*x))/c
```

Giac [F]

$$\int \frac{\log(cx)}{1-cx}\,dx = \int -\frac{\log{(cx)}}{cx-1}\,dx$$

```
[In] integrate(log(c*x)/(-c*x+1),x, algorithm="giac")
```

```
[Out] integrate(-log(c*x)/(c*x - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\log(cx)}{1-cx}\,dx = \frac{\mathrm{Li}_2(c\,x)}{c}$$

```
[In] int(-log(c*x)/(c*x - 1),x)
```

```
[Out] dilog(c*x)/c
```

3.75 $\int \frac{\log\left(\frac{x}{c}\right)}{c-x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 10

$$\int \frac{\log\left(\frac{x}{c}\right)}{c-x}\,dx = \text{PolyLog}\left(2,1-\frac{x}{c}\right)$$

```
[Out] polylog(2,1-x/c)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2352}

$$\int \frac{\log\left(\frac{x}{c}\right)}{c-x}\,dx = \text{PolyLog}\left(2,1-\frac{x}{c}\right)$$

```
[In] Int[Log[x/c]/(c - x),x]
```

```
[Out] PolyLog[2, 1 - x/c]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\text{integral} = \text{Li}_2\left(1-\frac{x}{c}\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{\log \left(\frac{x}{c}\right)}{c-x}\,dx = \text{PolyLog}\left(2,\frac{c-x}{c}\right)$$

```
[In] Integrate[Log[x/c]/(c - x),x]
[Out] PolyLog[2, (c - x)/c]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\text{dilog}\left(\frac{x}{c}\right)$	7
default	$\text{dilog}\left(\frac{x}{c}\right)$	7
risch	$\text{dilog}\left(\frac{x}{c}\right)$	7
parts	$\text{dilog}\left(\frac{x}{c}\right)$	7

```
[In] int(ln(x/c)/(c-x),x,method=_RETURNVERBOSE)
[Out] dilog(x/c)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{\log \left(\frac{x}{c}\right)}{c-x}\,dx = \text{Li}_2\left(-\frac{x}{c}+1\right)$$

```
[In] integrate(log(x/c)/(c-x),x, algorithm="fricas")
[Out] dilog(-x/c + 1)
```

Sympy [F]

$$\int \frac{\log\left(\frac{x}{c}\right)}{c-x}\,dx = -\int \frac{\log\left(\frac{x}{c}\right)}{-c+x}\,dx$$

```
[In] integrate(ln(x/c)/(c-x),x)
[Out] -Integral(log(x/c)/(-c + x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(9) = 18$.

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 4.50

$$\int \frac{\log\left(\frac{x}{c}\right)}{c-x}\,dx = \log(c-x)\log(x) - \log(c-x)\log\left(\frac{x}{c}\right) - \log(x)\log\left(-\frac{x}{c}+1\right) - \mathrm{Li}_2\left(\frac{x}{c}\right)$$

```
[In] integrate(log(x/c)/(c-x),x, algorithm="maxima")
[Out] log(c - x)*log(x) - log(c - x)*log(x/c) - log(x)*log(-x/c + 1) - dilog(x/c)
```

Giac [F]

$$\int \frac{\log\left(\frac{x}{c}\right)}{c-x}\,dx = \int \frac{\log\left(\frac{x}{c}\right)}{c-x}\,dx$$

```
[In] integrate(log(x/c)/(c-x),x, algorithm="giac")
[Out] integrate(log(x/c)/(c - x), x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\log\left(\frac{x}{c}\right)}{c-x}\,dx = \mathrm{Li}_2\left(\frac{x}{c}\right)$$

```
[In] int(log(x/c)/(c - x),x)
[Out] dilog(x/c)
```

3.76 $\int x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 109

$$\int x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx=\frac{2}{27}b^2dn^2x^3+\frac{1}{32}b^2en^2x^4-\frac{2}{9}bdnx^3\left(a+b\log\left(cx^n\right)\right)-\frac{1}{8}benx^4\left(a+b\log\left(cx^n\right)\right)+\frac{1}{3}dx^3\left(a+b\log\left(cx^n\right)\right)^2+\frac{1}{4}ex^4\left(a+b\log\left(cx^n\right)\right)^2$$

```
[Out] 2/27*b^2*d*n^2*x^3+1/32*b^2*e*n^2*x^4-2/9*b*d*n*x^3*(a+b*ln(c*x^n))-1/8*b*e
*n*x^4*(a+b*ln(c*x^n))+1/3*d*x^3*(a+b*ln(c*x^n))^2+1/4*e*x^4*(a+b*ln(c*x^n)
)^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2395, 2342, 2341}

$$\int x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx=\frac{1}{3}dx^3\left(a+b\log\left(cx^n\right)\right)^2-\frac{2}{9}bdnx^3\left(a+b\log\left(cx^n\right)\right)+\frac{1}{4}ex^4\left(a+b\log\left(cx^n\right)\right)^2-\frac{1}{8}benx^4\left(a+b\log\left(cx^n\right)\right)+\frac{2}{27}b^2dn^2x^3+\frac{1}{32}b^2en^2x^4$$

```
[In] Int[x^2*(d + e*x)*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (2*b^2*d*n^2*x^3)/27 + (b^2*e*n^2*x^4)/32 - (2*b*d*n*x^3*(a + b*Log[c*x^n])
)/9 - (b*e*n*x^4*(a + b*Log[c*x^n]))/8 + (d*x^3*(a + b*Log[c*x^n])^2)/3 + (
e*x^4*(a + b*Log[c*x^n])^2)/4
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(dx^2(a+b\log(cx^n))^2+ex^3(a+b\log(cx^n))^2\right)\,dx\\
&= d\int x^2(a+b\log(cx^n))^2\,dx+e\int x^3(a+b\log(cx^n))^2\,dx\\
&= \frac{1}{3}dx^3(a+b\log(cx^n))^2+\frac{1}{4}ex^4(a+b\log(cx^n))^2\\
&\quad -\frac{1}{3}(2bdn)\int x^2(a+b\log(cx^n))\,dx-\frac{1}{2}(ben)\int x^3(a+b\log(cx^n))\,dx\\
&= \frac{2}{27}b^2dn^2x^3+\frac{1}{32}b^2en^2x^4-\frac{2}{9}bdnx^3(a+b\log(cx^n))\\
&\quad -\frac{1}{8}benx^4(a+b\log(cx^n))+\frac{1}{3}dx^3(a+b\log(cx^n))^2+\frac{1}{4}ex^4(a+b\log(cx^n))^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx=\frac{1}{864}x^3\big(27benx(-4a+bn-4b\log\left(cx^n\right))+64bdn(-3a+bn-3b\log\left(cx^n\right))+288d(a+b\log\left(cx^n\right))^2+216ex(a+b\log\left(cx^n\right))^2\big)$$

```
[In] Integrate[x^2*(d + e*x)*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (x^3*(27*b*e*n*x*(-4*a + b*n - 4*b*Log[c*x^n]) + 64*b*d*n*(-3*a + b*n - 3*b
*Log[c*x^n]) + 288*d*(a + b*Log[c*x^n])^2 + 216*e*x*(a + b*Log[c*x^n])^2))/
864
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.43

method	result
parallelrisch	$\frac{x^4\ln(cx^n)^2b^2e}{4}-\frac{\ln(cx^n)x^4b^2ne}{8}+\frac{b^2en^2x^4}{32}+\frac{\ln(cx^n)x^4abe}{2}-\frac{aben\,x^4}{8}+\frac{x^3\ln(cx^n)^2b^2d}{3}-\frac{2\ln(cx^n)x^3b^2nd}{9}+$
risch	Expression too large to display

```
[In] int(x^2*(e*x+d)*(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln(c*x^n)^2*b^2*e-1/8*ln(c*x^n)*x^4*b^2*n*e+1/32*b^2*e*n^2*x^4+1/2*
ln(c*x^n)*x^4*a*b*e-1/8*a*b*e*n*x^4+1/3*x^3*ln(c*x^n)^2*b^2*d-2/9*ln(c*x^n)
*x^3*b^2*n*d+2/27*b^2*d*n^2*x^3+1/4*a^2*e*x^4+2/3*ln(c*x^n)*x^3*a*b*d-2/9*a
*b*d*n*x^3+1/3*a^2*d*x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(97) = 194.

Time = 0.31 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.01

$$\begin{aligned}\int x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx &= \frac{1}{32}\left(b^2en^2-4\,aben+8\,a^2e\right)x^4+\frac{1}{27}\left(2\,b^2dn^2-6\,abdn+9\,a^2d\right)x^3\\&\quad+\frac{1}{12}\left(3\,b^2ex^4+4\,b^2dx^3\right)\log\left(c\right)^2+\frac{1}{12}\left(3\,b^2en^2x^4+4\,b^2dn^2x^3\right)\log\left(x\right)^2\\&\quad-\frac{1}{72}\left(9\left(b^2en-4\,abe\right)x^4+16\left(b^2dn-3\,abd\right)x^3\right)\log\left(c\right)\\&\quad-\frac{1}{72}\left(9\left(b^2en^2-4\,aben\right)x^4+16\left(b^2dn^2-3\,abdn\right)x^3-12\left(3\,b^2enx^4+4\,b^2dnx^3\right)\log\left(c\right)\right)\log\left(x\right)\end{aligned}$$

```
[In] integrate(x^2*(e*x+d)*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/32*(b^2*e*n^2 - 4*a*b*e*n + 8*a^2*e)*x^4 + 1/27*(2*b^2*d*n^2 - 6*a*b*d*n
+ 9*a^2*d)*x^3 + 1/12*(3*b^2*e*x^4 + 4*b^2*d*x^3)*log(c)^2 + 1/12*(3*b^2*e*
n^2*x^4 + 4*b^2*d*n^2*x^3)*log(x)^2 - 1/72*(9*(b^2*e*n - 4*a*b*e)*x^4 + 16*
(b^2*d*n - 3*a*b*d)*x^3)*log(c) - 1/72*(9*(b^2*e*n^2 - 4*a*b*e*n)*x^4 + 16*
(b^2*d*n^2 - 3*a*b*d*n)*x^3 - 12*(3*b^2*e*n*x^4 + 4*b^2*d*n*x^3)*log(c))*lo
g(x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.70

$$\int x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx=\frac{a^2dx^3}{3}+\frac{a^2ex^4}{4}-\frac{2abdnx^3}{9}+\frac{2abdx^3\log\left(cx^n\right)}{3}-\frac{abenx^4}{8}+\frac{abex^4\log\left(cx^n\right)}{2}+\frac{2b^2dn^2x^3}{27}-\frac{2b^2dnx^3\log\left(cx^n\right)}{9}+\frac{b^2dx^3\log\left(cx^n\right)^2}{3}+\frac{b^2en^2x^4}{32}-\frac{b^2enx^4\log\left(cx^n\right)}{8}+\frac{b^2ex^4\log\left(cx^n\right)^2}{4}$$

```
[In] integrate(x**2*(e*x+d)*(a+b*ln(c*x**n))**2,x)
```

```
[Out] a**2*d*x**3/3 + a**2*e*x**4/4 - 2*a*b*d*n*x**3/9 + 2*a*b*d*x**3*log(c*x**n)
/3 - a*b*e*n*x**4/8 + a*b*e*x**4*log(c*x**n)/2 + 2*b**2*d*n**2*x**3/27 - 2*
b**2*d*n*x**3*log(c*x**n)/9 + b**2*d*x**3*log(c*x**n)**2/3 + b**2*e*n**2*x*
*4/32 - b**2*e*n*x**4*log(c*x**n)/8 + b**2*e*x**4*log(c*x**n)**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

$$\int x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx=\frac{1}{4}\,b^2ex^4\log\left(cx^n\right)^2-\frac{1}{8}\,abenx^4+\frac{1}{2}\,abex^4\log\left(cx^n\right)+\frac{1}{3}\,b^2dx^3\log\left(cx^n\right)^2-\frac{2}{9}\,abdnx^3+\frac{1}{4}\,a^2ex^4+\frac{2}{3}\,abdx^3\log\left(cx^n\right)+\frac{1}{3}\,a^2dx^3+\frac{2}{27}\,\left(n^2x^3-3\,nx^3\log\left(cx^n\right)\right)b^2d+\frac{1}{32}\,\left(n^2x^4-4\,nx^4\log\left(cx^n\right)\right)b^2e$$

```
[In] integrate(x^2*(e*x+d)*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*e*x^4*log(c*x^n)^2 - 1/8*a*b*e*n*x^4 + 1/2*a*b*e*x^4*log(c*x^n) + 1
/3*b^2*d*x^3*log(c*x^n)^2 - 2/9*a*b*d*n*x^3 + 1/4*a^2*e*x^4 + 2/3*a*b*d*x^3
*log(c*x^n) + 1/3*a^2*d*x^3 + 2/27*(n^2*x^3 - 3*n*x^3*log(c*x^n))*b^2*d + 1
/32*(n^2*x^4 - 4*n*x^4*log(c*x^n))*b^2*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(97) = 194$.

Time = 0.47 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.21

$$\begin{aligned}\int x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx &= \frac{1}{4}\,b^2en^2x^4\log\left(x\right)^2-\frac{1}{8}\,b^2en^2x^4\log\left(x\right)\\&+\frac{1}{2}\,b^2enx^4\log\left(c\right)\log\left(x\right)+\frac{1}{3}\,b^2dn^2x^3\log\left(x\right)^2\\&+\frac{1}{32}\,b^2en^2x^4-\frac{1}{8}\,b^2enx^4\log\left(c\right)+\frac{1}{4}\,b^2ex^4\log\left(c\right)^2\\&-\frac{2}{9}\,b^2dn^2x^3\log\left(x\right)+\frac{1}{2}\,abenx^4\log\left(x\right)\\&+\frac{2}{3}\,b^2dnx^3\log\left(c\right)\log\left(x\right)+\frac{2}{27}\,b^2dn^2x^3\\&-\frac{1}{8}\,abenx^4-\frac{2}{9}\,b^2dnx^3\log\left(c\right)+\frac{1}{2}\,abex^4\log\left(c\right)\\&+\frac{1}{3}\,b^2dx^3\log\left(c\right)^2+\frac{2}{3}\,abdnx^3\log\left(x\right)-\frac{2}{9}\,abdnx^3\\&+\frac{1}{4}\,a^2ex^4+\frac{2}{3}\,abdx^3\log\left(c\right)+\frac{1}{3}\,a^2dx^3\end{aligned}$$

```
[In] integrate(x^2*(e*x+d)*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/4*b^2*e*n^2*x^4*log(x)^2 - 1/8*b^2*e*n^2*x^4*log(x) + 1/2*b^2*e*n*x^4*log
(c)*log(x) + 1/3*b^2*d*n^2*x^3*log(x)^2 + 1/32*b^2*e*n^2*x^4 - 1/8*b^2*e*n*
x^4*log(c) + 1/4*b^2*e*x^4*log(c)^2 - 2/9*b^2*d*n^2*x^3*log(x) + 1/2*a*b*e*
n*x^4*log(x) + 2/3*b^2*d*n*x^3*log(c)*log(x) + 2/27*b^2*d*n^2*x^3 - 1/8*a*b
*e*n*x^4 - 2/9*b^2*d*n*x^3*log(c) + 1/2*a*b*e*x^4*log(c) + 1/3*b^2*d*x^3*lo
g(c)^2 + 2/3*a*b*d*n*x^3*log(x) - 2/9*a*b*d*n*x^3 + 1/4*a^2*e*x^4 + 2/3*a*b
*d*x^3*log(c) + 1/3*a^2*d*x^3
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06

$$\int x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx=\ln\left(c\,x^n\right)^2\left(\frac{e\,b^2\,x^4}{4}+\frac{d\,b^2\,x^3}{3}\right)+\ln\left(c\,x^n\right)\left(\frac{b\,e\,\left(4\,a-b\,n\right)\,x^4}{8}+\frac{2\,b\,d\,\left(3\,a-b\,n\right)\,x^3}{9}\right)+\frac{d\,x^3\,\left(9\,a^2-6\,a\,b\,n+2\,b^2\,n^2\right)}{27}+\frac{e\,x^4\,\left(8\,a^2-4\,a\,b\,n+b^2\,n^2\right)}{32}$$

```
[In] int(x^2*(a + b*log(c*x^n))^2*(d + e*x),x)
```

```
[Out] log(c*x^n)^2*((b^2*d*x^3)/3 + (b^2*e*x^4)/4) + log(c*x^n)*((2*b*d*x^3*(3*a
- b*n))/9 + (b*e*x^4*(4*a - b*n))/8) + (d*x^3*(9*a^2 + 2*b^2*n^2 - 6*a*b*n)
)/27 + (e*x^4*(8*a^2 + b^2*n^2 - 4*a*b*n))/32
```

3.77 $\int x(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int x(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx=\frac{1}{4}b^2dn^2x^2+\frac{2}{27}b^2en^2x^3-\frac{1}{2}bdnx^2\left(a+b\log\left(cx^n\right)\right)-\frac{2}{9}benx^3\left(a+b\log\left(cx^n\right)\right)+\frac{1}{2}dx^2\left(a+b\log\left(cx^n\right)\right)^2+\frac{1}{3}ex^3\left(a+b\log\left(cx^n\right)\right)^2$$

```
[Out] 1/4*b^2*d*n^2*x^2+2/27*b^2*e*n^2*x^3-1/2*b*d*n*x^2*(a+b*ln(c*x^n))-2/9*b*e*
n*x^3*(a+b*ln(c*x^n))+1/2*d*x^2*(a+b*ln(c*x^n))^2+1/3*e*x^3*(a+b*ln(c*x^n))
^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2395, 2342, 2341}

$$\int x(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx=\frac{1}{2}dx^2\left(a+b\log\left(cx^n\right)\right)^2-\frac{1}{2}bdnx^2\left(a+b\log\left(cx^n\right)\right)+\frac{1}{3}ex^3\left(a+b\log\left(cx^n\right)\right)^2-\frac{2}{9}benx^3\left(a+b\log\left(cx^n\right)\right)+\frac{1}{4}b^2dn^2x^2+\frac{2}{27}b^2en^2x^3$$

```
[In] Int[x*(d + e*x)*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (b^2*d*n^2*x^2)/4 + (2*b^2*e*n^2*x^3)/27 - (b*d*n*x^2*(a + b*Log[c*x^n]))/2
 - (2*b*e*n*x^3*(a + b*Log[c*x^n]))/9 + (d*x^2*(a + b*Log[c*x^n])^2)/2 + (e
*x^3*(a + b*Log[c*x^n])^2)/3
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(dx(a+b\log(cx^n))^2+ex^2(a+b\log(cx^n))^2\right)\,dx\\
&= d\int x(a+b\log(cx^n))^2\,dx+e\int x^2(a+b\log(cx^n))^2\,dx\\
&= \frac{1}{2}dx^2(a+b\log(cx^n))^2+\frac{1}{3}ex^3(a+b\log(cx^n))^2\\
&\quad-(bdn)\int x(a+b\log(cx^n))\,dx-\frac{1}{3}(2ben)\int x^2(a+b\log(cx^n))\,dx\\
&= \frac{1}{4}b^2dn^2x^2+\frac{2}{27}b^2en^2x^3-\frac{1}{2}bdnx^2(a+b\log(cx^n))-\frac{2}{9}benx^3(a+b\log(cx^n))\\
&\quad+\frac{1}{2}dx^2(a+b\log(cx^n))^2+\frac{1}{3}ex^3(a+b\log(cx^n))^2
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int x(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx=\frac{1}{108}x^2\left(8benx(-3a+bn-3b\log\left(cx^n\right))+27bdn(-2a+bn-2b\log\left(cx^n\right))+54d(a+b\log\left(cx^n\right))^2+36ex(a+b\log\left(cx^n\right))^2\right)$$

```
[In] Integrate[x*(d + e*x)*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (x^2*(8*b*e*n*x*(-3*a + b*n - 3*b*Log[c*x^n]) + 27*b*d*n*(-2*a + b*n - 2*b*
Log[c*x^n]) + 54*d*(a + b*Log[c*x^n])^2 + 36*e*x*(a + b*Log[c*x^n])^2))/108
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.42

method	result
parallelrisch	$\frac{x^3b^2\ln(c\,x^n)^2e}{3}-\frac{2\ln(c\,x^n)x^3b^2ne}{9}+\frac{2b^2e\,n^2x^3}{27}+\frac{2x^3ab\ln(c\,x^n)e}{3}-\frac{2bn\,x^3ae}{9}+\frac{x^2b^2\ln(c\,x^n)^2d}{2}-\frac{\ln(c\,x^n)x^2b^2nd}{2}$
risch	Expression too large to display

```
[In] int(x*(e*x+d)*(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*b^2*ln(c*x^n)^2*e-2/9*ln(c*x^n)*x^3*b^2*n*e+2/27*b^2*e*n^2*x^3+2/3*
x^3*a*b*ln(c*x^n)*e-2/9*b*n*x^3*a*e+1/2*x^2*b^2*ln(c*x^n)^2*d-1/2*ln(c*x^n)
*x^2*b^2*n*d+1/4*b^2*d*n^2*x^2+1/3*x^3*a^2*e+x^2*a*b*ln(c*x^n)*d-1/2*b*n*a*
d*x^2+1/2*x^2*a^2*d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(97) = 194.

Time = 0.27 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.01

$$\int x(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx=\frac{1}{27}\left(2\,b^2en^2-6\,aben+9\,a^2e\right)x^3+\frac{1}{4}\left(b^2dn^2-2\,abdn+2\,a^2d\right)x^2+\frac{1}{6}\left(2\,b^2ex^3+3\,b^2dx^2\right)\log\left(c\right)^2+\frac{1}{6}\left(2\,b^2en^2x^3+3\,b^2dn^2x^2\right)\log\left(x\right)^2-\frac{1}{18}\left(4\left(b^2en-3\,abe\right)x^3+9\left(b^2dn-2\,abd\right)x^2\right)\log\left(c\right)-\frac{1}{18}\left(4\left(b^2en^2-3\,aben\right)x^3+9\left(b^2dn^2-2\,abdn\right)x^2-6\left(2\,b^2enx^3+3\,b^2dnx^2\right)\log\left(c\right)\right)\log\left(x\right)$$

```
[In] integrate(x*(e*x+d)*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/27*(2*b^2*e*n^2 - 6*a*b*e*n + 9*a^2*e)*x^3 + 1/4*(b^2*d*n^2 - 2*a*b*d*n +
 2*a^2*d)*x^2 + 1/6*(2*b^2*e*x^3 + 3*b^2*d*x^2)*log(c)^2 + 1/6*(2*b^2*e*n^2
*x^3 + 3*b^2*d*n^2*x^2)*log(x)^2 - 1/18*(4*(b^2*e*n - 3*a*b*e)*x^3 + 9*(b^2
*d*n - 2*a*b*d)*x^2)*log(c) - 1/18*(4*(b^2*e*n^2 - 3*a*b*e*n)*x^3 + 9*(b^2*
d*n^2 - 2*a*b*d*n)*x^2 - 6*(2*b^2*e*n*x^3 + 3*b^2*d*n*x^2)*log(c))*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.69

$$\int x(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx = \frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}-\frac{abdnx^2}{2}+abdx^2\log\left(cx^n\right)-\frac{2abenx^3}{9}+\frac{2abex^3\log\left(cx^n\right)}{3}+\frac{b^2dn^2x^2}{4}-\frac{b^2dnx^2\log\left(cx^n\right)}{2}+\frac{b^2dx^2\log\left(cx^n\right)^2}{2}+\frac{2b^2en^2x^3}{27}-\frac{2b^2enx^3\log\left(cx^n\right)}{9}+\frac{b^2ex^3\log\left(cx^n\right)^2}{3}$$

```
[In] integrate(x*(e*x+d)*(a+b*ln(c*x**n))**2,x)
```

```
[Out] a**2*d*x**2/2 + a**2*e*x**3/3 - a*b*d*n*x**2/2 + a*b*d*x**2*log(c*x**n) - 2
*a*b*e*n*x**3/9 + 2*a*b*e*x**3*log(c*x**n)/3 + b**2*d*n**2*x**2/4 - b**2*d*
n*x**2*log(c*x**n)/2 + b**2*d*x**2*log(c*x**n)**2/2 + 2*b**2*e*n**2*x**3/27
 - 2*b**2*e*n*x**3*log(c*x**n)/9 + b**2*e*x**3*log(c*x**n)**2/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.38

$$\int x(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx = \frac{1}{3}\,b^2ex^3\log\left(cx^n\right)^2-\frac{2}{9}\,abenx^3+\frac{2}{3}\,abex^3\log\left(cx^n\right)+\frac{1}{2}\,b^2dx^2\log\left(cx^n\right)^2-\frac{1}{2}\,abdnx^2+\frac{1}{3}\,a^2ex^3+abdx^2\log\left(cx^n\right)+\frac{1}{2}\,a^2dx^2+\frac{1}{4}\,\left(n^2x^2-2\,nx^2\log\left(cx^n\right)\right)b^2d+\frac{2}{27}\,\left(n^2x^3-3\,nx^3\log\left(cx^n\right)\right)b^2e$$

```
[In] integrate(x*(e*x+d)*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*e*x^3*log(c*x^n)^2 - 2/9*a*b*e*n*x^3 + 2/3*a*b*e*x^3*log(c*x^n) + 1
/2*b^2*d*x^2*log(c*x^n)^2 - 1/2*a*b*d*n*x^2 + 1/3*a^2*e*x^3 + a*b*d*x^2*log
```

```
(c*x^n) + 1/2*a^2*d*x^2 + 1/4*(n^2*x^2 - 2*n*x^2*log(c*x^n))*b^2*d + 2/27*(
n^2*x^3 - 3*n*x^3*log(c*x^n))*b^2*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(97) = 194$.

Time = 0.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.18

$$\int x(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx = \frac{1}{3}\,b^2en^2x^3\log\left(x\right)^2 - \frac{2}{9}\,b^2en^2x^3\log\left(x\right) + \frac{2}{3}\,b^2enx^3\log\left(c\right)\log\left(x\right) + \frac{1}{2}\,b^2dn^2x^2\log\left(x\right)^2 + \frac{2}{27}\,b^2en^2x^3 - \frac{2}{9}\,b^2enx^3\log\left(c\right) + \frac{1}{3}\,b^2ex^3\log\left(c\right)^2 - \frac{1}{2}\,b^2dn^2x^2\log\left(x\right) + \frac{2}{3}\,abenx^3\log\left(x\right) + b^2dnx^2\log\left(c\right)\log\left(x\right) + \frac{1}{4}\,b^2dn^2x^2 - \frac{2}{9}\,abenx^3 - \frac{1}{2}\,b^2dnx^2\log\left(c\right) + \frac{2}{3}\,abex^3\log\left(c\right) + \frac{1}{2}\,b^2dx^2\log\left(c\right)^2 + abdnx^2\log\left(x\right) - \frac{1}{2}\,abdnx^2 + \frac{1}{3}\,a^2ex^3 + abdx^2\log\left(c\right) + \frac{1}{2}\,a^2dx^2$$

```
[In] integrate(x*(e*x+d)*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*e*n^2*x^3*log(x)^2 - 2/9*b^2*e*n^2*x^3*log(x) + 2/3*b^2*e*n*x^3*log
(c)*log(x) + 1/2*b^2*d*n^2*x^2*log(x)^2 + 2/27*b^2*e*n^2*x^3 - 2/9*b^2*e*n*
x^3*log(c) + 1/3*b^2*e*x^3*log(c)^2 - 1/2*b^2*d*n^2*x^2*log(x) + 2/3*a*b*e*
n*x^3*log(x) + b^2*d*n*x^2*log(c)*log(x) + 1/4*b^2*d*n^2*x^2 - 2/9*a*b*e*n*
x^3 - 1/2*b^2*d*n*x^2*log(c) + 2/3*a*b*e*x^3*log(c) + 1/2*b^2*d*x^2*log(c)^
2 + a*b*d*n*x^2*log(x) - 1/2*a*b*d*n*x^2 + 1/3*a^2*e*x^3 + a*b*d*x^2*log(c)
 + 1/2*a^2*d*x^2
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06

$$\int x(d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx = \ln\left(c\,x^n\right)^2\left(\frac{e\,b^2\,x^3}{3}+\frac{d\,b^2\,x^2}{2}\right) + \ln\left(c\,x^n\right)\left(\frac{2\,b\,e\,\left(3\,a-b\,n\right)\,x^3}{9}+\frac{b\,d\,\left(2\,a-b\,n\right)\,x^2}{2}\right) + \frac{d\,x^2\,\left(2\,a^2-2\,a\,b\,n+b^2\,n^2\right)}{4} + \frac{e\,x^3\,\left(9\,a^2-6\,a\,b\,n+2\,b^2\,n^2\right)}{27}$$

```
[In] int(x*(a + b*log(c*x^n))^2*(d + e*x),x)
```

```
[Out] log(c*x^n)^2*((b^2*d*x^2)/2 + (b^2*e*x^3)/3) + log(c*x^n)*((b*d*x^2*(2*a -
b*n))/2 + (2*b*e*x^3*(3*a - b*n))/9) + (d*x^2*(2*a^2 + b^2*n^2 - 2*a*b*n))/
4 + (e*x^3*(9*a^2 + 2*b^2*n^2 - 6*a*b*n))/27
```

3.78 $\int (d+ex) (a+b\log (cx^n))^2 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 101

$$\begin{aligned}\int (d+ex) (a+b\log (cx^n))^2 \, dx = & -2abdnx+2b^2dn^2x+\frac{1}{4}b^2en^2x^2 \\ & -2b^2dnx\log (cx^n)-\frac{1}{2}benx^2(a+b\log (cx^n)) \\ & +dx(a+b\log (cx^n))^2+\frac{1}{2}ex^2(a+b\log (cx^n))^2\end{aligned}$$

```
[Out] -2*a*b*d*n*x+2*b^2*d*n^2*x+1/4*b^2*e*n^2*x^2-2*b^2*d*n*x*ln(c*x^n)-1/2*b*e*
n*x^2*(a+b*ln(c*x^n))+d*x*(a+b*ln(c*x^n))^2+1/2*e*x^2*(a+b*ln(c*x^n))^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2367, 2333, 2332, 2342, 2341}

$$\begin{aligned}\int (d+ex) (a+b\log (cx^n))^2 \, dx = & \, dx(a+b\log (cx^n))^2-\frac{1}{2}benx^2(a+b\log (cx^n)) \\ & +\frac{1}{2}ex^2(a+b\log (cx^n))^2-2abdnx \\ & -2b^2dnx\log (cx^n)+2b^2dn^2x+\frac{1}{4}b^2en^2x^2\end{aligned}$$

```
[In] Int[(d + e*x)*(a + b*Log[c*x^n])^2,x]
```

```
[Out] -2*a*b*d*n*x + 2*b^2*d*n^2*x + (b^2*e*n^2*x^2)/4 - 2*b^2*d*n*x*Log[c*x^n] -
 (b*e*n*x^2*(a + b*Log[c*x^n]))/2 + d*x*(a + b*Log[c*x^n])^2 + (e*x^2*(a +
b*Log[c*x^n])^2)/2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(d(a+b\log(cx^n))^2+ex(a+b\log(cx^n))^2\right)\,dx\\
&= d\int (a+b\log(cx^n))^2\,dx+e\int x(a+b\log(cx^n))^2\,dx\\
&= dx(a+b\log(cx^n))^2+\frac{1}{2}ex^2(a+b\log(cx^n))^2\\
&\quad-(2bdn)\int (a+b\log(cx^n))\,dx-(ben)\int x(a+b\log(cx^n))\,dx\\
&= -2abdnx+\frac{1}{4}b^2en^2x^2-\frac{1}{2}benx^2(a+b\log(cx^n))+dx(a+b\log(cx^n))^2\\
&\quad+\frac{1}{2}ex^2(a+b\log(cx^n))^2-\left(2b^2dn\right)\int \log(cx^n)\,dx
\end{aligned}$$

$$= -2abdnx + 2b^2dn^2x + \frac{1}{4}b^2en^2x^2 - 2b^2dnx\log(cx^n) - \frac{1}{2}benx^2(a + b\log(cx^n)) + dx(a + b\log(cx^n))^2 + \frac{1}{2}ex^2(a + b\log(cx^n))^2$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int (d+ex)\left(a+b\log(cx^n)\right)^2 dx = \frac{1}{4}x\big(benx(-2a + bn - 2b\log(cx^n)) + 4d(a + b\log(cx^n))^2 + 2ex(a + b\log(cx^n))^2 - 8bdn(a - bn + b\log(cx^n))\big)$$

```
[In] Integrate[(d + e*x)*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (x*(b*e*n*x*(-2*a + b*n - 2*b*Log[c*x^n]) + 4*d*(a + b*Log[c*x^n])^2 + 2*e*
x*(a + b*Log[c*x^n])^2 - 8*b*d*n*(a - b*n + b*Log[c*x^n])))/4
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.40

method	result
parallelrisch	$\frac{b^2\ln(c\,x^n)^2 e\,x^2}{2} - \frac{\ln(c\,x^n)x^2b^2ne}{2} + \frac{b^2e\,n^2x^2}{4} + ab\ln(c\,x^n)\,e\,x^2 - \frac{bnae\,x^2}{2} + x\,b^2\ln(c\,x^n)^2\,d - 2b^2dnx\,\mathrm{l}$
risch	Expression too large to display

```
[In] int((e*x+d)*(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2*ln(c*x^n)^2*e*x^2-1/2*ln(c*x^n)*x^2*b^2*n*e+1/4*b^2*e*n^2*x^2+a*b*l
n(c*x^n)*e*x^2-1/2*b*n*a*e*x^2+x*b^2*ln(c*x^n)^2*d-2*b^2*d*n*x*ln(c*x^n)+2*
b^2*d*n^2*x+1/2*a^2*e*x^2+2*x*a*b*ln(c*x^n)*d-2*a*b*d*n*x+a^2*d*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(95) = 190.

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.98

$$\int (d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx$$
$$=\frac{1}{4}\left(b^2en^2-2\,aben+2\,a^2e\right)x^2+\frac{1}{2}\left(b^2ex^2+2\,b^2dx\right)\log\left(c\right)^2$$
$$+\frac{1}{2}\left(b^2en^2x^2+2\,b^2dn^2x\right)\log\left(x\right)^2+\left(2\,b^2dn^2-2\,abdn+a^2d\right)x$$
$$-\frac{1}{2}\left(\left(b^2en-2\,abe\right)x^2+4\left(b^2dn-abd\right)x\right)\log\left(c\right)$$
$$-\frac{1}{2}\left(\left(b^2en^2-2\,aben\right)x^2+4\left(b^2dn^2-abdn\right)x-2\left(b^2enx^2+2\,b^2dnx\right)\log\left(c\right)\right)\log\left(x\right)$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*e*n^2 - 2*a*b*e*n + 2*a^2*e)*x^2 + 1/2*(b^2*e*x^2 + 2*b^2*d*x)*log
(c)^2 + 1/2*(b^2*e*n^2*x^2 + 2*b^2*d*n^2*x)*log(x)^2 + (2*b^2*d*n^2 - 2*a*b
*d*n + a^2*d)*x - 1/2*((b^2*e*n - 2*a*b*e)*x^2 + 4*(b^2*d*n - a*b*d)*x)*log
(c) - 1/2*((b^2*e*n^2 - 2*a*b*e*n)*x^2 + 4*(b^2*d*n^2 - a*b*d*n)*x - 2*(b^2
*e*n*x^2 + 2*b^2*d*n*x)*log(c))*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.61

$$\int (d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx=a^2dx+\frac{a^2ex^2}{2}-2abdnx+2abdx\log\left(cx^n\right)$$
$$-\frac{abenx^2}{2}+abex^2\log\left(cx^n\right)+2b^2dn^2x$$
$$-2b^2dnx\log\left(cx^n\right)+b^2dx\log\left(cx^n\right)^2+\frac{b^2en^2x^2}{4}$$
$$-\frac{b^2enx^2\log\left(cx^n\right)}{2}+\frac{b^2ex^2\log\left(cx^n\right)^2}{2}$$

```
[In] integrate((e*x+d)*(a+b*ln(c*x**n))**2,x)
```

```
[Out] a**2*d*x + a**2*e*x**2/2 - 2*a*b*d*n*x + 2*a*b*d*x*log(c*x**n) - a*b*e*n*x*
*2/2 + a*b*e*x**2*log(c*x**n) + 2*b**2*d*n**2*x - 2*b**2*d*n*x*log(c*x**n)
+ b**2*d*x*log(c*x**n)**2 + b**2*e*n**2*x**2/4 - b**2*e*n*x**2*log(c*x**n)/
2 + b**2*e*x**2*log(c*x**n)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.35

$$\int (d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx = \frac{1}{2}\,b^2ex^2\log\left(cx^n\right)^2 - \frac{1}{2}\,abenx^2 + abex^2\log\left(cx^n\right) + b^2dx\log\left(cx^n\right)^2 - 2\,abdnx + \frac{1}{2}\,a^2ex^2 + 2\,abdx\log\left(cx^n\right) + 2\left(n^2x - nx\log\left(cx^n\right)\right)b^2d + \frac{1}{4}\left(n^2x^2 - 2\,nx^2\log\left(cx^n\right)\right)b^2e + a^2dx$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*e*x^2*log(c*x^n)^2 - 1/2*a*b*e*n*x^2 + a*b*e*x^2*log(c*x^n) + b^2*d
*x*log(c*x^n)^2 - 2*a*b*d*n*x + 1/2*a^2*e*x^2 + 2*a*b*d*x*log(c*x^n) + 2*(n
^2*x - n*x*log(c*x^n))*b^2*d + 1/4*(n^2*x^2 - 2*n*x^2*log(c*x^n))*b^2*e + a
^2*d*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(95) = 190.

Time = 0.34 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.13

$$\int (d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx = \frac{1}{2}\,b^2en^2x^2\log\left(x\right)^2 - \frac{1}{2}\,b^2en^2x^2\log\left(x\right) + b^2enx^2\log\left(c\right)\log\left(x\right) + b^2dn^2x\log\left(x\right)^2 + \frac{1}{4}\,b^2en^2x^2 - \frac{1}{2}\,b^2enx^2\log\left(c\right) + \frac{1}{2}\,b^2ex^2\log\left(c\right)^2 - 2\,b^2dn^2x\log\left(x\right) + abenx^2\log\left(x\right) + 2\,b^2dnx\log\left(c\right)\log\left(x\right) + 2\,b^2dn^2x - \frac{1}{2}\,abenx^2 - 2\,b^2dnx\log\left(c\right) + abex^2\log\left(c\right) + b^2dx\log\left(c\right)^2 + 2\,abdnx\log\left(x\right) - 2\,abdnx + \frac{1}{2}\,a^2ex^2 + 2\,abdx\log\left(c\right) + a^2dx$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*e*n^2*x^2*log(x)^2 - 1/2*b^2*e*n^2*x^2*log(x) + b^2*e*n*x^2*log(c)*
log(x) + b^2*d*n^2*x*log(x)^2 + 1/4*b^2*e*n^2*x^2 - 1/2*b^2*e*n*x^2*log(c)
+ 1/2*b^2*e*x^2*log(c)^2 - 2*b^2*d*n^2*x*log(x) + a*b*e*n*x^2*log(x) + 2*b^
2*d*n*x*log(c)*log(x) + 2*b^2*d*n^2*x - 1/2*a*b*e*n*x^2 - 2*b^2*d*n*x*log(c
) + a*b*e*x^2*log(c) + b^2*d*x*log(c)^2 + 2*a*b*d*n*x*log(x) - 2*a*b*d*n*x
+ 1/2*a^2*e*x^2 + 2*a*b*d*x*log(c) + a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int (d+ex)\left(a+b\log\left(cx^n\right)\right)^2\,dx = \ln\left(c\,x^n\right)\,\left(\frac{b\,e\,\left(2\,a-b\,n\right)\,x^2}{2}+2\,b\,d\,\left(a-b\,n\right)\,x\right) + \ln\left(c\,x^n\right)^2\,\left(\frac{e\,b^2\,x^2}{2}+d\,b^2\,x\right) + \frac{e\,x^2\,\left(2\,a^2-2\,a\,b\,n+b^2\,n^2\right)}{4}+d\,x\,\left(a^2-2\,a\,b\,n+2\,b^2\,n^2\right)$$

```
[In] int((a + b*log(c*x^n))^2*(d + e*x),x)
```

```
[Out] log(c*x^n)*((b*e*x^2*(2*a - b*n))/2 + 2*b*d*x*(a - b*n)) + log(c*x^n)^2*((b
^2*e*x^2)/2 + b^2*d*x) + (e*x^2*(2*a^2 + b^2*n^2 - 2*a*b*n))/4 + d*x*(a^2 +
 2*b^2*n^2 - 2*a*b*n)
```

3.79 $\int \frac{(d+ex)(a+b\log(cx^n))^2}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{(d+ex)(a+b\log(cx^n))^2}{x}\,dx = -2abenx+2b^2en^2x-2b^2enx\log(cx^n) + ex(a+b\log(cx^n))^2+\frac{d(a+b\log(cx^n))^3}{3bn}$$

```
[Out] -2*a*b*e*n*x+2*b^2*e*n^2*x-2*b^2*e*n*x*ln(c*x^n)+e*x*(a+b*ln(c*x^n))^2+1/3*
d*(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2388, 2339, 30, 2333, 2332}

$$\int \frac{(d+ex)(a+b\log(cx^n))^2}{x}\,dx = \frac{d(a+b\log(cx^n))^3}{3bn}+ex(a+b\log(cx^n))^2 - 2abenx-2b^2enx\log(cx^n)+2b^2en^2x$$

```
[In] Int[((d + e*x)*(a + b*Log[c*x^n])^2)/x,x]
```

```
[Out] -2*a*b*e*n*x + 2*b^2*e*n^2*x - 2*b^2*e*n*x*Log[c*x^n] + e*x*(a + b*Log[c*x^
n])^2 + (d*(a + b*Log[c*x^n])^3)/(3*b*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int \frac{(a+b\log(cx^n))^2}{x}\,dx + e\int (a+b\log(cx^n))^2\,dx\\
&= ex(a+b\log(cx^n))^2+\frac{d\text{Subst}\left(\int x^2\,dx, x, a+b\log(cx^n)\right)}{bn}-(2ben)\int (a+b\log(cx^n))\,dx\\
&= -2abenx+ex(a+b\log(cx^n))^2+\frac{d(a+b\log(cx^n))^3}{3bn}-\left(2b^2en\right)\int \log(cx^n)\,dx\\
&= -2abenx+2b^2en^2x-2b^2enx\log(cx^n)+ex(a+b\log(cx^n))^2+\frac{d(a+b\log(cx^n))^3}{3bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx = ex\left(a+b\log\left(cx^n\right)\right)^2+\frac{d\left(a+b\log\left(cx^n\right)\right)^3}{3bn} - 2benx\left(a-bn+b\log\left(cx^n\right)\right)$$

[In] Integrate[((d + e*x)*(a + b*Log[c*x^n])^2)/x,x]

[Out] e*x*(a + b*Log[c*x^n])^2 + (d*(a + b*Log[c*x^n])^3)/(3*b*n) - 2*b*e*n*x*(a - b*n + b*Log[c*x^n])

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.63

method	result
parallelrisch	$\frac{3x\ln(cx^n)^2b^2en-6x\ln(cx^n)b^2en^2+6xb^2en^3+6x\ln(cx^n)aben-6xaben^2+b^2d\ln(cx^n)^3+3\ln(x)a^2dn+3xa^2en+3abd\ln(cx}{3n}$
risch	Expression too large to display

[In] int((e*x+d)*(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)

[Out] 1/3*(3*x*ln(c*x^n)^2*b^2*e*n-6*x*ln(c*x^n)*b^2*e*n^2+6*x*b^2*e*n^3+6*x*ln(c*x^n)*a*b*e*n-6*x*a*b*e*n^2+b^2*d*ln(c*x^n)^3+3*ln(x)*a^2*d*n+3*x*a^2*e*n+3*a*b*d*ln(c*x^n)^2)/n

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(68) = 136.

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.06

$$\begin{aligned}\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx &= \frac{1}{3}b^2dn^2\log(x)^3+b^2ex\log(c)^2-2\left(b^2en-abe\right)x\log(c)\\ &\quad+\left(b^2en^2x+b^2dn\log(c)+abdn\right)\log(x)^2+\left(2b^2en^2-2aben+a^2e\right)x\\ &\quad+\left(b^2d\log(c)^2+a^2d-2\left(b^2en^2-aben\right)x+2\left(b^2enx+abd\right)\log(c)\right)\log(x)\end{aligned}$$

[In] integrate((e*x+d)*(a+b*log(c*x^n))^2/x,x, algorithm="fricas")

[Out] 1/3*b^2*d*n^2*log(x)^3 + b^2*e*x*log(c)^2 - 2*(b^2*e*n - a*b*e)*x*log(c) + (b^2*e*n^2*x + b^2*d*n*log(c) + a*b*d*n)*log(x)^2 + (2*b^2*e*n^2 - 2*a*b*e*n + a^2*e)*x + (b^2*d*log(c)^2 + a^2*d - 2*(b^2*e*n^2 - a*b*e*n)*x + 2*(b^2*e*n*x + a*b*d)*log(c))*log(x)

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.97

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx$$
$$=\begin{cases} \frac{a^2d\log(cx^n)}{n}+a^2ex+\frac{abd\log(cx^n)^2}{n}-2abenx+2abex\log\left(cx^n\right)+\frac{b^2d\log(cx^n)^3}{3n}+2b^2en^2x-2b^2enx\log\left(cx^n\right)+ \\ (a+b\log(c))^2\,(d\log(x)+ex) \end{cases}$$

```
[In] integrate((e*x+d)*(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Piecewise((a**2*d*log(c*x**n)/n + a**2*e*x + a*b*d*log(c*x**n)**2/n - 2*a*b
*e*n*x + 2*a*b*e*x*log(c*x**n) + b**2*d*log(c*x**n)**3/(3*n) + 2*b**2*e*n**
2*x - 2*b**2*e*n*x*log(c*x**n) + b**2*e*x*log(c*x**n)**2, Ne(n, 0)), ((a +
b*log(c))**2*(d*log(x) + e*x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx = b^2ex\log\left(cx^n\right)^2-2\,abenx+2\,abex\log\left(cx^n\right) + \frac{b^2d\log\left(cx^n\right)^3}{3\,n}+2\left(n^2x-nx\log\left(cx^n\right)\right)b^2e + a^2ex+\frac{abd\log\left(cx^n\right)^2}{n}+a^2d\log\left(x\right)$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*e*x*log(c*x^n)^2 - 2*a*b*e*n*x + 2*a*b*e*x*log(c*x^n) + 1/3*b^2*d*log(c
*x^n)^3/n + 2*(n^2*x - n*x*log(c*x^n))*b^2*e + a^2*e*x + a*b*d*log(c*x^n)^2
/n + a^2*d*log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(68) = 136.

Time = 0.35 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.06

$$\int \frac{(d+ex)(a+b\log(cx^n))^2}{x}\,dx$$
$$= \frac{1}{3}b^2dn^2\log(x)^3 - 2\left(b^2en^2 - b^2en\log(c) - aben\right)x\log(x)$$
$$+ \left(b^2en^2x + b^2dn\log(c) + abdn\right)\log(x)^2$$
$$+ \left(2b^2en^2 - 2b^2en\log(c) + b^2e\log(c)^2 - 2aben + 2abe\log(c) + a^2e\right)x$$
$$+ \left(b^2d\log(c)^2 + 2abd\log(c) + a^2d\right)\log(x)$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] 1/3*b^2*d*n^2*log(x)^3 - 2*(b^2*e*n^2 - b^2*e*n*log(c) - a*b*e*n)*x*log(x)
+ (b^2*e*n^2*x + b^2*d*n*log(c) + a*b*d*n)*log(x)^2 + (2*b^2*e*n^2 - 2*b^2*
e*n*log(c) + b^2*e*log(c)^2 - 2*a*b*e*n + 2*a*b*e*log(c) + a^2*e)*x + (b^2*
d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)(a+b\log(cx^n))^2}{x}\,dx = \ln(c\,x^n)^2\left(b^2\,e\,x + \frac{a\,b\,d}{n}\right) + a^2\,d\,\ln(x)$$
$$+ e\,x\left(a^2 - 2\,a\,b\,n + 2\,b^2\,n^2\right)$$
$$+ \frac{b^2\,d\ln(c\,x^n)^3}{3\,n} + 2\,b\,e\,x\,\ln(c\,x^n)\,(a - b\,n)$$

```
[In] int(((a + b*log(c*x^n))^2*(d + e*x))/x,x)
```

```
[Out] log(c*x^n)^2*(b^2*e*x + (a*b*d)/n) + a^2*d*log(x) + e*x*(a^2 + 2*b^2*n^2 -
2*a*b*n) + (b^2*d*log(c*x^n)^3)/(3*n) + 2*b*e*x*log(c*x^n)*(a - b*n)
```

3.80 $\int \frac{(d+ex)(a+b\log(cx^n))^2}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)^2}{x^2}\,dx=-\frac{2b^2dn^2}{x}-\frac{2bdn(a+b\log\left(cx^n\right))}{x}-\frac{d(a+b\log\left(cx^n\right))^2}{x}+\frac{e(a+b\log\left(cx^n\right))^3}{3bn}$$

```
[Out] -2*b^2*d*n^2/x-2*b*d*n*(a+b*ln(c*x^n))/x-d*(a+b*ln(c*x^n))^2/x+1/3*e*(a+b*l
n(c*x^n))^3/b/n
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2395, 2342, 2341, 2339, 30}

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)^2}{x^2}\,dx=-\frac{d(a+b\log\left(cx^n\right))^2}{x}-\frac{2bdn(a+b\log\left(cx^n\right))}{x}+\frac{e(a+b\log\left(cx^n\right))^3}{3bn}-\frac{2b^2dn^2}{x}$$

```
[In] Int[((d + e*x)*(a + b*Log[c*x^n])^2)/x^2,x]
```

```
[Out] (-2*b^2*d*n^2)/x - (2*b*d*n*(a + b*Log[c*x^n]))/x - (d*(a + b*Log[c*x^n])^2
)/x + (e*(a + b*Log[c*x^n])^3)/(3*b*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d(a+b\log(cx^n))^2}{x^2}+\frac{e(a+b\log(cx^n))^2}{x}\right)dx\\
&= d\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx+e\int \frac{(a+b\log(cx^n))^2}{x}\,dx\\
&= -\frac{d(a+b\log(cx^n))^2}{x}+\frac{e\text{Subst}\left(\int x^2\,dx,x,a+b\log(cx^n)\right)}{bn}+(2bdn)\int \frac{a+b\log(cx^n)}{x^2}\,dx\\
&= -\frac{2b^2dn^2}{x}-\frac{2bdn(a+b\log(cx^n))}{x}-\frac{d(a+b\log(cx^n))^2}{x}+\frac{e(a+b\log(cx^n))^3}{3bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)\,(a+b\log{(cx^n)})^2}{x^2}\,dx = -\frac{d(a+b\log{(cx^n)})^2}{x} + \frac{e(a+b\log{(cx^n)})^3}{3bn} - \frac{2bdn(a+bn+b\log{(cx^n)})}{x}$$

[In] Integrate[((d + e*x)*(a + b*Log[c*x^n])^2)/x^2,x]

[Out] -((d*(a + b*Log[c*x^n])^2)/x) + (e*(a + b*Log[c*x^n])^3)/(3*b*n) - (2*b*d*n*(a + b*n + b*Log[c*x^n]))/x

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.58

method	result
parallelrisch	$\frac{e\,b^2\ln(c\,x^n)^3x+3\ln(x)x\,a^2en+3abe\ln(c\,x^n)^2x-3\ln(c\,x^n)^2b^2dn-6\ln(c\,x^n)b^2d\,n^2-6b^2d\,n^3-6\ln(c\,x^n)abdn-6abd\,n^2-3a^2dn}{3xn}$
risch	Expression too large to display

[In] int((e*x+d)*(a+b*ln(c*x^n))^2/x^2,x,method=_RETURNVERBOSE)

[Out] 1/3/x*(e*b^2*ln(c*x^n)^3*x+3*ln(x)*x*a^2*e*n+3*a*b*e*ln(c*x^n)^2*x-3*ln(c*x^n)^2*b^2*d*n-6*ln(c*x^n)*b^2*d*n^2-6*b^2*d*n^3-6*ln(c*x^n)*a*b*d*n-6*a*b*d*n^2-3*a^2*d*n)/n

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(70) = 140.

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.07

$$\int \frac{(d+ex)\,(a+b\log{(cx^n)})^2}{x^2}\,dx = \frac{b^2en^2x\log{(x)}^3 - 6\,b^2dn^2 - 3\,b^2d\log{(c)}^2 - 6\,abdn - 3\,a^2d + 3\,(b^2enx\log{(c)} - b^2dn^2 + abenx)\log{(x)}^2 - 6}{3\,x}$$

[In] integrate((e*x+d)*(a+b*log(c*x^n))^2/x^2,x, algorithm="fricas")

[Out] 1/3*(b^2*e*n^2*x*log(x)^3 - 6*b^2*d*n^2 - 3*b^2*d*log(c)^2 - 6*a*b*d*n - 3*a^2*d + 3*(b^2*e*n*x*log(c) - b^2*d*n^2 + a*b*e*n*x)*log(x)^2 - 6*(b^2*d*n + a*b*d)*log(c) + 3*(b^2*e*x*log(c)^2 - 2*b^2*d*n^2 - 2*a*b*d*n + a^2*e*x - 2*(b^2*d*n - a*b*e*x)*log(c))*log(x))/x

Sympy [A] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.96

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)^2}{x^2}\,dx = -\frac{a^2d}{x}+a^2e\log(x)-\frac{2abdn}{x}-\frac{2abd\log\left(cx^n\right)}{x} - 2abe\left(\begin{cases}-\log(c)\log(x) & \text{for } n=0\\ -\frac{\log(cx^n)^2}{2n} & \text{otherwise}\end{cases}\right) - \frac{2b^2dn^2}{x}-\frac{2b^2dn\log\left(cx^n\right)}{x}-\frac{b^2d\log\left(cx^n\right)^2}{x} - b^2e\left(\begin{cases}-\log(c)^2\log(x) & \text{for } n=0\\ -\frac{\log(cx^n)^3}{3n} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((e*x+d)*(a+b*ln(c*x**n))**2/x**2,x)
```

```
[Out] -a**2*d/x + a**2*e*log(x) - 2*a*b*d*n/x - 2*a*b*d*log(c*x**n)/x - 2*a*b*e*P
iecewise((-log(c)*log(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), True)) - 2*b**
2*d*n**2/x - 2*b**2*d*n*log(c*x**n)/x - b**2*d*log(c*x**n)**2/x - b**2*e*Pi
ecewise((-log(c)**2*log(x), Eq(n, 0)), (-log(c*x**n)**3/(3*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)^2}{x^2}\,dx = \frac{b^2e\log\left(cx^n\right)^3}{3\,n}-2\,b^2d\left(\frac{n^2}{x}+\frac{n\log\left(cx^n\right)}{x}\right) + \frac{abe\log\left(cx^n\right)^2}{n}-\frac{b^2d\log\left(cx^n\right)^2}{x}+a^2e\log(x) - \frac{2\,abdn}{x}-\frac{2\,abd\log\left(cx^n\right)}{x}-\frac{a^2d}{x}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*e*log(c*x^n)^3/n - 2*b^2*d*(n^2/x + n*log(c*x^n)/x) + a*b*e*log(c*x
^n)^2/n - b^2*d*log(c*x^n)^2/x + a^2*e*log(x) - 2*a*b*d*n/x - 2*a*b*d*log(c
*x^n)/x - a^2*d/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(70) = 140.

Time = 0.33 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.25

$$\int \frac{(d+ex)\,(a+b\log(cx^n))^2}{x^2}\,dx = \frac{1}{3}\,b^2en^2\log(x)^3 + b^2en\log(c)\log(x)^2 - b^2dn^2\left(\frac{\log(x)^2}{x} + \frac{2\,\log(x)}{x} + \frac{2}{x}\right) - 2\,b^2dn\left(\frac{\log(x)}{x} + \frac{1}{x}\right)\log(c) + aben\log(x)^2 + b^2e\log(c)^2\log(|x|) - 2\,abdn\left(\frac{\log(x)}{x} + \frac{1}{x}\right) + 2\,abe\log(c)\log(|x|) - \frac{b^2d\log(c)^2}{x} + a^2e\log(|x|) - \frac{2\,abd\log(c)}{x} - \frac{a^2d}{x}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^2/x^2,x, algorithm="giac")
[Out] 1/3*b^2*e*n^2*log(x)^3 + b^2*e*n*log(c)*log(x)^2 - b^2*d*n^2*(log(x)^2/x +
2*log(x)/x + 2/x) - 2*b^2*d*n*(log(x)/x + 1/x)*log(c) + a*b*e*n*log(x)^2 +
b^2*e*log(c)^2*log(abs(x)) - 2*a*b*d*n*(log(x)/x + 1/x) + 2*a*b*e*log(c)*lo
g(abs(x)) - b^2*d*log(c)^2/x + a^2*e*log(abs(x)) - 2*a*b*d*log(c)/x - a^2*d
/x
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.92

$$\int \frac{(d+ex)\,(a+b\log(cx^n))^2}{x^2}\,dx = \ln(x)\,\left(e\,a^2 + 2\,e\,a\,b\,n + 2\,e\,b^2\,n^2\right) - \frac{d\,a^2 + 2\,d\,a\,b\,n + 2\,d\,b^2\,n^2}{x} - \ln(c\,x^n)^2\left(\frac{b^2\,d + b^2\,e\,x}{x} - \frac{b\,e\,(a+b\,n)}{n}\right) - \frac{\ln(c\,x^n)\,\left(2\,b\,d\,(a+b\,n) + 2\,b\,e\,x\,(a+b\,n)\right)}{x} + \frac{b^2\,e\ln(c\,x^n)^3}{3\,n}$$

```
[In] int(((a + b*log(c*x^n))^2*(d + e*x))/x^2,x)
[Out] log(x)*(a^2*e + 2*b^2*e*n^2 + 2*a*b*e*n) - (a^2*d + 2*b^2*d*n^2 + 2*a*b*d*n
)/x - log(c*x^n)^2*((b^2*d + b^2*e*x)/x - (b*e*(a + b*n))/n) - (log(c*x^n)*
(2*b*d*(a + b*n) + 2*b*e*x*(a + b*n)))/x + (b^2*e*log(c*x^n)^3)/(3*n)
```

3.81 $\int \frac{(d+ex)(a+b\log(cx^n))^2}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{(d+ex)\,(a+b\log(cx^n))^2}{x^3}\,dx = -\frac{b^2dn^2}{4x^2}-\frac{2b^2en^2}{x}-\frac{bdn(a+b\log(cx^n))}{2x^2}-\frac{2ben(a+b\log(cx^n))}{x}-\frac{d(a+b\log(cx^n))^2}{2x^2}-\frac{e(a+b\log(cx^n))^2}{x}$$

```
[Out] -1/4*b^2*d*n^2/x^2-2*b^2*e*n^2/x-1/2*b*d*n*(a+b*ln(c*x^n))/x^2-2*b*e*n*(a+b
*ln(c*x^n))/x-1/2*d*(a+b*ln(c*x^n))^2/x^2-e*(a+b*ln(c*x^n))^2/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2395, 2342, 2341}

$$\int \frac{(d+ex)\,(a+b\log(cx^n))^2}{x^3}\,dx = -\frac{bdn(a+b\log(cx^n))}{2x^2}-\frac{d(a+b\log(cx^n))^2}{2x^2}-\frac{2ben(a+b\log(cx^n))}{x}-\frac{e(a+b\log(cx^n))^2}{x}-\frac{b^2dn^2}{4x^2}-\frac{2b^2en^2}{x}$$

```
[In] Int[((d + e*x)*(a + b*Log[c*x^n])^2)/x^3,x]
```

```
[Out] -1/4*(b^2*d*n^2)/x^2 - (2*b^2*e*n^2)/x - (b*d*n*(a + b*Log[c*x^n]))/(2*x^2)
 - (2*b*e*n*(a + b*Log[c*x^n]))/x - (d*(a + b*Log[c*x^n])^2)/(2*x^2) - (e*(
a + b*Log[c*x^n])^2)/x
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d(a+b\log(cx^n))^2}{x^3}+\frac{e(a+b\log(cx^n))^2}{x^2}\right)dx\\
&= d\int \frac{(a+b\log(cx^n))^2}{x^3}\,dx+e\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx\\
&= -\frac{d(a+b\log(cx^n))^2}{2x^2}-\frac{e(a+b\log(cx^n))^2}{x}\\
&\quad+(bdn)\int \frac{a+b\log(cx^n)}{x^3}\,dx+(2ben)\int \frac{a+b\log(cx^n)}{x^2}\,dx\\
&= -\frac{b^2dn^2}{4x^2}-\frac{2b^2en^2}{x}-\frac{bdn(a+b\log(cx^n))}{2x^2}-\frac{2ben(a+b\log(cx^n))}{x}\\
&\quad-\frac{d(a+b\log(cx^n))^2}{2x^2}-\frac{e(a+b\log(cx^n))^2}{x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)\,(a+b\log(cx^n))^2}{x^3}\,dx = -\frac{2a^2(d+2ex)+2abn(d+4ex)+b^2n^2(d+8ex)+2b(2a(d+2ex)+bn(d+4ex))\log(cx^n)+2b^2(d+2}{4x^2}$$

```
[In] Integrate[((d + e*x)*(a + b*Log[c*x^n])^2)/x^3,x]
[Out] -1/4*(2*a^2*(d + 2*e*x) + 2*a*b*n*(d + 4*e*x) + b^2*n^2*(d + 8*e*x) + 2*b*(
2*a*(d + 2*e*x) + b*n*(d + 4*e*x))*Log[c*x^n] + 2*b^2*(d + 2*e*x)*Log[c*x^n
]^2)/x^2
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.26

method	result
parallelrisch	$-\frac{4b^2\ln(c\,x^n)^2ex+8b^2enx\ln(c\,x^n)+8b^2e\,n^2x+8ab\ln(c\,x^n)ex+8abenx+2b^2\ln(c\,x^n)^2d+2\ln(c\,x^n)b^2nd+b^2d\,n^2+4a^2ex+4ab\,l}{4x^2}$
risch	Expression too large to display

```
[In] int((e*x+d)*(a+b*ln(c*x^n))^2/x^3,x,method=_RETURNVERBOSE)
[Out] -1/4/x^2*(4*b^2*ln(c*x^n)^2*e*x+8*b^2*e*n*x*ln(c*x^n)+8*b^2*e*n^2*x+8*a*b*l
n(c*x^n)*e*x+8*a*b*e*n*x+2*b^2*ln(c*x^n)^2*d+2*ln(c*x^n)*b^2*n*d+b^2*d*n^2+
4*a^2*e*x+4*a*b*ln(c*x^n)*d+2*a*b*d*n+2*a^2*d)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex)\,(a+b\log(cx^n))^2}{x^3}\,dx = -\frac{b^2dn^2+2\,abdn+2\,a^2d+2\,(2\,b^2ex+b^2d)\log(c)^2+2\,(2\,b^2en^2x+b^2dn^2)\log(x)^2+4\,(2\,b^2en^2+2\,aben}{}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^2/x^3,x, algorithm="fricas")
[Out] -1/4*(b^2*d*n^2 + 2*a*b*d*n + 2*a^2*d + 2*(2*b^2*e*x + b^2*d)*log(c)^2 + 2*
(2*b^2*e*n^2*x + b^2*d*n^2)*log(x)^2 + 4*(2*b^2*e*n^2 + 2*a*b*e*n + a^2*e)*
x + 2*(b^2*d*n + 2*a*b*d + 4*(b^2*e*n + a*b*e)*x)*log(c) + 2*(b^2*d*n^2 + 2
*a*b*d*n + 4*(b^2*e*n^2 + a*b*e*n)*x + 2*(2*b^2*e*n*x + b^2*d*n)*log(c))*lo
g(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)\,(a+b\log{(cx^n)})^2}{x^3}\,dx = -\frac{a^2d}{2x^2} - \frac{a^2e}{x} - \frac{abdn}{2x^2} - \frac{abd\log{(cx^n)}}{x^2} - \frac{2aben}{x} - \frac{2abe\log{(cx^n)}}{x} - \frac{b^2dn^2}{4x^2} - \frac{b^2dn\log{(cx^n)}}{2x^2} - \frac{b^2d\log{(cx^n)}^2}{2x^2} - \frac{2b^2en^2}{x} - \frac{2b^2en\log{(cx^n)}}{x} - \frac{b^2e\log{(cx^n)}^2}{x}$$

```
[In] integrate((e*x+d)*(a+b*ln(c*x**n))**2/x**3,x)
```

```
[Out] -a**2*d/(2*x**2) - a**2*e/x - a*b*d*n/(2*x**2) - a*b*d*log(c*x**n)/x**2 - 2
*a*b*e*n/x - 2*a*b*e*log(c*x**n)/x - b**2*d*n**2/(4*x**2) - b**2*d*n*log(c*
x**n)/(2*x**2) - b**2*d*log(c*x**n)**2/(2*x**2) - 2*b**2*e*n**2/x - 2*b**2*
e*n*log(c*x**n)/x - b**2*e*log(c*x**n)**2/x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.46

$$\int \frac{(d+ex)\,(a+b\log{(cx^n)})^2}{x^3}\,dx = -2\,b^2e\left(\frac{n^2}{x} + \frac{n\log{(cx^n)}}{x}\right) - \frac{1}{4}\,b^2d\left(\frac{n^2}{x^2} + \frac{2\,n\log{(cx^n)}}{x^2}\right) - \frac{b^2e\log{(cx^n)}^2}{x} - \frac{2\,aben}{x} - \frac{2\,abe\log{(cx^n)}}{x} - \frac{b^2d\log{(cx^n)}^2}{2\,x^2} - \frac{abdn}{2\,x^2} - \frac{a^2e}{x} - \frac{abd\log{(cx^n)}}{x^2} - \frac{a^2d}{2\,x^2}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -2*b^2*e*(n^2/x + n*log(c*x^n)/x) - 1/4*b^2*d*(n^2/x^2 + 2*n*log(c*x^n)/x^2
) - b^2*e*log(c*x^n)^2/x - 2*a*b*e*n/x - 2*a*b*e*log(c*x^n)/x - 1/2*b^2*d*l
og(c*x^n)^2/x^2 - 1/2*a*b*d*n/x^2 - a^2*e/x - a*b*d*log(c*x^n)/x^2 - 1/2*a^
2*d/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.86

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)^2}{x^3}\,dx = -\frac{\left(2\,b^2en^2x+b^2dn^2\right)\log\left(x\right)^2}{2\,x^2} - \frac{\left(4\,b^2en^2x+4\,b^2enx\log\left(c\right)+b^2dn^2+4\,abenx+2\,b^2dn\log\left(c\right)+2\,abdn\right)\log\left(x\right)}{2\,x^2} - \frac{8\,b^2en^2x+8\,b^2enx\log\left(c\right)+4\,b^2ex\log\left(c\right)^2+b^2dn^2+8\,abenx+2\,b^2dn\log\left(c\right)+8\,abex\log\left(c\right)+2\,b^2d\log}{4\,x^2}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*e*n^2*x + b^2*d*n^2)*log(x)^2/x^2 - 1/2*(4*b^2*e*n^2*x + 4*b^2*
e*n*x*log(c) + b^2*d*n^2 + 4*a*b*e*n*x + 2*b^2*d*n*log(c) + 2*a*b*d*n)*log(
x)/x^2 - 1/4*(8*b^2*e*n^2*x + 8*b^2*e*n*x*log(c) + 4*b^2*e*x*log(c)^2 + b^2
*d*n^2 + 8*a*b*e*n*x + 2*b^2*d*n*log(c) + 8*a*b*e*x*log(c) + 2*b^2*d*log(c)
^2 + 2*a*b*d*n + 4*a^2*e*x + 4*a*b*d*log(c) + 2*a^2*d)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)^2}{x^3}\,dx = -\frac{x\,\left(2\,e\,a^2+4\,e\,a\,b\,n+4\,e\,b^2\,n^2\right)+a^2\,d+\frac{b^2\,d\,n^2}{2}+a\,b\,d\,n}{2\,x^2} - \frac{\ln\left(c\,x^n\right)\,\left(\frac{b\,d\,\left(2\,a+b\,n\right)}{2}+2\,b\,e\,x\,\left(a+b\,n\right)\right)}{x^2} - \frac{{\ln\left(c\,x^n\right)}^2\,\left(\frac{b^2\,d}{2}+b^2\,e\,x\right)}{x^2}$$

```
[In] int(((a + b*log(c*x^n))^2*(d + e*x))/x^3,x)
```

```
[Out] - (x*(2*a^2*e + 4*b^2*e*n^2 + 4*a*b*e*n) + a^2*d + (b^2*d*n^2)/2 + a*b*d*n)
/(2*x^2) - (log(c*x^n)*((b*d*(2*a + b*n))/2 + 2*b*e*x*(a + b*n)))/x^2 - (lo
g(c*x^n)^2*((b^2*d)/2 + b^2*e*x))/x^2
```

3.82 $\int \frac{(d+ex)(a+b\log(cx^n))^2}{x^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{(d+ex)(a+b\log(cx^n))^2}{x^4} dx = -\frac{2b^2dn^2}{27x^3} - \frac{b^2en^2}{4x^2} - \frac{2bdn(a+b\log(cx^n))}{9x^3} - \frac{ben(a+b\log(cx^n))}{2x^2} - \frac{d(a+b\log(cx^n))^2}{3x^3} - \frac{e(a+b\log(cx^n))^2}{2x^2}$$

```
[Out] -2/27*b^2*d*n^2/x^3-1/4*b^2*e*n^2/x^2-2/9*b*d*n*(a+b*ln(c*x^n))/x^3-1/2*b*e
*n*(a+b*ln(c*x^n))/x^2-1/3*d*(a+b*ln(c*x^n))^2/x^3-1/2*e*(a+b*ln(c*x^n))^2/
x^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2395, 2342, 2341}

$$\int \frac{(d+ex)(a+b\log(cx^n))^2}{x^4} dx = -\frac{2bdn(a+b\log(cx^n))}{9x^3} - \frac{d(a+b\log(cx^n))^2}{3x^3} - \frac{ben(a+b\log(cx^n))}{2x^2} - \frac{e(a+b\log(cx^n))^2}{2x^2} - \frac{2b^2dn^2}{27x^3} - \frac{b^2en^2}{4x^2}$$

```
[In] Int[((d + e*x)*(a + b*Log[c*x^n])^2)/x^4,x]
```

```
[Out] (-2*b^2*d*n^2)/(27*x^3) - (b^2*e*n^2)/(4*x^2) - (2*b*d*n*(a + b*Log[c*x^n]))/(9*x^3) - (b*e*n*(a + b*Log[c*x^n]))/(2*x^2) - (d*(a + b*Log[c*x^n])^2)/(3*x^3) - (e*(a + b*Log[c*x^n])^2)/(2*x^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d(a+b\log(cx^n))^2}{x^4}+\frac{e(a+b\log(cx^n))^2}{x^3}\right)dx\\
&= d\int \frac{(a+b\log(cx^n))^2}{x^4}\,dx+e\int \frac{(a+b\log(cx^n))^2}{x^3}\,dx\\
&= -\frac{d(a+b\log(cx^n))^2}{3x^3}-\frac{e(a+b\log(cx^n))^2}{2x^2}\\
&\quad +\frac{1}{3}(2bdn)\int \frac{a+b\log(cx^n)}{x^4}\,dx+(ben)\int \frac{a+b\log(cx^n)}{x^3}\,dx\\
&= -\frac{2b^2dn^2}{27x^3}-\frac{b^2en^2}{4x^2}-\frac{2bdn(a+b\log(cx^n))}{9x^3}-\frac{ben(a+b\log(cx^n))}{2x^2}\\
&\quad -\frac{d(a+b\log(cx^n))^2}{3x^3}-\frac{e(a+b\log(cx^n))^2}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)(a+b\log(cx^n))^2}{x^4}\,dx = -\frac{36d(a+b\log(cx^n))^2+54ex(a+b\log(cx^n))^2+27benx(2a+bn+2b\log(cx^n))+8bdn(3a+bn+3b\log}{108x^3}$$

```
[In] Integrate[((d + e*x)*(a + b*Log[c*x^n])^2)/x^4,x]
```

```
[Out] -1/108*(36*d*(a + b*Log[c*x^n])^2 + 54*e*x*(a + b*Log[c*x^n])^2 + 27*b*e*n*
x*(2*a + b*n + 2*b*Log[c*x^n]) + 8*b*d*n*(3*a + b*n + 3*b*Log[c*x^n]))/x^3
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.20

method	result
parallelrisch	$-\frac{54b^2\ln(cx^n)^2ex+54b^2enx\ln(cx^n)+27b^2en^2x+108ab\ln(cx^n)ex+54abenx+36b^2\ln(cx^n)^2d+24\ln(cx^n)b^2nd+8b^2dn^2+54a}{108x^3}$
risch	Expression too large to display

```
[In] int((e*x+d)*(a+b*ln(c*x^n))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/108/x^3*(54*b^2*ln(c*x^n)^2*e*x+54*b^2*e*n*x*ln(c*x^n)+27*b^2*e*n^2*x+10
8*a*b*ln(c*x^n)*e*x+54*a*b*e*n*x+36*b^2*ln(c*x^n)^2*d+24*ln(c*x^n)*b^2*n*d+
8*b^2*d*n^2+54*a^2*e*x+72*a*b*ln(c*x^n)*d+24*a*b*d*n+36*a^2*d)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex)(a+b\log(cx^n))^2}{x^4}\,dx = -\frac{8\,b^2dn^2+24\,abdn+36\,a^2d+18\,(3\,b^2ex+2\,b^2d)\log(c)^2+18\,(3\,b^2en^2x+2\,b^2dn^2)\log(x)^2+27\,(b^2en^2+}{}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/108*(8*b^2*d*n^2 + 24*a*b*d*n + 36*a^2*d + 18*(3*b^2*e*x + 2*b^2*d)*log(
c)^2 + 18*(3*b^2*e*n^2*x + 2*b^2*d*n^2)*log(x)^2 + 27*(b^2*e*n^2 + 2*a*b*e*
n + 2*a^2*e)*x + 6*(4*b^2*d*n + 12*a*b*d + 9*(b^2*e*n + 2*a*b*e)*x)*log(c)
+ 6*(4*b^2*d*n^2 + 12*a*b*d*n + 9*(b^2*e*n^2 + 2*a*b*e*n)*x + 6*(3*b^2*e*n*
x + 2*b^2*d*n)*log(c))*log(x))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.70

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)^2}{x^4}\,dx = -\frac{a^2d}{3x^3} - \frac{a^2e}{2x^2} - \frac{2abdn}{9x^3} - \frac{2abd\log\left(cx^n\right)}{3x^3} - \frac{aben}{2x^2} - \frac{abe\log\left(cx^n\right)}{x^2} - \frac{2b^2dn^2}{27x^3} - \frac{2b^2dn\log\left(cx^n\right)}{9x^3} - \frac{b^2d\log\left(cx^n\right)^2}{3x^3} - \frac{b^2en^2}{4x^2} - \frac{b^2en\log\left(cx^n\right)}{2x^2} - \frac{b^2e\log\left(cx^n\right)^2}{2x^2}$$

```
[In] integrate((e*x+d)*(a+b*ln(c*x**n))**2/x**4,x)
```

```
[Out] -a**2*d/(3*x**3) - a**2*e/(2*x**2) - 2*a*b*d*n/(9*x**3) - 2*a*b*d*log(c*x**
n)/(3*x**3) - a*b*e*n/(2*x**2) - a*b*e*log(c*x**n)/x**2 - 2*b**2*d*n**2/(27
*x**3) - 2*b**2*d*n*log(c*x**n)/(9*x**3) - b**2*d*log(c*x**n)**2/(3*x**3) -
 b**2*e*n**2/(4*x**2) - b**2*e*n*log(c*x**n)/(2*x**2) - b**2*e*log(c*x**n)*
*2/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)^2}{x^4}\,dx = -\frac{1}{4}\,b^2e\left(\frac{n^2}{x^2} + \frac{2\,n\log\left(cx^n\right)}{x^2}\right) - \frac{2}{27}\,b^2d\left(\frac{n^2}{x^3} + \frac{3\,n\log\left(cx^n\right)}{x^3}\right) - \frac{b^2e\log\left(cx^n\right)^2}{2\,x^2} - \frac{aben}{2\,x^2} - \frac{abe\log\left(cx^n\right)}{x^2} - \frac{b^2d\log\left(cx^n\right)^2}{3\,x^3} - \frac{2\,abdn}{9\,x^3} - \frac{a^2e}{2\,x^2} - \frac{2\,abd\log\left(cx^n\right)}{3\,x^3} - \frac{a^2d}{3\,x^3}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*e*(n^2/x^2 + 2*n*log(c*x^n)/x^2) - 2/27*b^2*d*(n^2/x^3 + 3*n*log(c
*x^n)/x^3) - 1/2*b^2*e*log(c*x^n)^2/x^2 - 1/2*a*b*e*n/x^2 - a*b*e*log(c*x^n
)/x^2 - 1/3*b^2*d*log(c*x^n)^2/x^3 - 2/9*a*b*d*n/x^3 - 1/2*a^2*e/x^2 - 2/3*
a*b*d*log(c*x^n)/x^3 - 1/3*a^2*d/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(97) = 194$.

Time = 0.33 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.79

$$\int \frac{(d+ex)\,(a+b\log(cx^n))^2}{x^4}\,dx = -\frac{(3\,b^2en^2x+2\,b^2dn^2)\log(x)^2}{6\,x^3} - \frac{(9\,b^2en^2x+18\,b^2enx\log(c)+4\,b^2dn^2+18\,abenx+12\,b^2dn\log(c)+12\,abdn)\log(x)}{18\,x^3} - \frac{27\,b^2en^2x+54\,b^2enx\log(c)+54\,b^2ex\log(c)^2+8\,b^2dn^2+54\,abenx+24\,b^2dn\log(c)+108\,abex\log(c)+}{108\,x^3}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^2/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^2*e*n^2*x + 2*b^2*d*n^2)*log(x)^2/x^3 - 1/18*(9*b^2*e*n^2*x + 18*
b^2*e*n*x*log(c) + 4*b^2*d*n^2 + 18*a*b*e*n*x + 12*b^2*d*n*log(c) + 12*a*b*
d*n)*log(x)/x^3 - 1/108*(27*b^2*e*n^2*x + 54*b^2*e*n*x*log(c) + 54*b^2*e*x*
log(c)^2 + 8*b^2*d*n^2 + 54*a*b*e*n*x + 24*b^2*d*n*log(c) + 108*a*b*e*x*log
(c) + 36*b^2*d*log(c)^2 + 24*a*b*d*n + 54*a^2*e*x + 72*a*b*d*log(c) + 36*a^
2*d)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)\,(a+b\log(cx^n))^2}{x^4}\,dx = -\frac{x\left(9\,e\,a^2+9\,e\,a\,b\,n+\frac{9\,e\,b^2\,n^2}{2}\right)+6\,a^2\,d+\frac{4\,b^2\,d\,n^2}{3}+4\,a\,b\,d\,n}{18\,x^3} - \frac{\ln(c\,x^n)\,\left(\frac{2\,b\,d\,(3\,a+b\,n)}{3}+\frac{3\,b\,e\,x\,(2\,a+b\,n)}{2}\right)}{3\,x^3} - \frac{\ln(c\,x^n)^2\,\left(\frac{b^2\,d}{3}+\frac{b^2\,e\,x}{2}\right)}{x^3}$$

```
[In] int(((a + b*log(c*x^n))^2*(d + e*x))/x^4,x)
```

```
[Out] - (x*(9*a^2*e + (9*b^2*e*n^2)/2 + 9*a*b*e*n) + 6*a^2*d + (4*b^2*d*n^2)/3 +
4*a*b*d*n)/(18*x^3) - (log(c*x^n)*((2*b*d*(3*a + b*n))/3 + (3*b*e*x*(2*a +
b*n))/2))/(3*x^3) - (log(c*x^n)^2*((b^2*d)/3 + (b^2*e*x)/2))/x^3
```

3.83 $\int \frac{(d+ex)(a+b\log(cx^n))^2}{x^5} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{(d+ex)(a+b\log(cx^n))^2}{x^5} \, dx = -\frac{b^2dn^2}{32x^4} - \frac{2b^2en^2}{27x^3} - \frac{bdn(a+b\log(cx^n))}{8x^4} - \frac{2ben(a+b\log(cx^n))}{9x^3} - \frac{d(a+b\log(cx^n))^2}{4x^4} - \frac{e(a+b\log(cx^n))^2}{3x^3}$$

```
[Out] -1/32*b^2*d*n^2/x^4-2/27*b^2*e*n^2/x^3-1/8*b*d*n*(a+b*ln(c*x^n))/x^4-2/9*b*
e*n*(a+b*ln(c*x^n))/x^3-1/4*d*(a+b*ln(c*x^n))^2/x^4-1/3*e*(a+b*ln(c*x^n))^2
/x^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2395, 2342, 2341}

$$\int \frac{(d+ex)(a+b\log(cx^n))^2}{x^5} \, dx = -\frac{bdn(a+b\log(cx^n))}{8x^4} - \frac{d(a+b\log(cx^n))^2}{4x^4} - \frac{2ben(a+b\log(cx^n))}{9x^3} - \frac{e(a+b\log(cx^n))^2}{3x^3} - \frac{b^2dn^2}{32x^4} - \frac{2b^2en^2}{27x^3}$$

```
[In] Int[((d + e*x)*(a + b*Log[c*x^n])^2)/x^5,x]
```

```
[Out] -1/32*(b^2*d*n^2)/x^4 - (2*b^2*e*n^2)/(27*x^3) - (b*d*n*(a + b*Log[c*x^n]))
/(8*x^4) - (2*b*e*n*(a + b*Log[c*x^n]))/(9*x^3) - (d*(a + b*Log[c*x^n])^2)/
(4*x^4) - (e*(a + b*Log[c*x^n])^2)/(3*x^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d(a+b\log(cx^n))^2}{x^5}+\frac{e(a+b\log(cx^n))^2}{x^4}\right)dx \\
&= d\int \frac{(a+b\log(cx^n))^2}{x^5}\,dx+e\int \frac{(a+b\log(cx^n))^2}{x^4}\,dx \\
&= -\frac{d(a+b\log(cx^n))^2}{4x^4}-\frac{e(a+b\log(cx^n))^2}{3x^3} \\
&\quad +\frac{1}{2}(bdn)\int \frac{a+b\log(cx^n)}{x^5}\,dx+\frac{1}{3}(2ben)\int \frac{a+b\log(cx^n)}{x^4}\,dx \\
&= -\frac{b^2dn^2}{32x^4}-\frac{2b^2en^2}{27x^3}-\frac{bdn(a+b\log(cx^n))}{8x^4}-\frac{2ben(a+b\log(cx^n))}{9x^3} \\
&\quad -\frac{d(a+b\log(cx^n))^2}{4x^4}-\frac{e(a+b\log(cx^n))^2}{3x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)^2}{x^5}\,dx = -\frac{216d(a+b\log\left(cx^n\right))^2+288ex(a+b\log\left(cx^n\right))^2+64benx(3a+bn+3b\log\left(cx^n\right))+27bdn(4a+bn+ }{864x^4}$$

```
[In] Integrate[((d + e*x)*(a + b*Log[c*x^n])^2)/x^5,x]
```

```
[Out] -1/864*(216*d*(a + b*Log[c*x^n])^2 + 288*e*x*(a + b*Log[c*x^n])^2 + 64*b*e*
n*x*(3*a + b*n + 3*b*Log[c*x^n]) + 27*b*d*n*(4*a + b*n + 4*b*Log[c*x^n]))/x
^4
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.20

method	result
parallelrisch	$-\frac{288b^2\ln(c\,x^n)^2ex+192b^2enx\ln(c\,x^n)+64b^2e\,n^2x+576ab\ln(c\,x^n)ex+192abenx+216b^2\ln(c\,x^n)^2d+108\ln(c\,x^n)b^2nd+27b^2d}{864x^4}$
risch	Expression too large to display

```
[In] int((e*x+d)*(a+b*ln(c*x^n))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/864/x^4*(288*b^2*ln(c*x^n)^2*e*x+192*b^2*e*n*x*ln(c*x^n)+64*b^2*e*n^2*x+
576*a*b*ln(c*x^n)*e*x+192*a*b*e*n*x+216*b^2*ln(c*x^n)^2*d+108*ln(c*x^n)*b^2
*n*d+27*b^2*d*n^2+288*a^2*e*x+432*a*b*ln(c*x^n)*d+108*a*b*d*n+216*a^2*d)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex)\left(a+b\log\left(cx^n\right)\right)^2}{x^5}\,dx = -\frac{27\,b^2dn^2+108\,abdn+216\,a^2d+72\left(4\,b^2ex+3\,b^2d\right)\log\left(c\right)^2+72\left(4\,b^2en^2x+3\,b^2dn^2\right)\log\left(x\right)^2+32\left(2\,b^2\right.}{}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/864*(27*b^2*d*n^2 + 108*a*b*d*n + 216*a^2*d + 72*(4*b^2*e*x + 3*b^2*d)*l
og(c)^2 + 72*(4*b^2*e*n^2*x + 3*b^2*d*n^2)*log(x)^2 + 32*(2*b^2*e*n^2 + 6*a
*b*e*n + 9*a^2*e)*x + 12*(9*b^2*d*n + 36*a*b*d + 16*(b^2*e*n + 3*a*b*e)*x)*
```

```
log(c) + 12*(9*b^2*d*n^2 + 36*a*b*d*n + 16*(b^2*e*n^2 + 3*a*b*e*n)*x + 12*(
4*b^2*e*n*x + 3*b^2*d*n)*log(c))*log(x))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex)\,(a+b\log{(cx^n)})^2}{x^5}\,dx = -\frac{a^2d}{4x^4} - \frac{a^2e}{3x^3} - \frac{abdn}{8x^4} - \frac{abd\log{(cx^n)}}{2x^4} - \frac{2aben}{9x^3} - \frac{2abe\log{(cx^n)}}{3x^3} - \frac{b^2dn^2}{32x^4} - \frac{b^2dn\log{(cx^n)}}{8x^4} - \frac{b^2d\log{(cx^n)}^2}{4x^4} - \frac{2b^2en^2}{27x^3} - \frac{2b^2en\log{(cx^n)}}{9x^3} - \frac{b^2e\log{(cx^n)}^2}{3x^3}$$

```
[In] integrate((e*x+d)*(a+b*ln(c*x**n))**2/x**5,x)
```

```
[Out] -a**2*d/(4*x**4) - a**2*e/(3*x**3) - a*b*d*n/(8*x**4) - a*b*d*log(c*x**n)/(
2*x**4) - 2*a*b*e*n/(9*x**3) - 2*a*b*e*log(c*x**n)/(3*x**3) - b**2*d*n**2/(
32*x**4) - b**2*d*n*log(c*x**n)/(8*x**4) - b**2*d*log(c*x**n)**2/(4*x**4) -
 2*b**2*e*n**2/(27*x**3) - 2*b**2*e*n*log(c*x**n)/(9*x**3) - b**2*e*log(c*x
**n)**2/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)\,(a+b\log{(cx^n)})^2}{x^5}\,dx = -\frac{2}{27}\,b^2e\left(\frac{n^2}{x^3} + \frac{3\,n\log{(cx^n)}}{x^3}\right) - \frac{1}{32}\,b^2d\left(\frac{n^2}{x^4} + \frac{4\,n\log{(cx^n)}}{x^4}\right) - \frac{b^2e\log{(cx^n)}^2}{3\,x^3} - \frac{2\,aben}{9\,x^3} - \frac{2\,abe\log{(cx^n)}}{3\,x^3} - \frac{b^2d\log{(cx^n)}^2}{4\,x^4} - \frac{abdn}{8\,x^4} - \frac{a^2e}{3\,x^3} - \frac{abd\log{(cx^n)}}{2\,x^4} - \frac{a^2d}{4\,x^4}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^2/x^5,x, algorithm="maxima")
```

```
[Out] -2/27*b^2*e*(n^2/x^3 + 3*n*log(c*x^n)/x^3) - 1/32*b^2*d*(n^2/x^4 + 4*n*log(
c*x^n)/x^4) - 1/3*b^2*e*log(c*x^n)^2/x^3 - 2/9*a*b*e*n/x^3 - 2/3*a*b*e*log(
c*x^n)/x^3 - 1/4*b^2*d*log(c*x^n)^2/x^4 - 1/8*a*b*d*n/x^4 - 1/3*a^2*e/x^3 -
 1/2*a*b*d*log(c*x^n)/x^4 - 1/4*a^2*d/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(97) = 194$.

Time = 0.44 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.79

$$\int \frac{(d+ex)(a+b\log(cx^n))^2}{x^5}\,dx = -\frac{(4\,b^2en^2x+3\,b^2dn^2)\log(x)^2}{12\,x^4} - \frac{(16\,b^2en^2x+48\,b^2enx\log(c)+9\,b^2dn^2+48\,abenx+36\,b^2dn\log(c)+36\,abdn)\log(x)}{72\,x^4} - \frac{64\,b^2en^2x+192\,b^2enx\log(c)+288\,b^2ex\log(c)^2+27\,b^2dn^2+192\,abenx+108\,b^2dn\log(c)+576\,abex\log}{864\,x^4}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^2/x^5,x, algorithm="giac")
```

```
[Out] -1/12*(4*b^2*e*n^2*x + 3*b^2*d*n^2)*log(x)^2/x^4 - 1/72*(16*b^2*e*n^2*x + 4
8*b^2*e*n*x*log(c) + 9*b^2*d*n^2 + 48*a*b*e*n*x + 36*b^2*d*n*log(c) + 36*a*
b*d*n)*log(x)/x^4 - 1/864*(64*b^2*e*n^2*x + 192*b^2*e*n*x*log(c) + 288*b^2*
e*x*log(c)^2 + 27*b^2*d*n^2 + 192*a*b*e*n*x + 108*b^2*d*n*log(c) + 576*a*b*
e*x*log(c) + 216*b^2*d*log(c)^2 + 108*a*b*d*n + 288*a^2*e*x + 432*a*b*d*log
(c) + 216*a^2*d)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)(a+b\log(cx^n))^2}{x^5}\,dx = -\frac{x\left(24\,e\,a^2+16\,e\,a\,b\,n+\frac{16\,e\,b^2\,n^2}{3}\right)+18\,a^2\,d+\frac{9\,b^2\,d\,n^2}{4}+9\,a\,b\,d\,n}{72\,x^4} - \frac{\ln(c\,x^n)\left(\frac{3\,b\,d\,(4\,a+b\,n)}{4}+\frac{4\,b\,e\,x\,(3\,a+b\,n)}{3}\right)}{6\,x^4} - \frac{\ln(c\,x^n)^2\left(\frac{b^2\,d}{4}+\frac{b^2\,e\,x}{3}\right)}{x^4}$$

```
[In] int(((a + b*log(c*x^n))^2*(d + e*x))/x^5,x)
```

```
[Out] - (x*(24*a^2*e + (16*b^2*e*n^2)/3 + 16*a*b*e*n) + 18*a^2*d + (9*b^2*d*n^2)/
4 + 9*a*b*d*n)/(72*x^4) - (log(c*x^n)*((3*b*d*(4*a + b*n))/4 + (4*b*e*x*(3*
a + b*n))/3))/(6*x^4) - (log(c*x^n)^2*((b^2*d)/4 + (b^2*e*x)/3))/x^4
```

3.84 $\int x^2(d+ex)^2 (a+b\log(cx^n))^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 178

$$\int x^2(d+ex)^2 (a+b\log(cx^n))^2 \, dx = \frac{2}{27}b^2d^2n^2x^3+\frac{1}{16}b^2den^2x^4+\frac{2}{125}b^2e^2n^2x^5 - \frac{2}{9}bd^2nx^3(a+b\log(cx^n))-\frac{1}{4}bdenx^4(a+b\log(cx^n)) - \frac{2}{25}be^2nx^5(a+b\log(cx^n))+\frac{1}{3}d^2x^3(a+b\log(cx^n))^2 + \frac{1}{2}dex^4(a+b\log(cx^n))^2+\frac{1}{5}e^2x^5(a+b\log(cx^n))^2$$

```
[Out] 2/27*b^2*d^2*n^2*x^3+1/16*b^2*d*e*n^2*x^4+2/125*b^2*e^2*n^2*x^5-2/9*b*d^2*n
*x^3*(a+b*ln(c*x^n))-1/4*b*d*e*n*x^4*(a+b*ln(c*x^n))-2/25*b*e^2*n*x^5*(a+b*
ln(c*x^n))+1/3*d^2*x^3*(a+b*ln(c*x^n))^2+1/2*d*e*x^4*(a+b*ln(c*x^n))^2+1/5*
e^2*x^5*(a+b*ln(c*x^n))^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2395, 2342, 2341}

$$\int x^2(d+ex)^2 (a+b\log(cx^n))^2 \, dx = \frac{1}{3}d^2x^3(a+b\log(cx^n))^2-\frac{2}{9}bd^2nx^3(a+b\log(cx^n)) + \frac{1}{2}dex^4(a+b\log(cx^n))^2-\frac{1}{4}bdenx^4(a+b\log(cx^n)) + \frac{1}{5}e^2x^5(a+b\log(cx^n))^2-\frac{2}{25}be^2nx^5(a+b\log(cx^n)) + \frac{2}{27}b^2d^2n^2x^3+\frac{1}{16}b^2den^2x^4+\frac{2}{125}b^2e^2n^2x^5$$

```
[In] Int[x^2*(d + e*x)^2*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (2*b^2*d^2*n^2*x^3)/27 + (b^2*d*e*n^2*x^4)/16 + (2*b^2*e^2*n^2*x^5)/125 - (
2*b*d^2*n*x^3*(a + b*Log[c*x^n]))/9 - (b*d*e*n*x^4*(a + b*Log[c*x^n]))/4 -
(2*b*e^2*n*x^5*(a + b*Log[c*x^n]))/25 + (d^2*x^3*(a + b*Log[c*x^n])^2)/3 +
(d*e*x^4*(a + b*Log[c*x^n])^2)/2 + (e^2*x^5*(a + b*Log[c*x^n])^2)/5
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(d^2x^2(a+b\log(cx^n))^2+2dex^3(a+b\log(cx^n))^2+e^2x^4(a+b\log(cx^n))^2\right)\,dx \\
&= d^2\int x^2(a+b\log(cx^n))^2\,dx+(2de)\int x^3(a+b\log(cx^n))^2\,dx+e^2\int x^4(a+b\log(cx^n))^2\,dx \\
&= \frac{1}{3}d^2x^3(a+b\log(cx^n))^2+\frac{1}{2}dex^4(a+b\log(cx^n))^2 \\
&\quad +\frac{1}{5}e^2x^5(a+b\log(cx^n))^2-\frac{1}{3}\left(2bd^2n\right)\int x^2(a+b\log(cx^n))\,dx \\
&\quad -(bden)\int x^3(a+b\log(cx^n))\,dx-\frac{1}{5}\left(2be^2n\right)\int x^4(a+b\log(cx^n))\,dx \\
&= \frac{2}{27}b^2d^2n^2x^3+\frac{1}{16}b^2den^2x^4+\frac{2}{125}b^2e^2n^2x^5-\frac{2}{9}bd^2nx^3(a+b\log(cx^n)) \\
&\quad -\frac{1}{4}bdenx^4(a+b\log(cx^n))-\frac{2}{25}be^2nx^5(a+b\log(cx^n)) \\
&\quad +\frac{1}{3}d^2x^3(a+b\log(cx^n))^2+\frac{1}{2}dex^4(a+b\log(cx^n))^2+\frac{1}{5}e^2x^5(a+b\log(cx^n))^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.84

$$\int x^2(d+ex)^2\left(a+b\log\left(cx^n\right)\right)^2\,dx=\frac{2}{125}be^2nx^5(-5a+bn-5b\log\left(cx^n\right))+\frac{1}{16}bdenx^4(-4a+bn-4b\log\left(cx^n\right))+\frac{2}{27}bd^2nx^3(-3a+bn-3b\log\left(cx^n\right))+\frac{1}{3}d^2x^3(a+b\log\left(cx^n\right))^2+\frac{1}{2}dex^4(a+b\log\left(cx^n\right))^2+\frac{1}{5}e^2x^5(a+b\log\left(cx^n\right))^2$$

```
[In] Integrate[x^2*(d + e*x)^2*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (2*b*e^2*n*x^5*(-5*a + b*n - 5*b*Log[c*x^n]))/125 + (b*d*e*n*x^4*(-4*a + b*
n - 4*b*Log[c*x^n]))/16 + (2*b*d^2*n*x^3*(-3*a + b*n - 3*b*Log[c*x^n]))/27
+ (d^2*x^3*(a + b*Log[c*x^n])^2)/3 + (d*e*x^4*(a + b*Log[c*x^n])^2)/2 + (e^
2*x^5*(a + b*Log[c*x^n])^2)/5
```

Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.47

method	result
parallelrisch	$\frac{x^5\ln(cx^n)^2b^2e^2}{5}-\frac{2\ln(cx^n)x^5ne^2b^2}{25}+\frac{2b^2e^2n^2x^5}{125}+\frac{2\ln(cx^n)x^5ae^2b}{5}-\frac{2abe^2nx^5}{25}+\frac{x^4\ln(cx^n)^2b^2de}{2}-\frac{\ln(cx^n)x^4}{4}$
risch	Expression too large to display

```
[In] int(x^2*(e*x+d)^2*(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*ln(c*x^n)^2*b^2*e^2-2/25*ln(c*x^n)*x^5*n*e^2*b^2+2/125*b^2*e^2*n^2*
x^5+2/5*ln(c*x^n)*x^5*a*e^2*b-2/25*a*b*e^2*n*x^5+1/2*x^4*ln(c*x^n)^2*b^2*d*
e-1/4*ln(c*x^n)*x^4*n*d*e*b^2+1/16*b^2*d*e*n^2*x^4+1/5*a^2*e^2*x^5+ln(c*x^n
)*x^4*a*d*e*b-1/4*a*b*d*e*n*x^4+1/3*x^3*ln(c*x^n)^2*b^2*d^2-2/9*ln(c*x^n)*x
^3*n*b^2*d^2+2/27*b^2*d^2*n^2*x^3+1/2*a^2*d*e*x^4+2/3*ln(c*x^n)*x^3*a*b*d^2
-2/9*a*b*d^2*n*x^3+1/3*a^2*d^2*x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(160) = 320$.

Time = 0.27 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.04

$$\int x^2(d+ex)^2\left(a+b\log\left(cx^n\right)\right)^2\,dx$$
$$=\frac{1}{125}\left(2\,b^2e^2n^2-10\,abe^2n+25\,a^2e^2\right)x^5+\frac{1}{16}\left(b^2den^2-4\,abden+8\,a^2de\right)x^4$$
$$+\frac{1}{27}\left(2\,b^2d^2n^2-6\,abd^2n+9\,a^2d^2\right)x^3+\frac{1}{30}\left(6\,b^2e^2x^5+15\,b^2dex^4+10\,b^2d^2x^3\right)\log\left(c\right)^2$$
$$+\frac{1}{30}\left(6\,b^2e^2n^2x^5+15\,b^2den^2x^4+10\,b^2d^2n^2x^3\right)\log\left(x\right)^2$$
$$-\frac{1}{900}\left(72\left(b^2e^2n-5\,abe^2\right)x^5+225\left(b^2den-4\,abde\right)x^4+200\left(b^2d^2n-3\,abd^2\right)x^3\right)\log\left(c\right)$$
$$-\frac{1}{900}\left(72\left(b^2e^2n^2-5\,abe^2n\right)x^5+225\left(b^2den^2-4\,abden\right)x^4+200\left(b^2d^2n^2-3\,abd^2n\right)x^3-60\left(6\,b^2e^2nx^5\right.\right.$$

```
[In] integrate(x^2*(e*x+d)^2*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/125*(2*b^2*e^2*n^2 - 10*a*b*e^2*n + 25*a^2*e^2)*x^5 + 1/16*(b^2*d*e*n^2 -
 4*a*b*d*e*n + 8*a^2*d*e)*x^4 + 1/27*(2*b^2*d^2*n^2 - 6*a*b*d^2*n + 9*a^2*d
^2)*x^3 + 1/30*(6*b^2*e^2*x^5 + 15*b^2*d*e*x^4 + 10*b^2*d^2*x^3)*log(c)^2 +
 1/30*(6*b^2*e^2*n^2*x^5 + 15*b^2*d*e*n^2*x^4 + 10*b^2*d^2*n^2*x^3)*log(x)^
2 - 1/900*(72*(b^2*e^2*n - 5*a*b*e^2)*x^5 + 225*(b^2*d*e*n - 4*a*b*d*e)*x^4
 + 200*(b^2*d^2*n - 3*a*b*d^2)*x^3)*log(c) - 1/900*(72*(b^2*e^2*n^2 - 5*a*b
*e^2*n)*x^5 + 225*(b^2*d*e*n^2 - 4*a*b*d*e*n)*x^4 + 200*(b^2*d^2*n^2 - 3*a*
b*d^2*n)*x^3 - 60*(6*b^2*e^2*n*x^5 + 15*b^2*d*e*n*x^4 + 10*b^2*d^2*n*x^3)*l
og(c))*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.75

$$\int x^2(d+ex)^2\left(a+b\log\left(cx^n\right)\right)^2\,dx=\frac{a^2d^2x^3}{3}+\frac{a^2dex^4}{2}+\frac{a^2e^2x^5}{5}-\frac{2abd^2nx^3}{9}$$
$$+\frac{2abd^2x^3\log\left(cx^n\right)}{3}-\frac{abdenx^4}{4}+abdex^4\log\left(cx^n\right)$$
$$-\frac{2abe^2nx^5}{25}+\frac{2abe^2x^5\log\left(cx^n\right)}{5}+\frac{2b^2d^2n^2x^3}{27}$$
$$-\frac{2b^2d^2nx^3\log\left(cx^n\right)}{9}+\frac{b^2d^2x^3\log\left(cx^n\right)^2}{3}$$
$$+\frac{b^2den^2x^4}{16}-\frac{b^2denx^4\log\left(cx^n\right)}{4}+\frac{b^2dex^4\log\left(cx^n\right)^2}{2}$$
$$+\frac{2b^2e^2n^2x^5}{125}-\frac{2b^2e^2nx^5\log\left(cx^n\right)}{25}+\frac{b^2e^2x^5\log\left(cx^n\right)^2}{5}$$

```
[In] integrate(x**2*(e*x+d)**2*(a+b*ln(c*x**n))**2,x)
```

```
[Out] a**2*d**2*x**3/3 + a**2*d*e*x**4/2 + a**2*e**2*x**5/5 - 2*a*b*d**2*n*x**3/9
 + 2*a*b*d**2*x**3*log(c*x**n)/3 - a*b*d*e*n*x**4/4 + a*b*d*e*x**4*log(c*x*
*n) - 2*a*b*e**2*n*x**5/25 + 2*a*b*e**2*x**5*log(c*x**n)/5 + 2*b**2*d**2*n*
*2*x**3/27 - 2*b**2*d**2*n*x**3*log(c*x**n)/9 + b**2*d**2*x**3*log(c*x**n)*
*2/3 + b**2*d*e*n**2*x**4/16 - b**2*d*e*n*x**4*log(c*x**n)/4 + b**2*d*e*x**
4*log(c*x**n)**2/2 + 2*b**2*e**2*n**2*x**5/125 - 2*b**2*e**2*n*x**5*log(c*x
**n)/25 + b**2*e**2*x**5*log(c*x**n)**2/5
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.40

$$\begin{aligned}\int x^2(d+ex)^2\left(a+b\log\left(cx^n\right)\right)^2\,dx=&\frac{1}{5}\,b^2e^2x^5\log\left(cx^n\right)^2-\frac{2}{25}\,abe^2nx^5+\frac{2}{5}\,abe^2x^5\log\left(cx^n\right)\\&+\frac{1}{2}\,b^2dex^4\log\left(cx^n\right)^2-\frac{1}{4}\,abdenx^4+\frac{1}{5}\,a^2e^2x^5\\&+abdex^4\log\left(cx^n\right)+\frac{1}{3}\,b^2d^2x^3\log\left(cx^n\right)^2\\&-\frac{2}{9}\,abd^2nx^3+\frac{1}{2}\,a^2dex^4+\frac{2}{3}\,abd^2x^3\log\left(cx^n\right)\\&+\frac{1}{3}\,a^2d^2x^3+\frac{2}{27}\left(n^2x^3-3\,nx^3\log\left(cx^n\right)\right)b^2d^2\\&+\frac{1}{16}\left(n^2x^4-4\,nx^4\log\left(cx^n\right)\right)b^2de\\&+\frac{2}{125}\left(n^2x^5-5\,nx^5\log\left(cx^n\right)\right)b^2e^2\end{aligned}$$

```
[In] integrate(x^2*(e*x+d)^2*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*e^2*x^5*log(c*x^n)^2 - 2/25*a*b*e^2*n*x^5 + 2/5*a*b*e^2*x^5*log(c*x
^n) + 1/2*b^2*d*e*x^4*log(c*x^n)^2 - 1/4*a*b*d*e*n*x^4 + 1/5*a^2*e^2*x^5 +
a*b*d*e*x^4*log(c*x^n) + 1/3*b^2*d^2*x^3*log(c*x^n)^2 - 2/9*a*b*d^2*n*x^3 +
 1/2*a^2*d*e*x^4 + 2/3*a*b*d^2*x^3*log(c*x^n) + 1/3*a^2*d^2*x^3 + 2/27*(n^2
*x^3 - 3*n*x^3*log(c*x^n))*b^2*d^2 + 1/16*(n^2*x^4 - 4*n*x^4*log(c*x^n))*b^
2*d*e + 2/125*(n^2*x^5 - 5*n*x^5*log(c*x^n))*b^2*e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(160) = 320$.

Time = 0.38 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.29

$$\int x^2(d+ex)^2\left(a+b\log\left(cx^n\right)\right)^2\,dx = \frac{1}{5}\,b^2e^2n^2x^5\log\left(x\right)^2 - \frac{2}{25}\,b^2e^2n^2x^5\log\left(x\right) + \frac{2}{5}\,b^2e^2nx^5\log\left(c\right)\log\left(x\right) + \frac{1}{2}\,b^2den^2x^4\log\left(x\right)^2 + \frac{2}{125}\,b^2e^2n^2x^5 - \frac{2}{25}\,b^2e^2nx^5\log\left(c\right) + \frac{1}{5}\,b^2e^2x^5\log\left(c\right)^2 - \frac{1}{4}\,b^2den^2x^4\log\left(x\right) + \frac{2}{5}\,abe^2nx^5\log\left(x\right) + b^2denx^4\log\left(c\right)\log\left(x\right) + \frac{1}{3}\,b^2d^2n^2x^3\log\left(x\right)^2 + \frac{1}{16}\,b^2den^2x^4 - \frac{2}{25}\,abe^2nx^5 - \frac{1}{4}\,b^2denx^4\log\left(c\right) + \frac{2}{5}\,abe^2x^5\log\left(c\right) + \frac{1}{2}\,b^2dex^4\log\left(c\right)^2 - \frac{2}{9}\,b^2d^2n^2x^3\log\left(x\right) + abdenx^4\log\left(x\right) + \frac{2}{3}\,b^2d^2nx^3\log\left(c\right)\log\left(x\right) + \frac{2}{27}\,b^2d^2n^2x^3 - \frac{1}{4}\,abdenx^4 + \frac{1}{5}\,a^2e^2x^5 - \frac{2}{9}\,b^2d^2nx^3\log\left(c\right) + abdex^4\log\left(c\right) + \frac{1}{3}\,b^2d^2x^3\log\left(c\right)^2 + \frac{2}{3}\,abd^2nx^3\log\left(x\right) - \frac{2}{9}\,abd^2nx^3 + \frac{1}{2}\,a^2dex^4 + \frac{2}{3}\,abd^2x^3\log\left(c\right) + \frac{1}{3}\,a^2d^2x^3$$

```
[In] integrate(x^2*(e*x+d)^2*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*e^2*n^2*x^5*log(x)^2 - 2/25*b^2*e^2*n^2*x^5*log(x) + 2/5*b^2*e^2*n*
x^5*log(c)*log(x) + 1/2*b^2*d*e*n^2*x^4*log(x)^2 + 2/125*b^2*e^2*n^2*x^5 -
2/25*b^2*e^2*n*x^5*log(c) + 1/5*b^2*e^2*x^5*log(c)^2 - 1/4*b^2*d*e*n^2*x^4*
log(x) + 2/5*a*b*e^2*n*x^5*log(x) + b^2*d*e*n*x^4*log(c)*log(x) + 1/3*b^2*d
^2*n^2*x^3*log(x)^2 + 1/16*b^2*d*e*n^2*x^4 - 2/25*a*b*e^2*n*x^5 - 1/4*b^2*d
*e*n*x^4*log(c) + 2/5*a*b*e^2*x^5*log(c) + 1/2*b^2*d*e*x^4*log(c)^2 - 2/9*b
^2*d^2*n^2*x^3*log(x) + a*b*d*e*n*x^4*log(x) + 2/3*b^2*d^2*n*x^3*log(c)*log
(x) + 2/27*b^2*d^2*n^2*x^3 - 1/4*a*b*d*e*n*x^4 + 1/5*a^2*e^2*x^5 - 2/9*b^2*
d^2*n*x^3*log(c) + a*b*d*e*x^4*log(c) + 1/3*b^2*d^2*x^3*log(c)^2 + 2/3*a*b*
d^2*n*x^3*log(x) - 2/9*a*b*d^2*n*x^3 + 1/2*a^2*d*e*x^4 + 2/3*a*b*d^2*x^3*lo
g(c) + 1/3*a^2*d^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01

$$\int x^2(d+ex)^2 \left(a+b\log \left(cx^n\right)\right)^2 \, dx = \ln \left(c\,x^n\right)\,\left(\frac{2\,b\,\left(3\,a-b\,n\right)\,d^2\,x^3}{9}+\frac{b\,\left(4\,a-b\,n\right)\,d\,e\,x^4}{4}+\frac{2\,b\,\left(5\,a-b\,n\right)\,e^2\,x^5}{25}\right)+{\ln \left(c\,x^n\right)}^2\,\left(\frac{b^2\,d^2\,x^3}{3}+\frac{b^2\,d\,e\,x^4}{2}+\frac{b^2\,e^2\,x^5}{5}\right)+\frac{d^2\,x^3\,\left(9\,a^2-6\,a\,b\,n+2\,b^2\,n^2\right)}{27}+\frac{e^2\,x^5\,\left(25\,a^2-10\,a\,b\,n+2\,b^2\,n^2\right)}{125}+\frac{d\,e\,x^4\,\left(8\,a^2-4\,a\,b\,n+b^2\,n^2\right)}{16}$$

```
[In] int(x^2*(a + b*log(c*x^n))^2*(d + e*x)^2,x)
```

```
[Out] log(c*x^n)*((2*b*d^2*x^3*(3*a - b*n))/9 + (2*b*e^2*x^5*(5*a - b*n))/25 + (b
*d*e*x^4*(4*a - b*n))/4) + log(c*x^n)^2*((b^2*d^2*x^3)/3 + (b^2*e^2*x^5)/5
+ (b^2*d*e*x^4)/2) + (d^2*x^3*(9*a^2 + 2*b^2*n^2 - 6*a*b*n))/27 + (e^2*x^5*
(25*a^2 + 2*b^2*n^2 - 10*a*b*n))/125 + (d*e*x^4*(8*a^2 + b^2*n^2 - 4*a*b*n)
)/16
```

3.85 $\int x(d+ex)^2 (a+b\log(cx^n))^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 178

$$\int x(d+ex)^2 (a+b\log(cx^n))^2 \, dx = \frac{1}{4}b^2d^2n^2x^2 + \frac{4}{27}b^2den^2x^3 + \frac{1}{32}b^2e^2n^2x^4 - \frac{1}{2}bd^2nx^2(a+b\log(cx^n)) - \frac{4}{9}bdenx^3(a+b\log(cx^n)) - \frac{1}{8}be^2nx^4(a+b\log(cx^n)) + \frac{1}{2}d^2x^2(a+b\log(cx^n))^2 + \frac{2}{3}dex^3(a+b\log(cx^n))^2 + \frac{1}{4}e^2x^4(a+b\log(cx^n))^2$$

```
[Out] 1/4*b^2*d^2*n^2*x^2+4/27*b^2*d*e*n^2*x^3+1/32*b^2*e^2*n^2*x^4-1/2*b*d^2*n*x
^2*(a+b*ln(c*x^n))-4/9*b*d*e*n*x^3*(a+b*ln(c*x^n))-1/8*b*e^2*n*x^4*(a+b*ln(
c*x^n))+1/2*d^2*x^2*(a+b*ln(c*x^n))^2+2/3*d*e*x^3*(a+b*ln(c*x^n))^2+1/4*e^2
*x^4*(a+b*ln(c*x^n))^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2395, 2342, 2341}

$$\int x(d+ex)^2 (a+b\log(cx^n))^2 \, dx = \frac{1}{2}d^2x^2(a+b\log(cx^n))^2 - \frac{1}{2}bd^2nx^2(a+b\log(cx^n)) + \frac{2}{3}dex^3(a+b\log(cx^n))^2 - \frac{4}{9}bdenx^3(a+b\log(cx^n)) + \frac{1}{4}e^2x^4(a+b\log(cx^n))^2 - \frac{1}{8}be^2nx^4(a+b\log(cx^n)) + \frac{1}{4}b^2d^2n^2x^2 + \frac{4}{27}b^2den^2x^3 + \frac{1}{32}b^2e^2n^2x^4$$

```
[In] Int[x*(d + e*x)^2*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (b^2*d^2*n^2*x^2)/4 + (4*b^2*d*e*n^2*x^3)/27 + (b^2*e^2*n^2*x^4)/32 - (b*d^
2*n*x^2*(a + b*Log[c*x^n]))/2 - (4*b*d*e*n*x^3*(a + b*Log[c*x^n]))/9 - (b*e
^2*n*x^4*(a + b*Log[c*x^n]))/8 + (d^2*x^2*(a + b*Log[c*x^n])^2)/2 + (2*d*e*
x^3*(a + b*Log[c*x^n])^2)/3 + (e^2*x^4*(a + b*Log[c*x^n])^2)/4
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(d^2x(a+b\log(cx^n))^2+2dex^2(a+b\log(cx^n))^2+e^2x^3(a+b\log(cx^n))^2\right)\,dx\\
&= d^2\int x(a+b\log(cx^n))^2\,dx+(2de)\int x^2(a+b\log(cx^n))^2\,dx+e^2\int x^3(a+b\log(cx^n))^2\,dx\\
&= \frac{1}{2}d^2x^2(a+b\log(cx^n))^2+\frac{2}{3}dex^3(a+b\log(cx^n))^2\\
&\quad+\frac{1}{4}e^2x^4(a+b\log(cx^n))^2-\left(bd^2n\right)\int x(a+b\log(cx^n))\,dx\\
&\quad-\frac{1}{3}(4bden)\int x^2(a+b\log(cx^n))\,dx-\frac{1}{2}\left(be^2n\right)\int x^3(a+b\log(cx^n))\,dx\\
&= \frac{1}{4}b^2d^2n^2x^2+\frac{4}{27}b^2den^2x^3+\frac{1}{32}b^2e^2n^2x^4-\frac{1}{2}bd^2nx^2(a+b\log(cx^n))\\
&\quad-\frac{4}{9}bdenx^3(a+b\log(cx^n))-\frac{1}{8}be^2nx^4(a+b\log(cx^n))\\
&\quad+\frac{1}{2}d^2x^2(a+b\log(cx^n))^2+\frac{2}{3}dex^3(a+b\log(cx^n))^2+\frac{1}{4}e^2x^4(a+b\log(cx^n))^2
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.75

$$\int x(d+ex)^2 \left(a+b\log\left(cx^n\right)\right)^2 \, dx = \frac{1}{864}x^2\big(27be^2nx^2(-4a+bn-4b\log\left(cx^n\right)) + 128bdenx(-3a+bn-3b\log\left(cx^n\right)) + 216bd^2n(-2a+bn-2b\log\left(cx^n\right)) + 432d^2(a+b\log\left(cx^n\right))^2 + 576dex(a+b\log\left(cx^n\right))^2 + 216e^2x^2(a+b\log\left(cx^n\right))^2\big)$$

```
[In] Integrate[x*(d + e*x)^2*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (x^2*(27*b*e^2*n*x^2*(-4*a + b*n - 4*b*Log[c*x^n]) + 128*b*d*e*n*x*(-3*a +
b*n - 3*b*Log[c*x^n]) + 216*b*d^2*n*(-2*a + b*n - 2*b*Log[c*x^n]) + 432*d^2
*(a + b*Log[c*x^n])^2 + 576*d*e*x*(a + b*Log[c*x^n])^2 + 216*e^2*x^2*(a + b
*Log[c*x^n])^2))/864
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.47

method	result
parallelrisch	$\frac{x^4b^2\ln(c\,x^n)^2e^2}{4}-\frac{\ln(c\,x^n)x^4n\,e^2b^2}{8}+\frac{b^2e^2n^2x^4}{32}+\frac{x^4ab\ln(c\,x^n)e^2}{2}-\frac{ab\,e^2n\,x^4}{8}+\frac{2x^3b^2\ln(c\,x^n)^2de}{3}-\frac{4\ln(c\,x^n)x^3n}{9}$
risch	Expression too large to display

```
[In] int(x*(e*x+d)^2*(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*b^2*ln(c*x^n)^2*e^2-1/8*ln(c*x^n)*x^4*n*e^2*b^2+1/32*b^2*e^2*n^2*x^
4+1/2*x^4*a*b*ln(c*x^n)*e^2-1/8*a*b*e^2*n*x^4+2/3*x^3*b^2*ln(c*x^n)^2*d*e-4
/9*ln(c*x^n)*x^3*n*d*e*b^2+4/27*b^2*d*e*n^2*x^3+1/4*x^4*a^2*e^2+4/3*x^3*a*b
*ln(c*x^n)*d*e-4/9*a*b*d*e*n*x^3+1/2*x^2*b^2*ln(c*x^n)^2*d^2-1/2*ln(c*x^n)*
x^2*n*b^2*d^2+1/4*b^2*d^2*n^2*x^2+2/3*x^3*a^2*d*e+x^2*a*b*ln(c*x^n)*d^2-1/2
*a*b*d^2*n*x^2+1/2*x^2*a^2*d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(160) = 320.

Time = 0.27 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.04

$$\int x(d+ex)^2 (a+b\log(cx^n))^2 \, dx$$
$$= \frac{1}{32} \left(b^2e^2n^2 - 4\,abe^2n + 8\,a^2e^2\right)x^4 + \frac{2}{27} \left(2\,b^2den^2 - 6\,abden + 9\,a^2de\right)x^3$$
$$+ \frac{1}{4} \left(b^2d^2n^2 - 2\,abd^2n + 2\,a^2d^2\right)x^2 + \frac{1}{12} \left(3\,b^2e^2x^4 + 8\,b^2dex^3 + 6\,b^2d^2x^2\right)\log(c)^2$$
$$+ \frac{1}{12} \left(3\,b^2e^2n^2x^4 + 8\,b^2den^2x^3 + 6\,b^2d^2n^2x^2\right)\log(x)^2$$
$$- \frac{1}{72} \left(9\left(b^2e^2n - 4\,abe^2\right)x^4 + 32\left(b^2den - 3\,abde\right)x^3 + 36\left(b^2d^2n - 2\,abd^2\right)x^2\right)\log(c)$$
$$- \frac{1}{72} \left(9\left(b^2e^2n^2 - 4\,abe^2n\right)x^4 + 32\left(b^2den^2 - 3\,abden\right)x^3 + 36\left(b^2d^2n^2 - 2\,abd^2n\right)x^2 - 12\left(3\,b^2e^2nx^4 + 8\,b^2d\right.\right.$$

```
[In] integrate(x*(e*x+d)^2*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/32*(b^2*e^2*n^2 - 4*a*b*e^2*n + 8*a^2*e^2)*x^4 + 2/27*(2*b^2*d*e*n^2 - 6*
a*b*d*e*n + 9*a^2*d*e)*x^3 + 1/4*(b^2*d^2*n^2 - 2*a*b*d^2*n + 2*a^2*d^2)*x^
2 + 1/12*(3*b^2*e^2*x^4 + 8*b^2*d*e*x^3 + 6*b^2*d^2*x^2)*log(c)^2 + 1/12*(3
*b^2*e^2*n^2*x^4 + 8*b^2*d*e*n^2*x^3 + 6*b^2*d^2*n^2*x^2)*log(x)^2 - 1/72*(
9*(b^2*e^2*n - 4*a*b*e^2)*x^4 + 32*(b^2*d*e*n - 3*a*b*d*e)*x^3 + 36*(b^2*d^
2*n - 2*a*b*d^2)*x^2)*log(c) - 1/72*(9*(b^2*e^2*n^2 - 4*a*b*e^2*n)*x^4 + 32
*(b^2*d*e*n^2 - 3*a*b*d*e*n)*x^3 + 36*(b^2*d^2*n^2 - 2*a*b*d^2*n)*x^2 - 12*
(3*b^2*e^2*n*x^4 + 8*b^2*d*e*n*x^3 + 6*b^2*d^2*n*x^2)*log(c))*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.73

$$\int x(d+ex)^2 (a+b\log(cx^n))^2 \, dx = \frac{a^2d^2x^2}{2} + \frac{2a^2dex^3}{3} + \frac{a^2e^2x^4}{4} - \frac{abd^2nx^2}{2} + abd^2x^2\log(cx^n) - \frac{4abdenx^3}{9} + \frac{4abdex^3\log(cx^n)}{3} - \frac{abe^2nx^4}{8} + \frac{abe^2x^4\log(cx^n)}{2} + \frac{b^2d^2n^2x^2}{4} - \frac{b^2d^2nx^2\log(cx^n)}{2} + \frac{b^2d^2x^2\log(cx^n)^2}{2} + \frac{4b^2den^2x^3}{27} - \frac{4b^2denx^3\log(cx^n)}{9} + \frac{2b^2dex^3\log(cx^n)^2}{3} + \frac{b^2e^2n^2x^4}{32} - \frac{b^2e^2nx^4\log(cx^n)}{8} + \frac{b^2e^2x^4\log(cx^n)^2}{4}$$

```
[In] integrate(x*(e*x+d)**2*(a+b*ln(c*x**n))**2,x)
```

```
[Out] a**2*d**2*x**2/2 + 2*a**2*d*e*x**3/3 + a**2*e**2*x**4/4 - a*b*d**2*n*x**2/2
 + a*b*d**2*x**2*log(c*x**n) - 4*a*b*d*e*n*x**3/9 + 4*a*b*d*e*x**3*log(c*x*
*n)/3 - a*b*e**2*n*x**4/8 + a*b*e**2*x**4*log(c*x**n)/2 + b**2*d**2*n**2*x*
*2/4 - b**2*d**2*n*x**2*log(c*x**n)/2 + b**2*d**2*x**2*log(c*x**n)**2/2 + 4
*b**2*d*e*n**2*x**3/27 - 4*b**2*d*e*n*x**3*log(c*x**n)/9 + 2*b**2*d*e*x**3*
log(c*x**n)**2/3 + b**2*e**2*n**2*x**4/32 - b**2*e**2*n*x**4*log(c*x**n)/8
+ b**2*e**2*x**4*log(c*x**n)**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.40

$$
\begin{aligned}
\int x(d+ex)^2\left(a+b\log\left(cx^n\right)\right)^2\,dx =& \frac{1}{4}\,b^2e^2x^4\log\left(cx^n\right)^2 - \frac{1}{8}\,abe^2nx^4 + \frac{1}{2}\,abe^2x^4\log\left(cx^n\right) \\
&+ \frac{2}{3}\,b^2dex^3\log\left(cx^n\right)^2 - \frac{4}{9}\,abdenx^3 + \frac{1}{4}\,a^2e^2x^4 \\
&+ \frac{4}{3}\,abdex^3\log\left(cx^n\right) + \frac{1}{2}\,b^2d^2x^2\log\left(cx^n\right)^2 \\
&- \frac{1}{2}\,abd^2nx^2 + \frac{2}{3}\,a^2dex^3 + abd^2x^2\log\left(cx^n\right) \\
&+ \frac{1}{2}\,a^2d^2x^2 + \frac{1}{4}\,\left(n^2x^2 - 2\,nx^2\log\left(cx^n\right)\right)b^2d^2 \\
&+ \frac{4}{27}\,\left(n^2x^3 - 3\,nx^3\log\left(cx^n\right)\right)b^2de \\
&+ \frac{1}{32}\,\left(n^2x^4 - 4\,nx^4\log\left(cx^n\right)\right)b^2e^2
\end{aligned}
$$

```
[In] integrate(x*(e*x+d)^2*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*e^2*x^4*log(c*x^n)^2 - 1/8*a*b*e^2*n*x^4 + 1/2*a*b*e^2*x^4*log(c*x^
n) + 2/3*b^2*d*e*x^3*log(c*x^n)^2 - 4/9*a*b*d*e*n*x^3 + 1/4*a^2*e^2*x^4 + 4
/3*a*b*d*e*x^3*log(c*x^n) + 1/2*b^2*d^2*x^2*log(c*x^n)^2 - 1/2*a*b*d^2*n*x^
2 + 2/3*a^2*d*e*x^3 + a*b*d^2*x^2*log(c*x^n) + 1/2*a^2*d^2*x^2 + 1/4*(n^2*x
^2 - 2*n*x^2*log(c*x^n))*b^2*d^2 + 4/27*(n^2*x^3 - 3*n*x^3*log(c*x^n))*b^2*
d*e + 1/32*(n^2*x^4 - 4*n*x^4*log(c*x^n))*b^2*e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(160) = 320.

Time = 0.36 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.29

$$\begin{aligned}\int x(d+ex)^2\left(a+b\log\left(cx^n\right)\right)^2\,dx &= \frac{1}{4}\,b^2e^2n^2x^4\log\left(x\right)^2-\frac{1}{8}\,b^2e^2n^2x^4\log\left(x\right)\\&+\frac{1}{2}\,b^2e^2nx^4\log\left(c\right)\log\left(x\right)+\frac{2}{3}\,b^2den^2x^3\log\left(x\right)^2\\&+\frac{1}{32}\,b^2e^2n^2x^4-\frac{1}{8}\,b^2e^2nx^4\log\left(c\right)+\frac{1}{4}\,b^2e^2x^4\log\left(c\right)^2\\&-\frac{4}{9}\,b^2den^2x^3\log\left(x\right)+\frac{1}{2}\,abe^2nx^4\log\left(x\right)\\&+\frac{4}{3}\,b^2denx^3\log\left(c\right)\log\left(x\right)+\frac{1}{2}\,b^2d^2n^2x^2\log\left(x\right)^2\\&+\frac{4}{27}\,b^2den^2x^3-\frac{1}{8}\,abe^2nx^4-\frac{4}{9}\,b^2denx^3\log\left(c\right)\\&+\frac{1}{2}\,abe^2x^4\log\left(c\right)+\frac{2}{3}\,b^2dex^3\log\left(c\right)^2\\&-\frac{1}{2}\,b^2d^2n^2x^2\log\left(x\right)+\frac{4}{3}\,abdenx^3\log\left(x\right)\\&+b^2d^2nx^2\log\left(c\right)\log\left(x\right)+\frac{1}{4}\,b^2d^2n^2x^2-\frac{4}{9}\,abdenx^3\\&+\frac{1}{4}\,a^2e^2x^4-\frac{1}{2}\,b^2d^2nx^2\log\left(c\right)+\frac{4}{3}\,abdex^3\log\left(c\right)\\&+\frac{1}{2}\,b^2d^2x^2\log\left(c\right)^2+abd^2nx^2\log\left(x\right)-\frac{1}{2}\,abd^2nx^2\\&+\frac{2}{3}\,a^2dex^3+abd^2x^2\log\left(c\right)+\frac{1}{2}\,a^2d^2x^2\end{aligned}$$

```
[In] integrate(x*(e*x+d)^2*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/4*b^2*e^2*n^2*x^4*log(x)^2 - 1/8*b^2*e^2*n^2*x^4*log(x) + 1/2*b^2*e^2*n*x
^4*log(c)*log(x) + 2/3*b^2*d*e*n^2*x^3*log(x)^2 + 1/32*b^2*e^2*n^2*x^4 - 1/
8*b^2*e^2*n*x^4*log(c) + 1/4*b^2*e^2*x^4*log(c)^2 - 4/9*b^2*d*e*n^2*x^3*log
(x) + 1/2*a*b*e^2*n*x^4*log(x) + 4/3*b^2*d*e*n*x^3*log(c)*log(x) + 1/2*b^2*
d^2*n^2*x^2*log(x)^2 + 4/27*b^2*d*e*n^2*x^3 - 1/8*a*b*e^2*n*x^4 - 4/9*b^2*d
*e*n*x^3*log(c) + 1/2*a*b*e^2*x^4*log(c) + 2/3*b^2*d*e*x^3*log(c)^2 - 1/2*b
^2*d^2*n^2*x^2*log(x) + 4/3*a*b*d*e*n*x^3*log(x) + b^2*d^2*n*x^2*log(c)*log
(x) + 1/4*b^2*d^2*n^2*x^2 - 4/9*a*b*d*e*n*x^3 + 1/4*a^2*e^2*x^4 - 1/2*b^2*d
^2*n*x^2*log(c) + 4/3*a*b*d*e*x^3*log(c) + 1/2*b^2*d^2*x^2*log(c)^2 + a*b*d
^2*n*x^2*log(x) - 1/2*a*b*d^2*n*x^2 + 2/3*a^2*d*e*x^3 + a*b*d^2*x^2*log(c)
+ 1/2*a^2*d^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.01

$$\int x(d+ex)^2\left(a+b\log\left(cx^n\right)\right)^2\,dx=\ln\left(c\,x^n\right)\,\left(\frac{b\,\left(2\,a-b\,n\right)\,d^2\,x^2}{2}+\frac{4\,b\,\left(3\,a-b\,n\right)\,d\,e\,x^3}{9}+\frac{b\,\left(4\,a-b\,n\right)\,e^2\,x^4}{8}\right)+{\ln\left(c\,x^n\right)}^2\,\left(\frac{b^2\,d^2\,x^2}{2}+\frac{2\,b^2\,d\,e\,x^3}{3}+\frac{b^2\,e^2\,x^4}{4}\right)+\frac{d^2\,x^2\,\left(2\,a^2-2\,a\,b\,n+b^2\,n^2\right)}{4}+\frac{e^2\,x^4\,\left(8\,a^2-4\,a\,b\,n+b^2\,n^2\right)}{32}+\frac{2\,d\,e\,x^3\,\left(9\,a^2-6\,a\,b\,n+2\,b^2\,n^2\right)}{27}$$

```
[In] int(x*(a + b*log(c*x^n))^2*(d + e*x)^2,x)
```

```
[Out] log(c*x^n)*((b*d^2*x^2*(2*a - b*n))/2 + (b*e^2*x^4*(4*a - b*n))/8 + (4*b*d*
e*x^3*(3*a - b*n))/9) + log(c*x^n)^2*((b^2*d^2*x^2)/2 + (b^2*e^2*x^4)/4 + (
2*b^2*d*e*x^3)/3) + (d^2*x^2*(2*a^2 + b^2*n^2 - 2*a*b*n))/4 + (e^2*x^4*(8*a
^2 + b^2*n^2 - 4*a*b*n))/32 + (2*d*e*x^3*(9*a^2 + 2*b^2*n^2 - 6*a*b*n))/27
```

3.86 $\int (d+ex)^2 (a+b\log(cx^n))^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 173

$$\begin{aligned}\int (d+ex)^2 (a+b\log(cx^n))^2 \, dx = {} & 2b^2d^2n^2x+\frac{1}{2}b^2den^2x^2+\frac{2}{27}b^2e^2n^2x^3 \\ & +\frac{b^2d^3n^2\log^2(x)}{3e}-2bd^2nx(a+b\log(cx^n)) \\ & -bdenx^2(a+b\log(cx^n))-\frac{2}{9}be^2nx^3(a+b\log(cx^n)) \\ & -\frac{2bd^3n\log(x)(a+b\log(cx^n))}{3e} \\ & +\frac{(d+ex)^3(a+b\log(cx^n))^2}{3e}\end{aligned}$$

```
[Out] 2*b^2*d^2*n^2*x+1/2*b^2*d*e*n^2*x^2+2/27*b^2*e^2*n^2*x^3+1/3*b^2*d^3*n^2*ln
(x)^2/e-2*b*d^2*n*x*(a+b*ln(c*x^n))-b*d*e*n*x^2*(a+b*ln(c*x^n))-2/9*b*e^2*n
*x^3*(a+b*ln(c*x^n))-2/3*b*d^3*n*ln(x)*(a+b*ln(c*x^n))/e+1/3*(e*x+d)^3*(a+b
*ln(c*x^n))^2/e
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{2356, 45, 2372, 2338\}$

$$\int (d+ex)^2 (a+b\log(cx^n))^2 \, dx = -\frac{2bd^3n\log(x)(a+b\log(cx^n))}{3e} - 2bd^2nx(a+b\log(cx^n)) + \frac{(d+ex)^3(a+b\log(cx^n))^2}{3e} - bdenx^2(a+b\log(cx^n)) - \frac{2}{9}be^2nx^3(a+b\log(cx^n)) + \frac{b^2d^3n^2\log^2(x)}{3e} + 2b^2d^2n^2x + \frac{1}{2}b^2den^2x^2 + \frac{2}{27}b^2e^2n^2x^3$$

```
[In] Int[(d + e*x)^2*(a + b*Log[c*x^n])^2,x]
```

```
[Out] 2*b^2*d^2*n^2*x + (b^2*d*e*n^2*x^2)/2 + (2*b^2*e^2*n^2*x^3)/27 + (b^2*d^3*n
^2*Log[x]^2)/(3*e) - 2*b*d^2*n*x*(a + b*Log[c*x^n]) - b*d*e*n*x^2*(a + b*Lo
g[c*x^n]) - (2*b*e^2*n*x^3*(a + b*Log[c*x^n]))/9 - (2*b*d^3*n*Log[x]*(a + b
*Log[c*x^n]))/(3*e) + ((d + e*x)^3*(a + b*Log[c*x^n])^2)/(3*e)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+ex)^3 (a+b\log(cx^n))^2}{3e} - \frac{(2bn)\int \frac{(d+ex)^3(a+b\log(cx^n))}{x}\,dx}{3e}\\
&= -2bd^2nx(a+b\log(cx^n)) - bdenx^2(a+b\log(cx^n)) - \frac{2}{9}be^2nx^3(a+b\log(cx^n))\\
&\quad - \frac{2bd^3n\log(x)(a+b\log(cx^n))}{3e} + \frac{(d+ex)^3(a+b\log(cx^n))^2}{3e}\\
&\quad + \frac{(2b^2n^2)\int \left(\frac{1}{6}e(18d^2+9dex+2e^2x^2) + \frac{d^3\log(x)}{x}\right)dx}{3e}\\
&= -2bd^2nx(a+b\log(cx^n)) - bdenx^2(a+b\log(cx^n)) - \frac{2}{9}be^2nx^3(a+b\log(cx^n))\\
&\quad - \frac{2bd^3n\log(x)(a+b\log(cx^n))}{3e} + \frac{(d+ex)^3(a+b\log(cx^n))^2}{3e}\\
&\quad + \frac{1}{9}(b^2n^2)\int (18d^2+9dex+2e^2x^2)\,dx + \frac{(2b^2d^3n^2)\int \frac{\log(x)}{x}\,dx}{3e}\\
&= 2b^2d^2n^2x + \frac{1}{2}b^2den^2x^2 + \frac{2}{27}b^2e^2n^2x^3 + \frac{b^2d^3n^2\log^2(x)}{3e}\\
&\quad - 2bd^2nx(a+b\log(cx^n)) - bdenx^2(a+b\log(cx^n)) - \frac{2}{9}be^2nx^3(a+b\log(cx^n))\\
&\quad - \frac{2bd^3n\log(x)(a+b\log(cx^n))}{3e} + \frac{(d+ex)^3(a+b\log(cx^n))^2}{3e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.78

$$
\begin{aligned}
\int (d+ex)^2 (a+b\log(cx^n))^2\,dx &= \frac{2}{27}be^2nx^3(-3a+bn-3b\log(cx^n))\\
&\quad + \frac{1}{2}bdenx^2(-2a+bn-2b\log(cx^n))\\
&\quad + d^2x(a+b\log(cx^n))^2 + dex^2(a+b\log(cx^n))^2\\
&\quad + \frac{1}{3}e^2x^3(a+b\log(cx^n))^2 - 2bd^2nx(a-bn+b\log(cx^n))
\end{aligned}
$$

```
[In] Integrate[(d + e*x)^2*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (2*b*e^2*n*x^3*(-3*a + b*n - 3*b*Log[c*x^n]))/27 + (b*d*e*n*x^2*(-2*a + b*n
 - 2*b*Log[c*x^n]))/2 + d^2*x*(a + b*Log[c*x^n])^2 + d*e*x^2*(a + b*Log[c*x
^n])^2 + (e^2*x^3*(a + b*Log[c*x^n])^2)/3 - 2*b*d^2*n*x*(a - b*n + b*Log[c*
x^n])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.43

method	result
parallelrisch	$\frac{b^2\ln(c\,x^n)^2 e^2 x^3}{3}-\frac{2\ln(c\,x^n)x^3 n\,e^2 b^2}{9}+\frac{2b^2 e^2 n^2 x^3}{27}+\frac{2ab\ln(c\,x^n)e^2x^3}{3}-\frac{2bna\,e^2x^3}{9}+b^2\ln\left(c\,x^n\right)^2 de\,x^2-x^2$
risch	Expression too large to display

```
[In] int((e*x+d)^2*(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^2*ln(c*x^n)^2*e^2*x^3-2/9*ln(c*x^n)*x^3*n*e^2*b^2+2/27*b^2*e^2*n^2*x^
3+2/3*a*b*ln(c*x^n)*e^2*x^3-2/9*b*n*a*e^2*x^3+b^2*ln(c*x^n)^2*d*e*x^2-x^2*l
n(c*x^n)*b^2*d*e*n+1/2*b^2*d*e*n^2*x^2+1/3*a^2*e^2*x^3+2*a*b*ln(c*x^n)*d*e*
x^2-b*n*a*d*e*x^2+x*b^2*ln(c*x^n)^2*d^2-2*x*ln(c*x^n)*b^2*d^2*n+2*b^2*d^2*n
^2*x+a^2*d*e*x^2+2*x*a*b*ln(c*x^n)*d^2-2*b*n*a*d^2*x+x*a^2*d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(161) = 322.

Time = 0.27 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.01

$$\begin{aligned}\int (d+ex)^2\left(a+b\log\left(cx^n\right)\right)^2\,dx &= \frac{1}{27}\left(2\,b^2e^2n^2-6\,abe^2n+9\,a^2e^2\right)x^3\\&+\frac{1}{2}\left(b^2den^2-2\,abden+2\,a^2de\right)x^2+\frac{1}{3}\left(b^2e^2x^3+3\,b^2dex^2+3\,b^2d^2x\right)\log\left(c\right)^2\\&+\frac{1}{3}\left(b^2e^2n^2x^3+3\,b^2den^2x^2+3\,b^2d^2n^2x\right)\log\left(x\right)^2+\left(2\,b^2d^2n^2-2\,abd^2n+a^2d^2\right)x\\&-\frac{1}{9}\left(2\left(b^2e^2n-3\,abe^2\right)x^3+9\left(b^2den-2\,abde\right)x^2+18\left(b^2d^2n-abd^2\right)x\right)\log\left(c\right)\\&-\frac{1}{9}\left(2\left(b^2e^2n^2-3\,abe^2n\right)x^3+9\left(b^2den^2-2\,abden\right)x^2+18\left(b^2d^2n^2-abd^2n\right)x-6\left(b^2e^2nx^3+3\,b^2denx^2\right.\right.\end{aligned}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/27*(2*b^2*e^2*n^2 - 6*a*b*e^2*n + 9*a^2*e^2)*x^3 + 1/2*(b^2*d*e*n^2 - 2*a
*b*d*e*n + 2*a^2*d*e)*x^2 + 1/3*(b^2*e^2*x^3 + 3*b^2*d*e*x^2 + 3*b^2*d^2*x)
*log(c)^2 + 1/3*(b^2*e^2*n^2*x^3 + 3*b^2*d*e*n^2*x^2 + 3*b^2*d^2*n^2*x)*log
(x)^2 + (2*b^2*d^2*n^2 - 2*a*b*d^2*n + a^2*d^2)*x - 1/9*(2*(b^2*e^2*n - 3*a
*b*e^2)*x^3 + 9*(b^2*d*e*n - 2*a*b*d*e)*x^2 + 18*(b^2*d^2*n - a*b*d^2)*x)*l
og(c) - 1/9*(2*(b^2*e^2*n^2 - 3*a*b*e^2*n)*x^3 + 9*(b^2*d*e*n^2 - 2*a*b*d*e
*n)*x^2 + 18*(b^2*d^2*n^2 - a*b*d^2*n)*x - 6*(b^2*e^2*n*x^3 + 3*b^2*d*e*n*x
^2 + 3*b^2*d^2*n*x)*log(c))*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.65

$$\int (d+ex)^2 (a+b\log (cx^n))^2 \, dx = a^2d^2x + a^2dex^2 + \frac{a^2e^2x^3}{3} - 2abd^2nx + 2abd^2x\log (cx^n) - abdenx^2 + 2abdex^2\log (cx^n) - \frac{2abe^2nx^3}{9} + \frac{2abe^2x^3\log (cx^n)}{3} + 2b^2d^2n^2x - 2b^2d^2nx\log (cx^n) + b^2d^2x\log (cx^n)^2 + \frac{b^2den^2x^2}{2} - b^2denx^2\log (cx^n) + b^2dex^2\log (cx^n)^2 + \frac{2b^2e^2n^2x^3}{27} - \frac{2b^2e^2nx^3\log (cx^n)}{9} + \frac{b^2e^2x^3\log (cx^n)^2}{3}$$

```
[In] integrate((e*x+d)**2*(a+b*ln(c*x**n))**2,x)
```

```
[Out] a**2*d**2*x + a**2*d*e*x**2 + a**2*e**2*x**3/3 - 2*a*b*d**2*n*x + 2*a*b*d**
2*x*log(c*x**n) - a*b*d*e*n*x**2 + 2*a*b*d*e*x**2*log(c*x**n) - 2*a*b*e**2*
n*x**3/9 + 2*a*b*e**2*x**3*log(c*x**n)/3 + 2*b**2*d**2*n**2*x - 2*b**2*d**2
*n*x*log(c*x**n) + b**2*d**2*x*log(c*x**n)**2 + b**2*d*e*n**2*x**2/2 - b**2
*d*e*n*x**2*log(c*x**n) + b**2*d*e*x**2*log(c*x**n)**2 + 2*b**2*e**2*n**2*x
**3/27 - 2*b**2*e**2*n*x**3*log(c*x**n)/9 + b**2*e**2*x**3*log(c*x**n)**2/3
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.36

$$\int (d+ex)^2 (a+b\log (cx^n))^2 \, dx = \frac{1}{3}\,b^2e^2x^3\log (cx^n)^2 - \frac{2}{9}\,abe^2nx^3 + \frac{2}{3}\,abe^2x^3\log (cx^n) + b^2dex^2\log (cx^n)^2 - abdenx^2 + \frac{1}{3}\,a^2e^2x^3 + 2\,abdex^2\log (cx^n) + b^2d^2x\log (cx^n)^2 - 2\,abd^2nx + a^2dex^2 + 2\,abd^2x\log (cx^n) + 2\left(n^2x - nx\log (cx^n)\right)b^2d^2 + \frac{1}{2}\left(n^2x^2 - 2\,nx^2\log (cx^n)\right)b^2de + \frac{2}{27}\left(n^2x^3 - 3\,nx^3\log (cx^n)\right)b^2e^2 + a^2d^2x$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*e^2*x^3*log(c*x^n)^2 - 2/9*a*b*e^2*n*x^3 + 2/3*a*b*e^2*x^3*log(c*x^
n) + b^2*d*e*x^2*log(c*x^n)^2 - a*b*d*e*n*x^2 + 1/3*a^2*e^2*x^3 + 2*a*b*d*e
```

```
*x^2*log(c*x^n) + b^2*d^2*x*log(c*x^n)^2 - 2*a*b*d^2*n*x + a^2*d*e*x^2 + 2*
a*b*d^2*x*log(c*x^n) + 2*(n^2*x - n*x*log(c*x^n))*b^2*d^2 + 1/2*(n^2*x^2 -
2*n*x^2*log(c*x^n))*b^2*d*e + 2/27*(n^2*x^3 - 3*n*x^3*log(c*x^n))*b^2*e^2 +
 a^2*d^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(161) = 322$.

Time = 0.36 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.23

$$\begin{aligned}\int (d+ex)^2 (a+b\log(cx^n))^2 \, dx = {} & \frac{1}{3} b^2e^2n^2x^3 \log(x)^2 - \frac{2}{9} b^2e^2n^2x^3 \log(x) \\ & + \frac{2}{3} b^2e^2nx^3 \log(c) \log(x) + b^2den^2x^2 \log(x)^2 \\ & + \frac{2}{27} b^2e^2n^2x^3 - \frac{2}{9} b^2e^2nx^3 \log(c) + \frac{1}{3} b^2e^2x^3 \log(c)^2 \\ & - b^2den^2x^2 \log(x) + \frac{2}{3} abe^2nx^3 \log(x) \\ & + 2 b^2denx^2 \log(c) \log(x) + b^2d^2n^2x \log(x)^2 \\ & + \frac{1}{2} b^2den^2x^2 - \frac{2}{9} abe^2nx^3 - b^2denx^2 \log(c) \\ & + \frac{2}{3} abe^2x^3 \log(c) + b^2dex^2 \log(c)^2 - 2 b^2d^2n^2x \log(x) \\ & + 2 abdenx^2 \log(x) + 2 b^2d^2nx \log(c) \log(x) \\ & + 2 b^2d^2n^2x - abdenx^2 + \frac{1}{3} a^2e^2x^3 - 2 b^2d^2nx \log(c) \\ & + 2 abdex^2 \log(c) + b^2d^2x \log(c)^2 + 2 abd^2nx \log(x) \\ & - 2 abd^2nx + a^2dex^2 + 2 abd^2x \log(c) + a^2d^2x\end{aligned}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*e^2*n^2*x^3*log(x)^2 - 2/9*b^2*e^2*n^2*x^3*log(x) + 2/3*b^2*e^2*n*x
^3*log(c)*log(x) + b^2*d*e*n^2*x^2*log(x)^2 + 2/27*b^2*e^2*n^2*x^3 - 2/9*b^
2*e^2*n*x^3*log(c) + 1/3*b^2*e^2*x^3*log(c)^2 - b^2*d*e*n^2*x^2*log(x) + 2/
3*a*b*e^2*n*x^3*log(x) + 2*b^2*d*e*n*x^2*log(c)*log(x) + b^2*d^2*n^2*x*log(
x)^2 + 1/2*b^2*d*e*n^2*x^2 - 2/9*a*b*e^2*n*x^3 - b^2*d*e*n*x^2*log(c) + 2/3
*a*b*e^2*x^3*log(c) + b^2*d*e*x^2*log(c)^2 - 2*b^2*d^2*n^2*x*log(x) + 2*a*b
*d*e*n*x^2*log(x) + 2*b^2*d^2*n*x*log(c)*log(x) + 2*b^2*d^2*n^2*x - a*b*d*e
*n*x^2 + 1/3*a^2*e^2*x^3 - 2*b^2*d^2*n*x*log(c) + 2*a*b*d*e*x^2*log(c) + b^
2*d^2*x*log(c)^2 + 2*a*b*d^2*n*x*log(x) - 2*a*b*d^2*n*x + a^2*d*e*x^2 + 2*a
*b*d^2*x*log(c) + a^2*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.96

$$\int (d+ex)^2 (a+b\log(cx^n))^2 \, dx = \ln(c\,x^n)^2 \left(b^2\,d^2\,x+b^2\,d\,e\,x^2+\frac{b^2\,e^2\,x^3}{3}\right) + \ln(c\,x^n) \left(2\,b\,(a-b\,n)\,d^2\,x+b\,(2\,a-b\,n)\,d\,e\,x^2 + \frac{2\,b\,(3\,a-b\,n)\,e^2\,x^3}{9}\right) + d^2\,x\,\left(a^2-2\,a\,b\,n+2\,b^2\,n^2\right) + \frac{e^2\,x^3\,(9\,a^2-6\,a\,b\,n+2\,b^2\,n^2)}{27} + \frac{d\,e\,x^2\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{2}$$

```
[In] int((a + b*log(c*x^n))^2*(d + e*x)^2,x)
```

```
[Out] log(c*x^n)^2*(b^2*d^2*x + (b^2*e^2*x^3)/3 + b^2*d*e*x^2) + log(c*x^n)*((2*b
*e^2*x^3*(3*a - b*n))/9 + 2*b*d^2*x*(a - b*n) + b*d*e*x^2*(2*a - b*n)) + d^
2*x*(a^2 + 2*b^2*n^2 - 2*a*b*n) + (e^2*x^3*(9*a^2 + 2*b^2*n^2 - 6*a*b*n))/2
7 + (d*e*x^2*(2*a^2 + b^2*n^2 - 2*a*b*n))/2
```

3.87 $\int \frac{(d+ex)^2(a+b\log(cx^n))^2}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{(d+ex)^2(a+b\log(cx^n))^2}{x}\,dx = -4abdenx+4b^2den^2x+\frac{1}{4}b^2e^2n^2x^2-4b^2denx\log(cx^n) - \frac{1}{2}be^2nx^2(a+b\log(cx^n))+2dex(a+b\log(cx^n))^2 + \frac{1}{2}e^2x^2(a+b\log(cx^n))^2+\frac{d^2(a+b\log(cx^n))^3}{3bn}$$

```
[Out] -4*a*b*d*e*n*x+4*b^2*d*e*n^2*x+1/4*b^2*e^2*n^2*x^2-4*b^2*d*e*n*x*ln(c*x^n)-
1/2*b*e^2*n*x^2*(a+b*ln(c*x^n))+2*d*e*x*(a+b*ln(c*x^n))^2+1/2*e^2*x^2*(a+b*
ln(c*x^n))^2+1/3*d^2*(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2388, 2339, 30, 2333, 2332, 2367, 2342, 2341}

$$\int \frac{(d+ex)^2(a+b\log(cx^n))^2}{x}\,dx = \frac{d^2(a+b\log(cx^n))^3}{3bn}+2dex(a+b\log(cx^n))^2 + \frac{1}{2}e^2x^2(a+b\log(cx^n))^2-\frac{1}{2}be^2nx^2(a+b\log(cx^n)) - 4abdenx-4b^2denx\log(cx^n)+4b^2den^2x+\frac{1}{4}b^2e^2n^2x^2$$

```
[In] Int[((d + e*x)^2*(a + b*Log[c*x^n])^2)/x,x]
```

```
[Out] -4*a*b*d*e*n*x + 4*b^2*d*e*n^2*x + (b^2*e^2*n^2*x^2)/4 - 4*b^2*d*e*n*x*Log[
c*x^n] - (b*e^2*n*x^2*(a + b*Log[c*x^n]))/2 + 2*d*e*x*(a + b*Log[c*x^n])^2
+ (e^2*x^2*(a + b*Log[c*x^n])^2)/2 + (d^2*(a + b*Log[c*x^n])^3)/(3*b*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int \frac{(d+ex)\,(a+b\log(cx^n))^2}{x}\,dx + e\int (d+ex)\,(a+b\log(cx^n))^2\,dx\\
&= d^2\int \frac{(a+b\log(cx^n))^2}{x}\,dx + e\int \left(d(a+b\log(cx^n))^2 + ex(a+b\log(cx^n))^2\right)\,dx\\
&\quad + (de)\int (a+b\log(cx^n))^2\,dx\\
&= dex(a+b\log(cx^n))^2 + (de)\int (a+b\log(cx^n))^2\,dx + e^2\int x(a+b\log(cx^n))^2\,dx\\
&\quad + \frac{d^2\text{Subst}\left(\int x^2\,dx, x, a+b\log(cx^n)\right)}{bn} - (2bden)\int (a+b\log(cx^n))\,dx\\
&= -2abdenx + 2dex(a+b\log(cx^n))^2 + \frac{1}{2}e^2x^2(a+b\log(cx^n))^2 + \frac{d^2(a+b\log(cx^n))^3}{3bn}\\
&\quad -(2bden)\int (a+b\log(cx^n))\,dx - \left(2b^2den\right)\int \log(cx^n)\,dx - \left(be^2n\right)\int x(a+b\log(cx^n))\,dx\\
&= -4abdenx + 2b^2den^2x + \frac{1}{4}b^2e^2n^2x^2 - 2b^2denx\log(cx^n)\\
&\quad - \frac{1}{2}be^2nx^2(a+b\log(cx^n)) + 2dex(a+b\log(cx^n))^2\\
&\quad + \frac{1}{2}e^2x^2(a+b\log(cx^n))^2 + \frac{d^2(a+b\log(cx^n))^3}{3bn} - \left(2b^2den\right)\int \log(cx^n)\,dx\\
&= -4abdenx + 4b^2den^2x + \frac{1}{4}b^2e^2n^2x^2 - 4b^2denx\log(cx^n) - \frac{1}{2}be^2nx^2(a+b\log(cx^n))\\
&\quad + 2dex(a+b\log(cx^n))^2 + \frac{1}{2}e^2x^2(a+b\log(cx^n))^2 + \frac{d^2(a+b\log(cx^n))^3}{3bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x} \, dx = \frac{1}{4}be^2nx^2(-2a+bn-2b\log(cx^n)) + 2dex(a+b\log(cx^n))^2 + \frac{1}{2}e^2x^2(a+b\log(cx^n))^2 + \frac{d^2(a+b\log(cx^n))^3}{3bn} - 4bdenx(a-bn+b\log(cx^n))$$

```
[In] Integrate[((d + e*x)^2*(a + b*Log[c*x^n])^2)/x,x]
```

```
[Out] (b*e^2*n*x^2*(-2*a + b*n - 2*b*Log[c*x^n]))/4 + 2*d*e*x*(a + b*Log[c*x^n])^
2 + (e^2*x^2*(a + b*Log[c*x^n])^2)/2 + (d^2*(a + b*Log[c*x^n])^3)/(3*b*n) -
 4*b*d*e*n*x*(a - b*n + b*Log[c*x^n])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.63

method	result
parallelrisch	$\frac{6x^2\ln(cx^n)^2b^2e^2n-6x^2\ln(cx^n)b^2e^2n^2+3x^2b^2e^2n^3+12x^2\ln(cx^n)ab\,e^2n-6x^2ab\,e^2n^2+24x\ln(cx^n)^2b^2den-48x\ln(cx^n)b^2de\,n}{12n}$
risch	Expression too large to display

```
[In] int((e*x+d)^2*(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(6*x^2*ln(c*x^n)^2*b^2*e^2*n-6*x^2*ln(c*x^n)*b^2*e^2*n^2+3*x^2*b^2*e^2
*n^3+12*x^2*ln(c*x^n)*a*b*e^2*n-6*x^2*a*b*e^2*n^2+24*x*ln(c*x^n)^2*b^2*d*e*
n-48*x*ln(c*x^n)*b^2*d*e*n^2+48*x*b^2*d*e*n^3+6*x^2*a^2*e^2*n+48*x*ln(c*x^n
)*a*b*d*e*n-48*x*a*b*d*e*n^2+4*b^2*d^2*ln(c*x^n)^3+12*ln(x)*a^2*d^2*n+24*x*
a^2*d*e*n+12*a*b*d^2*ln(c*x^n)^2)/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(129) = 258.

Time = 0.29 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.14

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x} dx$$
$$= \frac{1}{3} b^2 d^2 n^2 \log(x)^3 + \frac{1}{4} \left(b^2 e^2 n^2 - 2 abe^2 n + 2 a^2 e^2\right) x^2 + \frac{1}{2} \left(b^2 e^2 x^2 + 4 b^2 dex\right) \log(c)^2$$
$$+ \frac{1}{2} \left(b^2 e^2 n^2 x^2 + 4 b^2 den^2 x + 2 b^2 d^2 n \log(c) + 2 abd^2 n\right) \log(x)^2$$
$$+ 2 \left(2 b^2 den^2 - 2 abden + a^2 de\right) x - \frac{1}{2} \left(\left(b^2 e^2 n - 2 abe^2\right) x^2 + 8 \left(b^2 den - abde\right) x\right) \log(c)$$
$$+ \frac{1}{2} \left(2 b^2 d^2 \log(c)^2 + 2 a^2 d^2 - \left(b^2 e^2 n^2 - 2 abe^2 n\right) x^2 - 8 \left(b^2 den^2 - abden\right) x + 2 \left(b^2 e^2 n x^2 + 4 b^2 denx + 2 c\right.\right.$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] 1/3*b^2*d^2*n^2*log(x)^3 + 1/4*(b^2*e^2*n^2 - 2*a*b*e^2*n + 2*a^2*e^2)*x^2
+ 1/2*(b^2*e^2*x^2 + 4*b^2*d*e*x)*log(c)^2 + 1/2*(b^2*e^2*n^2*x^2 + 4*b^2*d
*e*n^2*x + 2*b^2*d^2*n*log(c) + 2*a*b*d^2*n)*log(x)^2 + 2*(2*b^2*d*e*n^2 -
2*a*b*d*e*n + a^2*d*e)*x - 1/2*((b^2*e^2*n - 2*a*b*e^2)*x^2 + 8*(b^2*d*e*n
- a*b*d*e)*x)*log(c) + 1/2*(2*b^2*d^2*log(c)^2 + 2*a^2*d^2 - (b^2*e^2*n^2 -
 2*a*b*e^2*n)*x^2 - 8*(b^2*d*e*n^2 - a*b*d*e*n)*x + 2*(b^2*e^2*n*x^2 + 4*b^
2*d*e*n*x + 2*a*b*d^2)*log(c))*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.96

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x} dx$$
$$= \begin{cases} \frac{a^2 d^2 \log(cx^n)}{n} + 2a^2 dex + \frac{a^2 e^2 x^2}{2} + \frac{abd^2 \log(cx^n)^2}{n} - 4abdenx + 4abdex \log(cx^n) - \frac{abe^2 n x^2}{2} + abe^2 x^2 \log(cx^n) \\ (a+b\log(c))^2 \left(d^2 \log(x) + 2dex + \frac{e^2 x^2}{2}\right) \end{cases}$$

```
[In] integrate((e*x+d)**2*(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Piecewise((a**2*d**2*log(c*x**n)/n + 2*a**2*d*e*x + a**2*e**2*x**2/2 + a*b*
d**2*log(c*x**n)**2/n - 4*a*b*d*e*n*x + 4*a*b*d*e*x*log(c*x**n) - a*b*e**2*
n*x**2/2 + a*b*e**2*x**2*log(c*x**n) + b**2*d**2*log(c*x**n)**3/(3*n) + 4*b
**2*d*e*n**2*x - 4*b**2*d*e*n*x*log(c*x**n) + 2*b**2*d*e*x*log(c*x**n)**2 +
 b**2*e**2*n**2*x**2/4 - b**2*e**2*n*x**2*log(c*x**n)/2 + b**2*e**2*x**2*lo
g(c*x**n)**2/2, Ne(n, 0)), ((a + b*log(c))**2*(d**2*log(x) + 2*d*e*x + e**2
*x**2/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.45

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x} dx = \frac{1}{2} b^2e^2x^2 \log(cx^n)^2 - \frac{1}{2} abe^2nx^2 + abe^2x^2 \log(cx^n) + 2 b^2dex \log(cx^n)^2 - 4 abdenx + \frac{1}{2} a^2e^2x^2 + 4 abdex \log(cx^n) + \frac{b^2d^2 \log(cx^n)^3}{3 n} + 4 \left(n^2x - nx \log(cx^n)\right) b^2de + \frac{1}{4} \left(n^2x^2 - 2 nx^2 \log(cx^n)\right) b^2e^2 + 2 a^2dex + \frac{abd^2 \log(cx^n)^2}{n} + a^2d^2 \log(x)$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*e^2*x^2*log(c*x^n)^2 - 1/2*a*b*e^2*n*x^2 + a*b*e^2*x^2*log(c*x^n) +
 2*b^2*d*e*x*log(c*x^n)^2 - 4*a*b*d*e*n*x + 1/2*a^2*e^2*x^2 + 4*a*b*d*e*x*l
og(c*x^n) + 1/3*b^2*d^2*log(c*x^n)^3/n + 4*(n^2*x - n*x*log(c*x^n))*b^2*d*e
 + 1/4*(n^2*x^2 - 2*n*x^2*log(c*x^n))*b^2*e^2 + 2*a^2*d*e*x + a*b*d^2*log(c
*x^n)^2/n + a^2*d^2*log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(129) = 258.

Time = 0.39 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.08

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x} dx = \frac{1}{3} b^2d^2n^2 \log(x)^3 + \frac{1}{4} \left(b^2e^2n^2 - 2 b^2e^2n \log(c) + 2 b^2e^2 \log(c)^2 - 2 abe^2n + 4 abe^2 \log(c) + 2 a^2e^2\right) x^2 + \frac{1}{2} \left(b^2e^2n^2x^2 + 4 b^2den^2x + 2 b^2d^2n \log(c) + 2 abd^2n\right) \log(x)^2 + 2 \left(2 b^2den^2 - 2 b^2den \log(c) + b^2de \log(c)^2 - 2 abden + 2 abde \log(c) + a^2de\right) x + \left(b^2d^2 \log(c)^2 + 2 abd^2 \log(c) + a^2d^2\right) \log(x) - \frac{1}{2} \left(\left(b^2e^2n^2 - 2 b^2e^2n \log(c) - 2 abe^2n\right) x^2 + 8 \left(b^2den^2 - b^2den \log(c) - abden\right) x\right) \log(x)$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] 1/3*b^2*d^2*n^2*log(x)^3 + 1/4*(b^2*e^2*n^2 - 2*b^2*e^2*n*log(c) + 2*b^2*e^
2*log(c)^2 - 2*a*b*e^2*n + 4*a*b*e^2*log(c) + 2*a^2*e^2)*x^2 + 1/2*(b^2*e^2
```

```
*n^2*x^2 + 4*b^2*d*e*n^2*x + 2*b^2*d^2*n*log(c) + 2*a*b*d^2*n)*log(x)^2 + 2
*(2*b^2*d*e*n^2 - 2*b^2*d*e*n*log(c) + b^2*d*e*log(c)^2 - 2*a*b*d*e*n + 2*a
*b*d*e*log(c) + a^2*d*e)*x + (b^2*d^2*log(c)^2 + 2*a*b*d^2*log(c) + a^2*d^2
)*log(x) - 1/2*((b^2*e^2*n^2 - 2*b^2*e^2*n*log(c) - 2*a*b*e^2*n)*x^2 + 8*(b
^2*d*e*n^2 - b^2*d*e*n*log(c) - a*b*d*e*n)*x)*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^2\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx = \ln\left(c\,x^n\right)^2\left(\frac{b^2\,e^2\,x^2}{2}+2\,b^2\,d\,e\,x+\frac{a\,b\,d^2}{n}\right) + \ln\left(c\,x^n\right)\left(\frac{b\left(2\,a-b\,n\right)\,e^2\,x^2}{2}+4\,b\,d\left(a-b\,n\right)\,e\,x\right) + a^2\,d^2\,\ln\left(x\right)+\frac{e^2\,x^2\left(2\,a^2-2\,a\,b\,n+b^2\,n^2\right)}{4} + 2\,d\,e\,x\left(a^2-2\,a\,b\,n+2\,b^2\,n^2\right)+\frac{b^2\,d^2\ln\left(c\,x^n\right)^3}{3\,n}$$

```
[In] int(((a + b*log(c*x^n))^2*(d + e*x)^2)/x,x)
```

```
[Out] log(c*x^n)^2*((b^2*e^2*x^2)/2 + 2*b^2*d*e*x + (a*b*d^2)/n) + log(c*x^n)*((b
*e^2*x^2*(2*a - b*n))/2 + 4*b*d*e*x*(a - b*n)) + a^2*d^2*log(x) + (e^2*x^2*
(2*a^2 + b^2*n^2 - 2*a*b*n))/4 + 2*d*e*x*(a^2 + 2*b^2*n^2 - 2*a*b*n) + (b^2
*d^2*log(c*x^n)^3)/(3*n)
```

3.88 $\int \frac{(d+ex)^2(a+b\log(cx^n))^2}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 133

$$\int \frac{(d+ex)^2\,(a+b\log{(cx^n)})^2}{x^2}\,dx = -\frac{2b^2d^2n^2}{x} - 2abe^2nx + 2b^2e^2n^2x - 2b^2e^2nx\log{(cx^n)} - \frac{2bd^2n(a+b\log{(cx^n)})}{x} - \frac{d^2(a+b\log{(cx^n)})^2}{x} + e^2x(a+b\log{(cx^n)})^2 + \frac{2de(a+b\log{(cx^n)})^3}{3bn}$$

```
[Out] -2*b^2*d^2*n^2/x-2*a*b*e^2*n*x+2*b^2*e^2*n^2*x-2*b^2*e^2*n*x*ln(c*x^n)-2*b*
d^2*n*(a+b*ln(c*x^n))/x-d^2*(a+b*ln(c*x^n))^2/x+e^2*x*(a+b*ln(c*x^n))^2+2/3
*d*e*(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2395, 2333, 2332, 2342, 2341, 2339, 30}

$$\int \frac{(d+ex)^2\,(a+b\log{(cx^n)})^2}{x^2}\,dx = -\frac{d^2(a+b\log{(cx^n)})^2}{x} - \frac{2bd^2n(a+b\log{(cx^n)})}{x} + \frac{2de(a+b\log{(cx^n)})^3}{3bn} + e^2x(a+b\log{(cx^n)})^2 - 2abe^2nx - 2b^2e^2nx\log{(cx^n)} - \frac{2b^2d^2n^2}{x} + 2b^2e^2n^2x$$

```
[In] Int[((d + e*x)^2*(a + b*Log[c*x^n])^2)/x^2,x]
```

```
[Out] (-2*b^2*d^2*n^2)/x - 2*a*b*e^2*n*x + 2*b^2*e^2*n^2*x - 2*b^2*e^2*n*x*Log[c*
x^n] - (2*b*d^2*n*(a + b*Log[c*x^n]))/x - (d^2*(a + b*Log[c*x^n])^2)/x + e^
2*x*(a + b*Log[c*x^n])^2 + (2*d*e*(a + b*Log[c*x^n])^3)/(3*b*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(e^2(a+b\log(cx^n))^2+\frac{d^2(a+b\log(cx^n))^2}{x^2}+\frac{2de(a+b\log(cx^n))^2}{x}\right)dx\\
&= d^2\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx+(2de)\int \frac{(a+b\log(cx^n))^2}{x}\,dx+e^2\int (a+b\log(cx^n))^2\,dx\\
&= -\frac{d^2(a+b\log(cx^n))^2}{x}+e^2x(a+b\log(cx^n))^2\\
&\quad+\frac{(2de)\operatorname{Subst}\left(\int x^2\,dx,x,a+b\log(cx^n)\right)}{bn}\\
&\quad+\left(2bd^2n\right)\int \frac{a+b\log(cx^n)}{x^2}\,dx-\left(2be^2n\right)\int (a+b\log(cx^n))\,dx\\
&= -\frac{2b^2d^2n^2}{x}-2abe^2nx-\frac{2bd^2n(a+b\log(cx^n))}{x}-\frac{d^2(a+b\log(cx^n))^2}{x}\\
&\quad+e^2x(a+b\log(cx^n))^2+\frac{2de(a+b\log(cx^n))^3}{3bn}-\left(2b^2e^2n\right)\int \log(cx^n)\,dx\\
&= -\frac{2b^2d^2n^2}{x}-2abe^2nx+2b^2e^2n^2x-2b^2e^2nx\log(cx^n)-\frac{2bd^2n(a+b\log(cx^n))}{x}\\
&\quad-\frac{d^2(a+b\log(cx^n))^2}{x}+e^2x(a+b\log(cx^n))^2+\frac{2de(a+b\log(cx^n))^3}{3bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.80

$$\begin{aligned}
\int \frac{(d+ex)^2(a+b\log(cx^n))^2}{x^2}\,dx &= -\frac{d^2(a+b\log(cx^n))^2}{x}+e^2x(a+b\log(cx^n))^2\\
&\quad+\frac{2de(a+b\log(cx^n))^3}{3bn}-2be^2nx(a-bn+b\log(cx^n))\\
&\quad-\frac{2bd^2n(a+bn+b\log(cx^n))}{x}
\end{aligned}$$

```
[In] Integrate[((d + e*x)^2*(a + b*Log[c*x^n])^2)/x^2,x]
```

```
[Out] -((d^2*(a + b*Log[c*x^n])^2)/x) + e^2*x*(a + b*Log[c*x^n])^2 + (2*d*e*(a +
b*Log[c*x^n])^3)/(3*b*n) - 2*b*e^2*n*x*(a - b*n + b*Log[c*x^n]) - (2*b*d^2*
n*(a + b*n + b*Log[c*x^n]))/x
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.70

method	result
parallelrisch	$\frac{3x^2 \ln(c\,x^n)^2 b^2 e^2 n - 6x^2 \ln(c\,x^n) b^2 e^2 n^2 + 6x^2 b^2 e^2 n^3 + 6x^2 \ln(c\,x^n) ab\,e^2 n - 6x^2 ab\,e^2 n^2 + 2de\,b^2 \ln(c\,x^n)^3 x + 6\ln(x) x\,a^2 den + 3x^2}{3xn}$
risch	Expression too large to display

```
[In] int((e*x+d)^2*(a+b*ln(c*x^n))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/x*(3*x^2*ln(c*x^n)^2*b^2*e^2*n-6*x^2*ln(c*x^n)*b^2*e^2*n^2+6*x^2*b^2*e^
2*n^3+6*x^2*ln(c*x^n)*a*b*e^2*n-6*x^2*a*b*e^2*n^2+2*d*e*b^2*ln(c*x^n)^3*x+6
*ln(x)*x*a^2*d*e*n+3*x^2*a^2*e^2*n+6*a*d*e*b*ln(c*x^n)^2*x-3*ln(c*x^n)^2*b^
2*d^2*n-6*ln(c*x^n)*b^2*d^2*n^2-6*b^2*d^2*n^3-6*ln(c*x^n)*a*b*d^2*n-6*a*b*d
^2*n^2-3*a^2*d^2*n)/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(131) = 262.

Time = 0.29 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.19

$$\int \frac{(d+ex)^2\,(a+b\log(cx^n))^2}{x^2}\,dx$$
$$= \frac{2\,b^2den^2x\log(x)^3 - 6\,b^2d^2n^2 - 6\,abd^2n - 3\,a^2d^2 + 3\,(2\,b^2e^2n^2 - 2\,abe^2n + a^2e^2)x^2 + 3\,(b^2e^2x^2 - b^2d^2)\log}{}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*b^2*d*e*n^2*x*log(x)^3 - 6*b^2*d^2*n^2 - 6*a*b*d^2*n - 3*a^2*d^2 + 3
*(2*b^2*e^2*n^2 - 2*a*b*e^2*n + a^2*e^2)*x^2 + 3*(b^2*e^2*x^2 - b^2*d^2)*lo
g(c)^2 + 3*(b^2*e^2*n^2*x^2 + 2*b^2*d*e*n*x*log(c) - b^2*d^2*n^2 + 2*a*b*d*
e*n*x)*log(x)^2 - 6*(b^2*d^2*n + a*b*d^2 + (b^2*e^2*n - a*b*e^2)*x^2)*log(c
) + 6*(b^2*d*e*x*log(c)^2 - b^2*d^2*n^2 - a*b*d^2*n + a^2*d*e*x - (b^2*e^2*
n^2 - a*b*e^2*n)*x^2 + (b^2*e^2*n*x^2 - b^2*d^2*n + 2*a*b*d*e*x)*log(c))*lo
g(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.92

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^2}\,dx$$
$$= \begin{cases} -\frac{a^2d^2}{x} + \frac{2a^2de\log(cx^n)}{n} + a^2e^2x - \frac{2abd^2n}{x} - \frac{2abd^2\log(cx^n)}{x} + \frac{2abde\log(cx^n)^2}{n} - 2abe^2nx + 2abe^2x\log(cx^n) - \frac{2b^2d^2}{x} \\ (a+b\log(c))^2\left(-\frac{d^2}{x} + 2de\log(x) + e^2x\right) \end{cases}$$

```
[In] integrate((e*x+d)**2*(a+b*ln(c*x**n))**2/x**2,x)
```

```
[Out] Piecewise((-a**2*d**2/x + 2*a**2*d*e*log(c*x**n)/n + a**2*e**2*x - 2*a*b*d*
*2*n/x - 2*a*b*d**2*log(c*x**n)/x + 2*a*b*d*e*log(c*x**n)**2/n - 2*a*b*e**2
*n*x + 2*a*b*e**2*x*log(c*x**n) - 2*b**2*d**2*n**2/x - 2*b**2*d**2*n*log(c*
x**n)/x - b**2*d**2*log(c*x**n)**2/x + 2*b**2*d*e*log(c*x**n)**3/(3*n) + 2*
b**2*e**2*n**2*x - 2*b**2*e**2*n*x*log(c*x**n) + b**2*e**2*x*log(c*x**n)**2
, Ne(n, 0)), ((a + b*log(c))**2*(-d**2/x + 2*d*e*log(x) + e**2*x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^2}\,dx = b^2e^2x\log(cx^n)^2 - 2\,abe^2nx + 2\,abe^2x\log(cx^n) + \frac{2\,b^2de\log(cx^n)^3}{3\,n} + 2\left(n^2x - nx\log(cx^n)\right)b^2e^2 - 2\,b^2d^2\left(\frac{n^2}{x} + \frac{n\log(cx^n)}{x}\right) + a^2e^2x + \frac{2\,abde\log(cx^n)^2}{n} - \frac{b^2d^2\log(cx^n)^2}{x} + 2\,a^2de\log(x) - \frac{2\,abd^2n}{x} - \frac{2\,abd^2\log(cx^n)}{x} - \frac{a^2d^2}{x}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] b^2*e^2*x*log(c*x^n)^2 - 2*a*b*e^2*n*x + 2*a*b*e^2*x*log(c*x^n) + 2/3*b^2*d
*e*log(c*x^n)^3/n + 2*(n^2*x - n*x*log(c*x^n))*b^2*e^2 - 2*b^2*d^2*(n^2/x +
 n*log(c*x^n)/x) + a^2*e^2*x + 2*a*b*d*e*log(c*x^n)^2/n - b^2*d^2*log(c*x^n
)^2/x + 2*a^2*d*e*log(x) - 2*a*b*d^2*n/x - 2*a*b*d^2*log(c*x^n)/x - a^2*d^2
/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(131) = 262$.

Time = 0.36 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.06

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^2}\,dx = \frac{2}{3}\,b^2den^2\log(x)^3 + 2\,b^2den\log(c)\log(x)^2 + \left(x\log(x)^2 - 2\,x\log(x) + 2\,x\right)b^2e^2n^2 - b^2d^2n^2\left(\frac{\log(x)^2}{x} + \frac{2\,\log(x)}{x} + \frac{2}{x}\right) + 2\,(x\log(x) - x)b^2e^2n\log(c) - 2\,b^2d^2n\left(\frac{\log(x)}{x} + \frac{1}{x}\right)\log(c) + b^2e^2x\log(c)^2 + 2\,abden\log(x)^2 + 2\,b^2de\log(c)^2\log(|x|) + 2\,(x\log(x) - x)abe^2n - 2\,abd^2n\left(\frac{\log(x)}{x} + \frac{1}{x}\right) + 2\,abe^2x\log(c) + 4\,abde\log(c)\log(|x|) + a^2e^2x - \frac{b^2d^2\log(c)^2}{x} + 2\,a^2de\log(|x|) - \frac{2\,abd^2\log(c)}{x} - \frac{a^2d^2}{x}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2/x^2,x, algorithm="giac")
```

```
[Out] 2/3*b^2*d*e*n^2*log(x)^3 + 2*b^2*d*e*n*log(c)*log(x)^2 + (x*log(x)^2 - 2*x*
log(x) + 2*x)*b^2*e^2*n^2 - b^2*d^2*n^2*(log(x)^2/x + 2*log(x)/x + 2/x) + 2
*(x*log(x) - x)*b^2*e^2*n*log(c) - 2*b^2*d^2*n*(log(x)/x + 1/x)*log(c) + b^
2*e^2*x*log(c)^2 + 2*a*b*d*e*n*log(x)^2 + 2*b^2*d*e*log(c)^2*log(abs(x)) +
2*(x*log(x) - x)*a*b*e^2*n - 2*a*b*d^2*n*(log(x)/x + 1/x) + 2*a*b*e^2*x*log
(c) + 4*a*b*d*e*log(c)*log(abs(x)) + a^2*e^2*x - b^2*d^2*log(c)^2/x + 2*a^2
*d*e*log(abs(x)) - 2*a*b*d^2*log(c)/x - a^2*d^2/x
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.71

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^2}\,dx = \ln(x)\,\left(2\,d\,e\,a^2 + 4\,d\,e\,a\,b\,n + 4\,d\,e\,b^2\,n^2\right) - \frac{a^2\,d^2 + 2\,a\,b\,d^2\,n + 2\,b^2\,d^2\,n^2}{x} - \ln(c\,x^n)\,\left(\frac{2\,b\,(a+b\,n)\,d^2 + 4\,b\,(a+b\,n)\,d\,e\,x + 2\,b\,(a-b\,n)\,e^2\,x^2}{x} - 4\,b\,e^2\,x\,(a-b\,n)\right) + \ln(c\,x^n)^2\left(2\,b^2\,e^2\,x - \frac{b^2\,d^2 + 2\,b^2\,d\,e\,x + b^2\,e^2\,x^2}{x} + \frac{2\,b\,d\,e\,(a+b\,n)}{n}\right) + e^2\,x\,\left(a^2 - 2\,a\,b\,n + 2\,b^2\,n^2\right) + \frac{2\,b^2\,d\,e\ln(c\,x^n)^3}{3\,n}$$

```
[In] int(((a + b*log(c*x^n))^2*(d + e*x)^2)/x^2,x)

[Out] log(x)*(2*a^2*d*e + 4*b^2*d*e*n^2 + 4*a*b*d*e*n) - (a^2*d^2 + 2*b^2*d^2*n^2
 + 2*a*b*d^2*n)/x - log(c*x^n)*((2*b*d^2*(a + b*n) + 2*b*e^2*x^2*(a - b*n)
+ 4*b*d*e*x*(a + b*n))/x - 4*b*e^2*x*(a - b*n)) + log(c*x^n)^2*(2*b^2*e^2*x
 - (b^2*d^2 + b^2*e^2*x^2 + 2*b^2*d*e*x)/x + (2*b*d*e*(a + b*n))/n) + e^2*x
*(a^2 + 2*b^2*n^2 - 2*a*b*n) + (2*b^2*d*e*log(c*x^n)^3)/(3*n)
```

3.89 $\int \frac{(d+ex)^2(a+b\log(cx^n))^2}{x^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^3} dx = -\frac{b^2d^2n^2}{4x^2} - \frac{4b^2den^2}{x} - \frac{bd^2n(a+b\log(cx^n))}{2x^2} - \frac{4bden(a+b\log(cx^n))}{x} - \frac{d^2(a+b\log(cx^n))^2}{2x^2} - \frac{2de(a+b\log(cx^n))^2}{x} + \frac{e^2(a+b\log(cx^n))^3}{3bn}$$

```
[Out] -1/4*b^2*d^2*n^2/x^2-4*b^2*d*e*n^2/x-1/2*b*d^2*n*(a+b*ln(c*x^n))/x^2-4*b*d*
e*n*(a+b*ln(c*x^n))/x-1/2*d^2*(a+b*ln(c*x^n))^2/x^2-2*d*e*(a+b*ln(c*x^n))^2
/x+1/3*e^2*(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2395, 2342, 2341, 2339, 30}

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^3} dx = -\frac{d^2(a+b\log(cx^n))^2}{2x^2} - \frac{bd^2n(a+b\log(cx^n))}{2x^2} - \frac{2de(a+b\log(cx^n))^2}{x} - \frac{4bden(a+b\log(cx^n))}{x} + \frac{e^2(a+b\log(cx^n))^3}{3bn} - \frac{b^2d^2n^2}{4x^2} - \frac{4b^2den^2}{x}$$

```
[In] Int[((d + e*x)^2*(a + b*Log[c*x^n])^2)/x^3,x]
```

```
[Out] -1/4*(b^2*d^2*n^2)/x^2 - (4*b^2*d*e*n^2)/x - (b*d^2*n*(a + b*Log[c*x^n]))/(
2*x^2) - (4*b*d*e*n*(a + b*Log[c*x^n]))/x - (d^2*(a + b*Log[c*x^n])^2)/(2*x
^2) - (2*d*e*(a + b*Log[c*x^n])^2)/x + (e^2*(a + b*Log[c*x^n])^3)/(3*b*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d^2(a+b\log(cx^n))^2}{x^3}+\frac{2de(a+b\log(cx^n))^2}{x^2}+\frac{e^2(a+b\log(cx^n))^2}{x}\right)dx \\
&= d^2\int \frac{(a+b\log(cx^n))^2}{x^3}\,dx+(2de)\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx+e^2\int \frac{(a+b\log(cx^n))^2}{x}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))^2}{2x^2}-\frac{2de(a+b\log(cx^n))^2}{x}+\frac{e^2\text{Subst}\left(\int x^2\,dx,x,a+b\log(cx^n)\right)}{bn} \\
&\quad +\left(bd^2n\right)\int \frac{a+b\log(cx^n)}{x^3}\,dx+(4bden)\int \frac{a+b\log(cx^n)}{x^2}\,dx
\end{aligned}
$$

$$= -\frac{b^2d^2n^2}{4x^2} - \frac{4b^2den^2}{x} - \frac{bd^2n(a+b\log{(cx^n)})}{2x^2} - \frac{4bden(a+b\log{(cx^n)})}{x} - \frac{d^2(a+b\log{(cx^n)})^2}{2x^2} - \frac{2de(a+b\log{(cx^n)})^2}{x} + \frac{e^2(a+b\log{(cx^n)})^3}{3bn}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^2 (a+b\log{(cx^n)})^2}{x^3}\,dx = -\frac{d^2(a+b\log{(cx^n)})^2}{2x^2} - \frac{2de(a+b\log{(cx^n)})^2}{x} + \frac{e^2(a+b\log{(cx^n)})^3}{3bn} - \frac{4bden(a+bn+b\log{(cx^n)})}{x} - \frac{bd^2n(2a+bn+2b\log{(cx^n)})}{4x^2}$$

```
[In] Integrate[((d + e*x)^2*(a + b*Log[c*x^n])^2)/x^3,x]
```

```
[Out] -1/2*(d^2*(a + b*Log[c*x^n])^2)/x^2 - (2*d*e*(a + b*Log[c*x^n])^2)/x + (e^2
*(a + b*Log[c*x^n])^3)/(3*b*n) - (4*b*d*e*n*(a + b*n + b*Log[c*x^n]))/x - (
b*d^2*n*(2*a + b*n + 2*b*Log[c*x^n]))/(4*x^2)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.58

method	result
parallelrisch	$\frac{4e^2b^2\ln(cx^n)^3x^2+12\ln(x)x^2a^2e^2n+12ae^2b\ln(cx^n)^2x^2-24x\ln(cx^n)^2b^2den-48x\ln(cx^n)b^2den^2-48xb^2den^3-48x\ln(cx^n}{12x^2n}$
risch	Expression too large to display

```
[In] int((e*x+d)^2*(a+b*ln(c*x^n))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/x^2*(4*e^2*b^2*ln(c*x^n)^3*x^2+12*ln(x)*x^2*a^2*e^2*n+12*a*e^2*b*ln(c*
x^n)^2*x^2-24*x*ln(c*x^n)^2*b^2*d*e*n-48*x*ln(c*x^n)*b^2*d*e*n^2-48*x*b^2*d
*e*n^3-48*x*ln(c*x^n)*a*b*d*e*n-48*x*a*b*d*e*n^2-6*ln(c*x^n)^2*b^2*d^2*n-6*
ln(c*x^n)*b^2*d^2*n^2-3*b^2*d^2*n^3-24*x*a^2*d*e*n-12*ln(c*x^n)*a*b*d^2*n-6
*a*b*d^2*n^2-6*a^2*d^2*n)/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(129) = 258.

Time = 0.31 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.12

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^3} dx$$
$$= \frac{4 b^2e^2n^2x^2 \log(x)^3 - 3 b^2d^2n^2 - 6 abd^2n - 6 a^2d^2 - 6 (4 b^2dex + b^2d^2) \log(c)^2 + 6 (2 b^2e^2nx^2 \log(c) - 4 b^2de}{}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^2*e^2*n^2*x^2*log(x)^3 - 3*b^2*d^2*n^2 - 6*a*b*d^2*n - 6*a^2*d^2
- 6*(4*b^2*d*e*x + b^2*d^2)*log(c)^2 + 6*(2*b^2*e^2*n*x^2*log(c) - 4*b^2*d*
e*n^2*x + 2*a*b*e^2*n*x^2 - b^2*d^2*n^2)*log(x)^2 - 24*(2*b^2*d*e*n^2 + 2*a
*b*d*e*n + a^2*d*e)*x - 6*(b^2*d^2*n + 2*a*b*d^2 + 8*(b^2*d*e*n + a*b*d*e)*
x)*log(c) + 6*(2*b^2*e^2*x^2*log(c)^2 - b^2*d^2*n^2 + 2*a^2*e^2*x^2 - 2*a*b
*d^2*n - 8*(b^2*d*e*n^2 + a*b*d*e*n)*x - 2*(4*b^2*d*e*n*x - 2*a*b*e^2*x^2 +
 b^2*d^2*n)*log(c))*log(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^3} dx = -\frac{a^2d^2}{2x^2} - \frac{2a^2de}{x} + a^2e^2\log(x) - \frac{abd^2n}{2x^2} - \frac{abd^2\log(cx^n)}{x^2} - \frac{4abden}{x} - \frac{4abde\log(cx^n)}{x} - 2abe^2\left(\begin{cases} -\log(c)\log(x) & \text{for } n=0 \\ -\frac{\log(cx^n)^2}{2n} & \text{otherwise} \end{cases}\right) - \frac{b^2d^2n^2}{4x^2} - \frac{b^2d^2n\log(cx^n)}{2x^2} - \frac{b^2d^2\log(cx^n)^2}{2x^2} - \frac{4b^2den^2}{x} - \frac{4b^2den\log(cx^n)}{x} - \frac{2b^2de\log(cx^n)^2}{x} - b^2e^2\left(\begin{cases} -\log(c)^2\log(x) & \text{for } n=0 \\ -\frac{\log(cx^n)^3}{3n} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x+d)**2*(a+b*ln(c*x**n))**2/x**3,x)
```

```
[Out] -a**2*d**2/(2*x**2) - 2*a**2*d*e/x + a**2*e**2*log(x) - a*b*d**2*n/(2*x**2)
 - a*b*d**2*log(c*x**n)/x**2 - 4*a*b*d*e*n/x - 4*a*b*d*e*log(c*x**n)/x - 2*
a*b*e**2*Piecewise((-log(c)*log(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), True
```

```
)) - b**2*d**2*n**2/(4*x**2) - b**2*d**2*n*log(c*x**n)/(2*x**2) - b**2*d**2
*log(c*x**n)**2/(2*x**2) - 4*b**2*d*e*n**2/x - 4*b**2*d*e*n*log(c*x**n)/x -
 2*b**2*d*e*log(c*x**n)**2/x - b**2*e**2*Piecewise((-log(c)**2*log(x), Eq(n
, 0)), (-log(c*x**n)**3/(3*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^3}\,dx = \frac{b^2e^2\log(cx^n)^3}{3n} - 4b^2de\left(\frac{n^2}{x} + \frac{n\log(cx^n)}{x}\right) - \frac{1}{4}b^2d^2\left(\frac{n^2}{x^2} + \frac{2n\log(cx^n)}{x^2}\right) + \frac{abe^2\log(cx^n)^2}{n} - \frac{2b^2de\log(cx^n)^2}{x} + a^2e^2\log(x) - \frac{4abden}{x} - \frac{4abde\log(cx^n)}{x} - \frac{b^2d^2\log(cx^n)^2}{2x^2} - \frac{abd^2n}{2x^2} - \frac{2a^2de}{x} - \frac{abd^2\log(cx^n)}{x^2} - \frac{a^2d^2}{2x^2}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*e^2*log(c*x^n)^3/n - 4*b^2*d*e*(n^2/x + n*log(c*x^n)/x) - 1/4*b^2*d
^2*(n^2/x^2 + 2*n*log(c*x^n)/x^2) + a*b*e^2*log(c*x^n)^2/n - 2*b^2*d*e*log(
c*x^n)^2/x + a^2*e^2*log(x) - 4*a*b*d*e*n/x - 4*a*b*d*e*log(c*x^n)/x - 1/2*
b^2*d^2*log(c*x^n)^2/x^2 - 1/2*a*b*d^2*n/x^2 - 2*a^2*d*e/x - a*b*d^2*log(c*
x^n)/x^2 - 1/2*a^2*d^2/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(129) = 258$.

Time = 0.36 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.12

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^3}\,dx = \frac{1}{3}\,b^2e^2n^2\log(x)^3 + b^2e^2n\log(c)\log(x)^2 - 2\,b^2den^2\left(\frac{\log(x)^2}{x} + \frac{2\,\log(x)}{x} + \frac{2}{x}\right) - \frac{1}{4}\,b^2d^2n^2\left(\frac{2\,\log(x)^2}{x^2} + \frac{2\,\log(x)}{x^2} + \frac{1}{x^2}\right) - 4\,b^2den\left(\frac{\log(x)}{x} + \frac{1}{x}\right)\log(c) - \frac{1}{2}\,b^2d^2n\left(\frac{2\,\log(x)}{x^2} + \frac{1}{x^2}\right)\log(c) + abe^2n\log(x)^2 + b^2e^2\log(c)^2\log(|x|) - 4\,abden\left(\frac{\log(x)}{x} + \frac{1}{x}\right) - \frac{1}{2}\,abd^2n\left(\frac{2\,\log(x)}{x^2} + \frac{1}{x^2}\right) + 2\,abe^2\log(c)\log(|x|) - \frac{2\,b^2de\log(c)^2}{x} + a^2e^2\log(|x|) - \frac{4\,abde\log(c)}{x} - \frac{b^2d^2\log(c)^2}{2\,x^2} - \frac{2\,a^2de}{x} - \frac{abd^2\log(c)}{x^2} - \frac{a^2d^2}{2\,x^2}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2/x^3,x, algorithm="giac")
```

```
[Out] 1/3*b^2*e^2*n^2*log(x)^3 + b^2*e^2*n*log(c)*log(x)^2 - 2*b^2*d*e*n^2*(log(x
)^2/x + 2*log(x)/x + 2/x) - 1/4*b^2*d^2*n^2*(2*log(x)^2/x^2 + 2*log(x)/x^2
+ 1/x^2) - 4*b^2*d*e*n*(log(x)/x + 1/x)*log(c) - 1/2*b^2*d^2*n*(2*log(x)/x^
2 + 1/x^2)*log(c) + a*b*e^2*n*log(x)^2 + b^2*e^2*log(c)^2*log(abs(x)) - 4*a
*b*d*e*n*(log(x)/x + 1/x) - 1/2*a*b*d^2*n*(2*log(x)/x^2 + 1/x^2) + 2*a*b*e^
2*log(c)*log(abs(x)) - 2*b^2*d*e*log(c)^2/x + a^2*e^2*log(abs(x)) - 4*a*b*d
*e*log(c)/x - 1/2*b^2*d^2*log(c)^2/x^2 - 2*a^2*d*e/x - a*b*d^2*log(c)/x^2 -
 1/2*a^2*d^2/x^2
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^3}\,dx$$
$$= \ln(x) \left(a^2 e^2 + 3\,a\,b\,e^2\,n + \frac{9\,b^2\,e^2\,n^2}{2}\right) - \frac{x\,(4\,d\,e\,a^2 + 8\,d\,e\,a\,b\,n + 8\,d\,e\,b^2\,n^2) + a^2\,d^2 + \frac{b^2\,d^2\,n^2}{2} + a\,b\,d^2\,n}{2\,x^2} - \ln(c\,x^n)^2 \left(\frac{\frac{b^2\,d^2}{2} + 2\,b^2\,d\,e\,x + \frac{3\,b^2\,e^2\,x^2}{2}}{x^2} - \frac{b\,e^2\,(2\,a+3\,b\,n)}{2\,n}\right) - \frac{\ln(c\,x^n)\,\left(\frac{b\,(2\,a+b\,n)\,d^2}{2} + 4\,b\,(a+b\,n)\,d\,e\,x + \frac{3\,b\,(2\,a+3\,b\,n)\,e^2\,x^2}{2}\right)}{x^2} + \frac{b^2\,e^2\,\ln(c\,x^n)^3}{3\,n}$$

```
[In] int(((a + b*log(c*x^n))^2*(d + e*x)^2)/x^3,x)
```

```
[Out] log(x)*(a^2*e^2 + (9*b^2*e^2*n^2)/2 + 3*a*b*e^2*n) - (x*(4*a^2*d*e + 8*b^2*
d*e*n^2 + 8*a*b*d*e*n) + a^2*d^2 + (b^2*d^2*n^2)/2 + a*b*d^2*n)/(2*x^2) - l
og(c*x^n)^2*(((b^2*d^2)/2 + (3*b^2*e^2*x^2)/2 + 2*b^2*d*e*x)/x^2 - (b*e^2*(
2*a + 3*b*n))/(2*n)) - (log(c*x^n)*((b*d^2*(2*a + b*n))/2 + (3*b*e^2*x^2*(2
*a + 3*b*n))/2 + 4*b*d*e*x*(a + b*n)))/x^2 + (b^2*e^2*log(c*x^n)^3)/(3*n)
```

3.90 $\int \frac{(d+ex)^2(a+b\log(cx^n))^2}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 168

$$\int \frac{(d+ex)^2\,(a+b\log(cx^n))^2}{x^4}\,dx = -\frac{2b^2d^2n^2}{27x^3}-\frac{b^2den^2}{2x^2}-\frac{2b^2e^2n^2}{x}-\frac{2bd^2n(a+b\log(cx^n))}{9x^3}-\frac{bden(a+b\log(cx^n))}{x^2}-\frac{2be^2n(a+b\log(cx^n))}{x}-\frac{d^2(a+b\log(cx^n))^2}{3x^3}-\frac{de(a+b\log(cx^n))^2}{x^2}-\frac{e^2(a+b\log(cx^n))^2}{x}$$

```
[Out] -2/27*b^2*d^2*n^2/x^3-1/2*b^2*d*e*n^2/x^2-2*b^2*e^2*n^2/x-2/9*b*d^2*n*(a+b*
ln(c*x^n))/x^3-b*d*e*n*(a+b*ln(c*x^n))/x^2-2*b*e^2*n*(a+b*ln(c*x^n))/x-1/3*
d^2*(a+b*ln(c*x^n))^2/x^3-d*e*(a+b*ln(c*x^n))^2/x^2-e^2*(a+b*ln(c*x^n))^2/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2395, 2342, 2341}

$$\int \frac{(d+ex)^2\,(a+b\log(cx^n))^2}{x^4}\,dx = -\frac{d^2(a+b\log(cx^n))^2}{3x^3}-\frac{2bd^2n(a+b\log(cx^n))}{9x^3}-\frac{de(a+b\log(cx^n))^2}{x^2}-\frac{bden(a+b\log(cx^n))}{x^2}-\frac{e^2(a+b\log(cx^n))^2}{x}-\frac{2be^2n(a+b\log(cx^n))}{x}-\frac{2b^2d^2n^2}{27x^3}-\frac{b^2den^2}{2x^2}-\frac{2b^2e^2n^2}{x}$$

```
[In] Int[((d + e*x)^2*(a + b*Log[c*x^n])^2)/x^4,x]
```

```
[Out] (-2*b^2*d^2*n^2)/(27*x^3) - (b^2*d*e*n^2)/(2*x^2) - (2*b^2*e^2*n^2)/x - (2*
b*d^2*n*(a + b*Log[c*x^n]))/(9*x^3) - (b*d*e*n*(a + b*Log[c*x^n]))/x^2 - (2
*b*e^2*n*(a + b*Log[c*x^n]))/x - (d^2*(a + b*Log[c*x^n])^2)/(3*x^3) - (d*e*
(a + b*Log[c*x^n])^2)/x^2 - (e^2*(a + b*Log[c*x^n])^2)/x
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d^2(a+b\log(cx^n))^2}{x^4}+\frac{2de(a+b\log(cx^n))^2}{x^3}+\frac{e^2(a+b\log(cx^n))^2}{x^2}\right)dx\\
&= d^2\int \frac{(a+b\log(cx^n))^2}{x^4}\,dx+(2de)\int \frac{(a+b\log(cx^n))^2}{x^3}\,dx+e^2\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx\\
&= -\frac{d^2(a+b\log(cx^n))^2}{3x^3}-\frac{de(a+b\log(cx^n))^2}{x^2}\\
&\quad -\frac{e^2(a+b\log(cx^n))^2}{x}+\frac{1}{3}(2bd^2n)\int \frac{a+b\log(cx^n)}{x^4}\,dx\\
&\quad +(2bden)\int \frac{a+b\log(cx^n)}{x^3}\,dx+(2be^2n)\int \frac{a+b\log(cx^n)}{x^2}\,dx\\
&= -\frac{2b^2d^2n^2}{27x^3}-\frac{b^2den^2}{2x^2}-\frac{2b^2e^2n^2}{x}-\frac{2bd^2n(a+b\log(cx^n))}{9x^3}-\frac{bden(a+b\log(cx^n))}{x^2}\\
&\quad -\frac{2be^2n(a+b\log(cx^n))}{x}-\frac{d^2(a+b\log(cx^n))^2}{3x^3}-\frac{de(a+b\log(cx^n))^2}{x^2}-\frac{e^2(a+b\log(cx^n))^2}{x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^2\,(a+b\log{(cx^n)})^2}{x^4}\,dx =$$
$$-\frac{18d^2(a+b\log{(cx^n)})^2+54dex(a+b\log{(cx^n)})^2+54e^2x^2(a+b\log{(cx^n)})^2+108be^2nx^2(a+bn+b\log{(}}{54x^3}$$

```
[In] Integrate[((d + e*x)^2*(a + b*Log[c*x^n])^2)/x^4,x]
```

```
[Out] -1/54*(18*d^2*(a + b*Log[c*x^n])^2 + 54*d*e*x*(a + b*Log[c*x^n])^2 + 54*e^2
*x^2*(a + b*Log[c*x^n])^2 + 108*b*e^2*n*x^2*(a + b*n + b*Log[c*x^n]) + 27*b
*d*e*n*x*(2*a + b*n + 2*b*Log[c*x^n]) + 4*b*d^2*n*(3*a + b*n + 3*b*Log[c*x^
n]))/x^3
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.42

method	result
parallelrisch	$-\frac{54b^2\ln(cx^n)^2e^2x^2+108x^2\ln(cx^n)b^2e^2n+108b^2e^2n^2x^2+108ab\ln(cx^n)e^2x^2+108bnx^2ae^2+54b^2\ln(cx^n)^2dex+54b^2denx\ln(}{}$
risch	Expression too large to display

```
[In] int((e*x+d)^2*(a+b*ln(c*x^n))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/54/x^3*(54*b^2*ln(c*x^n)^2*e^2*x^2+108*x^2*ln(c*x^n)*b^2*e^2*n+108*b^2*e
^2*n^2*x^2+108*a*b*ln(c*x^n)*e^2*x^2+108*b*n*x^2*a*e^2+54*b^2*ln(c*x^n)^2*d
*e*x+54*b^2*d*e*n*x*ln(c*x^n)+27*b^2*d*e*n^2*x+54*a^2*e^2*x^2+108*a*b*ln(c*
x^n)*d*e*x+54*a*b*d*e*n*x+18*b^2*ln(c*x^n)^2*d^2+12*ln(c*x^n)*n*b^2*d^2+4*b
^2*d^2*n^2+54*a^2*d*e*x+36*a*b*ln(c*x^n)*d^2+12*b*d^2*n*a+18*a^2*d^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(160) = 320.

Time = 0.31 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.94

$$\int \frac{(d+ex)^2\,(a+b\log{(cx^n)})^2}{x^4}\,dx =$$
$$-\frac{4\,b^2d^2n^2+12\,abd^2n+18\,a^2d^2+54\,(2\,b^2e^2n^2+2\,abe^2n+a^2e^2)x^2+18\,(3\,b^2e^2x^2+3\,b^2dex+b^2d^2)\log{(c)}}{}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/54*(4*b^2*d^2*n^2 + 12*a*b*d^2*n + 18*a^2*d^2 + 54*(2*b^2*e^2*n^2 + 2*a*
b*e^2*n + a^2*e^2)*x^2 + 18*(3*b^2*e^2*x^2 + 3*b^2*d*e*x + b^2*d^2)*log(c)^
2 + 18*(3*b^2*e^2*n^2*x^2 + 3*b^2*d*e*n^2*x + b^2*d^2*n^2)*log(x)^2 + 27*(b
^2*d*e*n^2 + 2*a*b*d*e*n + 2*a^2*d*e)*x + 6*(2*b^2*d^2*n + 6*a*b*d^2 + 18*(
b^2*e^2*n + a*b*e^2)*x^2 + 9*(b^2*d*e*n + 2*a*b*d*e)*x)*log(c) + 6*(2*b^2*d
^2*n^2 + 6*a*b*d^2*n + 18*(b^2*e^2*n^2 + a*b*e^2*n)*x^2 + 9*(b^2*d*e*n^2 +
2*a*b*d*e*n)*x + 6*(3*b^2*e^2*n*x^2 + 3*b^2*d*e*n*x + b^2*d^2*n)*log(c))*lo
g(x))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.71

$$\int \frac{(d+ex)^2\,(a+b\log{(cx^n)})^2}{x^4}\,dx = -\frac{a^2d^2}{3x^3} - \frac{a^2de}{x^2} - \frac{a^2e^2}{x} - \frac{2abd^2n}{9x^3} - \frac{2abd^2\log{(cx^n)}}{3x^3} - \frac{abden}{x^2} - \frac{2abde\log{(cx^n)}}{x^2} - \frac{2abe^2n}{x} - \frac{2abe^2\log{(cx^n)}}{x} - \frac{2b^2d^2n^2}{27x^3} - \frac{2b^2d^2n\log{(cx^n)}}{9x^3} - \frac{b^2d^2\log{(cx^n)}^2}{3x^3} - \frac{b^2den^2}{2x^2} - \frac{b^2den\log{(cx^n)}}{x^2} - \frac{b^2de\log{(cx^n)}^2}{x^2} - \frac{2b^2e^2n^2}{x} - \frac{2b^2e^2n\log{(cx^n)}}{x} - \frac{b^2e^2\log{(cx^n)}^2}{x}$$

```
[In] integrate((e*x+d)**2*(a+b*ln(c*x**n))**2/x**4,x)
```

```
[Out] -a**2*d**2/(3*x**3) - a**2*d*e/x**2 - a**2*e**2/x - 2*a*b*d**2*n/(9*x**3) -
 2*a*b*d**2*log(c*x**n)/(3*x**3) - a*b*d*e*n/x**2 - 2*a*b*d*e*log(c*x**n)/x
**2 - 2*a*b*e**2*n/x - 2*a*b*e**2*log(c*x**n)/x - 2*b**2*d**2*n**2/(27*x**3
) - 2*b**2*d**2*n*log(c*x**n)/(9*x**3) - b**2*d**2*log(c*x**n)**2/(3*x**3)
- b**2*d*e*n**2/(2*x**2) - b**2*d*e*n*log(c*x**n)/x**2 - b**2*d*e*log(c*x**
n)**2/x**2 - 2*b**2*e**2*n**2/x - 2*b**2*e**2*n*log(c*x**n)/x - b**2*e**2*l
og(c*x**n)**2/x
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^4} dx = -2b^2e^2\left(\frac{n^2}{x} + \frac{n\log(cx^n)}{x}\right) - \frac{1}{2}b^2de\left(\frac{n^2}{x^2} + \frac{2n\log(cx^n)}{x^2}\right) - \frac{2}{27}b^2d^2\left(\frac{n^2}{x^3} + \frac{3n\log(cx^n)}{x^3}\right) - \frac{b^2e^2\log(cx^n)^2}{x} - \frac{2abe^2n}{x} - \frac{2abe^2\log(cx^n)}{x} - \frac{b^2de\log(cx^n)^2}{x^2} - \frac{abden}{x^2} - \frac{a^2e^2}{x} - \frac{2abde\log(cx^n)}{x^2} - \frac{b^2d^2\log(cx^n)^2}{3x^3} - \frac{2abd^2n}{9x^3} - \frac{a^2de}{x^2} - \frac{2abd^2\log(cx^n)}{3x^3} - \frac{a^2d^2}{3x^3}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2/x^4,x, algorithm="maxima")
```

```
[Out] -2*b^2*e^2*(n^2/x + n*log(c*x^n)/x) - 1/2*b^2*d*e*(n^2/x^2 + 2*n*log(c*x^n)
/x^2) - 2/27*b^2*d^2*(n^2/x^3 + 3*n*log(c*x^n)/x^3) - b^2*e^2*log(c*x^n)^2/
x - 2*a*b*e^2*n/x - 2*a*b*e^2*log(c*x^n)/x - b^2*d*e*log(c*x^n)^2/x^2 - a*b
*d*e*n/x^2 - a^2*e^2/x - 2*a*b*d*e*log(c*x^n)/x^2 - 1/3*b^2*d^2*log(c*x^n)^
2/x^3 - 2/9*a*b*d^2*n/x^3 - a^2*d*e/x^2 - 2/3*a*b*d^2*log(c*x^n)/x^3 - 1/3*
a^2*d^2/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(160) = 320.

Time = 0.36 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.11

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^4} dx = -\frac{(3b^2e^2n^2x^2 + 3b^2den^2x + b^2d^2n^2)\log(x)^2}{3x^3} - \frac{(18b^2e^2n^2x^2 + 18b^2e^2nx^2\log(c) + 9b^2den^2x + 18abe^2nx^2 + 18b^2denx\log(c) + 2b^2d^2n^2 + 18abdenx + \cdots}{9x^3} - \frac{108b^2e^2n^2x^2 + 108b^2e^2nx^2\log(c) + 54b^2e^2x^2\log(c)^2 + 27b^2den^2x + 108abe^2nx^2 + 54b^2denx\log(c) + \cdots}{\cdots}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*e^2*n^2*x^2 + 3*b^2*d*e*n^2*x + b^2*d^2*n^2)*log(x)^2/x^3 - 1/9
*(18*b^2*e^2*n^2*x^2 + 18*b^2*e^2*n*x^2*log(c) + 9*b^2*d*e*n^2*x + 18*a*b*e
^2*n*x^2 + 18*b^2*d*e*n*x*log(c) + 2*b^2*d^2*n^2 + 18*a*b*d*e*n*x + 6*b^2*d
^2*n*log(c) + 6*a*b*d^2*n)*log(x)/x^3 - 1/54*(108*b^2*e^2*n^2*x^2 + 108*b^2
*e^2*n*x^2*log(c) + 54*b^2*e^2*x^2*log(c)^2 + 27*b^2*d*e*n^2*x + 108*a*b*e^
```

```
2*n*x^2 + 54*b^2*d*e*n*x*log(c) + 108*a*b*e^2*x^2*log(c) + 54*b^2*d*e*x*log
(c)^2 + 4*b^2*d^2*n^2 + 54*a*b*d*e*n*x + 54*a^2*e^2*x^2 + 12*b^2*d^2*n*log(
c) + 108*a*b*d*e*x*log(c) + 18*b^2*d^2*log(c)^2 + 12*a*b*d^2*n + 54*a^2*d*e
*x + 36*a*b*d^2*log(c) + 18*a^2*d^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^2\,(a+b\log(cx^n))^2}{x^4}\,dx = -\frac{x\left(9\,d\,e\,a^2+9\,d\,e\,a\,b\,n+\frac{9\,d\,e\,b^2\,n^2}{2}\right)+x^2\,\left(9\,a^2\,e^2+18\,a\,b\,e^2\,n+18\,b^2\,e^2\,n^2\right)+3\,a^2\,d^2+\frac{2\,b^2\,d^2\,n^2}{3}+2\,a\,b\,d^2}{9\,x^3} - \frac{\ln\left(c\,x^n\right)^2\,\left(\frac{b^2\,d^2}{3}+b^2\,d\,e\,x+b^2\,e^2\,x^2\right)}{x^3} - \frac{\ln\left(c\,x^n\right)\,\left(\frac{2\,b\,\left(3\,a+b\,n\right)\,d^2}{3}+3\,b\,\left(2\,a+b\,n\right)\,d\,e\,x+6\,b\,\left(a+b\,n\right)\,e^2\,x^2\right)}{3\,x^3}$$

```
[In] int(((a + b*log(c*x^n))^2*(d + e*x)^2)/x^4,x)
```

```
[Out] - (x*(9*a^2*d*e + (9*b^2*d*e*n^2)/2 + 9*a*b*d*e*n) + x^2*(9*a^2*e^2 + 18*b^
2*e^2*n^2 + 18*a*b*e^2*n) + 3*a^2*d^2 + (2*b^2*d^2*n^2)/3 + 2*a*b*d^2*n)/(9
*x^3) - (log(c*x^n)^2*((b^2*d^2)/3 + b^2*e^2*x^2 + b^2*d*e*x))/x^3 - (log(c
*x^n)*((2*b*d^2*(3*a + b*n))/3 + 6*b*e^2*x^2*(a + b*n) + 3*b*d*e*x*(2*a + b
*n)))/(3*x^3)
```

3.91 $\int \frac{(d+ex)^2(a+b\log(cx^n))^2}{x^5} dx$

Optimal result

Integrand size = 23, antiderivative size = 178

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^5} dx = -\frac{b^2d^2n^2}{32x^4} - \frac{4b^2den^2}{27x^3} - \frac{b^2e^2n^2}{4x^2} - \frac{bd^2n(a+b\log(cx^n))}{8x^4} - \frac{4bden(a+b\log(cx^n))}{9x^3} - \frac{be^2n(a+b\log(cx^n))}{2x^2} - \frac{d^2(a+b\log(cx^n))^2}{4x^4} - \frac{2de(a+b\log(cx^n))^2}{3x^3} - \frac{e^2(a+b\log(cx^n))^2}{2x^2}$$

```
[Out] -1/32*b^2*d^2*n^2/x^4-4/27*b^2*d*e*n^2/x^3-1/4*b^2*e^2*n^2/x^2-1/8*b*d^2*n*
(a+b*ln(c*x^n))/x^4-4/9*b*d*e*n*(a+b*ln(c*x^n))/x^3-1/2*b*e^2*n*(a+b*ln(c*x
^n))/x^2-1/4*d^2*(a+b*ln(c*x^n))^2/x^4-2/3*d*e*(a+b*ln(c*x^n))^2/x^3-1/2*e^
2*(a+b*ln(c*x^n))^2/x^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2395, 2342, 2341}

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^5} dx = -\frac{d^2(a+b\log(cx^n))^2}{4x^4} - \frac{bd^2n(a+b\log(cx^n))}{8x^4} - \frac{2de(a+b\log(cx^n))^2}{3x^3} - \frac{4bden(a+b\log(cx^n))}{9x^3} - \frac{e^2(a+b\log(cx^n))^2}{2x^2} - \frac{be^2n(a+b\log(cx^n))}{2x^2} - \frac{b^2d^2n^2}{32x^4} - \frac{4b^2den^2}{27x^3} - \frac{b^2e^2n^2}{4x^2}$$

```
[In] Int[((d + e*x)^2*(a + b*Log[c*x^n])^2)/x^5,x]
```

```
[Out] -1/32*(b^2*d^2*n^2)/x^4 - (4*b^2*d*e*n^2)/(27*x^3) - (b^2*e^2*n^2)/(4*x^2)
- (b*d^2*n*(a + b*Log[c*x^n]))/(8*x^4) - (4*b*d*e*n*(a + b*Log[c*x^n]))/(9*
x^3) - (b*e^2*n*(a + b*Log[c*x^n]))/(2*x^2) - (d^2*(a + b*Log[c*x^n])^2)/(4
*x^4) - (2*d*e*(a + b*Log[c*x^n])^2)/(3*x^3) - (e^2*(a + b*Log[c*x^n])^2)/(
2*x^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d^2(a+b\log(cx^n))^2}{x^5} + \frac{2de(a+b\log(cx^n))^2}{x^4} + \frac{e^2(a+b\log(cx^n))^2}{x^3} \right) dx \\
&= d^2 \int \frac{(a+b\log(cx^n))^2}{x^5}\, dx + (2de) \int \frac{(a+b\log(cx^n))^2}{x^4}\, dx + e^2 \int \frac{(a+b\log(cx^n))^2}{x^3}\, dx \\
&= -\frac{d^2(a+b\log(cx^n))^2}{4x^4} - \frac{2de(a+b\log(cx^n))^2}{3x^3} \\
&\quad - \frac{e^2(a+b\log(cx^n))^2}{2x^2} + \frac{1}{2}(bd^2n) \int \frac{a+b\log(cx^n)}{x^5}\, dx \\
&\quad + \frac{1}{3}(4bden) \int \frac{a+b\log(cx^n)}{x^4}\, dx + (be^2n) \int \frac{a+b\log(cx^n)}{x^3}\, dx
\end{aligned}$$

$$= -\frac{b^2d^2n^2}{32x^4} - \frac{4b^2den^2}{27x^3} - \frac{b^2e^2n^2}{4x^2} - \frac{bd^2n(a+b\log(cx^n))}{8x^4} - \frac{4bden(a+b\log(cx^n))}{9x^3} - \frac{be^2n(a+b\log(cx^n))}{2x^2} - \frac{d^2(a+b\log(cx^n))^2}{4x^4} - \frac{2de(a+b\log(cx^n))^2}{3x^3} - \frac{e^2(a+b\log(cx^n))^2}{2x^2}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^2(a+b\log(cx^n))^2}{x^5}\,dx = -\frac{216d^2(a+b\log(cx^n))^2 + 576dex(a+b\log(cx^n))^2 + 432e^2x^2(a+b\log(cx^n))^2 + 216be^2nx^2(2a+bn+2}{864x^4}$$

```
[In] Integrate[((d + e*x)^2*(a + b*Log[c*x^n])^2)/x^5,x]
```

```
[Out] -1/864*(216*d^2*(a + b*Log[c*x^n])^2 + 576*d*e*x*(a + b*Log[c*x^n])^2 + 432
*e^2*x^2*(a + b*Log[c*x^n])^2 + 216*b*e^2*n*x^2*(2*a + b*n + 2*b*Log[c*x^n]
) + 128*b*d*e*n*x*(3*a + b*n + 3*b*Log[c*x^n]) + 27*b*d^2*n*(4*a + b*n + 4*
b*Log[c*x^n]))/x^4
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.34

method	result
parallelrisch	$-\frac{432b^2\ln(cx^n)^2e^2x^2+432x^2\ln(cx^n)b^2e^2n+216b^2e^2n^2x^2+864ab\ln(cx^n)e^2x^2+432bn\,x^2a\,e^2+576b^2\ln(cx^n)^2dex+384b^2denx}{}$
risch	Expression too large to display

```
[In] int((e*x+d)^2*(a+b*ln(c*x^n))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/864/x^4*(432*b^2*ln(c*x^n)^2*e^2*x^2+432*x^2*ln(c*x^n)*b^2*e^2*n+216*b^2
*e^2*n^2*x^2+864*a*b*ln(c*x^n)*e^2*x^2+432*b*n*x^2*a*e^2+576*b^2*ln(c*x^n)^
2*d*e*x+384*b^2*d*e*n*x*ln(c*x^n)+128*b^2*d*e*n^2*x+432*a^2*e^2*x^2+1152*a*
b*ln(c*x^n)*d*e*x+384*a*b*d*e*n*x+216*b^2*ln(c*x^n)^2*d^2+108*ln(c*x^n)*n*b
^2*d^2+27*b^2*d^2*n^2+576*a^2*d*e*x+432*a*b*ln(c*x^n)*d^2+108*b*d^2*n*a+216
*a^2*d^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(160) = 320$.

Time = 0.30 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.87

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^5}\,dx =$$
$$-\frac{27\,b^2d^2n^2+108\,abd^2n+216\,a^2d^2+216\,(b^2e^2n^2+2\,abe^2n+2\,a^2e^2)x^2+72\,(6\,b^2e^2x^2+8\,b^2dex+3\,b^2d^2}{}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/864*(27*b^2*d^2*n^2 + 108*a*b*d^2*n + 216*a^2*d^2 + 216*(b^2*e^2*n^2 + 2
*a*b*e^2*n + 2*a^2*e^2)*x^2 + 72*(6*b^2*e^2*x^2 + 8*b^2*d*e*x + 3*b^2*d^2)*
log(c)^2 + 72*(6*b^2*e^2*n^2*x^2 + 8*b^2*d*e*n^2*x + 3*b^2*d^2*n^2)*log(x)^
2 + 64*(2*b^2*d*e*n^2 + 6*a*b*d*e*n + 9*a^2*d*e)*x + 12*(9*b^2*d^2*n + 36*a
*b*d^2 + 36*(b^2*e^2*n + 2*a*b*e^2)*x^2 + 32*(b^2*d*e*n + 3*a*b*d*e)*x)*log
(c) + 12*(9*b^2*d^2*n^2 + 36*a*b*d^2*n + 36*(b^2*e^2*n^2 + 2*a*b*e^2*n)*x^2
 + 32*(b^2*d*e*n^2 + 3*a*b*d*e*n)*x + 12*(6*b^2*e^2*n*x^2 + 8*b^2*d*e*n*x +
 3*b^2*d^2*n)*log(c))*log(x))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.74

$$\begin{aligned}\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^5}\,dx = & -\frac{a^2d^2}{4x^4}-\frac{2a^2de}{3x^3}-\frac{a^2e^2}{2x^2}-\frac{abd^2n}{8x^4}-\frac{abd^2\log(cx^n)}{2x^4}\\ & -\frac{4abden}{9x^3}-\frac{4abde\log(cx^n)}{3x^3}-\frac{abe^2n}{2x^2}-\frac{abe^2\log(cx^n)}{x^2}\\ & -\frac{b^2d^2n^2}{32x^4}-\frac{b^2d^2n\log(cx^n)}{8x^4}-\frac{b^2d^2\log(cx^n)^2}{4x^4}\\ & -\frac{4b^2den^2}{27x^3}-\frac{4b^2den\log(cx^n)}{9x^3}-\frac{2b^2de\log(cx^n)^2}{3x^3}\\ & -\frac{b^2e^2n^2}{4x^2}-\frac{b^2e^2n\log(cx^n)}{2x^2}-\frac{b^2e^2\log(cx^n)^2}{2x^2}\end{aligned}$$

```
[In] integrate((e*x+d)**2*(a+b*ln(c*x**n))**2/x**5,x)
```

```
[Out] -a**2*d**2/(4*x**4) - 2*a**2*d*e/(3*x**3) - a**2*e**2/(2*x**2) - a*b*d**2*n
/(8*x**4) - a*b*d**2*log(c*x**n)/(2*x**4) - 4*a*b*d*e*n/(9*x**3) - 4*a*b*d*
e*log(c*x**n)/(3*x**3) - a*b*e**2*n/(2*x**2) - a*b*e**2*log(c*x**n)/x**2 -
b**2*d**2*n**2/(32*x**4) - b**2*d**2*n*log(c*x**n)/(8*x**4) - b**2*d**2*log
(c*x**n)**2/(4*x**4) - 4*b**2*d*e*n**2/(27*x**3) - 4*b**2*d*e*n*log(c*x**n)
/(9*x**3) - 2*b**2*d*e*log(c*x**n)**2/(3*x**3) - b**2*e**2*n**2/(4*x**2) -
b**2*e**2*n*log(c*x**n)/(2*x**2) - b**2*e**2*log(c*x**n)**2/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^5} dx = -\frac{1}{4} b^2e^2\left(\frac{n^2}{x^2}+\frac{2\,n\log(cx^n)}{x^2}\right) - \frac{4}{27} b^2de\left(\frac{n^2}{x^3}+\frac{3\,n\log(cx^n)}{x^3}\right) - \frac{1}{32} b^2d^2\left(\frac{n^2}{x^4}+\frac{4\,n\log(cx^n)}{x^4}\right) - \frac{b^2e^2\log(cx^n)^2}{2\,x^2} - \frac{abe^2n}{2\,x^2} - \frac{abe^2\log(cx^n)}{x^2} - \frac{2\,b^2de\log(cx^n)^2}{3\,x^3} - \frac{4\,abden}{9\,x^3} - \frac{a^2e^2}{2\,x^2} - \frac{4\,abde\log(cx^n)}{3\,x^3} - \frac{b^2d^2\log(cx^n)^2}{4\,x^4} - \frac{abd^2n}{8\,x^4} - \frac{2\,a^2de}{3\,x^3} - \frac{abd^2\log(cx^n)}{2\,x^4} - \frac{a^2d^2}{4\,x^4}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*e^2*(n^2/x^2 + 2*n*log(c*x^n)/x^2) - 4/27*b^2*d*e*(n^2/x^3 + 3*n*l
og(c*x^n)/x^3) - 1/32*b^2*d^2*(n^2/x^4 + 4*n*log(c*x^n)/x^4) - 1/2*b^2*e^2*
log(c*x^n)^2/x^2 - 1/2*a*b*e^2*n/x^2 - a*b*e^2*log(c*x^n)/x^2 - 2/3*b^2*d*e
*log(c*x^n)^2/x^3 - 4/9*a*b*d*e*n/x^3 - 1/2*a^2*e^2/x^2 - 4/3*a*b*d*e*log(c
*x^n)/x^3 - 1/4*b^2*d^2*log(c*x^n)^2/x^4 - 1/8*a*b*d^2*n/x^4 - 2/3*a^2*d*e/
x^3 - 1/2*a*b*d^2*log(c*x^n)/x^4 - 1/4*a^2*d^2/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(160) = 320.

Time = 0.31 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.99

$$\int \frac{(d+ex)^2 (a+b\log(cx^n))^2}{x^5} dx = -\frac{(6\,b^2e^2n^2x^2+8\,b^2den^2x+3\,b^2d^2n^2)\log(x)^2}{12\,x^4} - \frac{(36\,b^2e^2n^2x^2+72\,b^2e^2nx^2\log(c)+32\,b^2den^2x+72\,abe^2nx^2+96\,b^2denx\log(c)+9\,b^2d^2n^2+96\,abdenx}{72\,x^4} - \frac{216\,b^2e^2n^2x^2+432\,b^2e^2nx^2\log(c)+432\,b^2e^2x^2\log(c)^2+128\,b^2den^2x+432\,abe^2nx^2+384\,b^2denx\log(c}{}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^2/x^5,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^2*e^2*n^2*x^2 + 8*b^2*d*e*n^2*x + 3*b^2*d^2*n^2)*log(x)^2/x^4 -
1/72*(36*b^2*e^2*n^2*x^2 + 72*b^2*e^2*n*x^2*log(c) + 32*b^2*d*e*n^2*x + 72*
```

```
a*b*e^2*n*x^2 + 96*b^2*d*e*n*x*log(c) + 9*b^2*d^2*n^2 + 96*a*b*d*e*n*x + 36
*b^2*d^2*n*log(c) + 36*a*b*d^2*n)*log(x)/x^4 - 1/864*(216*b^2*e^2*n^2*x^2 +
 432*b^2*e^2*n*x^2*log(c) + 432*b^2*e^2*x^2*log(c)^2 + 128*b^2*d*e*n^2*x +
432*a*b*e^2*n*x^2 + 384*b^2*d*e*n*x*log(c) + 864*a*b*e^2*x^2*log(c) + 576*b
^2*d*e*x*log(c)^2 + 27*b^2*d^2*n^2 + 384*a*b*d*e*n*x + 432*a^2*e^2*x^2 + 10
8*b^2*d^2*n*log(c) + 1152*a*b*d*e*x*log(c) + 216*b^2*d^2*log(c)^2 + 108*a*b
*d^2*n + 576*a^2*d*e*x + 432*a*b*d^2*log(c) + 216*a^2*d^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^2\left(a+b\log\left(cx^n\right)\right)^2}{x^5}\,dx =$$
$$-\frac{x\left(48\,d\,e\,a^2+32\,d\,e\,a\,b\,n+\frac{32\,d\,e\,b^2\,n^2}{3}\right)+x^2\left(36\,a^2\,e^2+36\,a\,b\,e^2\,n+18\,b^2\,e^2\,n^2\right)+18\,a^2\,d^2+\frac{9\,b^2\,d^2\,n^2}{4}+9}{72\,x^4}$$
$$-\frac{\ln\left(c\,x^n\right)^2\left(\frac{b^2\,d^2}{4}+\frac{2\,b^2\,d\,e\,x}{3}+\frac{b^2\,e^2\,x^2}{2}\right)}{x^4}$$
$$-\frac{\ln\left(c\,x^n\right)\left(\frac{3\,b\,(4\,a+b\,n)\,d^2}{4}+\frac{8\,b\,(3\,a+b\,n)\,d\,e\,x}{3}+3\,b\,(2\,a+b\,n)\,e^2\,x^2\right)}{6\,x^4}$$

```
[In] int(((a + b*log(c*x^n))^2*(d + e*x)^2)/x^5,x)
```

```
[Out] - (x*(48*a^2*d*e + (32*b^2*d*e*n^2)/3 + 32*a*b*d*e*n) + x^2*(36*a^2*e^2 + 1
8*b^2*e^2*n^2 + 36*a*b*e^2*n) + 18*a^2*d^2 + (9*b^2*d^2*n^2)/4 + 9*a*b*d^2*
n)/(72*x^4) - (log(c*x^n)^2*((b^2*d^2)/4 + (b^2*e^2*x^2)/2 + (2*b^2*d*e*x)/
3))/x^4 - (log(c*x^n)*((3*b*d^2*(4*a + b*n))/4 + 3*b*e^2*x^2*(2*a + b*n) +
(8*b*d*e*x*(3*a + b*n))/3))/(6*x^4)
```

3.92 $\int \frac{x^3(a+b\log(cx^n))^2}{d+ex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 271

$$\begin{aligned}\int \frac{x^3(a+b\log(cx^n))^2}{d+ex}\,dx = &-\frac{2abd^2nx}{e^3}+\frac{2b^2d^2n^2x}{e^3}-\frac{b^2dn^2x^2}{4e^2}+\frac{2b^2n^2x^3}{27e}-\frac{2b^2d^2nx\log(cx^n)}{e^3}\\&+\frac{bdnx^2(a+b\log(cx^n))}{2e^2}-\frac{2bnx^3(a+b\log(cx^n))}{9e}\\&+\frac{d^2x(a+b\log(cx^n))^2}{e^3}-\frac{dx^2(a+b\log(cx^n))^2}{2e^2}\\&+\frac{x^3(a+b\log(cx^n))^2}{3e}-\frac{d^3(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^4}\\&-\frac{2bd^3n(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4}\\&+\frac{2b^2d^3n^2\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^4}\end{aligned}$$

```
[Out] -2*a*b*d^2*n*x/e^3+2*b^2*d^2*n^2*x/e^3-1/4*b^2*d*n^2*x^2/e^2+2/27*b^2*n^2*x
^3/e-2*b^2*d^2*n*x*ln(c*x^n)/e^3+1/2*b*d*n*x^2*(a+b*ln(c*x^n))/e^2-2/9*b*n*
x^3*(a+b*ln(c*x^n))/e+d^2*x*(a+b*ln(c*x^n))^2/e^3-1/2*d*x^2*(a+b*ln(c*x^n))
^2/e^2+1/3*x^3*(a+b*ln(c*x^n))^2/e-d^3*(a+b*ln(c*x^n))^2*ln(1+e*x/d)/e^4-2*
b*d^3*n*(a+b*ln(c*x^n))*polylog(2,-e*x/d)/e^4+2*b^2*d^3*n^2*polylog(3,-e*x/
d)/e^4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2395, 2333, 2332, 2342, 2341, 2354, 2421, 6724}

$$\int \frac{x^3(a+b\log(cx^n))^2}{d+ex}\,dx = -\frac{2bd^3n\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)(a+b\log(cx^n))}{e^4} - \frac{d^3\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))^2}{e^4} + \frac{d^2x(a+b\log(cx^n))^2}{e^3} - \frac{dx^2(a+b\log(cx^n))^2}{2e^2} + \frac{bdnx^2(a+b\log(cx^n))}{2e^2} + \frac{x^3(a+b\log(cx^n))^2}{3e} - \frac{2bnx^3(a+b\log(cx^n))}{9e} - \frac{2abd^2nx}{e^3} - \frac{2b^2d^2nx\log(cx^n)}{e^3} + \frac{2b^2d^3n^2\,\text{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^4} + \frac{2b^2d^2n^2x}{e^3} - \frac{b^2dn^2x^2}{4e^2} + \frac{2b^2n^2x^3}{27e}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n])^2)/(d + e*x),x]
```

```
[Out] (-2*a*b*d^2*n*x)/e^3 + (2*b^2*d^2*n^2*x)/e^3 - (b^2*d*n^2*x^2)/(4*e^2) + (2
*b^2*n^2*x^3)/(27*e) - (2*b^2*d^2*n*x*Log[c*x^n])/e^3 + (b*d*n*x^2*(a + b*L
og[c*x^n]))/(2*e^2) - (2*b*n*x^3*(a + b*Log[c*x^n]))/(9*e) + (d^2*x*(a + b*
Log[c*x^n])^2)/e^3 - (d*x^2*(a + b*Log[c*x^n])^2)/(2*e^2) + (x^3*(a + b*Log
[c*x^n])^2)/(3*e) - (d^3*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d])/e^4 - (2*b*
d^3*n*(a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)])/e^4 + (2*b^2*d^3*n^2*PolyL
og[3, -((e*x)/d)])/e^4
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2(a+b\log(cx^n))^2}{e^3} - \frac{dx(a+b\log(cx^n))^2}{e^2} + \frac{x^2(a+b\log(cx^n))^2}{e} - \frac{d^3(a+b\log(cx^n))^2}{e^3(d+ex)} \right) dx$$

$$= \frac{d^2 \int (a + b\log(cx^n))^2\, dx}{e^3} - \frac{d^3 \int \frac{(a+b\log(cx^n))^2}{d+ex}\, dx}{e^3} - \frac{d \int x(a + b\log(cx^n))^2\, dx}{e^2} + \frac{\int x^2(a + b\log(cx^n))^2\, dx}{e}$$

$$= \frac{d^2x(a + b\log(cx^n))^2}{e^3} - \frac{dx^2(a + b\log(cx^n))^2}{2e^2} + \frac{x^3(a + b\log(cx^n))^2}{3e} - \frac{d^3(a + b\log(cx^n))^2 \log\left(1 + \frac{ex}{d}\right)}{e^4} + \frac{(2bd^3n) \int \frac{(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{x}\, dx}{e^4} - \frac{(2bd^2n) \int (a + b\log(cx^n))\, dx}{e^3} + \frac{(bdn) \int x(a + b\log(cx^n))\, dx}{e^2} - \frac{(2bn) \int x^2(a + b\log(cx^n))\, dx}{3e}$$

$$= -\frac{2abd^2nx}{e^3} - \frac{b^2dn^2x^2}{4e^2} + \frac{2b^2n^2x^3}{27e} + \frac{bdnx^2(a + b\log(cx^n))}{2e^2} - \frac{2bnx^3(a + b\log(cx^n))}{9e} + \frac{d^2x(a + b\log(cx^n))^2}{e^3} - \frac{dx^2(a + b\log(cx^n))^2}{2e^2} + \frac{x^3(a + b\log(cx^n))^2}{3e} - \frac{d^3(a + b\log(cx^n))^2 \log\left(1 + \frac{ex}{d}\right)}{e^4} - \frac{2bd^3n(a + b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^4} - \frac{(2b^2d^2n) \int \log(cx^n)\, dx}{e^3} + \frac{(2b^2d^3n^2) \int \frac{\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{x}\, dx}{e^4}$$

$$= -\frac{2abd^2nx}{e^3} + \frac{2b^2d^2n^2x}{e^3} - \frac{b^2dn^2x^2}{4e^2} + \frac{2b^2n^2x^3}{27e} - \frac{2b^2d^2nx\log(cx^n)}{e^3} + \frac{bdnx^2(a + b\log(cx^n))}{2e^2} - \frac{2bnx^3(a + b\log(cx^n))}{9e} + \frac{d^2x(a + b\log(cx^n))^2}{e^3} - \frac{dx^2(a + b\log(cx^n))^2}{2e^2} + \frac{x^3(a + b\log(cx^n))^2}{3e} - \frac{d^3(a + b\log(cx^n))^2 \log\left(1 + \frac{ex}{d}\right)}{e^4} - \frac{2bd^3n(a + b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^4} + \frac{2b^2d^3n^2\operatorname{Li}_3\left(-\frac{ex}{d}\right)}{e^4}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.78

$$\int \frac{x^3(a + b\log(cx^n))^2}{d + ex}\, dx = -\frac{-108d^2ex(a + b\log(cx^n))^2 + 54de^2x^2(a + b\log(cx^n))^2 - 36e^3x^3(a + b\log(cx^n))^2 + 216bd^2enx(a - bn}{}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n])^2)/(d + e*x),x]
```

```
[Out] -1/108*(-108*d^2*e*x*(a + b*Log[c*x^n])^2 + 54*d*e^2*x^2*(a + b*Log[c*x^n])
^2 - 36*e^3*x^3*(a + b*Log[c*x^n])^2 + 216*b*d^2*e*n*x*(a - b*n + b*Log[c*x
```

```
^n]) - 8*b*e^3*n*x^3*(b*n - 3*(a + b*Log[c*x^n])) + 27*b*d*e^2*n*x^2*(b*n -
 2*(a + b*Log[c*x^n])) + 108*d^3*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d] + 21
6*b*d^3*n*((a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)] - b*n*PolyLog[3, -((e*
x)/d)]))/e^4
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 732, normalized size of antiderivative = 2.70

method	result
risch	$\frac{b^2 \ln(x^n)^2 x^3}{3e} - \frac{b^2 \ln(x^n)^2 d\, x^2}{2e^2} + \frac{b^2 \ln(x^n)^2 x\, d^2}{e^3} - \frac{b^2 \ln(x^n)^2 d^3 \ln(ex+d)}{e^4} - \frac{2b^2 n \ln(x^n) x^3}{9e} + \frac{b^2 n \ln(x^n) d\, x^2}{2e^2} - \frac{2b^2 n \ln(x^n) x}{e^3}$

```
[In] int(x^3*(a+b*ln(c*x^n))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^2*ln(x^n)^2/e*x^3-1/2*b^2*ln(x^n)^2/e^2*d*x^2+b^2*ln(x^n)^2/e^3*x*d^2
-b^2*ln(x^n)^2*d^3/e^4*ln(e*x+d)-2/9*b^2*n*ln(x^n)/e*x^3+1/2*b^2*n*ln(x^n)/
e^2*d*x^2-2*b^2*n*ln(x^n)/e^3*x*d^2+2/27*b^2*n^2*x^3/e-1/4*b^2*d*n^2*x^2/e^
2+2*b^2*d^2*n^2*x/e^3-2*b^2*d^3/e^4*ln(x)*ln(e*x+d)*ln(-e*x/d)*n^2-2*b^2*d^
3/e^4*ln(x)*dilog(-e*x/d)*n^2+2*b^2*n*d^3/e^4*ln(x^n)*ln(e*x+d)*ln(-e*x/d)+
2*b^2*n*d^3/e^4*ln(x^n)*dilog(-e*x/d)+b^2*d^3/e^4*n^2*ln(e*x+d)*ln(x)^2-b^2
*d^3/e^4*n^2*ln(x)^2*ln(1+e*x/d)-2*b^2*d^3/e^4*n^2*ln(x)*polylog(2,-e*x/d)+
2*b^2*d^3*n^2*polylog(3,-e*x/d)/e^4+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c
*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I
*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(1/3*ln(x^n)/e*x^3-1/2*ln(x^n)/e^2*d
*x^2+ln(x^n)/e^3*x*d^2-ln(x^n)*d^3/e^4*ln(e*x+d)-n*(1/6/e^4*(2/3*(e*x+d)^3-
7/2*d*(e*x+d)^2+11*d^2*(e*x+d))-d^3/e^4*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d)
)))+1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(
I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln
(c)+2*a)^2*(1/e^3*(1/3*e^2*x^3-1/2*d*e*x^2+d^2*x)-d^3/e^4*ln(e*x+d))
```

Fricas [F]

$$\int \frac{x^3(a+b\log{(cx^n)})^2}{d+ex}\,dx = \int \frac{(b\log{(cx^n)}+a)^2x^3}{ex+d}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*log(c*x^n)^2 + 2*a*b*x^3*log(c*x^n) + a^2*x^3)/(e*x + d),
 x)
```

Sympy [F]

$$\int \frac{x^3(a+b\log(cx^n))^2}{d+ex}\,dx = \int \frac{x^3(a+b\log(cx^n))^2}{d+ex}\,dx$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))**2/(e*x+d),x)
```

```
[Out] Integral(x**3*(a + b*log(c*x**n))**2/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x^3(a+b\log(cx^n))^2}{d+ex}\,dx = \int \frac{(b\log(cx^n)+a)^2x^3}{ex+d}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/6*a^2*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) +
 integrate((b^2*x^3*log(x^n)^2 + 2*(b^2*log(c) + a*b)*x^3*log(x^n) + (b^2*l
og(c)^2 + 2*a*b*log(c))*x^3)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^3(a+b\log(cx^n))^2}{d+ex}\,dx = \int \frac{(b\log(cx^n)+a)^2x^3}{ex+d}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x^3/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(cx^n))^2}{d+ex}\,dx = \int \frac{x^3\left(a+b\ln(c\,x^n)\right)^2}{d+e\,x}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n))^2)/(d + e*x),x)
```

```
[Out] int((x^3*(a + b*log(c*x^n))^2)/(d + e*x), x)
```

3.93 $\int \frac{x^2(a+b\log(cx^n))^2}{d+ex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 200

$$\int \frac{x^2(a+b\log(cx^n))^2}{d+ex}\,dx = \frac{2abdnx}{e^2}-\frac{2b^2dn^2x}{e^2}+\frac{b^2n^2x^2}{4e}+\frac{2b^2dnx\log(cx^n)}{e^2} - \frac{bnx^2(a+b\log(cx^n))}{2e}-\frac{dx(a+b\log(cx^n))^2}{e^2} + \frac{x^2(a+b\log(cx^n))^2}{2e}+\frac{d^2(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^3} + \frac{2bd^2n(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^3} - \frac{2b^2d^2n^2\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^3}$$

```
[Out] 2*a*b*d*n*x/e^2-2*b^2*d*n^2*x/e^2+1/4*b^2*n^2*x^2/e+2*b^2*d*n*x*ln(c*x^n)/e
^2-1/2*b*n*x^2*(a+b*ln(c*x^n))/e-d*x*(a+b*ln(c*x^n))^2/e^2+1/2*x^2*(a+b*ln(
c*x^n))^2/e+d^2*(a+b*ln(c*x^n))^2*ln(1+e*x/d)/e^3+2*b*d^2*n*(a+b*ln(c*x^n))
*polylog(2,-e*x/d)/e^3-2*b^2*d^2*n^2*polylog(3,-e*x/d)/e^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used

$= \{2395, 2333, 2332, 2342, 2341, 2354, 2421, 6724\}$

$$\int \frac{x^2(a+b\log(cx^n))^2}{d+ex}\,dx = \frac{2bd^2n\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)(a+b\log(cx^n))}{e^3} + \frac{d^2\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))^2}{e^3} - \frac{dx(a+b\log(cx^n))^2}{e^2} - \frac{bnx^2(a+b\log(cx^n))}{2e} + \frac{x^2(a+b\log(cx^n))^2}{2e} + \frac{2abdnx}{e^2} + \frac{2b^2dnx\log(cx^n)}{e^2} - \frac{2b^2d^2n^2\,\mathrm{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^3} - \frac{2b^2dn^2x}{e^2} + \frac{b^2n^2x^2}{4e}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n])^2)/(d + e*x),x]
```

```
[Out] (2*a*b*d*n*x)/e^2 - (2*b^2*d*n^2*x)/e^2 + (b^2*n^2*x^2)/(4*e) + (2*b^2*d*n*
x*Log[c*x^n])/e^2 - (b*n*x^2*(a + b*Log[c*x^n]))/(2*e) - (d*x*(a + b*Log[c*
x^n])^2)/e^2 + (x^2*(a + b*Log[c*x^n])^2)/(2*e) + (d^2*(a + b*Log[c*x^n])^2
*Log[1 + (e*x)/d])/e^3 + (2*b*d^2*n*(a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d
)])/e^3 - (2*b^2*d^2*n^2*PolyLog[3, -((e*x)/d)])/e^3
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{d(a+b\log(cx^n))^2}{e^2}+\frac{x(a+b\log(cx^n))^2}{e}+\frac{d^2(a+b\log(cx^n))^2}{e^2(d+ex)}\right)dx\\
&= -\frac{d\int (a+b\log(cx^n))^2\,dx}{e^2}+\frac{d^2\int \frac{(a+b\log(cx^n))^2}{d+ex}\,dx}{e^2}+\frac{\int x(a+b\log(cx^n))^2\,dx}{e}\\
&= -\frac{dx(a+b\log(cx^n))^2}{e^2}+\frac{x^2(a+b\log(cx^n))^2}{2e}\\
&\quad+\frac{d^2(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^3}-\frac{(2bd^2n)\int \frac{(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^3}\\
&\quad+\frac{(2bdn)\int (a+b\log(cx^n))\,dx}{e^2}-\frac{(bn)\int x(a+b\log(cx^n))\,dx}{e}
\end{aligned}
$$

$$= \frac{2abdnx}{e^2} + \frac{b^2n^2x^2}{4e} - \frac{bnx^2(a + b\log(cx^n))}{2e} - \frac{dx(a + b\log(cx^n))^2}{e^2} + \frac{x^2(a + b\log(cx^n))^2}{2e} + \frac{d^2(a + b\log(cx^n))^2\log\left(1 + \frac{ex}{d}\right)}{e^3} + \frac{2bd^2n(a + b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^3} + \frac{(2b^2dn)\int\log(cx^n)\,dx}{e^2} - \frac{(2b^2d^2n^2)\int\frac{\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{x}\,dx}{e^3}$$

$$= \frac{2abdnx}{e^2} - \frac{2b^2dn^2x}{e^2} + \frac{b^2n^2x^2}{4e} + \frac{2b^2dnx\log(cx^n)}{e^2} - \frac{bnx^2(a + b\log(cx^n))}{2e} - \frac{dx(a + b\log(cx^n))^2}{e^2} + \frac{x^2(a + b\log(cx^n))^2}{2e} + \frac{d^2(a + b\log(cx^n))^2\log\left(1 + \frac{ex}{d}\right)}{e^3} + \frac{2bd^2n(a + b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^3} - \frac{2b^2d^2n^2\operatorname{Li}_3\left(-\frac{ex}{d}\right)}{e^3}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.79

$$\int \frac{x^2(a + b\log(cx^n))^2}{d + ex}\,dx$$
$$= \frac{-4dex(a + b\log(cx^n))^2 + 2e^2x^2(a + b\log(cx^n))^2 + 8bdenx(a - bn + b\log(cx^n)) + be^2nx^2(bn - 2(a + b\log \cdots}{\cdots}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n])^2)/(d + e*x),x]
```

```
[Out] (-4*d*e*x*(a + b*Log[c*x^n])^2 + 2*e^2*x^2*(a + b*Log[c*x^n])^2 + 8*b*d*e*n
*x*(a - b*n + b*Log[c*x^n]) + b*e^2*n*x^2*(b*n - 2*(a + b*Log[c*x^n])) + 4*
d^2*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d] + 8*b*d^2*n*((a + b*Log[c*x^n])*P
olyLog[2, -((e*x)/d)] - b*n*PolyLog[3, -((e*x)/d)]))/(4*e^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.49 (sec) , antiderivative size = 637, normalized size of antiderivative = 3.18

method	result
risch	$\frac{b^2\ln(x^n)^2x^2}{2e} - \frac{b^2\ln(x^n)^2dx}{e^2} + \frac{b^2\ln(x^n)^2d^2\ln(ex+d)}{e^3} + \frac{2b^2d^2\ln(ex+d)\ln\left(-\frac{ex}{d}\right)\ln(x)n^2}{e^3} - \frac{2b^2n\,d^2\ln(x^n)\ln(ex+d)\ln\left(-\frac{ex}{d}\right)\cdots}{e^3}$

```
[In] int(x^2*(a+b*ln(c*x^n))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2*ln(x^n)^2/e*x^2-b^2*ln(x^n)^2/e^2*d*x+b^2*ln(x^n)^2*d^2/e^3*ln(e*x+
d)+2*b^2*d^2/e^3*ln(e*x+d)*ln(-e*x/d)*ln(x)*n^2-2*b^2*n*d^2/e^3*ln(x^n)*ln(
e*x+d)*ln(-e*x/d)+2*b^2*d^2/e^3*dilog(-e*x/d)*ln(x)*n^2-2*b^2*n*d^2/e^3*ln(
x^n)*dilog(-e*x/d)-b^2*d^2/e^3*n^2*ln(e*x+d)*ln(x)^2+b^2*d^2/e^3*n^2*ln(x)^
2*ln(1+e*x/d)+2*b^2*d^2/e^3*n^2*ln(x)*polylog(2,-e*x/d)-2*b^2*d^2*n^2*polyl
og(3,-e*x/d)/e^3-1/2*b^2*n*ln(x^n)/e*x^2+2*b^2*n*ln(x^n)/e^2*d*x+1/4*b^2*n^
2*x^2/e-2*b^2*d*n^2*x/e^2+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*
Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn
(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(1/2*ln(x^n)/e*x^2-ln(x^n)/e^2*d*x+ln(x^n)*d^2
/e^3*ln(e*x+d)-n*(d^2/e^3*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))+1/2/e^3*(1/2
*(e*x+d)^2-3*d*(e*x+d))))+1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+
I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*
csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*(1/e^2*(1/2*e*x^2-d*x)+d^2/e^3*ln(e*x+d))
```

Fricas [F]

$$\int \frac{x^2(a+b\log(cx^n))^2}{d+ex}\,dx = \int \frac{(b\log(cx^n)+a)^2x^2}{ex+d}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*log(c*x^n)^2 + 2*a*b*x^2*log(c*x^n) + a^2*x^2)/(e*x + d),
 x)
```

Sympy [F]

$$\int \frac{x^2(a+b\log(cx^n))^2}{d+ex}\,dx = \int \frac{x^2(a+b\log(cx^n))^2}{d+ex}\,dx$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2/(e*x+d),x)
```

```
[Out] Integral(x**2*(a + b*log(c*x**n))**2/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x^2(a+b\log(cx^n))^2}{d+ex}\,dx = \int \frac{(b\log(cx^n)+a)^2x^2}{ex+d}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + integrate((b^2*x^2
*log(x^n)^2 + 2*(b^2*log(c) + a*b)*x^2*log(x^n) + (b^2*log(c)^2 + 2*a*b*log
(c))*x^2)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^2(a+b\log(cx^n))^2}{d+ex}\,dx=\int \frac{(b\log(cx^n)+a)^2x^2}{ex+d}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))^2/(e*x+d),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*x^2/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(cx^n))^2}{d+ex}\,dx=\int \frac{x^2\,(a+b\,\ln(c\,x^n))^2}{d+e\,x}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n))^2)/(d + e*x),x)
[Out] int((x^2*(a + b*log(c*x^n))^2)/(d + e*x), x)
```

3.94 $\int \frac{x(a+b\log(cx^n))^2}{d+ex}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 130

$$\int \frac{x(a+b\log(cx^n))^2}{d+ex}\,dx = -\frac{2abnx}{e}+\frac{2b^2n^2x}{e}-\frac{2b^2nx\log(cx^n)}{e}+\frac{x(a+b\log(cx^n))^2}{e}-\frac{d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^2}-\frac{2bdn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^2}+\frac{2b^2dn^2\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^2}$$

```
[Out] -2*a*b*n*x/e+2*b^2*n^2*x/e-2*b^2*n*x*ln(c*x^n)/e+x*(a+b*ln(c*x^n))^2/e-d*(a
+b*ln(c*x^n))^2*ln(1+e*x/d)/e^2-2*b*d*n*(a+b*ln(c*x^n))*polylog(2,-e*x/d)/e
^2+2*b^2*d*n^2*polylog(3,-e*x/d)/e^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2395, 2333, 2332, 2354, 2421, 6724}

$$\int \frac{x(a+b\log(cx^n))^2}{d+ex}\,dx = -\frac{2bdn\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)(a+b\log(cx^n))}{e^2}-\frac{d\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))^2}{e^2}+\frac{x(a+b\log(cx^n))^2}{e}-\frac{2abnx}{e}-\frac{2b^2nx\log(cx^n)}{e}+\frac{2b^2dn^2\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^2}+\frac{2b^2n^2x}{e}$$

```
[In] Int[(x*(a + b*Log[c*x^n])^2)/(d + e*x),x]
```

```
[Out] (-2*a*b*n*x)/e + (2*b^2*n^2*x)/e - (2*b^2*n*x*Log[c*x^n])/e + (x*(a + b*Log
[c*x^n])^2)/e - (d*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d])/e^2 - (2*b*d*n*(a
 + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)])/e^2 + (2*b^2*d*n^2*PolyLog[3, -((e
*x)/d)])/e^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a+b\log(cx^n))^2}{e} - \frac{d(a+b\log(cx^n))^2}{e(d+ex)}\right) dx$$

$$= \frac{\int (a+b\log(cx^n))^2\, dx}{e} - \frac{d\int \frac{(a+b\log(cx^n))^2}{d+ex}\, dx}{e}$$

$$= \frac{x(a+b\log(cx^n))^2}{e} - \frac{d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^2} + \frac{(2bdn)\int \frac{(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{x}\, dx}{e^2} - \frac{(2bn)\int (a+b\log(cx^n))\, dx}{e}$$

$$= -\frac{2abnx}{e} + \frac{x(a+b\log(cx^n))^2}{e} - \frac{d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^2} - \frac{2bdn(a+b\log(cx^n))\,\text{Li}_2\left(-\frac{ex}{d}\right)}{e^2} - \frac{(2b^2n)\int \log(cx^n)\, dx}{e} + \frac{(2b^2dn^2)\int \frac{\text{Li}_2\left(-\frac{ex}{d}\right)}{x}\, dx}{e^2}$$

$$= -\frac{2abnx}{e} + \frac{2b^2n^2x}{e} - \frac{2b^2nx\log(cx^n)}{e} + \frac{x(a+b\log(cx^n))^2}{e} - \frac{d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^2} - \frac{2bdn(a+b\log(cx^n))\,\text{Li}_2\left(-\frac{ex}{d}\right)}{e^2} + \frac{2b^2dn^2\text{Li}_3\left(-\frac{ex}{d}\right)}{e^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

$$\int \frac{x(a+b\log(cx^n))^2}{d+ex}\, dx$$
$$= \frac{ex(a+b\log(cx^n))^2 - 2benx(a-bn+b\log(cx^n)) - d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right) - 2bdn\big((a+b\log(cx^n)\ldots}{e^2}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n])^2)/(d + e*x),x]
```

```
[Out] (e*x*(a + b*Log[c*x^n])^2 - 2*b*e*n*x*(a - b*n + b*Log[c*x^n]) - d*(a + b*L
og[c*x^n])^2*Log[1 + (e*x)/d] - 2*b*d*n*((a + b*Log[c*x^n])*PolyLog[2, -((e
*x)/d)] - b*n*PolyLog[3, -((e*x)/d)]))/e^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 528, normalized size of antiderivative = 4.06

method	result
risch	$\frac{b^2 \ln(x^n)^2 x}{e} - \frac{b^2 \ln(x^n)^2 d \ln(ex+d)}{e^2} - \frac{2b^2 n \ln(x^n) x}{e} + \frac{2b^2 n^2 x}{e} - \frac{2b^2 d \ln(ex+d) \ln(-\frac{ex}{d}) \ln(x) n^2}{e^2} + \frac{2b^2 n d \ln(x^n) \ln(ex+d) \mathrm{l}}{e^2}$

[In] int(x*(a+b*ln(c*x^n))^2/(e*x+d),x,method=_RETURNVERBOSE)

[Out] b^2*ln(x^n)^2/e*x-b^2*ln(x^n)^2*d/e^2*ln(e*x+d)-2*b^2*n*ln(x^n)/e*x+2*b^2*n^2*x/e-2*b^2*d/e^2*ln(e*x+d)*ln(-e*x/d)*ln(x)*n^2+2*b^2*n*d/e^2*ln(x^n)*ln(e*x+d)*ln(-e*x/d)-2*b^2*d/e^2*dilog(-e*x/d)*ln(x)*n^2+2*b^2*n*d/e^2*ln(x^n)*dilog(-e*x/d)+b^2*d/e^2*n^2*ln(e*x+d)*ln(x)^2-b^2*d/e^2*n^2*ln(x)^2*ln(1+e*x/d)-2*b^2*d/e^2*n^2*ln(x)*polylog(2,-e*x/d)+2*b^2*d*n^2*polylog(3,-e*x/d)/e^2+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(ln(x^n)/e*x-ln(x^n)*d/e^2*ln(e*x+d)-n*((e*x+d)/e^2-d/e^2*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))))+1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*(x/e-d/e^2*ln(e*x+d))

Fricas [F]

$$\int \frac{x(a+b\log(cx^n))^2}{d+ex}\,dx = \int \frac{(b\log(cx^n)+a)^2x}{ex+d}\,dx$$

[In] integrate(x*(a+b*log(c*x^n))^2/(e*x+d),x, algorithm="fricas")

[Out] integral((b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)/(e*x + d), x)

Sympy [F]

$$\int \frac{x(a+b\log(cx^n))^2}{d+ex}\,dx = \int \frac{x(a+b\log(cx^n))^2}{d+ex}\,dx$$

[In] integrate(x*(a+b*ln(c*x**n))**2/(e*x+d),x)

[Out] Integral(x*(a + b*log(c*x**n))**2/(d + e*x), x)

Maxima [F]

$$\int \frac{x(a+b\log(cx^n))^2}{d+ex}\,dx = \int \frac{(b\log(cx^n)+a)^2 x}{ex+d}\,dx$$

[In] integrate(x*(a+b*log(c*x^n))^2/(e*x+d),x, algorithm="maxima")

[Out] a^2*(x/e - d*log(e*x + d)/e^2) + integrate((b^2*x*log(x^n)^2 + 2*(b^2*log(c) + a*b)*x*log(x^n) + (b^2*log(c)^2 + 2*a*b*log(c))*x)/(e*x + d), x)

Giac [F]

$$\int \frac{x(a+b\log(cx^n))^2}{d+ex}\,dx = \int \frac{(b\log(cx^n)+a)^2 x}{ex+d}\,dx$$

[In] integrate(x*(a+b*log(c*x^n))^2/(e*x+d),x, algorithm="giac")

[Out] integrate((b*log(c*x^n) + a)^2*x/(e*x + d), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(cx^n))^2}{d+ex}\,dx = \int \frac{x\,(a+b\,\ln(c\,x^n))^2}{d+e\,x}\,dx$$

[In] int((x*(a + b*log(c*x^n))^2)/(d + e*x),x)

[Out] int((x*(a + b*log(c*x^n))^2)/(d + e*x), x)

3.95 $\int \frac{(a+b\log(cx^n))^2}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int \frac{(a+b\log(cx^n))^2}{d+ex}\,dx = \frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e} + \frac{2bn(a+b\log(cx^n))\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e} - \frac{2b^2n^2\,\text{PolyLog}\left(3,-\frac{ex}{d}\right)}{e}$$

```
[Out] (a+b*ln(c*x^n))^2*ln(1+e*x/d)/e+2*b*n*(a+b*ln(c*x^n))*polylog(2,-e*x/d)/e-2
*b^2*n^2*polylog(3,-e*x/d)/e
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2354, 2421, 6724}

$$\int \frac{(a+b\log(cx^n))^2}{d+ex}\,dx = \frac{2bn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)(a+b\log(cx^n))}{e} + \frac{\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))^2}{e} - \frac{2b^2n^2\,\text{PolyLog}\left(3,-\frac{ex}{d}\right)}{e}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(d + e*x),x]
```

```
[Out] ((a + b*Log[c*x^n])^2*Log[1 + (e*x)/d])/e + (2*b*n*(a + b*Log[c*x^n])*PolyL
og[2, -((e*x)/d)])/e - (2*b^2*n^2*PolyLog[3, -((e*x)/d)])/e
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
```

```
Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e} - \frac{(2bn)\int\frac{(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e} \\
&= \frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e} + \frac{2bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e} - \frac{(2b^2n^2)\int\frac{\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{x}\,dx}{e} \\
&= \frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e} + \frac{2bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e} - \frac{2b^2n^2\operatorname{Li}_3\left(-\frac{ex}{d}\right)}{e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\begin{aligned}
&\int\frac{(a+b\log(cx^n))^2}{d+ex}\,dx \\
&= \frac{(a+b\log(cx^n))^2\log\left(\frac{d+ex}{d}\right)}{e} \\
&\quad - \frac{2bn\left(-\left((a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)\right)+bn\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)\right)}{e}
\end{aligned}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(d + e*x),x]
```

```
[Out] ((a + b*Log[c*x^n])^2*Log[(d + e*x)/d])/e - (2*b*n*(-((a + b*Log[c*x^n])*Po
lyLog[2, -((e*x)/d)]) + b*n*PolyLog[3, -((e*x)/d)]))/e
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 445, normalized size of antiderivative = 6.18

method	result
risch	$\frac{b^2\ln(x^n)^2\ln(ex+d)}{e}+\frac{2b^2n^2\ln(x)\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{e}+\frac{2b^2n^2\ln(x)\operatorname{dilog}\left(-\frac{ex}{d}\right)}{e}-\frac{2b^2n\ln(x^n)\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{e}-\frac{2b^2n\ln(x}{}$

```
[In] int((a+b*ln(c*x^n))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*ln(x^n)^2*ln(e*x+d)/e+2*b^2/e*n^2*ln(x)*ln(e*x+d)*ln(-e*x/d)+2*b^2/e*n^
2*ln(x)*dilog(-e*x/d)-2*b^2/e*n*ln(x^n)*ln(e*x+d)*ln(-e*x/d)-2*b^2/e*n*ln(x
^n)*dilog(-e*x/d)-b^2/e*n^2*ln(e*x+d)*ln(x)^2+b^2/e*n^2*ln(x)^2*ln(1+e*x/d)
+2*b^2/e*n^2*ln(x)*polylog(2,-e*x/d)-2*b^2*n^2*polylog(3,-e*x/d)/e+(-I*b*Pi
*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*P
i*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(ln(x
^n)*ln(e*x+d)/e-1/e*n*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d)))+1/4*(-I*b*Pi*cs
gn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*c
sgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*ln(e*x+d
)/e
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2}{d+ex}\,dx=\int \frac{(b\log(cx^n)+a)^2}{ex+d}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2}{d+ex}\,dx=\int \frac{(a+b\log(cx^n))^2}{d+ex}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**2/(e*x+d),x)
```

```
[Out] Integral((a + b*log(c*x**n))**2/(d + e*x), x)
```

Maxima [F]

$$\int \frac{(a + b\log(cx^n))^2}{d + ex} \, dx = \int \frac{(b\log(cx^n) + a)^2}{ex + d} \, dx$$

```
[In] integrate((a+b*log(c*x^n))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*log(e*x + d)/e + integrate((b^2*log(c)^2 + b^2*log(x^n)^2 + 2*a*b*log(c
) + 2*(b^2*log(c) + a*b)*log(x^n))/(e*x + d), x)
```

Giac [F]

$$\int \frac{(a + b\log(cx^n))^2}{d + ex} \, dx = \int \frac{(b\log(cx^n) + a)^2}{ex + d} \, dx$$

```
[In] integrate((a+b*log(c*x^n))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(cx^n))^2}{d + ex} \, dx = \int \frac{(a + b\,\ln(c\,x^n))^2}{d + e\,x} \, dx$$

```
[In] int((a + b*log(c*x^n))^2/(d + e*x),x)
```

```
[Out] int((a + b*log(c*x^n))^2/(d + e*x), x)
```

3.96 $\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx = -\frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d} + \frac{2bn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)}{d} + \frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{d}{ex}\right)}{d}$$

```
[Out] -ln(1+d/e/x)*(a+b*ln(c*x^n))^2/d+2*b*n*(a+b*ln(c*x^n))*polylog(2,-d/e/x)/d+
2*b^2*n^2*polylog(3,-d/e/x)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2379, 2421, 6724}

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx = \frac{2bn\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)(a+b\log(cx^n))}{d} - \frac{\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))^2}{d} + \frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{d}{ex}\right)}{d}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(x*(d + e*x)),x]
```

```
[Out] -((Log[1 + d/(e*x)]*(a + b*Log[c*x^n])^2)/d) + (2*b*n*(a + b*Log[c*x^n])*Po
lyLog[2, -(d/(e*x))])/d + (2*b^2*n^2*PolyLog[3, -(d/(e*x))])/d
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
```

```
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d}+\frac{(2bn)\int\frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{x}\,dx}{d}\\
&= -\frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d}+\frac{2bn(a+b\log(cx^n))\,\mathrm{Li}_2\left(-\frac{d}{ex}\right)}{d}-\frac{(2b^2n^2)\int\frac{\mathrm{Li}_2\left(-\frac{d}{ex}\right)}{x}\,dx}{d}\\
&= -\frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d}+\frac{2bn(a+b\log(cx^n))\,\mathrm{Li}_2\left(-\frac{d}{ex}\right)}{d}+\frac{2b^2n^2\mathrm{Li}_3\left(-\frac{d}{ex}\right)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int\frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx=\frac{(a+b\log(cx^n))^3}{3bdn}-\frac{(a+b\log(cx^n))^2\log\left(\frac{d+ex}{d}\right)}{d}-\frac{2bn\left((a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)-bn\,\mathrm{PolyLog}\left(3,-\frac{ex}{d}\right)\right)}{d}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(x*(d + e*x)),x]
```

```
[Out] (a + b*Log[c*x^n])^3/(3*b*d*n) - ((a + b*Log[c*x^n])^2*Log[(d + e*x)/d])/d
- (2*b*n*((a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)] - b*n*PolyLog[3, -((e*x
)/d)]))/d
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 528, normalized size of antiderivative = 6.68

method	result
risch	$-\frac{b^2 \ln(x^n)^2 \ln(ex+d)}{d} + \frac{b^2 \ln(x^n)^2 \ln(x)}{d} - \frac{b^2 n \ln(x^n) \ln(x)^2}{d} + \frac{b^2 \ln(x)^3 n^2}{3d} - \frac{2b^2 \ln(x) \ln(ex+d) \ln(-\frac{ex}{d}) n^2}{d} - \frac{2b^2 \ln(x) \text{di}$

```
[In] int((a+b*ln(c*x^n))^2/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -b^2*ln(x^n)^2/d*ln(e*x+d)+b^2*ln(x^n)^2/d*ln(x)-b^2*n/d*ln(x^n)*ln(x)^2+1/
3*b^2/d*ln(x)^3*n^2-2*b^2/d*ln(x)*ln(e*x+d)*ln(-e*x/d)*n^2-2*b^2/d*ln(x)*di
log(-e*x/d)*n^2+2*b^2*n/d*ln(x^n)*ln(e*x+d)*ln(-e*x/d)+2*b^2*n/d*ln(x^n)*di
log(-e*x/d)+b^2/d*n^2*ln(e*x+d)*ln(x)^2-b^2/d*n^2*ln(x)^2*ln(1+e*x/d)-2*b^2
/d*n^2*ln(x)*polylog(2,-e*x/d)+2*b^2/d*n^2*polylog(3,-e*x/d)+(-I*b*Pi*csgn(
I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn
(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(-ln(x^n)/d
*ln(e*x+d)+ln(x^n)/d*ln(x)-n*(1/2/d*ln(x)^2-1/d*ln(e*x+d)*ln(-e*x/d)-1/d*di
log(-e*x/d)))+1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(
I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n
)^3+2*b*ln(c)+2*a)^2*(-1/d*ln(e*x+d)+1/d*ln(x))
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e*x^2 + d*x), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx = \int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**2/x/(e*x+d),x)
```

```
[Out] Integral((a + b*log(c*x**n))**2/(x*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -a^2*(log(e*x + d)/d - log(x)/d) + integrate((b^2*log(c)^2 + b^2*log(x^n)^2
 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*log(x^n))/(e*x^2 + d*x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{x\,(d+e\,x)}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(x*(d + e*x)),x)
```

```
[Out] int((a + b*log(c*x^n))^2/(x*(d + e*x)), x)
```

3.97 $\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)}\,dx = -\frac{2b^2n^2}{dx}-\frac{2bn(a+b\log(cx^n))}{dx}-\frac{(a+b\log(cx^n))^2}{dx}+\frac{e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^2}-\frac{2ben(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^2}-\frac{2b^2en^2\operatorname{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^2}$$

```
[Out] -2*b^2*n^2/d/x-2*b*n*(a+b*ln(c*x^n))/d/x-(a+b*ln(c*x^n))^2/d/x+e*ln(1+d/e/x
)*(a+b*ln(c*x^n))^2/d^2-2*b*e*n*(a+b*ln(c*x^n))*polylog(2,-d/e/x)/d^2-2*b^2
*e*n^2*polylog(3,-d/e/x)/d^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2380, 2342, 2341, 2379, 2421, 6724}

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)}\,dx = -\frac{2ben\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)(a+b\log(cx^n))}{d^2}+\frac{e\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))^2}{d^2}-\frac{2bn(a+b\log(cx^n))}{dx}-\frac{(a+b\log(cx^n))^2}{dx}-\frac{2b^2en^2\operatorname{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^2}-\frac{2b^2n^2}{dx}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(x^2*(d + e*x)),x]
```

```
[Out] (-2*b^2*n^2)/(d*x) - (2*b*n*(a + b*Log[c*x^n]))/(d*x) - (a + b*Log[c*x^n])^
2/(d*x) + (e*Log[1 + d/(e*x)]*(a + b*Log[c*x^n])^2)/d^2 - (2*b*e*n*(a + b*L
og[c*x^n])*PolyLog[2, -(d/(e*x))])/d^2 - (2*b^2*e*n^2*PolyLog[3, -(d/(e*x))
])/d^2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx}{d} - \frac{e\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx}{d}$$

$$= -\frac{(a+b\log(cx^n))^2}{dx} + \frac{e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^2} + \frac{(2bn)\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d} - \frac{(2ben)\int \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{x}\,dx}{d^2}$$

$$= -\frac{2b^2n^2}{dx} - \frac{2bn(a+b\log(cx^n))}{dx} - \frac{(a+b\log(cx^n))^2}{dx} + \frac{e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^2} - \frac{2ben(a+b\log(cx^n))\,\text{Li}_2\left(-\frac{d}{ex}\right)}{d^2} + \frac{(2b^2en^2)\int \frac{\text{Li}_2\left(-\frac{d}{ex}\right)}{x}\,dx}{d^2}$$

$$= -\frac{2b^2n^2}{dx} - \frac{2bn(a+b\log(cx^n))}{dx} - \frac{(a+b\log(cx^n))^2}{dx} + \frac{e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^2} - \frac{2ben(a+b\log(cx^n))\,\text{Li}_2\left(-\frac{d}{ex}\right)}{d^2} - \frac{2b^2en^2\text{Li}_3\left(-\frac{d}{ex}\right)}{d^2}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)}\,dx = -\frac{\frac{3d(a+b\log(cx^n))^2}{x} + \frac{e(a+b\log(cx^n))^3}{bn} + \frac{6bdn(a+bn+b\log(cx^n))}{x} - 3e(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right) - 6ben\big((a+b\log}{3d^2}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(x^2*(d + e*x)),x]
```

```
[Out] -1/3*((3*d*(a + b*Log[c*x^n])^2)/x + (e*(a + b*Log[c*x^n])^3)/(b*n) + (6*b*
d*n*(a + b*n + b*Log[c*x^n]))/x - 3*e*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d]
 - 6*b*e*n*((a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)] - b*n*PolyLog[3, -((e
*x)/d)]))/d^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.41 (sec) , antiderivative size = 615, normalized size of antiderivative = 4.56

method	result
risch	$\frac{b^2 \ln(x^n)^2 e \ln(ex+d)}{d^2} - \frac{b^2 \ln(x^n)^2}{dx} - \frac{b^2 \ln(x^n)^2 e \ln(x)}{d^2} + \frac{2b^2 e \ln(x) \ln(ex+d) \ln(-\frac{ex}{d}) n^2}{d^2} + \frac{2b^2 e \ln(x) \operatorname{dilog}(-\frac{ex}{d}) n^2}{d^2} - 2b^2 ne$

```
[In] int((a+b*ln(c*x^n))^2/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*ln(x^n)^2*e/d^2*ln(e*x+d)-b^2*ln(x^n)^2/d/x-b^2*ln(x^n)^2*e/d^2*ln(x)+2
*b^2*e/d^2*ln(x)*ln(e*x+d)*ln(-e*x/d)*n^2+2*b^2*e/d^2*ln(x)*dilog(-e*x/d)*n
^2-2*b^2*n*e/d^2*ln(x^n)*ln(e*x+d)*ln(-e*x/d)-2*b^2*n*e/d^2*ln(x^n)*dilog(-
e*x/d)-b^2*e/d^2*n^2*ln(e*x+d)*ln(x)^2+b^2*e/d^2*n^2*ln(x)^2*ln(1+e*x/d)+2*
b^2*e/d^2*n^2*ln(x)*polylog(2,-e*x/d)-2*b^2*e/d^2*n^2*polylog(3,-e*x/d)-2*b
^2*n*ln(x^n)/d/x-2*b^2*n^2/d/x+b^2*n*e/d^2*ln(x^n)*ln(x)^2-1/3*b^2*e/d^2*ln
(x)^3*n^2+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csg
n(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*
ln(c)+2*a)*b*(ln(x^n)*e/d^2*ln(e*x+d)-ln(x^n)/d/x-ln(x^n)*e/d^2*ln(x)-n*(e/
d^2*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))+1/d/x-1/2*e/d^2*ln(x)^2))+1/4*(-I*
b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I
*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*(
e/d^2*ln(e*x+d)-1/d/x-e/d^2*ln(x))
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e*x^3 + d*x^2), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)}\,dx = \int \frac{(a+b\log(cx^n))^2}{x^2\,(d+ex)}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**2/x**2/(e*x+d),x)
```

```
[Out] Integral((a + b*log(c*x**n))**2/(x**2*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) + integrate((b^2*log(c)^2
 + b^2*log(x^n)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*log(x^n))/(e*x^3 +
d*x^2), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{x^2\,(d+e\,x)}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(x^2*(d + e*x)),x)
```

```
[Out] int((a + b*log(c*x^n))^2/(x^2*(d + e*x)), x)
```

3.98 $\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 204

$$\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)}\,dx = -\frac{b^2n^2}{4dx^2}+\frac{2b^2en^2}{d^2x}-\frac{bn(a+b\log(cx^n))}{2dx^2}+\frac{2ben(a+b\log(cx^n))}{d^2x}-\frac{(a+b\log(cx^n))^2}{2dx^2}+\frac{e(a+b\log(cx^n))^2}{d^2x}-\frac{e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^3}+\frac{2be^2n(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^3}+\frac{2b^2e^2n^2\operatorname{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^3}$$

```
[Out] -1/4*b^2*n^2/d/x^2+2*b^2*e*n^2/d^2/x-1/2*b*n*(a+b*ln(c*x^n))/d/x^2+2*b*e*n*
(a+b*ln(c*x^n))/d^2/x-1/2*(a+b*ln(c*x^n))^2/d/x^2+e*(a+b*ln(c*x^n))^2/d^2/x
-e^2*ln(1+d/e/x)*(a+b*ln(c*x^n))^2/d^3+2*b*e^2*n*(a+b*ln(c*x^n))*polylog(2,
-d/e/x)/d^3+2*b^2*e^2*n^2*polylog(3,-d/e/x)/d^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

$= \{2380, 2342, 2341, 2379, 2421, 6724\}$

$$\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)}\,dx = \frac{2be^2n\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)(a+b\log(cx^n))}{d^3} - \frac{e^2\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))^2}{d^3} + \frac{e(a+b\log(cx^n))^2}{d^2x} + \frac{2ben(a+b\log(cx^n))}{d^2x} - \frac{(a+b\log(cx^n))^2}{2dx^2} - \frac{bn(a+b\log(cx^n))}{2dx^2} + \frac{2b^2e^2n^2\,\text{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^3} + \frac{2b^2en^2}{d^2x} - \frac{b^2n^2}{4dx^2}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(x^3*(d + e*x)),x]
```

```
[Out] -1/4*(b^2*n^2)/(d*x^2) + (2*b^2*e*n^2)/(d^2*x) - (b*n*(a + b*Log[c*x^n]))/(
2*d*x^2) + (2*b*e*n*(a + b*Log[c*x^n]))/(d^2*x) - (a + b*Log[c*x^n])^2/(2*d
*x^2) + (e*(a + b*Log[c*x^n])^2)/(d^2*x) - (e^2*Log[1 + d/(e*x)]*(a + b*Log
[c*x^n])^2)/d^3 + (2*b*e^2*n*(a + b*Log[c*x^n])*PolyLog[2, -(d/(e*x))])/d^3
 + (2*b^2*e^2*n^2*PolyLog[3, -(d/(e*x))])/d^3
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{(a+b\log(cx^n))^2}{x^3}\,dx}{d} - \frac{e\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)}\,dx}{d} \\
&= -\frac{(a+b\log(cx^n))^2}{2dx^2} - \frac{e\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx}{d^2} + \frac{(bn)\int \frac{a+b\log(cx^n)}{x^3}\,dx}{d} \\
&= -\frac{b^2n^2}{4dx^2} - \frac{bn(a+b\log(cx^n))}{2dx^2} - \frac{(a+b\log(cx^n))^2}{2dx^2} \\
&\quad + \frac{e(a+b\log(cx^n))^2}{d^2x} - \frac{e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^3} \\
&\quad - \frac{(2ben)\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d^2} + \frac{(2be^2n)\int \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{x}\,dx}{d^3} \\
&= -\frac{b^2n^2}{4dx^2} + \frac{2b^2en^2}{d^2x} - \frac{bn(a+b\log(cx^n))}{2dx^2} + \frac{2ben(a+b\log(cx^n))}{d^2x} \\
&\quad - \frac{(a+b\log(cx^n))^2}{2dx^2} + \frac{e(a+b\log(cx^n))^2}{d^2x} - \frac{e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^3} \\
&\quad + \frac{2be^2n(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^3} - \frac{(2b^2e^2n^2)\int \frac{\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{x}\,dx}{d^3} \\
&= -\frac{b^2n^2}{4dx^2} + \frac{2b^2en^2}{d^2x} - \frac{bn(a+b\log(cx^n))}{2dx^2} + \frac{2ben(a+b\log(cx^n))}{d^2x} \\
&\quad - \frac{(a+b\log(cx^n))^2}{2dx^2} + \frac{e(a+b\log(cx^n))^2}{d^2x} - \frac{e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^3} \\
&\quad + \frac{2be^2n(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^3} + \frac{2b^2e^2n^2\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)}\,dx$$
$$= \frac{-\frac{6d^2(a+b\log(cx^n))^2}{x^2}+\frac{12de(a+b\log(cx^n))^2}{x}+\frac{4e^2(a+b\log(cx^n))^3}{bn}+\frac{24bden(a+bn+b\log(cx^n))}{x}-\frac{3bd^2n(2a+bn+2b\log(cx^n))}{x^2}-12e}{12d^3}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(x^3*(d + e*x)),x]
```

```
[Out] ((-6*d^2*(a + b*Log[c*x^n])^2)/x^2 + (12*d*e*(a + b*Log[c*x^n])^2)/x + (4*e
^2*(a + b*Log[c*x^n])^3)/(b*n) + (24*b*d*e*n*(a + b*n + b*Log[c*x^n]))/x -
(3*b*d^2*n*(2*a + b*n + 2*b*Log[c*x^n]))/x^2 - 12*e^2*(a + b*Log[c*x^n])^2*
Log[1 + (e*x)/d] - 24*b*e^2*n*((a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)] -
b*n*PolyLog[3, -((e*x)/d)]))/(12*d^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 731, normalized size of antiderivative = 3.58

method	result
risch	$-\frac{b^2\ln(x^n)^2e^2\ln(ex+d)}{d^3}-\frac{b^2\ln(x^n)^2}{2d\,x^2}+\frac{b^2\ln(x^n)^2e^2\ln(x)}{d^3}+\frac{b^2\ln(x^n)^2e}{d^2x}+\frac{2b^2n\ln(x^n)e}{d^2x}-\frac{b^2n\ln(x^n)}{2d\,x^2}+\frac{2b^2e\,n^2}{d^2x}-\frac{b^2n}{4d\,x}$

```
[In] int((a+b*ln(c*x^n))^2/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -b^2*ln(x^n)^2*e^2/d^3*ln(e*x+d)-1/2*b^2*ln(x^n)^2/d/x^2+b^2*ln(x^n)^2*e^2/
d^3*ln(x)+b^2*ln(x^n)^2*e/d^2/x+2*b^2*n*ln(x^n)*e/d^2/x-1/2*b^2*n*ln(x^n)/d
/x^2+2*b^2*e*n^2/d^2/x-1/4*b^2*n^2/d/x^2-b^2*n*e^2/d^3*ln(x^n)*ln(x)^2+1/3*
b^2*e^2/d^3*ln(x)^3*n^2-2*b^2*e^2/d^3*ln(x)*ln(e*x+d)*ln(-e*x/d)*n^2-2*b^2*
e^2/d^3*ln(x)*dilog(-e*x/d)*n^2+2*b^2*n*e^2/d^3*ln(x^n)*ln(e*x+d)*ln(-e*x/d
)+2*b^2*n*e^2/d^3*ln(x^n)*dilog(-e*x/d)+b^2*e^2/d^3*n^2*ln(e*x+d)*ln(x)^2-b
^2*e^2/d^3*n^2*ln(x)^2*ln(1+e*x/d)-2*b^2*e^2/d^3*n^2*ln(x)*polylog(2,-e*x/d
)+2*b^2*e^2/d^3*n^2*polylog(3,-e*x/d)+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I
*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2
-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(-ln(x^n)*e^2/d^3*ln(e*x+d)-1/2*ln
(x^n)/d/x^2+ln(x^n)*e^2/d^3*ln(x)+ln(x^n)*e/d^2/x-1/2*n*(1/d^2*(-2*e/x+1/2*
d/x^2)+e^2/d^3*ln(x)^2-2*e^2/d^3*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))))+1/4
*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n
)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a
)^2*(-e^2/d^3*ln(e*x+d)-1/2/d/x^2+e^2/d^3*ln(x)+e/d^2/x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^3/(e*x+d),x, algorithm="fricas")
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e*x^4 + d*x^3), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)}\,dx = \int \frac{(a+b\log(cx^n))^2}{x^3\,(d+ex)}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**2/x**3/(e*x+d),x)
[Out] Integral((a + b*log(c*x**n))**2/(x**3*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^3/(e*x+d),x, algorithm="maxima")
[Out] -1/2*a^2*(2*e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2)
) + integrate((b^2*log(c)^2 + b^2*log(x^n)^2 + 2*a*b*log(c) + 2*(b^2*log(c)
 + a*b)*log(x^n))/(e*x^4 + d*x^3), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^3/(e*x+d),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2/((e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log{(cx^n)})^2}{x^3(d+ex)}\,dx = \int \frac{(a+b\,\ln{(c\,x^n)})^2}{x^3\,(d+e\,x)}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(x^3*(d + e*x)),x)
[Out] int((a + b*log(c*x^n))^2/(x^3*(d + e*x)), x)
```

3.99 $\int \frac{(a+b\log(cx^n))^2}{x^4(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 273

$$\int \frac{(a+b\log(cx^n))^2}{x^4(d+ex)}\,dx = -\frac{2b^2n^2}{27dx^3}+\frac{b^2en^2}{4d^2x^2}-\frac{2b^2e^2n^2}{d^3x}-\frac{2bn(a+b\log(cx^n))}{9dx^3} + \frac{ben(a+b\log(cx^n))}{2d^2x^2}-\frac{2be^2n(a+b\log(cx^n))}{d^3x} - \frac{(a+b\log(cx^n))^2}{3dx^3}+\frac{e(a+b\log(cx^n))^2}{2d^2x^2} - \frac{e^2(a+b\log(cx^n))^2}{d^3x}+\frac{e^3\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^4} - \frac{2be^3n(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^4} - \frac{2b^2e^3n^2\operatorname{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^4}$$

```
[Out] -2/27*b^2*n^2/d/x^3+1/4*b^2*e*n^2/d^2/x^2-2*b^2*e^2*n^2/d^3/x-2/9*b*n*(a+b*
ln(c*x^n))/d/x^3+1/2*b*e*n*(a+b*ln(c*x^n))/d^2/x^2-2*b*e^2*n*(a+b*ln(c*x^n)
)/d^3/x-1/3*(a+b*ln(c*x^n))^2/d/x^3+1/2*e*(a+b*ln(c*x^n))^2/d^2/x^2-e^2*(a+
b*ln(c*x^n))^2/d^3/x+e^3*ln(1+d/e/x)*(a+b*ln(c*x^n))^2/d^4-2*b*e^3*n*(a+b*l
n(c*x^n))*polylog(2,-d/e/x)/d^4-2*b^2*e^3*n^2*polylog(3,-d/e/x)/d^4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2380, 2342, 2341, 2379, 2421, 6724}

$$\int \frac{(a+b\log(cx^n))^2}{x^4(d+ex)}\,dx = -\frac{2be^3n\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4} + \frac{e^3\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))^2}{d^4} - \frac{e^2(a+b\log(cx^n))^2}{d^3x} - \frac{2be^2n(a+b\log(cx^n))}{d^3x} + \frac{e(a+b\log(cx^n))^2}{2d^2x^2} + \frac{ben(a+b\log(cx^n))}{2d^2x^2} - \frac{(a+b\log(cx^n))^2}{3dx^3} - \frac{2bn(a+b\log(cx^n))}{9dx^3} - \frac{2b^2e^3n^2\,\text{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^4} - \frac{2b^2e^2n^2}{d^3x} + \frac{b^2en^2}{4d^2x^2} - \frac{2b^2n^2}{27dx^3}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(x^4*(d + e*x)),x]
```

```
[Out] (-2*b^2*n^2)/(27*d*x^3) + (b^2*e*n^2)/(4*d^2*x^2) - (2*b^2*e^2*n^2)/(d^3*x)
 - (2*b*n*(a + b*Log[c*x^n]))/(9*d*x^3) + (b*e*n*(a + b*Log[c*x^n]))/(2*d^2
*x^2) - (2*b*e^2*n*(a + b*Log[c*x^n]))/(d^3*x) - (a + b*Log[c*x^n])^2/(3*d*
x^3) + (e*(a + b*Log[c*x^n])^2)/(2*d^2*x^2) - (e^2*(a + b*Log[c*x^n])^2)/(d
^3*x) + (e^3*Log[1 + d/(e*x)]*(a + b*Log[c*x^n])^2)/d^4 - (2*b*e^3*n*(a + b
*Log[c*x^n])*PolyLog[2, -(d/(e*x))])/d^4 - (2*b^2*e^3*n^2*PolyLog[3, -(d/(e
*x))])/d^4
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{(a+b\log(cx^n))^2}{x^4}\,dx}{d} - \frac{e\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)}\,dx}{d}\\
&= -\frac{(a+b\log(cx^n))^2}{3dx^3} - \frac{e\int \frac{(a+b\log(cx^n))^2}{x^3}\,dx}{d^2} + \frac{e^2\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)}\,dx}{d^2} + \frac{(2bn)\int \frac{a+b\log(cx^n)}{x^4}\,dx}{3d}\\
&= -\frac{2b^2n^2}{27dx^3} - \frac{2bn(a+b\log(cx^n))}{9dx^3} - \frac{(a+b\log(cx^n))^2}{3dx^3} + \frac{e(a+b\log(cx^n))^2}{2d^2x^2}\\
&\quad + \frac{e^2\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx}{d^3} - \frac{e^3\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx}{d^3} - \frac{(ben)\int \frac{a+b\log(cx^n)}{x^3}\,dx}{d^2}\\
&= -\frac{2b^2n^2}{27dx^3} + \frac{b^2en^2}{4d^2x^2} - \frac{2bn(a+b\log(cx^n))}{9dx^3} + \frac{ben(a+b\log(cx^n))}{2d^2x^2} - \frac{(a+b\log(cx^n))^2}{3dx^3}\\
&\quad + \frac{e(a+b\log(cx^n))^2}{2d^2x^2} - \frac{e^2(a+b\log(cx^n))^2}{d^3x} + \frac{e^3\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^4}\\
&\quad + \frac{(2be^2n)\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d^3} - \frac{(2be^3n)\int \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{x}\,dx}{d^4}
\end{aligned}$$

$$= -\frac{2b^2n^2}{27dx^3} + \frac{b^2en^2}{4d^2x^2} - \frac{2b^2e^2n^2}{d^3x} - \frac{2bn(a+b\log(cx^n))}{9dx^3} + \frac{ben(a+b\log(cx^n))}{2d^2x^2} - \frac{2be^2n(a+b\log(cx^n))}{d^3x} - \frac{(a+b\log(cx^n))^2}{3dx^3} + \frac{e(a+b\log(cx^n))^2}{2d^2x^2} - \frac{e^2(a+b\log(cx^n))^2}{d^3x} + \frac{e^3\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^4} - \frac{2be^3n(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^4} + \frac{(2b^2e^3n^2)\int\frac{\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{x}\,dx}{d^4}$$

$$= -\frac{2b^2n^2}{27dx^3} + \frac{b^2en^2}{4d^2x^2} - \frac{2b^2e^2n^2}{d^3x} - \frac{2bn(a+b\log(cx^n))}{9dx^3} + \frac{ben(a+b\log(cx^n))}{2d^2x^2} - \frac{2be^2n(a+b\log(cx^n))}{d^3x} - \frac{(a+b\log(cx^n))^2}{3dx^3} + \frac{e(a+b\log(cx^n))^2}{2d^2x^2} - \frac{e^2(a+b\log(cx^n))^2}{d^3x} + \frac{e^3\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^4} - \frac{2be^3n(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^4} - \frac{2b^2e^3n^2\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.87

$$\int\frac{(a+b\log(cx^n))^2}{x^4(d+ex)}\,dx$$

$$= \frac{-\frac{36d^3(a+b\log(cx^n))^2}{x^3} + \frac{54d^2e(a+b\log(cx^n))^2}{x^2} - \frac{108de^2(a+b\log(cx^n))^2}{x} - \frac{36e^3(a+b\log(cx^n))^3}{bn} - \frac{216bde^2n(a+bn+b\log(cx^n))}{x} + 27b}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(x^4*(d + e*x)),x]
```

```
[Out] ((-36*d^3*(a + b*Log[c*x^n])^2)/x^3 + (54*d^2*e*(a + b*Log[c*x^n])^2)/x^2 -
 (108*d*e^2*(a + b*Log[c*x^n])^2)/x - (36*e^3*(a + b*Log[c*x^n])^3)/(b*n) -
 (216*b*d*e^2*n*(a + b*n + b*Log[c*x^n]))/x + (27*b*d^2*e*n*(2*a + b*n + 2*
b*Log[c*x^n]))/x^2 - (8*b*d^3*n*(3*a + b*n + 3*b*Log[c*x^n]))/x^3 + 108*e^3
*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d] + 216*b*e^3*n*((a + b*Log[c*x^n])*Po
lyLog[2, -((e*x)/d)] - b*n*PolyLog[3, -((e*x)/d)]))/(108*d^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.53 (sec) , antiderivative size = 828, normalized size of antiderivative = 3.03

method	result
risch	$\frac{b^2 \ln(x^n)^2 e^3 \ln(ex+d)}{d^4} - \frac{b^2 \ln(x^n)^2}{3d\,x^3} - \frac{b^2 \ln(x^n)^2 e^2}{d^3 x} + \frac{b^2 \ln(x^n)^2 e}{2d^2 x^2} - \frac{b^2 \ln(x^n)^2 e^3 \ln(x)}{d^4} - \frac{2b^2 n \ln(x^n) e^2}{d^3 x} + \frac{b^2 n \ln(x^n) e}{2d^2 x^2} - 2$

```
[In] int((a+b*ln(c*x^n))^2/x^4/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*ln(x^n)^2*e^3/d^4*ln(e*x+d)-1/3*b^2*ln(x^n)^2/d/x^3-b^2*ln(x^n)^2*e^2/d
^3/x+1/2*b^2*ln(x^n)^2*e/d^2/x^2-b^2*ln(x^n)^2*e^3/d^4*ln(x)-2*b^2*n*ln(x^n
)*e^2/d^3/x+1/2*b^2*n*ln(x^n)*e/d^2/x^2-2/9*b^2*n*ln(x^n)/d/x^3-2*b^2*e^2*n
^2/d^3/x+1/4*b^2*e*n^2/d^2/x^2-2/27*b^2*n^2/d/x^3+b^2*n*e^3/d^4*ln(x^n)*ln(
x)^2-1/3*b^2*e^3/d^4*ln(x)^3*n^2+2*b^2*e^3/d^4*ln(x)*ln(e*x+d)*ln(-e*x/d)*n
^2+2*b^2*e^3/d^4*ln(x)*dilog(-e*x/d)*n^2-2*b^2*n*e^3/d^4*ln(x^n)*ln(e*x+d)*
ln(-e*x/d)-2*b^2*n*e^3/d^4*ln(x^n)*dilog(-e*x/d)-b^2*e^3/d^4*n^2*ln(e*x+d)*
ln(x)^2+b^2*e^3/d^4*n^2*ln(x)^2*ln(1+e*x/d)+2*b^2*e^3/d^4*n^2*ln(x)*polylog
(2,-e*x/d)-2*b^2*e^3/d^4*n^2*polylog(3,-e*x/d)+(-I*b*Pi*csgn(I*c)*csgn(I*x^
n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I
*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(ln(x^n)*e^3/d^4*ln(e*x+d
)-1/3*ln(x^n)/d/x^3-ln(x^n)*e^2/d^3/x+1/2*ln(x^n)*e/d^2/x^2-ln(x^n)*e^3/d^4
*ln(x)-1/6*n*(-1/d^3*(-6*e^2/x+3/2*d*e/x^2-2/3*d^2/x^3)-3*e^3/d^4*ln(x)^2+6
*e^3/d^4*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))))+1/4*(-I*b*Pi*csgn(I*c)*csgn
(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*c
sgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*(e^3/d^4*ln(e*x+d)-1
/3/d/x^3-e^2/d^3/x+1/2*e/d^2/x^2-e^3/d^4*ln(x))
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^4(d+ex)}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e*x^5 + d*x^4), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^4(d+ex)}\,dx = \int \frac{(a+b\log(cx^n))^2}{x^4\,(d+ex)}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**2/x**4/(e*x+d),x)
[Out] Integral((a + b*log(c*x**n))**2/(x**4*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^4(d+ex)}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^4/(e*x+d),x, algorithm="maxima")
[Out] 1/6*a^2*(6*e^3*log(e*x + d)/d^4 - 6*e^3*log(x)/d^4 - (6*e^2*x^2 - 3*d*e*x +
 2*d^2)/(d^3*x^3)) + integrate((b^2*log(c)^2 + b^2*log(x^n)^2 + 2*a*b*log(c
) + 2*(b^2*log(c) + a*b)*log(x^n))/(e*x^5 + d*x^4), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^4(d+ex)}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^4/(e*x+d),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2/((e*x + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{x^4(d+ex)}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{x^4\,(d+e\,x)}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(x^4*(d + e*x)),x)
[Out] int((a + b*log(c*x^n))^2/(x^4*(d + e*x)), x)
```

3.100 $\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 281

$$\begin{aligned}\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^2}\,dx &= \frac{4abdnx}{e^3}-\frac{4b^2dn^2x}{e^3}+\frac{b^2n^2x^2}{4e^2}+\frac{4b^2dnx\log(cx^n)}{e^3}\\&-\frac{bnx^2(a+b\log(cx^n))}{2e^2}-\frac{2dx(a+b\log(cx^n))^2}{e^3}\\&+\frac{x^2(a+b\log(cx^n))^2}{2e^2}-\frac{d^2x(a+b\log(cx^n))^2}{e^3(d+ex)}\\&+\frac{2bd^2n(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^4}\\&+\frac{3d^2(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^4}+\frac{2b^2d^2n^2\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4}\\&+\frac{6bd^2n(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4}\\&-\frac{6b^2d^2n^2\,\mathrm{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^4}\end{aligned}$$

```
[Out] 4*a*b*d*n*x/e^3-4*b^2*d*n^2*x/e^3+1/4*b^2*n^2*x^2/e^2+4*b^2*d*n*x*ln(c*x^n)
/e^3-1/2*b*n*x^2*(a+b*ln(c*x^n))/e^2-2*d*x*(a+b*ln(c*x^n))^2/e^3+1/2*x^2*(a
+b*ln(c*x^n))^2/e^2-d^2*x*(a+b*ln(c*x^n))^2/e^3/(e*x+d)+2*b*d^2*n*(a+b*ln(c
*x^n))*ln(1+e*x/d)/e^4+3*d^2*(a+b*ln(c*x^n))^2*ln(1+e*x/d)/e^4+2*b^2*d^2*n^
2*polylog(2,-e*x/d)/e^4+6*b*d^2*n*(a+b*ln(c*x^n))*polylog(2,-e*x/d)/e^4-6*b
^2*d^2*n^2*polylog(3,-e*x/d)/e^4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2395, 2333, 2332, 2342, 2341, 2355, 2354, 2438, 2421, 6724}

$$\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \frac{6bd^2n\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)(a+b\log(cx^n))}{e^4} + \frac{2bd^2n\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{e^4} + \frac{3d^2\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))^2}{e^4} - \frac{d^2x(a+b\log(cx^n))^2}{e^3(d+ex)} - \frac{2dx(a+b\log(cx^n))^2}{e^3} - \frac{bnx^2(a+b\log(cx^n))}{2e^2} + \frac{x^2(a+b\log(cx^n))^2}{2e^2} + \frac{4abdnx}{e^3} + \frac{4b^2dnx\log(cx^n)}{e^3} + \frac{2b^2d^2n^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4} - \frac{6b^2d^2n^2\,\text{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^4} - \frac{4b^2dn^2x}{e^3} + \frac{b^2n^2x^2}{4e^2}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n])^2)/(d + e*x)^2,x]
```

```
[Out] (4*a*b*d*n*x)/e^3 - (4*b^2*d*n^2*x)/e^3 + (b^2*n^2*x^2)/(4*e^2) + (4*b^2*d*
n*x*Log[c*x^n])/e^3 - (b*n*x^2*(a + b*Log[c*x^n]))/(2*e^2) - (2*d*x*(a + b*
Log[c*x^n])^2)/e^3 + (x^2*(a + b*Log[c*x^n])^2)/(2*e^2) - (d^2*x*(a + b*Log
[c*x^n])^2)/(e^3*(d + e*x)) + (2*b*d^2*n*(a + b*Log[c*x^n])*Log[1 + (e*x)/d
])/e^4 + (3*d^2*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d])/e^4 + (2*b^2*d^2*n^2
*PolyLog[2, -((e*x)/d)])/e^4 + (6*b*d^2*n*(a + b*Log[c*x^n])*PolyLog[2, -((
e*x)/d)])/e^4 - (6*b^2*d^2*n^2*PolyLog[3, -((e*x)/d)])/e^4
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{2d(a+b\log(cx^n))^2}{e^3}+\frac{x(a+b\log(cx^n))^2}{e^2}-\frac{d^3(a+b\log(cx^n))^2}{e^3(d+ex)^2}+\frac{3d^2(a+b\log(cx^n))^2}{e^3(d+ex)}\right)dx$$

$$=-\frac{(2d)\int(a+b\log(cx^n))^2\,dx}{e^3}+\frac{(3d^2)\int\frac{(a+b\log(cx^n))^2}{d+ex}\,dx}{e^3}-\frac{d^3\int\frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx}{e^3}+\frac{\int x(a+b\log(cx^n))^2\,dx}{e^2}$$

$$=-\frac{2dx(a+b\log(cx^n))^2}{e^3}+\frac{x^2(a+b\log(cx^n))^2}{2e^2}-\frac{d^2x(a+b\log(cx^n))^2}{e^3(d+ex)}+\frac{3d^2(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^4}-\frac{(6bd^2n)\int\frac{(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^4}+\frac{(4bdn)\int(a+b\log(cx^n))\,dx}{e^3}+\frac{(2bd^2n)\int\frac{a+b\log(cx^n)}{d+ex}\,dx}{e^3}-\frac{(bn)\int x(a+b\log(cx^n))\,dx}{e^2}$$

$$=\frac{4abdnx}{e^3}+\frac{b^2n^2x^2}{4e^2}-\frac{bnx^2(a+b\log(cx^n))}{2e^2}-\frac{2dx(a+b\log(cx^n))^2}{e^3}+\frac{x^2(a+b\log(cx^n))^2}{2e^2}-\frac{d^2x(a+b\log(cx^n))^2}{e^3(d+ex)}+\frac{2bd^2n(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^4}+\frac{3d^2(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^4}+\frac{6bd^2n(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^4}+\frac{(4b^2dn)\int\log(cx^n)\,dx}{e^3}-\frac{(2b^2d^2n^2)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^4}-\frac{(6b^2d^2n^2)\int\frac{\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{x}\,dx}{e^4}$$

$$=\frac{4abdnx}{e^3}-\frac{4b^2dn^2x}{e^3}+\frac{b^2n^2x^2}{4e^2}+\frac{4b^2dnx\log(cx^n)}{e^3}-\frac{bnx^2(a+b\log(cx^n))}{2e^2}-\frac{2dx(a+b\log(cx^n))^2}{e^3}+\frac{x^2(a+b\log(cx^n))^2}{2e^2}-\frac{d^2x(a+b\log(cx^n))^2}{e^3(d+ex)}+\frac{2bd^2n(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^4}+\frac{3d^2(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^4}+\frac{2b^2d^2n^2\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^4}+\frac{6bd^2n(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^4}-\frac{6b^2d^2n^2\operatorname{Li}_3\left(-\frac{ex}{d}\right)}{e^4}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.85

$$\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^2}\,dx$$
$$= \frac{-8dex(a+b\log(cx^n))^2+2e^2x^2(a+b\log(cx^n))^2+\frac{4d^3(a+b\log(cx^n))^2}{d+ex}+16bdenx(a-bn+b\log(cx^n))+be^2n}{}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n])^2)/(d + e*x)^2,x]
```

```
[Out] (-8*d*e*x*(a + b*Log[c*x^n])^2 + 2*e^2*x^2*(a + b*Log[c*x^n])^2 + (4*d^3*(a
 + b*Log[c*x^n])^2)/(d + e*x) + 16*b*d*e*n*x*(a - b*n + b*Log[c*x^n]) + b*e
^2*n*x^2*(b*n - 2*(a + b*Log[c*x^n])) + 12*d^2*(a + b*Log[c*x^n])^2*Log[1 +
 (e*x)/d] + 4*d^2*(-((a + b*Log[c*x^n])*(a + b*Log[c*x^n] - 2*b*n*Log[1 + (
e*x)/d])) + 2*b^2*n^2*PolyLog[2, -((e*x)/d)]) + 24*b*d^2*n*((a + b*Log[c*x^
n])*PolyLog[2, -((e*x)/d)] - b*n*PolyLog[3, -((e*x)/d)]))/(4*e^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 824, normalized size of antiderivative = 2.93

method	result	size
risch	Expression too large to display	824

```
[In] int(x^3*(a+b*ln(c*x^n))^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2*ln(x^n)^2/e^2*x^2-2*b^2*ln(x^n)^2/e^3*d*x+3*b^2*ln(x^n)^2/e^4*d^2*l
n(e*x+d)+b^2*ln(x^n)^2*d^3/e^4/(e*x+d)+6*b^2/e^4*d^2*ln(x)*ln(e*x+d)*ln(-e*
x/d)*n^2+6*b^2/e^4*d^2*ln(x)*dilog(-e*x/d)*n^2-6*b^2*n/e^4*d^2*ln(x^n)*ln(e
*x+d)*ln(-e*x/d)-6*b^2*n/e^4*d^2*ln(x^n)*dilog(-e*x/d)-3*b^2/e^4*d^2*n^2*ln
(e*x+d)*ln(x)^2+3*b^2/e^4*d^2*n^2*ln(x)^2*ln(1+e*x/d)+6*b^2/e^4*d^2*n^2*ln(
x)*polylog(2,-e*x/d)-6*b^2*d^2*n^2*polylog(3,-e*x/d)/e^4-1/2*b^2*n*ln(x^n)/
e^2*x^2+4*b^2*n*ln(x^n)/e^3*d*x+2*b^2*n*ln(x^n)/e^4*d^2*ln(e*x+d)-2*b^2*n/e
^4*ln(x^n)*d^2*ln(x)+1/4*b^2*n^2*x^2/e^2-4*b^2*d*n^2*x/e^3+b^2/e^4*n^2*d^2*
ln(x)^2-2*b^2/e^4*n^2*ln(-e*x/d)*ln(e*x+d)*d^2-2*b^2/e^4*n^2*dilog(-e*x/d)*
d^2+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*
x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+
2*a)*b*(1/2*ln(x^n)/e^2*x^2-2*ln(x^n)/e^3*d*x+3*ln(x^n)/e^4*d^2*ln(e*x+d)+l
n(x^n)*d^3/e^4/(e*x+d)-n*(3/e^4*d^2*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))+1/
2/e^4*(1/2*(e*x+d)^2-5*d*(e*x+d)-2*d^2*ln(e*x+d)+2*d^2*ln(e*x))))+1/4*(-I*b
*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*
```

```
b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*(1
/e^3*(1/2*e*x^2-2*d*x)+3/e^4*d^2*ln(e*x+d)+d^3/e^4/(e*x+d))
```

Fricas [F]

$$\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2x^3}{(ex+d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^2/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((b^2*x^3*log(c*x^n)^2 + 2*a*b*x^3*log(c*x^n) + a^2*x^3)/(e^2*x^2 +
 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^2}\,dx$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))**2/(e*x+d)**2,x)
[Out] Integral(x**3*(a + b*log(c*x**n))**2/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2x^3}{(ex+d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^2/(e*x+d)^2,x, algorithm="maxima")
[Out] 1/2*(2*d^3/(e^5*x + d*e^4) + 6*d^2*log(e*x + d)/e^4 + (e*x^2 - 4*d*x)/e^3)*
a^2 + integrate((b^2*x^3*log(x^n)^2 + 2*(b^2*log(c) + a*b)*x^3*log(x^n) + (
b^2*log(c)^2 + 2*a*b*log(c))*x^3)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Giac [F]

$$\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2x^3}{(ex+d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^2/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*x^3/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{x^3\,(a+b\,\ln(c\,x^n))^2}{(d+e\,x)^2}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n))^2)/(d + e*x)^2,x)
[Out] int((x^3*(a + b*log(c*x^n))^2)/(d + e*x)^2, x)
```

3.101 $\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 203

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = -\frac{2abnx}{e^2}+\frac{2b^2n^2x}{e^2}-\frac{2b^2nx\log(cx^n)}{e^2}+\frac{x(a+b\log(cx^n))^2}{e^2}+\frac{dx(a+b\log(cx^n))^2}{e^2(d+ex)}-\frac{2bdn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^3}-\frac{2d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^3}-\frac{2b^2dn^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^3}-\frac{4bdn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^3}+\frac{4b^2dn^2\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^3}$$

```
[Out] -2*a*b*n*x/e^2+2*b^2*n^2*x/e^2-2*b^2*n*x*ln(c*x^n)/e^2+x*(a+b*ln(c*x^n))^2/
e^2+d*x*(a+b*ln(c*x^n))^2/e^2/(e*x+d)-2*b*d*n*(a+b*ln(c*x^n))*ln(1+e*x/d)/e
^3-2*d*(a+b*ln(c*x^n))^2*ln(1+e*x/d)/e^3-2*b^2*d*n^2*polylog(2,-e*x/d)/e^3-
4*b*d*n*(a+b*ln(c*x^n))*polylog(2,-e*x/d)/e^3+4*b^2*d*n^2*polylog(3,-e*x/d)
/e^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2395, 2333, 2332, 2355, 2354, 2438, 2421, 6724}

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = -\frac{4bdn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)(a+b\log(cx^n))}{e^3} - \frac{2bdn\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{e^3} - \frac{2d\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))^2}{e^3} + \frac{dx(a+b\log(cx^n))^2}{e^2(d+ex)} + \frac{x(a+b\log(cx^n))^2}{e^2} - \frac{2abnx}{e^2} - \frac{2b^2nx\log(cx^n)}{e^2} - \frac{2b^2dn^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^3} + \frac{4b^2dn^2\,\text{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^3} + \frac{2b^2n^2x}{e^2}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n])^2)/(d + e*x)^2,x]
```

```
[Out] (-2*a*b*n*x)/e^2 + (2*b^2*n^2*x)/e^2 - (2*b^2*n*x*Log[c*x^n])/e^2 + (x*(a +
 b*Log[c*x^n])^2)/e^2 + (d*x*(a + b*Log[c*x^n])^2)/(e^2*(d + e*x)) - (2*b*d
*n*(a + b*Log[c*x^n])*Log[1 + (e*x)/d])/e^3 - (2*d*(a + b*Log[c*x^n])^2*Log
[1 + (e*x)/d])/e^3 - (2*b^2*d*n^2*PolyLog[2, -((e*x)/d)])/e^3 - (4*b*d*n*(a
 + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)])/e^3 + (4*b^2*d*n^2*PolyLog[3, -((e
*x)/d)])/e^3
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{(a+b\log(cx^n))^2}{e^2}+\frac{d^2(a+b\log(cx^n))^2}{e^2(d+ex)^2}-\frac{2d(a+b\log(cx^n))^2}{e^2(d+ex)}\right)dx\\
&= \frac{\int (a+b\log(cx^n))^2\,dx}{e^2}-\frac{(2d)\int \frac{(a+b\log(cx^n))^2}{d+ex}\,dx}{e^2}+\frac{d^2\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx}{e^2}\\
&= \frac{x(a+b\log(cx^n))^2}{e^2}+\frac{dx(a+b\log(cx^n))^2}{e^2(d+ex)}-\frac{2d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^3}\\
&\quad+\frac{(4bdn)\int \frac{(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^3}-\frac{(2bn)\int (a+b\log(cx^n))\,dx}{e^2}\\
&\quad-\frac{(2bdn)\int \frac{a+b\log(cx^n)}{d+ex}\,dx}{e^2}
\end{aligned}
$$

$$= -\frac{2abnx}{e^2} + \frac{x(a+b\log(cx^n))^2}{e^2} + \frac{dx(a+b\log(cx^n))^2}{e^2(d+ex)} - \frac{2bdn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^3} - \frac{2d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^3} - \frac{4bdn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^3} - \frac{(2b^2n)\int\log(cx^n)\,dx}{e^2} + \frac{(2b^2dn^2)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}dx}{e^3} + \frac{(4b^2dn^2)\int\frac{\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{x}dx}{e^3}$$

$$= -\frac{2abnx}{e^2} + \frac{2b^2n^2x}{e^2} - \frac{2b^2nx\log(cx^n)}{e^2} + \frac{x(a+b\log(cx^n))^2}{e^2} + \frac{dx(a+b\log(cx^n))^2}{e^2(d+ex)} - \frac{2bdn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^3} - \frac{2d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^3} - \frac{2b^2dn^2\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^3} - \frac{4bdn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^3} + \frac{4b^2dn^2\operatorname{Li}_3\left(-\frac{ex}{d}\right)}{e^3}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.92

$$\int\frac{x^2(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \frac{d(a+b\log(cx^n))^2 + ex(a+b\log(cx^n))^2 - \frac{d^2(a+b\log(cx^n))^2}{d+ex} - 2benx(a-bn+b\log(cx^n)) - 2bdn(a+b\log(}{}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n])^2)/(d + e*x)^2,x]
```

```
[Out] (d*(a + b*Log[c*x^n])^2 + e*x*(a + b*Log[c*x^n])^2 - (d^2*(a + b*Log[c*x^n]
)^2)/(d + e*x) - 2*b*e*n*x*(a - b*n + b*Log[c*x^n]) - 2*b*d*n*(a + b*Log[c*
x^n])*Log[1 + (e*x)/d] - 2*d*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d] - 2*b^2*
d*n^2*PolyLog[2, -((e*x)/d)] - 4*b*d*n*(a + b*Log[c*x^n])*PolyLog[2, -((e*x
)/d)] + 4*b^2*d*n^2*PolyLog[3, -((e*x)/d)])/e^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 700, normalized size of antiderivative = 3.45

method	result
risch	$\frac{b^2\ln(x^n)^2x}{e^2} - \frac{2b^2\ln(x^n)^2d\ln(ex+d)}{e^3} - \frac{b^2\ln(x^n)^2d^2}{e^3(ex+d)} + \frac{2b^2n\ln(x)\ln(x^n)d}{e^3} - \frac{2b^2n\ln(x^n)d\ln(ex+d)}{e^3} - \frac{2b^2n\ln(x^n)x}{e^2} + \frac{2b^2n}{e^2}$

```
[In] int(x^2*(a+b*ln(c*x^n))^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*ln(x^n)^2/e^2*x-2*b^2*ln(x^n)^2/e^3*d*ln(e*x+d)-b^2*ln(x^n)^2/e^3*d^2/(
e*x+d)+2*b^2*n/e^3*ln(x)*ln(x^n)*d-2*b^2*n*ln(x^n)/e^3*d*ln(e*x+d)-2*b^2*n*
ln(x^n)/e^2*x+2*b^2*n^2*x/e^2+2*b^2/e^3*n^2*ln(e*x+d)*ln(-e*x/d)*d+2*b^2/e^
3*n^2*dilog(-e*x/d)*d-b^2/e^3*n^2*d*ln(x)^2-4*b^2/e^3*d*ln(x)*ln(e*x+d)*ln(
-e*x/d)*n^2-4*b^2/e^3*d*ln(x)*dilog(-e*x/d)*n^2+4*b^2*n/e^3*d*ln(x^n)*ln(e*
x+d)*ln(-e*x/d)+4*b^2*n/e^3*d*ln(x^n)*dilog(-e*x/d)+2*b^2/e^3*d*n^2*ln(e*x+
d)*ln(x)^2-2*b^2/e^3*d*n^2*ln(x)^2*ln(1+e*x/d)-4*b^2/e^3*d*n^2*ln(x)*polylo
g(2,-e*x/d)+4*b^2*d*n^2*polylog(3,-e*x/d)/e^3+(-I*b*Pi*csgn(I*c)*csgn(I*x^n
)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*
c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(ln(x^n)/e^2*x-2*ln(x^n)/e
^3*d*ln(e*x+d)-ln(x^n)/e^3*d^2/(e*x+d)-n*(1/e^3*(e*x+d+d*ln(e*x+d)-d*ln(e*x
))-2/e^3*d*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))))+1/4*(-I*b*Pi*csgn(I*c)*cs
gn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)
*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*(x/e^2-2/e^3*d*ln(
e*x+d)-1/e^3*d^2/(e*x+d))
```

Fricas [F]

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2x^2}{(ex+d)^2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*log(c*x^n)^2 + 2*a*b*x^2*log(c*x^n) + a^2*x^2)/(e^2*x^2 +
 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^2}\,dx$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2/(e*x+d)**2,x)
```

```
[Out] Integral(x**2*(a + b*log(c*x**n))**2/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2x^2}{(ex+d)^2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -a^2*(d^2/(e^4*x + d*e^3) - x/e^2 + 2*d*log(e*x + d)/e^3) + integrate((b^2*
x^2*log(x^n)^2 + 2*(b^2*log(c) + a*b)*x^2*log(x^n) + (b^2*log(c)^2 + 2*a*b*
log(c))*x^2)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Giac [F]

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2x^2}{(ex+d)^2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x^2/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,x^n))^2}{(d+e\,x)^2}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n))^2)/(d + e*x)^2,x)
```

```
[Out] int((x^2*(a + b*log(c*x^n))^2)/(d + e*x)^2, x)
```

3.102 $\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = -\frac{x(a+b\log(cx^n))^2}{e(d+ex)}+\frac{2bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^2}+\frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^2}+\frac{2b^2n^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^2}+\frac{2bn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^2}-\frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^2}$$

```
[Out] -x*(a+b*ln(c*x^n))^2/e/(e*x+d)+2*b*n*(a+b*ln(c*x^n))*ln(1+e*x/d)/e^2+(a+b*l
n(c*x^n))^2*ln(1+e*x/d)/e^2+2*b^2*n^2*polylog(2,-e*x/d)/e^2+2*b*n*(a+b*ln(c
*x^n))*polylog(2,-e*x/d)/e^2-2*b^2*n^2*polylog(3,-e*x/d)/e^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2395, 2355, 2354, 2438, 2421, 6724}

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \frac{2bn\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)(a+b\log(cx^n))}{e^2}+\frac{2bn\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{e^2}+\frac{\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))^2}{e^2}-\frac{x(a+b\log(cx^n))^2}{e(d+ex)}+\frac{2b^2n^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^2}-\frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^2}$$

```
[In] Int[(x*(a + b*Log[c*x^n])^2)/(d + e*x)^2,x]
```

```
[Out] -((x*(a + b*Log[c*x^n])^2)/(e*(d + e*x))) + (2*b*n*(a + b*Log[c*x^n])*Log[1
 + (e*x)/d])/e^2 + ((a + b*Log[c*x^n])^2*Log[1 + (e*x)/d])/e^2 + (2*b^2*n^2
*PolyLog[2, -((e*x)/d)])/e^2 + (2*b*n*(a + b*Log[c*x^n])*PolyLog[2, -((e*x)
/d)])/e^2 - (2*b^2*n^2*PolyLog[3, -((e*x)/d)])/e^2
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{d(a+b\log(cx^n))^2}{e(d+ex)^2} + \frac{(a+b\log(cx^n))^2}{e(d+ex)}\right) dx$$

$$= \frac{\int \frac{(a+b\log(cx^n))^2}{d+ex}\,dx}{e} - \frac{d\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx}{e}$$

$$= -\frac{x(a+b\log(cx^n))^2}{e(d+ex)} + \frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^2} - \frac{(2bn)\int \frac{(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^2} + \frac{(2bn)\int \frac{a+b\log(cx^n)}{d+ex}\,dx}{e}$$

$$= -\frac{x(a+b\log(cx^n))^2}{e(d+ex)} + \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^2} + \frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^2} + \frac{2bn(a+b\log(cx^n))\,\text{Li}_2\left(-\frac{ex}{d}\right)}{e^2} - \frac{(2b^2n^2)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^2} - \frac{(2b^2n^2)\int \frac{\text{Li}_2\left(-\frac{ex}{d}\right)}{x}\,dx}{e^2}$$

$$= -\frac{x(a+b\log(cx^n))^2}{e(d+ex)} + \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^2} + \frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^2} + \frac{2b^2n^2\text{Li}_2\left(-\frac{ex}{d}\right)}{e^2} + \frac{2bn(a+b\log(cx^n))\,\text{Li}_2\left(-\frac{ex}{d}\right)}{e^2} - \frac{2b^2n^2\text{Li}_3\left(-\frac{ex}{d}\right)}{e^2}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \frac{-(a+b\log(cx^n))^2 + \frac{d(a+b\log(cx^n))^2}{d+ex} + 2bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right) + (a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right) +}{e^2}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n])^2)/(d + e*x)^2,x]
```

```
[Out] (-(a + b*Log[c*x^n])^2 + (d*(a + b*Log[c*x^n])^2)/(d + e*x) + 2*b*n*(a + b*
Log[c*x^n])*Log[1 + (e*x)/d] + (a + b*Log[c*x^n])^2*Log[1 + (e*x)/d] + 2*b^
2*n^2*PolyLog[2, -((e*x)/d)] + 2*b*n*(a + b*Log[c*x^n])*PolyLog[2, -((e*x)/
d)] - 2*b^2*n^2*PolyLog[3, -((e*x)/d)])/e^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 609, normalized size of antiderivative = 4.26

method	result
risch	$\frac{b^2 \ln(x^n)^2 \ln(ex+d)}{e^2} + \frac{b^2 \ln(x^n)^2 d}{e^2(ex+d)} + \frac{2b^2 \ln(x) \ln(ex+d) \ln(-\frac{ex}{d}) n^2}{e^2} + \frac{2b^2 \ln(x) \operatorname{dilog}(-\frac{ex}{d}) n^2}{e^2} - \frac{2b^2 n \ln(x^n) \ln(ex+d) \ln(-\frac{ex}{d}}{e^2}$

```
[In] int(x*(a+b*ln(c*x^n))^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*ln(x^n)^2/e^2*ln(e*x+d)+b^2*ln(x^n)^2/e^2*d/(e*x+d)+2*b^2/e^2*ln(x)*ln(
e*x+d)*ln(-e*x/d)*n^2+2*b^2/e^2*ln(x)*dilog(-e*x/d)*n^2-2*b^2*n/e^2*ln(x^n)
*ln(e*x+d)*ln(-e*x/d)-2*b^2*n/e^2*ln(x^n)*dilog(-e*x/d)-b^2/e^2*n^2*ln(e*x+
d)*ln(x)^2+b^2/e^2*n^2*ln(x)^2*ln(1+e*x/d)+2*b^2/e^2*n^2*ln(x)*polylog(2,-e
*x/d)-2*b^2*n^2*polylog(3,-e*x/d)/e^2+2*b^2*n*ln(x^n)/e^2*ln(e*x+d)-2*b^2*n
/e^2*ln(x^n)*ln(x)+b^2/e^2*n^2*ln(x)^2-2*b^2/e^2*n^2*ln(e*x+d)*ln(-e*x/d)-2
*b^2/e^2*n^2*dilog(-e*x/d)+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b
*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csg
n(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(ln(x^n)/e^2*ln(e*x+d)+ln(x^n)/e^2*d/(e*x+d)-
n*(1/e^2*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))-1/e^2*ln(e*x+d)+1/e^2*ln(e*x)
))+1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I
*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(
c)+2*a)^2*(1/e^2*ln(e*x+d)+1/e^2*d/(e*x+d))
```

Fricas [F]

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2x}{(ex+d)^2}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)/(e^2*x^2 + 2*d*e
*x + d^2), x)
```

Sympy [F]

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{x(a+b\log(cx^n))^2}{(d+ex)^2}\,dx$$

```
[In] integrate(x*(a+b*ln(c*x**n))**2/(e*x+d)**2,x)
[Out] Integral(x*(a + b*log(c*x**n))**2/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2x}{(ex+d)^2}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))^2/(e*x+d)^2,x, algorithm="maxima")
[Out] a^2*(d/(e^3*x + d*e^2) + log(e*x + d)/e^2) + integrate((b^2*x*log(x^n)^2 +
2*(b^2*log(c) + a*b)*x*log(x^n) + (b^2*log(c)^2 + 2*a*b*log(c))*x)/(e^2*x^2
 + 2*d*e*x + d^2), x)
```

Giac [F]

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2x}{(ex+d)^2}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))^2/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*x/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{x\,(a+b\,\ln(c\,x^n))^2}{(d+e\,x)^2}\,dx$$

```
[In] int((x*(a + b*log(c*x^n))^2)/(d + e*x)^2,x)
[Out] int((x*(a + b*log(c*x^n))^2)/(d + e*x)^2, x)
```

3.103 $\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 77

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2} dx = \frac{x(a+b\log(cx^n))^2}{d(d+ex)} - \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{de} - \frac{2b^2n^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{de}$$

```
[Out] x*(a+b*ln(c*x^n))^2/d/(e*x+d)-2*b*n*(a+b*ln(c*x^n))*ln(1+e*x/d)/d/e-2*b^2*n
^2*polylog(2,-e*x/d)/d/e
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2355, 2354, 2438}

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2} dx = -\frac{2bn\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{de} + \frac{x(a+b\log(cx^n))^2}{d(d+ex)} - \frac{2b^2n^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{de}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(d + e*x)^2,x]
```

```
[Out] (x*(a + b*Log[c*x^n])^2)/(d*(d + e*x)) - (2*b*n*(a + b*Log[c*x^n])*Log[1 +
(e*x)/d])/(d*e) - (2*b^2*n^2*PolyLog[2, -((e*x)/d)])/(d*e)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
```

```
Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a+b\log(cx^n))^2}{d(d+ex)} - \frac{(2bn)\int \frac{a+b\log(cx^n)}{d+ex}\,dx}{d} \\
&= \frac{x(a+b\log(cx^n))^2}{d(d+ex)} - \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{de} + \frac{(2b^2n^2)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{de} \\
&= \frac{x(a+b\log(cx^n))^2}{d(d+ex)} - \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{de} - \frac{2b^2n^2\mathrm{Li}_2\left(-\frac{ex}{d}\right)}{de}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \frac{(a+b\log(cx^n))\left(aex+bex\log(cx^n)-2bn(d+ex)\log\left(1+\frac{ex}{d}\right)\right)-2b^2n^2(d+ex)\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{de(d+ex)}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(d + e*x)^2,x]
```

```
[Out] ((a + b*Log[c*x^n])*(a*e*x + b*e*x*Log[c*x^n] - 2*b*n*(d + e*x)*Log[1 + (e*
x)/d]) - 2*b^2*n^2*(d + e*x)*PolyLog[2, -((e*x)/d)])/(d*e*(d + e*x))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 369, normalized size of antiderivative = 4.79

method	result
risch	$-\frac{b^2 \ln(x^n)^2}{e(ex+d)} - \frac{2b^2 n \ln(x^n) \ln(ex+d)}{ed} + \frac{2b^2 n \ln(x^n) \ln(x)}{ed} - \frac{b^2 n^2 \ln(x)^2}{ed} + \frac{2b^2 n^2 \ln(ex+d) \ln(-\frac{ex}{d})}{ed} + \frac{2b^2 n^2 \operatorname{dilog}(-\frac{ex}{d})}{ed} +$

[In] int((a+b*ln(c*x^n))^2/(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] -b^2*ln(x^n)^2/e/(e*x+d)-2*b^2/e*n*ln(x^n)/d*ln(e*x+d)+2*b^2/e*n*ln(x^n)/d*ln(x)-b^2/e*n^2/d*ln(x)^2+2*b^2/e*n^2/d*ln(e*x+d)*ln(-e*x/d)+2*b^2/e*n^2/d*dilog(-e*x/d)+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(-ln(x^n)/e/(e*x+d)+1/e*n*(-1/d*ln(e*x+d)+1/d*ln(x)))-1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2/(e*x+d)/e

Fricas [F]

$$\int \frac{(a + b \log(cx^n))^2}{(d + ex)^2} dx = \int \frac{(b \log(cx^n) + a)^2}{(ex + d)^2} dx$$

[In] integrate((a+b*log(c*x^n))^2/(e*x+d)^2,x, algorithm="fricas")

[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e^2*x^2 + 2*d*e*x + d^2), x)

Sympy [F]

$$\int \frac{(a + b \log(cx^n))^2}{(d + ex)^2} dx = \int \frac{(a + b \log(cx^n))^2}{(d + ex)^2} dx$$

[In] integrate((a+b*ln(c*x**n))**2/(e*x+d)**2,x)

[Out] Integral((a + b*log(c*x**n))**2/(d + e*x)**2, x)

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*n*(log(e*x + d)/(d*e) - log(x)/(d*e)) - b^2*(log(x^n)^2/(e^2*x + d*e
) - integrate((e*x*log(c)^2 + 2*(d*n + (e*n + e*log(c))*x)*log(x^n))/(e^3*x
^3 + 2*d*e^2*x^2 + d^2*e*x), x)) - 2*a*b*log(c*x^n)/(e^2*x + d*e) - a^2/(e^
2*x + d*e)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{(d+e\,x)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(d + e*x)^2,x)
```

```
[Out] int((a + b*log(c*x^n))^2/(d + e*x)^2, x)
```

3.104 $\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 151

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^2}\,dx = -\frac{ex(a+b\log(cx^n))^2}{d^2(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^2} + \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^2} + \frac{2bn(a+b\log(cx^n))\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^2} + \frac{2b^2n^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^2} + \frac{2b^2n^2\,\text{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^2}$$

```
[Out] -e*x*(a+b*ln(c*x^n))^2/d^2/(e*x+d)-ln(1+d/e/x)*(a+b*ln(c*x^n))^2/d^2+2*b*n*
(a+b*ln(c*x^n))*ln(1+e*x/d)/d^2+2*b*n*(a+b*ln(c*x^n))*polylog(2,-d/e/x)/d^2
+2*b^2*n^2*polylog(2,-e*x/d)/d^2+2*b^2*n^2*polylog(3,-d/e/x)/d^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2389, 2379, 2421, 6724, 2355, 2354, 2438}

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^2}\,dx = \frac{2bn\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)(a+b\log(cx^n))}{d^2} + \frac{2bn\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{d^2} - \frac{\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))^2}{d^2} - \frac{ex(a+b\log(cx^n))^2}{d^2(d+ex)} + \frac{2b^2n^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^2} + \frac{2b^2n^2\,\text{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^2}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(x*(d + e*x)^2),x]
```

```
[Out] -((e*x*(a + b*Log[c*x^n])^2)/(d^2*(d + e*x))) - (Log[1 + d/(e*x)]*(a + b*Lo
g[c*x^n])^2)/d^2 + (2*b*n*(a + b*Log[c*x^n])*Log[1 + (e*x)/d])/d^2 + (2*b*n
*(a + b*Log[c*x^n])*PolyLog[2, -(d/(e*x))])/d^2 + (2*b^2*n^2*PolyLog[2, -((
e*x)/d)])/d^2 + (2*b^2*n^2*PolyLog[3, -(d/(e*x))])/d^2
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx}{d} - \frac{e\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx}{d} \\
&= -\frac{ex(a+b\log(cx^n))^2}{d^2(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^2} \\
&\quad + \frac{(2bn)\int \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{x}\,dx}{d^2} + \frac{(2ben)\int \frac{a+b\log(cx^n)}{d+ex}\,dx}{d^2} \\
&= -\frac{ex(a+b\log(cx^n))^2}{d^2(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^2} + \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^2} \\
&\quad + \frac{2bn(a+b\log(cx^n))\,\text{Li}_2\left(-\frac{d}{ex}\right)}{d^2} - \frac{(2b^2n^2)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{d^2} - \frac{(2b^2n^2)\int \frac{\text{Li}_2\left(-\frac{d}{ex}\right)}{x}\,dx}{d^2} \\
&= -\frac{ex(a+b\log(cx^n))^2}{d^2(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^2} \\
&\quad + \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^2} + \frac{2bn(a+b\log(cx^n))\,\text{Li}_2\left(-\frac{d}{ex}\right)}{d^2} \\
&\quad + \frac{2b^2n^2\text{Li}_2\left(-\frac{ex}{d}\right)}{d^2} + \frac{2b^2n^2\text{Li}_3\left(-\frac{d}{ex}\right)}{d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.10

$$
\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^2}\,dx = \frac{-3(a+b\log(cx^n))^2 + \frac{3d(a+b\log(cx^n))^2}{d+ex} + \frac{(a+b\log(cx^n))^3}{bn} + 6bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right) - 3(a+b\log(cx^n)}{3}
$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(x*(d + e*x)^2),x]
```

```
[Out] (-3*(a + b*Log[c*x^n])^2 + (3*d*(a + b*Log[c*x^n])^2)/(d + e*x) + (a + b*Lo
g[c*x^n])^3/(b*n) + 6*b*n*(a + b*Log[c*x^n])*Log[1 + (e*x)/d] - 3*(a + b*Lo
g[c*x^n])^2*Log[1 + (e*x)/d] + 6*b^2*n^2*PolyLog[2, -((e*x)/d)] - 6*b*n*(a
+ b*Log[c*x^n])*PolyLog[2, -((e*x)/d)] + 6*b^2*n^2*PolyLog[3, -((e*x)/d)])/
(3*d^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 683, normalized size of antiderivative = 4.52

method	result
risch	$-\frac{b^{2}\ln(x^{n})^{2}\ln(ex+d)}{d^{2}}+\frac{b^{2}\ln(x^{n})^{2}}{d(ex+d)}+\frac{b^{2}\ln(x^{n})^{2}\ln(x)}{d^{2}}+\frac{2b^{2}n\ln(x^{n})\ln(ex+d)}{d^{2}}-\frac{2b^{2}n\ln(x^{n})\ln(x)}{d^{2}}+\frac{b^{2}n^{2}\ln(x)^{2}}{d^{2}}-\frac{2b^{2}n^{2}}{}$

```
[In] int((a+b*ln(c*x^n))^2/x/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b^2*ln(x^n)^2/d^2*ln(e*x+d)+b^2*ln(x^n)^2/d/(e*x+d)+b^2*ln(x^n)^2/d^2*ln(x
)+2*b^2*n*ln(x^n)/d^2*ln(e*x+d)-2*b^2*n*ln(x^n)/d^2*ln(x)+b^2/d^2*n^2*ln(x)
^2-2*b^2/d^2*n^2*ln(e*x+d)*ln(-e*x/d)-2*b^2/d^2*n^2*dilog(-e*x/d)-b^2*n/d^2
*ln(x^n)*ln(x)^2+1/3*b^2/d^2*ln(x)^3*n^2-2*b^2/d^2*ln(x)*ln(e*x+d)*ln(-e*x/
d)*n^2-2*b^2/d^2*ln(x)*dilog(-e*x/d)*n^2+2*b^2*n/d^2*ln(x^n)*ln(e*x+d)*ln(-
e*x/d)+2*b^2*n/d^2*ln(x^n)*dilog(-e*x/d)+b^2/d^2*n^2*ln(e*x+d)*ln(x)^2-b^2/
d^2*n^2*ln(x)^2*ln(1+e*x/d)-2*b^2/d^2*n^2*ln(x)*polylog(2,-e*x/d)+2*b^2/d^2
*n^2*polylog(3,-e*x/d)+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*
csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*
c*x^n)^3+2*b*ln(c)+2*a)*b*(-ln(x^n)/d^2*ln(e*x+d)+ln(x^n)/d/(e*x+d)+ln(x^n)
/d^2*ln(x)-n*(1/2/d^2*ln(x)^2-1/d^2*ln(e*x+d)+1/d^2*ln(x)-1/d^2*ln(e*x+d)*l
n(-e*x/d)-1/d^2*dilog(-e*x/d)))+1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c
*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I
*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*(-1/d^2*ln(e*x+d)+1/d/(e*x+d)+1/d^2*
ln(x))
```

Fricas [F]

$$\int \frac{(a+b\log(cx^{n}))^{2}}{x(d+ex)^{2}}\,dx=\int \frac{(b\log(cx^{n})+a)^{2}}{(ex+d)^{2}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e^2*x^3 + 2*d*e*x^2 +
 d^2*x), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^2}\,dx = \int \frac{(a+b\log(cx^n))^2}{x(d+ex)^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**2/x/(e*x+d)**2,x)
[Out] Integral((a + b*log(c*x**n))**2/(x*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^2x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(e*x+d)^2,x, algorithm="maxima")
[Out] a^2*(1/(d*e*x + d^2) - log(e*x + d)/d^2 + log(x)/d^2) + integrate((b^2*log(
c)^2 + b^2*log(x^n)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*log(x^n))/(e^2*
x^3 + 2*d*e*x^2 + d^2*x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^2x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2/((e*x + d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^2}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{x\,(d+e\,x)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(x*(d + e*x)^2),x)
[Out] int((a + b*log(c*x^n))^2/(x*(d + e*x)^2), x)
```

3.105 $\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 211

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^2}\,dx = & -\frac{2b^2n^2}{d^2x}-\frac{2bn(a+b\log(cx^n))}{d^2x}-\frac{(a+b\log(cx^n))^2}{d^2x} \\ & +\frac{e^2x(a+b\log(cx^n))^2}{d^3(d+ex)}+\frac{2e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^3} \\ & -\frac{2ben(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^3} \\ & -\frac{4ben(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^3} \\ & -\frac{2b^2en^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^3}-\frac{4b^2en^2\operatorname{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^3}\end{aligned}$$

```
[Out] -2*b^2*n^2/d^2/x-2*b*n*(a+b*ln(c*x^n))/d^2/x-(a+b*ln(c*x^n))^2/d^2/x+e^2*x*
(a+b*ln(c*x^n))^2/d^3/(e*x+d)+2*e*ln(1+d/e/x)*(a+b*ln(c*x^n))^2/d^3-2*b*e*n
*(a+b*ln(c*x^n))*ln(1+e*x/d)/d^3-4*b*e*n*(a+b*ln(c*x^n))*polylog(2,-d/e/x)/
d^3-2*b^2*e*n^2*polylog(2,-e*x/d)/d^3-4*b^2*e*n^2*polylog(3,-d/e/x)/d^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used

= {2395, 2342, 2341, 2355, 2354, 2438, 2379, 2421, 6724}

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^2}\,dx = \frac{e^2x(a+b\log(cx^n))^2}{d^3(d+ex)} - \frac{4ben\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)(a+b\log(cx^n))}{d^3} - \frac{2ben\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{d^3} + \frac{2e\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))^2}{d^3} - \frac{2bn(a+b\log(cx^n))}{d^2x} - \frac{(a+b\log(cx^n))^2}{d^2x} - \frac{2b^2en^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^3} - \frac{4b^2en^2\,\text{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^3} - \frac{2b^2n^2}{d^2x}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(x^2*(d + e*x)^2),x]
```

```
[Out] (-2*b^2*n^2)/(d^2*x) - (2*b*n*(a + b*Log[c*x^n]))/(d^2*x) - (a + b*Log[c*x^
n])^2/(d^2*x) + (e^2*x*(a + b*Log[c*x^n])^2)/(d^3*(d + e*x)) + (2*e*Log[1 +
 d/(e*x)]*(a + b*Log[c*x^n])^2)/d^3 - (2*b*e*n*(a + b*Log[c*x^n])*Log[1 + (
e*x)/d])/d^3 - (4*b*e*n*(a + b*Log[c*x^n])*PolyLog[2, -(d/(e*x))])/d^3 - (2
*b^2*e*n^2*PolyLog[2, -((e*x)/d)])/d^3 - (4*b^2*e*n^2*PolyLog[3, -(d/(e*x))
])/d^3
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
```

```
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(a+b\log(cx^n))^2}{d^2x^2}+\frac{e^2(a+b\log(cx^n))^2}{d^2(d+ex)^2}-\frac{2e(a+b\log(cx^n))^2}{d^2x(d+ex)}\right)dx \\
&= \frac{\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx}{d^2}-\frac{(2e)\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx}{d^2}+\frac{e^2\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx}{d^2}
\end{aligned}$$

$$= -\frac{(a+b\log(cx^n))^2}{d^2x} + \frac{e^2x(a+b\log(cx^n))^2}{d^3(d+ex)} + \frac{2e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^3} + \frac{(2bn)\int\frac{a+b\log(cx^n)}{x^2}\,dx}{d^2} - \frac{(4ben)\int\frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{x}\,dx}{d^3} - \frac{(2be^2n)\int\frac{a+b\log(cx^n)}{d+ex}\,dx}{d^3}$$

$$= -\frac{2b^2n^2}{d^2x} - \frac{2bn(a+b\log(cx^n))}{d^2x} - \frac{(a+b\log(cx^n))^2}{d^2x} + \frac{e^2x(a+b\log(cx^n))^2}{d^3(d+ex)} + \frac{2e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^3} - \frac{2ben(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^3} - \frac{4ben(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^3} + \frac{(2b^2en^2)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{d^3} + \frac{(4b^2en^2)\int\frac{\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{x}\,dx}{d^3}$$

$$= -\frac{2b^2n^2}{d^2x} - \frac{2bn(a+b\log(cx^n))}{d^2x} - \frac{(a+b\log(cx^n))^2}{d^2x} + \frac{e^2x(a+b\log(cx^n))^2}{d^3(d+ex)} + \frac{2e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^3} - \frac{2ben(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^3} - \frac{4ben(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^3} - \frac{2b^2en^2\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{d^3} - \frac{4b^2en^2\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.06

$$\int\frac{(a+b\log(cx^n))^2}{x^2(d+ex)^2}\,dx = -\frac{\frac{6b^2dn^2}{x} + \frac{6bdn(a+b\log(cx^n))}{x} - 3e(a+b\log(cx^n))^2 + \frac{3d(a+b\log(cx^n))^2}{x} + \frac{3de(a+b\log(cx^n))^2}{d+ex} + \frac{2e(a+b\log(cx^n))^3}{bn} + 6ben}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(x^2*(d + e*x)^2),x]
```

```
[Out] -1/3*((6*b^2*d*n^2)/x + (6*b*d*n*(a + b*Log[c*x^n]))/x - 3*e*(a + b*Log[c*x
^n])^2 + (3*d*(a + b*Log[c*x^n])^2)/x + (3*d*e*(a + b*Log[c*x^n])^2)/(d + e
*x) + (2*e*(a + b*Log[c*x^n])^3)/(b*n) + 6*b*e*n*(a + b*Log[c*x^n])*Log[1 +
 (e*x)/d] - 6*e*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d] + 6*b^2*e*n^2*PolyLog
[2, -((e*x)/d)] - 12*b*e*n*(a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)] + 12*b
^2*e*n^2*PolyLog[3, -((e*x)/d)])/d^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 790, normalized size of antiderivative = 3.74

method	result	size
risch	Expression too large to display	790

```
[In] int((a+b*ln(c*x^n))^2/x^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b^2*ln(x^n)^2/d^2*e/(e*x+d)+2*b^2*ln(x^n)^2/d^3*e*ln(e*x+d)-b^2*ln(x^n)^2/
d^2/x-2*b^2*ln(x^n)^2/d^3*e*ln(x)-2*b^2*n*ln(x^n)/d^3*e*ln(e*x+d)-2*b^2*n*l
n(x^n)/d^2/x+2*b^2*n*ln(x^n)/d^3*e*ln(x)-b^2/d^3*n^2*e*ln(x)^2-2*b^2*n^2/d^
2/x+2*b^2/d^3*n^2*e*ln(e*x+d)*ln(-e*x/d)+2*b^2/d^3*n^2*e*dilog(-e*x/d)+2*b^
2*n/d^3*e*ln(x^n)*ln(x)^2-2/3*b^2/d^3*e*ln(x)^3*n^2+4*b^2/d^3*e*ln(x)*ln(e*
x+d)*ln(-e*x/d)*n^2+4*b^2/d^3*e*ln(x)*dilog(-e*x/d)*n^2-4*b^2*n/d^3*e*ln(x^
n)*ln(e*x+d)*ln(-e*x/d)-4*b^2*n/d^3*e*ln(x^n)*dilog(-e*x/d)-2*b^2/d^3*e*n^2
*ln(e*x+d)*ln(x)^2+2*b^2/d^3*e*n^2*ln(x)^2*ln(1+e*x/d)+4*b^2/d^3*e*n^2*ln(x
)*polylog(2,-e*x/d)-4*b^2/d^3*e*n^2*polylog(3,-e*x/d)+(-I*b*Pi*csgn(I*c)*cs
gn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)
*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(-ln(x^n)/d^2*e/(e
*x+d)+2*ln(x^n)/d^3*e*ln(e*x+d)-ln(x^n)/d^2/x-2*ln(x^n)/d^3*e*ln(x)-n*(-1/d
^3*e*ln(x)^2+2/d^3*e*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))+1/d^3*e*ln(e*x+d)
+1/d^2/x-1/d^3*e*ln(x)))+1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I
*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*c
sgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*(-1/d^2*e/(e*x+d)+2/d^3*e*ln(e*x+d)-1/d^2/x
-2/d^3*e*ln(x))
```

Fricas [F]

$$\int \frac{(a+b\log{(cx^n)})^2}{x^2(d+ex)^2}\,dx=\int \frac{(b\log{(cx^n)}+a)^2}{(ex+d)^2x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e^2*x^4 + 2*d*e*x^3 +
 d^2*x^2), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^2}\,dx = \int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**2/x**2/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*log(c*x**n))**2/(x**2*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^2x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -a^2*((2*e*x + d)/(d^2*e*x^2 + d^3*x) - 2*e*log(e*x + d)/d^3 + 2*e*log(x)/d
^3) + integrate((b^2*log(c)^2 + b^2*log(x^n)^2 + 2*a*b*log(c) + 2*(b^2*log(
c) + a*b)*log(x^n))/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^2x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2/((e*x + d)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^2}\,dx = \int \frac{(a+b\ln(c\,x^n))^2}{x^2\,(d+e\,x)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(x^2*(d + e*x)^2),x)
```

```
[Out] int((a + b*log(c*x^n))^2/(x^2*(d + e*x)^2), x)
```

3.106 $\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 285

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)^2}\,dx &= -\frac{b^2n^2}{4d^2x^2}+\frac{4b^2en^2}{d^3x}-\frac{bn(a+b\log(cx^n))}{2d^2x^2}+\frac{4ben(a+b\log(cx^n))}{d^3x}\\ &\quad-\frac{(a+b\log(cx^n))^2}{2d^2x^2}+\frac{2e(a+b\log(cx^n))^2}{d^3x}\\ &\quad-\frac{e^3x(a+b\log(cx^n))^2}{d^4(d+ex)}-\frac{3e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^4}\\ &\quad+\frac{2be^2n(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^4}\\ &\quad+\frac{6be^2n(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^4}\\ &\quad+\frac{2b^2e^2n^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^4}+\frac{6b^2e^2n^2\operatorname{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^4}\end{aligned}$$

```
[Out] -1/4*b^2*n^2/d^2/x^2+4*b^2*e*n^2/d^3/x-1/2*b*n*(a+b*ln(c*x^n))/d^2/x^2+4*b*
e*n*(a+b*ln(c*x^n))/d^3/x-1/2*(a+b*ln(c*x^n))^2/d^2/x^2+2*e*(a+b*ln(c*x^n))
^2/d^3/x-e^3*x*(a+b*ln(c*x^n))^2/d^4/(e*x+d)-3*e^2*ln(1+d/e/x)*(a+b*ln(c*x^
n))^2/d^4+2*b*e^2*n*(a+b*ln(c*x^n))*ln(1+e*x/d)/d^4+6*b*e^2*n*(a+b*ln(c*x^n
))*polylog(2,-d/e/x)/d^4+2*b^2*e^2*n^2*polylog(2,-e*x/d)/d^4+6*b^2*e^2*n^2*
polylog(3,-d/e/x)/d^4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2395, 2342, 2341, 2355, 2354, 2438, 2379, 2421, 6724}

$$\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)^2}\,dx = -\frac{e^3x(a+b\log(cx^n))^2}{d^4(d+ex)}+\frac{6be^2n\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4} -\frac{3e^2\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))^2}{d^4} +\frac{2be^2n\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{d^4}+\frac{2e(a+b\log(cx^n))^2}{d^3x} +\frac{4ben(a+b\log(cx^n))}{d^3x}-\frac{(a+b\log(cx^n))^2}{2d^2x^2} -\frac{bn(a+b\log(cx^n))}{2d^2x^2}+\frac{2b^2e^2n^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^4} +\frac{6b^2e^2n^2\,\text{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^4}+\frac{4b^2en^2}{d^3x}-\frac{b^2n^2}{4d^2x^2}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(x^3*(d + e*x)^2),x]
```

```
[Out] -1/4*(b^2*n^2)/(d^2*x^2) + (4*b^2*e*n^2)/(d^3*x) - (b*n*(a + b*Log[c*x^n]))
/(2*d^2*x^2) + (4*b*e*n*(a + b*Log[c*x^n]))/(d^3*x) - (a + b*Log[c*x^n])^2/
(2*d^2*x^2) + (2*e*(a + b*Log[c*x^n])^2)/(d^3*x) - (e^3*x*(a + b*Log[c*x^n]
)^2)/(d^4*(d + e*x)) - (3*e^2*Log[1 + d/(e*x)]*(a + b*Log[c*x^n])^2)/d^4 +
(2*b*e^2*n*(a + b*Log[c*x^n])*Log[1 + (e*x)/d])/d^4 + (6*b*e^2*n*(a + b*Log
[c*x^n])*PolyLog[2, -(d/(e*x))])/d^4 + (2*b^2*e^2*n^2*PolyLog[2, -((e*x)/d)
])/d^4 + (6*b^2*e^2*n^2*PolyLog[3, -(d/(e*x))])/d^4
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
```

```
Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a+b\log(cx^n))^2}{d^2x^3} - \frac{2e(a+b\log(cx^n))^2}{d^3x^2} - \frac{e^3(a+b\log(cx^n))^2}{d^3(d+ex)^2} + \frac{3e^2(a+b\log(cx^n))^2}{d^3x(d+ex)}\right) dx$$

$$= \frac{\int \frac{(a+b\log(cx^n))^2}{x^3}\,dx}{d^2} - \frac{(2e)\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx}{d^3} + \frac{(3e^2)\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx}{d^3} - \frac{e^3\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx}{d^3}$$

$$= -\frac{(a+b\log(cx^n))^2}{2d^2x^2} + \frac{2e(a+b\log(cx^n))^2}{d^3x} - \frac{e^3x(a+b\log(cx^n))^2}{d^4(d+ex)} - \frac{3e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^4} + \frac{(bn)\int \frac{a+b\log(cx^n)}{x^3}\,dx}{d^2} - \frac{(4ben)\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d^3} + \frac{(6be^2n)\int \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{x}\,dx}{d^4} + \frac{(2be^3n)\int \frac{a+b\log(cx^n)}{d+ex}\,dx}{d^4}$$

$$= -\frac{b^2n^2}{4d^2x^2} + \frac{4b^2en^2}{d^3x} - \frac{bn(a+b\log(cx^n))}{2d^2x^2} + \frac{4ben(a+b\log(cx^n))}{d^3x} - \frac{(a+b\log(cx^n))^2}{2d^2x^2} + \frac{2e(a+b\log(cx^n))^2}{d^3x} - \frac{e^3x(a+b\log(cx^n))^2}{d^4(d+ex)} - \frac{3e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^4} + \frac{2be^2n(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^4} + \frac{6be^2n(a+b\log(cx^n))\,\text{Li}_2\left(-\frac{d}{ex}\right)}{d^4} - \frac{(2b^2e^2n^2)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{d^4} - \frac{(6b^2e^2n^2)\int \frac{\text{Li}_2\left(-\frac{d}{ex}\right)}{x}\,dx}{d^4}$$

$$= -\frac{b^2n^2}{4d^2x^2} + \frac{4b^2en^2}{d^3x} - \frac{bn(a+b\log(cx^n))}{2d^2x^2} + \frac{4ben(a+b\log(cx^n))}{d^3x} - \frac{(a+b\log(cx^n))^2}{2d^2x^2} + \frac{2e(a+b\log(cx^n))^2}{d^3x} - \frac{e^3x(a+b\log(cx^n))^2}{d^4(d+ex)} - \frac{3e^2\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^4} + \frac{2be^2n(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^4} + \frac{6be^2n(a+b\log(cx^n))\,\text{Li}_2\left(-\frac{d}{ex}\right)}{d^4} + \frac{2b^2e^2n^2\text{Li}_2\left(-\frac{ex}{d}\right)}{d^4} + \frac{6b^2e^2n^2\text{Li}_3\left(-\frac{d}{ex}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)^2}\,dx = \frac{-\frac{2d^2(a+b\log(cx^n))^2}{x^2}+\frac{8de(a+b\log(cx^n))^2}{x}+\frac{4de^2(a+b\log(cx^n))^2}{d+ex}+\frac{4e^2(a+b\log(cx^n))^3}{bn}+\frac{16bden(a+bn+b\log(cx^n))}{x}-\frac{bd^2n(2a+bn\cdots}{x\cdots}}{\cdots}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(x^3*(d + e*x)^2),x]
```

```
[Out] ((-2*d^2*(a + b*Log[c*x^n])^2)/x^2 + (8*d*e*(a + b*Log[c*x^n])^2)/x + (4*d*
e^2*(a + b*Log[c*x^n])^2)/(d + e*x) + (4*e^2*(a + b*Log[c*x^n])^3)/(b*n) +
(16*b*d*e*n*(a + b*n + b*Log[c*x^n]))/x - (b*d^2*n*(2*a + b*n + 2*b*Log[c*x
^n]))/x^2 - 12*e^2*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d] + 4*e^2*(-((a + b*
Log[c*x^n])*(a + b*Log[c*x^n] - 2*b*n*Log[1 + (e*x)/d])) + 2*b^2*n^2*PolyLo
g[2, -((e*x)/d)]) - 24*b*e^2*n*((a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)] -
 b*n*PolyLog[3, -((e*x)/d)]))/(4*d^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.48 (sec) , antiderivative size = 924, normalized size of antiderivative = 3.24

method	result	size
risch	Expression too large to display	924

```
[In] int((a+b*ln(c*x^n))^2/x^3/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2*ln(x^n)^2/d^2/x^2+2*b^2*ln(x^n)^2/d^3*e/x-6*b^2/d^4*e^2*ln(x)*ln(e
*x+d)*ln(-e*x/d)*n^2+6*b^2*n/d^4*e^2*ln(x^n)*ln(e*x+d)*ln(-e*x/d)+b^2*ln(x^
n)^2/d^3*e^2/(e*x+d)+4*b^2*n*ln(x^n)/d^3*e/x+4*b^2*e*n^2/d^3/x+(-I*b*Pi*csg
n(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*cs
gn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(-3*ln(x^
n)/d^4*e^2*ln(e*x+d)+ln(x^n)/d^3*e^2/(e*x+d)-1/2*ln(x^n)/d^2/x^2+3*ln(x^n)/
d^4*e^2*ln(x)+2*ln(x^n)/d^3*e/x-1/2*n*(-2/d^4*e^2*ln(e*x+d)+1/2/d^2/x^2-4/d
^3*e/x+2/d^4*e^2*ln(x)+3/d^4*e^2*ln(x)^2-6/d^4*e^2*(dilog(-e*x/d)+ln(e*x+d)
*ln(-e*x/d))))+2*b^2*n*ln(x^n)/d^4*e^2*ln(e*x+d)-2*b^2*n*ln(x^n)/d^4*e^2*ln
(x)-2*b^2/d^4*n^2*e^2*ln(e*x+d)*ln(-e*x/d)-3*b^2*n/d^4*e^2*ln(x^n)*ln(x)^2-
6*b^2/d^4*e^2*ln(x)*dilog(-e*x/d)*n^2+6*b^2*n/d^4*e^2*ln(x^n)*dilog(-e*x/d)
+3*b^2/d^4*e^2*n^2*ln(e*x+d)*ln(x)^2-3*b^2/d^4*e^2*n^2*ln(x)^2*ln(1+e*x/d)-
6*b^2/d^4*e^2*n^2*ln(x)*polylog(2,-e*x/d)+1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n
)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*
c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*(-3/d^4*e^2*ln(e*x+d)+1/d^
```

```
3*e^2/(e*x+d)-1/2/d^2/x^2+3/d^4*e^2*ln(x)+2/d^3*e/x)-3*b^2*ln(x^n)^2/d^4*e^
2*ln(e*x+d)+3*b^2*ln(x^n)^2/d^4*e^2*ln(x)-1/2*b^2*n*ln(x^n)/d^2/x^2+b^2/d^4
*n^2*e^2*ln(x)^2-2*b^2/d^4*n^2*e^2*dilog(-e*x/d)+b^2/d^4*e^2*ln(x)^3*n^2+6*
b^2/d^4*e^2*n^2*polylog(3,-e*x/d)-1/4*b^2*n^2/d^2/x^2
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^2x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e^2*x^5 + 2*d*e*x^4 +
 d^2*x^3), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)^2}\,dx = \int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**2/x**3/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*log(c*x**n))**2/(x**3*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^2x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*((6*e^2*x^2 + 3*d*e*x - d^2)/(d^3*e*x^3 + d^4*x^2) - 6*e^2*log(e*x
+ d)/d^4 + 6*e^2*log(x)/d^4) + integrate((b^2*log(c)^2 + b^2*log(x^n)^2 + 2
*a*b*log(c) + 2*(b^2*log(c) + a*b)*log(x^n))/(e^2*x^5 + 2*d*e*x^4 + d^2*x^3
), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^2x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^3/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2/((e*x + d)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{x^3(d+ex)^2}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{x^3\,(d+e\,x)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(x^3*(d + e*x)^2),x)
[Out] int((a + b*log(c*x^n))^2/(x^3*(d + e*x)^2), x)
```

3.107 $\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 296

$$\begin{aligned}\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = & -\frac{2abnx}{e^3}+\frac{2b^2n^2x}{e^3}-\frac{2b^2nx\log(cx^n)}{e^3}+\frac{bdnx(a+b\log(cx^n))}{e^3(d+ex)}\\ & -\frac{d(a+b\log(cx^n))^2}{2e^4}+\frac{x(a+b\log(cx^n))^2}{e^3}\\ & +\frac{d^3(a+b\log(cx^n))^2}{2e^4(d+ex)^2}+\frac{3dx(a+b\log(cx^n))^2}{e^3(d+ex)}\\ & -\frac{b^2dn^2\log(d+ex)}{e^4}-\frac{5bdn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^4}\\ & -\frac{3d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^4}-\frac{5b^2dn^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4}\\ & -\frac{6bdn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4}\\ & +\frac{6b^2dn^2\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^4}\end{aligned}$$

```
[Out] -2*a*b*n*x/e^3+2*b^2*n^2*x/e^3-2*b^2*n*x*ln(c*x^n)/e^3+b*d*n*x*(a+b*ln(c*x^
n))/e^3/(e*x+d)-1/2*d*(a+b*ln(c*x^n))^2/e^4+x*(a+b*ln(c*x^n))^2/e^3+1/2*d^3
*(a+b*ln(c*x^n))^2/e^4/(e*x+d)^2+3*d*x*(a+b*ln(c*x^n))^2/e^3/(e*x+d)-b^2*d*
n^2*ln(e*x+d)/e^4-5*b*d*n*(a+b*ln(c*x^n))*ln(1+e*x/d)/e^4-3*d*(a+b*ln(c*x^n
))^2*ln(1+e*x/d)/e^4-5*b^2*d*n^2*polylog(2,-e*x/d)/e^4-6*b*d*n*(a+b*ln(c*x^
n))*polylog(2,-e*x/d)/e^4+6*b^2*d*n^2*polylog(3,-e*x/d)/e^4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.10, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2395, 2333, 2332, 2356, 2389, 2379, 2438, 2351, 31, 2355, 2354, 2421, 6724}

$$\begin{aligned}\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^3}dx = &\frac{d^3(a+b\log(cx^n))^2}{2e^4(d+ex)^2}-\frac{6bdn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)(a+b\log(cx^n))}{e^4}\\&+\frac{bdn\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{e^4}\\&-\frac{3d\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))^2}{e^4}\\&-\frac{6bdn\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{e^4}\\&+\frac{3dx(a+b\log(cx^n))^2}{e^3(d+ex)}+\frac{bdnx(a+b\log(cx^n))}{e^3(d+ex)}\\&+\frac{x(a+b\log(cx^n))^2}{e^3}-\frac{2abnx}{e^3}-\frac{2b^2nx\log(cx^n)}{e^3}\\&-\frac{b^2dn^2\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{e^4}-\frac{6b^2dn^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4}\\&+\frac{6b^2dn^2\,\text{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^4}-\frac{b^2dn^2\log(d+ex)}{e^4}+\frac{2b^2n^2x}{e^3}\end{aligned}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n])^2)/(d + e*x)^3,x]
```

```
[Out] (-2*a*b*n*x)/e^3 + (2*b^2*n^2*x)/e^3 - (2*b^2*n*x*Log[c*x^n])/e^3 + (b*d*n*
x*(a + b*Log[c*x^n]))/(e^3*(d + e*x)) + (b*d*n*Log[1 + d/(e*x)]*(a + b*Log[
c*x^n]))/e^4 + (x*(a + b*Log[c*x^n])^2)/e^3 + (d^3*(a + b*Log[c*x^n])^2)/(2
*e^4*(d + e*x)^2) + (3*d*x*(a + b*Log[c*x^n])^2)/(e^3*(d + e*x)) - (b^2*d*n
^2*Log[d + e*x])/e^4 - (6*b*d*n*(a + b*Log[c*x^n])*Log[1 + (e*x)/d])/e^4 -
(3*d*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d])/e^4 - (b^2*d*n^2*PolyLog[2, -(d
/(e*x))])/e^4 - (6*b^2*d*n^2*PolyLog[2, -((e*x)/d)])/e^4 - (6*b*d*n*(a + b*
Log[c*x^n])*PolyLog[2, -((e*x)/d)])/e^4 + (6*b^2*d*n^2*PolyLog[3, -((e*x)/d
)])/e^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
```

```
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a+b\log(cx^n))^2}{e^3} - \frac{d^3(a+b\log(cx^n))^2}{e^3(d+ex)^3} + \frac{3d^2(a+b\log(cx^n))^2}{e^3(d+ex)^2} - \frac{3d(a+b\log(cx^n))^2}{e^3(d+ex)}\right) dx$$

$$= \frac{\int (a+b\log(cx^n))^2\,dx}{e^3} - \frac{(3d)\int \frac{(a+b\log(cx^n))^2}{d+ex}\,dx}{e^3} + \frac{(3d^2)\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx}{e^3} - \frac{d^3\int \frac{(a+b\log(cx^n))^2}{(d+ex)^3}\,dx}{e^3}$$

$$
\begin{aligned}
&= \frac{x(a+b\log(cx^n))^2}{e^3} + \frac{d^3(a+b\log(cx^n))^2}{2e^4(d+ex)^2} + \frac{3dx(a+b\log(cx^n))^2}{e^3(d+ex)} \\
&\quad - \frac{3d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^4} + \frac{(6bdn)\int \frac{(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^4} \\
&\quad - \frac{(bd^3n)\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx}{e^4} - \frac{(2bn)\int (a+b\log(cx^n))\,dx}{e^3} - \frac{(6bdn)\int \frac{a+b\log(cx^n)}{d+ex}\,dx}{e^3} \\
&= -\frac{2abnx}{e^3} + \frac{x(a+b\log(cx^n))^2}{e^3} + \frac{d^3(a+b\log(cx^n))^2}{2e^4(d+ex)^2} + \frac{3dx(a+b\log(cx^n))^2}{e^3(d+ex)} \\
&\quad - \frac{6bdn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^4} - \frac{3d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^4} \\
&\quad - \frac{6bdn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^4} - \frac{(bd^2n)\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{e^4} - \frac{(2b^2n)\int \log(cx^n)\,dx}{e^3} \\
&\quad + \frac{(bd^2n)\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{e^3} + \frac{(6b^2dn^2)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^4} + \frac{(6b^2dn^2)\int \frac{\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{x}\,dx}{e^4} \\
&= -\frac{2abnx}{e^3} + \frac{2b^2n^2x}{e^3} - \frac{2b^2nx\log(cx^n)}{e^3} + \frac{bdnx(a+b\log(cx^n))}{e^3(d+ex)} \\
&\quad + \frac{bdn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{e^4} + \frac{x(a+b\log(cx^n))^2}{e^3} \\
&\quad + \frac{d^3(a+b\log(cx^n))^2}{2e^4(d+ex)^2} + \frac{3dx(a+b\log(cx^n))^2}{e^3(d+ex)} \\
&\quad - \frac{6bdn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^4} - \frac{3d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^4} \\
&\quad - \frac{6b^2dn^2\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^4} - \frac{6bdn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^4} \\
&\quad + \frac{6b^2dn^2\operatorname{Li}_3\left(-\frac{ex}{d}\right)}{e^4} - \frac{(b^2dn^2)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{e^4} - \frac{(b^2dn^2)\int \frac{1}{d+ex}\,dx}{e^3} \\
&= -\frac{2abnx}{e^3} + \frac{2b^2n^2x}{e^3} - \frac{2b^2nx\log(cx^n)}{e^3} + \frac{bdnx(a+b\log(cx^n))}{e^3(d+ex)} \\
&\quad + \frac{bdn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{e^4} + \frac{x(a+b\log(cx^n))^2}{e^3} + \frac{d^3(a+b\log(cx^n))^2}{2e^4(d+ex)^2} \\
&\quad + \frac{3dx(a+b\log(cx^n))^2}{e^3(d+ex)} - \frac{b^2dn^2\log(d+ex)}{e^4} - \frac{6bdn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^4} \\
&\quad - \frac{3d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^4} - \frac{b^2dn^2\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{e^4} - \frac{6b^2dn^2\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^4} \\
&\quad - \frac{6bdn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^4} + \frac{6b^2dn^2\operatorname{Li}_3\left(-\frac{ex}{d}\right)}{e^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.87

$$\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \frac{-\frac{2bd^2n(a+b\log(cx^n))}{d+ex}+5d(a+b\log(cx^n))^2+2ex(a+b\log(cx^n))^2+\frac{d^3(a+b\log(cx^n))^2}{(d+ex)^2}-\frac{6d^2(a+b\log(cx^n))^2}{d+ex}-4ben}{}$$

[In] Integrate[(x^3*(a + b*Log[c*x^n])^2)/(d + e*x)^3,x]

[Out] ((-2*b*d^2*n*(a + b*Log[c*x^n]))/(d + e*x) + 5*d*(a + b*Log[c*x^n])^2 + 2*e*x*(a + b*Log[c*x^n])^2 + (d^3*(a + b*Log[c*x^n])^2)/(d + e*x)^2 - (6*d^2*(a + b*Log[c*x^n])^2)/(d + e*x) - 4*b*e*n*x*(a - b*n + b*Log[c*x^n]) + 2*b^2*d*n^2*(Log[x] - Log[d + e*x]) - 10*b*d*n*(a + b*Log[c*x^n])*Log[1 + (e*x)/d] - 6*d*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d] - 10*b^2*d*n^2*PolyLog[2, -((e*x)/d)] - 12*b*d*n*(a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)] + 12*b^2*d*n^2*PolyLog[3, -((e*x)/d)])/(2*e^4)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 827, normalized size of antiderivative = 2.79

method	result	size
risch	Expression too large to display	827

[In] int(x^3*(a+b*ln(c*x^n))^2/(e*x+d)^3,x,method=_RETURNVERBOSE)

[Out] b^2*ln(x^n)^2*x/e^3-3*b^2*ln(x^n)^2/e^4*d*ln(e*x+d)-3*b^2*ln(x^n)^2/e^4*d^2/(e*x+d)+1/2*b^2*ln(x^n)^2*d^3/e^4/(e*x+d)^2-2*b^2*n*ln(x^n)*x/e^3-5*b^2*n*ln(x^n)/e^4*d*ln(e*x+d)-b^2*n*ln(x^n)/e^4*d^2/(e*x+d)+5*b^2*n/e^4*ln(x)*ln(x^n)*d+2*b^2*n^2*x/e^3-b^2*d*n^2*ln(e*x+d)/e^4+b^2/e^4*n^2*d*ln(x)-5/2*b^2/e^4*n^2*d*ln(x)^2+5*b^2/e^4*n^2*ln(e*x+d)*ln(-e*x/d)*d+5*b^2/e^4*n^2*dilog(-e*x/d)*d-6*b^2/e^4*d*ln(e*x+d)*ln(-e*x/d)*ln(x)*n^2+6*b^2*n/e^4*d*ln(x^n)*ln(e*x+d)*ln(-e*x/d)-6*b^2/e^4*d*dilog(-e*x/d)*ln(x)*n^2+6*b^2*n/e^4*d*ln(x^n)*dilog(-e*x/d)+3*b^2/e^4*d*n^2*ln(e*x+d)*ln(x)^2-3*b^2/e^4*d*n^2*ln(x)^2*ln(1+e*x/d)-6*b^2/e^4*d*n^2*ln(x)*polylog(2,-e*x/d)+6*b^2*d*n^2*polylog(3,-e*x/d)/e^4+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(ln(x^n)*x/e^3-3*ln(x^n)/e^4*d*ln(e*x+d)-3*ln(x^n)/e^4*d^2/(e*x+d)+1/2*ln(x^n)*d^3/e^4/(e*x+d)^2-1/2*n*(1/e^4*(2*e*x+2*d+5*d*ln(e*x+d)+d^2/(e*x+d)-5*d*ln(e*x))-6/e^4*d*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))))+1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n

```
)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a
)^2*(x/e^3-3/e^4*d*ln(e*x+d)-3/e^4*d^2/(e*x+d)+1/2*d^3/e^4/(e*x+d)^2)
```

Fricas [F]

$$\int \frac{x^3(a+b\log{(cx^n)})^2}{(d+ex)^3}\,dx = \int \frac{(b\log{(cx^n)}+a)^2x^3}{(ex+d)^3}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*log(c*x^n)^2 + 2*a*b*x^3*log(c*x^n) + a^2*x^3)/(e^3*x^3 +
 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\log{(cx^n)})^2}{(d+ex)^3}\,dx = \int \frac{x^3(a+b\log{(cx^n)})^2}{(d+ex)^3}\,dx$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))**2/(e*x+d)**3,x)
```

```
[Out] Integral(x**3*(a + b*log(c*x**n))**2/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{x^3(a+b\log{(cx^n)})^2}{(d+ex)^3}\,dx = \int \frac{(b\log{(cx^n)}+a)^2x^3}{(ex+d)^3}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*((6*d^2*e*x + 5*d^3)/(e^6*x^2 + 2*d*e^5*x + d^2*e^4) - 2*x/e^3 + 6
*d*log(e*x + d)/e^4) + integrate((b^2*x^3*log(x^n)^2 + 2*(b^2*log(c) + a*b)
*x^3*log(x^n) + (b^2*log(c)^2 + 2*a*b*log(c))*x^3)/(e^3*x^3 + 3*d*e^2*x^2 +
 3*d^2*e*x + d^3), x)
```

Giac [F]

$$\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)^2x^3}{(ex+d)^3}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^2/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*x^3/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \int \frac{x^3\,(a+b\,\ln(c\,x^n))^2}{(d+e\,x)^3}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n))^2)/(d + e*x)^3,x)
[Out] int((x^3*(a + b*log(c*x^n))^2)/(d + e*x)^3, x)
```

3.108 $\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 232

$$\begin{aligned}\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = &-\frac{bnx(a+b\log(cx^n))}{e^2(d+ex)}+\frac{(a+b\log(cx^n))^2}{2e^3}\\&-\frac{d^2(a+b\log(cx^n))^2}{2e^3(d+ex)^2}-\frac{2x(a+b\log(cx^n))^2}{e^2(d+ex)}\\&+\frac{b^2n^2\log(d+ex)}{e^3}+\frac{3bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^3}\\&+\frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^3}+\frac{3b^2n^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^3}\\&+\frac{2bn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^3}\\&-\frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^3}\end{aligned}$$

```
[Out] -b*n*x*(a+b*ln(c*x^n))/e^2/(e*x+d)+1/2*(a+b*ln(c*x^n))^2/e^3-1/2*d^2*(a+b*l
n(c*x^n))^2/e^3/(e*x+d)^2-2*x*(a+b*ln(c*x^n))^2/e^2/(e*x+d)+b^2*n^2*ln(e*x+
d)/e^3+3*b*n*(a+b*ln(c*x^n))*ln(1+e*x/d)/e^3+(a+b*ln(c*x^n))^2*ln(1+e*x/d)/
e^3+3*b^2*n^2*polylog(2,-e*x/d)/e^3+2*b*n*(a+b*ln(c*x^n))*polylog(2,-e*x/d)
/e^3-2*b^2*n^2*polylog(3,-e*x/d)/e^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2395, 2356, 2389, 2379, 2438, 2351, 31, 2355, 2354, 2421, 6724}

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = -\frac{d^2(a+b\log(cx^n))^2}{2e^3(d+ex)^2}+\frac{2bn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)(a+b\log(cx^n))}{e^3} - \frac{bn\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{e^3} + \frac{4bn\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{e^3} + \frac{\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))^2}{e^3} - \frac{bnx(a+b\log(cx^n))}{e^2(d+ex)} - \frac{2x(a+b\log(cx^n))^2}{e^2(d+ex)} + \frac{b^2n^2\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{e^3} + \frac{4b^2n^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^3} - \frac{2b^2n^2\,\text{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^3} + \frac{b^2n^2\log(d+ex)}{e^3}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n])^2)/(d + e*x)^3,x]
```

```
[Out] -((b*n*x*(a + b*Log[c*x^n]))/(e^2*(d + e*x))) - (b*n*Log[1 + d/(e*x)]*(a +
b*Log[c*x^n]))/e^3 - (d^2*(a + b*Log[c*x^n])^2)/(2*e^3*(d + e*x)^2) - (2*x*
(a + b*Log[c*x^n])^2)/(e^2*(d + e*x)) + (b^2*n^2*Log[d + e*x])/e^3 + (4*b*n
*(a + b*Log[c*x^n])*Log[1 + (e*x)/d])/e^3 + ((a + b*Log[c*x^n])^2*Log[1 + (
e*x)/d])/e^3 + (b^2*n^2*PolyLog[2, -(d/(e*x))])/e^3 + (4*b^2*n^2*PolyLog[2,
 -((e*x)/d)])/e^3 + (2*b*n*(a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)])/e^3 -
 (2*b^2*n^2*PolyLog[3, -((e*x)/d)])/e^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
```

```
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
```

```
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2(a+b\log(cx^n))^2}{e^2(d+ex)^3} - \frac{2d(a+b\log(cx^n))^2}{e^2(d+ex)^2} + \frac{(a+b\log(cx^n))^2}{e^2(d+ex)}\right) dx$$

$$= \frac{\int \frac{(a+b\log(cx^n))^2}{d+ex}\,dx}{e^2} - \frac{(2d)\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx}{e^2} + \frac{d^2\int \frac{(a+b\log(cx^n))^2}{(d+ex)^3}\,dx}{e^2}$$

$$= -\frac{d^2(a+b\log(cx^n))^2}{2e^3(d+ex)^2} - \frac{2x(a+b\log(cx^n))^2}{e^2(d+ex)} + \frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^3} - \frac{(2bn)\int \frac{(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^3} + \frac{(bd^2n)\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx}{e^3} + \frac{(4bn)\int \frac{a+b\log(cx^n)}{d+ex}\,dx}{e^2}$$

$$= -\frac{d^2(a+b\log(cx^n))^2}{2e^3(d+ex)^2} - \frac{2x(a+b\log(cx^n))^2}{e^2(d+ex)} + \frac{4bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^3} + \frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^3} + \frac{2bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^3} + \frac{(bdn)\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{e^3} - \frac{(bdn)\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{e^2} - \frac{(2b^2n^2)\int \frac{\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{x}\,dx}{e^3} - \frac{(4b^2n^2)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^3}$$

$$= -\frac{bnx(a+b\log(cx^n))}{e^2(d+ex)} - \frac{bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{e^3} - \frac{d^2(a+b\log(cx^n))^2}{2e^3(d+ex)^2} - \frac{2x(a+b\log(cx^n))^2}{e^2(d+ex)} + \frac{4bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^3} + \frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^3} + \frac{4b^2n^2\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^3} + \frac{2bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^3} - \frac{2b^2n^2\operatorname{Li}_3\left(-\frac{ex}{d}\right)}{e^3} + \frac{(b^2n^2)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{e^3} + \frac{(b^2n^2)\int \frac{1}{d+ex}\,dx}{e^2}$$

$$= -\frac{bnx(a+b\log(cx^n))}{e^2(d+ex)} - \frac{bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{e^3} - \frac{d^2(a+b\log(cx^n))^2}{2e^3(d+ex)^2} - \frac{2x(a+b\log(cx^n))^2}{e^2(d+ex)} + \frac{b^2n^2\log(d+ex)}{e^3} + \frac{4bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^3} + \frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^3} + \frac{b^2n^2\text{Li}_2\left(-\frac{d}{ex}\right)}{e^3} + \frac{4b^2n^2\text{Li}_2\left(-\frac{ex}{d}\right)}{e^3} + \frac{2bn(a+b\log(cx^n))\text{Li}_2\left(-\frac{ex}{d}\right)}{e^3} - \frac{2b^2n^2\text{Li}_3\left(-\frac{ex}{d}\right)}{e^3}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \frac{\frac{2bdn(a+b\log(cx^n))}{d+ex} - 3(a+b\log(cx^n))^2 - \frac{d^2(a+b\log(cx^n))^2}{(d+ex)^2} + \frac{4d(a+b\log(cx^n))^2}{d+ex} - 2b^2n^2(\log(x)-\log(d+ex)) + 6bn}{}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n])^2)/(d + e*x)^3,x]
```

```
[Out] ((2*b*d*n*(a + b*Log[c*x^n]))/(d + e*x) - 3*(a + b*Log[c*x^n])^2 - (d^2*(a
+ b*Log[c*x^n])^2)/(d + e*x)^2 + (4*d*(a + b*Log[c*x^n])^2)/(d + e*x) - 2*b
^2*n^2*(Log[x] - Log[d + e*x]) + 6*b*n*(a + b*Log[c*x^n])*Log[1 + (e*x)/d]
+ 2*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d] + 6*b^2*n^2*PolyLog[2, -((e*x)/d)
] + 4*b*n*(a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)] - 4*b^2*n^2*PolyLog[3,
-((e*x)/d)])/(2*e^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 738, normalized size of antiderivative = 3.18

method	result
risch	$\frac{b^2\ln(x^n)^2\ln(ex+d)}{e^3} + \frac{2b^2\ln(x^n)^2 d}{e^3(ex+d)} - \frac{b^2\ln(x^n)^2 d^2}{2e^3(ex+d)^2} + \frac{b^2n\ln(x^n)d}{e^3(ex+d)} + \frac{3b^2n\ln(x^n)\ln(ex+d)}{e^3} - \frac{3b^2n\ln(x^n)\ln(x)}{e^3} + \frac{b^2n^2\ln(ex}{e^3}$

```
[In] int(x^2*(a+b*ln(c*x^n))^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*ln(x^n)^2/e^3*ln(e*x+d)+2*b^2*ln(x^n)^2/e^3*d/(e*x+d)-1/2*b^2*ln(x^n)^2
/e^3*d^2/(e*x+d)^2+b^2*n*ln(x^n)/e^3*d/(e*x+d)+3*b^2*n*ln(x^n)/e^3*ln(e*x+d
)-3*b^2*n/e^3*ln(x^n)*ln(x)+b^2*n^2*ln(e*x+d)/e^3-b^2/e^3*n^2*ln(x)+3/2*b^2
/e^3*n^2*ln(x)^2-3*b^2/e^3*n^2*ln(e*x+d)*ln(-e*x/d)-3*b^2/e^3*n^2*dilog(-e*
```

```
x/d)+2*b^2/e^3*ln(e*x+d)*ln(-e*x/d)*ln(x)*n^2-2*b^2*n/e^3*ln(x^n)*ln(e*x+d)
*ln(-e*x/d)+2*b^2/e^3*dilog(-e*x/d)*ln(x)*n^2-2*b^2*n/e^3*ln(x^n)*dilog(-e*
x/d)-b^2/e^3*n^2*ln(e*x+d)*ln(x)^2+b^2/e^3*n^2*ln(x)^2*ln(1+e*x/d)+2*b^2/e^
3*n^2*ln(x)*polylog(2,-e*x/d)-2*b^2*n^2*polylog(3,-e*x/d)/e^3+(-I*b*Pi*csgn
(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csg
n(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(ln(x^n)/e
^3*ln(e*x+d)+2*ln(x^n)/e^3*d/(e*x+d)-1/2*ln(x^n)/e^3*d^2/(e*x+d)^2-1/2*n*(-
1/e^3*d/(e*x+d)-3/e^3*ln(e*x+d)+3/e^3*ln(e*x)+2/e^3*ln(e*x+d)*ln(-e*x/d)+2/
e^3*dilog(-e*x/d)))+1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi
*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I
*c*x^n)^3+2*b*ln(c)+2*a)^2*(1/e^3*ln(e*x+d)+2/e^3*d/(e*x+d)-1/2/e^3*d^2/(e*
x+d)^2)
```

Fricas [F]

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)^2x^2}{(ex+d)^3}\,dx$$

[In] `integrate(x^2*(a+b*log(c*x^n))^2/(e*x+d)^3,x, algorithm="fricas")`

[Out] `integral((b^2*x^2*log(c*x^n)^2 + 2*a*b*x^2*log(c*x^n) + a^2*x^2)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)`

Sympy [F]

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^3}\,dx$$

[In] `integrate(x**2*(a+b*ln(c*x**n))**2/(e*x+d)**3,x)`

[Out] `Integral(x**2*(a + b*log(c*x**n))**2/(d + e*x)**3, x)`

Maxima [F]

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)^2x^2}{(ex+d)^3}\,dx$$

[In] `integrate(x^2*(a+b*log(c*x^n))^2/(e*x+d)^3,x, algorithm="maxima")`

[Out] `1/2*a^2*((4*d*e*x + 3*d^2)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3) + 2*log(e*x + d)/e^3) + integrate((b^2*x^2*log(x^n)^2 + 2*(b^2*log(c) + a*b)*x^2*log(x^n) + (b^2*log(c)^2 + 2*a*b*log(c))*x^2)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)`

Giac [F]

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)^2x^2}{(ex+d)^3}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))^2/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*x^2/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,x^n))^2}{(d+e\,x)^3}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n))^2)/(d + e*x)^3,x)
[Out] int((x^2*(a + b*log(c*x^n))^2)/(d + e*x)^3, x)
```

3.109 $\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \frac{bnx(a+b\log(cx^n))}{de(d+ex)} + \frac{x^2(a+b\log(cx^n))^2}{2d(d+ex)^2} - \frac{bn(a+bn+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{de^2} - \frac{b^2n^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{de^2}$$

```
[Out] b*n*x*(a+b*ln(c*x^n))/d/e/(e*x+d)+1/2*x^2*(a+b*ln(c*x^n))^2/d/(e*x+d)^2-b*n
*(a+b*n+b*ln(c*x^n))*ln(1+e*x/d)/d/e^2-b^2*n^2*polylog(2,-e*x/d)/d/e^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2381, 2384, 2354, 2438}

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = -\frac{bn\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n)+bn)}{de^2} + \frac{bnx(a+b\log(cx^n))}{de(d+ex)} + \frac{x^2(a+b\log(cx^n))^2}{2d(d+ex)^2} - \frac{b^2n^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{de^2}$$

```
[In] Int[(x*(a + b*Log[c*x^n])^2)/(d + e*x)^3,x]
```

```
[Out] (b*n*x*(a + b*Log[c*x^n]))/(d*e*(d + e*x)) + (x^2*(a + b*Log[c*x^n])^2)/(2*
d*(d + e*x)^2) - (b*n*(a + b*n + b*Log[c*x^n])*Log[1 + (e*x)/d])/(d*e^2) -
(b^2*n^2*PolyLog[2, -((e*x)/d)])/(d*e^2)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^2(a+b\log(cx^n))^2}{2d(d+ex)^2} - \frac{(bn)\int \frac{x(a+b\log(cx^n))}{(d+ex)^2}\,dx}{d} \\
&= \frac{bnx(a+b\log(cx^n))}{de(d+ex)} + \frac{x^2(a+b\log(cx^n))^2}{2d(d+ex)^2} - \frac{(bn)\int \frac{a+bn+b\log(cx^n)}{d+ex}\,dx}{de} \\
&= \frac{bnx(a+b\log(cx^n))}{de(d+ex)} + \frac{x^2(a+b\log(cx^n))^2}{2d(d+ex)^2} \\
&\quad - \frac{bn(a+bn+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{de^2} + \frac{(b^2n^2)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{de^2} \\
&= \frac{bnx(a+b\log(cx^n))}{de(d+ex)} + \frac{x^2(a+b\log(cx^n))^2}{2d(d+ex)^2} \\
&\quad - \frac{bn(a+bn+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{de^2} - \frac{b^2n^2\mathrm{Li}_2\left(-\frac{ex}{d}\right)}{de^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.38

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^3}\,dx$$
$$= \frac{-\frac{2bn(a+b\log(cx^n))}{d+ex}+\frac{(a+b\log(cx^n))^2}{d}+\frac{d(a+b\log(cx^n))^2}{(d+ex)^2}-\frac{2(a+b\log(cx^n))^2}{d+ex}+\frac{2b^2n^2(\log(x)-\log(d+ex))}{d}-\frac{2bn(a+b\log(cx^n))\log(}{d}}{2e^2}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n])^2)/(d + e*x)^3,x]
```

```
[Out] ((-2*b*n*(a + b*Log[c*x^n]))/(d + e*x) + (a + b*Log[c*x^n])^2/d + (d*(a + b
*Log[c*x^n])^2)/(d + e*x)^2 - (2*(a + b*Log[c*x^n])^2)/(d + e*x) + (2*b^2*n
^2*(Log[x] - Log[d + e*x]))/d - (2*b*n*(a + b*Log[c*x^n])*Log[1 + (e*x)/d])
/d - (2*b^2*n^2*PolyLog[2, -((e*x)/d)])/d)/(2*e^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 484, normalized size of antiderivative = 4.32

method	result
risch	$-\frac{b^2\ln(x^n)^2}{e^2(ex+d)}+\frac{b^2\ln(x^n)^2 d}{2e^2(ex+d)^2}-\frac{b^2 n\ln(x^n)}{e^2(ex+d)}-\frac{b^2 n\ln(x^n)\ln(ex+d)}{e^2 d}+\frac{b^2 n\ln(x^n)\ln(x)}{e^2 d}-\frac{b^2n^2\ln(x)^2}{2e^2 d}-\frac{b^2n^2\ln(ex+d)}{e^2 d}+b^2$

```
[In] int(x*(a+b*ln(c*x^n))^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b^2*ln(x^n)^2/e^2/(e*x+d)+1/2*b^2*ln(x^n)^2/e^2*d/(e*x+d)^2-b^2*n*ln(x^n)/
e^2/(e*x+d)-b^2*n*ln(x^n)/e^2/d*ln(e*x+d)+b^2*n*ln(x^n)/e^2/d*ln(x)-1/2*b^2
*n^2/e^2/d*ln(x)^2-b^2*n^2/e^2/d*ln(e*x+d)+b^2*n^2/e^2/d*ln(x)+b^2*n^2/e^2/
d*ln(e*x+d)*ln(-e*x/d)+b^2*n^2/e^2/d*dilog(-e*x/d)+(-I*b*Pi*csgn(I*c)*csgn(
I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*cs
gn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(-ln(x^n)/e^2/(e*x+d)
+1/2*ln(x^n)/e^2*d/(e*x+d)^2-1/2*n/e^2*(1/d*ln(e*x+d)+1/(e*x+d)-1/d*ln(x)))
+1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c
*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)
+2*a)^2*(-1/e^2/(e*x+d)+1/2/e^2*d/(e*x+d)^2)
```

Fricas [F]

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)^2x}{(ex+d)^3}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)/(e^3*x^3 + 3*d*e
^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F]

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \int \frac{x(a+b\log(cx^n))^2}{(d+ex)^3}\,dx$$

```
[In] integrate(x*(a+b*ln(c*x**n))**2/(e*x+d)**3,x)
```

```
[Out] Integral(x*(a + b*log(c*x**n))**2/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)^2x}{(ex+d)^3}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -a*b*n*(1/(e^3*x + d*e^2) + log(e*x + d)/(d*e^2) - log(x)/(d*e^2)) - 1/2*((
2*e*x + d)*log(x^n)^2/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) - 2*integrate((e^2*x^
2*log(c)^2 + (3*d*e*n*x + d^2*n + 2*(e^2*n + e^2*log(c))*x^2)*log(x^n))/(e^
5*x^4 + 3*d*e^4*x^3 + 3*d^2*e^3*x^2 + d^3*e^2*x), x))*b^2 - (2*e*x + d)*a*b
*log(c*x^n)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) - 1/2*(2*e*x + d)*a^2/(e^4*x^2
+ 2*d*e^3*x + d^2*e^2)
```

Giac [F]

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)^2x}{(ex+d)^3}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))^2/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*x/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \int \frac{x\,(a+b\,\ln(c\,x^n))^2}{(d+e\,x)^3}\,dx$$

```
[In] int((x*(a + b*log(c*x^n))^2)/(d + e*x)^3,x)
[Out] int((x*(a + b*log(c*x^n))^2)/(d + e*x)^3, x)
```

3.110 $\int \frac{(a+b\log(cx^n))^2}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 126

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = -\frac{bnx(a+b\log(cx^n))}{d^2(d+ex)} - \frac{bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^2e} - \frac{(a+b\log(cx^n))^2}{2e(d+ex)^2} + \frac{b^2n^2\log(d+ex)}{d^2e} + \frac{b^2n^2\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^2e}$$

```
[Out] -b*n*x*(a+b*ln(c*x^n))/d^2/(e*x+d)-b*n*ln(1+d/e/x)*(a+b*ln(c*x^n))/d^2/e-1/
2*(a+b*ln(c*x^n))^2/e/(e*x+d)^2+b^2*n^2*ln(e*x+d)/d^2/e+b^2*n^2*polylog(2,-
d/e/x)/d^2/e
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2356, 2389, 2379, 2438, 2351, 31}

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = -\frac{bn\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^2e} - \frac{bnx(a+b\log(cx^n))}{d^2(d+ex)} - \frac{(a+b\log(cx^n))^2}{2e(d+ex)^2} + \frac{b^2n^2\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^2e} + \frac{b^2n^2\log(d+ex)}{d^2e}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(d + e*x)^3,x]
```

```
[Out] -((b*n*x*(a + b*Log[c*x^n]))/(d^2*(d + e*x))) - (b*n*Log[1 + d/(e*x)]*(a +
b*Log[c*x^n]))/(d^2*e) - (a + b*Log[c*x^n])^2/(2*e*(d + e*x)^2) + (b^2*n^2*
Log[d + e*x])/(d^2*e) + (b^2*n^2*PolyLog[2, -(d/(e*x))])/(d^2*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(a+b\log(cx^n))^2}{2e(d+ex)^2}+\frac{(bn)\int\frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx}{e}\\ &= -\frac{(a+b\log(cx^n))^2}{2e(d+ex)^2}-\frac{(bn)\int\frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{d}+\frac{(bn)\int\frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{de}\end{aligned}$$

$$= -\frac{bnx(a+b\log(cx^n))}{d^2(d+ex)} - \frac{bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^2e} - \frac{(a+b\log(cx^n))^2}{2e(d+ex)^2} + \frac{(b^2n^2)\int\frac{1}{d+ex}dx}{d^2} + \frac{(b^2n^2)\int\frac{\log\left(1+\frac{d}{ex}\right)}{x}dx}{d^2e}$$

$$= -\frac{bnx(a+b\log(cx^n))}{d^2(d+ex)} - \frac{bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^2e} - \frac{(a+b\log(cx^n))^2}{2e(d+ex)^2} + \frac{b^2n^2\log(d+ex)}{d^2e} + \frac{b^2n^2\mathrm{Li}_2\left(-\frac{d}{ex}\right)}{d^2e}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16

$$\int\frac{(a+b\log(cx^n))^2}{(d+ex)^3}dx = -\frac{(a+b\log(cx^n))^2}{2e(d+ex)^2} + \frac{bn\left(\frac{a+b\log(cx^n)}{d(d+ex)} + \frac{(a+b\log(cx^n))^2}{2bd^2n} - \frac{bn\left(\frac{\log(x)}{d}-\frac{\log(d+ex)}{d}\right)}{d} - \frac{(a+b\log(cx^n))\log\left(\frac{d+ex}{d}\right)}{d^2} - \frac{bn\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^2}\right)}{e}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(d + e*x)^3,x]
```

```
[Out] -1/2*(a + b*Log[c*x^n])^2/(e*(d + e*x)^2) + (b*n*((a + b*Log[c*x^n])/(d*(d
+ e*x)) + (a + b*Log[c*x^n])^2/(2*b*d^2*n) - (b*n*(Log[x]/d - Log[d + e*x]/
d))/d - ((a + b*Log[c*x^n])*Log[(d + e*x)/d])/d^2 - (b*n*PolyLog[2, -((e*x)
/d)])/d^2))/e
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.48 (sec) , antiderivative size = 435, normalized size of antiderivative = 3.45

method	result
risch	$-\frac{b^2\ln(x^n)^2}{2e(ex+d)^2} - \frac{b^2n\ln(x^n)\ln(ex+d)}{e\,d^2} + \frac{b^2n\ln(x^n)}{ed(ex+d)} + \frac{b^2n\ln(x^n)\ln(x)}{e\,d^2} - \frac{b^2n^2\ln(x)^2}{2e\,d^2} + \frac{b^2n^2\ln(ex+d)}{d^2e} - \frac{b^2n^2\ln(x)}{e\,d^2} + \frac{b^2n^2}{}$

```
[In] int((a+b*ln(c*x^n))^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2*ln(x^n)^2/e/(e*x+d)^2-b^2/e*n*ln(x^n)/d^2*ln(e*x+d)+b^2*n*ln(x^n)/
e/d/(e*x+d)+b^2/e*n*ln(x^n)/d^2*ln(x)-1/2*b^2/e*n^2/d^2*ln(x)^2+b^2*n^2*ln(
e*x+d)/d^2/e-b^2/e*n^2/d^2*ln(x)+b^2/e*n^2/d^2*ln(e*x+d)*ln(-e*x/d)+b^2/e*n
^2/d^2*dilog(-e*x/d)+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*cs
gn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*
```

```
x^n)^3+2*b*ln(c)+2*a)*b*(-1/2*ln(x^n)/e/(e*x+d)^2+1/2/e*n*(-1/d^2*ln(e*x+d)
+1/d/(e*x+d)+1/d^2*ln(x)))-1/8*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)
+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi
*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2/(e*x+d)^2/e
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^3}\,dx=\int \frac{(b\log(cx^n)+a)^2}{(ex+d)^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e^3*x^3 + 3*d*e^2*x^2
 + 3*d^2*e*x + d^3), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^3}\,dx=\int \frac{(a+b\log(cx^n))^2}{(d+ex)^3}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**2/(e*x+d)**3,x)
[Out] Integral((a + b*log(c*x**n))**2/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^3}\,dx=\int \frac{(b\log(cx^n)+a)^2}{(ex+d)^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/(e*x+d)^3,x, algorithm="maxima")
[Out] a*b*n*(1/(d*e^2*x + d^2*e) - log(e*x + d)/(d^2*e) + log(x)/(d^2*e)) - 1/2*b
^2*(log(x^n)^2/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 2*integrate((e*x*log(c)^2 +
(d*n + (e*n + 2*e*log(c))*x)*log(x^n))/(e^4*x^4 + 3*d*e^3*x^3 + 3*d^2*e^2*x
^2 + d^3*e*x), x)) - a*b*log(c*x^n)/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 1/2*a^2
/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{(d+e\,x)^3}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(d + e*x)^3,x)
[Out] int((a + b*log(c*x^n))^2/(d + e*x)^3, x)
```

3.111 $\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 257

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^3}\,dx = \frac{benx(a+b\log(cx^n))}{d^3(d+ex)} - \frac{(a+b\log(cx^n))^2}{2d^3} + \frac{(a+b\log(cx^n))^2}{2d(d+ex)^2} - \frac{ex(a+b\log(cx^n))^2}{d^3(d+ex)} + \frac{(a+b\log(cx^n))^3}{3bd^3n} - \frac{b^2n^2\log(d+ex)}{d^3} + \frac{3bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^3} - \frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{d^3} + \frac{3b^2n^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^3} - \frac{2bn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^3} + \frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{d^3}$$

```
[Out] b*e*n*x*(a+b*ln(c*x^n))/d^3/(e*x+d)-1/2*(a+b*ln(c*x^n))^2/d^3+1/2*(a+b*ln(c
*x^n))^2/d/(e*x+d)^2-e*x*(a+b*ln(c*x^n))^2/d^3/(e*x+d)+1/3*(a+b*ln(c*x^n))^
3/b/d^3/n-b^2*n^2*ln(e*x+d)/d^3+3*b*n*(a+b*ln(c*x^n))*ln(1+e*x/d)/d^3-(a+b*
ln(c*x^n))^2*ln(1+e*x/d)/d^3+3*b^2*n^2*polylog(2,-e*x/d)/d^3-2*b*n*(a+b*ln(
c*x^n))*polylog(2,-e*x/d)/d^3+2*b^2*n^2*polylog(3,-e*x/d)/d^3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31}

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^3}\,dx = \frac{2bn\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)(a+b\log(cx^n))}{d^3} + \frac{bn\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^3} + \frac{benx(a+b\log(cx^n))}{d^3(d+ex)} + \frac{2bn\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{d^3} - \frac{\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))^2}{d^3} - \frac{ex(a+b\log(cx^n))^2}{d^3(d+ex)} + \frac{(a+b\log(cx^n))^2}{2d(d+ex)^2} - \frac{b^2n^2\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^3} + \frac{2b^2n^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^3} + \frac{2b^2n^2\,\text{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^3} - \frac{b^2n^2\log(d+ex)}{d^3}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(x*(d + e*x)^3),x]
```

```
[Out] (b*e*n*x*(a + b*Log[c*x^n]))/(d^3*(d + e*x)) + (b*n*Log[1 + d/(e*x)]*(a + b
*Log[c*x^n]))/d^3 + (a + b*Log[c*x^n])^2/(2*d*(d + e*x)^2) - (e*x*(a + b*Lo
g[c*x^n])^2)/(d^3*(d + e*x)) - (Log[1 + d/(e*x)]*(a + b*Log[c*x^n])^2)/d^3
- (b^2*n^2*Log[d + e*x])/d^3 + (2*b*n*(a + b*Log[c*x^n])*Log[1 + (e*x)/d])/
d^3 - (b^2*n^2*PolyLog[2, -(d/(e*x))])/d^3 + (2*b*n*(a + b*Log[c*x^n])*Poly
Log[2, -(d/(e*x))])/d^3 + (2*b^2*n^2*PolyLog[2, -((e*x)/d)])/d^3 + (2*b^2*n
^2*PolyLog[3, -(d/(e*x))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^2}\,dx}{d} - \frac{e\int \frac{(a+b\log(cx^n))^2}{(d+ex)^3}\,dx}{d}\\
&= \frac{(a+b\log(cx^n))^2}{2d(d+ex)^2} + \frac{\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx}{d^2} - \frac{e\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx}{d^2} - \frac{(bn)\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx}{d}\\
&= \frac{(a+b\log(cx^n))^2}{2d(d+ex)^2} - \frac{ex(a+b\log(cx^n))^2}{d^3(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^3}\\
&\quad + \frac{(2bn)\int \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{x}\,dx}{d^3} - \frac{(bn)\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d^2}\\
&\quad + \frac{(2ben)\int \frac{a+b\log(cx^n)}{d+ex}\,dx}{d^3} + \frac{(ben)\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{d^2}\\
&= \frac{benx(a+b\log(cx^n))}{d^3(d+ex)} + \frac{bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^3}\\
&\quad + \frac{(a+b\log(cx^n))^2}{2d(d+ex)^2} - \frac{ex(a+b\log(cx^n))^2}{d^3(d+ex)}\\
&\quad - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^3} + \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^3}\\
&\quad + \frac{2bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^3} - \frac{(b^2n^2)\int \frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^3}\\
&\quad - \frac{(2b^2n^2)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{d^3} - \frac{(2b^2n^2)\int \frac{\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{x}\,dx}{d^3} - \frac{(b^2en^2)\int \frac{1}{d+ex}\,dx}{d^3}\\
&= \frac{benx(a+b\log(cx^n))}{d^3(d+ex)} + \frac{bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^3}\\
&\quad + \frac{(a+b\log(cx^n))^2}{2d(d+ex)^2} - \frac{ex(a+b\log(cx^n))^2}{d^3(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^3}\\
&\quad - \frac{b^2n^2\log(d+ex)}{d^3} + \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^3} - \frac{b^2n^2\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^3}\\
&\quad + \frac{2bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^3} + \frac{2b^2n^2\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{d^3} + \frac{2b^2n^2\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^3}\,dx = \frac{-\frac{6bdn(a+b\log(cx^n))}{d+ex}-9(a+b\log(cx^n))^2+\frac{3d^2(a+b\log(cx^n))^2}{(d+ex)^2}+\frac{6d(a+b\log(cx^n))^2}{d+ex}+\frac{2(a+b\log(cx^n))^3}{bn}+6b^2n^2(\log(x)-\ldots}{\ldots}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(x*(d + e*x)^3),x]
```

```
[Out] ((-6*b*d*n*(a + b*Log[c*x^n]))/(d + e*x) - 9*(a + b*Log[c*x^n])^2 + (3*d^2*
(a + b*Log[c*x^n])^2)/(d + e*x)^2 + (6*d*(a + b*Log[c*x^n])^2)/(d + e*x) +
(2*(a + b*Log[c*x^n])^3)/(b*n) + 6*b^2*n^2*(Log[x] - Log[d + e*x]) + 18*b*n
*(a + b*Log[c*x^n])*Log[1 + (e*x)/d] - 6*(a + b*Log[c*x^n])^2*Log[1 + (e*x)
/d] + 18*b^2*n^2*PolyLog[2, -((e*x)/d)] - 12*b*n*(a + b*Log[c*x^n])*PolyLog
[2, -((e*x)/d)] + 12*b^2*n^2*PolyLog[3, -((e*x)/d)])/(6*d^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 793, normalized size of antiderivative = 3.09

method	result	size
risch	Expression too large to display	793

```
[In] int((a+b*ln(c*x^n))^2/x/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b^2*ln(x^n)^2/d^3*ln(e*x+d)+b^2*ln(x^n)^2/d^2/(e*x+d)+1/2*b^2*ln(x^n)^2/d/
(e*x+d)^2+b^2*ln(x^n)^2/d^3*ln(x)-b^2*n*ln(x^n)/d^2/(e*x+d)+3*b^2*n*ln(x^n)
/d^3*ln(e*x+d)-3*b^2*n*ln(x^n)/d^3*ln(x)-b^2*n^2*ln(e*x+d)/d^3+b^2/d^3*n^2*
ln(x)+3/2*b^2/d^3*n^2*ln(x)^2-3*b^2/d^3*n^2*ln(e*x+d)*ln(-e*x/d)-3*b^2/d^3*
n^2*dilog(-e*x/d)-b^2*n/d^3*ln(x^n)*ln(x)^2+1/3*b^2/d^3*ln(x)^3*n^2-2*b^2/d
^3*ln(e*x+d)*ln(-e*x/d)*ln(x)*n^2+2*b^2*n/d^3*ln(x^n)*ln(e*x+d)*ln(-e*x/d)-
2*b^2/d^3*dilog(-e*x/d)*ln(x)*n^2+2*b^2*n/d^3*ln(x^n)*dilog(-e*x/d)+b^2/d^3
*n^2*ln(e*x+d)*ln(x)^2-b^2/d^3*n^2*ln(x)^2*ln(1+e*x/d)-2*b^2/d^3*n^2*ln(x)*
polylog(2,-e*x/d)+2*b^2*n^2*polylog(3,-e*x/d)/d^3+(-I*b*Pi*csgn(I*c)*csgn(I
*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csg
n(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(-ln(x^n)/d^3*ln(e*x+d
)+ln(x^n)/d^2/(e*x+d)+1/2*ln(x^n)/d/(e*x+d)^2+ln(x^n)/d^3*ln(x)-1/2*n*(1/d^
2/(e*x+d)-3/d^3*ln(e*x+d)+3/d^3*ln(x)+1/d^3*ln(x)^2-2/d^3*ln(e*x+d)*ln(-e*x
/d)-2/d^3*dilog(-e*x/d)))+1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+
I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*
```

```
csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*(-1/d^3*ln(e*x+d)+1/d^2/(e*x+d)+1/2/d/(e*x
+d)^2+1/d^3*ln(x))
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^3x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e^3*x^4 + 3*d*e^2*x^3
 + 3*d^2*e*x^2 + d^3*x), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^3}\,dx = \int \frac{(a+b\log(cx^n))^2}{x(d+ex)^3}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**2/x/(e*x+d)**3,x)
[Out] Integral((a + b*log(c*x**n))**2/(x*(d + e*x)**3), x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^3x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(e*x+d)^3,x, algorithm="maxima")
[Out] 1/2*a^2*((2*e*x + 3*d)/(d^2*e^2*x^2 + 2*d^3*e*x + d^4) - 2*log(e*x + d)/d^3
 + 2*log(x)/d^3) + integrate((b^2*log(c)^2 + b^2*log(x^n)^2 + 2*a*b*log(c)
+ 2*(b^2*log(c) + a*b)*log(x^n))/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3
*x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^3x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2/((e*x + d)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^3}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{x\,(d+e\,x)^3}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(x*(d + e*x)^3),x)
[Out] int((a + b*log(c*x^n))^2/(x*(d + e*x)^3), x)
```

3.112 $\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 322

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^3}\,dx =& -\frac{2b^2n^2}{d^3x}-\frac{2bn(a+b\log(cx^n))}{d^3x}-\frac{be^2nx(a+b\log(cx^n))}{d^4(d+ex)}\\&+\frac{e(a+b\log(cx^n))^2}{2d^4}-\frac{(a+b\log(cx^n))^2}{d^3x}-\frac{e(a+b\log(cx^n))^2}{2d^2(d+ex)^2}\\&+\frac{2e^2x(a+b\log(cx^n))^2}{d^4(d+ex)}-\frac{e(a+b\log(cx^n))^3}{bd^4n}\\&+\frac{b^2en^2\log(d+ex)}{d^4}-\frac{5ben(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^4}\\&+\frac{3e(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{d^4}-\frac{5b^2en^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^4}\\&+\frac{6ben(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^4}\\&-\frac{6b^2en^2\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{d^4}\end{aligned}$$

```
[Out] -2*b^2*n^2/d^3/x-2*b*n*(a+b*ln(c*x^n))/d^3/x-b*e^2*n*x*(a+b*ln(c*x^n))/d^4/
(e*x+d)+1/2*e*(a+b*ln(c*x^n))^2/d^4-(a+b*ln(c*x^n))^2/d^3/x-1/2*e*(a+b*ln(c
*x^n))^2/d^2/(e*x+d)^2+2*e^2*x*(a+b*ln(c*x^n))^2/d^4/(e*x+d)-e*(a+b*ln(c*x^
n))^3/b/d^4/n+b^2*e*n^2*ln(e*x+d)/d^4-5*b*e*n*(a+b*ln(c*x^n))*ln(1+e*x/d)/d
^4+3*e*(a+b*ln(c*x^n))^2*ln(1+e*x/d)/d^4-5*b^2*e*n^2*polylog(2,-e*x/d)/d^4+
6*b*e*n*(a+b*ln(c*x^n))*polylog(2,-e*x/d)/d^4-6*b^2*e*n^2*polylog(3,-e*x/d)
/d^4
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2395, 2342, 2341, 2356, 2389, 2379, 2438, 2351, 31, 2355, 2354, 2421, 6724}

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^3}\,dx = \frac{2e^2x(a+b\log(cx^n))^2}{d^4(d+ex)} - \frac{be^2nx(a+b\log(cx^n))}{d^4(d+ex)} - \frac{6ben\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4} + \frac{3e\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))^2}{d^4} - \frac{ben\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{d^4} - \frac{4ben\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{d^4} - \frac{(a+b\log(cx^n))^2}{d^3x} - \frac{2bn(a+b\log(cx^n))}{d^3x} - \frac{e(a+b\log(cx^n))^2}{2d^2(d+ex)^2} + \frac{b^2en^2\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^4} - \frac{4b^2en^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^4} - \frac{6b^2en^2\,\text{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^4} + \frac{b^2en^2\log(d+ex)}{d^4} - \frac{2b^2n^2}{d^3x}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(x^2*(d + e*x)^3),x]
```

```
[Out] (-2*b^2*n^2)/(d^3*x) - (2*b*n*(a + b*Log[c*x^n]))/(d^3*x) - (b*e^2*n*x*(a +
 b*Log[c*x^n]))/(d^4*(d + e*x)) - (b*e*n*Log[1 + d/(e*x)]*(a + b*Log[c*x^n]
))/d^4 - (a + b*Log[c*x^n])^2/(d^3*x) - (e*(a + b*Log[c*x^n])^2)/(2*d^2*(d
+ e*x)^2) + (2*e^2*x*(a + b*Log[c*x^n])^2)/(d^4*(d + e*x)) + (3*e*Log[1 + d
/(e*x)]*(a + b*Log[c*x^n])^2)/d^4 + (b^2*e*n^2*Log[d + e*x])/d^4 - (4*b*e*n
*(a + b*Log[c*x^n])*Log[1 + (e*x)/d])/d^4 + (b^2*e*n^2*PolyLog[2, -(d/(e*x)
)])/d^4 - (6*b*e*n*(a + b*Log[c*x^n])*PolyLog[2, -(d/(e*x))])/d^4 - (4*b^2*
e*n^2*PolyLog[2, -((e*x)/d)])/d^4 - (6*b^2*e*n^2*PolyLog[3, -(d/(e*x))])/d^
4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a+b\log(cx^n))^2}{d^3x^2} + \frac{e^2(a+b\log(cx^n))^2}{d^2(d+ex)^3} + \frac{2e^2(a+b\log(cx^n))^2}{d^3(d+ex)^2} - \frac{3e(a+b\log(cx^n))^2}{d^3x(d+ex)}\right) dx$$

$$= \frac{\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx}{d^3} - \frac{(3e)\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx}{d^3} + \frac{(2e^2)\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx}{d^3} + \frac{e^2\int \frac{(a+b\log(cx^n))^2}{(d+ex)^3}\,dx}{d^2}$$

$$= -\frac{(a+b\log(cx^n))^2}{d^3x} - \frac{e(a+b\log(cx^n))^2}{2d^2(d+ex)^2} + \frac{2e^2x(a+b\log(cx^n))^2}{d^4(d+ex)} + \frac{3e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^4} + \frac{(2bn)\int\frac{a+b\log(cx^n)}{x^2}\,dx}{d^3} - \frac{(6ben)\int\frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{x}\,dx}{d^4} + \frac{(ben)\int\frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx}{d^2} - \frac{(4be^2n)\int\frac{a+b\log(cx^n)}{d+ex}\,dx}{d^4}$$

$$= -\frac{2b^2n^2}{d^3x} - \frac{2bn(a+b\log(cx^n))}{d^3x} - \frac{(a+b\log(cx^n))^2}{d^3x} - \frac{e(a+b\log(cx^n))^2}{2d^2(d+ex)^2} + \frac{2e^2x(a+b\log(cx^n))^2}{d^4(d+ex)} + \frac{3e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^4} - \frac{4ben(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^4} - \frac{6ben(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^4} + \frac{(ben)\int\frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d^3} - \frac{(be^2n)\int\frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{d^3} + \frac{(4b^2en^2)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{d^4} + \frac{(6b^2en^2)\int\frac{\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{x}\,dx}{d^4}$$

$$= -\frac{2b^2n^2}{d^3x} - \frac{2bn(a+b\log(cx^n))}{d^3x} - \frac{be^2nx(a+b\log(cx^n))}{d^4(d+ex)} - \frac{ben\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4} - \frac{(a+b\log(cx^n))^2}{d^3x} - \frac{e(a+b\log(cx^n))^2}{2d^2(d+ex)^2} + \frac{2e^2x(a+b\log(cx^n))^2}{d^4(d+ex)} + \frac{3e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^4} - \frac{4ben(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^4} - \frac{6ben(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^4} - \frac{4b^2en^2\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{d^4} - \frac{6b^2en^2\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{d^4} + \frac{(b^2en^2)\int\frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^4} + \frac{(b^2e^2n^2)\int\frac{1}{d+ex}\,dx}{d^4}$$

$$= -\frac{2b^2n^2}{d^3x} - \frac{2bn(a+b\log(cx^n))}{d^3x} - \frac{be^2nx(a+b\log(cx^n))}{d^4(d+ex)} - \frac{ben\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4} - \frac{(a+b\log(cx^n))^2}{d^3x} - \frac{e(a+b\log(cx^n))^2}{2d^2(d+ex)^2} + \frac{2e^2x(a+b\log(cx^n))^2}{d^4(d+ex)} + \frac{3e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^4} + \frac{b^2en^2\log(d+ex)}{d^4} - \frac{4ben(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^4} + \frac{b^2en^2\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^4} - \frac{6ben(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^4} - \frac{4b^2en^2\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{d^4} - \frac{6b^2en^2\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^3}\,dx = -\frac{\frac{4b^2dn^2}{x}+\frac{4bdn(a+b\log(cx^n))}{x}-\frac{2bden(a+b\log(cx^n))}{d+ex}-5e(a+b\log(cx^n))^2+\frac{2d(a+b\log(cx^n))^2}{x}+\frac{d^2e(a+b\log(cx^n))^2}{(d+ex)^2}+4\cdots}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(x^2*(d + e*x)^3),x]
```

```
[Out] -1/2*((4*b^2*d*n^2)/x + (4*b*d*n*(a + b*Log[c*x^n]))/x - (2*b*d*e*n*(a + b*
Log[c*x^n]))/(d + e*x) - 5*e*(a + b*Log[c*x^n])^2 + (2*d*(a + b*Log[c*x^n])
^2)/x + (d^2*e*(a + b*Log[c*x^n])^2)/(d + e*x)^2 + (4*d*e*(a + b*Log[c*x^n]
)^2)/(d + e*x) + (2*e*(a + b*Log[c*x^n])^3)/(b*n) + 2*b^2*e*n^2*(Log[x] - L
og[d + e*x]) + 10*b*e*n*(a + b*Log[c*x^n])*Log[1 + (e*x)/d] - 6*e*(a + b*Lo
g[c*x^n])^2*Log[1 + (e*x)/d] + 10*b^2*e*n^2*PolyLog[2, -((e*x)/d)] - 12*b*e
*n*(a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)] + 12*b^2*e*n^2*PolyLog[3, -((e
*x)/d)])/d^4
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 908, normalized size of antiderivative = 2.82

method	result	size
risch	Expression too large to display	908

```
[In] int((a+b*ln(c*x^n))^2/x^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^2*n*ln(x^n)/d^3/x-b^2*ln(x^n)^2/d^3/x+6*b^2/d^4*e*ln(x)*ln(e*x+d)*ln(-
e*x/d)*n^2-6*b^2*n/d^4*e*ln(x^n)*ln(e*x+d)*ln(-e*x/d)-6*b^2*e*n^2*polylog(3
,-e*x/d)/d^4+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*
csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2
*b*ln(c)+2*a)*b*(-1/2*ln(x^n)/d^2/(e*x+d)^2*e+3*ln(x^n)/d^4*e*ln(e*x+d)-2*l
n(x^n)/d^3*e/(e*x+d)-ln(x^n)/d^3/x-3*ln(x^n)/d^4*e*ln(x)-1/2*n*(-3/d^4*e*ln
(x)^2+6/d^4*e*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))-1/d^3*e/(e*x+d)+5/d^4*e*
ln(e*x+d)+2/d^3/x-5/d^4*e*ln(x)))-2*b^2*ln(x^n)^2/d^3*e/(e*x+d)-1/2*b^2*ln(
x^n)^2/d^2/(e*x+d)^2*e-3*b^2*ln(x^n)^2/d^4*e*ln(x)-b^2/d^4*n^2*e*ln(x)-5/2*
b^2/d^4*n^2*e*ln(x)^2+5*b^2/d^4*n^2*e*dilog(-e*x/d)-b^2/d^4*e*ln(x)^3*n^2+3
*b^2*ln(x^n)^2/d^4*e*ln(e*x+d)+b^2*n*ln(x^n)/d^3*e/(e*x+d)+1/4*(-I*b*Pi*csg
n(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*cs
gn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*(-1/2/d^2
/(e*x+d)^2*e+3/d^4*e*ln(e*x+d)-2/d^3*e/(e*x+d)-1/d^3/x-3/d^4*e*ln(x))+6*b^2
```

```
/d^4*e*ln(x)*dilog(-e*x/d)*n^2-6*b^2*n/d^4*e*ln(x^n)*dilog(-e*x/d)-3*b^2/d^
4*e*n^2*ln(e*x+d)*ln(x)^2+3*b^2/d^4*e*n^2*ln(x)^2*ln(1+e*x/d)+6*b^2/d^4*e*n
^2*ln(x)*polylog(2,-e*x/d)-5*b^2*n*ln(x^n)/d^4*e*ln(e*x+d)+5*b^2*n*ln(x^n)/
d^4*e*ln(x)+5*b^2/d^4*n^2*e*ln(e*x+d)*ln(-e*x/d)+3*b^2*n/d^4*e*ln(x^n)*ln(x
)^2+b^2*e*n^2*ln(e*x+d)/d^4-2*b^2*n^2/d^3/x
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^3x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e^3*x^5 + 3*d*e^2*x^4
 + 3*d^2*e*x^3 + d^3*x^2), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^3}\,dx = \int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^3}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**2/x**2/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*log(c*x**n))**2/(x**2*(d + e*x)**3), x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^3x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*((6*e^2*x^2 + 9*d*e*x + 2*d^2)/(d^3*e^2*x^3 + 2*d^4*e*x^2 + d^5*x)
 - 6*e*log(e*x + d)/d^4 + 6*e*log(x)/d^4) + integrate((b^2*log(c)^2 + b^2*l
og(x^n)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*log(x^n))/(e^3*x^5 + 3*d*e^
2*x^4 + 3*d^2*e*x^3 + d^3*x^2), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^3x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^2/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2/((e*x + d)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^3}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{x^2\,(d+e\,x)^3}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(x^2*(d + e*x)^3),x)
[Out] int((a + b*log(c*x^n))^2/(x^2*(d + e*x)^3), x)
```

3.113 $\int \frac{x^4(a+b\log(cx^n))^2}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 398

$$\begin{aligned}\int \frac{x^4(a+b\log(cx^n))^2}{(d+ex)^4}\,dx =& -\frac{2abnx}{e^4}+\frac{2b^2n^2x}{e^4}-\frac{b^2d^2n^2}{3e^5(d+ex)}-\frac{b^2dn^2\log(x)}{3e^5} \\ &-\frac{2b^2nx\log(cx^n)}{e^4}+\frac{bd^3n(a+b\log(cx^n))}{3e^5(d+ex)^2} \\ &+\frac{10bdnx(a+b\log(cx^n))}{3e^4(d+ex)}-\frac{5d(a+b\log(cx^n))^2}{3e^5} \\ &+\frac{x(a+b\log(cx^n))^2}{e^4}-\frac{d^4(a+b\log(cx^n))^2}{3e^5(d+ex)^3} \\ &+\frac{2d^3(a+b\log(cx^n))^2}{e^5(d+ex)^2}+\frac{6dx(a+b\log(cx^n))^2}{e^4(d+ex)} \\ &-\frac{3b^2dn^2\log(d+ex)}{e^5}-\frac{26bdn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{3e^5} \\ &-\frac{4d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^5}-\frac{26b^2dn^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{3e^5} \\ &-\frac{8bdn(a+b\log(cx^n))\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^5} \\ &+\frac{8b^2dn^2\,\text{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^5}\end{aligned}$$

```
[Out] -2*a*b*n*x/e^4+2*b^2*n^2*x/e^4-1/3*b^2*d^2*n^2/e^5/(e*x+d)-1/3*b^2*d*n^2*ln
(x)/e^5-2*b^2*n*x*ln(c*x^n)/e^4+1/3*b*d^3*n*(a+b*ln(c*x^n))/e^5/(e*x+d)^2+1
0/3*b*d*n*x*(a+b*ln(c*x^n))/e^4/(e*x+d)-5/3*d*(a+b*ln(c*x^n))^2/e^5+x*(a+b*
ln(c*x^n))^2/e^4-1/3*d^4*(a+b*ln(c*x^n))^2/e^5/(e*x+d)^3+2*d^3*(a+b*ln(c*x^
n))^2/e^5/(e*x+d)^2+6*d*x*(a+b*ln(c*x^n))^2/e^4/(e*x+d)-3*b^2*d*n^2*ln(e*x+
d)/e^5-26/3*b*d*n*(a+b*ln(c*x^n))*ln(1+e*x/d)/e^5-4*d*(a+b*ln(c*x^n))^2*ln(
```

```
1+e*x/d)/e^5-26/3*b^2*d*n^2*polylog(2,-e*x/d)/e^5-8*b*d*n*(a+b*ln(c*x^n))*p
olylog(2,-e*x/d)/e^5+8*b^2*d*n^2*polylog(3,-e*x/d)/e^5
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.08, number of steps used = 27, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {2395, 2333, 2332, 2356, 2389, 2379, 2438, 2351, 31, 46, 2355, 2354, 2421, 6724}

$$\begin{aligned}\int \frac{x^4(a+b\log(cx^n))^2}{(d+ex)^4}dx = &-\frac{d^4(a+b\log(cx^n))^2}{3e^5(d+ex)^3}+\frac{2d^3(a+b\log(cx^n))^2}{e^5(d+ex)^2}\\&+\frac{bd^3n(a+b\log(cx^n))}{3e^5(d+ex)^2}\\&-\frac{8bdn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)(a+b\log(cx^n))}{e^5}\\&+\frac{10bdn\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{3e^5}\\&-\frac{4d\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))^2}{e^5}\\&-\frac{12bdn\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{e^5}+\frac{6dx(a+b\log(cx^n))^2}{e^4(d+ex)}\\&+\frac{10bdnx(a+b\log(cx^n))}{3e^4(d+ex)}+\frac{x(a+b\log(cx^n))^2}{e^4}-\frac{2abnx}{e^4}\\&-\frac{2b^2nx\log(cx^n)}{e^4}-\frac{b^2d^2n^2}{3e^5(d+ex)}-\frac{10b^2dn^2\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{3e^5}\\&-\frac{12b^2dn^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^5}+\frac{8b^2dn^2\,\text{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^5}\\&-\frac{b^2dn^2\log(x)}{3e^5}-\frac{3b^2dn^2\log(d+ex)}{e^5}+\frac{2b^2n^2x}{e^4}\end{aligned}$$

```
[In] Int[(x^4*(a + b*Log[c*x^n])^2)/(d + e*x)^4,x]
```

```
[Out] (-2*a*b*n*x)/e^4 + (2*b^2*n^2*x)/e^4 - (b^2*d^2*n^2)/(3*e^5*(d + e*x)) - (b
^2*d*n^2*Log[x])/(3*e^5) - (2*b^2*n*x*Log[c*x^n])/e^4 + (b*d^3*n*(a + b*Log
[c*x^n]))/(3*e^5*(d + e*x)^2) + (10*b*d*n*x*(a + b*Log[c*x^n]))/(3*e^4*(d +
 e*x)) + (10*b*d*n*Log[1 + d/(e*x)]*(a + b*Log[c*x^n]))/(3*e^5) + (x*(a + b
*Log[c*x^n])^2)/e^4 - (d^4*(a + b*Log[c*x^n])^2)/(3*e^5*(d + e*x)^3) + (2*d
^3*(a + b*Log[c*x^n])^2)/(e^5*(d + e*x)^2) + (6*d*x*(a + b*Log[c*x^n])^2)/(
e^4*(d + e*x)) - (3*b^2*d*n^2*Log[d + e*x])/e^5 - (12*b*d*n*(a + b*Log[c*x^
n])*Log[1 + (e*x)/d])/e^5 - (4*d*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d])/e^5
 - (10*b^2*d*n^2*PolyLog[2, -(d/(e*x))])/(3*e^5) - (12*b^2*d*n^2*PolyLog[2,
 -((e*x)/d)])/e^5 - (8*b*d*n*(a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)])/e^5
 + (8*b^2*d*n^2*PolyLog[3, -((e*x)/d)])/e^5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
```

```
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{(a+b\log(cx^n))^2}{e^4}+\frac{d^4(a+b\log(cx^n))^2}{e^4(d+ex)^4}-\frac{4d^3(a+b\log(cx^n))^2}{e^4(d+ex)^3}\right. \\
&\qquad \left.+\frac{6d^2(a+b\log(cx^n))^2}{e^4(d+ex)^2}-\frac{4d(a+b\log(cx^n))^2}{e^4(d+ex)}\right)dx \\
&= \frac{\int (a+b\log(cx^n))^2\,dx}{e^4}-\frac{(4d)\int \frac{(a+b\log(cx^n))^2}{d+ex}\,dx}{e^4}+\frac{(6d^2)\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx}{e^4} \\
&\quad -\frac{(4d^3)\int \frac{(a+b\log(cx^n))^2}{(d+ex)^3}\,dx}{e^4}+\frac{d^4\int \frac{(a+b\log(cx^n))^2}{(d+ex)^4}\,dx}{e^4} \\
&= \frac{x(a+b\log(cx^n))^2}{e^4}-\frac{d^4(a+b\log(cx^n))^2}{3e^5(d+ex)^3}+\frac{2d^3(a+b\log(cx^n))^2}{e^5(d+ex)^2} \\
&\quad +\frac{6dx(a+b\log(cx^n))^2}{e^4(d+ex)}-\frac{4d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^5} \\
&\quad +\frac{(8bdn)\int \frac{(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^5}-\frac{(4bd^3n)\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx}{e^5} \\
&\quad +\frac{(2bd^4n)\int \frac{a+b\log(cx^n)}{x(d+ex)^3}\,dx}{3e^5}-\frac{(2bn)\int (a+b\log(cx^n))\,dx}{e^4}-\frac{(12bdn)\int \frac{a+b\log(cx^n)}{d+ex}\,dx}{e^4} \\
&= -\frac{2abnx}{e^4}+\frac{x(a+b\log(cx^n))^2}{e^4}-\frac{d^4(a+b\log(cx^n))^2}{3e^5(d+ex)^3}+\frac{2d^3(a+b\log(cx^n))^2}{e^5(d+ex)^2} \\
&\quad +\frac{6dx(a+b\log(cx^n))^2}{e^4(d+ex)}-\frac{12bdn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^5} \\
&\quad -\frac{4d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^5}-\frac{8bdn(a+b\log(cx^n))\,\mathrm{Li}_2\left(-\frac{ex}{d}\right)}{e^5} \\
&\quad -\frac{(4bd^2n)\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{e^5}+\frac{(2bd^3n)\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx}{3e^5} \\
&\quad -\frac{(2b^2n)\int \log(cx^n)\,dx}{e^4}+\frac{(4bd^2n)\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{e^4}-\frac{(2bd^3n)\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx}{3e^4} \\
&\quad +\frac{(8b^2dn^2)\int \frac{\mathrm{Li}_2\left(-\frac{ex}{d}\right)}{x}\,dx}{e^5}+\frac{(12b^2dn^2)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^5}
\end{aligned}
$$

$$= -\frac{2abnx}{e^4} + \frac{2b^2n^2x}{e^4} - \frac{2b^2nx\log(cx^n)}{e^4} + \frac{bd^3n(a+b\log(cx^n))}{3e^5(d+ex)^2} + \frac{4bdnx(a+b\log(cx^n))}{e^4(d+ex)} + \frac{4bdn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{e^5} + \frac{x(a+b\log(cx^n))^2}{e^4} - \frac{d^4(a+b\log(cx^n))^2}{3e^5(d+ex)^3} + \frac{2d^3(a+b\log(cx^n))^2}{e^5(d+ex)^2} + \frac{6dx(a+b\log(cx^n))^2}{e^4(d+ex)} - \frac{12bdn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^5} - \frac{4d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^5} - \frac{12b^2dn^2\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^5} - \frac{8bdn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^5} + \frac{8b^2dn^2\operatorname{Li}_3\left(-\frac{ex}{d}\right)}{e^5} + \frac{(2bd^2n)\int\frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{3e^5} - \frac{(2bd^2n)\int\frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{3e^4} - \frac{(4b^2dn^2)\int\frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{e^5} - \frac{(b^2d^3n^2)\int\frac{1}{x(d+ex)^2}\,dx}{3e^5} - \frac{(4b^2dn^2)\int\frac{1}{d+ex}\,dx}{e^4}$$

$$= -\frac{2abnx}{e^4} + \frac{2b^2n^2x}{e^4} - \frac{2b^2nx\log(cx^n)}{e^4} + \frac{bd^3n(a+b\log(cx^n))}{3e^5(d+ex)^2} + \frac{10bdnx(a+b\log(cx^n))}{3e^4(d+ex)} + \frac{10bdn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{3e^5} + \frac{x(a+b\log(cx^n))^2}{e^4} - \frac{d^4(a+b\log(cx^n))^2}{3e^5(d+ex)^3} + \frac{2d^3(a+b\log(cx^n))^2}{e^5(d+ex)^2} + \frac{6dx(a+b\log(cx^n))^2}{e^4(d+ex)} - \frac{4b^2dn^2\log(d+ex)}{e^5} - \frac{12bdn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^5} - \frac{4d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^5} - \frac{4b^2dn^2\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{e^5} - \frac{12b^2dn^2\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^5} - \frac{8bdn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^5} + \frac{8b^2dn^2\operatorname{Li}_3\left(-\frac{ex}{d}\right)}{e^5} + \frac{(2b^2dn^2)\int\frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{3e^5} - \frac{(b^2d^3n^2)\int\left(\frac{1}{d^2x}-\frac{e}{d(d+ex)^2}-\frac{e}{d^2(d+ex)}\right)dx}{3e^5} + \frac{(2b^2dn^2)\int\frac{1}{d+ex}\,dx}{3e^4}$$

$$= -\frac{2abnx}{e^4} + \frac{2b^2n^2x}{e^4} - \frac{b^2d^2n^2}{3e^5(d+ex)} - \frac{b^2dn^2\log(x)}{3e^5} - \frac{2b^2nx\log(cx^n)}{e^4} + \frac{bd^3n(a+b\log(cx^n))}{3e^5(d+ex)^2} + \frac{10bdnx(a+b\log(cx^n))}{3e^4(d+ex)} + \frac{10bdn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{3e^5} + \frac{x(a+b\log(cx^n))^2}{e^4} - \frac{d^4(a+b\log(cx^n))^2}{3e^5(d+ex)^3} + \frac{2d^3(a+b\log(cx^n))^2}{e^5(d+ex)^2} + \frac{6dx(a+b\log(cx^n))^2}{e^4(d+ex)} - \frac{3b^2dn^2\log(d+ex)}{e^5} - \frac{12bdn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^5} - \frac{4d(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^5} - \frac{10b^2dn^2\text{Li}_2\left(-\frac{d}{ex}\right)}{3e^5} - \frac{12b^2dn^2\text{Li}_2\left(-\frac{ex}{d}\right)}{e^5} - \frac{8bdn(a+b\log(cx^n))\text{Li}_2\left(-\frac{ex}{d}\right)}{e^5} + \frac{8b^2dn^2\text{Li}_3\left(-\frac{ex}{d}\right)}{e^5}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.86

$$\int \frac{x^4(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = -\frac{-\frac{bd^3n(a+b\log(cx^n))}{(d+ex)^2} + \frac{10bd^2n(a+b\log(cx^n))}{d+ex} - 13d(a+b\log(cx^n))^2 - 3ex(a+b\log(cx^n))^2 + \frac{d^4(a+b\log(cx^n))^2}{(d+ex)^3} - \cdots}{\cdots}$$

```
[In] Integrate[(x^4*(a + b*Log[c*x^n])^2)/(d + e*x)^4,x]
```

```
[Out] -1/3*(-((b*d^3*n*(a + b*Log[c*x^n]))/(d + e*x)^2) + (10*b*d^2*n*(a + b*Log[
c*x^n]))/(d + e*x) - 13*d*(a + b*Log[c*x^n])^2 - 3*e*x*(a + b*Log[c*x^n])^2
 + (d^4*(a + b*Log[c*x^n])^2)/(d + e*x)^3 - (6*d^3*(a + b*Log[c*x^n])^2)/(d
 + e*x)^2 + (18*d^2*(a + b*Log[c*x^n])^2)/(d + e*x) + 6*b*e*n*x*(a - b*n +
b*Log[c*x^n]) - 10*b^2*d*n^2*(Log[x] - Log[d + e*x]) + (b^2*d*n^2*(d + (d +
 e*x)*Log[x] - (d + e*x)*Log[d + e*x]))/(d + e*x) + 26*b*d*n*(a + b*Log[c*x
^n])*Log[1 + (e*x)/d] + 12*d*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d] + 26*b^2
*d*n^2*PolyLog[2, -((e*x)/d)] + 24*b*d*n*(a + b*Log[c*x^n])*PolyLog[2, -((e
*x)/d)] - 24*b^2*d*n^2*PolyLog[3, -((e*x)/d)])/e^5
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 943, normalized size of antiderivative = 2.37

method	result	size
risch	Expression too large to display	943

```
[In] int(x^4*(a+b*ln(c*x^n))^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -6*b^2*ln(x^n)^2/e^5*d^2/(e*x+d)+2*b^2*ln(x^n)^2/e^5*d^3/(e*x+d)^2-2*b^2*n*
ln(x^n)*x/e^4-13/3*b^2/e^5*n^2*d*ln(x)^2+26/3*b^2/e^5*n^2*dilog(-e*x/d)*d-1
/3*b^2*ln(x^n)^2*d^4/e^5/(e*x+d)^3-4*b^2*ln(x^n)^2/e^5*d*ln(e*x+d)-8*b^2/e^
5*d*n^2*ln(x)*polylog(2,-e*x/d)+1/3*b^2*n*ln(x^n)/e^5*d^3/(e*x+d)^2-26/3*b^
2*n*ln(x^n)/e^5*d*ln(e*x+d)-10/3*b^2*n*ln(x^n)/e^5*d^2/(e*x+d)+(-I*b*Pi*csg
n(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*cs
gn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(ln(x^n)*
x/e^4-1/3*ln(x^n)*d^4/e^5/(e*x+d)^3-4*ln(x^n)/e^5*d*ln(e*x+d)-6*ln(x^n)/e^5
*d^2/(e*x+d)+2*ln(x^n)/e^5*d^3/(e*x+d)^2-1/3*n*(1/e^5*(3*e*x+3*d-1/2*d^3/(e
*x+d)^2+13*d*ln(e*x+d)+5*d^2/(e*x+d)-13*d*ln(e*x))-12/e^5*d*(dilog(-e*x/d)+
ln(e*x+d)*ln(-e*x/d))))-1/3*b^2*d^2*n^2/e^5/(e*x+d)+3*b^2*d*n^2*ln(x)/e^5-3
*b^2*d*n^2*ln(e*x+d)/e^5+8*b^2*d*n^2*polylog(3,-e*x/d)/e^5+1/4*(-I*b*Pi*csg
n(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*cs
gn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*(x/e^4-1/
3/e^5*d^4/(e*x+d)^3-4/e^5*d*ln(e*x+d)-6/e^5*d^2/(e*x+d)+2/e^5*d^3/(e*x+d)^2
)+b^2*ln(x^n)^2*x/e^4+26/3*b^2*n/e^5*ln(x)*ln(x^n)*d+26/3*b^2/e^5*n^2*ln(e*
x+d)*ln(-e*x/d)*d-8*b^2/e^5*d*ln(x)*dilog(-e*x/d)*n^2+8*b^2*n/e^5*d*ln(x^n)
*dilog(-e*x/d)+4*b^2/e^5*d*n^2*ln(e*x+d)*ln(x)^2-4*b^2/e^5*d*n^2*ln(x)^2*ln
(1+e*x/d)-8*b^2/e^5*d*ln(x)*ln(e*x+d)*ln(-e*x/d)*n^2+8*b^2*n/e^5*d*ln(x^n)*
ln(e*x+d)*ln(-e*x/d)+2*b^2*n^2*x/e^4
```

Fricas [F]

$$\int \frac{x^4(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2x^4}{(ex+d)^4}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*x^n))^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*log(c*x^n)^2 + 2*a*b*x^4*log(c*x^n) + a^2*x^4)/(e^4*x^4 +
 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{x^4(a+b\log(cx^n))^2}{(d+ex)^4}\,dx$$

[In] integrate(x**4*(a+b*ln(c*x**n))**2/(e*x+d)**4,x)

[Out] Integral(x**4*(a + b*log(c*x**n))**2/(d + e*x)**4, x)

Maxima [F]

$$\int \frac{x^4(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2x^4}{(ex+d)^4}\,dx$$

[In] integrate(x^4*(a+b*log(c*x^n))^2/(e*x+d)^4,x, algorithm="maxima")

[Out] -1/3*a^2*((18*d^2*e^2*x^2 + 30*d^3*e*x + 13*d^4)/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5) - 3*x/e^4 + 12*d*log(e*x + d)/e^5) + integrate((b^2*x^4*log(x^n)^2 + 2*(b^2*log(c) + a*b)*x^4*log(x^n) + (b^2*log(c)^2 + 2*a*b*log(c))*x^4)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)

Giac [F]

$$\int \frac{x^4(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2x^4}{(ex+d)^4}\,dx$$

[In] integrate(x^4*(a+b*log(c*x^n))^2/(e*x+d)^4,x, algorithm="giac")

[Out] integrate((b*log(c*x^n) + a)^2*x^4/(e*x + d)^4, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{x^4\,(a+b\,\ln(c\,x^n))^2}{(d+e\,x)^4}\,dx$$

[In] int((x^4*(a + b*log(c*x^n))^2)/(d + e*x)^4,x)

[Out] int((x^4*(a + b*log(c*x^n))^2)/(d + e*x)^4, x)

3.114 $\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 333

$$
\begin{aligned}
\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^4}\,dx &= \frac{b^2dn^2}{3e^4(d+ex)} + \frac{b^2n^2\log(x)}{3e^4} - \frac{bd^2n(a+b\log(cx^n))}{3e^4(d+ex)^2} \\
&\quad - \frac{7bnx(a+b\log(cx^n))}{3e^3(d+ex)} + \frac{7(a+b\log(cx^n))^2}{6e^4} \\
&\quad + \frac{d^3(a+b\log(cx^n))^2}{3e^4(d+ex)^3} - \frac{3d^2(a+b\log(cx^n))^2}{2e^4(d+ex)^2} \\
&\quad - \frac{3x(a+b\log(cx^n))^2}{e^3(d+ex)} + \frac{2b^2n^2\log(d+ex)}{e^4} \\
&\quad + \frac{11bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{3e^4} \\
&\quad + \frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^4} + \frac{11b^2n^2\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{3e^4} \\
&\quad + \frac{2bn(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4} \\
&\quad - \frac{2b^2n^2\,\mathrm{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^4}
\end{aligned}
$$

```
[Out] 1/3*b^2*d*n^2/e^4/(e*x+d)+1/3*b^2*n^2*ln(x)/e^4-1/3*b*d^2*n*(a+b*ln(c*x^n))
/e^4/(e*x+d)^2-7/3*b*n*x*(a+b*ln(c*x^n))/e^3/(e*x+d)+7/6*(a+b*ln(c*x^n))^2/
e^4+1/3*d^3*(a+b*ln(c*x^n))^2/e^4/(e*x+d)^3-3/2*d^2*(a+b*ln(c*x^n))^2/e^4/(
e*x+d)^2-3*x*(a+b*ln(c*x^n))^2/e^3/(e*x+d)+2*b^2*n^2*ln(e*x+d)/e^4+11/3*b*n
*(a+b*ln(c*x^n))*ln(1+e*x/d)/e^4+(a+b*ln(c*x^n))^2*ln(1+e*x/d)/e^4+11/3*b^2
*n^2*polylog(2,-e*x/d)/e^4+2*b*n*(a+b*ln(c*x^n))*polylog(2,-e*x/d)/e^4-2*b^
2*n^2*polylog(3,-e*x/d)/e^4
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.09, number of steps used = 24, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {2395, 2356, 2389, 2379, 2438, 2351, 31, 46, 2355, 2354, 2421, 6724}

$$\begin{aligned}\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^4}\,dx &= \frac{d^3(a+b\log(cx^n))^2}{3e^4(d+ex)^3}-\frac{3d^2(a+b\log(cx^n))^2}{2e^4(d+ex)^2}\\ &-\frac{bd^2n(a+b\log(cx^n))}{3e^4(d+ex)^2}+\frac{2bn\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)(a+b\log(cx^n))}{e^4}\\ &-\frac{7bn\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{3e^4}\\ &+\frac{\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))^2}{e^4}\\ &+\frac{6bn\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{e^4}-\frac{3x(a+b\log(cx^n))^2}{e^3(d+ex)}\\ &-\frac{7bnx(a+b\log(cx^n))}{3e^3(d+ex)}+\frac{7b^2n^2\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{3e^4}\\ &+\frac{6b^2n^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{e^4}-\frac{2b^2n^2\,\text{PolyLog}\left(3,-\frac{ex}{d}\right)}{e^4}\\ &+\frac{b^2dn^2}{3e^4(d+ex)}+\frac{2b^2n^2\log(d+ex)}{e^4}+\frac{b^2n^2\log(x)}{3e^4}\end{aligned}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n])^2)/(d + e*x)^4,x]
```

```
[Out] (b^2*d*n^2)/(3*e^4*(d + e*x)) + (b^2*n^2*Log[x])/(3*e^4) - (b*d^2*n*(a + b*
Log[c*x^n]))/(3*e^4*(d + e*x)^2) - (7*b*n*x*(a + b*Log[c*x^n]))/(3*e^3*(d +
 e*x)) - (7*b*n*Log[1 + d/(e*x)]*(a + b*Log[c*x^n]))/(3*e^4) + (d^3*(a + b*
Log[c*x^n])^2)/(3*e^4*(d + e*x)^3) - (3*d^2*(a + b*Log[c*x^n])^2)/(2*e^4*(d
 + e*x)^2) - (3*x*(a + b*Log[c*x^n])^2)/(e^3*(d + e*x)) + (2*b^2*n^2*Log[d
+ e*x])/e^4 + (6*b*n*(a + b*Log[c*x^n])*Log[1 + (e*x)/d])/e^4 + ((a + b*Log
[c*x^n])^2*Log[1 + (e*x)/d])/e^4 + (7*b^2*n^2*PolyLog[2, -(d/(e*x))])/(3*e^
4) + (6*b^2*n^2*PolyLog[2, -((e*x)/d)])/e^4 + (2*b*n*(a + b*Log[c*x^n])*Pol
yLog[2, -((e*x)/d)])/e^4 - (2*b^2*n^2*PolyLog[3, -((e*x)/d)])/e^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{d^3(a+b\log(cx^n))^2}{e^3(d+ex)^4}+\frac{3d^2(a+b\log(cx^n))^2}{e^3(d+ex)^3}-\frac{3d(a+b\log(cx^n))^2}{e^3(d+ex)^2}+\frac{(a+b\log(cx^n))^2}{e^3(d+ex)}\right)dx$$

$$=\frac{\int\frac{(a+b\log(cx^n))^2}{d+ex}dx}{e^3}-\frac{(3d)\int\frac{(a+b\log(cx^n))^2}{(d+ex)^2}dx}{e^3}+\frac{(3d^2)\int\frac{(a+b\log(cx^n))^2}{(d+ex)^3}dx}{e^3}-\frac{d^3\int\frac{(a+b\log(cx^n))^2}{(d+ex)^4}dx}{e^3}$$

$$=\frac{d^3(a+b\log(cx^n))^2}{3e^4(d+ex)^3}-\frac{3d^2(a+b\log(cx^n))^2}{2e^4(d+ex)^2}-\frac{3x(a+b\log(cx^n))^2}{e^3(d+ex)}+\frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^4}-\frac{(2bn)\int\frac{(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{x}dx}{e^4}+\frac{(3bd^2n)\int\frac{a+b\log(cx^n)}{x(d+ex)^2}dx}{e^4}-\frac{(2bd^3n)\int\frac{a+b\log(cx^n)}{x(d+ex)^3}dx}{3e^4}+\frac{(6bn)\int\frac{a+b\log(cx^n)}{d+ex}dx}{e^3}$$

$$= \frac{d^3(a+b\log(cx^n))^2}{3e^4(d+ex)^3} - \frac{3d^2(a+b\log(cx^n))^2}{2e^4(d+ex)^2} - \frac{3x(a+b\log(cx^n))^2}{e^3(d+ex)} + \frac{6bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^4} + \frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^4} + \frac{2bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^4} + \frac{(3bdn)\int\frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{e^4} - \frac{(2bd^2n)\int\frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx}{3e^4} - \frac{(3bdn)\int\frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{e^3} + \frac{(2bd^2n)\int\frac{a+b\log(cx^n)}{(d+ex)^3}\,dx}{3e^3} - \frac{(2b^2n^2)\int\frac{\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{x}\,dx}{e^4} - \frac{(6b^2n^2)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{e^4}$$

$$= -\frac{bd^2n(a+b\log(cx^n))}{3e^4(d+ex)^2} - \frac{3bnx(a+b\log(cx^n))}{e^3(d+ex)} - \frac{3bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{e^4} + \frac{d^3(a+b\log(cx^n))^2}{3e^4(d+ex)^3} - \frac{3d^2(a+b\log(cx^n))^2}{2e^4(d+ex)^2} - \frac{3x(a+b\log(cx^n))^2}{e^3(d+ex)} + \frac{6bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^4} + \frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^4} + \frac{6b^2n^2\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^4} + \frac{2bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^4} - \frac{2b^2n^2\operatorname{Li}_3\left(-\frac{ex}{d}\right)}{e^4} - \frac{(2bdn)\int\frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{3e^4} + \frac{(2bdn)\int\frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{3e^3} + \frac{(3b^2n^2)\int\frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{e^4} + \frac{(b^2d^2n^2)\int\frac{1}{x(d+ex)^2}\,dx}{3e^4} + \frac{(3b^2n^2)\int\frac{1}{d+ex}\,dx}{e^3}$$

$$= -\frac{bd^2n(a+b\log(cx^n))}{3e^4(d+ex)^2} - \frac{7bnx(a+b\log(cx^n))}{3e^3(d+ex)} - \frac{7bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{3e^4} + \frac{d^3(a+b\log(cx^n))^2}{3e^4(d+ex)^3} - \frac{3d^2(a+b\log(cx^n))^2}{2e^4(d+ex)^2} - \frac{3x(a+b\log(cx^n))^2}{e^3(d+ex)} + \frac{3b^2n^2\log(d+ex)}{e^4} + \frac{6bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^4} + \frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^4} + \frac{3b^2n^2\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{e^4} + \frac{6b^2n^2\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^4} + \frac{2bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{e^4} - \frac{2b^2n^2\operatorname{Li}_3\left(-\frac{ex}{d}\right)}{e^4} - \frac{(2b^2n^2)\int\frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{3e^4} + \frac{(b^2d^2n^2)\int\left(\frac{1}{d^2x}-\frac{e}{d(d+ex)^2}-\frac{e}{d^2(d+ex)}\right)dx}{3e^4} - \frac{(2b^2n^2)\int\frac{1}{d+ex}\,dx}{3e^3}$$

$$= \frac{b^2dn^2}{3e^4(d+ex)} + \frac{b^2n^2\log(x)}{3e^4} - \frac{bd^2n(a+b\log(cx^n))}{3e^4(d+ex)^2} - \frac{7bnx(a+b\log(cx^n))}{3e^3(d+ex)} - \frac{7bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{3e^4} + \frac{d^3(a+b\log(cx^n))^2}{3e^4(d+ex)^3} - \frac{3d^2(a+b\log(cx^n))^2}{2e^4(d+ex)^2} - \frac{3x(a+b\log(cx^n))^2}{e^3(d+ex)} + \frac{2b^2n^2\log(d+ex)}{e^4} + \frac{6bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{e^4} + \frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{e^4} + \frac{7b^2n^2\text{Li}_2\left(-\frac{d}{ex}\right)}{3e^4} + \frac{6b^2n^2\text{Li}_2\left(-\frac{ex}{d}\right)}{e^4} + \frac{2bn(a+b\log(cx^n))\text{Li}_2\left(-\frac{ex}{d}\right)}{e^4} - \frac{2b^2n^2\text{Li}_3\left(-\frac{ex}{d}\right)}{e^4}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.89

$$\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \frac{-\frac{2bd^2n(a+b\log(cx^n))}{(d+ex)^2} + \frac{14bdn(a+b\log(cx^n))}{d+ex} - 11(a+b\log(cx^n))^2 + \frac{2d^3(a+b\log(cx^n))^2}{(d+ex)^3} - \frac{9d^2(a+b\log(cx^n))^2}{(d+ex)^2} + \frac{18d(a+b\log(}{d+ex}}{}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n])^2)/(d + e*x)^4,x]
```

```
[Out] ((-2*b*d^2*n*(a + b*Log[c*x^n]))/(d + e*x)^2 + (14*b*d*n*(a + b*Log[c*x^n])
)/(d + e*x) - 11*(a + b*Log[c*x^n])^2 + (2*d^3*(a + b*Log[c*x^n])^2)/(d + e
*x)^3 - (9*d^2*(a + b*Log[c*x^n])^2)/(d + e*x)^2 + (18*d*(a + b*Log[c*x^n])
^2)/(d + e*x) - 14*b^2*n^2*(Log[x] - Log[d + e*x]) + (2*b^2*n^2*(d + (d + e
*x)*Log[x] - (d + e*x)*Log[d + e*x]))/(d + e*x) + 22*b*n*(a + b*Log[c*x^n])
*Log[1 + (e*x)/d] + 6*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d] + 22*b^2*n^2*Po
lyLog[2, -((e*x)/d)] + 12*b*n*(a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)] - 1
2*b^2*n^2*PolyLog[3, -((e*x)/d)])/(6*e^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.60 (sec) , antiderivative size = 854, normalized size of antiderivative = 2.56

method	result	size
risch	Expression too large to display	854

```
[In] int(x^3*(a+b*ln(c*x^n))^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^2*ln(x^n)^2/e^4*d^3/(e*x+d)^3+b^2*ln(x^n)^2/e^4*ln(e*x+d)+3*b^2*ln(x^
n)^2/e^4*d/(e*x+d)-3/2*b^2*ln(x^n)^2/e^4*d^2/(e*x+d)^2+7/3*b^2*n*ln(x^n)/e^
```

```
4*d/(e*x+d)-1/3*b^2*n*ln(x^n)/e^4*d^2/(e*x+d)^2+11/3*b^2*n*ln(x^n)/e^4*ln(e
*x+d)-11/3*b^2*n/e^4*ln(x^n)*ln(x)+11/6*b^2/e^4*n^2*ln(x)^2-11/3*b^2/e^4*n^
2*ln(e*x+d)*ln(-e*x/d)-11/3*b^2/e^4*n^2*dilog(-e*x/d)+1/3*b^2*d*n^2/e^4/(e*
x+d)+2*b^2*n^2*ln(e*x+d)/e^4-2*b^2*n^2*ln(x)/e^4+2*b^2/e^4*ln(x)*ln(e*x+d)*
ln(-e*x/d)*n^2+2*b^2/e^4*ln(x)*dilog(-e*x/d)*n^2-2*b^2*n/e^4*ln(x^n)*ln(e*x
+d)*ln(-e*x/d)-2*b^2*n/e^4*ln(x^n)*dilog(-e*x/d)-b^2/e^4*n^2*ln(e*x+d)*ln(x
)^2+b^2/e^4*n^2*ln(x)^2*ln(1+e*x/d)+2*b^2/e^4*n^2*ln(x)*polylog(2,-e*x/d)-2
*b^2*n^2*polylog(3,-e*x/d)/e^4+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)
+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi
*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(1/3*ln(x^n)/e^4*d^3/(e*x+d)^3+ln(x^n)/e^
4*ln(e*x+d)+3*ln(x^n)/e^4*d/(e*x+d)-3/2*ln(x^n)/e^4*d^2/(e*x+d)^2-1/6*n*(-7
/e^4*d/(e*x+d)-11/e^4*ln(e*x+d)+1/e^4*d^2/(e*x+d)^2+11/e^4*ln(e*x)+6/e^4*ln
(e*x+d)*ln(-e*x/d)+6/e^4*dilog(-e*x/d)))+1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)
*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c
*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*(1/3/e^4*d^3/(e*x+d)^3+1/e^
4*ln(e*x+d)+3/e^4*d/(e*x+d)-3/2/e^4*d^2/(e*x+d)^2)
```

Fricas [F]

$$\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^4}\,dx=\int \frac{(b\log(cx^n)+a)^2x^3}{(ex+d)^4}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^2/(e*x+d)^4,x, algorithm="fricas")
[Out] integral((b^2*x^3*log(c*x^n)^2 + 2*a*b*x^3*log(c*x^n) + a^2*x^3)/(e^4*x^4 +
 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^4}\,dx=\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^4}\,dx$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))**2/(e*x+d)**4,x)
[Out] Integral(x**3*(a + b*log(c*x**n))**2/(d + e*x)**4, x)
```

Maxima [F]

$$\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2x^3}{(ex+d)^4}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*((18*d*e^2*x^2 + 27*d^2*e*x + 11*d^3)/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^
2*e^5*x + d^3*e^4) + 6*log(e*x + d)/e^4) + integrate((b^2*x^3*log(x^n)^2 +
2*(b^2*log(c) + a*b)*x^3*log(x^n) + (b^2*log(c)^2 + 2*a*b*log(c))*x^3)/(e^4
*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Giac [F]

$$\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2x^3}{(ex+d)^4}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^2/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x^3/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{x^3\,(a+b\,\ln(c\,x^n))^2}{(d+e\,x)^4}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n))^2)/(d + e*x)^4,x)
```

```
[Out] int((x^3*(a + b*log(c*x^n))^2)/(d + e*x)^4, x)
```

3.115 $\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^4} dx$

Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^4} dx = \frac{bnx^2(a+b\log(cx^n))}{3de(d+ex)^2} + \frac{x^3(a+b\log(cx^n))^2}{3d(d+ex)^3} + \frac{bnx(2a+bn+2b\log(cx^n))}{3de^2(d+ex)} - \frac{bn(2a+3bn+2b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{3de^3} - \frac{2b^2n^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{3de^3}$$

```
[Out] 1/3*b*n*x^2*(a+b*ln(c*x^n))/d/e/(e*x+d)^2+1/3*x^3*(a+b*ln(c*x^n))^2/d/(e*x+
d)^3+1/3*b*n*x*(2*a+b*n+2*b*ln(c*x^n))/d/e^2/(e*x+d)-1/3*b*n*(2*a+3*b*n+2*b
*ln(c*x^n))*ln(1+e*x/d)/d/e^3-2/3*b^2*n^2*polylog(2,-e*x/d)/d/e^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2381, 2384, 2354, 2438}

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^4} dx = -\frac{bn\log\left(\frac{ex}{d}+1\right)(2a+2b\log(cx^n)+3bn)}{3de^3} + \frac{bnx(2a+2b\log(cx^n)+bn)}{3de^2(d+ex)} + \frac{x^3(a+b\log(cx^n))^2}{3d(d+ex)^3} + \frac{bnx^2(a+b\log(cx^n))}{3de(d+ex)^2} - \frac{2b^2n^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{3de^3}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n])^2)/(d + e*x)^4,x]
```

```
[Out] (b*n*x^2*(a + b*Log[c*x^n]))/(3*d*e*(d + e*x)^2) + (x^3*(a + b*Log[c*x^n])^
2)/(3*d*(d + e*x)^3) + (b*n*x*(2*a + b*n + 2*b*Log[c*x^n]))/(3*d*e^2*(d + e
*x)) - (b*n*(2*a + 3*b*n + 2*b*Log[c*x^n])*Log[1 + (e*x)/d])/(3*d*e^3) - (2
*b^2*n^2*PolyLog[2, -((e*x)/d)])/(3*d*e^3)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^3(a+b\log(cx^n))^2}{3d(d+ex)^3} - \frac{(2bn)\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^3}\,dx}{3d} \\
&= \frac{bnx^2(a+b\log(cx^n))}{3de(d+ex)^2} + \frac{x^3(a+b\log(cx^n))^2}{3d(d+ex)^3} - \frac{(bn)\int \frac{x(2a+bn+2b\log(cx^n))}{(d+ex)^2}\,dx}{3de} \\
&= \frac{bnx^2(a+b\log(cx^n))}{3de(d+ex)^2} + \frac{x^3(a+b\log(cx^n))^2}{3d(d+ex)^3} \\
&\quad + \frac{bnx(2a+bn+2b\log(cx^n))}{3de^2(d+ex)} - \frac{(bn)\int \frac{2a+3bn+2b\log(cx^n)}{d+ex}\,dx}{3de^2}
\end{aligned}
$$

$$= \frac{bnx^2(a+b\log(cx^n))}{3de(d+ex)^2} + \frac{x^3(a+b\log(cx^n))^2}{3d(d+ex)^3} + \frac{bnx(2a+bn+2b\log(cx^n))}{3de^2(d+ex)} - \frac{bn(2a+3bn+2b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{3de^3} + \frac{(2b^2n^2)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{3de^3}$$

$$= \frac{bnx^2(a+b\log(cx^n))}{3de(d+ex)^2} + \frac{x^3(a+b\log(cx^n))^2}{3d(d+ex)^3} + \frac{bnx(2a+bn+2b\log(cx^n))}{3de^2(d+ex)} - \frac{bn(2a+3bn+2b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{3de^3} - \frac{2b^2n^2\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{3de^3}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 371 vs. $2(161) = 322$.

Time = 0.32 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.30

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^4}\,dx =$$

$$-\frac{-\frac{a^2}{d} + \frac{a^2d^2}{(d+ex)^3} - \frac{3a^2d}{(d+ex)^2} - \frac{abdn}{(d+ex)^2} + \frac{3a^2}{d+ex} + \frac{4abn}{d+ex} + \frac{b^2n^2}{d+ex} - \frac{3b^2n^2\log(x)}{d} - \frac{2ab\log(cx^n)}{d} + \frac{2abd^2\log(cx^n)}{(d+ex)^3} - \frac{6abd\log}{(d+e}}{}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n])^2)/(d + e*x)^4,x]
```

```
[Out] -1/3*(-(a^2/d) + (a^2*d^2)/(d + e*x)^3 - (3*a^2*d)/(d + e*x)^2 - (a*b*d*n)/
(d + e*x)^2 + (3*a^2)/(d + e*x) + (4*a*b*n)/(d + e*x) + (b^2*n^2)/(d + e*x)
 - (3*b^2*n^2*Log[x])/d - (2*a*b*Log[c*x^n])/d + (2*a*b*d^2*Log[c*x^n])/(d
+ e*x)^3 - (6*a*b*d*Log[c*x^n])/(d + e*x)^2 - (b^2*d*n*Log[c*x^n])/(d + e*x
)^2 + (6*a*b*Log[c*x^n])/(d + e*x) + (4*b^2*n*Log[c*x^n])/(d + e*x) - (b^2*
Log[c*x^n]^2)/d + (b^2*d^2*Log[c*x^n]^2)/(d + e*x)^3 - (3*b^2*d*Log[c*x^n]^
2)/(d + e*x)^2 + (3*b^2*Log[c*x^n]^2)/(d + e*x) + (3*b^2*n^2*Log[d + e*x])/
d + (2*a*b*n*Log[1 + (e*x)/d])/d + (2*b^2*n*Log[c*x^n]*Log[1 + (e*x)/d])/d
+ (2*b^2*n^2*PolyLog[2, -((e*x)/d)])/d)/e^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 593, normalized size of antiderivative = 3.68

method	result
risch	$-\frac{b^2\ln(x^n)^2d^2}{3e^3(ex+d)^3} - \frac{b^2\ln(x^n)^2}{e^3(ex+d)} + \frac{b^2\ln(x^n)^2d}{e^3(ex+d)^2} - \frac{4b^2n\ln(x^n)}{3e^3(ex+d)} + \frac{b^2n\ln(x^n)d}{3e^3(ex+d)^2} - \frac{2b^2n\ln(x^n)\ln(ex+d)}{3e^3d} + \frac{2b^2n\ln(x^n)\ln(x)}{3e^3d} -$

```
[In] int(x^2*(a+b*ln(c*x^n))^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

[Out] -1/3*b^2*ln(x^n)^2/e^3*d^2/(e*x+d)^3-b^2*ln(x^n)^2/e^3/(e*x+d)+b^2*ln(x^n)^2/e^3*d/(e*x+d)^2-4/3*b^2*n*ln(x^n)/e^3/(e*x+d)+1/3*b^2*n*ln(x^n)/e^3*d/(e*x+d)^2-2/3*b^2*n*ln(x^n)/e^3/d*ln(e*x+d)+2/3*b^2*n*ln(x^n)/e^3/d*ln(x)-1/3*b^2*n^2/e^3/d*ln(x)^2+2/3*b^2*n^2/e^3/d*ln(e*x+d)*ln(-e*x/d)+2/3*b^2*n^2/e^3/d*dilog(-e*x/d)-1/3*b^2*n^2/e^3/(e*x+d)-b^2*n^2/e^3/d*ln(e*x+d)+b^2*n^2/e^3/d*ln(x)+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(-1/3*ln(x^n)/e^3*d^2/(e*x+d)^3-ln(x^n)/e^3/(e*x+d)+ln(x^n)/e^3*d/(e*x+d)^2-1/3*n/e^3*(-1/2*d/(e*x+d)^2+1/d*ln(e*x+d)+2/(e*x+d)-1/d*ln(x)))+1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*(-1/3/e^3*d^2/(e*x+d)^3-1/e^3/(e*x+d)+1/e^3*d/(e*x+d)^2)

Fricas [F]

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2x^2}{(ex+d)^4}\,dx$$

[In] integrate(x^2*(a+b*log(c*x^n))^2/(e*x+d)^4,x, algorithm="fricas")

[Out] integral((b^2*x^2*log(c*x^n)^2 + 2*a*b*x^2*log(c*x^n) + a^2*x^2)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)

Sympy [F]

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^4}\,dx$$

[In] integrate(x**2*(a+b*ln(c*x**n))**2/(e*x+d)**4,x)

[Out] Integral(x**2*(a + b*log(c*x**n))**2/(d + e*x)**4, x)

Maxima [F]

$$\int \frac{x^2(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2x^2}{(ex+d)^4}\,dx$$

[In] integrate(x^2*(a+b*log(c*x^n))^2/(e*x+d)^4,x, algorithm="maxima")

[Out] -1/3*a*b*n*((4*e*x + 3*d)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3) + 2*log(e*x + d)/(d*e^3) - 2*log(x)/(d*e^3)) - 1/3*((3*e^2*x^2 + 3*d*e*x + d^2)*log(x^n)^2/(

```
e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3) - 3*integrate(1/3*(3*e^3*x^3
*log(c)^2 + 2*(6*d*e^2*n*x^2 + 4*d^2*e*n*x + d^3*n + 3*(e^3*n + e^3*log(c))
*x^3)*log(x^n))/(e^7*x^5 + 4*d*e^6*x^4 + 6*d^2*e^5*x^3 + 4*d^3*e^4*x^2 + d^
4*e^3*x), x))*b^2 - 2/3*(3*e^2*x^2 + 3*d*e*x + d^2)*a*b*log(c*x^n)/(e^6*x^3
 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3) - 1/3*(3*e^2*x^2 + 3*d*e*x + d^2)*a
^2/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Giac [F]

$$\int \frac{x^2(a+b\log{(cx^n)})^2}{(d+ex)^4}\,dx = \int \frac{{(b\log{(cx^n)}+a)}^2x^2}{{(ex+d)}^4}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))^2/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*x^2/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log{(cx^n)})^2}{(d+ex)^4}\,dx = \int \frac{x^2\,{\left(a+b\,\ln{\left(c\,x^n\right)}\right)}^2}{{\left(d+e\,x\right)}^4}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n))^2)/(d + e*x)^4,x)
[Out] int((x^2*(a + b*log(c*x^n))^2)/(d + e*x)^4, x)
```

3.116 $\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 210

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \frac{b^2n^2}{3de^2(d+ex)} - \frac{bn(a+b\log(cx^n))}{3e^2(d+ex)^2} + \frac{bn(a+b\log(cx^n))}{3de^2(d+ex)} + \frac{(a+b\log(cx^n))^2}{6d^2e^2} + \frac{d(a+b\log(cx^n))^2}{3e^2(d+ex)^3} - \frac{(a+b\log(cx^n))^2}{2e^2(d+ex)^2} - \frac{bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{3d^2e^2} - \frac{b^2n^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{3d^2e^2}$$

```
[Out] 1/3*b^2*n^2/d/e^2/(e*x+d)-1/3*b*n*(a+b*ln(c*x^n))/e^2/(e*x+d)^2+1/3*b*n*(a+
b*ln(c*x^n))/d/e^2/(e*x+d)+1/6*(a+b*ln(c*x^n))^2/d^2/e^2+1/3*d*(a+b*ln(c*x^
n))^2/e^2/(e*x+d)^3-1/2*(a+b*ln(c*x^n))^2/e^2/(e*x+d)^2-1/3*b*n*(a+b*ln(c*x
^n))*ln(1+e*x/d)/d^2/e^2-1/3*b^2*n^2*polylog(2,-e*x/d)/d^2/e^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2383, 2381, 2384, 2354, 2438, 2373, 45}

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = -\frac{bn\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n)+bn)}{3d^2e^2} + \frac{bnx(a+b\log(cx^n))}{3d^2e(d+ex)} - \frac{bnx^2(a+b\log(cx^n))}{3d^2(d+ex)^2} + \frac{x^2(a+b\log(cx^n))^2}{6d^2(d+ex)^2} + \frac{x^2(a+b\log(cx^n))^2}{3d(d+ex)^3} - \frac{b^2n^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{3d^2e^2} + \frac{b^2n^2\log(d+ex)}{3d^2e^2} + \frac{b^2n^2}{3de^2(d+ex)}$$

```
[In] Int[(x*(a + b*Log[c*x^n])^2)/(d + e*x)^4,x]
```

```
[Out] (b^2*n^2)/(3*d*e^2*(d + e*x)) - (b*n*x^2*(a + b*Log[c*x^n]))/(3*d^2*(d + e*
x)^2) + (b*n*x*(a + b*Log[c*x^n]))/(3*d^2*e*(d + e*x)) + (x^2*(a + b*Log[c*
x^n])^2)/(3*d*(d + e*x)^3) + (x^2*(a + b*Log[c*x^n])^2)/(6*d^2*(d + e*x)^2)
 + (b^2*n^2*Log[d + e*x])/(3*d^2*e^2) - (b*n*(a + b*n + b*Log[c*x^n])*Log[1
 + (e*x)/d])/(3*d^2*e^2) - (b^2*n^2*PolyLog[2, -((e*x)/d)])/(3*d^2*e^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^2(a+b\log(cx^n))^2}{3d(d+ex)^3}+\frac{\int\frac{x(a+b\log(cx^n))^2}{(d+ex)^3}\,dx}{3d}-\frac{(2bn)\int\frac{x(a+b\log(cx^n))}{(d+ex)^3}\,dx}{3d}\\
&= -\frac{bnx^2(a+b\log(cx^n))}{3d^2(d+ex)^2}+\frac{x^2(a+b\log(cx^n))^2}{3d(d+ex)^3}+\frac{x^2(a+b\log(cx^n))^2}{6d^2(d+ex)^2}\\
&\quad-\frac{(bn)\int\frac{x(a+b\log(cx^n))}{(d+ex)^2}\,dx}{3d^2}+\frac{(b^2n^2)\int\frac{x}{(d+ex)^2}\,dx}{3d^2}\\
&= -\frac{bnx^2(a+b\log(cx^n))}{3d^2(d+ex)^2}+\frac{bnx(a+b\log(cx^n))}{3d^2e(d+ex)}\\
&\quad+\frac{x^2(a+b\log(cx^n))^2}{3d(d+ex)^3}+\frac{x^2(a+b\log(cx^n))^2}{6d^2(d+ex)^2}\\
&\quad-\frac{(bn)\int\frac{a+bn+b\log(cx^n)}{d+ex}\,dx}{3d^2e}+\frac{(b^2n^2)\int\left(-\frac{d}{e(d+ex)^2}+\frac{1}{e(d+ex)}\right)dx}{3d^2}\\
&= \frac{b^2n^2}{3de^2(d+ex)}-\frac{bnx^2(a+b\log(cx^n))}{3d^2(d+ex)^2}+\frac{bnx(a+b\log(cx^n))}{3d^2e(d+ex)}\\
&\quad+\frac{x^2(a+b\log(cx^n))^2}{3d(d+ex)^3}+\frac{x^2(a+b\log(cx^n))^2}{6d^2(d+ex)^2}+\frac{b^2n^2\log(d+ex)}{3d^2e^2}\\
&\quad-\frac{bn(a+bn+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{3d^2e^2}+\frac{(b^2n^2)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{3d^2e^2}\\
&= \frac{b^2n^2}{3de^2(d+ex)}-\frac{bnx^2(a+b\log(cx^n))}{3d^2(d+ex)^2}+\frac{bnx(a+b\log(cx^n))}{3d^2e(d+ex)}\\
&\quad+\frac{x^2(a+b\log(cx^n))^2}{3d(d+ex)^3}+\frac{x^2(a+b\log(cx^n))^2}{6d^2(d+ex)^2}+\frac{b^2n^2\log(d+ex)}{3d^2e^2}\\
&\quad-\frac{bn(a+bn+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{3d^2e^2}-\frac{b^2n^2\text{Li}_2\left(-\frac{ex}{d}\right)}{3d^2e^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.34

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \frac{2b^2d^3n^2+2abd^2enx+4b^2d^2en^2x+3a^2de^2x^2+2abde^2nx^2+2b^2de^2n^2x^2+a^2e^3x^3+b^2e^2x^2(3d+ex)\log^2}{}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n])^2)/(d + e*x)^4,x]
```

```
[Out] (2*b^2*d^3*n^2 + 2*a*b*d^2*e*n*x + 4*b^2*d^2*e*n^2*x + 3*a^2*d*e^2*x^2 + 2*
a*b*d*e^2*n*x^2 + 2*b^2*d*e^2*n^2*x^2 + a^2*e^3*x^3 + b^2*e^2*x^2*(3*d + e*
x)*Log[c*x^n]^2 - 2*a*b*d^3*n*Log[1 + (e*x)/d] - 6*a*b*d^2*e*n*x*Log[1 + (e
*x)/d] - 6*a*b*d*e^2*n*x^2*Log[1 + (e*x)/d] - 2*a*b*e^3*n*x^3*Log[1 + (e*x)
/d] - 2*b*Log[c*x^n]*(-(e*x*(b*d*n*(d + e*x) + a*e*x*(3*d + e*x))) + b*n*(d
 + e*x)^3*Log[1 + (e*x)/d]) - 2*b^2*n^2*(d + e*x)^3*PolyLog[2, -((e*x)/d)])
/(6*d^2*e^2*(d + e*x)^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.42

method	result
risch	$-\frac{b^2\ln(x^n)^2}{2e^2(ex+d)^2}+\frac{b^2\ln(x^n)^2d}{3e^2(ex+d)^3}-\frac{b^2n\ln(x^n)}{3e^2(ex+d)^2}-\frac{b^2n\ln(x^n)\ln(ex+d)}{3e^2d^2}+\frac{b^2n\ln(x^n)}{3e^2d(ex+d)}+\frac{b^2n\ln(x^n)\ln(x)}{3e^2d^2}-\frac{b^2n^2\ln(x)^2}{6e^2d^2}+$

```
[In] int(x*(a+b*ln(c*x^n))^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2*ln(x^n)^2/e^2/(e*x+d)^2+1/3*b^2*ln(x^n)^2/e^2*d/(e*x+d)^3-1/3*b^2*
n*ln(x^n)/e^2/(e*x+d)^2-1/3*b^2*n*ln(x^n)/e^2/d^2*ln(e*x+d)+1/3*b^2*n*ln(x^
n)/e^2/d/(e*x+d)+1/3*b^2*n*ln(x^n)/e^2/d^2*ln(x)-1/6*b^2*n^2/e^2/d^2*ln(x)^
2+1/3*b^2*n^2/d/e^2/(e*x+d)+1/3*b^2*n^2/e^2/d^2*ln(e*x+d)*ln(-e*x/d)+1/3*b^
2*n^2/e^2/d^2*dilog(-e*x/d)+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*
b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*cs
gn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(-1/2*ln(x^n)/e^2/(e*x+d)^2+1/3*ln(x^n)/e^2*
d/(e*x+d)^3-1/6*n/e^2*(1/d^2*ln(e*x+d)-1/d/(e*x+d)+1/(e*x+d)^2-1/d^2*ln(x))
)+1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*
c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c
)+2*a)^2*(-1/2/e^2/(e*x+d)^2+1/3/e^2*d/(e*x+d)^3)
```

Fricas [F]

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2x}{(ex+d)^4}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)/(e^4*x^4 + 4*d*e
^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F]

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{x(a+b\log(cx^n))^2}{(d+ex)^4}\,dx$$

```
[In] integrate(x*(a+b*ln(c*x**n))**2/(e*x+d)**4,x)
```

```
[Out] Integral(x*(a + b*log(c*x**n))**2/(d + e*x)**4, x)
```

Maxima [F]

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2x}{(ex+d)^4}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/3*a*b*n*(x/(d*e^3*x^2 + 2*d^2*e^2*x + d^3*e) - log(e*x + d)/(d^2*e^2) + l
og(x)/(d^2*e^2)) - 1/6*((3*e*x + d)*log(x^n)^2/(e^5*x^3 + 3*d*e^4*x^2 + 3*d
^2*e^3*x + d^3*e^2) - 6*integrate(1/3*(3*e^2*x^2*log(c)^2 + (4*d*e*n*x + d^
2*n + 3*(e^2*n + 2*e^2*log(c))*x^2)*log(x^n))/(e^6*x^5 + 4*d*e^5*x^4 + 6*d^
2*e^4*x^3 + 4*d^3*e^3*x^2 + d^4*e^2*x), x))*b^2 - 1/3*(3*e*x + d)*a*b*log(c
*x^n)/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2) - 1/6*(3*e*x + d)*a^2
/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2)
```

Giac [F]

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2x}{(ex+d)^4}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))^2/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*x/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{x\,(a+b\,\ln(c\,x^n))^2}{(d+e\,x)^4}\,dx$$

```
[In] int((x*(a + b*log(c*x^n))^2)/(d + e*x)^4,x)
[Out] int((x*(a + b*log(c*x^n))^2)/(d + e*x)^4, x)
```

3.117 $\int \frac{(a+b\log(cx^n))^2}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 203

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = -\frac{b^2n^2}{3d^2e(d+ex)} - \frac{b^2n^2\log(x)}{3d^3e} + \frac{bn(a+b\log(cx^n))}{3de(d+ex)^2} - \frac{2bnx(a+b\log(cx^n))}{3d^3(d+ex)} - \frac{2bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{3d^3e} - \frac{(a+b\log(cx^n))^2}{3e(d+ex)^3} + \frac{b^2n^2\log(d+ex)}{d^3e} + \frac{2b^2n^2\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)}{3d^3e}$$

```
[Out] -1/3*b^2*n^2/d^2/e/(e*x+d)-1/3*b^2*n^2*ln(x)/d^3/e+1/3*b*n*(a+b*ln(c*x^n))/
d/e/(e*x+d)^2-2/3*b*n*x*(a+b*ln(c*x^n))/d^3/(e*x+d)-2/3*b*n*ln(1+d/e/x)*(a+
b*ln(c*x^n))/d^3/e-1/3*(a+b*ln(c*x^n))^2/e/(e*x+d)^3+b^2*n^2*ln(e*x+d)/d^3/
e+2/3*b^2*n^2*polylog(2,-d/e/x)/d^3/e
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2356, 2389, 2379, 2438, 2351, 31, 46}

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = -\frac{2bn\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{3d^3e} - \frac{2bnx(a+b\log(cx^n))}{3d^3(d+ex)} + \frac{bn(a+b\log(cx^n))}{3de(d+ex)^2} - \frac{(a+b\log(cx^n))^2}{3e(d+ex)^3} + \frac{2b^2n^2\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)}{3d^3e} - \frac{b^2n^2\log(x)}{3d^3e} + \frac{b^2n^2\log(d+ex)}{d^3e} - \frac{b^2n^2}{3d^2e(d+ex)}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(d + e*x)^4,x]
```

```
[Out] -1/3*(b^2*n^2)/(d^2*e*(d + e*x)) - (b^2*n^2*Log[x])/(3*d^3*e) + (b*n*(a + b
*Log[c*x^n]))/(3*d*e*(d + e*x)^2) - (2*b*n*x*(a + b*Log[c*x^n]))/(3*d^3*(d
+ e*x)) - (2*b*n*Log[1 + d/(e*x)]*(a + b*Log[c*x^n]))/(3*d^3*e) - (a + b*Lo
g[c*x^n])^2/(3*e*(d + e*x)^3) + (b^2*n^2*Log[d + e*x])/(d^3*e) + (2*b^2*n^2
*PolyLog[2, -(d/(e*x))])/(3*d^3*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
```

```
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\log(cx^n))^2}{3e(d+ex)^3}+\frac{(2bn)\int\frac{a+b\log(cx^n)}{x(d+ex)^3}\,dx}{3e}\\
&= -\frac{(a+b\log(cx^n))^2}{3e(d+ex)^3}-\frac{(2bn)\int\frac{a+b\log(cx^n)}{(d+ex)^3}\,dx}{3d}+\frac{(2bn)\int\frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx}{3de}\\
&= \frac{bn(a+b\log(cx^n))}{3de(d+ex)^2}-\frac{(a+b\log(cx^n))^2}{3e(d+ex)^3}-\frac{(2bn)\int\frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{3d^2}\\
&\quad+\frac{(2bn)\int\frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{3d^2e}-\frac{(b^2n^2)\int\frac{1}{x(d+ex)^2}\,dx}{3de}\\
&= \frac{bn(a+b\log(cx^n))}{3de(d+ex)^2}-\frac{2bnx(a+b\log(cx^n))}{3d^3(d+ex)}\\
&\quad-\frac{2bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{3d^3e}-\frac{(a+b\log(cx^n))^2}{3e(d+ex)^3}+\frac{(2b^2n^2)\int\frac{1}{d+ex}\,dx}{3d^3}\\
&\quad+\frac{(2b^2n^2)\int\frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{3d^3e}-\frac{(b^2n^2)\int\left(\frac{1}{d^2x}-\frac{e}{d(d+ex)^2}-\frac{e}{d^2(d+ex)}\right)dx}{3de}\\
&= -\frac{b^2n^2}{3d^2e(d+ex)}-\frac{b^2n^2\log(x)}{3d^3e}+\frac{bn(a+b\log(cx^n))}{3de(d+ex)^2}\\
&\quad-\frac{2bnx(a+b\log(cx^n))}{3d^3(d+ex)}-\frac{2bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{3d^3e}\\
&\quad-\frac{(a+b\log(cx^n))^2}{3e(d+ex)^3}+\frac{b^2n^2\log(d+ex)}{d^3e}+\frac{2b^2n^2\text{Li}_2\left(-\frac{d}{ex}\right)}{3d^3e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = -\frac{(a+b\log(cx^n))^2}{3e(d+ex)^3} + \frac{2bn\left(\frac{a+b\log(cx^n)}{2d(d+ex)^2}+\frac{a+b\log(cx^n)}{d^2(d+ex)}+\frac{(a+b\log(cx^n))^2}{2bd^3n}-\frac{bn\left(\frac{1}{d(d+ex)}+\frac{\log(x)}{d^2}-\frac{\log(d+ex)}{d^2}\right)}{2d}-\frac{bn\left(\frac{\log(x)}{d}-\frac{\log(d+ex)}{d}\right)}{d^2}-\frac{(a+b\log(cx^n}{}\right.}{3e}$$

[In] Integrate[(a + b*Log[c*x^n])^2/(d + e*x)^4,x]

[Out] -1/3*(a + b*Log[c*x^n])^2/(e*(d + e*x)^3) + (2*b*n*((a + b*Log[c*x^n])/(2*d *(d + e*x)^2) + (a + b*Log[c*x^n])/(d^2*(d + e*x)) + (a + b*Log[c*x^n])^2/(2*b*d^3*n) - (b*n*(1/(d*(d + e*x)) + Log[x]/d^2 - Log[d + e*x]/d^2))/(2*d) - (b*n*(Log[x]/d - Log[d + e*x]/d))/d^2 - ((a + b*Log[c*x^n])*Log[(d + e*x) /d])/d^3 - (b*n*PolyLog[2, -((e*x)/d)])/d^3))/(3*e)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.44

method	result
risch	$-\frac{b^2\ln(x^n)^2}{3e(ex+d)^3}-\frac{2b^2n\ln(x^n)\ln(ex+d)}{3e\,d^3}+\frac{2b^2n\ln(x^n)}{3e\,d^2(ex+d)}+\frac{b^2n\ln(x^n)}{3ed(ex+d)^2}+\frac{2b^2n\ln(x^n)\ln(x)}{3e\,d^3}-\frac{b^2n^2}{3d^2e(ex+d)}+\frac{b^2n^2\ln(ex+d)}{d^3e}$

[In] int((a+b*ln(c*x^n))^2/(e*x+d)^4,x,method=_RETURNVERBOSE)

[Out] -1/3*b^2*ln(x^n)^2/e/(e*x+d)^3-2/3*b^2/e*n*ln(x^n)/d^3*ln(e*x+d)+2/3*b^2*n* ln(x^n)/e/d^2/(e*x+d)+1/3*b^2*n*ln(x^n)/e/d/(e*x+d)^2+2/3*b^2/e*n*ln(x^n)/d ^3*ln(x)-1/3*b^2*n^2/d^2/e/(e*x+d)+b^2*n^2*ln(e*x+d)/d^3/e-b^2*n^2*ln(x)/d^ 3/e-1/3*b^2/e*n^2/d^3*ln(x)^2+2/3*b^2/e*n^2/d^3*ln(e*x+d)*ln(-e*x/d)+2/3*b^ 2/e*n^2/d^3*dilog(-e*x/d)+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b* Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn (I*c*x^n)^3+2*b*ln(c)+2*a)*b*(-1/3*ln(x^n)/e/(e*x+d)^3+1/3/e*n*(-1/d^3*ln(e *x+d)+1/d^2/(e*x+d)+1/2/d/(e*x+d)^2+1/d^3*ln(x)))-1/12*(-I*b*Pi*csgn(I*c)*c sgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2/(e*x+d)^3/e

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/(e*x+d)^4,x, algorithm="fricas")
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e^4*x^4 + 4*d*e^3*x^3
 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{(a+b\log(cx^n))^2}{(d+ex)^4}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**2/(e*x+d)**4,x)
[Out] Integral((a + b*log(c*x**n))**2/(d + e*x)**4, x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/(e*x+d)^4,x, algorithm="maxima")
[Out] 1/3*a*b*n*((2*e*x + 3*d)/(d^2*e^3*x^2 + 2*d^3*e^2*x + d^4*e) - 2*log(e*x +
d)/(d^3*e) + 2*log(x)/(d^3*e)) - 1/3*b^2*(log(x^n)^2/(e^4*x^3 + 3*d*e^3*x^2
 + 3*d^2*e^2*x + d^3*e) - 3*integrate(1/3*(3*e*x*log(c)^2 + 2*(d*n + (e*n +
 3*e*log(c))*x)*log(x^n))/(e^5*x^5 + 4*d*e^4*x^4 + 6*d^2*e^3*x^3 + 4*d^3*e^
2*x^2 + d^4*e*x), x)) - 2/3*a*b*log(c*x^n)/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e
^2*x + d^3*e) - 1/3*a^2/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex)^4}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{(d+e\,x)^4}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(d + e*x)^4,x)
[Out] int((a + b*log(c*x^n))^2/(d + e*x)^4, x)
```

3.118 $\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 351

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^4}\,dx &= \frac{b^2n^2}{3d^3(d+ex)}+\frac{b^2n^2\log(x)}{3d^4}-\frac{bn(a+b\log(cx^n))}{3d^2(d+ex)^2}\\&+\frac{5benx(a+b\log(cx^n))}{3d^4(d+ex)}-\frac{5(a+b\log(cx^n))^2}{6d^4}+\frac{(a+b\log(cx^n))^2}{3d(d+ex)^3}\\&+\frac{(a+b\log(cx^n))^2}{2d^2(d+ex)^2}-\frac{ex(a+b\log(cx^n))^2}{d^4(d+ex)}+\frac{(a+b\log(cx^n))^3}{3bd^4n}\\&-\frac{2b^2n^2\log(d+ex)}{d^4}+\frac{11bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{3d^4}\\&-\frac{(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{d^4}+\frac{11b^2n^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{3d^4}\\&-\frac{2bn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^4}+\frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{d^4}\end{aligned}$$

```
[Out] 1/3*b^2*n^2/d^3/(e*x+d)+1/3*b^2*n^2*ln(x)/d^4-1/3*b*n*(a+b*ln(c*x^n))/d^2/(
e*x+d)^2+5/3*b*e*n*x*(a+b*ln(c*x^n))/d^4/(e*x+d)-5/6*(a+b*ln(c*x^n))^2/d^4+
1/3*(a+b*ln(c*x^n))^2/d/(e*x+d)^3+1/2*(a+b*ln(c*x^n))^2/d^2/(e*x+d)^2-e*x*(
a+b*ln(c*x^n))^2/d^4/(e*x+d)+1/3*(a+b*ln(c*x^n))^3/b/d^4/n-2*b^2*n^2*ln(e*x
+d)/d^4+11/3*b*n*(a+b*ln(c*x^n))*ln(1+e*x/d)/d^4-(a+b*ln(c*x^n))^2*ln(1+e*x
/d)/d^4+11/3*b^2*n^2*polylog(2,-e*x/d)/d^4-2*b*n*(a+b*ln(c*x^n))*polylog(2,
-e*x/d)/d^4+2*b^2*n^2*polylog(3,-e*x/d)/d^4
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.03, number of steps used = 25, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31, 46}

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^4}dx = \frac{2bn\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)(a+b\log(cx^n))}{d^4} + \frac{5bn\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{3d^4} + \frac{5benx(a+b\log(cx^n))}{3d^4(d+ex)} + \frac{2bn\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{d^4} - \frac{\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))^2}{d^4} - \frac{ex(a+b\log(cx^n))^2}{d^4(d+ex)} - \frac{bn(a+b\log(cx^n))}{3d^2(d+ex)^2} + \frac{(a+b\log(cx^n))^2}{2d^2(d+ex)^2} + \frac{(a+b\log(cx^n))^2}{3d(d+ex)^3} - \frac{5b^2n^2\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{3d^4} + \frac{2b^2n^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^4} + \frac{2b^2n^2\,\text{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^4} - \frac{2b^2n^2\log(d+ex)}{d^4} + \frac{b^2n^2\log(x)}{3d^4} + \frac{b^2n^2}{3d^3(d+ex)}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(x*(d + e*x)^4),x]
```

```
[Out] (b^2*n^2)/(3*d^3*(d + e*x)) + (b^2*n^2*Log[x])/(3*d^4) - (b*n*(a + b*Log[c*
x^n]))/(3*d^2*(d + e*x)^2) + (5*b*e*n*x*(a + b*Log[c*x^n]))/(3*d^4*(d + e*x
)) + (5*b*n*Log[1 + d/(e*x)]*(a + b*Log[c*x^n]))/(3*d^4) + (a + b*Log[c*x^n
])^2/(3*d*(d + e*x)^3) + (a + b*Log[c*x^n])^2/(2*d^2*(d + e*x)^2) - (e*x*(a
 + b*Log[c*x^n])^2)/(d^4*(d + e*x)) - (Log[1 + d/(e*x)]*(a + b*Log[c*x^n])^
2)/d^4 - (2*b^2*n^2*Log[d + e*x])/d^4 + (2*b*n*(a + b*Log[c*x^n])*Log[1 + (
e*x)/d])/d^4 - (5*b^2*n^2*PolyLog[2, -(d/(e*x))])/(3*d^4) + (2*b*n*(a + b*L
og[c*x^n])*PolyLog[2, -(d/(e*x))])/d^4 + (2*b^2*n^2*PolyLog[2, -((e*x)/d)])
/d^4 + (2*b^2*n^2*PolyLog[3, -(d/(e*x))])/d^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^3}\,dx}{d} - \frac{e\int \frac{(a+b\log(cx^n))^2}{(d+ex)^4}\,dx}{d} \\
&= \frac{(a+b\log(cx^n))^2}{3d(d+ex)^3} + \frac{\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^2}\,dx}{d^2} - \frac{e\int \frac{(a+b\log(cx^n))^2}{(d+ex)^3}\,dx}{d^2} - \frac{(2bn)\int \frac{a+b\log(cx^n)}{x(d+ex)^3}\,dx}{3d} \\
&= \frac{(a+b\log(cx^n))^2}{3d(d+ex)^3} + \frac{(a+b\log(cx^n))^2}{2d^2(d+ex)^2} + \frac{\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx}{d^3} - \frac{e\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx}{d^3} \\
&\quad - \frac{(2bn)\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx}{3d^2} - \frac{(bn)\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx}{d^2} + \frac{(2ben)\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx}{3d^2} \\
&= -\frac{bn(a+b\log(cx^n))}{3d^2(d+ex)^2} + \frac{(a+b\log(cx^n))^2}{3d(d+ex)^3} + \frac{(a+b\log(cx^n))^2}{2d^2(d+ex)^2} - \frac{ex(a+b\log(cx^n))^2}{d^4(d+ex)} \\
&\quad - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^4} + \frac{(2bn)\int \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{x}\,dx}{d^4} \\
&\quad - \frac{(2bn)\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{3d^3} - \frac{(bn)\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d^3} + \frac{(2ben)\int \frac{a+b\log(cx^n)}{d+ex}\,dx}{d^4} \\
&\quad + \frac{(2ben)\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{3d^3} + \frac{(ben)\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{d^3} + \frac{(b^2n^2)\int \frac{1}{x(d+ex)^2}\,dx}{3d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bn(a+b\log(cx^n))}{3d^2(d+ex)^2} + \frac{5benx(a+b\log(cx^n))}{3d^4(d+ex)} + \frac{5bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{3d^4} \\
&\quad + \frac{(a+b\log(cx^n))^2}{3d(d+ex)^3} + \frac{(a+b\log(cx^n))^2}{2d^2(d+ex)^2} - \frac{ex(a+b\log(cx^n))^2}{d^4(d+ex)} \\
&\quad - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^4} + \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^4} \\
&\quad + \frac{2bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^4} - \frac{(2b^2n^2)\int\frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{3d^4} \\
&\quad - \frac{(b^2n^2)\int\frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^4} - \frac{(2b^2n^2)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{d^4} - \frac{(2b^2n^2)\int\frac{\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{x}\,dx}{d^4} \\
&\quad + \frac{(b^2n^2)\int\left(\frac{1}{d^2x} - \frac{e}{d(d+ex)^2} - \frac{e}{d^2(d+ex)}\right)dx}{3d^2} - \frac{(2b^2en^2)\int\frac{1}{d+ex}\,dx}{3d^4} - \frac{(b^2en^2)\int\frac{1}{d+ex}\,dx}{d^4} \\
&= \frac{b^2n^2}{3d^3(d+ex)} + \frac{b^2n^2\log(x)}{3d^4} - \frac{bn(a+b\log(cx^n))}{3d^2(d+ex)^2} + \frac{5benx(a+b\log(cx^n))}{3d^4(d+ex)} \\
&\quad + \frac{5bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{3d^4} + \frac{(a+b\log(cx^n))^2}{3d(d+ex)^3} \\
&\quad + \frac{(a+b\log(cx^n))^2}{2d^2(d+ex)^2} - \frac{ex(a+b\log(cx^n))^2}{d^4(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^4} \\
&\quad - \frac{2b^2n^2\log(d+ex)}{d^4} + \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^4} - \frac{5b^2n^2\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{3d^4} \\
&\quad + \frac{2bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^4} + \frac{2b^2n^2\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{d^4} + \frac{2b^2n^2\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{d^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.91

$$
\int\frac{(a+b\log(cx^n))^2}{x(d+ex)^4}\,dx
= \frac{-\frac{2bd^2n(a+b\log(cx^n))}{(d+ex)^2} - \frac{10bdn(a+b\log(cx^n))}{d+ex} - 11(a+b\log(cx^n))^2 + \frac{2d^3(a+b\log(cx^n))^2}{(d+ex)^3} + \frac{3d^2(a+b\log(cx^n))^2}{(d+ex)^2} + \frac{6d(a+b\log(c}{d+ex}}{}
$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(x*(d + e*x)^4),x]
```

```
[Out] ((-2*b*d^2*n*(a + b*Log[c*x^n]))/(d + e*x)^2 - (10*b*d*n*(a + b*Log[c*x^n])
)/(d + e*x) - 11*(a + b*Log[c*x^n])^2 + (2*d^3*(a + b*Log[c*x^n])^2)/(d + e
*x)^3 + (3*d^2*(a + b*Log[c*x^n])^2)/(d + e*x)^2 + (6*d*(a + b*Log[c*x^n])^
2)/(d + e*x) + (2*(a + b*Log[c*x^n])^3)/(b*n) + 10*b^2*n^2*(Log[x] - Log[d
+ e*x]) + (2*b^2*n^2*(d + (d + e*x)*Log[x] - (d + e*x)*Log[d + e*x]))/(d +
e*x) + 22*b*n*(a + b*Log[c*x^n])*Log[1 + (e*x)/d] - 6*(a + b*Log[c*x^n])^2*
Log[1 + (e*x)/d] + 22*b^2*n^2*PolyLog[2, -((e*x)/d)] - 12*b*n*(a + b*Log[c*
x^n])*PolyLog[2, -((e*x)/d)] + 12*b^2*n^2*PolyLog[3, -((e*x)/d)])/(6*d^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.68 (sec) , antiderivative size = 894, normalized size of antiderivative = 2.55

method	result	size
risch	Expression too large to display	894

```
[In] int((a+b*ln(c*x^n))^2/x/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*
x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+
2*a)^2*(-1/d^4*ln(e*x+d)+1/d^3/(e*x+d)+1/2/d^2/(e*x+d)^2+1/3/d/(e*x+d)^3+1/
d^4*ln(x))-11/3*b^2/d^4*n^2*ln(e*x+d)*ln(-e*x/d)-b^2*n/d^4*ln(x^n)*ln(x)^2-
2*b^2/d^4*ln(x)*dilog(-e*x/d)*n^2+2*b^2*n/d^4*ln(x^n)*dilog(-e*x/d)+b^2/d^4
*n^2*ln(e*x+d)*ln(x)^2-b^2/d^4*n^2*ln(x)^2*ln(1+e*x/d)-2*b^2/d^4*n^2*ln(x)*
polylog(2,-e*x/d)+11/3*b^2*n*ln(x^n)/d^4*ln(e*x+d)-11/3*b^2*n*ln(x^n)/d^4*l
n(x)+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c
*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)
+2*a)*b*(-ln(x^n)/d^4*ln(e*x+d)+ln(x^n)/d^3/(e*x+d)+1/2*ln(x^n)/d^2/(e*x+d)
^2+1/3*ln(x^n)/d/(e*x+d)^3+ln(x^n)/d^4*ln(x)-1/6*n*(5/d^3/(e*x+d)+1/d^2/(e*
x+d)^2-11/d^4*ln(e*x+d)+11/d^4*ln(x)+3/d^4*ln(x)^2-6/d^4*ln(e*x+d)*ln(-e*x/
d)-6/d^4*dilog(-e*x/d)))+2*b^2*n/d^4*ln(x^n)*ln(e*x+d)*ln(-e*x/d)-2*b^2/d^4
*ln(x)*ln(e*x+d)*ln(-e*x/d)*n^2-5/3*b^2*n*ln(x^n)/d^3/(e*x+d)-1/3*b^2*n*ln(
x^n)/d^2/(e*x+d)^2-b^2*ln(x^n)^2/d^4*ln(e*x+d)+b^2*ln(x^n)^2/d^3/(e*x+d)+1/
2*b^2*ln(x^n)^2/d^2/(e*x+d)^2+1/3*b^2*ln(x^n)^2/d/(e*x+d)^3+b^2*ln(x^n)^2/d
^4*ln(x)+11/6*b^2/d^4*n^2*ln(x)^2-11/3*b^2/d^4*n^2*dilog(-e*x/d)+1/3*b^2/d^
4*ln(x)^3*n^2+1/3*b^2*n^2/d^3/(e*x+d)+2*b^2*n^2*ln(x)/d^4-2*b^2*n^2*ln(e*x+
d)/d^4+2*b^2*n^2*polylog(3,-e*x/d)/d^4
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^4}\,dx=\int \frac{(b\log(cx^n)+a)^2}{(ex+d)^4x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e^4*x^5 + 4*d*e^3*x^4
 + 6*d^2*e^2*x^3 + 4*d^3*e*x^2 + d^4*x), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^4}\,dx = \int \frac{(a+b\log(cx^n))^2}{x\,(d+ex)^4}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**2/x/(e*x+d)**4,x)
[Out] Integral((a + b*log(c*x**n))**2/(x*(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^4x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(e*x+d)^4,x, algorithm="maxima")
[Out] 1/6*a^2*((6*e^2*x^2 + 15*d*e*x + 11*d^2)/(d^3*e^3*x^3 + 3*d^4*e^2*x^2 + 3*d
^5*e*x + d^6) - 6*log(e*x + d)/d^4 + 6*log(x)/d^4) + integrate((b^2*log(c)^
2 + b^2*log(x^n)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*log(x^n))/(e^4*x^5
 + 4*d*e^3*x^4 + 6*d^2*e^2*x^3 + 4*d^3*e*x^2 + d^4*x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^4x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2/((e*x + d)^4*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^4}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{x\,(d+e\,x)^4}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(x*(d + e*x)^4),x)
[Out] int((a + b*log(c*x^n))^2/(x*(d + e*x)^4), x)
```

3.119 $\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 420

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^4}\,dx = & -\frac{2b^2n^2}{d^4x}-\frac{b^2en^2}{3d^4(d+ex)}-\frac{b^2en^2\log(x)}{3d^5}-\frac{2bn(a+b\log(cx^n))}{d^4x}\\ & +\frac{ben(a+b\log(cx^n))}{3d^3(d+ex)^2}-\frac{8be^2nx(a+b\log(cx^n))}{3d^5(d+ex)}\\ & +\frac{4e(a+b\log(cx^n))^2}{3d^5}-\frac{(a+b\log(cx^n))^2}{d^4x}\\ & -\frac{e(a+b\log(cx^n))^2}{3d^2(d+ex)^3}-\frac{e(a+b\log(cx^n))^2}{d^3(d+ex)^2}\\ & +\frac{3e^2x(a+b\log(cx^n))^2}{d^5(d+ex)}-\frac{4e(a+b\log(cx^n))^3}{3bd^5n}\\ & +\frac{3b^2en^2\log(d+ex)}{d^5}-\frac{26ben(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{3d^5}\\ & +\frac{4e(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{d^5}-\frac{26b^2en^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{3d^5}\\ & +\frac{8ben(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^5}\\ & -\frac{8b^2en^2\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{d^5}\end{aligned}$$

```
[Out] -2*b^2*n^2/d^4/x-1/3*b^2*e*n^2/d^4/(e*x+d)-1/3*b^2*e*n^2*ln(x)/d^5-2*b*n*(a
+b*ln(c*x^n))/d^4/x+1/3*b*e*n*(a+b*ln(c*x^n))/d^3/(e*x+d)^2-8/3*b*e^2*n*x*(
a+b*ln(c*x^n))/d^5/(e*x+d)+4/3*e*(a+b*ln(c*x^n))^2/d^5-(a+b*ln(c*x^n))^2/d^
4/x-1/3*e*(a+b*ln(c*x^n))^2/d^2/(e*x+d)^3-e*(a+b*ln(c*x^n))^2/d^3/(e*x+d)^2
+3*e^2*x*(a+b*ln(c*x^n))^2/d^5/(e*x+d)-4/3*e*(a+b*ln(c*x^n))^3/b/d^5/n+3*b^
2*e*n^2*ln(e*x+d)/d^5-26/3*b*e*n*(a+b*ln(c*x^n))*ln(1+e*x/d)/d^5+4*e*(a+b*l
```

```
n(c*x^n))^2*ln(1+e*x/d)/d^5-26/3*b^2*e*n^2*polylog(2,-e*x/d)/d^5+8*b*e*n*(a
+b*ln(c*x^n))*polylog(2,-e*x/d)/d^5-8*b^2*e*n^2*polylog(3,-e*x/d)/d^5
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.03, number of steps used = 26, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {2395, 2342, 2341, 2356, 2389, 2379, 2438, 2351, 31, 46, 2355, 2354, 2421, 6724}

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^4}\,dx=&\ \frac{3e^2x(a+b\log(cx^n))^2}{d^5(d+ex)}-\frac{8be^2nx(a+b\log(cx^n))}{3d^5(d+ex)}\\&-\frac{8ben\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)(a+b\log(cx^n))}{d^5}\\&+\frac{4e\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))^2}{d^5}\\&-\frac{8ben\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))}{3d^5}\\&-\frac{6ben\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{d^5}-\frac{(a+b\log(cx^n))^2}{d^4x}\\&-\frac{2bn(a+b\log(cx^n))}{d^4x}-\frac{e(a+b\log(cx^n))^2}{d^3(d+ex)^2}+\frac{ben(a+b\log(cx^n))}{3d^3(d+ex)^2}\\&-\frac{e(a+b\log(cx^n))^2}{3d^2(d+ex)^3}+\frac{8b^2en^2\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)}{3d^5}\\&-\frac{6b^2en^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^5}-\frac{8b^2en^2\,\text{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^5}\\&-\frac{b^2en^2\log(x)}{3d^5}+\frac{3b^2en^2\log(d+ex)}{d^5}-\frac{b^2en^2}{3d^4(d+ex)}-\frac{2b^2n^2}{d^4x}\end{aligned}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(x^2*(d + e*x)^4),x]
```

```
[Out] (-2*b^2*n^2)/(d^4*x) - (b^2*e*n^2)/(3*d^4*(d + e*x)) - (b^2*e*n^2*Log[x])/(
3*d^5) - (2*b*n*(a + b*Log[c*x^n]))/(d^4*x) + (b*e*n*(a + b*Log[c*x^n]))/(3
*d^3*(d + e*x)^2) - (8*b*e^2*n*x*(a + b*Log[c*x^n]))/(3*d^5*(d + e*x)) - (8
*b*e*n*Log[1 + d/(e*x)]*(a + b*Log[c*x^n]))/(3*d^5) - (a + b*Log[c*x^n])^2/
(d^4*x) - (e*(a + b*Log[c*x^n])^2)/(3*d^2*(d + e*x)^3) - (e*(a + b*Log[c*x^
n])^2)/(d^3*(d + e*x)^2) + (3*e^2*x*(a + b*Log[c*x^n])^2)/(d^5*(d + e*x)) +
 (4*e*Log[1 + d/(e*x)]*(a + b*Log[c*x^n])^2)/d^5 + (3*b^2*e*n^2*Log[d + e*x
])/d^5 - (6*b*e*n*(a + b*Log[c*x^n])*Log[1 + (e*x)/d])/d^5 + (8*b^2*e*n^2*P
olyLog[2, -(d/(e*x))])/(3*d^5) - (8*b*e*n*(a + b*Log[c*x^n])*PolyLog[2, -(d
/(e*x))])/d^5 - (6*b^2*e*n^2*PolyLog[2, -((e*x)/d)])/d^5 - (8*b^2*e*n^2*Pol
yLog[3, -(d/(e*x))])/d^5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a+b\log(cx^n))^2}{d^4x^2} + \frac{e^2(a+b\log(cx^n))^2}{d^2(d+ex)^4} + \frac{2e^2(a+b\log(cx^n))^2}{d^3(d+ex)^3} + \frac{3e^2(a+b\log(cx^n))^2}{d^4(d+ex)^2} - \frac{4e(a+b\log(cx^n))^2}{d^4x(d+ex)} \right) dx$$

$$= \frac{\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx}{d^4} - \frac{(4e)\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx}{d^4} + \frac{(3e^2)\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx}{d^4} + \frac{(2e^2)\int \frac{(a+b\log(cx^n))^2}{(d+ex)^3}\,dx}{d^3} + \frac{e^2\int \frac{(a+b\log(cx^n))^2}{(d+ex)^4}\,dx}{d^2}$$

$$= -\frac{(a+b\log(cx^n))^2}{d^4x} - \frac{e(a+b\log(cx^n))^2}{3d^2(d+ex)^3} - \frac{e(a+b\log(cx^n))^2}{d^3(d+ex)^2} + \frac{3e^2x(a+b\log(cx^n))^2}{d^5(d+ex)} + \frac{4e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^5} + \frac{(2bn)\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d^4} - \frac{(8ben)\int \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{x}\,dx}{d^5} + \frac{(2ben)\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx}{d^3} + \frac{(2ben)\int \frac{a+b\log(cx^n)}{x(d+ex)^3}\,dx}{3d^2} - \frac{(6be^2n)\int \frac{a+b\log(cx^n)}{d+ex}\,dx}{d^5}$$

$$= -\frac{2b^2n^2}{d^4x} - \frac{2bn(a+b\log(cx^n))}{d^4x} - \frac{(a+b\log(cx^n))^2}{d^4x} - \frac{e(a+b\log(cx^n))^2}{3d^2(d+ex)^3} - \frac{e(a+b\log(cx^n))^2}{d^3(d+ex)^2} + \frac{3e^2x(a+b\log(cx^n))^2}{d^5(d+ex)} + \frac{4e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^5} - \frac{6ben(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^5} - \frac{8ben(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^5} + \frac{(2ben)\int \frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{d^4} + \frac{(2ben)\int \frac{a+b\log(cx^n)}{x(d+ex)^2}\,dx}{3d^3} - \frac{(2be^2n)\int \frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{d^4} - \frac{(2be^2n)\int \frac{a+b\log(cx^n)}{(d+ex)^3}\,dx}{3d^3} + \frac{(6b^2en^2)\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{d^5} + \frac{(8b^2en^2)\int \frac{\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{x}\,dx}{d^5}$$

$$= -\frac{2b^2n^2}{d^4x} - \frac{2bn(a+b\log(cx^n))}{d^4x} + \frac{ben(a+b\log(cx^n))}{3d^3(d+ex)^2} - \frac{2be^2nx(a+b\log(cx^n))}{d^5(d+ex)}$$
$$- \frac{2ben\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{d^5} - \frac{(a+b\log(cx^n))^2}{d^4x}$$
$$- \frac{e(a+b\log(cx^n))^2}{3d^2(d+ex)^3} - \frac{e(a+b\log(cx^n))^2}{d^3(d+ex)^2} + \frac{3e^2x(a+b\log(cx^n))^2}{d^5(d+ex)}$$
$$+ \frac{4e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^5} - \frac{6ben(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^5}$$
$$- \frac{8ben(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^5} - \frac{6b^2en^2\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{d^5}$$
$$- \frac{8b^2en^2\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{d^5} + \frac{(2ben)\int\frac{a+b\log(cx^n)}{x(d+ex)}\,dx}{3d^4} - \frac{(2be^2n)\int\frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{3d^4}$$
$$+ \frac{(2b^2en^2)\int\frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{d^5} - \frac{(b^2en^2)\int\frac{1}{x(d+ex)^2}\,dx}{3d^3} + \frac{(2b^2e^2n^2)\int\frac{1}{d+ex}\,dx}{d^5}$$
$$= -\frac{2b^2n^2}{d^4x} - \frac{2bn(a+b\log(cx^n))}{d^4x} + \frac{ben(a+b\log(cx^n))}{3d^3(d+ex)^2} - \frac{8be^2nx(a+b\log(cx^n))}{3d^5(d+ex)}$$
$$- \frac{8ben\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{3d^5} - \frac{(a+b\log(cx^n))^2}{d^4x} - \frac{e(a+b\log(cx^n))^2}{3d^2(d+ex)^3}$$
$$- \frac{e(a+b\log(cx^n))^2}{d^3(d+ex)^2} + \frac{3e^2x(a+b\log(cx^n))^2}{d^5(d+ex)} + \frac{4e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^5}$$
$$+ \frac{2b^2en^2\log(d+ex)}{d^5} - \frac{6ben(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^5}$$
$$+ \frac{2b^2en^2\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^5} - \frac{8ben(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^5}$$
$$- \frac{6b^2en^2\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{d^5} - \frac{8b^2en^2\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{d^5} + \frac{(2b^2en^2)\int\frac{\log\left(1+\frac{d}{ex}\right)}{x}\,dx}{3d^5}$$
$$- \frac{(b^2en^2)\int\left(\frac{1}{d^2x} - \frac{e}{d(d+ex)^2} - \frac{e}{d^2(d+ex)}\right)dx}{3d^3} + \frac{(2b^2e^2n^2)\int\frac{1}{d+ex}\,dx}{3d^5}$$

$$= -\frac{2b^2n^2}{d^4x} - \frac{b^2en^2}{3d^4(d+ex)} - \frac{b^2en^2\log(x)}{3d^5} - \frac{2bn(a+b\log(cx^n))}{d^4x} + \frac{ben(a+b\log(cx^n))}{3d^3(d+ex)^2}$$
$$- \frac{8be^2nx(a+b\log(cx^n))}{3d^5(d+ex)} - \frac{8ben\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{3d^5}$$
$$- \frac{(a+b\log(cx^n))^2}{d^4x} - \frac{e(a+b\log(cx^n))^2}{3d^2(d+ex)^3} - \frac{e(a+b\log(cx^n))^2}{d^3(d+ex)^2}$$
$$+ \frac{3e^2x(a+b\log(cx^n))^2}{d^5(d+ex)} + \frac{4e\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^5}$$
$$+ \frac{3b^2en^2\log(d+ex)}{d^5} - \frac{6ben(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^5} + \frac{8b^2en^2\mathrm{Li}_2\left(-\frac{d}{ex}\right)}{3d^5}$$
$$- \frac{8ben(a+b\log(cx^n))\mathrm{Li}_2\left(-\frac{d}{ex}\right)}{d^5} - \frac{6b^2en^2\mathrm{Li}_2\left(-\frac{ex}{d}\right)}{d^5} - \frac{8b^2en^2\mathrm{Li}_3\left(-\frac{d}{ex}\right)}{d^5}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^4}\,dx =$$
$$-\frac{\frac{6b^2dn^2}{x} + \frac{6bdn(a+b\log(cx^n))}{x} - \frac{bd^2en(a+b\log(cx^n))}{(d+ex)^2} - \frac{8bden(a+b\log(cx^n))}{d+ex} - 13e(a+b\log(cx^n))^2 + \frac{3d(a+b\log(cx^n))^2}{x} + \cdots}{\cdots}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(x^2*(d + e*x)^4),x]
```

```
[Out] -1/3*((6*b^2*d*n^2)/x + (6*b*d*n*(a + b*Log[c*x^n]))/x - (b*d^2*e*n*(a + b*
Log[c*x^n]))/(d + e*x)^2 - (8*b*d*e*n*(a + b*Log[c*x^n]))/(d + e*x) - 13*e*
(a + b*Log[c*x^n])^2 + (3*d*(a + b*Log[c*x^n])^2)/x + (d^3*e*(a + b*Log[c*x
^n])^2)/(d + e*x)^3 + (3*d^2*e*(a + b*Log[c*x^n])^2)/(d + e*x)^2 + (9*d*e*(
a + b*Log[c*x^n])^2)/(d + e*x) + (4*e*(a + b*Log[c*x^n])^3)/(b*n) + 8*b^2*e
*n^2*(Log[x] - Log[d + e*x]) + (b^2*e*n^2*(d + (d + e*x)*Log[x] - (d + e*x)
*Log[d + e*x]))/(d + e*x) + 26*b*e*n*(a + b*Log[c*x^n])*Log[1 + (e*x)/d] -
12*e*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d] + 26*b^2*e*n^2*PolyLog[2, -((e*x
)/d)] - 24*b*e*n*(a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)] + 24*b^2*e*n^2*P
olyLog[3, -((e*x)/d)])/d^5
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.66 (sec) , antiderivative size = 1015, normalized size of antiderivative = 2.42

method	result	size
risch	Expression too large to display	1015

```
[In] int((a+b*ln(c*x^n))^2/x^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^2*n*ln(x^n)/d^4/x-3*b^2*ln(x^n)^2/d^4*e/(e*x+d)-b^2*ln(x^n)^2/d^3/(e*x
+d)^2*e-1/3*b^2*ln(x^n)^2/d^2/(e*x+d)^3*e+1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n
)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*
c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*(-1/3/d^2/(e*x+d)^3*e+4/d^
5*e*ln(e*x+d)-3/d^4*e/(e*x+d)-1/d^3/(e*x+d)^2*e-1/d^4/x-4/d^5*e*ln(x))+4*b^
2*ln(x^n)^2/d^5*e*ln(e*x+d)-26/3*b^2*n*ln(x^n)/d^5*e*ln(e*x+d)+26/3*b^2*n*l
n(x^n)/d^5*e*ln(x)+26/3*b^2/d^5*n^2*e*ln(e*x+d)*ln(-e*x/d)+4*b^2*n/d^5*e*ln
(x^n)*ln(x)^2+8*b^2/d^5*e*ln(x)*dilog(-e*x/d)*n^2-8*b^2*n/d^5*e*ln(x^n)*dil
og(-e*x/d)-4*b^2/d^5*e*n^2*ln(e*x+d)*ln(x)^2+4*b^2/d^5*e*n^2*ln(x)^2*ln(1+e
*x/d)+8*b^2/d^5*e*n^2*ln(x)*polylog(2,-e*x/d)-4*b^2*ln(x^n)^2/d^5*e*ln(x)-1
3/3*b^2/d^5*n^2*e*ln(x)^2+26/3*b^2/d^5*n^2*e*dilog(-e*x/d)-4/3*b^2/d^5*e*ln
(x)^3*n^2+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csg
n(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*
ln(c)+2*a)*b*(-1/3*ln(x^n)/d^2/(e*x+d)^3*e+4*ln(x^n)/d^5*e*ln(e*x+d)-3*ln(x
^n)/d^4*e/(e*x+d)-ln(x^n)/d^3/(e*x+d)^2*e-ln(x^n)/d^4/x-4*ln(x^n)/d^5*e*ln(
x)-1/3*n*(-6/d^5*e*ln(x)^2+12/d^5*e*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))-4/
d^4*e/(e*x+d)+13/d^5*e*ln(e*x+d)-1/2/d^3/(e*x+d)^2*e+3/d^4/x-13/d^5*e*ln(x)
))+8/3*b^2*n*ln(x^n)/d^4*e/(e*x+d)+1/3*b^2*n*ln(x^n)/d^3/(e*x+d)^2*e+8*b^2/
d^5*e*ln(x)*ln(e*x+d)*ln(-e*x/d)*n^2-8*b^2*n/d^5*e*ln(x^n)*ln(e*x+d)*ln(-e*
x/d)-1/3*b^2*e*n^2/d^4/(e*x+d)-3*b^2*e*n^2*ln(x)/d^5+3*b^2*e*n^2*ln(e*x+d)/
d^5-8*b^2*e*n^2*polylog(3,-e*x/d)/d^5-2*b^2*n^2/d^4/x-b^2*ln(x^n)^2/d^4/x
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^4}\,dx=\int \frac{(b\log(cx^n)+a)^2}{(ex+d)^4x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e^4*x^6 + 4*d*e^3*x^5
 + 6*d^2*e^2*x^4 + 4*d^3*e*x^3 + d^4*x^2), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^4}\,dx = \int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^4}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**2/x**2/(e*x+d)**4,x)
[Out] Integral((a + b*log(c*x**n))**2/(x**2*(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^4x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^2/(e*x+d)^4,x, algorithm="maxima")
[Out] -1/3*a^2*((12*e^3*x^3 + 30*d*e^2*x^2 + 22*d^2*e*x + 3*d^3)/(d^4*e^3*x^4 + 3
*d^5*e^2*x^3 + 3*d^6*e*x^2 + d^7*x) - 12*e*log(e*x + d)/d^5 + 12*e*log(x)/d
^5) + integrate((b^2*log(c)^2 + b^2*log(x^n)^2 + 2*a*b*log(c) + 2*(b^2*log(
c) + a*b)*log(x^n))/(e^4*x^6 + 4*d*e^3*x^5 + 6*d^2*e^2*x^4 + 4*d^3*e*x^3 +
d^4*x^2), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex+d)^4x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x^2/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2/((e*x + d)^4*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{x^2(d+ex)^4}\,dx = \int \frac{(a+b\ln(c\,x^n))^2}{x^2\,(d+e\,x)^4}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(x^2*(d + e*x)^4),x)
[Out] int((a + b*log(c*x^n))^2/(x^2*(d + e*x)^4), x)
```

3.120 $\int \frac{x\log^2(x)}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 107

$$\int \frac{x\log^2(x)}{(d+ex)^4}\,dx = -\frac{x}{3d^2e(d+ex)}+\frac{x\log(x)}{3de(d+ex)^2}+\frac{x^2(3d+ex)\log^2(x)}{6d^2(d+ex)^3} -\frac{\log(x)\log\left(1+\frac{ex}{d}\right)}{3d^2e^2}-\frac{\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{3d^2e^2}$$

```
[Out] -1/3*x/d^2/e/(e*x+d)+1/3*x*ln(x)/d/e/(e*x+d)^2+1/6*x^2*(e*x+3*d)*ln(x)^2/d^
2/(e*x+d)^3-1/3*ln(x)*ln(1+e*x/d)/d^2/e^2-1/3*polylog(2,-e*x/d)/d^2/e^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.47, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2383, 2381, 2384, 2354, 2438, 2373, 45}

$$\int \frac{x\log^2(x)}{(d+ex)^4}\,dx = -\frac{\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{3d^2e^2}+\frac{\log(d+ex)}{3d^2e^2} -\frac{(\log(x)+1)\log\left(\frac{ex}{d}+1\right)}{3d^2e^2}+\frac{x^2\log^2(x)}{6d^2(d+ex)^2}-\frac{x^2\log(x)}{3d^2(d+ex)^2} +\frac{x\log(x)}{3d^2e(d+ex)}+\frac{1}{3de^2(d+ex)}+\frac{x^2\log^2(x)}{3d(d+ex)^3}$$

```
[In] Int[(x*Log[x]^2)/(d + e*x)^4,x]
```

```
[Out] 1/(3*d*e^2*(d + e*x)) - (x^2*Log[x])/(3*d^2*(d + e*x)^2) + (x*Log[x])/(3*d^
2*e*(d + e*x)) + (x^2*Log[x]^2)/(3*d*(d + e*x)^3) + (x^2*Log[x]^2)/(6*d^2*(
```

```
d + e*x)^2) + Log[d + e*x]/(3*d^2*e^2) - ((1 + Log[x])*Log[1 + (e*x)/d])/(3
*d^2*e^2) - PolyLog[2, -((e*x)/d)]/(3*d^2*e^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
```

```
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2\log^2(x)}{3d(d+ex)^3}+\frac{\int\frac{x\log^2(x)}{(d+ex)^3}\,dx}{3d}-\frac{2\int\frac{x\log(x)}{(d+ex)^3}\,dx}{3d}\\
&= -\frac{x^2\log(x)}{3d^2(d+ex)^2}+\frac{x^2\log^2(x)}{3d(d+ex)^3}+\frac{x^2\log^2(x)}{6d^2(d+ex)^2}+\frac{\int\frac{x}{(d+ex)^2}\,dx}{3d^2}-\frac{\int\frac{x\log(x)}{(d+ex)^2}\,dx}{3d^2}\\
&= -\frac{x^2\log(x)}{3d^2(d+ex)^2}+\frac{x\log(x)}{3d^2e(d+ex)}+\frac{x^2\log^2(x)}{3d(d+ex)^3}\\
&\quad+\frac{x^2\log^2(x)}{6d^2(d+ex)^2}+\frac{\int\left(-\frac{d}{e(d+ex)^2}+\frac{1}{e(d+ex)}\right)dx}{3d^2}-\frac{\int\frac{1+\log(x)}{d+ex}\,dx}{3d^2e}\\
&= \frac{1}{3de^2(d+ex)}-\frac{x^2\log(x)}{3d^2(d+ex)^2}+\frac{x\log(x)}{3d^2e(d+ex)}+\frac{x^2\log^2(x)}{3d(d+ex)^3}\\
&\quad+\frac{x^2\log^2(x)}{6d^2(d+ex)^2}+\frac{\log(d+ex)}{3d^2e^2}-\frac{(1+\log(x))\log\left(1+\frac{ex}{d}\right)}{3d^2e^2}+\frac{\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{3d^2e^2}\\
&= \frac{1}{3de^2(d+ex)}-\frac{x^2\log(x)}{3d^2(d+ex)^2}+\frac{x\log(x)}{3d^2e(d+ex)}+\frac{x^2\log^2(x)}{3d(d+ex)^3}\\
&\quad+\frac{x^2\log^2(x)}{6d^2(d+ex)^2}+\frac{\log(d+ex)}{3d^2e^2}-\frac{(1+\log(x))\log\left(1+\frac{ex}{d}\right)}{3d^2e^2}-\frac{\text{Li}_2\left(-\frac{ex}{d}\right)}{3d^2e^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int\frac{x\log^2(x)}{(d+ex)^4}\,dx = \frac{2d(d+ex)^2+e^2x^2(3d+ex)\log^2(x)-2(d+ex)\log(x)\left(-dex+(d+ex)^2\log\left(1+\frac{ex}{d}\right)\right)-2(d+ex)^3\,\text{Poly}}{6d^2e^2(d+ex)^3}$$

```
[In] Integrate[(x*Log[x]^2)/(d + e*x)^4,x]
[Out] (2*d*(d + e*x)^2 + e^2*x^2*(3*d + e*x)*Log[x]^2 - 2*(d + e*x)*Log[x]*(-(d*e
*x) + (d + e*x)^2*Log[1 + (e*x)/d]) - 2*(d + e*x)^3*PolyLog[2, -((e*x)/d)])
/(6*d^2*e^2*(d + e*x)^3)
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.68

method	result
parts	$-\frac{\ln(x)^2}{2e^2(ex+d)^2}+\frac{\ln(x)^2 d}{3e^2(ex+d)^3}+\frac{\ln(x)^2}{6d^2e^2}-\frac{\frac{\mathrm{dilog}\left(\frac{ex+d}{d}\right)}{e}+\frac{\ln(x)\ln\left(\frac{ex+d}{d}\right)}{e}}{3e\,d^2}+\frac{-\frac{\ln(ex+d)}{3d^2e}+\frac{1}{3de(ex+d)}+\frac{\ln(x)x(ex+2d)}{3d^2(ex+d)^2}}{e}-\frac{-\frac{\ln(ex+\cdots}{de}\cdots}{\cdots}$

```
[In] int(x*ln(x)^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(x)^2/e^2/(e*x+d)^2+1/3*ln(x)^2/e^2*d/(e*x+d)^3+1/6/d^2/e^2*ln(x)^2-
1/3/e/d^2*(dilog((e*x+d)/d)/e+ln(x)*ln((e*x+d)/d)/e)+2/3/e*(-1/2/d^2*ln(e*x
+d)/e+1/2/d/e/(e*x+d)+1/2*ln(x)*x*(e*x+2*d)/d^2/(e*x+d)^2)-1/3/e/d*(-1/d/e*
ln(e*x+d)+ln(x)*x/d/(e*x+d))
```

Fricas [F]

$$\int \frac{x\log^2(x)}{(d+ex)^4}\,dx=\int \frac{x\log{(x)}^2}{(ex+d)^4}\,dx$$

```
[In] integrate(x*log(x)^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral(x*log(x)^2/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^
4), x)
```

Sympy [A] (verification not implemented)

Time = 20.97 (sec) , antiderivative size = 374, normalized size of antiderivative = 3.50

$$\int \frac{x\log^2(x)}{(d+ex)^4}\,dx = \frac{(-d-3ex)\log(x)^2}{6d^3e^2+18d^2e^3x+18de^4x^2+6e^5x^3} + \frac{\left(\begin{cases}\frac{x}{d^3} & \text{for } e=0\\ -\frac{1}{2e(d+ex)^2} & \text{otherwise}\end{cases}\right)\log(x)}{e} - \frac{\begin{cases}\frac{x}{d^3} & \text{for } e=0\\ -\frac{1}{2d^2e+2de^2x}-\frac{\log(x)}{2d^2e}+\frac{\log\left(\frac{d}{e}+x\right)}{2d^2e} & \text{otherwise}\end{cases}}{e} + \frac{\begin{cases}-\frac{1}{e^3x} & \text{for } d=0\\ -\frac{1}{2de^2+2e^3x}-\frac{\log(d+ex)}{2de^2} & \text{otherwise}\end{cases}}{3d} - \frac{\left(\begin{cases}\frac{1}{e^3x} & \text{for } d=0\\ -\frac{1}{2d\left(\frac{d}{x}+e\right)^2} & \text{otherwise}\end{cases}\right)\log(x)}{3d} - \frac{2\left(\begin{cases}-\frac{1}{e^2x} & \text{for } d=0\\ -\frac{\log(d^2+dex)}{de} & \text{otherwise}\end{cases}\right)}{3de} + \frac{2\left(\begin{cases}\frac{1}{e^2x} & \text{for } d=0\\ -\frac{1}{\frac{d^2}{x}+de} & \text{otherwise}\end{cases}\right)\log(x)}{3de} + \frac{\begin{cases}-\frac{1}{ex} & \text{for } d = \\ \frac{\begin{cases}\operatorname{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(e)\log(x)+\operatorname{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{for } |x|<1\\ -\log(e)\log\left(\frac{1}{x}\right)+\operatorname{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log(e)+\operatorname{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{otherwise}\end{cases}}{d} & \text{otherv}\end{cases}}{3de^2} - \frac{\left(\begin{cases}\frac{1}{ex} & \text{for } d=0\\ \frac{\log\left(\frac{d}{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)\log(x)}{3de^2}$$

```
[In] integrate(x*ln(x)**2/(e*x+d)**4,x)
```

```
[Out] (-d - 3*e*x)*log(x)**2/(6*d**3*e**2 + 18*d**2*e**3*x + 18*d*e**4*x**2 + 6*e
**5*x**3) + Piecewise((x/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x)**2), True))*lo
g(x)/e - Piecewise((x/d**3, Eq(e, 0)), (-1/(2*d**2*e + 2*d*e**2*x) - log(x)
/(2*d**2*e) + log(d/e + x)/(2*d**2*e), True))/e + Piecewise((-1/(e**3*x), E
q(d, 0)), (-1/(2*d*e**2 + 2*e**3*x) - log(d + e*x)/(2*d*e**2), True))/(3*d)
 - Piecewise((1/(e**3*x), Eq(d, 0)), (-1/(2*d*(d/x + e)**2), True))*log(x)/
```

```
(3*d) - 2*Piecewise((-1/(e**2*x), Eq(d, 0)), (-log(d**2 + d*e*x)/(d*e), Tru
e))/(3*d*e) + 2*Piecewise((1/(e**2*x), Eq(d, 0)), (-1/(d**2/x + d*e), True)
)*log(x)/(3*d*e) + Piecewise((-1/(e*x), Eq(d, 0)), (Piecewise((polylog(2, d
*exp_polar(I*pi)/(e*x)), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(e)*log(x) + p
olylog(2, d*exp_polar(I*pi)/(e*x)), Abs(x) < 1), (-log(e)*log(1/x) + polylo
g(2, d*exp_polar(I*pi)/(e*x)), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0,
0), ()), x)*log(e) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(e) + polylo
g(2, d*exp_polar(I*pi)/(e*x)), True))/d, True))/(3*d*e**2) - Piecewise((1/(
e*x), Eq(d, 0)), (log(d/x + e)/d, True))*log(x)/(3*d*e**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.23

$$\int \frac{x\log^2(x)}{(d+ex)^4}\,dx = -\frac{d^2\log(x)^2 - 2\,(e^2\log(x) + e^2)x^2 - 2\,d^2 + \left(3\,de\log(x)^2 - 2\,de\log(x) - 4\,de\right)x}{6\,(de^5x^3 + 3\,d^2e^4x^2 + 3\,d^3e^3x + d^4e^2)} + \frac{\log(x)^2}{6\,d^2e^2} - \frac{\log\left(\frac{ex}{d} + 1\right)\log(x) + \mathrm{Li}_2\left(-\frac{ex}{d}\right)}{3\,d^2e^2}$$

```
[In] integrate(x*log(x)^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(d^2*log(x)^2 - 2*(e^2*log(x) + e^2)*x^2 - 2*d^2 + (3*d*e*log(x)^2 - 2
*d*e*log(x) - 4*d*e)*x)/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4*e^2)
 + 1/6*log(x)^2/(d^2*e^2) - 1/3*(log(e*x/d + 1)*log(x) + dilog(-e*x/d))/(d^
2*e^2)
```

Giac [F]

$$\int \frac{x\log^2(x)}{(d+ex)^4}\,dx = \int \frac{x\log(x)^2}{(ex+d)^4}\,dx$$

```
[In] integrate(x*log(x)^2/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate(x*log(x)^2/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \log^2(x)}{(d+ex)^4} \, dx = \int \frac{x \ln(x)^2}{(d+e\,x)^4} \, dx$$

```
[In] int((x*log(x)^2)/(d + e*x)^4,x)
[Out] int((x*log(x)^2)/(d + e*x)^4, x)
```

3.121 $\int \frac{(a+b\log(cx^n))^3}{x(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)}\,dx = -\frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^3}{d} + \frac{3bn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)}{d} + \frac{6b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{d}{ex}\right)}{d} + \frac{6b^3n^3\operatorname{PolyLog}\left(4,-\frac{d}{ex}\right)}{d}$$

```
[Out] -ln(1+d/e/x)*(a+b*ln(c*x^n))^3/d+3*b*n*(a+b*ln(c*x^n))^2*polylog(2,-d/e/x)/
d+6*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-d/e/x)/d+6*b^3*n^3*polylog(4,-d/e/x)
/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2379, 2421, 2430, 6724}

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)}\,dx = \frac{6b^2n^2\operatorname{PolyLog}\left(3,-\frac{d}{ex}\right)(a+b\log(cx^n))}{d} + \frac{3bn\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d} - \frac{\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))^3}{d} + \frac{6b^3n^3\operatorname{PolyLog}\left(4,-\frac{d}{ex}\right)}{d}$$

```
[In] Int[(a + b*Log[c*x^n])^3/(x*(d + e*x)),x]
```

```
[Out] -((Log[1 + d/(e*x)]*(a + b*Log[c*x^n])^3)/d) + (3*b*n*(a + b*Log[c*x^n])^2*
PolyLog[2, -(d/(e*x))])/d + (6*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(d/(e
*x))])/d + (6*b^3*n^3*PolyLog[4, -(d/(e*x))])/d
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log\left(1+\frac{d}{ex}\right)\left(a+b\log\left(cx^n\right)\right)^3}{d}+\frac{(3bn)\int\frac{\log\left(1+\frac{d}{ex}\right)\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx}{d}\\
&= -\frac{\log\left(1+\frac{d}{ex}\right)\left(a+b\log\left(cx^n\right)\right)^3}{d}+\frac{3bn\left(a+b\log\left(cx^n\right)\right)^2\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d}\\
&\quad-\frac{\left(6b^2n^2\right)\int\frac{\left(a+b\log\left(cx^n\right)\right)\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{x}\,dx}{d}\\
&= -\frac{\log\left(1+\frac{d}{ex}\right)\left(a+b\log\left(cx^n\right)\right)^3}{d}+\frac{3bn\left(a+b\log\left(cx^n\right)\right)^2\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d}\\
&\quad+\frac{6b^2n^2\left(a+b\log\left(cx^n\right)\right)\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{d}-\frac{\left(6b^3n^3\right)\int\frac{\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{x}\,dx}{d}
\end{aligned}
$$

$$= -\frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^3}{d} + \frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d} + \frac{6b^2n^2(a+b\log(cx^n))\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{d} + \frac{6b^3n^3\operatorname{Li}_4\left(-\frac{d}{ex}\right)}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 243 vs. 2(113) = 226.

Time = 0.14 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.15

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)}\,dx = \frac{4\log(x)(a-bn\log(x)+b\log(cx^n))^3 - 4(a-bn\log(x)+b\log(cx^n))^3\log(d+ex) + 6bn(a-bn\log(x)+}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])^3/(x*(d + e*x)),x]
```

```
[Out] (4*Log[x]*(a - b*n*Log[x] + b*Log[c*x^n])^3 - 4*(a - b*n*Log[x] + b*Log[c*x
^n])^3*Log[d + e*x] + 6*b*n*(a - b*n*Log[x] + b*Log[c*x^n])^2*(Log[x]^2 - 2
*(Log[x]*Log[1 + (e*x)/d] + PolyLog[2, -((e*x)/d)])) - 4*b^2*n^2*(-a + b*n*
Log[x] - b*Log[c*x^n])*(Log[x]^2*(Log[x] - 3*Log[1 + (e*x)/d]) - 6*Log[x]*P
olyLog[2, -((e*x)/d)] + 6*PolyLog[3, -((e*x)/d)]) + b^3*n^3*(Log[x]^4 - 4*L
og[x]^3*Log[1 + (e*x)/d] - 12*Log[x]^2*PolyLog[2, -((e*x)/d)] + 24*Log[x]*P
olyLog[3, -((e*x)/d)] - 24*PolyLog[4, -((e*x)/d)]))/(4*d)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 967, normalized size of antiderivative = 8.56

method	result	size
risch	Expression too large to display	967

```
[In] int((a+b*ln(c*x^n))^3/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -b^3*ln(x^n)^3/d*ln(e*x+d)+b^3*ln(x^n)^3/d*ln(x)-3/2*b^3*n/d*ln(x^n)^2*ln(x
)^2+b^3/d*n^2*ln(x^n)*ln(x)^3-1/4*b^3/d*ln(x)^4*n^3+3*b^3/d*ln(x)^2*ln(e*x+
d)*ln(-e*x/d)*n^3+3*b^3/d*ln(x)^2*dilog(-e*x/d)*n^3-6*b^3/d*ln(x)*ln(x^n)*l
n(e*x+d)*ln(-e*x/d)*n^2-6*b^3/d*ln(x)*ln(x^n)*dilog(-e*x/d)*n^2+3*b^3*n/d*l
n(x^n)^2*ln(e*x+d)*ln(-e*x/d)+3*b^3*n/d*ln(x^n)^2*dilog(-e*x/d)-2*b^3/d*n^3
*ln(e*x+d)*ln(x)^3+2*b^3/d*n^3*ln(x)^3*ln(1+e*x/d)+3*b^3/d*n^3*ln(x)^2*poly
log(2,-e*x/d)-6*b^3/d*n^3*polylog(4,-e*x/d)+3*b^3/d*n^2*ln(x)^2*ln(x^n)*ln(
e*x+d)-3*b^3/d*n^2*ln(x)^2*ln(x^n)*ln(1+e*x/d)-6*b^3/d*n^2*ln(x)*ln(x^n)*po
lylog(2,-e*x/d)+6*b^3/d*n^2*ln(x^n)*polylog(3,-e*x/d)+1/8*(-I*b*Pi*csgn(I*c
```

```
)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*
x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^3*(-1/d*ln(e*x+d
)+1/d*ln(x))+3/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I
*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)
^3+2*b*ln(c)+2*a)*b^2*(-ln(x^n)^2/d*ln(e*x+d)+ln(x^n)^2/d*ln(x)-2*n*(1/2/d*
ln(x^n)*ln(x)^2-1/6/d*ln(x)^3*n-1/d*((ln(x^n)-n*ln(x))*(dilog(-e*x/d)+ln(e*
x+d)*ln(-e*x/d))+n*(1/2*ln(e*x+d)*ln(x)^2-1/2*ln(x)^2*ln(1+e*x/d)-ln(x)*pol
ylog(2,-e*x/d)+polylog(3,-e*x/d)))))+3/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csg
n(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n
)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*b*(-ln(x^n)/d*ln(e*x+d)+ln(x^n)
/d*ln(x)-n*(1/2/d*ln(x)^2-1/d*ln(e*x+d)*ln(-e*x/d)-1/d*dilog(-e*x/d)))
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)}\,dx=\int \frac{(b\log(cx^n)+a)^3}{(ex+d)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^3/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)/(e*x^2 + d*x), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)}\,dx=\int \frac{(a+b\log(cx^n))^3}{x\,(d+ex)}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**3/x/(e*x+d),x)
```

```
[Out] Integral((a + b*log(c*x**n))**3/(x*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)}\,dx=\int \frac{(b\log(cx^n)+a)^3}{(ex+d)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^3/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -a^3*(log(e*x + d)/d - log(x)/d) + integrate((b^3*log(c)^3 + b^3*log(x^n)^3
 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + 3*(b^3*log(c) + a*b^2)*log(x^n)^2 +
3*(b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*log(x^n))/(e*x^2 + d*x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)}\,dx = \int \frac{(b\log(cx^n)+a)^3}{(ex+d)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^3/x/(e*x+d),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^3/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^3}{x\,(d+e\,x)}\,dx$$

```
[In] int((a + b*log(c*x^n))^3/(x*(d + e*x)),x)
[Out] int((a + b*log(c*x^n))^3/(x*(d + e*x)), x)
```

3.122 $\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 217

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^2}\,dx &= -\frac{ex(a+b\log(cx^n))^3}{d^2(d+ex)}-\frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^3}{d^2}\\&\quad+\frac{3bn(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{d^2}\\&\quad+\frac{3bn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)}{d^2}\\&\quad+\frac{6b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^2}\\&\quad+\frac{6b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^2}\\&\quad-\frac{6b^3n^3\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{d^2}+\frac{6b^3n^3\operatorname{PolyLog}\left(4,-\frac{d}{ex}\right)}{d^2}\end{aligned}$$

```
[Out] -e*x*(a+b*ln(c*x^n))^3/d^2/(e*x+d)-ln(1+d/e/x)*(a+b*ln(c*x^n))^3/d^2+3*b*n*
(a+b*ln(c*x^n))^2*ln(1+e*x/d)/d^2+3*b*n*(a+b*ln(c*x^n))^2*polylog(2,-d/e/x)
/d^2+6*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-e*x/d)/d^2+6*b^2*n^2*(a+b*ln(c*x^
n))*polylog(3,-d/e/x)/d^2-6*b^3*n^3*polylog(3,-e*x/d)/d^2+6*b^3*n^3*polylog
(4,-d/e/x)/d^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2389, 2379, 2421, 2430, 6724, 2355, 2354}

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^2}\,dx = \frac{6b^2n^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)(a+b\log(cx^n))}{d^2} + \frac{6b^2n^2\,\text{PolyLog}\left(3,-\frac{d}{ex}\right)(a+b\log(cx^n))}{d^2} + \frac{3bn\,\text{PolyLog}\left(2,-\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^2} + \frac{3bn\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))^2}{d^2} - \frac{\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))^3}{d^2} - \frac{ex(a+b\log(cx^n))^3}{d^2(d+ex)} - \frac{6b^3n^3\,\text{PolyLog}\left(3,-\frac{ex}{d}\right)}{d^2} + \frac{6b^3n^3\,\text{PolyLog}\left(4,-\frac{d}{ex}\right)}{d^2}$$

```
[In] Int[(a + b*Log[c*x^n])^3/(x*(d + e*x)^2),x]
```

```
[Out] -((e*x*(a + b*Log[c*x^n])^3)/(d^2*(d + e*x))) - (Log[1 + d/(e*x)]*(a + b*Lo
g[c*x^n])^3)/d^2 + (3*b*n*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d])/d^2 + (3*b
*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(d/(e*x))])/d^2 + (6*b^2*n^2*(a + b*Log
[c*x^n])*PolyLog[2, -((e*x)/d)])/d^2 + (6*b^2*n^2*(a + b*Log[c*x^n])*PolyLo
g[3, -(d/(e*x))])/d^2 - (6*b^3*n^3*PolyLog[3, -((e*x)/d)])/d^2 + (6*b^3*n^3
*PolyLog[4, -(d/(e*x))])/d^2
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
```

```
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{(a+b\log(cx^n))^3}{x(d+ex)}\,dx}{d} - \frac{e\int \frac{(a+b\log(cx^n))^3}{(d+ex)^2}\,dx}{d} \\
&= -\frac{ex(a+b\log(cx^n))^3}{d^2(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^3}{d^2} \\
&\quad + \frac{(3bn)\int \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{x}\,dx}{d^2} + \frac{(3ben)\int \frac{(a+b\log(cx^n))^2}{d+ex}\,dx}{d^2}
\end{aligned}$$

$$= -\frac{ex(a+b\log(cx^n))^3}{d^2(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^3}{d^2} + \frac{3bn(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{d^2} + \frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^2} - \frac{(6b^2n^2)\int\frac{(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{x}dx}{d^2} - \frac{(6b^2n^2)\int\frac{(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{x}dx}{d^2}$$

$$= -\frac{ex(a+b\log(cx^n))^3}{d^2(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^3}{d^2} + \frac{3bn(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{d^2} + \frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^2} + \frac{6b^2n^2(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{d^2} + \frac{6b^2n^2(a+b\log(cx^n))\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{d^2} - \frac{(6b^3n^3)\int\frac{\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{x}dx}{d^2} - \frac{(6b^3n^3)\int\frac{\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{x}dx}{d^2}$$

$$= -\frac{ex(a+b\log(cx^n))^3}{d^2(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^3}{d^2} + \frac{3bn(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{d^2} + \frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^2} + \frac{6b^2n^2(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{d^2} + \frac{6b^2n^2(a+b\log(cx^n))\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{d^2} - \frac{6b^3n^3\operatorname{Li}_3\left(-\frac{ex}{d}\right)}{d^2} + \frac{6b^3n^3\operatorname{Li}_4\left(-\frac{d}{ex}\right)}{d^2}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.99

$$\int\frac{(a+b\log(cx^n))^3}{x(d+ex)^2}\,dx = \frac{4d(a-bn\log(x)+b\log(cx^n))^3+4(d+ex)\log(x)(a-bn\log(x)+b\log(cx^n))^3-4(d+ex)(a-bn\log(\ldots}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])^3/(x*(d + e*x)^2),x]
```

```
[Out] (4*d*(a - b*n*Log[x] + b*Log[c*x^n])^3 + 4*(d + e*x)*Log[x]*(a - b*n*Log[x]
 + b*Log[c*x^n])^3 - 4*(d + e*x)*(a - b*n*Log[x] + b*Log[c*x^n])^3*Log[d +
e*x] + 6*b*n*(a - b*n*Log[x] + b*Log[c*x^n])^2*(-2*e*x*Log[x] + (d + e*x)*L
og[x]^2 + 2*(d + e*x)*Log[d + e*x] - 2*(d + e*x)*(Log[x]*Log[1 + (e*x)/d] +
 PolyLog[2, -((e*x)/d)])) + 4*b^2*n^2*(a - b*n*Log[x] + b*Log[c*x^n])*(Log[
x]*((d + e*x)*Log[x]^2 + 6*(d + e*x)*Log[1 + (e*x)/d] - 3*Log[x]*(e*x + (d
+ e*x)*Log[1 + (e*x)/d])) - 6*(d + e*x)*(-1 + Log[x])*PolyLog[2, -((e*x)/d)
```

```
] + 6*(d + e*x)*PolyLog[3, -((e*x)/d)]) + b^3*n^3*((d + e*x)*Log[x]^4 - 4*(
Log[x]^2*(e*x*Log[x] - 3*(d + e*x)*Log[1 + (e*x)/d]) - 6*(d + e*x)*Log[x]*P
olyLog[2, -((e*x)/d)] + 6*(d + e*x)*PolyLog[3, -((e*x)/d)]) - 4*(d + e*x)*(
Log[x]^3*Log[1 + (e*x)/d] + 3*Log[x]^2*PolyLog[2, -((e*x)/d)] - 6*Log[x]*Po
lyLog[3, -((e*x)/d)] + 6*PolyLog[4, -((e*x)/d)])))/(4*d^2*(d + e*x))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 1373, normalized size of antiderivative = 6.33

method	result	size
risch	Expression too large to display	1373

```
[In] int((a+b*ln(c*x^n))^3/x/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 6*b^3/d^2*ln(e*x+d)*ln(-e*x/d)*ln(x)*n^3-6*b^3/d^2*n^2*ln(x^n)*ln(e*x+d)*ln
(-e*x/d)+3*b^3/d^2*ln(e*x+d)*ln(-e*x/d)*ln(x)^2*n^3-6*b^3/d^2*ln(x^n)*dilog
(-e*x/d)*ln(x)*n^2+3*b^3/d^2*n^2*ln(x^n)*ln(e*x+d)*ln(x)^2-6*b^3/d^2*ln(x^n
)*ln(e*x+d)*ln(-e*x/d)*ln(x)*n^2+3/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*
c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-
I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b^2*(-ln(x^n)^2/d^2*ln(e*x+d)+ln(x^n)
^2/d/(e*x+d)+ln(x^n)^2/d^2*ln(x)-2*n*(-ln(x^n)/d^2*ln(e*x+d)+ln(x^n)/d^2*ln
(x)-1/2/d^2*n*ln(x)^2+1/d^2*n*ln(e*x+d)*ln(-e*x/d)+1/d^2*n*dilog(-e*x/d)+1/
2/d^2*ln(x^n)*ln(x)^2-1/6/d^2*ln(x)^3*n-1/d^2*((ln(x^n)-n*ln(x))*(dilog(-e*
x/d)+ln(e*x+d)*ln(-e*x/d))+n*(1/2*ln(e*x+d)*ln(x)^2-1/2*ln(x)^2*ln(1+e*x/d)
-ln(x)*polylog(2,-e*x/d)+polylog(3,-e*x/d)))))-2*b^3/d^2*n^3*ln(e*x+d)*ln(x
)^3+2*b^3/d^2*n^3*ln(x)^3*ln(1+e*x/d)+3*b^3*n/d^2*ln(x^n)^2*dilog(-e*x/d)+3
*b^3*n*ln(x^n)^2/d^2*ln(e*x+d)-3*b^3*n*ln(x^n)^2/d^2*ln(x)+3*b^3/d^2*n^3*ln
(x)^2*polylog(2,-e*x/d)+6*b^3/d^2*n^2*ln(x^n)*polylog(3,-e*x/d)+b^3/d^2*n^2
*ln(x^n)*ln(x)^3-3/2*b^3*n/d^2*ln(x^n)^2*ln(x)^2+3*b^3/d^2*n^2*ln(x^n)*ln(x
)^2+6*b^3/d^2*dilog(-e*x/d)*ln(x)*n^3-6*b^3/d^2*n^2*ln(x^n)*dilog(-e*x/d)-3
*b^3/d^2*n^3*ln(e*x+d)*ln(x)^2+3*b^3/d^2*n^3*ln(x)^2*ln(1+e*x/d)+6*b^3/d^2*
n^3*ln(x)*polylog(2,-e*x/d)+3*b^3/d^2*dilog(-e*x/d)*ln(x)^2*n^3+3/4*(-I*b*P
i*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*
Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*b*(-
ln(x^n)/d^2*ln(e*x+d)+ln(x^n)/d/(e*x+d)+ln(x^n)/d^2*ln(x)-n*(1/2/d^2*ln(x)^
2-1/d^2*ln(e*x+d)+1/d^2*ln(x)-1/d^2*ln(e*x+d)*ln(-e*x/d)-1/d^2*dilog(-e*x/d
)))-3*b^3/d^2*n^2*ln(x^n)*ln(1+e*x/d)*ln(x)^2-6*b^3/d^2*n^2*ln(x^n)*polylog
(2,-e*x/d)*ln(x)+3*b^3*n/d^2*ln(x^n)^2*ln(e*x+d)*ln(-e*x/d)-b^3*ln(x^n)^3/d
^2*ln(e*x+d)+b^3*ln(x^n)^3/d/(e*x+d)+b^3*ln(x^n)^3/d^2*ln(x)-b^3/d^2*ln(x)^
3*n^3-1/4*b^3/d^2*ln(x)^4*n^3-6*b^3/d^2*n^3*polylog(4,-e*x/d)+1/8*(-I*b*Pi*
csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi
*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^3*(-1/d^
2*ln(e*x+d)+1/d/(e*x+d)+1/d^2*ln(x))-6*b^3*n^3*polylog(3,-e*x/d)/d^2
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^2}\,dx=\int \frac{(b\log(cx^n)+a)^3}{(ex+d)^2x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^3/x/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)/(e^2*x^3 + 2*d*e*x^2 + d^2*x), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^2}\,dx=\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**3/x/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*log(c*x**n))**3/(x*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^2}\,dx=\int \frac{(b\log(cx^n)+a)^3}{(ex+d)^2x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^3/x/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] a^3*(1/(d*e*x + d^2) - log(e*x + d)/d^2 + log(x)/d^2) + integrate((b^3*log(
c)^3 + b^3*log(x^n)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + 3*(b^3*log(c) +
 a*b^2)*log(x^n)^2 + 3*(b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*log(x^n))/(e
^2*x^3 + 2*d*e*x^2 + d^2*x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^2}\,dx=\int \frac{(b\log(cx^n)+a)^3}{(ex+d)^2x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^3/x/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3/((e*x + d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^2}\,dx = \int \frac{\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x\,\left(d+e\,x\right)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))^3/(x*(d + e*x)^2),x)
[Out] int((a + b*log(c*x^n))^3/(x*(d + e*x)^2), x)
```

3.123 $\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 361

$$
\begin{aligned}
\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^3}\,dx &= \frac{3benx(a+b\log(cx^n))^2}{2d^3(d+ex)} - \frac{(a+b\log(cx^n))^3}{2d^3} \\
&+ \frac{(a+b\log(cx^n))^3}{2d(d+ex)^2} - \frac{ex(a+b\log(cx^n))^3}{d^3(d+ex)} \\
&+ \frac{(a+b\log(cx^n))^4}{4bd^3n} - \frac{3b^2n^2(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^3} \\
&+ \frac{9bn(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{2d^3} \\
&- \frac{(a+b\log(cx^n))^3\log\left(1+\frac{ex}{d}\right)}{d^3} - \frac{3b^3n^3\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^3} \\
&+ \frac{9b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^3} \\
&- \frac{3bn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^3} \\
&- \frac{9b^3n^3\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{d^3} \\
&+ \frac{6b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{d^3} \\
&- \frac{6b^3n^3\operatorname{PolyLog}\left(4,-\frac{ex}{d}\right)}{d^3}
\end{aligned}
$$

```
[Out] 3/2*b*e*n*x*(a+b*ln(c*x^n))^2/d^3/(e*x+d)-1/2*(a+b*ln(c*x^n))^3/d^3+1/2*(a+
b*ln(c*x^n))^3/d/(e*x+d)^2-e*x*(a+b*ln(c*x^n))^3/d^3/(e*x+d)+1/4*(a+b*ln(c*
x^n))^4/b/d^3/n-3*b^2*n^2*(a+b*ln(c*x^n))*ln(1+e*x/d)/d^3+9/2*b*n*(a+b*ln(c
*x^n))^2*ln(1+e*x/d)/d^3-(a+b*ln(c*x^n))^3*ln(1+e*x/d)/d^3-3*b^3*n^3*polylo
```

```
g(2,-e*x/d)/d^3+9*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-e*x/d)/d^3-3*b*n*(a+b*
ln(c*x^n))^2*polylog(2,-e*x/d)/d^3-9*b^3*n^3*polylog(3,-e*x/d)/d^3+6*b^2*n^
2*(a+b*ln(c*x^n))*polylog(3,-e*x/d)/d^3-6*b^3*n^3*polylog(4,-e*x/d)/d^3
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.13, number of steps used = 18, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2389, 2379, 2421, 2430, 6724, 2355, 2354, 2356, 2438}

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^3}\,dx = -\frac{3b^2n^2\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)(a+b\log(cx^n))}{d^3} + \frac{6b^2n^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)(a+b\log(cx^n))}{d^3} + \frac{6b^2n^2\operatorname{PolyLog}\left(3,-\frac{d}{ex}\right)(a+b\log(cx^n))}{d^3} - \frac{3b^2n^2\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))}{d^3} + \frac{3bn\operatorname{PolyLog}\left(2,-\frac{d}{ex}\right)(a+b\log(cx^n))^2}{d^3} + \frac{3bn\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))^2}{2d^3} + \frac{3benx(a+b\log(cx^n))^2}{2d^3(d+ex)} + \frac{3bn\log\left(\frac{ex}{d}+1\right)(a+b\log(cx^n))^2}{d^3} - \frac{\log\left(\frac{d}{ex}+1\right)(a+b\log(cx^n))^3}{d^3} - \frac{ex(a+b\log(cx^n))^3}{d^3(d+ex)} + \frac{(a+b\log(cx^n))^3}{2d(d+ex)^2} - \frac{3b^3n^3\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^3} - \frac{3b^3n^3\operatorname{PolyLog}\left(3,-\frac{d}{ex}\right)}{d^3} - \frac{6b^3n^3\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{d^3} + \frac{6b^3n^3\operatorname{PolyLog}\left(4,-\frac{d}{ex}\right)}{d^3}$$

```
[In] Int[(a + b*Log[c*x^n])^3/(x*(d + e*x)^3),x]
```

```
[Out] (3*b*e*n*x*(a + b*Log[c*x^n])^2)/(2*d^3*(d + e*x)) + (3*b*n*Log[1 + d/(e*x)
]*(a + b*Log[c*x^n])^2)/(2*d^3) + (a + b*Log[c*x^n])^3/(2*d*(d + e*x)^2) -
(e*x*(a + b*Log[c*x^n])^3)/(d^3*(d + e*x)) - (Log[1 + d/(e*x)]*(a + b*Log[c
*x^n])^3)/d^3 - (3*b^2*n^2*(a + b*Log[c*x^n])*Log[1 + (e*x)/d])/d^3 + (3*b*
n*(a + b*Log[c*x^n])^2*Log[1 + (e*x)/d])/d^3 - (3*b^2*n^2*(a + b*Log[c*x^n]
)*PolyLog[2, -(d/(e*x))])/d^3 + (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(d/
(e*x))])/d^3 - (3*b^3*n^3*PolyLog[2, -((e*x)/d)])/d^3 + (6*b^2*n^2*(a + b*L
og[c*x^n])*PolyLog[2, -((e*x)/d)])/d^3 - (3*b^3*n^3*PolyLog[3, -(d/(e*x))])
```

```
/d^3 + (6*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(d/(e*x))])/d^3 - (6*b^3*n
^3*PolyLog[3, -((e*x)/d)])/d^3 + (6*b^3*n^3*PolyLog[4, -(d/(e*x))])/d^3
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^2}\,dx}{d} - \frac{e\int \frac{(a+b\log(cx^n))^3}{(d+ex)^3}\,dx}{d}\\
&= \frac{(a+b\log(cx^n))^3}{2d(d+ex)^2} + \frac{\int \frac{(a+b\log(cx^n))^3}{x(d+ex)}\,dx}{d^2} - \frac{e\int \frac{(a+b\log(cx^n))^3}{(d+ex)^2}\,dx}{d^2} - \frac{(3bn)\int \frac{(a+b\log(cx^n))^2}{x(d+ex)^2}\,dx}{2d}\\
&= \frac{(a+b\log(cx^n))^3}{2d(d+ex)^2} - \frac{ex(a+b\log(cx^n))^3}{d^3(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^3}{d^3}\\
&\quad + \frac{(3bn)\int \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{x}\,dx}{d^3} - \frac{(3bn)\int \frac{(a+b\log(cx^n))^2}{x(d+ex)}\,dx}{2d^2}\\
&\quad + \frac{(3ben)\int \frac{(a+b\log(cx^n))^2}{d+ex}\,dx}{d^3} + \frac{(3ben)\int \frac{(a+b\log(cx^n))^2}{(d+ex)^2}\,dx}{2d^2}\\
&= \frac{3benx(a+b\log(cx^n))^2}{2d^3(d+ex)} + \frac{3bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{2d^3}\\
&\quad + \frac{(a+b\log(cx^n))^3}{2d(d+ex)^2} - \frac{ex(a+b\log(cx^n))^3}{d^3(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^3}{d^3}\\
&\quad + \frac{3bn(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{d^3} + \frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^3}\\
&\quad - \frac{(3b^2n^2)\int \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))}{x}\,dx}{d^3} - \frac{(6b^2n^2)\int \frac{(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{d^3}\\
&\quad - \frac{(6b^2n^2)\int \frac{(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{x}\,dx}{d^3} - \frac{(3b^2en^2)\int \frac{a+b\log(cx^n)}{d+ex}\,dx}{d^3}
\end{aligned}
$$

$$= \frac{3benx(a+b\log(cx^n))^2}{2d^3(d+ex)} + \frac{3bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{2d^3} + \frac{(a+b\log(cx^n))^3}{2d(d+ex)^2} - \frac{ex(a+b\log(cx^n))^3}{d^3(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^3}{d^3} - \frac{3b^2n^2(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^3} + \frac{3bn(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{d^3} - \frac{3b^2n^2(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^3} + \frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^3} + \frac{6b^2n^2(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{d^3} + \frac{6b^2n^2(a+b\log(cx^n))\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{d^3} + \frac{(3b^3n^3)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{d^3} + \frac{(3b^3n^3)\int\frac{\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{x}\,dx}{d^3} - \frac{(6b^3n^3)\int\frac{\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{x}\,dx}{d^3} - \frac{(6b^3n^3)\int\frac{\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{x}\,dx}{d^3}$$

$$= \frac{3benx(a+b\log(cx^n))^2}{2d^3(d+ex)} + \frac{3bn\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^2}{2d^3} + \frac{(a+b\log(cx^n))^3}{2d(d+ex)^2} - \frac{ex(a+b\log(cx^n))^3}{d^3(d+ex)} - \frac{\log\left(1+\frac{d}{ex}\right)(a+b\log(cx^n))^3}{d^3} - \frac{3b^2n^2(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^3} + \frac{3bn(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{d^3} - \frac{3b^2n^2(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^3} + \frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_2\left(-\frac{d}{ex}\right)}{d^3} - \frac{3b^3n^3\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{d^3} + \frac{6b^2n^2(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{ex}{d}\right)}{d^3} - \frac{3b^3n^3\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{d^3} + \frac{6b^2n^2(a+b\log(cx^n))\operatorname{Li}_3\left(-\frac{d}{ex}\right)}{d^3} - \frac{6b^3n^3\operatorname{Li}_3\left(-\frac{ex}{d}\right)}{d^3} + \frac{6b^3n^3\operatorname{Li}_4\left(-\frac{d}{ex}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^3}\,dx = \frac{2d^2(a-bn\log(x)+b\log(cx^n))^3 + 4d(d+ex)(a-bn\log(x)+b\log(cx^n))^3 + 4(d+ex)^2\log(x)(a-bn\log}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])^3/(x*(d + e*x)^3),x]
```

```
[Out] (2*d^2*(a - b*n*Log[x] + b*Log[c*x^n])^3 + 4*d*(d + e*x)*(a - b*n*Log[x] +
b*Log[c*x^n])^3 + 4*(d + e*x)^2*Log[x]*(a - b*n*Log[x] + b*Log[c*x^n])^3 -
4*(d + e*x)^2*(a - b*n*Log[x] + b*Log[c*x^n])^3*Log[d + e*x] + 6*b*n*(a - b
```

```
*n*Log[x] + b*Log[c*x^n])^2*((d + e*x)^2*Log[x]^2 + (d + e*x)*(-d + 3*(d +
e*x)*Log[d + e*x]) - Log[x]*(e*x*(4*d + 3*e*x) + 2*(d + e*x)^2*Log[1 + (e*x
)/d]) - 2*(d + e*x)^2*PolyLog[2, -((e*x)/d)]) + 2*b^2*n^2*(a - b*n*Log[x] +
 b*Log[c*x^n])*(-3*e*x*(2*d + e*x)*Log[x]^2 + 2*(d + e*x)^2*Log[x]^3 - 6*(d
 + e*x)^2*Log[d + e*x] + 6*(d + e*x)*Log[x]*(e*x + (d + e*x)*Log[1 + (e*x)/
d]) + 6*(d + e*x)^2*PolyLog[2, -((e*x)/d)] - 6*(d + e*x)*(Log[x]*(e*x*Log[x
] - 2*(d + e*x)*Log[1 + (e*x)/d]) - 2*(d + e*x)*PolyLog[2, -((e*x)/d)]) - 6
*(d + e*x)^2*(Log[x]^2*Log[1 + (e*x)/d] + 2*Log[x]*PolyLog[2, -((e*x)/d)] -
 2*PolyLog[3, -((e*x)/d)])) + b^3*n^3*((d + e*x)^2*Log[x]^4 - 4*(d + e*x)*(
Log[x]^2*(e*x*Log[x] - 3*(d + e*x)*Log[1 + (e*x)/d]) - 6*(d + e*x)*Log[x]*P
olyLog[2, -((e*x)/d)] + 6*(d + e*x)*PolyLog[3, -((e*x)/d)]) - 2*(Log[x]*(e*
x*(2*d + e*x)*Log[x]^2 + 6*(d + e*x)^2*Log[1 + (e*x)/d] - 3*(d + e*x)*Log[x
]*(e*x + (d + e*x)*Log[1 + (e*x)/d])) - 6*(d + e*x)^2*(-1 + Log[x])*PolyLog
[2, -((e*x)/d)] + 6*(d + e*x)^2*PolyLog[3, -((e*x)/d)]) - 4*(d + e*x)^2*(Lo
g[x]^3*Log[1 + (e*x)/d] + 3*Log[x]^2*PolyLog[2, -((e*x)/d)] - 6*Log[x]*Poly
Log[3, -((e*x)/d)] + 6*PolyLog[4, -((e*x)/d)])))/(4*d^3*(d + e*x)^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.63 (sec) , antiderivative size = 1607, normalized size of antiderivative = 4.45

method	result	size
risch	Expression too large to display	1607

```
[In] int((a+b*ln(c*x^n))^3/x/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -6*b^3/d^3*ln(x)*ln(x^n)*ln(e*x+d)*ln(-e*x/d)*n^2+3/4*(-I*b*Pi*csgn(I*c)*cs
gn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)
*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*b*(-ln(x^n)/d^3*ln
(e*x+d)+ln(x^n)/d^2/(e*x+d)+1/2*ln(x^n)/d/(e*x+d)^2+ln(x^n)/d^3*ln(x)-1/2*n
*(1/d^2/(e*x+d)-3/d^3*ln(e*x+d)+3/d^3*ln(x)+1/d^3*ln(x)^2-2/d^3*ln(e*x+d)*l
n(-e*x/d)-2/d^3*dilog(-e*x/d)))+3/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c
*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I
*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b^2*(-ln(x^n)^2/d^3*ln(e*x+d)+ln(x^n)^
2/d^2/(e*x+d)+1/2*ln(x^n)^2/d/(e*x+d)^2+ln(x^n)^2/d^3*ln(x)-n*(1/d^2*(ln(x^
n)/(e*x+d)-3*ln(x^n)/d*ln(e*x+d)+3*ln(x^n)/d*ln(x)-n*(-1/d*ln(e*x+d)+1/d*ln
(x)+3/2/d*ln(x)^2-3/d*ln(e*x+d)*ln(-e*x/d)-3/d*dilog(-e*x/d)))+1/d^3*ln(x^n
)*ln(x)^2-1/3/d^3*ln(x)^3*n-2/d^3*((ln(x^n)-n*ln(x))*(dilog(-e*x/d)+ln(e*x+
d)*ln(-e*x/d))+n*(1/2*ln(e*x+d)*ln(x)^2-1/2*ln(x)^2*ln(1+e*x/d)-ln(x)*polyl
og(2,-e*x/d)+polylog(3,-e*x/d)))))+1/8*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(
I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^
2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^3*(-1/d^3*ln(e*x+d)+1/d^2/(e*x+d)+1
/2/d/(e*x+d)^2+1/d^3*ln(x))-9/2*b^3/d^3*n^3*ln(e*x+d)*ln(x)^2-3/2*b^3/d^3*n
^3*ln(x)^2+3*b^3/d^3*n^3*dilog(-e*x/d)-3/2*b^3/d^3*ln(x)^3*n^3-1/4*b^3/d^3*
```

```
ln(x)^4*n^3-b^3*ln(x^n)^3/d^3*ln(e*x+d)+b^3*ln(x^n)^3/d^2/(e*x+d)+1/2*b^3*l
n(x^n)^3/d/(e*x+d)^2+b^3*ln(x^n)^3/d^3*ln(x)+9*b^3/d^3*ln(x)*ln(e*x+d)*ln(-
e*x/d)*n^3-9*b^3/d^3*n^2*ln(x^n)*ln(e*x+d)*ln(-e*x/d)+3*b^3/d^3*ln(x)^2*ln(
e*x+d)*ln(-e*x/d)*n^3-6*b^3/d^3*ln(x)*ln(x^n)*dilog(-e*x/d)*n^2+3*b^3/d^3*n
^2*ln(x)^2*ln(x^n)*ln(e*x+d)-3*b^3/d^3*n^2*ln(x)^2*ln(x^n)*ln(1+e*x/d)-6*b^
3/d^3*n^2*ln(x)*ln(x^n)*polylog(2,-e*x/d)+3*b^3*n/d^3*ln(x^n)^2*ln(e*x+d)*l
n(-e*x/d)+9/2*b^3/d^3*n^3*ln(x)^2*ln(1+e*x/d)+9*b^3/d^3*n^3*ln(x)*polylog(2
,-e*x/d)+9/2*b^3/d^3*n^2*ln(x^n)*ln(x)^2-3/2*b^3*n/d^3*ln(x^n)^2*ln(x)^2+9/
2*b^3*n*ln(x^n)^2/d^3*ln(e*x+d)-3/2*b^3*n*ln(x^n)^2/d^2/(e*x+d)-9/2*b^3*n*l
n(x^n)^2/d^3*ln(x)+3*b^3/d^3*ln(x)^2*dilog(-e*x/d)*n^3-2*b^3/d^3*n^3*ln(e*x
+d)*ln(x)^3+2*b^3/d^3*n^3*ln(x)^3*ln(1+e*x/d)+3*b^3/d^3*n^3*ln(x)^2*polylog
(2,-e*x/d)+6*b^3/d^3*n^2*ln(x^n)*polylog(3,-e*x/d)+b^3/d^3*n^2*ln(x^n)*ln(x
)^3+3*b^3*n/d^3*ln(x^n)^2*dilog(-e*x/d)-3*b^3/d^3*n^2*ln(x^n)*ln(e*x+d)+3*b
^3/d^3*n^2*ln(x^n)*ln(x)+3*b^3/d^3*n^3*ln(e*x+d)*ln(-e*x/d)+9*b^3/d^3*ln(x)
*dilog(-e*x/d)*n^3-9*b^3/d^3*n^2*ln(x^n)*dilog(-e*x/d)-9*b^3*n^3*polylog(3,
-e*x/d)/d^3-6*b^3*n^3*polylog(4,-e*x/d)/d^3
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)^3}{(ex+d)^3x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^3/x/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^3}\,dx = \int \frac{(a+b\log(cx^n))^3}{x(d+ex)^3}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**3/x/(e*x+d)**3,x)
[Out] Integral((a + b*log(c*x**n))**3/(x*(d + e*x)**3), x)
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)^3}{(ex+d)^3x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^3/x/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*((2*e*x + 3*d)/(d^2*e^2*x^2 + 2*d^3*e*x + d^4) - 2*log(e*x + d)/d^3
 + 2*log(x)/d^3) + integrate((b^3*log(c)^3 + b^3*log(x^n)^3 + 3*a*b^2*log(c
)^2 + 3*a^2*b*log(c) + 3*(b^3*log(c) + a*b^2)*log(x^n)^2 + 3*(b^3*log(c)^2
+ 2*a*b^2*log(c) + a^2*b)*log(x^n))/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 +
d^3*x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^3}\,dx = \int \frac{(b\log(cx^n)+a)^3}{(ex+d)^3x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^3/x/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3/((e*x + d)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3}{x(d+ex)^3}\,dx = \int \frac{(a+b\ln(c\,x^n))^3}{x\,(d+e\,x)^3}\,dx$$

```
[In] int((a + b*log(c*x^n))^3/(x*(d + e*x)^3),x)
```

```
[Out] int((a + b*log(c*x^n))^3/(x*(d + e*x)^3), x)
```

3.124 $\int (d+ex)\sqrt{a+b\log(cx^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 189

$$\int (d+ex)\sqrt{a+b\log(cx^n)}\,dx = -\frac{1}{2}\sqrt{b}de^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}x(cx^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right) - \frac{1}{4}\sqrt{b}ee^{-\frac{2a}{bn}}\sqrt{n}\sqrt{\frac{\pi}{2}}x^2(cx^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right) + dx\sqrt{a+b\log(cx^n)} + \frac{1}{2}ex^2\sqrt{a+b\log(cx^n)}$$

```
[Out] -1/8*e*x^2*erfi(2^(1/2)*(a+b*ln(c*x^n))^(1/2)/b^(1/2)/n^(1/2))*b^(1/2)*n^(1
/2)*2^(1/2)*Pi^(1/2)/exp(2*a/b/n)/((c*x^n)^(2/n))-1/2*d*x*erfi((a+b*ln(c*x^
n))^(1/2)/b^(1/2)/n^(1/2))*b^(1/2)*n^(1/2)*Pi^(1/2)/exp(a/b/n)/((c*x^n)^(1/
n))+d*x*(a+b*ln(c*x^n))^(1/2)+1/2*e*x^2*(a+b*ln(c*x^n))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2367, 2333, 2337, 2211, 2235, 2342, 2347}

$$\int (d+ex)\sqrt{a+b\log(cx^n)}\,dx = -\frac{1}{2}\sqrt{\pi}\sqrt{b}d\sqrt{n}xe^{-\frac{a}{bn}}(cx^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right) + dx\sqrt{a+b\log(cx^n)} - \frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e\sqrt{n}x^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right) + \frac{1}{2}ex^2\sqrt{a+b\log(cx^n)}$$

```
[In] Int[(d + e*x)*Sqrt[a + b*Log[c*x^n]],x]
```

```
[Out] -1/2*(Sqrt[b]*d*Sqrt[n]*Sqrt[Pi]*x*Erfi[Sqrt[a + b*Log[c*x^n]]/(Sqrt[b]*Sqr
t[n])])/(E^(a/(b*n))*(c*x^n)^n^(-1)) - (Sqrt[b]*e*Sqrt[n]*Sqrt[Pi/2]*x^2*Er
fi[(Sqrt[2]*Sqrt[a + b*Log[c*x^n]])/(Sqrt[b]*Sqrt[n])])/(4*E^((2*a)/(b*n))*
(c*x^n)^(2/n)) + d*x*Sqrt[a + b*Log[c*x^n]] + (e*x^2*Sqrt[a + b*Log[c*x^n]]
)/2
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
```

```
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(d\sqrt{a+b\log\left(cx^n\right)}+ex\sqrt{a+b\log\left(cx^n\right)}\right)\,dx\\
&= d\int \sqrt{a+b\log\left(cx^n\right)}\,dx+e\int x\sqrt{a+b\log\left(cx^n\right)}\,dx\\
&= dx\sqrt{a+b\log\left(cx^n\right)}+\frac{1}{2}ex^2\sqrt{a+b\log\left(cx^n\right)}\\
&\quad -\frac{1}{2}(bdn)\int \frac{1}{\sqrt{a+b\log\left(cx^n\right)}}\,dx-\frac{1}{4}(ben)\int \frac{x}{\sqrt{a+b\log\left(cx^n\right)}}\,dx\\
&= dx\sqrt{a+b\log\left(cx^n\right)}+\frac{1}{2}ex^2\sqrt{a+b\log\left(cx^n\right)}\\
&\quad -\frac{1}{4}\left(bex^2\left(cx^n\right)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(cx^n\right)\right)\\
&\quad -\frac{1}{2}\left(bdx\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(cx^n\right)\right)\\
&= dx\sqrt{a+b\log\left(cx^n\right)}+\frac{1}{2}ex^2\sqrt{a+b\log\left(cx^n\right)}\\
&\quad -\frac{1}{2}\left(ex^2\left(cx^n\right)^{-2/n}\right)\text{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx,x,\sqrt{a+b\log\left(cx^n\right)}\right)\\
&\quad -\left(dx\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx,x,\sqrt{a+b\log\left(cx^n\right)}\right)\\
&= -\frac{1}{2}\sqrt{b}de^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}x\left(cx^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log\left(cx^n\right)}}{\sqrt{b}\sqrt{n}}\right)\\
&\quad -\frac{1}{4}\sqrt{b}ee^{-\frac{2a}{bn}}\sqrt{n}\sqrt{\frac{\pi}{2}}x^2\left(cx^n\right)^{-2/n}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log\left(cx^n\right)}}{\sqrt{b}\sqrt{n}}\right)\\
&\quad +dx\sqrt{a+b\log\left(cx^n\right)}+\frac{1}{2}ex^2\sqrt{a+b\log\left(cx^n\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.89

$$\int (d+ex)\sqrt{a+b\log(cx^n)}\,dx$$
$$= \frac{1}{8}x\left(-4\sqrt{b}de^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}(cx^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)\right.$$
$$\left.-\sqrt{b}ee^{-\frac{2a}{bn}}\sqrt{n}\sqrt{2\pi}x(cx^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)+4(2d+ex)\sqrt{a+b\log(cx^n)}\right)$$

```
[In] Integrate[(d + e*x)*Sqrt[a + b*Log[c*x^n]],x]
```

```
[Out] (x*((-4*Sqrt[b]*d*Sqrt[n]*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*x^n]]/(Sqrt[b]*Sqr
t[n])])/(E^(a/(b*n))*(c*x^n)^n^(-1)) - (Sqrt[b]*e*Sqrt[n]*Sqrt[2*Pi]*x*Erfi
[(Sqrt[2]*Sqrt[a + b*Log[c*x^n]])/(Sqrt[b]*Sqrt[n])])/(E^((2*a)/(b*n))*(c*x
^n)^(2/n)) + 4*(2*d + e*x)*Sqrt[a + b*Log[c*x^n]]))/8
```

Maple [F]

$$\int (ex+d)\sqrt{a+b\ln(c\,x^n)}dx$$

```
[In] int((e*x+d)*(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] int((e*x+d)*(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (d+ex)\sqrt{a+b\log(cx^n)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (d+ex)\sqrt{a+b\log{(cx^n)}}\,dx = \int \sqrt{a+b\log{(cx^n)}}(d+ex)\ dx$$

```
[In] integrate((e*x+d)*(a+b*ln(c*x**n))**(1/2),x)
[Out] Integral(sqrt(a + b*log(c*x**n))*(d + e*x), x)
```

Maxima [F]

$$\int (d+ex)\sqrt{a+b\log{(cx^n)}}\,dx = \int (ex+d)\sqrt{b\log{(cx^n)}+a}\,dx$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)*sqrt(b*log(c*x^n) + a), x)
```

Giac [F]

$$\int (d+ex)\sqrt{a+b\log{(cx^n)}}\,dx = \int (ex+d)\sqrt{b\log{(cx^n)}+a}\,dx$$

```
[In] integrate((e*x+d)*(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)*sqrt(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\sqrt{a+b\log{(cx^n)}}\,dx = \int \sqrt{a+b\,\ln{(c\,x^n)}}\,(d+e\,x)\ dx$$

```
[In] int((a + b*log(c*x^n))^(1/2)*(d + e*x),x)
[Out] int((a + b*log(c*x^n))^(1/2)*(d + e*x), x)
```

3.125 $\int (d+ex)^2 \sqrt{a+b\log(cx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 298

$$\begin{aligned}\int (d+ex)^2\sqrt{a+b\log(cx^n)}\,dx &= -\frac{1}{2}\sqrt{b}d^2e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}x(cx^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)\\ &\quad -\frac{1}{2}\sqrt{b}dee^{-\frac{2a}{bn}}\sqrt{n}\sqrt{\frac{\pi}{2}}x^2(cx^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)\\ &\quad -\frac{1}{6}\sqrt{b}e^2e^{-\frac{3a}{bn}}\sqrt{n}\sqrt{\frac{\pi}{3}}x^3(cx^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)+d^2x\sqrt{a+b\log(cx^n)}+dex^2\sqrt{a+b\log(cx^n)}\end{aligned}$$

```
[Out] -1/18*e^2*x^3*erfi(3^(1/2)*(a+b*ln(c*x^n))^(1/2)/b^(1/2)/n^(1/2))*b^(1/2)*n
^(1/2)*3^(1/2)*Pi^(1/2)/exp(3*a/b/n)/((c*x^n)^(3/n))-1/4*d*e*x^2*erfi(2^(1/
2)*(a+b*ln(c*x^n))^(1/2)/b^(1/2)/n^(1/2))*b^(1/2)*n^(1/2)*2^(1/2)*Pi^(1/2)/
exp(2*a/b/n)/((c*x^n)^(2/n))-1/2*d^2*x*erfi((a+b*ln(c*x^n))^(1/2)/b^(1/2)/n
^(1/2))*b^(1/2)*n^(1/2)*Pi^(1/2)/exp(a/b/n)/((c*x^n)^(1/n))+d^2*x*(a+b*ln(c
*x^n))^(1/2)+d*e*x^2*(a+b*ln(c*x^n))^(1/2)+1/3*e^2*x^3*(a+b*ln(c*x^n))^(1/2
)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used

$= \{2367, 2333, 2337, 2211, 2235, 2342, 2347\}$

$$\int (d+ex)^2 \sqrt{a+b\log(cx^n)}\,dx$$
$$= -\frac{1}{2}\sqrt{\pi}\sqrt{b}d^2\sqrt{n}xe^{-\frac{a}{bn}}(cx^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right) + d^2x\sqrt{a+b\log(cx^n)}$$
$$- \frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}de\sqrt{n}x^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right) + dex^2\sqrt{a+b\log(cx^n)}$$
$$- \frac{1}{6}\sqrt{\frac{\pi}{3}}\sqrt{b}e^2\sqrt{n}x^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right) + \frac{1}{3}e^2x^3\sqrt{a+b\log(cx^n)}$$

```
[In] Int[(d + e*x)^2*Sqrt[a + b*Log[c*x^n]],x]
```

```
[Out] -1/2*(Sqrt[b]*d^2*Sqrt[n]*Sqrt[Pi]*x*Erfi[Sqrt[a + b*Log[c*x^n]]/(Sqrt[b]*S
qrt[n])])/(E^(a/(b*n))*(c*x^n)^n^(-1)) - (Sqrt[b]*d*e*Sqrt[n]*Sqrt[Pi/2]*x^
2*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*x^n]])/(Sqrt[b]*Sqrt[n])])/(2*E^((2*a)/(b*
n))*(c*x^n)^(2/n)) - (Sqrt[b]*e^2*Sqrt[n]*Sqrt[Pi/3]*x^3*Erfi[(Sqrt[3]*Sqrt
[a + b*Log[c*x^n]])/(Sqrt[b]*Sqrt[n])])/(6*E^((3*a)/(b*n))*(c*x^n)^(3/n)) +
 d^2*x*Sqrt[a + b*Log[c*x^n]] + d*e*x^2*Sqrt[a + b*Log[c*x^n]] + (e^2*x^3*S
qrt[a + b*Log[c*x^n]])/3
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rubi steps

$$\text{integral} = \int \left(d^2\sqrt{a+b\log(cx^n)}+2dex\sqrt{a+b\log(cx^n)}+e^2x^2\sqrt{a+b\log(cx^n)}\right)dx$$

$$= d^2\int\sqrt{a+b\log(cx^n)}\,dx+(2de)\int x\sqrt{a+b\log(cx^n)}\,dx+e^2\int x^2\sqrt{a+b\log(cx^n)}\,dx$$

$$= d^2x\sqrt{a+b\log(cx^n)}+dex^2\sqrt{a+b\log(cx^n)} + \frac{1}{3}e^2x^3\sqrt{a+b\log(cx^n)}-\frac{1}{2}(bd^2n)\int\frac{1}{\sqrt{a+b\log(cx^n)}}\,dx - \frac{1}{2}(bden)\int\frac{x}{\sqrt{a+b\log(cx^n)}}\,dx-\frac{1}{6}(be^2n)\int\frac{x^2}{\sqrt{a+b\log(cx^n)}}\,dx$$

$$= d^2x\sqrt{a+b\log(cx^n)}+dex^2\sqrt{a+b\log(cx^n)}+\frac{1}{3}e^2x^3\sqrt{a+b\log(cx^n)} - \frac{1}{6}\left(be^2x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{n}}}{\sqrt{a+bx}}\,dx,x,\log(cx^n)\right) - \frac{1}{2}\left(bdex^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx,x,\log(cx^n)\right) - \frac{1}{2}\left(bd^2x(cx^n)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx,x,\log(cx^n)\right)$$

$$
\begin{aligned}
&= d^2x\sqrt{a+b\log(cx^n)} + dex^2\sqrt{a+b\log(cx^n)} + \frac{1}{3}e^2x^3\sqrt{a+b\log(cx^n)} \\
&\quad - \frac{1}{3}\left(e^2x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int e^{-\frac{3a}{bn}+\frac{3x^2}{bn}}\,dx, x, \sqrt{a+b\log(cx^n)}\right) \\
&\quad - \left(dex^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log(cx^n)}\right) \\
&\quad - \left(d^2x(cx^n)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx, x, \sqrt{a+b\log(cx^n)}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{2}\sqrt{b}d^2e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}x(cx^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right) \\
&\quad - \frac{1}{2}\sqrt{b}dee^{-\frac{2a}{bn}}\sqrt{n}\sqrt{\frac{\pi}{2}}x^2(cx^n)^{-2/n}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right) \\
&\quad - \frac{1}{6}\sqrt{b}e^2e^{-\frac{3a}{bn}}\sqrt{n}\sqrt{\frac{\pi}{3}}x^3(cx^n)^{-3/n}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right) + d^2x\sqrt{a+b\log(cx^n)} + dex^2\sqrt{a+b
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.96

$$
\int (d+ex)^2\sqrt{a+b\log(cx^n)}\,dx
$$

$$
\begin{aligned}
&= \frac{1}{36}x\left(-18\sqrt{b}d^2e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}(cx^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)\right. \\
&\quad -9\sqrt{b}dee^{-\frac{2a}{bn}}\sqrt{n}\sqrt{2\pi}x(cx^n)^{-2/n}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right) - 2\sqrt{b}e^2e^{-\frac{3a}{bn}}\sqrt{n}\sqrt{3\pi}x^2(cx^n)^{-3/n}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a}
\end{aligned}
$$

```
[In] Integrate[(d + e*x)^2*Sqrt[a + b*Log[c*x^n]],x]
```

```
[Out] (x*((-18*Sqrt[b]*d^2*Sqrt[n]*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[c*x^n]]/(Sqrt[b]*
Sqrt[n])])/(E^(a/(b*n))*(c*x^n)^n^(-1)) - (9*Sqrt[b]*d*e*Sqrt[n]*Sqrt[2*Pi]
*x*Erfi[(Sqrt[2]*Sqrt[a + b*Log[c*x^n]])/(Sqrt[b]*Sqrt[n])])/(E^((2*a)/(b*n
))*(c*x^n)^(2/n)) - (2*Sqrt[b]*e^2*Sqrt[n]*Sqrt[3*Pi]*x^2*Erfi[(Sqrt[3]*Sqr
t[a + b*Log[c*x^n]])/(Sqrt[b]*Sqrt[n])])/(E^((3*a)/(b*n))*(c*x^n)^(3/n)) +
36*d^2*Sqrt[a + b*Log[c*x^n]] + 36*d*e*x*Sqrt[a + b*Log[c*x^n]] + 12*e^2*x^
2*Sqrt[a + b*Log[c*x^n]]))/36
```

Maple [F]

$$\int (ex+d)^2 \sqrt{a+b\ln (c\,x^n)}dx$$

```
[In] int((e*x+d)^2*(a+b*ln(c*x^n))^(1/2),x)
[Out] int((e*x+d)^2*(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (d+ex)^2\sqrt{a+b\log (cx^n)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (d+ex)^2\sqrt{a+b\log (cx^n)}\,dx = \int \sqrt{a+b\log (cx^n)}(d+ex)^2\,dx$$

```
[In] integrate((e*x+d)**2*(a+b*ln(c*x**n))**(1/2),x)
[Out] Integral(sqrt(a + b*log(c*x**n))*(d + e*x)**2, x)
```

Maxima [F]

$$\int (d+ex)^2\sqrt{a+b\log (cx^n)}\,dx = \int (ex+d)^2\sqrt{b\log (cx^n)+a}\,dx$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^2*sqrt(b*log(c*x^n) + a), x)
```

Giac [F]

$$\int (d+ex)^2\sqrt{a+b\log(cx^n)}\,dx = \int (ex+d)^2\sqrt{b\log(cx^n)+a}\,dx$$

```
[In] integrate((e*x+d)^2*(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)^2*sqrt(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2\sqrt{a+b\log(cx^n)}\,dx = \int \sqrt{a+b\,\ln(c\,x^n)}\,(d+e\,x)^2\,dx$$

```
[In] int((a + b*log(c*x^n))^(1/2)*(d + e*x)^2,x)
[Out] int((a + b*log(c*x^n))^(1/2)*(d + e*x)^2, x)
```

3.126 $\int (d+ex)^3\sqrt{a+b\log(cx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 402

$$\int (d+ex)^3\sqrt{a+b\log(cx^n)}\,dx = -\frac{1}{2}\sqrt{b}d^3e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}x(cx^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)$$
$$-\frac{1}{16}\sqrt{b}e^3e^{-\frac{4a}{bn}}\sqrt{n}\sqrt{\pi}x^4(cx^n)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)$$
$$-\frac{3}{4}\sqrt{b}d^2ee^{-\frac{2a}{bn}}\sqrt{n}\sqrt{\frac{\pi}{2}}x^2(cx^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)-\frac{1}{2}\sqrt{b}de^2e^{-\frac{3a}{bn}}\sqrt{n}\sqrt{\frac{\pi}{3}}x^3(cx^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\cdots\right.$$

```
[Out] -1/6*d*e^2*x^3*erfi(3^(1/2)*(a+b*ln(c*x^n))^(1/2)/b^(1/2)/n^(1/2))*b^(1/2)*
n^(1/2)*3^(1/2)*Pi^(1/2)/exp(3*a/b/n)/((c*x^n)^(3/n))-3/8*d^2*e*x^2*erfi(2^
(1/2)*(a+b*ln(c*x^n))^(1/2)/b^(1/2)/n^(1/2))*b^(1/2)*n^(1/2)*2^(1/2)*Pi^(1/
2)/exp(2*a/b/n)/((c*x^n)^(2/n))-1/2*d^3*x*erfi((a+b*ln(c*x^n))^(1/2)/b^(1/2
)/n^(1/2))*b^(1/2)*n^(1/2)*Pi^(1/2)/exp(a/b/n)/((c*x^n)^(1/n))-1/16*e^3*x^4
*erfi(2*(a+b*ln(c*x^n))^(1/2)/b^(1/2)/n^(1/2))*b^(1/2)*n^(1/2)*Pi^(1/2)/exp
(4*a/b/n)/((c*x^n)^(4/n))+d^3*x*(a+b*ln(c*x^n))^(1/2)+3/2*d^2*e*x^2*(a+b*ln
(c*x^n))^(1/2)+d*e^2*x^3*(a+b*ln(c*x^n))^(1/2)+1/4*e^3*x^4*(a+b*ln(c*x^n))^
(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2367, 2333, 2337, 2211, 2235, 2342, 2347}

$$\int (d+ex)^3\sqrt{a+b\log(cx^n)}\,dx$$
$$= -\frac{1}{2}\sqrt{\pi}\sqrt{b}d^3\sqrt{n}xe^{-\frac{a}{bn}}(cx^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)+d^3x\sqrt{a+b\log(cx^n)}$$
$$-\frac{3}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}d^2e\sqrt{n}x^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)+\frac{3}{2}d^2ex^2\sqrt{a+b\log(cx^n)}$$
$$-\frac{1}{2}\sqrt{\frac{\pi}{3}}\sqrt{b}de^2\sqrt{n}x^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)+de^2x^3\sqrt{a+b\log(cx^n)}-\frac{1}{16}\sqrt{\pi}\sqrt{b}e^3\sqrt{n}x^4$$

```
[In] Int[(d + e*x)^3*Sqrt[a + b*Log[c*x^n]],x]
```

```
[Out] -1/2*(Sqrt[b]*d^3*Sqrt[n]*Sqrt[Pi]*x*Erfi[Sqrt[a + b*Log[c*x^n]]/(Sqrt[b]*S
qrt[n])])/(E^(a/(b*n))*(c*x^n)^n^(-1)) - (Sqrt[b]*e^3*Sqrt[n]*Sqrt[Pi]*x^4*
Erfi[(2*Sqrt[a + b*Log[c*x^n]])/(Sqrt[b]*Sqrt[n])])/(16*E^((4*a)/(b*n))*(c*
x^n)^(4/n)) - (3*Sqrt[b]*d^2*e*Sqrt[n]*Sqrt[Pi/2]*x^2*Erfi[(Sqrt[2]*Sqrt[a
+ b*Log[c*x^n]])/(Sqrt[b]*Sqrt[n])])/(4*E^((2*a)/(b*n))*(c*x^n)^(2/n)) - (S
qrt[b]*d*e^2*Sqrt[n]*Sqrt[Pi/3]*x^3*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*x^n]])/(
Sqrt[b]*Sqrt[n])])/(2*E^((3*a)/(b*n))*(c*x^n)^(3/n)) + d^3*x*Sqrt[a + b*Log
[c*x^n]] + (3*d^2*e*x^2*Sqrt[a + b*Log[c*x^n]])/2 + d*e^2*x^3*Sqrt[a + b*Lo
g[c*x^n]] + (e^3*x^4*Sqrt[a + b*Log[c*x^n]])/4
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rubi steps

$$\text{integral} = \int \left(d^3\sqrt{a+b\log(cx^n)} + 3d^2ex\sqrt{a+b\log(cx^n)} + 3de^2x^2\sqrt{a+b\log(cx^n)} + e^3x^3\sqrt{a+b\log(cx^n)}\right) dx$$

$$= d^3\int \sqrt{a+b\log(cx^n)}\,dx + \left(3d^2e\right)\int x\sqrt{a+b\log(cx^n)}\,dx + \left(3de^2\right)\int x^2\sqrt{a+b\log(cx^n)}\,dx + e^3\int x^3\sqrt{a+b\log(cx^n)}\,dx$$

$$= d^3x\sqrt{a+b\log(cx^n)} + \frac{3}{2}d^2ex^2\sqrt{a+b\log(cx^n)} + de^2x^3\sqrt{a+b\log(cx^n)} + \frac{1}{4}e^3x^4\sqrt{a+b\log(cx^n)} - \frac{1}{2}\left(bd^3n\right)\int \frac{1}{\sqrt{a+b\log(cx^n)}}\,dx - \frac{1}{4}\left(3bd^2en\right)\int \frac{x}{\sqrt{a+b\log(cx^n)}}\,dx - \frac{1}{2}\left(bde^2n\right)\int \frac{x^2}{\sqrt{a+b\log(cx^n)}}\,dx - \frac{1}{8}\left(be^3n\right)\int \frac{x^3}{\sqrt{a+b\log(cx^n)}}\,dx$$

$$= d^3x\sqrt{a+b\log(cx^n)} + \frac{3}{2}d^2ex^2\sqrt{a+b\log(cx^n)} + de^2x^3\sqrt{a+b\log(cx^n)}$$
$$+\frac{1}{4}e^3x^4\sqrt{a+b\log(cx^n)} - \frac{1}{8}\left(be^3x^4(cx^n)^{-4/n}\right)\text{Subst}\left(\int\frac{e^{\frac{4x}{n}}}{\sqrt{a+bx}}\,dx, x, \log(cx^n)\right)$$
$$-\frac{1}{2}\left(bde^2x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{n}}}{\sqrt{a+bx}}\,dx, x, \log(cx^n)\right)$$
$$-\frac{1}{4}\left(3bd^2ex^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{\sqrt{a+bx}}\,dx, x, \log(cx^n)\right) - \frac{1}{2}\left(bd^3x(cx^n)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\right.$$

$$= d^3x\sqrt{a+b\log(cx^n)} + \frac{3}{2}d^2ex^2\sqrt{a+b\log(cx^n)}$$
$$+ de^2x^3\sqrt{a+b\log(cx^n)} + \frac{1}{4}e^3x^4\sqrt{a+b\log(cx^n)}$$
$$-\frac{1}{4}\left(e^3x^4(cx^n)^{-4/n}\right)\text{Subst}\left(\int e^{-\frac{4a}{bn}+\frac{4x^2}{bn}}\,dx, x, \sqrt{a+b\log(cx^n)}\right)$$
$$-\left(de^2x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int e^{-\frac{3a}{bn}+\frac{3x^2}{bn}}\,dx, x, \sqrt{a+b\log(cx^n)}\right)$$
$$-\frac{1}{2}\left(3d^2ex^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int e^{-\frac{2a}{bn}+\frac{2x^2}{bn}}\,dx, x, \sqrt{a+b\log(cx^n)}\right) - \left(d^3x(cx^n)^{-1/n}\right)\text{Subst}\left(\int e\right.$$

$$= -\frac{1}{2}\sqrt{b}d^3e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}x(cx^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)$$
$$-\frac{1}{16}\sqrt{b}e^3e^{-\frac{4a}{bn}}\sqrt{n}\sqrt{\pi}x^4(cx^n)^{-4/n}\,\text{erfi}\left(\frac{2\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)$$
$$-\frac{3}{4}\sqrt{b}d^2ee^{-\frac{2a}{bn}}\sqrt{n}\sqrt{\frac{\pi}{2}}x^2(cx^n)^{-2/n}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right) - \frac{1}{2}\sqrt{b}de^2e^{-\frac{3a}{bn}}\sqrt{n}\sqrt{\frac{\pi}{3}}x^3(cx^n)^{-3/n}\,\text{e}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.91

$$\int(d+ex)^3\sqrt{a+b\log(cx^n)}\,dx$$
$$= \frac{1}{48}e^{-\frac{4a}{bn}}x(cx^n)^{-4/n}\left(-24\sqrt{b}d^3e^{\frac{3a}{bn}}\sqrt{n}\sqrt{\pi}(cx^n)^{3/n}\,\text{erfi}\left(\frac{\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right) - 3\sqrt{b}e^3\sqrt{n}\sqrt{\pi}x^3\text{erfi}\left(\frac{2\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right.\right.$$

```
[In] Integrate[(d + e*x)^3*Sqrt[a + b*Log[c*x^n]],x]
```

```
[Out] (x*(-24*Sqrt[b]*d^3*E^((3*a)/(b*n))*Sqrt[n]*Sqrt[Pi]*(c*x^n)^(3/n)*Erfi[Sqr
t[a + b*Log[c*x^n]]/(Sqrt[b]*Sqrt[n])] - 3*Sqrt[b]*e^3*Sqrt[n]*Sqrt[Pi]*x^3
*Erfi[(2*Sqrt[a + b*Log[c*x^n]])/(Sqrt[b]*Sqrt[n])] + 2*E^(a/(b*n))*(c*x^n)
```

```
^n^(-1)*(-9*Sqrt[b]*d^2*e*E^(a/(b*n))*Sqrt[n]*Sqrt[2*Pi]*x*(c*x^n)^n^(-1)*E
rfi[(Sqrt[2]*Sqrt[a + b*Log[c*x^n]])/(Sqrt[b]*Sqrt[n])] - 4*Sqrt[b]*d*e^2*S
qrt[n]*Sqrt[3*Pi]*x^2*Erfi[(Sqrt[3]*Sqrt[a + b*Log[c*x^n]])/(Sqrt[b]*Sqrt[n
])] + 6*E^((3*a)/(b*n))*(c*x^n)^(3/n)*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^
3*x^3)*Sqrt[a + b*Log[c*x^n]]))/(48*E^((4*a)/(b*n))*(c*x^n)^(4/n))
```

Maple [F]

$$\int \left(e x +d \right)^{3} \sqrt{a +b \ln \left(c \,x^{n}\right)}d x$$

```
[In] int((e*x+d)^3*(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] int((e*x+d)^3*(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (d+ex)^3\sqrt{a+b\log(cx^n)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (d+ex)^3\sqrt{a+b\log(cx^n)}\,dx = \int \sqrt{a+b\log(cx^n)}(d+ex)^3\,dx$$

```
[In] integrate((e*x+d)**3*(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*log(c*x**n))*(d + e*x)**3, x)
```

Maxima [F]

$$\int (d+ex)^3\sqrt{a+b\log(cx^n)}\,dx=\int (ex+d)^3\sqrt{b\log(cx^n)+a}\,dx$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^3*sqrt(b*log(c*x^n) + a), x)
```

Giac [F]

$$\int (d+ex)^3\sqrt{a+b\log(cx^n)}\,dx=\int (ex+d)^3\sqrt{b\log(cx^n)+a}\,dx$$

```
[In] integrate((e*x+d)^3*(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)^3*sqrt(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3\sqrt{a+b\log(cx^n)}\,dx=\int \sqrt{a+b\,\ln(c\,x^n)}\,(d+e\,x)^3\,dx$$

```
[In] int((a + b*log(c*x^n))^(1/2)*(d + e*x)^3,x)
[Out] int((a + b*log(c*x^n))^(1/2)*(d + e*x)^3, x)
```

3.127 $\int \frac{\sqrt{a+b\log(cx^n)}}{d+ex}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{a+b\log(cx^n)}}{d+ex}\,dx = \text{Int}\left(\frac{\sqrt{a+b\log(cx^n)}}{d+ex},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*x^n))^(1/2)/(e*x+d),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\log(cx^n)}}{d+ex}\,dx = \int \frac{\sqrt{a+b\log(cx^n)}}{d+ex}\,dx$$

```
[In] Int[Sqrt[a + b*Log[c*x^n]]/(d + e*x),x]
[Out] Defer[Int][Sqrt[a + b*Log[c*x^n]]/(d + e*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{a+b\log(cx^n)}}{d+ex}\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+b\log(cx^n)}}{d+ex}\,dx=\int \frac{\sqrt{a+b\log(cx^n)}}{d+ex}\,dx$$

```
[In] Integrate[Sqrt[a + b*Log[c*x^n]]/(d + e*x),x]
[Out] Integrate[Sqrt[a + b*Log[c*x^n]]/(d + e*x), x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\ln(c\,x^n)}}{ex+d}dx$$

```
[In] int((a+b*ln(c*x^n))^(1/2)/(e*x+d),x)
[Out] int((a+b*ln(c*x^n))^(1/2)/(e*x+d),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log(cx^n)}}{d+ex}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*x^n))^(1/2)/(e*x+d),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+b\log(cx^n)}}{d+ex}\,dx=\int \frac{\sqrt{a+b\log(cx^n)}}{d+ex}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**(1/2)/(e*x+d),x)
[Out] Integral(sqrt(a + b*log(c*x**n))/(d + e*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log(cx^n)}}{d+ex}\,dx = \int \frac{\sqrt{b\log(cx^n)+a}}{ex+d}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(b*log(c*x^n) + a)/(e*x + d), x)
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log(cx^n)}}{d+ex}\,dx = \int \frac{\sqrt{b\log(cx^n)+a}}{ex+d}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^(1/2)/(e*x+d),x, algorithm="giac")
[Out] integrate(sqrt(b*log(c*x^n) + a)/(e*x + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log(cx^n)}}{d+ex}\,dx = \int \frac{\sqrt{a+b\,\ln(c\,x^n)}}{d+e\,x}\,dx$$

```
[In] int((a + b*log(c*x^n))^(1/2)/(d + e*x),x)
[Out] int((a + b*log(c*x^n))^(1/2)/(d + e*x), x)
```

3.128 $\int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^2}\,dx = \frac{x\sqrt{a+b\log(cx^n)}}{d(d+ex)} - \frac{bn\text{Int}\left(\frac{1}{(d+ex)\sqrt{a+b\log(cx^n)}},x\right)}{2d}$$

```
[Out] x*(a+b*ln(c*x^n))^(1/2)/d/(e*x+d)-1/2*b*n*Unintegrable(1/(e*x+d)/(a+b*ln(c*
x^n))^(1/2),x)/d
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^2}\,dx = \int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^2}\,dx$$

```
[In] Int[Sqrt[a + b*Log[c*x^n]]/(d + e*x)^2,x]
```

```
[Out] (x*Sqrt[a + b*Log[c*x^n]])/(d*(d + e*x)) - (b*n*Defer[Int][1/((d + e*x)*Sqr
t[a + b*Log[c*x^n]]), x])/(2*d)
```

Rubi steps

$$\text{integral} = \frac{x\sqrt{a+b\log(cx^n)}}{d(d+ex)} - \frac{(bn)\int \frac{1}{(d+ex)\sqrt{a+b\log(cx^n)}}\,dx}{2d}$$

Mathematica [N/A]

Not integrable

Time = 5.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^2}\,dx = \int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^2}\,dx$$

```
[In] Integrate[Sqrt[a + b*Log[c*x^n]]/(d + e*x)^2,x]
[Out] Integrate[Sqrt[a + b*Log[c*x^n]]/(d + e*x)^2, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\ln(c\,x^n)}}{(ex+d)^2}dx$$

```
[In] int((a+b*ln(c*x^n))^(1/2)/(e*x+d)^2,x)
[Out] int((a+b*ln(c*x^n))^(1/2)/(e*x+d)^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*x^n))^(1/2)/(e*x+d)^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^2}\,dx = \int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**(1/2)/(e*x+d)**2,x)
[Out] Integral(sqrt(a + b*log(c*x**n))/(d + e*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^2}\,dx = \int \frac{\sqrt{b\log(cx^n)+a}}{(ex+d)^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^(1/2)/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate(sqrt(b*log(c*x^n) + a)/(e*x + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^2}\,dx = \int \frac{\sqrt{b\log(cx^n)+a}}{(ex+d)^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^(1/2)/(e*x+d)^2,x, algorithm="giac")
[Out] integrate(sqrt(b*log(c*x^n) + a)/(e*x + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^2}\,dx = \int \frac{\sqrt{a+b\,\ln(c\,x^n)}}{(d+e\,x)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))^(1/2)/(d + e*x)^2,x)
[Out] int((a + b*log(c*x^n))^(1/2)/(d + e*x)^2, x)
```

3.129 $\int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^3}\,dx = -\frac{\sqrt{a+b\log(cx^n)}}{2e(d+ex)^2} + \frac{bn\text{Int}\left(\frac{1}{x(d+ex)^2\sqrt{a+b\log(cx^n)}},x\right)}{4e}$$

```
[Out] -1/2*(a+b*ln(c*x^n))^(1/2)/e/(e*x+d)^2+1/4*b*n*Unintegrable(1/x/(e*x+d)^2/(
a+b*ln(c*x^n))^(1/2),x)/e
```

Rubi [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^3}\,dx = \int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^3}\,dx$$

```
[In] Int[Sqrt[a + b*Log[c*x^n]]/(d + e*x)^3,x]
```

```
[Out] -1/2*Sqrt[a + b*Log[c*x^n]]/(e*(d + e*x)^2) + (b*n*Defer[Int][1/(x*(d + e*x
)^2*Sqrt[a + b*Log[c*x^n]]), x])/(4*e)
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{a+b\log(cx^n)}}{2e(d+ex)^2} + \frac{(bn)\int \frac{1}{x(d+ex)^2\sqrt{a+b\log(cx^n)}}\,dx}{4e}$$

Mathematica [N/A]

Not integrable

Time = 13.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^3}\,dx = \int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^3}\,dx$$

```
[In] Integrate[Sqrt[a + b*Log[c*x^n]]/(d + e*x)^3,x]
[Out] Integrate[Sqrt[a + b*Log[c*x^n]]/(d + e*x)^3, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\ln(c\,x^{n})}}{(ex+d)^{3}}dx$$

```
[In] int((a+b*ln(c*x^n))^(1/2)/(e*x+d)^3,x)
[Out] int((a+b*ln(c*x^n))^(1/2)/(e*x+d)^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*x^n))^(1/2)/(e*x+d)^3,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^3}\,dx = \int \frac{\sqrt{a+b\log(cx^n)}}{(d+ex)^{3}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**(1/2)/(e*x+d)**3,x)
[Out] Integral(sqrt(a + b*log(c*x**n))/(d + e*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\log{(cx^n)}}}{(d + ex)^3}\,dx = \int \frac{\sqrt{b\log{(cx^n)} + a}}{(ex + d)^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^(1/2)/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate(sqrt(b*log(c*x^n) + a)/(e*x + d)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\log{(cx^n)}}}{(d + ex)^3}\,dx = \int \frac{\sqrt{b\log{(cx^n)} + a}}{(ex + d)^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^(1/2)/(e*x+d)^3,x, algorithm="giac")
[Out] integrate(sqrt(b*log(c*x^n) + a)/(e*x + d)^3, x)
```

Mupad [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\log{(cx^n)}}}{(d + ex)^3}\,dx = \int \frac{\sqrt{a + b\,\ln{(c\,x^n)}}}{(d + e\,x)^3}\,dx$$

```
[In] int((a + b*log(c*x^n))^(1/2)/(d + e*x)^3,x)
[Out] int((a + b*log(c*x^n))^(1/2)/(d + e*x)^3, x)
```

3.130 $\int x^3\sqrt{d+ex}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 242

$$\int x^3\sqrt{d+ex}(a+b\log(cx^n))\,dx=\frac{64bd^4n\sqrt{d+ex}}{315e^4}+\frac{64bd^3n(d+ex)^{3/2}}{945e^4}-\frac{356bd^2n(d+ex)^{5/2}}{1575e^4}+\frac{80bdn(d+ex)^{7/2}}{441e^4}-\frac{4bn(d+ex)^{9/2}}{81e^4}-\frac{64bd^{9/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{315e^4}-\frac{2d^3(d+ex)^{3/2}(a+b\log(cx^n))}{3e^4}+\frac{6d^2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4}-\frac{6d(d+ex)^{7/2}(a+b\log(cx^n))}{7e^4}+\frac{2(d+ex)^{9/2}(a+b\log(cx^n))}{9e^4}$$

```
[Out] 64/945*b*d^3*n*(e*x+d)^(3/2)/e^4-356/1575*b*d^2*n*(e*x+d)^(5/2)/e^4+80/441*
b*d*n*(e*x+d)^(7/2)/e^4-4/81*b*n*(e*x+d)^(9/2)/e^4-64/315*b*d^(9/2)*n*arcta
nh((e*x+d)^(1/2)/d^(1/2))/e^4-2/3*d^3*(e*x+d)^(3/2)*(a+b*ln(c*x^n))/e^4+6/5
*d^2*(e*x+d)^(5/2)*(a+b*ln(c*x^n))/e^4-6/7*d*(e*x+d)^(7/2)*(a+b*ln(c*x^n))/
e^4+2/9*(e*x+d)^(9/2)*(a+b*ln(c*x^n))/e^4+64/315*b*d^4*n*(e*x+d)^(1/2)/e^4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {45, 2392, 12, 1634, 52, 65, 214}

$$\int x^3\sqrt{d+ex}(a+b\log(cx^n))\,dx = -\frac{2d^3(d+ex)^{3/2}(a+b\log(cx^n))}{3e^4}+\frac{6d^2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4}-\frac{6d(d+ex)^{7/2}(a+b\log(cx^n))}{7e^4}+\frac{2(d+ex)^{9/2}(a+b\log(cx^n))}{9e^4}-\frac{64bd^{9/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{315e^4}+\frac{64bd^4n\sqrt{d+ex}}{315e^4}+\frac{64bd^3n(d+ex)^{3/2}}{945e^4}-\frac{356bd^2n(d+ex)^{5/2}}{1575e^4}+\frac{80bdn(d+ex)^{7/2}}{441e^4}-\frac{4bn(d+ex)^{9/2}}{81e^4}$$

```
[In] Int[x^3*Sqrt[d + e*x]*(a + b*Log[c*x^n]),x]
```

```
[Out] (64*b*d^4*n*Sqrt[d + e*x])/(315*e^4) + (64*b*d^3*n*(d + e*x)^(3/2))/(945*e^
4) - (356*b*d^2*n*(d + e*x)^(5/2))/(1575*e^4) + (80*b*d*n*(d + e*x)^(7/2))/
(441*e^4) - (4*b*n*(d + e*x)^(9/2))/(81*e^4) - (64*b*d^(9/2)*n*ArcTanh[Sqrt
[d + e*x]/Sqrt[d]])/(315*e^4) - (2*d^3*(d + e*x)^(3/2)*(a + b*Log[c*x^n]))/
(3*e^4) + (6*d^2*(d + e*x)^(5/2)*(a + b*Log[c*x^n]))/(5*e^4) - (6*d*(d + e*
x)^(7/2)*(a + b*Log[c*x^n]))/(7*e^4) + (2*(d + e*x)^(9/2)*(a + b*Log[c*x^n]
))/(9*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} = & -\frac{2d^3(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)}{3e^4}+\frac{6d^2(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^4} \\
& -\frac{6d(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^4}+\frac{2(d+ex)^{9/2}\left(a+b\log\left(cx^n\right)\right)}{9e^4} \\
& -(bn)\int\frac{2(d+ex)^{3/2}\left(-16d^3+24d^2ex-30de^2x^2+35e^3x^3\right)}{315e^4x}\,dx
\end{aligned}
$$

$$=-\frac{2d^3(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)}{3e^4}+\frac{6d^2(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^4}-\frac{6d(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^4}+\frac{2(d+ex)^{9/2}\left(a+b\log\left(cx^n\right)\right)}{9e^4}-\frac{(2bn)\int\frac{(d+ex)^{3/2}\left(-16d^3+24d^2ex-30de^2x^2+35e^3x^3\right)}{x}\,dx}{315e^4}$$

$$=-\frac{2d^3(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)}{3e^4}+\frac{6d^2(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^4}-\frac{6d(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^4}+\frac{2(d+ex)^{9/2}\left(a+b\log\left(cx^n\right)\right)}{9e^4}-\frac{(2bn)\int\left(89d^2e(d+ex)^{3/2}-\frac{16d^3(d+ex)^{3/2}}{x}-100de(d+ex)^{5/2}+35e(d+ex)^{7/2}\right)dx}{315e^4}$$

$$=-\frac{356bd^2n(d+ex)^{5/2}}{1575e^4}+\frac{80bdn(d+ex)^{7/2}}{441e^4}-\frac{4bn(d+ex)^{9/2}}{81e^4}-\frac{2d^3(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)}{3e^4}+\frac{6d^2(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^4}-\frac{6d(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^4}+\frac{2(d+ex)^{9/2}\left(a+b\log\left(cx^n\right)\right)}{9e^4}+\frac{(32bd^3n)\int\frac{(d+ex)^{3/2}}{x}\,dx}{315e^4}$$

$$=\frac{64bd^3n(d+ex)^{3/2}}{945e^4}-\frac{356bd^2n(d+ex)^{5/2}}{1575e^4}+\frac{80bdn(d+ex)^{7/2}}{441e^4}-\frac{4bn(d+ex)^{9/2}}{81e^4}-\frac{2d^3(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)}{3e^4}+\frac{6d^2(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^4}-\frac{6d(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^4}+\frac{2(d+ex)^{9/2}\left(a+b\log\left(cx^n\right)\right)}{9e^4}+\frac{(32bd^4n)\int\frac{\sqrt{d+ex}}{x}\,dx}{315e^4}$$

$$=\frac{64bd^4n\sqrt{d+ex}}{315e^4}+\frac{64bd^3n(d+ex)^{3/2}}{945e^4}-\frac{356bd^2n(d+ex)^{5/2}}{1575e^4}+\frac{80bdn(d+ex)^{7/2}}{441e^4}-\frac{4bn(d+ex)^{9/2}}{81e^4}-\frac{2d^3(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)}{3e^4}+\frac{6d^2(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^4}-\frac{6d(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^4}+\frac{2(d+ex)^{9/2}\left(a+b\log\left(cx^n\right)\right)}{9e^4}+\frac{(32bd^5n)\int\frac{1}{x\sqrt{d+ex}}\,dx}{315e^4}$$

$$= \frac{64bd^4n\sqrt{d+ex}}{315e^4} + \frac{64bd^3n(d+ex)^{3/2}}{945e^4} - \frac{356bd^2n(d+ex)^{5/2}}{1575e^4} + \frac{80bdn(d+ex)^{7/2}}{441e^4} - \frac{4bn(d+ex)^{9/2}}{81e^4} - \frac{2d^3(d+ex)^{3/2}(a+b\log(cx^n))}{3e^4} + \frac{6d^2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4} - \frac{6d(d+ex)^{7/2}(a+b\log(cx^n))}{7e^4} + \frac{2(d+ex)^{9/2}(a+b\log(cx^n))}{9e^4} + \frac{(64bd^5n)\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{315e^5}$$

$$= \frac{64bd^4n\sqrt{d+ex}}{315e^4} + \frac{64bd^3n(d+ex)^{3/2}}{945e^4} - \frac{356bd^2n(d+ex)^{5/2}}{1575e^4} + \frac{80bdn(d+ex)^{7/2}}{441e^4} - \frac{4bn(d+ex)^{9/2}}{81e^4} - \frac{64bd^{9/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{315e^4} - \frac{2d^3(d+ex)^{3/2}(a+b\log(cx^n))}{3e^4} + \frac{6d^2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4} - \frac{6d(d+ex)^{7/2}(a+b\log(cx^n))}{7e^4} + \frac{2(d+ex)^{9/2}(a+b\log(cx^n))}{9e^4}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.76

$$\int x^3\sqrt{d+ex}(a+b\log(cx^n))\,dx = -\frac{2\left(10080bd^{9/2}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) + \sqrt{d+ex}(315a(16d^4-8d^3ex+6d^2e^2x^2-5de^3x^3-35e^4x^4)+2bn(-$$

```
[In] Integrate[x^3*Sqrt[d + e*x]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-2*(10080*b*d^(9/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + Sqrt[d + e*x]*(315*
a*(16*d^4 - 8*d^3*e*x + 6*d^2*e^2*x^2 - 5*d*e^3*x^3 - 35*e^4*x^4) + 2*b*n*(
-4388*d^4 + 934*d^3*e*x - 543*d^2*e^2*x^2 + 400*d*e^3*x^3 + 1225*e^4*x^4) +
 315*b*(16*d^4 - 8*d^3*e*x + 6*d^2*e^2*x^2 - 5*d*e^3*x^3 - 35*e^4*x^4)*Log[
c*x^n]))/(99225*e^4)
```

Maple [F]

$$\int x^{3}(a+b\ln(c\,x^{n}))\,\sqrt{ex+d}dx$$

[In] int(x^3*(a+b*ln(c*x^n))*(e*x+d)^(1/2),x)

[Out] int(x^3*(a+b*ln(c*x^n))*(e*x+d)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.05

$$\int x^{3}\sqrt{d+ex}(a+b\log(cx^{n}))\;dx$$
$$=\left[\frac{2\left(5040\,bd^{\frac{9}{2}}n\log\left(\frac{ex-2\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)+(8776\,bd^{4}n-5040\,ad^{4}-1225\,(2\,be^{4}n-9\,ae^{4})x^{4}-25\,(32\,bde^{3}n-\right.}{}\right.$$

[In] integrate(x^3*(a+b*log(c*x^n))*(e*x+d)^(1/2),x, algorithm="fricas")

[Out] [2/99225*(5040*b*d^(9/2)*n*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + (8776*b*d^4*n - 5040*a*d^4 - 1225*(2*b*e^4*n - 9*a*e^4)*x^4 - 25*(32*b*d*e^3*n - 63*a*d*e^3)*x^3 + 6*(181*b*d^2*e^2*n - 315*a*d^2*e^2)*x^2 - 4*(467*b*d^3*e*n - 630*a*d^3*e)*x + 315*(35*b*e^4*x^4 + 5*b*d*e^3*x^3 - 6*b*d^2*e^2*x^2 + 8*b*d^3*e*x - 16*b*d^4)*log(c) + 315*(35*b*e^4*n*x^4 + 5*b*d*e^3*n*x^3 - 6*b*d^2*e^2*n*x^2 + 8*b*d^3*e*n*x - 16*b*d^4*n)*log(x))*sqrt(e*x + d))/e^4, 2/99225*(10080*b*sqrt(-d)*d^4*n*arctan(sqrt(e*x + d)*sqrt(-d)/d) + (8776*b*d^4*n - 5040*a*d^4 - 1225*(2*b*e^4*n - 9*a*e^4)*x^4 - 25*(32*b*d*e^3*n - 63*a*d*e^3)*x^3 + 6*(181*b*d^2*e^2*n - 315*a*d^2*e^2)*x^2 - 4*(467*b*d^3*e*n - 630*a*d^3*e)*x + 315*(35*b*e^4*x^4 + 5*b*d*e^3*x^3 - 6*b*d^2*e^2*x^2 + 8*b*d^3*e*x - 16*b*d^4)*log(c) + 315*(35*b*e^4*n*x^4 + 5*b*d*e^3*n*x^3 - 6*b*d^2*e^2*n*x^2 + 8*b*d^3*e*n*x - 16*b*d^4*n)*log(x))*sqrt(e*x + d))/e^4]

Sympy [A] (verification not implemented)

Time = 70.60 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.43

$$\int x^3\sqrt{d+ex}(a+b\log(cx^n))\,dx$$
$$= a\left(\begin{cases}-\frac{2d^3(d+ex)^{\frac{3}{2}}}{3e^4}+\frac{6d^2(d+ex)^{\frac{5}{2}}}{5e^4}-\frac{6d(d+ex)^{\frac{7}{2}}}{7e^4}+\frac{2(d+ex)^{\frac{9}{2}}}{9e^4} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^4}{4} & \text{otherwise}\end{cases}\right)$$
$$-bn\left(\begin{cases}-\frac{17552d^{\frac{9}{2}}\sqrt{1+\frac{ex}{d}}}{99225e^4}-\frac{32d^{\frac{9}{2}}\log\left(\frac{ex}{d}\right)}{315e^4}+\frac{64d^{\frac{9}{2}}\log\left(\sqrt{1+\frac{ex}{d}}+1\right)}{315e^4}+\frac{3736d^{\frac{7}{2}}x\sqrt{1+\frac{ex}{d}}}{99225e^3}-\frac{724d^{\frac{5}{2}}x^2\sqrt{1+\frac{ex}{d}}}{33075e^2}+\frac{64d^{\frac{3}{2}}x^3\sqrt{1+\frac{ex}{d}}}{3969e} \\ \frac{\sqrt{d}x^4}{16}\end{cases}\right)$$
$$+b\left(\begin{cases}-\frac{2d^3(d+ex)^{\frac{3}{2}}}{3e^4}+\frac{6d^2(d+ex)^{\frac{5}{2}}}{5e^4}-\frac{6d(d+ex)^{\frac{7}{2}}}{7e^4}+\frac{2(d+ex)^{\frac{9}{2}}}{9e^4} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^4}{4} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))*(e*x+d)**(1/2),x)
```

```
[Out] a*Piecewise((-2*d**3*(d + e*x)**(3/2)/(3*e**4) + 6*d**2*(d + e*x)**(5/2)/(5
*e**4) - 6*d*(d + e*x)**(7/2)/(7*e**4) + 2*(d + e*x)**(9/2)/(9*e**4), Ne(e,
 0)), (sqrt(d)*x**4/4, True)) - b*n*Piecewise((-17552*d**(9/2)*sqrt(1 + e*x
/d)/(99225*e**4) - 32*d**(9/2)*log(e*x/d)/(315*e**4) + 64*d**(9/2)*log(sqrt
(1 + e*x/d) + 1)/(315*e**4) + 3736*d**(7/2)*x*sqrt(1 + e*x/d)/(99225*e**3)
- 724*d**(5/2)*x**2*sqrt(1 + e*x/d)/(33075*e**2) + 64*d**(3/2)*x**3*sqrt(1
+ e*x/d)/(3969*e) + 4*sqrt(d)*x**4*sqrt(1 + e*x/d)/81, (e > -oo) & (e < oo)
 & Ne(e, 0)), (sqrt(d)*x**4/16, True)) + b*Piecewise((-2*d**3*(d + e*x)**(3
/2)/(3*e**4) + 6*d**2*(d + e*x)**(5/2)/(5*e**4) - 6*d*(d + e*x)**(7/2)/(7*e
**4) + 2*(d + e*x)**(9/2)/(9*e**4), Ne(e, 0)), (sqrt(d)*x**4/4, True))*log(
c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.94

$$\int x^3\sqrt{d+ex}(a+b\log(cx^n))\,dx$$
$$=\frac{4}{99225}\left(\frac{2520\,d^{\frac{9}{2}}\log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{e^4}-\frac{1225\,(ex+d)^{\frac{9}{2}}-4500\,(ex+d)^{\frac{7}{2}}d+5607\,(ex+d)^{\frac{5}{2}}d^2-1680\,(ex+d)}{e^4}\right.$$
$$+\frac{2}{315}\,b\left(\frac{35\,(ex+d)^{\frac{9}{2}}}{e^4}-\frac{135\,(ex+d)^{\frac{7}{2}}d}{e^4}+\frac{189\,(ex+d)^{\frac{5}{2}}d^2}{e^4}-\frac{105\,(ex+d)^{\frac{3}{2}}d^3}{e^4}\right)\log(cx^n)$$
$$+\frac{2}{315}\,a\left(\frac{35\,(ex+d)^{\frac{9}{2}}}{e^4}-\frac{135\,(ex+d)^{\frac{7}{2}}d}{e^4}+\frac{189\,(ex+d)^{\frac{5}{2}}d^2}{e^4}-\frac{105\,(ex+d)^{\frac{3}{2}}d^3}{e^4}\right)$$

```
[In] integrate(x^3*(a+b*log(c*x^n))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 4/99225*(2520*d^(9/2)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d
)))/e^4 - (1225*(e*x + d)^(9/2) - 4500*(e*x + d)^(7/2)*d + 5607*(e*x + d)^(
5/2)*d^2 - 1680*(e*x + d)^(3/2)*d^3 - 5040*sqrt(e*x + d)*d^4)/e^4)*b*n + 2/
315*b*(35*(e*x + d)^(9/2)/e^4 - 135*(e*x + d)^(7/2)*d/e^4 + 189*(e*x + d)^(
5/2)*d^2/e^4 - 105*(e*x + d)^(3/2)*d^3/e^4)*log(c*x^n) + 2/315*a*(35*(e*x +
 d)^(9/2)/e^4 - 135*(e*x + d)^(7/2)*d/e^4 + 189*(e*x + d)^(5/2)*d^2/e^4 - 1
05*(e*x + d)^(3/2)*d^3/e^4)
```

Giac [F]

$$\int x^3\sqrt{d+ex}(a+b\log(cx^n))\,dx=\int\sqrt{ex+d}(b\log(cx^n)+a)x^3\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*log(c*x^n) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d+ex}(a+b\log(cx^n))\,dx=\int x^3\,(a+b\,\ln(c\,x^n))\,\sqrt{d+e\,x}\,dx$$

```
[In] int(x^3*(a + b*log(c*x^n))*(d + e*x)^(1/2),x)
[Out] int(x^3*(a + b*log(c*x^n))*(d + e*x)^(1/2), x)
```

3.131 $\int x^2\sqrt{d+ex}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 192

$$\int x^2\sqrt{d+ex}(a+b\log(cx^n))\,dx = -\frac{32bd^3n\sqrt{d+ex}}{105e^3}-\frac{32bd^2n(d+ex)^{3/2}}{315e^3}+\frac{36bdn(d+ex)^{5/2}}{175e^3}-\frac{4bn(d+ex)^{7/2}}{49e^3}+\frac{32bd^{7/2}n\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{105e^3}+\frac{2d^2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3}-\frac{4d(d+ex)^{5/2}(a+b\log(cx^n))}{5e^3}+\frac{2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^3}$$

```
[Out] -32/315*b*d^2*n*(e*x+d)^(3/2)/e^3+36/175*b*d*n*(e*x+d)^(5/2)/e^3-4/49*b*n*(
e*x+d)^(7/2)/e^3+32/105*b*d^(7/2)*n*arctanh((e*x+d)^(1/2)/d^(1/2))/e^3+2/3*
d^2*(e*x+d)^(3/2)*(a+b*ln(c*x^n))/e^3-4/5*d*(e*x+d)^(5/2)*(a+b*ln(c*x^n))/e
^3+2/7*(e*x+d)^(7/2)*(a+b*ln(c*x^n))/e^3-32/105*b*d^3*n*(e*x+d)^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {45, 2392, 12, 911, 1275, 214}

$$\int x^2\sqrt{d+ex}(a+b\log(cx^n))\,dx=\frac{2d^2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3}-\frac{4d(d+ex)^{5/2}(a+b\log(cx^n))}{5e^3}+\frac{2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^3}+\frac{32bd^{7/2}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{105e^3}-\frac{32bd^3n\sqrt{d+ex}}{105e^3}-\frac{32bd^2n(d+ex)^{3/2}}{315e^3}+\frac{36bdn(d+ex)^{5/2}}{175e^3}-\frac{4bn(d+ex)^{7/2}}{49e^3}$$

```
[In] Int[x^2*Sqrt[d + e*x]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-32*b*d^3*n*Sqrt[d + e*x])/(105*e^3) - (32*b*d^2*n*(d + e*x)^(3/2))/(315*e
^3) + (36*b*d*n*(d + e*x)^(5/2))/(175*e^3) - (4*b*n*(d + e*x)^(7/2))/(49*e^
3) + (32*b*d^(7/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(105*e^3) + (2*d^2*(d
+ e*x)^(3/2)*(a + b*Log[c*x^n]))/(3*e^3) - (4*d*(d + e*x)^(5/2)*(a + b*Log[
c*x^n]))/(5*e^3) + (2*(d + e*x)^(7/2)*(a + b*Log[c*x^n]))/(7*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2d^2(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)}{3e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^3} \\
&\quad+\frac{2(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^3}-(bn)\int\frac{2(d+ex)^{3/2}\left(8d^2-12dex+15e^2x^2\right)}{105e^3x}\,dx \\
&= \frac{2d^2(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)}{3e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^3} \\
&\quad+\frac{2(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^3}-\frac{(2bn)\int\frac{(d+ex)^{3/2}\left(8d^2-12dex+15e^2x^2\right)}{x}\,dx}{105e^3} \\
&= \frac{2d^2(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)}{3e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^3} \\
&\quad+\frac{2(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^3} \\
&\quad-\frac{(4bn)\text{Subst}\left(\int\frac{x^4\left(35d^2-42dx^2+15x^4\right)}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{105e^4}
\end{aligned}$$

$$= \frac{2d^2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3} - \frac{4d(d+ex)^{5/2}(a+b\log(cx^n))}{5e^3} + \frac{2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^3} - \frac{(4bn)\text{Subst}\left(\int\left(8d^3e+8d^2ex^2-27dex^4+15ex^6+\frac{8d^4}{-\frac{d}{e}+\frac{x^2}{e}}\right)dx, x, \sqrt{d+ex}\right)}{105e^4}$$

$$= -\frac{32bd^3n\sqrt{d+ex}}{105e^3} - \frac{32bd^2n(d+ex)^{3/2}}{315e^3} + \frac{36bdn(d+ex)^{5/2}}{175e^3} - \frac{4bn(d+ex)^{7/2}}{49e^3} + \frac{2d^2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3} - \frac{4d(d+ex)^{5/2}(a+b\log(cx^n))}{5e^3} + \frac{2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^3} - \frac{(32bd^4n)\,\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}dx, x, \sqrt{d+ex}\right)}{105e^4}$$

$$= -\frac{32bd^3n\sqrt{d+ex}}{105e^3} - \frac{32bd^2n(d+ex)^{3/2}}{315e^3} + \frac{36bdn(d+ex)^{5/2}}{175e^3} - \frac{4bn(d+ex)^{7/2}}{49e^3} + \frac{32bd^{7/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{105e^3} + \frac{2d^2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3} - \frac{4d(d+ex)^{5/2}(a+b\log(cx^n))}{5e^3} + \frac{2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^3}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.79

$$\int x^2\sqrt{d+ex}(a+b\log(cx^n))\,dx$$

$$= \frac{3360bd^{7/2}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) + 2\sqrt{d+ex}(105a(8d^3-4d^2ex+3de^2x^2+15e^3x^3) - 2bn(778d^3-179d^2ex+\ldots}{11025e^3}$$

```
[In] Integrate[x^2*Sqrt[d + e*x]*(a + b*Log[c*x^n]),x]
```

```
[Out] (3360*b*d^(7/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + 2*Sqrt[d + e*x]*(105*a*(
8*d^3 - 4*d^2*e*x + 3*d*e^2*x^2 + 15*e^3*x^3) - 2*b*n*(778*d^3 - 179*d^2*e*
x + 108*d*e^2*x^2 + 225*e^3*x^3) + 105*b*(8*d^3 - 4*d^2*e*x + 3*d*e^2*x^2 +
 15*e^3*x^3)*Log[c*x^n]))/(11025*e^3)
```

Maple [F]

$$\int x^2(a+b\ln(c\,x^n))\,\sqrt{ex+d}dx$$

[In] int(x^2*(a+b*ln(c*x^n))*(e*x+d)^(1/2),x)

[Out] int(x^2*(a+b*ln(c*x^n))*(e*x+d)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.06

$$\int x^2\sqrt{d+ex}(a+b\log(cx^n))\,dx$$

$$= \left[\frac{2\left(840\,bd^{\frac{7}{2}}n\log\left(\frac{ex+2\,\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)-(1556\,bd^3n-840\,ad^3+225\,(2\,be^3n-7\,ae^3)x^3+9\,(24\,bde^2n-35\,ad\cdots\right.}{\cdots}\right.$$

$$-\frac{2\left(1680\,b\sqrt{-d}d^3n\arctan\left(\frac{\sqrt{ex+d}\sqrt{-d}}{d}\right)+(1556\,bd^3n-840\,ad^3+225\,(2\,be^3n-7\,ae^3)x^3+9\,(24\,bde^2n-\cdots\right.}{\cdots}$$

[In] integrate(x^2*(a+b*log(c*x^n))*(e*x+d)^(1/2),x, algorithm="fricas")

[Out] [2/11025*(840*b*d^(7/2)*n*log((e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - (1556*b*d^3*n - 840*a*d^3 + 225*(2*b*e^3*n - 7*a*e^3)*x^3 + 9*(24*b*d*e^2*n - 35*a*d*e^2)*x^2 - 2*(179*b*d^2*e*n - 210*a*d^2*e)*x - 105*(15*b*e^3*x^3 + 3*b*d*e^2*x^2 - 4*b*d^2*e*x + 8*b*d^3)*log(c) - 105*(15*b*e^3*n*x^3 + 3*b*d*e^2*n*x^2 - 4*b*d^2*e*n*x + 8*b*d^3*n)*log(x))*sqrt(e*x + d))/e^3, -2/11025*(1680*b*sqrt(-d)*d^3*n*arctan(sqrt(e*x + d)*sqrt(-d)/d) + (1556*b*d^3*n - 840*a*d^3 + 225*(2*b*e^3*n - 7*a*e^3)*x^3 + 9*(24*b*d*e^2*n - 35*a*d*e^2)*x^2 - 2*(179*b*d^2*e*n - 210*a*d^2*e)*x - 105*(15*b*e^3*x^3 + 3*b*d*e^2*x^2 - 4*b*d^2*e*x + 8*b*d^3)*log(c) - 105*(15*b*e^3*n*x^3 + 3*b*d*e^2*n*x^2 - 4*b*d^2*e*n*x + 8*b*d^3*n)*log(x))*sqrt(e*x + d))/e^3]

Sympy [A] (verification not implemented)

Time = 51.04 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.48

$$\int x^2\sqrt{d+ex}(a+b\log{(cx^n)})\,dx = a\left(\begin{cases}\frac{2d^2(d+ex)^{\frac{3}{2}}}{3e^3}-\frac{4d(d+ex)^{\frac{5}{2}}}{5e^3}+\frac{2(d+ex)^{\frac{7}{2}}}{7e^3} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^3}{3} & \text{otherwise}\end{cases}\right)$$
$$-bn\left(\begin{cases}\frac{3112d^{\frac{7}{2}}\sqrt{1+\frac{ex}{d}}}{11025e^3}+\frac{16d^{\frac{7}{2}}\log{\left(\frac{ex}{d}\right)}}{105e^3}-\frac{32d^{\frac{7}{2}}\log{\left(\sqrt{1+\frac{ex}{d}}+1\right)}}{105e^3}-\frac{716d^{\frac{5}{2}}x\sqrt{1+\frac{ex}{d}}}{11025e^2}+\frac{48d^{\frac{3}{2}}x^2\sqrt{1+\frac{ex}{d}}}{1225e}+\frac{4\sqrt{d}x^3\sqrt{1+\frac{ex}{d}}}{49} & \text{for } \\ \frac{\sqrt{d}x^3}{9} & \text{othe}\end{cases}\right.$$
$$+b\left(\begin{cases}\frac{2d^2(d+ex)^{\frac{3}{2}}}{3e^3}-\frac{4d(d+ex)^{\frac{5}{2}}}{5e^3}+\frac{2(d+ex)^{\frac{7}{2}}}{7e^3} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^3}{3} & \text{otherwise}\end{cases}\right)\log{(cx^n)}$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))*(e*x+d)**(1/2),x)
```

```
[Out] a*Piecewise((2*d**2*(d + e*x)**(3/2)/(3*e**3) - 4*d*(d + e*x)**(5/2)/(5*e**
3) + 2*(d + e*x)**(7/2)/(7*e**3), Ne(e, 0)), (sqrt(d)*x**3/3, True)) - b*n*
Piecewise((3112*d**(7/2)*sqrt(1 + e*x/d)/(11025*e**3) + 16*d**(7/2)*log(e*x
/d)/(105*e**3) - 32*d**(7/2)*log(sqrt(1 + e*x/d) + 1)/(105*e**3) - 716*d**(
5/2)*x*sqrt(1 + e*x/d)/(11025*e**2) + 48*d**(3/2)*x**2*sqrt(1 + e*x/d)/(122
5*e) + 4*sqrt(d)*x**3*sqrt(1 + e*x/d)/49, (e > -oo) & (e < oo) & Ne(e, 0)),
 (sqrt(d)*x**3/9, True)) + b*Piecewise((2*d**2*(d + e*x)**(3/2)/(3*e**3) -
4*d*(d + e*x)**(5/2)/(5*e**3) + 2*(d + e*x)**(7/2)/(7*e**3), Ne(e, 0)), (sq
rt(d)*x**3/3, True))*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{d+ex}(a+b\log{(cx^n)})\,dx =$$
$$-\frac{4}{11025}\left(\frac{420\,d^{\frac{7}{2}}\log{\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}}{e^3}+\frac{225\,(ex+d)^{\frac{7}{2}}-567\,(ex+d)^{\frac{5}{2}}d+280\,(ex+d)^{\frac{3}{2}}d^2+840\,\sqrt{ex+d}d^3}{e^3}\right.$$
$$+\frac{2}{105}\,b\left(\frac{15\,(ex+d)^{\frac{7}{2}}}{e^3}-\frac{42\,(ex+d)^{\frac{5}{2}}d}{e^3}+\frac{35\,(ex+d)^{\frac{3}{2}}d^2}{e^3}\right)\log{(cx^n)}$$
$$+\frac{2}{105}\,a\left(\frac{15\,(ex+d)^{\frac{7}{2}}}{e^3}-\frac{42\,(ex+d)^{\frac{5}{2}}d}{e^3}+\frac{35\,(ex+d)^{\frac{3}{2}}d^2}{e^3}\right)$$

```
[In] integrate(x^2*(a+b*log(c*x^n))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -4/11025*(420*d^(7/2)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d
)))/e^3 + (225*(e*x + d)^(7/2) - 567*(e*x + d)^(5/2)*d + 280*(e*x + d)^(3/2
)*d^2 + 840*sqrt(e*x + d)*d^3)/e^3)*b*n + 2/105*b*(15*(e*x + d)^(7/2)/e^3 -
 42*(e*x + d)^(5/2)*d/e^3 + 35*(e*x + d)^(3/2)*d^2/e^3)*log(c*x^n) + 2/105*
a*(15*(e*x + d)^(7/2)/e^3 - 42*(e*x + d)^(5/2)*d/e^3 + 35*(e*x + d)^(3/2)*d
^2/e^3)
```

Giac [F]

$$\int x^2\sqrt{d+ex}(a+b\log(cx^n))\,dx=\int\sqrt{ex+d}(b\log(cx^n)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*log(c*x^n) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d+ex}(a+b\log(cx^n))\,dx=\int x^2\left(a+b\,\ln\left(c\,x^n\right)\right)\sqrt{d+e\,x}\,dx$$

```
[In] int(x^2*(a + b*log(c*x^n))*(d + e*x)^(1/2),x)
[Out] int(x^2*(a + b*log(c*x^n))*(d + e*x)^(1/2), x)
```

3.132 $\int x\sqrt{d+ex}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 142

$$\int x\sqrt{d+ex}(a+b\log(cx^n))\,dx=\frac{8bd^2n\sqrt{d+ex}}{15e^2}+\frac{8bdn(d+ex)^{3/2}}{45e^2}-\frac{4bn(d+ex)^{5/2}}{25e^2}-\frac{8bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{15e^2}-\frac{2d(d+ex)^{3/2}(a+b\log(cx^n))}{3e^2}+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^2}$$

```
[Out] 8/45*b*d*n*(e*x+d)^(3/2)/e^2-4/25*b*n*(e*x+d)^(5/2)/e^2-8/15*b*d^(5/2)*n*ar
ctanh((e*x+d)^(1/2)/d^(1/2))/e^2-2/3*d*(e*x+d)^(3/2)*(a+b*ln(c*x^n))/e^2+2/
5*(e*x+d)^(5/2)*(a+b*ln(c*x^n))/e^2+8/15*b*d^2*n*(e*x+d)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {45, 2392, 12, 81, 52, 65, 214}

$$\int x\sqrt{d+ex}(a+b\log(cx^n))\,dx=-\frac{2d(d+ex)^{3/2}(a+b\log(cx^n))}{3e^2}+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^2}-\frac{8bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{15e^2}+\frac{8bd^2n\sqrt{d+ex}}{15e^2}+\frac{8bdn(d+ex)^{3/2}}{45e^2}-\frac{4bn(d+ex)^{5/2}}{25e^2}$$

```
[In] Int[x*Sqrt[d + e*x]*(a + b*Log[c*x^n]),x]
```

```
[Out] (8*b*d^2*n*Sqrt[d + e*x])/(15*e^2) + (8*b*d*n*(d + e*x)^(3/2))/(45*e^2) - (
4*b*n*(d + e*x)^(5/2))/(25*e^2) - (8*b*d^(5/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt
[d]])/(15*e^2) - (2*d*(d + e*x)^(3/2)*(a + b*Log[c*x^n]))/(3*e^2) + (2*(d +
 e*x)^(5/2)*(a + b*Log[c*x^n]))/(5*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d(d+ex)^{3/2}(a+b\log(cx^n))}{3e^2}+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^2} \\
&\quad -(bn)\int\frac{2(d+ex)^{3/2}(-2d+3ex)}{15e^2x}\,dx \\
&= -\frac{2d(d+ex)^{3/2}(a+b\log(cx^n))}{3e^2} \\
&\quad +\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^2}-\frac{(2bn)\int\frac{(d+ex)^{3/2}(-2d+3ex)}{x}\,dx}{15e^2} \\
&= -\frac{4bn(d+ex)^{5/2}}{25e^2}-\frac{2d(d+ex)^{3/2}(a+b\log(cx^n))}{3e^2} \\
&\quad +\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^2}+\frac{(4bdn)\int\frac{(d+ex)^{3/2}}{x}\,dx}{15e^2} \\
&= \frac{8bdn(d+ex)^{3/2}}{45e^2}-\frac{4bn(d+ex)^{5/2}}{25e^2}-\frac{2d(d+ex)^{3/2}(a+b\log(cx^n))}{3e^2} \\
&\quad +\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^2}+\frac{(4bd^2n)\int\frac{\sqrt{d+ex}}{x}\,dx}{15e^2} \\
&= \frac{8bd^2n\sqrt{d+ex}}{15e^2}+\frac{8bdn(d+ex)^{3/2}}{45e^2}-\frac{4bn(d+ex)^{5/2}}{25e^2}-\frac{2d(d+ex)^{3/2}(a+b\log(cx^n))}{3e^2} \\
&\quad +\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^2}+\frac{(4bd^3n)\int\frac{1}{x\sqrt{d+ex}}\,dx}{15e^2} \\
&= \frac{8bd^2n\sqrt{d+ex}}{15e^2}+\frac{8bdn(d+ex)^{3/2}}{45e^2}-\frac{4bn(d+ex)^{5/2}}{25e^2}-\frac{2d(d+ex)^{3/2}(a+b\log(cx^n))}{3e^2} \\
&\quad +\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^2}+\frac{(8bd^3n)\,\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{15e^3} \\
&= \frac{8bd^2n\sqrt{d+ex}}{15e^2}+\frac{8bdn(d+ex)^{3/2}}{45e^2}-\frac{4bn(d+ex)^{5/2}}{25e^2}-\frac{8bd^{5/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{15e^2} \\
&\quad -\frac{2d(d+ex)^{3/2}(a+b\log(cx^n))}{3e^2}+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

$$\int x\sqrt{d+ex}(a+b\log(cx^n))\,dx$$
$$=\frac{-120bd^{5/2}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)+2\sqrt{d+ex}(2bn(31d^2-8dex-9e^2x^2)+15a(-2d^2+dex+3e^2x^2)+15b(-2}{225e^2}$$

```
[In] Integrate[x*Sqrt[d + e*x]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-120*b*d^(5/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + 2*Sqrt[d + e*x]*(2*b*n*(
31*d^2 - 8*d*e*x - 9*e^2*x^2) + 15*a*(-2*d^2 + d*e*x + 3*e^2*x^2) + 15*b*(-
2*d^2 + d*e*x + 3*e^2*x^2)*Log[c*x^n]))/(225*e^2)
```

Maple [F]

$$\int x(a+b\ln(c\,x^n))\sqrt{ex+d}dx$$

```
[In] int(x*(a+b*ln(c*x^n))*(e*x+d)^(1/2),x)
```

```
[Out] int(x*(a+b*ln(c*x^n))*(e*x+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.05

$$\int x\sqrt{d+ex}(a+b\log(cx^n))\,dx$$
$$=\left[\frac{2\left(30\,bd^{\frac{5}{2}}n\log\left(\frac{ex-2\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)+(62\,bd^2n-30\,ad^2-9\,(2\,be^2n-5\,ae^2)x^2-(16\,bden-15\,ade)x+15}{225\,e^2}\right.$$

```
[In] integrate(x*(a+b*log(c*x^n))*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/225*(30*b*d^(5/2)*n*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + (62*b
*d^2*n - 30*a*d^2 - 9*(2*b*e^2*n - 5*a*e^2)*x^2 - (16*b*d*e*n - 15*a*d*e)*x
 + 15*(3*b*e^2*x^2 + b*d*e*x - 2*b*d^2)*log(c) + 15*(3*b*e^2*n*x^2 + b*d*e*
n*x - 2*b*d^2*n)*log(x))*sqrt(e*x + d))/e^2, 2/225*(60*b*sqrt(-d)*d^2*n*arc
tan(sqrt(e*x + d)*sqrt(-d)/d) + (62*b*d^2*n - 30*a*d^2 - 9*(2*b*e^2*n - 5*a
*e^2)*x^2 - (16*b*d*e*n - 15*a*d*e)*x + 15*(3*b*e^2*x^2 + b*d*e*x - 2*b*d^2
)*log(c) + 15*(3*b*e^2*n*x^2 + b*d*e*n*x - 2*b*d^2*n)*log(x))*sqrt(e*x + d)
)/e^2]
```

Sympy [A] (verification not implemented)

Time = 56.22 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.56

$$\int x\sqrt{d+ex}(a+b\log{(cx^n)})\,dx = a\left(\begin{cases}-\frac{2d(d+ex)^{\frac{3}{2}}}{3e^2}+\frac{2(d+ex)^{\frac{5}{2}}}{5e^2} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^2}{2} & \text{otherwise}\end{cases}\right) - bn\left(\begin{cases}-\frac{124d^{\frac{5}{2}}\sqrt{1+\frac{ex}{d}}}{225e^2}-\frac{4d^{\frac{5}{2}}\log{\left(\frac{ex}{d}\right)}}{15e^2}+\frac{8d^{\frac{5}{2}}\log{\left(\sqrt{1+\frac{ex}{d}}+1\right)}}{15e^2}+\frac{32d^{\frac{3}{2}}x\sqrt{1+\frac{ex}{d}}}{225e}+\frac{4\sqrt{d}x^2\sqrt{1+\frac{ex}{d}}}{25} & \text{for } e>-\infty\wedge e<\infty\\ \frac{\sqrt{d}x^2}{4} & \text{otherwise}\end{cases}\right) + b\left(\begin{cases}-\frac{2d(d+ex)^{\frac{3}{2}}}{3e^2}+\frac{2(d+ex)^{\frac{5}{2}}}{5e^2} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^2}{2} & \text{otherwise}\end{cases}\right)\log{(cx^n)}$$

[In] integrate(x*(a+b*ln(c*x**n))*(e*x+d)**(1/2),x)

[Out] a*Piecewise((-2*d*(d + e*x)**(3/2)/(3*e**2) + 2*(d + e*x)**(5/2)/(5*e**2), Ne(e, 0)), (sqrt(d)*x**2/2, True)) - b*n*Piecewise((-124*d**(5/2)*sqrt(1 + e*x/d)/(225*e**2) - 4*d**(5/2)*log(e*x/d)/(15*e**2) + 8*d**(5/2)*log(sqrt(1 + e*x/d) + 1)/(15*e**2) + 32*d**(3/2)*x*sqrt(1 + e*x/d)/(225*e) + 4*sqrt(d)*x**2*sqrt(1 + e*x/d)/25, (e > -oo) & (e < oo) & Ne(e, 0)), (sqrt(d)*x**2/4, True)) + b*Piecewise((-2*d*(d + e*x)**(3/2)/(3*e**2) + 2*(d + e*x)**(5/2)/(5*e**2), Ne(e, 0)), (sqrt(d)*x**2/2, True))*log(c*x**n)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01

$$\int x\sqrt{d+ex}(a+b\log{(cx^n)})\,dx = \frac{4}{225}\left(\frac{15\,d^{\frac{5}{2}}\log{\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}}{e^2}-\frac{9\,(ex+d)^{\frac{5}{2}}-10\,(ex+d)^{\frac{3}{2}}d-30\,\sqrt{ex+d}d^2}{e^2}\right)bn + \frac{2}{15}\,b\left(\frac{3\,(ex+d)^{\frac{5}{2}}}{e^2}-\frac{5\,(ex+d)^{\frac{3}{2}}d}{e^2}\right)\log{(cx^n)}+\frac{2}{15}\,a\left(\frac{3\,(ex+d)^{\frac{5}{2}}}{e^2}-\frac{5\,(ex+d)^{\frac{3}{2}}d}{e^2}\right)$$

[In] integrate(x*(a+b*log(c*x^n))*(e*x+d)^(1/2),x, algorithm="maxima")

[Out] 4/225*(15*d^(5/2)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/e^2 - (9*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d - 30*sqrt(e*x + d)*d^2)/e^2)*b*n + 2/15*b*(3*(e*x + d)^(5/2)/e^2 - 5*(e*x + d)^(3/2)*d/e^2)*log(c*x^n) + 2/15*a*(3*(e*x + d)^(5/2)/e^2 - 5*(e*x + d)^(3/2)*d/e^2)

Giac [F]

$$\int x\sqrt{d+ex}(a+b\log(cx^n))\,dx = \int \sqrt{ex+d}(b\log(cx^n)+a)x\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*log(c*x^n) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+ex}(a+b\log(cx^n))\,dx = \int x\,(a+b\,\ln(c\,x^n))\,\sqrt{d+e\,x}\,dx$$

```
[In] int(x*(a + b*log(c*x^n))*(d + e*x)^(1/2),x)
[Out] int(x*(a + b*log(c*x^n))*(d + e*x)^(1/2), x)
```

3.133 $\int \sqrt{d+ex}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 94

$$\int \sqrt{d+ex}(a+b\log(cx^n))\,dx = -\frac{4bdn\sqrt{d+ex}}{3e}-\frac{4bn(d+ex)^{3/2}}{9e}+\frac{4bd^{3/2}n\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{3e}+\frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e}$$

```
[Out] -4/9*b*n*(e*x+d)^(3/2)/e+4/3*b*d^(3/2)*n*arctanh((e*x+d)^(1/2)/d^(1/2))/e+2
/3*(e*x+d)^(3/2)*(a+b*ln(c*x^n))/e-4/3*b*d*n*(e*x+d)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2356, 52, 65, 214}

$$\int \sqrt{d+ex}(a+b\log(cx^n))\,dx = \frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e}+\frac{4bd^{3/2}n\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{3e}-\frac{4bdn\sqrt{d+ex}}{3e}-\frac{4bn(d+ex)^{3/2}}{9e}$$

```
[In] Int[Sqrt[d + e*x]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-4*b*d*n*Sqrt[d + e*x])/(3*e) - (4*b*n*(d + e*x)^(3/2))/(9*e) + (4*b*d^(3/
2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(3*e) + (2*(d + e*x)^(3/2)*(a + b*Log[
c*x^n]))/(3*e)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(d+ex)^{3/2}\,(a+b\log(cx^n))}{3e} - \frac{(2bn)\int\frac{(d+ex)^{3/2}}{x}\,dx}{3e}\\
&= -\frac{4bn(d+ex)^{3/2}}{9e} + \frac{2(d+ex)^{3/2}\,(a+b\log(cx^n))}{3e} - \frac{(2bdn)\int\frac{\sqrt{d+ex}}{x}\,dx}{3e}\\
&= -\frac{4bdn\sqrt{d+ex}}{3e} - \frac{4bn(d+ex)^{3/2}}{9e} + \frac{2(d+ex)^{3/2}\,(a+b\log(cx^n))}{3e} - \frac{(2bd^2n)\int\frac{1}{x\sqrt{d+ex}}\,dx}{3e}\\
&= -\frac{4bdn\sqrt{d+ex}}{3e} - \frac{4bn(d+ex)^{3/2}}{9e} + \frac{2(d+ex)^{3/2}\,(a+b\log(cx^n))}{3e}\\
&\quad - \frac{(4bd^2n)\,\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{3e^2}\\
&= -\frac{4bdn\sqrt{d+ex}}{3e} - \frac{4bn(d+ex)^{3/2}}{9e} + \frac{4bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{3e} + \frac{2(d+ex)^{3/2}\,(a+b\log(cx^n))}{3e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

$$\int \sqrt{d+ex}(a+b\log(cx^n))\,dx$$
$$=\frac{2\left(6bd^{3/2}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)+\sqrt{d+ex}(3a(d+ex)-2bn(4d+ex)+3b(d+ex)\log(cx^n))\right)}{9e}$$

```
[In] Integrate[Sqrt[d + e*x]*(a + b*Log[c*x^n]),x]
[Out] (2*(6*b*d^(3/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + Sqrt[d + e*x]*(3*a*(d +
e*x) - 2*b*n*(4*d + e*x) + 3*b*(d + e*x)*Log[c*x^n])))/(9*e)
```

Maple [F]

$$\int (a+b\ln(c\,x^n))\sqrt{ex+d}dx$$

```
[In] int((a+b*ln(c*x^n))*(e*x+d)^(1/2),x)
[Out] int((a+b*ln(c*x^n))*(e*x+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.96

$$\int \sqrt{d+ex}(a+b\log(cx^n))\,dx$$
$$=\left[\frac{2\left(3\,bd^{\frac{3}{2}}n\log\left(\frac{ex+2\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)-(8\,bdn-3\,ad+(2\,ben-3\,ae)x-3\,(bex+bd)\log(c)-3\,(benx+bd\right.}{9\,e}\right.$$
$$-\frac{2\left(6\,b\sqrt{-d}dn\arctan\left(\frac{\sqrt{ex+d}\sqrt{-d}}{d}\right)+(8\,bdn-3\,ad+(2\,ben-3\,ae)x-3\,(bex+bd)\log(c)-3\,(benx+\right.}{9\,e}$$

```
[In] integrate((a+b*log(c*x^n))*(e*x+d)^(1/2),x, algorithm="fricas")
[Out] [2/9*(3*b*d^(3/2)*n*log((e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - (8*b*d*n
 - 3*a*d + (2*b*e*n - 3*a*e)*x - 3*(b*e*x + b*d)*log(c) - 3*(b*e*n*x + b*d*
n)*log(x))*sqrt(e*x + d))/e, -2/9*(6*b*sqrt(-d)*d*n*arctan(sqrt(e*x + d)*sq
rt(-d)/d) + (8*b*d*n - 3*a*d + (2*b*e*n - 3*a*e)*x - 3*(b*e*x + b*d)*log(c)
 - 3*(b*e*n*x + b*d*n)*log(x))*sqrt(e*x + d))/e]
```

Sympy [A] (verification not implemented)

Time = 29.54 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.53

$$\int \sqrt{d+ex}(a+b\log(cx^n))\,dx = a\left(\begin{cases}\frac{2(d+ex)^{\frac{3}{2}}}{3e} & \text{for } e\neq 0\\ \sqrt{d}x & \text{otherwise}\end{cases}\right) - bn\left(\begin{cases}\frac{16d^{\frac{3}{2}}\sqrt{1+\frac{ex}{d}}}{9e}+\frac{2d^{\frac{3}{2}}\log\left(\frac{ex}{d}\right)}{3e}-\frac{4d^{\frac{3}{2}}\log\left(\sqrt{1+\frac{ex}{d}}+1\right)}{3e}+\frac{4\sqrt{d}x\sqrt{1+\frac{ex}{d}}}{9} & \text{for } e>-\infty\wedge e<\infty\wedge e\neq 0\\ \sqrt{d}x & \text{otherwise}\end{cases}\right) + b\left(\begin{cases}\frac{2(d+ex)^{\frac{3}{2}}}{3e} & \text{for } e\neq 0\\ \sqrt{d}x & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate((a+b*ln(c*x**n))*(e*x+d)**(1/2),x)
```

```
[Out] a*Piecewise((2*(d + e*x)**(3/2)/(3*e), Ne(e, 0)), (sqrt(d)*x, True)) - b*n*
Piecewise((16*d**(3/2)*sqrt(1 + e*x/d)/(9*e) + 2*d**(3/2)*log(e*x/d)/(3*e)
- 4*d**(3/2)*log(sqrt(1 + e*x/d) + 1)/(3*e) + 4*sqrt(d)*x*sqrt(1 + e*x/d)/9
, (e > -oo) & (e < oo) & Ne(e, 0)), (sqrt(d)*x, True)) + b*Piecewise((2*(d
+ e*x)**(3/2)/(3*e), Ne(e, 0)), (sqrt(d)*x, True))*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int \sqrt{d+ex}(a+b\log(cx^n))\,dx = \frac{2\,(ex+d)^{\frac{3}{2}}b\log(cx^n)}{3\,e} - \frac{2\left(3\,d^{\frac{3}{2}}\log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)+2\,(ex+d)^{\frac{3}{2}}+6\,\sqrt{ex+d}d\right)bn}{9\,e} + \frac{2\,(ex+d)^{\frac{3}{2}}a}{3\,e}$$

```
[In] integrate((a+b*log(c*x^n))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(e*x + d)^(3/2)*b*log(c*x^n)/e - 2/9*(3*d^(3/2)*log((sqrt(e*x + d) - sq
rt(d))/(sqrt(e*x + d) + sqrt(d))) + 2*(e*x + d)^(3/2) + 6*sqrt(e*x + d)*d)*
b*n/e + 2/3*(e*x + d)^(3/2)*a/e
```

Giac [F]

$$\int \sqrt{d+ex}(a+b\log(cx^n))\,dx = \int \sqrt{ex+d}(b\log(cx^n)+a)\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}(a+b\log(cx^n))\,dx = \int (a+b\,\ln(c\,x^n))\,\sqrt{d+e\,x}\,dx$$

```
[In] int((a + b*log(c*x^n))*(d + e*x)^(1/2),x)
[Out] int((a + b*log(c*x^n))*(d + e*x)^(1/2), x)
```

3.134 $\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 211

$$\begin{aligned}\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x}\,dx &= -4bn\sqrt{d+ex}+4b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\\
&\quad+2b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2+2\sqrt{d+ex}(a+b\log(cx^n))\\
&\quad-2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))\\
&\quad-4b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)\\
&\quad-2b\sqrt{d}n\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)\end{aligned}$$

```
[Out] 4*b*n*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)+2*b*n*arctanh((e*x+d)^(1/2)/d^
(1/2))^2*d^(1/2)-2*arctanh((e*x+d)^(1/2)/d^(1/2))*(a+b*ln(c*x^n))*d^(1/2)-4
*b*n*arctanh((e*x+d)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)-(e*x+d)^(1/2)))*d
^(1/2)-2*b*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-(e*x+d)^(1/2)))*d^(1/2)-4*b*n*(
e*x+d)^(1/2)+2*(a+b*ln(c*x^n))*(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2388, 65, 214, 2390, 12, 6131, 6055, 2449, 2352, 2356, 52}

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x} dx = -2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n)) + 2\sqrt{d+ex}(a+b\log(cx^n)) + 2b\sqrt{d}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2 + 4b\sqrt{d}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) - 4b\sqrt{d}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right) - 2b\sqrt{d}n\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right) - 4bn\sqrt{d+ex}$$

```
[In] Int[(Sqrt[d + e*x]*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] -4*b*n*Sqrt[d + e*x] + 4*b*Sqrt[d]*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + 2*b*S
qrt[d]*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]^2 + 2*Sqrt[d + e*x]*(a + b*Log[c*x^
n]) - 2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]*(a + b*Log[c*x^n]) - 4*b*Sqr
t[d]*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e
*x])] - 2*b*Sqrt[d]*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
```

```
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int \frac{a+b\log(cx^n)}{x\sqrt{d+ex}}\,dx + e\int \frac{a+b\log(cx^n)}{\sqrt{d+ex}}\,dx\\
&= 2\sqrt{d+ex}(a+b\log(cx^n)) - 2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))\\
&\quad - (2bn)\int \frac{\sqrt{d+ex}}{x}\,dx - (bdn)\int -\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}x}\,dx\\
&= -4bn\sqrt{d+ex} + 2\sqrt{d+ex}(a+b\log(cx^n))\\
&\quad - 2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))\\
&\quad + \left(2b\sqrt{d}n\right)\int \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx - (2bdn)\int \frac{1}{x\sqrt{d+ex}}\,dx\\
&= -4bn\sqrt{d+ex} + 2\sqrt{d+ex}(a+b\log(cx^n))\\
&\quad - 2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))\\
&\quad + \left(4b\sqrt{d}n\right)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx, x, \sqrt{d+ex}\right)\\
&\quad - \frac{(4bdn)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{e}\\
&= -4bn\sqrt{d+ex} + 4b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) + 2b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2\\
&\quad + 2\sqrt{d+ex}(a+b\log(cx^n)) - 2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))\\
&\quad - (4bn)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx, x, \sqrt{d+ex}\right)
\end{aligned}$$

$$= -4bn\sqrt{d+ex} + 4b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) + 2b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2 + 2\sqrt{d+ex}(a+b\log(cx^n)) - 2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n)) - 4b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right) + (4bn)\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}\,dx, x, \sqrt{d+ex}\right)$$

$$= -4bn\sqrt{d+ex} + 4b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) + 2b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2 + 2\sqrt{d+ex}(a+b\log(cx^n)) - 2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n)) - 4b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right) - \left(4b\sqrt{d}n\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)$$

$$= -4bn\sqrt{d+ex} + 4b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) + 2b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2 + 2\sqrt{d+ex}(a+b\log(cx^n)) - 2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n)) - 4b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right) - 2b\sqrt{d}n\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.57

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x}\,dx = 2a\sqrt{d+ex} - 4bn\sqrt{d+ex} + 4b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) + 2b\sqrt{d+ex}\log(cx^n) + \sqrt{d}(a+b\log(cx^n))\log\left(\sqrt{d}-\sqrt{d+ex}\right) - \sqrt{d}(a+b\log(cx^n))\log\left(\sqrt{d}+\sqrt{d+ex}\right) - \frac{1}{2}b\sqrt{d}n\left(\log\left(\sqrt{d}-\sqrt{d+ex}\right)\left(\log\left(\sqrt{d}-\sqrt{d+ex}\right) + 2\log\left(\frac{1}{2}\left(1+\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\right)\right) + 2\operatorname{PolyLog}\left(2,\frac{1}{2}-\frac{\sqrt{d+ex}}{2\sqrt{d}}\right)\right) + \frac{1}{2}b\sqrt{d}n\left(\log\left(\sqrt{d}+\sqrt{d+ex}\right)\left(\log\left(\sqrt{d}+\sqrt{d+ex}\right) + 2\log\left(\frac{1}{2}-\frac{\sqrt{d+ex}}{2\sqrt{d}}\right)\right) + 2\operatorname{PolyLog}\left(2,\frac{1}{2}\left(1+\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\right)\right)$$

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] 2*a*Sqrt[d + e*x] - 4*b*n*Sqrt[d + e*x] + 4*b*Sqrt[d]*n*ArcTanh[Sqrt[d + e*
x]/Sqrt[d]] + 2*b*Sqrt[d + e*x]*Log[c*x^n] + Sqrt[d]*(a + b*Log[c*x^n])*Log
[Sqrt[d] - Sqrt[d + e*x]] - Sqrt[d]*(a + b*Log[c*x^n])*Log[Sqrt[d] + Sqrt[d
 + e*x]] - (b*Sqrt[d]*n*(Log[Sqrt[d] - Sqrt[d + e*x]]*(Log[Sqrt[d] - Sqrt[d
 + e*x]] + 2*Log[(1 + Sqrt[d + e*x]/Sqrt[d])/2]) + 2*PolyLog[2, 1/2 - Sqrt[
d + e*x]/(2*Sqrt[d])]))/2 + (b*Sqrt[d]*n*(Log[Sqrt[d] + Sqrt[d + e*x]]*(Log
[Sqrt[d] + Sqrt[d + e*x]] + 2*Log[1/2 - Sqrt[d + e*x]/(2*Sqrt[d])]) + 2*Pol
yLog[2, (1 + Sqrt[d + e*x]/Sqrt[d])/2]))/2
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))\sqrt{ex+d}}{x}dx$$

```
[In] int((a+b*ln(c*x^n))*(e*x+d)^(1/2)/x,x)
[Out] int((a+b*ln(c*x^n))*(e*x+d)^(1/2)/x,x)
```

Fricas [F]

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{ex+d}(b\log(cx^n)+a)}{x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x+d)^(1/2)/x,x, algorithm="fricas")
[Out] integral((sqrt(e*x + d)*b*log(c*x^n) + sqrt(e*x + d)*a)/x, x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x}\,dx = \int \frac{(a+b\log(cx^n))\sqrt{d+ex}}{x}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))*(e*x+d)**(1/2)/x,x)
[Out] Integral((a + b*log(c*x**n))*sqrt(d + e*x)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{ex+d}(b\log(cx^n)+a)}{x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x+d)^(1/2)/x,x, algorithm="maxima")
[Out] (sqrt(d)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d))) + 2*sqrt(
e*x + d))*a + b*integrate(sqrt(e*x + d)*(log(c) + log(x^n))/x, x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{ex+d}(b\log(cx^n)+a)}{x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x+d)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*log(c*x^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x}\,dx = \int \frac{(a+b\ln(c\,x^n))\,\sqrt{d+e\,x}}{x}\,dx$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^(1/2))/x,x)
[Out] int(((a + b*log(c*x^n))*(d + e*x)^(1/2))/x, x)
```

3.135 $\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 221

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x^2}\,dx = -\frac{bn\sqrt{d+ex}}{x} - \frac{ben\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{ben\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{\sqrt{d}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x} - \frac{e\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} - \frac{2ben\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{\sqrt{d}} - \frac{ben\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{\sqrt{d}}$$

```
[Out] -b*e*n*arctanh((e*x+d)^(1/2)/d^(1/2))/d^(1/2)+b*e*n*arctanh((e*x+d)^(1/2)/d
^(1/2))^2/d^(1/2)-e*arctanh((e*x+d)^(1/2)/d^(1/2))*(a+b*ln(c*x^n))/d^(1/2)-
2*b*e*n*arctanh((e*x+d)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)-(e*x+d)^(1/2))
)/d^(1/2)-b*e*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-(e*x+d)^(1/2)))/d^(1/2)-b*n*
(e*x+d)^(1/2)/x-(a+b*ln(c*x^n))*(e*x+d)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {43, 65, 214, 2392, 14, 6131, 6055, 2449, 2352}

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x^2}\,dx = -\frac{e\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x} + \frac{ben\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{\sqrt{d}} - \frac{ben\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{2ben\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{\sqrt{d}} - \frac{ben\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{\sqrt{d}} - \frac{bn\sqrt{d+ex}}{x}$$

```
[In] Int[(Sqrt[d + e*x]*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -((b*n*Sqrt[d + e*x])/x) - (b*e*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] +
 (b*e*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]^2)/Sqrt[d] - (Sqrt[d + e*x]*(a + b*L
og[c*x^n]))/x - (e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]*(a + b*Log[c*x^n]))/Sqrt[
d] - (2*b*e*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqr
t[d + e*x])])/Sqrt[d] - (b*e*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d
 + e*x])])/Sqrt[d]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{d+ex}(a+b\log(cx^n))}{x} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} - (bn)\int \frac{-\sqrt{d+ex} - \frac{ex\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{x^2}\,dx$$

$$= -\frac{\sqrt{d+ex}(a+b\log(cx^n))}{x} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} - (bn)\int \left(-\frac{\sqrt{d+ex}}{x^2} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}x}\right)dx$$

$$= -\frac{\sqrt{d+ex}(a+b\log(cx^n))}{x} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} + (bn)\int \frac{\sqrt{d+ex}}{x^2}\,dx + \frac{(ben)\int \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx}{\sqrt{d}}$$

$$= -\frac{bn\sqrt{d+ex}}{x} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} + \frac{1}{2}(ben)\int \frac{1}{x\sqrt{d+ex}}\,dx + \frac{(2ben)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx, x, \sqrt{d+ex}\right)}{\sqrt{d}}$$

$$= -\frac{bn\sqrt{d+ex}}{x} + \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{\sqrt{d}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} + (bn)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right) - \frac{(2ben)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx, x, \sqrt{d+ex}\right)}{d}$$

$$= -\frac{bn\sqrt{d+ex}}{x} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{\sqrt{d}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} - \frac{2ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{\sqrt{d}} + \frac{(2ben)\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}dx, x, \sqrt{d+ex}\right)}{d}$$

$$= -\frac{bn\sqrt{d+ex}}{x} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{\sqrt{d}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} - \frac{2ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{\sqrt{d}} - \frac{(2ben)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}dx, x, \frac{1}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)}{\sqrt{d}}$$

$$= -\frac{bn\sqrt{d+ex}}{x} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{\sqrt{d}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} - \frac{2ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{\sqrt{d}} - \frac{ben\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)}{\sqrt{d}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x^2}dx = -\frac{4a\sqrt{d}\sqrt{d+ex} + 4b\sqrt{d}n\sqrt{d+ex} + 4benx\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) + 4b\sqrt{d}\sqrt{d+ex}\log(cx^n) - 2aex\log\left(\sqrt{d} - \right.}{}$$

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -1/4*(4*a*Sqrt[d]*Sqrt[d + e*x] + 4*b*Sqrt[d]*n*Sqrt[d + e*x] + 4*b*e*n*x*A
rcTanh[Sqrt[d + e*x]/Sqrt[d]] + 4*b*Sqrt[d]*Sqrt[d + e*x]*Log[c*x^n] - 2*a*
e*x*Log[Sqrt[d] - Sqrt[d + e*x]] - 2*b*e*x*Log[c*x^n]*Log[Sqrt[d] - Sqrt[d
+ e*x]] + b*e*n*x*Log[Sqrt[d] - Sqrt[d + e*x]]^2 + 2*a*e*x*Log[Sqrt[d] + Sq
rt[d + e*x]] + 2*b*e*x*Log[c*x^n]*Log[Sqrt[d] + Sqrt[d + e*x]] - b*e*n*x*Lo
g[Sqrt[d] + Sqrt[d + e*x]]^2 - 2*b*e*n*x*Log[Sqrt[d] + Sqrt[d + e*x]]*Log[1
/2 - Sqrt[d + e*x]/(2*Sqrt[d])] + 2*b*e*n*x*Log[Sqrt[d] - Sqrt[d + e*x]]*Lo
g[(1 + Sqrt[d + e*x]/Sqrt[d])/2] + 2*b*e*n*x*PolyLog[2, 1/2 - Sqrt[d + e*x]
/(2*Sqrt[d])] - 2*b*e*n*x*PolyLog[2, (1 + Sqrt[d + e*x]/Sqrt[d])/2])/(Sqrt[
d]*x)
```

Maple [F]

$$\int \frac{(a+b\ln (c\,x^{n}))\,\sqrt{ex+d}}{x^{2}}dx$$

```
[In] int((a+b*ln(c*x^n))*(e*x+d)^(1/2)/x^2,x)
```

```
[Out] int((a+b*ln(c*x^n))*(e*x+d)^(1/2)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x^2}\,dx=\int \frac{\sqrt{ex+d}(b\log(cx^n)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x+d)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((sqrt(e*x + d)*b*log(c*x^n) + sqrt(e*x + d)*a)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x^2}\,dx=\int \frac{(a+b\log(cx^n))\,\sqrt{d+ex}}{x^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))*(e*x+d)**(1/2)/x**2,x)
```

```
[Out] Integral((a + b*log(c*x**n))*sqrt(d + e*x)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x^2}\,dx = \int \frac{\sqrt{ex+d}(b\log(cx^n)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x+d)^(1/2)/x^2,x, algorithm="maxima")
[Out] 1/2*(e*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/sqrt(d) - 2
*sqrt(e*x + d)/x)*a + b*integrate(sqrt(e*x + d)*(log(c) + log(x^n))/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x^2}\,dx = \int \frac{\sqrt{ex+d}(b\log(cx^n)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x+d)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*log(c*x^n) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x^2}\,dx = \int \frac{(a+b\,\ln(c\,x^n))\,\sqrt{d+e\,x}}{x^2}\,dx$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^(1/2))/x^2,x)
[Out] int(((a + b*log(c*x^n))*(d + e*x)^(1/2))/x^2, x)
```

3.136 $\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 298

$$\begin{aligned}\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x^3}\,dx =& -\frac{bn\sqrt{d+ex}}{4x^2}-\frac{3ben\sqrt{d+ex}}{8dx}\\ &-\frac{be^2n\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{8d^{3/2}}-\frac{be^2n\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4d^{3/2}}\\ &-\frac{\sqrt{d+ex}(a+b\log(cx^n))}{2x^2}-\frac{e\sqrt{d+ex}(a+b\log(cx^n))}{4dx}\\ &+\frac{e^2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{3/2}}\\ &+\frac{be^2n\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{2d^{3/2}}\\ &+\frac{be^2n\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{4d^{3/2}}\end{aligned}$$

```
[Out] -1/8*b*e^2*n*arctanh((e*x+d)^(1/2)/d^(1/2))/d^(3/2)-1/4*b*e^2*n*arctanh((e*
x+d)^(1/2)/d^(1/2))^2/d^(3/2)+1/4*e^2*arctanh((e*x+d)^(1/2)/d^(1/2))*(a+b*l
n(c*x^n))/d^(3/2)+1/2*b*e^2*n*arctanh((e*x+d)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(
d^(1/2)-(e*x+d)^(1/2)))/d^(3/2)+1/4*b*e^2*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-
(e*x+d)^(1/2)))/d^(3/2)-1/4*b*n*(e*x+d)^(1/2)/x^2-3/8*b*e*n*(e*x+d)^(1/2)/d
/x-1/2*(a+b*ln(c*x^n))*(e*x+d)^(1/2)/x^2-1/4*e*(a+b*ln(c*x^n))*(e*x+d)^(1/2
)/d/x
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {43, 44, 65, 214, 2392, 12, 14, 6131, 6055, 2449, 2352}

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x^3}\,dx = \frac{e^2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{3/2}} - \frac{e\sqrt{d+ex}(a+b\log(cx^n))}{4dx} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{2x^2} - \frac{be^2n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4d^{3/2}} - \frac{be^2n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{8d^{3/2}} + \frac{be^2n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{2d^{3/2}} + \frac{be^2n\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{4d^{3/2}} - \frac{bn\sqrt{d+ex}}{4x^2} - \frac{3ben\sqrt{d+ex}}{8dx}$$

```
[In] Int[(Sqrt[d + e*x]*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/4*(b*n*Sqrt[d + e*x])/x^2 - (3*b*e*n*Sqrt[d + e*x])/(8*d*x) - (b*e^2*n*A
rcTanh[Sqrt[d + e*x]/Sqrt[d]])/(8*d^(3/2)) - (b*e^2*n*ArcTanh[Sqrt[d + e*x]
/Sqrt[d]]^2)/(4*d^(3/2)) - (Sqrt[d + e*x]*(a + b*Log[c*x^n]))/(2*x^2) - (e*
Sqrt[d + e*x]*(a + b*Log[c*x^n]))/(4*d*x) + (e^2*ArcTanh[Sqrt[d + e*x]/Sqrt
[d]]*(a + b*Log[c*x^n]))/(4*d^(3/2)) + (b*e^2*n*ArcTanh[Sqrt[d + e*x]/Sqrt[
d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x])])/(2*d^(3/2)) + (b*e^2*n*Poly
Log[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x])])/(4*d^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
```

```
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+ex}(a+b\log(cx^n))}{2x^2} - \frac{e\sqrt{d+ex}(a+b\log(cx^n))}{4dx} \\
&\quad + \frac{e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{3/2}} \\
&\quad - (bn)\int \frac{-\sqrt{d}\sqrt{d+ex}(2d+ex)+e^2x^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4d^{3/2}x^3}\,dx \\
&= -\frac{\sqrt{d+ex}(a+b\log(cx^n))}{2x^2} - \frac{e\sqrt{d+ex}(a+b\log(cx^n))}{4dx} \\
&\quad + \frac{e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{3/2}} \\
&\quad - \frac{(bn)\int \frac{-\sqrt{d}\sqrt{d+ex}(2d+ex)+e^2x^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x^3}\,dx}{4d^{3/2}} \\
&= -\frac{\sqrt{d+ex}(a+b\log(cx^n))}{2x^2} - \frac{e\sqrt{d+ex}(a+b\log(cx^n))}{4dx} \\
&\quad + \frac{e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{3/2}} \\
&\quad - \frac{(bn)\int\left(-\frac{2d^{3/2}\sqrt{d+ex}}{x^3} - \frac{\sqrt{d}e\sqrt{d+ex}}{x^2} + \frac{e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\right)dx}{4d^{3/2}} \\
&= -\frac{\sqrt{d+ex}(a+b\log(cx^n))}{2x^2} - \frac{e\sqrt{d+ex}(a+b\log(cx^n))}{4dx} \\
&\quad + \frac{e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{3/2}} \\
&\quad + \frac{1}{2}(bn)\int \frac{\sqrt{d+ex}}{x^3}\,dx + \frac{(ben)\int \frac{\sqrt{d+ex}}{x^2}\,dx}{4d} - \frac{(be^2n)\int \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx}{4d^{3/2}}
\end{aligned}$$

$$= -\frac{bn\sqrt{d+ex}}{4x^2} - \frac{ben\sqrt{d+ex}}{4dx} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{2x^2} - \frac{e\sqrt{d+ex}(a+b\log(cx^n))}{4dx} + \frac{e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{3/2}} + \frac{1}{8}(ben)\int\frac{1}{x^2\sqrt{d+ex}}\,dx - \frac{(be^2n)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx,x,\sqrt{d+ex}\right)}{2d^{3/2}} + \frac{(be^2n)\int\frac{1}{x\sqrt{d+ex}}\,dx}{8d}$$

$$= -\frac{bn\sqrt{d+ex}}{4x^2} - \frac{3ben\sqrt{d+ex}}{8dx} - \frac{be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4d^{3/2}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{2x^2} - \frac{e\sqrt{d+ex}(a+b\log(cx^n))}{4dx} + \frac{e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{3/2}} + \frac{(ben)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{4d} + \frac{(be^2n)\,\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx,x,\sqrt{d+ex}\right)}{2d^2} - \frac{(be^2n)\int\frac{1}{x\sqrt{d+ex}}\,dx}{16d}$$

$$= -\frac{bn\sqrt{d+ex}}{4x^2} - \frac{3ben\sqrt{d+ex}}{8dx} - \frac{be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4d^{3/2}} - \frac{be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4d^{3/2}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{2x^2} - \frac{e\sqrt{d+ex}(a+b\log(cx^n))}{4dx} + \frac{e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{3/2}} + \frac{be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{2d^{3/2}} - \frac{(ben)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{8d} - \frac{(be^2n)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}\,dx,x,\sqrt{d+ex}\right)}{2d^2}$$

$$= -\frac{bn\sqrt{d+ex}}{4x^2} - \frac{3ben\sqrt{d+ex}}{8dx} - \frac{be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{8d^{3/2}} - \frac{be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4d^{3/2}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{2x^2} - \frac{e\sqrt{d+ex}(a+b\log(cx^n))}{4dx} + \frac{e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{3/2}} + \frac{be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{2d^{3/2}} + \frac{(be^2n)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)}{2d^{3/2}}$$

$$= -\frac{bn\sqrt{d+ex}}{4x^2} - \frac{3ben\sqrt{d+ex}}{8dx} - \frac{be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{8d^{3/2}} - \frac{be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4d^{3/2}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{2x^2} - \frac{e\sqrt{d+ex}(a+b\log(cx^n))}{4dx} + \frac{e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{3/2}} + \frac{be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{2d^{3/2}} + \frac{be^2n\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)}{4d^{3/2}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.68

$$\int\frac{\sqrt{d+ex}(a+b\log(cx^n))}{x^3}\,dx = -\frac{8ad^{3/2}\sqrt{d+ex} + 4bd^{3/2}n\sqrt{d+ex} + 4a\sqrt{d}ex\sqrt{d+ex} + 6b\sqrt{d}enx\sqrt{d+ex} + 2be^2nx^2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) + \cdots}{\cdots}$$

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/16*(8*a*d^(3/2)*Sqrt[d + e*x] + 4*b*d^(3/2)*n*Sqrt[d + e*x] + 4*a*Sqrt[d
]*e*x*Sqrt[d + e*x] + 6*b*Sqrt[d]*e*n*x*Sqrt[d + e*x] + 2*b*e^2*n*x^2*ArcTa
nh[Sqrt[d + e*x]/Sqrt[d]] + 8*b*d^(3/2)*Sqrt[d + e*x]*Log[c*x^n] + 4*b*Sqrt
[d]*e*x*Sqrt[d + e*x]*Log[c*x^n] + 2*a*e^2*x^2*Log[Sqrt[d] - Sqrt[d + e*x]]
 + 2*b*e^2*x^2*Log[c*x^n]*Log[Sqrt[d] - Sqrt[d + e*x]] - b*e^2*n*x^2*Log[Sq
rt[d] - Sqrt[d + e*x]]^2 - 2*a*e^2*x^2*Log[Sqrt[d] + Sqrt[d + e*x]] - 2*b*e
^2*x^2*Log[c*x^n]*Log[Sqrt[d] + Sqrt[d + e*x]] + b*e^2*n*x^2*Log[Sqrt[d] +
Sqrt[d + e*x]]^2 + 2*b*e^2*n*x^2*Log[Sqrt[d] + Sqrt[d + e*x]]*Log[1/2 - Sqr
t[d + e*x]/(2*Sqrt[d])] - 2*b*e^2*n*x^2*Log[Sqrt[d] - Sqrt[d + e*x]]*Log[(1
 + Sqrt[d + e*x]/Sqrt[d])/2] - 2*b*e^2*n*x^2*PolyLog[2, 1/2 - Sqrt[d + e*x]
/(2*Sqrt[d])] + 2*b*e^2*n*x^2*PolyLog[2, (1 + Sqrt[d + e*x]/Sqrt[d])/2])/(d
^(3/2)*x^2)
```

Maple [F]

$$\int \frac{(a + b \ln (c x^n)) \sqrt{ex + d}}{x^3} dx$$

[In] int((a+b*ln(c*x^n))*(e*x+d)^(1/2)/x^3,x)

[Out] int((a+b*ln(c*x^n))*(e*x+d)^(1/2)/x^3,x)

Fricas [F]

$$\int \frac{\sqrt{d + ex}(a + b \log (cx^n))}{x^3} dx = \int \frac{\sqrt{ex + d}(b \log (cx^n) + a)}{x^3} dx$$

[In] integrate((a+b*log(c*x^n))*(e*x+d)^(1/2)/x^3,x, algorithm="fricas")

[Out] integral((sqrt(e*x + d)*b*log(c*x^n) + sqrt(e*x + d)*a)/x^3, x)

Sympy [F]

$$\int \frac{\sqrt{d + ex}(a + b \log (cx^n))}{x^3} dx = \int \frac{(a + b \log (cx^n)) \sqrt{d + ex}}{x^3} dx$$

[In] integrate((a+b*ln(c*x**n))*(e*x+d)**(1/2)/x**3,x)

[Out] Integral((a + b*log(c*x**n))*sqrt(d + e*x)/x**3, x)

Maxima [F]

$$\int \frac{\sqrt{d + ex}(a + b \log (cx^n))}{x^3} dx = \int \frac{\sqrt{ex + d}(b \log (cx^n) + a)}{x^3} dx$$

[In] integrate((a+b*log(c*x^n))*(e*x+d)^(1/2)/x^3,x, algorithm="maxima")

[Out] -1/8*(e^2*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/d^(3/2) + 2*((e*x + d)^(3/2)*e^2 + sqrt(e*x + d)*d*e^2)/((e*x + d)^2*d - 2*(e*x + d)*d^2 + d^3))*a + b*integrate(sqrt(e*x + d)*(log(c) + log(x^n))/x^3, x)

Giac [F]

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\sqrt{ex+d}(b\log(cx^n)+a)}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x+d)^(1/2)/x^3,x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*log(c*x^n) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x^3}\,dx = \int \frac{(a+b\,\ln(c\,x^n))\,\sqrt{d+e\,x}}{x^3}\,dx$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^(1/2))/x^3,x)
[Out] int(((a + b*log(c*x^n))*(d + e*x)^(1/2))/x^3, x)
```

3.137 $\int x^3(d+ex)^{3/2}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 263

$$\int x^3(d+ex)^{3/2}(a+b\log(cx^n))\,dx = \frac{64bd^5n\sqrt{d+ex}}{1155e^4} + \frac{64bd^4n(d+ex)^{3/2}}{3465e^4} + \frac{64bd^3n(d+ex)^{5/2}}{5775e^4} - \frac{172bd^2n(d+ex)^{7/2}}{1617e^4} + \frac{32bdn(d+ex)^{9/2}}{297e^4} - \frac{4bn(d+ex)^{11/2}}{121e^4} - \frac{64bd^{11/2}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{1155e^4} - \frac{2d^3(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4} + \frac{6d^2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^4} - \frac{2d(d+ex)^{9/2}(a+b\log(cx^n))}{3e^4} + \frac{2(d+ex)^{11/2}(a+b\log(cx^n))}{11e^4}$$

```
[Out] 64/3465*b*d^4*n*(e*x+d)^(3/2)/e^4+64/5775*b*d^3*n*(e*x+d)^(5/2)/e^4-172/161
7*b*d^2*n*(e*x+d)^(7/2)/e^4+32/297*b*d*n*(e*x+d)^(9/2)/e^4-4/121*b*n*(e*x+d
)^(11/2)/e^4-64/1155*b*d^(11/2)*n*arctanh((e*x+d)^(1/2)/d^(1/2))/e^4-2/5*d^
3*(e*x+d)^(5/2)*(a+b*ln(c*x^n))/e^4+6/7*d^2*(e*x+d)^(7/2)*(a+b*ln(c*x^n))/e
^4-2/3*d*(e*x+d)^(9/2)*(a+b*ln(c*x^n))/e^4+2/11*(e*x+d)^(11/2)*(a+b*ln(c*x^
n))/e^4+64/1155*b*d^5*n*(e*x+d)^(1/2)/e^4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {45, 2392, 12, 1634, 52, 65, 214}

$$\int x^3(d+ex)^{3/2}(a+b\log(cx^n))\,dx = -\frac{2d^3(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4} + \frac{6d^2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^4} - \frac{2d(d+ex)^{9/2}(a+b\log(cx^n))}{3e^4} + \frac{2(d+ex)^{11/2}(a+b\log(cx^n))}{11e^4} - \frac{64bd^{11/2}n\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{1155e^4} + \frac{64bd^5n\sqrt{d+ex}}{1155e^4} + \frac{64bd^4n(d+ex)^{3/2}}{3465e^4} + \frac{64bd^3n(d+ex)^{5/2}}{5775e^4} - \frac{172bd^2n(d+ex)^{7/2}}{1617e^4} + \frac{32bdn(d+ex)^{9/2}}{297e^4} - \frac{4bn(d+ex)^{11/2}}{121e^4}$$

```
[In] Int[x^3*(d + e*x)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (64*b*d^5*n*Sqrt[d + e*x])/(1155*e^4) + (64*b*d^4*n*(d + e*x)^(3/2))/(3465*
e^4) + (64*b*d^3*n*(d + e*x)^(5/2))/(5775*e^4) - (172*b*d^2*n*(d + e*x)^(7/
2))/(1617*e^4) + (32*b*d*n*(d + e*x)^(9/2))/(297*e^4) - (4*b*n*(d + e*x)^(1
1/2))/(121*e^4) - (64*b*d^(11/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(1155*e^
4) - (2*d^3*(d + e*x)^(5/2)*(a + b*Log[c*x^n]))/(5*e^4) + (6*d^2*(d + e*x)^
(7/2)*(a + b*Log[c*x^n]))/(7*e^4) - (2*d*(d + e*x)^(9/2)*(a + b*Log[c*x^n])
)/(3*e^4) + (2*(d + e*x)^(11/2)*(a + b*Log[c*x^n]))/(11*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d^3(d+ex)^{5/2}\,(a+b\log{(cx^n)})}{5e^4}+\frac{6d^2(d+ex)^{7/2}\,(a+b\log{(cx^n)})}{7e^4}\\
&\quad-\frac{2d(d+ex)^{9/2}\,(a+b\log{(cx^n)})}{3e^4}+\frac{2(d+ex)^{11/2}\,(a+b\log{(cx^n)})}{11e^4}\\
&\quad-(bn)\int\frac{2(d+ex)^{5/2}\,(-16d^3+40d^2ex-70de^2x^2+105e^3x^3)}{1155e^4x}\,dx\\
&= -\frac{2d^3(d+ex)^{5/2}\,(a+b\log{(cx^n)})}{5e^4}+\frac{6d^2(d+ex)^{7/2}\,(a+b\log{(cx^n)})}{7e^4}\\
&\quad-\frac{2d(d+ex)^{9/2}\,(a+b\log{(cx^n)})}{3e^4}+\frac{2(d+ex)^{11/2}\,(a+b\log{(cx^n)})}{11e^4}\\
&\quad-\frac{(2bn)\int\frac{(d+ex)^{5/2}(-16d^3+40d^2ex-70de^2x^2+105e^3x^3)}{x}\,dx}{1155e^4}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2d^3(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^4}+\frac{6d^2(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^4} \\
&\quad -\frac{2d(d+ex)^{9/2}\left(a+b\log\left(cx^n\right)\right)}{3e^4}+\frac{2(d+ex)^{11/2}\left(a+b\log\left(cx^n\right)\right)}{11e^4} \\
&\quad -\frac{(2bn)\int\left(215d^2e(d+ex)^{5/2}-\frac{16d^3(d+ex)^{5/2}}{x}-280de(d+ex)^{7/2}+105e(d+ex)^{9/2}\right)dx}{1155e^4} \\
&= -\frac{172bd^2n(d+ex)^{7/2}}{1617e^4}+\frac{32bdn(d+ex)^{9/2}}{297e^4} \\
&\quad -\frac{4bn(d+ex)^{11/2}}{121e^4}-\frac{2d^3(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^4} \\
&\quad +\frac{6d^2(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^4}-\frac{2d(d+ex)^{9/2}\left(a+b\log\left(cx^n\right)\right)}{3e^4} \\
&\quad +\frac{2(d+ex)^{11/2}\left(a+b\log\left(cx^n\right)\right)}{11e^4}+\frac{(32bd^3n)\int\frac{(d+ex)^{5/2}}{x}\,dx}{1155e^4} \\
&= \frac{64bd^3n(d+ex)^{5/2}}{5775e^4}-\frac{172bd^2n(d+ex)^{7/2}}{1617e^4}+\frac{32bdn(d+ex)^{9/2}}{297e^4} \\
&\quad -\frac{4bn(d+ex)^{11/2}}{121e^4}-\frac{2d^3(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^4} \\
&\quad +\frac{6d^2(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^4}-\frac{2d(d+ex)^{9/2}\left(a+b\log\left(cx^n\right)\right)}{3e^4} \\
&\quad +\frac{2(d+ex)^{11/2}\left(a+b\log\left(cx^n\right)\right)}{11e^4}+\frac{(32bd^4n)\int\frac{(d+ex)^{3/2}}{x}\,dx}{1155e^4} \\
&= \frac{64bd^4n(d+ex)^{3/2}}{3465e^4}+\frac{64bd^3n(d+ex)^{5/2}}{5775e^4}-\frac{172bd^2n(d+ex)^{7/2}}{1617e^4}+\frac{32bdn(d+ex)^{9/2}}{297e^4} \\
&\quad -\frac{4bn(d+ex)^{11/2}}{121e^4}-\frac{2d^3(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^4}+\frac{6d^2(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^4} \\
&\quad -\frac{2d(d+ex)^{9/2}\left(a+b\log\left(cx^n\right)\right)}{3e^4}+\frac{2(d+ex)^{11/2}\left(a+b\log\left(cx^n\right)\right)}{11e^4}+\frac{(32bd^5n)\int\frac{\sqrt{d+ex}}{x}\,dx}{1155e^4} \\
&= \frac{64bd^5n\sqrt{d+ex}}{1155e^4}+\frac{64bd^4n(d+ex)^{3/2}}{3465e^4}+\frac{64bd^3n(d+ex)^{5/2}}{5775e^4}-\frac{172bd^2n(d+ex)^{7/2}}{1617e^4} \\
&\quad +\frac{32bdn(d+ex)^{9/2}}{297e^4}-\frac{4bn(d+ex)^{11/2}}{121e^4}-\frac{2d^3(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^4} \\
&\quad +\frac{6d^2(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^4}-\frac{2d(d+ex)^{9/2}\left(a+b\log\left(cx^n\right)\right)}{3e^4} \\
&\quad +\frac{2(d+ex)^{11/2}\left(a+b\log\left(cx^n\right)\right)}{11e^4}+\frac{(32bd^6n)\int\frac{1}{x\sqrt{d+ex}}\,dx}{1155e^4}
\end{aligned}
$$

$$= \frac{64bd^5n\sqrt{d+ex}}{1155e^4} + \frac{64bd^4n(d+ex)^{3/2}}{3465e^4} + \frac{64bd^3n(d+ex)^{5/2}}{5775e^4} - \frac{172bd^2n(d+ex)^{7/2}}{1617e^4} + \frac{32bdn(d+ex)^{9/2}}{297e^4} - \frac{4bn(d+ex)^{11/2}}{121e^4} - \frac{2d^3(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4} + \frac{6d^2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^4} - \frac{2d(d+ex)^{9/2}(a+b\log(cx^n))}{3e^4} + \frac{2(d+ex)^{11/2}(a+b\log(cx^n))}{11e^4} + \frac{(64bd^6n)\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{1155e^5}$$

$$= \frac{64bd^5n\sqrt{d+ex}}{1155e^4} + \frac{64bd^4n(d+ex)^{3/2}}{3465e^4} + \frac{64bd^3n(d+ex)^{5/2}}{5775e^4} - \frac{172bd^2n(d+ex)^{7/2}}{1617e^4} + \frac{32bdn(d+ex)^{9/2}}{297e^4} - \frac{4bn(d+ex)^{11/2}}{121e^4} - \frac{64bd^{11/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{1155e^4} - \frac{2d^3(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4} + \frac{6d^2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^4} - \frac{2d(d+ex)^{9/2}(a+b\log(cx^n))}{3e^4} + \frac{2(d+ex)^{11/2}(a+b\log(cx^n))}{11e^4}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.71

$$\int x^3(d+ex)^{3/2}(a+b\log(cx^n))\,dx = \frac{-221760bd^{11/2}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) + 2\sqrt{d+ex}(-3465a(d+ex)^2(16d^3-40d^2ex+70de^2x$$

```
[In] Integrate[x^3*(d + e*x)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (-221760*b*d^(11/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + 2*Sqrt[d + e*x]*(-34
65*a*(d + e*x)^2*(16*d^3 - 40*d^2*e*x + 70*d*e^2*x^2 - 105*e^3*x^3) + 2*b*n
*(53308*d^5 - 12794*d^4*e*x + 7863*d^3*e^2*x^2 - 5975*d^2*e^3*x^3 - 57575*d
*e^4*x^4 - 33075*e^5*x^5) - 3465*b*(d + e*x)^2*(16*d^3 - 40*d^2*e*x + 70*d*
e^2*x^2 - 105*e^3*x^3)*Log[c*x^n]))/(4002075*e^4)
```

Maple [F]

$$\int x^{3}(ex+d)^{\frac{3}{2}}\left(a+b\ln\left(c\,x^{n}\right)\right)dx$$

```
[In] int(x^3*(e*x+d)^(3/2)*(a+b*ln(c*x^n)),x)
[Out] int(x^3*(e*x+d)^(3/2)*(a+b*ln(c*x^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 595, normalized size of antiderivative = 2.26

$$\int x^3(d+ex)^{3/2}(a+b\log\left(cx^n\right))\,dx = \left[\frac{2\left(55440\,bd^{\frac{11}{2}}n\log\left(\frac{ex-2\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)+(106616\,bd^5n-55440\,ad^5-33075\,(2\,be^5n-11\,a\right.}{}\right.$$

```
[In] integrate(x^3*(e*x+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] [2/4002075*(55440*b*d^(11/2)*n*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x)
 + (106616*b*d^5*n - 55440*a*d^5 - 33075*(2*b*e^5*n - 11*a*e^5)*x^5 - 2450*
(47*b*d*e^4*n - 198*a*d*e^4)*x^4 - 25*(478*b*d^2*e^3*n - 693*a*d^2*e^3)*x^3
 + 6*(2621*b*d^3*e^2*n - 3465*a*d^3*e^2)*x^2 - 4*(6397*b*d^4*e*n - 6930*a*d
^4*e)*x + 3465*(105*b*e^5*x^5 + 140*b*d*e^4*x^4 + 5*b*d^2*e^3*x^3 - 6*b*d^3
*e^2*x^2 + 8*b*d^4*e*x - 16*b*d^5)*log(c) + 3465*(105*b*e^5*n*x^5 + 140*b*d
*e^4*n*x^4 + 5*b*d^2*e^3*n*x^3 - 6*b*d^3*e^2*n*x^2 + 8*b*d^4*e*n*x - 16*b*d
^5*n)*log(x))*sqrt(e*x + d))/e^4, 2/4002075*(110880*b*sqrt(-d)*d^5*n*arctan
(sqrt(e*x + d)*sqrt(-d)/d) + (106616*b*d^5*n - 55440*a*d^5 - 33075*(2*b*e^5
*n - 11*a*e^5)*x^5 - 2450*(47*b*d*e^4*n - 198*a*d*e^4)*x^4 - 25*(478*b*d^2*
e^3*n - 693*a*d^2*e^3)*x^3 + 6*(2621*b*d^3*e^2*n - 3465*a*d^3*e^2)*x^2 - 4*
(6397*b*d^4*e*n - 6930*a*d^4*e)*x + 3465*(105*b*e^5*x^5 + 140*b*d*e^4*x^4 +
 5*b*d^2*e^3*x^3 - 6*b*d^3*e^2*x^2 + 8*b*d^4*e*x - 16*b*d^5)*log(c) + 3465*
(105*b*e^5*n*x^5 + 140*b*d*e^4*n*x^4 + 5*b*d^2*e^3*n*x^3 - 6*b*d^3*e^2*n*x^
2 + 8*b*d^4*e*n*x - 16*b*d^5*n)*log(x))*sqrt(e*x + d))/e^4]
```

Sympy [F(-1)]

Timed out.

$$\int x^3(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx=\text{Timed out}$$

```
[In] integrate(x**3*(e*x+d)**(3/2)*(a+b*ln(c*x**n)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.91

$$\int x^3(d+ex)^{3/2}(a+b\log\left(cx^n\right))\,dx=\frac{4}{4002075}\left(\frac{27720\,d^{\frac{11}{2}}\log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{e^4}-\frac{33075\,(ex+d)^{\frac{11}{2}}-107800\,(ex+d)^{\frac{9}{2}}d+106425}{}\right.$$
$$+\frac{2}{1155}\left(\frac{105\,(ex+d)^{\frac{11}{2}}}{e^4}-\frac{385\,(ex+d)^{\frac{9}{2}}d}{e^4}+\frac{495\,(ex+d)^{\frac{7}{2}}d^2}{e^4}-\frac{231\,(ex+d)^{\frac{5}{2}}d^3}{e^4}\right)b\log\left(cx^n\right)$$
$$+\frac{2}{1155}\left(\frac{105\,(ex+d)^{\frac{11}{2}}}{e^4}-\frac{385\,(ex+d)^{\frac{9}{2}}d}{e^4}+\frac{495\,(ex+d)^{\frac{7}{2}}d^2}{e^4}-\frac{231\,(ex+d)^{\frac{5}{2}}d^3}{e^4}\right)a$$

```
[In] integrate(x^3*(e*x+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 4/4002075*(27720*d^(11/2)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sq
rt(d)))/e^4 - (33075*(e*x + d)^(11/2) - 107800*(e*x + d)^(9/2)*d + 106425*(
e*x + d)^(7/2)*d^2 - 11088*(e*x + d)^(5/2)*d^3 - 18480*(e*x + d)^(3/2)*d^4
- 55440*sqrt(e*x + d)*d^5)/e^4)*b*n + 2/1155*(105*(e*x + d)^(11/2)/e^4 - 38
5*(e*x + d)^(9/2)*d/e^4 + 495*(e*x + d)^(7/2)*d^2/e^4 - 231*(e*x + d)^(5/2)
*d^3/e^4)*b*log(c*x^n) + 2/1155*(105*(e*x + d)^(11/2)/e^4 - 385*(e*x + d)^(
9/2)*d/e^4 + 495*(e*x + d)^(7/2)*d^2/e^4 - 231*(e*x + d)^(5/2)*d^3/e^4)*a
```

Giac [F]

$$\int x^3(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx=\int (ex+d)^{\frac{3}{2}}(b\log\left(cx^n\right)+a)x^3\,dx$$

```
[In] integrate(x^3*(e*x+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate((e*x + d)^(3/2)*(b*log(c*x^n) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx=\int x^3\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,{\left(d+e\,x\right)}^{3/2}\,dx$$

```
[In] int(x^3*(a + b*log(c*x^n))*(d + e*x)^(3/2),x)
[Out] int(x^3*(a + b*log(c*x^n))*(d + e*x)^(3/2), x)
```

3.138 $\int x^2(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 213

$$\int x^2(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{32bd^4n\sqrt{d+ex}}{315e^3}-\frac{32bd^3n(d+ex)^{3/2}}{945e^3}-\frac{32bd^2n(d+ex)^{5/2}}{1575e^3}+\frac{44bdn(d+ex)^{7/2}}{441e^3}-\frac{4bn(d+ex)^{9/2}}{81e^3}+\frac{32bd^{9/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{315e^3}+\frac{2d^2(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^3}-\frac{4d(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^3}+\frac{2(d+ex)^{9/2}\left(a+b\log\left(cx^n\right)\right)}{9e^3}$$

```
[Out] -32/945*b*d^3*n*(e*x+d)^(3/2)/e^3-32/1575*b*d^2*n*(e*x+d)^(5/2)/e^3+44/441*
b*d*n*(e*x+d)^(7/2)/e^3-4/81*b*n*(e*x+d)^(9/2)/e^3+32/315*b*d^(9/2)*n*arcta
nh((e*x+d)^(1/2)/d^(1/2))/e^3+2/5*d^2*(e*x+d)^(5/2)*(a+b*ln(c*x^n))/e^3-4/7
*d*(e*x+d)^(7/2)*(a+b*ln(c*x^n))/e^3+2/9*(e*x+d)^(9/2)*(a+b*ln(c*x^n))/e^3-
32/315*b*d^4*n*(e*x+d)^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.261$, Rules used

= {45, 2392, 12, 911, 1275, 214}

$$\int x^2(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{2d^2(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^3}-\frac{4d(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^3}+\frac{2(d+ex)^{9/2}\left(a+b\log\left(cx^n\right)\right)}{9e^3}+\frac{32bd^{9/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{315e^3}-\frac{32bd^4n\sqrt{d+ex}}{315e^3}-\frac{32bd^3n(d+ex)^{3/2}}{945e^3}-\frac{32bd^2n(d+ex)^{5/2}}{1575e^3}+\frac{44bdn(d+ex)^{7/2}}{441e^3}-\frac{4bn(d+ex)^{9/2}}{81e^3}$$

```
[In] Int[x^2*(d + e*x)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (-32*b*d^4*n*Sqrt[d + e*x])/(315*e^3) - (32*b*d^3*n*(d + e*x)^(3/2))/(945*e
^3) - (32*b*d^2*n*(d + e*x)^(5/2))/(1575*e^3) + (44*b*d*n*(d + e*x)^(7/2))/
(441*e^3) - (4*b*n*(d + e*x)^(9/2))/(81*e^3) + (32*b*d^(9/2)*n*ArcTanh[Sqrt
[d + e*x]/Sqrt[d]])/(315*e^3) + (2*d^2*(d + e*x)^(5/2)*(a + b*Log[c*x^n]))/
(5*e^3) - (4*d*(d + e*x)^(7/2)*(a + b*Log[c*x^n]))/(7*e^3) + (2*(d + e*x)^(
9/2)*(a + b*Log[c*x^n]))/(9*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2d^2(d+ex)^{5/2}\,(a+b\log(cx^n))}{5e^3} - \frac{4d(d+ex)^{7/2}\,(a+b\log(cx^n))}{7e^3} \\
&\quad + \frac{2(d+ex)^{9/2}\,(a+b\log(cx^n))}{9e^3} - (bn)\int \frac{2(d+ex)^{5/2}\,(8d^2-20dex+35e^2x^2)}{315e^3x}\,dx \\
&= \frac{2d^2(d+ex)^{5/2}\,(a+b\log(cx^n))}{5e^3} - \frac{4d(d+ex)^{7/2}\,(a+b\log(cx^n))}{7e^3} \\
&\quad + \frac{2(d+ex)^{9/2}\,(a+b\log(cx^n))}{9e^3} - \frac{(2bn)\int \frac{(d+ex)^{5/2}(8d^2-20dex+35e^2x^2)}{x}\,dx}{315e^3} \\
&= \frac{2d^2(d+ex)^{5/2}\,(a+b\log(cx^n))}{5e^3} - \frac{4d(d+ex)^{7/2}\,(a+b\log(cx^n))}{7e^3} \\
&\quad + \frac{2(d+ex)^{9/2}\,(a+b\log(cx^n))}{9e^3} \\
&\quad - \frac{(4bn)\text{Subst}\left(\int \frac{x^6(63d^2-90dx^2+35x^4)}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{315e^4} \\
&= \frac{2d^2(d+ex)^{5/2}\,(a+b\log(cx^n))}{5e^3} \\
&\quad - \frac{4d(d+ex)^{7/2}\,(a+b\log(cx^n))}{7e^3} + \frac{2(d+ex)^{9/2}\,(a+b\log(cx^n))}{9e^3} \\
&\quad - \frac{(4bn)\text{Subst}\left(\int \left(8d^4e+8d^3ex^2+8d^2ex^4-55dex^6+35ex^8+\frac{8d^5}{-\frac{d}{e}+\frac{x^2}{e}}\right)dx,x,\sqrt{d+ex}\right)}{315e^4}
\end{aligned}
$$

$$= -\frac{32bd^4n\sqrt{d+ex}}{315e^3} - \frac{32bd^3n(d+ex)^{3/2}}{945e^3} - \frac{32bd^2n(d+ex)^{5/2}}{1575e^3} + \frac{44bdn(d+ex)^{7/2}}{441e^3} - \frac{4bn(d+ex)^{9/2}}{81e^3} + \frac{2d^2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^3} - \frac{4d(d+ex)^{7/2}(a+b\log(cx^n))}{7e^3} + \frac{2(d+ex)^{9/2}(a+b\log(cx^n))}{9e^3} - \frac{(32bd^5n)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{315e^4}$$

$$= -\frac{32bd^4n\sqrt{d+ex}}{315e^3} - \frac{32bd^3n(d+ex)^{3/2}}{945e^3} - \frac{32bd^2n(d+ex)^{5/2}}{1575e^3} + \frac{44bdn(d+ex)^{7/2}}{441e^3} - \frac{4bn(d+ex)^{9/2}}{81e^3} + \frac{32bd^{9/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{315e^3} + \frac{2d^2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^3} - \frac{4d(d+ex)^{7/2}(a+b\log(cx^n))}{7e^3} + \frac{2(d+ex)^{9/2}(a+b\log(cx^n))}{9e^3}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.72

$$\int x^2(d+ex)^{3/2}(a+b\log(cx^n))\,dx = \frac{2\left(5040bd^{9/2}n\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) + \sqrt{d+ex}(315a(d+ex)^2(8d^2-20dex+35e^2x^2) - 2bn(26$$

```
[In] Integrate[x^2*(d + e*x)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (2*(5040*b*d^(9/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + Sqrt[d + e*x]*(315*a*
(d + e*x)^2*(8*d^2 - 20*d*e*x + 35*e^2*x^2) - 2*b*n*(2614*d^4 - 677*d^3*e*x
 + 429*d^2*e^2*x^2 + 2425*d*e^3*x^3 + 1225*e^4*x^4) + 315*b*(d + e*x)^2*(8*
d^2 - 20*d*e*x + 35*e^2*x^2)*Log[c*x^n])))/(99225*e^3)
```

Maple [F]

$$\int x^2(ex+d)^{\frac{3}{2}}(a+b\ln(c\,x^n))\,dx$$

```
[In] int(x^2*(e*x+d)^(3/2)*(a+b*ln(c*x^n)),x)
```

```
[Out] int(x^2*(e*x+d)^(3/2)*(a+b*ln(c*x^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.33

$$\int x^2(d+ex)^{3/2}(a+b\log(cx^n))\,dx = \left[\frac{2\left(2520\,bd^{\frac{9}{2}}n\log\left(\frac{ex+2\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)-(5228\,bd^4n-2520\,ad^4+1225\,(2\,be^4n-9\,ae^4)x^4\right.}{}\right.$$

$$-\frac{2\left(5040\,b\sqrt{-d}d^4n\arctan\left(\frac{\sqrt{ex+d}\sqrt{-d}}{d}\right)+(5228\,bd^4n-2520\,ad^4+1225\,(2\,be^4n-9\,ae^4)x^4+50\,(97\,bde^3n-\right.}{}$$

```
[In] integrate(x^2*(e*x+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] [2/99225*(2520*b*d^(9/2)*n*log((e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - (
5228*b*d^4*n - 2520*a*d^4 + 1225*(2*b*e^4*n - 9*a*e^4)*x^4 + 50*(97*b*d*e^3
*n - 315*a*d*e^3)*x^3 + 3*(286*b*d^2*e^2*n - 315*a*d^2*e^2)*x^2 - 2*(677*b*
d^3*e*n - 630*a*d^3*e)*x - 315*(35*b*e^4*x^4 + 50*b*d*e^3*x^3 + 3*b*d^2*e^2
*x^2 - 4*b*d^3*e*x + 8*b*d^4)*log(c) - 315*(35*b*e^4*n*x^4 + 50*b*d*e^3*n*x
^3 + 3*b*d^2*e^2*n*x^2 - 4*b*d^3*e*n*x + 8*b*d^4*n)*log(x))*sqrt(e*x + d))/
e^3, -2/99225*(5040*b*sqrt(-d)*d^4*n*arctan(sqrt(e*x + d)*sqrt(-d)/d) + (52
28*b*d^4*n - 2520*a*d^4 + 1225*(2*b*e^4*n - 9*a*e^4)*x^4 + 50*(97*b*d*e^3*n
 - 315*a*d*e^3)*x^3 + 3*(286*b*d^2*e^2*n - 315*a*d^2*e^2)*x^2 - 2*(677*b*d^
3*e*n - 630*a*d^3*e)*x - 315*(35*b*e^4*x^4 + 50*b*d*e^3*x^3 + 3*b*d^2*e^2*x
^2 - 4*b*d^3*e*x + 8*b*d^4)*log(c) - 315*(35*b*e^4*n*x^4 + 50*b*d*e^3*n*x^3
 + 3*b*d^2*e^2*n*x^2 - 4*b*d^3*e*n*x + 8*b*d^4*n)*log(x))*sqrt(e*x + d))/e^
3]
```

Sympy [A] (verification not implemented)

Time = 129.07 (sec) , antiderivative size = 643, normalized size of antiderivative = 3.02

$$\int x^2(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx = ad\left(\begin{cases}\frac{2d^2(d+ex)^{\frac{3}{2}}}{3e^3}-\frac{4d(d+ex)^{\frac{5}{2}}}{5e^3}+\frac{2(d+ex)^{\frac{7}{2}}}{7e^3} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^3}{3} & \text{otherwise}\end{cases}\right)$$

$$+ae\left(\begin{cases}-\frac{2d^3(d+ex)^{\frac{3}{2}}}{3e^4}+\frac{6d^2(d+ex)^{\frac{5}{2}}}{5e^4}-\frac{6d(d+ex)^{\frac{7}{2}}}{7e^4}+\frac{2(d+ex)^{\frac{9}{2}}}{9e^4} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^4}{4} & \text{otherwise}\end{cases}\right)$$

$$-bdn\left(\begin{cases}\frac{3112d^{\frac{7}{2}}\sqrt{1+\frac{ex}{d}}}{11025e^3}+\frac{16d^{\frac{7}{2}}\log\left(\frac{ex}{d}\right)}{105e^3}-\frac{32d^{\frac{7}{2}}\log\left(\sqrt{1+\frac{ex}{d}}+1\right)}{105e^3}-\frac{716d^{\frac{5}{2}}x\sqrt{1+\frac{ex}{d}}}{11025e^2}+\frac{48d^{\frac{3}{2}}x^2\sqrt{1+\frac{ex}{d}}}{1225e}+\frac{4\sqrt{d}x^3\sqrt{1+\frac{ex}{d}}}{49} & \text{for } e>\\ \frac{\sqrt{d}x^3}{9} & \text{otherw}\end{cases}\right.$$

$$+bd\left(\begin{cases}\frac{2d^2(d+ex)^{\frac{3}{2}}}{3e^3}-\frac{4d(d+ex)^{\frac{5}{2}}}{5e^3}+\frac{2(d+ex)^{\frac{7}{2}}}{7e^3} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^3}{3} & \text{otherwise}\end{cases}\right)\log\left(cx^n\right)$$

$$-ben\left(\begin{cases}-\frac{17552d^{\frac{9}{2}}\sqrt{1+\frac{ex}{d}}}{99225e^4}-\frac{32d^{\frac{9}{2}}\log\left(\frac{ex}{d}\right)}{315e^4}+\frac{64d^{\frac{9}{2}}\log\left(\sqrt{1+\frac{ex}{d}}+1\right)}{315e^4}+\frac{3736d^{\frac{7}{2}}x\sqrt{1+\frac{ex}{d}}}{99225e^3}-\frac{724d^{\frac{5}{2}}x^2\sqrt{1+\frac{ex}{d}}}{33075e^2}+\frac{64d^{\frac{3}{2}}x^3\sqrt{1+\frac{ex}{d}}}{3969e}+\cdots\\ \frac{\sqrt{d}x^4}{16}\end{cases}\right.$$

$$+be\left(\begin{cases}-\frac{2d^3(d+ex)^{\frac{3}{2}}}{3e^4}+\frac{6d^2(d+ex)^{\frac{5}{2}}}{5e^4}-\frac{6d(d+ex)^{\frac{7}{2}}}{7e^4}+\frac{2(d+ex)^{\frac{9}{2}}}{9e^4} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^4}{4} & \text{otherwise}\end{cases}\right)\log\left(cx^n\right)$$

```
[In] integrate(x**2*(e*x+d)**(3/2)*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d*Piecewise((2*d**2*(d + e*x)**(3/2)/(3*e**3) - 4*d*(d + e*x)**(5/2)/(5*e
**3) + 2*(d + e*x)**(7/2)/(7*e**3), Ne(e, 0)), (sqrt(d)*x**3/3, True)) + a*
e*Piecewise((-2*d**3*(d + e*x)**(3/2)/(3*e**4) + 6*d**2*(d + e*x)**(5/2)/(5
*e**4) - 6*d*(d + e*x)**(7/2)/(7*e**4) + 2*(d + e*x)**(9/2)/(9*e**4), Ne(e,
 0)), (sqrt(d)*x**4/4, True)) - b*d*n*Piecewise((3112*d**(7/2)*sqrt(1 + e*x
/d)/(11025*e**3) + 16*d**(7/2)*log(e*x/d)/(105*e**3) - 32*d**(7/2)*log(sqrt
(1 + e*x/d) + 1)/(105*e**3) - 716*d**(5/2)*x*sqrt(1 + e*x/d)/(11025*e**2) +
 48*d**(3/2)*x**2*sqrt(1 + e*x/d)/(1225*e) + 4*sqrt(d)*x**3*sqrt(1 + e*x/d)
/49, (e > -oo) & (e < oo) & Ne(e, 0)), (sqrt(d)*x**3/9, True)) + b*d*Piecew
ise((2*d**2*(d + e*x)**(3/2)/(3*e**3) - 4*d*(d + e*x)**(5/2)/(5*e**3) + 2*(
d + e*x)**(7/2)/(7*e**3), Ne(e, 0)), (sqrt(d)*x**3/3, True))*log(c*x**n) -
b*e*n*Piecewise((-17552*d**(9/2)*sqrt(1 + e*x/d)/(99225*e**4) - 32*d**(9/2)
*log(e*x/d)/(315*e**4) + 64*d**(9/2)*log(sqrt(1 + e*x/d) + 1)/(315*e**4) +
3736*d**(7/2)*x*sqrt(1 + e*x/d)/(99225*e**3) - 724*d**(5/2)*x**2*sqrt(1 + e
*x/d)/(33075*e**2) + 64*d**(3/2)*x**3*sqrt(1 + e*x/d)/(3969*e) + 4*sqrt(d)*
x**4*sqrt(1 + e*x/d)/81, (e > -oo) & (e < oo) & Ne(e, 0)), (sqrt(d)*x**4/16
```

```
, True)) + b*e*Piecewise((-2*d**3*(d + e*x)**(3/2)/(3*e**4) + 6*d**2*(d + e
*x)**(5/2)/(5*e**4) - 6*d*(d + e*x)**(7/2)/(7*e**4) + 2*(d + e*x)**(9/2)/(9
*e**4), Ne(e, 0)), (sqrt(d)*x**4/4, True))*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.92

$$\int x^2(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx =$$

$$-\frac{4}{99225}\left(\frac{1260\,d^{\frac{9}{2}}\log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{e^3}+\frac{1225\,(ex+d)^{\frac{9}{2}}-2475\,(ex+d)^{\frac{7}{2}}d+504\,(ex+d)^{\frac{5}{2}}d^2+840\,(ex+d)^{\frac{3}{2}}}{e^3}\right.$$

$$+\frac{2}{315}\left(\frac{35\,(ex+d)^{\frac{9}{2}}}{e^3}-\frac{90\,(ex+d)^{\frac{7}{2}}d}{e^3}+\frac{63\,(ex+d)^{\frac{5}{2}}d^2}{e^3}\right)b\log\left(cx^n\right)$$

$$+\frac{2}{315}\left(\frac{35\,(ex+d)^{\frac{9}{2}}}{e^3}-\frac{90\,(ex+d)^{\frac{7}{2}}d}{e^3}+\frac{63\,(ex+d)^{\frac{5}{2}}d^2}{e^3}\right)a$$

```
[In] integrate(x^2*(e*x+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -4/99225*(1260*d^(9/2)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(
d)))/e^3 + (1225*(e*x + d)^(9/2) - 2475*(e*x + d)^(7/2)*d + 504*(e*x + d)^(
5/2)*d^2 + 840*(e*x + d)^(3/2)*d^3 + 2520*sqrt(e*x + d)*d^4)/e^3)*b*n + 2/3
15*(35*(e*x + d)^(9/2)/e^3 - 90*(e*x + d)^(7/2)*d/e^3 + 63*(e*x + d)^(5/2)*
d^2/e^3)*b*log(c*x^n) + 2/315*(35*(e*x + d)^(9/2)/e^3 - 90*(e*x + d)^(7/2)*
d/e^3 + 63*(e*x + d)^(5/2)*d^2/e^3)*a
```

Giac [F]

$$\int x^2(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx=\int(ex+d)^{\frac{3}{2}}(b\log\left(cx^n\right)+a)x^2\,dx$$

```
[In] integrate(x^2*(e*x+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(b*log(c*x^n) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx=\int x^2\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,{\left(d+e\,x\right)}^{3/2}\,dx$$

```
[In] int(x^2*(a + b*log(c*x^n))*(d + e*x)^(3/2),x)
[Out] int(x^2*(a + b*log(c*x^n))*(d + e*x)^(3/2), x)
```

3.139 $\int x(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 163

$$\int x(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{8bd^3n\sqrt{d+ex}}{35e^2}+\frac{8bd^2n(d+ex)^{3/2}}{105e^2}+\frac{8bdn(d+ex)^{5/2}}{175e^2}-\frac{4bn(d+ex)^{7/2}}{49e^2}-\frac{8bd^{7/2}n\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{35e^2}-\frac{2d(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^2}+\frac{2(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^2}$$

```
[Out] 8/105*b*d^2*n*(e*x+d)^(3/2)/e^2+8/175*b*d*n*(e*x+d)^(5/2)/e^2-4/49*b*n*(e*x
+d)^(7/2)/e^2-8/35*b*d^(7/2)*n*arctanh((e*x+d)^(1/2)/d^(1/2))/e^2-2/5*d*(e*
x+d)^(5/2)*(a+b*ln(c*x^n))/e^2+2/7*(e*x+d)^(7/2)*(a+b*ln(c*x^n))/e^2+8/35*b
*d^3*n*(e*x+d)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {45, 2392, 12, 81, 52, 65, 214}

$$\int x(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{2d(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^2}+\frac{2(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^2}-\frac{8bd^{7/2}n\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{35e^2}+\frac{8bd^3n\sqrt{d+ex}}{35e^2}+\frac{8bd^2n(d+ex)^{3/2}}{105e^2}+\frac{8bdn(d+ex)^{5/2}}{175e^2}-\frac{4bn(d+ex)^{7/2}}{49e^2}$$

```
[In] Int[x*(d + e*x)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (8*b*d^3*n*Sqrt[d + e*x])/(35*e^2) + (8*b*d^2*n*(d + e*x)^(3/2))/(105*e^2)
+ (8*b*d*n*(d + e*x)^(5/2))/(175*e^2) - (4*b*n*(d + e*x)^(7/2))/(49*e^2) -
(8*b*d^(7/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(35*e^2) - (2*d*(d + e*x)^(5
/2)*(a + b*Log[c*x^n]))/(5*e^2) + (2*(d + e*x)^(7/2)*(a + b*Log[c*x^n]))/(7
*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^2}+\frac{2(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^2} \\
&\quad -(bn)\int\frac{2(d+ex)^{5/2}(-2d+5ex)}{35e^2x}\,dx \\
&= -\frac{2d(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^2} \\
&\quad +\frac{2(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^2}-\frac{(2bn)\int\frac{(d+ex)^{5/2}(-2d+5ex)}{x}\,dx}{35e^2} \\
&= -\frac{4bn(d+ex)^{7/2}}{49e^2}-\frac{2d(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^2} \\
&\quad +\frac{2(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^2}+\frac{(4bdn)\int\frac{(d+ex)^{5/2}}{x}\,dx}{35e^2} \\
&= \frac{8bdn(d+ex)^{5/2}}{175e^2}-\frac{4bn(d+ex)^{7/2}}{49e^2}-\frac{2d(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^2} \\
&\quad +\frac{2(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^2}+\frac{(4bd^2n)\int\frac{(d+ex)^{3/2}}{x}\,dx}{35e^2} \\
&= \frac{8bd^2n(d+ex)^{3/2}}{105e^2}+\frac{8bdn(d+ex)^{5/2}}{175e^2} \\
&\quad -\frac{4bn(d+ex)^{7/2}}{49e^2}-\frac{2d(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^2} \\
&\quad +\frac{2(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^2}+\frac{(4bd^3n)\int\frac{\sqrt{d+ex}}{x}\,dx}{35e^2} \\
&= \frac{8bd^3n\sqrt{d+ex}}{35e^2}+\frac{8bd^2n(d+ex)^{3/2}}{105e^2}+\frac{8bdn(d+ex)^{5/2}}{175e^2} \\
&\quad -\frac{4bn(d+ex)^{7/2}}{49e^2}-\frac{2d(d+ex)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^2} \\
&\quad +\frac{2(d+ex)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^2}+\frac{(4bd^4n)\int\frac{1}{x\sqrt{d+ex}}\,dx}{35e^2}
\end{aligned}$$

$$= \frac{8bd^3n\sqrt{d+ex}}{35e^2} + \frac{8bd^2n(d+ex)^{3/2}}{105e^2} + \frac{8bdn(d+ex)^{5/2}}{175e^2} - \frac{4bn(d+ex)^{7/2}}{49e^2} - \frac{2d(d+ex)^{5/2}(a+b\log(cx^n))}{5e^2} + \frac{2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^2} + \frac{(8bd^4n)\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{35e^3}$$

$$= \frac{8bd^3n\sqrt{d+ex}}{35e^2} + \frac{8bd^2n(d+ex)^{3/2}}{105e^2} + \frac{8bdn(d+ex)^{5/2}}{175e^2} - \frac{4bn(d+ex)^{7/2}}{49e^2} - \frac{8bd^{7/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{35e^2} - \frac{2d(d+ex)^{5/2}(a+b\log(cx^n))}{5e^2} + \frac{2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^2}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.74

$$\int x(d+ex)^{3/2}(a+b\log(cx^n))\,dx =$$

$$-\frac{2\left(420bd^{7/2}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)+\sqrt{d+ex}(105a(2d-5ex)(d+ex)^2+2bn(-247d^3+71d^2ex+183de^2x^2+7}{3675e^2}$$

```
[In] Integrate[x*(d + e*x)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (-2*(420*b*d^(7/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + Sqrt[d + e*x]*(105*a*
(2*d - 5*e*x)*(d + e*x)^2 + 2*b*n*(-247*d^3 + 71*d^2*e*x + 183*d*e^2*x^2 +
75*e^3*x^3) + 105*b*(2*d - 5*e*x)*(d + e*x)^2*Log[c*x^n])))/(3675*e^2)
```

Maple [F]

$$\int x(ex+d)^{\frac{3}{2}}(a+b\ln(c\,x^n))\,dx$$

```
[In] int(x*(e*x+d)^(3/2)*(a+b*ln(c*x^n)),x)
```

```
[Out] int(x*(e*x+d)^(3/2)*(a+b*ln(c*x^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.40

$$\int x(d+ex)^{3/2}(a+b\log(cx^n))\,dx = \left[\frac{2\left(210\,bd^{\frac{7}{2}}n\log\left(\frac{ex-2\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)+(494\,bd^3n-210\,ad^3-75\,(2\,be^3n-7\,ae^3)x^3-6\,(6\right.}{}\right.$$

```
[In] integrate(x*(e*x+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] [2/3675*(210*b*d^(7/2)*n*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + (49
4*b*d^3*n - 210*a*d^3 - 75*(2*b*e^3*n - 7*a*e^3)*x^3 - 6*(61*b*d*e^2*n - 14
0*a*d*e^2)*x^2 - (142*b*d^2*e*n - 105*a*d^2*e)*x + 105*(5*b*e^3*x^3 + 8*b*d
*e^2*x^2 + b*d^2*e*x - 2*b*d^3)*log(c) + 105*(5*b*e^3*n*x^3 + 8*b*d*e^2*n*x
^2 + b*d^2*e*n*x - 2*b*d^3*n)*log(x))*sqrt(e*x + d))/e^2, 2/3675*(420*b*sqr
t(-d)*d^3*n*arctan(sqrt(e*x + d)*sqrt(-d)/d) + (494*b*d^3*n - 210*a*d^3 - 7
5*(2*b*e^3*n - 7*a*e^3)*x^3 - 6*(61*b*d*e^2*n - 140*a*d*e^2)*x^2 - (142*b*d
^2*e*n - 105*a*d^2*e)*x + 105*(5*b*e^3*x^3 + 8*b*d*e^2*x^2 + b*d^2*e*x - 2*
b*d^3)*log(c) + 105*(5*b*e^3*n*x^3 + 8*b*d*e^2*n*x^2 + b*d^2*e*n*x - 2*b*d^
3*n)*log(x))*sqrt(e*x + d))/e^2]
```

Sympy [A] (verification not implemented)

Time = 113.00 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.17

$$\int x(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx = ad\left(\begin{cases}-\frac{2d(d+ex)^{\frac{3}{2}}}{3e^2}+\frac{2(d+ex)^{\frac{5}{2}}}{5e^2} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ae\left(\begin{cases}\frac{2d^2(d+ex)^{\frac{3}{2}}}{3e^3}-\frac{4d(d+ex)^{\frac{5}{2}}}{5e^3}+\frac{2(d+ex)^{\frac{7}{2}}}{7e^3} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^3}{3} & \text{otherwise}\end{cases}\right)$$
$$-bdn\left(\begin{cases}-\frac{124d^{\frac{5}{2}}\sqrt{1+\frac{ex}{d}}}{225e^2}-\frac{4d^{\frac{5}{2}}\log\left(\frac{ex}{d}\right)}{15e^2}+\frac{8d^{\frac{5}{2}}\log\left(\sqrt{1+\frac{ex}{d}}+1\right)}{15e^2}+\frac{32d^{\frac{3}{2}}x\sqrt{1+\frac{ex}{d}}}{225e}+\frac{4\sqrt{d}x^2\sqrt{1+\frac{ex}{d}}}{25} & \text{for } e>-\infty\wedge e<\infty\wedge\\ \frac{\sqrt{d}x^2}{4} & \text{otherwise}\end{cases}\right.$$
$$+bd\left(\begin{cases}-\frac{2d(d+ex)^{\frac{3}{2}}}{3e^2}+\frac{2(d+ex)^{\frac{5}{2}}}{5e^2} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^2}{2} & \text{otherwise}\end{cases}\right)\log\left(cx^n\right)$$
$$-ben\left(\begin{cases}\frac{3112d^{\frac{7}{2}}\sqrt{1+\frac{ex}{d}}}{11025e^3}+\frac{16d^{\frac{7}{2}}\log\left(\frac{ex}{d}\right)}{105e^3}-\frac{32d^{\frac{7}{2}}\log\left(\sqrt{1+\frac{ex}{d}}+1\right)}{105e^3}-\frac{716d^{\frac{5}{2}}x\sqrt{1+\frac{ex}{d}}}{11025e^2}+\frac{48d^{\frac{3}{2}}x^2\sqrt{1+\frac{ex}{d}}}{1225e}+\frac{4\sqrt{d}x^3\sqrt{1+\frac{ex}{d}}}{49} & \text{for } e>\\ \frac{\sqrt{d}x^3}{9} & \text{otherw}\end{cases}\right.$$
$$+be\left(\begin{cases}\frac{2d^2(d+ex)^{\frac{3}{2}}}{3e^3}-\frac{4d(d+ex)^{\frac{5}{2}}}{5e^3}+\frac{2(d+ex)^{\frac{7}{2}}}{7e^3} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^3}{3} & \text{otherwise}\end{cases}\right)\log\left(cx^n\right)$$

```
[In] integrate(x*(e*x+d)**(3/2)*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d*Piecewise((-2*d*(d + e*x)**(3/2)/(3*e**2) + 2*(d + e*x)**(5/2)/(5*e**2)
, Ne(e, 0)), (sqrt(d)*x**2/2, True)) + a*e*Piecewise((2*d**2*(d + e*x)**(3/
2)/(3*e**3) - 4*d*(d + e*x)**(5/2)/(5*e**3) + 2*(d + e*x)**(7/2)/(7*e**3),
Ne(e, 0)), (sqrt(d)*x**3/3, True)) - b*d*n*Piecewise((-124*d**(5/2)*sqrt(1
+ e*x/d)/(225*e**2) - 4*d**(5/2)*log(e*x/d)/(15*e**2) + 8*d**(5/2)*log(sqrt
(1 + e*x/d) + 1)/(15*e**2) + 32*d**(3/2)*x*sqrt(1 + e*x/d)/(225*e) + 4*sqrt
(d)*x**2*sqrt(1 + e*x/d)/25, (e > -oo) & (e < oo) & Ne(e, 0)), (sqrt(d)*x**
2/4, True)) + b*d*Piecewise((-2*d*(d + e*x)**(3/2)/(3*e**2) + 2*(d + e*x)**
(5/2)/(5*e**2), Ne(e, 0)), (sqrt(d)*x**2/2, True))*log(c*x**n) - b*e*n*Piec
ewise((3112*d**(7/2)*sqrt(1 + e*x/d)/(11025*e**3) + 16*d**(7/2)*log(e*x/d)/
(105*e**3) - 32*d**(7/2)*log(sqrt(1 + e*x/d) + 1)/(105*e**3) - 716*d**(5/2)
*x*sqrt(1 + e*x/d)/(11025*e**2) + 48*d**(3/2)*x**2*sqrt(1 + e*x/d)/(1225*e)
 + 4*sqrt(d)*x**3*sqrt(1 + e*x/d)/49, (e > -oo) & (e < oo) & Ne(e, 0)), (sq
rt(d)*x**3/9, True)) + b*e*Piecewise((2*d**2*(d + e*x)**(3/2)/(3*e**3) - 4*
d*(d + e*x)**(5/2)/(5*e**3) + 2*(d + e*x)**(7/2)/(7*e**3), Ne(e, 0)), (sqrt
(d)*x**3/3, True))*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.95

$$\int x(d+ex)^{3/2}(a+b\log(cx^n))\,dx = \frac{4}{3675}\left(\frac{105\,d^{\frac{7}{2}}\log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{e^2} - \frac{75\,(ex+d)^{\frac{7}{2}} - 42\,(ex+d)^{\frac{5}{2}}d - 70\,(ex+d)^{\frac{3}{2}}d^2 - 210\,\sqrt{ex+d}\,d^3}{e^2}\right)bn + \frac{2}{35}\left(\frac{5\,(ex+d)^{\frac{7}{2}}}{e^2} - \frac{7\,(ex+d)^{\frac{5}{2}}d}{e^2}\right)b\log(cx^n) + \frac{2}{35}\left(\frac{5\,(ex+d)^{\frac{7}{2}}}{e^2} - \frac{7\,(ex+d)^{\frac{5}{2}}d}{e^2}\right)a$$

```
[In] integrate(x*(e*x+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 4/3675*(105*d^(7/2)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d))
)/e^2 - (75*(e*x + d)^(7/2) - 42*(e*x + d)^(5/2)*d - 70*(e*x + d)^(3/2)*d^2
 - 210*sqrt(e*x + d)*d^3)/e^2)*b*n + 2/35*(5*(e*x + d)^(7/2)/e^2 - 7*(e*x +
 d)^(5/2)*d/e^2)*b*log(c*x^n) + 2/35*(5*(e*x + d)^(7/2)/e^2 - 7*(e*x + d)^(
5/2)*d/e^2)*a
```

Giac [F]

$$\int x(d+ex)^{3/2}(a+b\log(cx^n))\,dx = \int (ex+d)^{\frac{3}{2}}(b\log(cx^n)+a)x\,dx$$

```
[In] integrate(x*(e*x+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(b*log(c*x^n) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex)^{3/2}(a+b\log(cx^n))\,dx = \int x\,(a+b\,\ln(c\,x^n))\,(d+e\,x)^{3/2}\,dx$$

```
[In] int(x*(a + b*log(c*x^n))*(d + e*x)^(3/2),x)
```

```
[Out] int(x*(a + b*log(c*x^n))*(d + e*x)^(3/2), x)
```

3.140 $\int (d+ex)^{3/2} (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 115

$$\int (d+ex)^{3/2} (a+b\log(cx^n))\,dx = -\frac{4bd^2n\sqrt{d+ex}}{5e}-\frac{4bdn(d+ex)^{3/2}}{15e} -\frac{4bn(d+ex)^{5/2}}{25e}+\frac{4bd^{5/2}n\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{5e}+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e}$$

```
[Out] -4/15*b*d*n*(e*x+d)^(3/2)/e-4/25*b*n*(e*x+d)^(5/2)/e+4/5*b*d^(5/2)*n*arctan
h((e*x+d)^(1/2)/d^(1/2))/e+2/5*(e*x+d)^(5/2)*(a+b*ln(c*x^n))/e-4/5*b*d^2*n*
(e*x+d)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2356, 52, 65, 214}

$$\int (d+ex)^{3/2} (a+b\log(cx^n))\,dx = \frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e} +\frac{4bd^{5/2}n\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{5e}-\frac{4bd^2n\sqrt{d+ex}}{5e}-\frac{4bdn(d+ex)^{3/2}}{15e}-\frac{4bn(d+ex)^{5/2}}{25e}$$

```
[In] Int[(d + e*x)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (-4*b*d^2*n*Sqrt[d + e*x])/(5*e) - (4*b*d*n*(d + e*x)^(3/2))/(15*e) - (4*b*
n*(d + e*x)^(5/2))/(25*e) + (4*b*d^(5/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/
(5*e) + (2*(d + e*x)^(5/2)*(a + b*Log[c*x^n]))/(5*e)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(d+ex)^{5/2}\,(a+b\log(cx^n))}{5e}-\frac{(2bn)\int\frac{(d+ex)^{5/2}}{x}\,dx}{5e}\\
&=-\frac{4bn(d+ex)^{5/2}}{25e}+\frac{2(d+ex)^{5/2}\,(a+b\log(cx^n))}{5e}-\frac{(2bdn)\int\frac{(d+ex)^{3/2}}{x}\,dx}{5e}\\
&=-\frac{4bdn(d+ex)^{3/2}}{15e}-\frac{4bn(d+ex)^{5/2}}{25e}+\frac{2(d+ex)^{5/2}\,(a+b\log(cx^n))}{5e}-\frac{(2bd^2n)\int\frac{\sqrt{d+ex}}{x}\,dx}{5e}\\
&=-\frac{4bd^2n\sqrt{d+ex}}{5e}-\frac{4bdn(d+ex)^{3/2}}{15e}-\frac{4bn(d+ex)^{5/2}}{25e}\\
&\quad+\frac{2(d+ex)^{5/2}\,(a+b\log(cx^n))}{5e}-\frac{(2bd^3n)\int\frac{1}{x\sqrt{d+ex}}\,dx}{5e}
\end{aligned}
$$

$$= -\frac{4bd^2n\sqrt{d+ex}}{5e} - \frac{4bdn(d+ex)^{3/2}}{15e} - \frac{4bn(d+ex)^{5/2}}{25e} + \frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e} - \frac{(4bd^3n)\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{5e^2}$$

$$= -\frac{4bd^2n\sqrt{d+ex}}{5e} - \frac{4bdn(d+ex)^{3/2}}{15e} - \frac{4bn(d+ex)^{5/2}}{25e} + \frac{4bd^{5/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{5e} + \frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

$$\int (d+ex)^{3/2}(a+b\log(cx^n))\,dx = \frac{2\left(-\frac{2}{15}bn\sqrt{d+ex}(23d^2+11dex+3e^2x^2)+2bd^{5/2}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)+(d+ex)^{5/2}(a+b\,\text{l}\right.}{5e}$$

```
[In] Integrate[(d + e*x)^(3/2)*(a + b*Log[c*x^n]),x]
[Out] (2*((-2*b*n*Sqrt[d + e*x]*(23*d^2 + 11*d*e*x + 3*e^2*x^2))/15 + 2*b*d^(5/2)
*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + (d + e*x)^(5/2)*(a + b*Log[c*x^n])))/(5
*e)
```

Maple [F]

$$\int (ex+d)^{\frac{3}{2}}(a+b\ln(c\,x^n))\,dx$$

```
[In] int((e*x+d)^(3/2)*(a+b*ln(c*x^n)),x)
[Out] int((e*x+d)^(3/2)*(a+b*ln(c*x^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.50

$$\int (d+ex)^{3/2} (a+b\log(cx^n))\,dx = \left[\frac{2\left(15\,bd^{\frac{5}{2}}n\log\left(\frac{ex+2\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right) - (46\,bd^2n - 15\,ad^2 + 3\,(2\,be^2n - 5\,ae^2)x^2 + 2\,(11\,bde\right.}{}\right.$$

$$-\frac{2\left(30\,b\sqrt{-d}d^2n\arctan\left(\frac{\sqrt{ex+d}\sqrt{-d}}{d}\right) + (46\,bd^2n - 15\,ad^2 + 3\,(2\,be^2n - 5\,ae^2)x^2 + 2\,(11\,bden - 15\,ade)x - \right.}{75\,e}$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] [2/75*(15*b*d^(5/2)*n*log((e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - (46*b*
d^2*n - 15*a*d^2 + 3*(2*b*e^2*n - 5*a*e^2)*x^2 + 2*(11*b*d*e*n - 15*a*d*e)*
x - 15*(b*e^2*x^2 + 2*b*d*e*x + b*d^2)*log(c) - 15*(b*e^2*n*x^2 + 2*b*d*e*n
*x + b*d^2*n)*log(x))*sqrt(e*x + d))/e, -2/75*(30*b*sqrt(-d)*d^2*n*arctan(s
qrt(e*x + d)*sqrt(-d)/d) + (46*b*d^2*n - 15*a*d^2 + 3*(2*b*e^2*n - 5*a*e^2)
*x^2 + 2*(11*b*d*e*n - 15*a*d*e)*x - 15*(b*e^2*x^2 + 2*b*d*e*x + b*d^2)*log
(c) - 15*(b*e^2*n*x^2 + 2*b*d*e*n*x + b*d^2*n)*log(x))*sqrt(e*x + d))/e]
```

Sympy [A] (verification not implemented)

Time = 88.80 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.28

$$\int (d+ex)^{3/2} (a+b\log(cx^n))\,dx = ad\left(\begin{cases} \frac{2(d+ex)^{\frac{3}{2}}}{3e} & \text{for}\ e\neq 0 \\ \sqrt{d}x & \text{otherwise} \end{cases}\right)$$
$$+ ae\left(\begin{cases} -\frac{2d(d+ex)^{\frac{3}{2}}}{3e^2} + \frac{2(d+ex)^{\frac{5}{2}}}{5e^2} & \text{for}\ e\neq 0 \\ \frac{\sqrt{d}x^2}{2} & \text{otherwise} \end{cases}\right)$$
$$-bdn\left(\begin{cases} \frac{16d^{\frac{3}{2}}\sqrt{1+\frac{ex}{d}}}{9e} + \frac{2d^{\frac{3}{2}}\log\left(\frac{ex}{d}\right)}{3e} - \frac{4d^{\frac{3}{2}}\log\left(\sqrt{1+\frac{ex}{d}}+1\right)}{3e} + \frac{4\sqrt{d}x\sqrt{1+\frac{ex}{d}}}{9} & \text{for}\ e>-\infty\wedge e<\infty\wedge e\neq 0 \\ \sqrt{d}x & \text{otherwise} \end{cases}\right)$$
$$+ bd\left(\begin{cases} \frac{2(d+ex)^{\frac{3}{2}}}{3e} & \text{for}\ e\neq 0 \\ \sqrt{d}x & \text{otherwise} \end{cases}\right)\log(cx^n)$$
$$-ben\left(\begin{cases} -\frac{124d^{\frac{5}{2}}\sqrt{1+\frac{ex}{d}}}{225e^2} - \frac{4d^{\frac{5}{2}}\log\left(\frac{ex}{d}\right)}{15e^2} + \frac{8d^{\frac{5}{2}}\log\left(\sqrt{1+\frac{ex}{d}}+1\right)}{15e^2} + \frac{32d^{\frac{3}{2}}x\sqrt{1+\frac{ex}{d}}}{225e} + \frac{4\sqrt{d}x^2\sqrt{1+\frac{ex}{d}}}{25} & \text{for}\ e>-\infty\wedge e<\infty \\ \frac{\sqrt{d}x^2}{4} & \text{otherwise} \end{cases}\right.$$
$$+ be\left(\begin{cases} -\frac{2d(d+ex)^{\frac{3}{2}}}{3e^2} + \frac{2(d+ex)^{\frac{5}{2}}}{5e^2} & \text{for}\ e\neq 0 \\ \frac{\sqrt{d}x^2}{2} & \text{otherwise} \end{cases}\right)\log(cx^n)$$

```
[In] integrate((e*x+d)**(3/2)*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d*Piecewise((2*(d + e*x)**(3/2)/(3*e), Ne(e, 0)), (sqrt(d)*x, True)) + a*
e*Piecewise((-2*d*(d + e*x)**(3/2)/(3*e**2) + 2*(d + e*x)**(5/2)/(5*e**2),
Ne(e, 0)), (sqrt(d)*x**2/2, True)) - b*d*n*Piecewise((16*d**(3/2)*sqrt(1 +
e*x/d)/(9*e) + 2*d**(3/2)*log(e*x/d)/(3*e) - 4*d**(3/2)*log(sqrt(1 + e*x/d)
 + 1)/(3*e) + 4*sqrt(d)*x*sqrt(1 + e*x/d)/9, (e > -oo) & (e < oo) & Ne(e, 0
)), (sqrt(d)*x, True)) + b*d*Piecewise((2*(d + e*x)**(3/2)/(3*e), Ne(e, 0))
, (sqrt(d)*x, True))*log(c*x**n) - b*e*n*Piecewise((-124*d**(5/2)*sqrt(1 +
e*x/d)/(225*e**2) - 4*d**(5/2)*log(e*x/d)/(15*e**2) + 8*d**(5/2)*log(sqrt(1
 + e*x/d) + 1)/(15*e**2) + 32*d**(3/2)*x*sqrt(1 + e*x/d)/(225*e) + 4*sqrt(d
)*x**2*sqrt(1 + e*x/d)/25, (e > -oo) & (e < oo) & Ne(e, 0)), (sqrt(d)*x**2/
4, True)) + b*e*Piecewise((-2*d*(d + e*x)**(3/2)/(3*e**2) + 2*(d + e*x)**(5
/2)/(5*e**2), Ne(e, 0)), (sqrt(d)*x**2/2, True))*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int (d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{2\,(ex+d)^{\frac{5}{2}}b\log\left(cx^n\right)}{5\,e}+\frac{2\,(ex+d)^{\frac{5}{2}}a}{5\,e}-\frac{2\left(15\,d^{\frac{5}{2}}\log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)+6\,(ex+d)^{\frac{5}{2}}+10\,(ex+d)^{\frac{3}{2}}d+30\,\sqrt{ex+d}d^2\right)bn}{75\,e}$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 2/5*(e*x + d)^(5/2)*b*log(c*x^n)/e + 2/5*(e*x + d)^(5/2)*a/e - 2/75*(15*d^(
5/2)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d))) + 6*(e*x + d)
^(5/2) + 10*(e*x + d)^(3/2)*d + 30*sqrt(e*x + d)*d^2)*b*n/e
```

Giac [F]

$$\int (d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx=\int (ex+d)^{\frac{3}{2}}\left(b\log\left(cx^n\right)+a\right)dx$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{3/2}(a+b\log(cx^n))\,dx = \int (a+b\,\ln(c\,x^n))\,(d+e\,x)^{3/2}\,dx$$

```
[In] int((a + b*log(c*x^n))*(d + e*x)^(3/2),x)
[Out] int((a + b*log(c*x^n))*(d + e*x)^(3/2), x)
```

3.141 $\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 255

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x}\,dx = -\frac{16}{3}bdn\sqrt{d+ex} - \frac{4}{9}bn(d+ex)^{3/2} + \frac{16}{3}bd^{3/2}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) + 2bd^{3/2}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2 + 2d\sqrt{d+ex}(a+b\log(cx^n)) + \frac{2}{3}(d+ex)^{3/2}(a+$$

```
[Out] -4/9*b*n*(e*x+d)^(3/2)+16/3*b*d^(3/2)*n*arctanh((e*x+d)^(1/2)/d^(1/2))+2*b*
d^(3/2)*n*arctanh((e*x+d)^(1/2)/d^(1/2))^2+2/3*(e*x+d)^(3/2)*(a+b*ln(c*x^n)
)-2*d^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*(a+b*ln(c*x^n))-4*b*d^(3/2)*n*ar
ctanh((e*x+d)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)-(e*x+d)^(1/2)))-2*b*d^(3
/2)*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-(e*x+d)^(1/2)))-16/3*b*d*n*(e*x+d)^(1/
2)+2*d*(a+b*ln(c*x^n))*(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2388, 65, 214, 2390, 12, 6131, 6055, 2449, 2352, 2356, 52}

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x}\,dx = -2d^{3/2}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n)) + \frac{2}{3}(d+ex)^{3/2}(a+b\log(cx^n)) + 2d\sqrt{d+ex}(a+b\log(cx^n)) + 2bd^{3/2}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2 + \frac{16}{3}bd^{3/2}n\text{arctanh}\Bigg($$

```
[In] Int[((d + e*x)^(3/2)*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] (-16*b*d*n*Sqrt[d + e*x])/3 - (4*b*n*(d + e*x)^(3/2))/9 + (16*b*d^(3/2)*n*A
rcTanh[Sqrt[d + e*x]/Sqrt[d]])/3 + 2*b*d^(3/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt
[d]]^2 + 2*d*Sqrt[d + e*x]*(a + b*Log[c*x^n]) + (2*(d + e*x)^(3/2)*(a + b*L
og[c*x^n]))/3 - 2*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]*(a + b*Log[c*x^n])
 - 4*b*d^(3/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] -
Sqrt[d + e*x])] - 2*b*d^(3/2)*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[
d + e*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
```

```
NeQ[q, 1]))
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int \frac{\sqrt{d+ex}(a+b\log(cx^n))}{x}\,dx + e\int \sqrt{d+ex}(a+b\log(cx^n))\,dx \\
&= \frac{2}{3}(d+ex)^{3/2}(a+b\log(cx^n)) + d^2\int \frac{a+b\log(cx^n)}{x\sqrt{d+ex}}\,dx + (de)\int \frac{a+b\log(cx^n)}{\sqrt{d+ex}}\,dx \\
&\quad - \frac{1}{3}(2bn)\int \frac{(d+ex)^{3/2}}{x}\,dx
\end{aligned}$$

$$= -\frac{4}{9}bn(d+ex)^{3/2} + 2d\sqrt{d+ex}(a+b\log(cx^n)) + \frac{2}{3}(d+ex)^{3/2}(a+b\log(cx^n))$$

$$-2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n)) - \frac{1}{3}(2bdn)\int\frac{\sqrt{d+ex}}{x}\,dx - (2bdn)\int\frac{\sqrt{d+ex}}{x}\,dx - ($$

$$= -\frac{16}{3}bdn\sqrt{d+ex} - \frac{4}{9}bn(d+ex)^{3/2} + 2d\sqrt{d+ex}(a+b\log(cx^n)) + \frac{2}{3}(d+ex)^{3/2}(a+b\log(cx^n))$$

$$-2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n)) + \left(2bd^{3/2}n\right)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx - \frac{1}{3}\left(2bd^2n\right)\int\frac{}{x\sqrt{}}$$

$$= -\frac{16}{3}bdn\sqrt{d+ex} - \frac{4}{9}bn(d+ex)^{3/2} + 2d\sqrt{d+ex}(a+b\log(cx^n)) + \frac{2}{3}(d+ex)^{3/2}(a+b\log(cx^n))$$

$$-2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n)) + \left(4bd^{3/2}n\right)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx, x, \sqrt{d+ex}\right.$$

$$= -\frac{16}{3}bdn\sqrt{d+ex} - \frac{4}{9}bn(d+ex)^{3/2} + \frac{16}{3}bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)$$

$$+2bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2 + 2d\sqrt{d+ex}(a+b\log(cx^n)) + \frac{2}{3}(d+ex)^{3/2}(a+b\log(cx^n)) - 2d^{3/2}\,\mathrm{t}$$

$$= -\frac{16}{3}bdn\sqrt{d+ex} - \frac{4}{9}bn(d+ex)^{3/2} + \frac{16}{3}bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)$$

$$+2bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2 + 2d\sqrt{d+ex}(a+b\log(cx^n)) + \frac{2}{3}(d+ex)^{3/2}(a+b\log(cx^n)) - 2d^{3/2}\,\mathrm{t}$$

$$= -\frac{16}{3}bdn\sqrt{d+ex} - \frac{4}{9}bn(d+ex)^{3/2} + \frac{16}{3}bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)$$

$$+2bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2 + 2d\sqrt{d+ex}(a+b\log(cx^n)) + \frac{2}{3}(d+ex)^{3/2}(a+b\log(cx^n)) - 2d^{3/2}\,\mathrm{t}$$

$$= -\frac{16}{3}bdn\sqrt{d+ex} - \frac{4}{9}bn(d+ex)^{3/2} + \frac{16}{3}bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)$$

$$+2bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2 + 2d\sqrt{d+ex}(a+b\log(cx^n)) + \frac{2}{3}(d+ex)^{3/2}(a+b\log(cx^n)) - 2d^{3/2}\,\mathrm{t}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x}\,dx = 2ad\sqrt{d+ex}-4bdn\sqrt{d+ex} -\frac{4}{9}bn\sqrt{d+ex}(4d+ex)+\frac{16}{3}bd^{3/2}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)+2bd\sqrt{d+ex}\log(cx^n) +\frac{2}{3}(d+ex)^{3/2}(a+b\log(cx^n))+d^{3/2}(a+b\log(cx^n))\log\left(\sqrt{d}-\sqrt{d+ex}\right)-d^{3/2}(a+b\log(cx^n))\log\left(\sqrt{d}+\sqrt{d+\right.$$

```
[In] Integrate[((d + e*x)^(3/2)*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] 2*a*d*Sqrt[d + e*x] - 4*b*d*n*Sqrt[d + e*x] - (4*b*n*Sqrt[d + e*x]*(4*d + e
*x))/9 + (16*b*d^(3/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/3 + 2*b*d*Sqrt[d +
 e*x]*Log[c*x^n] + (2*(d + e*x)^(3/2)*(a + b*Log[c*x^n]))/3 + d^(3/2)*(a +
b*Log[c*x^n])*Log[Sqrt[d] - Sqrt[d + e*x]] - d^(3/2)*(a + b*Log[c*x^n])*Log
[Sqrt[d] + Sqrt[d + e*x]] + (b*d^(3/2)*n*Log[Sqrt[d] + Sqrt[d + e*x]]*(Log[
Sqrt[d] + Sqrt[d + e*x]] + 2*Log[1/2 - Sqrt[d + e*x]/(2*Sqrt[d])]))/2 - (b*
d^(3/2)*n*Log[Sqrt[d] - Sqrt[d + e*x]]*(Log[Sqrt[d] - Sqrt[d + e*x]] + 2*Lo
g[(1 + Sqrt[d + e*x]/Sqrt[d])/2]))/2 - b*d^(3/2)*n*PolyLog[2, 1/2 - Sqrt[d
+ e*x]/(2*Sqrt[d])] + b*d^(3/2)*n*PolyLog[2, (1 + Sqrt[d + e*x]/Sqrt[d])/2]
```

Maple [F]

$$\int \frac{(ex+d)^{\frac{3}{2}}(a+b\ln(c\,x^n))}{x}dx$$

```
[In] int((e*x+d)^(3/2)*(a+b*ln(c*x^n))/x,x)
```

```
[Out] int((e*x+d)^(3/2)*(a+b*ln(c*x^n))/x,x)
```

Fricas [F]

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(b\log(cx^n)+a)}{x}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] integral(((b*e*x + b*d)*sqrt(e*x + d)*log(c*x^n) + (a*e*x + a*d)*sqrt(e*x +
 d))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(3/2)*(a+b*ln(c*x**n))/x,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(b\log(cx^n)+a)}{x}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*log(c*x^n))/x,x, algorithm="maxima")
[Out] 1/3*(3*d^(3/2)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d))) + 2
*(e*x + d)^(3/2) + 6*sqrt(e*x + d)*d)*a + b*integrate((e*x*log(c) + d*log(c
) + (e*x + d)*log(x^n))*sqrt(e*x + d)/x, x)
```

Giac [F]

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(b\log(cx^n)+a)}{x}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*log(c*x^n))/x,x, algorithm="giac")
[Out] integrate((e*x + d)^(3/2)*(b*log(c*x^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x}\,dx = \int \frac{(a+b\,\ln(c\,x^n))\,(d+e\,x)^{3/2}}{x}\,dx$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^(3/2))/x,x)
[Out] int(((a + b*log(c*x^n))*(d + e*x)^(3/2))/x, x)
```

3.142 $\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 259

$$\begin{aligned}\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x^2}\,dx &= -4ben\sqrt{d+ex}-\frac{bdn\sqrt{d+ex}}{x}\\&+3b\sqrt{d}en\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)+3b\sqrt{d}en\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2\\&+3e\sqrt{d+ex}(a+b\log(cx^n))-\frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x}\\&-3\sqrt{d}e\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))\\&-6b\sqrt{d}en\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)\\&-3b\sqrt{d}en\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)\end{aligned}$$

```
[Out] -(e*x+d)^(3/2)*(a+b*ln(c*x^n))/x+3*b*e*n*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(
1/2)+3*b*e*n*arctanh((e*x+d)^(1/2)/d^(1/2))^2*d^(1/2)-3*e*arctanh((e*x+d)^(
1/2)/d^(1/2))*(a+b*ln(c*x^n))*d^(1/2)-6*b*e*n*arctanh((e*x+d)^(1/2)/d^(1/2)
)*ln(2*d^(1/2)/(d^(1/2)-(e*x+d)^(1/2)))*d^(1/2)-3*b*e*n*polylog(2,1-2*d^(1/
2)/(d^(1/2)-(e*x+d)^(1/2)))*d^(1/2)-4*b*e*n*(e*x+d)^(1/2)-b*d*n*(e*x+d)^(1/
2)/x+3*e*(a+b*ln(c*x^n))*(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {43, 52, 65, 214, 2392, 14, 6131, 6055, 2449, 2352}

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x^2}\,dx =$$
$$-3\sqrt{d}e\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n)) - \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x}$$
$$+3e\sqrt{d+ex}(a+b\log(cx^n)) + 3b\sqrt{d}en\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2$$
$$+3b\sqrt{d}en\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) - 6b\sqrt{d}en\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)$$
$$-3b\sqrt{d}en\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right) - 4ben\sqrt{d+ex} - \frac{bdn\sqrt{d+ex}}{x}$$

```
[In] Int[((d + e*x)^(3/2)*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -4*b*e*n*Sqrt[d + e*x] - (b*d*n*Sqrt[d + e*x])/x + 3*b*Sqrt[d]*e*n*ArcTanh[
Sqrt[d + e*x]/Sqrt[d]] + 3*b*Sqrt[d]*e*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]^2 +
 3*e*Sqrt[d + e*x]*(a + b*Log[c*x^n]) - ((d + e*x)^(3/2)*(a + b*Log[c*x^n])
)/x - 3*Sqrt[d]*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]*(a + b*Log[c*x^n]) - 6*b*S
qrt[d]*e*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d
 + e*x])] - 3*b*Sqrt[d]*e*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d +
e*x])]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3e\sqrt{d+ex}(a+b\log(cx^n)) - \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x} \\
&\quad - 3\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n)) \\
&\quad - (bn)\int \frac{-\left((d-2ex)\sqrt{d+ex}\right) - 3\sqrt{d}ex\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x^2}\,dx \\
&= 3e\sqrt{d+ex}(a+b\log(cx^n)) - \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x} \\
&\quad - 3\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n)) \\
&\quad - (bn)\int \left(-\frac{d\sqrt{d+ex}}{x^2} + \frac{2e\sqrt{d+ex}}{x} - \frac{3\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\right)dx \\
&= 3e\sqrt{d+ex}(a+b\log(cx^n)) - \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x} \\
&\quad - 3\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n)) + (bdn)\int \frac{\sqrt{d+ex}}{x^2}\,dx \\
&\quad - (2ben)\int \frac{\sqrt{d+ex}}{x}\,dx + \left(3b\sqrt{d}en\right)\int \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx \\
&= -4ben\sqrt{d+ex} - \frac{bdn\sqrt{d+ex}}{x} + 3e\sqrt{d+ex}(a+b\log(cx^n)) \\
&\quad - \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x} - 3\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n)) \\
&\quad + \left(6b\sqrt{d}en\right)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx, x, \sqrt{d+ex}\right) \\
&\quad + \frac{1}{2}(bden)\int \frac{1}{x\sqrt{d+ex}}\,dx - (2bden)\int \frac{1}{x\sqrt{d+ex}}\,dx
\end{aligned}$$

$$= -4ben\sqrt{d+ex} - \frac{bdn\sqrt{d+ex}}{x} + 3b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2 + 3e\sqrt{d+ex}(a+b\log(cx^n)) - \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x} - 3\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n)) + (bdn)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right) - (4bdn)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right) - (6ben)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx, x, \sqrt{d+ex}\right)$$

$$= -4ben\sqrt{d+ex} - \frac{bdn\sqrt{d+ex}}{x} + 3b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) + 3b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2 + 3e\sqrt{d+ex}(a+b\log(cx^n)) - \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x} - 3\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n)) - 6b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right) + (6ben)\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}\,dx, x, \sqrt{d+ex}\right)$$

$$= -4ben\sqrt{d+ex} - \frac{bdn\sqrt{d+ex}}{x} + 3b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) + 3b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2 + 3e\sqrt{d+ex}(a+b\log(cx^n)) - \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x} - 3\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n)) - 6b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right) - \left(6b\sqrt{d}en\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)$$

$$= -4ben\sqrt{d+ex} - \frac{bdn\sqrt{d+ex}}{x} + 3b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) + 3b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2 + 3e\sqrt{d+ex}(a+b\log(cx^n)) - \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x} - 3\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n)) - 6b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right) - 3b\sqrt{d}en\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.85

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x^2}\,dx = \frac{-4ad\sqrt{d+ex} - 4bdn\sqrt{d+ex} + 8aex\sqrt{d+ex} - 16benx\sqrt{d+ex} + 12}{}$$

```
[In] Integrate[((d + e*x)^(3/2)*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] (-4*a*d*Sqrt[d + e*x] - 4*b*d*n*Sqrt[d + e*x] + 8*a*e*x*Sqrt[d + e*x] - 16*
b*e*n*x*Sqrt[d + e*x] + 12*b*Sqrt[d]*e*n*x*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] -
 4*b*d*Sqrt[d + e*x]*Log[c*x^n] + 8*b*e*x*Sqrt[d + e*x]*Log[c*x^n] + 6*a*Sq
rt[d]*e*x*Log[Sqrt[d] - Sqrt[d + e*x]] + 6*b*Sqrt[d]*e*x*Log[c*x^n]*Log[Sqr
t[d] - Sqrt[d + e*x]] - 3*b*Sqrt[d]*e*n*x*Log[Sqrt[d] - Sqrt[d + e*x]]^2 -
6*a*Sqrt[d]*e*x*Log[Sqrt[d] + Sqrt[d + e*x]] - 6*b*Sqrt[d]*e*x*Log[c*x^n]*L
og[Sqrt[d] + Sqrt[d + e*x]] + 3*b*Sqrt[d]*e*n*x*Log[Sqrt[d] + Sqrt[d + e*x]
]^2 + 6*b*Sqrt[d]*e*n*x*Log[Sqrt[d] + Sqrt[d + e*x]]*Log[1/2 - Sqrt[d + e*x
]/(2*Sqrt[d])] - 6*b*Sqrt[d]*e*n*x*Log[Sqrt[d] - Sqrt[d + e*x]]*Log[(1 + Sq
rt[d + e*x]/Sqrt[d])/2] - 6*b*Sqrt[d]*e*n*x*PolyLog[2, 1/2 - Sqrt[d + e*x]/
(2*Sqrt[d])] + 6*b*Sqrt[d]*e*n*x*PolyLog[2, (1 + Sqrt[d + e*x]/Sqrt[d])/2])
/(4*x)
```

Maple [F]

$$\int \frac{(ex+d)^{\frac{3}{2}}(a+b\ln(c\,x^n))}{x^2}dx$$

```
[In] int((e*x+d)^(3/2)*(a+b*ln(c*x^n))/x^2,x)
```

```
[Out] int((e*x+d)^(3/2)*(a+b*ln(c*x^n))/x^2,x)
```

Fricas [F]

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(b\log(cx^n)+a)}{x^2}\,dx$$

[In] integrate((e*x+d)^(3/2)*(a+b*log(c*x^n))/x^2,x, algorithm="fricas")

[Out] integral(((b*e*x + b*d)*sqrt(e*x + d)*log(c*x^n) + (a*e*x + a*d)*sqrt(e*x + d))/x^2, x)

Sympy [F]

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x^2}\,dx = \int \frac{(a+b\log(cx^n))(d+ex)^{\frac{3}{2}}}{x^2}\,dx$$

[In] integrate((e*x+d)**(3/2)*(a+b*ln(c*x**n))/x**2,x)

[Out] Integral((a + b*log(c*x**n))*(d + e*x)**(3/2)/x**2, x)

Maxima [F]

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(b\log(cx^n)+a)}{x^2}\,dx$$

[In] integrate((e*x+d)^(3/2)*(a+b*log(c*x^n))/x^2,x, algorithm="maxima")

[Out] 1/2*(3*sqrt(d)*e*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d))) + 4*sqrt(e*x + d)*e - 2*sqrt(e*x + d)*d/x)*a + b*integrate((e*x*log(c) + d*log(c) + (e*x + d)*log(x^n))*sqrt(e*x + d)/x^2, x)

Giac [F]

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(b\log(cx^n)+a)}{x^2}\,dx$$

[In] integrate((e*x+d)^(3/2)*(a+b*log(c*x^n))/x^2,x, algorithm="giac")

[Out] integrate((e*x + d)^(3/2)*(b*log(c*x^n) + a)/x^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,x^n\right)\right)\,\left(d+e\,x\right)^{3/2}}{x^2}\,dx$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^(3/2))/x^2,x)
[Out] int(((a + b*log(c*x^n))*(d + e*x)^(3/2))/x^2, x)
```

3.143 $\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 293

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x^3}\,dx = -\frac{bdn\sqrt{d+ex}}{4x^2}-\frac{11ben\sqrt{d+ex}}{8x} -\frac{9be^2n\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{8\sqrt{d}}+\frac{3be^2n\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4\sqrt{d}}-\frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4x} -\frac{(d+ex)^{3/2}(a+b\log(cx^n))}{2x^2}-\frac{3e^2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4\sqrt{d}} -\frac{3be^2n\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{2\sqrt{d}}-\frac{3be^2n\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{4\sqrt{d}}$$

```
[Out] -1/2*(e*x+d)^(3/2)*(a+b*ln(c*x^n))/x^2-9/8*b*e^2*n*arctanh((e*x+d)^(1/2)/d^
(1/2))/d^(1/2)+3/4*b*e^2*n*arctanh((e*x+d)^(1/2)/d^(1/2))^2/d^(1/2)-3/4*e^2
*arctanh((e*x+d)^(1/2)/d^(1/2))*(a+b*ln(c*x^n))/d^(1/2)-3/2*b*e^2*n*arctanh
((e*x+d)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)-(e*x+d)^(1/2)))/d^(1/2)-3/4*b
*e^2*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-(e*x+d)^(1/2)))/d^(1/2)-1/4*b*d*n*(e*
x+d)^(1/2)/x^2-11/8*b*e*n*(e*x+d)^(1/2)/x-3/4*e*(a+b*ln(c*x^n))*(e*x+d)^(1/
2)/x
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {43, 65, 214, 2392, 12, 14, 44, 6131, 6055, 2449, 2352}

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x^3}\,dx = -\frac{3e^2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4\sqrt{d}} - \frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4x} - \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{2x^2} + \frac{3be^2n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4\sqrt{d}} - \frac{9be^2n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{8\sqrt{d}} - \frac{3be^2n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{2\sqrt{d}} - \frac{3be^2n\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{4\sqrt{d}} - \frac{bdn\sqrt{d+ex}}{4x^2} - \frac{11ben\sqrt{d+ex}}{8x}$$

```
[In] Int[((d + e*x)^(3/2)*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/4*(b*d*n*Sqrt[d + e*x])/x^2 - (11*b*e*n*Sqrt[d + e*x])/(8*x) - (9*b*e^2*
n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(8*Sqrt[d]) + (3*b*e^2*n*ArcTanh[Sqrt[d +
 e*x]/Sqrt[d]]^2)/(4*Sqrt[d]) - (3*e*Sqrt[d + e*x]*(a + b*Log[c*x^n]))/(4*x
) - ((d + e*x)^(3/2)*(a + b*Log[c*x^n]))/(2*x^2) - (3*e^2*ArcTanh[Sqrt[d +
e*x]/Sqrt[d]]*(a + b*Log[c*x^n]))/(4*Sqrt[d]) - (3*b*e^2*n*ArcTanh[Sqrt[d +
 e*x]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x])])/(2*Sqrt[d]) - (3
*b*e^2*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x])])/(4*Sqrt[d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
```

```
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4x}-\frac{(d+ex)^{3/2}(a+b\log(cx^n))}{2x^2}\\
&\quad-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4\sqrt{d}}\\
&\quad-(bn)\int\frac{-\sqrt{d+ex}(2d+5ex)-\frac{3e^2x^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{4x^3}\,dx\\
&= -\frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4x}-\frac{(d+ex)^{3/2}(a+b\log(cx^n))}{2x^2}\\
&\quad-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4\sqrt{d}}\\
&\quad-\frac{1}{4}(bn)\int\frac{-\sqrt{d+ex}(2d+5ex)-\frac{3e^2x^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{x^3}\,dx\\
&= -\frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4x}-\frac{(d+ex)^{3/2}(a+b\log(cx^n))}{2x^2}\\
&\quad-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4\sqrt{d}}\\
&\quad-\frac{1}{4}(bn)\int\left(-\frac{2d\sqrt{d+ex}}{x^3}-\frac{5e\sqrt{d+ex}}{x^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}x}\right)dx\\
&= -\frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4x}-\frac{(d+ex)^{3/2}(a+b\log(cx^n))}{2x^2}\\
&\quad-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4\sqrt{d}}\\
&\quad+\frac{1}{2}(bdn)\int\frac{\sqrt{d+ex}}{x^3}\,dx+\frac{1}{4}(5ben)\int\frac{\sqrt{d+ex}}{x^2}\,dx+\frac{(3be^2n)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx}{4\sqrt{d}}
\end{aligned}
$$

$$= -\frac{bdn\sqrt{d+ex}}{4x^2} - \frac{5ben\sqrt{d+ex}}{4x} - \frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4x} - \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{2x^2} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4\sqrt{d}} + \frac{1}{8}(bden)\int\frac{1}{x^2\sqrt{d+ex}}\,dx + \frac{1}{8}(5be^2n)\int\frac{1}{x\sqrt{d+ex}}\,dx + \frac{(3be^2n)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx, x, \sqrt{d+ex}\right)}{2\sqrt{d}}$$

$$= -\frac{bdn\sqrt{d+ex}}{4x^2} - \frac{11ben\sqrt{d+ex}}{8x} + \frac{3be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4\sqrt{d}} - \frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4x} - \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{2x^2} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4\sqrt{d}} + \frac{1}{4}(5ben)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right) - \frac{1}{16}(be^2n)\int\frac{1}{x\sqrt{d+ex}}\,dx - \frac{(3be^2n)\,\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx, x, \sqrt{d+ex}\right)}{2d}$$

$$= -\frac{bdn\sqrt{d+ex}}{4x^2} - \frac{11ben\sqrt{d+ex}}{8x} - \frac{5be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4\sqrt{d}} + \frac{3be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4\sqrt{d}} - \frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4x} - \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{2x^2} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4\sqrt{d}} - \frac{3be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{2\sqrt{d}} - \frac{1}{8}(ben)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right) + \frac{(3be^2n)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}\,dx, x, \sqrt{d+ex}\right)}{2d}$$

$$= -\frac{bdn\sqrt{d+ex}}{4x^2} - \frac{11ben\sqrt{d+ex}}{8x} - \frac{9be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{8\sqrt{d}} + \frac{3be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4\sqrt{d}} - \frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4x} - \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{2x^2} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4\sqrt{d}} - \frac{3be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{2\sqrt{d}} - \frac{(3be^2n)\operatorname{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)}{2\sqrt{d}}$$

$$= -\frac{bdn\sqrt{d+ex}}{4x^2} - \frac{11ben\sqrt{d+ex}}{8x} - \frac{9be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{8\sqrt{d}} + \frac{3be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4\sqrt{d}} - \frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4x} - \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{2x^2} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4\sqrt{d}} - \frac{3be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{2\sqrt{d}} - \frac{3be^2n\operatorname{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)}{4\sqrt{d}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.71

$$\int\frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x^3}\,dx = -\frac{8ad^{3/2}\sqrt{d+ex}+4bd^{3/2}n\sqrt{d+ex}+20a\sqrt{d}ex\sqrt{d+ex}+22b\sqrt{d}enx\sqrt{d+ex}+18be^2nx^2\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{}$$

```
[In] Integrate[((d + e*x)^(3/2)*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/16*(8*a*d^(3/2)*Sqrt[d + e*x] + 4*b*d^(3/2)*n*Sqrt[d + e*x] + 20*a*Sqrt[
d]*e*x*Sqrt[d + e*x] + 22*b*Sqrt[d]*e*n*x*Sqrt[d + e*x] + 18*b*e^2*n*x^2*Ar
cTanh[Sqrt[d + e*x]/Sqrt[d]] + 8*b*d^(3/2)*Sqrt[d + e*x]*Log[c*x^n] + 20*b*
Sqrt[d]*e*x*Sqrt[d + e*x]*Log[c*x^n] - 6*a*e^2*x^2*Log[Sqrt[d] - Sqrt[d + e
*x]] - 6*b*e^2*x^2*Log[c*x^n]*Log[Sqrt[d] - Sqrt[d + e*x]] + 3*b*e^2*n*x^2*
Log[Sqrt[d] - Sqrt[d + e*x]]^2 + 6*a*e^2*x^2*Log[Sqrt[d] + Sqrt[d + e*x]] +
 6*b*e^2*x^2*Log[c*x^n]*Log[Sqrt[d] + Sqrt[d + e*x]] - 3*b*e^2*n*x^2*Log[Sq
rt[d] + Sqrt[d + e*x]]^2 - 6*b*e^2*n*x^2*Log[Sqrt[d] + Sqrt[d + e*x]]*Log[1
```

```
/2 - Sqrt[d + e*x]/(2*Sqrt[d])] + 6*b*e^2*n*x^2*Log[Sqrt[d] - Sqrt[d + e*x]
]*Log[(1 + Sqrt[d + e*x]/Sqrt[d])/2] + 6*b*e^2*n*x^2*PolyLog[2, 1/2 - Sqrt[
d + e*x]/(2*Sqrt[d])] - 6*b*e^2*n*x^2*PolyLog[2, (1 + Sqrt[d + e*x]/Sqrt[d]
)/2])/(Sqrt[d]*x^2)
```

Maple [F]

$$\int \frac{(ex+d)^{\frac{3}{2}} (a+b\ln(c\,x^n))}{x^3}dx$$

[In] int((e*x+d)^(3/2)*(a+b*ln(c*x^n))/x^3,x)

[Out] int((e*x+d)^(3/2)*(a+b*ln(c*x^n))/x^3,x)

Fricas [F]

$$\int \frac{(d+ex)^{3/2} (a+b\log(cx^n))}{x^3}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(b\log(cx^n)+a)}{x^3}\,dx$$

[In] integrate((e*x+d)^(3/2)*(a+b*log(c*x^n))/x^3,x, algorithm="fricas")

[Out] integral(((b*e*x + b*d)*sqrt(e*x + d)*log(c*x^n) + (a*e*x + a*d)*sqrt(e*x + d))/x^3, x)

Sympy [F]

$$\int \frac{(d+ex)^{3/2} (a+b\log(cx^n))}{x^3}\,dx = \int \frac{(a+b\log(cx^n))(d+ex)^{\frac{3}{2}}}{x^3}\,dx$$

[In] integrate((e*x+d)**(3/2)*(a+b*ln(c*x**n))/x**3,x)

[Out] Integral((a + b*log(c*x**n))*(d + e*x)**(3/2)/x**3, x)

Maxima [F]

$$\int \frac{(d+ex)^{3/2} (a+b\log(cx^n))}{x^3}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(b\log(cx^n)+a)}{x^3}\,dx$$

[In] integrate((e*x+d)^(3/2)*(a+b*log(c*x^n))/x^3,x, algorithm="maxima")

[Out] 1/8*(3*e^2*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/sqrt(d) - 2*(5*(e*x + d)^(3/2)*e^2 - 3*sqrt(e*x + d)*d*e^2)/((e*x + d)^2 - 2*(e*x + d)*d + d^2))*a + b*integrate((e*x*log(c) + d*log(c) + (e*x + d)*log(x^n))*sqrt(e*x + d)/x^3, x)

Giac [F]

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x^3}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(b\log(cx^n)+a)}{x^3}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*log(c*x^n))/x^3,x, algorithm="giac")
[Out] integrate((e*x + d)^(3/2)*(b*log(c*x^n) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(a+b\log(cx^n))}{x^3}\,dx = \int \frac{(a+b\,\ln(c\,x^n))\,(d+e\,x)^{3/2}}{x^3}\,dx$$

```
[In] int(((a + b*log(c*x^n))*(d + e*x)^(3/2))/x^3,x)
[Out] int(((a + b*log(c*x^n))*(d + e*x)^(3/2))/x^3, x)
```

3.144 $\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 217

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx = \frac{64bd^3n\sqrt{d+ex}}{35e^4}-\frac{76bd^2n(d+ex)^{3/2}}{105e^4}+\frac{64bdn(d+ex)^{5/2}}{175e^4} -\frac{4bn(d+ex)^{7/2}}{49e^4}-\frac{64bd^{7/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{35e^4} -\frac{2d^3\sqrt{d+ex}(a+b\log(cx^n))}{e^4}+\frac{2d^2(d+ex)^{3/2}(a+b\log(cx^n))}{e^4} -\frac{6d(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4}+\frac{2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^4}$$

```
[Out] -76/105*b*d^2*n*(e*x+d)^(3/2)/e^4+64/175*b*d*n*(e*x+d)^(5/2)/e^4-4/49*b*n*(
e*x+d)^(7/2)/e^4-64/35*b*d^(7/2)*n*arctanh((e*x+d)^(1/2)/d^(1/2))/e^4+2*d^2
*(e*x+d)^(3/2)*(a+b*ln(c*x^n))/e^4-6/5*d*(e*x+d)^(5/2)*(a+b*ln(c*x^n))/e^4+
2/7*(e*x+d)^(7/2)*(a+b*ln(c*x^n))/e^4+64/35*b*d^3*n*(e*x+d)^(1/2)/e^4-2*d^3
*(a+b*ln(c*x^n))*(e*x+d)^(1/2)/e^4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used

= {45, 2392, 12, 1634, 52, 65, 214}

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx = -\frac{2d^3\sqrt{d+ex}(a+b\log(cx^n))}{e^4} + \frac{2d^2(d+ex)^{3/2}(a+b\log(cx^n))}{e^4} - \frac{6d(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4} + \frac{2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^4} - \frac{64bd^{7/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{35e^4} + \frac{64bd^3n\sqrt{d+ex}}{35e^4} - \frac{76bd^2n(d+ex)^{3/2}}{105e^4} + \frac{64bdn(d+ex)^{5/2}}{175e^4} - \frac{4bn(d+ex)^{7/2}}{49e^4}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n]))/Sqrt[d + e*x],x]
```

```
[Out] (64*b*d^3*n*Sqrt[d + e*x])/(35*e^4) - (76*b*d^2*n*(d + e*x)^(3/2))/(105*e^4
) + (64*b*d*n*(d + e*x)^(5/2))/(175*e^4) - (4*b*n*(d + e*x)^(7/2))/(49*e^4)
 - (64*b*d^(7/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(35*e^4) - (2*d^3*Sqrt[d
 + e*x]*(a + b*Log[c*x^n]))/e^4 + (2*d^2*(d + e*x)^(3/2)*(a + b*Log[c*x^n])
)/e^4 - (6*d*(d + e*x)^(5/2)*(a + b*Log[c*x^n]))/(5*e^4) + (2*(d + e*x)^(7/
2)*(a + b*Log[c*x^n]))/(7*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2d^3\sqrt{d+ex}(a+b\log(cx^n))}{e^4}+\frac{2d^2(d+ex)^{3/2}(a+b\log(cx^n))}{e^4} \\
&\quad -\frac{6d(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4}+\frac{2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^4} \\
&\quad -(bn)\int\frac{2\sqrt{d+ex}(-16d^3+8d^2ex-6de^2x^2+5e^3x^3)}{35e^4x}\,dx \\
&= -\frac{2d^3\sqrt{d+ex}(a+b\log(cx^n))}{e^4}+\frac{2d^2(d+ex)^{3/2}(a+b\log(cx^n))}{e^4} \\
&\quad -\frac{6d(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4}+\frac{2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^4} \\
&\quad -\frac{(2bn)\int\frac{\sqrt{d+ex}(-16d^3+8d^2ex-6de^2x^2+5e^3x^3)}{x}\,dx}{35e^4} \\
&= -\frac{2d^3\sqrt{d+ex}(a+b\log(cx^n))}{e^4}+\frac{2d^2(d+ex)^{3/2}(a+b\log(cx^n))}{e^4} \\
&\quad -\frac{6d(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4}+\frac{2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^4} \\
&\quad -\frac{(2bn)\int\left(19d^2e\sqrt{d+ex}-\frac{16d^3\sqrt{d+ex}}{x}-16de(d+ex)^{3/2}+5e(d+ex)^{5/2}\right)dx}{35e^4}
\end{aligned}
$$

$$= -\frac{76bd^2n(d+ex)^{3/2}}{105e^4} + \frac{64bdn(d+ex)^{5/2}}{175e^4} - \frac{4bn(d+ex)^{7/2}}{49e^4} - \frac{2d^3\sqrt{d+ex}(a+b\log(cx^n))}{e^4} + \frac{2d^2(d+ex)^{3/2}(a+b\log(cx^n))}{e^4} - \frac{6d(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4} + \frac{2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^4} + \frac{(32bd^3n)\int \frac{\sqrt{d+ex}}{x}\,dx}{35e^4}$$

$$= \frac{64bd^3n\sqrt{d+ex}}{35e^4} - \frac{76bd^2n(d+ex)^{3/2}}{105e^4} + \frac{64bdn(d+ex)^{5/2}}{175e^4} - \frac{4bn(d+ex)^{7/2}}{49e^4} - \frac{2d^3\sqrt{d+ex}(a+b\log(cx^n))}{e^4} + \frac{2d^2(d+ex)^{3/2}(a+b\log(cx^n))}{e^4} - \frac{6d(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4} + \frac{2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^4} + \frac{(32bd^4n)\int \frac{1}{x\sqrt{d+ex}}\,dx}{35e^4}$$

$$= \frac{64bd^3n\sqrt{d+ex}}{35e^4} - \frac{76bd^2n(d+ex)^{3/2}}{105e^4} + \frac{64bdn(d+ex)^{5/2}}{175e^4} - \frac{4bn(d+ex)^{7/2}}{49e^4} - \frac{2d^3\sqrt{d+ex}(a+b\log(cx^n))}{e^4} + \frac{2d^2(d+ex)^{3/2}(a+b\log(cx^n))}{e^4} - \frac{6d(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4} + \frac{2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^4} + \frac{(64bd^4n)\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{35e^5}$$

$$= \frac{64bd^3n\sqrt{d+ex}}{35e^4} - \frac{76bd^2n(d+ex)^{3/2}}{105e^4} + \frac{64bdn(d+ex)^{5/2}}{175e^4} - \frac{4bn(d+ex)^{7/2}}{49e^4} - \frac{64bd^{7/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{35e^4} - \frac{2d^3\sqrt{d+ex}(a+b\log(cx^n))}{e^4} + \frac{2d^2(d+ex)^{3/2}(a+b\log(cx^n))}{e^4} - \frac{6d(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4} + \frac{2(d+ex)^{7/2}(a+b\log(cx^n))}{7e^4}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.69

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx = -\frac{2\left(3360bd^{7/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)+\sqrt{d+ex}(105a(16d^3-8d^2ex+6de^2x^2-5e^3x^3)+2bn(-1276d^3+218d\right.}{3675e^4}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/Sqrt[d + e*x],x]
```

```
[Out] (-2*(3360*b*d^(7/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + Sqrt[d + e*x]*(105*a
*(16*d^3 - 8*d^2*e*x + 6*d*e^2*x^2 - 5*e^3*x^3) + 2*b*n*(-1276*d^3 + 218*d^
2*e*x - 111*d*e^2*x^2 + 75*e^3*x^3) + 105*b*(16*d^3 - 8*d^2*e*x + 6*d*e^2*x
^2 - 5*e^3*x^3)*Log[c*x^n])))/(3675*e^4)
```

Maple [F]

$$\int \frac{x^3(a+b\ln(c\,x^n))}{\sqrt{ex+d}}dx$$

```
[In] int(x^3*(a+b*ln(c*x^n))/(e*x+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*ln(c*x^n))/(e*x+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.82

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx = \left[\frac{2\left(1680\,bd^{\frac{7}{2}}n\log\left(\frac{ex-2\,\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)+(2552\,bd^3n-1680\,ad^3-75\,(2\,be^3n-7\,ae^3)x^3+6\,(37\,bde^2n-105\right.}{}\right.$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/3675*(1680*b*d^(7/2)*n*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + (2
552*b*d^3*n - 1680*a*d^3 - 75*(2*b*e^3*n - 7*a*e^3)*x^3 + 6*(37*b*d*e^2*n -
 105*a*d*e^2)*x^2 - 4*(109*b*d^2*e*n - 210*a*d^2*e)*x + 105*(5*b*e^3*x^3 -
6*b*d*e^2*x^2 + 8*b*d^2*e*x - 16*b*d^3)*log(c) + 105*(5*b*e^3*n*x^3 - 6*b*d
*e^2*n*x^2 + 8*b*d^2*e*n*x - 16*b*d^3*n)*log(x))*sqrt(e*x + d))/e^4, 2/3675
*(3360*b*sqrt(-d)*d^3*n*arctan(sqrt(e*x + d)*sqrt(-d)/d) + (2552*b*d^3*n -
```

```
1680*a*d^3 - 75*(2*b*e^3*n - 7*a*e^3)*x^3 + 6*(37*b*d*e^2*n - 105*a*d*e^2)*
x^2 - 4*(109*b*d^2*e*n - 210*a*d^2*e)*x + 105*(5*b*e^3*x^3 - 6*b*d*e^2*x^2
+ 8*b*d^2*e*x - 16*b*d^3)*log(c) + 105*(5*b*e^3*n*x^3 - 6*b*d*e^2*n*x^2 + 8
*b*d^2*e*n*x - 16*b*d^3*n)*log(x))*sqrt(e*x + d))/e^4]
```

Sympy [A] (verification not implemented)

Time = 57.10 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.82

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx = a\left(\begin{cases} -\frac{2d^3\sqrt{d+ex}}{e^4}+\frac{2d^2(d+ex)^{\frac{3}{2}}}{e^4}-\frac{6d(d+ex)^{\frac{5}{2}}}{5e^4}+\frac{2(d+ex)^{\frac{7}{2}}}{7e^4} & \text{for } e\neq 0 \\ \frac{x^4}{4\sqrt{d}} & \text{otherwise}\end{cases}\right) - bn\left(\begin{cases} \frac{9596d^{\frac{7}{2}}\sqrt{1+\frac{ex}{d}}}{3675e^4}+\frac{38d^{\frac{7}{2}}\log\left(\frac{ex}{d}\right)}{35e^4}-\frac{76d^{\frac{7}{2}}\log\left(\sqrt{1+\frac{ex}{d}}+1\right)}{35e^4}+\frac{4d^{\frac{7}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}\sqrt{x}}\right)}{e^4}+\frac{872d^{\frac{5}{2}}x\sqrt{1+\frac{ex}{d}}}{3675e^3}-\frac{148d^{\frac{3}{2}}x^2\sqrt{1+\frac{ex}{d}}}{1225e^2}+ \\ \frac{x^4}{16\sqrt{d}} \end{cases}\right) + b\left(\begin{cases} -\frac{2d^3\sqrt{d+ex}}{e^4}+\frac{2d^2(d+ex)^{\frac{3}{2}}}{e^4}-\frac{6d(d+ex)^{\frac{5}{2}}}{5e^4}+\frac{2(d+ex)^{\frac{7}{2}}}{7e^4} & \text{for } e\neq 0 \\ \frac{x^4}{4\sqrt{d}} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))/(e*x+d)**(1/2),x)
```

```
[Out] a*Piecewise((-2*d**3*sqrt(d + e*x)/e**4 + 2*d**2*(d + e*x)**(3/2)/e**4 - 6*
d*(d + e*x)**(5/2)/(5*e**4) + 2*(d + e*x)**(7/2)/(7*e**4), Ne(e, 0)), (x**4
/(4*sqrt(d)), True)) - b*n*Piecewise((9596*d**(7/2)*sqrt(1 + e*x/d)/(3675*e
**4) + 38*d**(7/2)*log(e*x/d)/(35*e**4) - 76*d**(7/2)*log(sqrt(1 + e*x/d) +
 1)/(35*e**4) + 4*d**(7/2)*asinh(sqrt(d)/(sqrt(e)*sqrt(x)))/e**4 + 872*d**(
5/2)*x*sqrt(1 + e*x/d)/(3675*e**3) - 148*d**(3/2)*x**2*sqrt(1 + e*x/d)/(122
5*e**2) + 4*sqrt(d)*x**3*sqrt(1 + e*x/d)/(49*e) - 4*d**4/(e**(9/2)*sqrt(x)*
sqrt(d/(e*x) + 1)) - 4*d**3*sqrt(x)/(e**(7/2)*sqrt(d/(e*x) + 1)), (e > -oo)
 & (e < oo) & Ne(e, 0)), (x**4/(16*sqrt(d)), True)) + b*Piecewise((-2*d**3*
sqrt(d + e*x)/e**4 + 2*d**2*(d + e*x)**(3/2)/e**4 - 6*d*(d + e*x)**(5/2)/(5
*e**4) + 2*(d + e*x)**(7/2)/(7*e**4), Ne(e, 0)), (x**4/(4*sqrt(d)), True))*
log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.99

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx$$
$$=\frac{4}{3675}bn\left(\frac{840\,d^{\frac{7}{2}}\log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{e^4}-\frac{75\,(ex+d)^{\frac{7}{2}}-336\,(ex+d)^{\frac{5}{2}}d+665\,(ex+d)^{\frac{3}{2}}d^2-1680\,\sqrt{ex+d}d^3}{e^4}\right)$$
$$+\frac{2}{35}b\left(\frac{5\,(ex+d)^{\frac{7}{2}}}{e^4}-\frac{21\,(ex+d)^{\frac{5}{2}}d}{e^4}+\frac{35\,(ex+d)^{\frac{3}{2}}d^2}{e^4}-\frac{35\,\sqrt{ex+d}d^3}{e^4}\right)\log(cx^n)$$
$$+\frac{2}{35}a\left(\frac{5\,(ex+d)^{\frac{7}{2}}}{e^4}-\frac{21\,(ex+d)^{\frac{5}{2}}d}{e^4}+\frac{35\,(ex+d)^{\frac{3}{2}}d^2}{e^4}-\frac{35\,\sqrt{ex+d}d^3}{e^4}\right)$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3675*b*n*(840*d^(7/2)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt
(d)))/e^4 - (75*(e*x + d)^(7/2) - 336*(e*x + d)^(5/2)*d + 665*(e*x + d)^(3/
2)*d^2 - 1680*sqrt(e*x + d)*d^3)/e^4) + 2/35*b*(5*(e*x + d)^(7/2)/e^4 - 21*
(e*x + d)^(5/2)*d/e^4 + 35*(e*x + d)^(3/2)*d^2/e^4 - 35*sqrt(e*x + d)*d^3/e
^4)*log(c*x^n) + 2/35*a*(5*(e*x + d)^(7/2)/e^4 - 21*(e*x + d)^(5/2)*d/e^4 +
 35*(e*x + d)^(3/2)*d^2/e^4 - 35*sqrt(e*x + d)*d^3/e^4)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.16

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx=\frac{64\,bd^4n\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{35\,\sqrt{-d}e^4}$$
$$+\frac{2}{35}\left(\frac{5\,(ex+d)^{\frac{7}{2}}bn}{e^4}-\frac{21\,(ex+d)^{\frac{5}{2}}bdn}{e^4}+\frac{35\,(ex+d)^{\frac{3}{2}}bd^2n}{e^4}-\frac{35\,\sqrt{ex+d}bd^3n}{e^4}\right)\log(ex)$$
$$-\frac{2\,(7\,bn\log(e)+2\,bn-7\,b\log(c)-7\,a)(ex+d)^{\frac{7}{2}}}{49\,e^4}$$
$$+\frac{2\,(105\,bdn\log(e)+32\,bdn-105\,bd\log(c)-105\,ad)(ex+d)^{\frac{5}{2}}}{175\,e^4}$$
$$-\frac{2\,(105\,bd^2n\log(e)+38\,bd^2n-105\,bd^2\log(c)-105\,ad^2)(ex+d)^{\frac{3}{2}}}{105\,e^4}$$
$$+\frac{2\,(35\,bd^3n\log(e)+32\,bd^3n-35\,bd^3\log(c)-35\,ad^3)\sqrt{ex+d}}{35\,e^4}$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 64/35*b*d^4*n*arctan(sqrt(e*x + d)/sqrt(-d))/(sqrt(-d)*e^4) + 2/35*(5*(e*x
+ d)^(7/2)*b*n/e^4 - 21*(e*x + d)^(5/2)*b*d*n/e^4 + 35*(e*x + d)^(3/2)*b*d^
2*n/e^4 - 35*sqrt(e*x + d)*b*d^3*n/e^4)*log(e*x) - 2/49*(7*b*n*log(e) + 2*b
*n - 7*b*log(c) - 7*a)*(e*x + d)^(7/2)/e^4 + 2/175*(105*b*d*n*log(e) + 32*b
*d*n - 105*b*d*log(c) - 105*a*d)*(e*x + d)^(5/2)/e^4 - 2/105*(105*b*d^2*n*l
og(e) + 38*b*d^2*n - 105*b*d^2*log(c) - 105*a*d^2)*(e*x + d)^(3/2)/e^4 + 2/
35*(35*b*d^3*n*log(e) + 32*b*d^3*n - 35*b*d^3*log(c) - 35*a*d^3)*sqrt(e*x +
 d)/e^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log{(cx^n)})}{\sqrt{d+ex}}\,dx = \int \frac{x^3\,(a+b\,\ln{(c\,x^n)})}{\sqrt{d+e\,x}}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n)))/(d + e*x)^(1/2),x)
```

```
[Out] int((x^3*(a + b*log(c*x^n)))/(d + e*x)^(1/2), x)
```

3.145 $\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 169

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx = -\frac{32bd^2n\sqrt{d+ex}}{15e^3}+\frac{28bdn(d+ex)^{3/2}}{45e^3}-\frac{4bn(d+ex)^{5/2}}{25e^3}+\frac{32bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{15e^3}+\frac{2d^2\sqrt{d+ex}(a+b\log(cx^n))}{e^3}-\frac{4d(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3}+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^3}$$

```
[Out] 28/45*b*d*n*(e*x+d)^(3/2)/e^3-4/25*b*n*(e*x+d)^(5/2)/e^3+32/15*b*d^(5/2)*n*
arctanh((e*x+d)^(1/2)/d^(1/2))/e^3-4/3*d*(e*x+d)^(3/2)*(a+b*ln(c*x^n))/e^3+
2/5*(e*x+d)^(5/2)*(a+b*ln(c*x^n))/e^3-32/15*b*d^2*n*(e*x+d)^(1/2)/e^3+2*d^2
*(a+b*ln(c*x^n))*(e*x+d)^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {45, 2392, 12, 911, 1275, 214}

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx = \frac{2d^2\sqrt{d+ex}(a+b\log(cx^n))}{e^3}-\frac{4d(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3}+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^3}+\frac{32bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{15e^3}-\frac{32bd^2n\sqrt{d+ex}}{15e^3}+\frac{28bdn(d+ex)^{3/2}}{45e^3}-\frac{4bn(d+ex)^{5/2}}{25e^3}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/Sqrt[d + e*x],x]
```

```
[Out] (-32*b*d^2*n*Sqrt[d + e*x])/(15*e^3) + (28*b*d*n*(d + e*x)^(3/2))/(45*e^3)
- (4*b*n*(d + e*x)^(5/2))/(25*e^3) + (32*b*d^(5/2)*n*ArcTanh[Sqrt[d + e*x]/
Sqrt[d]])/(15*e^3) + (2*d^2*Sqrt[d + e*x]*(a + b*Log[c*x^n]))/e^3 - (4*d*(d
 + e*x)^(3/2)*(a + b*Log[c*x^n]))/(3*e^3) + (2*(d + e*x)^(5/2)*(a + b*Log[c
*x^n]))/(5*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
```

```
InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2d^2\sqrt{d+ex}(a+b\log(cx^n))}{e^3}-\frac{4d(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3} \\
&\quad+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^3}-(bn)\int\frac{2\sqrt{d+ex}(8d^2-4dex+3e^2x^2)}{15e^3x}\,dx \\
&= \frac{2d^2\sqrt{d+ex}(a+b\log(cx^n))}{e^3}-\frac{4d(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3} \\
&\quad+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^3}-\frac{(2bn)\int\frac{\sqrt{d+ex}(8d^2-4dex+3e^2x^2)}{x}\,dx}{15e^3} \\
&= \frac{2d^2\sqrt{d+ex}(a+b\log(cx^n))}{e^3}-\frac{4d(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3} \\
&\quad+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^3}-\frac{(4bn)\text{Subst}\left(\int\frac{x^2(15d^2-10dx^2+3x^4)}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{15e^4} \\
&= \frac{2d^2\sqrt{d+ex}(a+b\log(cx^n))}{e^3}-\frac{4d(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3} \\
&\quad+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^3} \\
&\quad-\frac{(4bn)\text{Subst}\left(\int\left(8d^2e-7dex^2+3ex^4+\frac{8d^3}{-\frac{d}{e}+\frac{x^2}{e}}\right)dx,x,\sqrt{d+ex}\right)}{15e^4} \\
&= -\frac{32bd^2n\sqrt{d+ex}}{15e^3}+\frac{28bdn(d+ex)^{3/2}}{45e^3}-\frac{4bn(d+ex)^{5/2}}{25e^3} \\
&\quad+\frac{2d^2\sqrt{d+ex}(a+b\log(cx^n))}{e^3}-\frac{4d(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3} \\
&\quad+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^3}-\frac{(32bd^3n)\,\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{15e^4} \\
&= -\frac{32bd^2n\sqrt{d+ex}}{15e^3}+\frac{28bdn(d+ex)^{3/2}}{45e^3}-\frac{4bn(d+ex)^{5/2}}{25e^3} \\
&\quad+\frac{32bd^{5/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{15e^3}+\frac{2d^2\sqrt{d+ex}(a+b\log(cx^n))}{e^3} \\
&\quad-\frac{4d(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3}+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.70

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx$$
$$= \frac{480bd^{5/2}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)+2\sqrt{d+ex}(15a(8d^2-4dex+3e^2x^2)-2bn(94d^2-17dex+9e^2x^2)+15b(8d}{225e^3}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/Sqrt[d + e*x],x]
```

```
[Out] (480*b*d^(5/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + 2*Sqrt[d + e*x]*(15*a*(8*
d^2 - 4*d*e*x + 3*e^2*x^2) - 2*b*n*(94*d^2 - 17*d*e*x + 9*e^2*x^2) + 15*b*(
8*d^2 - 4*d*e*x + 3*e^2*x^2)*Log[c*x^n]))/(225*e^3)
```

Maple [F]

$$\int \frac{x^2(a+b\ln(c\,x^n))}{\sqrt{ex+d}}dx$$

```
[In] int(x^2*(a+b*ln(c*x^n))/(e*x+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*ln(c*x^n))/(e*x+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.75

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx$$
$$= \left[\frac{2\left(120\,bd^{\frac{5}{2}}n\log\left(\frac{ex+2\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)-(188\,bd^2n-120\,ad^2+9\,(2\,be^2n-5\,ae^2)x^2-2\,(17\,bden-30\,ade)x}{225\,e^3}\right.$$
$$-\frac{2\left(240\,b\sqrt{-d}d^2n\arctan\left(\frac{\sqrt{ex+d}\sqrt{-d}}{d}\right)+(188\,bd^2n-120\,ad^2+9\,(2\,be^2n-5\,ae^2)x^2-2\,(17\,bden-30\,ad\right.}{225\,e^3}$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/225*(120*b*d^(5/2)*n*log((e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - (188
*b*d^2*n - 120*a*d^2 + 9*(2*b*e^2*n - 5*a*e^2)*x^2 - 2*(17*b*d*e*n - 30*a*d
*e)*x - 15*(3*b*e^2*x^2 - 4*b*d*e*x + 8*b*d^2)*log(c) - 15*(3*b*e^2*n*x^2 -
 4*b*d*e*n*x + 8*b*d^2*n)*log(x))*sqrt(e*x + d))/e^3, -2/225*(240*b*sqrt(-d
```

```
)*d^2*n*arctan(sqrt(e*x + d)*sqrt(-d)/d) + (188*b*d^2*n - 120*a*d^2 + 9*(2*
b*e^2*n - 5*a*e^2)*x^2 - 2*(17*b*d*e*n - 30*a*d*e)*x - 15*(3*b*e^2*x^2 - 4*
b*d*e*x + 8*b*d^2)*log(c) - 15*(3*b*e^2*n*x^2 - 4*b*d*e*n*x + 8*b*d^2*n)*lo
g(x))*sqrt(e*x + d))/e^3]
```

Sympy [A] (verification not implemented)

Time = 42.28 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.99

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx = a\left(\begin{cases}\frac{2d^2\sqrt{d+ex}}{e^3}-\frac{4d(d+ex)^{\frac{3}{2}}}{3e^3}+\frac{2(d+ex)^{\frac{5}{2}}}{5e^3} & \text{for } e\neq 0\\ \frac{x^3}{3\sqrt{d}} & \text{otherwise}\end{cases}\right)$$
$$-bn\left(\begin{cases}-\frac{524d^{\frac{5}{2}}\sqrt{1+\frac{ex}{d}}}{225e^3}-\frac{14d^{\frac{5}{2}}\log\left(\frac{ex}{d}\right)}{15e^3}+\frac{28d^{\frac{5}{2}}\log\left(\sqrt{1+\frac{ex}{d}}+1\right)}{15e^3}-\frac{4d^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}\sqrt{x}}\right)}{e^3}-\frac{68d^{\frac{3}{2}}x\sqrt{1+\frac{ex}{d}}}{225e^2}+\frac{4\sqrt{d}x^2\sqrt{1+\frac{ex}{d}}}{25e}+\frac{\cdots}{e^{\frac{7}{2}}\cdots}\\ \frac{x^3}{9\sqrt{d}}\end{cases}\right.$$
$$+b\left(\begin{cases}\frac{2d^2\sqrt{d+ex}}{e^3}-\frac{4d(d+ex)^{\frac{3}{2}}}{3e^3}+\frac{2(d+ex)^{\frac{5}{2}}}{5e^3} & \text{for } e\neq 0\\ \frac{x^3}{3\sqrt{d}} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(e*x+d)**(1/2),x)
```

```
[Out] a*Piecewise((2*d**2*sqrt(d + e*x)/e**3 - 4*d*(d + e*x)**(3/2)/(3*e**3) + 2*
(d + e*x)**(5/2)/(5*e**3), Ne(e, 0)), (x**3/(3*sqrt(d)), True)) - b*n*Piece
wise((-524*d**(5/2)*sqrt(1 + e*x/d)/(225*e**3) - 14*d**(5/2)*log(e*x/d)/(15
*e**3) + 28*d**(5/2)*log(sqrt(1 + e*x/d) + 1)/(15*e**3) - 4*d**(5/2)*asinh(
sqrt(d)/(sqrt(e)*sqrt(x)))/e**3 - 68*d**(3/2)*x*sqrt(1 + e*x/d)/(225*e**2)
+ 4*sqrt(d)*x**2*sqrt(1 + e*x/d)/(25*e) + 4*d**3/(e**(7/2)*sqrt(x)*sqrt(d/(
e*x) + 1)) + 4*d**2*sqrt(x)/(e**(5/2)*sqrt(d/(e*x) + 1)), (e > -oo) & (e <
oo) & Ne(e, 0)), (x**3/(9*sqrt(d)), True)) + b*Piecewise((2*d**2*sqrt(d + e
*x)/e**3 - 4*d*(d + e*x)**(3/2)/(3*e**3) + 2*(d + e*x)**(5/2)/(5*e**3), Ne(
e, 0)), (x**3/(3*sqrt(d)), True))*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx$$
$$= -\frac{4}{225}bn\left(\frac{60\,d^{\frac{5}{2}}\log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{e^3} + \frac{9\,(ex+d)^{\frac{5}{2}} - 35\,(ex+d)^{\frac{3}{2}}d + 120\,\sqrt{ex+d}d^2}{e^3}\right)$$
$$+ \frac{2}{15}b\left(\frac{3\,(ex+d)^{\frac{5}{2}}}{e^3} - \frac{10\,(ex+d)^{\frac{3}{2}}d}{e^3} + \frac{15\,\sqrt{ex+d}d^2}{e^3}\right)\log(cx^n)$$
$$+ \frac{2}{15}a\left(\frac{3\,(ex+d)^{\frac{5}{2}}}{e^3} - \frac{10\,(ex+d)^{\frac{3}{2}}d}{e^3} + \frac{15\,\sqrt{ex+d}d^2}{e^3}\right)$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -4/225*b*n*(60*d^(5/2)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(
d)))/e^3 + (9*(e*x + d)^(5/2) - 35*(e*x + d)^(3/2)*d + 120*sqrt(e*x + d)*d^
2)/e^3) + 2/15*b*(3*(e*x + d)^(5/2)/e^3 - 10*(e*x + d)^(3/2)*d/e^3 + 15*sqr
t(e*x + d)*d^2/e^3)*log(c*x^n) + 2/15*a*(3*(e*x + d)^(5/2)/e^3 - 10*(e*x +
d)^(3/2)*d/e^3 + 15*sqrt(e*x + d)*d^2/e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.14

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx$$
$$= -\frac{32\,bd^3n\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{15\,\sqrt{-d}e^3}$$
$$+ \frac{2}{15}\left(\frac{3\,(ex+d)^{\frac{5}{2}}bn}{e^3} - \frac{10\,(ex+d)^{\frac{3}{2}}bdn}{e^3} + \frac{15\,\sqrt{ex+d}bd^2n}{e^3}\right)\log(ex)$$
$$- \frac{2\,(5\,bn\log(e) + 2\,bn - 5\,b\log(c) - 5\,a)(ex+d)^{\frac{5}{2}}}{25\,e^3}$$
$$+ \frac{4\,(15\,bdn\log(e) + 7\,bdn - 15\,bd\log(c) - 15\,ad)(ex+d)^{\frac{3}{2}}}{45\,e^3}$$
$$- \frac{2\,(15\,bd^2n\log(e) + 16\,bd^2n - 15\,bd^2\log(c) - 15\,ad^2)\sqrt{ex+d}}{15\,e^3}$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] -32/15*b*d^3*n*arctan(sqrt(e*x + d)/sqrt(-d))/(sqrt(-d)*e^3) + 2/15*(3*(e*x
 + d)^(5/2)*b*n/e^3 - 10*(e*x + d)^(3/2)*b*d*n/e^3 + 15*sqrt(e*x + d)*b*d^2
*n/e^3)*log(e*x) - 2/25*(5*b*n*log(e) + 2*b*n - 5*b*log(c) - 5*a)*(e*x + d)
^(5/2)/e^3 + 4/45*(15*b*d*n*log(e) + 7*b*d*n - 15*b*d*log(c) - 15*a*d)*(e*x
 + d)^(3/2)/e^3 - 2/15*(15*b*d^2*n*log(e) + 16*b*d^2*n - 15*b*d^2*log(c) -
15*a*d^2)*sqrt(e*x + d)/e^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx=\int \frac{x^2\,(a+b\,\ln(c\,x^n))}{\sqrt{d+e\,x}}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e*x)^(1/2),x)
```

```
[Out] int((x^2*(a + b*log(c*x^n)))/(d + e*x)^(1/2), x)
```

3.146 $\int \frac{x(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx = \frac{8bdn\sqrt{d+ex}}{3e^2}-\frac{4bn(d+ex)^{3/2}}{9e^2}-\frac{8bd^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{3e^2}-\frac{2d\sqrt{d+ex}(a+b\log(cx^n))}{e^2}+\frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^2}$$

```
[Out] -4/9*b*n*(e*x+d)^(3/2)/e^2-8/3*b*d^(3/2)*n*arctanh((e*x+d)^(1/2)/d^(1/2))/e
^2+2/3*(e*x+d)^(3/2)*(a+b*ln(c*x^n))/e^2+8/3*b*d*n*(e*x+d)^(1/2)/e^2-2*d*(a
+b*ln(c*x^n))*(e*x+d)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {45, 2392, 12, 81, 52, 65, 214}

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx = -\frac{2d\sqrt{d+ex}(a+b\log(cx^n))}{e^2}+\frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^2}-\frac{8bd^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{3e^2}+\frac{8bdn\sqrt{d+ex}}{3e^2}-\frac{4bn(d+ex)^{3/2}}{9e^2}$$

```
[In] Int[(x*(a + b*Log[c*x^n]))/Sqrt[d + e*x],x]
```

```
[Out] (8*b*d*n*Sqrt[d + e*x])/(3*e^2) - (4*b*n*(d + e*x)^(3/2))/(9*e^2) - (8*b*d^
(3/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(3*e^2) - (2*d*Sqrt[d + e*x]*(a + b
*Log[c*x^n]))/e^2 + (2*(d + e*x)^(3/2)*(a + b*Log[c*x^n]))/(3*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
```

```
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d\sqrt{d+ex}(a+b\log(cx^n))}{e^2}+\frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^2} \\
&\quad -(bn)\int\frac{2(-2d+ex)\sqrt{d+ex}}{3e^2x}\,dx \\
&= -\frac{2d\sqrt{d+ex}(a+b\log(cx^n))}{e^2}+\frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^2}-\frac{(2bn)\int\frac{(-2d+ex)\sqrt{d+ex}}{x}\,dx}{3e^2} \\
&= -\frac{4bn(d+ex)^{3/2}}{9e^2}-\frac{2d\sqrt{d+ex}(a+b\log(cx^n))}{e^2} \\
&\quad +\frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^2}+\frac{(4bdn)\int\frac{\sqrt{d+ex}}{x}\,dx}{3e^2} \\
&= \frac{8bdn\sqrt{d+ex}}{3e^2}-\frac{4bn(d+ex)^{3/2}}{9e^2}-\frac{2d\sqrt{d+ex}(a+b\log(cx^n))}{e^2} \\
&\quad +\frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^2}+\frac{(4bd^2n)\int\frac{1}{x\sqrt{d+ex}}\,dx}{3e^2} \\
&= \frac{8bdn\sqrt{d+ex}}{3e^2}-\frac{4bn(d+ex)^{3/2}}{9e^2}-\frac{2d\sqrt{d+ex}(a+b\log(cx^n))}{e^2} \\
&\quad +\frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^2}+\frac{(8bd^2n)\,\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{3e^3} \\
&= \frac{8bdn\sqrt{d+ex}}{3e^2}-\frac{4bn(d+ex)^{3/2}}{9e^2}-\frac{8bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{3e^2} \\
&\quad -\frac{2d\sqrt{d+ex}(a+b\log(cx^n))}{e^2}+\frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67

$$\int\frac{x(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx = -\frac{2\left(12bd^{3/2}n\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)+\sqrt{d+ex}(6ad-10bdn-3aex+2benx+b(6d-3ex)\log(cx^n))\right)}{9e^2}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/Sqrt[d + e*x],x]
[Out] (-2*(12*b*d^(3/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + Sqrt[d + e*x]*(6*a*d -
 10*b*d*n - 3*a*e*x + 2*b*e*n*x + b*(6*d - 3*e*x)*Log[c*x^n])))/(9*e^2)
```

Maple [F]

$$\int \frac{x(a+b\ln(c\,x^n))}{\sqrt{ex+d}}dx$$

[In] int(x*(a+b*ln(c*x^n))/(e*x+d)^(1/2),x)

[Out] int(x*(a+b*ln(c*x^n))/(e*x+d)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.59

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx$$
$$=\left[\frac{2\left(6\,bd^{\frac{3}{2}}n\log\left(\frac{ex-2\,\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)+(10\,bdn-6\,ad-(2\,ben-3\,ae)x+3\,(bex-2\,bd)\log(c)+3\,(benx-2\right.}{9\,e^2}\right.$$

[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] [2/9*(6*b*d^(3/2)*n*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + (10*b*d*n - 6*a*d - (2*b*e*n - 3*a*e)*x + 3*(b*e*x - 2*b*d)*log(c) + 3*(b*e*n*x - 2*b*d*n)*log(x))*sqrt(e*x + d))/e^2, 2/9*(12*b*sqrt(-d)*d*n*arctan(sqrt(e*x + d)*sqrt(-d)/d) + (10*b*d*n - 6*a*d - (2*b*e*n - 3*a*e)*x + 3*(b*e*x - 2*b*d)*log(c) + 3*(b*e*n*x - 2*b*d*n)*log(x))*sqrt(e*x + d))/e^2]

Sympy [A] (verification not implemented)

Time = 32.67 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.29

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx = a\left(\begin{cases}-\frac{2d\sqrt{d+ex}}{e^2}+\frac{2(d+ex)^{\frac{3}{2}}}{3e^2} & \text{for } e\neq 0\\ \frac{x^2}{2\sqrt{d}} & \text{otherwise}\end{cases}\right)$$
$$-bn\left(\begin{cases}\frac{16d^{\frac{3}{2}}\sqrt{1+\frac{ex}{d}}}{9e^2}+\frac{2d^{\frac{3}{2}}\log\left(\frac{ex}{d}\right)}{3e^2}-\frac{4d^{\frac{3}{2}}\log\left(\sqrt{1+\frac{ex}{d}}+1\right)}{3e^2}+\frac{4d^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}\sqrt{x}}\right)}{e^2}+\frac{4\sqrt{d}x\sqrt{1+\frac{ex}{d}}}{9e}-\frac{4d^2}{e^{\frac{5}{2}}\sqrt{x}\sqrt{\frac{d}{ex}+1}}-\frac{4d\sqrt{x}}{e^{\frac{3}{2}}\sqrt{\frac{d}{ex}+}}\\ \frac{x^2}{4\sqrt{d}}\end{cases}\right.$$
$$+b\left(\begin{cases}-\frac{2d\sqrt{d+ex}}{e^2}+\frac{2(d+ex)^{\frac{3}{2}}}{3e^2} & \text{for } e\neq 0\\ \frac{x^2}{2\sqrt{d}} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

[In] integrate(x*(a+b*ln(c*x**n))/(e*x+d)**(1/2),x)

```
[Out] a*Piecewise((-2*d*sqrt(d + e*x)/e**2 + 2*(d + e*x)**(3/2)/(3*e**2), Ne(e, 0
)), (x**2/(2*sqrt(d)), True)) - b*n*Piecewise((16*d**(3/2)*sqrt(1 + e*x/d)/
(9*e**2) + 2*d**(3/2)*log(e*x/d)/(3*e**2) - 4*d**(3/2)*log(sqrt(1 + e*x/d)
+ 1)/(3*e**2) + 4*d**(3/2)*asinh(sqrt(d)/(sqrt(e)*sqrt(x)))/e**2 + 4*sqrt(d
)*x*sqrt(1 + e*x/d)/(9*e) - 4*d**2/(e**(5/2)*sqrt(x)*sqrt(d/(e*x) + 1)) - 4
*d*sqrt(x)/(e**(3/2)*sqrt(d/(e*x) + 1)), (e > -oo) & (e < oo) & Ne(e, 0)),
(x**2/(4*sqrt(d)), True)) + b*Piecewise((-2*d*sqrt(d + e*x)/e**2 + 2*(d + e
*x)**(3/2)/(3*e**2), Ne(e, 0)), (x**2/(2*sqrt(d)), True))*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx = \frac{4}{9}\,bn\left(\frac{3\,d^{\frac{3}{2}}\log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{e^2} - \frac{(ex+d)^{\frac{3}{2}} - 6\sqrt{ex+d}d}{e^2}\right) + \frac{2}{3}\,b\left(\frac{(ex+d)^{\frac{3}{2}}}{e^2} - \frac{3\sqrt{ex+d}d}{e^2}\right)\log(cx^n) + \frac{2}{3}\,a\left(\frac{(ex+d)^{\frac{3}{2}}}{e^2} - \frac{3\sqrt{ex+d}d}{e^2}\right)$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 4/9*b*n*(3*d^(3/2)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))
/e^2 - ((e*x + d)^(3/2) - 6*sqrt(e*x + d)*d)/e^2) + 2/3*b*((e*x + d)^(3/2)/
e^2 - 3*sqrt(e*x + d)*d/e^2)*log(c*x^n) + 2/3*a*((e*x + d)^(3/2)/e^2 - 3*sq
rt(e*x + d)*d/e^2)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.10

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx = \frac{8\,bd^2n\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{3\sqrt{-d}e^2} + \frac{2}{3}\left(\frac{(ex+d)^{\frac{3}{2}}bn}{e^2} - \frac{3\sqrt{ex+d}bdn}{e^2}\right)\log(ex) - \frac{2\,(3\,bn\log(e) + 2\,bn - 3\,b\log(c) - 3\,a)(ex+d)^{\frac{3}{2}}}{9\,e^2} + \frac{2\,(3\,bdn\log(e) + 4\,bdn - 3\,bd\log(c) - 3\,ad)\sqrt{ex+d}}{3\,e^2}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*b*d^2*n*arctan(sqrt(e*x + d)/sqrt(-d))/(sqrt(-d)*e^2) + 2/3*((e*x + d)^
(3/2)*b*n/e^2 - 3*sqrt(e*x + d)*b*d*n/e^2)*log(e*x) - 2/9*(3*b*n*log(e) + 2
*b*n - 3*b*log(c) - 3*a)*(e*x + d)^(3/2)/e^2 + 2/3*(3*b*d*n*log(e) + 4*b*d*
n - 3*b*d*log(c) - 3*a*d)*sqrt(e*x + d)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d+ex}}\,dx=\int \frac{x\,(a+b\,\ln(c\,x^n))}{\sqrt{d+e\,x}}\,dx$$

```
[In] int((x*(a + b*log(c*x^n)))/(d + e*x)^(1/2),x)
```

```
[Out] int((x*(a + b*log(c*x^n)))/(d + e*x)^(1/2), x)
```

3.147 $\int \frac{a+b\log(cx^n)}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{a+b\log(cx^n)}{\sqrt{d+ex}}\,dx=-\frac{4bn\sqrt{d+ex}}{e}+\frac{4b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{e}+\frac{2\sqrt{d+ex}(a+b\log(cx^n))}{e}$$

```
[Out] 4*b*n*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)/e-4*b*n*(e*x+d)^(1/2)/e+2*(a+b
*ln(c*x^n))*(e*x+d)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2356, 52, 65, 214}

$$\int \frac{a+b\log(cx^n)}{\sqrt{d+ex}}\,dx=\frac{2\sqrt{d+ex}(a+b\log(cx^n))}{e}+\frac{4b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{e}-\frac{4bn\sqrt{d+ex}}{e}$$

```
[In] Int[(a + b*Log[c*x^n])/Sqrt[d + e*x],x]
```

```
[Out] (-4*b*n*Sqrt[d + e*x])/e + (4*b*Sqrt[d]*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/e
 + (2*Sqrt[d + e*x]*(a + b*Log[c*x^n]))/e
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\sqrt{d+ex}(a+b\log(cx^n))}{e} - \frac{(2bn)\int\frac{\sqrt{d+ex}}{x}\,dx}{e}\\
&= -\frac{4bn\sqrt{d+ex}}{e} + \frac{2\sqrt{d+ex}(a+b\log(cx^n))}{e} - \frac{(2bdn)\int\frac{1}{x\sqrt{d+ex}}\,dx}{e}\\
&= -\frac{4bn\sqrt{d+ex}}{e} + \frac{2\sqrt{d+ex}(a+b\log(cx^n))}{e} - \frac{(4bdn)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{e^2}\\
&= -\frac{4bn\sqrt{d+ex}}{e} + \frac{4b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{e} + \frac{2\sqrt{d+ex}(a+b\log(cx^n))}{e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \frac{a+b\log\left(cx^n\right)}{\sqrt{d+ex}}\,dx=\frac{4b\sqrt{d}n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)+2\sqrt{d+ex}\left(a-2bn+b\log\left(cx^n\right)\right)}{e}$$

```
[In] Integrate[(a + b*Log[c*x^n])/Sqrt[d + e*x],x]
```

```
[Out] (4*b*Sqrt[d]*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + 2*Sqrt[d + e*x]*(a - 2*b*n
+ b*Log[c*x^n]))/e
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{2\sqrt{ex+d}\,a+2b\left(\ln\left(c\,x^{n}\right)\sqrt{ex+d}+2n\left(-\sqrt{ex+d}+\sqrt{d}\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)\right)\right)}{e}$	62
default	$\frac{2\sqrt{ex+d}\,a+2b\left(\ln\left(c\,x^{n}\right)\sqrt{ex+d}+2n\left(-\sqrt{ex+d}+\sqrt{d}\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)\right)\right)}{e}$	62
parts	$\frac{2a\sqrt{ex+d}}{e}+\frac{2b\left(\ln\left(c\,x^{n}\right)\sqrt{ex+d}-2n\left(\sqrt{ex+d}-\sqrt{d}\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)\right)\right)}{e}$	64

```
[In] int((a+b*ln(c*x^n))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*((e*x+d)^(1/2)*a+b*(ln(c*x^n)*(e*x+d)^(1/2)+2*n*(-(e*x+d)^(1/2)+d^(1/2)
*arctanh((e*x+d)^(1/2)/d^(1/2)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.68

$$\int \frac{a+b\log\left(cx^n\right)}{\sqrt{d+ex}}\,dx = \left[\frac{2\left(b\sqrt{d}n\log\left(\frac{ex+2\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)+\left(bn\log\left(x\right)-2\,bn+b\log\left(c\right)+a\right)\sqrt{ex+d}\right)}{e},\ -\frac{2\left(2\,b\sqrt{-d}n\arctan\left(\frac{\sqrt{ex+d}\sqrt{-d}}{d}\right)-\left(bn\log\left(x\right)-2\,bn+b\log\left(c\right)+a\right)\sqrt{ex+d}\right)}{e}\right]$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*(b*sqrt(d)*n*log((e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + (b*n*log(x)
- 2*b*n + b*log(c) + a)*sqrt(e*x + d))/e, -2*(2*b*sqrt(-d)*n*arctan(sqrt(e*
x + d)*sqrt(-d)/d) - (b*n*log(x) - 2*b*n + b*log(c) + a)*sqrt(e*x + d))/e]
```

Sympy [A] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.90

$$\int \frac{a+b\log(cx^n)}{\sqrt{d+ex}}\,dx = a\left(\begin{cases}\frac{2\sqrt{d+ex}}{e} & \text{for } e\neq 0\\ \frac{x}{\sqrt{d}} & \text{otherwise}\end{cases}\right) - bn\left(\begin{cases}-\frac{4\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}\sqrt{x}}\right)}{e}+\frac{4d}{e^{\frac{3}{2}}\sqrt{x}\sqrt{\frac{d}{ex}+1}}+\frac{4\sqrt{x}}{\sqrt{e}\sqrt{\frac{d}{ex}+1}} & \text{for } e>-\infty\wedge e<\infty\wedge e\neq 0\\ \frac{x}{\sqrt{d}} & \text{otherwise}\end{cases}\right) + b\left(\begin{cases}\frac{2\sqrt{d+ex}}{e} & \text{for } e\neq 0\\ \frac{x}{\sqrt{d}} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate((a+b*ln(c*x**n))/(e*x+d)**(1/2),x)
```

```
[Out] a*Piecewise((2*sqrt(d + e*x)/e, Ne(e, 0)), (x/sqrt(d), True)) - b*n*Piecewi
se((-4*sqrt(d)*asinh(sqrt(d)/(sqrt(e)*sqrt(x)))/e + 4*d/(e**(3/2)*sqrt(x)*s
qrt(d/(e*x) + 1)) + 4*sqrt(x)/(sqrt(e)*sqrt(d/(e*x) + 1)), (e > -oo) & (e <
 oo) & Ne(e, 0)), (x/sqrt(d), True)) + b*Piecewise((2*sqrt(d + e*x)/e, Ne(e
, 0)), (x/sqrt(d), True))*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int \frac{a+b\log(cx^n)}{\sqrt{d+ex}}\,dx = -\frac{2\left(\sqrt{d}\log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)+2\sqrt{ex+d}\right)bn}{e} + \frac{2\sqrt{ex+d}b\log(cx^n)}{e} + \frac{2\sqrt{ex+d}a}{e}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(sqrt(d)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d))) + 2*sq
rt(e*x + d))*b*n/e + 2*sqrt(e*x + d)*b*log(c*x^n)/e + 2*sqrt(e*x + d)*a/e
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log(cx^n)}{\sqrt{d+ex}}\,dx = -\frac{2\left(\left(\frac{2\,d\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\sqrt{ex+d}\log(x)+2\sqrt{ex+d}\right)bn-\sqrt{ex+d}b\log(c)-\sqrt{ex+d}a\right)}{e}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d)^(1/2),x, algorithm="giac")
[Out] -2*((2*d*arctan(sqrt(e*x + d)/sqrt(-d))/sqrt(-d) - sqrt(e*x + d)*log(x) + 2
*sqrt(e*x + d))*b*n - sqrt(e*x + d)*b*log(c) - sqrt(e*x + d)*a)/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{\sqrt{d+ex}}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{\sqrt{d+e\,x}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(d + e*x)^(1/2),x)
[Out] int((a + b*log(c*x^n))/(d + e*x)^(1/2), x)
```

3.148 $\int \frac{a+b\log(cx^n)}{x\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 152

$$\int \frac{a+b\log(cx^n)}{x\sqrt{d+ex}}\,dx = \frac{2bn\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{\sqrt{d}} - \frac{2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} - \frac{4bn\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{\sqrt{d}} - \frac{2bn\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{\sqrt{d}}$$

```
[Out] 2*b*n*arctanh((e*x+d)^(1/2)/d^(1/2))^2/d^(1/2)-2*arctanh((e*x+d)^(1/2)/d^(1
/2))*(a+b*ln(c*x^n))/d^(1/2)-4*b*n*arctanh((e*x+d)^(1/2)/d^(1/2))*ln(2*d^(1
/2)/(d^(1/2)-(e*x+d)^(1/2)))/d^(1/2)-2*b*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-(
e*x+d)^(1/2)))/d^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {65, 214, 2390, 12, 6131, 6055, 2449, 2352}

$$\int \frac{a+b\log(cx^n)}{x\sqrt{d+ex}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}}+\frac{2bn\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{\sqrt{d}}-\frac{4bn\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{\sqrt{d}}-\frac{2bn\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{\sqrt{d}}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*Sqrt[d + e*x]),x]
```

```
[Out] (2*b*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]^2)/Sqrt[d] - (2*ArcTanh[Sqrt[d + e*x]
/Sqrt[d]]*(a + b*Log[c*x^n]))/Sqrt[d] - (4*b*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d
]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x])])/Sqrt[d] - (2*b*n*PolyLog[2,
1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x])])/Sqrt[d]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} - (bn)\int -\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}x}\,dx \\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} + \frac{(2bn)\int \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx}{\sqrt{d}} \\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} + \frac{(4bn)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx, x, \sqrt{d+ex}\right)}{\sqrt{d}} \\
&= \frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{\sqrt{d}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} \\
&\quad - \frac{(4bn)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx, x, \sqrt{d+ex}\right)}{d}
\end{aligned}$$

$$= \frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{\sqrt{d}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} - \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{\sqrt{d}} + \frac{(4bn)\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}\,dx, x, \sqrt{d+ex}\right)}{d}$$

$$= \frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{\sqrt{d}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} - \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{\sqrt{d}} - \frac{(4bn)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)}{\sqrt{d}}$$

$$= \frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{\sqrt{d}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} - \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{\sqrt{d}} - \frac{2bn\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)}{\sqrt{d}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.64

$$\int \frac{a+b\log(cx^n)}{x\sqrt{d+ex}}\,dx = \frac{2(a+b\log(cx^n))\log\left(\sqrt{d}-\sqrt{d+ex}\right) - 2(a+b\log(cx^n))\log\left(\sqrt{d}+\sqrt{d+ex}\right) - bn\left(\log\left(\sqrt{d}-\sqrt{d+e\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*Sqrt[d + e*x]),x]
```

```
[Out] (2*(a + b*Log[c*x^n])*Log[Sqrt[d] - Sqrt[d + e*x]] - 2*(a + b*Log[c*x^n])*L
og[Sqrt[d] + Sqrt[d + e*x]] - b*n*(Log[Sqrt[d] - Sqrt[d + e*x]]*(Log[Sqrt[d
] - Sqrt[d + e*x]] + 2*Log[(1 + Sqrt[d + e*x]/Sqrt[d])/2]) + 2*PolyLog[2, 1
/2 - Sqrt[d + e*x]/(2*Sqrt[d])]) + b*n*(Log[Sqrt[d] + Sqrt[d + e*x]]*(Log[S
qrt[d] + Sqrt[d + e*x]] + 2*Log[1/2 - Sqrt[d + e*x]/(2*Sqrt[d])]) + 2*PolyL
og[2, (1 + Sqrt[d + e*x]/Sqrt[d])/2]))/(2*Sqrt[d])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\,x^{n}\right)}{x\sqrt{ex+d}}dx$$

```
[In] int((a+b*ln(c*x^n))/x/(e*x+d)^(1/2),x)
[Out] int((a+b*ln(c*x^n))/x/(e*x+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x\sqrt{d+ex}}\,dx = \int \frac{b\log\left(cx^{n}\right)+a}{\sqrt{ex+dx}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^(1/2),x, algorithm="fricas")
[Out] integral((sqrt(e*x + d)*b*log(c*x^n) + sqrt(e*x + d)*a)/(e*x^2 + d*x), x)
```

Sympy [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x\sqrt{d+ex}}\,dx = \int \frac{a+b\log\left(cx^{n}\right)}{x\sqrt{d+ex}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x/(e*x+d)**(1/2),x)
[Out] Integral((a + b*log(c*x**n))/(x*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x\sqrt{d+ex}}\,dx = \int \frac{b\log\left(cx^{n}\right)+a}{\sqrt{ex+dx}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^(1/2),x, algorithm="maxima")
[Out] b*integrate((log(c) + log(x^n))/(sqrt(e*x + d)*x), x) + a*log((sqrt(e*x + d
) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/sqrt(d)
```

Giac [F]

$$\int \frac{a + b\log(cx^n)}{x\sqrt{d + ex}}\,dx = \int \frac{b\log(cx^n) + a}{\sqrt{ex + dx}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/(sqrt(e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{x\sqrt{d + ex}}\,dx = \int \frac{a + b\,\ln(c\,x^n)}{x\,\sqrt{d + e\,x}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x)^(1/2)),x)
[Out] int((a + b*log(c*x^n))/(x*(d + e*x)^(1/2)), x)
```

3.149 $\int \frac{a+b\log(cx^n)}{x^2\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 226

$$\begin{aligned}\int \frac{a+b\log(cx^n)}{x^2\sqrt{d+ex}}\,dx &= -\frac{bn\sqrt{d+ex}}{dx} - \frac{ben\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{ben\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{d^{3/2}}\\ &\quad - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{dx} + \frac{e\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}}\\ &\quad + \frac{2ben\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{3/2}}\\ &\quad + \frac{ben\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{3/2}}\end{aligned}$$

```
[Out] -b*e*n*arctanh((e*x+d)^(1/2)/d^(1/2))/d^(3/2)-b*e*n*arctanh((e*x+d)^(1/2)/d
^(1/2))^2/d^(3/2)+e*arctanh((e*x+d)^(1/2)/d^(1/2))*(a+b*ln(c*x^n))/d^(3/2)+
2*b*e*n*arctanh((e*x+d)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)-(e*x+d)^(1/2))
)/d^(3/2)+b*e*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-(e*x+d)^(1/2)))/d^(3/2)-b*n*
(e*x+d)^(1/2)/d/x-(a+b*ln(c*x^n))*(e*x+d)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules

used = {44, 65, 214, 2392, 14, 43, 6131, 6055, 2449, 2352}

$$\int \frac{a+b\log(cx^n)}{x^2\sqrt{d+ex}}\,dx = \frac{e\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{dx} - \frac{ben\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{ben\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{2ben\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{3/2}} + \frac{ben\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{3/2}} - \frac{bn\sqrt{d+ex}}{dx}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^2*Sqrt[d + e*x]),x]
```

```
[Out] -((b*n*Sqrt[d + e*x])/(d*x)) - (b*e*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/d^(3/
2) - (b*e*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]^2)/d^(3/2) - (Sqrt[d + e*x]*(a +
 b*Log[c*x^n]))/(d*x) + (e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]*(a + b*Log[c*x^n]
))/d^(3/2) + (2*b*e*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[
d] - Sqrt[d + e*x])])/d^(3/2) + (b*e*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d]
- Sqrt[d + e*x])])/d^(3/2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+ex}(a+b\log(cx^n))}{dx}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} \\
&\quad -(bn)\int\frac{-\frac{\sqrt{d+ex}}{d}+\frac{ex\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{3/2}}}{x^2}\,dx \\
&= -\frac{\sqrt{d+ex}(a+b\log(cx^n))}{dx}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} \\
&\quad -(bn)\int\left(-\frac{\sqrt{d+ex}}{dx^2}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{3/2}x}\right)dx \\
&= -\frac{\sqrt{d+ex}(a+b\log(cx^n))}{dx}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} \\
&\quad +\frac{(bn)\int\frac{\sqrt{d+ex}}{x^2}\,dx}{d}-\frac{(ben)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx}{d^{3/2}} \\
&= -\frac{bn\sqrt{d+ex}}{dx}-\frac{\sqrt{d+ex}(a+b\log(cx^n))}{dx}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} \\
&\quad -\frac{(2ben)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx,x,\sqrt{d+ex}\right)}{d^{3/2}}+\frac{(ben)\int\frac{1}{x\sqrt{d+ex}}\,dx}{2d} \\
&= -\frac{bn\sqrt{d+ex}}{dx}-\frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{d^{3/2}}-\frac{\sqrt{d+ex}(a+b\log(cx^n))}{dx} \\
&\quad +\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}}+\frac{(bn)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{d} \\
&\quad +\frac{(2ben)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx,x,\sqrt{d+ex}\right)}{d^2}
\end{aligned}
$$

$$= -\frac{bn\sqrt{d+ex}}{dx} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{dx} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} + \frac{2ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{3/2}} - \frac{(2ben)\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}dx, x, \sqrt{d+ex}\right)}{d^2}$$

$$= -\frac{bn\sqrt{d+ex}}{dx} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{dx} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} + \frac{2ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{3/2}} + \frac{(2ben)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}dx, x, \frac{1}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)}{d^{3/2}}$$

$$= -\frac{bn\sqrt{d+ex}}{dx} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{dx} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} + \frac{2ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{3/2}} + \frac{ben\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)}{d^{3/2}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.73

$$\int \frac{a+b\log(cx^n)}{x^2\sqrt{d+ex}}dx = -\frac{4a\sqrt{d}\sqrt{d+ex} + 4b\sqrt{d}n\sqrt{d+ex} + 4benx\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) + 4b\sqrt{d}\sqrt{d+ex}\log(cx^n) + 2aex\log\left(\sqrt{d}-\right.}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*Sqrt[d + e*x]),x]
```

```
[Out] -1/4*(4*a*Sqrt[d]*Sqrt[d + e*x] + 4*b*Sqrt[d]*n*Sqrt[d + e*x] + 4*b*e*n*x*A
rcTanh[Sqrt[d + e*x]/Sqrt[d]] + 4*b*Sqrt[d]*Sqrt[d + e*x]*Log[c*x^n] + 2*a*
e*x*Log[Sqrt[d] - Sqrt[d + e*x]] + 2*b*e*x*Log[c*x^n]*Log[Sqrt[d] - Sqrt[d
```

```
+ e*x]] - b*e*n*x*Log[Sqrt[d] - Sqrt[d + e*x]]^2 - 2*a*e*x*Log[Sqrt[d] + Sq
rt[d + e*x]] - 2*b*e*x*Log[c*x^n]*Log[Sqrt[d] + Sqrt[d + e*x]] + b*e*n*x*Lo
g[Sqrt[d] + Sqrt[d + e*x]]^2 + 2*b*e*n*x*Log[Sqrt[d] + Sqrt[d + e*x]]*Log[1
/2 - Sqrt[d + e*x]/(2*Sqrt[d])] - 2*b*e*n*x*Log[Sqrt[d] - Sqrt[d + e*x]]*Lo
g[(1 + Sqrt[d + e*x]/Sqrt[d])/2] - 2*b*e*n*x*PolyLog[2, 1/2 - Sqrt[d + e*x]
/(2*Sqrt[d])] + 2*b*e*n*x*PolyLog[2, (1 + Sqrt[d + e*x]/Sqrt[d])/2])/(d^(3/
2)*x)
```

Maple [F]

$$\int \frac{a+b\ln \left(c\,x^{n}\right)}{x^{2}\sqrt{e x +d}}d x$$

[In] `int((a+b*ln(c*x^n))/x^2/(e*x+d)^(1/2),x)`

[Out] `int((a+b*ln(c*x^n))/x^2/(e*x+d)^(1/2),x)`

Fricas [F]

$$\int \frac{a+b\log (cx^n)}{x^2\sqrt{d+ex}}\,dx=\int \frac{b\log (cx^n)+a}{\sqrt{ex+d}x^2}\,dx$$

[In] `integrate((a+b*log(c*x^n))/x^2/(e*x+d)^(1/2),x, algorithm="fricas")`

[Out] `integral((sqrt(e*x + d)*b*log(c*x^n) + sqrt(e*x + d)*a)/(e*x^3 + d*x^2), x)`

Sympy [F]

$$\int \frac{a+b\log (cx^n)}{x^2\sqrt{d+ex}}\,dx=\int \frac{a+b\log (cx^n)}{x^2\sqrt{d+ex}}\,dx$$

[In] `integrate((a+b*ln(c*x**n))/x**2/(e*x+d)**(1/2),x)`

[Out] `Integral((a + b*log(c*x**n))/(x**2*sqrt(d + e*x)), x)`

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{x^2\sqrt{d+ex}}\,dx = \int \frac{b\log(cx^n)+a}{\sqrt{ex+dx^2}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*sqrt(e*x + d)*e/((e*x + d)*d - d^2) + e*log((sqrt(e*x + d) - sqrt
(d))/(sqrt(e*x + d) + sqrt(d)))/d^(3/2)) + b*integrate((log(c) + log(x^n))/
(sqrt(e*x + d)*x^2), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^2\sqrt{d+ex}}\,dx = \int \frac{b\log(cx^n)+a}{\sqrt{ex+dx^2}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/(sqrt(e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^2\sqrt{d+ex}}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^2\,\sqrt{d+e\,x}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*x)^(1/2)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*x)^(1/2)), x)
```

3.150 $\int \frac{a+b\log(cx^n)}{x^3\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 304

$$\int \frac{a+b\log(cx^n)}{x^3\sqrt{d+ex}}\,dx = -\frac{bn\sqrt{d+ex}}{4dx^2}+\frac{5ben\sqrt{d+ex}}{8d^2x}+\frac{7be^2n\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{8d^{5/2}} + \frac{3be^2n\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4d^{5/2}}-\frac{\sqrt{d+ex}(a+b\log(cx^n))}{2dx^2} + \frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4d^2x}-\frac{3e^2\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{5/2}} - \frac{3be^2n\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{2d^{5/2}} - \frac{3be^2n\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{4d^{5/2}}$$

```
[Out] 7/8*b*e^2*n*arctanh((e*x+d)^(1/2)/d^(1/2))/d^(5/2)+3/4*b*e^2*n*arctanh((e*x
+d)^(1/2)/d^(1/2))^2/d^(5/2)-3/4*e^2*arctanh((e*x+d)^(1/2)/d^(1/2))*(a+b*ln
(c*x^n))/d^(5/2)-3/2*b*e^2*n*arctanh((e*x+d)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d
^(1/2)-(e*x+d)^(1/2)))/d^(5/2)-3/4*b*e^2*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-(
e*x+d)^(1/2)))/d^(5/2)-1/4*b*n*(e*x+d)^(1/2)/d/x^2+5/8*b*e*n*(e*x+d)^(1/2)/
d^2/x-1/2*(a+b*ln(c*x^n))*(e*x+d)^(1/2)/d/x^2+3/4*e*(a+b*ln(c*x^n))*(e*x+d)
^(1/2)/d^2/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {44, 65, 214, 2392, 12, 14, 43, 6131, 6055, 2449, 2352}

$$\int \frac{a+b\log(cx^n)}{x^3\sqrt{d+ex}}\,dx = -\frac{3e^2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{5/2}}+\frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4d^2x} -\frac{\sqrt{d+ex}(a+b\log(cx^n))}{2dx^2}+\frac{3be^2n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4d^{5/2}} +\frac{7be^2n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{8d^{5/2}}-\frac{3be^2n\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{2d^{5/2}} -\frac{3be^2n\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{4d^{5/2}}+\frac{5ben\sqrt{d+ex}}{8d^2x}-\frac{bn\sqrt{d+ex}}{4dx^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^3*Sqrt[d + e*x]),x]
```

```
[Out] -1/4*(b*n*Sqrt[d + e*x])/(d*x^2) + (5*b*e*n*Sqrt[d + e*x])/(8*d^2*x) + (7*b
*e^2*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(8*d^(5/2)) + (3*b*e^2*n*ArcTanh[Sqr
t[d + e*x]/Sqrt[d]]^2)/(4*d^(5/2)) - (Sqrt[d + e*x]*(a + b*Log[c*x^n]))/(2*
d*x^2) + (3*e*Sqrt[d + e*x]*(a + b*Log[c*x^n]))/(4*d^2*x) - (3*e^2*ArcTanh[
Sqrt[d + e*x]/Sqrt[d]]*(a + b*Log[c*x^n]))/(4*d^(5/2)) - (3*b*e^2*n*ArcTanh
[Sqrt[d + e*x]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x])])/(2*d^(5
/2)) - (3*b*e^2*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x])])/(4
*d^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
```

```
0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{d+ex}(a+b\log(cx^n))}{2dx^2} + \frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4d^2x} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{5/2}} - (bn)\int \frac{\sqrt{d}\sqrt{d+ex}(-2d+3ex) - 3e^2x^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4d^{5/2}x^3}\,dx$$

$$= -\frac{\sqrt{d+ex}(a+b\log(cx^n))}{2dx^2} + \frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4d^2x} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{5/2}} - \frac{(bn)\int \frac{\sqrt{d}\sqrt{d+ex}(-2d+3ex)-3e^2x^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x^3}\,dx}{4d^{5/2}}$$

$$= -\frac{\sqrt{d+ex}(a+b\log(cx^n))}{2dx^2} + \frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4d^2x} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{5/2}} - \frac{(bn)\int \left(-\frac{2d^{3/2}\sqrt{d+ex}}{x^3} + \frac{3\sqrt{d}e\sqrt{d+ex}}{x^2} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\right)dx}{4d^{5/2}}$$

$$= -\frac{\sqrt{d+ex}(a+b\log(cx^n))}{2dx^2} + \frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4d^2x} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{5/2}} + \frac{(bn)\int \frac{\sqrt{d+ex}}{x^3}\,dx}{2d} - \frac{(3ben)\int \frac{\sqrt{d+ex}}{x^2}\,dx}{4d^2} + \frac{(3be^2n)\int \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx}{4d^{5/2}}$$

$$= -\frac{bn\sqrt{d+ex}}{4dx^2} + \frac{3ben\sqrt{d+ex}}{4d^2x} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{2dx^2} + \frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4d^2x} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{5/2}} + \frac{(ben)\int\frac{1}{x^2\sqrt{d+ex}}\,dx}{8d} + \frac{(3be^2n)\,\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx, x, \sqrt{d+ex}\right)}{2d^{5/2}} - \frac{(3be^2n)\int\frac{1}{x\sqrt{d+ex}}\,dx}{8d^2}$$

$$= -\frac{bn\sqrt{d+ex}}{4dx^2} + \frac{5ben\sqrt{d+ex}}{8d^2x} + \frac{3be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4d^{5/2}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{2dx^2} + \frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4d^2x} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{5/2}} - \frac{(3ben)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{4d^2} - \frac{(3be^2n)\,\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx, x, \sqrt{d+ex}\right)}{2d^3} - \frac{(be^2n)\int\frac{1}{x\sqrt{d+ex}}\,dx}{16d^2}$$

$$= -\frac{bn\sqrt{d+ex}}{4dx^2} + \frac{5ben\sqrt{d+ex}}{8d^2x} + \frac{3be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4d^{5/2}} + \frac{3be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4d^{5/2}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{2dx^2} + \frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4d^2x} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{5/2}} - \frac{3be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{2d^{5/2}} - \frac{(ben)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{8d^2} + \frac{(3be^2n)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}\,dx, x, \sqrt{d+ex}\right)}{2d^3}$$

$$= -\frac{bn\sqrt{d+ex}}{4dx^2} + \frac{5ben\sqrt{d+ex}}{8d^2x} + \frac{7be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{8d^{5/2}} + \frac{3be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4d^{5/2}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{2dx^2} + \frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4d^2x} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{5/2}} - \frac{3be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{2d^{5/2}} - \frac{(3be^2n)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)}{2d^{5/2}}$$

$$= -\frac{bn\sqrt{d+ex}}{4dx^2} + \frac{5ben\sqrt{d+ex}}{8d^2x} + \frac{7be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{8d^{5/2}} + \frac{3be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{4d^{5/2}} - \frac{\sqrt{d+ex}(a+b\log(cx^n))}{2dx^2} + \frac{3e\sqrt{d+ex}(a+b\log(cx^n))}{4d^2x} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{4d^{5/2}} - \frac{3be^2n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{2d^{5/2}} - \frac{3be^2n\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)}{4d^{5/2}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.65

$$\int \frac{a+b\log(cx^n)}{x^3\sqrt{d+ex}}\,dx = \frac{-8ad^{3/2}\sqrt{d+ex} - 4bd^{3/2}n\sqrt{d+ex} + 12a\sqrt{d}ex\sqrt{d+ex} + 10b\sqrt{d}enx\sqrt{d+ex} + 14be^2nx^2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right.}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*Sqrt[d + e*x]),x]
```

```
[Out] (-8*a*d^(3/2)*Sqrt[d + e*x] - 4*b*d^(3/2)*n*Sqrt[d + e*x] + 12*a*Sqrt[d]*e*
x*Sqrt[d + e*x] + 10*b*Sqrt[d]*e*n*x*Sqrt[d + e*x] + 14*b*e^2*n*x^2*ArcTanh
[Sqrt[d + e*x]/Sqrt[d]] - 8*b*d^(3/2)*Sqrt[d + e*x]*Log[c*x^n] + 12*b*Sqrt[
d]*e*x*Sqrt[d + e*x]*Log[c*x^n] + 6*a*e^2*x^2*Log[Sqrt[d] - Sqrt[d + e*x]]
+ 6*b*e^2*x^2*Log[c*x^n]*Log[Sqrt[d] - Sqrt[d + e*x]] - 3*b*e^2*n*x^2*Log[S
qrt[d] - Sqrt[d + e*x]]^2 - 6*a*e^2*x^2*Log[Sqrt[d] + Sqrt[d + e*x]] - 6*b*
e^2*x^2*Log[c*x^n]*Log[Sqrt[d] + Sqrt[d + e*x]] + 3*b*e^2*n*x^2*Log[Sqrt[d]
 + Sqrt[d + e*x]]^2 + 6*b*e^2*n*x^2*Log[Sqrt[d] + Sqrt[d + e*x]]*Log[1/2 -
Sqrt[d + e*x]/(2*Sqrt[d])] - 6*b*e^2*n*x^2*Log[Sqrt[d] - Sqrt[d + e*x]]*Log
[(1 + Sqrt[d + e*x]/Sqrt[d])/2] - 6*b*e^2*n*x^2*PolyLog[2, 1/2 - Sqrt[d + e
*x]/(2*Sqrt[d])] + 6*b*e^2*n*x^2*PolyLog[2, (1 + Sqrt[d + e*x]/Sqrt[d])/2])
/(16*d^(5/2)*x^2)
```

Maple [F]

$$\int \frac{a+b\ln\left(c\,x^{n}\right)}{x^{3}\sqrt{ex+d}}dx$$

```
[In] int((a+b*ln(c*x^n))/x^3/(e*x+d)^(1/2),x)
[Out] int((a+b*ln(c*x^n))/x^3/(e*x+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{3}\sqrt{d+ex}}\,dx = \int \frac{b\log\left(cx^{n}\right)+a}{\sqrt{ex+d}x^{3}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d)^(1/2),x, algorithm="fricas")
[Out] integral((sqrt(e*x + d)*b*log(c*x^n) + sqrt(e*x + d)*a)/(e*x^4 + d*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{3}\sqrt{d+ex}}\,dx = \int \frac{a+b\log\left(cx^{n}\right)}{x^{3}\sqrt{d+ex}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**3/(e*x+d)**(1/2),x)
[Out] Integral((a + b*log(c*x**n))/(x**3*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{3}\sqrt{d+ex}}\,dx = \int \frac{b\log\left(cx^{n}\right)+a}{\sqrt{ex+d}x^{3}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d)^(1/2),x, algorithm="maxima")
[Out] 1/8*a*(3*e^2*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/d^(5/
2) + 2*(3*(e*x + d)^(3/2)*e^2 - 5*sqrt(e*x + d)*d*e^2)/((e*x + d)^2*d^2 - 2
*(e*x + d)*d^3 + d^4)) + b*integrate((log(c) + log(x^n))/(sqrt(e*x + d)*x^3
), x)
```

Giac [F]

$$\int \frac{a + b\log(cx^n)}{x^3\sqrt{d + ex}}\,dx = \int \frac{b\log(cx^n) + a}{\sqrt{ex + d}x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/(sqrt(e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{x^3\sqrt{d + ex}}\,dx = \int \frac{a + b\,\ln(c\,x^n)}{x^3\,\sqrt{d + e\,x}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e*x)^(1/2)),x)
[Out] int((a + b*log(c*x^n))/(x^3*(d + e*x)^(1/2)), x)
```

3.151 $\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 194

$$\begin{aligned}\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx &= -\frac{44bd^2n\sqrt{d+ex}}{5e^4}+\frac{16bdn(d+ex)^{3/2}}{15e^4}-\frac{4bn(d+ex)^{5/2}}{25e^4}\\&\quad+\frac{64bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{5e^4}+\frac{2d^3(a+b\log(cx^n))}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}(a+b\log(cx^n))}{e^4}\\&\quad-\frac{2d(d+ex)^{3/2}(a+b\log(cx^n))}{e^4}+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4}\end{aligned}$$

```
[Out] 16/15*b*d*n*(e*x+d)^(3/2)/e^4-4/25*b*n*(e*x+d)^(5/2)/e^4+64/5*b*d^(5/2)*n*a
rctanh((e*x+d)^(1/2)/d^(1/2))/e^4-2*d*(e*x+d)^(3/2)*(a+b*ln(c*x^n))/e^4+2/5
*(e*x+d)^(5/2)*(a+b*ln(c*x^n))/e^4+2*d^3*(a+b*ln(c*x^n))/e^4/(e*x+d)^(1/2)-
44/5*b*d^2*n*(e*x+d)^(1/2)/e^4+6*d^2*(a+b*ln(c*x^n))*(e*x+d)^(1/2)/e^4
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {45, 2392, 12, 1634, 65, 214}

$$\begin{aligned}\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx &= \frac{2d^3(a+b\log(cx^n))}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}(a+b\log(cx^n))}{e^4}\\&\quad-\frac{2d(d+ex)^{3/2}(a+b\log(cx^n))}{e^4}+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4}\\&\quad+\frac{64bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{5e^4}-\frac{44bd^2n\sqrt{d+ex}}{5e^4}+\frac{16bdn(d+ex)^{3/2}}{15e^4}-\frac{4bn(d+ex)^{5/2}}{25e^4}\end{aligned}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n]))/(d + e*x)^(3/2),x]
```

```
[Out] (-44*b*d^2*n*Sqrt[d + e*x])/(5*e^4) + (16*b*d*n*(d + e*x)^(3/2))/(15*e^4) -
 (4*b*n*(d + e*x)^(5/2))/(25*e^4) + (64*b*d^(5/2)*n*ArcTanh[Sqrt[d + e*x]/S
qrt[d]])/(5*e^4) + (2*d^3*(a + b*Log[c*x^n]))/(e^4*Sqrt[d + e*x]) + (6*d^2*
Sqrt[d + e*x]*(a + b*Log[c*x^n]))/e^4 - (2*d*(d + e*x)^(3/2)*(a + b*Log[c*x
^n]))/e^4 + (2*(d + e*x)^(5/2)*(a + b*Log[c*x^n]))/(5*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
```

```
IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2d^3(a+b\log(cx^n))}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}(a+b\log(cx^n))}{e^4}-\frac{2d(d+ex)^{3/2}(a+b\log(cx^n))}{e^4} \\
&\quad+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4}-(bn)\int\frac{2(16d^3+8d^2ex-2de^2x^2+e^3x^3)}{5e^4x\sqrt{d+ex}}\,dx \\
&= \frac{2d^3(a+b\log(cx^n))}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}(a+b\log(cx^n))}{e^4}-\frac{2d(d+ex)^{3/2}(a+b\log(cx^n))}{e^4} \\
&\quad+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4}-\frac{(2bn)\int\frac{16d^3+8d^2ex-2de^2x^2+e^3x^3}{x\sqrt{d+ex}}\,dx}{5e^4} \\
&= \frac{2d^3(a+b\log(cx^n))}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}(a+b\log(cx^n))}{e^4} \\
&\quad-\frac{2d(d+ex)^{3/2}(a+b\log(cx^n))}{e^4}+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4} \\
&\quad-\frac{(2bn)\int\left(\frac{11d^2e}{\sqrt{d+ex}}+\frac{16d^3}{x\sqrt{d+ex}}-4de\sqrt{d+ex}+e(d+ex)^{3/2}\right)dx}{5e^4} \\
&= -\frac{44bd^2n\sqrt{d+ex}}{5e^4}+\frac{16bdn(d+ex)^{3/2}}{15e^4}-\frac{4bn(d+ex)^{5/2}}{25e^4}+\frac{2d^3(a+b\log(cx^n))}{e^4\sqrt{d+ex}} \\
&\quad+\frac{6d^2\sqrt{d+ex}(a+b\log(cx^n))}{e^4}-\frac{2d(d+ex)^{3/2}(a+b\log(cx^n))}{e^4} \\
&\quad+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4}-\frac{(32bd^3n)\int\frac{1}{x\sqrt{d+ex}}\,dx}{5e^4} \\
&= -\frac{44bd^2n\sqrt{d+ex}}{5e^4}+\frac{16bdn(d+ex)^{3/2}}{15e^4}-\frac{4bn(d+ex)^{5/2}}{25e^4}+\frac{2d^3(a+b\log(cx^n))}{e^4\sqrt{d+ex}} \\
&\quad+\frac{6d^2\sqrt{d+ex}(a+b\log(cx^n))}{e^4}-\frac{2d(d+ex)^{3/2}(a+b\log(cx^n))}{e^4} \\
&\quad+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4}-\frac{(64bd^3n)\,\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{5e^5} \\
&= -\frac{44bd^2n\sqrt{d+ex}}{5e^4}+\frac{16bdn(d+ex)^{3/2}}{15e^4}-\frac{4bn(d+ex)^{5/2}}{25e^4} \\
&\quad+\frac{64bd^{5/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{5e^4}+\frac{2d^3(a+b\log(cx^n))}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}(a+b\log(cx^n))}{e^4} \\
&\quad-\frac{2d(d+ex)^{3/2}(a+b\log(cx^n))}{e^4}+\frac{2(d+ex)^{5/2}(a+b\log(cx^n))}{5e^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = \frac{480ad^3-592bd^3n+240ad^2ex-536bd^2enx-60ade^2x^2+44bde^2nx^2+30ae^3x^3}{}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/(d + e*x)^(3/2),x]
```

```
[Out] (480*a*d^3 - 592*b*d^3*n + 240*a*d^2*e*x - 536*b*d^2*e*n*x - 60*a*d*e^2*x^2
 + 44*b*d*e^2*n*x^2 + 30*a*e^3*x^3 - 12*b*e^3*n*x^3 + 960*b*d^(5/2)*n*Sqrt[
d + e*x]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + 30*b*(16*d^3 + 8*d^2*e*x - 2*d*e^
2*x^2 + e^3*x^3)*Log[c*x^n])/(75*e^4*Sqrt[d + e*x])
```

Maple [F]

$$\int \frac{x^3(a+b\ln(c\,x^n))}{(ex+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^3*(a+b*ln(c*x^n))/(e*x+d)^(3/2),x)
```

```
[Out] int(x^3*(a+b*ln(c*x^n))/(e*x+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.24

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = \left[\frac{2\left(240\,(bd^2enx+bd^3n)\sqrt{d}\log\left(\frac{ex+2\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)-(296\,bd^3n-240\,ad^3+3\,(2\,b}{}\right.$$

$$-\frac{2\left(480\,(bd^2enx+bd^3n)\sqrt{-d}\arctan\left(\frac{\sqrt{ex+d}\sqrt{-d}}{d}\right)+(296\,bd^3n-240\,ad^3+3\,(2\,be^3n-5\,ae^3)x^3-2\,(11\,bde^2n}{}$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/75*(240*(b*d^2*e*n*x + b*d^3*n)*sqrt(d)*log((e*x + 2*sqrt(e*x + d)*sqrt(
d) + 2*d)/x) - (296*b*d^3*n - 240*a*d^3 + 3*(2*b*e^3*n - 5*a*e^3)*x^3 - 2*(
11*b*d*e^2*n - 15*a*d*e^2)*x^2 + 4*(67*b*d^2*e*n - 30*a*d^2*e)*x - 15*(b*e^
3*x^3 - 2*b*d*e^2*x^2 + 8*b*d^2*e*x + 16*b*d^3)*log(c) - 15*(b*e^3*n*x^3 -
2*b*d*e^2*n*x^2 + 8*b*d^2*e*n*x + 16*b*d^3*n)*log(x))*sqrt(e*x + d))/(e^5*x
 + d*e^4), -2/75*(480*(b*d^2*e*n*x + b*d^3*n)*sqrt(-d)*arctan(sqrt(e*x + d)
*sqrt(-d)/d) + (296*b*d^3*n - 240*a*d^3 + 3*(2*b*e^3*n - 5*a*e^3)*x^3 - 2*(
```

```
11*b*d*e^2*n - 15*a*d*e^2)*x^2 + 4*(67*b*d^2*e*n - 30*a*d^2*e)*x - 15*(b*e^
3*x^3 - 2*b*d*e^2*x^2 + 8*b*d^2*e*x + 16*b*d^3)*log(c) - 15*(b*e^3*n*x^3 -
2*b*d*e^2*n*x^2 + 8*b*d^2*e*n*x + 16*b*d^3*n)*log(x))*sqrt(e*x + d))/(e^5*x
 + d*e^4)]
```

Sympy [A] (verification not implemented)

Time = 116.99 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.90

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = a\left(\begin{cases} \frac{2d^3}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}}{e^4} - \frac{2d(d+ex)^{\frac{3}{2}}}{e^4} + \frac{2(d+ex)^{\frac{5}{2}}}{5e^4} & \text{for } e \neq 0 \\ \frac{x^4}{4d^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) - bn\left(\begin{cases} -\frac{308d^{\frac{5}{2}}\sqrt{1+\frac{ex}{d}}}{75e^4} - \frac{8d^{\frac{5}{2}}\log\left(\frac{ex}{d}\right)}{5e^4} + \frac{16d^{\frac{5}{2}}\log\left(\sqrt{1+\frac{ex}{d}}+1\right)}{5e^4} - \frac{16d^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}\sqrt{x}}\right)}{e^4} - \frac{56d^{\frac{3}{2}}x\sqrt{1+\frac{ex}{d}}}{75e^3} + \frac{4\sqrt{d}x^2\sqrt{1+\frac{ex}{d}}}{25e^2} + \frac{\ldots}{e^{\frac{9}{2}}\ldots} \\ \frac{x^4}{16d^{\frac{3}{2}}} \end{cases}\right) + b\left(\begin{cases} \frac{2d^3}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}}{e^4} - \frac{2d(d+ex)^{\frac{3}{2}}}{e^4} + \frac{2(d+ex)^{\frac{5}{2}}}{5e^4} & \text{for } e \neq 0 \\ \frac{x^4}{4d^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)\log(cx^n)$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))/(e*x+d)**(3/2),x)
```

```
[Out] a*Piecewise((2*d**3/(e**4*sqrt(d + e*x)) + 6*d**2*sqrt(d + e*x)/e**4 - 2*d*
(d + e*x)**(3/2)/e**4 + 2*(d + e*x)**(5/2)/(5*e**4), Ne(e, 0)), (x**4/(4*d*
*(3/2)), True)) - b*n*Piecewise((-308*d**(5/2)*sqrt(1 + e*x/d)/(75*e**4) -
8*d**(5/2)*log(e*x/d)/(5*e**4) + 16*d**(5/2)*log(sqrt(1 + e*x/d) + 1)/(5*e*
*4) - 16*d**(5/2)*asinh(sqrt(d)/(sqrt(e)*sqrt(x)))/e**4 - 56*d**(3/2)*x*sqr
t(1 + e*x/d)/(75*e**3) + 4*sqrt(d)*x**2*sqrt(1 + e*x/d)/(25*e**2) + 12*d**3
/(e**(9/2)*sqrt(x)*sqrt(d/(e*x) + 1)) + 12*d**2*sqrt(x)/(e**(7/2)*sqrt(d/(e
*x) + 1)), (e > -oo) & (e < oo) & Ne(e, 0)), (x**4/(16*d**(3/2)), True)) +
b*Piecewise((2*d**3/(e**4*sqrt(d + e*x)) + 6*d**2*sqrt(d + e*x)/e**4 - 2*d*
(d + e*x)**(3/2)/e**4 + 2*(d + e*x)**(5/2)/(5*e**4), Ne(e, 0)), (x**4/(4*d*
*(3/2)), True))*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.03

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = -\frac{4}{75}\,bn\left(\frac{120\,d^{\frac{5}{2}}\log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{e^4}+\frac{3\,(ex+d)^{\frac{5}{2}}-20\,(ex+d)^{\frac{3}{2}}d+165\,\sqrt{ex+d}d^2}{e^4}\right) + \frac{2}{5}\,b\left(\frac{(ex+d)^{\frac{5}{2}}}{e^4}-\frac{5\,(ex+d)^{\frac{3}{2}}d}{e^4}+\frac{15\,\sqrt{ex+d}d^2}{e^4}+\frac{5\,d^3}{\sqrt{ex+d}e^4}\right)\log(cx^n) + \frac{2}{5}\,a\left(\frac{(ex+d)^{\frac{5}{2}}}{e^4}-\frac{5\,(ex+d)^{\frac{3}{2}}d}{e^4}+\frac{15\,\sqrt{ex+d}d^2}{e^4}+\frac{5\,d^3}{\sqrt{ex+d}e^4}\right)$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^(3/2),x, algorithm="maxima")
[Out] -4/75*b*n*(120*d^(5/2)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(
d)))/e^4 + (3*(e*x + d)^(5/2) - 20*(e*x + d)^(3/2)*d + 165*sqrt(e*x + d)*d^
2)/e^4) + 2/5*b*((e*x + d)^(5/2)/e^4 - 5*(e*x + d)^(3/2)*d/e^4 + 15*sqrt(e*
x + d)*d^2/e^4 + 5*d^3/(sqrt(e*x + d)*e^4))*log(c*x^n) + 2/5*a*((e*x + d)^(
5/2)/e^4 - 5*(e*x + d)^(3/2)*d/e^4 + 15*sqrt(e*x + d)*d^2/e^4 + 5*d^3/(sqrt
(e*x + d)*e^4))
```

Giac [F]

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^3/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx=\int \frac{x^3\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{{\left(d+e\,x\right)}^{3/2}}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n)))/(d + e*x)^(3/2),x)
[Out] int((x^3*(a + b*log(c*x^n)))/(d + e*x)^(3/2), x)
```

3.152 $\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 146

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = \frac{20bdn\sqrt{d+ex}}{3e^3}-\frac{4bn(d+ex)^{3/2}}{9e^3}-\frac{32bd^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{3e^3}-\frac{2d^2(a+b\log(cx^n))}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}(a+b\log(cx^n))}{e^3}+\frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3}$$

```
[Out] -4/9*b*n*(e*x+d)^(3/2)/e^3-32/3*b*d^(3/2)*n*arctanh((e*x+d)^(1/2)/d^(1/2))/
e^3+2/3*(e*x+d)^(3/2)*(a+b*ln(c*x^n))/e^3-2*d^2*(a+b*ln(c*x^n))/e^3/(e*x+d)
^(1/2)+20/3*b*d*n*(e*x+d)^(1/2)/e^3-4*d*(a+b*ln(c*x^n))*(e*x+d)^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {45, 2392, 12, 911, 1167, 214}

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = -\frac{2d^2(a+b\log(cx^n))}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}(a+b\log(cx^n))}{e^3}+\frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3}-\frac{32bd^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{3e^3}+\frac{20bdn\sqrt{d+ex}}{3e^3}-\frac{4bn(d+ex)^{3/2}}{9e^3}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/(d + e*x)^(3/2),x]
```

```
[Out] (20*b*d*n*Sqrt[d + e*x])/(3*e^3) - (4*b*n*(d + e*x)^(3/2))/(9*e^3) - (32*b*
d^(3/2)*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(3*e^3) - (2*d^2*(a + b*Log[c*x^n
```

```
]))/(e^3*Sqrt[d + e*x]) - (4*d*Sqrt[d + e*x]*(a + b*Log[c*x^n]))/e^3 + (2*(
d + e*x)^(3/2)*(a + b*Log[c*x^n]))/(3*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = -\frac{2d^2(a+b\log(cx^n))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\log(cx^n))}{e^3} + \frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3} - (bn)\int \frac{2(-8d^2-4dex+e^2x^2)}{3e^3x\sqrt{d+ex}}\,dx$$

$$= -\frac{2d^2(a+b\log(cx^n))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\log(cx^n))}{e^3} + \frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3} - \frac{(2bn)\int \frac{-8d^2-4dex+e^2x^2}{x\sqrt{d+ex}}\,dx}{3e^3}$$

$$= -\frac{2d^2(a+b\log(cx^n))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\log(cx^n))}{e^3} + \frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3} - \frac{(4bn)\text{Subst}\left(\int \frac{-3d^2-6dx^2+x^4}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{3e^4}$$

$$= -\frac{2d^2(a+b\log(cx^n))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\log(cx^n))}{e^3} + \frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3} - \frac{(4bn)\text{Subst}\left(\int \left(-5de+ex^2-\frac{8d^2}{-\frac{d}{e}+\frac{x^2}{e}}\right)dx, x, \sqrt{d+ex}\right)}{3e^4}$$

$$= \frac{20bdn\sqrt{d+ex}}{3e^3} - \frac{4bn(d+ex)^{3/2}}{9e^3} - \frac{2d^2(a+b\log(cx^n))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\log(cx^n))}{e^3} + \frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3} + \frac{(32bd^2n)\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{3e^4}$$

$$= \frac{20bdn\sqrt{d+ex}}{3e^3} - \frac{4bn(d+ex)^{3/2}}{9e^3} - \frac{32bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{3e^3} - \frac{2d^2(a+b\log(cx^n))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\log(cx^n))}{e^3} + \frac{2(d+ex)^{3/2}(a+b\log(cx^n))}{3e^3}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.85

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = \frac{-48ad^2+56bd^2n-24adex+52bdenx+6ae^2x^2-4be^2nx^2-96bd^{3/2}n\sqrt{d+ex}}{9e^3\sqrt{d+ex}}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*x)^(3/2),x]
```

```
[Out] (-48*a*d^2 + 56*b*d^2*n - 24*a*d*e*x + 52*b*d*e*n*x + 6*a*e^2*x^2 - 4*b*e^2
*n*x^2 - 96*b*d^(3/2)*n*Sqrt[d + e*x]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] - 6*b*
(8*d^2 + 4*d*e*x - e^2*x^2)*Log[c*x^n])/(9*e^3*Sqrt[d + e*x])
```

Maple [F]

$$\int \frac{x^2(a+b\ln(c\,x^n))}{(ex+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^2*(a+b*ln(c*x^n))/(e*x+d)^(3/2),x)
```

```
[Out] int(x^2*(a+b*ln(c*x^n))/(e*x+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.26

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = \left[\frac{2\left(24\left(bdenx+bd^2n\right)\sqrt{d}\log\left(\frac{ex-2\sqrt{ex+d}\sqrt{d}+2d}{x}\right)+\left(28\,bd^2n-24\,ad^2-\left(2\,be^2n\right.\right.\right.}{}\right.$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/9*(24*(b*d*e*n*x + b*d^2*n)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) +
 2*d)/x) + (28*b*d^2*n - 24*a*d^2 - (2*b*e^2*n - 3*a*e^2)*x^2 + 2*(13*b*d*e
*n - 6*a*d*e)*x + 3*(b*e^2*x^2 - 4*b*d*e*x - 8*b*d^2)*log(c) + 3*(b*e^2*n*x
^2 - 4*b*d*e*n*x - 8*b*d^2*n)*log(x))*sqrt(e*x + d))/(e^4*x + d*e^3), 2/9*(
48*(b*d*e*n*x + b*d^2*n)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) + (28*b*
d^2*n - 24*a*d^2 - (2*b*e^2*n - 3*a*e^2)*x^2 + 2*(13*b*d*e*n - 6*a*d*e)*x +
 3*(b*e^2*x^2 - 4*b*d*e*x - 8*b*d^2)*log(c) + 3*(b*e^2*n*x^2 - 4*b*d*e*n*x
- 8*b*d^2*n)*log(x))*sqrt(e*x + d))/(e^4*x + d*e^3)]
```

Sympy [A] (verification not implemented)

Time = 135.75 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.11

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = a\left(\begin{cases}-\frac{2d^2}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}}{e^3}+\frac{2(d+ex)^{\frac{3}{2}}}{3e^3} & \text{for } e\neq 0\\ \frac{x^3}{3d^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) - bn\left(\begin{cases}\frac{16d^{\frac{3}{2}}\sqrt{1+\frac{ex}{d}}}{9e^3}+\frac{2d^{\frac{3}{2}}\log\left(\frac{ex}{d}\right)}{3e^3}-\frac{4d^{\frac{3}{2}}\log\left(\sqrt{1+\frac{ex}{d}}+1\right)}{3e^3}+\frac{12d^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}\sqrt{x}}\right)}{e^3}+\frac{4\sqrt{d}x\sqrt{1+\frac{ex}{d}}}{9e^2}-\frac{8d^2}{e^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{d}{ex}+1}}-\frac{8d\sqrt{x}}{e^{\frac{5}{2}}\sqrt{\frac{d}{ex}+1}} & \\ \frac{x^3}{9d^{\frac{3}{2}}} & \end{cases}\right) + b\left(\begin{cases}-\frac{2d^2}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}}{e^3}+\frac{2(d+ex)^{\frac{3}{2}}}{3e^3} & \text{for } e\neq 0\\ \frac{x^3}{3d^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(e*x+d)**(3/2),x)
```

```
[Out] a*Piecewise((-2*d**2/(e**3*sqrt(d + e*x)) - 4*d*sqrt(d + e*x)/e**3 + 2*(d +
 e*x)**(3/2)/(3*e**3), Ne(e, 0)), (x**3/(3*d**(3/2)), True)) - b*n*Piecewis
e((16*d**(3/2)*sqrt(1 + e*x/d)/(9*e**3) + 2*d**(3/2)*log(e*x/d)/(3*e**3) -
4*d**(3/2)*log(sqrt(1 + e*x/d) + 1)/(3*e**3) + 12*d**(3/2)*asinh(sqrt(d)/(s
qrt(e)*sqrt(x)))/e**3 + 4*sqrt(d)*x*sqrt(1 + e*x/d)/(9*e**2) - 8*d**2/(e**(
7/2)*sqrt(x)*sqrt(d/(e*x) + 1)) - 8*d*sqrt(x)/(e**(5/2)*sqrt(d/(e*x) + 1)),
 (e > -oo) & (e < oo) & Ne(e, 0)), (x**3/(9*d**(3/2)), True)) + b*Piecewise
((-2*d**2/(e**3*sqrt(d + e*x)) - 4*d*sqrt(d + e*x)/e**3 + 2*(d + e*x)**(3/2
)/(3*e**3), Ne(e, 0)), (x**3/(3*d**(3/2)), True))*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.08

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = \frac{4}{9}bn\left(\frac{12\,d^{\frac{3}{2}}\log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{e^3}-\frac{(ex+d)^{\frac{3}{2}}-15\sqrt{ex+d}d}{e^3}\right) + \frac{2}{3}b\left(\frac{(ex+d)^{\frac{3}{2}}}{e^3}-\frac{6\sqrt{ex+d}d}{e^3}-\frac{3\,d^2}{\sqrt{ex+d}e^3}\right)\log(cx^n) + \frac{2}{3}a\left(\frac{(ex+d)^{\frac{3}{2}}}{e^3}-\frac{6\sqrt{ex+d}d}{e^3}-\frac{3\,d^2}{\sqrt{ex+d}e^3}\right)$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 4/9*b*n*(12*d^(3/2)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d))
)/e^3 - ((e*x + d)^(3/2) - 15*sqrt(e*x + d)*d)/e^3) + 2/3*b*((e*x + d)^(3/2
)/e^3 - 6*sqrt(e*x + d)*d/e^3 - 3*d^2/(sqrt(e*x + d)*e^3))*log(c*x^n) + 2/3
*a*((e*x + d)^(3/2)/e^3 - 6*sqrt(e*x + d)*d/e^3 - 3*d^2/(sqrt(e*x + d)*e^3)
)
```

Giac [F]

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^2/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,x^n))}{(d+e\,x)^{3/2}}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e*x)^(3/2),x)
[Out] int((x^2*(a + b*log(c*x^n)))/(d + e*x)^(3/2), x)
```

3.153 $\int \frac{x(a+b\log(cx^n))}{(d+ex)^{3/2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^{3/2}} dx = -\frac{4bn\sqrt{d+ex}}{e^2} + \frac{8b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{e^2} + \frac{2d(a+b\log(cx^n))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\log(cx^n))}{e^2}$$

```
[Out] 8*b*n*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)/e^2+2*d*(a+b*ln(c*x^n))/e^2/(e
*x+d)^(1/2)-4*b*n*(e*x+d)^(1/2)/e^2+2*(a+b*ln(c*x^n))*(e*x+d)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {45, 2392, 12, 81, 65, 214}

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^{3/2}} dx = \frac{2\sqrt{d+ex}(a+b\log(cx^n))}{e^2} + \frac{2d(a+b\log(cx^n))}{e^2\sqrt{d+ex}} + \frac{8b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{e^2} - \frac{4bn\sqrt{d+ex}}{e^2}$$

```
[In] Int[(x*(a + b*Log[c*x^n]))/(d + e*x)^(3/2),x]
```

```
[Out] (-4*b*n*Sqrt[d + e*x])/e^2 + (8*b*Sqrt[d]*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])
/e^2 + (2*d*(a + b*Log[c*x^n]))/(e^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a +
 b*Log[c*x^n]))/e^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \frac{2d(a+b\log(cx^n))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\log(cx^n))}{e^2} - (bn)\int \frac{2(2d+ex)}{e^2x\sqrt{d+ex}}\,dx$$

$$= \frac{2d(a+b\log(cx^n))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\log(cx^n))}{e^2} - \frac{(2bn)\int \frac{2d+ex}{x\sqrt{d+ex}}\,dx}{e^2}$$

$$= -\frac{4bn\sqrt{d+ex}}{e^2} + \frac{2d(a+b\log(cx^n))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\log(cx^n))}{e^2} - \frac{(4bdn)\int \frac{1}{x\sqrt{d+ex}}\,dx}{e^2}$$

$$= -\frac{4bn\sqrt{d+ex}}{e^2} + \frac{2d(a+b\log(cx^n))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\log(cx^n))}{e^2} - \frac{(8bdn)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{e^3}$$

$$= -\frac{4bn\sqrt{d+ex}}{e^2} + \frac{8b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{e^2} + \frac{2d(a+b\log(cx^n))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\log(cx^n))}{e^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = \frac{2\left(2ad-2bdn+aex-2benx+4b\sqrt{d}n\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)+b(2d+ex)\log(\right.}{e^2\sqrt{d+ex}}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e*x)^(3/2),x]
```

```
[Out] (2*(2*a*d - 2*b*d*n + a*e*x - 2*b*e*n*x + 4*b*Sqrt[d]*n*Sqrt[d + e*x]*ArcTa
nh[Sqrt[d + e*x]/Sqrt[d]] + b*(2*d + e*x)*Log[c*x^n]))/(e^2*Sqrt[d + e*x])
```

Maple [F]

$$\int \frac{x(a+b\ln(c\,x^n))}{(ex+d)^{\frac{3}{2}}}dx$$

```
[In] int(x*(a+b*ln(c*x^n))/(e*x+d)^(3/2),x)
```

```
[Out] int(x*(a+b*ln(c*x^n))/(e*x+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.37

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = \left[\frac{2\left(2\,(benx+bdn)\sqrt{d}\log\left(\frac{ex+2\,\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right) - (2\,bdn - 2\,ad + (2\,ben - ae)x - \right.}{e^3x+de^2}\right.$$
$$-\frac{2\left(4\,(benx+bdn)\sqrt{-d}\arctan\left(\frac{\sqrt{ex+d}\sqrt{-d}}{d}\right) + (2\,bdn - 2\,ad + (2\,ben - ae)x - (bex+2\,bd)\log(c) - (benx\right.}{e^3x+de^2}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [2*(2*(b*e*n*x + b*d*n)*sqrt(d)*log((e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x
) - (2*b*d*n - 2*a*d + (2*b*e*n - a*e)*x - (b*e*x + 2*b*d)*log(c) - (b*e*n*
x + 2*b*d*n)*log(x))*sqrt(e*x + d))/(e^3*x + d*e^2), -2*(4*(b*e*n*x + b*d*n
)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) + (2*b*d*n - 2*a*d + (2*b*e*n -
 a*e)*x - (b*e*x + 2*b*d)*log(c) - (b*e*n*x + 2*b*d*n)*log(x))*sqrt(e*x + d
))/(e^3*x + d*e^2)]
```

Sympy [A] (verification not implemented)

Time = 91.74 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.88

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = a\left(\begin{cases}\frac{2d}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}}{e^2} & \text{for } e\neq 0\\ \frac{x^2}{2d^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) - bn\left(\begin{cases}-\frac{8\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}\sqrt{x}}\right)}{e^2} + \frac{4d}{e^{\frac{5}{2}}\sqrt{x}\sqrt{\frac{d}{ex}+1}} + \frac{4\sqrt{x}}{e^{\frac{3}{2}}\sqrt{\frac{d}{ex}+1}} & \text{for } e>-\infty\wedge e<\infty\wedge e\neq 0\\ \frac{x^2}{4d^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + b\left(\begin{cases}\frac{2d}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}}{e^2} & \text{for } e\neq 0\\ \frac{x^2}{2d^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate(x*(a+b*ln(c*x**n))/(e*x+d)**(3/2),x)
```

```
[Out] a*Piecewise((2*d/(e**2*sqrt(d + e*x)) + 2*sqrt(d + e*x)/e**2, Ne(e, 0)), (x
**2/(2*d**(3/2)), True)) - b*n*Piecewise((-8*sqrt(d)*asinh(sqrt(d)/(sqrt(e)
*sqrt(x)))/e**2 + 4*d/(e**(5/2)*sqrt(x)*sqrt(d/(e*x) + 1)) + 4*sqrt(x)/(e**
(3/2)*sqrt(d/(e*x) + 1)), (e > -oo) & (e < oo) & Ne(e, 0)), (x**2/(4*d**(3/
2)), True)) + b*Piecewise((2*d/(e**2*sqrt(d + e*x)) + 2*sqrt(d + e*x)/e**2,
 Ne(e, 0)), (x**2/(2*d**(3/2)), True))*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.19

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = -4\,bn\left(\frac{\sqrt{d}\log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{e^2}+\frac{\sqrt{ex+d}}{e^2}\right) + 2\,b\left(\frac{\sqrt{ex+d}}{e^2}+\frac{d}{\sqrt{ex+d}e^2}\right)\log(cx^n) + 2\,a\left(\frac{\sqrt{ex+d}}{e^2}+\frac{d}{\sqrt{ex+d}e^2}\right)$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*b*n*(sqrt(d)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/e^
2 + sqrt(e*x + d)/e^2) + 2*b*(sqrt(e*x + d)/e^2 + d/(sqrt(e*x + d)*e^2))*lo
g(c*x^n) + 2*a*(sqrt(e*x + d)/e^2 + d/(sqrt(e*x + d)*e^2))
```

Giac [F]

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = \int \frac{(b\log(cx^n)+a)x}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(cx^n))}{(d+ex)^{3/2}}\,dx = \int \frac{x\,(a+b\,\ln(c\,x^n))}{(d+e\,x)^{3/2}}\,dx$$

```
[In] int((x*(a + b*log(c*x^n)))/(d + e*x)^(3/2),x)
```

```
[Out] int((x*(a + b*log(c*x^n)))/(d + e*x)^(3/2), x)
```

3.154 $\int \frac{a+b\log(cx^n)}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{a+b\log(cx^n)}{(d+ex)^{3/2}}\,dx = -\frac{4bn\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}e} - \frac{2(a+b\log(cx^n))}{e\sqrt{d+ex}}$$

```
[Out] -4*b*n*arctanh((e*x+d)^(1/2)/d^(1/2))/e/d^(1/2)-2*(a+b*ln(c*x^n))/e/(e*x+d)
^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2356, 65, 214}

$$\int \frac{a+b\log(cx^n)}{(d+ex)^{3/2}}\,dx = -\frac{2(a+b\log(cx^n))}{e\sqrt{d+ex}} - \frac{4bn\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}e}$$

```
[In] Int[(a + b*Log[c*x^n])/(d + e*x)^(3/2),x]
```

```
[Out] (-4*b*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(Sqrt[d]*e) - (2*(a + b*Log[c*x^n])
)/(e*Sqrt[d + e*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2(a+b\log(cx^n))}{e\sqrt{d+ex}}+\frac{(2bn)\int\frac{1}{x\sqrt{d+ex}}\,dx}{e}\\ &= -\frac{2(a+b\log(cx^n))}{e\sqrt{d+ex}}+\frac{(4bn)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{e^2}\\ &= -\frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}e}-\frac{2(a+b\log(cx^n))}{e\sqrt{d+ex}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int\frac{a+b\log(cx^n)}{(d+ex)^{3/2}}\,dx=-\frac{4bn\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}e}-\frac{2(a+b\log(cx^n))}{e\sqrt{d+ex}}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*x)^(3/2),x]
[Out] (-4*b*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(Sqrt[d]*e) - (2*(a + b*Log[c*x^n])
)/(e*Sqrt[d + e*x])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\,x^{n}\right)}{\left(ex+d\right)^{\frac{3}{2}}}dx$$

[In] int((a+b*ln(c*x^n))/(e*x+d)^(3/2),x)

[Out] int((a+b*ln(c*x^n))/(e*x+d)^(3/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.92

$$\int \frac{a+b\log\left(cx^n\right)}{(d+ex)^{3/2}}\,dx = \left[\frac{2\left(\left(benx+bdn\right)\sqrt{d}\log\left(\frac{ex-2\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)-\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)\sqrt{ex+d}\right)}{de^2x+d^2e}\right.$$

[In] integrate((a+b*log(c*x^n))/(e*x+d)^(3/2),x, algorithm="fricas")

[Out] [2*((b*e*n*x + b*d*n)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - (b*d*n*log(x) + b*d*log(c) + a*d)*sqrt(e*x + d))/(d*e^2*x + d^2*e), 2*(2*(b*e*n*x + b*d*n)*sqrt(-d)*arctan(sqrt(e*x + d)*sqrt(-d)/d) - (b*d*n*log(x) + b*d*log(c) + a*d)*sqrt(e*x + d))/(d*e^2*x + d^2*e)]

Sympy [A] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.66

$$\begin{aligned}\int \frac{a+b\log\left(cx^n\right)}{(d+ex)^{3/2}}\,dx &= a\left(\begin{cases}-\frac{2}{e\sqrt{d+ex}} & \text{for}\ e\neq 0\\ \frac{x}{d^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)\\ &- bn\left(\begin{cases}\frac{4\,\text{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}\sqrt{x}}\right)}{\sqrt{d}e} & \text{for}\ e>-\infty\wedge e<\infty\wedge e\neq 0\\ \frac{x}{d^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)\\ &+ b\left(\begin{cases}-\frac{2}{e\sqrt{d+ex}} & \text{for}\ e\neq 0\\ \frac{x}{d^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)\log\left(cx^n\right)\end{aligned}$$

[In] integrate((a+b*ln(c*x**n))/(e*x+d)**(3/2),x)

[Out] a*Piecewise((-2/(e*sqrt(d + e*x)), Ne(e, 0)), (x/d**(3/2), True)) - b*n*Piecewise((4*asinh(sqrt(d)/(sqrt(e)*sqrt(x)))/(sqrt(d)*e), (e > -oo) & (e < oo) & Ne(e, 0)), (x/d**(3/2), True)) + b*Piecewise((-2/(e*sqrt(d + e*x)), Ne(e, 0)), (x/d**(3/2), True))*log(c*x**n)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

$$\int \frac{a+b\log(cx^n)}{(d+ex)^{3/2}}\,dx = \frac{2\,bn\log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{\sqrt{d}e} - \frac{2\,b\log(cx^n)}{\sqrt{ex+d}e} - \frac{2\,a}{\sqrt{ex+d}e}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b*n*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/(sqrt(d)*e)
- 2*b*log(c*x^n)/(sqrt(e*x + d)*e) - 2*a/(sqrt(e*x + d)*e)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

$$\int \frac{a+b\log(cx^n)}{(d+ex)^{3/2}}\,dx = \frac{4\,bn\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{\sqrt{-d}e} - \frac{2\,bn\log(ex)}{\sqrt{ex+d}e} + \frac{2\,(bn\log(e) - b\log(c) - a)}{\sqrt{ex+d}e}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] 4*b*n*arctan(sqrt(e*x + d)/sqrt(-d))/(sqrt(-d)*e) - 2*b*n*log(e*x)/(sqrt(e*
x + d)*e) + 2*(b*n*log(e) - b*log(c) - a)/(sqrt(e*x + d)*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{(d+ex)^{3/2}}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{(d+e\,x)^{3/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(d + e*x)^(3/2),x)
```

```
[Out] int((a + b*log(c*x^n))/(d + e*x)^(3/2), x)
```

3.155 $\int \frac{a+b\log(cx^n)}{x(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 201

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^{3/2}}\,dx = \frac{4bn\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{2bn\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{2(a+b\log(cx^n))}{d\sqrt{d+ex}} - \frac{2\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} - \frac{4bn\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{3/2}} - \frac{2bn\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{3/2}}$$

```
[Out] 4*b*n*arctanh((e*x+d)^(1/2)/d^(1/2))/d^(3/2)+2*b*n*arctanh((e*x+d)^(1/2)/d^
(1/2))^2/d^(3/2)-2*arctanh((e*x+d)^(1/2)/d^(1/2))*(a+b*ln(c*x^n))/d^(3/2)-4
*b*n*arctanh((e*x+d)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)-(e*x+d)^(1/2)))/d
^(3/2)-2*b*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-(e*x+d)^(1/2)))/d^(3/2)+2*(a+b*
ln(c*x^n))/d/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules

used = {2389, 65, 214, 2390, 12, 6131, 6055, 2449, 2352, 2356}

$$\int \frac{a+b\log\left(cx^{n}\right)}{x(d+ex)^{3/2}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(a+b\log\left(cx^{n}\right)\right)}{d^{3/2}} + \frac{2\left(a+b\log\left(cx^{n}\right)\right)}{d\sqrt{d+ex}} + \frac{2bn\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^{2}}{d^{3/2}} + \frac{4bn\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4bn\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{3/2}} - \frac{2bn\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{3/2}}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x)^(3/2)),x]
```

```
[Out] (4*b*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/d^(3/2) + (2*b*n*ArcTanh[Sqrt[d + e*
x]/Sqrt[d]]^2)/d^(3/2) + (2*(a + b*Log[c*x^n]))/(d*Sqrt[d + e*x]) - (2*ArcT
anh[Sqrt[d + e*x]/Sqrt[d]]*(a + b*Log[c*x^n]))/d^(3/2) - (4*b*n*ArcTanh[Sqr
t[d + e*x]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x])])/d^(3/2) - (
2*b*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x])])/d^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
```

```
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{a+b\log(cx^n)}{x\sqrt{d+ex}}\,dx}{d} - \frac{e\int \frac{a+b\log(cx^n)}{(d+ex)^{3/2}}\,dx}{d}$$

$$= \frac{2(a+b\log(cx^n))}{d\sqrt{d+ex}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} - \frac{(bn)\int -\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}x}\,dx}{d} - \frac{(2bn)\int \frac{1}{x\sqrt{d+ex}}\,dx}{d}$$

$$= \frac{2(a+b\log(cx^n))}{d\sqrt{d+ex}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} + \frac{(2bn)\int \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx}{d^{3/2}} - \frac{(4bn)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex}\right)}{de}$$

$$= \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{2(a+b\log(cx^n))}{d\sqrt{d+ex}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} + \frac{(4bn)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx, x, \sqrt{d+ex}\right)}{d^{3/2}}$$

$$= \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{2(a+b\log(cx^n))}{d\sqrt{d+ex}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} - \frac{(4bn)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx, x, \sqrt{d+ex}\right)}{d^2}$$

$$= \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{2(a+b\log(cx^n))}{d\sqrt{d+ex}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} - \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{3/2}} + \frac{(4bn)\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}\,dx, x, \sqrt{d+ex}\right)}{d^2}$$

$$= \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{2(a+b\log(cx^n))}{d\sqrt{d+ex}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} - \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{3/2}} - \frac{(4bn)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)}{d^{3/2}}$$

$$= \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{2(a+b\log(cx^n))}{d\sqrt{d+ex}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} - \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{3/2}} - \frac{2bn\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)}{d^{3/2}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.47

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^{3/2}}\,dx = \frac{8bn\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) + \frac{4\sqrt{d}(a+b\log(cx^n))}{\sqrt{d+ex}} + 2(a+b\log(cx^n))\log\left(\sqrt{d}-\sqrt{d+ex}\right) - 2(}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x)^(3/2)),x]
```

```
[Out] (8*b*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + (4*Sqrt[d]*(a + b*Log[c*x^n]))/Sqrt
[d + e*x] + 2*(a + b*Log[c*x^n])*Log[Sqrt[d] - Sqrt[d + e*x]] - 2*(a + b*Lo
g[c*x^n])*Log[Sqrt[d] + Sqrt[d + e*x]] - b*n*(Log[Sqrt[d] - Sqrt[d + e*x]]*
(Log[Sqrt[d] - Sqrt[d + e*x]] + 2*Log[(1 + Sqrt[d + e*x]/Sqrt[d])/2]) + 2*P
olyLog[2, 1/2 - Sqrt[d + e*x]/(2*Sqrt[d])]) + b*n*(Log[Sqrt[d] + Sqrt[d + e
*x]]*(Log[Sqrt[d] + Sqrt[d + e*x]] + 2*Log[1/2 - Sqrt[d + e*x]/(2*Sqrt[d])]
) + 2*PolyLog[2, (1 + Sqrt[d + e*x]/Sqrt[d])/2]))/(2*d^(3/2))
```

Maple [F]

$$\int \frac{a+b\ln(c\,x^n)}{x\,(ex+d)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*ln(c*x^n))/x/(e*x+d)^(3/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/x/(e*x+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^{3/2}}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(e*x + d)*b*log(c*x^n) + sqrt(e*x + d)*a)/(e^2*x^3 + 2*d*e*x^
2 + d^2*x), x)
```

Sympy [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^{3/2}}\,dx = \int \frac{a+b\log(cx^n)}{x(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x/(e*x+d)**(3/2),x)
[Out] Integral((a + b*log(c*x**n))/(x*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^{3/2}}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^(3/2),x, algorithm="maxima")
[Out] a*(log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/d^(3/2) + 2/(sq
rt(e*x + d)*d)) + b*integrate((log(c) + log(x^n))/((e*x^2 + d*x)*sqrt(e*x +
 d)), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^{3/2}}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x + d)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x(d+ex)^{3/2}}\,dx = \int \frac{a+b\ln(c\,x^n)}{x\,(d+e\,x)^{3/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x)^(3/2)),x)
[Out] int((a + b*log(c*x^n))/(x*(d + e*x)^(3/2)), x)
```

3.156 $\int \frac{a+b\log(cx^n)}{x^2(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 253

$$\begin{aligned}\int \frac{a+b\log(cx^n)}{x^2(d+ex)^{3/2}}\,dx &= -\frac{bn\sqrt{d+ex}}{d^2x} - \frac{5ben\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{3ben\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{d^{5/2}} \\ &\quad - \frac{3e(a+b\log(cx^n))}{d^2\sqrt{d+ex}} - \frac{a+b\log(cx^n)}{dx\sqrt{d+ex}} + \frac{3e\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{5/2}} \\ &\quad + \frac{6ben\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{5/2}} + \frac{3ben\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{5/2}}\end{aligned}$$

```
[Out] -5*b*e*n*arctanh((e*x+d)^(1/2)/d^(1/2))/d^(5/2)-3*b*e*n*arctanh((e*x+d)^(1/
2)/d^(1/2))^2/d^(5/2)+3*e*arctanh((e*x+d)^(1/2)/d^(1/2))*(a+b*ln(c*x^n))/d^
(5/2)+6*b*e*n*arctanh((e*x+d)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)-(e*x+d)^
(1/2)))/d^(5/2)+3*b*e*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-(e*x+d)^(1/2)))/d^(5
/2)-3*e*(a+b*ln(c*x^n))/d^2/(e*x+d)^(1/2)+(-a-b*ln(c*x^n))/d/x/(e*x+d)^(1/2
)-b*n*(e*x+d)^(1/2)/d^2/x
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules

used = {44, 53, 65, 214, 2392, 12, 14, 43, 52, 6131, 6055, 2449, 2352}

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^{3/2}}\,dx = \frac{3e\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{5/2}} - \frac{3e(a+b\log(cx^n))}{d^2\sqrt{d+ex}} - \frac{a+b\log(cx^n)}{dx\sqrt{d+ex}} - \frac{3ben\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{d^{5/2}} - \frac{5ben\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{6ben\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{5/2}} + \frac{3ben\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{5/2}} - \frac{bn\sqrt{d+ex}}{d^2x}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^2*(d + e*x)^(3/2)),x]
```

```
[Out] -((b*n*Sqrt[d + e*x])/(d^2*x)) - (5*b*e*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/d
^(5/2) - (3*b*e*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]^2)/d^(5/2) - (3*e*(a + b*L
og[c*x^n]))/(d^2*Sqrt[d + e*x]) - (a + b*Log[c*x^n])/(d*x*Sqrt[d + e*x]) +
(3*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]*(a + b*Log[c*x^n]))/d^(5/2) + (6*b*e*n*
ArcTanh[Sqrt[d + e*x]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x])])/
d^(5/2) + (3*b*e*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x])])/d
^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
```

```
IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3e(a+b\log(cx^n))}{d^2\sqrt{d+ex}} - \frac{a+b\log(cx^n)}{dx\sqrt{d+ex}} + \frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{5/2}} \\
&\quad - (bn)\int \frac{-\frac{\sqrt{d}(d+3ex)}{\sqrt{d+ex}} + 3ex\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{5/2}x^2}\,dx \\
&= -\frac{3e(a+b\log(cx^n))}{d^2\sqrt{d+ex}} - \frac{a+b\log(cx^n)}{dx\sqrt{d+ex}} + \frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{5/2}} \\
&\quad - \frac{(bn)\int \frac{-\frac{\sqrt{d}(d+3ex)}{\sqrt{d+ex}} + 3ex\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x^2}\,dx}{d^{5/2}} \\
&= -\frac{3e(a+b\log(cx^n))}{d^2\sqrt{d+ex}} - \frac{a+b\log(cx^n)}{dx\sqrt{d+ex}} + \frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{5/2}} \\
&\quad - \frac{(bn)\int \left(\frac{2e^2}{\sqrt{d}\sqrt{d+ex}} - \frac{\sqrt{d}\sqrt{d+ex}}{x^2} - \frac{2e\sqrt{d+ex}}{\sqrt{d}x} + \frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\right)dx}{d^{5/2}}
\end{aligned}$$

$$=-\frac{4ben\sqrt{d+ex}}{d^3}-\frac{3e(a+b\log(cx^n))}{d^2\sqrt{d+ex}}-\frac{a+b\log(cx^n)}{dx\sqrt{d+ex}}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{5/2}}+\frac{(bn)\int\frac{\sqrt{d+ex}}{x^2}\,dx}{d^2}+\frac{(2ben)\int\frac{\sqrt{d+ex}}{x}\,dx}{d^3}-\frac{(3ben)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx}{d^{5/2}}$$

$$=-\frac{bn\sqrt{d+ex}}{d^2x}-\frac{3e(a+b\log(cx^n))}{d^2\sqrt{d+ex}}-\frac{a+b\log(cx^n)}{dx\sqrt{d+ex}}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{5/2}}-\frac{(6ben)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx,x,\sqrt{d+ex}\right)}{d^{5/2}}+\frac{(ben)\int\frac{1}{x\sqrt{d+ex}}\,dx}{2d^2}+\frac{(2ben)\int\frac{1}{x\sqrt{d+ex}}\,dx}{d^2}$$

$$=-\frac{bn\sqrt{d+ex}}{d^2x}-\frac{3ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{d^{5/2}}-\frac{3e(a+b\log(cx^n))}{d^2\sqrt{d+ex}}-\frac{a+b\log(cx^n)}{dx\sqrt{d+ex}}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{5/2}}+\frac{(bn)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{d^2}+\frac{(4bn)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex}\right)}{d^2}+\frac{(6ben)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx,x,\sqrt{d+ex}\right)}{d^3}$$

$$=-\frac{bn\sqrt{d+ex}}{d^2x}-\frac{5ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{5/2}}-\frac{3ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{d^{5/2}}-\frac{3e(a+b\log(cx^n))}{d^2\sqrt{d+ex}}-\frac{a+b\log(cx^n)}{dx\sqrt{d+ex}}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{5/2}}+\frac{6ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{5/2}}-\frac{(6ben)\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}\,dx,x,\sqrt{d+ex}\right)}{d^3}$$

$$=-\frac{bn\sqrt{d+ex}}{d^2x}-\frac{5ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{5/2}}-\frac{3ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{d^{5/2}}-\frac{3e(a+b\log(cx^n))}{d^2\sqrt{d+ex}}-\frac{a+b\log(cx^n)}{dx\sqrt{d+ex}}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{5/2}}+\frac{6ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{5/2}}+\frac{(6ben)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)}{d^{5/2}}$$

$$= -\frac{bn\sqrt{d+ex}}{d^2x} - \frac{5ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{3ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)^2}{d^{5/2}} - \frac{3e(a+b\log(cx^n))}{d^2\sqrt{d+ex}} - \frac{a+b\log(cx^n)}{dx\sqrt{d+ex}} + \frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{5/2}} + \frac{6ben\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right)}{d^{5/2}} + \frac{3ben\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex}}{\sqrt{d}}}\right)}{d^{5/2}}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.00

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^{3/2}}\,dx = 2e\left(-\frac{2bn\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{bn\left(\frac{1}{\sqrt{d}-\sqrt{d+ex}} - \frac{\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}\right)}{4d^2} - \frac{bn\left(\frac{1}{\sqrt{d}+\sqrt{d+ex}} + \frac{\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}\right)}{4d^2} - \frac{a+b\log(cx^n)}{d^2\sqrt{d+ex}} + \frac{a+b\log(cx^n)}{4d^2\left(\sqrt{d}-\sqrt{d+ex}\right)} - \frac{a+b\log(cx^n)}{4d^2\left(\sqrt{d}+\sqrt{d+ex}\right)} - \frac{3(a+b\log(cx^n))\log\left(\sqrt{d}-\sqrt{d+ex}\right)}{4d^{5/2}} + \frac{3(a+b\log(cx^n))\log\left(\sqrt{d}+\sqrt{d+ex}\right)}{4d^{5/2}} + \frac{3bn\left(\log^2\left(\sqrt{d}-\sqrt{d+ex}\right)+2\log\left(\sqrt{d}-\sqrt{d+ex}\right)\log\left(\frac{\sqrt{d}+\sqrt{d+ex}}{2\sqrt{d}}\right)+2\,\text{PolyLog}\left(2,\frac{\sqrt{d}-\sqrt{d+ex}}{2\sqrt{d}}\right)\right)}{8d^{5/2}} - \frac{3bn\left(2\log\left(\frac{\sqrt{d}-\sqrt{d+ex}}{2\sqrt{d}}\right)\log\left(\sqrt{d}+\sqrt{d+ex}\right)+\log^2\left(\sqrt{d}+\sqrt{d+ex}\right)+2\,\text{PolyLog}\left(2,\frac{\sqrt{d}+\sqrt{d+ex}}{2\sqrt{d}}\right)\right)}{8d^{5/2}}\right)$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*x)^(3/2)),x]
```

```
[Out] 2*e*((-2*b*n*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/d^(5/2) + (b*n*((Sqrt[d] - Sqr
t[d + e*x])^(-1) - ArcTanh[Sqrt[d + e*x]/Sqrt[d]]/Sqrt[d]))/(4*d^2) - (b*n*
((Sqrt[d] + Sqrt[d + e*x])^(-1) + ArcTanh[Sqrt[d + e*x]/Sqrt[d]]/Sqrt[d]))/
(4*d^2) - (a + b*Log[c*x^n])/(d^2*Sqrt[d + e*x]) + (a + b*Log[c*x^n])/(4*d^
2*(Sqrt[d] - Sqrt[d + e*x])) - (a + b*Log[c*x^n])/(4*d^2*(Sqrt[d] + Sqrt[d
+ e*x])) - (3*(a + b*Log[c*x^n])*Log[Sqrt[d] - Sqrt[d + e*x]])/(4*d^(5/2))
+ (3*(a + b*Log[c*x^n])*Log[Sqrt[d] + Sqrt[d + e*x]])/(4*d^(5/2)) + (3*b*n*
(Log[Sqrt[d] - Sqrt[d + e*x]]^2 + 2*Log[Sqrt[d] - Sqrt[d + e*x]]*Log[(Sqrt[
d] + Sqrt[d + e*x])/(2*Sqrt[d])] + 2*PolyLog[2, (Sqrt[d] - Sqrt[d + e*x])/(
```

```
2*Sqrt[d])]))/(8*d^(5/2)) - (3*b*n*(2*Log[(Sqrt[d] - Sqrt[d + e*x])/(2*Sqrt
[d])]*Log[Sqrt[d] + Sqrt[d + e*x]] + Log[Sqrt[d] + Sqrt[d + e*x]]^2 + 2*Pol
yLog[2, (Sqrt[d] + Sqrt[d + e*x])/(2*Sqrt[d])]))/(8*d^(5/2)))
```

Maple [F]

$$\int \frac{a+b\ln\left(c\,x^{n}\right)}{x^{2}\left(ex+d\right)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*ln(c*x^n))/x^2/(e*x+d)^(3/2),x)
[Out] int((a+b*ln(c*x^n))/x^2/(e*x+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{2}(d+ex)^{3/2}}\,dx=\int \frac{b\log\left(cx^{n}\right)+a}{\left(ex+d\right)^{\frac{3}{2}}x^{2}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d)^(3/2),x, algorithm="fricas")
[Out] integral((sqrt(e*x + d)*b*log(c*x^n) + sqrt(e*x + d)*a)/(e^2*x^4 + 2*d*e*x^
3 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{2}(d+ex)^{3/2}}\,dx=\int \frac{a+b\log\left(cx^{n}\right)}{x^{2}\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**2/(e*x+d)**(3/2),x)
[Out] Integral((a + b*log(c*x**n))/(x**2*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{2}(d+ex)^{3/2}}\,dx=\int \frac{b\log\left(cx^{n}\right)+a}{\left(ex+d\right)^{\frac{3}{2}}x^{2}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d)^(3/2),x, algorithm="maxima")
[Out] -1/2*a*(2*(3*(e*x + d)*e - 2*d*e)/((e*x + d)^(3/2)*d^2 - sqrt(e*x + d)*d^3)
 + 3*e*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/d^(5/2)) +
b*integrate((log(c) + log(x^n))/((e*x^3 + d*x^2)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{b\log(cx^n)+a}{(ex+d)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x + d)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^2\,(d+e\,x)^{3/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*x)^(3/2)),x)
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*x)^(3/2)), x)
```

3.157 $\int \frac{x^2}{(d+ex)(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2}{(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx = \operatorname{Int}\left(\frac{x^2}{(d+ex)\left(a+b\log\left(cx^n\right)\right)},x\right)$$

```
[Out] Unintegrable(x^2/(e*x+d)/(a+b*ln(c*x^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{x^2}{(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx$$

```
[In] Int[x^2/((d + e*x)*(a + b*Log[c*x^n])),x]
```

```
[Out] Defer[Int][x^2/((d + e*x)*(a + b*Log[c*x^n])), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{x^2}{(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx$$

```
[In] Integrate[x^2/((d + e*x)*(a + b*Log[c*x^n])),x]
[Out] Integrate[x^2/((d + e*x)*(a + b*Log[c*x^n])), x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(ex+d)\left(a+b\ln\left(c\,x^n\right)\right)}dx$$

```
[In] int(x^2/(e*x+d)/(a+b*ln(c*x^n)),x)
[Out] int(x^2/(e*x+d)/(a+b*ln(c*x^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{x^2}{(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{x^2}{(ex+d)(b\log\left(cx^n\right)+a)}\,dx$$

```
[In] integrate(x^2/(e*x+d)/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(x^2/(a*e*x + a*d + (b*e*x + b*d)*log(c*x^n)), x)
```

Sympy [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{x^2}{\left(a+b\log\left(cx^n\right)\right)(d+ex)}\,dx$$

```
[In] integrate(x**2/(e*x+d)/(a+b*ln(c*x**n)),x)
[Out] Integral(x**2/((a + b*log(c*x**n))*(d + e*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(d+ex)(a+b\log(cx^n))}\,dx = \int \frac{x^2}{(ex+d)(b\log(cx^n)+a)}\,dx$$

```
[In] integrate(x^2/(e*x+d)/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(x^2/((e*x + d)*(b*log(c*x^n) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(d+ex)(a+b\log(cx^n))}\,dx = \int \frac{x^2}{(ex+d)(b\log(cx^n)+a)}\,dx$$

```
[In] integrate(x^2/(e*x+d)/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(x^2/((e*x + d)*(b*log(c*x^n) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(d+ex)(a+b\log(cx^n))}\,dx = \int \frac{x^2}{(a+b\,\ln(c\,x^n))\,(d+e\,x)}\,dx$$

```
[In] int(x^2/((a + b*log(c*x^n))*(d + e*x)),x)
[Out] int(x^2/((a + b*log(c*x^n))*(d + e*x)), x)
```

3.158 $\int \frac{x}{(d+ex)(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{x}{(d+ex)(a+b\log(cx^n))}\,dx = \text{Int}\left(\frac{x}{(d+ex)(a+b\log(cx^n))},x\right)$$

```
[Out] Unintegrable(x/(e*x+d)/(a+b*ln(c*x^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{(d+ex)(a+b\log(cx^n))}\,dx = \int \frac{x}{(d+ex)(a+b\log(cx^n))}\,dx$$

```
[In] Int[x/((d + e*x)*(a + b*Log[c*x^n])),x]
[Out] Defer[Int][x/((d + e*x)*(a + b*Log[c*x^n])), x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{(d+ex)(a+b\log(cx^n))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x}{(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{x}{(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx$$

```
[In] Integrate[x/((d + e*x)*(a + b*Log[c*x^n])),x]
[Out] Integrate[x/((d + e*x)*(a + b*Log[c*x^n])), x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x}{(ex+d)\left(a+b\ln\left(c\,x^n\right)\right)}dx$$

```
[In] int(x/(e*x+d)/(a+b*ln(c*x^n)),x)
[Out] int(x/(e*x+d)/(a+b*ln(c*x^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{x}{(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{x}{(ex+d)(b\log\left(cx^n\right)+a)}\,dx$$

```
[In] integrate(x/(e*x+d)/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(x/(a*e*x + a*d + (b*e*x + b*d)*log(c*x^n)), x)
```

Sympy [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{x}{\left(a+b\log\left(cx^n\right)\right)(d+ex)}\,dx$$

```
[In] integrate(x/(e*x+d)/(a+b*ln(c*x**n)),x)
[Out] Integral(x/((a + b*log(c*x**n))*(d + e*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x}{(d+ex)\,(a+b\log{(cx^n)})}\,dx = \int \frac{x}{(ex+d)(b\log{(cx^n)}+a)}\,dx$$

```
[In] integrate(x/(e*x+d)/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(x/((e*x + d)*(b*log(c*x^n) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x}{(d+ex)\,(a+b\log{(cx^n)})}\,dx = \int \frac{x}{(ex+d)(b\log{(cx^n)}+a)}\,dx$$

```
[In] integrate(x/(e*x+d)/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(x/((e*x + d)*(b*log(c*x^n) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x}{(d+ex)\,(a+b\log{(cx^n)})}\,dx = \int \frac{x}{(a+b\,\ln{(c\,x^n)})\,\,(d+e\,x)}\,dx$$

```
[In] int(x/((a + b*log(c*x^n))*(d + e*x)),x)
[Out] int(x/((a + b*log(c*x^n))*(d + e*x)), x)
```

3.159 $\int \frac{1}{(d+ex)(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx = \operatorname{Int}\left(\frac{1}{(d+ex)\left(a+b\log\left(cx^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x+d)/(a+b*ln(c*x^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{1}{(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx$$

```
[In] Int[1/((d + e*x)*(a + b*Log[c*x^n])),x]
```

```
[Out] Defer[Int][1/((d + e*x)*(a + b*Log[c*x^n])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex)(a+b\log(cx^n))}\,dx = \int \frac{1}{(d+ex)(a+b\log(cx^n))}\,dx$$

```
[In] Integrate[1/((d + e*x)*(a + b*Log[c*x^n])),x]
[Out] Integrate[1/((d + e*x)*(a + b*Log[c*x^n])), x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ex+d)(a+b\ln(c\,x^n))}dx$$

```
[In] int(1/(e*x+d)/(a+b*ln(c*x^n)),x)
[Out] int(1/(e*x+d)/(a+b*ln(c*x^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(d+ex)(a+b\log(cx^n))}\,dx = \int \frac{1}{(ex+d)(b\log(cx^n)+a)}\,dx$$

```
[In] integrate(1/(e*x+d)/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(1/(a*e*x + a*d + (b*e*x + b*d)*log(c*x^n)), x)
```

Sympy [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(d+ex)(a+b\log(cx^n))}\,dx = \int \frac{1}{(a+b\log(cx^n))(d+ex)}\,dx$$

```
[In] integrate(1/(e*x+d)/(a+b*ln(c*x**n)),x)
[Out] Integral(1/((a + b*log(c*x**n))*(d + e*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex)\,(a+b\log{(cx^n)})}\,dx = \int \frac{1}{(ex+d)(b\log{(cx^n)}+a)}\,dx$$

```
[In] integrate(1/(e*x+d)/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(1/((e*x + d)*(b*log(c*x^n) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex)\,(a+b\log{(cx^n)})}\,dx = \int \frac{1}{(ex+d)(b\log{(cx^n)}+a)}\,dx$$

```
[In] integrate(1/(e*x+d)/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(1/((e*x + d)*(b*log(c*x^n) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex)\,(a+b\log{(cx^n)})}\,dx = \int \frac{1}{(a+b\,\ln{(c\,x^n)})\,(d+e\,x)}\,dx$$

```
[In] int(1/((a + b*log(c*x^n))*(d + e*x)),x)
[Out] int(1/((a + b*log(c*x^n))*(d + e*x)), x)
```

3.160 $\int \frac{1}{x(d+ex)(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{x(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx=\text{Int}\left(\frac{1}{x(d+ex)\left(a+b\log\left(cx^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(e*x+d)/(a+b*ln(c*x^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{1}{x(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx$$

```
[In] Int[1/(x*(d + e*x)*(a + b*Log[c*x^n])),x]
[Out] Defer[Int][1/(x*(d + e*x)*(a + b*Log[c*x^n])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{1}{x(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx$$

```
[In] Integrate[1/(x*(d + e*x)*(a + b*Log[c*x^n])),x]
[Out] Integrate[1/(x*(d + e*x)*(a + b*Log[c*x^n])), x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(ex+d\right)\left(a+b\ln\left(c\,x^{n}\right)\right)}dx$$

```
[In] int(1/x/(e*x+d)/(a+b*ln(c*x^n)),x)
[Out] int(1/x/(e*x+d)/(a+b*ln(c*x^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{1}{x(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{1}{(ex+d)(b\log\left(cx^n\right)+a)x}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(1/(a*e*x^2 + a*d*x + (b*e*x^2 + b*d*x)*log(c*x^n)), x)
```

Sympy [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{1}{x\left(a+b\log\left(cx^n\right)\right)(d+ex)}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(a+b*ln(c*x**n)),x)
[Out] Integral(1/(x*(a + b*log(c*x**n))*(d + e*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(d+ex)(a+b\log(cx^n))}\,dx = \int \frac{1}{(ex+d)(b\log(cx^n)+a)x}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(1/((e*x + d)*(b*log(c*x^n) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(d+ex)(a+b\log(cx^n))}\,dx = \int \frac{1}{(ex+d)(b\log(cx^n)+a)x}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(1/((e*x + d)*(b*log(c*x^n) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(d+ex)(a+b\log(cx^n))}\,dx = \int \frac{1}{x\,(a+b\,\ln(c\,x^n))\,(d+e\,x)}\,dx$$

```
[In] int(1/(x*(a + b*log(c*x^n))*(d + e*x)),x)
[Out] int(1/(x*(a + b*log(c*x^n))*(d + e*x)), x)
```

3.161 $\int \frac{1}{x^2(d+ex)(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx=\operatorname{Int}\left(\frac{1}{x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/(e*x+d)/(a+b*ln(c*x^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{1}{x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx$$

```
[In] Int[1/(x^2*(d + e*x)*(a + b*Log[c*x^n])),x]
```

```
[Out] Defer[Int][1/(x^2*(d + e*x)*(a + b*Log[c*x^n])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{1}{x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx$$

```
[In] Integrate[1/(x^2*(d + e*x)*(a + b*Log[c*x^n])),x]
[Out] Integrate[1/(x^2*(d + e*x)*(a + b*Log[c*x^n])), x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(ex+d\right)\left(a+b\ln\left(c\,x^n\right)\right)}dx$$

```
[In] int(1/x^2/(e*x+d)/(a+b*ln(c*x^n)),x)
[Out] int(1/x^2/(e*x+d)/(a+b*ln(c*x^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \frac{1}{x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{1}{(ex+d)(b\log\left(cx^n\right)+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x+d)/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(1/(a*e*x^3 + a*d*x^2 + (b*e*x^3 + b*d*x^2)*log(c*x^n)), x)
```

Sympy [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{1}{x^2\left(a+b\log\left(cx^n\right)\right)\left(d+ex\right)}\,dx$$

```
[In] integrate(1/x**2/(e*x+d)/(a+b*ln(c*x**n)),x)
[Out] Integral(1/(x**2*(a + b*log(c*x**n))*(d + e*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{1}{(ex+d)(b\log\left(cx^n\right)+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x+d)/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(1/((e*x + d)*(b*log(c*x^n) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{1}{(ex+d)(b\log\left(cx^n\right)+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x+d)/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(1/((e*x + d)*(b*log(c*x^n) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2(d+ex)\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{1}{x^2\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,\left(d+e\,x\right)}\,dx$$

```
[In] int(1/(x^2*(a + b*log(c*x^n))*(d + e*x)),x)
[Out] int(1/(x^2*(a + b*log(c*x^n))*(d + e*x)), x)
```

3.162 $\int (fx)^m(d+ex)^3 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 211

$$\int (fx)^m(d+ex)^3 (a+b\log(cx^n))\,dx = -\frac{bd^3n(fx)^{1+m}}{f(1+m)^2}-\frac{3bd^2en(fx)^{2+m}}{f^2(2+m)^2}-\frac{3bde^2n(fx)^{3+m}}{f^3(3+m)^2}-\frac{be^3n(fx)^{4+m}}{f^4(4+m)^2}+\frac{d^3(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)}+\frac{3d^2e(fx)^{2+m}(a+b\log(cx^n))}{f^2(2+m)}+\frac{3de^2(fx)^{3+m}(a+b\log(cx^n))}{f^3(3+m)}+\frac{e^3(fx)^{4+m}(a+b\log(cx^n))}{f^4(4+m)}$$

```
[Out] -b*d^3*n*(f*x)^(1+m)/f/(1+m)^2-3*b*d^2*e*n*(f*x)^(2+m)/f^2/(2+m)^2-3*b*d*e^
2*n*(f*x)^(3+m)/f^3/(3+m)^2-b*e^3*n*(f*x)^(4+m)/f^4/(4+m)^2+d^3*(f*x)^(1+m)
*(a+b*ln(c*x^n))/f/(1+m)+3*d^2*e*(f*x)^(2+m)*(a+b*ln(c*x^n))/f^2/(2+m)+3*d*
e^2*(f*x)^(3+m)*(a+b*ln(c*x^n))/f^3/(3+m)+e^3*(f*x)^(4+m)*(a+b*ln(c*x^n))/f
^4/(4+m)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used

$= \{45, 2392, 14\}$

$$\int (fx)^m(d+ex)^3 (a+b\log(cx^n))\, dx = \frac{d^3(fx)^{m+1}(a+b\log(cx^n))}{f(m+1)} + \frac{3d^2e(fx)^{m+2}(a+b\log(cx^n))}{f^2(m+2)} + \frac{3de^2(fx)^{m+3}(a+b\log(cx^n))}{f^3(m+3)} + \frac{e^3(fx)^{m+4}(a+b\log(cx^n))}{f^4(m+4)} - \frac{bd^3n(fx)^{m+1}}{f(m+1)^2} - \frac{3bd^2en(fx)^{m+2}}{f^2(m+2)^2} - \frac{3bde^2n(fx)^{m+3}}{f^3(m+3)^2} - \frac{be^3n(fx)^{m+4}}{f^4(m+4)^2}$$

```
[In] Int[(f*x)^m*(d + e*x)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -((b*d^3*n*(f*x)^(1 + m))/(f*(1 + m)^2)) - (3*b*d^2*e*n*(f*x)^(2 + m))/(f^2
*(2 + m)^2) - (3*b*d*e^2*n*(f*x)^(3 + m))/(f^3*(3 + m)^2) - (b*e^3*n*(f*x)^
(4 + m))/(f^4*(4 + m)^2) + (d^3*(f*x)^(1 + m)*(a + b*Log[c*x^n]))/(f*(1 + m
)) + (3*d^2*e*(f*x)^(2 + m)*(a + b*Log[c*x^n]))/(f^2*(2 + m)) + (3*d*e^2*(f
*x)^(3 + m)*(a + b*Log[c*x^n]))/(f^3*(3 + m)) + (e^3*(f*x)^(4 + m)*(a + b*L
og[c*x^n]))/(f^4*(4 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^3(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)} + \frac{3d^2e(fx)^{2+m}(a+b\log(cx^n))}{f^2(2+m)} \\
&\quad + \frac{3de^2(fx)^{3+m}(a+b\log(cx^n))}{f^3(3+m)} + \frac{e^3(fx)^{4+m}(a+b\log(cx^n))}{f^4(4+m)} \\
&\quad - (bn)\int (fx)^m\left(\frac{d^3}{1+m} + \frac{3d^2ex}{2+m} + \frac{3de^2x^2}{3+m} + \frac{e^3x^3}{4+m}\right)dx \\
&= \frac{d^3(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)} + \frac{3d^2e(fx)^{2+m}(a+b\log(cx^n))}{f^2(2+m)} \\
&\quad + \frac{3de^2(fx)^{3+m}(a+b\log(cx^n))}{f^3(3+m)} + \frac{e^3(fx)^{4+m}(a+b\log(cx^n))}{f^4(4+m)} \\
&\quad - (bn)\int\left(\frac{d^3(fx)^m}{1+m} + \frac{3d^2e(fx)^{1+m}}{f(2+m)} + \frac{3de^2(fx)^{2+m}}{f^2(3+m)} + \frac{e^3(fx)^{3+m}}{f^3(4+m)}\right)dx \\
&= -\frac{bd^3n(fx)^{1+m}}{f(1+m)^2} - \frac{3bd^2en(fx)^{2+m}}{f^2(2+m)^2} - \frac{3bde^2n(fx)^{3+m}}{f^3(3+m)^2} - \frac{be^3n(fx)^{4+m}}{f^4(4+m)^2} \\
&\quad + \frac{d^3(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)} + \frac{3d^2e(fx)^{2+m}(a+b\log(cx^n))}{f^2(2+m)} \\
&\quad + \frac{3de^2(fx)^{3+m}(a+b\log(cx^n))}{f^3(3+m)} + \frac{e^3(fx)^{4+m}(a+b\log(cx^n))}{f^4(4+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.72

$$\int (fx)^m(d+ex)^3(a+b\log(cx^n))\,dx = x(fx)^m\left(-\frac{bd^3n}{(1+m)^2} - \frac{3bd^2enx}{(2+m)^2} - \frac{3bde^2nx^2}{(3+m)^2} - \frac{be^3nx^3}{(4+m)^2} + \frac{d^3(a+b\log(cx^n))}{1+m} + \frac{3d^2ex(a+b\log(cx^n))}{2+m} + \frac{3de^2x^2(a+b\log(cx^n))}{3+m} + \frac{e^3x^3(a+b\log(cx^n))}{4+m}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] x*(f*x)^m*(-((b*d^3*n)/(1 + m)^2) - (3*b*d^2*e*n*x)/(2 + m)^2 - (3*b*d*e^2*
n*x^2)/(3 + m)^2 - (b*e^3*n*x^3)/(4 + m)^2 + (d^3*(a + b*Log[c*x^n]))/(1 +
m) + (3*d^2*e*x*(a + b*Log[c*x^n]))/(2 + m) + (3*d*e^2*x^2*(a + b*Log[c*x^n
]))/(3 + m) + (e^3*x^3*(a + b*Log[c*x^n]))/(4 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1745 vs. 2(211) = 422.

Time = 2.22 (sec) , antiderivative size = 1746, normalized size of antiderivative = 8.27

method	result	size
parallelrisch	Expression too large to display	1746
risch	Expression too large to display	4955

```
[In] int((f*x)^m*(e*x+d)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -(984*x^2*(f*x)^m*b*d^2*e*m^3*n-2208*x^3*(f*x)^m*ln(c*x^n)*b*d*e^2*m+672*x^
3*(f*x)^m*b*d*e^2*m*n-4686*x^2*(f*x)^m*ln(c*x^n)*b*d^2*e*m^2+1659*x^2*(f*x)
^m*b*d^2*e*m^2*n-3168*x^2*(f*x)^m*ln(c*x^n)*b*d^2*e*m+1368*x^2*(f*x)^m*b*d^
2*e*m*n-3*x^3*(f*x)^m*ln(c*x^n)*b*d*e^2*m^7-51*x^3*(f*x)^m*ln(c*x^n)*b*d*e^
2*m^6+3*x^3*(f*x)^m*b*d*e^2*m^6*n-3*x^2*(f*x)^m*ln(c*x^n)*b*d^2*e*m^7-357*x
^3*(f*x)^m*ln(c*x^n)*b*d*e^2*m^5+42*x^3*(f*x)^m*b*d*e^2*m^5*n-54*x^2*(f*x)^
m*ln(c*x^n)*b*d^2*e*m^6+3*x^2*(f*x)^m*b*d^2*e*m^6*n-1329*x^3*(f*x)^m*ln(c*x
^n)*b*d*e^2*m^4+231*x^3*(f*x)^m*b*d*e^2*m^4*n-402*x^2*(f*x)^m*ln(c*x^n)*b*d
^2*e*m^5+48*x^2*(f*x)^m*b*d^2*e*m^5*n-2832*x^3*(f*x)^m*ln(c*x^n)*b*d*e^2*m^
3+636*x^3*(f*x)^m*b*d*e^2*m^3*n-1596*x^2*(f*x)^m*ln(c*x^n)*b*d^2*e*m^4+306*
x^2*(f*x)^m*b*d^2*e*m^4*n-144*x^4*(f*x)^m*a*e^3-576*x*(f*x)^m*a*d^3-3444*x^
3*(f*x)^m*ln(c*x^n)*b*d*e^2*m^2+924*x^3*(f*x)^m*b*d*e^2*m^2*n-3627*x^2*(f*x
)^m*ln(c*x^n)*b*d^2*e*m^3+576*x*(f*x)^m*b*d^3*n-576*b*d^3*ln(c*x^n)*(f*x)^m
*x-144*e^3*b*ln(c*x^n)*(f*x)^m*x^4-3627*x^2*(f*x)^m*a*d^2*e*m^3-1624*x*(f*x
)^m*ln(c*x^n)*b*d^3*m^3+516*x*(f*x)^m*b*d^3*m^3*n-2208*x^3*(f*x)^m*a*d*e^2*
m+192*x^3*(f*x)^m*b*d*e^2*n-4686*x^2*(f*x)^m*a*d^2*e*m^2-2356*x*(f*x)^m*ln(
c*x^n)*b*d^3*m^2+1108*x*(f*x)^m*b*d^3*m^2*n-864*e*d^2*b*ln(c*x^n)*(f*x)^m*x
^2-576*e^2*d*b*ln(c*x^n)*(f*x)^m*x^3-x^4*(f*x)^m*a*e^3*m^7-16*x^4*(f*x)^m*a
*e^3*m^6-106*x^4*(f*x)^m*a*e^3*m^5-376*x^4*(f*x)^m*a*e^3*m^4-x*(f*x)^m*a*d^
3*m^7-769*x^4*(f*x)^m*a*e^3*m^3-19*x*(f*x)^m*a*d^3*m^6-904*x^4*(f*x)^m*a*e^
3*m^2-151*x*(f*x)^m*a*d^3*m^5-564*x^4*(f*x)^m*a*e^3*m+36*x^4*(f*x)^m*b*e^3*
n-649*x*(f*x)^m*a*d^3*m^4-1624*x*(f*x)^m*a*d^3*m^3-576*x^3*(f*x)^m*a*d*e^2-
2356*x*(f*x)^m*a*d^3*m^2-864*x^2*(f*x)^m*a*d^2*e-1824*x*(f*x)^m*a*d^3*m-316
8*x^2*(f*x)^m*a*d^2*e*m+432*x^2*(f*x)^m*b*d^2*e*n-1824*x*(f*x)^m*ln(c*x^n)*
b*d^3*m+1248*x*(f*x)^m*b*d^3*m*n-x^4*(f*x)^m*ln(c*x^n)*b*e^3*m^7-16*x^4*(f*
x)^m*ln(c*x^n)*b*e^3*m^6+x^4*(f*x)^m*b*e^3*m^6*n-106*x^4*(f*x)^m*ln(c*x^n)*
b*e^3*m^5+12*x^4*(f*x)^m*b*e^3*m^5*n-3*x^3*(f*x)^m*a*d*e^2*m^7-376*x^4*(f*x
)^m*ln(c*x^n)*b*e^3*m^4+58*x^4*(f*x)^m*b*e^3*m^4*n-51*x^3*(f*x)^m*a*d*e^2*m
^6-3*x^2*(f*x)^m*a*d^2*e*m^7-x*(f*x)^m*ln(c*x^n)*b*d^3*m^7-769*x^4*(f*x)^m*
ln(c*x^n)*b*e^3*m^3+144*x^4*(f*x)^m*b*e^3*m^3*n-357*x^3*(f*x)^m*a*d*e^2*m^5
-54*x^2*(f*x)^m*a*d^2*e*m^6-19*x*(f*x)^m*ln(c*x^n)*b*d^3*m^6+x*(f*x)^m*b*d^
3*m^6*n-904*x^4*(f*x)^m*ln(c*x^n)*b*e^3*m^2+193*x^4*(f*x)^m*b*e^3*m^2*n-132
9*x^3*(f*x)^m*a*d*e^2*m^4-402*x^2*(f*x)^m*a*d^2*e*m^5-151*x*(f*x)^m*ln(c*x^
n)*b*d^3*m^5+18*x*(f*x)^m*b*d^3*m^5*n-564*x^4*(f*x)^m*ln(c*x^n)*b*e^3*m+132
```

```
*x^4*(f*x)^m*b*e^3*m*n-2832*x^3*(f*x)^m*a*d*e^2*m^3-1596*x^2*(f*x)^m*a*d^2*
e*m^4-649*x*(f*x)^m*ln(c*x^n)*b*d^3*m^4+133*x*(f*x)^m*b*d^3*m^4*n-3444*x^3*
(f*x)^m*a*d*e^2*m^2)/(4+m)^2/(3+m)^2/(2+m)^2/(1+m)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1222 vs. $2(211) = 422$.

Time = 0.32 (sec) , antiderivative size = 1222, normalized size of antiderivative = 5.79

$$\int (fx)^m (d+ex)^3 (a+b\log(cx^n))\,dx = \text{Too large to display}$$

```
[In] integrate((f*x)^m*(e*x+d)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] ((a*e^3*m^7 + 16*a*e^3*m^6 + 106*a*e^3*m^5 + 376*a*e^3*m^4 + 769*a*e^3*m^3
+ 904*a*e^3*m^2 + 564*a*e^3*m + 144*a*e^3 - (b*e^3*m^6 + 12*b*e^3*m^5 + 58*
b*e^3*m^4 + 144*b*e^3*m^3 + 193*b*e^3*m^2 + 132*b*e^3*m + 36*b*e^3)*n)*x^4
+ 3*(a*d*e^2*m^7 + 17*a*d*e^2*m^6 + 119*a*d*e^2*m^5 + 443*a*d*e^2*m^4 + 944
*a*d*e^2*m^3 + 1148*a*d*e^2*m^2 + 736*a*d*e^2*m + 192*a*d*e^2 - (b*d*e^2*m^
6 + 14*b*d*e^2*m^5 + 77*b*d*e^2*m^4 + 212*b*d*e^2*m^3 + 308*b*d*e^2*m^2 + 2
24*b*d*e^2*m + 64*b*d*e^2)*n)*x^3 + 3*(a*d^2*e*m^7 + 18*a*d^2*e*m^6 + 134*a
*d^2*e*m^5 + 532*a*d^2*e*m^4 + 1209*a*d^2*e*m^3 + 1562*a*d^2*e*m^2 + 1056*a
*d^2*e*m + 288*a*d^2*e - (b*d^2*e*m^6 + 16*b*d^2*e*m^5 + 102*b*d^2*e*m^4 +
328*b*d^2*e*m^3 + 553*b*d^2*e*m^2 + 456*b*d^2*e*m + 144*b*d^2*e)*n)*x^2 + (
a*d^3*m^7 + 19*a*d^3*m^6 + 151*a*d^3*m^5 + 649*a*d^3*m^4 + 1624*a*d^3*m^3 +
 2356*a*d^3*m^2 + 1824*a*d^3*m + 576*a*d^3 - (b*d^3*m^6 + 18*b*d^3*m^5 + 13
3*b*d^3*m^4 + 516*b*d^3*m^3 + 1108*b*d^3*m^2 + 1248*b*d^3*m + 576*b*d^3)*n)
*x + ((b*e^3*m^7 + 16*b*e^3*m^6 + 106*b*e^3*m^5 + 376*b*e^3*m^4 + 769*b*e^3
*m^3 + 904*b*e^3*m^2 + 564*b*e^3*m + 144*b*e^3)*x^4 + 3*(b*d*e^2*m^7 + 17*b
*d*e^2*m^6 + 119*b*d*e^2*m^5 + 443*b*d*e^2*m^4 + 944*b*d*e^2*m^3 + 1148*b*d
*e^2*m^2 + 736*b*d*e^2*m + 192*b*d*e^2)*x^3 + 3*(b*d^2*e*m^7 + 18*b*d^2*e*m
^6 + 134*b*d^2*e*m^5 + 532*b*d^2*e*m^4 + 1209*b*d^2*e*m^3 + 1562*b*d^2*e*m^
2 + 1056*b*d^2*e*m + 288*b*d^2*e)*x^2 + (b*d^3*m^7 + 19*b*d^3*m^6 + 151*b*d
^3*m^5 + 649*b*d^3*m^4 + 1624*b*d^3*m^3 + 2356*b*d^3*m^2 + 1824*b*d^3*m + 5
76*b*d^3)*x)*log(c) + ((b*e^3*m^7 + 16*b*e^3*m^6 + 106*b*e^3*m^5 + 376*b*e^
3*m^4 + 769*b*e^3*m^3 + 904*b*e^3*m^2 + 564*b*e^3*m + 144*b*e^3)*n*x^4 + 3*
(b*d*e^2*m^7 + 17*b*d*e^2*m^6 + 119*b*d*e^2*m^5 + 443*b*d*e^2*m^4 + 944*b*d
*e^2*m^3 + 1148*b*d*e^2*m^2 + 736*b*d*e^2*m + 192*b*d*e^2)*n*x^3 + 3*(b*d^2
*e*m^7 + 18*b*d^2*e*m^6 + 134*b*d^2*e*m^5 + 532*b*d^2*e*m^4 + 1209*b*d^2*e*
m^3 + 1562*b*d^2*e*m^2 + 1056*b*d^2*e*m + 288*b*d^2*e)*n*x^2 + (b*d^3*m^7 +
 19*b*d^3*m^6 + 151*b*d^3*m^5 + 649*b*d^3*m^4 + 1624*b*d^3*m^3 + 2356*b*d^3
*m^2 + 1824*b*d^3*m + 576*b*d^3)*n*x)*log(x))*e^(m*log(f) + m*log(x))/(m^8
+ 20*m^7 + 170*m^6 + 800*m^5 + 2273*m^4 + 3980*m^3 + 4180*m^2 + 2400*m + 57
6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6156 vs. 2(206) = 412.

Time = 4.99 (sec) , antiderivative size = 6156, normalized size of antiderivative = 29.18

$$\int (fx)^m(d+ex)^3(a+b\log(cx^n))\,dx = \text{Too large to display}$$

```
[In] integrate((f*x)**m*(e*x+d)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise(((-a*d**3/(3*x**3) - 3*a*d**2*e/(2*x**2) - 3*a*d*e**2/x + a*e**3*
log(x) + b*d**3*(-n/(9*x**3) - log(c*x**n)/(3*x**3)) + 3*b*d**2*e*(-n/(4*x*
*2) - log(c*x**n)/(2*x**2)) + 3*b*d*e**2*(-n/x - log(c*x**n)/x) - b*e**3*Pi
ecewise((-log(c)*log(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), True)))/f**4, E
q(m, -4)), ((-a*d**3/(2*x**2) - 3*a*d**2*e/x + 3*a*d*e**2*log(c*x**n)/n + a
*e**3*x - b*d**3*n/(4*x**2) - b*d**3*log(c*x**n)/(2*x**2) - 3*b*d**2*e*n/x
- 3*b*d**2*e*log(c*x**n)/x + 3*b*d*e**2*log(c*x**n)**2/(2*n) - b*e**3*n*x +
 b*e**3*x*log(c*x**n))/f**3, Eq(m, -3)), ((-a*d**3/x + 3*a*d**2*e*log(c*x**
n)/n + 3*a*d*e**2*x + a*e**3*x**2/2 - b*d**3*n/x - b*d**3*log(c*x**n)/x + 3
*b*d**2*e*log(c*x**n)**2/(2*n) - 3*b*d*e**2*n*x + 3*b*d*e**2*x*log(c*x**n)
- b*e**3*n*x**2/4 + b*e**3*x**2*log(c*x**n)/2)/f**2, Eq(m, -2)), ((a*d**3*l
og(c*x**n)/n + 3*a*d**2*e*x + 3*a*d*e**2*x**2/2 + a*e**3*x**3/3 + b*d**3*lo
g(c*x**n)**2/(2*n) - 3*b*d**2*e*n*x + 3*b*d**2*e*x*log(c*x**n) - 3*b*d*e**2
*n*x**2/4 + 3*b*d*e**2*x**2*log(c*x**n)/2 - b*e**3*n*x**3/9 + b*e**3*x**3*l
og(c*x**n)/3)/f, Eq(m, -1)), (a*d**3*m**7*x*(f*x)**m/(m**8 + 20*m**7 + 170*
m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 19*a*
d**3*m**6*x*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 39
80*m**3 + 4180*m**2 + 2400*m + 576) + 151*a*d**3*m**5*x*(f*x)**m/(m**8 + 20
*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m +
576) + 649*a*d**3*m**4*x*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2
273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 1624*a*d**3*m**3*x*(f*x)
**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m*
*2 + 2400*m + 576) + 2356*a*d**3*m**2*x*(f*x)**m/(m**8 + 20*m**7 + 170*m**6
 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 1824*a*d*
*3*m*x*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m*
*3 + 4180*m**2 + 2400*m + 576) + 576*a*d**3*x*(f*x)**m/(m**8 + 20*m**7 + 17
0*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 3*a
*d**2*e*m**7*x**2*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**
4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 54*a*d**2*e*m**6*x**2*(f*x)**m/
(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 +
 2400*m + 576) + 402*a*d**2*e*m**5*x**2*(f*x)**m/(m**8 + 20*m**7 + 170*m**6
 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 1596*a*d*
*2*e*m**4*x**2*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 +
 3980*m**3 + 4180*m**2 + 2400*m + 576) + 3627*a*d**2*e*m**3*x**2*(f*x)**m/(
m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 +
```

2400*m + 576) + 4686*a*d**2*e*m**2*x**2*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 3168*a*d**2*e*m*x**2*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 864*a*d**2*e*x**2*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 3*a*d*e**2*m**7*x**3*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 51*a*d*e**2*m**6*x**3*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 357*a*d*e**2*m**5*x**3*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 1329*a*d*e**2*m**4*x**3*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 2832*a*d*e**2*m**3*x**3*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 3444*a*d*e**2*m**2*x**3*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 2208*a*d*e**2*m*x**3*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 576*a*d*e**2*x**3*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + a*e**3*m**7*x**4*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 16*a*e**3*m**6*x**4*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 106*a*e**3*m**5*x**4*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 376*a*e**3*m**4*x**4*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 769*a*e**3*m**3*x**4*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 904*a*e**3*m**2*x**4*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 564*a*e**3*m*x**4*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 144*a*e**3*x**4*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + b*d**3*m**7*x*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) - b*d**3*m**6*n*x*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 19*b*d**3*m**6*x*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) - 18*b*d**3*m**5*n*x*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 151*b*d**3*m**5*x*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) - 133*b*d**3*m**4*n*x*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 649*b*d**3*m**4*x*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) - 516*b*d**3*m**3*n*x*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 1624*b*d**3*m**3*x*(f*x)**m*log(c*x**n)/(m**8 +

```
 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m
 + 576) - 1108*b*d**3*m**2*n*x*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m*
*5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 2356*b*d**3*m**2*x
*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3
980*m**3 + 4180*m**2 + 2400*m + 576) - 1248*b*d**3*m*n*x*(f*x)**m/(m**8 + 2
0*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m +
 576) + 1824*b*d**3*m*x*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 8
00*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) - 576*b*d**3*n*
x*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 +
4180*m**2 + 2400*m + 576) + 576*b*d**3*x*(f*x)**m*log(c*x**n)/(m**8 + 20*m*
*7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576
) + 3*b*d**2*e*m**7*x**2*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**6 +
800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) - 3*b*d**2*e*m
**6*n*x**2*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 398
0*m**3 + 4180*m**2 + 2400*m + 576) + 54*b*d**2*e*m**6*x**2*(f*x)**m*log(c*x
**n)/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m
**2 + 2400*m + 576) - 48*b*d**2*e*m**5*n*x**2*(f*x)**m/(m**8 + 20*m**7 + 17
0*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 402
*b*d**2*e*m**5*x**2*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 800*m
**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) - 306*b*d**2*e*m**4
*n*x**2*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m
**3 + 4180*m**2 + 2400*m + 576) + 1596*b*d**2*e*m**4*x**2*(f*x)**m*log(c*x*
*n)/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m*
*2 + 2400*m + 576) - 984*b*d**2*e*m**3*n*x**2*(f*x)**m/(m**8 + 20*m**7 + 17
0*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 362
7*b*d**2*e*m**3*x**2*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 800*
m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) - 1659*b*d**2*e*m*
*2*n*x**2*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980
*m**3 + 4180*m**2 + 2400*m + 576) + 4686*b*d**2*e*m**2*x**2*(f*x)**m*log(c*
x**n)/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*
m**2 + 2400*m + 576) - 1368*b*d**2*e*m*n*x**2*(f*x)**m/(m**8 + 20*m**7 + 17
0*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 316
8*b*d**2*e*m*x**2*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 800*m**
5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) - 432*b*d**2*e*n*x**2
*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4
180*m**2 + 2400*m + 576) + 864*b*d**2*e*x**2*(f*x)**m*log(c*x**n)/(m**8 + 2
0*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m +
 576) + 3*b*d*e**2*m**7*x**3*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**
6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) - 3*b*d*e*
*2*m**6*n*x**3*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 +
 3980*m**3 + 4180*m**2 + 2400*m + 576) + 51*b*d*e**2*m**6*x**3*(f*x)**m*log
(c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 41
80*m**2 + 2400*m + 576) - 42*b*d*e**2*m**5*n*x**3*(f*x)**m/(m**8 + 20*m**7
+ 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) +
 357*b*d*e**2*m**5*x**3*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 8
```

```
00*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) - 231*b*d*e**2*
m**4*n*x**3*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 39
80*m**3 + 4180*m**2 + 2400*m + 576) + 1329*b*d*e**2*m**4*x**3*(f*x)**m*log(
c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 418
0*m**2 + 2400*m + 576) - 636*b*d*e**2*m**3*n*x**3*(f*x)**m/(m**8 + 20*m**7
+ 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) +
 2832*b*d*e**2*m**3*x**3*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**6 +
800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) - 924*b*d*e**2
*m**2*n*x**3*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3
980*m**3 + 4180*m**2 + 2400*m + 576) + 3444*b*d*e**2*m**2*x**3*(f*x)**m*log
(c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 41
80*m**2 + 2400*m + 576) - 672*b*d*e**2*m*n*x**3*(f*x)**m/(m**8 + 20*m**7 +
170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 2
208*b*d*e**2*m*x**3*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 800*m
**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) - 192*b*d*e**2*n*x*
*3*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 +
 4180*m**2 + 2400*m + 576) + 576*b*d*e**2*x**3*(f*x)**m*log(c*x**n)/(m**8 +
 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m
 + 576) + b*e**3*m**7*x**4*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**6
+ 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) - b*e**3*m**
6*n*x**4*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*
m**3 + 4180*m**2 + 2400*m + 576) + 16*b*e**3*m**6*x**4*(f*x)**m*log(c*x**n)
/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2
+ 2400*m + 576) - 12*b*e**3*m**5*n*x**4*(f*x)**m/(m**8 + 20*m**7 + 170*m**6
 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 106*b*e**
3*m**5*x**4*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 22
73*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) - 58*b*e**3*m**4*n*x**4*(f*
x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*
m**2 + 2400*m + 576) + 376*b*e**3*m**4*x**4*(f*x)**m*log(c*x**n)/(m**8 + 20
*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m +
576) - 144*b*e**3*m**3*n*x**4*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**
5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 769*b*e**3*m**3*x**
4*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 +
3980*m**3 + 4180*m**2 + 2400*m + 576) - 193*b*e**3*m**2*n*x**4*(f*x)**m/(m*
*8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 24
00*m + 576) + 904*b*e**3*m**2*x**4*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 1
70*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) - 13
2*b*e**3*m*n*x**4*(f*x)**m/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**
4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 564*b*e**3*m*x**4*(f*x)**m*log(
c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 418
0*m**2 + 2400*m + 576) - 36*b*e**3*n*x**4*(f*x)**m/(m**8 + 20*m**7 + 170*m*
*6 + 800*m**5 + 2273*m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576) + 144*b*e
**3*x**4*(f*x)**m*log(c*x**n)/(m**8 + 20*m**7 + 170*m**6 + 800*m**5 + 2273*
m**4 + 3980*m**3 + 4180*m**2 + 2400*m + 576), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.28

$$\int (fx)^m(d+ex)^3 (a+b\log (cx^n)) \ dx = \frac{be^3f^mx^4x^m\log (cx^n)}{m+4}+\frac{ae^3f^mx^4x^m}{m+4}-\frac{be^3f^mnx^4x^m}{(m+4)^2}+\frac{3\,bde^2f^mx^3x^m\log (cx^n)}{m+3}+\frac{3\,ade^2f^mx^3x^m}{m+3}-\frac{3\,bde^2f^mnx^3x^m}{(m+3)^2}+\frac{3\,bd^2ef^mx^2x^m\log (cx^n)}{m+2}+\frac{3\,ad^2ef^mx^2x^m}{m+2}-\frac{3\,bd^2ef^mnx^2x^m}{(m+2)^2}-\frac{bd^3f^mnxx^m}{(m+1)^2}+\frac{(fx)^{m+1}\,bd^3\log (cx^n)}{f(m+1)}+\frac{(fx)^{m+1}\,ad^3}{f(m+1)}$$

```
[In] integrate((f*x)^m*(e*x+d)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] b*e^3*f^m*x^4*x^m*log(c*x^n)/(m + 4) + a*e^3*f^m*x^4*x^m/(m + 4) - b*e^3*f^
m*n*x^4*x^m/(m + 4)^2 + 3*b*d*e^2*f^m*x^3*x^m*log(c*x^n)/(m + 3) + 3*a*d*e^
2*f^m*x^3*x^m/(m + 3) - 3*b*d*e^2*f^m*n*x^3*x^m/(m + 3)^2 + 3*b*d^2*e*f^m*x
^2*x^m*log(c*x^n)/(m + 2) + 3*a*d^2*e*f^m*x^2*x^m/(m + 2) - 3*b*d^2*e*f^m*n
*x^2*x^m/(m + 2)^2 - b*d^3*f^m*n*x*x^m/(m + 1)^2 + (f*x)^(m + 1)*b*d^3*log(
c*x^n)/(f*(m + 1)) + (f*x)^(m + 1)*a*d^3/(f*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. 2(211) = 422.

Time = 0.33 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.54

$$\int (fx)^m(d+ex)^3(a+b\log(cx^n))\,dx = \frac{be^3f^3f^mx^4x^m\log(c)}{f^3m+4f^3}+\frac{ae^3f^3f^mx^4x^m}{f^3m+4f^3}+\frac{be^3f^mmnx^4x^m\log(x)}{m^2+8m+16}+\frac{3bde^2f^2f^mx^3x^m\log(c)}{f^2m+3f^2}+\frac{3bde^2f^mmnx^3x^m\log(x)}{m^2+6m+9}+\frac{4be^3f^mnx^4x^m\log(x)}{m^2+8m+16}+\frac{3ade^2f^2f^mx^3x^m}{f^2m+3f^2}-\frac{be^3f^mnx^4x^m}{m^2+8m+16}+\frac{3bd^2ef^mmnx^2x^m\log(x)}{m^2+4m+4}+\frac{9bde^2f^mnx^3x^m\log(x)}{m^2+6m+9}-\frac{3bde^2f^mnx^3x^m}{m^2+6m+9}+\frac{bd^3f^mmnxx^m\log(x)}{m^2+2m+1}+\frac{6bd^2ef^mnx^2x^m\log(x)}{m^2+4m+4}-\frac{3bd^2ef^mnx^2x^m}{m^2+4m+4}+\frac{3bd^2ef^mx^2x^m\log(c)}{m+2}+\frac{bd^3f^mnxx^m\log(x)}{m^2+2m+1}-\frac{bd^3f^mnxx^m}{m^2+2m+1}+\frac{3ad^2ef^mx^2x^m}{m+2}+\frac{(fx)^m bd^3x\log(c)}{m+1}+\frac{(fx)^m ad^3x}{m+1}$$

```
[In] integrate((f*x)^m*(e*x+d)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*e^3*f^3*f^m*x^4*x^m*log(c)/(f^3*m + 4*f^3) + a*e^3*f^3*f^m*x^4*x^m/(f^3*m
 + 4*f^3) + b*e^3*f^m*m*n*x^4*x^m*log(x)/(m^2 + 8*m + 16) + 3*b*d*e^2*f^2*f
^m*x^3*x^m*log(c)/(f^2*m + 3*f^2) + 3*b*d*e^2*f^m*m*n*x^3*x^m*log(x)/(m^2 +
 6*m + 9) + 4*b*e^3*f^m*n*x^4*x^m*log(x)/(m^2 + 8*m + 16) + 3*a*d*e^2*f^2*f
^m*x^3*x^m/(f^2*m + 3*f^2) - b*e^3*f^m*n*x^4*x^m/(m^2 + 8*m + 16) + 3*b*d^2
*e*f^m*m*n*x^2*x^m*log(x)/(m^2 + 4*m + 4) + 9*b*d*e^2*f^m*n*x^3*x^m*log(x)/
(m^2 + 6*m + 9) - 3*b*d*e^2*f^m*n*x^3*x^m/(m^2 + 6*m + 9) + b*d^3*f^m*m*n*x
*x^m*log(x)/(m^2 + 2*m + 1) + 6*b*d^2*e*f^m*n*x^2*x^m*log(x)/(m^2 + 4*m + 4
) - 3*b*d^2*e*f^m*n*x^2*x^m/(m^2 + 4*m + 4) + 3*b*d^2*e*f^m*x^2*x^m*log(c)/
(m + 2) + b*d^3*f^m*n*x*x^m*log(x)/(m^2 + 2*m + 1) - b*d^3*f^m*n*x*x^m/(m^2
 + 2*m + 1) + 3*a*d^2*e*f^m*x^2*x^m/(m + 2) + (f*x)^m*b*d^3*x*log(c)/(m + 1
) + (f*x)^m*a*d^3*x/(m + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex)^3 (a+b\log(cx^n)) \, dx = \int (f\,x)^m \, (a+b\,\ln(c\,x^n)) \, (d+e\,x)^3 \, dx$$

```
[In] int((f*x)^m*(a + b*log(c*x^n))*(d + e*x)^3,x)
[Out] int((f*x)^m*(a + b*log(c*x^n))*(d + e*x)^3, x)
```

3.163 $\int (fx)^m(d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 153

$$\int (fx)^m(d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{bd^2n(fx)^{1+m}}{f(1+m)^2}-\frac{2bden(fx)^{2+m}}{f^2(2+m)^2}-\frac{be^2n(fx)^{3+m}}{f^3(3+m)^2}+\frac{d^2(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)}+\frac{2de(fx)^{2+m}\left(a+b\log\left(cx^n\right)\right)}{f^2(2+m)}+\frac{e^2(fx)^{3+m}\left(a+b\log\left(cx^n\right)\right)}{f^3(3+m)}$$

```
[Out] -b*d^2*n*(f*x)^(1+m)/f/(1+m)^2-2*b*d*e*n*(f*x)^(2+m)/f^2/(2+m)^2-b*e^2*n*(f
*x)^(3+m)/f^3/(3+m)^2+d^2*(f*x)^(1+m)*(a+b*ln(c*x^n))/f/(1+m)+2*d*e*(f*x)^(
2+m)*(a+b*ln(c*x^n))/f^2/(2+m)+e^2*(f*x)^(3+m)*(a+b*ln(c*x^n))/f^3/(3+m)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {45, 2392, 12, 14}

$$\int (fx)^m(d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{d^2(fx)^{m+1}\left(a+b\log\left(cx^n\right)\right)}{f(m+1)}+\frac{2de(fx)^{m+2}\left(a+b\log\left(cx^n\right)\right)}{f^2(m+2)}+\frac{e^2(fx)^{m+3}\left(a+b\log\left(cx^n\right)\right)}{f^3(m+3)}-\frac{bd^2n(fx)^{m+1}}{f(m+1)^2}-\frac{2bden(fx)^{m+2}}{f^2(m+2)^2}-\frac{be^2n(fx)^{m+3}}{f^3(m+3)^2}$$

```
[In] Int[(f*x)^m*(d + e*x)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -((b*d^2*n*(f*x)^(1 + m))/(f*(1 + m)^2)) - (2*b*d*e*n*(f*x)^(2 + m))/(f^2*(
2 + m)^2) - (b*e^2*n*(f*x)^(3 + m))/(f^3*(3 + m)^2) + (d^2*(f*x)^(1 + m)*(a
 + b*Log[c*x^n]))/(f*(1 + m)) + (2*d*e*(f*x)^(2 + m)*(a + b*Log[c*x^n]))/(f
^2*(2 + m)) + (e^2*(f*x)^(3 + m)*(a + b*Log[c*x^n]))/(f^3*(3 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

integral

$$= \frac{d^2(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)} + \frac{2de(fx)^{2+m}(a+b\log(cx^n))}{f^2(2+m)} + \frac{e^2(fx)^{3+m}(a+b\log(cx^n))}{f^3(3+m)} - (bn)\int \frac{(fx)^m\left(d^2(2+m)(3+m)+2de(1+m)(3+m)x+e^2(1+m)(2+m)x^2\right)}{(1+m)(2+m)(3+m)}\,dx$$

$$= \frac{d^2(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)} + \frac{2de(fx)^{2+m}(a+b\log(cx^n))}{f^2(2+m)} + \frac{e^2(fx)^{3+m}(a+b\log(cx^n))}{f^3(3+m)} - \frac{(bn)\int (fx)^m\left(d^2(2+m)(3+m)+2de(1+m)(3+m)x+e^2(1+m)(2+m)x^2\right)\,dx}{6+11m+6m^2+m^3}$$

$$
\begin{aligned}
&= \frac{d^2(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)}+\frac{2de(fx)^{2+m}\left(a+b\log\left(cx^n\right)\right)}{f^2(2+m)}+\frac{e^2(fx)^{3+m}\left(a+b\log\left(cx^n\right)\right)}{f^3(3+m)} \\
&\quad -\frac{(bn)\int\left(d^2(2+m)(3+m)(fx)^m+\frac{2de(1+m)(3+m)(fx)^{1+m}}{f}+\frac{e^2(1+m)(2+m)(fx)^{2+m}}{f^2}\right)dx}{6+11m+6m^2+m^3} \\
&= -\frac{bd^2n(fx)^{1+m}}{f(1+m)^2}-\frac{2bden(fx)^{2+m}}{f^2(2+m)^2}-\frac{be^2n(fx)^{3+m}}{f^3(3+m)^2}+\frac{d^2(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)} \\
&\quad +\frac{2de(fx)^{2+m}\left(a+b\log\left(cx^n\right)\right)}{f^2(2+m)}+\frac{e^2(fx)^{3+m}\left(a+b\log\left(cx^n\right)\right)}{f^3(3+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.71

$$
\int(fx)^m(d+ex)^2\left(a+b\log\left(cx^n\right)\right)dx = x(fx)^m\left(-\frac{bd^2n}{(1+m)^2}-\frac{2bdenx}{(2+m)^2}-\frac{be^2nx^2}{(3+m)^2}+\frac{d^2(a+b\log\left(cx^n\right))}{1+m}+\frac{2dex(a+b\log\left(cx^n\right))}{2+m}+\frac{e^2x^2(a+b\log\left(cx^n\right))}{3+m}\right)
$$

```
[In] Integrate[(f*x)^m*(d + e*x)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] x*(f*x)^m*(-((b*d^2*n)/(1 + m)^2) - (2*b*d*e*n*x)/(2 + m)^2 - (b*e^2*n*x^2)
/(3 + m)^2 + (d^2*(a + b*Log[c*x^n]))/(1 + m) + (2*d*e*x*(a + b*Log[c*x^n])
)/(2 + m) + (e^2*x^2*(a + b*Log[c*x^n]))/(3 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. 2(153) = 306.

Time = 1.05 (sec) , antiderivative size = 911, normalized size of antiderivative = 5.95

method	result
parallelrisch	$-\frac{-12x^3(fx)^m a e^2-36x(fx)^m a d^2-36x(fx)^m \ln(cx^n) b d^2-12x^3(fx)^m \ln(cx^n) b e^2-x^3(fx)^m a e^2 m^5-9x^3(fx)^m a e^2 m^4-31x}{}$
risch	Expression too large to display

```
[In] int((f*x)^m*(e*x+d)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -(-12*x^3*(f*x)^m*a*e^2-36*x*(f*x)^m*a*d^2-36*x*(f*x)^m*ln(c*x^n)*b*d^2-12*
x^3*(f*x)^m*ln(c*x^n)*b*e^2-x^3*(f*x)^m*a*e^2*m^5-9*x^3*(f*x)^m*a*e^2*m^4-3
1*x^3*(f*x)^m*a*e^2*m^3-x*(f*x)^m*a*d^2*m^5-51*x^3*(f*x)^m*a*e^2*m^2-11*x*(
f*x)^m*a*d^2*m^4-40*x^3*(f*x)^m*a*e^2*m+4*x^3*(f*x)^m*b*e^2*n-47*x*(f*x)^m*
a*d^2*m^3-97*x*(f*x)^m*a*d^2*m^2-36*x^2*(f*x)^m*a*d*e-96*x*(f*x)^m*a*d^2*m+
```

```
36*x*(f*x)^m*b*d^2*n-x^3*(f*x)^m*ln(c*x^n)*b*e^2*m^5-36*x^2*(f*x)^m*ln(c*x^
n)*b*d*e-9*x^3*(f*x)^m*ln(c*x^n)*b*e^2*m^4+x^3*(f*x)^m*b*e^2*m^4*n-31*x^3*(
f*x)^m*ln(c*x^n)*b*e^2*m^3+6*x^3*(f*x)^m*b*e^2*m^3*n-2*x^2*(f*x)^m*a*d*e*m^
5-x*(f*x)^m*ln(c*x^n)*b*d^2*m^5-51*x^3*(f*x)^m*ln(c*x^n)*b*e^2*m^2+13*x^3*(
f*x)^m*b*e^2*m^2*n-20*x^2*(f*x)^m*a*d*e*m^4-11*x*(f*x)^m*ln(c*x^n)*b*d^2*m^
4+x*(f*x)^m*b*d^2*m^4*n-40*x^3*(f*x)^m*ln(c*x^n)*b*e^2*m+12*x^3*(f*x)^m*b*e
^2*m*n-76*x^2*(f*x)^m*a*d*e*m^3-47*x*(f*x)^m*ln(c*x^n)*b*d^2*m^3+10*x*(f*x)
^m*b*d^2*m^3*n-136*x^2*(f*x)^m*a*d*e*m^2-97*x*(f*x)^m*ln(c*x^n)*b*d^2*m^2+3
7*x*(f*x)^m*b*d^2*m^2*n-114*x^2*(f*x)^m*a*d*e*m+18*x^2*(f*x)^m*b*d*e*n-96*x
*(f*x)^m*ln(c*x^n)*b*d^2*m+60*x*(f*x)^m*b*d^2*m*n-2*x^2*(f*x)^m*ln(c*x^n)*b
*d*e*m^5-20*x^2*(f*x)^m*ln(c*x^n)*b*d*e*m^4+2*x^2*(f*x)^m*b*d*e*m^4*n-76*x^
2*(f*x)^m*ln(c*x^n)*b*d*e*m^3+16*x^2*(f*x)^m*b*d*e*m^3*n-136*x^2*(f*x)^m*ln
(c*x^n)*b*d*e*m^2+44*x^2*(f*x)^m*b*d*e*m^2*n-114*x^2*(f*x)^m*ln(c*x^n)*b*d*
e*m+48*x^2*(f*x)^m*b*d*e*m*n)/(3+m)^2/(1+m)^2/(2+m)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(153) = 306$.

Time = 0.32 (sec) , antiderivative size = 633, normalized size of antiderivative = 4.14

$$\int (fx)^m(d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{\left(\left(ae^2m^5+9\,ae^2m^4+31\,ae^2m^3+51\,ae^2m^2+40\,ae^2m+12\,ae^2-\left(be^2m^4+6\,be^2m^3+13\,be^2m^2+12\,be^2n\right.\right.\right.}{}$$

```
[In] integrate((f*x)^m*(e*x+d)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] ((a*e^2*m^5 + 9*a*e^2*m^4 + 31*a*e^2*m^3 + 51*a*e^2*m^2 + 40*a*e^2*m + 12*a
*e^2 - (b*e^2*m^4 + 6*b*e^2*m^3 + 13*b*e^2*m^2 + 12*b*e^2*m + 4*b*e^2)*n)*x
^3 + 2*(a*d*e*m^5 + 10*a*d*e*m^4 + 38*a*d*e*m^3 + 68*a*d*e*m^2 + 57*a*d*e*m
 + 18*a*d*e - (b*d*e*m^4 + 8*b*d*e*m^3 + 22*b*d*e*m^2 + 24*b*d*e*m + 9*b*d*
e)*n)*x^2 + (a*d^2*m^5 + 11*a*d^2*m^4 + 47*a*d^2*m^3 + 97*a*d^2*m^2 + 96*a*
d^2*m + 36*a*d^2 - (b*d^2*m^4 + 10*b*d^2*m^3 + 37*b*d^2*m^2 + 60*b*d^2*m +
36*b*d^2)*n)*x + ((b*e^2*m^5 + 9*b*e^2*m^4 + 31*b*e^2*m^3 + 51*b*e^2*m^2 +
40*b*e^2*m + 12*b*e^2)*x^3 + 2*(b*d*e*m^5 + 10*b*d*e*m^4 + 38*b*d*e*m^3 + 6
8*b*d*e*m^2 + 57*b*d*e*m + 18*b*d*e)*x^2 + (b*d^2*m^5 + 11*b*d^2*m^4 + 47*b
*d^2*m^3 + 97*b*d^2*m^2 + 96*b*d^2*m + 36*b*d^2)*x)*log(c) + ((b*e^2*m^5 +
9*b*e^2*m^4 + 31*b*e^2*m^3 + 51*b*e^2*m^2 + 40*b*e^2*m + 12*b*e^2)*n*x^3 +
2*(b*d*e*m^5 + 10*b*d*e*m^4 + 38*b*d*e*m^3 + 68*b*d*e*m^2 + 57*b*d*e*m + 18
*b*d*e)*n*x^2 + (b*d^2*m^5 + 11*b*d^2*m^4 + 47*b*d^2*m^3 + 97*b*d^2*m^2 + 9
6*b*d^2*m + 36*b*d^2)*n*x)*log(x))*e^(m*log(f) + m*log(x))/(m^6 + 12*m^5 +
58*m^4 + 144*m^3 + 193*m^2 + 132*m + 36)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2791 vs. $2(146) = 292$.

Time = 3.51 (sec) , antiderivative size = 2791, normalized size of antiderivative = 18.24

$$\int (fx)^m(d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x)**m*(e*x+d)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise(((-a*d**2/(2*x**2) - 2*a*d*e/x + a*e**2*log(x) + b*d**2*(-n/(4*x*
*2) - log(c*x**n)/(2*x**2)) + 2*b*d*e*(-n/x - log(c*x**n)/x) - b*e**2*Piece
wise((-log(c)*log(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), True)))/f**3, Eq(m
, -3)), ((-a*d**2/x + 2*a*d*e*log(c*x**n)/n + a*e**2*x - b*d**2*n/x - b*d**
2*log(c*x**n)/x + b*d*e*log(c*x**n)**2/n - b*e**2*n*x + b*e**2*x*log(c*x**n
))/f**2, Eq(m, -2)), ((a*d**2*log(c*x**n)/n + 2*a*d*e*x + a*e**2*x**2/2 + b
*d**2*log(c*x**n)**2/(2*n) - 2*b*d*e*n*x + 2*b*d*e*x*log(c*x**n) - b*e**2*n
*x**2/4 + b*e**2*x**2*log(c*x**n)/2)/f, Eq(m, -1)), (a*d**2*m**5*x*(f*x)**m
/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) + 11*a*d**2*
m**4*x*(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 3
6) + 47*a*d**2*m**3*x*(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m
**2 + 132*m + 36) + 97*a*d**2*m**2*x*(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 1
44*m**3 + 193*m**2 + 132*m + 36) + 96*a*d**2*m*x*(f*x)**m/(m**6 + 12*m**5 +
 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) + 36*a*d**2*x*(f*x)**m/(m**6 +
 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) + 2*a*d*e*m**5*x**2*
(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) + 20
*a*d*e*m**4*x**2*(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 +
 132*m + 36) + 76*a*d*e*m**3*x**2*(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 144*
m**3 + 193*m**2 + 132*m + 36) + 136*a*d*e*m**2*x**2*(f*x)**m/(m**6 + 12*m**
5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) + 114*a*d*e*m*x**2*(f*x)**m
/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) + 36*a*d*e*x
**2*(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36)
+ a*e**2*m**5*x**3*(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2
 + 132*m + 36) + 9*a*e**2*m**4*x**3*(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 14
4*m**3 + 193*m**2 + 132*m + 36) + 31*a*e**2*m**3*x**3*(f*x)**m/(m**6 + 12*m
**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) + 51*a*e**2*m**2*x**3*(f*
x)**m/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) + 40*a*
e**2*m*x**3*(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*
m + 36) + 12*a*e**2*x**3*(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 19
3*m**2 + 132*m + 36) + b*d**2*m**5*x*(f*x)**m*log(c*x**n)/(m**6 + 12*m**5 +
 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) - b*d**2*m**4*n*x*(f*x)**m/(m*
*6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) + 11*b*d**2*m**4
*x*(f*x)**m*log(c*x**n)/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 1
32*m + 36) - 10*b*d**2*m**3*n*x*(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 144*m*
*3 + 193*m**2 + 132*m + 36) + 47*b*d**2*m**3*x*(f*x)**m*log(c*x**n)/(m**6 +
```

```
 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) - 37*b*d**2*m**2*n*x
*(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) + 9
7*b*d**2*m**2*x*(f*x)**m*log(c*x**n)/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 +
 193*m**2 + 132*m + 36) - 60*b*d**2*m*n*x*(f*x)**m/(m**6 + 12*m**5 + 58*m**
4 + 144*m**3 + 193*m**2 + 132*m + 36) + 96*b*d**2*m*x*(f*x)**m*log(c*x**n)/
(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) - 36*b*d**2*n
*x*(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) +
 36*b*d**2*x*(f*x)**m*log(c*x**n)/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 19
3*m**2 + 132*m + 36) + 2*b*d*e*m**5*x**2*(f*x)**m*log(c*x**n)/(m**6 + 12*m*
*5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) - 2*b*d*e*m**4*n*x**2*(f*x
)**m/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) + 20*b*d
*e*m**4*x**2*(f*x)**m*log(c*x**n)/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 19
3*m**2 + 132*m + 36) - 16*b*d*e*m**3*n*x**2*(f*x)**m/(m**6 + 12*m**5 + 58*m
**4 + 144*m**3 + 193*m**2 + 132*m + 36) + 76*b*d*e*m**3*x**2*(f*x)**m*log(c
*x**n)/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) - 44*b
*d*e*m**2*n*x**2*(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 +
 132*m + 36) + 136*b*d*e*m**2*x**2*(f*x)**m*log(c*x**n)/(m**6 + 12*m**5 + 5
8*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) - 48*b*d*e*m*n*x**2*(f*x)**m/(m*
*6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) + 114*b*d*e*m*x*
*2*(f*x)**m*log(c*x**n)/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 1
32*m + 36) - 18*b*d*e*n*x**2*(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 144*m**3
+ 193*m**2 + 132*m + 36) + 36*b*d*e*x**2*(f*x)**m*log(c*x**n)/(m**6 + 12*m*
*5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) + b*e**2*m**5*x**3*(f*x)**
m*log(c*x**n)/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36)
 - b*e**2*m**4*n*x**3*(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m
**2 + 132*m + 36) + 9*b*e**2*m**4*x**3*(f*x)**m*log(c*x**n)/(m**6 + 12*m**5
 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) - 6*b*e**2*m**3*n*x**3*(f*x)
**m/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) + 31*b*e*
*2*m**3*x**3*(f*x)**m*log(c*x**n)/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 19
3*m**2 + 132*m + 36) - 13*b*e**2*m**2*n*x**3*(f*x)**m/(m**6 + 12*m**5 + 58*
m**4 + 144*m**3 + 193*m**2 + 132*m + 36) + 51*b*e**2*m**2*x**3*(f*x)**m*log
(c*x**n)/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) - 12
*b*e**2*m*n*x**3*(f*x)**m/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 +
 132*m + 36) + 40*b*e**2*m*x**3*(f*x)**m*log(c*x**n)/(m**6 + 12*m**5 + 58*m
**4 + 144*m**3 + 193*m**2 + 132*m + 36) - 4*b*e**2*n*x**3*(f*x)**m/(m**6 +
12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m + 36) + 12*b*e**2*x**3*(f*x
)**m*log(c*x**n)/(m**6 + 12*m**5 + 58*m**4 + 144*m**3 + 193*m**2 + 132*m +
36), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.27

$$\int (fx)^m(d+ex)^2\,(a+b\log{(cx^n)})\;dx = \frac{be^2f^mx^3x^m\log{(cx^n)}}{m+3} + \frac{ae^2f^mx^3x^m}{m+3} - \frac{be^2f^mnx^3x^m}{(m+3)^2} + \frac{2\,bdef^mx^2x^m\log{(cx^n)}}{m+2} + \frac{2\,adef^mx^2x^m}{m+2} - \frac{2\,bdef^mnx^2x^m}{(m+2)^2} - \frac{bd^2f^mnxx^m}{(m+1)^2} + \frac{(fx)^{m+1}\,bd^2\log{(cx^n)}}{f(m+1)} + \frac{(fx)^{m+1}\,ad^2}{f(m+1)}$$

```
[In] integrate((f*x)^m*(e*x+d)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] b*e^2*f^m*x^3*x^m*log(c*x^n)/(m + 3) + a*e^2*f^m*x^3*x^m/(m + 3) - b*e^2*f^
m*n*x^3*x^m/(m + 3)^2 + 2*b*d*e*f^m*x^2*x^m*log(c*x^n)/(m + 2) + 2*a*d*e*f^
m*x^2*x^m/(m + 2) - 2*b*d*e*f^m*n*x^2*x^m/(m + 2)^2 - b*d^2*f^m*n*x*x^m/(m
+ 1)^2 + (f*x)^(m + 1)*b*d^2*log(c*x^n)/(f*(m + 1)) + (f*x)^(m + 1)*a*d^2/(
f*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(153) = 306.

Time = 0.34 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.44

$$\int (fx)^m(d+ex)^2\,(a+b\log{(cx^n)})\;dx = \frac{be^2f^2f^mx^3x^m\log{(c)}}{f^2m+3\,f^2} + \frac{be^2f^mmnx^3x^m\log{(x)}}{m^2+6\,m+9} + \frac{ae^2f^2f^mx^3x^m}{f^2m+3\,f^2} + \frac{2\,bdef^mmnx^2x^m\log{(x)}}{m^2+4\,m+4} + \frac{3\,be^2f^mnx^3x^m\log{(x)}}{m^2+6\,m+9} - \frac{be^2f^mnx^3x^m}{m^2+6\,m+9} + \frac{bd^2f^mmnxx^m\log{(x)}}{m^2+2\,m+1} + \frac{4\,bdef^mnx^2x^m\log{(x)}}{m^2+4\,m+4} - \frac{2\,bdef^mnx^2x^m}{m^2+4\,m+4} + \frac{2\,bdef^mx^2x^m\log{(c)}}{m+2} + \frac{bd^2f^mnxx^m\log{(x)}}{m^2+2\,m+1} - \frac{bd^2f^mnxx^m}{m^2+2\,m+1} + \frac{2\,adef^mx^2x^m}{m+2} + \frac{(fx)^m\,bd^2x\log{(c)}}{m+1} + \frac{(fx)^m\,ad^2x}{m+1}$$

```
[In] integrate((f*x)^m*(e*x+d)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*e^2*f^2*f^m*x^3*x^m*log(c)/(f^2*m + 3*f^2) + b*e^2*f^m*m*n*x^3*x^m*log(x)
/(m^2 + 6*m + 9) + a*e^2*f^2*f^m*x^3*x^m/(f^2*m + 3*f^2) + 2*b*d*e*f^m*m*n*
x^2*x^m*log(x)/(m^2 + 4*m + 4) + 3*b*e^2*f^m*n*x^3*x^m*log(x)/(m^2 + 6*m +
9) - b*e^2*f^m*n*x^3*x^m/(m^2 + 6*m + 9) + b*d^2*f^m*m*n*x*x^m*log(x)/(m^2
+ 2*m + 1) + 4*b*d*e*f^m*n*x^2*x^m*log(x)/(m^2 + 4*m + 4) - 2*b*d*e*f^m*n*x
^2*x^m/(m^2 + 4*m + 4) + 2*b*d*e*f^m*x^2*x^m*log(c)/(m + 2) + b*d^2*f^m*n*x
*x^m*log(x)/(m^2 + 2*m + 1) - b*d^2*f^m*n*x*x^m/(m^2 + 2*m + 1) + 2*a*d*e*f
^m*x^2*x^m/(m + 2) + (f*x)^m*b*d^2*x*log(c)/(m + 1) + (f*x)^m*a*d^2*x/(m +
1)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m(d+ex)^2\left(a+b\log\left(cx^n\right)\right)\,dx=\int (f\,x)^m\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,{\left(d+e\,x\right)}^2\,dx$$

```
[In] int((f*x)^m*(a + b*log(c*x^n))*(d + e*x)^2,x)
```

```
[Out] int((f*x)^m*(a + b*log(c*x^n))*(d + e*x)^2, x)
```

3.164 $\int (fx)^m(d+ex) (a+b\log (cx^n)) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int (fx)^m(d+ex) (a+b\log (cx^n)) \, dx=-\frac{bdn(fx)^{1+m}}{f(1+m)^2}-\frac{ben(fx)^{2+m}}{f^2(2+m)^2}+\frac{d(fx)^{1+m} (a+b\log (cx^n))}{f(1+m)}+\frac{e(fx)^{2+m} (a+b\log (cx^n))}{f^2(2+m)}$$

```
[Out] -b*d*n*(f*x)^(1+m)/f/(1+m)^2-b*e*n*(f*x)^(2+m)/f^2/(2+m)^2+d*(f*x)^(1+m)*(a
+b*ln(c*x^n))/f/(1+m)+e*(f*x)^(2+m)*(a+b*ln(c*x^n))/f^2/(2+m)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {45, 2392}

$$\int (fx)^m(d+ex) (a+b\log (cx^n)) \, dx=\frac{d(fx)^{m+1} (a+b\log (cx^n))}{f(m+1)}+\frac{e(fx)^{m+2} (a+b\log (cx^n))}{f^2(m+2)}-\frac{bdn(fx)^{m+1}}{f(m+1)^2}-\frac{ben(fx)^{m+2}}{f^2(m+2)^2}$$

```
[In] Int[(f*x)^m*(d + e*x)*(a + b*Log[c*x^n]),x]
```

```
[Out] -((b*d*n*(f*x)^(1 + m))/(f*(1 + m)^2)) - (b*e*n*(f*x)^(2 + m))/(f^2*(2 + m)
^2) + (d*(f*x)^(1 + m)*(a + b*Log[c*x^n]))/(f*(1 + m)) + (e*(f*x)^(2 + m)*(
a + b*Log[c*x^n]))/(f^2*(2 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(fx)^{1+m}\,(a+b\log(cx^n))}{f(1+m)}+\frac{e(fx)^{2+m}\,(a+b\log(cx^n))}{f^2(2+m)} \\
&\quad-(bn)\int(fx)^m\left(\frac{d}{1+m}+\frac{ex}{2+m}\right)\,dx \\
&= \frac{d(fx)^{1+m}\,(a+b\log(cx^n))}{f(1+m)}+\frac{e(fx)^{2+m}\,(a+b\log(cx^n))}{f^2(2+m)}-(bn)\int\left(\frac{d(fx)^m}{1+m}+\frac{e(fx)^{1+m}}{f(2+m)}\right)\,dx \\
&= -\frac{bdn(fx)^{1+m}}{f(1+m)^2}-\frac{ben(fx)^{2+m}}{f^2(2+m)^2}+\frac{d(fx)^{1+m}\,(a+b\log(cx^n))}{f(1+m)}+\frac{e(fx)^{2+m}\,(a+b\log(cx^n))}{f^2(2+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int(fx)^m(d+ex)\,(a+b\log(cx^n))\,dx = x(fx)^m\left(-\frac{bdn}{(1+m)^2}-\frac{benx}{(2+m)^2}+\frac{d(a+b\log(cx^n))}{1+m}+\frac{ex(a+b\log(cx^n))}{2+m}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x)*(a + b*Log[c*x^n]),x]
[Out] x*(f*x)^m*(-((b*d*n)/(1 + m)^2) - (b*e*n*x)/(2 + m)^2 + (d*(a + b*Log[c*x^n
]))/(1 + m) + (e*x*(a + b*Log[c*x^n]))/(2 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(95) = 190.

Time = 0.22 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.72

method	result
parallelrisch	$-\frac{-2x^2(fx)^m ae-4x(fx)^m ad-x^2(fx)^m ae\,m^3-4x^2(fx)^m ae\,m^2-x(fx)^m ad\,m^3-5x^2(fx)^m aem+x^2(fx)^m ben-5x(fx)^m ad\,m^2}{}$
risch	Expression too large to display

```
[In] int((f*x)^m*(e*x+d)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -(-2*x^2*(f*x)^m*a*e-4*x*(f*x)^m*a*d-x^2*(f*x)^m*a*e*m^3-4*x^2*(f*x)^m*a*e*
m^2-x*(f*x)^m*a*d*m^3-5*x^2*(f*x)^m*a*e*m+x^2*(f*x)^m*b*e*n-5*x*(f*x)^m*a*d
*m^2-8*x*(f*x)^m*a*d*m+4*x*(f*x)^m*b*d*n-4*x*(f*x)^m*ln(c*x^n)*b*d-2*x^2*(f
*x)^m*ln(c*x^n)*b*e-x^2*(f*x)^m*ln(c*x^n)*b*e*m^3-4*x^2*(f*x)^m*ln(c*x^n)*b
*e*m^2+x^2*(f*x)^m*b*e*m^2*n-x*(f*x)^m*ln(c*x^n)*b*d*m^3-5*x^2*(f*x)^m*ln(c
*x^n)*b*e*m+2*x^2*(f*x)^m*b*e*m*n-5*x*(f*x)^m*ln(c*x^n)*b*d*m^2+x*(f*x)^m*b
*d*m^2*n-8*x*(f*x)^m*ln(c*x^n)*b*d*m+4*x*(f*x)^m*b*d*m*n)/(m^2+4*m+4)/(m^2+
2*m+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(95) = 190.

Time = 0.33 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.47

$$\int (fx)^m(d+ex)\,(a+b\log{(cx^n)})\;dx$$
$$=\frac{((aem^3+4\,aem^2+5\,aem+2\,ae-(bem^2+2\,bem+be)n)x^2+(adm^3+5\,adm^2+8\,adm+4\,ad-(bdm^2}{}$$

```
[In] integrate((f*x)^m*(e*x+d)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] ((a*e*m^3 + 4*a*e*m^2 + 5*a*e*m + 2*a*e - (b*e*m^2 + 2*b*e*m + b*e)*n)*x^2
+ (a*d*m^3 + 5*a*d*m^2 + 8*a*d*m + 4*a*d - (b*d*m^2 + 4*b*d*m + 4*b*d)*n)*x
 + ((b*e*m^3 + 4*b*e*m^2 + 5*b*e*m + 2*b*e)*x^2 + (b*d*m^3 + 5*b*d*m^2 + 8*
b*d*m + 4*b*d)*x)*log(c) + ((b*e*m^3 + 4*b*e*m^2 + 5*b*e*m + 2*b*e)*n*x^2 +
 (b*d*m^3 + 5*b*d*m^2 + 8*b*d*m + 4*b*d)*n*x)*log(x))*e^(m*log(f) + m*log(x
))/(m^4 + 6*m^3 + 13*m^2 + 12*m + 4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. 2(87) = 174.

Time = 2.39 (sec) , antiderivative size = 899, normalized size of antiderivative = 9.46

$$\int (fx)^m(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\begin{cases}\frac{-\frac{ad}{x}+ae\log(x)+bd\left(-\frac{n}{x}-\frac{\log(cx^n)}{x}\right)-be\left(\begin{cases}-\log(c)\log(x) & \text{for } n=0\\ -\frac{\log(cx^n)^2}{2n} & \text{otherwise}\end{cases}\right)}{f^2} \\ \frac{\frac{ad\log(cx^n)}{n}+aex+\frac{bd\log(cx^n)^2}{2n}-benx+bex\log(cx^n)}{f} \\ \frac{adm^3x(fx)^m}{m^4+6m^3+13m^2+12m+4}+\frac{5adm^2x(fx)^m}{m^4+6m^3+13m^2+12m+4}+\frac{8admx(fx)^m}{m^4+6m^3+13m^2+12m+4}+\frac{4adx(fx)^m}{m^4+6m^3+13m^2+12m+4}+\frac{aem^3x^2(fx)}{m^4+6m^3+13m^2+}\end{cases}$$

```
[In] integrate((f*x)**m*(e*x+d)*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise(((-a*d/x + a*e*log(x) + b*d*(-n/x - log(c*x**n)/x) - b*e*Piecewis
e((-log(c)*log(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), True)))/f**2, Eq(m, -
2)), ((a*d*log(c*x**n)/n + a*e*x + b*d*log(c*x**n)**2/(2*n) - b*e*n*x + b*e
*x*log(c*x**n))/f, Eq(m, -1)), (a*d*m**3*x*(f*x)**m/(m**4 + 6*m**3 + 13*m**
2 + 12*m + 4) + 5*a*d*m**2*x*(f*x)**m/(m**4 + 6*m**3 + 13*m**2 + 12*m + 4)
+ 8*a*d*m*x*(f*x)**m/(m**4 + 6*m**3 + 13*m**2 + 12*m + 4) + 4*a*d*x*(f*x)**
m/(m**4 + 6*m**3 + 13*m**2 + 12*m + 4) + a*e*m**3*x**2*(f*x)**m/(m**4 + 6*m
**3 + 13*m**2 + 12*m + 4) + 4*a*e*m**2*x**2*(f*x)**m/(m**4 + 6*m**3 + 13*m*
*2 + 12*m + 4) + 5*a*e*m*x**2*(f*x)**m/(m**4 + 6*m**3 + 13*m**2 + 12*m + 4)
 + 2*a*e*x**2*(f*x)**m/(m**4 + 6*m**3 + 13*m**2 + 12*m + 4) + b*d*m**3*x*(f
*x)**m*log(c*x**n)/(m**4 + 6*m**3 + 13*m**2 + 12*m + 4) - b*d*m**2*n*x*(f*x
)**m/(m**4 + 6*m**3 + 13*m**2 + 12*m + 4) + 5*b*d*m**2*x*(f*x)**m*log(c*x**
n)/(m**4 + 6*m**3 + 13*m**2 + 12*m + 4) - 4*b*d*m*n*x*(f*x)**m/(m**4 + 6*m*
*3 + 13*m**2 + 12*m + 4) + 8*b*d*m*x*(f*x)**m*log(c*x**n)/(m**4 + 6*m**3 +
13*m**2 + 12*m + 4) - 4*b*d*n*x*(f*x)**m/(m**4 + 6*m**3 + 13*m**2 + 12*m +
4) + 4*b*d*x*(f*x)**m*log(c*x**n)/(m**4 + 6*m**3 + 13*m**2 + 12*m + 4) + b*
e*m**3*x**2*(f*x)**m*log(c*x**n)/(m**4 + 6*m**3 + 13*m**2 + 12*m + 4) - b*e
*m**2*n*x**2*(f*x)**m/(m**4 + 6*m**3 + 13*m**2 + 12*m + 4) + 4*b*e*m**2*x**
2*(f*x)**m*log(c*x**n)/(m**4 + 6*m**3 + 13*m**2 + 12*m + 4) - 2*b*e*m*n*x**
2*(f*x)**m/(m**4 + 6*m**3 + 13*m**2 + 12*m + 4) + 5*b*e*m*x**2*(f*x)**m*log
(c*x**n)/(m**4 + 6*m**3 + 13*m**2 + 12*m + 4) - b*e*n*x**2*(f*x)**m/(m**4 +
 6*m**3 + 13*m**2 + 12*m + 4) + 2*b*e*x**2*(f*x)**m*log(c*x**n)/(m**4 + 6*m
**3 + 13*m**2 + 12*m + 4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25

$$\int (fx)^m(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{bef^m x^2 x^m \log\left(cx^n\right)}{m+2} + \frac{aef^m x^2 x^m}{m+2} - \frac{bef^m n x^2 x^m}{(m+2)^2} - \frac{bdf^m n x x^m}{(m+1)^2} + \frac{(fx)^{m+1}\,bd\log\left(cx^n\right)}{f(m+1)} + \frac{(fx)^{m+1}\,ad}{f(m+1)}$$

```
[In] integrate((f*x)^m*(e*x+d)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] b*e*f^m*x^2*x^m*log(c*x^n)/(m + 2) + a*e*f^m*x^2*x^m/(m + 2) - b*e*f^m*n*x^
2*x^m/(m + 2)^2 - b*d*f^m*n*x*x^m/(m + 1)^2 + (f*x)^(m + 1)*b*d*log(c*x^n)/
(f*(m + 1)) + (f*x)^(m + 1)*a*d/(f*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(95) = 190.

Time = 0.33 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.23

$$\int (fx)^m(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{bef^m mnx^2x^m\log(x)}{m^2+4\,m+4} + \frac{bdf^m mnxx^m\log(x)}{m^2+2\,m+1} + \frac{2\,bef^m nx^2x^m\log(x)}{m^2+4\,m+4} - \frac{bef^m nx^2x^m}{m^2+4\,m+4} + \frac{bef^m x^2x^m\log(c)}{m+2} + \frac{bdf^m nxx^m\log(x)}{m^2+2\,m+1} - \frac{bdf^m nxx^m}{m^2+2\,m+1} + \frac{aef^m x^2x^m}{m+2} + \frac{(fx)^m\,bdx\log(c)}{m+1} + \frac{(fx)^m\,adx}{m+1}$$

```
[In] integrate((f*x)^m*(e*x+d)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*e*f^m*m*n*x^2*x^m*log(x)/(m^2 + 4*m + 4) + b*d*f^m*m*n*x*x^m*log(x)/(m^2
+ 2*m + 1) + 2*b*e*f^m*n*x^2*x^m*log(x)/(m^2 + 4*m + 4) - b*e*f^m*n*x^2*x^m
/(m^2 + 4*m + 4) + b*e*f^m*x^2*x^m*log(c)/(m + 2) + b*d*f^m*n*x*x^m*log(x)/
(m^2 + 2*m + 1) - b*d*f^m*n*x*x^m/(m^2 + 2*m + 1) + a*e*f^m*x^2*x^m/(m + 2)
 + (f*x)^m*b*d*x*log(c)/(m + 1) + (f*x)^m*a*d*x/(m + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m(d+ex)\left(a+b\log\left(cx^n\right)\right)\,dx=\int (f\,x)^m\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,\left(d+e\,x\right)\,dx$$

```
[In] int((f*x)^m*(a + b*log(c*x^n))*(d + e*x),x)
[Out] int((f*x)^m*(a + b*log(c*x^n))*(d + e*x), x)
```

3.165 $\int (fx)^m (a+b\log(cx^n)) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int (fx)^m (a+b\log(cx^n)) \, dx = -\frac{bn(fx)^{1+m}}{f(1+m)^2} + \frac{(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)}$$

```
[Out] -b*n*(f*x)^(1+m)/f/(1+m)^2+(f*x)^(1+m)*(a+b*ln(c*x^n))/f/(1+m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2341}

$$\int (fx)^m (a+b\log(cx^n)) \, dx = \frac{(fx)^{m+1}(a+b\log(cx^n))}{f(m+1)} - \frac{bn(fx)^{m+1}}{f(m+1)^2}$$

```
[In] Int[(f*x)^m*(a + b*Log[c*x^n]),x]
```

```
[Out] -((b*n*(f*x)^(1 + m))/(f*(1 + m)^2)) + ((f*x)^(1 + m)*(a + b*Log[c*x^n]))/(
f*(1 + m))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{bn(fx)^{1+m}}{f(1+m)^2} + \frac{(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int (fx)^m (a+b\log(cx^n))\,dx=\frac{x(fx)^m(a+am-bn+b(1+m)\log(cx^n))}{(1+m)^2}$$

```
[In] Integrate[(f*x)^m*(a + b*Log[c*x^n]),x]
[Out] (x*(f*x)^m*(a + a*m - b*n + b*(1 + m)*Log[c*x^n]))/(1 + m)^2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

method	result
parallelrisch	$-\frac{-x(fx)^m\ln(c\,x^n)bm-x(fx)^m\ln(c\,x^n)b-x(fx)^m am+x(fx)^m bn-x(fx)^m a}{(1+m)^2}$
risch	$\frac{bx\,x^m f^m \mathrm{e}^{\frac{i\,\mathrm{csgn}(ifx)\pi m(\mathrm{csgn}(ifx)-\mathrm{csgn}(ix))(-\mathrm{csgn}(ifx)+\mathrm{csgn}(if))}{2}}\ln(x^n)}{1+m}-\frac{\left(i\pi b\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)m-i\pi b\ \mathrm{csgn}(i\right.}{}$

```
[In] int((f*x)^m*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] -(-x*(f*x)^m*ln(c*x^n)*b*m-x*(f*x)^m*ln(c*x^n)*b-x*(f*x)^m*a*m+x*(f*x)^m*b*
n-x*(f*x)^m*a)/(1+m)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int (fx)^m (a+b\log(cx^n))\,dx =\frac{((bm+b)nx\log(x)+(bm+b)x\log(c)+(am-bn+a)x)e^{(m\log(f)+m\log(x))}}{m^2+2\,m+1}$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] ((b*m + b)*n*x*log(x) + (b*m + b)*x*log(c) + (a*m - b*n + a)*x)*e^(m*log(f)
 + m*log(x))/(m^2 + 2*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(37) = 74$.

Time = 2.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.07

$$\int (fx)^m (a + b\log(cx^n))\,dx = \begin{cases} \frac{amx(fx)^m}{m^2+2m+1} + \frac{ax(fx)^m}{m^2+2m+1} + \frac{bmx(fx)^m\log(cx^n)}{m^2+2m+1} - \frac{bnx(fx)^m}{m^2+2m+1} + \frac{bx(fx)^m\log(cx^n)}{m^2+2m+1} & \text{for } m \neq -1 \\ \frac{\begin{cases} a\log(x) & \text{for } b = 0 \\ -(-a - b\log(c))\log(x) & \text{for } n = 0 \\ \frac{(-a-b\log(cx^n))^2}{2bn} & \text{otherwise} \end{cases}}{f} & \text{otherwise} \end{cases}$$

```
[In] integrate((f*x)**m*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((a*m*x*(f*x)**m/(m**2 + 2*m + 1) + a*x*(f*x)**m/(m**2 + 2*m + 1)
+ b*m*x*(f*x)**m*log(c*x**n)/(m**2 + 2*m + 1) - b*n*x*(f*x)**m/(m**2 + 2*m
+ 1) + b*x*(f*x)**m*log(c*x**n)/(m**2 + 2*m + 1), Ne(m, -1)), (Piecewise((a
*log(x), Eq(b, 0)), (-(-a - b*log(c))*log(x), Eq(n, 0)), ((-a - b*log(c*x**
n))**2/(2*b*n), True))/f, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int (fx)^m (a + b\log(cx^n))\,dx = -\frac{bf^m nxx^m}{(m+1)^2} + \frac{(fx)^{m+1} b\log(cx^n)}{f(m+1)} + \frac{(fx)^{m+1} a}{f(m+1)}$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -b*f^m*n*x*x^m/(m + 1)^2 + (f*x)^(m + 1)*b*log(c*x^n)/(f*(m + 1)) + (f*x)^(
m + 1)*a/(f*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(46) = 92$.

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int (fx)^m (a + b\log(cx^n)) \, dx = \frac{bf^m mnx x^m \log(x)}{m^2 + 2m + 1} + \frac{bf^m nx x^m \log(x)}{m^2 + 2m + 1} - \frac{bf^m nx x^m}{m^2 + 2m + 1} + \frac{(fx)^m bx \log(c)}{m + 1} + \frac{(fx)^m ax}{m + 1}$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*f^m*m*n*x*x^m*log(x)/(m^2 + 2*m + 1) + b*f^m*n*x*x^m*log(x)/(m^2 + 2*m +
1) - b*f^m*n*x*x^m/(m^2 + 2*m + 1) + (f*x)^m*b*x*log(c)/(m + 1) + (f*x)^m*a
*x/(m + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (a + b\log(cx^n)) \, dx = \int (f\,x)^m \left(a + b\,\ln\left(c\,x^n\right)\right) \, dx$$

```
[In] int((f*x)^m*(a + b*log(c*x^n)),x)
```

```
[Out] int((f*x)^m*(a + b*log(c*x^n)), x)
```

3.166 $\int \frac{(fx)^m(a+b\log(cx^n))}{d+ex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m\,(a+b\log\left(cx^n\right))}{d+ex}\,dx = \operatorname{Int}\left(\frac{(fx)^m\,(a+b\log\left(cx^n\right))}{d+ex},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*ln(c*x^n))/(e*x+d),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\,(a+b\log\left(cx^n\right))}{d+ex}\,dx = \int \frac{(fx)^m\,(a+b\log\left(cx^n\right))}{d+ex}\,dx$$

```
[In] Int[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x),x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{(fx)^m\,(a+b\log\left(cx^n\right))}{d+ex}\,dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 72 vs. 2(26) = 52.

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.13

$$\int \frac{(fx)^m (a+b\log(cx^n))}{d+ex}\,dx$$
$$= \frac{x(fx)^m \left(-bn\,{}_3F_2\left(1,1+m,1+m;2+m,2+m;-\frac{ex}{d}\right)+(1+m)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\right.\right.}{d(1+m)^2}$$

```
[In] Integrate[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x),x]
```

```
[Out] (x*(f*x)^m*(-(b*n*HypergeometricPFQ[{1, 1 + m, 1 + m}, {2 + m, 2 + m}, -((e
*x)/d)]) + (1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((e*x)/d)]*(a + b*Lo
g[c*x^n])))/(d*(1 + m)^2)
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a+b\ln(cx^n))}{ex+d}dx$$

```
[In] int((f*x)^m*(a+b*ln(c*x^n))/(e*x+d),x)
```

```
[Out] int((f*x)^m*(a+b*ln(c*x^n))/(e*x+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{(fx)^m (a+b\log(cx^n))}{d+ex}\,dx = \int \frac{(b\log(cx^n)+a)(fx)^m}{ex+d}\,dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(((f*x)^m*b*log(c*x^n) + (f*x)^m*a)/(e*x + d), x)
```

Sympy [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(fx)^m (a + b\log(cx^n))}{d + ex}\,dx = \int \frac{(fx)^m (a + b\log(cx^n))}{d + ex}\,dx$$

```
[In] integrate((f*x)**m*(a+b*ln(c*x**n))/(e*x+d),x)
[Out] Integral((f*x)**m*(a + b*log(c*x**n))/(d + e*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\log(cx^n))}{d + ex}\,dx = \int \frac{(b\log(cx^n) + a)(fx)^m}{ex + d}\,dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(e*x+d),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*(f*x)^m/(e*x + d), x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\log(cx^n))}{d + ex}\,dx = \int \frac{(b\log(cx^n) + a)(fx)^m}{ex + d}\,dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(e*x+d),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*(f*x)^m/(e*x + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a+b\log(cx^n))}{d+ex}\,dx = \int \frac{(f\,x)^m\,(a+b\,\ln(c\,x^n))}{d+e\,x}\,dx$$

```
[In] int(((f*x)^m*(a + b*log(c*x^n)))/(d + e*x),x)
[Out] int(((f*x)^m*(a + b*log(c*x^n)))/(d + e*x), x)
```

3.167 $\int \frac{(fx)^m(a+b\log(cx^n))}{(d+ex)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m (a+b\log(cx^n))}{(d+ex)^2} dx = \text{Int}\left(\frac{(fx)^m (a+b\log(cx^n))}{(d+ex)^2}, x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*ln(c*x^n))/(e*x+d)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m (a+b\log(cx^n))}{(d+ex)^2} dx = \int \frac{(fx)^m (a+b\log(cx^n))}{(d+ex)^2} dx$$

```
[In] Int[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x)^2,x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(fx)^m (a+b\log(cx^n))}{(d+ex)^2} dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 72 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.13

$$\int \frac{(fx)^m (a+b\log(cx^n))}{(d+ex)^2}\,dx$$
$$= \frac{x(fx)^m \left(-bn\,{}_3F_2\left(2,1+m,1+m;2+m,2+m;-\frac{ex}{d}\right)+(1+m)\,\text{Hypergeometric2F1}\left(2,1+m,2+m,-\right.\right.}{d^2(1+m)^2}$$

```
[In] Integrate[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x)^2,x]
```

```
[Out] (x*(f*x)^m*(-(b*n*HypergeometricPFQ[{2, 1 + m, 1 + m}, {2 + m, 2 + m}, -((e
*x)/d)]) + (1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((e*x)/d)]*(a + b*Lo
g[c*x^n])))/(d^2*(1 + m)^2)
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a+b\ln(c\,x^n))}{(ex+d)^2}dx$$

```
[In] int((f*x)^m*(a+b*ln(c*x^n))/(e*x+d)^2,x)
```

```
[Out] int((f*x)^m*(a+b*ln(c*x^n))/(e*x+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{(fx)^m (a+b\log(cx^n))}{(d+ex)^2}\,dx = \int \frac{(b\log(cx^n)+a)(fx)^m}{(ex+d)^2}\,dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(((f*x)^m*b*log(c*x^n) + (f*x)^m*a)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 4.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m (a + b\log(cx^n))}{(d + ex)^2} \, dx = \int \frac{(fx)^m (a + b\log(cx^n))}{(d + ex)^2} \, dx$$

```
[In] integrate((f*x)**m*(a+b*ln(c*x**n))/(e*x+d)**2,x)
[Out] Integral((f*x)**m*(a + b*log(c*x**n))/(d + e*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\log(cx^n))}{(d + ex)^2} \, dx = \int \frac{(b\log(cx^n) + a)(fx)^m}{(ex + d)^2} \, dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*(f*x)^m/(e*x + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\log(cx^n))}{(d + ex)^2} \, dx = \int \frac{(b\log(cx^n) + a)(fx)^m}{(ex + d)^2} \, dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*(f*x)^m/(e*x + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\log(cx^n))}{(d + ex)^2} \, dx = \int \frac{(f\,x)^m \,(a + b\,\ln(c\,x^n))}{(d + e\,x)^2} \, dx$$

```
[In] int(((f*x)^m*(a + b*log(c*x^n)))/(d + e*x)^2,x)
[Out] int(((f*x)^m*(a + b*log(c*x^n)))/(d + e*x)^2, x)
```

3.168 $\int x(a+bx)^m \log(cx^n)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int x(a+bx)^m \log(cx^n)\,dx = \mathrm{Int}(x(a+bx)^m \log(cx^n), x)$$

```
[Out] Unintegrable(x*(b*x+a)^m*ln(c*x^n),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x(a+bx)^m \log(cx^n)\,dx = \int x(a+bx)^m \log(cx^n)\,dx$$

```
[In] Int[x*(a + b*x)^m*Log[c*x^n],x]
```

```
[Out] Defer[Int][x*(a + b*x)^m*Log[c*x^n], x]
```

Rubi steps

$$\text{integral} = \int x(a+bx)^m \log(cx^n)\,dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 173 vs. 2(18) = 36.

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 11.53

$$\int x(a+bx)^m\log(cx^n)\,dx$$
$$=\frac{(a+bx)^m\left(1+\frac{bx}{a}\right)^{-m}\left(-n\left(2abx\left(1+\frac{bx}{a}\right)^m+b^2x^2\left(1+\frac{bx}{a}\right)^m+a^2\left(-1+\left(1+\frac{bx}{a}\right)^m\right)\right)+ab(2+m)nx\,{}_3F_2(\right.}{b^2(1+m)}$$

```
[In] Integrate[x*(a + b*x)^m*Log[c*x^n],x]
```

```
[Out] ((a + b*x)^m*(-(n*(2*a*b*x*(1 + (b*x)/a)^m + b^2*x^2*(1 + (b*x)/a)^m + a^2*
(-1 + (1 + (b*x)/a)^m))) + a*b*(2 + m)*n*x*HypergeometricPFQ[{1, 1, -1 - m}
, {2, 2}, -((b*x)/a)] + (a*b*m*x*(1 + (b*x)/a)^m + b^2*(1 + m)*x^2*(1 + (b*
x)/a)^m - a^2*(-1 + (1 + (b*x)/a)^m))*Log[c*x^n]))/(b^2*(1 + m)*(2 + m)*(1
+ (b*x)/a)^m)
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x(bx+a)^m\ln(c\,x^n)\,dx$$

```
[In] int(x*(b*x+a)^m*ln(c*x^n),x)
```

```
[Out] int(x*(b*x+a)^m*ln(c*x^n),x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int x(a+bx)^m\log(cx^n)\,dx=\int(bx+a)^m x\log(cx^n)\,dx$$

```
[In] integrate(x*(b*x+a)^m*log(c*x^n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m*x*log(c*x^n), x)
```

Sympy [N/A]

Not integrable

Time = 7.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x(a+bx)^m \log(cx^n) \, dx = \int x(a+bx)^m \log(cx^n) \, dx$$

```
[In] integrate(x*(b*x+a)**m*ln(c*x**n),x)
[Out] Integral(x*(a + b*x)**m*log(c*x**n), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 7.47

$$\int x(a+bx)^m \log(cx^n) \, dx = \int (bx+a)^m x \log(cx^n) \, dx$$

```
[In] integrate(x*(b*x+a)^m*log(c*x^n),x, algorithm="maxima")
[Out] (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*log(x^n)/((m^2 + 3*m + 2)*b^2
) + integrate(-(a*b*m*n*x + (m*n - (m^2 + 3*m + 2)*log(c) + n)*b^2*x^2 - a^
2*n)*(b*x + a)^m/x, x)/((m^2 + 3*m + 2)*b^2)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int x(a+bx)^m \log(cx^n) \, dx = \int (bx+a)^m x \log(cx^n) \, dx$$

```
[In] integrate(x*(b*x+a)^m*log(c*x^n),x, algorithm="giac")
[Out] integrate((b*x + a)^m*x*log(c*x^n), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int x(a+bx)^m \log (cx^n) \, dx = \int x \, \ln (c\,x^n) \, (a+b\,x)^m \, dx$$

```
[In] int(x*log(c*x^n)*(a + b*x)^m,x)
[Out] int(x*log(c*x^n)*(a + b*x)^m, x)
```

3.169 $\int (a+bx)^m \log(cx^n)\, dx$

Optimal result

Integrand size = 14, antiderivative size = 68

$$\int (a+bx)^m \log(cx^n)\, dx = \frac{n(a+bx)^{2+m} \text{Hypergeometric2F1}\left(1,2+m,3+m,1+\frac{bx}{a}\right)}{ab(2+3m+m^2)} + \frac{(a+bx)^{1+m}\log(cx^n)}{b(1+m)}$$

```
[Out] n*(b*x+a)^(2+m)*hypergeom([1, 2+m],[3+m],1+b*x/a)/a/b/(m^2+3*m+2)+(b*x+a)^(
1+m)*ln(c*x^n)/b/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2356, 67}

$$\int (a+bx)^m \log(cx^n)\, dx = \frac{(a+bx)^{m+1}\log(cx^n)}{b(m+1)} + \frac{n(a+bx)^{m+2} \text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{bx}{a}+1\right)}{ab(m^2+3m+2)}$$

```
[In] Int[(a + b*x)^m*Log[c*x^n],x]
```

```
[Out] (n*(a + b*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, 1 + (b*x)/a])/(a*b*
(2 + 3*m + m^2)) + ((a + b*x)^(1 + m)*Log[c*x^n])/(b*(1 + m))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
```

```
d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(a+bx)^{1+m}\log(cx^n)}{b(1+m)} - \frac{n\int\frac{(a+bx)^{1+m}}{x}\,dx}{b(1+m)}\\ &= \frac{n(a+bx)^{2+m}\,{}_2F_1\left(1,2+m;3+m;1+\frac{bx}{a}\right)}{ab\,(2+3m+m^2)} + \frac{(a+bx)^{1+m}\log(cx^n)}{b(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int (a+bx)^m\log(cx^n)\ dx = \frac{(a+bx)^{1+m}\left(n(a+bx)\,\text{Hypergeometric2F1}\left(1,2+m,3+m,1+\frac{bx}{a}\right)+a(2+m)\log(cx^n)\right)}{ab(1+m)(2+m)}$$

```
[In] Integrate[(a + b*x)^m*Log[c*x^n],x]
[Out] ((a + b*x)^(1 + m)*(n*(a + b*x)*Hypergeometric2F1[1, 2 + m, 3 + m, 1 + (b*x
)/a] + a*(2 + m)*Log[c*x^n]))/(a*b*(1 + m)*(2 + m))
```

Maple [F]

$$\int (bx+a)^m \ln(c\,x^n)\,dx$$

```
[In] int((b*x+a)^m*ln(c*x^n),x)
[Out] int((b*x+a)^m*ln(c*x^n),x)
```

Fricas [F]

$$\int (a+bx)^m \log (cx^n) \, dx = \int (bx+a)^m \log (cx^n) \, dx$$

```
[In] integrate((b*x+a)^m*log(c*x^n),x, algorithm="fricas")
[Out] integral((b*x + a)^m*log(c*x^n), x)
```

Sympy [A] (verification not implemented)

Time = 6.24 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.50

$$\int (a+bx)^m \log (cx^n) \, dx =$$

$$-n \left(\begin{cases} a^m x & \text{fo} \\ -\frac{b^{m+2}m\left(\frac{a}{b}+x\right)^{m+2}\Phi\left(1+\frac{bx}{a},1,m+2\right)\Gamma(m+2)}{abm\Gamma(m+3)+ab\Gamma(m+3)} - \frac{2b^{m+2}\left(\frac{a}{b}+x\right)^{m+2}\Phi\left(1+\frac{bx}{a},1,m+2\right)\Gamma(m+2)}{abm\Gamma(m+3)+ab\Gamma(m+3)} & \text{fo} \\ \frac{\begin{cases} -\operatorname{Li}_2\left(\frac{bxe^{i\pi}}{a}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1 \\ \log(a)\log(x) - \operatorname{Li}_2\left(\frac{bxe^{i\pi}}{a}\right) & \text{for } |x|<1 \\ -\log(a)\log\left(\frac{1}{x}\right) - \operatorname{Li}_2\left(\frac{bxe^{i\pi}}{a}\right) & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| x\right)\log(a) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| x\right)\log(a) - \operatorname{Li}_2\left(\frac{bxe^{i\pi}}{a}\right) & \text{otherwise} \end{cases}}{b} & \text{o} \end{cases} \right) + \left(\begin{cases} a^m x & \text{for } b=0 \\ \frac{\begin{cases} \frac{(a+bx)^{m+1}}{m+1} & \text{for } m \neq -1 \\ \log(a+bx) & \text{otherwise} \end{cases}}{b} & \text{otherwise} \end{cases} \right) \log(cx^n)$$

```
[In] integrate((b*x+a)**m*ln(c*x**n),x)
[Out] -n*Piecewise((a**m*x, Eq(b, 0) | (Eq(b, 0) & Ne(m, -1))), (-b**(m + 2)*m*(a
/b + x)**(m + 2)*lerchphi(1 + b*x/a, 1, m + 2)*gamma(m + 2)/(a*b*m*gamma(m
+ 3) + a*b*gamma(m + 3)) - 2*b**(m + 2)*(a/b + x)**(m + 2)*lerchphi(1 + b*x
/a, 1, m + 2)*gamma(m + 2)/(a*b*m*gamma(m + 3) + a*b*gamma(m + 3)), (m > -o
o) & (m < oo) & Ne(m, -1)), (Piecewise((-polylog(2, b*x*exp_polar(I*pi)/a),
 (Abs(x) < 1) & (1/Abs(x) < 1)), (log(a)*log(x) - polylog(2, b*x*exp_polar(
I*pi)/a), Abs(x) < 1), (-log(a)*log(1/x) - polylog(2, b*x*exp_polar(I*pi)/a
), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(a) + meijerg
```

```
(((1, 1), ()), ((), (0, 0)), x)*log(a) - polylog(2, b*x*exp_polar(I*pi)/a),
 True))/b, True)) + Piecewise((a**m*x, Eq(b, 0)), (Piecewise(((a + b*x)**(m
 + 1)/(m + 1), Ne(m, -1)), (log(a + b*x), True))/b, True))*log(c*x**n)
```

Maxima [F]

$$\int (a+bx)^m \log(cx^n)\,dx = \int (bx+a)^m \log(cx^n)\,dx$$

```
[In] integrate((b*x+a)^m*log(c*x^n),x, algorithm="maxima")
[Out] (b*x + a)*(b*x + a)^m*log(x^n)/(b*(m + 1)) + integrate((((m + 1)*log(c) - n
)*b*x - a*n)*(b*x + a)^m/x, x)/(b*(m + 1))
```

Giac [F]

$$\int (a+bx)^m \log(cx^n)\,dx = \int (bx+a)^m \log(cx^n)\,dx$$

```
[In] integrate((b*x+a)^m*log(c*x^n),x, algorithm="giac")
[Out] integrate((b*x + a)^m*log(c*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m \log(cx^n)\,dx = \int \ln(c\,x^n)\,(a+b\,x)^m\,dx$$

```
[In] int(log(c*x^n)*(a + b*x)^m,x)
[Out] int(log(c*x^n)*(a + b*x)^m, x)
```

3.170 $\int \frac{(a+bx)^m \log(cx^n)}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{(a+bx)^m \log(cx^n)}{x} dx = \text{Int}\left(\frac{(a+bx)^m \log(cx^n)}{x}, x\right)$$

```
[Out] Unintegrable((b*x+a)^m*ln(c*x^n)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+bx)^m \log(cx^n)}{x} dx = \int \frac{(a+bx)^m \log(cx^n)}{x} dx$$

```
[In] Int[((a + b*x)^m*Log[c*x^n])/x,x]
```

```
[Out] Defer[Int][((a + b*x)^m*Log[c*x^n])/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+bx)^m \log(cx^n)}{x} dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 89 vs. $2(20) = 40$.

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 5.24

$$\int \frac{(a+bx)^m \log(cx^n)}{x}\,dx = \frac{\left(1+\frac{a}{bx}\right)^{-m}(a+bx)^m\left(-n\,{}_3F_2\left(-m,-m,-m;1-m,1-m;-\frac{a}{bx}\right)+m\,\text{Hypergeometric2F1}\left(-m,-m,1\right.\right.}{m^2}$$

```
[In] Integrate[((a + b*x)^m*Log[c*x^n])/x,x]
[Out] ((a + b*x)^m*(-(n*HypergeometricPFQ[{-m, -m, -m}, {1 - m, 1 - m}, -(a/(b*x)
)]) + m*Hypergeometric2F1[-m, -m, 1 - m, -(a/(b*x))]*Log[c*x^n]))/(m^2*(1 +
 a/(b*x))^m)
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{(bx+a)^m \ln(c\,x^n)}{x}dx$$

```
[In] int((b*x+a)^m*ln(c*x^n)/x,x)
[Out] int((b*x+a)^m*ln(c*x^n)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^m \log(cx^n)}{x}\,dx = \int \frac{(bx+a)^m \log(cx^n)}{x}\,dx$$

```
[In] integrate((b*x+a)^m*log(c*x^n)/x,x, algorithm="fricas")
[Out] integral((b*x + a)^m*log(c*x^n)/x, x)
```

Sympy [N/A]

Not integrable

Time = 4.98 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^m \log(cx^n)}{x}\,dx = \int \frac{(a+bx)^m \log(cx^n)}{x}\,dx$$

```
[In] integrate((b*x+a)**m*ln(c*x**n)/x,x)
[Out] Integral((a + b*x)**m*log(c*x**n)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^m \log(cx^n)}{x}\,dx = \int \frac{(bx+a)^m \log(cx^n)}{x}\,dx$$

```
[In] integrate((b*x+a)^m*log(c*x^n)/x,x, algorithm="maxima")
[Out] integrate((b*x + a)^m*log(c*x^n)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^m \log(cx^n)}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((b*x+a)^m*log(c*x^n)/x,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0,1
,0]%%%} / %%%{1,[0,0,1]%%%} Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^m \log(cx^n)}{x} \, dx = \int \frac{\ln(c\,x^n)\,\left(a+b\,x\right)^m}{x} \, dx$$

```
[In] int((log(c*x^n)*(a + b*x)^m)/x,x)
[Out] int((log(c*x^n)*(a + b*x)^m)/x, x)
```

3.171 $\int x^5(d+ex^2)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int x^5(d+ex^2)(a+b\log(cx^n))\,dx=-\frac{1}{36}bdnx^6-\frac{1}{64}benx^8+\frac{1}{24}(4dx^6+3ex^8)(a+b\log(cx^n))$$

```
[Out] -1/36*b*d*n*x^6-1/64*b*e*n*x^8+1/24*(3*e*x^8+4*d*x^6)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {14, 2371}

$$\int x^5(d+ex^2)(a+b\log(cx^n))\,dx=\frac{1}{24}(4dx^6+3ex^8)(a+b\log(cx^n))-\frac{1}{36}bdnx^6-\frac{1}{64}benx^8$$

```
[In] Int[x^5*(d + e*x^2)*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/36*(b*d*n*x^6) - (b*e*n*x^8)/64 + ((4*d*x^6 + 3*e*x^8)*(a + b*Log[c*x^n]
))/24
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
```

```
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{24}\left(4dx^6 + 3ex^8\right)\left(a + b\log\left(cx^n\right)\right) - (bn)\int\left(\frac{dx^5}{6} + \frac{ex^7}{8}\right)dx$$
$$= -\frac{1}{36}bdnx^6 - \frac{1}{64}benx^8 + \frac{1}{24}\left(4dx^6 + 3ex^8\right)\left(a + b\log\left(cx^n\right)\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int x^5\left(d + ex^2\right)\left(a + b\log\left(cx^n\right)\right)\,dx = \frac{1}{6}adx^6 - \frac{1}{36}bdnx^6 + \frac{1}{8}aex^8 - \frac{1}{64}benx^8 + \frac{1}{6}bdx^6\log\left(cx^n\right) + \frac{1}{8}bex^8\log\left(cx^n\right)$$

```
[In] Integrate[x^5*(d + e*x^2)*(a + b*Log[c*x^n]),x]
[Out] (a*d*x^6)/6 - (b*d*n*x^6)/36 + (a*e*x^8)/8 - (b*e*n*x^8)/64 + (b*d*x^6*Log[
c*x^n])/6 + (b*e*x^8*Log[c*x^n])/8
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

method	result
parallelrisch	$\frac{x^8\ln(c\,x^n)be}{8} - \frac{ben\,x^8}{64} + \frac{ae\,x^8}{8} + \frac{x^6\ln(c\,x^n)bd}{6} - \frac{bdn\,x^6}{36} + \frac{ad\,x^6}{6}$
risch	$\frac{b\,x^6(3e\,x^2+4d)\ln(x^n)}{24} - \frac{i\pi be\,x^8\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)}{16} + \frac{i\pi be\,x^8\operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^n)^2}{16} + \frac{i\pi be\,x^8\operatorname{csgn}(ix^n)\operatorname{csgn}(}{16}$

```
[In] int(x^5*(e*x^2+d)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] 1/8*x^8*ln(c*x^n)*b*e-1/64*b*e*n*x^8+1/8*a*e*x^8+1/6*x^6*ln(c*x^n)*b*d-1/36
*b*d*n*x^6+1/6*a*d*x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int x^5(d+ex^2)\,(a+b\log(cx^n))\,dx = -\frac{1}{64}\,(ben-8\,ae)x^8-\frac{1}{36}\,(bdn-6\,ad)x^6+\frac{1}{24}\,\left(3\,bex^8+4\,bdx^6\right)\log(c)+\frac{1}{24}\,\left(3\,benx^8+4\,bdnx^6\right)\log(x)$$

```
[In] integrate(x^5*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/64*(b*e*n - 8*a*e)*x^8 - 1/36*(b*d*n - 6*a*d)*x^6 + 1/24*(3*b*e*x^8 + 4*
b*d*x^6)*log(c) + 1/24*(3*b*e*n*x^8 + 4*b*d*n*x^6)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int x^5(d+ex^2)\,(a+b\log(cx^n))\,dx = \frac{adx^6}{6}+\frac{aex^8}{8}-\frac{bdnx^6}{36}+\frac{bdx^6\log(cx^n)}{6}-\frac{benx^8}{64}+\frac{bex^8\log(cx^n)}{8}$$

```
[In] integrate(x**5*(e*x**2+d)*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d*x**6/6 + a*e*x**8/8 - b*d*n*x**6/36 + b*d*x**6*log(c*x**n)/6 - b*e*n*x*
*8/64 + b*e*x**8*log(c*x**n)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int x^5(d+ex^2)\,(a+b\log(cx^n))\,dx = -\frac{1}{64}\,benx^8+\frac{1}{8}\,bex^8\log(cx^n)+\frac{1}{8}\,aex^8-\frac{1}{36}\,bdnx^6+\frac{1}{6}\,bdx^6\log(cx^n)+\frac{1}{6}\,adx^6$$

```
[In] integrate(x^5*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/64*b*e*n*x^8 + 1/8*b*e*x^8*log(c*x^n) + 1/8*a*e*x^8 - 1/36*b*d*n*x^6 + 1
/6*b*d*x^6*log(c*x^n) + 1/6*a*d*x^6
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int x^5\left(d+ex^2\right)\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{1}{8}\,benx^8\log\left(x\right)-\frac{1}{64}\,benx^8+\frac{1}{8}\,bex^8\log\left(c\right)+\frac{1}{8}\,aex^8+\frac{1}{6}\,bdnx^6\log\left(x\right)-\frac{1}{36}\,bdnx^6+\frac{1}{6}\,bdx^6\log\left(c\right)+\frac{1}{6}\,adx^6$$

```
[In] integrate(x^5*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/8*b*e*n*x^8*log(x) - 1/64*b*e*n*x^8 + 1/8*b*e*x^8*log(c) + 1/8*a*e*x^8 +
1/6*b*d*n*x^6*log(x) - 1/36*b*d*n*x^6 + 1/6*b*d*x^6*log(c) + 1/6*a*d*x^6
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^5\left(d+ex^2\right)\left(a+b\log\left(cx^n\right)\right)\,dx=\ln\left(c\,x^n\right)\,\left(\frac{b\,e\,x^8}{8}+\frac{b\,d\,x^6}{6}\right)+\frac{d\,x^6\,\left(6\,a-b\,n\right)}{36}+\frac{e\,x^8\,\left(8\,a-b\,n\right)}{64}$$

```
[In] int(x^5*(d + e*x^2)*(a + b*log(c*x^n)),x)
```

```
[Out] log(c*x^n)*((b*d*x^6)/6 + (b*e*x^8)/8) + (d*x^6*(6*a - b*n))/36 + (e*x^8*(8
*a - b*n))/64
```

3.172 $\int x^3(d+ex^2)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int x^3(d+ex^2)(a+b\log(cx^n))\,dx=-\frac{1}{16}bdnx^4-\frac{1}{36}benx^6+\frac{1}{12}(3dx^4+2ex^6)(a+b\log(cx^n))$$

```
[Out] -1/16*b*d*n*x^4-1/36*b*e*n*x^6+1/12*(2*e*x^6+3*d*x^4)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {14, 2371}

$$\int x^3(d+ex^2)(a+b\log(cx^n))\,dx=\frac{1}{12}(3dx^4+2ex^6)(a+b\log(cx^n))-\frac{1}{16}bdnx^4-\frac{1}{36}benx^6$$

```
[In] Int[x^3*(d + e*x^2)*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/16*(b*d*n*x^4) - (b*e*n*x^6)/36 + ((3*d*x^4 + 2*e*x^6)*(a + b*Log[c*x^n]
))/12
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
```

```
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{12}\left(3dx^4+2ex^6\right)(a+b\log(cx^n))-(bn)\int\left(\frac{dx^3}{4}+\frac{ex^5}{6}\right)dx \\ &= -\frac{1}{16}bdnx^4-\frac{1}{36}benx^6+\frac{1}{12}\left(3dx^4+2ex^6\right)(a+b\log(cx^n)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\begin{aligned} \int x^3\left(d+ex^2\right)(a+b\log(cx^n))\,dx &= \frac{1}{4}adx^4-\frac{1}{16}bdnx^4+\frac{1}{6}aex^6-\frac{1}{36}benx^6 \\ &\quad+\frac{1}{4}bdx^4\log(cx^n)+\frac{1}{6}bex^6\log(cx^n) \end{aligned}$$

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*Log[c*x^n]),x]
[Out] (a*d*x^4)/4 - (b*d*n*x^4)/16 + (a*e*x^6)/6 - (b*e*n*x^6)/36 + (b*d*x^4*Log[
c*x^n])/4 + (b*e*x^6*Log[c*x^n])/6
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

method	result
parallelrisch	$\frac{x^6\ln(cx^n)be}{6}-\frac{ben\,x^6}{36}+\frac{ae\,x^6}{6}+\frac{x^4\ln(cx^n)bd}{4}-\frac{bdn\,x^4}{16}+\frac{ad\,x^4}{4}$
risch	$\frac{b\,x^4(2e\,x^2+3d)\ln(x^n)}{12}-\frac{i\pi be\,x^6\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)}{12}+\frac{i\pi be\,x^6\operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^n)^2}{12}+\frac{i\pi be\,x^6\operatorname{csgn}(ix^n)\operatorname{csgn}(}{12}$

```
[In] int(x^3*(e*x^2+d)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] 1/6*x^6*ln(c*x^n)*b*e-1/36*b*e*n*x^6+1/6*a*e*x^6+1/4*x^4*ln(c*x^n)*b*d-1/16
*b*d*n*x^4+1/4*a*d*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int x^3\left(d+ex^2\right)\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{36}\,(ben-6\,ae)x^6-\frac{1}{16}\,(bdn-4\,ad)x^4+\frac{1}{12}\,\left(2\,bex^6+3\,bdx^4\right)\log\left(c\right)+\frac{1}{12}\,\left(2\,benx^6+3\,bdnx^4\right)\log\left(x\right)$$

```
[In] integrate(x^3*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/36*(b*e*n - 6*a*e)*x^6 - 1/16*(b*d*n - 4*a*d)*x^4 + 1/12*(2*b*e*x^6 + 3*
b*d*x^4)*log(c) + 1/12*(2*b*e*n*x^6 + 3*b*d*n*x^4)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int x^3\left(d+ex^2\right)\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{adx^4}{4}+\frac{aex^6}{6}-\frac{bdnx^4}{16}+\frac{bdx^4\log\left(cx^n\right)}{4}-\frac{benx^6}{36}+\frac{bex^6\log\left(cx^n\right)}{6}$$

```
[In] integrate(x**3*(e*x**2+d)*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d*x**4/4 + a*e*x**6/6 - b*d*n*x**4/16 + b*d*x**4*log(c*x**n)/4 - b*e*n*x*
*6/36 + b*e*x**6*log(c*x**n)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int x^3\left(d+ex^2\right)\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{36}\,benx^6+\frac{1}{6}\,bex^6\log\left(cx^n\right)+\frac{1}{6}\,aex^6-\frac{1}{16}\,bdnx^4+\frac{1}{4}\,bdx^4\log\left(cx^n\right)+\frac{1}{4}\,adx^4$$

```
[In] integrate(x^3*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/36*b*e*n*x^6 + 1/6*b*e*x^6*log(c*x^n) + 1/6*a*e*x^6 - 1/16*b*d*n*x^4 + 1
/4*b*d*x^4*log(c*x^n) + 1/4*a*d*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int x^3(d+ex^2)(a+b\log(cx^n))\,dx = \frac{1}{6}\,benx^6\log(x) - \frac{1}{36}\,benx^6 + \frac{1}{6}\,bex^6\log(c) + \frac{1}{6}\,aex^6 + \frac{1}{4}\,bdnx^4\log(x) - \frac{1}{16}\,bdnx^4 + \frac{1}{4}\,bdx^4\log(c) + \frac{1}{4}\,adx^4$$

```
[In] integrate(x^3*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/6*b*e*n*x^6*log(x) - 1/36*b*e*n*x^6 + 1/6*b*e*x^6*log(c) + 1/6*a*e*x^6 +
1/4*b*d*n*x^4*log(x) - 1/16*b*d*n*x^4 + 1/4*b*d*x^4*log(c) + 1/4*a*d*x^4
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^3(d+ex^2)(a+b\log(cx^n))\,dx = \ln(c\,x^n)\left(\frac{b\,e\,x^6}{6} + \frac{b\,d\,x^4}{4}\right) + \frac{d\,x^4\,(4\,a-b\,n)}{16} + \frac{e\,x^6\,(6\,a-b\,n)}{36}$$

```
[In] int(x^3*(d + e*x^2)*(a + b*log(c*x^n)),x)
```

```
[Out] log(c*x^n)*((b*d*x^4)/4 + (b*e*x^6)/6) + (d*x^4*(4*a - b*n))/16 + (e*x^6*(6
*a - b*n))/36
```

3.173 $\int x(d+ex^2)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int x(d+ex^2)(a+b\log(cx^n))\,dx=-\frac{1}{4}bdnx^2-\frac{1}{16}benx^4+\frac{1}{4}(2dx^2+ex^4)(a+b\log(cx^n))$$

```
[Out] -1/4*b*d*n*x^2-1/16*b*e*n*x^4+1/4*(e*x^4+2*d*x^2)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {14, 2371, 12}

$$\int x(d+ex^2)(a+b\log(cx^n))\,dx=\frac{1}{4}(2dx^2+ex^4)(a+b\log(cx^n))-\frac{1}{4}bdnx^2-\frac{1}{16}benx^4$$

```
[In] Int[x*(d + e*x^2)*(a + b*Log[c*x^n]),x]
[Out] -1/4*(b*d*n*x^2) - (b*e*n*x^4)/16 + ((2*d*x^2 + e*x^4)*(a + b*Log[c*x^n]))/
4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\left(2dx^2+ex^4\right)\left(a+b\log\left(cx^n\right)\right)-(bn)\int\frac{1}{4}x\left(2d+ex^2\right)\,dx\\
&= \frac{1}{4}\left(2dx^2+ex^4\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{4}(bn)\int x\left(2d+ex^2\right)\,dx\\
&= \frac{1}{4}\left(2dx^2+ex^4\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{4}(bn)\int\left(2dx+ex^3\right)\,dx\\
&= -\frac{1}{4}bdnx^2-\frac{1}{16}benx^4+\frac{1}{4}\left(2dx^2+ex^4\right)\left(a+b\log\left(cx^n\right)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.47

$$\int x\left(d+ex^2\right)\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{1}{2}adx^2-\frac{1}{4}bdnx^2+\frac{1}{4}aex^4-\frac{1}{16}benx^4+\frac{1}{2}bdx^2\log\left(cx^n\right)+\frac{1}{4}bex^4\log\left(cx^n\right)$$

```
[In] Integrate[x*(d + e*x^2)*(a + b*Log[c*x^n]),x]
[Out] (a*d*x^2)/2 - (b*d*n*x^2)/4 + (a*e*x^4)/4 - (b*e*n*x^4)/16 + (b*d*x^2*Log[c
*x^n])/2 + (b*e*x^4*Log[c*x^n])/4
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

method	result	size
parallelrisch	$\frac{x^4 be\ln(cx^n)}{4}-\frac{ben\,x^4}{16}+\frac{x^4 ae}{4}+\frac{x^2\ln(cx^n)bd}{2}-\frac{bdn\,x^2}{4}+\frac{ad\,x^2}{2}$	58
risch	Expression too large to display	2346

```
[In] int(x*(e*x^2+d)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4*b*e*ln(c*x^n)-1/16*b*e*n*x^4+1/4*x^4*a*e+1/2*x^2*ln(c*x^n)*b*d-1/4*
b*d*n*x^2+1/2*a*d*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int x(d+ex^2)\,(a+b\log{(cx^n)})\;dx = -\frac{1}{16}\,(ben-4\,ae)x^4 - \frac{1}{4}\,(bdn-2\,ad)x^2 + \frac{1}{4}\,(bex^4+2\,bdx^2)\log{(c)} + \frac{1}{4}\,(benx^4+2\,bdnx^2)\log{(x)}$$

```
[In] integrate(x*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] -1/16*(b*e*n - 4*a*e)*x^4 - 1/4*(b*d*n - 2*a*d)*x^2 + 1/4*(b*e*x^4 + 2*b*d*
x^2)*log(c) + 1/4*(b*e*n*x^4 + 2*b*d*n*x^2)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int x(d+ex^2)\,(a+b\log{(cx^n)})\;dx = \frac{adx^2}{2} + \frac{aex^4}{4} - \frac{bdnx^2}{4} + \frac{bdx^2\log{(cx^n)}}{2} - \frac{benx^4}{16} + \frac{bex^4\log{(cx^n)}}{4}$$

```
[In] integrate(x*(e*x**2+d)*(a+b*ln(c*x**n)),x)
[Out] a*d*x**2/2 + a*e*x**4/4 - b*d*n*x**2/4 + b*d*x**2*log(c*x**n)/2 - b*e*n*x**
4/16 + b*e*x**4*log(c*x**n)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int x(d+ex^2)\,(a+b\log{(cx^n)})\;dx = -\frac{1}{16}\,benx^4 + \frac{1}{4}\,bex^4\log{(cx^n)} + \frac{1}{4}\,aex^4 - \frac{1}{4}\,bdnx^2 + \frac{1}{2}\,bdx^2\log{(cx^n)} + \frac{1}{2}\,adx^2$$

```
[In] integrate(x*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] -1/16*b*e*n*x^4 + 1/4*b*e*x^4*log(c*x^n) + 1/4*a*e*x^4 - 1/4*b*d*n*x^2 + 1/
2*b*d*x^2*log(c*x^n) + 1/2*a*d*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.47

$$\int x(d+ex^2)(a+b\log(cx^n))\,dx = \frac{1}{4}benx^4\log(x) - \frac{1}{16}benx^4 + \frac{1}{4}bex^4\log(c) + \frac{1}{4}aex^4 + \frac{1}{2}bdnx^2\log(x) - \frac{1}{4}bdnx^2 + \frac{1}{2}bdx^2\log(c) + \frac{1}{2}adx^2$$

```
[In] integrate(x*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*e*n*x^4*log(x) - 1/16*b*e*n*x^4 + 1/4*b*e*x^4*log(c) + 1/4*a*e*x^4 +
1/2*b*d*n*x^2*log(x) - 1/4*b*d*n*x^2 + 1/2*b*d*x^2*log(c) + 1/2*a*d*x^2
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int x(d+ex^2)(a+b\log(cx^n))\,dx = \ln(c\,x^n)\left(\frac{b\,e\,x^4}{4} + \frac{b\,d\,x^2}{2}\right) + \frac{d\,x^2\,(2\,a-b\,n)}{4} + \frac{e\,x^4\,(4\,a-b\,n)}{16}$$

```
[In] int(x*(d + e*x^2)*(a + b*log(c*x^n)),x)
```

```
[Out] log(c*x^n)*((b*d*x^2)/2 + (b*e*x^4)/4) + (d*x^2*(2*a - b*n))/4 + (e*x^4*(4*
a - b*n))/16
```

3.174 $\int \frac{(d+ex^2)(a+b\log(cx^n))}{x} dx$

Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x} dx = -\frac{1}{4}benx^2 + \frac{1}{2}ex^2(a+b\log(cx^n)) + \frac{d(a+b\log(cx^n))^2}{2bn}$$

```
[Out] -1/4*b*e*n*x^2+1/2*e*x^2*(a+b*ln(c*x^n))+1/2*d*(a+b*ln(c*x^n))^2/b/n
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {14, 2393, 2338, 2341}

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x} dx = \frac{d(a+b\log(cx^n))^2}{2bn} + \frac{1}{2}ex^2(a+b\log(cx^n)) - \frac{1}{4}benx^2$$

```
[In] Int[((d + e*x^2)*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] -1/4*(b*e*n*x^2) + (e*x^2*(a + b*Log[c*x^n]))/2 + (d*(a + b*Log[c*x^n])^2)/
(2*b*n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{d(a+b\log(cx^n))}{x} + ex(a+b\log(cx^n))\right) dx \\ &= d\int \frac{a+b\log(cx^n)}{x}\,dx + e\int x(a+b\log(cx^n))\,dx \\ &= -\frac{1}{4}benx^2 + \frac{1}{2}ex^2(a+b\log(cx^n)) + \frac{d(a+b\log(cx^n))^2}{2bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x}\,dx = \frac{1}{2}aex^2 - \frac{1}{4}benx^2 + ad\log(x) + \frac{1}{2}bex^2\log(cx^n) + \frac{bd\log^2(cx^n)}{2n}$$

```
[In] Integrate[((d + e*x^2)*(a + b*Log[c*x^n]))/x,x]
[Out] (a*e*x^2)/2 - (b*e*n*x^2)/4 + a*d*Log[x] + (b*e*x^2*Log[c*x^n])/2 + (b*d*Lo
g[c*x^n]^2)/(2*n)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

method	result
parallelrisch	$\frac{2x^2 \ln(c\,x^n)ben - x^2be\,n^2+2x^2aen+4\ln(x)adn+2bd\ln(c\,x^n)^2}{4n}$
risch	$\left(\frac{be\,x^2}{2}+bd\ln\left(x\right)\right)\ln\left(x^n\right)-\frac{bdn\ln(x)^2}{2}-\frac{i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)be\,x^2}{4}+\frac{i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2 be\,x^2}{4}-$

```
[In] int((e*x^2+d)*(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
[Out] 1/4*(2*x^2*ln(c*x^n)*b*e*n-x^2*b*e*n^2+2*x^2*a*e*n+4*ln(x)*a*d*n+2*b*d*ln(c
*x^n)^2)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex^2)\,(a+b\log{(cx^n)})}{x}\,dx = \frac{1}{2}\,bex^2\log{(c)} + \frac{1}{2}\,bdn\log{(x)}^2 - \frac{1}{4}\,(ben-2\,ae)x^2 + \frac{1}{2}\,(benx^2+2\,bd\log{(c)}+2\,ad)\log{(x)}$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x,x, algorithm="fricas")
[Out] 1/2*b*e*x^2*log(c) + 1/2*b*d*n*log(x)^2 - 1/4*(b*e*n - 2*a*e)*x^2 + 1/2*(b*
e*n*x^2 + 2*b*d*log(c) + 2*a*d)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex^2)\,(a+b\log{(cx^n)})}{x}\,dx = \begin{cases} \frac{ad\log{(cx^n)}}{n} + \frac{aex^2}{2} + \frac{bd\log{(cx^n)}^2}{2n} - \frac{benx^2}{4} + \frac{bex^2\log{(cx^n)}}{2} & \text{for } n \neq 0 \\ (a+b\log{(c)})\left(d\log{(x)}+\frac{ex^2}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((e*x**2+d)*(a+b*ln(c*x**n))/x,x)
[Out] Piecewise((a*d*log(c*x**n)/n + a*e*x**2/2 + b*d*log(c*x**n)**2/(2*n) - b*e*
n*x**2/4 + b*e*x**2*log(c*x**n)/2, Ne(n, 0)), ((a + b*log(c))*(d*log(x) + e
*x**2/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x}\,dx = -\frac{1}{4}\,benx^2 + \frac{1}{2}\,bex^2\log(cx^n) + \frac{1}{2}\,aex^2 + \frac{bd\log(cx^n)^2}{2\,n} + ad\log(x)$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] -1/4*b*e*n*x^2 + 1/2*b*e*x^2*log(c*x^n) + 1/2*a*e*x^2 + 1/2*b*d*log(c*x^n)^
2/n + a*d*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x}\,dx = \frac{1}{2}\,benx^2\log(x) + \frac{1}{2}\,bdn\log(x)^2 - \frac{1}{4}\,(ben - 2\,be\log(c) - 2\,ae)x^2 + (bd\log(c) + ad)\log(x)$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] 1/2*b*e*n*x^2*log(x) + 1/2*b*d*n*log(x)^2 - 1/4*(b*e*n - 2*b*e*log(c) - 2*a
*e)*x^2 + (b*d*log(c) + a*d)*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x}\,dx = a\,d\,\ln(x) + \frac{e\,x^2\,(2\,a-b\,n)}{4} + \frac{b\,e\,x^2\,\ln(c\,x^n)}{2} + \frac{b\,d\ln(c\,x^n)^2}{2\,n}$$

```
[In] int(((d + e*x^2)*(a + b*log(c*x^n)))/x,x)
```

```
[Out] a*d*log(x) + (e*x^2*(2*a - b*n))/4 + (b*e*x^2*log(c*x^n))/2 + (b*d*log(c*x^
n)^2)/(2*n)
```

3.175 $\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^3} dx = -\frac{bdn}{4x^2} - \frac{d(a+b\log(cx^n))}{2x^2} + \frac{e(a+b\log(cx^n))^2}{2bn}$$

```
[Out] -1/4*b*d*n/x^2-1/2*d*(a+b*ln(c*x^n))/x^2+1/2*e*(a+b*ln(c*x^n))^2/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {14, 2372, 2338}

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^3} dx = -\frac{d(a+b\log(cx^n))}{2x^2} + e\log(x)(a+b\log(cx^n)) - \frac{bdn}{4x^2} - \frac{1}{2}ben\log^2(x)$$

```
[In] Int[((d + e*x^2)*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/4*(b*d*n)/x^2 - (b*e*n*Log[x]^2)/2 - (d*(a + b*Log[c*x^n]))/(2*x^2) + e*
Log[x]*(a + b*Log[c*x^n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{d(a+b\log(cx^n))}{2x^2}+e\log(x)(a+b\log(cx^n))-(bn)\int\left(-\frac{d}{2x^3}+\frac{e\log(x)}{x}\right)dx\\ &= -\frac{bdn}{4x^2}-\frac{d(a+b\log(cx^n))}{2x^2}+e\log(x)(a+b\log(cx^n))-(ben)\int\frac{\log(x)}{x}\,dx\\ &= -\frac{bdn}{4x^2}-\frac{1}{2}ben\log^2(x)-\frac{d(a+b\log(cx^n))}{2x^2}+e\log(x)(a+b\log(cx^n))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10

$$\int\frac{(d+ex^2)(a+b\log(cx^n))}{x^3}\,dx=-\frac{ad}{2x^2}-\frac{bdn}{4x^2}+ae\log(x)-\frac{bd\log(cx^n)}{2x^2}+\frac{be\log^2(cx^n)}{2n}$$

```
[In] Integrate[((d + e*x^2)*(a + b*Log[c*x^n]))/x^3,x]
[Out] -1/2*(a*d)/x^2 - (b*d*n)/(4*x^2) + a*e*Log[x] - (b*d*Log[c*x^n])/(2*x^2) +
(b*e*Log[c*x^n]^2)/(2*n)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

method	result
parallelrisch	$\frac{4\ln(x)x^2aen+2be\ln(cx^n)^2x^2-2\ln(cx^n)bdn-bd\,n^2-2adn}{4x^2n}$
risch	$-\frac{b(-2e\ln(x)x^2+d)\ln(x^n)}{2x^2}-\frac{2i\ln(x)\pi be\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)x^2-2i\ln(x)\pi be\,\mathrm{csgn}(ic)\,\mathrm{csgn}(icx^n)^2x^2-2i\ln(x)\pi b}{}$

```
[In] int((e*x^2+d)*(a+b*ln(c*x^n))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/x^2*(4*ln(x)*x^2*a*e*n+2*b*e*ln(c*x^n)^2*x^2-2*ln(c*x^n)*b*d*n-b*d*n^2-
2*a*d*n)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^3}\,dx = \frac{2\,benx^2\log(x)^2 - bdn - 2\,bd\log(c) - 2\,ad + 2\,(2\,bex^2\log(c) + 2\,aex^2 - bdn)\log(x)}{4\,x^2}$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*e*n*x^2*log(x)^2 - b*d*n - 2*b*d*log(c) - 2*a*d + 2*(2*b*e*x^2*log
(c) + 2*a*e*x^2 - b*d*n)*log(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^3}\,dx = -\frac{ad}{2x^2} + ae\log(x) + bd\left(-\frac{n}{4x^2} - \frac{\log(cx^n)}{2x^2}\right) - be\left(\begin{cases} -\log(c)\log(x) & \text{for } n = 0 \\ -\frac{\log(cx^n)^2}{2n} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x**2+d)*(a+b*ln(c*x**n))/x**3,x)
```

```
[Out] -a*d/(2*x**2) + a*e*log(x) + b*d*(-n/(4*x**2) - log(c*x**n)/(2*x**2)) - b*e
*Piecewise((-log(c)*log(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^3}\,dx = \frac{be\log(cx^n)^2}{2\,n} + ae\log(x) - \frac{bdn}{4\,x^2} - \frac{bd\log(cx^n)}{2\,x^2} - \frac{ad}{2\,x^2}$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*e*log(c*x^n)^2/n + a*e*log(x) - 1/4*b*d*n/x^2 - 1/2*b*d*log(c*x^n)/x^
2 - 1/2*a*d/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^3}\,dx = \frac{1}{2}ben\log(x)^2 - \frac{1}{4}bdn\left(\frac{2\log(x)}{x^2}+\frac{1}{x^2}\right) + be\log(c)\log(|x|) + ae\log(|x|) - \frac{bd\log(c)}{2x^2} - \frac{ad}{2x^2}$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b*e*n*log(x)^2 - 1/4*b*d*n*(2*log(x)/x^2 + 1/x^2) + b*e*log(c)*log(abs(
x)) + a*e*log(abs(x)) - 1/2*b*d*log(c)/x^2 - 1/2*a*d/x^2
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^3}\,dx = \ln(x)\left(a\,e+\frac{b\,e\,n}{2}\right) - \frac{\frac{a\,d}{2}+\frac{b\,d\,n}{4}}{x^2} - \frac{\ln(c\,x^n)\left(\frac{b\,e\,x^2}{2}+\frac{b\,d}{2}\right)}{x^2} + \frac{b\,e\ln(c\,x^n)^2}{2\,n}$$

```
[In] int(((d + e*x^2)*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] log(x)*(a*e + (b*e*n)/2) - ((a*d)/2 + (b*d*n)/4)/x^2 - (log(c*x^n)*((b*d)/2
 + (b*e*x^2)/2))/x^2 + (b*e*log(c*x^n)^2)/(2*n)
```

3.176 $\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^5}\,dx = -\frac{bdn}{16x^4}-\frac{ben}{4x^2}-\frac{d(a+b\log(cx^n))}{4x^4}-\frac{e(a+b\log(cx^n))}{2x^2}$$

```
[Out] -1/16*b*d*n/x^4-1/4*b*e*n/x^2-1/4*d*(a+b*ln(c*x^n))/x^4-1/2*e*(a+b*ln(c*x^n
))/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {14, 2372, 12}

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^5}\,dx = -\frac{d(a+b\log(cx^n))}{4x^4}-\frac{e(a+b\log(cx^n))}{2x^2}-\frac{bdn}{16x^4}-\frac{ben}{4x^2}$$

```
[In] Int[((d + e*x^2)*(a + b*Log[c*x^n]))/x^5,x]
```

```
[Out] -1/16*(b*d*n)/x^4 - (b*e*n)/(4*x^2) - (d*(a + b*Log[c*x^n]))/(4*x^4) - (e*(
a + b*Log[c*x^n]))/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{d(a+b\log(cx^n))}{4x^4}-\frac{e(a+b\log(cx^n))}{2x^2}-(bn)\int\frac{-d-2ex^2}{4x^5}\,dx\\ &= -\frac{d(a+b\log(cx^n))}{4x^4}-\frac{e(a+b\log(cx^n))}{2x^2}-\frac{1}{4}(bn)\int\frac{-d-2ex^2}{x^5}\,dx\\ &= -\frac{d(a+b\log(cx^n))}{4x^4}-\frac{e(a+b\log(cx^n))}{2x^2}-\frac{1}{4}(bn)\int\left(-\frac{d}{x^5}-\frac{2e}{x^3}\right)dx\\ &= -\frac{bdn}{16x^4}-\frac{ben}{4x^2}-\frac{d(a+b\log(cx^n))}{4x^4}-\frac{e(a+b\log(cx^n))}{2x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int\frac{(d+ex^2)(a+b\log(cx^n))}{x^5}\,dx=-\frac{ad}{4x^4}-\frac{bdn}{16x^4}-\frac{ae}{2x^2}-\frac{ben}{4x^2}-\frac{bd\log(cx^n)}{4x^4}-\frac{be\log(cx^n)}{2x^2}$$

```
[In] Integrate[((d + e*x^2)*(a + b*Log[c*x^n]))/x^5,x]
```

```
[Out] -1/4*(a*d)/x^4 - (b*d*n)/(16*x^4) - (a*e)/(2*x^2) - (b*e*n)/(4*x^2) - (b*d*
Log[c*x^n])/(4*x^4) - (b*e*Log[c*x^n])/(2*x^2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

method	result
parallelrisch	$-\frac{8be\,x^2\ln(c\,x^n)+4ben\,x^2+8ae\,x^2+4b\ln(c\,x^n)d+bdn+4ad}{16x^4}$
risch	$-\frac{b(2e\,x^2+d)\ln(x^n)}{4x^4}-\frac{-4i\pi\ \text{csgn}(ic)\,\text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)be\,x^2+4i\pi\ \text{csgn}(ic)\,\text{csgn}(ic\,x^n)^2be\,x^2+4i\pi\ \text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)}{}$

```
[In] int((e*x^2+d)*(a+b*ln(c*x^n))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/x^4*(8*b*e*x^2*ln(c*x^n)+4*b*e*n*x^2+8*a*e*x^2+4*b*ln(c*x^n)*d+b*d*n+
4*a*d)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^5}\,dx = -\frac{bdn+4(ben+2ae)x^2+4ad+4(2bex^2+bd)\log(c)+4(2benx^2+bdn)\log(x)}{16x^4}$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(b*d*n + 4*(b*e*n + 2*a*e)*x^2 + 4*a*d + 4*(2*b*e*x^2 + b*d)*log(c) +
 4*(2*b*e*n*x^2 + b*d*n)*log(x))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^5}\,dx = -\frac{ad}{4x^4}-\frac{ae}{2x^2}-\frac{bdn}{16x^4}-\frac{bd\log(cx^n)}{4x^4}-\frac{ben}{4x^2}-\frac{be\log(cx^n)}{2x^2}$$

```
[In] integrate((e*x**2+d)*(a+b*ln(c*x**n))/x**5,x)
```

```
[Out] -a*d/(4*x**4) - a*e/(2*x**2) - b*d*n/(16*x**4) - b*d*log(c*x**n)/(4*x**4) -
 b*e*n/(4*x**2) - b*e*log(c*x**n)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^5}\,dx = -\frac{ben}{4x^2}-\frac{be\log(cx^n)}{2x^2}-\frac{ae}{2x^2}-\frac{bdn}{16x^4}-\frac{bd\log(cx^n)}{4x^4}-\frac{ad}{4x^4}$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*b*e*n/x^2 - 1/2*b*e*log(c*x^n)/x^2 - 1/2*a*e/x^2 - 1/16*b*d*n/x^4 - 1/
4*b*d*log(c*x^n)/x^4 - 1/4*a*d/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

$$\int \frac{(d + ex^2) (a + b \log (cx^n))}{x^5} dx = -\frac{(2 benx^2 + bdn) \log (x)}{4 x^4} - \frac{4 benx^2 + 8 bex^2 \log (c) + 8 aex^2 + bdn + 4 bd \log (c) + 4 ad}{16 x^4}$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*e*n*x^2 + b*d*n)*log(x)/x^4 - 1/16*(4*b*e*n*x^2 + 8*b*e*x^2*log(c
) + 8*a*e*x^2 + b*d*n + 4*b*d*log(c) + 4*a*d)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{(d + ex^2) (a + b \log (cx^n))}{x^5} dx = -\frac{(2 a e + b e n) x^2 + a d + \frac{b d n}{4}}{4 x^4} - \frac{\ln (c x^n) \left(\frac{b e x^2}{2} + \frac{b d}{4}\right)}{x^4}$$

```
[In] int(((d + e*x^2)*(a + b*log(c*x^n)))/x^5,x)
```

```
[Out] - (a*d + x^2*(2*a*e + b*e*n) + (b*d*n)/4)/(4*x^4) - (log(c*x^n)*((b*d)/4 +
(b*e*x^2)/2))/x^4
```

3.177 $\int x^4(d+ex^2)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int x^4(d+ex^2)(a+b\log(cx^n))\,dx=-\frac{1}{25}bdnx^5-\frac{1}{49}benx^7+\frac{1}{35}(7dx^5+5ex^7)(a+b\log(cx^n))$$

```
[Out] -1/25*b*d*n*x^5-1/49*b*e*n*x^7+1/35*(5*e*x^7+7*d*x^5)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {14, 2371}

$$\int x^4(d+ex^2)(a+b\log(cx^n))\,dx=\frac{1}{35}(7dx^5+5ex^7)(a+b\log(cx^n))-\frac{1}{25}bdnx^5-\frac{1}{49}benx^7$$

```
[In] Int[x^4*(d + e*x^2)*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/25*(b*d*n*x^5) - (b*e*n*x^7)/49 + ((7*d*x^5 + 5*e*x^7)*(a + b*Log[c*x^n]
))/35
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
```

```
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{35}\left(7dx^5+5ex^7\right)\left(a+b\log\left(cx^n\right)\right)-(bn)\int\left(\frac{dx^4}{5}+\frac{ex^6}{7}\right)dx\\&=-\frac{1}{25}bdnx^5-\frac{1}{49}benx^7+\frac{1}{35}\left(7dx^5+5ex^7\right)\left(a+b\log\left(cx^n\right)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int x^4\left(d+ex^2\right)\left(a+b\log\left(cx^n\right)\right)dx=\frac{1}{5}adx^5-\frac{1}{25}bdnx^5+\frac{1}{7}aex^7-\frac{1}{49}benx^7+\frac{1}{5}bdx^5\log\left(cx^n\right)+\frac{1}{7}bex^7\log\left(cx^n\right)$$

```
[In] Integrate[x^4*(d + e*x^2)*(a + b*Log[c*x^n]),x]
[Out] (a*d*x^5)/5 - (b*d*n*x^5)/25 + (a*e*x^7)/7 - (b*e*n*x^7)/49 + (b*d*x^5*Log[
c*x^n])/5 + (b*e*x^7*Log[c*x^n])/7
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

method	result
parallelrisch	$\frac{x^7 be\ln(c\,x^n)}{7}-\frac{ben\,x^7}{49}+\frac{x^7ae}{7}+\frac{x^5b\ln(c\,x^n)d}{5}-\frac{bdn\,x^5}{25}+\frac{x^5ad}{5}$
risch	$\frac{b\,x^5(5e\,x^2+7d)\ln(x^n)}{35}-\frac{i\pi be\,x^7\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)}{14}+\frac{i\pi be\,x^7\operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^n)^2}{14}+\frac{i\pi be\,x^7\operatorname{csgn}(ix^n)\operatorname{csgn}(}{14}$

```
[In] int(x^4*(e*x^2+d)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] 1/7*x^7*b*e*ln(c*x^n)-1/49*b*e*n*x^7+1/7*x^7*a*e+1/5*x^5*b*ln(c*x^n)*d-1/25
*b*d*n*x^5+1/5*x^5*a*d
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int x^4(d+ex^2)\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{49}\,(ben-7\,ae)x^7-\frac{1}{25}\,(bdn-5\,ad)x^5+\frac{1}{35}\,\left(5\,bex^7+7\,bdx^5\right)\log\left(c\right)+\frac{1}{35}\,\left(5\,benx^7+7\,bdnx^5\right)\log\left(x\right)$$

```
[In] integrate(x^4*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/49*(b*e*n - 7*a*e)*x^7 - 1/25*(b*d*n - 5*a*d)*x^5 + 1/35*(5*b*e*x^7 + 7*
b*d*x^5)*log(c) + 1/35*(5*b*e*n*x^7 + 7*b*d*n*x^5)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int x^4(d+ex^2)\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{adx^5}{5}+\frac{aex^7}{7}-\frac{bdnx^5}{25}+\frac{bdx^5\log\left(cx^n\right)}{5}-\frac{benx^7}{49}+\frac{bex^7\log\left(cx^n\right)}{7}$$

```
[In] integrate(x**4*(e*x**2+d)*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d*x**5/5 + a*e*x**7/7 - b*d*n*x**5/25 + b*d*x**5*log(c*x**n)/5 - b*e*n*x*
*7/49 + b*e*x**7*log(c*x**n)/7
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int x^4(d+ex^2)\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{49}\,benx^7+\frac{1}{7}\,bex^7\log\left(cx^n\right)+\frac{1}{7}\,aex^7-\frac{1}{25}\,bdnx^5+\frac{1}{5}\,bdx^5\log\left(cx^n\right)+\frac{1}{5}\,adx^5$$

```
[In] integrate(x^4*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/49*b*e*n*x^7 + 1/7*b*e*x^7*log(c*x^n) + 1/7*a*e*x^7 - 1/25*b*d*n*x^5 + 1
/5*b*d*x^5*log(c*x^n) + 1/5*a*d*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int x^4(d+ex^2)(a+b\log(cx^n))\,dx = \frac{1}{7}benx^7\log(x) - \frac{1}{49}benx^7 + \frac{1}{7}bex^7\log(c) + \frac{1}{7}aex^7 + \frac{1}{5}bdnx^5\log(x) - \frac{1}{25}bdnx^5 + \frac{1}{5}bdx^5\log(c) + \frac{1}{5}adx^5$$

```
[In] integrate(x^4*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/7*b*e*n*x^7*log(x) - 1/49*b*e*n*x^7 + 1/7*b*e*x^7*log(c) + 1/7*a*e*x^7 +
1/5*b*d*n*x^5*log(x) - 1/25*b*d*n*x^5 + 1/5*b*d*x^5*log(c) + 1/5*a*d*x^5
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^4(d+ex^2)(a+b\log(cx^n))\,dx = \ln(c\,x^n)\left(\frac{b\,e\,x^7}{7} + \frac{b\,d\,x^5}{5}\right) + \frac{d\,x^5\,(5\,a-b\,n)}{25} + \frac{e\,x^7\,(7\,a-b\,n)}{49}$$

```
[In] int(x^4*(d + e*x^2)*(a + b*log(c*x^n)),x)
```

```
[Out] log(c*x^n)*((b*d*x^5)/5 + (b*e*x^7)/7) + (d*x^5*(5*a - b*n))/25 + (e*x^7*(7
*a - b*n))/49
```

3.178 $\int x^2(d+ex^2)\,(a+b\log{(cx^n)})\,\,dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int x^2\left(d+ex^2\right)\left(a+b\log{(cx^n)}\right)\,dx=-\frac{1}{9}bdnx^3-\frac{1}{25}benx^5+\frac{1}{15}\left(5dx^3+3ex^5\right)\left(a+b\log{(cx^n)}\right)$$

```
[Out] -1/9*b*d*n*x^3-1/25*b*e*n*x^5+1/15*(3*e*x^5+5*d*x^3)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {14, 2371}

$$\int x^2\left(d+ex^2\right)\left(a+b\log{(cx^n)}\right)\,dx=\frac{1}{15}\left(5dx^3+3ex^5\right)\left(a+b\log{(cx^n)}\right)-\frac{1}{9}bdnx^3-\frac{1}{25}benx^5$$

```
[In] Int[x^2*(d + e*x^2)*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/9*(b*d*n*x^3) - (b*e*n*x^5)/25 + ((5*d*x^3 + 3*e*x^5)*(a + b*Log[c*x^n])
)/15
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
```

```
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{15}\left(5dx^3+3ex^5\right)\left(a+b\log\left(cx^n\right)\right)-(bn)\int\left(\frac{dx^2}{3}+\frac{ex^4}{5}\right)\,dx\\ &= -\frac{1}{9}bdnx^3-\frac{1}{25}benx^5+\frac{1}{15}\left(5dx^3+3ex^5\right)\left(a+b\log\left(cx^n\right)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int x^2\left(d+ex^2\right)\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{1}{3}adx^3-\frac{1}{9}bdnx^3+\frac{1}{5}aex^5-\frac{1}{25}benx^5+\frac{1}{3}bdx^3\log\left(cx^n\right)+\frac{1}{5}bex^5\log\left(cx^n\right)$$

```
[In] Integrate[x^2*(d + e*x^2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (a*d*x^3)/3 - (b*d*n*x^3)/9 + (a*e*x^5)/5 - (b*e*n*x^5)/25 + (b*d*x^3*Log[c
*x^n])/3 + (b*e*x^5*Log[c*x^n])/5
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

method	result
parallelrisch	$\frac{x^5\ln(c\,x^n)be}{5}-\frac{ben\,x^5}{25}+\frac{ae\,x^5}{5}+\frac{x^3\ln(c\,x^n)bd}{3}-\frac{bdn\,x^3}{9}+\frac{x^3ad}{3}$
risch	$\frac{b\,x^3\left(3e\,x^2+5d\right)\ln(x^n)}{15}-\frac{i\pi be\,x^5\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)}{10}+\frac{i\pi be\,x^5\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2}{10}+\frac{i\pi be\,x^5\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(}{10}$

```
[In] int(x^2*(e*x^2+d)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*ln(c*x^n)*b*e-1/25*b*e*n*x^5+1/5*a*e*x^5+1/3*x^3*ln(c*x^n)*b*d-1/9*
b*d*n*x^3+1/3*x^3*a*d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int x^2\left(d+ex^2\right)\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{25}\,(ben-5\,ae)x^5-\frac{1}{9}\,(bdn-3\,ad)x^3+\frac{1}{15}\,\left(3\,bex^5+5\,bdx^3\right)\log\left(c\right)+\frac{1}{15}\,\left(3\,benx^5+5\,bdnx^3\right)\log\left(x\right)$$

```
[In] integrate(x^2*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/25*(b*e*n - 5*a*e)*x^5 - 1/9*(b*d*n - 3*a*d)*x^3 + 1/15*(3*b*e*x^5 + 5*b
*d*x^3)*log(c) + 1/15*(3*b*e*n*x^5 + 5*b*d*n*x^3)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int x^2\left(d+ex^2\right)\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{adx^3}{3}+\frac{aex^5}{5}-\frac{bdnx^3}{9}+\frac{bdx^3\log\left(cx^n\right)}{3}-\frac{benx^5}{25}+\frac{bex^5\log\left(cx^n\right)}{5}$$

```
[In] integrate(x**2*(e*x**2+d)*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d*x**3/3 + a*e*x**5/5 - b*d*n*x**3/9 + b*d*x**3*log(c*x**n)/3 - b*e*n*x**
5/25 + b*e*x**5*log(c*x**n)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int x^2\left(d+ex^2\right)\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{25}\,benx^5+\frac{1}{5}\,bex^5\log\left(cx^n\right)+\frac{1}{5}\,aex^5-\frac{1}{9}\,bdnx^3+\frac{1}{3}\,bdx^3\log\left(cx^n\right)+\frac{1}{3}\,adx^3$$

```
[In] integrate(x^2*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/25*b*e*n*x^5 + 1/5*b*e*x^5*log(c*x^n) + 1/5*a*e*x^5 - 1/9*b*d*n*x^3 + 1/
3*b*d*x^3*log(c*x^n) + 1/3*a*d*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int x^2(d+ex^2)\,(a+b\log{(cx^n)})\,\,dx = \frac{1}{5}\,benx^5\log{(x)} - \frac{1}{25}\,benx^5 + \frac{1}{5}\,bex^5\log{(c)} + \frac{1}{5}\,aex^5 + \frac{1}{3}\,bdnx^3\log{(x)} - \frac{1}{9}\,bdnx^3 + \frac{1}{3}\,bdx^3\log{(c)} + \frac{1}{3}\,adx^3$$

```
[In] integrate(x^2*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/5*b*e*n*x^5*log(x) - 1/25*b*e*n*x^5 + 1/5*b*e*x^5*log(c) + 1/5*a*e*x^5 +
1/3*b*d*n*x^3*log(x) - 1/9*b*d*n*x^3 + 1/3*b*d*x^3*log(c) + 1/3*a*d*x^3
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^2(d+ex^2)\,(a+b\log{(cx^n)})\,\,dx = \ln{(c\,x^n)}\,\left(\frac{b\,e\,x^5}{5}+\frac{b\,d\,x^3}{3}\right) + \frac{d\,x^3\,(3\,a-b\,n)}{9} + \frac{e\,x^5\,(5\,a-b\,n)}{25}$$

```
[In] int(x^2*(d + e*x^2)*(a + b*log(c*x^n)),x)
```

```
[Out] log(c*x^n)*((b*d*x^3)/3 + (b*e*x^5)/5) + (d*x^3*(3*a - b*n))/9 + (e*x^5*(5*
a - b*n))/25
```

3.179 $\int (d+ex^2)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 48

$$\int (d+ex^2)(a+b\log(cx^n))\,dx=-bdnx-\frac{1}{9}benx^3+dx(a+b\log(cx^n))+\frac{1}{3}ex^3(a+b\log(cx^n))$$

```
[Out] -b*d*n*x-1/9*b*e*n*x^3+d*x*(a+b*ln(c*x^n))+1/3*e*x^3*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2350}

$$\int (d+ex^2)(a+b\log(cx^n))\,dx=dx(a+b\log(cx^n))+\frac{1}{3}ex^3(a+b\log(cx^n))-bdnx-\frac{1}{9}benx^3$$

```
[In] Int[(d + e*x^2)*(a + b*Log[c*x^n]),x]
```

```
[Out] -(b*d*n*x) - (b*e*n*x^3)/9 + d*x*(a + b*Log[c*x^n]) + (e*x^3*(a + b*Log[c*x
^n]))/3
```

Rule 2350

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Dist[a + b*Log[c*x^n], u
, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=dx(a+b\log(cx^n))+\frac{1}{3}ex^3(a+b\log(cx^n))-(bn)\int\left(d+\frac{ex^2}{3}\right)dx\\&=-bdnx-\frac{1}{9}benx^3+dx(a+b\log(cx^n))+\frac{1}{3}ex^3(a+b\log(cx^n))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \left(d+ex^2\right)\left(a+b\log\left(cx^n\right)\right)\,dx = adx-bdnx+\frac{1}{3}aex^3-\frac{1}{9}benx^3+bdx\log\left(cx^n\right)+\frac{1}{3}bex^3\log\left(cx^n\right)$$

```
[In] Integrate[(d + e*x^2)*(a + b*Log[c*x^n]),x]
```

```
[Out] a*d*x - b*d*n*x + (a*e*x^3)/3 - (b*e*n*x^3)/9 + b*d*x*Log[c*x^n] + (b*e*x^3
*Log[c*x^n])/3
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{x^{3}be\ln\left(c\,x^{n}\right)}{3}-\frac{ben\,x^{3}}{9}+\frac{x^{3}ae}{3}+x\ln\left(c\,x^{n}\right)bd-bdnx+xad$
risch	$\frac{bx\left(e\,x^{2}+3d\right)\ln\left(x^{n}\right)}{3}-\frac{i\pi be\,x^{3}\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(ic\,x^{n}\right)}{6}+\frac{i\pi be\,x^{3}\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\,x^{n}\right)^{2}}{6}+\frac{i\pi be\,x^{3}\,\mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(ic\dots\right.}{6}$

```
[In] int((e*x^2+d)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*b*e*ln(c*x^n)-1/9*b*e*n*x^3+1/3*x^3*a*e+x*ln(c*x^n)*b*d-b*d*n*x+x*a
*d
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int \left(d+ex^2\right)\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{1}{9}\,(ben-3\,ae)x^3-(bdn-ad)x+\frac{1}{3}\,\left(bex^3+3\,bdx\right)\log\left(c\right)+\frac{1}{3}\,\left(benx^3+3\,bdnx\right)\log\left(x\right)$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/9*(b*e*n - 3*a*e)*x^3 - (b*d*n - a*d)*x + 1/3*(b*e*x^3 + 3*b*d*x)*log(c)
 + 1/3*(b*e*n*x^3 + 3*b*d*n*x)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

$$\int (d+ex^2) (a+b\log (cx^n)) \, dx = adx+\frac{aex^3}{3}-bdnx+bdx\log (cx^n)-\frac{benx^3}{9}+\frac{bex^3\log (cx^n)}{3}$$

```
[In] integrate((e*x**2+d)*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d*x + a*e*x**3/3 - b*d*n*x + b*d*x*log(c*x**n) - b*e*n*x**3/9 + b*e*x**3*
log(c*x**n)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int (d+ex^2) (a+b\log (cx^n)) \, dx = -\frac{1}{9}\,benx^3+\frac{1}{3}\,bex^3\log (cx^n)+\frac{1}{3}\,aex^3 - bdnx+bdx\log (cx^n)+adx$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/9*b*e*n*x^3 + 1/3*b*e*x^3*log(c*x^n) + 1/3*a*e*x^3 - b*d*n*x + b*d*x*log
(c*x^n) + a*d*x
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int (d+ex^2) (a+b\log (cx^n)) \, dx = \frac{1}{3}\,benx^3\log (x)-\frac{1}{9}\,benx^3+\frac{1}{3}\,bex^3\log (c)+\frac{1}{3}\,aex^3 + bdnx\log (x)-bdnx+bdx\log (c)+adx$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*e*n*x^3*log(x) - 1/9*b*e*n*x^3 + 1/3*b*e*x^3*log(c) + 1/3*a*e*x^3 + b
*d*n*x*log(x) - b*d*n*x + b*d*x*log(c) + a*d*x
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \left(d+ex^2\right)\left(a+b\log\left(cx^n\right)\right)\,dx=\ln\left(c\,x^n\right)\,\left(\frac{b\,e\,x^3}{3}+b\,d\,x\right)+d\,x\,\left(a-b\,n\right)+\frac{e\,x^3\,\left(3\,a-b\,n\right)}{9}$$

```
[In] int((d + e*x^2)*(a + b*log(c*x^n)),x)
[Out] log(c*x^n)*(b*d*x + (b*e*x^3)/3) + d*x*(a - b*n) + (e*x^3*(3*a - b*n))/9
```

3.180 $\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 44

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^2} dx = -\frac{bdn}{x} - benx - \frac{d(a+b\log(cx^n))}{x} + ex(a+b\log(cx^n))$$

```
[Out] -b*d*n/x-b*e*n*x-d*(a+b*ln(c*x^n))/x+e*x*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {14, 2372}

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^2} dx = -\frac{d(a+b\log(cx^n))}{x} + ex(a+b\log(cx^n)) - \frac{bdn}{x} - benx$$

```
[In] Int[((d + e*x^2)*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -((b*d*n)/x) - b*e*n*x - (d*(a + b*Log[c*x^n]))/x + e*x*(a + b*Log[c*x^n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
```

```
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{d(a+b\log(cx^n))}{x}+ex(a+b\log(cx^n))-(bn)\int\left(e-\frac{d}{x^2}\right)dx\\ &= -\frac{bdn}{x}-benx-\frac{d(a+b\log(cx^n))}{x}+ex(a+b\log(cx^n))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int\frac{(d+ex^2)(a+b\log(cx^n))}{x^2}\,dx=-\frac{ad}{x}-\frac{bdn}{x}+aex-benx-\frac{bd\log(cx^n)}{x}+bex\log(cx^n)$$

```
[In] Integrate[((d + e*x^2)*(a + b*Log[c*x^n]))/x^2,x]
[Out] -((a*d)/x) - (b*d*n)/x + a*e*x - b*e*n*x - (b*d*Log[c*x^n])/x + b*e*x*Log[c
*x^n]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14

method	result
parallelrisch	$-\frac{-be\,x^2\ln(c\,x^n)+ben\,x^2-ae\,x^2+b\ln(c\,x^n)d+bdn+ad}{x}$
risch	$-\frac{b(-e\,x^2+d)\ln(x^n)}{x}-\frac{i\pi\ \text{csgn}(ic)\,\text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)be\,x^2-i\pi\ \text{csgn}(ic)\,\text{csgn}(ic\,x^n)^2be\,x^2-i\pi\ \text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)^2be\,x}{}$

```
[In] int((e*x^2+d)*(a+b*ln(c*x^n))/x^2,x,method=_RETURNVERBOSE)
[Out] -1/x*(-b*e*x^2*ln(c*x^n)+b*e*n*x^2-a*e*x^2+b*ln(c*x^n)*d+b*d*n+a*d)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^2}\,dx = -\frac{bdn+(ben-ae)x^2+ad-(bex^2-bd)\log(c)-(benx^2-bdn)\log(x)}{x}$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
```

```
[Out] -(b*d*n + (b*e*n - a*e)*x^2 + a*d - (b*e*x^2 - b*d)*log(c) - (b*e*n*x^2 - b
*d*n)*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^2}\,dx = -\frac{ad}{x}+aex-\frac{bdn}{x}-\frac{bd\log(cx^n)}{x}-benx+bex\log(cx^n)$$

```
[In] integrate((e*x**2+d)*(a+b*ln(c*x**n))/x**2,x)
```

```
[Out] -a*d/x + a*e*x - b*d*n/x - b*d*log(c*x**n)/x - b*e*n*x + b*e*x*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^2}\,dx = -benx+bex\log(cx^n)+aex-\frac{bdn}{x}-\frac{bd\log(cx^n)}{x}-\frac{ad}{x}$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
```

```
[Out] -b*e*n*x + b*e*x*log(c*x^n) + a*e*x - b*d*n/x - b*d*log(c*x^n)/x - a*d/x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^2}\,dx = -(ben - be\log(c) - ae)x + \left(benx - \frac{bdn}{x}\right)\log(x) - \frac{bdn + bd\log(c) + ad}{x}$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x^2,x, algorithm="giac")
[Out] -(b*e*n - b*e*log(c) - a*e)*x + (b*e*n*x - b*d*n/x)*log(x) - (b*d*n + b*d*l
og(c) + a*d)/x
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^2}\,dx = e\,x\,(a-b\,n) - \ln(c\,x^n)\,\left(\frac{b\,e\,x^2+b\,d}{x} - 2\,b\,e\,x\right) - \frac{a\,d+b\,d\,n}{x}$$

```
[In] int(((d + e*x^2)*(a + b*log(c*x^n)))/x^2,x)
[Out] e*x*(a - b*n) - log(c*x^n)*((b*d + b*e*x^2)/x - 2*b*e*x) - (a*d + b*d*n)/x
```

3.181 $\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^4} dx = -\frac{bdn}{9x^3} - \frac{ben}{x} - \frac{d(a+b\log(cx^n))}{3x^3} - \frac{e(a+b\log(cx^n))}{x}$$

```
[Out] -1/9*b*d*n/x^3-b*e*n/x-1/3*d*(a+b*ln(c*x^n))/x^3-e*(a+b*ln(c*x^n))/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {14, 2372, 12}

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^4} dx = -\frac{d(a+b\log(cx^n))}{3x^3} - \frac{e(a+b\log(cx^n))}{x} - \frac{bdn}{9x^3} - \frac{ben}{x}$$

```
[In] Int[((d + e*x^2)*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] -1/9*(b*d*n)/x^3 - (b*e*n)/x - (d*(a + b*Log[c*x^n]))/(3*x^3) - (e*(a + b*L
og[c*x^n]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(a+b\log(cx^n))}{3x^3} - \frac{e(a+b\log(cx^n))}{x} - (bn)\int \frac{-d-3ex^2}{3x^4}\,dx \\
&= -\frac{d(a+b\log(cx^n))}{3x^3} - \frac{e(a+b\log(cx^n))}{x} - \frac{1}{3}(bn)\int \frac{-d-3ex^2}{x^4}\,dx \\
&= -\frac{d(a+b\log(cx^n))}{3x^3} - \frac{e(a+b\log(cx^n))}{x} - \frac{1}{3}(bn)\int \left(-\frac{d}{x^4} - \frac{3e}{x^2}\right)dx \\
&= -\frac{bdn}{9x^3} - \frac{ben}{x} - \frac{d(a+b\log(cx^n))}{3x^3} - \frac{e(a+b\log(cx^n))}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^4}\,dx = -\frac{ad}{3x^3} - \frac{bdn}{9x^3} - \frac{ae}{x} - \frac{ben}{x} - \frac{bd\log(cx^n)}{3x^3} - \frac{be\log(cx^n)}{x}$$

```
[In] Integrate[((d + e*x^2)*(a + b*Log[c*x^n]))/x^4,x]
[Out] -1/3*(a*d)/x^3 - (b*d*n)/(9*x^3) - (a*e)/x - (b*e*n)/x - (b*d*Log[c*x^n])/(
3*x^3) - (b*e*Log[c*x^n])/x
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

method	result
parallelrisch	$-\frac{9be\,x^2\ln(c\,x^n)+9ben\,x^2+9ae\,x^2+3b\ln(c\,x^n)d+bdn+3ad}{9x^3}$
risch	$-\frac{b(3e\,x^2+d)\ln(x^n)}{3x^3} - \frac{-9i\pi\ \text{csgn}(ic)\,\text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)be\,x^2+9i\pi\ \text{csgn}(ic)\,\text{csgn}(ic\,x^n)^2be\,x^2+9i\pi\ \text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)}{}$

```
[In] int((e*x^2+d)*(a+b*ln(c*x^n))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/x^3*(9*b*e*x^2*ln(c*x^n)+9*b*e*n*x^2+9*a*e*x^2+3*b*ln(c*x^n)*d+b*d*n+3
*a*d)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^4}\,dx = -\frac{bdn+9(ben+ae)x^2+3\,ad+3\,(3\,bex^2+bd)\log(c)+3\,(3\,benx^2+bdn)\log(x)}{9\,x^3}$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(b*d*n + 9*(b*e*n + a*e)*x^2 + 3*a*d + 3*(3*b*e*x^2 + b*d)*log(c) + 3*
(3*b*e*n*x^2 + b*d*n)*log(x))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^4}\,dx = -\frac{ad}{3x^3}-\frac{ae}{x}-\frac{bdn}{9x^3}-\frac{bd\log(cx^n)}{3x^3}-\frac{ben}{x}-\frac{be\log(cx^n)}{x}$$

```
[In] integrate((e*x**2+d)*(a+b*ln(c*x**n))/x**4,x)
```

```
[Out] -a*d/(3*x**3) - a*e/x - b*d*n/(9*x**3) - b*d*log(c*x**n)/(3*x**3) - b*e*n/x
 - b*e*log(c*x**n)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^4}\,dx = -\frac{ben}{x}-\frac{be\log(cx^n)}{x}-\frac{ae}{x}-\frac{bdn}{9\,x^3}-\frac{bd\log(cx^n)}{3\,x^3}-\frac{ad}{3\,x^3}$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x^4,x, algorithm="maxima")
```

```
[Out] -b*e*n/x - b*e*log(c*x^n)/x - a*e/x - 1/9*b*d*n/x^3 - 1/3*b*d*log(c*x^n)/x^
3 - 1/3*a*d/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^4}\,dx = -\frac{(3\,benx^2+bdn)\log(x)}{3\,x^3} - \frac{9\,benx^2+9\,bex^2\log(c)+9\,aex^2+bdn+3\,bd\log(c)+3\,ad}{9\,x^3}$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b*e*n*x^2 + b*d*n)*log(x)/x^3 - 1/9*(9*b*e*n*x^2 + 9*b*e*x^2*log(c)
 + 9*a*e*x^2 + b*d*n + 3*b*d*log(c) + 3*a*d)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^4}\,dx = -\frac{(3\,a\,e+3\,b\,e\,n)\,x^2+a\,d+\frac{b\,d\,n}{3}}{3\,x^3} - \frac{\ln(c\,x^n)\,\left(b\,e\,x^2+\frac{b\,d}{3}\right)}{x^3}$$

```
[In] int(((d + e*x^2)*(a + b*log(c*x^n)))/x^4,x)
```

```
[Out] - (a*d + x^2*(3*a*e + 3*b*e*n) + (b*d*n)/3)/(3*x^3) - (log(c*x^n)*((b*d)/3
+ b*e*x^2))/x^3
```

3.182 $\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^6}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^6}\,dx = -\frac{bdn}{25x^5} - \frac{ben}{9x^3} - \frac{d(a+b\log(cx^n))}{5x^5} - \frac{e(a+b\log(cx^n))}{3x^3}$$

```
[Out] -1/25*b*d*n/x^5-1/9*b*e*n/x^3-1/5*d*(a+b*ln(c*x^n))/x^5-1/3*e*(a+b*ln(c*x^n
))/x^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {14, 2372, 12}

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^6}\,dx = -\frac{d(a+b\log(cx^n))}{5x^5} - \frac{e(a+b\log(cx^n))}{3x^3} - \frac{bdn}{25x^5} - \frac{ben}{9x^3}$$

```
[In] Int[((d + e*x^2)*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] -1/25*(b*d*n)/x^5 - (b*e*n)/(9*x^3) - (d*(a + b*Log[c*x^n]))/(5*x^5) - (e*(
a + b*Log[c*x^n]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(a+b\log(cx^n))}{5x^5}-\frac{e(a+b\log(cx^n))}{3x^3}-(bn)\int\frac{-3d-5ex^2}{15x^6}\,dx\\
&= -\frac{d(a+b\log(cx^n))}{5x^5}-\frac{e(a+b\log(cx^n))}{3x^3}-\frac{1}{15}(bn)\int\frac{-3d-5ex^2}{x^6}\,dx\\
&= -\frac{d(a+b\log(cx^n))}{5x^5}-\frac{e(a+b\log(cx^n))}{3x^3}-\frac{1}{15}(bn)\int\left(-\frac{3d}{x^6}-\frac{5e}{x^4}\right)dx\\
&= -\frac{bdn}{25x^5}-\frac{ben}{9x^3}-\frac{d(a+b\log(cx^n))}{5x^5}-\frac{e(a+b\log(cx^n))}{3x^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int\frac{(d+ex^2)(a+b\log(cx^n))}{x^6}\,dx=-\frac{ad}{5x^5}-\frac{bdn}{25x^5}-\frac{ae}{3x^3}-\frac{ben}{9x^3}-\frac{bd\log(cx^n)}{5x^5}-\frac{be\log(cx^n)}{3x^3}$$

```
[In] Integrate[((d + e*x^2)*(a + b*Log[c*x^n]))/x^6,x]
[Out] -1/5*(a*d)/x^5 - (b*d*n)/(25*x^5) - (a*e)/(3*x^3) - (b*e*n)/(9*x^3) - (b*d*
Log[c*x^n])/(5*x^5) - (b*e*Log[c*x^n])/(3*x^3)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

method	result
parallelrisch	$-\frac{75be\,x^2\ln(c\,x^n)+25ben\,x^2+75ae\,x^2+45b\ln(c\,x^n)d+9bdn+45ad}{225x^5}$
risch	$-\frac{b(5e\,x^2+3d)\ln(x^n)}{15x^5}-\frac{-75i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)be\,x^2+75i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2be\,x^2+75i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(i}{}$

```
[In] int((e*x^2+d)*(a+b*ln(c*x^n))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/225/x^5*(75*b*e*x^2*ln(c*x^n)+25*b*e*n*x^2+75*a*e*x^2+45*b*ln(c*x^n)*d+9
*b*d*n+45*a*d)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^6}\,dx = -\frac{9\,bdn+25\,(ben+3\,ae)x^2+45\,ad+15\,(5\,bex^2+3\,bd)\log(c)+15\,(5\,benx^2+3\,bdn)\log(x)}{225\,x^5}$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*(9*b*d*n + 25*(b*e*n + 3*a*e)*x^2 + 45*a*d + 15*(5*b*e*x^2 + 3*b*d)*
log(c) + 15*(5*b*e*n*x^2 + 3*b*d*n)*log(x))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^6}\,dx = -\frac{ad}{5x^5}-\frac{ae}{3x^3}-\frac{bdn}{25x^5}-\frac{bd\log(cx^n)}{5x^5}-\frac{ben}{9x^3}-\frac{be\log(cx^n)}{3x^3}$$

```
[In] integrate((e*x**2+d)*(a+b*ln(c*x**n))/x**6,x)
```

```
[Out] -a*d/(5*x**5) - a*e/(3*x**3) - b*d*n/(25*x**5) - b*d*log(c*x**n)/(5*x**5) -
 b*e*n/(9*x**3) - b*e*log(c*x**n)/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^6}\,dx = -\frac{ben}{9\,x^3}-\frac{be\log(cx^n)}{3\,x^3}-\frac{ae}{3\,x^3}-\frac{bdn}{25\,x^5}-\frac{bd\log(cx^n)}{5\,x^5}-\frac{ad}{5\,x^5}$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x^6,x, algorithm="maxima")
```

```
[Out] -1/9*b*e*n/x^3 - 1/3*b*e*log(c*x^n)/x^3 - 1/3*a*e/x^3 - 1/25*b*d*n/x^5 - 1/
5*b*d*log(c*x^n)/x^5 - 1/5*a*d/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^6}\,dx = -\frac{(5\,benx^2+3\,bdn)\log(x)}{15\,x^5} - \frac{25\,benx^2+75\,bex^2\log(c)+75\,aex^2+9\,bdn+45\,bd\log(c)+45\,ad}{225\,x^5}$$

```
[In] integrate((e*x^2+d)*(a+b*log(c*x^n))/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(5*b*e*n*x^2 + 3*b*d*n)*log(x)/x^5 - 1/225*(25*b*e*n*x^2 + 75*b*e*x^2
*log(c) + 75*a*e*x^2 + 9*b*d*n + 45*b*d*log(c) + 45*a*d)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex^2)(a+b\log(cx^n))}{x^6}\,dx = -\frac{\left(5\,a\,e+\frac{5\,b\,e\,n}{3}\right)\,x^2+3\,a\,d+\frac{3\,b\,d\,n}{5}}{15\,x^5} - \frac{\ln(c\,x^n)\,\left(\frac{b\,e\,x^2}{3}+\frac{b\,d}{5}\right)}{x^5}$$

```
[In] int(((d + e*x^2)*(a + b*log(c*x^n)))/x^6,x)
```

```
[Out] - (3*a*d + x^2*(5*a*e + (5*b*e*n)/3) + (3*b*d*n)/5)/(15*x^5) - (log(c*x^n)*
((b*d)/5 + (b*e*x^2)/3))/x^5
```

3.183 $\int x^5(d+ex^2)^2 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int x^5(d+ex^2)^2 (a+b\log(cx^n))\,dx = -\frac{1}{36}bd^2nx^6 - \frac{1}{32}bdenx^8 - \frac{1}{100}be^2nx^{10} + \frac{1}{60}(10d^2x^6+15dex^8+6e^2x^{10})(a+b\log(cx^n))$$

```
[Out] -1/36*b*d^2*n*x^6-1/32*b*d*e*n*x^8-1/100*b*e^2*n*x^10+1/60*(6*e^2*x^10+15*d
*e*x^8+10*d^2*x^6)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {272, 45, 2371, 12, 14}

$$\int x^5(d+ex^2)^2 (a+b\log(cx^n))\,dx = \frac{1}{60}(10d^2x^6+15dex^8+6e^2x^{10})(a+b\log(cx^n)) - \frac{1}{36}bd^2nx^6 - \frac{1}{32}bdenx^8 - \frac{1}{100}be^2nx^{10}$$

```
[In] Int[x^5*(d + e*x^2)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/36*(b*d^2*n*x^6) - (b*d*e*n*x^8)/32 - (b*e^2*n*x^10)/100 + ((10*d^2*x^6
+ 15*d*e*x^8 + 6*e^2*x^10)*(a + b*Log[c*x^n]))/60
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{60}\left(10d^2x^6+15dex^8+6e^2x^{10}\right)\left(a+b\log\left(cx^n\right)\right) \\
&\quad -(bn)\int\frac{1}{60}x^5\left(10d^2+15dex^2+6e^2x^4\right)\,dx \\
&= \frac{1}{60}\left(10d^2x^6+15dex^8+6e^2x^{10}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{60}(bn)\int x^5\left(10d^2+15dex^2+6e^2x^4\right)\,dx \\
&= \frac{1}{60}\left(10d^2x^6+15dex^8+6e^2x^{10}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{60}(bn)\int\left(10d^2x^5+15dex^7+6e^2x^9\right)\,dx \\
&= -\frac{1}{36}bd^2nx^6-\frac{1}{32}bdenx^8-\frac{1}{100}be^2nx^{10}+\frac{1}{60}\left(10d^2x^6+15dex^8+6e^2x^{10}\right)\left(a+b\log\left(cx^n\right)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

$$\int x^5(d+ex^2)^2(a+b\log(cx^n))\,dx$$
$$=\frac{x^6(-200bd^2n-225bdenx^2-72be^2nx^4+1200d^2(a+b\log(cx^n))+1800dex^2(a+b\log(cx^n))+720e^2x^4(a$$
$$7200$$

```
[In] Integrate[x^5*(d + e*x^2)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^6*(-200*b*d^2*n - 225*b*d*e*n*x^2 - 72*b*e^2*n*x^4 + 1200*d^2*(a + b*Log
[c*x^n]) + 1800*d*e*x^2*(a + b*Log[c*x^n]) + 720*e^2*x^4*(a + b*Log[c*x^n])
))/7200
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36

method	result
parallelrisch	$\frac{x^{10}\ln(c\,x^n)b\,e^2}{10}-\frac{b\,e^2n\,x^{10}}{100}+\frac{a\,e^2x^{10}}{10}+\frac{x^8\ln(c\,x^n)bde}{4}-\frac{bden\,x^8}{32}+\frac{ade\,x^8}{4}+\frac{x^6\ln(c\,x^n)b\,d^2}{6}-\frac{b\,d^2n\,x^6}{36}+\frac{a\,d^2x^6}{6}$
risch	$\frac{b\,x^6(6e^2x^4+15de\,x^2+10d^2)\ln(x^n)}{60}-\frac{i\pi bde\,x^8\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)}{8}+\frac{i\pi bde\,x^8\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2}{8}+\frac{i\pi b\,d^2x^6}{}$

```
[In] int(x^5*(e*x^2+d)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*x^10*ln(c*x^n)*b*e^2-1/100*b*e^2*n*x^10+1/10*a*e^2*x^10+1/4*x^8*ln(c*x
^n)*b*d*e-1/32*b*d*e*n*x^8+1/4*a*d*e*x^8+1/6*x^6*ln(c*x^n)*b*d^2-1/36*b*d^2
*n*x^6+1/6*a*d^2*x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.59

$$\begin{aligned}\int x^5(d+ex^2)^2(a+b\log(cx^n))\,dx =& -\frac{1}{100}\,(be^2n-10\,ae^2)x^{10}\\&-\frac{1}{32}\,(bden-8\,ade)x^8-\frac{1}{36}\,(bd^2n-6\,ad^2)x^6\\&+\frac{1}{60}\,(6\,be^2x^{10}+15\,bdex^8+10\,bd^2x^6)\log(c)\\&+\frac{1}{60}\,(6\,be^2nx^{10}+15\,bdenx^8+10\,bd^2nx^6)\log(x)\end{aligned}$$

```
[In] integrate(x^5*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/100*(b*e^2*n - 10*a*e^2)*x^10 - 1/32*(b*d*e*n - 8*a*d*e)*x^8 - 1/36*(b*d
^2*n - 6*a*d^2)*x^6 + 1/60*(6*b*e^2*x^10 + 15*b*d*e*x^8 + 10*b*d^2*x^6)*log
(c) + 1/60*(6*b*e^2*n*x^10 + 15*b*d*e*n*x^8 + 10*b*d^2*n*x^6)*log(x)
```

Sympy [A] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.57

$$\int x^5(d+ex^2)^2(a+b\log(cx^n))\,dx = \frac{ad^2x^6}{6}+\frac{adex^8}{4}+\frac{ae^2x^{10}}{10}-\frac{bd^2nx^6}{36}+\frac{bd^2x^6\log(cx^n)}{6}-\frac{bdenx^8}{32}+\frac{bdex^8\log(cx^n)}{4}-\frac{be^2nx^{10}}{100}+\frac{be^2x^{10}\log(cx^n)}{10}$$

```
[In] integrate(x**5*(e*x**2+d)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**2*x**6/6 + a*d*e*x**8/4 + a*e**2*x**10/10 - b*d**2*n*x**6/36 + b*d**2*
x**6*log(c*x**n)/6 - b*d*e*n*x**8/32 + b*d*e*x**8*log(c*x**n)/4 - b*e**2*n*
x**10/100 + b*e**2*x**10*log(c*x**n)/10
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int x^5(d+ex^2)^2(a+b\log(cx^n))\,dx = -\frac{1}{100}\,be^2nx^{10}+\frac{1}{10}\,be^2x^{10}\log(cx^n)+\frac{1}{10}\,ae^2x^{10}-\frac{1}{32}\,bdenx^8+\frac{1}{4}\,bdex^8\log(cx^n)+\frac{1}{4}\,adex^8-\frac{1}{36}\,bd^2nx^6+\frac{1}{6}\,bd^2x^6\log(cx^n)+\frac{1}{6}\,ad^2x^6$$

```
[In] integrate(x^5*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/100*b*e^2*n*x^10 + 1/10*b*e^2*x^10*log(c*x^n) + 1/10*a*e^2*x^10 - 1/32*b
*d*e*n*x^8 + 1/4*b*d*e*x^8*log(c*x^n) + 1/4*a*d*e*x^8 - 1/36*b*d^2*n*x^6 +
1/6*b*d^2*x^6*log(c*x^n) + 1/6*a*d^2*x^6
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.66

$$\int x^5 (d+ex^2)^2 (a+b\log(cx^n))\,dx = \frac{1}{10}\,be^2nx^{10}\log(x) - \frac{1}{100}\,be^2nx^{10} + \frac{1}{10}\,be^2x^{10}\log(c) + \frac{1}{10}\,ae^2x^{10} + \frac{1}{4}\,bdenx^8\log(x) - \frac{1}{32}\,bdenx^8 + \frac{1}{4}\,bdex^8\log(c) + \frac{1}{4}\,adex^8 + \frac{1}{6}\,bd^2nx^6\log(x) - \frac{1}{36}\,bd^2nx^6 + \frac{1}{6}\,bd^2x^6\log(c) + \frac{1}{6}\,ad^2x^6$$

```
[In] integrate(x^5*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/10*b*e^2*n*x^10*log(x) - 1/100*b*e^2*n*x^10 + 1/10*b*e^2*x^10*log(c) + 1/
10*a*e^2*x^10 + 1/4*b*d*e*n*x^8*log(x) - 1/32*b*d*e*n*x^8 + 1/4*b*d*e*x^8*l
og(c) + 1/4*a*d*e*x^8 + 1/6*b*d^2*n*x^6*log(x) - 1/36*b*d^2*n*x^6 + 1/6*b*d
^2*x^6*log(c) + 1/6*a*d^2*x^6
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int x^5 (d+ex^2)^2 (a+b\log(cx^n))\,dx = \ln(c\,x^n)\,\left(\frac{b\,d^2\,x^6}{6} + \frac{b\,d\,e\,x^8}{4} + \frac{b\,e^2\,x^{10}}{10}\right) + \frac{d^2\,x^6\,(6\,a-b\,n)}{36} + \frac{e^2\,x^{10}\,(10\,a-b\,n)}{100} + \frac{d\,e\,x^8\,(8\,a-b\,n)}{32}$$

```
[In] int(x^5*(d + e*x^2)^2*(a + b*log(c*x^n)),x)
```

```
[Out] log(c*x^n)*((b*d^2*x^6)/6 + (b*e^2*x^10)/10 + (b*d*e*x^8)/4) + (d^2*x^6*(6*
a - b*n))/36 + (e^2*x^10*(10*a - b*n))/100 + (d*e*x^8*(8*a - b*n))/32
```

3.184 $\int x^3(d+ex^2)^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int x^3(d+ex^2)^2(a+b\log(cx^n))\,dx = -\frac{1}{16}bd^2nx^4-\frac{1}{18}bdenx^6-\frac{1}{64}be^2nx^8+\frac{1}{24}(6d^2x^4+8dex^6+3e^2x^8)(a+b\log(cx^n))$$

```
[Out] -1/16*b*d^2*n*x^4-1/18*b*d*e*n*x^6-1/64*b*e^2*n*x^8+1/24*(3*e^2*x^8+8*d*e*x
^6+6*d^2*x^4)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {272, 45, 2371, 12, 14}

$$\int x^3(d+ex^2)^2(a+b\log(cx^n))\,dx = \frac{1}{24}(6d^2x^4+8dex^6+3e^2x^8)(a+b\log(cx^n))-\frac{1}{16}bd^2nx^4-\frac{1}{18}bdenx^6-\frac{1}{64}be^2nx^8$$

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/16*(b*d^2*n*x^4) - (b*d*e*n*x^6)/18 - (b*e^2*n*x^8)/64 + ((6*d^2*x^4 + 8
*d*e*x^6 + 3*e^2*x^8)*(a + b*Log[c*x^n]))/24
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{24}\left(6d^2x^4+8dex^6+3e^2x^8\right)(a+b\log(cx^n))-(bn)\int\frac{1}{24}x^3\left(6d^2+8dex^2+3e^2x^4\right)\,dx\\
&= \frac{1}{24}\left(6d^2x^4+8dex^6+3e^2x^8\right)(a+b\log(cx^n))-\frac{1}{24}(bn)\int x^3\left(6d^2+8dex^2+3e^2x^4\right)\,dx\\
&= \frac{1}{24}\left(6d^2x^4+8dex^6+3e^2x^8\right)(a+b\log(cx^n))-\frac{1}{24}(bn)\int\left(6d^2x^3+8dex^5+3e^2x^7\right)\,dx\\
&= -\frac{1}{16}bd^2nx^4-\frac{1}{18}bdenx^6-\frac{1}{64}be^2nx^8+\frac{1}{24}\left(6d^2x^4+8dex^6+3e^2x^8\right)(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18

$$\int x^3 \left(d+ex^2\right)^2 \left(a+b\log\left(cx^n\right)\right)\,dx = \frac{1}{576}x^4\left(24a\left(6d^2+8dex^2+3e^2x^4\right) - bn\left(36d^2+32dex^2+9e^2x^4\right) + 24b\left(6d^2+8dex^2+3e^2x^4\right)\log\left(cx^n\right)\right)$$

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^4*(24*a*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) - b*n*(36*d^2 + 32*d*e*x^2 + 9*e
^2*x^4) + 24*b*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4)*Log[c*x^n]))/576
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36

method	result
parallelrisch	$\frac{x^8\ln(cx^n)be^2}{8}-\frac{be^2nx^8}{64}+\frac{ae^2x^8}{8}+\frac{x^6\ln(cx^n)bde}{3}-\frac{bdenx^6}{18}+\frac{adex^6}{3}+\frac{x^4\ln(cx^n)bd^2}{4}-\frac{bd^2nx^4}{16}+\frac{ad^2x^4}{4}$
risch	$\frac{bx^4(3e^2x^4+8dex^2+6d^2)\ln(x^n)}{24}+\frac{i\pi bd^2x^4\,\mathrm{csgn}(ic)\,\mathrm{csgn}(icx^n)^2}{8}-\frac{i\pi bdex^6\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)}{6}+\frac{i\pi be^2x^8\,\mathrm{cs}}{}$

```
[In] int(x^3*(e*x^2+d)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^8*ln(c*x^n)*b*e^2-1/64*b*e^2*n*x^8+1/8*a*e^2*x^8+1/3*x^6*ln(c*x^n)*b*
d*e-1/18*b*d*e*n*x^6+1/3*a*d*e*x^6+1/4*x^4*ln(c*x^n)*b*d^2-1/16*b*d^2*n*x^4
+1/4*a*d^2*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.59

$$\int x^3 \left(d+ex^2\right)^2 \left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{1}{64}\left(be^2n-8\,ae^2\right)x^8 - \frac{1}{18}\left(bden-6\,ade\right)x^6 - \frac{1}{16}\left(bd^2n-4\,ad^2\right)x^4 + \frac{1}{24}\left(3\,be^2x^8+8\,bdex^6+6\,bd^2x^4\right)\log\left(c\right) + \frac{1}{24}\left(3\,be^2nx^8+8\,bdenx^6+6\,bd^2nx^4\right)\log\left(x\right)$$

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/64*(b*e^2*n - 8*a*e^2)*x^8 - 1/18*(b*d*e*n - 6*a*d*e)*x^6 - 1/16*(b*d^2*
n - 4*a*d^2)*x^4 + 1/24*(3*b*e^2*x^8 + 8*b*d*e*x^6 + 6*b*d^2*x^4)*log(c) +
1/24*(3*b*e^2*n*x^8 + 8*b*d*e*n*x^6 + 6*b*d^2*n*x^4)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.57

$$\int x^3(d+ex^2)^2(a+b\log(cx^n))\,dx=\frac{ad^2x^4}{4}+\frac{adex^6}{3}+\frac{ae^2x^8}{8}-\frac{bd^2nx^4}{16}+\frac{bd^2x^4\log(cx^n)}{4}-\frac{bdenx^6}{18}+\frac{bdex^6\log(cx^n)}{3}-\frac{be^2nx^8}{64}+\frac{be^2x^8\log(cx^n)}{8}$$

```
[In] integrate(x**3*(e*x**2+d)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 - b*d**2*n*x**4/16 + b*d**2*x*
*4*log(c*x**n)/4 - b*d*e*n*x**6/18 + b*d*e*x**6*log(c*x**n)/3 - b*e**2*n*x*
*8/64 + b*e**2*x**8*log(c*x**n)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int x^3(d+ex^2)^2(a+b\log(cx^n))\,dx=-\frac{1}{64}be^2nx^8+\frac{1}{8}be^2x^8\log(cx^n)+\frac{1}{8}ae^2x^8-\frac{1}{18}bdenx^6+\frac{1}{3}bdex^6\log(cx^n)+\frac{1}{3}adex^6-\frac{1}{16}bd^2nx^4+\frac{1}{4}bd^2x^4\log(cx^n)+\frac{1}{4}ad^2x^4$$

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/64*b*e^2*n*x^8 + 1/8*b*e^2*x^8*log(c*x^n) + 1/8*a*e^2*x^8 - 1/18*b*d*e*n
*x^6 + 1/3*b*d*e*x^6*log(c*x^n) + 1/3*a*d*e*x^6 - 1/16*b*d^2*n*x^4 + 1/4*b*
d^2*x^4*log(c*x^n) + 1/4*a*d^2*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.66

$$\int x^3 (d + ex^2)^2 (a + b\log(cx^n)) \, dx = \frac{1}{8} be^2nx^8 \log(x) - \frac{1}{64} be^2nx^8 + \frac{1}{8} be^2x^8 \log(c) + \frac{1}{8} ae^2x^8 + \frac{1}{3} bdenx^6 \log(x) - \frac{1}{18} bdenx^6 + \frac{1}{3} bdex^6 \log(c) + \frac{1}{3} adex^6 + \frac{1}{4} bd^2nx^4 \log(x) - \frac{1}{16} bd^2nx^4 + \frac{1}{4} bd^2x^4 \log(c) + \frac{1}{4} ad^2x^4$$

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/8*b*e^2*n*x^8*log(x) - 1/64*b*e^2*n*x^8 + 1/8*b*e^2*x^8*log(c) + 1/8*a*e^
2*x^8 + 1/3*b*d*e*n*x^6*log(x) - 1/18*b*d*e*n*x^6 + 1/3*b*d*e*x^6*log(c) +
1/3*a*d*e*x^6 + 1/4*b*d^2*n*x^4*log(x) - 1/16*b*d^2*n*x^4 + 1/4*b*d^2*x^4*l
og(c) + 1/4*a*d^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int x^3 (d+ex^2)^2 (a+b\log(cx^n)) \, dx = \ln(c\,x^n) \left(\frac{b\,d^2\,x^4}{4} + \frac{b\,d\,e\,x^6}{3} + \frac{b\,e^2\,x^8}{8}\right) + \frac{d^2\,x^4\,(4\,a - b\,n)}{16} + \frac{e^2\,x^8\,(8\,a - b\,n)}{64} + \frac{d\,e\,x^6\,(6\,a - b\,n)}{18}$$

```
[In] int(x^3*(d + e*x^2)^2*(a + b*log(c*x^n)),x)
```

```
[Out] log(c*x^n)*((b*d^2*x^4)/4 + (b*e^2*x^8)/8 + (b*d*e*x^6)/3) + (d^2*x^4*(4*a
- b*n))/16 + (e^2*x^8*(8*a - b*n))/64 + (d*e*x^6*(6*a - b*n))/18
```

3.185 $\int x(d+ex^2)^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 76

$$\int x(d+ex^2)^2(a+b\log(cx^n))\,dx = -\frac{1}{4}bd^2nx^2-\frac{1}{8}bdenx^4-\frac{1}{36}be^2nx^6-\frac{bd^3n\log(x)}{6e}+\frac{(d+ex^2)^3(a+b\log(cx^n))}{6e}$$

```
[Out] -1/4*b*d^2*n*x^2-1/8*b*d*e*n*x^4-1/36*b*e^2*n*x^6-1/6*b*d^3*n*ln(x)/e+1/6*(
e*x^2+d)^3*(a+b*ln(c*x^n))/e
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {267, 2371, 12, 272, 45}

$$\int x(d+ex^2)^2(a+b\log(cx^n))\,dx = \frac{(d+ex^2)^3(a+b\log(cx^n))}{6e}-\frac{bd^3n\log(x)}{6e}-\frac{1}{4}bd^2nx^2-\frac{1}{8}bdenx^4-\frac{1}{36}be^2nx^6$$

```
[In] Int[x*(d + e*x^2)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/4*(b*d^2*n*x^2) - (b*d*e*n*x^4)/8 - (b*e^2*n*x^6)/36 - (b*d^3*n*Log[x])/
(6*e) + ((d + e*x^2)^3*(a + b*Log[c*x^n]))/(6*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex^2)^3\,(a+b\log(cx^n))}{6e} - (bn)\int \frac{(d+ex^2)^3}{6ex}\,dx \\
&= \frac{(d+ex^2)^3\,(a+b\log(cx^n))}{6e} - \frac{(bn)\int \frac{(d+ex^2)^3}{x}\,dx}{6e} \\
&= \frac{(d+ex^2)^3\,(a+b\log(cx^n))}{6e} - \frac{(bn)\text{Subst}\left(\int \frac{(d+ex)^3}{x}\,dx, x, x^2\right)}{12e} \\
&= \frac{(d+ex^2)^3\,(a+b\log(cx^n))}{6e} - \frac{(bn)\text{Subst}\left(\int \left(3d^2e+\frac{d^3}{x}+3de^2x+e^3x^2\right)\,dx, x, x^2\right)}{12e} \\
&= -\frac{1}{4}bd^2nx^2 - \frac{1}{8}bdenx^4 - \frac{1}{36}be^2nx^6 - \frac{bd^3n\log(x)}{6e} + \frac{(d+ex^2)^3\,(a+b\log(cx^n))}{6e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int x(d+ex^2)^2(a+b\log(cx^n))\,dx = \frac{1}{72}x^2\left(12a(3d^2+3dex^2+e^2x^4)-bn(18d^2+9dex^2+2e^2x^4)\right.$$
$$\left.+12b(3d^2+3dex^2+e^2x^4)\log(cx^n)\right)$$

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^2*(12*a*(3*d^2 + 3*d*e*x^2 + e^2*x^4) - b*n*(18*d^2 + 9*d*e*x^2 + 2*e^2*
x^4) + 12*b*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*Log[c*x^n]))/72
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.33

method	result
parallelrisch	$\frac{x^6\ln(cx^n)be^2}{6}-\frac{be^2nx^6}{36}+\frac{ae^2x^6}{6}+\frac{x^4b\ln(cx^n)de}{2}-\frac{bdenx^4}{8}+\frac{x^4ade}{2}+\frac{x^2b\ln(cx^n)d^2}{2}-\frac{bd^2nx^2}{4}+\frac{ad^2x^2}{2}$
risch	$\frac{(ex^2+d)^3b\ln(x^n)}{6e}+\frac{i\pi be^2x^6\,\mathrm{csgn}(ic)\,\mathrm{csgn}(icx^n)^2}{12}+\frac{i\pi bdex^4\,\mathrm{csgn}(ic)\,\mathrm{csgn}(icx^n)^2}{4}-\frac{i\pi bdex^4\,\mathrm{csgn}(icx^n)^3}{4}-\frac{i\pi bd^2x^2\,\mathrm{c}}{}$

```
[In] int(x*(e*x^2+d)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^6*ln(c*x^n)*b*e^2-1/36*b*e^2*n*x^6+1/6*a*e^2*x^6+1/2*x^4*b*ln(c*x^n)*
d*e-1/8*b*d*e*n*x^4+1/2*x^4*a*d*e+1/2*x^2*b*ln(c*x^n)*d^2-1/4*b*d^2*n*x^2+1
/2*a*d^2*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.53

$$\int x(d+ex^2)^2(a+b\log(cx^n))\,dx = -\frac{1}{36}(be^2n-6ae^2)x^6 - \frac{1}{8}(bden-4ade)x^4 - \frac{1}{4}(bd^2n-2ad^2)x^2 + \frac{1}{6}(be^2x^6+3bdex^4+3bd^2x^2)\log(c) + \frac{1}{6}(be^2nx^6+3bdenx^4+3bd^2nx^2)\log(x)$$

```
[In] integrate(x*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/36*(b*e^2*n - 6*a*e^2)*x^6 - 1/8*(b*d*e*n - 4*a*d*e)*x^4 - 1/4*(b*d^2*n
- 2*a*d^2)*x^2 + 1/6*(b*e^2*x^6 + 3*b*d*e*x^4 + 3*b*d^2*x^2)*log(c) + 1/6*(
b*e^2*n*x^6 + 3*b*d*e*n*x^4 + 3*b*d^2*n*x^2)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.53

$$\int x(d+ex^2)^2 (a+b\log(cx^n)) \, dx = \frac{ad^2x^2}{2} + \frac{adex^4}{2} + \frac{ae^2x^6}{6} - \frac{bd^2nx^2}{4} + \frac{bd^2x^2\log(cx^n)}{2} - \frac{bdenx^4}{8} + \frac{bdex^4\log(cx^n)}{2} - \frac{be^2nx^6}{36} + \frac{be^2x^6\log(cx^n)}{6}$$

```
[In] integrate(x*(e*x**2+d)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 - b*d**2*n*x**2/4 + b*d**2*x**
2*log(c*x**n)/2 - b*d*e*n*x**4/8 + b*d*e*x**4*log(c*x**n)/2 - b*e**2*n*x**6
/36 + b*e**2*x**6*log(c*x**n)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.32

$$\int x(d+ex^2)^2 (a+b\log(cx^n)) \, dx = -\frac{1}{36}be^2nx^6 + \frac{1}{6}be^2x^6\log(cx^n) + \frac{1}{6}ae^2x^6 - \frac{1}{8}bdenx^4 + \frac{1}{2}bdex^4\log(cx^n) + \frac{1}{2}adex^4 - \frac{1}{4}bd^2nx^2 + \frac{1}{2}bd^2x^2\log(cx^n) + \frac{1}{2}ad^2x^2$$

```
[In] integrate(x*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/36*b*e^2*n*x^6 + 1/6*b*e^2*x^6*log(c*x^n) + 1/6*a*e^2*x^6 - 1/8*b*d*e*n*
x^4 + 1/2*b*d*e*x^4*log(c*x^n) + 1/2*a*d*e*x^4 - 1/4*b*d^2*n*x^2 + 1/2*b*d^
2*x^2*log(c*x^n) + 1/2*a*d^2*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.62

$$\int x(d+ex^2)^2 (a+b\log(cx^n)) \, dx = \frac{1}{6}be^2nx^6\log(x) - \frac{1}{36}be^2nx^6 + \frac{1}{6}be^2x^6\log(c) + \frac{1}{6}ae^2x^6 + \frac{1}{2}bdenx^4\log(x) - \frac{1}{8}bdenx^4 + \frac{1}{2}bdex^4\log(c) + \frac{1}{2}adex^4 + \frac{1}{2}bd^2nx^2\log(x) - \frac{1}{4}bd^2nx^2 + \frac{1}{2}bd^2x^2\log(c) + \frac{1}{2}ad^2x^2$$

```
[In] integrate(x*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/6*b*e^2*n*x^6*log(x) - 1/36*b*e^2*n*x^6 + 1/6*b*e^2*x^6*log(c) + 1/6*a*e^
2*x^6 + 1/2*b*d*e*n*x^4*log(x) - 1/8*b*d*e*n*x^4 + 1/2*b*d*e*x^4*log(c) + 1
/2*a*d*e*x^4 + 1/2*b*d^2*n*x^2*log(x) - 1/4*b*d^2*n*x^2 + 1/2*b*d^2*x^2*log
(c) + 1/2*a*d^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int x\left(d+ex^2\right)^2\left(a+b\log\left(cx^n\right)\right)\,dx=\ln\left(c\,x^n\right)\,\left(\frac{b\,d^2\,x^2}{2}+\frac{b\,d\,e\,x^4}{2}+\frac{b\,e^2\,x^6}{6}\right)+\frac{d^2\,x^2\,\left(2\,a-b\,n\right)}{4}+\frac{e^2\,x^6\,\left(6\,a-b\,n\right)}{36}+\frac{d\,e\,x^4\,\left(4\,a-b\,n\right)}{8}$$

```
[In] int(x*(d + e*x^2)^2*(a + b*log(c*x^n)),x)
```

```
[Out] log(c*x^n)*((b*d^2*x^2)/2 + (b*e^2*x^6)/6 + (b*d*e*x^4)/2) + (d^2*x^2*(2*a
- b*n))/4 + (e^2*x^6*(6*a - b*n))/36 + (d*e*x^4*(4*a - b*n))/8
```

3.186 $\int \frac{(d+ex^2)^2(a+b\log(cx^n))}{x} dx$

Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x} dx = -\frac{1}{2}bdenx^2 - \frac{1}{16}be^2nx^4 - \frac{1}{2}bd^2n\log^2(x) + dex^2(a+b\log(cx^n)) + \frac{1}{4}e^2x^4(a+b\log(cx^n)) + d^2\log(x)(a+b\log(cx^n))$$

```
[Out] -1/2*b*d*e*n*x^2-1/16*b*e^2*n*x^4-1/2*b*d^2*n*ln(x)^2+d*e*x^2*(a+b*ln(c*x^n
))+1/4*e^2*x^4*(a+b*ln(c*x^n))+d^2*ln(x)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {272, 45, 2372, 2338}

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x} dx = d^2\log(x)(a+b\log(cx^n)) + dex^2(a+b\log(cx^n)) + \frac{1}{4}e^2x^4(a+b\log(cx^n)) - \frac{1}{2}bd^2n\log^2(x) - \frac{1}{2}bdenx^2 - \frac{1}{16}be^2nx^4$$

```
[In] Int[((d + e*x^2)^2*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] -1/2*(b*d*e*n*x^2) - (b*e^2*n*x^4)/16 - (b*d^2*n*Log[x]^2)/2 + d*e*x^2*(a +
 b*Log[c*x^n]) + (e^2*x^4*(a + b*Log[c*x^n]))/4 + d^2*Log[x]*(a + b*Log[c*x
^n])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= dex^2(a+b\log(cx^n))+\frac{1}{4}e^2x^4(a+b\log(cx^n)) \\
&\quad +d^2\log(x)(a+b\log(cx^n))-(bn)\int\left(dex+\frac{e^2x^3}{4}+\frac{d^2\log(x)}{x}\right)dx \\
&= -\frac{1}{2}bdenx^2-\frac{1}{16}be^2nx^4+dex^2(a+b\log(cx^n))+\frac{1}{4}e^2x^4(a+b\log(cx^n)) \\
&\quad +d^2\log(x)(a+b\log(cx^n))-\left(bd^2n\right)\int\frac{\log(x)}{x}\,dx \\
&= -\frac{1}{2}bdenx^2-\frac{1}{16}be^2nx^4-\frac{1}{2}bd^2n\log^2(x)+dex^2(a+b\log(cx^n)) \\
&\quad +\frac{1}{4}e^2x^4(a+b\log(cx^n))+d^2\log(x)(a+b\log(cx^n))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x}\,dx = \frac{1}{16}\left(-8bdenx^2 - be^2nx^4 + 16dex^2(a+b\log(cx^n)) + 4e^2x^4(a+b\log(cx^n)) + \frac{8d^2(a+b\log(cx^n))^2}{bn}\right)$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] (-8*b*d*e*n*x^2 - b*e^2*n*x^4 + 16*d*e*x^2*(a + b*Log[c*x^n]) + 4*e^2*x^4*(
a + b*Log[c*x^n]) + (8*d^2*(a + b*Log[c*x^n])^2)/(b*n))/16
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

method	result	size
parallelrisch	$\frac{4x^4\ln(cx^n)b\,e^2n - x^4b\,e^2n^2 + 4x^4a\,e^2n + 16x^2\ln(cx^n)bden - 8x^2bde\,n^2 + 16x^2aden + 16\ln(x)a\,d^2n + 8b\,d^2\ln(cx^n)^2}{16n}$	103
risch	Expression too large to display	3072

```
[In] int((e*x^2+d)^2*(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(4*x^4*ln(c*x^n)*b*e^2*n-x^4*b*e^2*n^2+4*x^4*a*e^2*n+16*x^2*ln(c*x^n)*
b*d*e*n-8*x^2*b*d*e*n^2+16*x^2*a*d*e*n+16*ln(x)*a*d^2*n+8*b*d^2*ln(c*x^n)^2
)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x}\,dx = \frac{1}{2}\,bd^2n\log(x)^2 - \frac{1}{16}\left(be^2n - 4\,ae^2\right)x^4 - \frac{1}{2}\left(bden - 2\,ade\right)x^2 + \frac{1}{4}\left(be^2x^4 + 4\,bdex^2\right)\log(c) + \frac{1}{4}\left(be^2nx^4 + 4\,bdenx^2 + 4\,bd^2\log(c) + 4\,ad^2\right)\log(x)$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] 1/2*b*d^2*n*log(x)^2 - 1/16*(b*e^2*n - 4*a*e^2)*x^4 - 1/2*(b*d*e*n - 2*a*d*
e)*x^2 + 1/4*(b*e^2*x^4 + 4*b*d*e*x^2)*log(c) + 1/4*(b*e^2*n*x^4 + 4*b*d*e*
n*x^2 + 4*b*d^2*log(c) + 4*a*d^2)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex^2)^2\,(a+b\log(cx^n))}{x}\,dx = \begin{cases} \frac{ad^2\log(cx^n)}{n} + adex^2 + \frac{ae^2x^4}{4} + \frac{bd^2\log(cx^n)^2}{2n} - \frac{bdenx^2}{2} + bdex^2\log(cx^n) - \frac{be^2nx^4}{16} + \frac{be^2x^4\log(cx^n)}{4} & \text{for } n \neq 0 \\ (a+b\log(c))\left(d^2\log(x) + dex^2 + \frac{e^2x^4}{4}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((e*x**2+d)**2*(a+b*ln(c*x**n))/x,x)
[Out] Piecewise((a*d**2*log(c*x**n)/n + a*d*e*x**2 + a*e**2*x**4/4 + b*d**2*log(c
*x**n)**2/(2*n) - b*d*e*n*x**2/2 + b*d*e*x**2*log(c*x**n) - b*e**2*n*x**4/1
6 + b*e**2*x**4*log(c*x**n)/4, Ne(n, 0)), ((a + b*log(c))*(d**2*log(x) + d*
e*x**2 + e**2*x**4/4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^2\,(a+b\log(cx^n))}{x}\,dx = -\frac{1}{16}\,be^2nx^4 + \frac{1}{4}\,be^2x^4\log(cx^n) + \frac{1}{4}\,ae^2x^4 - \frac{1}{2}\,bdenx^2 + bdex^2\log(cx^n) + adex^2 + \frac{bd^2\log(cx^n)^2}{2\,n} + ad^2\log(x)$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x,x, algorithm="maxima")
[Out] -1/16*b*e^2*n*x^4 + 1/4*b*e^2*x^4*log(c*x^n) + 1/4*a*e^2*x^4 - 1/2*b*d*e*n*
x^2 + b*d*e*x^2*log(c*x^n) + a*d*e*x^2 + 1/2*b*d^2*log(c*x^n)^2/n + a*d^2*l
og(x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex^2)^2\,(a+b\log(cx^n))}{x}\,dx = \frac{1}{2}\,bd^2n\log(x)^2 - \frac{1}{16}\,(be^2n - 4\,be^2\log(c) - 4\,ae^2)x^4 - \frac{1}{2}\,(bden - 2\,bde\log(c) - 2\,ade)x^2 + \frac{1}{4}\,(be^2nx^4 + 4\,bdenx^2)\log(x) + (bd^2\log(c) + ad^2)\log(x)$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] 1/2*b*d^2*n*log(x)^2 - 1/16*(b*e^2*n - 4*b*e^2*log(c) - 4*a*e^2)*x^4 - 1/2*
(b*d*e*n - 2*b*d*e*log(c) - 2*a*d*e)*x^2 + 1/4*(b*e^2*n*x^4 + 4*b*d*e*n*x^2
)*log(x) + (b*d^2*log(c) + a*d^2)*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x} dx = \ln(c\,x^n) \left(\frac{b\,e^2\,x^4}{4} + b\,d\,e\,x^2\right) + \frac{e^2\,x^4\,(4\,a-b\,n)}{16} + a\,d^2\,\ln(x) + \frac{b\,d^2\ln(c\,x^n)^2}{2\,n} + \frac{d\,e\,x^2\,(2\,a-b\,n)}{2}$$

```
[In] int(((d + e*x^2)^2*(a + b*log(c*x^n)))/x,x)
```

```
[Out] log(c*x^n)*((b*e^2*x^4)/4 + b*d*e*x^2) + (e^2*x^4*(4*a - b*n))/16 + a*d^2*l
og(x) + (b*d^2*log(c*x^n)^2)/(2*n) + (d*e*x^2*(2*a - b*n))/2
```

3.187 $\int \frac{(d+ex^2)^2(a+b\log(cx^n))}{x^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^3} dx = -\frac{bd^2n}{4x^2} - \frac{1}{4}be^2nx^2 - bden\log^2(x) - \frac{d^2(a+b\log(cx^n))}{2x^2} + \frac{1}{2}e^2x^2(a+b\log(cx^n)) + 2de\log(x)(a+b\log(cx^n))$$

```
[Out] -1/4*b*d^2*n/x^2-1/4*b*e^2*n*x^2-b*d*e*n*ln(x)^2-1/2*d^2*(a+b*ln(c*x^n))/x^
2+1/2*e^2*x^2*(a+b*ln(c*x^n))+2*d*e*ln(x)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {272, 45, 2372, 12, 14, 2338}

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^3} dx = -\frac{d^2(a+b\log(cx^n))}{2x^2} + 2de\log(x)(a+b\log(cx^n)) + \frac{1}{2}e^2x^2(a+b\log(cx^n)) - \frac{bd^2n}{4x^2} - bden\log^2(x) - \frac{1}{4}be^2nx^2$$

```
[In] Int[((d + e*x^2)^2*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/4*(b*d^2*n)/x^2 - (b*e^2*n*x^2)/4 - b*d*e*n*Log[x]^2 - (d^2*(a + b*Log[c
*x^n]))/(2*x^2) + (e^2*x^2*(a + b*Log[c*x^n]))/2 + 2*d*e*Log[x]*(a + b*Log[
c*x^n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{2x^2}+\frac{1}{2}e^2x^2(a+b\log(cx^n)) \\
&\quad +2de\log(x)(a+b\log(cx^n))-(bn)\int\frac{-d^2+e^2x^4+4dex^2\log(x)}{2x^3}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{2x^2}+\frac{1}{2}e^2x^2(a+b\log(cx^n)) \\
&\quad +2de\log(x)(a+b\log(cx^n))-\frac{1}{2}(bn)\int\frac{-d^2+e^2x^4+4dex^2\log(x)}{x^3}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{d^2(a+b\log(cx^n))}{2x^2} + \frac{1}{2}e^2x^2(a+b\log(cx^n)) \\
&\quad + 2de\log(x)\,(a+b\log(cx^n)) - \frac{1}{2}(bn)\int\left(\frac{-d^2+e^2x^4}{x^3}+\frac{4de\log(x)}{x}\right)dx \\
&= -\frac{d^2(a+b\log(cx^n))}{2x^2} + \frac{1}{2}e^2x^2(a+b\log(cx^n)) + 2de\log(x)\,(a+b\log(cx^n)) \\
&\quad - \frac{1}{2}(bn)\int\frac{-d^2+e^2x^4}{x^3}\,dx - (2bden)\int\frac{\log(x)}{x}\,dx \\
&= -bden\log^2(x) - \frac{d^2(a+b\log(cx^n))}{2x^2} + \frac{1}{2}e^2x^2(a+b\log(cx^n)) \\
&\quad + 2de\log(x)\,(a+b\log(cx^n)) - \frac{1}{2}(bn)\int\left(-\frac{d^2}{x^3}+e^2x\right)dx \\
&= -\frac{bd^2n}{4x^2} - \frac{1}{4}be^2nx^2 - bden\log^2(x) - \frac{d^2(a+b\log(cx^n))}{2x^2} \\
&\quad + \frac{1}{2}e^2x^2(a+b\log(cx^n)) + 2de\log(x)\,(a+b\log(cx^n))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.91

$$
\int\frac{(d+ex^2)^2\,(a+b\log(cx^n))}{x^3}\,dx = \frac{1}{4}\left(-\frac{bd^2n}{x^2} - be^2nx^2 - \frac{2d^2(a+b\log(cx^n))}{x^2} + 2e^2x^2(a+b\log(cx^n)) + \frac{4de(a+b\log(cx^n))^2}{bn}\right)
$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] (-((b*d^2*n)/x^2) - b*e^2*n*x^2 - (2*d^2*(a + b*Log[c*x^n]))/x^2 + 2*e^2*x^
2*(a + b*Log[c*x^n]) + (4*d*e*(a + b*Log[c*x^n])^2)/(b*n))/4
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{2x^4\ln(c\,x^n)b\,e^2n-x^4b\,e^2n^2+2x^4a\,e^2n+8\ln(x)x^2aden+4bde\ln(c\,x^n)^2x^2-2\ln(c\,x^n)b\,d^2n-b\,d^2n^2-2a\,d^2n}{4x^2n}$
risch	$-\frac{b(-e^2x^4-4de\ln(x)x^2+d^2)\ln(x^n)}{2x^2}-\frac{-4i\ln(x)\pi bde\ \mathrm{csgn}(ix^n)\ \mathrm{csgn}(ic\,x^n)^2x^2+i\pi b\,d^2\ \mathrm{csgn}(ic)\ \mathrm{csgn}(ic\,x^n)^2-i\pi b\,e^2x^4\ \mathrm{csgn}(i}{}$

```
[In] int((e*x^2+d)^2*(a+b*ln(c*x^n))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/x^2*(2*x^4*ln(c*x^n)*b*e^2*n-x^4*b*e^2*n^2+2*x^4*a*e^2*n+8*ln(x)*x^2*a*
d*e*n+4*b*d*e*ln(c*x^n)^2*x^2-2*ln(c*x^n)*b*d^2*n-b*d^2*n^2-2*a*d^2*n)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^3} \, dx$$
$$= \frac{4\,bdenx^2\log(x)^2 - (be^2n - 2\,ae^2)x^4 - bd^2n - 2\,ad^2 + 2\,(be^2x^4 - bd^2)\log(c) + 2\,(be^2nx^4 + 4\,bdex^2\log(c)}{4\,x^2}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*d*e*n*x^2*log(x)^2 - (b*e^2*n - 2*a*e^2)*x^4 - b*d^2*n - 2*a*d^2 +
 2*(b*e^2*x^4 - b*d^2)*log(c) + 2*(b*e^2*n*x^4 + 4*b*d*e*x^2*log(c) + 4*a*d
*e*x^2 - b*d^2*n)*log(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^3} \, dx$$
$$= \begin{cases} -\frac{ad^2}{2x^2} + \frac{2ade\log(cx^n)}{n} + \frac{ae^2x^2}{2} - \frac{bd^2n}{4x^2} - \frac{bd^2\log(cx^n)}{2x^2} + \frac{bde\log(cx^n)^2}{n} - \frac{be^2nx^2}{4} + \frac{be^2x^2\log(cx^n)}{2} & \text{for } n \neq 0 \\ (a+b\log(c))\left(-\frac{d^2}{2x^2} + 2de\log(x) + \frac{e^2x^2}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((e*x**2+d)**2*(a+b*ln(c*x**n))/x**3,x)
```

```
[Out] Piecewise((-a*d**2/(2*x**2) + 2*a*d*e*log(c*x**n)/n + a*e**2*x**2/2 - b*d**
2*n/(4*x**2) - b*d**2*log(c*x**n)/(2*x**2) + b*d*e*log(c*x**n)**2/n - b*e**
2*n*x**2/4 + b*e**2*x**2*log(c*x**n)/2, Ne(n, 0)), ((a + b*log(c))*(-d**2/(
2*x**2) + 2*d*e*log(x) + e**2*x**2/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^3}\,dx = -\frac{1}{4}\,be^2nx^2 + \frac{1}{2}\,be^2x^2\log(cx^n) + \frac{1}{2}\,ae^2x^2 + \frac{bde\log(cx^n)^2}{n} + 2\,ade\log(x) - \frac{bd^2n}{4\,x^2} - \frac{bd^2\log(cx^n)}{2\,x^2} - \frac{ad^2}{2\,x^2}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*e^2*n*x^2 + 1/2*b*e^2*x^2*log(c*x^n) + 1/2*a*e^2*x^2 + b*d*e*log(c*x
^n)^2/n + 2*a*d*e*log(x) - 1/4*b*d^2*n/x^2 - 1/2*b*d^2*log(c*x^n)/x^2 - 1/2
*a*d^2/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^3}\,dx = \frac{1}{2}\,be^2x^2\log(c) + bden\log(x)^2 + \frac{1}{4}\,(2\,x^2\log(x) - x^2)be^2n + \frac{1}{2}\,ae^2x^2 - \frac{1}{4}\,bd^2n\left(\frac{2\,\log(x)}{x^2} + \frac{1}{x^2}\right) + 2\,bde\log(c)\log(|x|) + 2\,ade\log(|x|) - \frac{bd^2\log(c)}{2\,x^2} - \frac{ad^2}{2\,x^2}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b*e^2*x^2*log(c) + b*d*e*n*log(x)^2 + 1/4*(2*x^2*log(x) - x^2)*b*e^2*n
+ 1/2*a*e^2*x^2 - 1/4*b*d^2*n*(2*log(x)/x^2 + 1/x^2) + 2*b*d*e*log(c)*log(a
bs(x)) + 2*a*d*e*log(abs(x)) - 1/2*b*d^2*log(c)/x^2 - 1/2*a*d^2/x^2
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^3}\,dx = \ln(x)\,(2\,a\,d\,e+b\,d\,e\,n)-\frac{\frac{a\,d^2}{2}+\frac{b\,d^2\,n}{4}}{x^2} - \ln(c\,x^n)\,\left(\frac{\frac{b\,d^2}{2}+b\,d\,e\,x^2+\frac{b\,e^2\,x^4}{2}}{x^2}-b\,e^2\,x^2\right) + \frac{e^2\,x^2\,(2\,a-b\,n)}{4}+\frac{b\,d\,e\ln(c\,x^n)^2}{n}$$

```
[In] int(((d + e*x^2)^2*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] log(x)*(2*a*d*e + b*d*e*n) - ((a*d^2)/2 + (b*d^2*n)/4)/x^2 - log(c*x^n)*(((
b*d^2)/2 + (b*e^2*x^4)/2 + b*d*e*x^2)/x^2 - b*e^2*x^2) + (e^2*x^2*(2*a - b*
n))/4 + (b*d*e*log(c*x^n)^2)/n
```

3.188 $\int \frac{(d+ex^2)^2(a+b\log(cx^n))}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{(d+ex^2)^2\,(a+b\log(cx^n))}{x^5}\,dx = -\frac{bd^2n}{16x^4}-\frac{bden}{2x^2}-\frac{1}{2}be^2n\log^2(x)-\frac{d^2(a+b\log(cx^n))}{4x^4} -\frac{de(a+b\log(cx^n))}{x^2}+e^2\log(x)\,(a+b\log(cx^n))$$

```
[Out] -1/16*b*d^2*n/x^4-1/2*b*d*e*n/x^2-1/2*b*e^2*n*ln(x)^2-1/4*d^2*(a+b*ln(c*x^n
))/x^4-d*e*(a+b*ln(c*x^n))/x^2+e^2*ln(x)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {272, 45, 2372, 14, 2338}

$$\int \frac{(d+ex^2)^2\,(a+b\log(cx^n))}{x^5}\,dx = -\frac{d^2(a+b\log(cx^n))}{4x^4}-\frac{de(a+b\log(cx^n))}{x^2} +e^2\log(x)\,(a+b\log(cx^n)) -\frac{bd^2n}{16x^4}-\frac{bden}{2x^2}-\frac{1}{2}be^2n\log^2(x)$$

```
[In] Int[((d + e*x^2)^2*(a + b*Log[c*x^n]))/x^5,x]
```

```
[Out] -1/16*(b*d^2*n)/x^4 - (b*d*e*n)/(2*x^2) - (b*e^2*n*Log[x]^2)/2 - (d^2*(a +
b*Log[c*x^n]))/(4*x^4) - (d*e*(a + b*Log[c*x^n]))/x^2 + e^2*Log[x]*(a + b*L
og[c*x^n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{4x^4} - \frac{de(a+b\log(cx^n))}{x^2} + e^2\log(x)\,(a+b\log(cx^n)) \\
&\quad -(bn)\int\left(-\frac{d(d+4ex^2)}{4x^5}+\frac{e^2\log(x)}{x}\right)dx \\
&= -\frac{d^2(a+b\log(cx^n))}{4x^4} - \frac{de(a+b\log(cx^n))}{x^2} + e^2\log(x)\,(a+b\log(cx^n)) \\
&\quad +\frac{1}{4}(bdn)\int\frac{d+4ex^2}{x^5}\,dx - (be^2n)\int\frac{\log(x)}{x}\,dx \\
&= -\frac{1}{2}be^2n\log^2(x) - \frac{d^2(a+b\log(cx^n))}{4x^4} - \frac{de(a+b\log(cx^n))}{x^2} \\
&\quad + e^2\log(x)\,(a+b\log(cx^n)) + \frac{1}{4}(bdn)\int\left(\frac{d}{x^5}+\frac{4e}{x^3}\right)dx
\end{aligned}$$

$$= -\frac{bd^2n}{16x^4} - \frac{bden}{2x^2} - \frac{1}{2}be^2n\log^2(x) - \frac{d^2(a + b\log(cx^n))}{4x^4} - \frac{de(a + b\log(cx^n))}{x^2} + e^2\log(x)(a + b\log(cx^n))$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{(d + ex^2)^2 (a + b\log(cx^n))}{x^5} dx = \frac{1}{16}\left(-\frac{bd^2n}{x^4} - \frac{8bden}{x^2} - \frac{4d^2(a + b\log(cx^n))}{x^4} - \frac{16de(a + b\log(cx^n))}{x^2} + \frac{8e^2(a + b\log(cx^n))^2}{bn}\right)$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*Log[c*x^n]))/x^5,x]
```

```
[Out] (-((b*d^2*n)/x^4) - (8*b*d*e*n)/x^2 - (4*d^2*(a + b*Log[c*x^n]))/x^4 - (16*
d*e*(a + b*Log[c*x^n]))/x^2 + (8*e^2*(a + b*Log[c*x^n])^2)/(b*n))/16
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{16\ln(x)x^4 a e^2 n+8e^2 b\ln(c x^n)^2 x^4-16x^2\ln(c x^n)bden-8x^2 bde n^2-16x^2 aden-4\ln(c x^n)b d^2 n-b d^2 n^2-4a d^2 n}{16x^4 n}$
risch	$-\frac{b(-4e^2\ln(x)x^4+4de x^2+d^2)\ln(x^n)}{4x^4} - \frac{2i\pi b d^2 \operatorname{csgn}(ic)\operatorname{csgn}(ic x^n)^2-8i\ln(x)\pi b e^2\operatorname{csgn}(ix^n)\operatorname{csgn}(ic x^n)^2 x^4-2i\pi b d^2 \operatorname{csgn}(ic$

```
[In] int((e*x^2+d)^2*(a+b*ln(c*x^n))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/x^4*(16*ln(x)*x^4*a*e^2*n+8*e^2*b*ln(c*x^n)^2*x^4-16*x^2*ln(c*x^n)*b*d
*e*n-8*x^2*b*d*e*n^2-16*x^2*a*d*e*n-4*ln(c*x^n)*b*d^2*n-b*d^2*n^2-4*a*d^2*n
)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^5} dx$$
$$= \frac{8\,be^2nx^4\log(x)^2 - bd^2n - 4\,ad^2 - 8\,(bden + 2\,ade)x^2 - 4\,(4\,bdex^2 + bd^2)\log(c) + 4\,(4\,be^2x^4\log(c) + 4\,ae}{16\,x^4}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^5,x, algorithm="fricas")
```

```
[Out] 1/16*(8*b*e^2*n*x^4*log(x)^2 - b*d^2*n - 4*a*d^2 - 8*(b*d*e*n + 2*a*d*e)*x^
2 - 4*(4*b*d*e*x^2 + b*d^2)*log(c) + 4*(4*b*e^2*x^4*log(c) + 4*a*e^2*x^4 -
4*b*d*e*n*x^2 - b*d^2*n)*log(x))/x^4
```

Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^5} dx = -\frac{ad^2}{4x^4} - \frac{ade}{x^2} + ae^2\log(x) + bd^2\left(-\frac{n}{16x^4} - \frac{\log(cx^n)}{4x^4}\right) + 2bde\left(-\frac{n}{4x^2} - \frac{\log(cx^n)}{2x^2}\right) - be^2\left(\begin{cases} -\log(c)\log(x) & \text{for } n = 0 \\ -\frac{\log(cx^n)^2}{2n} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x**2+d)**2*(a+b*ln(c*x**n))/x**5,x)
```

```
[Out] -a*d**2/(4*x**4) - a*d*e/x**2 + a*e**2*log(x) + b*d**2*(-n/(16*x**4) - log(
c*x**n)/(4*x**4)) + 2*b*d*e*(-n/(4*x**2) - log(c*x**n)/(2*x**2)) - b*e**2*P
iecewise((-log(c)*log(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^5} dx = \frac{be^2\log(cx^n)^2}{2\,n} + ae^2\log(x) - \frac{bden}{2\,x^2} - \frac{bde\log(cx^n)}{x^2} - \frac{ade}{x^2} - \frac{bd^2n}{16\,x^4} - \frac{bd^2\log(cx^n)}{4\,x^4} - \frac{ad^2}{4\,x^4}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*b*e^2*log(c*x^n)^2/n + a*e^2*log(x) - 1/2*b*d*e*n/x^2 - b*d*e*log(c*x^n
)/x^2 - a*d*e/x^2 - 1/16*b*d^2*n/x^4 - 1/4*b*d^2*log(c*x^n)/x^4 - 1/4*a*d^2
/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex^2)^2\,(a+b\log(cx^n))}{x^5}\,dx = \frac{1}{2}\,be^2n\log(x)^2 - \frac{1}{2}\,bden\left(\frac{2\,\log(x)}{x^2} + \frac{1}{x^2}\right) - \frac{1}{16}\,bd^2n\left(\frac{4\,\log(x)}{x^4} + \frac{1}{x^4}\right) + be^2\log(c)\log(|x|) + ae^2\log(|x|) - \frac{bde\log(c)}{x^2} - \frac{ade}{x^2} - \frac{bd^2\log(c)}{4\,x^4} - \frac{ad^2}{4\,x^4}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^5,x, algorithm="giac")
```

```
[Out] 1/2*b*e^2*n*log(x)^2 - 1/2*b*d*e*n*(2*log(x)/x^2 + 1/x^2) - 1/16*b*d^2*n*(4
*log(x)/x^4 + 1/x^4) + b*e^2*log(c)*log(abs(x)) + a*e^2*log(abs(x)) - b*d*e
*log(c)/x^2 - a*d*e/x^2 - 1/4*b*d^2*log(c)/x^4 - 1/4*a*d^2/x^4
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex^2)^2\,(a+b\log(cx^n))}{x^5}\,dx = \ln(x)\,\left(a\,e^2 + \frac{3\,b\,e^2\,n}{4}\right) - \frac{x^2\,(4\,a\,d\,e + 2\,b\,d\,e\,n) + a\,d^2 + \frac{b\,d^2\,n}{4}}{4\,x^4} - \frac{\ln(c\,x^n)\,\left(\frac{b\,d^2}{4} + b\,d\,e\,x^2 + \frac{3\,b\,e^2\,x^4}{4}\right)}{x^4} + \frac{b\,e^2\ln(c\,x^n)^2}{2\,n}$$

```
[In] int(((d + e*x^2)^2*(a + b*log(c*x^n)))/x^5,x)
```

```
[Out] log(x)*(a*e^2 + (3*b*e^2*n)/4) - (x^2*(4*a*d*e + 2*b*d*e*n) + a*d^2 + (b*d^
2*n)/4)/(4*x^4) - (log(c*x^n)*((b*d^2)/4 + (3*b*e^2*x^4)/4 + b*d*e*x^2))/x^
4 + (b*e^2*log(c*x^n)^2)/(2*n)
```

3.189 $\int x^4(d+ex^2)^2 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int x^4(d+ex^2)^2(a+b\log(cx^n))\,dx = -\frac{1}{25}bd^2nx^5-\frac{2}{49}bdenx^7-\frac{1}{81}be^2nx^9 + \frac{1}{315}(63d^2x^5+90dex^7+35e^2x^9)(a+b\log(cx^n))$$

```
[Out] -1/25*b*d^2*n*x^5-2/49*b*d*e*n*x^7-1/81*b*e^2*n*x^9+1/315*(35*e^2*x^9+90*d*
e*x^7+63*d^2*x^5)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {276, 2371}

$$\int x^4(d+ex^2)^2(a+b\log(cx^n))\,dx = \frac{1}{315}(63d^2x^5+90dex^7+35e^2x^9)(a+b\log(cx^n)) - \frac{1}{25}bd^2nx^5-\frac{2}{49}bdenx^7-\frac{1}{81}be^2nx^9$$

```
[In] Int[x^4*(d + e*x^2)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/25*(b*d^2*n*x^5) - (2*b*d*e*n*x^7)/49 - (b*e^2*n*x^9)/81 + ((63*d^2*x^5
+ 90*d*e*x^7 + 35*e^2*x^9)*(a + b*Log[c*x^n]))/315
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{315}\left(63d^2x^5+90dex^7+35e^2x^9\right)(a+b\log(cx^n)) \\ &\quad -(bn)\int\left(\frac{d^2x^4}{5}+\frac{2}{7}dex^6+\frac{e^2x^8}{9}\right)dx \\ &= -\frac{1}{25}bd^2nx^5-\frac{2}{49}bdenx^7-\frac{1}{81}be^2nx^9+\frac{1}{315}\left(63d^2x^5+90dex^7+35e^2x^9\right)(a+b\log(cx^n))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\begin{aligned}\int x^4\left(d+ex^2\right)^2(a+b\log(cx^n))\,dx &= -\frac{1}{25}bd^2nx^5-\frac{2}{49}bdenx^7-\frac{1}{81}be^2nx^9 \\ &\quad +\frac{1}{5}d^2x^5(a+b\log(cx^n)) \\ &\quad +\frac{2}{7}dex^7(a+b\log(cx^n))+\frac{1}{9}e^2x^9(a+b\log(cx^n))\end{aligned}$$

```
[In] Integrate[x^4*(d + e*x^2)^2*(a + b*Log[c*x^n]),x]
[Out] -1/25*(b*d^2*n*x^5) - (2*b*d*e*n*x^7)/49 - (b*e^2*n*x^9)/81 + (d^2*x^5*(a +
 b*Log[c*x^n]))/5 + (2*d*e*x^7*(a + b*Log[c*x^n]))/7 + (e^2*x^9*(a + b*Log[
c*x^n]))/9
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36

method	result
parallelrisch	$\frac{x^9 b\ln(c\,x^n)e^2}{9}-\frac{b\,e^2 n\,x^9}{81}+\frac{x^9 a\,e^2}{9}+\frac{2x^7 b\ln(c\,x^n)de}{7}-\frac{2bden\,x^7}{49}+\frac{2x^7 ade}{7}+\frac{x^5 b\ln(c\,x^n)d^2}{5}-\frac{b\,d^2 n\,x^5}{25}+\frac{x^5 a\,d^2}{5}$
risch	$\frac{b\,x^5\left(35e^2x^4+90de\,x^2+63d^2\right)\ln(x^n)}{315}+\frac{i\pi b\,d^2x^5\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2}{10}+\frac{i\pi b\,d^2x^5\operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^n)^2}{10}-\frac{i\pi bde\,x^7\operatorname{csgn}(ic$

```
[In] int(x^4*(e*x^2+d)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*x^9*b*ln(c*x^n)*e^2-1/81*b*e^2*n*x^9+1/9*x^9*a*e^2+2/7*x^7*b*ln(c*x^n)*
d*e-2/49*b*d*e*n*x^7+2/7*x^7*a*d*e+1/5*x^5*b*ln(c*x^n)*d^2-1/25*b*d^2*n*x^5
+1/5*x^5*a*d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.59

$$\int x^4\left(d+ex^2\right)^2\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{81}\left(be^2n-9\,ae^2\right)x^9-\frac{2}{49}\left(bden-7\,ade\right)x^7-\frac{1}{25}\left(bd^2n-5\,ad^2\right)x^5+\frac{1}{315}\left(35\,be^2x^9+90\,bdex^7+63\,bd^2x^5\right)\log\left(c\right)+\frac{1}{315}\left(35\,be^2nx^9+90\,bdenx^7+63\,bd^2nx^5\right)\log\left(x\right)$$

```
[In] integrate(x^4*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/81*(b*e^2*n - 9*a*e^2)*x^9 - 2/49*(b*d*e*n - 7*a*d*e)*x^7 - 1/25*(b*d^2*
n - 5*a*d^2)*x^5 + 1/315*(35*b*e^2*x^9 + 90*b*d*e*x^7 + 63*b*d^2*x^5)*log(c
) + 1/315*(35*b*e^2*n*x^9 + 90*b*d*e*n*x^7 + 63*b*d^2*n*x^5)*log(x)
```

Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.64

$$\int x^4\left(d+ex^2\right)^2\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{ad^2x^5}{5}+\frac{2adex^7}{7}+\frac{ae^2x^9}{9}-\frac{bd^2nx^5}{25}+\frac{bd^2x^5\log\left(cx^n\right)}{5}-\frac{2bdenx^7}{49}+\frac{2bdex^7\log\left(cx^n\right)}{7}-\frac{be^2nx^9}{81}+\frac{be^2x^9\log\left(cx^n\right)}{9}$$

```
[In] integrate(x**4*(e*x**2+d)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**2*x**5/5 + 2*a*d*e*x**7/7 + a*e**2*x**9/9 - b*d**2*n*x**5/25 + b*d**2*
x**5*log(c*x**n)/5 - 2*b*d*e*n*x**7/49 + 2*b*d*e*x**7*log(c*x**n)/7 - b*e**
2*n*x**9/81 + b*e**2*x**9*log(c*x**n)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int x^4(d+ex^2)^2\,(a+b\log{(cx^n)})\;dx = -\frac{1}{81}\,be^2nx^9+\frac{1}{9}\,be^2x^9\log{(cx^n)}+\frac{1}{9}\,ae^2x^9 -\frac{2}{49}\,bdenx^7+\frac{2}{7}\,bdex^7\log{(cx^n)}+\frac{2}{7}\,adex^7 -\frac{1}{25}\,bd^2nx^5+\frac{1}{5}\,bd^2x^5\log{(cx^n)}+\frac{1}{5}\,ad^2x^5$$

```
[In] integrate(x^4*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/81*b*e^2*n*x^9 + 1/9*b*e^2*x^9*log(c*x^n) + 1/9*a*e^2*x^9 - 2/49*b*d*e*n
*x^7 + 2/7*b*d*e*x^7*log(c*x^n) + 2/7*a*d*e*x^7 - 1/25*b*d^2*n*x^5 + 1/5*b*
d^2*x^5*log(c*x^n) + 1/5*a*d^2*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.66

$$\int x^4(d+ex^2)^2\,(a+b\log{(cx^n)})\;dx = \frac{1}{9}\,be^2nx^9\log{(x)}-\frac{1}{81}\,be^2nx^9+\frac{1}{9}\,be^2x^9\log{(c)} +\frac{1}{9}\,ae^2x^9+\frac{2}{7}\,bdenx^7\log{(x)}-\frac{2}{49}\,bdenx^7 +\frac{2}{7}\,bdex^7\log{(c)}+\frac{2}{7}\,adex^7+\frac{1}{5}\,bd^2nx^5\log{(x)} -\frac{1}{25}\,bd^2nx^5+\frac{1}{5}\,bd^2x^5\log{(c)}+\frac{1}{5}\,ad^2x^5$$

```
[In] integrate(x^4*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/9*b*e^2*n*x^9*log(x) - 1/81*b*e^2*n*x^9 + 1/9*b*e^2*x^9*log(c) + 1/9*a*e^
2*x^9 + 2/7*b*d*e*n*x^7*log(x) - 2/49*b*d*e*n*x^7 + 2/7*b*d*e*x^7*log(c) +
2/7*a*d*e*x^7 + 1/5*b*d^2*n*x^5*log(x) - 1/25*b*d^2*n*x^5 + 1/5*b*d^2*x^5*l
og(c) + 1/5*a*d^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int x^4 (d+ex^2)^2 (a+b\log(cx^n))\,dx = \ln(c\,x^n)\left(\frac{b\,d^2\,x^5}{5}+\frac{2\,b\,d\,e\,x^7}{7}+\frac{b\,e^2\,x^9}{9}\right)+\frac{d^2\,x^5\,(5\,a-b\,n)}{25}+\frac{e^2\,x^9\,(9\,a-b\,n)}{81}+\frac{2\,d\,e\,x^7\,(7\,a-b\,n)}{49}$$

```
[In] int(x^4*(d + e*x^2)^2*(a + b*log(c*x^n)),x)
```

```
[Out] log(c*x^n)*((b*d^2*x^5)/5 + (b*e^2*x^9)/9 + (2*b*d*e*x^7)/7) + (d^2*x^5*(5*
a - b*n))/25 + (e^2*x^9*(9*a - b*n))/81 + (2*d*e*x^7*(7*a - b*n))/49
```

3.190 $\int x^2(d+ex^2)^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int x^2(d+ex^2)^2(a+b\log(cx^n))\,dx = -\frac{1}{9}bd^2nx^3-\frac{2}{25}bdenx^5-\frac{1}{49}be^2nx^7+\frac{1}{105}(35d^2x^3+42dex^5+15e^2x^7)(a+b\log(cx^n))$$

```
[Out] -1/9*b*d^2*n*x^3-2/25*b*d*e*n*x^5-1/49*b*e^2*n*x^7+1/105*(15*e^2*x^7+42*d*e
*x^5+35*d^2*x^3)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {276, 2371}

$$\int x^2(d+ex^2)^2(a+b\log(cx^n))\,dx = \frac{1}{105}(35d^2x^3+42dex^5+15e^2x^7)(a+b\log(cx^n))-\frac{1}{9}bd^2nx^3-\frac{2}{25}bdenx^5-\frac{1}{49}be^2nx^7$$

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/9*(b*d^2*n*x^3) - (2*b*d*e*n*x^5)/25 - (b*e^2*n*x^7)/49 + ((35*d^2*x^3 +
 42*d*e*x^5 + 15*e^2*x^7)*(a + b*Log[c*x^n]))/105
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{105}\left(35d^2x^3+42dex^5+15e^2x^7\right)(a+b\log(cx^n)) \\ &\quad -(bn)\int\left(\frac{d^2x^2}{3}+\frac{2}{5}dex^4+\frac{e^2x^6}{7}\right)dx \\ &= -\frac{1}{9}bd^2nx^3-\frac{2}{25}bdenx^5-\frac{1}{49}be^2nx^7+\frac{1}{105}\left(35d^2x^3+42dex^5+15e^2x^7\right)(a+b\log(cx^n))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\begin{aligned}\int x^2\left(d+ex^2\right)^2(a+b\log(cx^n))\,dx &= -\frac{1}{9}bd^2nx^3-\frac{2}{25}bdenx^5-\frac{1}{49}be^2nx^7 \\ &\quad +\frac{1}{3}d^2x^3(a+b\log(cx^n)) \\ &\quad +\frac{2}{5}dex^5(a+b\log(cx^n))+\frac{1}{7}e^2x^7(a+b\log(cx^n))\end{aligned}$$

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*Log[c*x^n]),x]
[Out] -1/9*(b*d^2*n*x^3) - (2*b*d*e*n*x^5)/25 - (b*e^2*n*x^7)/49 + (d^2*x^3*(a +
b*Log[c*x^n]))/3 + (2*d*e*x^5*(a + b*Log[c*x^n]))/5 + (e^2*x^7*(a + b*Log[c
*x^n]))/7
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36

method	result
parallelrisch	$\frac{x^7 b\ln(c\,x^n)e^2}{7}-\frac{b\,e^2 n\,x^7}{49}+\frac{x^7 a\,e^2}{7}+\frac{2x^5\ln(c\,x^n)bde}{5}-\frac{2bden\,x^5}{25}+\frac{2ade\,x^5}{5}+\frac{x^3 b\ln(c\,x^n)d^2}{3}-\frac{b\,d^2 n\,x^3}{9}+\frac{a\,d^2 x^{3}}{3}$
risch	$\frac{b\,x^3\left(15e^2x^4+42de\,x^2+35d^2\right)\ln(x^n)}{105}+\frac{i\pi b\,d^2x^3\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2}{6}-\frac{i\pi b\,e^2x^7\,\mathrm{csgn}(ic\,x^n)^3}{14}+\frac{i\pi bde\,x^5\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(}{5}$

```
[In] int(x^2*(e*x^2+d)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*x^7*b*ln(c*x^n)*e^2-1/49*b*e^2*n*x^7+1/7*x^7*a*e^2+2/5*x^5*ln(c*x^n)*b*
d*e-2/25*b*d*e*n*x^5+2/5*a*d*e*x^5+1/3*x^3*b*ln(c*x^n)*d^2-1/9*b*d^2*n*x^3+
1/3*a*d^2*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.59

$$\int x^2(d+ex^2)^2(a+b\log(cx^n))\,dx = -\frac{1}{49}\left(be^2n-7\,ae^2\right)x^7 - \frac{2}{25}\left(bden-5\,ade\right)x^5 - \frac{1}{9}\left(bd^2n-3\,ad^2\right)x^3 + \frac{1}{105}\left(15\,be^2x^7+42\,bdex^5+35\,bd^2x^3\right)\log(c) + \frac{1}{105}\left(15\,be^2nx^7+42\,bdenx^5+35\,bd^2nx^3\right)\log(x)$$

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/49*(b*e^2*n - 7*a*e^2)*x^7 - 2/25*(b*d*e*n - 5*a*d*e)*x^5 - 1/9*(b*d^2*n
 - 3*a*d^2)*x^3 + 1/105*(15*b*e^2*x^7 + 42*b*d*e*x^5 + 35*b*d^2*x^3)*log(c)
 + 1/105*(15*b*e^2*n*x^7 + 42*b*d*e*n*x^5 + 35*b*d^2*n*x^3)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.64

$$\int x^2(d+ex^2)^2(a+b\log(cx^n))\,dx = \frac{ad^2x^3}{3} + \frac{2adex^5}{5} + \frac{ae^2x^7}{7} - \frac{bd^2nx^3}{9} + \frac{bd^2x^3\log(cx^n)}{3} - \frac{2bdenx^5}{25} + \frac{2bdex^5\log(cx^n)}{5} - \frac{be^2nx^7}{49} + \frac{be^2x^7\log(cx^n)}{7}$$

```
[In] integrate(x**2*(e*x**2+d)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a*e**2*x**7/7 - b*d**2*n*x**3/9 + b*d**2*x
**3*log(c*x**n)/3 - 2*b*d*e*n*x**5/25 + 2*b*d*e*x**5*log(c*x**n)/5 - b*e**2
*n*x**7/49 + b*e**2*x**7*log(c*x**n)/7
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int x^2 (d+ex^2)^2 (a+b\log(cx^n))\,dx = -\frac{1}{49}\,be^2nx^7 + \frac{1}{7}\,be^2x^7\log(cx^n) + \frac{1}{7}\,ae^2x^7 - \frac{2}{25}\,bdenx^5 + \frac{2}{5}\,bdex^5\log(cx^n) + \frac{2}{5}\,adex^5 - \frac{1}{9}\,bd^2nx^3 + \frac{1}{3}\,bd^2x^3\log(cx^n) + \frac{1}{3}\,ad^2x^3$$

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/49*b*e^2*n*x^7 + 1/7*b*e^2*x^7*log(c*x^n) + 1/7*a*e^2*x^7 - 2/25*b*d*e*n
*x^5 + 2/5*b*d*e*x^5*log(c*x^n) + 2/5*a*d*e*x^5 - 1/9*b*d^2*n*x^3 + 1/3*b*d
^2*x^3*log(c*x^n) + 1/3*a*d^2*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.66

$$\int x^2 (d+ex^2)^2 (a+b\log(cx^n))\,dx = \frac{1}{7}\,be^2nx^7\log(x) - \frac{1}{49}\,be^2nx^7 + \frac{1}{7}\,be^2x^7\log(c) + \frac{1}{7}\,ae^2x^7 + \frac{2}{5}\,bdenx^5\log(x) - \frac{2}{25}\,bdenx^5 + \frac{2}{5}\,bdex^5\log(c) + \frac{2}{5}\,adex^5 + \frac{1}{3}\,bd^2nx^3\log(x) - \frac{1}{9}\,bd^2nx^3 + \frac{1}{3}\,bd^2x^3\log(c) + \frac{1}{3}\,ad^2x^3$$

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/7*b*e^2*n*x^7*log(x) - 1/49*b*e^2*n*x^7 + 1/7*b*e^2*x^7*log(c) + 1/7*a*e^
2*x^7 + 2/5*b*d*e*n*x^5*log(x) - 2/25*b*d*e*n*x^5 + 2/5*b*d*e*x^5*log(c) +
2/5*a*d*e*x^5 + 1/3*b*d^2*n*x^3*log(x) - 1/9*b*d^2*n*x^3 + 1/3*b*d^2*x^3*lo
g(c) + 1/3*a*d^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int x^2 (d+ex^2)^2 (a+b\log(cx^n))\,dx = \ln(c\,x^n)\left(\frac{b\,d^2\,x^3}{3}+\frac{2\,b\,d\,e\,x^5}{5}+\frac{b\,e^2\,x^7}{7}\right)+\frac{d^2\,x^3\,(3\,a-b\,n)}{9}+\frac{e^2\,x^7\,(7\,a-b\,n)}{49}+\frac{2\,d\,e\,x^5\,(5\,a-b\,n)}{25}$$

```
[In] int(x^2*(d + e*x^2)^2*(a + b*log(c*x^n)),x)
```

```
[Out] log(c*x^n)*((b*d^2*x^3)/3 + (b*e^2*x^7)/7 + (2*b*d*e*x^5)/5) + (d^2*x^3*(3*
a - b*n))/9 + (e^2*x^7*(7*a - b*n))/49 + (2*d*e*x^5*(5*a - b*n))/25
```

3.191 $\int (d+ex^2)^2 (a+b\log(cx^n)) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 86

$$\int (d+ex^2)^2 (a+b\log(cx^n)) \, dx = -bd^2nx - \frac{2}{9}bdenx^3 - \frac{1}{25}be^2nx^5 + d^2x(a+b\log(cx^n)) + \frac{2}{3}dex^3(a+b\log(cx^n)) + \frac{1}{5}e^2x^5(a+b\log(cx^n))$$

```
[Out] -b*d^2*n*x-2/9*b*d*e*n*x^3-1/25*b*e^2*n*x^5+d^2*x*(a+b*ln(c*x^n))+2/3*d*e*x
^3*(a+b*ln(c*x^n))+1/5*e^2*x^5*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {200, 2350}

$$\int (d+ex^2)^2 (a+b\log(cx^n)) \, dx = d^2x(a+b\log(cx^n)) + \frac{2}{3}dex^3(a+b\log(cx^n)) + \frac{1}{5}e^2x^5(a+b\log(cx^n)) - bd^2nx - \frac{2}{9}bdenx^3 - \frac{1}{25}be^2nx^5$$

```
[In] Int[(d + e*x^2)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -(b*d^2*n*x) - (2*b*d*e*n*x^3)/9 - (b*e^2*n*x^5)/25 + d^2*x*(a + b*Log[c*x^
n]) + (2*d*e*x^3*(a + b*Log[c*x^n]))/3 + (e^2*x^5*(a + b*Log[c*x^n]))/5
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2350

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Dist[a + b*Log[c*x^n], u
, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= d^2x(a+b\log(cx^n))+\frac{2}{3}dex^3(a+b\log(cx^n)) \\ &\quad+\frac{1}{5}e^2x^5(a+b\log(cx^n))-(bn)\int\left(d^2+\frac{2}{3}dex^2+\frac{e^2x^4}{5}\right)dx \\ &= -bd^2nx-\frac{2}{9}bdenx^3-\frac{1}{25}be^2nx^5+d^2x(a+b\log(cx^n)) \\ &\quad+\frac{2}{3}dex^3(a+b\log(cx^n))+\frac{1}{5}e^2x^5(a+b\log(cx^n))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03

$$\begin{aligned}\int\left(d+ex^2\right)^2(a+b\log(cx^n))\,dx &= ad^2x-bd^2nx-\frac{2}{9}bdenx^3-\frac{1}{25}be^2nx^5+bd^2x\log(cx^n) \\ &\quad+\frac{2}{3}dex^3(a+b\log(cx^n))+\frac{1}{5}e^2x^5(a+b\log(cx^n))\end{aligned}$$

```
[In] Integrate[(d + e*x^2)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] a*d^2*x - b*d^2*n*x - (2*b*d*e*n*x^3)/9 - (b*e^2*n*x^5)/25 + b*d^2*x*Log[c*
x^n] + (2*d*e*x^3*(a + b*Log[c*x^n]))/3 + (e^2*x^5*(a + b*Log[c*x^n]))/5
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.08

method	result
parallelrisch	$\frac{x^5 b\ln(c\,x^n)e^2}{5}-\frac{b\,e^2n\,x^5}{25}+\frac{x^5a\,e^2}{5}+\frac{2x^3b\ln(c\,x^n)de}{3}-\frac{2bden\,x^3}{9}+\frac{2x^3ade}{3}+xb\ln(c\,x^n)\,d^2-b\,d^2nx+a\,d^2$
risch	$\frac{bx(3e^2x^4+10de\,x^2+15d^2)\ln(x^n)}{15}+\frac{i\pi b\,e^2x^5\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2}{10}-\frac{i\pi bde\,x^3\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)}{3}+\frac{i\pi b\,d^2\,\mathrm{csgn}(}{}$

```
[In] int((e*x^2+d)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*b*ln(c*x^n)*e^2-1/25*b*e^2*n*x^5+1/5*x^5*a*e^2+2/3*x^3*b*ln(c*x^n)*
d*e-2/9*b*d*e*n*x^3+2/3*x^3*a*d*e+x*b*ln(c*x^n)*d^2-b*d^2*n*x+a*d^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

$$\int (d+ex^2)^2 (a+b\log(cx^n))\, dx = -\frac{1}{25}\left(be^2n-5\,ae^2\right)x^5 - \frac{2}{9}\left(bden-3\,ade\right)x^3 - \left(bd^2n-ad^2\right)x + \frac{1}{15}\left(3\,be^2x^5+10\,bdex^3+15\,bd^2x\right)\log(c) + \frac{1}{15}\left(3\,be^2nx^5+10\,bdenx^3+15\,bd^2nx\right)\log(x)$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/25*(b*e^2*n - 5*a*e^2)*x^5 - 2/9*(b*d*e*n - 3*a*d*e)*x^3 - (b*d^2*n - a*
d^2)*x + 1/15*(3*b*e^2*x^5 + 10*b*d*e*x^3 + 15*b*d^2*x)*log(c) + 1/15*(3*b*
e^2*n*x^5 + 10*b*d*e*n*x^3 + 15*b*d^2*n*x)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.28

$$\int (d+ex^2)^2 (a+b\log(cx^n))\, dx = ad^2x + \frac{2adex^3}{3} + \frac{ae^2x^5}{5} - bd^2nx + bd^2x\log(cx^n) - \frac{2bdenx^3}{9} + \frac{2bdex^3\log(cx^n)}{3} - \frac{be^2nx^5}{25} + \frac{be^2x^5\log(cx^n)}{5}$$

```
[In] integrate((e*x**2+d)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 - b*d**2*n*x + b*d**2*x*log(c*x**
n) - 2*b*d*e*n*x**3/9 + 2*b*d*e*x**3*log(c*x**n)/3 - b*e**2*n*x**5/25 + b*e
**2*x**5*log(c*x**n)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07

$$\int (d+ex^2)^2 (a+b\log(cx^n))\, dx = -\frac{1}{25}\,be^2nx^5 + \frac{1}{5}\,be^2x^5\log(cx^n) + \frac{1}{5}\,ae^2x^5 - \frac{2}{9}\,bdenx^3 + \frac{2}{3}\,bdex^3\log(cx^n) + \frac{2}{3}\,adex^3 - bd^2nx + bd^2x\log(cx^n) + ad^2x$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/25*b*e^2*n*x^5 + 1/5*b*e^2*x^5*log(c*x^n) + 1/5*a*e^2*x^5 - 2/9*b*d*e*n*
x^3 + 2/3*b*d*e*x^3*log(c*x^n) + 2/3*a*d*e*x^3 - b*d^2*n*x + b*d^2*x*log(c*
x^n) + a*d^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

$$\int \left(d+ex^2\right)^2 (a+b\log(cx^n))\,dx = \frac{1}{5}be^2nx^5\log(x) - \frac{1}{25}be^2nx^5 + \frac{1}{5}be^2x^5\log(c) + \frac{1}{5}ae^2x^5 + \frac{2}{3}bdenx^3\log(x) - \frac{2}{9}bdenx^3 + \frac{2}{3}bdex^3\log(c) + \frac{2}{3}adex^3 + bd^2nx\log(x) - bd^2nx + bd^2x\log(c) + ad^2x$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/5*b*e^2*n*x^5*log(x) - 1/25*b*e^2*n*x^5 + 1/5*b*e^2*x^5*log(c) + 1/5*a*e^
2*x^5 + 2/3*b*d*e*n*x^3*log(x) - 2/9*b*d*e*n*x^3 + 2/3*b*d*e*x^3*log(c) + 2
/3*a*d*e*x^3 + b*d^2*n*x*log(x) - b*d^2*n*x + b*d^2*x*log(c) + a*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \left(d+ex^2\right)^2 (a+b\log(cx^n))\,dx = \ln(c\,x^n)\left(b\,d^2\,x + \frac{2\,b\,d\,e\,x^3}{3} + \frac{b\,e^2\,x^5}{5}\right) + \frac{e^2\,x^5\,(5\,a-b\,n)}{25} + d^2\,x\,(a-b\,n) + \frac{2\,d\,e\,x^3\,(3\,a-b\,n)}{9}$$

```
[In] int((d + e*x^2)^2*(a + b*log(c*x^n)),x)
```

```
[Out] log(c*x^n)*((b*e^2*x^5)/5 + b*d^2*x + (2*b*d*e*x^3)/3) + (e^2*x^5*(5*a - b*
n))/25 + d^2*x*(a - b*n) + (2*d*e*x^3*(3*a - b*n))/9
```

3.192 $\int \frac{(d+ex^2)^2(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^2}\,dx = -\frac{bd^2n}{x} - 2bdenx - \frac{1}{9}be^2nx^3 - \frac{d^2(a+b\log(cx^n))}{x} + 2dex(a+b\log(cx^n)) + \frac{1}{3}e^2x^3(a+b\log(cx^n))$$

```
[Out] -b*d^2*n/x-2*b*d*e*n*x-1/9*b*e^2*n*x^3-d^2*(a+b*ln(c*x^n))/x+2*d*e*x*(a+b*l
n(c*x^n))+1/3*e^2*x^3*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {276, 2372}

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^2}\,dx = -\frac{d^2(a+b\log(cx^n))}{x} + 2dex(a+b\log(cx^n)) + \frac{1}{3}e^2x^3(a+b\log(cx^n)) - \frac{bd^2n}{x} - 2bdenx - \frac{1}{9}be^2nx^3$$

```
[In] Int[((d + e*x^2)^2*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -((b*d^2*n)/x) - 2*b*d*e*n*x - (b*e^2*n*x^3)/9 - (d^2*(a + b*Log[c*x^n]))/x
 + 2*d*e*x*(a + b*Log[c*x^n]) + (e^2*x^3*(a + b*Log[c*x^n]))/3
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{x}+2dex(a+b\log(cx^n))\\&\quad+\frac{1}{3}e^2x^3(a+b\log(cx^n))-(bn)\int\left(2de-\frac{d^2}{x^2}+\frac{e^2x^2}{3}\right)dx\\&=-\frac{bd^2n}{x}-2bdenx-\frac{1}{9}be^2nx^3-\frac{d^2(a+b\log(cx^n))}{x}\\&\quad+2dex(a+b\log(cx^n))+\frac{1}{3}e^2x^3(a+b\log(cx^n))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\begin{aligned}\int\frac{(d+ex^2)^2(a+b\log(cx^n))}{x^2}dx &= -\frac{bd^2n}{x}+2adex-2bdenx-\frac{1}{9}be^2nx^3+2bdex\log(cx^n)\\&\quad-\frac{d^2(a+b\log(cx^n))}{x}+\frac{1}{3}e^2x^3(a+b\log(cx^n))\end{aligned}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*Log[c*x^n]))/x^2,x]
[Out] -((b*d^2*n)/x) + 2*a*d*e*x - 2*b*d*e*n*x - (b*e^2*n*x^3)/9 + 2*b*d*e*x*Log[
c*x^n] - (d^2*(a + b*Log[c*x^n]))/x + (e^2*x^3*(a + b*Log[c*x^n]))/3
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.16

method	result
parallelrisch	$-\frac{-3x^4b\ln(cx^n)e^2+be^2nx^4-3x^4ae^2-18b\ln(cx^n)dex^2+18bdenx^2-18adex^2+9b\ln(cx^n)d^2+9bd^2n+9ad^2}{9x}$
risch	$-\frac{b(-e^2x^4-6dex^2+3d^2)\ln(x^n)}{3x}-\frac{-18i\pi bdex^2\operatorname{csgn}(ic)\operatorname{csgn}(icx^n)^2-9i\pi bd^2\operatorname{csgn}(icx^n)^3+9i\pi bd^2\operatorname{csgn}(ic)\operatorname{csgn}(icx^n)^2-18}{}$

```
[In] int((e*x^2+d)^2*(a+b*ln(c*x^n))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/x*(-3*x^4*b*ln(c*x^n)*e^2+b*e^2*n*x^4-3*x^4*a*e^2-18*b*ln(c*x^n)*d*e*x
^2+18*b*d*e*n*x^2-18*a*d*e*x^2+9*b*ln(c*x^n)*d^2+9*b*d^2*n+9*a*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex^2)^2\,(a+b\log(cx^n))}{x^2}\,dx = -\frac{(be^2n-3\,ae^2)x^4+9\,bd^2n+9\,ad^2+18\,(bden-ade)x^2-3\,(be^2x^4+6\,bdex^2-3\,bd^2)\log(c)-3\,(be^2nx^4}{9\,x}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
```

```
[Out] -1/9*((b*e^2*n - 3*a*e^2)*x^4 + 9*b*d^2*n + 9*a*d^2 + 18*(b*d*e*n - a*d*e)*
x^2 - 3*(b*e^2*x^4 + 6*b*d*e*x^2 - 3*b*d^2)*log(c) - 3*(b*e^2*n*x^4 + 6*b*d
*e*n*x^2 - 3*b*d^2*n)*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex^2)^2\,(a+b\log(cx^n))}{x^2}\,dx = -\frac{ad^2}{x}+2adex+\frac{ae^2x^3}{3}-\frac{bd^2n}{x}-\frac{bd^2\log(cx^n)}{x}-2bdenx+2bdex\log(cx^n)-\frac{be^2nx^3}{9}+\frac{be^2x^3\log(cx^n)}{3}$$

```
[In] integrate((e*x**2+d)**2*(a+b*ln(c*x**n))/x**2,x)
```

```
[Out] -a*d**2/x + 2*a*d*e*x + a*e**2*x**3/3 - b*d**2*n/x - b*d**2*log(c*x**n)/x -
 2*b*d*e*n*x + 2*b*d*e*x*log(c*x**n) - b*e**2*n*x**3/9 + b*e**2*x**3*log(c*
x**n)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex^2)^2\,(a+b\log(cx^n))}{x^2}\,dx = -\frac{1}{9}\,be^2nx^3+\frac{1}{3}\,be^2x^3\log(cx^n)+\frac{1}{3}\,ae^2x^3-2\,bdenx+2\,bdex\log(cx^n)+2\,adex-\frac{bd^2n}{x}-\frac{bd^2\log(cx^n)}{x}-\frac{ad^2}{x}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
```

```
[Out] -1/9*b*e^2*n*x^3 + 1/3*b*e^2*x^3*log(c*x^n) + 1/3*a*e^2*x^3 - 2*b*d*e*n*x +
 2*b*d*e*x*log(c*x^n) + 2*a*d*e*x - b*d^2*n/x - b*d^2*log(c*x^n)/x - a*d^2/
x
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^2}\,dx = -\frac{1}{9}\left(be^2n - 3\,be^2\log(c) - 3\,ae^2\right)x^3 - 2\,(bden - bde\log(c) - ade)x + \frac{1}{3}\left(be^2nx^3 + 6\,bdenx - \frac{3\,bd^2n}{x}\right)\log(x) - \frac{bd^2n + bd^2\log(c) + ad^2}{x}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

```
[Out] -1/9*(b*e^2*n - 3*b*e^2*log(c) - 3*a*e^2)*x^3 - 2*(b*d*e*n - b*d*e*log(c) -
 a*d*e)*x + 1/3*(b*e^2*n*x^3 + 6*b*d*e*n*x - 3*b*d^2*n/x)*log(x) - (b*d^2*n
 + b*d^2*log(c) + a*d^2)/x
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^2}\,dx = \ln(c\,x^n)\left(\frac{\frac{4\,b\,e^2\,x^4}{3} + 4\,b\,d\,e\,x^2}{x} - \frac{b\,d^2 + 2\,b\,d\,e\,x^2 + b\,e^2\,x^4}{x}\right) - \frac{a\,d^2 + b\,d^2\,n}{x} + \frac{e^2\,x^3\,(3\,a - b\,n)}{9} + 2\,d\,e\,x\,(a - b\,n)$$

```
[In] int(((d + e*x^2)^2*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] log(c*x^n)*(((4*b*e^2*x^4)/3 + 4*b*d*e*x^2)/x - (b*d^2 + b*e^2*x^4 + 2*b*d*
e*x^2)/x) - (a*d^2 + b*d^2*n)/x + (e^2*x^3*(3*a - b*n))/9 + 2*d*e*x*(a - b*
n)
```

3.193 $\int \frac{(d+ex^2)^2(a+b\log(cx^n))}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{(d+ex^2)^2\,(a+b\log(cx^n))}{x^4}\,dx = -\frac{bd^2n}{9x^3}-\frac{2bden}{x}-be^2nx-\frac{d^2(a+b\log(cx^n))}{3x^3}-\frac{2de(a+b\log(cx^n))}{x}+e^2x(a+b\log(cx^n))$$

```
[Out] -1/9*b*d^2*n/x^3-2*b*d*e*n/x-b*e^2*n*x-1/3*d^2*(a+b*ln(c*x^n))/x^3-2*d*e*(a
+b*ln(c*x^n))/x+e^2*x*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {276, 2372}

$$\int \frac{(d+ex^2)^2\,(a+b\log(cx^n))}{x^4}\,dx = -\frac{d^2(a+b\log(cx^n))}{3x^3}-\frac{2de(a+b\log(cx^n))}{x}+e^2x(a+b\log(cx^n))-\frac{bd^2n}{9x^3}-\frac{2bden}{x}-be^2nx$$

```
[In] Int[((d + e*x^2)^2*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] -1/9*(b*d^2*n)/x^3 - (2*b*d*e*n)/x - b*e^2*n*x - (d^2*(a + b*Log[c*x^n]))/(
3*x^3) - (2*d*e*(a + b*Log[c*x^n]))/x + e^2*x*(a + b*Log[c*x^n])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{3x^3} - \frac{2de(a+b\log(cx^n))}{x} \\
&\quad + e^2x(a+b\log(cx^n)) - (bn)\int\left(e^2 - \frac{d^2}{3x^4} - \frac{2de}{x^2}\right)dx \\
&= -\frac{bd^2n}{9x^3} - \frac{2bden}{x} - be^2nx - \frac{d^2(a+b\log(cx^n))}{3x^3} - \frac{2de(a+b\log(cx^n))}{x} + e^2x(a+b\log(cx^n))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$
\int \frac{(d+ex^2)^2(a+b\log(cx^n))}{x^4}\,dx = -\frac{3a(d^2+6dex^2-3e^2x^4)+bn(d^2+18dex^2+9e^2x^4)+3b(d^2+6dex^2-3e^2x^4)\log(cx^n)}{9x^3}
$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*Log[c*x^n]))/x^4,x]
[Out] -1/9*(3*a*(d^2 + 6*d*e*x^2 - 3*e^2*x^4) + b*n*(d^2 + 18*d*e*x^2 + 9*e^2*x^4
) + 3*b*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*Log[c*x^n])/x^3
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.17

method	result
parallelrisch	$-\frac{-9x^4b\ln(cx^n)e^2+9be^2nx^4-9x^4ae^2+18b\ln(cx^n)dex^2+18bdenx^2+18adex^2+3b\ln(cx^n)d^2+bd^2n+3ad^2}{9x^3}$
risch	$-\frac{b(-3e^2x^4+6dex^2+d^2)\ln(x^n)}{3x^3} - \frac{-9i\pi be^2x^4\operatorname{csgn}(ic)\operatorname{csgn}(icx^n)^2+3i\pi bd^2\operatorname{csgn}(ic)\operatorname{csgn}(icx^n)^2-18i\pi bdex^2\operatorname{csgn}(icx^n)^3+}{}$

```
[In] int((e*x^2+d)^2*(a+b*ln(c*x^n))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/x^3*(-9*x^4*b*ln(c*x^n)*e^2+9*b*e^2*n*x^4-9*x^4*a*e^2+18*b*ln(c*x^n)*d
*e*x^2+18*b*d*e*n*x^2+18*a*d*e*x^2+3*b*ln(c*x^n)*d^2+b*d^2*n+3*a*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^4}\,dx = -\frac{9\,(be^2n-ae^2)x^4+bd^2n+3\,ad^2+18\,(bden+ade)x^2-3\,(3\,be^2x^4-6\,bdex^2-bd^2)\log(c)-3\,(3\,be^2nx^4}{9\,x^3}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(9*(b*e^2*n - a*e^2)*x^4 + b*d^2*n + 3*a*d^2 + 18*(b*d*e*n + a*d*e)*x^
2 - 3*(3*b*e^2*x^4 - 6*b*d*e*x^2 - b*d^2)*log(c) - 3*(3*b*e^2*n*x^4 - 6*b*d
*e*n*x^2 - b*d^2*n)*log(x))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^4}\,dx = -\frac{ad^2}{3x^3}-\frac{2ade}{x}+ae^2x-\frac{bd^2n}{9x^3}-\frac{bd^2\log(cx^n)}{3x^3}-\frac{2bden}{x}-\frac{2bde\log(cx^n)}{x}-be^2nx+be^2x\log(cx^n)$$

```
[In] integrate((e*x**2+d)**2*(a+b*ln(c*x**n))/x**4,x)
```

```
[Out] -a*d**2/(3*x**3) - 2*a*d*e/x + a*e**2*x - b*d**2*n/(9*x**3) - b*d**2*log(c*
x**n)/(3*x**3) - 2*b*d*e*n/x - 2*b*d*e*log(c*x**n)/x - b*e**2*n*x + b*e**2*
x*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^4}\,dx = -be^2nx+be^2x\log(cx^n)+ae^2x-\frac{2\,bden}{x}-\frac{2\,bde\log(cx^n)}{x}-\frac{2\,ade}{x}-\frac{bd^2n}{9\,x^3}-\frac{bd^2\log(cx^n)}{3\,x^3}-\frac{ad^2}{3\,x^3}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^4,x, algorithm="maxima")
```

```
[Out] -b*e^2*n*x + b*e^2*x*log(c*x^n) + a*e^2*x - 2*b*d*e*n/x - 2*b*d*e*log(c*x^n
)/x - 2*a*d*e/x - 1/9*b*d^2*n/x^3 - 1/3*b*d^2*log(c*x^n)/x^3 - 1/3*a*d^2/x^
3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.37

$$\int \frac{(d+ex^2)^2(a+b\log(cx^n))}{x^4}\,dx = -(be^2n-be^2\log(c)-ae^2)x+\frac{1}{3}\left(3\,be^2nx-\frac{6\,bdenx^2+bd^2n}{x^3}\right)\log(x) - \frac{18\,bdenx^2+18\,bdex^2\log(c)+18\,adex^2+bd^2n+3\,bd^2\log(c)+3\,ad^2}{9\,x^3}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^4,x, algorithm="giac")
```

```
[Out] -(b*e^2*n - b*e^2*log(c) - a*e^2)*x + 1/3*(3*b*e^2*n*x - (6*b*d*e*n*x^2 + b
*d^2*n)/x^3)*log(x) - 1/9*(18*b*d*e*n*x^2 + 18*b*d*e*x^2*log(c) + 18*a*d*e*
x^2 + b*d^2*n + 3*b*d^2*log(c) + 3*a*d^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex^2)^2(a+b\log(cx^n))}{x^4}\,dx = e^2\,x\,(a-b\,n)-\frac{x^2\,(6\,a\,d\,e+6\,b\,d\,e\,n)+a\,d^2+\frac{b\,d^2\,n}{3}}{3\,x^3} - \ln(c\,x^n)\left(\frac{\frac{b\,d^2}{3}+2\,b\,d\,e\,x^2+\frac{5\,b\,e^2\,x^4}{3}}{x^3}-\frac{8\,b\,e^2\,x}{3}\right)$$

```
[In] int(((d + e*x^2)^2*(a + b*log(c*x^n)))/x^4,x)
```

```
[Out] e^2*x*(a - b*n) - (x^2*(6*a*d*e + 6*b*d*e*n) + a*d^2 + (b*d^2*n)/3)/(3*x^3)
 - log(c*x^n)*(((b*d^2)/3 + (5*b*e^2*x^4)/3 + 2*b*d*e*x^2)/x^3 - (8*b*e^2*x
)/3)
```

3.194 $\int \frac{(d+ex^2)^2(a+b\log(cx^n))}{x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{(d+ex^2)^2(a+b\log(cx^n))}{x^6}\,dx = -\frac{bd^2n}{25x^5}-\frac{2bden}{9x^3}-\frac{be^2n}{x}-\frac{d^2(a+b\log(cx^n))}{5x^5}-\frac{2de(a+b\log(cx^n))}{3x^3}-\frac{e^2(a+b\log(cx^n))}{x}$$

```
[Out] -1/25*b*d^2*n/x^5-2/9*b*d*e*n/x^3-b*e^2*n/x-1/5*d^2*(a+b*ln(c*x^n))/x^5-2/3
*d*e*(a+b*ln(c*x^n))/x^3-e^2*(a+b*ln(c*x^n))/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2372, 12, 14}

$$\int \frac{(d+ex^2)^2(a+b\log(cx^n))}{x^6}\,dx = -\frac{d^2(a+b\log(cx^n))}{5x^5}-\frac{2de(a+b\log(cx^n))}{3x^3}-\frac{e^2(a+b\log(cx^n))}{x}-\frac{bd^2n}{25x^5}-\frac{2bden}{9x^3}-\frac{be^2n}{x}$$

```
[In] Int[((d + e*x^2)^2*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] -1/25*(b*d^2*n)/x^5 - (2*b*d*e*n)/(9*x^3) - (b*e^2*n)/x - (d^2*(a + b*Log[c
*x^n]))/(5*x^5) - (2*d*e*(a + b*Log[c*x^n]))/(3*x^3) - (e^2*(a + b*Log[c*x^
n]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{5x^5}-\frac{2de(a+b\log(cx^n))}{3x^3} \\
&\quad -\frac{e^2(a+b\log(cx^n))}{x}-(bn)\int\frac{-3d^2-10dex^2-15e^2x^4}{15x^6}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{5x^5}-\frac{2de(a+b\log(cx^n))}{3x^3} \\
&\quad -\frac{e^2(a+b\log(cx^n))}{x}-\frac{1}{15}(bn)\int\frac{-3d^2-10dex^2-15e^2x^4}{x^6}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{5x^5}-\frac{2de(a+b\log(cx^n))}{3x^3} \\
&\quad -\frac{e^2(a+b\log(cx^n))}{x}-\frac{1}{15}(bn)\int\left(-\frac{3d^2}{x^6}-\frac{10de}{x^4}-\frac{15e^2}{x^2}\right)dx \\
&= -\frac{bd^2n}{25x^5}-\frac{2bden}{9x^3}-\frac{be^2n}{x}-\frac{d^2(a+b\log(cx^n))}{5x^5}-\frac{2de(a+b\log(cx^n))}{3x^3}-\frac{e^2(a+b\log(cx^n))}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^2)^2\,(a+b\log{(cx^n)})}{x^6}\,dx = -\frac{15a(3d^2+10dex^2+15e^2x^4)+bn(9d^2+50dex^2+225e^2x^4)+15b(3d^2+10dex^2+15e^2x^4)\log{(cx^n)}}{225x^5}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] -1/225*(15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*n*(9*d^2 + 50*d*e*x^2 +
225*e^2*x^4) + 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*Log[c*x^n])/x^5
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07

method	result
parallelrisch	$-\frac{225x^4b\ln(c\,x^n)e^2+225b\,e^2n\,x^4+225x^4a\,e^2+150b\ln(c\,x^n)de\,x^2+50bden\,x^2+150ade\,x^2+45b\ln(c\,x^n)d^2+9b\,d^2n+45a\,d^2}{225x^5}$
risch	$-\frac{b(15e^2x^4+10de\,x^2+3d^2)\ln(x^n)}{15x^5}-\frac{-150i\pi bde\,x^2\operatorname{csgn}(ic\,x^n)^3-225i\pi b\,e^2x^4\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)+45i\pi b\,d^2\operatorname{csgn}(}{}$

```
[In] int((e*x^2+d)^2*(a+b*ln(c*x^n))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/225/x^5*(225*x^4*b*ln(c*x^n)*e^2+225*b*e^2*n*x^4+225*x^4*a*e^2+150*b*ln(
c*x^n)*d*e*x^2+50*b*d*e*n*x^2+150*a*d*e*x^2+45*b*ln(c*x^n)*d^2+9*b*d^2*n+45
*a*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex^2)^2\,(a+b\log{(cx^n)})}{x^6}\,dx = -\frac{225\,(be^2n+ae^2)x^4+9\,bd^2n+45\,ad^2+50\,(bden+3\,ade)x^2+15\,(15\,be^2x^4+10\,bdex^2+3\,bd^2)\log{(c)}+}{225\,x^5}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*(225*(b*e^2*n + a*e^2)*x^4 + 9*b*d^2*n + 45*a*d^2 + 50*(b*d*e*n + 3*
a*d*e)*x^2 + 15*(15*b*e^2*x^4 + 10*b*d*e*x^2 + 3*b*d^2)*log(c) + 15*(15*b*e
^2*n*x^4 + 10*b*d*e*n*x^2 + 3*b*d^2*n)*log(x))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex^2)^2\,(a+b\log{(cx^n)})}{x^6}\,dx = -\frac{ad^2}{5x^5} - \frac{2ade}{3x^3} - \frac{ae^2}{x} - \frac{bd^2n}{25x^5} - \frac{bd^2\log{(cx^n)}}{5x^5} - \frac{2bden}{9x^3} - \frac{2bde\log{(cx^n)}}{3x^3} - \frac{be^2n}{x} - \frac{be^2\log{(cx^n)}}{x}$$

```
[In] integrate((e*x**2+d)**2*(a+b*ln(c*x**n))/x**6,x)
```

```
[Out] -a*d**2/(5*x**5) - 2*a*d*e/(3*x**3) - a*e**2/x - b*d**2*n/(25*x**5) - b*d**
2*log(c*x**n)/(5*x**5) - 2*b*d*e*n/(9*x**3) - 2*b*d*e*log(c*x**n)/(3*x**3)
- b*e**2*n/x - b*e**2*log(c*x**n)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex^2)^2\,(a+b\log{(cx^n)})}{x^6}\,dx = -\frac{be^2n}{x} - \frac{be^2\log{(cx^n)}}{x} - \frac{ae^2}{x} - \frac{2\,bden}{9\,x^3} - \frac{2\,bde\log{(cx^n)}}{3\,x^3} - \frac{2\,ade}{3\,x^3} - \frac{bd^2n}{25\,x^5} - \frac{bd^2\log{(cx^n)}}{5\,x^5} - \frac{ad^2}{5\,x^5}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^6,x, algorithm="maxima")
```

```
[Out] -b*e^2*n/x - b*e^2*log(c*x^n)/x - a*e^2/x - 2/9*b*d*e*n/x^3 - 2/3*b*d*e*log
(c*x^n)/x^3 - 2/3*a*d*e/x^3 - 1/25*b*d^2*n/x^5 - 1/5*b*d^2*log(c*x^n)/x^5 -
 1/5*a*d^2/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex^2)^2\,(a+b\log{(cx^n)})}{x^6}\,dx = -\frac{(15\,be^2nx^4 + 10\,bdenx^2 + 3\,bd^2n)\log{(x)}}{15\,x^5} - \frac{225\,be^2nx^4 + 225\,be^2x^4\log{(c)} + 225\,ae^2x^4 + 50\,bdenx^2 + 150\,bdex^2\log{(c)} + 150\,adex^2 + 9\,bd^2n + 45\,bd^2}{225\,x^5}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*b*e^2*n*x^4 + 10*b*d*e*n*x^2 + 3*b*d^2*n)*log(x)/x^5 - 1/225*(225
*b*e^2*n*x^4 + 225*b*e^2*x^4*log(c) + 225*a*e^2*x^4 + 50*b*d*e*n*x^2 + 150*
b*d*e*x^2*log(c) + 150*a*d*e*x^2 + 9*b*d^2*n + 45*b*d^2*log(c) + 45*a*d^2)/
x^5
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex^2)^2 (a+b\log (cx^n))}{x^6}\,dx$$
$$= -\frac{x^4 (15\,a\,e^2+15\,b\,e^2\,n)+x^2 \left(10\,a\,d\,e+\frac{10\,b\,d\,e\,n}{3}\right)+3\,a\,d^2+\frac{3\,b\,d^2\,n}{5}}{15\,x^5}$$
$$-\frac{\ln (c\,x^n) \left(\frac{b\,d^2}{5}+\frac{2\,b\,d\,e\,x^2}{3}+b\,e^2\,x^4\right)}{x^5}$$

```
[In] int(((d + e*x^2)^2*(a + b*log(c*x^n)))/x^6,x)
```

```
[Out] - (x^4*(15*a*e^2 + 15*b*e^2*n) + x^2*(10*a*d*e + (10*b*d*e*n)/3) + 3*a*d^2
+ (3*b*d^2*n)/5)/(15*x^5) - (log(c*x^n)*((b*d^2)/5 + b*e^2*x^4 + (2*b*d*e*x
^2)/3))/x^5
```

3.195 $\int \frac{(d+ex^2)^2(a+b\log(cx^n))}{x^8}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \frac{(d+ex^2)^2(a+b\log(cx^n))}{x^8}\,dx = -\frac{bd^2n}{49x^7}-\frac{2bden}{25x^5}-\frac{be^2n}{9x^3}-\frac{d^2(a+b\log(cx^n))}{7x^7}-\frac{2de(a+b\log(cx^n))}{5x^5}-\frac{e^2(a+b\log(cx^n))}{3x^3}$$

```
[Out] -1/49*b*d^2*n/x^7-2/25*b*d*e*n/x^5-1/9*b*e^2*n/x^3-1/7*d^2*(a+b*ln(c*x^n))/
x^7-2/5*d*e*(a+b*ln(c*x^n))/x^5-1/3*e^2*(a+b*ln(c*x^n))/x^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {276, 2372, 12, 14}

$$\int \frac{(d+ex^2)^2(a+b\log(cx^n))}{x^8}\,dx = -\frac{d^2(a+b\log(cx^n))}{7x^7}-\frac{2de(a+b\log(cx^n))}{5x^5}-\frac{e^2(a+b\log(cx^n))}{3x^3}-\frac{bd^2n}{49x^7}-\frac{2bden}{25x^5}-\frac{be^2n}{9x^3}$$

```
[In] Int[((d + e*x^2)^2*(a + b*Log[c*x^n]))/x^8,x]
```

```
[Out] -1/49*(b*d^2*n)/x^7 - (2*b*d*e*n)/(25*x^5) - (b*e^2*n)/(9*x^3) - (d^2*(a +
b*Log[c*x^n]))/(7*x^7) - (2*d*e*(a + b*Log[c*x^n]))/(5*x^5) - (e^2*(a + b*L
og[c*x^n]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{7x^7} - \frac{2de(a+b\log(cx^n))}{5x^5} \\
&\quad - \frac{e^2(a+b\log(cx^n))}{3x^3} - (bn)\int \frac{-15d^2-42dex^2-35e^2x^4}{105x^8}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{7x^7} - \frac{2de(a+b\log(cx^n))}{5x^5} - \frac{e^2(a+b\log(cx^n))}{3x^3} \\
&\quad - \frac{1}{105}(bn)\int \frac{-15d^2-42dex^2-35e^2x^4}{x^8}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{7x^7} - \frac{2de(a+b\log(cx^n))}{5x^5} - \frac{e^2(a+b\log(cx^n))}{3x^3} \\
&\quad - \frac{1}{105}(bn)\int \left(-\frac{15d^2}{x^8} - \frac{42de}{x^6} - \frac{35e^2}{x^4}\right)dx \\
&= -\frac{bd^2n}{49x^7} - \frac{2bden}{25x^5} - \frac{be^2n}{9x^3} - \frac{d^2(a+b\log(cx^n))}{7x^7} - \frac{2de(a+b\log(cx^n))}{5x^5} - \frac{e^2(a+b\log(cx^n))}{3x^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^8}\,dx = -\frac{bd^2n}{49x^7}-\frac{2bden}{25x^5}-\frac{be^2n}{9x^3}-\frac{d^2(a+b\log(cx^n))}{7x^7} - \frac{2de(a+b\log(cx^n))}{5x^5}-\frac{e^2(a+b\log(cx^n))}{3x^3}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*Log[c*x^n]))/x^8,x]
```

```
[Out] -1/49*(b*d^2*n)/x^7 - (2*b*d*e*n)/(25*x^5) - (b*e^2*n)/(9*x^3) - (d^2*(a +
b*Log[c*x^n]))/(7*x^7) - (2*d*e*(a + b*Log[c*x^n]))/(5*x^5) - (e^2*(a + b*L
og[c*x^n]))/(3*x^3)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02

method	result
parallelrisch	$-\frac{3675x^4b\ln(cx^n)e^2+1225be^2nx^4+3675x^4ae^2+4410b\ln(cx^n)dex^2+882bdenx^2+4410adex^2+1575b\ln(cx^n)d^2+225bd^2n+1}{11025x^7}$
risch	$-\frac{b(35e^2x^4+42dex^2+15d^2)\ln(x^n)}{105x^7}-\frac{-1575i\pi bd^2\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)+1575i\pi bd^2\operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)^2-3675i\pi}{}$

```
[In] int((e*x^2+d)^2*(a+b*ln(c*x^n))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/11025/x^7*(3675*x^4*b*ln(c*x^n)*e^2+1225*b*e^2*n*x^4+3675*x^4*a*e^2+4410
*b*ln(c*x^n)*d*e*x^2+882*b*d*e*n*x^2+4410*a*d*e*x^2+1575*b*ln(c*x^n)*d^2+22
5*b*d^2*n+1575*a*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^8}\,dx = -\frac{1225\,(be^2n+3\,ae^2)x^4+225\,bd^2n+1575\,ad^2+882\,(bden+5\,ade)x^2+105\,(35\,be^2x^4+42\,bdex^2+15\,bd^2}{11025\,x^7}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^8,x, algorithm="fricas")
```

```
[Out] -1/11025*(1225*(b*e^2*n + 3*a*e^2)*x^4 + 225*b*d^2*n + 1575*a*d^2 + 882*(b*
d*e*n + 5*a*d*e)*x^2 + 105*(35*b*e^2*x^4 + 42*b*d*e*x^2 + 15*b*d^2)*log(c)
+ 105*(35*b*e^2*n*x^4 + 42*b*d*e*n*x^2 + 15*b*d^2*n)*log(x))/x^7
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^8}\,dx = -\frac{ad^2}{7x^7} - \frac{2ade}{5x^5} - \frac{ae^2}{3x^3} - \frac{bd^2n}{49x^7} - \frac{bd^2\log(cx^n)}{7x^7} - \frac{2bden}{25x^5} - \frac{2bde\log(cx^n)}{5x^5} - \frac{be^2n}{9x^3} - \frac{be^2\log(cx^n)}{3x^3}$$

```
[In] integrate((e*x**2+d)**2*(a+b*ln(c*x**n))/x**8,x)
```

```
[Out] -a*d**2/(7*x**7) - 2*a*d*e/(5*x**5) - a*e**2/(3*x**3) - b*d**2*n/(49*x**7)
- b*d**2*log(c*x**n)/(7*x**7) - 2*b*d*e*n/(25*x**5) - 2*b*d*e*log(c*x**n)/(
5*x**5) - b*e**2*n/(9*x**3) - b*e**2*log(c*x**n)/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^8}\,dx = -\frac{be^2n}{9\,x^3} - \frac{be^2\log(cx^n)}{3\,x^3} - \frac{ae^2}{3\,x^3} - \frac{2\,bden}{25\,x^5} - \frac{2\,bde\log(cx^n)}{5\,x^5} - \frac{2\,ade}{5\,x^5} - \frac{bd^2n}{49\,x^7} - \frac{bd^2\log(cx^n)}{7\,x^7} - \frac{ad^2}{7\,x^7}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^8,x, algorithm="maxima")
```

```
[Out] -1/9*b*e^2*n/x^3 - 1/3*b*e^2*log(c*x^n)/x^3 - 1/3*a*e^2/x^3 - 2/25*b*d*e*n/
x^5 - 2/5*b*d*e*log(c*x^n)/x^5 - 2/5*a*d*e/x^5 - 1/49*b*d^2*n/x^7 - 1/7*b*d
^2*log(c*x^n)/x^7 - 1/7*a*d^2/x^7
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^8}\,dx = -\frac{(35\,be^2nx^4 + 42\,bdenx^2 + 15\,bd^2n)\log(x)}{105\,x^7} - \frac{1225\,be^2nx^4 + 3675\,be^2x^4\log(c) + 3675\,ae^2x^4 + 882\,bdenx^2 + 4410\,bdex^2\log(c) + 4410\,adex^2 + 225\,bd}{11025\,x^7}$$

```
[In] integrate((e*x^2+d)^2*(a+b*log(c*x^n))/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b*e^2*n*x^4 + 42*b*d*e*n*x^2 + 15*b*d^2*n)*log(x)/x^7 - 1/11025*
(1225*b*e^2*n*x^4 + 3675*b*e^2*x^4*log(c) + 3675*a*e^2*x^4 + 882*b*d*e*n*x^
2 + 4410*b*d*e*x^2*log(c) + 4410*a*d*e*x^2 + 225*b*d^2*n + 1575*b*d^2*log(c
) + 1575*a*d^2)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex^2)^2 (a+b\log(cx^n))}{x^8} dx$$
$$= -\frac{x^4 \left(35\,a\,e^2+\frac{35\,b\,e^2\,n}{3}\right)+x^2 \left(42\,a\,d\,e+\frac{42\,b\,d\,e\,n}{5}\right)+15\,a\,d^2+\frac{15\,b\,d^2\,n}{7}}{105\,x^7}$$
$$-\frac{\ln(c\,x^n) \left(\frac{b\,d^2}{7}+\frac{2\,b\,d\,e\,x^2}{5}+\frac{b\,e^2\,x^4}{3}\right)}{x^7}$$

```
[In] int(((d + e*x^2)^2*(a + b*log(c*x^n)))/x^8,x)
```

```
[Out] - (x^4*(35*a*e^2 + (35*b*e^2*n)/3) + x^2*(42*a*d*e + (42*b*d*e*n)/5) + 15*a
*d^2 + (15*b*d^2*n)/7)/(105*x^7) - (log(c*x^n)*((b*d^2)/7 + (b*e^2*x^4)/3 +
 (2*b*d*e*x^2)/5))/x^7
```

3.196 $\int x^5(d+ex^2)^3 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int x^5 (d+ex^2)^3 (a+b\log(cx^n))\,dx = -\frac{1}{36}bd^3nx^6-\frac{3}{64}bd^2enx^8-\frac{3}{100}bde^2nx^{10}-\frac{1}{144}be^3nx^{12}+\frac{1}{120}(20d^3x^6+45d^2ex^8+36de^2x^{10}+10e^3x^{12})(a+b\log(cx^n))$$

```
[Out] -1/36*b*d^3*n*x^6-3/64*b*d^2*e*n*x^8-3/100*b*d*e^2*n*x^10-1/144*b*e^3*n*x^1
2+1/120*(10*e^3*x^12+36*d*e^2*x^10+45*d^2*e*x^8+20*d^3*x^6)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {272, 45, 2371, 12, 14}

$$\int x^5 (d+ex^2)^3 (a+b\log(cx^n))\,dx = \frac{1}{120}(20d^3x^6+45d^2ex^8+36de^2x^{10}+10e^3x^{12})(a+b\log(cx^n))-\frac{1}{36}bd^3nx^6-\frac{3}{64}bd^2enx^8-\frac{3}{100}bde^2nx^{10}-\frac{1}{144}be^3nx^{12}$$

```
[In] Int[x^5*(d + e*x^2)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/36*(b*d^3*n*x^6) - (3*b*d^2*e*n*x^8)/64 - (3*b*d*e^2*n*x^10)/100 - (b*e^
3*n*x^12)/144 + ((20*d^3*x^6 + 45*d^2*e*x^8 + 36*d*e^2*x^10 + 10*e^3*x^12)*
(a + b*Log[c*x^n]))/120
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{120}\left(20d^3x^6+45d^2ex^8+36de^2x^{10}+10e^3x^{12}\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int \frac{1}{120}x^5\left(20d^3+45d^2ex^2+36de^2x^4+10e^3x^6\right)\,dx \\
&= \frac{1}{120}\left(20d^3x^6+45d^2ex^8+36de^2x^{10}+10e^3x^{12}\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{120}(bn)\int x^5\left(20d^3+45d^2ex^2+36de^2x^4+10e^3x^6\right)\,dx \\
&= \frac{1}{120}\left(20d^3x^6+45d^2ex^8+36de^2x^{10}+10e^3x^{12}\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{120}(bn)\int \left(20d^3x^5+45d^2ex^7+36de^2x^9+10e^3x^{11}\right)\,dx
\end{aligned}
$$

$$= -\frac{1}{36}bd^3nx^6 - \frac{3}{64}bd^2enx^8 - \frac{3}{100}bde^2nx^{10} - \frac{1}{144}be^3nx^{12} + \frac{1}{120}\left(20d^3x^6 + 45d^2ex^8 + 36de^2x^{10} + 10e^3x^{12}\right)\left(a + b\log\left(cx^n\right)\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int x^5\left(d+ex^2\right)^3\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{x^6(120a(20d^3+45d^2ex^2+36de^2x^4+10e^3x^6)-bn(400d^3+675d^2ex^2+432de^2x^4+100e^3x^6)+120b(20d}{14400}$$

```
[In] Integrate[x^5*(d + e*x^2)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^6*(120*a*(20*d^3 + 45*d^2*e*x^2 + 36*d*e^2*x^4 + 10*e^3*x^6) - b*n*(400*
d^3 + 675*d^2*e*x^2 + 432*d*e^2*x^4 + 100*e^3*x^6) + 120*b*(20*d^3 + 45*d^2
*e*x^2 + 36*d*e^2*x^4 + 10*e^3*x^6)*Log[c*x^n]))/14400
```

Maple [A] (verified)

Time = 9.55 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.44

method	result
parallelrisch	$\frac{e^3b\ln(cx^n)x^{12}}{12}-\frac{be^3nx^{12}}{144}+\frac{ae^3x^{12}}{12}+\frac{3e^2db\ln(cx^n)x^{10}}{10}-\frac{3bde^2nx^{10}}{100}+\frac{3ade^2x^{10}}{10}+\frac{3ed^2b\ln(cx^n)x^8}{8}-\frac{3bd^2e}{64}$
risch	$\frac{bx^6\left(10e^3x^6+36e^2dx^4+45d^2ex^2+20d^3\right)\ln(x^n)}{120}+\frac{ae^3x^{12}}{12}+\frac{ad^3x^6}{6}-\frac{3i\pi bde^2x^{10}\,\mathrm{csgn}(icx^n)^3}{20}+\frac{i\pi be^3x^{12}\,\mathrm{csgn}(ic)\,\mathrm{cs}}{24}$

```
[In] int(x^5*(e*x^2+d)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*e^3*b*ln(c*x^n)*x^12-1/144*b*e^3*n*x^12+1/12*a*e^3*x^12+3/10*e^2*d*b*l
n(c*x^n)*x^10-3/100*b*d*e^2*n*x^10+3/10*a*d*e^2*x^10+3/8*e*d^2*b*ln(c*x^n)*
x^8-3/64*b*d^2*e*n*x^8+3/8*a*d^2*e*x^8+1/6*b*d^3*ln(c*x^n)*x^6-1/36*b*d^3*n
*x^6+1/6*a*d^3*x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.67

$$\int x^5 (d+ex^2)^3 (a+b\log(cx^n))\,dx$$
$$= -\frac{1}{144}(be^3n-12\,ae^3)x^{12} - \frac{3}{100}(bde^2n-10\,ade^2)x^{10} - \frac{3}{64}(bd^2en-8\,ad^2e)x^8 - \frac{1}{36}(bd^3n-6\,ad^3)x^6 + \frac{1}{120}(10\,be^3x^{12}+36\,bde^2x^{10}+45\,bd^2ex^8+20\,bd^3x^6)\log(c) + \frac{1}{120}(10\,be^3nx^{12}+36\,bde^2nx^{10}+45\,bd^2enx^8+20\,bd^3nx^6)\log(x)$$

```
[In] integrate(x^5*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/144*(b*e^3*n - 12*a*e^3)*x^12 - 3/100*(b*d*e^2*n - 10*a*d*e^2)*x^10 - 3/
64*(b*d^2*e*n - 8*a*d^2*e)*x^8 - 1/36*(b*d^3*n - 6*a*d^3)*x^6 + 1/120*(10*b
*e^3*x^12 + 36*b*d*e^2*x^10 + 45*b*d^2*e*x^8 + 20*b*d^3*x^6)*log(c) + 1/120
*(10*b*e^3*n*x^12 + 36*b*d*e^2*n*x^10 + 45*b*d^2*e*n*x^8 + 20*b*d^3*n*x^6)*
log(x)
```

Sympy [A] (verification not implemented)

Time = 2.96 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.75

$$\int x^5 (d+ex^2)^3 (a+b\log(cx^n))\,dx = \frac{ad^3x^6}{6} + \frac{3ad^2ex^8}{8} + \frac{3ade^2x^{10}}{10} + \frac{ae^3x^{12}}{12} - \frac{bd^3nx^6}{36} + \frac{bd^3x^6\log(cx^n)}{6} - \frac{3bd^2enx^8}{64} + \frac{3bd^2ex^8\log(cx^n)}{8} - \frac{3bde^2nx^{10}}{100} + \frac{3bde^2x^{10}\log(cx^n)}{10} - \frac{be^3nx^{12}}{144} + \frac{be^3x^{12}\log(cx^n)}{12}$$

```
[In] integrate(x**5*(e*x**2+d)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**3*x**6/6 + 3*a*d**2*e*x**8/8 + 3*a*d*e**2*x**10/10 + a*e**3*x**12/12 -
 b*d**3*n*x**6/36 + b*d**3*x**6*log(c*x**n)/6 - 3*b*d**2*e*n*x**8/64 + 3*b*
d**2*e*x**8*log(c*x**n)/8 - 3*b*d*e**2*n*x**10/100 + 3*b*d*e**2*x**10*log(c
*x**n)/10 - b*e**3*n*x**12/144 + b*e**3*x**12*log(c*x**n)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.43

$$\int x^5 \left(d+ex^2\right)^3 (a+b\log\left(cx^n\right))\,dx = -\frac{1}{144}\,be^3nx^{12} + \frac{1}{12}\,be^3x^{12}\log\left(cx^n\right) + \frac{1}{12}\,ae^3x^{12} - \frac{3}{100}\,bde^2nx^{10} + \frac{3}{10}\,bde^2x^{10}\log\left(cx^n\right) + \frac{3}{10}\,ade^2x^{10} - \frac{3}{64}\,bd^2enx^8 + \frac{3}{8}\,bd^2ex^8\log\left(cx^n\right) + \frac{3}{8}\,ad^2ex^8 - \frac{1}{36}\,bd^3nx^6 + \frac{1}{6}\,bd^3x^6\log\left(cx^n\right) + \frac{1}{6}\,ad^3x^6$$

```
[In] integrate(x^5*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/144*b*e^3*n*x^12 + 1/12*b*e^3*x^12*log(c*x^n) + 1/12*a*e^3*x^12 - 3/100*
b*d*e^2*n*x^10 + 3/10*b*d*e^2*x^10*log(c*x^n) + 3/10*a*d*e^2*x^10 - 3/64*b*
d^2*e*n*x^8 + 3/8*b*d^2*e*x^8*log(c*x^n) + 3/8*a*d^2*e*x^8 - 1/36*b*d^3*n*x
^6 + 1/6*b*d^3*x^6*log(c*x^n) + 1/6*a*d^3*x^6
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.77

$$\int x^5 \left(d+ex^2\right)^3 (a+b\log\left(cx^n\right))\,dx = \frac{1}{12}\,be^3nx^{12}\log\left(x\right) - \frac{1}{144}\,be^3nx^{12} + \frac{1}{12}\,be^3x^{12}\log\left(c\right) + \frac{1}{12}\,ae^3x^{12} + \frac{3}{10}\,bde^2nx^{10}\log\left(x\right) - \frac{3}{100}\,bde^2nx^{10} + \frac{3}{10}\,bde^2x^{10}\log\left(c\right) + \frac{3}{10}\,ade^2x^{10} + \frac{3}{8}\,bd^2enx^8\log\left(x\right) - \frac{3}{64}\,bd^2enx^8 + \frac{3}{8}\,bd^2ex^8\log\left(c\right) + \frac{3}{8}\,ad^2ex^8 + \frac{1}{6}\,bd^3nx^6\log\left(x\right) - \frac{1}{36}\,bd^3nx^6 + \frac{1}{6}\,bd^3x^6\log\left(c\right) + \frac{1}{6}\,ad^3x^6$$

```
[In] integrate(x^5*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/12*b*e^3*n*x^12*log(x) - 1/144*b*e^3*n*x^12 + 1/12*b*e^3*x^12*log(c) + 1/
12*a*e^3*x^12 + 3/10*b*d*e^2*n*x^10*log(x) - 3/100*b*d*e^2*n*x^10 + 3/10*b*
d*e^2*x^10*log(c) + 3/10*a*d*e^2*x^10 + 3/8*b*d^2*e*n*x^8*log(x) - 3/64*b*d
^2*e*n*x^8 + 3/8*b*d^2*e*x^8*log(c) + 3/8*a*d^2*e*x^8 + 1/6*b*d^3*n*x^6*log
(x) - 1/36*b*d^3*n*x^6 + 1/6*b*d^3*x^6*log(c) + 1/6*a*d^3*x^6
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13

$$\int x^5 \left(d+ex^2\right)^3 (a+b\log(cx^n)) \, dx = \ln(c\,x^n) \left(\frac{b\,d^3\,x^6}{6} + \frac{3\,b\,d^2\,e\,x^8}{8} + \frac{3\,b\,d\,e^2\,x^{10}}{10} + \frac{b\,e^3\,x^{12}}{12}\right) + \frac{d^3\,x^6\,(6\,a-b\,n)}{36} + \frac{e^3\,x^{12}\,(12\,a-b\,n)}{144} + \frac{3\,d^2\,e\,x^8\,(8\,a-b\,n)}{64} + \frac{3\,d\,e^2\,x^{10}\,(10\,a-b\,n)}{100}$$

```
[In] int(x^5*(d + e*x^2)^3*(a + b*log(c*x^n)),x)
```

```
[Out] log(c*x^n)*((b*d^3*x^6)/6 + (b*e^3*x^12)/12 + (3*b*d^2*e*x^8)/8 + (3*b*d*e^
2*x^10)/10) + (d^3*x^6*(6*a - b*n))/36 + (e^3*x^12*(12*a - b*n))/144 + (3*d
^2*e*x^8*(8*a - b*n))/64 + (3*d*e^2*x^10*(10*a - b*n))/100
```

3.197 $\int x^3(d+ex^2)^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 130

$$\int x^3(d+ex^2)^3(a+b\log(cx^n))\,dx=\frac{bd^4nx^2}{20e}+\frac{3}{80}bd^3nx^4+\frac{1}{60}bd^2enx^6+\frac{1}{320}bde^2nx^8-\frac{bn(d+ex^2)^5}{100e^2}+\frac{bd^5n\log(x)}{40e^2}-\frac{1}{40}\left(\frac{5d(d+ex^2)^4}{e^2}-\frac{4(d+ex^2)^5}{e^2}\right)(a+b\log(cx^n))$$

```
[Out] 1/20*b*d^4*n*x^2/e+3/80*b*d^3*n*x^4+1/60*b*d^2*e*n*x^6+1/320*b*d*e^2*n*x^8-
1/100*b*n*(e*x^2+d)^5/e^2+1/40*b*d^5*n*ln(x)/e^2-1/40*(5*d*(e*x^2+d)^4/e^2-
4*(e*x^2+d)^5/e^2)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {272, 45, 2371, 12, 457, 81}

$$\int x^3(d+ex^2)^3(a+b\log(cx^n))\,dx=-\frac{1}{40}\left(\frac{5d(d+ex^2)^4}{e^2}-\frac{4(d+ex^2)^5}{e^2}\right)(a+b\log(cx^n))+\frac{bd^5n\log(x)}{40e^2}+\frac{bd^4nx^2}{20e}+\frac{3}{80}bd^3nx^4+\frac{1}{60}bd^2enx^6+\frac{1}{320}bde^2nx^8-\frac{bn(d+ex^2)^5}{100e^2}$$

```
[In] Int[x^3*(d + e*x^2)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] (b*d^4*n*x^2)/(20*e) + (3*b*d^3*n*x^4)/80 + (b*d^2*e*n*x^6)/60 + (b*d*e^2*n
*x^8)/320 - (b*n*(d + e*x^2)^5)/(100*e^2) + (b*d^5*n*Log[x])/(40*e^2) - (((
5*d*(d + e*x^2)^4)/e^2 - (4*(d + e*x^2)^5)/e^2)*(a + b*Log[c*x^n]))/40
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{40}\left(\frac{5d(d+ex^2)^4}{e^2}-\frac{4(d+ex^2)^5}{e^2}\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int \frac{(d+ex^2)^4(-d+4ex^2)}{40e^2x}\,dx \\
&= -\frac{1}{40}\left(\frac{5d(d+ex^2)^4}{e^2}-\frac{4(d+ex^2)^5}{e^2}\right)(a+b\log(cx^n))-\frac{(bn)\int \frac{(d+ex^2)^4(-d+4ex^2)}{x}\,dx}{40e^2} \\
&= -\frac{1}{40}\left(\frac{5d(d+ex^2)^4}{e^2}-\frac{4(d+ex^2)^5}{e^2}\right)(a+b\log(cx^n))-\frac{(bn)\text{Subst}\left(\int \frac{(d+ex)^4(-d+4ex)}{x}\,dx,x,x^2\right)}{80e^2} \\
&= -\frac{bn(d+ex^2)^5}{100e^2}-\frac{1}{40}\left(\frac{5d(d+ex^2)^4}{e^2}-\frac{4(d+ex^2)^5}{e^2}\right)(a+b\log(cx^n)) \\
&\quad +\frac{(bdn)\text{Subst}\left(\int \frac{(d+ex)^4}{x}\,dx,x,x^2\right)}{80e^2} \\
&= -\frac{bn(d+ex^2)^5}{100e^2}-\frac{1}{40}\left(\frac{5d(d+ex^2)^4}{e^2}-\frac{4(d+ex^2)^5}{e^2}\right)(a+b\log(cx^n)) \\
&\quad +\frac{(bdn)\text{Subst}\left(\int \left(4d^3e+\frac{d^4}{x}+6d^2e^2x+4de^3x^2+e^4x^3\right)\,dx,x,x^2\right)}{80e^2} \\
&= \frac{bd^4nx^2}{20e}+\frac{3}{80}bd^3nx^4+\frac{1}{60}bd^2enx^6+\frac{1}{320}bde^2nx^8-\frac{bn(d+ex^2)^5}{100e^2} \\
&\quad +\frac{bd^5n\log(x)}{40e^2}-\frac{1}{40}\left(\frac{5d(d+ex^2)^4}{e^2}-\frac{4(d+ex^2)^5}{e^2}\right)(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.92

$$\int x^3(d+ex^2)^3(a+b\log(cx^n))\,dx = \frac{x^4(120a(10d^3+20d^2ex^2+15de^2x^4+4e^3x^6)-bn(300d^3+400d^2ex^2+225de^2x^4+48e^3x^6)+120b(10d^3 \cdots}{4800}$$

```
[In] Integrate[x^3*(d + e*x^2)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^4*(120*a*(10*d^3 + 20*d^2*e*x^2 + 15*d*e^2*x^4 + 4*e^3*x^6) - b*n*(300*d
^3 + 400*d^2*e*x^2 + 225*d*e^2*x^4 + 48*e^3*x^6) + 120*b*(10*d^3 + 20*d^2*e
*x^2 + 15*d*e^2*x^4 + 4*e^3*x^6)*Log[c*x^n]))/4800
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{x^{10}\ln(c\,x^{n})b\,e^{3}}{10}-\frac{b\,e^{3}n\,x^{10}}{100}+\frac{a\,e^{3}x^{10}}{10}+\frac{3x^{8}\ln(c\,x^{n})bd\,e^{2}}{8}-\frac{3bd\,e^{2}n\,x^{8}}{64}+\frac{3ad\,e^{2}x^{8}}{8}+\frac{x^{6}\ln(c\,x^{n})b\,d^{2}e}{2}-\frac{b\,d^{2}en\,x^{6}}{12}$
risch	$\frac{a\,e^{3}x^{10}}{10}+\frac{a\,d^{3}x^{4}}{4}+\frac{i\pi b\,e^{3}x^{10}\operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})^{2}}{20}-\frac{3i\pi bd\,e^{2}x^{8}\operatorname{csgn}(ic\,x^{n})^{3}}{16}+\frac{i\pi b\,d^{2}e\,x^{6}\operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^{n})^{2}}{4}+$

```
[In] int(x^3*(e*x^2+d)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*x^10*ln(c*x^n)*b*e^3-1/100*b*e^3*n*x^10+1/10*a*e^3*x^10+3/8*x^8*ln(c*x
^n)*b*d*e^2-3/64*b*d*e^2*n*x^8+3/8*a*d*e^2*x^8+1/2*x^6*ln(c*x^n)*b*d^2*e-1/
12*b*d^2*e*n*x^6+1/2*a*d^2*e*x^6+1/4*x^4*ln(c*x^n)*b*d^3-1/16*b*d^3*n*x^4+1
/4*a*d^3*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.28

$$\begin{aligned}\int x^3(d+ex^2)^3(a+b\log(cx^n))\,dx &= -\frac{1}{100}\left(be^3n-10\,ae^3\right)x^{10}-\frac{3}{64}\left(bde^2n-8\,ade^2\right)x^8-\frac{1}{12}\left(bd^2en-6\,ad^2e\right)x^6\\&\quad-\frac{1}{16}\left(bd^3n-4\,ad^3\right)x^4+\frac{1}{40}\left(4\,be^3x^{10}+15\,bde^2x^8+20\,bd^2ex^6+10\,bd^3x^4\right)\log(c)\\&\quad+\frac{1}{40}\left(4\,be^3nx^{10}+15\,bde^2nx^8+20\,bd^2enx^6+10\,bd^3nx^4\right)\log(x)\end{aligned}$$

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/100*(b*e^3*n - 10*a*e^3)*x^10 - 3/64*(b*d*e^2*n - 8*a*d*e^2)*x^8 - 1/12*
(b*d^2*e*n - 6*a*d^2*e)*x^6 - 1/16*(b*d^3*n - 4*a*d^3)*x^4 + 1/40*(4*b*e^3*
x^10 + 15*b*d*e^2*x^8 + 20*b*d^2*e*x^6 + 10*b*d^3*x^4)*log(c) + 1/40*(4*b*e
^3*n*x^10 + 15*b*d*e^2*n*x^8 + 20*b*d^2*e*n*x^6 + 10*b*d^3*n*x^4)*log(x)
```

Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.31

$$\int x^3(d+ex^2)^3(a+b\log(cx^n))\,dx = \frac{ad^3x^4}{4}+\frac{ad^2ex^6}{2}+\frac{3ade^2x^8}{8}+\frac{ae^3x^{10}}{10} - \frac{bd^3nx^4}{16}+\frac{bd^3x^4\log(cx^n)}{4}-\frac{bd^2enx^6}{12} + \frac{bd^2ex^6\log(cx^n)}{2}-\frac{3bde^2nx^8}{64} + \frac{3bde^2x^8\log(cx^n)}{8}-\frac{be^3nx^{10}}{100}+\frac{be^3x^{10}\log(cx^n)}{10}$$

```
[In] integrate(x**3*(e*x**2+d)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**3*x**4/4 + a*d**2*e*x**6/2 + 3*a*d*e**2*x**8/8 + a*e**3*x**10/10 - b*d
**3*n*x**4/16 + b*d**3*x**4*log(c*x**n)/4 - b*d**2*e*n*x**6/12 + b*d**2*e*x
**6*log(c*x**n)/2 - 3*b*d*e**2*n*x**8/64 + 3*b*d*e**2*x**8*log(c*x**n)/8 -
b*e**3*n*x**10/100 + b*e**3*x**10*log(c*x**n)/10
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10

$$\int x^3(d+ex^2)^3(a+b\log(cx^n))\,dx = -\frac{1}{100}\,be^3nx^{10}+\frac{1}{10}\,be^3x^{10}\log(cx^n)+\frac{1}{10}\,ae^3x^{10} - \frac{3}{64}\,bde^2nx^8+\frac{3}{8}\,bde^2x^8\log(cx^n)+\frac{3}{8}\,ade^2x^8 - \frac{1}{12}\,bd^2enx^6+\frac{1}{2}\,bd^2ex^6\log(cx^n)+\frac{1}{2}\,ad^2ex^6 - \frac{1}{16}\,bd^3nx^4+\frac{1}{4}\,bd^3x^4\log(cx^n)+\frac{1}{4}\,ad^3x^4$$

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/100*b*e^3*n*x^10 + 1/10*b*e^3*x^10*log(c*x^n) + 1/10*a*e^3*x^10 - 3/64*b
*d*e^2*n*x^8 + 3/8*b*d*e^2*x^8*log(c*x^n) + 3/8*a*d*e^2*x^8 - 1/12*b*d^2*e*
n*x^6 + 1/2*b*d^2*e*x^6*log(c*x^n) + 1/2*a*d^2*e*x^6 - 1/16*b*d^3*n*x^4 + 1
/4*b*d^3*x^4*log(c*x^n) + 1/4*a*d^3*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.36

$$\int x^3\left(d+ex^2\right)^3\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{1}{10}\,be^3nx^{10}\log\left(x\right)-\frac{1}{100}\,be^3nx^{10}+\frac{1}{10}\,be^3x^{10}\log\left(c\right)+\frac{1}{10}\,ae^3x^{10}+\frac{3}{8}\,bde^2nx^8\log\left(x\right)-\frac{3}{64}\,bde^2nx^8+\frac{3}{8}\,bde^2x^8\log\left(c\right)+\frac{3}{8}\,ade^2x^8+\frac{1}{2}\,bd^2enx^6\log\left(x\right)-\frac{1}{12}\,bd^2enx^6+\frac{1}{2}\,bd^2ex^6\log\left(c\right)+\frac{1}{2}\,ad^2ex^6+\frac{1}{4}\,bd^3nx^4\log\left(x\right)-\frac{1}{16}\,bd^3nx^4+\frac{1}{4}\,bd^3x^4\log\left(c\right)+\frac{1}{4}\,ad^3x^4$$

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/10*b*e^3*n*x^10*log(x) - 1/100*b*e^3*n*x^10 + 1/10*b*e^3*x^10*log(c) + 1/
10*a*e^3*x^10 + 3/8*b*d*e^2*n*x^8*log(x) - 3/64*b*d*e^2*n*x^8 + 3/8*b*d*e^2
*x^8*log(c) + 3/8*a*d*e^2*x^8 + 1/2*b*d^2*e*n*x^6*log(x) - 1/12*b*d^2*e*n*x
^6 + 1/2*b*d^2*e*x^6*log(c) + 1/2*a*d^2*e*x^6 + 1/4*b*d^3*n*x^4*log(x) - 1/
16*b*d^3*n*x^4 + 1/4*b*d^3*x^4*log(c) + 1/4*a*d^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.87

$$\int x^3\left(d+ex^2\right)^3\left(a+b\log\left(cx^n\right)\right)\,dx=\ln\left(c\,x^n\right)\,\left(\frac{b\,d^3\,x^4}{4}+\frac{b\,d^2\,e\,x^6}{2}+\frac{3\,b\,d\,e^2\,x^8}{8}+\frac{b\,e^3\,x^{10}}{10}\right)+\frac{d^3\,x^4\,\left(4\,a-b\,n\right)}{16}+\frac{e^3\,x^{10}\,\left(10\,a-b\,n\right)}{100}+\frac{d^2\,e\,x^6\,\left(6\,a-b\,n\right)}{12}+\frac{3\,d\,e^2\,x^8\,\left(8\,a-b\,n\right)}{64}$$

```
[In] int(x^3*(d + e*x^2)^3*(a + b*log(c*x^n)),x)
```

```
[Out] log(c*x^n)*((b*d^3*x^4)/4 + (b*e^3*x^10)/10 + (b*d^2*e*x^6)/2 + (3*b*d*e^2*
x^8)/8) + (d^3*x^4*(4*a - b*n))/16 + (e^3*x^10*(10*a - b*n))/100 + (d^2*e*x
^6*(6*a - b*n))/12 + (3*d*e^2*x^8*(8*a - b*n))/64
```

3.198 $\int x(d+ex^2)^3 (a+b\log(cx^n)) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int x(d+ex^2)^3 (a+b\log(cx^n)) \, dx = -\frac{1}{4}bd^3nx^2 - \frac{3}{16}bd^2enx^4 - \frac{1}{12}bde^2nx^6 - \frac{1}{64}be^3nx^8 - \frac{bd^4n\log(x)}{8e} + \frac{(d+ex^2)^4 (a+b\log(cx^n))}{8e}$$

```
[Out] -1/4*b*d^3*n*x^2-3/16*b*d^2*e*n*x^4-1/12*b*d*e^2*n*x^6-1/64*b*e^3*n*x^8-1/8
*b*d^4*n*ln(x)/e+1/8*(e*x^2+d)^4*(a+b*ln(c*x^n))/e
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {267, 2371, 12, 272, 45}

$$\int x(d+ex^2)^3 (a+b\log(cx^n)) \, dx = \frac{(d+ex^2)^4 (a+b\log(cx^n))}{8e} - \frac{bd^4n\log(x)}{8e} - \frac{1}{4}bd^3nx^2 - \frac{3}{16}bd^2enx^4 - \frac{1}{12}bde^2nx^6 - \frac{1}{64}be^3nx^8$$

```
[In] Int[x*(d + e*x^2)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/4*(b*d^3*n*x^2) - (3*b*d^2*e*n*x^4)/16 - (b*d*e^2*n*x^6)/12 - (b*e^3*n*x
^8)/64 - (b*d^4*n*Log[x])/(8*e) + ((d + e*x^2)^4*(a + b*Log[c*x^n]))/(8*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex^2)^4\,(a+b\log(cx^n))}{8e}-(bn)\int\frac{(d+ex^2)^4}{8ex}\,dx\\
&= \frac{(d+ex^2)^4\,(a+b\log(cx^n))}{8e}-\frac{(bn)\int\frac{(d+ex^2)^4}{x}\,dx}{8e}\\
&= \frac{(d+ex^2)^4\,(a+b\log(cx^n))}{8e}-\frac{(bn)\text{Subst}\left(\int\frac{(d+ex)^4}{x}\,dx,x,x^2\right)}{16e}\\
&= \frac{(d+ex^2)^4\,(a+b\log(cx^n))}{8e}-\frac{(bn)\text{Subst}\left(\int\left(4d^3e+\frac{d^4}{x}+6d^2e^2x+4de^3x^2+e^4x^3\right)\,dx,x,x^2\right)}{16e}\\
&= -\frac{1}{4}bd^3nx^2-\frac{3}{16}bd^2enx^4-\frac{1}{12}bde^2nx^6-\frac{1}{64}be^3nx^8-\frac{bd^4n\log(x)}{8e}+\frac{(d+ex^2)^4\,(a+b\log(cx^n))}{8e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.30

$$\int x(d+ex^2)^3(a+b\log(cx^n))\,dx=\frac{1}{192}x^2\big(24a(4d^3+6d^2ex^2+4de^2x^4+e^3x^6)-bn(48d^3+36d^2ex^2+16de^2x^4+3e^3x^6)+24b(4d^3+6d^2ex^2+4de^2x^4+e^3x^6)\log(cx^n)\big)$$

```
[In] Integrate[x*(d + e*x^2)^3*(a + b*Log[c*x^n]),x]
[Out] (x^2*(24*a*(4*d^3 + 6*d^2*e*x^2 + 4*d*e^2*x^4 + e^3*x^6) - b*n*(48*d^3 + 36
*d^2*e*x^2 + 16*d*e^2*x^4 + 3*e^3*x^6) + 24*b*(4*d^3 + 6*d^2*e*x^2 + 4*d*e^
2*x^4 + e^3*x^6)*Log[c*x^n]))/192
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.58

method	result
parallelrisch	$\frac{x^8\ln(cx^n)be^3}{8}-\frac{be^3nx^8}{64}+\frac{ae^3x^8}{8}+\frac{x^6\ln(cx^n)bde^2}{2}-\frac{bde^2nx^6}{12}+\frac{ade^2x^6}{2}+\frac{3x^4b\ln(cx^n)d^2e}{4}-\frac{3bd^2enx^4}{16}+$
risch	$\frac{(ex^2+d)^4b\ln(x^n)}{8e}+\frac{ae^3x^8}{8}-\frac{ie^3\pi bx^8\,\mathrm{csgn}(icx^n)^3}{16}+\frac{ade^2x^6}{2}-\frac{i\pi bd^3x^2\,\mathrm{csgn}(icx^n)^3}{4}-\frac{i\pi bde^2x^6\,\mathrm{csgn}(icx^n)^3}{4}+$

```
[In] int(x*(e*x^2+d)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] 1/8*x^8*ln(c*x^n)*b*e^3-1/64*b*e^3*n*x^8+1/8*a*e^3*x^8+1/2*x^6*ln(c*x^n)*b*
d*e^2-1/12*b*d*e^2*n*x^6+1/2*a*d*e^2*x^6+3/4*x^4*b*ln(c*x^n)*d^2*e-3/16*b*d
^2*e*n*x^4+3/4*x^4*a*d^2*e+1/2*x^2*b*ln(c*x^n)*d^3-1/4*b*d^3*n*x^2+1/2*a*d^
3*x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(79) = 158$.

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.81

$$\int x(d+ex^2)^3(a+b\log(cx^n))\,dx=-\frac{1}{64}(be^3n-8ae^3)x^8-\frac{1}{12}(bde^2n-6ade^2)x^6-\frac{3}{16}(bd^2en-4ad^2e)x^4-\frac{1}{4}(bd^3n-2ad^3)x^2+\frac{1}{8}(be^3x^8+4bde^2x^6+6bd^2ex^4+4bd^3x^2)\log(c)+\frac{1}{8}(be^3nx^8+4bde^2nx^6+6bd^2enx^4+4bd^3nx^2)\log(x)$$

```
[In] integrate(x*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/64*(b*e^3*n - 8*a*e^3)*x^8 - 1/12*(b*d*e^2*n - 6*a*d*e^2)*x^6 - 3/16*(b*
d^2*e*n - 4*a*d^2*e)*x^4 - 1/4*(b*d^3*n - 2*a*d^3)*x^2 + 1/8*(b*e^3*x^8 + 4
*b*d*e^2*x^6 + 6*b*d^2*e*x^4 + 4*b*d^3*x^2)*log(c) + 1/8*(b*e^3*n*x^8 + 4*b
*d*e^2*n*x^6 + 6*b*d^2*e*n*x^4 + 4*b*d^3*n*x^2)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.87

$$\int x(d+ex^2)^3(a+b\log(cx^n))\,dx = \frac{ad^3x^2}{2} + \frac{3ad^2ex^4}{4} + \frac{ade^2x^6}{2} + \frac{ae^3x^8}{8} - \frac{bd^3nx^2}{4} + \frac{bd^3x^2\log(cx^n)}{2} - \frac{3bd^2enx^4}{16} + \frac{3bd^2ex^4\log(cx^n)}{4} - \frac{bde^2nx^6}{12} + \frac{bde^2x^6\log(cx^n)}{2} - \frac{be^3nx^8}{64} + \frac{be^3x^8\log(cx^n)}{8}$$

```
[In] integrate(x*(e*x**2+d)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**3*x**2/2 + 3*a*d**2*e*x**4/4 + a*d*e**2*x**6/2 + a*e**3*x**8/8 - b*d**
3*n*x**2/4 + b*d**3*x**2*log(c*x**n)/2 - 3*b*d**2*e*n*x**4/16 + 3*b*d**2*e*
x**4*log(c*x**n)/4 - b*d*e**2*n*x**6/12 + b*d*e**2*x**6*log(c*x**n)/2 - b*e
**3*n*x**8/64 + b*e**3*x**8*log(c*x**n)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.57

$$\int x(d+ex^2)^3(a+b\log(cx^n))\,dx = -\frac{1}{64}be^3nx^8 + \frac{1}{8}be^3x^8\log(cx^n) + \frac{1}{8}ae^3x^8 - \frac{1}{12}bde^2nx^6 + \frac{1}{2}bde^2x^6\log(cx^n) + \frac{1}{2}ade^2x^6 - \frac{3}{16}bd^2enx^4 + \frac{3}{4}bd^2ex^4\log(cx^n) + \frac{3}{4}ad^2ex^4 - \frac{1}{4}bd^3nx^2 + \frac{1}{2}bd^3x^2\log(cx^n) + \frac{1}{2}ad^3x^2$$

```
[In] integrate(x*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/64*b*e^3*n*x^8 + 1/8*b*e^3*x^8*log(c*x^n) + 1/8*a*e^3*x^8 - 1/12*b*d*e^2
*n*x^6 + 1/2*b*d*e^2*x^6*log(c*x^n) + 1/2*a*d*e^2*x^6 - 3/16*b*d^2*e*n*x^4
+ 3/4*b*d^2*e*x^4*log(c*x^n) + 3/4*a*d^2*e*x^4 - 1/4*b*d^3*n*x^2 + 1/2*b*d^
3*x^2*log(c*x^n) + 1/2*a*d^3*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(79) = 158.

Time = 0.38 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.95

$$\int x(d+ex^2)^3(a+b\log(cx^n))\,dx = \frac{1}{8}be^3nx^8\log(x) - \frac{1}{64}be^3nx^8 + \frac{1}{8}be^3x^8\log(c) + \frac{1}{8}ae^3x^8 + \frac{1}{2}bde^2nx^6\log(x) - \frac{1}{12}bde^2nx^6 + \frac{1}{2}bde^2x^6\log(c) + \frac{1}{2}ade^2x^6 + \frac{3}{4}bd^2enx^4\log(x) - \frac{3}{16}bd^2enx^4 + \frac{3}{4}bd^2ex^4\log(c) + \frac{3}{4}ad^2ex^4 + \frac{1}{2}bd^3nx^2\log(x) - \frac{1}{4}bd^3nx^2 + \frac{1}{2}bd^3x^2\log(c) + \frac{1}{2}ad^3x^2$$

```
[In] integrate(x*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/8*b*e^3*n*x^8*log(x) - 1/64*b*e^3*n*x^8 + 1/8*b*e^3*x^8*log(c) + 1/8*a*e^
3*x^8 + 1/2*b*d*e^2*n*x^6*log(x) - 1/12*b*d*e^2*n*x^6 + 1/2*b*d*e^2*x^6*log
(c) + 1/2*a*d*e^2*x^6 + 3/4*b*d^2*e*n*x^4*log(x) - 3/16*b*d^2*e*n*x^4 + 3/4
*b*d^2*e*x^4*log(c) + 3/4*a*d^2*e*x^4 + 1/2*b*d^3*n*x^2*log(x) - 1/4*b*d^3*
n*x^2 + 1/2*b*d^3*x^2*log(c) + 1/2*a*d^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.24

$$\int x(d+ex^2)^3(a+b\log(cx^n))\,dx = \ln(c\,x^n)\left(\frac{b\,d^3\,x^2}{2} + \frac{3\,b\,d^2\,e\,x^4}{4} + \frac{b\,d\,e^2\,x^6}{2} + \frac{b\,e^3\,x^8}{8}\right) + \frac{d^3\,x^2\,(2\,a-b\,n)}{4} + \frac{e^3\,x^8\,(8\,a-b\,n)}{64} + \frac{3\,d^2\,e\,x^4\,(4\,a-b\,n)}{16} + \frac{d\,e^2\,x^6\,(6\,a-b\,n)}{12}$$

```
[In] int(x*(d + e*x^2)^3*(a + b*log(c*x^n)),x)
```

```
[Out] log(c*x^n)*((b*d^3*x^2)/2 + (b*e^3*x^8)/8 + (3*b*d^2*e*x^4)/4 + (b*d*e^2*x^
6)/2) + (d^3*x^2*(2*a - b*n))/4 + (e^3*x^8*(8*a - b*n))/64 + (3*d^2*e*x^4*(
4*a - b*n))/16 + (d*e^2*x^6*(6*a - b*n))/12
```

3.199 $\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 130

$$\int \frac{(d+ex^2)^3\,(a+b\log(cx^n))}{x}\,dx = -\frac{3}{4}bd^2enx^2 - \frac{3}{16}bde^2nx^4 - \frac{1}{36}be^3nx^6 - \frac{1}{2}bd^3n\log^2(x) + \frac{3}{2}d^2ex^2(a+b\log(cx^n)) + \frac{3}{4}de^2x^4(a+b\log(cx^n)) + \frac{1}{6}e^3x^6(a+b\log(cx^n)) + d^3\log(x)\,(a+b\log(cx^n))$$

```
[Out] -3/4*b*d^2*e*n*x^2-3/16*b*d*e^2*n*x^4-1/36*b*e^3*n*x^6-1/2*b*d^3*n*ln(x)^2+
3/2*d^2*e*x^2*(a+b*ln(c*x^n))+3/4*d*e^2*x^4*(a+b*ln(c*x^n))+1/6*e^3*x^6*(a+
b*ln(c*x^n))+d^3*ln(x)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {272, 45, 2372, 14, 2338}

$$\int \frac{(d+ex^2)^3\,(a+b\log(cx^n))}{x}\,dx = d^3\log(x)\,(a+b\log(cx^n)) + \frac{3}{2}d^2ex^2(a+b\log(cx^n)) + \frac{3}{4}de^2x^4(a+b\log(cx^n)) + \frac{1}{6}e^3x^6(a+b\log(cx^n)) - \frac{1}{2}bd^3n\log^2(x) - \frac{3}{4}bd^2enx^2 - \frac{3}{16}bde^2nx^4 - \frac{1}{36}be^3nx^6$$

```
[In] Int[((d + e*x^2)^3*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] (-3*b*d^2*e*n*x^2)/4 - (3*b*d*e^2*n*x^4)/16 - (b*e^3*n*x^6)/36 - (b*d^3*n*L
og[x]^2)/2 + (3*d^2*e*x^2*(a + b*Log[c*x^n]))/2 + (3*d*e^2*x^4*(a + b*Log[c
*x^n]))/4 + (e^3*x^6*(a + b*Log[c*x^n]))/6 + d^3*Log[x]*(a + b*Log[c*x^n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3}{2}d^2ex^2(a+b\log(cx^n))+\frac{3}{4}de^2x^4(a+b\log(cx^n))+\frac{1}{6}e^3x^6(a+b\log(cx^n)) \\
&\quad +d^3\log(x)\,(a+b\log(cx^n))-(bn)\int\left(\frac{1}{12}ex\left(18d^2+9dex^2+2e^2x^4\right)+\frac{d^3\log(x)}{x}\right)\,dx \\
&= \frac{3}{2}d^2ex^2(a+b\log(cx^n))+\frac{3}{4}de^2x^4(a+b\log(cx^n))+\frac{1}{6}e^3x^6(a+b\log(cx^n)) \\
&\quad +d^3\log(x)\,(a+b\log(cx^n))-\left(bd^3n\right)\int\frac{\log(x)}{x}\,dx-\frac{1}{12}(ben)\int x\left(18d^2+9dex^2\right. \\
&\qquad \left.+2e^2x^4\right)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{1}{2}bd^3n\log^2(x) + \frac{3}{2}d^2ex^2(a+b\log(cx^n)) + \frac{3}{4}de^2x^4(a+b\log(cx^n)) \\
&\quad + \frac{1}{6}e^3x^6(a+b\log(cx^n)) + d^3\log(x)(a+b\log(cx^n)) \\
&\quad - \frac{1}{12}(ben)\int \left(18d^2x + 9dex^3 + 2e^2x^5\right)\,dx \\
&= -\frac{3}{4}bd^2enx^2 - \frac{3}{16}bde^2nx^4 - \frac{1}{36}be^3nx^6 - \frac{1}{2}bd^3n\log^2(x) + \frac{3}{2}d^2ex^2(a+b\log(cx^n)) \\
&\quad + \frac{3}{4}de^2x^4(a+b\log(cx^n)) + \frac{1}{6}e^3x^6(a+b\log(cx^n)) + d^3\log(x)(a+b\log(cx^n))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.89

$$
\begin{aligned}
\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x}\,dx = \frac{1}{144}\Bigg(&-108bd^2enx^2 - 27bde^2nx^4 - 4be^3nx^6 \\
&+ 216d^2ex^2(a+b\log(cx^n)) + 108de^2x^4(a+b\log(cx^n)) \\
&+ 24e^3x^6(a+b\log(cx^n)) + \frac{72d^3(a+b\log(cx^n))^2}{bn}\Bigg)
\end{aligned}
$$

```
[In] Integrate[((d + e*x^2)^3*(a + b*Log[c*x^n]))/x,x]
[Out] (-108*b*d^2*e*n*x^2 - 27*b*d*e^2*n*x^4 - 4*b*e^3*n*x^6 + 216*d^2*e*x^2*(a +
 b*Log[c*x^n]) + 108*d*e^2*x^4*(a + b*Log[c*x^n]) + 24*e^3*x^6*(a + b*Log[c
*x^n]) + (72*d^3*(a + b*Log[c*x^n])^2)/(b*n))/144
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.15

method	result
parallelrisch	$\frac{24x^6\ln(cx^n)be^3n-4x^6be^3n^2+24x^6ae^3n+108x^4\ln(cx^n)bde^2n-27x^4bde^2n^2+108x^4ade^2n+216x^2\ln(cx^n)bd^2en-108x^2bd^2e\cdots}{144n}$
risch	$\frac{x^6ae^3}{6}+\frac{3\ln(c)bde^2x^4}{4}-\frac{3i\pi bde^2x^4\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)}{8}-\frac{3ie\pi bd^2x^2\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)}{4}-\frac{i\ln(x)\cdots}{}$

```
[In] int((e*x^2+d)^3*(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
[Out] 1/144*(24*x^6*ln(c*x^n)*b*e^3*n-4*x^6*b*e^3*n^2+24*x^6*a*e^3*n+108*x^4*ln(c
*x^n)*b*d*e^2*n-27*x^4*b*d*e^2*n^2+108*x^4*a*d*e^2*n+216*x^2*ln(c*x^n)*b*d^
2*e*n-108*x^2*b*d^2*e*n^2+216*x^2*a*d^2*e*n+144*ln(x)*a*d^3*n+72*b*d^3*ln(c
*x^n)^2)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x}\,dx$$
$$= -\frac{1}{36}\left(be^3n - 6\,ae^3\right)x^6 + \frac{1}{2}\,bd^3n\log(x)^2 - \frac{3}{16}\left(bde^2n - 4\,ade^2\right)x^4 - \frac{3}{4}\left(bd^2en - 2\,ad^2e\right)x^2 + \frac{1}{12}\left(2\,be^3x^6 + 9\,bde^2x^4 + 18\,bd^2ex^2\right)\log(c) + \frac{1}{12}\left(2\,be^3nx^6 + 9\,bde^2nx^4 + 18\,bd^2enx^2 + 12\,bd^3\log(c) + 12\,ad^3\right)\log(x)$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] -1/36*(b*e^3*n - 6*a*e^3)*x^6 + 1/2*b*d^3*n*log(x)^2 - 3/16*(b*d*e^2*n - 4*
a*d*e^2)*x^4 - 3/4*(b*d^2*e*n - 2*a*d^2*e)*x^2 + 1/12*(2*b*e^3*x^6 + 9*b*d*
e^2*x^4 + 18*b*d^2*e*x^2)*log(c) + 1/12*(2*b*e^3*n*x^6 + 9*b*d*e^2*n*x^4 +
18*b*d^2*e*n*x^2 + 12*b*d^3*log(c) + 12*a*d^3)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.63

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x}\,dx$$
$$= \begin{cases} \frac{ad^3\log(cx^n)}{n} + \frac{3ad^2ex^2}{2} + \frac{3ade^2x^4}{4} + \frac{ae^3x^6}{6} + \frac{bd^3\log(cx^n)^2}{2n} - \frac{3bd^2enx^2}{4} + \frac{3bd^2ex^2\log(cx^n)}{2} - \frac{3bde^2nx^4}{16} + \frac{3bde^2x^4\log(cx^n)}{4} \\ (a+b\log(c))\left(d^3\log(x) + \frac{3d^2ex^2}{2} + \frac{3de^2x^4}{4} + \frac{e^3x^6}{6}\right) \end{cases}$$

```
[In] integrate((e*x**2+d)**3*(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise((a*d**3*log(c*x**n)/n + 3*a*d**2*e*x**2/2 + 3*a*d*e**2*x**4/4 + a
*e**3*x**6/6 + b*d**3*log(c*x**n)**2/(2*n) - 3*b*d**2*e*n*x**2/4 + 3*b*d**2
*e*x**2*log(c*x**n)/2 - 3*b*d*e**2*n*x**4/16 + 3*b*d*e**2*x**4*log(c*x**n)/
4 - b*e**3*n*x**6/36 + b*e**3*x**6*log(c*x**n)/6, Ne(n, 0)), ((a + b*log(c)
)*(d**3*log(x) + 3*d**2*e*x**2/2 + 3*d*e**2*x**4/4 + e**3*x**6/6), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x}\,dx = -\frac{1}{36}\,be^3nx^6 + \frac{1}{6}\,be^3x^6\log(cx^n) + \frac{1}{6}\,ae^3x^6 - \frac{3}{16}\,bde^2nx^4 + \frac{3}{4}\,bde^2x^4\log(cx^n) + \frac{3}{4}\,ade^2x^4 - \frac{3}{4}\,bd^2enx^2 + \frac{3}{2}\,bd^2ex^2\log(cx^n) + \frac{3}{2}\,ad^2ex^2 + \frac{bd^3\log(cx^n)^2}{2\,n} + ad^3\log(x)$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] -1/36*b*e^3*n*x^6 + 1/6*b*e^3*x^6*log(c*x^n) + 1/6*a*e^3*x^6 - 3/16*b*d*e^2
*n*x^4 + 3/4*b*d*e^2*x^4*log(c*x^n) + 3/4*a*d*e^2*x^4 - 3/4*b*d^2*e*n*x^2 +
 3/2*b*d^2*e*x^2*log(c*x^n) + 3/2*a*d^2*e*x^2 + 1/2*b*d^3*log(c*x^n)^2/n +
a*d^3*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x}\,dx = -\frac{1}{36}\,\left(be^3n - 6\,be^3\log(c) - 6\,ae^3\right)x^6 + \frac{1}{2}\,bd^3n\log(x)^2 - \frac{3}{16}\,\left(bde^2n - 4\,bde^2\log(c) - 4\,ade^2\right)x^4 - \frac{3}{4}\,\left(bd^2en - 2\,bd^2e\log(c) - 2\,ad^2e\right)x^2 + \frac{1}{12}\,\left(2\,be^3nx^6 + 9\,bde^2nx^4 + 18\,bd^2enx^2\right)\log(x) + \left(bd^3\log(c) + ad^3\right)\log(x)$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] -1/36*(b*e^3*n - 6*b*e^3*log(c) - 6*a*e^3)*x^6 + 1/2*b*d^3*n*log(x)^2 - 3/1
6*(b*d*e^2*n - 4*b*d*e^2*log(c) - 4*a*d*e^2)*x^4 - 3/4*(b*d^2*e*n - 2*b*d^2
*e*log(c) - 2*a*d^2*e)*x^2 + 1/12*(2*b*e^3*n*x^6 + 9*b*d*e^2*n*x^4 + 18*b*d
^2*e*n*x^2)*log(x) + (b*d^3*log(c) + a*d^3)*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x}\,dx = \ln(c\,x^n)\left(\frac{3\,b\,d^2\,e\,x^2}{2}+\frac{3\,b\,d\,e^2\,x^4}{4}+\frac{b\,e^3\,x^6}{6}\right) + \frac{e^3\,x^6\,(6\,a-b\,n)}{36} + a\,d^3\,\ln(x) + \frac{b\,d^3\,\ln(c\,x^n)^2}{2\,n} + \frac{3\,d^2\,e\,x^2\,(2\,a-b\,n)}{4} + \frac{3\,d\,e^2\,x^4\,(4\,a-b\,n)}{16}$$

```
[In] int(((d + e*x^2)^3*(a + b*log(c*x^n)))/x,x)
```

```
[Out] log(c*x^n)*((b*e^3*x^6)/6 + (3*b*d^2*e*x^2)/2 + (3*b*d*e^2*x^4)/4) + (e^3*x
^6*(6*a - b*n))/36 + a*d^3*log(x) + (b*d^3*log(c*x^n)^2)/(2*n) + (3*d^2*e*x
^2*(2*a - b*n))/4 + (3*d*e^2*x^4*(4*a - b*n))/16
```

3.200 $\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^3}\,dx = -\frac{bd^3n}{4x^2} - \frac{3}{4}bde^2nx^2 - \frac{1}{16}be^3nx^4 - \frac{3}{2}bd^2en\log^2(x) - \frac{d^3(a+b\log(cx^n))}{2x^2} + \frac{3}{2}de^2x^2(a+b\log(cx^n)) + \frac{1}{4}e^3x^4(a+b\log(cx^n)) + 3d^2e\log(x)(a+b\log(cx^n))$$

```
[Out] -1/4*b*d^3*n/x^2-3/4*b*d*e^2*n*x^2-1/16*b*e^3*n*x^4-3/2*b*d^2*e*n*ln(x)^2-1
/2*d^3*(a+b*ln(c*x^n))/x^2+3/2*d*e^2*x^2*(a+b*ln(c*x^n))+1/4*e^3*x^4*(a+b*l
n(c*x^n))+3*d^2*e*ln(x)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {272, 45, 2372, 12, 14, 2338}

$$\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^3}\,dx = -\frac{d^3(a+b\log(cx^n))}{2x^2} + 3d^2e\log(x)(a+b\log(cx^n)) + \frac{3}{2}de^2x^2(a+b\log(cx^n)) + \frac{1}{4}e^3x^4(a+b\log(cx^n)) - \frac{bd^3n}{4x^2} - \frac{3}{2}bd^2en\log^2(x) - \frac{3}{4}bde^2nx^2 - \frac{1}{16}be^3nx^4$$

```
[In] Int[((d + e*x^2)^3*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/4*(b*d^3*n)/x^2 - (3*b*d*e^2*n*x^2)/4 - (b*e^3*n*x^4)/16 - (3*b*d^2*e*n*
Log[x]^2)/2 - (d^3*(a + b*Log[c*x^n]))/(2*x^2) + (3*d*e^2*x^2*(a + b*Log[c*
```

```
x^n]))/2 + (e^3*x^4*(a + b*Log[c*x^n]))/4 + 3*d^2*e*Log[x]*(a + b*Log[c*x^n
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\text{integral} = -\frac{d^3(a+b\log(cx^n))}{2x^2}+\frac{3}{2}de^2x^2(a+b\log(cx^n))+\frac{1}{4}e^3x^4(a+b\log(cx^n)) + 3d^2e\log(x)(a+b\log(cx^n))-(bn)\int\frac{-2d^3+6de^2x^4+e^3x^6+12d^2ex^2\log(x)}{4x^3}\,dx$$

$$= -\frac{d^3(a+b\log(cx^n))}{2x^2} + \frac{3}{2}de^2x^2(a+b\log(cx^n)) + \frac{1}{4}e^3x^4(a+b\log(cx^n)) + 3d^2e\log(x)(a+b\log(cx^n)) - \frac{1}{4}(bn)\int \frac{-2d^3+6de^2x^4+e^3x^6+12d^2ex^2\log(x)}{x^3}\,dx$$

$$= -\frac{d^3(a+b\log(cx^n))}{2x^2} + \frac{3}{2}de^2x^2(a+b\log(cx^n)) + \frac{1}{4}e^3x^4(a+b\log(cx^n)) + 3d^2e\log(x)(a+b\log(cx^n)) - \frac{1}{4}(bn)\int \left(\frac{-2d^3+6de^2x^4+e^3x^6}{x^3} + \frac{12d^2e\log(x)}{x}\right)dx$$

$$= -\frac{d^3(a+b\log(cx^n))}{2x^2} + \frac{3}{2}de^2x^2(a+b\log(cx^n)) + \frac{1}{4}e^3x^4(a+b\log(cx^n)) + 3d^2e\log(x)(a+b\log(cx^n)) - \frac{1}{4}(bn)\int \frac{-2d^3+6de^2x^4+e^3x^6}{x^3}\,dx - (3bd^2en)\int \frac{\log(x)}{x}\,dx$$

$$= -\frac{3}{2}bd^2en\log^2(x) - \frac{d^3(a+b\log(cx^n))}{2x^2} + \frac{3}{2}de^2x^2(a+b\log(cx^n)) + \frac{1}{4}e^3x^4(a+b\log(cx^n)) + 3d^2e\log(x)(a+b\log(cx^n)) - \frac{1}{4}(bn)\int \left(-\frac{2d^3}{x^3}+6de^2x+e^3x^3\right)dx$$

$$= -\frac{bd^3n}{4x^2} - \frac{3}{4}bde^2nx^2 - \frac{1}{16}be^3nx^4 - \frac{3}{2}bd^2en\log^2(x) - \frac{d^3(a+b\log(cx^n))}{2x^2} + \frac{3}{2}de^2x^2(a+b\log(cx^n)) + \frac{1}{4}e^3x^4(a+b\log(cx^n)) + 3d^2e\log(x)(a+b\log(cx^n))$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^3}\,dx = \frac{1}{16}\left(-\frac{4bd^3n}{x^2} - 12bde^2nx^2 - be^3nx^4 - \frac{8d^3(a+b\log(cx^n))}{x^2} + 24de^2x^2(a+b\log(cx^n)) + 4e^3x^4(a+b\log(cx^n)) + \frac{24d^2e(a+b\log(cx^n))^2}{bn}\right)$$

```
[In] Integrate[((d + e*x^2)^3*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] ((-4*b*d^3*n)/x^2 - 12*b*d*e^2*n*x^2 - b*e^3*n*x^4 - (8*d^3*(a + b*Log[c*x^
n]))/x^2 + 24*d*e^2*x^2*(a + b*Log[c*x^n]) + 4*e^3*x^4*(a + b*Log[c*x^n]) +
 (24*d^2*e*(a + b*Log[c*x^n])^2)/(b*n))/16
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{4x^6 \ln(c\,x^n) b\,e^3 n - x^6 b\,e^3 n^2 + 4x^6 a\,e^3 n + 24x^4 \ln(c\,x^n) bd\,e^2 n - 12x^4 bd\,e^2 n^2 + 24x^4 ad\,e^2 n + 48\ln(x) x^2 a\,d^2 en + 24e\,d^2 b \ln(c\,x^n)^2 x}{16x^2 n}$
risch	Expression too large to display

```
[In] int((e*x^2+d)^3*(a+b*ln(c*x^n))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/x^2*(4*x^6*ln(c*x^n)*b*e^3*n-x^6*b*e^3*n^2+4*x^6*a*e^3*n+24*x^4*ln(c*x
^n)*b*d*e^2*n-12*x^4*b*d*e^2*n^2+24*x^4*a*d*e^2*n+48*ln(x)*x^2*a*d^2*e*n+24
*e*d^2*b*ln(c*x^n)^2*x^2-8*ln(c*x^n)*b*d^3*n-4*b*d^3*n^2-8*a*d^3*n)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^3}\,dx$$
$$= \frac{24\,bd^2enx^2\log(x)^2 - (be^3n - 4\,ae^3)x^6 - 4\,bd^3n - 12\,(bde^2n - 2\,ade^2)x^4 - 8\,ad^3 + 4\,(be^3x^6 + 6\,bde^2x^4 - 2}{16\,x^2}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(24*b*d^2*e*n*x^2*log(x)^2 - (b*e^3*n - 4*a*e^3)*x^6 - 4*b*d^3*n - 12*
(b*d*e^2*n - 2*a*d*e^2)*x^4 - 8*a*d^3 + 4*(b*e^3*x^6 + 6*b*d*e^2*x^4 - 2*b*
d^3)*log(c) + 4*(b*e^3*n*x^6 + 6*b*d*e^2*n*x^4 + 12*b*d^2*e*x^2*log(c) + 12
*a*d^2*e*x^2 - 2*b*d^3*n)*log(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^3}\,dx$$
$$= \begin{cases} -\frac{ad^3}{2x^2} + \frac{3ad^2e\log(cx^n)}{n} + \frac{3ade^2x^2}{2} + \frac{ae^3x^4}{4} - \frac{bd^3n}{4x^2} - \frac{bd^3\log(cx^n)}{2x^2} + \frac{3bd^2e\log(cx^n)^2}{2n} - \frac{3bde^2nx^2}{4} + \frac{3bde^2x^2\log(cx^n)}{2} - \frac{be^3}{} \\ (a+b\log(c))\left(-\frac{d^3}{2x^2} + 3d^2e\log(x) + \frac{3de^2x^2}{2} + \frac{e^3x^4}{4}\right) \end{cases}$$

```
[In] integrate((e*x**2+d)**3*(a+b*ln(c*x**n))/x**3,x)
```

```
[Out] Piecewise((-a*d**3/(2*x**2) + 3*a*d**2*e*log(c*x**n)/n + 3*a*d*e**2*x**2/2
+ a*e**3*x**4/4 - b*d**3*n/(4*x**2) - b*d**3*log(c*x**n)/(2*x**2) + 3*b*d**
2*e*log(c*x**n)**2/(2*n) - 3*b*d*e**2*n*x**2/4 + 3*b*d*e**2*x**2*log(c*x**n
)/2 - b*e**3*n*x**4/16 + b*e**3*x**4*log(c*x**n)/4, Ne(n, 0)), ((a + b*log(
c))*(-d**3/(2*x**2) + 3*d**2*e*log(x) + 3*d*e**2*x**2/2 + e**3*x**4/4), Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^3} \, dx = -\frac{1}{16} be^3nx^4 + \frac{1}{4} be^3x^4 \log(cx^n) + \frac{1}{4} ae^3x^4 - \frac{3}{4} bde^2nx^2 + \frac{3}{2} bde^2x^2 \log(cx^n) + \frac{3}{2} ade^2x^2 + \frac{3\, bd^2e \log(cx^n)^2}{2\, n} + 3\, ad^2e \log(x) - \frac{bd^3n}{4\, x^2} - \frac{bd^3 \log(cx^n)}{2\, x^2} - \frac{ad^3}{2\, x^2}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*b*e^3*n*x^4 + 1/4*b*e^3*x^4*log(c*x^n) + 1/4*a*e^3*x^4 - 3/4*b*d*e^2*
n*x^2 + 3/2*b*d*e^2*x^2*log(c*x^n) + 3/2*a*d*e^2*x^2 + 3/2*b*d^2*e*log(c*x^
n)^2/n + 3*a*d^2*e*log(x) - 1/4*b*d^3*n/x^2 - 1/2*b*d^3*log(c*x^n)/x^2 - 1/
2*a*d^3/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^3} \, dx = \frac{1}{4} be^3x^4 \log(c) + \frac{1}{4} ae^3x^4 + \frac{3}{2} bde^2x^2 \log(c) + \frac{3}{2} bd^2en \log(x)^2 + \frac{3}{4} \left(2\, x^2 \log(x) - x^2\right) bde^2n + \frac{1}{16} \left(4\, x^4 \log(x) - x^4\right) be^3n + \frac{3}{2} ade^2x^2 - \frac{1}{4} bd^3n \left(\frac{2\, \log(x)}{x^2} + \frac{1}{x^2}\right) + 3\, bd^2e \log(c) \log(|x|) + 3\, ad^2e \log(|x|) - \frac{bd^3 \log(c)}{2\, x^2} - \frac{ad^3}{2\, x^2}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b*e^3*x^4*log(c) + 1/4*a*e^3*x^4 + 3/2*b*d*e^2*x^2*log(c) + 3/2*b*d^2*e
*n*log(x)^2 + 3/4*(2*x^2*log(x) - x^2)*b*d*e^2*n + 1/16*(4*x^4*log(x) - x^4
)*b*e^3*n + 3/2*a*d*e^2*x^2 - 1/4*b*d^3*n*(2*log(x)/x^2 + 1/x^2) + 3*b*d^2*
e*log(c)*log(abs(x)) + 3*a*d^2*e*log(abs(x)) - 1/2*b*d^3*log(c)/x^2 - 1/2*a
*d^3/x^2
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^3}\,dx = \ln(c\,x^n)\left(\frac{\frac{3\,b\,e^3\,x^6}{4}+3\,b\,d\,e^2\,x^4}{x^2} - \frac{\frac{b\,d^3}{2}+\frac{3\,b\,d^2\,e\,x^2}{2}+\frac{3\,b\,d\,e^2\,x^4}{2}+\frac{b\,e^3\,x^6}{2}}{x^2}\right) - \frac{\frac{a\,d^3}{2}+\frac{b\,d^3\,n}{4}}{x^2} + \ln(x)\left(3\,a\,d^2\,e+\frac{3\,b\,d^2\,e\,n}{2}\right) + \frac{e^3\,x^4\,(4\,a-b\,n)}{16} + \frac{3\,d\,e^2\,x^2\,(2\,a-b\,n)}{4} + \frac{3\,b\,d^2\,e\ln(c\,x^n)^2}{2\,n}$$

```
[In] int(((d + e*x^2)^3*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] log(c*x^n)*((((3*b*e^3*x^6)/4 + 3*b*d*e^2*x^4)/x^2 - ((b*d^3)/2 + (b*e^3*x^6
)/2 + (3*b*d^2*e*x^2)/2 + (3*b*d*e^2*x^4)/2)/x^2) - ((a*d^3)/2 + (b*d^3*n)/
4)/x^2 + log(x)*(3*a*d^2*e + (3*b*d^2*e*n)/2) + (e^3*x^4*(4*a - b*n))/16 +
(3*d*e^2*x^2*(2*a - b*n))/4 + (3*b*d^2*e*log(c*x^n)^2)/(2*n)
```

3.201 $\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^5}\,dx = -\frac{bd^3n}{16x^4}-\frac{3bd^2en}{4x^2}-\frac{1}{4}be^3nx^2-\frac{3}{2}bde^2n\log^2(x)-\frac{d^3(a+b\log(cx^n))}{4x^4}-\frac{3d^2e(a+b\log(cx^n))}{2x^2}+\frac{1}{2}e^3x^2(a+b\log(cx^n))+3de^2\log(x)(a+b\log(cx^n))$$

```
[Out] -1/16*b*d^3*n/x^4-3/4*b*d^2*e*n/x^2-1/4*b*e^3*n*x^2-3/2*b*d*e^2*n*ln(x)^2-1
/4*d^3*(a+b*ln(c*x^n))/x^4-3/2*d^2*e*(a+b*ln(c*x^n))/x^2+1/2*e^3*x^2*(a+b*l
n(c*x^n))+3*d*e^2*ln(x)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {272, 45, 2372, 12, 14, 2338}

$$\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^5}\,dx = -\frac{d^3(a+b\log(cx^n))}{4x^4}-\frac{3d^2e(a+b\log(cx^n))}{2x^2}+3de^2\log(x)(a+b\log(cx^n))+\frac{1}{2}e^3x^2(a+b\log(cx^n))-\frac{bd^3n}{16x^4}-\frac{3bd^2en}{4x^2}-\frac{3}{2}bde^2n\log^2(x)-\frac{1}{4}be^3nx^2$$

```
[In] Int[((d + e*x^2)^3*(a + b*Log[c*x^n]))/x^5,x]
```

```
[Out] -1/16*(b*d^3*n)/x^4 - (3*b*d^2*e*n)/(4*x^2) - (b*e^3*n*x^2)/4 - (3*b*d*e^2*
n*Log[x]^2)/2 - (d^3*(a + b*Log[c*x^n]))/(4*x^4) - (3*d^2*e*(a + b*Log[c*x^
```

```
n]))/(2*x^2) + (e^3*x^2*(a + b*Log[c*x^n]))/2 + 3*d*e^2*Log[x]*(a + b*Log[c
*x^n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{4x^4}-\frac{3d^2e(a+b\log(cx^n))}{2x^2}+\frac{1}{2}e^3x^2(a+b\log(cx^n))\\&\quad+3de^2\log(x)\,(a+b\log(cx^n))-(bn)\int\frac{-d^3-6d^2ex^2+2e^3x^6+12de^2x^4\log(x)}{4x^5}\,dx\end{aligned}$$

$$
\begin{aligned}
&= -\frac{d^3(a+b\log(cx^n))}{4x^4} - \frac{3d^2e(a+b\log(cx^n))}{2x^2} \\
&\quad + \frac{1}{2}e^3x^2(a+b\log(cx^n)) + 3de^2\log(x)(a+b\log(cx^n)) \\
&\quad - \frac{1}{4}(bn)\int \frac{-d^3-6d^2ex^2+2e^3x^6+12de^2x^4\log(x)}{x^5}\,dx \\
&= -\frac{d^3(a+b\log(cx^n))}{4x^4} - \frac{3d^2e(a+b\log(cx^n))}{2x^2} \\
&\quad + \frac{1}{2}e^3x^2(a+b\log(cx^n)) + 3de^2\log(x)(a+b\log(cx^n)) \\
&\quad - \frac{1}{4}(bn)\int \left(\frac{-d^3-6d^2ex^2+2e^3x^6}{x^5} + \frac{12de^2\log(x)}{x}\right)dx \\
&= -\frac{d^3(a+b\log(cx^n))}{4x^4} - \frac{3d^2e(a+b\log(cx^n))}{2x^2} \\
&\quad + \frac{1}{2}e^3x^2(a+b\log(cx^n)) + 3de^2\log(x)(a+b\log(cx^n)) \\
&\quad - \frac{1}{4}(bn)\int \frac{-d^3-6d^2ex^2+2e^3x^6}{x^5}\,dx - (3bde^2n)\int \frac{\log(x)}{x}\,dx \\
&= -\frac{3}{2}bde^2n\log^2(x) - \frac{d^3(a+b\log(cx^n))}{4x^4} - \frac{3d^2e(a+b\log(cx^n))}{2x^2} \\
&\quad + \frac{1}{2}e^3x^2(a+b\log(cx^n)) + 3de^2\log(x)(a+b\log(cx^n)) \\
&\quad - \frac{1}{4}(bn)\int \left(-\frac{d^3}{x^5} - \frac{6d^2e}{x^3} + 2e^3x\right)dx \\
&= -\frac{bd^3n}{16x^4} - \frac{3bd^2en}{4x^2} - \frac{1}{4}be^3nx^2 - \frac{3}{2}bde^2n\log^2(x) - \frac{d^3(a+b\log(cx^n))}{4x^4} \\
&\quad - \frac{3d^2e(a+b\log(cx^n))}{2x^2} + \frac{1}{2}e^3x^2(a+b\log(cx^n)) + 3de^2\log(x)(a+b\log(cx^n))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.88

$$
\begin{aligned}
\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^5}\,dx = \frac{1}{16}\Bigg(&-\frac{bd^3n}{x^4} - \frac{12bd^2en}{x^2} - 4be^3nx^2 - \frac{4d^3(a+b\log(cx^n))}{x^4} \\
&- \frac{24d^2e(a+b\log(cx^n))}{x^2} + 8e^3x^2(a+b\log(cx^n)) \\
&+ \frac{24de^2(a+b\log(cx^n))^2}{bn}\Bigg)
\end{aligned}
$$

```
[In] Integrate[((d + e*x^2)^3*(a + b*Log[c*x^n]))/x^5,x]
```

[Out] (-((b*d^3*n)/x^4) - (12*b*d^2*e*n)/x^2 - 4*b*e^3*n*x^2 - (4*d^3*(a + b*Log[
c*x^n]))/x^4 - (24*d^2*e*(a + b*Log[c*x^n]))/x^2 + 8*e^3*x^2*(a + b*Log[c*x
^n]) + (24*d*e^2*(a + b*Log[c*x^n])^2)/(b*n))/16

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{8x^6 \ln(c\,x^n) b\,e^3 n - 4x^6 b\,e^3 n^2 + 8x^6 a\,e^3 n + 48\ln(x) x^4 a d\,e^2 n + 24e^2 db \ln(c\,x^n)^2 x^4 - 24x^2 \ln(c\,x^n) b\,d^2 e n - 12x^2 b\,d^2 e\,n^2 - 24x^2 a\,d^2}{16x^4 n}$
risch	$-\frac{b(-2e^3x^6 - 12e^2 d\ln(x) x^4 + 6d^2 e\,x^2 + d^3)\ln(x^n)}{4x^4} - \frac{-48\ln(x) a d\,e^2 x^4 - 48\ln(x)\ln(c) b d\,e^2 x^4 + 24e^2 dbn \ln(x)^2 x^4 - 8x^6 a\,e^3 - 1}{}$

[In] int((e*x^2+d)^3*(a+b*ln(c*x^n))/x^5,x,method=_RETURNVERBOSE)

[Out] 1/16/x^4*(8*x^6*ln(c*x^n)*b*e^3*n-4*x^6*b*e^3*n^2+8*x^6*a*e^3*n+48*ln(x)*x^
4*a*d*e^2*n+24*e^2*d*b*ln(c*x^n)^2*x^4-24*x^2*ln(c*x^n)*b*d^2*e*n-12*x^2*b*
d^2*e*n^2-24*x^2*a*d^2*e*n-4*ln(c*x^n)*b*d^3*n-b*d^3*n^2-4*a*d^3*n)/n

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^5}\,dx$$
$$= \frac{24\,bde^2nx^4\log(x)^2 - 4\,(be^3n - 2\,ae^3)x^6 - bd^3n - 4\,ad^3 - 12\,(bd^2en + 2\,ad^2e)x^2 + 4\,(2\,be^3x^6 - 6\,bd^2ex^2 -}{16\,x^4}$$

[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^5,x, algorithm="fricas")

[Out] 1/16*(24*b*d*e^2*n*x^4*log(x)^2 - 4*(b*e^3*n - 2*a*e^3)*x^6 - b*d^3*n - 4*a
d^3 - 12(b*d^2*e*n + 2*a*d^2*e)*x^2 + 4*(2*b*e^3*x^6 - 6*b*d^2*e*x^2 - b*
d^3)*log(c) + 4*(2*b*e^3*n*x^6 + 12*b*d*e^2*x^4*log(c) + 12*a*d*e^2*x^4 - 6
*b*d^2*e*n*x^2 - b*d^3*n)*log(x))/x^4

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^5}\,dx$$
$$= \begin{cases} -\frac{ad^3}{4x^4} - \frac{3ad^2e}{2x^2} + \frac{3ade^2\log(cx^n)}{n} + \frac{ae^3x^2}{2} - \frac{bd^3n}{16x^4} - \frac{bd^3\log(cx^n)}{4x^4} - \frac{3bd^2en}{4x^2} - \frac{3bd^2e\log(cx^n)}{2x^2} + \frac{3bde^2\log(cx^n)^2}{2n} - \frac{be^3nx^2}{4} + \\ (a+b\log(c))\left(-\frac{d^3}{4x^4} - \frac{3d^2e}{2x^2} + 3de^2\log(x) + \frac{e^3x^2}{2}\right) \end{cases}$$

```
[In] integrate((e*x**2+d)**3*(a+b*ln(c*x**n))/x**5,x)
```

```
[Out] Piecewise((-a*d**3/(4*x**4) - 3*a*d**2*e/(2*x**2) + 3*a*d*e**2*log(c*x**n)/
n + a*e**3*x**2/2 - b*d**3*n/(16*x**4) - b*d**3*log(c*x**n)/(4*x**4) - 3*b*
d**2*e*n/(4*x**2) - 3*b*d**2*e*log(c*x**n)/(2*x**2) + 3*b*d*e**2*log(c*x**n
)**2/(2*n) - b*e**3*n*x**2/4 + b*e**3*x**2*log(c*x**n)/2, Ne(n, 0)), ((a +
b*log(c))*(-d**3/(4*x**4) - 3*d**2*e/(2*x**2) + 3*d*e**2*log(x) + e**3*x**2
/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^5}\,dx = -\frac{1}{4}\,be^3nx^2 + \frac{1}{2}\,be^3x^2\log(cx^n) + \frac{1}{2}\,ae^3x^2 + \frac{3\,bde^2\log(cx^n)^2}{2\,n} + 3\,ade^2\log(x) - \frac{3\,bd^2en}{4\,x^2} - \frac{3\,bd^2e\log(cx^n)}{2\,x^2} - \frac{3\,ad^2e}{2\,x^2} - \frac{bd^3n}{16\,x^4} - \frac{bd^3\log(cx^n)}{4\,x^4} - \frac{ad^3}{4\,x^4}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*b*e^3*n*x^2 + 1/2*b*e^3*x^2*log(c*x^n) + 1/2*a*e^3*x^2 + 3/2*b*d*e^2*l
og(c*x^n)^2/n + 3*a*d*e^2*log(x) - 3/4*b*d^2*e*n/x^2 - 3/2*b*d^2*e*log(c*x^
n)/x^2 - 3/2*a*d^2*e/x^2 - 1/16*b*d^3*n/x^4 - 1/4*b*d^3*log(c*x^n)/x^4 - 1/
4*a*d^3/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^5}\,dx = \frac{1}{2}\,be^3x^2\log(c) + \frac{3}{2}\,bde^2n\log(x)^2 + \frac{1}{4}\left(2\,x^2\log(x) - x^2\right)be^3n + \frac{1}{2}\,ae^3x^2 - \frac{3}{4}\,bd^2en\left(\frac{2\,\log(x)}{x^2} + \frac{1}{x^2}\right) - \frac{1}{16}\,bd^3n\left(\frac{4\,\log(x)}{x^4} + \frac{1}{x^4}\right) + 3\,bde^2\log(c)\log(|x|) + 3\,ade^2\log(|x|) - \frac{3\,bd^2e\log(c)}{2\,x^2} - \frac{3\,ad^2e}{2\,x^2} - \frac{bd^3\log(c)}{4\,x^4} - \frac{ad^3}{4\,x^4}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^5,x, algorithm="giac")
```

```
[Out] 1/2*b*e^3*x^2*log(c) + 3/2*b*d*e^2*n*log(x)^2 + 1/4*(2*x^2*log(x) - x^2)*b*
e^3*n + 1/2*a*e^3*x^2 - 3/4*b*d^2*e*n*(2*log(x)/x^2 + 1/x^2) - 1/16*b*d^3*n
*(4*log(x)/x^4 + 1/x^4) + 3*b*d*e^2*log(c)*log(abs(x)) + 3*a*d*e^2*log(abs(
x)) - 3/2*b*d^2*e*log(c)/x^2 - 3/2*a*d^2*e/x^2 - 1/4*b*d^3*log(c)/x^4 - 1/4
*a*d^3/x^4
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^5}\,dx = \ln(x)\,\left(3\,a\,d\,e^2 + \frac{9\,b\,d\,e^2\,n}{4}\right) - \ln(c\,x^n)\,\left(\frac{\frac{b\,d^3}{4} + \frac{3\,b\,d^2\,e\,x^2}{2} + \frac{9\,b\,d\,e^2\,x^4}{4} + b\,e^3\,x^6}{x^4} - \frac{3\,b\,e^3\,x^2}{2}\right) - \frac{a\,d^3 + x^2\,(6\,a\,d^2\,e + 3\,b\,d^2\,e\,n) + \frac{b\,d^3\,n}{4}}{4\,x^4} + \frac{e^3\,x^2\,(2\,a - b\,n)}{4} + \frac{3\,b\,d\,e^2\ln(c\,x^n)^2}{2\,n}$$

```
[In] int(((d + e*x^2)^3*(a + b*log(c*x^n)))/x^5,x)
```

```
[Out] log(x)*(3*a*d*e^2 + (9*b*d*e^2*n)/4) - log(c*x^n)*(((b*d^3)/4 + b*e^3*x^6 +
 (3*b*d^2*e*x^2)/2 + (9*b*d*e^2*x^4)/4)/x^4 - (3*b*e^3*x^2)/2) - (a*d^3 + x
^2*(6*a*d^2*e + 3*b*d^2*e*n) + (b*d^3*n)/4)/(4*x^4) + (e^3*x^2*(2*a - b*n))
/4 + (3*b*d*e^2*log(c*x^n)^2)/(2*n)
```

3.202 $\int x^4(d+ex^2)^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int x^4(d+ex^2)^3(a+b\log(cx^n))\,dx = -\frac{1}{25}bd^3nx^5-\frac{3}{49}bd^2enx^7-\frac{1}{27}bde^2nx^9-\frac{1}{121}be^3nx^{11}+\frac{(231d^3x^5+495d^2ex^7+385de^2x^9+105e^3x^{11})(a+b\log(cx^n))}{1155}$$

```
[Out] -1/25*b*d^3*n*x^5-3/49*b*d^2*e*n*x^7-1/27*b*d*e^2*n*x^9-1/121*b*e^3*n*x^11+
1/1155*(105*e^3*x^11+385*d*e^2*x^9+495*d^2*e*x^7+231*d^3*x^5)*(a+b*ln(c*x^n
))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {276, 2371}

$$\int x^4(d+ex^2)^3(a+b\log(cx^n))\,dx = \frac{(231d^3x^5+495d^2ex^7+385de^2x^9+105e^3x^{11})(a+b\log(cx^n))}{1155}-\frac{1}{25}bd^3nx^5-\frac{3}{49}bd^2enx^7-\frac{1}{27}bde^2nx^9-\frac{1}{121}be^3nx^{11}$$

```
[In] Int[x^4*(d + e*x^2)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/25*(b*d^3*n*x^5) - (3*b*d^2*e*n*x^7)/49 - (b*d*e^2*n*x^9)/27 - (b*e^3*n*
x^11)/121 + ((231*d^3*x^5 + 495*d^2*e*x^7 + 385*d*e^2*x^9 + 105*e^3*x^11)*(
a + b*Log[c*x^n]))/1155
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(231d^3x^5+495d^2ex^7+385de^2x^9+105e^3x^{11})\,(a+b\log(cx^n))}{1155} \\ &\quad -(bn)\int\left(\frac{d^3x^4}{5}+\frac{3}{7}d^2ex^6+\frac{1}{3}de^2x^8+\frac{e^3x^{10}}{11}\right)dx \\ &= -\frac{1}{25}bd^3nx^5-\frac{3}{49}bd^2enx^7-\frac{1}{27}bde^2nx^9-\frac{1}{121}be^3nx^{11} \\ &\quad +\frac{(231d^3x^5+495d^2ex^7+385de^2x^9+105e^3x^{11})\,(a+b\log(cx^n))}{1155}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.33

$$\begin{aligned}\int x^4(d+ex^2)^3\,(a+b\log(cx^n))\,dx &= -\frac{1}{25}bd^3nx^5-\frac{3}{49}bd^2enx^7-\frac{1}{27}bde^2nx^9-\frac{1}{121}be^3nx^{11} \\ &\quad +\frac{1}{5}d^3x^5(a+b\log(cx^n))+\frac{3}{7}d^2ex^7(a+b\log(cx^n)) \\ &\quad +\frac{1}{3}de^2x^9(a+b\log(cx^n))+\frac{1}{11}e^3x^{11}(a+b\log(cx^n))\end{aligned}$$

```
[In] Integrate[x^4*(d + e*x^2)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/25*(b*d^3*n*x^5) - (3*b*d^2*e*n*x^7)/49 - (b*d*e^2*n*x^9)/27 - (b*e^3*n*
x^11)/121 + (d^3*x^5*(a + b*Log[c*x^n]))/5 + (3*d^2*e*x^7*(a + b*Log[c*x^n]
))/7 + (d*e^2*x^9*(a + b*Log[c*x^n]))/3 + (e^3*x^11*(a + b*Log[c*x^n]))/11
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.44

method	result
parallelrisch	$\frac{x^{11} b \ln(c x^n) e^3}{11} - \frac{b e^3 n x^{11}}{121} + \frac{x^{11} a e^3}{11} + \frac{x^9 b \ln(c x^n) d e^2}{3} - \frac{b d e^2 n x^9}{27} + \frac{x^9 a d e^2}{3} + \frac{3 x^7 b \ln(c x^n) d^2 e}{7} - \frac{3 b d^2 e n x^7}{49} +$
risch	$\frac{x^{11} a e^3}{11} + \frac{x^5 a d^3}{5} + \frac{3 \ln(c) b d^2 e x^7}{7} + \frac{\ln(c) b d e^2 x^9}{3} - \frac{3 i \pi b d^2 e x^7 \operatorname{csgn}(i c x^n)^3}{14} + \frac{i \pi b e^3 x^{11} \operatorname{csgn}(i c) \operatorname{csgn}(i c x^n)^2}{22} + \frac{x^9 a d}{3}$

```
[In] int(x^4*(e*x^2+d)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*x^11*b*ln(c*x^n)*e^3-1/121*b*e^3*n*x^11+1/11*x^11*a*e^3+1/3*x^9*b*ln(c
*x^n)*d*e^2-1/27*b*d*e^2*n*x^9+1/3*x^9*a*d*e^2+3/7*x^7*b*ln(c*x^n)*d^2*e-3/
49*b*d^2*e*n*x^7+3/7*x^7*a*d^2*e+1/5*x^5*b*ln(c*x^n)*d^3-1/25*b*d^3*n*x^5+1
/5*x^5*a*d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.67

$$\begin{aligned}\int x^4 (d+ex^2)^3 (a+b\log(cx^n))\,dx &= -\frac{1}{121}\left(be^3n - 11\,ae^3\right)x^{11} - \frac{1}{27}\left(bde^2n - 9\,ade^2\right)x^9 - \frac{3}{49}\left(bd^2en - 7\,ad^2e\right)x^7 \\ &\quad - \frac{1}{25}\left(bd^3n - 5\,ad^3\right)x^5 + \frac{1}{1155}\left(105\,be^3x^{11} + 385\,bde^2x^9 + 495\,bd^2ex^7 + 231\,bd^3x^5\right)\log(c) \\ &\quad + \frac{1}{1155}\left(105\,be^3nx^{11} + 385\,bde^2nx^9 + 495\,bd^2enx^7 + 231\,bd^3nx^5\right)\log(x)\end{aligned}$$

```
[In] integrate(x^4*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/121*(b*e^3*n - 11*a*e^3)*x^11 - 1/27*(b*d*e^2*n - 9*a*d*e^2)*x^9 - 3/49*
(b*d^2*e*n - 7*a*d^2*e)*x^7 - 1/25*(b*d^3*n - 5*a*d^3)*x^5 + 1/1155*(105*b*
e^3*x^11 + 385*b*d*e^2*x^9 + 495*b*d^2*e*x^7 + 231*b*d^3*x^5)*log(c) + 1/11
55*(105*b*e^3*n*x^11 + 385*b*d*e^2*n*x^9 + 495*b*d^2*e*n*x^7 + 231*b*d^3*n*
x^5)*log(x)
```

Sympy [A] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.70

$$\int x^4(d+ex^2)^3(a+b\log(cx^n))\,dx = \frac{ad^3x^5}{5}+\frac{3ad^2ex^7}{7}+\frac{ade^2x^9}{3}+\frac{ae^3x^{11}}{11}-\frac{bd^3nx^5}{25}+\frac{bd^3x^5\log(cx^n)}{5}-\frac{3bd^2enx^7}{49}+\frac{3bd^2ex^7\log(cx^n)}{7}-\frac{bde^2nx^9}{27}+\frac{bde^2x^9\log(cx^n)}{3}-\frac{be^3nx^{11}}{121}+\frac{be^3x^{11}\log(cx^n)}{11}$$

```
[In] integrate(x**4*(e*x**2+d)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**3*x**5/5 + 3*a*d**2*e*x**7/7 + a*d*e**2*x**9/3 + a*e**3*x**11/11 - b*d
**3*n*x**5/25 + b*d**3*x**5*log(c*x**n)/5 - 3*b*d**2*e*n*x**7/49 + 3*b*d**2
*e*x**7*log(c*x**n)/7 - b*d*e**2*n*x**9/27 + b*d*e**2*x**9*log(c*x**n)/3 -
b*e**3*n*x**11/121 + b*e**3*x**11*log(c*x**n)/11
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.43

$$\int x^4(d+ex^2)^3(a+b\log(cx^n))\,dx = -\frac{1}{121}\,be^3nx^{11}+\frac{1}{11}\,be^3x^{11}\log(cx^n)+\frac{1}{11}\,ae^3x^{11}-\frac{1}{27}\,bde^2nx^9+\frac{1}{3}\,bde^2x^9\log(cx^n)+\frac{1}{3}\,ade^2x^9-\frac{3}{49}\,bd^2enx^7+\frac{3}{7}\,bd^2ex^7\log(cx^n)+\frac{3}{7}\,ad^2ex^7-\frac{1}{25}\,bd^3nx^5+\frac{1}{5}\,bd^3x^5\log(cx^n)+\frac{1}{5}\,ad^3x^5$$

```
[In] integrate(x^4*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/121*b*e^3*n*x^11 + 1/11*b*e^3*x^11*log(c*x^n) + 1/11*a*e^3*x^11 - 1/27*b
*d*e^2*n*x^9 + 1/3*b*d*e^2*x^9*log(c*x^n) + 1/3*a*d*e^2*x^9 - 3/49*b*d^2*e*
n*x^7 + 3/7*b*d^2*e*x^7*log(c*x^n) + 3/7*a*d^2*e*x^7 - 1/25*b*d^3*n*x^5 + 1
/5*b*d^3*x^5*log(c*x^n) + 1/5*a*d^3*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.77

$$\int x^4(d+ex^2)^3(a+b\log(cx^n))\,dx = \frac{1}{11}be^3nx^{11}\log(x) - \frac{1}{121}be^3nx^{11} + \frac{1}{11}be^3x^{11}\log(c) + \frac{1}{11}ae^3x^{11} + \frac{1}{3}bde^2nx^9\log(x) - \frac{1}{27}bde^2nx^9 + \frac{1}{3}bde^2x^9\log(c) + \frac{1}{3}ade^2x^9 + \frac{3}{7}bd^2enx^7\log(x) - \frac{3}{49}bd^2enx^7 + \frac{3}{7}bd^2ex^7\log(c) + \frac{3}{7}ad^2ex^7 + \frac{1}{5}bd^3nx^5\log(x) - \frac{1}{25}bd^3nx^5 + \frac{1}{5}bd^3x^5\log(c) + \frac{1}{5}ad^3x^5$$

```
[In] integrate(x^4*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/11*b*e^3*n*x^11*log(x) - 1/121*b*e^3*n*x^11 + 1/11*b*e^3*x^11*log(c) + 1/
11*a*e^3*x^11 + 1/3*b*d*e^2*n*x^9*log(x) - 1/27*b*d*e^2*n*x^9 + 1/3*b*d*e^2
*x^9*log(c) + 1/3*a*d*e^2*x^9 + 3/7*b*d^2*e*n*x^7*log(x) - 3/49*b*d^2*e*n*x
^7 + 3/7*b*d^2*e*x^7*log(c) + 3/7*a*d^2*e*x^7 + 1/5*b*d^3*n*x^5*log(x) - 1/
25*b*d^3*n*x^5 + 1/5*b*d^3*x^5*log(c) + 1/5*a*d^3*x^5
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13

$$\int x^4(d+ex^2)^3(a+b\log(cx^n))\,dx = \ln(c\,x^n)\left(\frac{b\,d^3\,x^5}{5} + \frac{3\,b\,d^2\,e\,x^7}{7} + \frac{b\,d\,e^2\,x^9}{3} + \frac{b\,e^3\,x^{11}}{11}\right) + \frac{d^3\,x^5\,(5\,a-b\,n)}{25} + \frac{e^3\,x^{11}\,(11\,a-b\,n)}{121} + \frac{3\,d^2\,e\,x^7\,(7\,a-b\,n)}{49} + \frac{d\,e^2\,x^9\,(9\,a-b\,n)}{27}$$

```
[In] int(x^4*(d + e*x^2)^3*(a + b*log(c*x^n)),x)
```

```
[Out] log(c*x^n)*((b*d^3*x^5)/5 + (b*e^3*x^11)/11 + (3*b*d^2*e*x^7)/7 + (b*d*e^2*
x^9)/3) + (d^3*x^5*(5*a - b*n))/25 + (e^3*x^11*(11*a - b*n))/121 + (3*d^2*e
*x^7*(7*a - b*n))/49 + (d*e^2*x^9*(9*a - b*n))/27
```

3.203 $\int x^2(d+ex^2)^3 (a+b\log(cx^n))\, dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int x^2(d+ex^2)^3 (a+b\log(cx^n))\, dx = -\frac{1}{9}bd^3nx^3 - \frac{3}{25}bd^2enx^5 - \frac{3}{49}bde^2nx^7 - \frac{1}{81}be^3nx^9 + \frac{1}{315}(105d^3x^3 + 189d^2ex^5 + 135de^2x^7 + 35e^3x^9)(a + b\log(cx^n))$$

```
[Out] -1/9*b*d^3*n*x^3-3/25*b*d^2*e*n*x^5-3/49*b*d*e^2*n*x^7-1/81*b*e^3*n*x^9+1/3
15*(35*e^3*x^9+135*d*e^2*x^7+189*d^2*e*x^5+105*d^3*x^3)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {276, 2371}

$$\int x^2(d+ex^2)^3 (a+b\log(cx^n))\, dx = \frac{1}{315}(105d^3x^3 + 189d^2ex^5 + 135de^2x^7 + 35e^3x^9)(a + b\log(cx^n)) - \frac{1}{9}bd^3nx^3 - \frac{3}{25}bd^2enx^5 - \frac{3}{49}bde^2nx^7 - \frac{1}{81}be^3nx^9$$

```
[In] Int[x^2*(d + e*x^2)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/9*(b*d^3*n*x^3) - (3*b*d^2*e*n*x^5)/25 - (3*b*d*e^2*n*x^7)/49 - (b*e^3*n
*x^9)/81 + ((105*d^3*x^3 + 189*d^2*e*x^5 + 135*d*e^2*x^7 + 35*e^3*x^9)*(a +
 b*Log[c*x^n]))/315
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{315}\left(105d^3x^3+189d^2ex^5+135de^2x^7+35e^3x^9\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int\left(\frac{d^3x^2}{3}+\frac{3}{5}d^2ex^4+\frac{3}{7}de^2x^6+\frac{e^3x^8}{9}\right)dx \\
&= -\frac{1}{9}bd^3nx^3-\frac{3}{25}bd^2enx^5-\frac{3}{49}bde^2nx^7-\frac{1}{81}be^3nx^9 \\
&\quad +\frac{1}{315}\left(105d^3x^3+189d^2ex^5+135de^2x^7+35e^3x^9\right)(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.33

$$\begin{aligned}
\int x^2\left(d+ex^2\right)^3(a+b\log(cx^n))\,dx &= -\frac{1}{9}bd^3nx^3-\frac{3}{25}bd^2enx^5-\frac{3}{49}bde^2nx^7-\frac{1}{81}be^3nx^9 \\
&\quad +\frac{1}{3}d^3x^3(a+b\log(cx^n))+\frac{3}{5}d^2ex^5(a+b\log(cx^n)) \\
&\quad +\frac{3}{7}de^2x^7(a+b\log(cx^n))+\frac{1}{9}e^3x^9(a+b\log(cx^n))
\end{aligned}$$

```
[In] Integrate[x^2*(d + e*x^2)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/9*(b*d^3*n*x^3) - (3*b*d^2*e*n*x^5)/25 - (3*b*d*e^2*n*x^7)/49 - (b*e^3*n
*x^9)/81 + (d^3*x^3*(a + b*Log[c*x^n]))/3 + (3*d^2*e*x^5*(a + b*Log[c*x^n])
)/5 + (3*d*e^2*x^7*(a + b*Log[c*x^n]))/7 + (e^3*x^9*(a + b*Log[c*x^n]))/9
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.44

method	result
parallelrisch	$\frac{x^9 b \ln(c x^n) e^3}{9} - \frac{b e^3 n x^9}{81} + \frac{x^9 a e^3}{9} + \frac{3x^7 b \ln(c x^n) d e^2}{7} - \frac{3bd e^2 n x^7}{49} + \frac{3x^7 a d e^2}{7} + \frac{3x^5 \ln(c x^n) b d^2 e}{5} - \frac{3b d^2 e n x^5}{25}$
risch	$\frac{a d^3 x^3}{3} + \frac{3a d^2 e x^5}{5} + \frac{x^9 a e^3}{9} + \frac{3\ln(c) b d e^2 x^7}{7} + \frac{i\pi b d^3 x^3 \operatorname{csgn}(ic) \operatorname{csgn}(ic x^n)^2}{6} - \frac{3i\pi b d e^2 x^7 \operatorname{csgn}(ic x^n)^3}{14} - \frac{i\pi b d^3 x^3}{}$

```
[In] int(x^2*(e*x^2+d)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*x^9*b*ln(c*x^n)*e^3-1/81*b*e^3*n*x^9+1/9*x^9*a*e^3+3/7*x^7*b*ln(c*x^n)*
d*e^2-3/49*b*d*e^2*n*x^7+3/7*x^7*a*d*e^2+3/5*x^5*ln(c*x^n)*b*d^2*e-3/25*b*d
^2*e*n*x^5+3/5*a*d^2*e*x^5+1/3*x^3*b*ln(c*x^n)*d^3-1/9*b*d^3*n*x^3+1/3*a*d^
3*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.67

$$\begin{aligned}\int x^2 (d+ex^2)^3 (a+b\log(cx^n))\,dx &= -\frac{1}{81}\left(be^3n-9\,ae^3\right)x^9 - \frac{3}{49}\left(bde^2n-7\,ade^2\right)x^7 - \frac{3}{25}\left(bd^2en-5\,ad^2e\right)x^5 \\ &\quad - \frac{1}{9}\left(bd^3n-3\,ad^3\right)x^3 + \frac{1}{315}\left(35\,be^3x^9+135\,bde^2x^7+189\,bd^2ex^5+105\,bd^3x^3\right)\log(c) \\ &\quad + \frac{1}{315}\left(35\,be^3nx^9+135\,bde^2nx^7+189\,bd^2enx^5+105\,bd^3nx^3\right)\log(x)\end{aligned}$$

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/81*(b*e^3*n - 9*a*e^3)*x^9 - 3/49*(b*d*e^2*n - 7*a*d*e^2)*x^7 - 3/25*(b*
d^2*e*n - 5*a*d^2*e)*x^5 - 1/9*(b*d^3*n - 3*a*d^3)*x^3 + 1/315*(35*b*e^3*x^
9 + 135*b*d*e^2*x^7 + 189*b*d^2*e*x^5 + 105*b*d^3*x^3)*log(c) + 1/315*(35*b
*e^3*n*x^9 + 135*b*d*e^2*n*x^7 + 189*b*d^2*e*n*x^5 + 105*b*d^3*n*x^3)*log(x
)
```

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.75

$$\int x^2(d+ex^2)^3(a+b\log(cx^n))\,dx = \frac{ad^3x^3}{3}+\frac{3ad^2ex^5}{5}+\frac{3ade^2x^7}{7}+\frac{ae^3x^9}{9} - \frac{bd^3nx^3}{9}+\frac{bd^3x^3\log(cx^n)}{3}-\frac{3bd^2enx^5}{25} + \frac{3bd^2ex^5\log(cx^n)}{5}-\frac{3bde^2nx^7}{49} + \frac{3bde^2x^7\log(cx^n)}{7}-\frac{be^3nx^9}{81}+\frac{be^3x^9\log(cx^n)}{9}$$

```
[In] integrate(x**2*(e*x**2+d)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**3*x**3/3 + 3*a*d**2*e*x**5/5 + 3*a*d*e**2*x**7/7 + a*e**3*x**9/9 - b*d
**3*n*x**3/9 + b*d**3*x**3*log(c*x**n)/3 - 3*b*d**2*e*n*x**5/25 + 3*b*d**2*
e*x**5*log(c*x**n)/5 - 3*b*d*e**2*n*x**7/49 + 3*b*d*e**2*x**7*log(c*x**n)/7
 - b*e**3*n*x**9/81 + b*e**3*x**9*log(c*x**n)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.43

$$\int x^2(d+ex^2)^3(a+b\log(cx^n))\,dx = -\frac{1}{81}be^3nx^9+\frac{1}{9}be^3x^9\log(cx^n)+\frac{1}{9}ae^3x^9 - \frac{3}{49}bde^2nx^7+\frac{3}{7}bde^2x^7\log(cx^n)+\frac{3}{7}ade^2x^7 - \frac{3}{25}bd^2enx^5+\frac{3}{5}bd^2ex^5\log(cx^n)+\frac{3}{5}ad^2ex^5 - \frac{1}{9}bd^3nx^3+\frac{1}{3}bd^3x^3\log(cx^n)+\frac{1}{3}ad^3x^3$$

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/81*b*e^3*n*x^9 + 1/9*b*e^3*x^9*log(c*x^n) + 1/9*a*e^3*x^9 - 3/49*b*d*e^2
*n*x^7 + 3/7*b*d*e^2*x^7*log(c*x^n) + 3/7*a*d*e^2*x^7 - 3/25*b*d^2*e*n*x^5
+ 3/5*b*d^2*e*x^5*log(c*x^n) + 3/5*a*d^2*e*x^5 - 1/9*b*d^3*n*x^3 + 1/3*b*d^
3*x^3*log(c*x^n) + 1/3*a*d^3*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.77

$$\int x^2\left(d+ex^2\right)^3\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{1}{9}\,be^3nx^9\log\left(x\right)-\frac{1}{81}\,be^3nx^9+\frac{1}{9}\,be^3x^9\log\left(c\right)+\frac{1}{9}\,ae^3x^9+\frac{3}{7}\,bde^2nx^7\log\left(x\right)-\frac{3}{49}\,bde^2nx^7+\frac{3}{7}\,bde^2x^7\log\left(c\right)+\frac{3}{7}\,ade^2x^7+\frac{3}{5}\,bd^2enx^5\log\left(x\right)-\frac{3}{25}\,bd^2enx^5+\frac{3}{5}\,bd^2ex^5\log\left(c\right)+\frac{3}{5}\,ad^2ex^5+\frac{1}{3}\,bd^3nx^3\log\left(x\right)-\frac{1}{9}\,bd^3nx^3+\frac{1}{3}\,bd^3x^3\log\left(c\right)+\frac{1}{3}\,ad^3x^3$$

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/9*b*e^3*n*x^9*log(x) - 1/81*b*e^3*n*x^9 + 1/9*b*e^3*x^9*log(c) + 1/9*a*e^
3*x^9 + 3/7*b*d*e^2*n*x^7*log(x) - 3/49*b*d*e^2*n*x^7 + 3/7*b*d*e^2*x^7*log
(c) + 3/7*a*d*e^2*x^7 + 3/5*b*d^2*e*n*x^5*log(x) - 3/25*b*d^2*e*n*x^5 + 3/5
*b*d^2*e*x^5*log(c) + 3/5*a*d^2*e*x^5 + 1/3*b*d^3*n*x^3*log(x) - 1/9*b*d^3*
n*x^3 + 1/3*b*d^3*x^3*log(c) + 1/3*a*d^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13

$$\int x^2\left(d+ex^2\right)^3\left(a+b\log\left(cx^n\right)\right)\,dx=\ln\left(c\,x^n\right)\,\left(\frac{b\,d^3\,x^3}{3}+\frac{3\,b\,d^2\,e\,x^5}{5}+\frac{3\,b\,d\,e^2\,x^7}{7}+\frac{b\,e^3\,x^9}{9}\right)+\frac{d^3\,x^3\,\left(3\,a-b\,n\right)}{9}+\frac{e^3\,x^9\,\left(9\,a-b\,n\right)}{81}+\frac{3\,d^2\,e\,x^5\,\left(5\,a-b\,n\right)}{25}+\frac{3\,d\,e^2\,x^7\,\left(7\,a-b\,n\right)}{49}$$

```
[In] int(x^2*(d + e*x^2)^3*(a + b*log(c*x^n)),x)
```

```
[Out] log(c*x^n)*((b*d^3*x^3)/3 + (b*e^3*x^9)/9 + (3*b*d^2*e*x^5)/5 + (3*b*d*e^2*
x^7)/7) + (d^3*x^3*(3*a - b*n))/9 + (e^3*x^9*(9*a - b*n))/81 + (3*d^2*e*x^5
*(5*a - b*n))/25 + (3*d*e^2*x^7*(7*a - b*n))/49
```

3.204 $\int (d+ex^2)^3 (a+b\log(cx^n))\, dx$

Optimal result

Integrand size = 20, antiderivative size = 121

$$\begin{aligned}\int (d+ex^2)^3 (a+b\log(cx^n))\, dx = & -bd^3nx-\frac{1}{3}bd^2enx^3-\frac{3}{25}bde^2nx^5-\frac{1}{49}be^3nx^7 \\ & +d^3x(a+b\log(cx^n))+d^2ex^3(a+b\log(cx^n)) \\ & +\frac{3}{5}de^2x^5(a+b\log(cx^n))+\frac{1}{7}e^3x^7(a+b\log(cx^n))\end{aligned}$$

```
[Out] -b*d^3*n*x-1/3*b*d^2*e*n*x^3-3/25*b*d*e^2*n*x^5-1/49*b*e^3*n*x^7+d^3*x*(a+b
*ln(c*x^n))+d^2*e*x^3*(a+b*ln(c*x^n))+3/5*d*e^2*x^5*(a+b*ln(c*x^n))+1/7*e^3
*x^7*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {200, 2350}

$$\begin{aligned}\int (d+ex^2)^3 (a+b\log(cx^n))\, dx = & \, d^3x(a+b\log(cx^n))+d^2ex^3(a+b\log(cx^n)) \\ & +\frac{3}{5}de^2x^5(a+b\log(cx^n))+\frac{1}{7}e^3x^7(a+b\log(cx^n)) \\ & -bd^3nx-\frac{1}{3}bd^2enx^3-\frac{3}{25}bde^2nx^5-\frac{1}{49}be^3nx^7\end{aligned}$$

```
[In] Int[(d + e*x^2)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -(b*d^3*n*x) - (b*d^2*e*n*x^3)/3 - (3*b*d*e^2*n*x^5)/25 - (b*e^3*n*x^7)/49
+ d^3*x*(a + b*Log[c*x^n]) + d^2*e*x^3*(a + b*Log[c*x^n]) + (3*d*e^2*x^5*(a
 + b*Log[c*x^n]))/5 + (e^3*x^7*(a + b*Log[c*x^n]))/7
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2350

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Dist[a + b*Log[c*x^n], u
, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= d^3x(a+b\log(cx^n))+d^2ex^3(a+b\log(cx^n))+\frac{3}{5}de^2x^5(a+b\log(cx^n))\\&\quad+\frac{1}{7}e^3x^7(a+b\log(cx^n))-(bn)\int\left(d^3+d^2ex^2+\frac{3}{5}de^2x^4+\frac{e^3x^6}{7}\right)dx\\&=-bd^3nx-\frac{1}{3}bd^2enx^3-\frac{3}{25}bde^2nx^5-\frac{1}{49}be^3nx^7+d^3x(a+b\log(cx^n))\\&\quad+d^2ex^3(a+b\log(cx^n))+\frac{3}{5}de^2x^5(a+b\log(cx^n))+\frac{1}{7}e^3x^7(a+b\log(cx^n))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\begin{aligned}\int(d+ex^2)^3(a+b\log(cx^n))\,dx &= ad^3x-bd^3nx-\frac{1}{3}bd^2enx^3-\frac{3}{25}bde^2nx^5-\frac{1}{49}be^3nx^7\\&\quad+bd^3x\log(cx^n)+d^2ex^3(a+b\log(cx^n))\\&\quad+\frac{3}{5}de^2x^5(a+b\log(cx^n))+\frac{1}{7}e^3x^7(a+b\log(cx^n))\end{aligned}$$

```
[In] Integrate[(d + e*x^2)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] a*d^3*x - b*d^3*n*x - (b*d^2*e*n*x^3)/3 - (3*b*d*e^2*n*x^5)/25 - (b*e^3*n*x
^7)/49 + b*d^3*x*Log[c*x^n] + d^2*e*x^3*(a + b*Log[c*x^n]) + (3*d*e^2*x^5*(
a + b*Log[c*x^n]))/5 + (e^3*x^7*(a + b*Log[c*x^n]))/7
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{x^{7}\ln(c\,x^{n})b\,e^{3}}{7}-\frac{b\,e^{3}n\,x^{7}}{49}+\frac{a\,e^{3}x^{7}}{7}+\frac{3x^{5}b\ln(c\,x^{n})d\,e^{2}}{5}-\frac{3bd\,e^{2}n\,x^{5}}{25}+\frac{3x^{5}ad\,e^{2}}{5}+x^{3}b\ln\left(c\,x^{n}\right)d^{2}e-\frac{b\,d^{2}en\,x^{3}}{3}$
risch	$\frac{a\,e^{3}x^{7}}{7}+\ln\left(c\right)b\,d^{2}e\,x^{3}-\frac{i\pi b\,d^{2}e\,x^{3}\operatorname{csgn}(ic)\operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})}{2}+\frac{i\pi b\,e^{3}x^{7}\operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^{n})^{2}}{14}+\frac{i\pi b\,e^{3}x^{7}\operatorname{csgn}(i}{1}$

```
[In] int((e*x^2+d)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*x^7*ln(c*x^n)*b*e^3-1/49*b*e^3*n*x^7+1/7*a*e^3*x^7+3/5*x^5*b*ln(c*x^n)*
d*e^2-3/25*b*d*e^2*n*x^5+3/5*x^5*a*d*e^2+x^3*b*ln(c*x^n)*d^2*e-1/3*b*d^2*e*
n*x^3+x^3*a*d^2*e+x*b*ln(c*x^n)*d^3-b*d^3*n*x+a*d^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.33

$$\int \left(d+ex^2\right)^3 \left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=-\frac{1}{49}\left(be^3n-7\,ae^3\right)x^7-\frac{3}{25}\left(bde^2n-5\,ade^2\right)x^5-\frac{1}{3}\left(bd^2en-3\,ad^2e\right)x^3$$
$$-\left(bd^3n-ad^3\right)x+\frac{1}{35}\left(5\,be^3x^7+21\,bde^2x^5+35\,bd^2ex^3+35\,bd^3x\right)\log\left(c\right)$$
$$+\frac{1}{35}\left(5\,be^3nx^7+21\,bde^2nx^5+35\,bd^2enx^3+35\,bd^3nx\right)\log\left(x\right)$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -1/49*(b*e^3*n - 7*a*e^3)*x^7 - 3/25*(b*d*e^2*n - 5*a*d*e^2)*x^5 - 1/3*(b*d
^2*e*n - 3*a*d^2*e)*x^3 - (b*d^3*n - a*d^3)*x + 1/35*(5*b*e^3*x^7 + 21*b*d*
e^2*x^5 + 35*b*d^2*e*x^3 + 35*b*d^3*x)*log(c) + 1/35*(5*b*e^3*n*x^7 + 21*b*
d*e^2*n*x^5 + 35*b*d^2*e*n*x^3 + 35*b*d^3*n*x)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.29

$$\int \left(d+ex^2\right)^3 \left(a+b\log\left(cx^n\right)\right)\,dx=ad^3x+ad^2ex^3+\frac{3ade^2x^5}{5}+\frac{ae^3x^7}{7}-bd^3nx+bd^3x\log\left(cx^n\right)$$
$$-\frac{bd^2enx^3}{3}+bd^2ex^3\log\left(cx^n\right)-\frac{3bde^2nx^5}{25}$$
$$+\frac{3bde^2x^5\log\left(cx^n\right)}{5}-\frac{be^3nx^7}{49}+\frac{be^3x^7\log\left(cx^n\right)}{7}$$

```
[In] integrate((e*x**2+d)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**3*x + a*d**2*e*x**3 + 3*a*d*e**2*x**5/5 + a*e**3*x**7/7 - b*d**3*n*x +
 b*d**3*x*log(c*x**n) - b*d**2*e*n*x**3/3 + b*d**2*e*x**3*log(c*x**n) - 3*b
*d*e**2*n*x**5/25 + 3*b*d*e**2*x**5*log(c*x**n)/5 - b*e**3*n*x**7/49 + b*e*
*3*x**7*log(c*x**n)/7
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10

$$\int (d+ex^2)^3 (a+b\log(cx^n))\, dx = -\frac{1}{49}\, be^3nx^7 + \frac{1}{7}\, be^3x^7\log(cx^n) + \frac{1}{7}\, ae^3x^7 - \frac{3}{25}\, bde^2nx^5 + \frac{3}{5}\, bde^2x^5\log(cx^n) + \frac{3}{5}\, ade^2x^5 - \frac{1}{3}\, bd^2enx^3 + bd^2ex^3\log(cx^n) + ad^2ex^3 - bd^3nx + bd^3x\log(cx^n) + ad^3x$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/49*b*e^3*n*x^7 + 1/7*b*e^3*x^7*log(c*x^n) + 1/7*a*e^3*x^7 - 3/25*b*d*e^2
*n*x^5 + 3/5*b*d*e^2*x^5*log(c*x^n) + 3/5*a*d*e^2*x^5 - 1/3*b*d^2*e*n*x^3 +
 b*d^2*e*x^3*log(c*x^n) + a*d^2*e*x^3 - b*d^3*n*x + b*d^3*x*log(c*x^n) + a*
d^3*x
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.35

$$\int (d+ex^2)^3 (a+b\log(cx^n))\, dx = \frac{1}{7}\, be^3nx^7\log(x) - \frac{1}{49}\, be^3nx^7 + \frac{1}{7}\, be^3x^7\log(c) + \frac{1}{7}\, ae^3x^7 + \frac{3}{5}\, bde^2nx^5\log(x) - \frac{3}{25}\, bde^2nx^5 + \frac{3}{5}\, bde^2x^5\log(c) + \frac{3}{5}\, ade^2x^5 + bd^2enx^3\log(x) - \frac{1}{3}\, bd^2enx^3 + bd^2ex^3\log(c) + ad^2ex^3 + bd^3nx\log(x) - bd^3nx + bd^3x\log(c) + ad^3x$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/7*b*e^3*n*x^7*log(x) - 1/49*b*e^3*n*x^7 + 1/7*b*e^3*x^7*log(c) + 1/7*a*e^
3*x^7 + 3/5*b*d*e^2*n*x^5*log(x) - 3/25*b*d*e^2*n*x^5 + 3/5*b*d*e^2*x^5*log
(c) + 3/5*a*d*e^2*x^5 + b*d^2*e*n*x^3*log(x) - 1/3*b*d^2*e*n*x^3 + b*d^2*e*
x^3*log(c) + a*d^2*e*x^3 + b*d^3*n*x*log(x) - b*d^3*n*x + b*d^3*x*log(c) +
a*d^3*x
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.86

$$\int \left(d+ex^2\right)^3 (a+b\log (cx^n))\, dx = \ln (c\,x^n)\,\left(b\,d^3\,x+b\,d^2\,e\,x^3+\frac{3\,b\,d\,e^2\,x^5}{5}+\frac{b\,e^3\,x^7}{7}\right)+\frac{e^3\,x^7\,(7\,a-b\,n)}{49}+d^3\,x\,(a-b\,n)+\frac{d^2\,e\,x^3\,(3\,a-b\,n)}{3}+\frac{3\,d\,e^2\,x^5\,(5\,a-b\,n)}{25}$$

```
[In] int((d + e*x^2)^3*(a + b*log(c*x^n)),x)
```

```
[Out] log(c*x^n)*((b*e^3*x^7)/7 + b*d^3*x + b*d^2*e*x^3 + (3*b*d*e^2*x^5)/5) + (e
^3*x^7*(7*a - b*n))/49 + d^3*x*(a - b*n) + (d^2*e*x^3*(3*a - b*n))/3 + (3*d
*e^2*x^5*(5*a - b*n))/25
```

3.205 $\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^2}\,dx = -\frac{bd^3n}{x} - 3bd^2enx - \frac{1}{3}bde^2nx^3 - \frac{1}{25}be^3nx^5 - \frac{d^3(a+b\log(cx^n))}{x} + 3d^2ex(a+b\log(cx^n)) + de^2x^3(a+b\log(cx^n)) + \frac{1}{5}e^3x^5(a+b\log(cx^n))$$

```
[Out] -b*d^3*n/x-3*b*d^2*e*n*x-1/3*b*d*e^2*n*x^3-1/25*b*e^3*n*x^5-d^3*(a+b*ln(c*x
^n))/x+3*d^2*e*x*(a+b*ln(c*x^n))+d*e^2*x^3*(a+b*ln(c*x^n))+1/5*e^3*x^5*(a+b
*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {276, 2372}

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^2}\,dx = -\frac{d^3(a+b\log(cx^n))}{x} + 3d^2ex(a+b\log(cx^n)) + de^2x^3(a+b\log(cx^n)) + \frac{1}{5}e^3x^5(a+b\log(cx^n)) - \frac{bd^3n}{x} - 3bd^2enx - \frac{1}{3}bde^2nx^3 - \frac{1}{25}be^3nx^5$$

```
[In] Int[((d + e*x^2)^3*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -((b*d^3*n)/x) - 3*b*d^2*e*n*x - (b*d*e^2*n*x^3)/3 - (b*e^3*n*x^5)/25 - (d^
3*(a + b*Log[c*x^n]))/x + 3*d^2*e*x*(a + b*Log[c*x^n]) + d*e^2*x^3*(a + b*L
og[c*x^n]) + (e^3*x^5*(a + b*Log[c*x^n]))/5
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{x}+3d^2ex(a+b\log(cx^n))+de^2x^3(a+b\log(cx^n))\\
&\quad+\frac{1}{5}e^3x^5(a+b\log(cx^n))-(bn)\int\left(3d^2e-\frac{d^3}{x^2}+de^2x^2+\frac{e^3x^4}{5}\right)dx\\
&= -\frac{bd^3n}{x}-3bd^2enx-\frac{1}{3}bde^2nx^3-\frac{1}{25}be^3nx^5-\frac{d^3(a+b\log(cx^n))}{x}\\
&\quad+3d^2ex(a+b\log(cx^n))+de^2x^3(a+b\log(cx^n))+\frac{1}{5}e^3x^5(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04

$$\begin{aligned}
\int\frac{(d+ex^2)^3(a+b\log(cx^n))}{x^2}\,dx &= -\frac{bd^3n}{x}+3ad^2ex-3bd^2enx-\frac{1}{3}bde^2nx^3\\
&\quad-\frac{1}{25}be^3nx^5+3bd^2ex\log(cx^n)-\frac{d^3(a+b\log(cx^n))}{x}\\
&\quad+de^2x^3(a+b\log(cx^n))+\frac{1}{5}e^3x^5(a+b\log(cx^n))
\end{aligned}$$

```
[In] Integrate[((d + e*x^2)^3*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -((b*d^3*n)/x) + 3*a*d^2*e*x - 3*b*d^2*e*n*x - (b*d*e^2*n*x^3)/3 - (b*e^3*n
*x^5)/25 + 3*b*d^2*e*x*Log[c*x^n] - (d^3*(a + b*Log[c*x^n]))/x + d*e^2*x^3*
(a + b*Log[c*x^n]) + (e^3*x^5*(a + b*Log[c*x^n]))/5
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.19

method	result
parallelrisch	$-\frac{-15x^{6}b\ln(c\,x^{n})e^{3}+3b\,e^{3}n\,x^{6}-15x^{6}a\,e^{3}-75x^{4}b\ln(c\,x^{n})d\,e^{2}+25bd\,e^{2}n\,x^{4}-75x^{4}ad\,e^{2}-225b\ln(c\,x^{n})d^{2}e\,x^{2}+225b\,d^{2}en\,x^{2}-2}{75x}$
risch	$-\frac{b(-e^{3}x^{6}-5e^{2}d\,x^{4}-15d^{2}e\,x^{2}+5d^{3})\ln(x^{n})}{5x}-\frac{-30x^{6}a\,e^{3}-150\ln(c)bd\,e^{2}x^{4}+75i\pi bd\,e^{2}x^{4}\operatorname{csgn}(ic)\operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})+22}{}$

```
[In] int((e*x^2+d)^3*(a+b*ln(c*x^n))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/75/x*(-15*x^6*b*ln(c*x^n)*e^3+3*b*e^3*n*x^6-15*x^6*a*e^3-75*x^4*b*ln(c*x
^n)*d*e^2+25*b*d*e^2*n*x^4-75*x^4*a*d*e^2-225*b*ln(c*x^n)*d^2*e*x^2+225*b*d
^2*e*n*x^2-225*a*d^2*e*x^2+75*b*ln(c*x^n)*d^3+75*b*d^3*n+75*a*d^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex^2)^3\,(a+b\log{(cx^n)})}{x^2}\,dx = -\frac{3\,(be^3n-5\,ae^3)x^6+75\,bd^3n+25\,(bde^2n-3\,ade^2)x^4+75\,ad^3+225\,(bd^2en-ad^2e)x^2-15\,(be^3x^6+5}{75\,x}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
```

```
[Out] -1/75*(3*(b*e^3*n - 5*a*e^3)*x^6 + 75*b*d^3*n + 25*(b*d*e^2*n - 3*a*d*e^2)*
x^4 + 75*a*d^3 + 225*(b*d^2*e*n - a*d^2*e)*x^2 - 15*(b*e^3*x^6 + 5*b*d*e^2*
x^4 + 15*b*d^2*e*x^2 - 5*b*d^3)*log(c) - 15*(b*e^3*n*x^6 + 5*b*d*e^2*n*x^4
+ 15*b*d^2*e*n*x^2 - 5*b*d^3*n)*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex^2)^3\,(a+b\log{(cx^n)})}{x^2}\,dx = -\frac{ad^3}{x}+3ad^2ex+ade^2x^3+\frac{ae^3x^5}{5}-\frac{bd^3n}{x} - \frac{bd^3\log{(cx^n)}}{x}-3bd^2enx+3bd^2ex\log{(cx^n)}-\frac{bde^2nx^3}{3} + bde^2x^3\log{(cx^n)}-\frac{be^3nx^5}{25}+\frac{be^3x^5\log{(cx^n)}}{5}$$

```
[In] integrate((e*x**2+d)**3*(a+b*ln(c*x**n))/x**2,x)
```

```
[Out] -a*d**3/x + 3*a*d**2*e*x + a*d*e**2*x**3 + a*e**3*x**5/5 - b*d**3*n/x - b*d
**3*log(c*x**n)/x - 3*b*d**2*e*n*x + 3*b*d**2*e*x*log(c*x**n) - b*d*e**2*n*
x**3/3 + b*d*e**2*x**3*log(c*x**n) - b*e**3*n*x**5/25 + b*e**3*x**5*log(c*x
**n)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^2} \, dx = -\frac{1}{25} be^3nx^5 + \frac{1}{5} be^3x^5 \log(cx^n) + \frac{1}{5} ae^3x^5 - \frac{1}{3} bde^2nx^3 + bde^2x^3 \log(cx^n) + ade^2x^3 - 3\,bd^2enx + 3\,bd^2ex \log(cx^n) + 3\,ad^2ex - \frac{bd^3n}{x} - \frac{bd^3 \log(cx^n)}{x} - \frac{ad^3}{x}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
```

```
[Out] -1/25*b*e^3*n*x^5 + 1/5*b*e^3*x^5*log(c*x^n) + 1/5*a*e^3*x^5 - 1/3*b*d*e^2*
n*x^3 + b*d*e^2*x^3*log(c*x^n) + a*d*e^2*x^3 - 3*b*d^2*e*n*x + 3*b*d^2*e*x*
log(c*x^n) + 3*a*d^2*e*x - b*d^3*n/x - b*d^3*log(c*x^n)/x - a*d^3/x
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^2} \, dx = -\frac{1}{25} \left(be^3n - 5\,be^3 \log(c) - 5\,ae^3\right)x^5 - \frac{1}{3} \left(bde^2n - 3\,bde^2 \log(c) - 3\,ade^2\right)x^3 - 3\left(bd^2en - bd^2e \log(c) - ad^2e\right)x + \frac{1}{5} \left(be^3nx^5 + 5\,bde^2nx^3 + 15\,bd^2enx - \frac{5\,bd^3n}{x}\right) \log(x) - \frac{bd^3n + bd^3 \log(c) + ad^3}{x}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

```
[Out] -1/25*(b*e^3*n - 5*b*e^3*log(c) - 5*a*e^3)*x^5 - 1/3*(b*d*e^2*n - 3*b*d*e^2
*log(c) - 3*a*d*e^2)*x^3 - 3*(b*d^2*e*n - b*d^2*e*log(c) - a*d^2*e)*x + 1/5
*(b*e^3*n*x^5 + 5*b*d*e^2*n*x^3 + 15*b*d^2*e*n*x - 5*b*d^3*n/x)*log(x) - (b
*d^3*n + b*d^3*log(c) + a*d^3)/x
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^2}\,dx = \ln(c\,x^n)\left(\frac{6\,b\,d^2\,e\,x^2+4\,b\,d\,e^2\,x^4+\frac{6\,b\,e^3\,x^6}{5}}{x} - \frac{b\,d^3+3\,b\,d^2\,e\,x^2+3\,b\,d\,e^2\,x^4+b\,e^3\,x^6}{x}\right) - \frac{a\,d^3+b\,d^3\,n}{x} + \frac{e^3\,x^5\,(5\,a-b\,n)}{25} + \frac{d\,e^2\,x^3\,(3\,a-b\,n)}{3} + 3\,d^2\,e\,x\,(a-b\,n)$$

```
[In] int(((d + e*x^2)^3*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] log(c*x^n)*(((6*b*e^3*x^6)/5 + 6*b*d^2*e*x^2 + 4*b*d*e^2*x^4)/x - (b*d^3 +
b*e^3*x^6 + 3*b*d^2*e*x^2 + 3*b*d*e^2*x^4)/x) - (a*d^3 + b*d^3*n)/x + (e^3*
x^5*(5*a - b*n))/25 + (d*e^2*x^3*(3*a - b*n))/3 + 3*d^2*e*x*(a - b*n)
```

3.206 $\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{(d+ex^2)^3\,(a+b\log(cx^n))}{x^4}\,dx = -\frac{bd^3n}{9x^3}-\frac{3bd^2en}{x}-3bde^2nx-\frac{1}{9}be^3nx^3-\frac{d^3(a+b\log(cx^n))}{3x^3}-\frac{3d^2e(a+b\log(cx^n))}{x}+3de^2x(a+b\log(cx^n))+\frac{1}{3}e^3x^3(a+b\log(cx^n))$$

```
[Out] -1/9*b*d^3*n/x^3-3*b*d^2*e*n/x-3*b*d*e^2*n*x-1/9*b*e^3*n*x^3-1/3*d^3*(a+b*l
n(c*x^n))/x^3-3*d^2*e*(a+b*ln(c*x^n))/x+3*d*e^2*x*(a+b*ln(c*x^n))+1/3*e^3*x
^3*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {276, 2372, 12}

$$\int \frac{(d+ex^2)^3\,(a+b\log(cx^n))}{x^4}\,dx = -\frac{d^3(a+b\log(cx^n))}{3x^3}-\frac{3d^2e(a+b\log(cx^n))}{x}+3de^2x(a+b\log(cx^n))+\frac{1}{3}e^3x^3(a+b\log(cx^n))-\frac{bd^3n}{9x^3}-\frac{3bd^2en}{x}-3bde^2nx-\frac{1}{9}be^3nx^3$$

```
[In] Int[((d + e*x^2)^3*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] -1/9*(b*d^3*n)/x^3 - (3*b*d^2*e*n)/x - 3*b*d*e^2*n*x - (b*e^3*n*x^3)/9 - (d
^3*(a + b*Log[c*x^n]))/(3*x^3) - (3*d^2*e*(a + b*Log[c*x^n]))/x + 3*d*e^2*x
*(a + b*Log[c*x^n]) + (e^3*x^3*(a + b*Log[c*x^n]))/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{3x^3}-\frac{3d^2e(a+b\log(cx^n))}{x}+3de^2x(a+b\log(cx^n)) \\
&\quad +\frac{1}{3}e^3x^3(a+b\log(cx^n))-(bn)\int\frac{1}{3}\left(9de^2-\frac{d^3}{x^4}-\frac{9d^2e}{x^2}+e^3x^2\right)\,dx \\
&= -\frac{d^3(a+b\log(cx^n))}{3x^3}-\frac{3d^2e(a+b\log(cx^n))}{x}+3de^2x(a+b\log(cx^n)) \\
&\quad +\frac{1}{3}e^3x^3(a+b\log(cx^n))-\frac{1}{3}(bn)\int\left(9de^2-\frac{d^3}{x^4}-\frac{9d^2e}{x^2}+e^3x^2\right)\,dx \\
&= -\frac{bd^3n}{9x^3}-\frac{3bd^2en}{x}-3bde^2nx-\frac{1}{9}be^3nx^3-\frac{d^3(a+b\log(cx^n))}{3x^3} \\
&\quad -\frac{3d^2e(a+b\log(cx^n))}{x}+3de^2x(a+b\log(cx^n))+\frac{1}{3}e^3x^3(a+b\log(cx^n))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^4} dx = -\frac{3a(d^3+9d^2ex^2-9de^2x^4-e^3x^6)+bn(d^3+27d^2ex^2+27de^2x^4+e^3x^6)+3b(d^3+9d^2ex^2-9de^2x^4-e^3}{9x^3}$$

[In] Integrate[((d + e*x^2)^3*(a + b*Log[c*x^n]))/x^4,x]

[Out] -1/9*(3*a*(d^3 + 9*d^2*e*x^2 - 9*d*e^2*x^4 - e^3*x^6) + b*n*(d^3 + 27*d^2*e *x^2 + 27*d*e^2*x^4 + e^3*x^6) + 3*b*(d^3 + 9*d^2*e*x^2 - 9*d*e^2*x^4 - e^3 *x^6)*Log[c*x^n])/x^3

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.14

method	result
parallelrisch	$-\frac{-3x^6 b\ln(c\,x^n)e^3+b\,e^3 n\,x^6-3x^6 a\,e^3-27x^4 b\ln(c\,x^n)d\,e^2+27bd\,e^2 n\,x^4-27x^4 ad\,e^2+27b\ln(c\,x^n)d^2 e\,x^2+27b\,d^2 en\,x^2+27a\,d^2 e\,x}{9x^3}$
risch	$-\frac{b(-e^3x^6-9e^2d\,x^4+9d^2e\,x^2+d^3)\ln(x^n)}{3x^3}-\frac{-6x^6 a\,e^3-54\ln(c)bd\,e^2x^4+27i\pi bd\,e^2x^4\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)-27ie\pi b\,d}{}$

[In] int((e*x^2+d)^3*(a+b*ln(c*x^n))/x^4,x,method=_RETURNVERBOSE)

[Out] -1/9/x^3*(-3*x^6*b*ln(c*x^n)*e^3+b*e^3*n*x^6-3*x^6*a*e^3-27*x^4*b*ln(c*x^n) *d*e^2+27*b*d*e^2*n*x^4-27*x^4*a*d*e^2+27*b*ln(c*x^n)*d^2*e*x^2+27*b*d^2*e* n*x^2+27*a*d^2*e*x^2+3*b*ln(c*x^n)*d^3+b*d^3*n+3*a*d^3)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^4} dx = -\frac{(be^3n-3\,ae^3)x^6+bd^3n+27\,(bde^2n-ade^2)x^4+3\,ad^3+27\,(bd^2en+ad^2e)x^2-3\,(be^3x^6+9\,bde^2x^4-9}{9\,x^3}$$

[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^4,x, algorithm="fricas")

[Out] -1/9*((b*e^3*n - 3*a*e^3)*x^6 + b*d^3*n + 27*(b*d*e^2*n - a*d*e^2)*x^4 + 3* a*d^3 + 27*(b*d^2*e*n + a*d^2*e)*x^2 - 3*(b*e^3*x^6 + 9*b*d*e^2*x^4 - 9*b*d ^2*e*x^2 - b*d^3)*log(c) - 3*(b*e^3*n*x^6 + 9*b*d*e^2*n*x^4 - 9*b*d^2*e*n*x ^2 - b*d^3*n)*log(x))/x^3

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^4}\,dx = -\frac{ad^3}{3x^3} - \frac{3ad^2e}{x} + 3ade^2x + \frac{ae^3x^3}{3} - \frac{bd^3n}{9x^3} - \frac{bd^3\log(cx^n)}{3x^3} - \frac{3bd^2en}{x} - \frac{3bd^2e\log(cx^n)}{x} - 3bde^2nx + 3bde^2x\log(cx^n) - \frac{be^3nx^3}{9} + \frac{be^3x^3\log(cx^n)}{3}$$

```
[In] integrate((e*x**2+d)**3*(a+b*ln(c*x**n))/x**4,x)
```

```
[Out] -a*d**3/(3*x**3) - 3*a*d**2*e/x + 3*a*d*e**2*x + a*e**3*x**3/3 - b*d**3*n/(
9*x**3) - b*d**3*log(c*x**n)/(3*x**3) - 3*b*d**2*e*n/x - 3*b*d**2*e*log(c*x
**n)/x - 3*b*d*e**2*n*x + 3*b*d*e**2*x*log(c*x**n) - b*e**3*n*x**3/9 + b*e*
*3*x**3*log(c*x**n)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^4}\,dx = -\frac{1}{9}\,be^3nx^3 + \frac{1}{3}\,be^3x^3\log(cx^n) + \frac{1}{3}\,ae^3x^3 - 3\,bde^2nx + 3\,bde^2x\log(cx^n) + 3\,ade^2x - \frac{3\,bd^2en}{x} - \frac{3\,bd^2e\log(cx^n)}{x} - \frac{3\,ad^2e}{x} - \frac{bd^3n}{9\,x^3} - \frac{bd^3\log(cx^n)}{3\,x^3} - \frac{ad^3}{3\,x^3}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^4,x, algorithm="maxima")
```

```
[Out] -1/9*b*e^3*n*x^3 + 1/3*b*e^3*x^3*log(c*x^n) + 1/3*a*e^3*x^3 - 3*b*d*e^2*n*x
 + 3*b*d*e^2*x*log(c*x^n) + 3*a*d*e^2*x - 3*b*d^2*e*n/x - 3*b*d^2*e*log(c*x
^n)/x - 3*a*d^2*e/x - 1/9*b*d^3*n/x^3 - 1/3*b*d^3*log(c*x^n)/x^3 - 1/3*a*d^
3/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^4}\,dx$$
$$= -\frac{1}{9}\left(be^3n-3\,be^3\log(c)-3\,ae^3\right)x^3-3\left(bde^2n-bde^2\log(c)-ade^2\right)x + \frac{1}{3}\left(be^3nx^3+9\,bde^2nx-\frac{9\,bd^2enx^2+bd^3n}{x^3}\right)\log(x) - \frac{27\,bd^2enx^2+27\,bd^2ex^2\log(c)+27\,ad^2ex^2+bd^3n+3\,bd^3\log(c)+3\,ad^3}{9\,x^3}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^4,x, algorithm="giac")
```

```
[Out] -1/9*(b*e^3*n - 3*b*e^3*log(c) - 3*a*e^3)*x^3 - 3*(b*d*e^2*n - b*d*e^2*log(
c) - a*d*e^2)*x + 1/3*(b*e^3*n*x^3 + 9*b*d*e^2*n*x - (9*b*d^2*e*n*x^2 + b*d
^3*n)/x^3)*log(x) - 1/9*(27*b*d^2*e*n*x^2 + 27*b*d^2*e*x^2*log(c) + 27*a*d^
2*e*x^2 + b*d^3*n + 3*b*d^3*log(c) + 3*a*d^3)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^4}\,dx = \ln(c\,x^n)\left(\frac{\frac{8\,b\,e^3\,x^6}{3}+8\,b\,d\,e^2\,x^4}{x^3} - \frac{\frac{b\,d^3}{3}+3\,b\,d^2\,e\,x^2+5\,b\,d\,e^2\,x^4+\frac{7\,b\,e^3\,x^6}{3}}{x^3}\right) - \frac{a\,d^3+x^2\,(9\,a\,d^2\,e+9\,b\,d^2\,e\,n)+\frac{b\,d^3\,n}{3}}{3\,x^3} + \frac{e^3\,x^3\,(3\,a-b\,n)}{9} + 3\,d\,e^2\,x\,(a-b\,n)$$

```
[In] int(((d + e*x^2)^3*(a + b*log(c*x^n)))/x^4,x)
```

```
[Out] log(c*x^n)*(((8*b*e^3*x^6)/3 + 8*b*d*e^2*x^4)/x^3 - ((b*d^3)/3 + (7*b*e^3*x
^6)/3 + 3*b*d^2*e*x^2 + 5*b*d*e^2*x^4)/x^3) - (a*d^3 + x^2*(9*a*d^2*e + 9*b
*d^2*e*n) + (b*d^3*n)/3)/(3*x^3) + (e^3*x^3*(3*a - b*n))/9 + 3*d*e^2*x*(a -
 b*n)
```

3.207 $\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^6} dx$

Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^6} dx = -\frac{bd^3n}{25x^5} - \frac{bd^2en}{3x^3} - \frac{3bde^2n}{x} - be^3nx - \frac{d^3(a+b\log(cx^n))}{5x^5} - \frac{d^2e(a+b\log(cx^n))}{x^3} - \frac{3de^2(a+b\log(cx^n))}{x} + e^3x(a+b\log(cx^n))$$

```
[Out] -1/25*b*d^3*n/x^5-1/3*b*d^2*e*n/x^3-3*b*d*e^2*n/x-b*e^3*n*x-1/5*d^3*(a+b*ln
(c*x^n))/x^5-d^2*e*(a+b*ln(c*x^n))/x^3-3*d*e^2*(a+b*ln(c*x^n))/x+e^3*x*(a+b
*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {276, 2372}

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^6} dx = -\frac{d^3(a+b\log(cx^n))}{5x^5} - \frac{d^2e(a+b\log(cx^n))}{x^3} - \frac{3de^2(a+b\log(cx^n))}{x} + e^3x(a+b\log(cx^n)) - \frac{bd^3n}{25x^5} - \frac{bd^2en}{3x^3} - \frac{3bde^2n}{x} - be^3nx$$

```
[In] Int[((d + e*x^2)^3*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] -1/25*(b*d^3*n)/x^5 - (b*d^2*e*n)/(3*x^3) - (3*b*d*e^2*n)/x - b*e^3*n*x - (
d^3*(a + b*Log[c*x^n]))/(5*x^5) - (d^2*e*(a + b*Log[c*x^n]))/x^3 - (3*d*e^2
*(a + b*Log[c*x^n]))/x + e^3*x*(a + b*Log[c*x^n])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{5x^5}-\frac{d^2e(a+b\log(cx^n))}{x^3}-\frac{3de^2(a+b\log(cx^n))}{x}\\
&\quad+e^3x(a+b\log(cx^n))-(bn)\int\left(e^3-\frac{d^3}{5x^6}-\frac{d^2e}{x^4}-\frac{3de^2}{x^2}\right)dx\\
&= -\frac{bd^3n}{25x^5}-\frac{bd^2en}{3x^3}-\frac{3bde^2n}{x}-be^3nx-\frac{d^3(a+b\log(cx^n))}{5x^5}\\
&\quad-\frac{d^2e(a+b\log(cx^n))}{x^3}-\frac{3de^2(a+b\log(cx^n))}{x}+e^3x(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int\frac{(d+ex^2)^3(a+b\log(cx^n))}{x^6}\,dx=$$
$$-\frac{15a(d^3+5d^2ex^2+15de^2x^4-5e^3x^6)+bn(3d^3+25d^2ex^2+225de^2x^4+75e^3x^6)+15b(d^3+5d^2ex^2+15}{75x^5}$$

```
[In] Integrate[((d + e*x^2)^3*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] -1/75*(15*a*(d^3 + 5*d^2*e*x^2 + 15*d*e^2*x^4 - 5*e^3*x^6) + b*n*(3*d^3 + 2
5*d^2*e*x^2 + 225*d*e^2*x^4 + 75*e^3*x^6) + 15*b*(d^3 + 5*d^2*e*x^2 + 15*d*
e^2*x^4 - 5*e^3*x^6)*Log[c*x^n])/x^5
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.19

method	result
parallelrisch	$-\frac{-75x^{6}b\ln(c\,x^{n})e^{3}+75b\,e^{3}n\,x^{6}-75x^{6}a\,e^{3}+225x^{4}b\ln(c\,x^{n})d\,e^{2}+225bd\,e^{2}n\,x^{4}+225x^{4}ad\,e^{2}+75b\ln(c\,x^{n})d^{2}e\,x^{2}+25b\,d^{2}en\,x^{2}}{75x^{5}}$
risch	$-\frac{b(-5e^{3}x^{6}+15e^{2}d\,x^{4}+5d^{2}e\,x^{2}+d^{3})\ln(x^{n})}{5x^{5}}-\frac{-150x^{6}a\,e^{3}+450\ln(c)bd\,e^{2}x^{4}-225i\pi bd\,e^{2}x^{4}\operatorname{csgn}(ic)\operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})-}{}$

```
[In] int((e*x^2+d)^3*(a+b*ln(c*x^n))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/75/x^5*(-75*x^6*b*ln(c*x^n)*e^3+75*b*e^3*n*x^6-75*x^6*a*e^3+225*x^4*b*ln
(c*x^n)*d*e^2+225*b*d*e^2*n*x^4+225*x^4*a*d*e^2+75*b*ln(c*x^n)*d^2*e*x^2+25
*b*d^2*e*n*x^2+75*a*d^2*e*x^2+15*b*ln(c*x^n)*d^3+3*b*d^3*n+15*a*d^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex^2)^3\,(a+b\log{(cx^n)})}{x^6}\,dx = -\frac{75\,(be^3n-ae^3)x^6+3\,bd^3n+225\,(bde^2n+ade^2)x^4+15\,ad^3+25\,(bd^2en+3\,ad^2e)x^2-15\,(5\,be^3x^6-15}{75\,x^5}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^6,x, algorithm="fricas")
```

```
[Out] -1/75*(75*(b*e^3*n - a*e^3)*x^6 + 3*b*d^3*n + 225*(b*d*e^2*n + a*d*e^2)*x^4
 + 15*a*d^3 + 25*(b*d^2*e*n + 3*a*d^2*e)*x^2 - 15*(5*b*e^3*x^6 - 15*b*d*e^2
*x^4 - 5*b*d^2*e*x^2 - b*d^3)*log(c) - 15*(5*b*e^3*n*x^6 - 15*b*d*e^2*n*x^4
 - 5*b*d^2*e*n*x^2 - b*d^3*n)*log(x))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex^2)^3\,(a+b\log{(cx^n)})}{x^6}\,dx = -\frac{ad^3}{5x^5}-\frac{ad^2e}{x^3}-\frac{3ade^2}{x}+ae^3x-\frac{bd^3n}{25x^5} - \frac{bd^3\log{(cx^n)}}{5x^5}-\frac{bd^2en}{3x^3}-\frac{bd^2e\log{(cx^n)}}{x^3}-\frac{3bde^2n}{x} - \frac{3bde^2\log{(cx^n)}}{x}-be^3nx+be^3x\log{(cx^n)}$$

```
[In] integrate((e*x**2+d)**3*(a+b*ln(c*x**n))/x**6,x)
```

```
[Out] -a*d**3/(5*x**5) - a*d**2*e/x**3 - 3*a*d*e**2/x + a*e**3*x - b*d**3*n/(25*x
**5) - b*d**3*log(c*x**n)/(5*x**5) - b*d**2*e*n/(3*x**3) - b*d**2*e*log(c*x
**n)/x**3 - 3*b*d*e**2*n/x - 3*b*d*e**2*log(c*x**n)/x - b*e**3*n*x + b*e**3
*x*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex^2)^3\,(a+b\log(cx^n))}{x^6}\,dx = -be^3nx + be^3x\log(cx^n) + ae^3x - \frac{3\,bde^2n}{x} - \frac{3\,bde^2\log(cx^n)}{x} - \frac{3\,ade^2}{x} - \frac{bd^2en}{3\,x^3} - \frac{bd^2e\log(cx^n)}{x^3} - \frac{ad^2e}{x^3} - \frac{bd^3n}{25\,x^5} - \frac{bd^3\log(cx^n)}{5\,x^5} - \frac{ad^3}{5\,x^5}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^6,x, algorithm="maxima")
```

```
[Out] -b*e^3*n*x + b*e^3*x*log(c*x^n) + a*e^3*x - 3*b*d*e^2*n/x - 3*b*d*e^2*log(c
*x^n)/x - 3*a*d*e^2/x - 1/3*b*d^2*e*n/x^3 - b*d^2*e*log(c*x^n)/x^3 - a*d^2*
e/x^3 - 1/25*b*d^3*n/x^5 - 1/5*b*d^3*log(c*x^n)/x^5 - 1/5*a*d^3/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex^2)^3\,(a+b\log(cx^n))}{x^6}\,dx = -\left(be^3n - be^3\log(c) - ae^3\right)x + \frac{1}{5}\left(5\,be^3nx - \frac{15\,bde^2nx^4 + 5\,bd^2enx^2 + bd^3n}{x^5}\right)\log(x) - \frac{225\,bde^2nx^4 + 225\,bde^2x^4\log(c) + 225\,ade^2x^4 + 25\,bd^2enx^2 + 75\,bd^2ex^2\log(c) + 75\,ad^2ex^2 + 3\,bd^3n + 1}{75\,x^5}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^6,x, algorithm="giac")
```

```
[Out] -(b*e^3*n - b*e^3*log(c) - a*e^3)*x + 1/5*(5*b*e^3*n*x - (15*b*d*e^2*n*x^4
+ 5*b*d^2*e*n*x^2 + b*d^3*n)/x^5)*log(x) - 1/75*(225*b*d*e^2*n*x^4 + 225*b*
d*e^2*x^4*log(c) + 225*a*d*e^2*x^4 + 25*b*d^2*e*n*x^2 + 75*b*d^2*e*x^2*log(
c) + 75*a*d^2*e*x^2 + 3*b*d^3*n + 15*b*d^3*log(c) + 15*a*d^3)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^6} dx$$
$$= e^3 x (a-b n) - \frac{a d^3 + x^2 \left(5 a d^2 e + \frac{5 b d^2 e n}{3}\right) + x^4 (15 a d e^2 + 15 b d e^2 n) + \frac{b d^3 n}{5}}{5 x^5}$$
$$- \ln(c x^n) \left(\frac{\frac{b d^3}{5} + b d^2 e x^2 + 3 b d e^2 x^4 + \frac{11 b e^3 x^6}{5}}{x^5} - \frac{16 b e^3 x}{5}\right)$$

```
[In] int(((d + e*x^2)^3*(a + b*log(c*x^n)))/x^6,x)
```

```
[Out] e^3*x*(a - b*n) - (a*d^3 + x^2*(5*a*d^2*e + (5*b*d^2*e*n)/3) + x^4*(15*a*d*
e^2 + 15*b*d*e^2*n) + (b*d^3*n)/5)/(5*x^5) - log(c*x^n)*(((b*d^3)/5 + (11*b
*e^3*x^6)/5 + b*d^2*e*x^2 + 3*b*d*e^2*x^4)/x^5 - (16*b*e^3*x)/5)
```

3.208 $\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^8} dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^8} dx = -\frac{bd^3n}{49x^7} - \frac{3bd^2en}{25x^5} - \frac{bde^2n}{3x^3} - \frac{be^3n}{x} - \frac{d^3(a+b\log(cx^n))}{7x^7} - \frac{3d^2e(a+b\log(cx^n))}{5x^5} - \frac{de^2(a+b\log(cx^n))}{x^3} - \frac{e^3(a+b\log(cx^n))}{x}$$

```
[Out] -1/49*b*d^3*n/x^7-3/25*b*d^2*e*n/x^5-1/3*b*d*e^2*n/x^3-b*e^3*n/x-1/7*d^3*(a
+b*ln(c*x^n))/x^7-3/5*d^2*e*(a+b*ln(c*x^n))/x^5-d*e^2*(a+b*ln(c*x^n))/x^3-e
^3*(a+b*ln(c*x^n))/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2372, 12, 14}

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^8} dx = -\frac{d^3(a+b\log(cx^n))}{7x^7} - \frac{3d^2e(a+b\log(cx^n))}{5x^5} - \frac{de^2(a+b\log(cx^n))}{x^3} - \frac{e^3(a+b\log(cx^n))}{x} - \frac{bd^3n}{49x^7} - \frac{3bd^2en}{25x^5} - \frac{bde^2n}{3x^3} - \frac{be^3n}{x}$$

```
[In] Int[((d + e*x^2)^3*(a + b*Log[c*x^n]))/x^8,x]
```

```
[Out] -1/49*(b*d^3*n)/x^7 - (3*b*d^2*e*n)/(25*x^5) - (b*d*e^2*n)/(3*x^3) - (b*e^3
*n)/x - (d^3*(a + b*Log[c*x^n]))/(7*x^7) - (3*d^2*e*(a + b*Log[c*x^n]))/(5*
x^5) - (d*e^2*(a + b*Log[c*x^n]))/x^3 - (e^3*(a + b*Log[c*x^n]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{7x^7}-\frac{3d^2e(a+b\log(cx^n))}{5x^5}-\frac{de^2(a+b\log(cx^n))}{x^3} \\
&\quad -\frac{e^3(a+b\log(cx^n))}{x}-(bn)\int\frac{-5d^3-21d^2ex^2-35de^2x^4-35e^3x^6}{35x^8}\,dx \\
&= -\frac{d^3(a+b\log(cx^n))}{7x^7}-\frac{3d^2e(a+b\log(cx^n))}{5x^5}-\frac{de^2(a+b\log(cx^n))}{x^3} \\
&\quad -\frac{e^3(a+b\log(cx^n))}{x}-\frac{1}{35}(bn)\int\frac{-5d^3-21d^2ex^2-35de^2x^4-35e^3x^6}{x^8}\,dx \\
&= -\frac{d^3(a+b\log(cx^n))}{7x^7}-\frac{3d^2e(a+b\log(cx^n))}{5x^5}-\frac{de^2(a+b\log(cx^n))}{x^3} \\
&\quad -\frac{e^3(a+b\log(cx^n))}{x}-\frac{1}{35}(bn)\int\left(-\frac{5d^3}{x^8}-\frac{21d^2e}{x^6}-\frac{35de^2}{x^4}-\frac{35e^3}{x^2}\right)dx
\end{aligned}$$

$$= -\frac{bd^3n}{49x^7} - \frac{3bd^2en}{25x^5} - \frac{bde^2n}{3x^3} - \frac{be^3n}{x} - \frac{d^3(a+b\log(cx^n))}{7x^7} - \frac{3d^2e(a+b\log(cx^n))}{5x^5} - \frac{de^2(a+b\log(cx^n))}{x^3} - \frac{e^3(a+b\log(cx^n))}{x}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^8}\,dx = -\frac{bd^3n}{49x^7} - \frac{3bd^2en}{25x^5} - \frac{bde^2n}{3x^3} - \frac{be^3n}{x} - \frac{d^3(a+b\log(cx^n))}{7x^7} - \frac{3d^2e(a+b\log(cx^n))}{5x^5} - \frac{de^2(a+b\log(cx^n))}{x^3} - \frac{e^3(a+b\log(cx^n))}{x}$$

```
[In] Integrate[((d + e*x^2)^3*(a + b*Log[c*x^n]))/x^8,x]
```

```
[Out] -1/49*(b*d^3*n)/x^7 - (3*b*d^2*e*n)/(25*x^5) - (b*d*e^2*n)/(3*x^3) - (b*e^3
*n)/x - (d^3*(a + b*Log[c*x^n]))/(7*x^7) - (3*d^2*e*(a + b*Log[c*x^n]))/(5*
x^5) - (d*e^2*(a + b*Log[c*x^n]))/x^3 - (e^3*(a + b*Log[c*x^n]))/x
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.10

method	result
parallelrisch	$-\frac{3675x^6b\ln(cx^n)e^3+3675be^3nx^6+3675x^6ae^3+3675x^4b\ln(cx^n)de^2+1225bde^2nx^4+3675x^4ade^2+2205b\ln(cx^n)d^2ex^2+44}{3675x^7}$
risch	$-\frac{b(35e^3x^6+35e^2dx^4+21d^2ex^2+5d^3)\ln(x^n)}{35x^7} - \frac{7350x^6ae^3+7350\ln(c)bde^2x^4-3675i\pi bde^2x^4\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)}{}$

```
[In] int((e*x^2+d)^3*(a+b*ln(c*x^n))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3675/x^7*(3675*x^6*b*ln(c*x^n)*e^3+3675*b*e^3*n*x^6+3675*x^6*a*e^3+3675*
x^4*b*ln(c*x^n)*d*e^2+1225*b*d*e^2*n*x^4+3675*x^4*a*d*e^2+2205*b*ln(c*x^n)*
d^2*e*x^2+441*b*d^2*e*n*x^2+2205*a*d^2*e*x^2+525*b*ln(c*x^n)*d^3+75*b*d^3*n
+525*a*d^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^8}\,dx = -\frac{3675\,(be^3n+ae^3)x^6+75\,bd^3n+1225\,(bde^2n+3\,ade^2)x^4+525\,ad^3+441\,(bd^2en+5\,ad^2e)x^2+105\,(35}{3}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^8,x, algorithm="fricas")
```

```
[Out] -1/3675*(3675*(b*e^3*n + a*e^3)*x^6 + 75*b*d^3*n + 1225*(b*d*e^2*n + 3*a*d*
e^2)*x^4 + 525*a*d^3 + 441*(b*d^2*e*n + 5*a*d^2*e)*x^2 + 105*(35*b*e^3*x^6
+ 35*b*d*e^2*x^4 + 21*b*d^2*e*x^2 + 5*b*d^3)*log(c) + 105*(35*b*e^3*n*x^6 +
 35*b*d*e^2*n*x^4 + 21*b*d^2*e*n*x^2 + 5*b*d^3*n)*log(x))/x^7
```

Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^8}\,dx = -\frac{ad^3}{7x^7} - \frac{3ad^2e}{5x^5} - \frac{ade^2}{x^3} - \frac{ae^3}{x} - \frac{bd^3n}{49x^7} - \frac{bd^3\log(cx^n)}{7x^7} - \frac{3bd^2en}{25x^5} - \frac{3bd^2e\log(cx^n)}{5x^5} - \frac{bde^2n}{3x^3} - \frac{bde^2\log(cx^n)}{x^3} - \frac{be^3n}{x} - \frac{be^3\log(cx^n)}{x}$$

```
[In] integrate((e*x**2+d)**3*(a+b*ln(c*x**n))/x**8,x)
```

```
[Out] -a*d**3/(7*x**7) - 3*a*d**2*e/(5*x**5) - a*d*e**2/x**3 - a*e**3/x - b*d**3*
n/(49*x**7) - b*d**3*log(c*x**n)/(7*x**7) - 3*b*d**2*e*n/(25*x**5) - 3*b*d*
*2*e*log(c*x**n)/(5*x**5) - b*d*e**2*n/(3*x**3) - b*d*e**2*log(c*x**n)/x**3
 - b*e**3*n/x - b*e**3*log(c*x**n)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^8}\,dx = -\frac{be^3n}{x} - \frac{be^3\log(cx^n)}{x} - \frac{ae^3}{x} - \frac{bde^2n}{3\,x^3} - \frac{bde^2\log(cx^n)}{x^3} - \frac{ade^2}{x^3} - \frac{3\,bd^2en}{25\,x^5} - \frac{3\,bd^2e\log(cx^n)}{5\,x^5} - \frac{3\,ad^2e}{5\,x^5} - \frac{bd^3n}{49\,x^7} - \frac{bd^3\log(cx^n)}{7\,x^7} - \frac{ad^3}{7\,x^7}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^8,x, algorithm="maxima")
```

```
[Out] -b*e^3*n/x - b*e^3*log(c*x^n)/x - a*e^3/x - 1/3*b*d*e^2*n/x^3 - b*d*e^2*log
(c*x^n)/x^3 - a*d*e^2/x^3 - 3/25*b*d^2*e*n/x^5 - 3/5*b*d^2*e*log(c*x^n)/x^5
 - 3/5*a*d^2*e/x^5 - 1/49*b*d^3*n/x^7 - 1/7*b*d^3*log(c*x^n)/x^7 - 1/7*a*d^
3/x^7
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^8} dx = -\frac{(35\,be^3nx^6+35\,bde^2nx^4+21\,bd^2enx^2+5\,bd^3n)\log(x)}{35\,x^7} - \frac{3675\,be^3nx^6+3675\,be^3x^6\log(c)+3675\,ae^3x^6+1225\,bde^2nx^4+3675\,bde^2x^4\log(c)+3675\,ade^2x^4+441}{3675\,x^7}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^8,x, algorithm="giac")
```

```
[Out] -1/35*(35*b*e^3*n*x^6 + 35*b*d*e^2*n*x^4 + 21*b*d^2*e*n*x^2 + 5*b*d^3*n)*lo
g(x)/x^7 - 1/3675*(3675*b*e^3*n*x^6 + 3675*b*e^3*x^6*log(c) + 3675*a*e^3*x^
6 + 1225*b*d*e^2*n*x^4 + 3675*b*d*e^2*x^4*log(c) + 3675*a*d*e^2*x^4 + 441*b
*d^2*e*n*x^2 + 2205*b*d^2*e*x^2*log(c) + 2205*a*d^2*e*x^2 + 75*b*d^3*n + 52
5*b*d^3*log(c) + 525*a*d^3)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^8} dx = -\frac{x^6\,(35\,a\,e^3+35\,b\,e^3\,n)+5\,a\,d^3+x^2\left(21\,a\,d^2\,e+\frac{21\,b\,d^2\,e\,n}{5}\right)+x^4\left(35\,a\,d\,e^2+\frac{35\,b\,d\,e^2\,n}{3}\right)+\frac{5\,b\,d^3\,n}{7}}{35\,x^7} - \frac{\ln(c\,x^n)\,\left(\frac{b\,d^3}{7}+\frac{3\,b\,d^2\,e\,x^2}{5}+b\,d\,e^2\,x^4+b\,e^3\,x^6\right)}{x^7}$$

```
[In] int(((d + e*x^2)^3*(a + b*log(c*x^n)))/x^8,x)
```

```
[Out] - (x^6*(35*a*e^3 + 35*b*e^3*n) + 5*a*d^3 + x^2*(21*a*d^2*e + (21*b*d^2*e*n)
/5) + x^4*(35*a*d*e^2 + (35*b*d*e^2*n)/3) + (5*b*d^3*n)/7)/(35*x^7) - (log(
c*x^n)*((b*d^3)/7 + b*e^3*x^6 + (3*b*d^2*e*x^2)/5 + b*d*e^2*x^4))/x^7
```

3.209 $\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^{10}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 133

$$\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^{10}}\,dx = -\frac{bd^3n}{81x^9}-\frac{3bd^2en}{49x^7}-\frac{3bde^2n}{25x^5}-\frac{be^3n}{9x^3} -\frac{d^3(a+b\log(cx^n))}{9x^9}-\frac{3d^2e(a+b\log(cx^n))}{7x^7} -\frac{3de^2(a+b\log(cx^n))}{5x^5}-\frac{e^3(a+b\log(cx^n))}{3x^3}$$

```
[Out] -1/81*b*d^3*n/x^9-3/49*b*d^2*e*n/x^7-3/25*b*d*e^2*n/x^5-1/9*b*e^3*n/x^3-1/9
*d^3*(a+b*ln(c*x^n))/x^9-3/7*d^2*e*(a+b*ln(c*x^n))/x^7-3/5*d*e^2*(a+b*ln(c*
x^n))/x^5-1/3*e^3*(a+b*ln(c*x^n))/x^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2372, 12, 14}

$$\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^{10}}\,dx = -\frac{d^3(a+b\log(cx^n))}{9x^9}-\frac{3d^2e(a+b\log(cx^n))}{7x^7} -\frac{3de^2(a+b\log(cx^n))}{5x^5}-\frac{e^3(a+b\log(cx^n))}{3x^3} -\frac{bd^3n}{81x^9}-\frac{3bd^2en}{49x^7}-\frac{3bde^2n}{25x^5}-\frac{be^3n}{9x^3}$$

```
[In] Int[((d + e*x^2)^3*(a + b*Log[c*x^n]))/x^10,x]
```

```
[Out] -1/81*(b*d^3*n)/x^9 - (3*b*d^2*e*n)/(49*x^7) - (3*b*d*e^2*n)/(25*x^5) - (b*
e^3*n)/(9*x^3) - (d^3*(a + b*Log[c*x^n]))/(9*x^9) - (3*d^2*e*(a + b*Log[c*x
^n]))/(7*x^7) - (3*d*e^2*(a + b*Log[c*x^n]))/(5*x^5) - (e^3*(a + b*Log[c*x^
n]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{9x^9}-\frac{3d^2e(a+b\log(cx^n))}{7x^7}-\frac{3de^2(a+b\log(cx^n))}{5x^5}\\
&\quad-\frac{e^3(a+b\log(cx^n))}{3x^3}-(bn)\int\frac{-35d^3-135d^2ex^2-189de^2x^4-105e^3x^6}{315x^{10}}\,dx\\
&= -\frac{d^3(a+b\log(cx^n))}{9x^9}-\frac{3d^2e(a+b\log(cx^n))}{7x^7}-\frac{3de^2(a+b\log(cx^n))}{5x^5}\\
&\quad-\frac{e^3(a+b\log(cx^n))}{3x^3}-\frac{1}{315}(bn)\int\frac{-35d^3-135d^2ex^2-189de^2x^4-105e^3x^6}{x^{10}}\,dx\\
&= -\frac{d^3(a+b\log(cx^n))}{9x^9}-\frac{3d^2e(a+b\log(cx^n))}{7x^7}-\frac{3de^2(a+b\log(cx^n))}{5x^5}\\
&\quad-\frac{e^3(a+b\log(cx^n))}{3x^3}-\frac{1}{315}(bn)\int\left(-\frac{35d^3}{x^{10}}-\frac{135d^2e}{x^8}-\frac{189de^2}{x^6}-\frac{105e^3}{x^4}\right)dx
\end{aligned}$$

$$= -\frac{bd^3n}{81x^9} - \frac{3bd^2en}{49x^7} - \frac{3bde^2n}{25x^5} - \frac{be^3n}{9x^3} - \frac{d^3(a+b\log(cx^n))}{9x^9} - \frac{3d^2e(a+b\log(cx^n))}{7x^7} - \frac{3de^2(a+b\log(cx^n))}{5x^5} - \frac{e^3(a+b\log(cx^n))}{3x^3}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^3(a+b\log(cx^n))}{x^{10}}\,dx = -\frac{bd^3n}{81x^9} - \frac{3bd^2en}{49x^7} - \frac{3bde^2n}{25x^5} - \frac{be^3n}{9x^3} - \frac{d^3(a+b\log(cx^n))}{9x^9} - \frac{3d^2e(a+b\log(cx^n))}{7x^7} - \frac{3de^2(a+b\log(cx^n))}{5x^5} - \frac{e^3(a+b\log(cx^n))}{3x^3}$$

```
[In] Integrate[((d + e*x^2)^3*(a + b*Log[c*x^n]))/x^10,x]
```

```
[Out] -1/81*(b*d^3*n)/x^9 - (3*b*d^2*e*n)/(49*x^7) - (3*b*d*e^2*n)/(25*x^5) - (b*
e^3*n)/(9*x^3) - (d^3*(a + b*Log[c*x^n]))/(9*x^9) - (3*d^2*e*(a + b*Log[c*x
^n]))/(7*x^7) - (3*d*e^2*(a + b*Log[c*x^n]))/(5*x^5) - (e^3*(a + b*Log[c*x^
n]))/(3*x^3)
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.05

method	result
parallelrisch	$-\frac{33075x^6b\ln(cx^n)e^3+11025be^3nx^6+33075x^6ae^3+59535x^4b\ln(cx^n)de^2+11907bde^2nx^4+59535x^4ade^2+42525b\ln(cx^n)d}{99225x^9}$
risch	$-\frac{b(105e^3x^6+189e^2dx^4+135d^2ex^2+35d^3)\ln(x^n)}{315x^9} - \frac{66150x^6ae^3+119070\ln(c)bde^2x^4-59535i\pi bde^2x^4\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)}{}$

```
[In] int((e*x^2+d)^3*(a+b*ln(c*x^n))/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/99225/x^9*(33075*x^6*b*ln(c*x^n)*e^3+11025*b*e^3*n*x^6+33075*x^6*a*e^3+5
9535*x^4*b*ln(c*x^n)*d*e^2+11907*b*d*e^2*n*x^4+59535*x^4*a*d*e^2+42525*b*ln
(c*x^n)*d^2*e*x^2+6075*b*d^2*e*n*x^2+42525*a*d^2*e*x^2+11025*b*ln(c*x^n)*d^
3+1225*b*d^3*n+11025*a*d^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^{10}}\,dx = -\frac{11025\,(be^3n+3\,ae^3)x^6+1225\,bd^3n+11907\,(bde^2n+5\,ade^2)x^4+11025\,ad^3+6075\,(bd^2en+7\,ad^2e)x^2 \cdots}{}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^10,x, algorithm="fricas")
```

```
[Out] -1/99225*(11025*(b*e^3*n + 3*a*e^3)*x^6 + 1225*b*d^3*n + 11907*(b*d*e^2*n +
 5*a*d*e^2)*x^4 + 11025*a*d^3 + 6075*(b*d^2*e*n + 7*a*d^2*e)*x^2 + 315*(105
*b*e^3*x^6 + 189*b*d*e^2*x^4 + 135*b*d^2*e*x^2 + 35*b*d^3)*log(c) + 315*(10
5*b*e^3*n*x^6 + 189*b*d*e^2*n*x^4 + 135*b*d^2*e*n*x^2 + 35*b*d^3*n)*log(x))
/x^9
```

Sympy [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^{10}}\,dx = -\frac{ad^3}{9x^9} - \frac{3ad^2e}{7x^7} - \frac{3ade^2}{5x^5} - \frac{ae^3}{3x^3} - \frac{bd^3n}{81x^9} - \frac{bd^3\log(cx^n)}{9x^9} - \frac{3bd^2en}{49x^7} - \frac{3bd^2e\log(cx^n)}{7x^7} - \frac{3bde^2n}{25x^5} - \frac{3bde^2\log(cx^n)}{5x^5} - \frac{be^3n}{9x^3} - \frac{be^3\log(cx^n)}{3x^3}$$

```
[In] integrate((e*x**2+d)**3*(a+b*ln(c*x**n))/x**10,x)
```

```
[Out] -a*d**3/(9*x**9) - 3*a*d**2*e/(7*x**7) - 3*a*d*e**2/(5*x**5) - a*e**3/(3*x*
*3) - b*d**3*n/(81*x**9) - b*d**3*log(c*x**n)/(9*x**9) - 3*b*d**2*e*n/(49*x
**7) - 3*b*d**2*e*log(c*x**n)/(7*x**7) - 3*b*d*e**2*n/(25*x**5) - 3*b*d*e**
2*log(c*x**n)/(5*x**5) - b*e**3*n/(9*x**3) - b*e**3*log(c*x**n)/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^{10}}\,dx = -\frac{be^3n}{9\,x^3} - \frac{be^3\log(cx^n)}{3\,x^3} - \frac{ae^3}{3\,x^3} - \frac{3\,bde^2n}{25\,x^5} - \frac{3\,bde^2\log(cx^n)}{5\,x^5} - \frac{3\,ade^2}{5\,x^5} - \frac{3\,bd^2en}{49\,x^7} - \frac{3\,bd^2e\log(cx^n)}{7\,x^7} - \frac{3\,ad^2e}{7\,x^7} - \frac{bd^3n}{81\,x^9} - \frac{bd^3\log(cx^n)}{9\,x^9} - \frac{ad^3}{9\,x^9}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^10,x, algorithm="maxima")
```

```
[Out] -1/9*b*e^3*n/x^3 - 1/3*b*e^3*log(c*x^n)/x^3 - 1/3*a*e^3/x^3 - 3/25*b*d*e^2*
n/x^5 - 3/5*b*d*e^2*log(c*x^n)/x^5 - 3/5*a*d*e^2/x^5 - 3/49*b*d^2*e*n/x^7 -
 3/7*b*d^2*e*log(c*x^n)/x^7 - 3/7*a*d^2*e/x^7 - 1/81*b*d^3*n/x^9 - 1/9*b*d^
3*log(c*x^n)/x^9 - 1/9*a*d^3/x^9
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^{10}}\,dx$$
$$= -\frac{(105\,be^3nx^6 + 189\,bde^2nx^4 + 135\,bd^2enx^2 + 35\,bd^3n)\log(x)}{315\,x^9} - \frac{11025\,be^3nx^6 + 33075\,be^3x^6\log(c) + 33075\,ae^3x^6 + 11907\,bde^2nx^4 + 59535\,bde^2x^4\log(c) + 59535\,ade^2 \cdots}{99225 \cdots}$$

```
[In] integrate((e*x^2+d)^3*(a+b*log(c*x^n))/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(105*b*e^3*n*x^6 + 189*b*d*e^2*n*x^4 + 135*b*d^2*e*n*x^2 + 35*b*d^3*
n)*log(x)/x^9 - 1/99225*(11025*b*e^3*n*x^6 + 33075*b*e^3*x^6*log(c) + 33075
*a*e^3*x^6 + 11907*b*d*e^2*n*x^4 + 59535*b*d*e^2*x^4*log(c) + 59535*a*d*e^2
*x^4 + 6075*b*d^2*e*n*x^2 + 42525*b*d^2*e*x^2*log(c) + 42525*a*d^2*e*x^2 +
1225*b*d^3*n + 11025*b*d^3*log(c) + 11025*a*d^3)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex^2)^3 (a+b\log(cx^n))}{x^{10}}\,dx = -\frac{x^6\,(105\,a\,e^3+35\,b\,e^3\,n)+35\,a\,d^3+x^2\,\left(135\,a\,d^2\,e+\frac{135\,b\,d^2\,e\,n}{7}\right)+x^4\,\left(189\,a\,d\,e^2+\frac{189\,b\,d\,e^2\,n}{5}\right)+\frac{35\,b\,d^3\,n}{9}}{315\,x^9} - \frac{\ln(c\,x^n)\,\left(\frac{b\,d^3}{9}+\frac{3\,b\,d^2\,e\,x^2}{7}+\frac{3\,b\,d\,e^2\,x^4}{5}+\frac{b\,e^3\,x^6}{3}\right)}{x^9}$$

```
[In] int(((d + e*x^2)^3*(a + b*log(c*x^n)))/x^10,x)

[Out] - (x^6*(105*a*e^3 + 35*b*e^3*n) + 35*a*d^3 + x^2*(135*a*d^2*e + (135*b*d^2*
e*n)/7) + x^4*(189*a*d*e^2 + (189*b*d*e^2*n)/5) + (35*b*d^3*n)/9)/(315*x^9)
 - (log(c*x^n)*((b*d^3)/9 + (b*e^3*x^6)/3 + (3*b*d^2*e*x^2)/7 + (3*b*d*e^2*
x^4)/5))/x^9
```

3.210 $\int \frac{x^5(a+b\log(cx^n))}{d+ex^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{x^5(a+b\log(cx^n))}{d+ex^2}\,dx = \frac{bdnx^2}{4e^2}-\frac{bnx^4}{16e}-\frac{dx^2(a+b\log(cx^n))}{2e^2}+\frac{x^4(a+b\log(cx^n))}{4e}+\frac{d^2(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{2e^3}+\frac{bd^2n\operatorname{PolyLog}\left(2,-\frac{ex^2}{d}\right)}{4e^3}$$

```
[Out] 1/4*b*d*n*x^2/e^2-1/16*b*n*x^4/e-1/2*d*x^2*(a+b*ln(c*x^n))/e^2+1/4*x^4*(a+b
*ln(c*x^n))/e+1/2*d^2*(a+b*ln(c*x^n))*ln(1+e*x^2/d)/e^3+1/4*b*d^2*n*polylog
(2,-e*x^2/d)/e^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {272, 45, 2393, 2341, 2375, 2438}

$$\int \frac{x^5(a+b\log(cx^n))}{d+ex^2}\,dx = \frac{d^2\log\left(\frac{ex^2}{d}+1\right)(a+b\log(cx^n))}{2e^3}-\frac{dx^2(a+b\log(cx^n))}{2e^2}+\frac{x^4(a+b\log(cx^n))}{4e}+\frac{bd^2n\operatorname{PolyLog}\left(2,-\frac{ex^2}{d}\right)}{4e^3}+\frac{bdnx^2}{4e^2}-\frac{bnx^4}{16e}$$

```
[In] Int[(x^5*(a + b*Log[c*x^n]))/(d + e*x^2),x]
```

```
[Out] (b*d*n*x^2)/(4*e^2) - (b*n*x^4)/(16*e) - (d*x^2*(a + b*Log[c*x^n]))/(2*e^2)
 + (x^4*(a + b*Log[c*x^n]))/(4*e) + (d^2*(a + b*Log[c*x^n])*Log[1 + (e*x^2)
/d])/(2*e^3) + (b*d^2*n*PolyLog[2, -((e*x^2)/d)])/(4*e^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{dx(a+b\log(cx^n))}{e^2}+\frac{x^3(a+b\log(cx^n))}{e}+\frac{d^2x(a+b\log(cx^n))}{e^2(d+ex^2)}\right)dx\\ &= -\frac{d\int x(a+b\log(cx^n))\,dx}{e^2}+\frac{d^2\int\frac{x(a+b\log(cx^n))}{d+ex^2}\,dx}{e^2}+\frac{\int x^3(a+b\log(cx^n))\,dx}{e}\end{aligned}$$

$$= \frac{bdnx^2}{4e^2} - \frac{bnx^4}{16e} - \frac{dx^2(a+b\log(cx^n))}{2e^2} + \frac{x^4(a+b\log(cx^n))}{4e} + \frac{d^2(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{2e^3} - \frac{(bd^2n)\int\frac{\log\left(1+\frac{ex^2}{d}\right)}{x}dx}{2e^3}$$

$$= \frac{bdnx^2}{4e^2} - \frac{bnx^4}{16e} - \frac{dx^2(a+b\log(cx^n))}{2e^2} + \frac{x^4(a+b\log(cx^n))}{4e} + \frac{d^2(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{2e^3} + \frac{bd^2n\text{Li}_2\left(-\frac{ex^2}{d}\right)}{4e^3}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.44

$$\int \frac{x^5(a+b\log(cx^n))}{d+ex^2}\,dx = \frac{4bdenx^2 - be^2nx^4 - 8dex^2(a+b\log(cx^n)) + 4e^2x^4(a+b\log(cx^n)) + 8d^2(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{16e^3}$$

```
[In] Integrate[(x^5*(a + b*Log[c*x^n]))/(d + e*x^2),x]
```

```
[Out] (4*b*d*e*n*x^2 - b*e^2*n*x^4 - 8*d*e*x^2*(a + b*Log[c*x^n]) + 4*e^2*x^4*(a
+ b*Log[c*x^n]) + 8*d^2*(a + b*Log[c*x^n])*Log[1 + (Sqrt[e]*x)/Sqrt[-d]] +
8*d^2*(a + b*Log[c*x^n])*Log[1 + (d*Sqrt[e]*x)/(-d)^(3/2)] + 8*b*d^2*n*Poly
Log[2, (Sqrt[e]*x)/Sqrt[-d]] + 8*b*d^2*n*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2
)])/(16*e^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.62 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.83

method	result
risch	$\frac{b\ln(x^n)x^4}{4e} - \frac{b\ln(x^n)dx^2}{2e^2} + \frac{b\ln(x^n)d^2\ln(ex^2+d)}{2e^3} - \frac{bnd^2\ln(x)\ln(ex^2+d)}{2e^3} + \frac{bnd^2\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2e^3} + \frac{bnd^2\ln(x)\ln\left(\right.}{2e^3}$

```
[In] int(x^5*(a+b*ln(c*x^n))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b*ln(x^n)/e*x^4-1/2*b*ln(x^n)/e^2*d*x^2+1/2*b*ln(x^n)*d^2/e^3*ln(e*x^2+
d)-1/2*b*n*d^2/e^3*ln(x)*ln(e*x^2+d)+1/2*b*n*d^2/e^3*ln(x)*ln((-e*x+(-d*e)^
(1/2))/(-d*e)^(1/2))+1/2*b*n*d^2/e^3*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/
2))+1/2*b*n*d^2/e^3*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*b*n*d^2/e^3
*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/16*b*n*x^4/e+1/4*b*d*n*x^2/e^2-1/
```

```
4*b*n*d^2/e^3+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*c
sgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*
csgn(I*c*x^n)^3+b*ln(c)+a)*(1/2/e^2*(1/2*e*x^4-d*x^2)+1/2*d^2/e^3*ln(e*x^2+
d))
```

Fricas [F]

$$\int \frac{x^5(a+b\log(cx^n))}{d+ex^2}\,dx=\int \frac{(b\log(cx^n)+a)x^5}{ex^2+d}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*x^5*log(c*x^n) + a*x^5)/(e*x^2 + d), x)
```

Sympy [A] (verification not implemented)

Time = 35.17 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.12

$$\int \frac{x^5(a+b\log(cx^n))}{d+ex^2}\,dx=\frac{ad^2\left(\begin{cases}\frac{x^2}{d} & \text{for } e=0\\ \frac{\log(d+ex^2)}{e} & \text{otherwise}\end{cases}\right)}{2e^2}-\frac{adx^2}{2e^2}+\frac{ax^4}{4e}-\frac{bd^2n\left(\begin{cases}\frac{x^2}{2d} & \\ \frac{\begin{cases}-\frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1\wedge|x|<1\\ \log(d)\log(x)-\frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right)-\frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log(d)-\frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{otherwise}\end{cases}}{e} & \end{cases}\right)}{2e^2}+\frac{bd^2\left(\begin{cases}\frac{x^2}{d} & \text{for } e=0\\ \frac{\log(d+ex^2)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{2e^2}+\frac{bdnx^2}{4e^2}-\frac{bdx^2\log(cx^n)}{2e^2}-\frac{bnx^4}{16e}+\frac{bx^4\log(cx^n)}{4e}$$

```
[In] integrate(x**5*(a+b*ln(c*x**n))/(e*x**2+d),x)
[Out] a*d**2*Piecewise((x**2/d, Eq(e, 0)), (log(d + e*x**2)/e, True))/(2*e**2) -
a*d*x**2/(2*e**2) + a*x**4/(4*e) - b*d**2*n*Piecewise((x**2/(2*d), Eq(e, 0)
```

```
), (Piecewise((-polylog(2, e*x**2*exp_polar(I*pi)/d)/2, (Abs(x) < 1) & (1/A
bs(x) < 1)), (log(d)*log(x) - polylog(2, e*x**2*exp_polar(I*pi)/d)/2, Abs(x
) < 1), (-log(d)*log(1/x) - polylog(2, e*x**2*exp_polar(I*pi)/d)/2, 1/Abs(x
) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1),
()), ((), (0, 0)), x)*log(d) - polylog(2, e*x**2*exp_polar(I*pi)/d)/2, True
))/e, True))/(2*e**2) + b*d**2*Piecewise((x**2/d, Eq(e, 0)), (log(d + e*x**
2)/e, True))*log(c*x**n)/(2*e**2) + b*d*n*x**2/(4*e**2) - b*d*x**2*log(c*x*
*n)/(2*e**2) - b*n*x**4/(16*e) + b*x**4*log(c*x**n)/(4*e)
```

Maxima [F]

$$\int \frac{x^5(a+b\log(cx^n))}{d+ex^2}\,dx=\int \frac{(b\log(cx^n)+a)x^5}{ex^2+d}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/4*a*(2*d^2*log(e*x^2 + d)/e^3 + (e*x^4 - 2*d*x^2)/e^2) + b*integrate((x^5
*log(c) + x^5*log(x^n))/(e*x^2 + d), x)
```

Giac [F]

$$\int \frac{x^5(a+b\log(cx^n))}{d+ex^2}\,dx=\int \frac{(b\log(cx^n)+a)x^5}{ex^2+d}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^5/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(cx^n))}{d+ex^2}\,dx=\int \frac{x^5\,(a+b\,\ln(c\,x^n))}{e\,x^2+d}\,dx$$

```
[In] int((x^5*(a + b*log(c*x^n)))/(d + e*x^2),x)
```

```
[Out] int((x^5*(a + b*log(c*x^n)))/(d + e*x^2), x)
```

3.211 $\int \frac{x^3(a+b\log(cx^n))}{d+ex^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex^2}\,dx = -\frac{bnx^2}{4e}+\frac{x^2(a+b\log(cx^n))}{2e} - \frac{d(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{2e^2}-\frac{bdn\operatorname{PolyLog}\left(2,-\frac{ex^2}{d}\right)}{4e^2}$$

```
[Out] -1/4*b*n*x^2/e+1/2*x^2*(a+b*ln(c*x^n))/e-1/2*d*(a+b*ln(c*x^n))*ln(1+e*x^2/d
)/e^2-1/4*b*d*n*polylog(2,-e*x^2/d)/e^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {272, 45, 2393, 2341, 2375, 2438}

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex^2}\,dx = -\frac{d\log\left(\frac{ex^2}{d}+1\right)(a+b\log(cx^n))}{2e^2}+\frac{x^2(a+b\log(cx^n))}{2e}-\frac{bdn\operatorname{PolyLog}\left(2,-\frac{ex^2}{d}\right)}{4e^2}-\frac{bnx^2}{4e}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n]))/(d + e*x^2),x]
```

```
[Out] -1/4*(b*n*x^2)/e + (x^2*(a + b*Log[c*x^n]))/(2*e) - (d*(a + b*Log[c*x^n])*L
og[1 + (e*x^2)/d])/(2*e^2) - (b*d*n*PolyLog[2, -((e*x^2)/d)])/(4*e^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{x(a+b\log(cx^n))}{e} - \frac{dx(a+b\log(cx^n))}{e(d+ex^2)}\right) dx$$

$$= \frac{\int x(a+b\log(cx^n))\,dx}{e} - \frac{d\int \frac{x(a+b\log(cx^n))}{d+ex^2}\,dx}{e}$$

$$= -\frac{bnx^2}{4e} + \frac{x^2(a+b\log(cx^n))}{2e} - \frac{d(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{2e^2} + \frac{(bdn)\int\frac{\log\left(1+\frac{ex^2}{d}\right)}{x}dx}{2e^2}$$

$$= -\frac{bnx^2}{4e} + \frac{x^2(a+b\log(cx^n))}{2e} - \frac{d(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{2e^2} - \frac{bdn\text{Li}_2\left(-\frac{ex^2}{d}\right)}{4e^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.63

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex^2}\,dx = -\frac{benx^2 - 2ex^2(a+b\log(cx^n)) + 2d(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right) + 2d(a+b\log(cx^n))\log\left(1+\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{4e^2}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/(d + e*x^2),x]
```

```
[Out] -1/4*(b*e*n*x^2 - 2*e*x^2*(a + b*Log[c*x^n]) + 2*d*(a + b*Log[c*x^n])*Log[1
 + (Sqrt[e]*x)/Sqrt[-d]] + 2*d*(a + b*Log[c*x^n])*Log[1 + (d*Sqrt[e]*x)/(-d
)^(3/2)] + 2*b*d*n*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]] + 2*b*d*n*PolyLog[2, (d
*Sqrt[e]*x)/(-d)^(3/2)])/e^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.42

method	result
risch	$\frac{b\ln(x^n)x^2}{2e} - \frac{b\ln(x^n)d\ln(e\,x^2+d)}{2e^2} - \frac{bn\,x^2}{4e} + \frac{bnd\ln(x)\ln(e\,x^2+d)}{2e^2} - \frac{bnd\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2e^2} - \frac{bnd\ln(x)\ln\left(\frac{ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2e^2} -$

```
[In] int(x^3*(a+b*ln(c*x^n))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*ln(x^n)/e*x^2-1/2*b*ln(x^n)*d/e^2*ln(e*x^2+d)-1/4*b*n*x^2/e+1/2*b*n*d
/e^2*ln(x)*ln(e*x^2+d)-1/2*b*n*d/e^2*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1
/2))-1/2*b*n*d/e^2*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*n*d/e^2*
dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*n*d/e^2*dilog((e*x+(-d*e)^(1/
2))/(-d*e)^(1/2))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*
Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b
*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/2*x^2/e-1/2*d/e^2*ln(e*x^2+d))
```

Fricas [F]

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex^2}\,dx=\int \frac{(b\log(cx^n)+a)x^3}{ex^2+d}\,dx$$

[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="fricas")

[Out] integral((b*x^3*log(c*x^n) + a*x^3)/(e*x^2 + d), x)

Sympy [A] (verification not implemented)

Time = 17.42 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.43

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex^2}\,dx=-\frac{ad\left(\begin{cases}\frac{x^2}{d} & \text{for } e=0\\ \frac{\log(d+ex^2)}{e} & \text{otherwise}\end{cases}\right)}{2e}+\frac{ax^2}{2e}+\frac{bdn\left(\begin{cases}\frac{x^2}{2d} & \\ \frac{\begin{cases}-\frac{\text{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1\wedge|x|<1\\ \log(d)\log(x)-\frac{\text{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right)-\frac{\text{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log(d)-\frac{\text{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{otherwise}\end{cases}}{e} & \end{cases}\right)}{2e}-\frac{bd\left(\begin{cases}\frac{x^2}{d} & \text{for } e=0\\ \frac{\log(d+ex^2)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{2e}-\frac{bnx^2}{4e}+\frac{bx^2\log(cx^n)}{2e}$$

[In] integrate(x**3*(a+b*ln(c*x**n))/(e*x**2+d),x)

[Out] -a*d*Piecewise((x**2/d, Eq(e, 0)), (log(d + e*x**2)/e, True))/(2*e) + a*x**2/(2*e) + b*d*n*Piecewise((x**2/(2*d), Eq(e, 0)), (Piecewise((-polylog(2, e*x**2*exp_polar(I*pi)/d)/2, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x**2*exp_polar(I*pi)/d)/2, Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x**2*exp_polar(I*pi)/d)/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x**2*exp_polar(I*pi)/d)/2, True))/e, True))/(2*e) - b*d*Piecewise((x**2/d, Eq(e, 0)), (log(d + e*x**2)/e, True))*log(c*x**n)/(2*e) - b*n*x**2/(4*e) + b*x**2*log(c*x**n)/(2*e)

Maxima [F]

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex^2}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{ex^2+d}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*(x^2/e - d*log(e*x^2 + d)/e^2) + b*integrate((x^3*log(c) + x^3*log(x^
n))/(e*x^2 + d), x)
```

Giac [F]

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex^2}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{ex^2+d}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^3/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex^2}\,dx = \int \frac{x^3\,(a+b\,\ln(c\,x^n))}{e\,x^2+d}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n)))/(d + e*x^2),x)
```

```
[Out] int((x^3*(a + b*log(c*x^n)))/(d + e*x^2), x)
```

3.212 $\int \frac{x(a+b\log(cx^n))}{d+ex^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{x(a+b\log(cx^n))}{d+ex^2}\,dx=\frac{(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{2e}+\frac{bn\operatorname{PolyLog}\left(2,-\frac{ex^2}{d}\right)}{4e}$$

```
[Out] 1/2*(a+b*ln(c*x^n))*ln(1+e*x^2/d)/e+1/4*b*n*polylog(2,-e*x^2/d)/e
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2375, 2438}

$$\int \frac{x(a+b\log(cx^n))}{d+ex^2}\,dx=\frac{\log\left(\frac{ex^2}{d}+1\right)(a+b\log(cx^n))}{2e}+\frac{bn\operatorname{PolyLog}\left(2,-\frac{ex^2}{d}\right)}{4e}$$

```
[In] Int[(x*(a + b*Log[c*x^n]))/(d + e*x^2),x]
```

```
[Out] ((a + b*Log[c*x^n])*Log[1 + (e*x^2)/d])/(2*e) + (b*n*PolyLog[2, -((e*x^2)/d
)])/(4*e)
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \frac{(a + b\log(cx^n))\log\left(1 + \frac{ex^2}{d}\right)}{2e} - \frac{(bn)\int\frac{\log\left(1+\frac{ex^2}{d}\right)}{x}\,dx}{2e}$$
$$= \frac{(a + b\log(cx^n))\log\left(1 + \frac{ex^2}{d}\right)}{2e} + \frac{bn\text{Li}_2\left(-\frac{ex^2}{d}\right)}{4e}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.92

$$\int\frac{x(a + b\log(cx^n))}{d + ex^2}\,dx$$
$$= \frac{(a + b\log(cx^n))\left(\log\left(1 + \frac{\sqrt{e}x}{\sqrt{-d}}\right) + \log\left(1 + \frac{d\sqrt{e}x}{(-d)^{3/2}}\right)\right) + bn\,\text{PolyLog}\left(2, \frac{\sqrt{e}x}{\sqrt{-d}}\right) + bn\,\text{PolyLog}\left(2, \frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{2e}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e*x^2),x]
```

```
[Out] ((a + b*Log[c*x^n])*(Log[1 + (Sqrt[e]*x)/Sqrt[-d]] + Log[1 + (d*Sqrt[e]*x)/
(-d)^(3/2)]) + b*n*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]] + b*n*PolyLog[2, (d*Sqr
t[e]*x)/(-d)^(3/2)])/(2*e)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 244, normalized size of antiderivative = 4.98

method	result
risch	$\frac{b\ln(x^n)\ln(e\,x^2+d)}{2e} - \frac{bn\ln(x)\ln(e\,x^2+d)}{2e} + \frac{bn\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2e} + \frac{bn\ln(x)\ln\left(\frac{ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2e} + \frac{bn\,\text{dilog}\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2e} + \frac{bn}{}$

```
[In] int(x*(a+b*ln(c*x^n))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*ln(x^n)/e*ln(e*x^2+d)-1/2*b/e*n*ln(x)*ln(e*x^2+d)+1/2*b/e*n*ln(x)*ln(
(-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*b/e*n*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d
*e)^(1/2))+1/2*b/e*n*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*b/e*n*dilo
g((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*c
```

```
sgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*cs
gn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)/e*ln(e*x^2+d)
```

Fricas [F]

$$\int \frac{x(a+b\log(cx^n))}{d+ex^2}\,dx = \int \frac{(b\log(cx^n)+a)x}{ex^2+d}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*x*log(c*x^n) + a*x)/(e*x^2 + d), x)
```

Sympy [A] (verification not implemented)

Time = 3.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.88

$$\int \frac{x(a+b\log(cx^n))}{d+ex^2}\,dx = \frac{a\log(d+ex^2)}{2e} - \frac{bn\left(\begin{cases} -\frac{\operatorname{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1 \wedge |x|<1 \\ \log(d)\log(x) - \frac{\operatorname{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } |x|<1 \\ -\log(d)\log\left(\frac{1}{x}\right) - \frac{\operatorname{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(d) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(d) - \frac{\operatorname{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{otherwise} \end{cases}\right)}{2e} + \frac{b\log(cx^n)\log(d+ex^2)}{2e}$$

```
[In] integrate(x*(a+b*ln(c*x**n))/(e*x**2+d),x)
[Out] a*log(d + e*x**2)/(2*e) - b*n*Piecewise((-polylog(2, e*x**2*exp_polar(I*pi)
/d)/2, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x**2*e
xp_polar(I*pi)/d)/2, Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x**2*exp
_polar(I*pi)/d)/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*
log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x**2*
exp_polar(I*pi)/d)/2, True))/(2*e) + b*log(c*x**n)*log(d + e*x**2)/(2*e)
```

Maxima [F]

$$\int \frac{x(a+b\log(cx^n))}{d+ex^2}\,dx = \int \frac{(b\log(cx^n)+a)x}{ex^2+d}\,dx$$

[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="maxima")

[Out] b*integrate((x*log(c) + x*log(x^n))/(e*x^2 + d), x) + 1/2*a*log(e*x^2 + d)/
e

Giac [F]

$$\int \frac{x(a+b\log(cx^n))}{d+ex^2}\,dx = \int \frac{(b\log(cx^n)+a)x}{ex^2+d}\,dx$$

[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="giac")

[Out] integrate((b*log(c*x^n) + a)*x/(e*x^2 + d), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(cx^n))}{d+ex^2}\,dx = \int \frac{x\,(a+b\,\ln(c\,x^n))}{e\,x^2+d}\,dx$$

[In] int((x*(a + b*log(c*x^n)))/(d + e*x^2),x)

[Out] int((x*(a + b*log(c*x^n)))/(d + e*x^2), x)

3.213 $\int \frac{a+b\log(cx^n)}{x(d+ex^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)}\,dx=-\frac{\log\left(1+\frac{d}{ex^2}\right)(a+b\log(cx^n))}{2d}+\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{d}{ex^2}\right)}{4d}$$

```
[Out] -1/2*ln(1+d/e/x^2)*(a+b*ln(c*x^n))/d+1/4*b*n*polylog(2,-d/e/x^2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)}\,dx=\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{d}{ex^2}\right)}{4d}-\frac{\log\left(\frac{d}{ex^2}+1\right)(a+b\log(cx^n))}{2d}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x^2)),x]
```

```
[Out] -1/2*(Log[1 + d/(e*x^2)]*(a + b*Log[c*x^n]))/d + (b*n*PolyLog[2, -(d/(e*x^2
))])/(4*d)
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\log\left(1+\frac{d}{ex^2}\right)(a+b\log(cx^n))}{2d}+\frac{(bn)\int\frac{\log\left(1+\frac{d}{ex^2}\right)}{x}\,dx}{2d}\\ &= -\frac{\log\left(1+\frac{d}{ex^2}\right)(a+b\log(cx^n))}{2d}+\frac{bn\mathrm{Li}_2\left(-\frac{d}{ex^2}\right)}{4d}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 126 vs. 2(49) = 98.

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.57

$$\int\frac{a+b\log(cx^n)}{x(d+ex^2)}\,dx = -\frac{-\left((a+b\log(cx^n))\left(a+b\log(cx^n)-bn\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)-bn\log\left(1+\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)\right)\right)+b^2n^2\operatorname{PolyLog}\left(2,\frac{\sqrt{e}\dots}{\sqrt{-\dots}}\right.}{2bdn}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x^2)),x]
[Out] -1/2*(-((a + b*Log[c*x^n])*(a + b*Log[c*x^n] - b*n*Log[1 + (Sqrt[e]*x)/Sqrt
[-d]] - b*n*Log[1 + (d*Sqrt[e]*x)/(-d)^(3/2)])) + b^2*n^2*PolyLog[2, (Sqrt[
e]*x)/Sqrt[-d]] + b^2*n^2*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)])/(b*d*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 274, normalized size of antiderivative = 5.59

method	result
risch	$-\frac{b\ln(x^n)\ln(e\,x^2+d)}{2d}+\frac{b\ln(x^n)\ln(x)}{d}-\frac{bn\ln(x)^2}{2d}+\frac{bn\ln(x)\ln(e\,x^2+d)}{2d}-\frac{bn\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2d}-\frac{bn\ln(x)\ln\left(\frac{ex+\sqrt{-de}}{\sqrt{-de}}\right.}{2d}$

```
[In] int((a+b*ln(c*x^n))/x/(e*x^2+d),x,method=_RETURNVERBOSE)
[Out] -1/2*b*ln(x^n)/d*ln(e*x^2+d)+b*ln(x^n)/d*ln(x)-1/2*b*n/d*ln(x)^2+1/2*b*n/d*
ln(x)*ln(e*x^2+d)-1/2*b*n/d*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*
b*n/d*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*n/d*dilog((-e*x+(-d*e
)^(1/2))/(-d*e)^(1/2))-1/2*b*n/d*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+(-1
/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c
```

```
*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b
*ln(c)+a)*(-1/2/d*ln(e*x^2+d)+1/d*ln(x))
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)}\,dx = \int \frac{b\log(cx^n)+a}{(ex^2+d)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(e*x^3 + d*x), x)
```

Sympy [A] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.94

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)}\,dx = \frac{a\log(x)}{d} - \frac{a\log(d+ex^2)}{2d} + \frac{bn\left(\begin{cases} \frac{\operatorname{Li}_2\left(\frac{de^{i\pi}}{ex^2}\right)}{2} & \text{for } \frac{1}{|x|}<1 \wedge |x|<1 \\ \log(e)\log(x)+\frac{\operatorname{Li}_2\left(\frac{de^{i\pi}}{ex^2}\right)}{2} & \text{for } |x|<1 \\ -\log(e)\log\left(\frac{1}{x}\right)+\frac{\operatorname{Li}_2\left(\frac{de^{i\pi}}{ex^2}\right)}{2} & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(e)+\frac{\operatorname{Li}_2\left(\frac{de^{i\pi}}{ex^2}\right)}{2} & \text{otherwise} \end{cases}\right)}{2d} - \frac{b\log(cx^n)\log\left(\frac{d}{x^2}+e\right)}{2d}$$

```
[In] integrate((a+b*ln(c*x**n))/x/(e*x**2+d),x)
[Out] a*log(x)/d - a*log(d + e*x**2)/(2*d) + b*n*Piecewise((polylog(2, d*exp_pola
r(I*pi)/(e*x**2))/2, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(e)*log(x) + polyl
og(2, d*exp_polar(I*pi)/(e*x**2))/2, Abs(x) < 1), (-log(e)*log(1/x) + polyl
og(2, d*exp_polar(I*pi)/(e*x**2))/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1)),
 ((0, 0), ()), x)*log(e) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(e) +
polylog(2, d*exp_polar(I*pi)/(e*x**2))/2, True))/(2*d) - b*log(c*x**n)*log(
d/x**2 + e)/(2*d)
```

Maxima [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x\left(d+ex^2\right)}\,dx = \int \frac{b\log\left(cx^n\right)+a}{(ex^2+d)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d),x, algorithm="maxima")
[Out] -1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate((log(c) + log(x^n))/(e
*x^3 + d*x), x)
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x\left(d+ex^2\right)}\,dx = \int \frac{b\log\left(cx^n\right)+a}{(ex^2+d)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x\left(d+ex^2\right)}\,dx = \int \frac{a+b\,\ln\left(c\,x^n\right)}{x\,\left(e\,x^2+d\right)}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x^2)),x)
[Out] int((a + b*log(c*x^n))/(x*(d + e*x^2)), x)
```

3.214 $\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)}\,dx=-\frac{bn}{4dx^2}-\frac{a+b\log(cx^n)}{2dx^2}+\frac{e\log\left(1+\frac{d}{ex^2}\right)(a+b\log(cx^n))}{2d^2}-\frac{ben\,\mathrm{PolyLog}\left(2,-\frac{d}{ex^2}\right)}{4d^2}$$

```
[Out] -1/4*b*n/d/x^2+1/2*(-a-b*ln(c*x^n))/d/x^2+1/2*e*ln(1+d/e/x^2)*(a+b*ln(c*x^n
))/d^2-1/4*b*e*n*polylog(2,-d/e/x^2)/d^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2380, 2341, 2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)}\,dx=\frac{e\log\left(\frac{d}{ex^2}+1\right)(a+b\log(cx^n))}{2d^2}-\frac{a+b\log(cx^n)}{2dx^2}-\frac{ben\,\mathrm{PolyLog}\left(2,-\frac{d}{ex^2}\right)}{4d^2}-\frac{bn}{4dx^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^3*(d + e*x^2)),x]
```

```
[Out] -1/4*(b*n)/(d*x^2) - (a + b*Log[c*x^n])/(2*d*x^2) + (e*Log[1 + d/(e*x^2)]*(
a + b*Log[c*x^n]))/(2*d^2) - (b*e*n*PolyLog[2, -(d/(e*x^2))])/(4*d^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{a+b\log(cx^n)}{x^3}\,dx}{d} - \frac{e\int \frac{a+b\log(cx^n)}{x(d+ex^2)}\,dx}{d}\\
&= -\frac{bn}{4dx^2} - \frac{a+b\log\left(cx^n\right)}{2dx^2} + \frac{e\log\left(1+\frac{d}{ex^2}\right)\left(a+b\log\left(cx^n\right)\right)}{2d^2} - \frac{(ben)\int \frac{\log\left(1+\frac{d}{ex^2}\right)}{x}\,dx}{2d^2}\\
&= -\frac{bn}{4dx^2} - \frac{a+b\log\left(cx^n\right)}{2dx^2} + \frac{e\log\left(1+\frac{d}{ex^2}\right)\left(a+b\log\left(cx^n\right)\right)}{2d^2} - \frac{ben\text{Li}_2\left(-\frac{d}{ex^2}\right)}{4d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.89

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^2\right)}\,dx$$
$$= \frac{-\frac{bdn}{x^2} - \frac{2d(a+b\log(cx^n))}{x^2} - \frac{2e(a+b\log(cx^n))^2}{bn} + 2e(a+b\log\left(cx^n\right))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right) + 2e(a+b\log\left(cx^n\right))\log\left(1+\frac{d}{(-}\right.}{4d^2}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*(d + e*x^2)),x]
```

```
[Out] (-((b*d*n)/x^2) - (2*d*(a + b*Log[c*x^n]))/x^2 - (2*e*(a + b*Log[c*x^n])^2)
/(b*n) + 2*e*(a + b*Log[c*x^n])*Log[1 + (Sqrt[e]*x)/Sqrt[-d]] + 2*e*(a + b*
Log[c*x^n])*Log[1 + (d*Sqrt[e]*x)/(-d)^(3/2)] + 2*b*e*n*PolyLog[2, (Sqrt[e]
*x)/Sqrt[-d]] + 2*b*e*n*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)])/(4*d^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.49 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.82

method	result
risch	$\frac{b\ln(x^n)e\ln(e\,x^2+d)}{2d^2}-\frac{b\ln(x^n)}{2d\,x^2}-\frac{b\ln(x^n)e\ln(x)}{d^2}-\frac{bne\ln(x)\ln(e\,x^2+d)}{2d^2}+\frac{bne\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2d^2}+\frac{bne\ln(x)\ln\left(\frac{ex+\sqrt{-d}}{\sqrt{-d}}\right.}{2d^2}$

[In] int((a+b*ln(c*x^n))/x^3/(e*x^2+d),x,method=_RETURNVERBOSE)

[Out] 1/2*b*ln(x^n)*e/d^2*ln(e*x^2+d)-1/2*b*ln(x^n)/d/x^2-b*ln(x^n)*e/d^2*ln(x)-1/2*b*n*e/d^2*ln(x)*ln(e*x^2+d)+1/2*b*n*e/d^2*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*b*n*e/d^2*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*b*n*e/d^2*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*b*n*e/d^2*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/4*b*n/d/x^2+1/2*b*n*e/d^2*ln(x)^2+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/2*e/d^2*ln(e*x^2+d)-1/2/d/x^2-e/d^2*ln(x))

Fricas [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^2\right)}\,dx=\int \frac{b\log\left(cx^n\right)+a}{(ex^2+d)x^3}\,dx$$

[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d),x, algorithm="fricas")

[Out] integral((b*log(c*x^n) + a)/(e*x^5 + d*x^3), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^2\right)}\,dx=\text{Timed out}$$

[In] integrate((a+b*ln(c*x**n))/x**3/(e*x**2+d),x)

[Out] Timed out

Maxima [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^2\right)}\,dx=\int \frac{b\log\left(cx^n\right)+a}{(ex^2+d)x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*(e*log(e*x^2 + d)/d^2 - 2*e*log(x)/d^2 - 1/(d*x^2)) + b*integrate((lo
g(c) + log(x^n))/(e*x^5 + d*x^3), x)
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^2\right)}\,dx=\int \frac{b\log\left(cx^n\right)+a}{(ex^2+d)x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^2\right)}\,dx=\int \frac{a+b\,\ln\left(c\,x^n\right)}{x^3\,\left(e\,x^2+d\right)}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e*x^2)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^3*(d + e*x^2)), x)
```

3.215 $\int \frac{a+b\log(cx^n)}{x^5(d+ex^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{a+b\log(cx^n)}{x^5(d+ex^2)}\,dx = -\frac{bn}{16dx^4}+\frac{ben}{4d^2x^2}-\frac{a+b\log(cx^n)}{4dx^4}+\frac{e(a+b\log(cx^n))}{2d^2x^2} -\frac{e^2\log\left(1+\frac{d}{ex^2}\right)(a+b\log(cx^n))}{2d^3}+\frac{be^2n\,\mathrm{PolyLog}\left(2,-\frac{d}{ex^2}\right)}{4d^3}$$

```
[Out] -1/16*b*n/d/x^4+1/4*b*e*n/d^2/x^2+1/4*(-a-b*ln(c*x^n))/d/x^4+1/2*e*(a+b*ln(
c*x^n))/d^2/x^2-1/2*e^2*ln(1+d/e/x^2)*(a+b*ln(c*x^n))/d^3+1/4*b*e^2*n*polyl
og(2,-d/e/x^2)/d^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2380, 2341, 2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x^5(d+ex^2)}\,dx = -\frac{e^2\log\left(\frac{d}{ex^2}+1\right)(a+b\log(cx^n))}{2d^3}+\frac{e(a+b\log(cx^n))}{2d^2x^2} -\frac{a+b\log(cx^n)}{4dx^4}+\frac{be^2n\,\mathrm{PolyLog}\left(2,-\frac{d}{ex^2}\right)}{4d^3}+\frac{ben}{4d^2x^2}-\frac{bn}{16dx^4}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^5*(d + e*x^2)),x]
```

```
[Out] -1/16*(b*n)/(d*x^4) + (b*e*n)/(4*d^2*x^2) - (a + b*Log[c*x^n])/(4*d*x^4) +
(e*(a + b*Log[c*x^n]))/(2*d^2*x^2) - (e^2*Log[1 + d/(e*x^2)]*(a + b*Log[c*x
^n]))/(2*d^3) + (b*e^2*n*PolyLog[2, -(d/(e*x^2))])/(4*d^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{a+b\log(cx^n)}{x^5}\,dx}{d} - \frac{e\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)}\,dx}{d} \\
&= -\frac{bn}{16dx^4} - \frac{a+b\log(cx^n)}{4dx^4} - \frac{e\int \frac{a+b\log(cx^n)}{x^3}\,dx}{d^2} + \frac{e^2\int \frac{a+b\log(cx^n)}{x(d+ex^2)}\,dx}{d^2} \\
&= -\frac{bn}{16dx^4} + \frac{ben}{4d^2x^2} - \frac{a+b\log(cx^n)}{4dx^4} + \frac{e(a+b\log(cx^n))}{2d^2x^2} \\
&\quad - \frac{e^2\log\left(1+\frac{d}{ex^2}\right)(a+b\log(cx^n))}{2d^3} + \frac{(be^2n)\int \frac{\log\left(1+\frac{d}{ex^2}\right)}{x}\,dx}{2d^3} \\
&= -\frac{bn}{16dx^4} + \frac{ben}{4d^2x^2} - \frac{a+b\log(cx^n)}{4dx^4} + \frac{e(a+b\log(cx^n))}{2d^2x^2} \\
&\quad - \frac{e^2\log\left(1+\frac{d}{ex^2}\right)(a+b\log(cx^n))}{2d^3} + \frac{be^2n\,\mathrm{Li}_2\left(-\frac{d}{ex^2}\right)}{4d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.62

$$\int \frac{a+b\log(cx^n)}{x^5(d+ex^2)}\,dx = -\frac{\frac{bd^2n}{x^4}-\frac{4bden}{x^2}+\frac{4d^2(a+b\log(cx^n))}{x^4}-\frac{8de(a+b\log(cx^n))}{x^2}-\frac{8e^2(a+b\log(cx^n))^2}{bn}+8e^2(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)+}{16d^3}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^5*(d + e*x^2)),x]
```

```
[Out] -1/16*((b*d^2*n)/x^4 - (4*b*d*e*n)/x^2 + (4*d^2*(a + b*Log[c*x^n]))/x^4 - (
8*d*e*(a + b*Log[c*x^n]))/x^2 - (8*e^2*(a + b*Log[c*x^n])^2)/(b*n) + 8*e^2*
(a + b*Log[c*x^n])*Log[1 + (Sqrt[e]*x)/Sqrt[-d]] + 8*e^2*(a + b*Log[c*x^n])
*Log[1 + (d*Sqrt[e]*x)/(-d)^(3/2)] + 8*b*e^2*n*PolyLog[2, (Sqrt[e]*x)/Sqrt[
-d]] + 8*b*e^2*n*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)])/d^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 369, normalized size of antiderivative = 3.05

method	result
risch	$-\frac{b\ln(x^n)e^2\ln(e\,x^2+d)}{2d^3}-\frac{b\ln(x^n)}{4d\,x^4}+\frac{b\ln(x^n)e^2\ln(x)}{d^3}+\frac{b\ln(x^n)e}{2d^2x^2}+\frac{ben}{4d^2x^2}-\frac{bn}{16d\,x^4}-\frac{bn\,e^2\ln(x)^2}{2d^3}+\frac{bn\,e^2\ln(x)\ln(e\,x^2}{2d^3}$

```
[In] int((a+b*ln(c*x^n))/x^5/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*ln(x^n)*e^2/d^3*ln(e*x^2+d)-1/4*b*ln(x^n)/d/x^4+b*ln(x^n)*e^2/d^3*ln
(x)+1/2*b*ln(x^n)*e/d^2/x^2+1/4*b*e*n/d^2/x^2-1/16*b*n/d/x^4-1/2*b*n*e^2/d^
3*ln(x)^2+1/2*b*n*e^2/d^3*ln(x)*ln(e*x^2+d)-1/2*b*n*e^2/d^3*ln(x)*ln((-e*x+
(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*n*e^2/d^3*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d
*e)^(1/2))-1/2*b*n*e^2/d^3*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*n*
e^2/d^3*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+(-1/2*I*b*Pi*csgn(I*c)*csgn(
I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I
*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-1/2*e^2/d^3*l
n(e*x^2+d)-1/4/d/x^4+e^2/d^3*ln(x)+1/2*e/d^2/x^2)
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x^5(d+ex^2)}\,dx = \int \frac{b\log(cx^n)+a}{(ex^2+d)x^5}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^5/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(e*x^7 + d*x^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^5(d+ex^2)}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*x**n))/x**5/(e*x**2+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{x^5(d+ex^2)}\,dx = \int \frac{b\log(cx^n)+a}{(ex^2+d)x^5}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^5/(e*x^2+d),x, algorithm="maxima")
[Out] -1/4*a*(2*e^2*log(e*x^2 + d)/d^3 - 4*e^2*log(x)/d^3 - (2*e*x^2 - d)/(d^2*x^
4)) + b*integrate((log(c) + log(x^n))/(e*x^7 + d*x^5), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^5(d+ex^2)}\,dx = \int \frac{b\log(cx^n)+a}{(ex^2+d)x^5}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^5/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^5(d+ex^2)}\,dx=\int \frac{a+b\,\ln(c\,x^n)}{x^5\,\left(e\,x^2+d\right)}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^5*(d + e*x^2)),x)
[Out] int((a + b*log(c*x^n))/(x^5*(d + e*x^2)), x)
```

3.216 $\int \frac{x^4(a+b\log(cx^n))}{d+ex^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 167

$$\int \frac{x^4(a+b\log(cx^n))}{d+ex^2} dx = -\frac{adx}{e^2}+\frac{bdnx}{e^2}-\frac{bnx^3}{9e}-\frac{bdx\log(cx^n)}{e^2}+\frac{x^3(a+b\log(cx^n))}{3e}+\frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{5/2}}-\frac{ibd^{3/2}n\,\text{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2e^{5/2}}+\frac{ibd^{3/2}n\,\text{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2e^{5/2}}$$

```
[Out] -a*d*x/e^2+b*d*n*x/e^2-1/9*b*n*x^3/e-b*d*x*ln(c*x^n)/e^2+1/3*x^3*(a+b*ln(c*
x^n))/e+d^(3/2)*arctan(x*e^(1/2)/d^(1/2))*(a+b*ln(c*x^n))/e^(5/2)-1/2*I*b*d
^(3/2)*n*polylog(2,-I*x*e^(1/2)/d^(1/2))/e^(5/2)+1/2*I*b*d^(3/2)*n*polylog(
2,I*x*e^(1/2)/d^(1/2))/e^(5/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {308, 211, 2393, 2332, 2341, 2361, 12, 4940, 2438}

$$\int \frac{x^4(a+b\log(cx^n))}{d+ex^2} dx = \frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{5/2}}+\frac{x^3(a+b\log(cx^n))}{3e}-\frac{adx}{e^2}-\frac{bdx\log(cx^n)}{e^2}-\frac{ibd^{3/2}n\,\text{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2e^{5/2}}+\frac{ibd^{3/2}n\,\text{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2e^{5/2}}+\frac{bdnx}{e^2}-\frac{bnx^3}{9e}$$

```
[In] Int[(x^4*(a + b*Log[c*x^n]))/(d + e*x^2),x]
```

```
[Out] -((a*d*x)/e^2) + (b*d*n*x)/e^2 - (b*n*x^3)/(9*e) - (b*d*x*Log[c*x^n])/e^2 +
 (x^3*(a + b*Log[c*x^n]))/(3*e) + (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(a +
 b*Log[c*x^n]))/e^(5/2) - ((I/2)*b*d^(3/2)*n*PolyLog[2, ((-I)*Sqrt[e]*x)/Sq
rt[d]])/e^(5/2) + ((I/2)*b*d^(3/2)*n*PolyLog[2, (I*Sqrt[e]*x)/Sqrt[d]])/e^(
5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
```

```
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{d(a+b\log(cx^n))}{e^2}+\frac{x^2(a+b\log(cx^n))}{e}+\frac{d^2(a+b\log(cx^n))}{e^2(d+ex^2)}\right)dx \\
&= -\frac{d\int(a+b\log(cx^n))\,dx}{e^2}+\frac{d^2\int\frac{a+b\log(cx^n)}{d+ex^2}\,dx}{e^2}+\frac{\int x^2(a+b\log(cx^n))\,dx}{e} \\
&= -\frac{adx}{e^2}-\frac{bnx^3}{9e}+\frac{x^3(a+b\log(cx^n))}{3e}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{5/2}} \\
&\quad-\frac{(bd)\int\log(cx^n)\,dx}{e^2}-\frac{(bd^2n)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}\,dx}{e^2} \\
&= -\frac{adx}{e^2}+\frac{bdnx}{e^2}-\frac{bnx^3}{9e}-\frac{bdx\log(cx^n)}{e^2}+\frac{x^3(a+b\log(cx^n))}{3e} \\
&\quad+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{5/2}}-\frac{(bd^{3/2}n)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{e^{5/2}} \\
&= -\frac{adx}{e^2}+\frac{bdnx}{e^2}-\frac{bnx^3}{9e}-\frac{bdx\log(cx^n)}{e^2} \\
&\quad+\frac{x^3(a+b\log(cx^n))}{3e}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{5/2}} \\
&\quad-\frac{(ibd^{3/2}n)\int\frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2e^{5/2}}+\frac{(ibd^{3/2}n)\int\frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2e^{5/2}} \\
&= -\frac{adx}{e^2}+\frac{bdnx}{e^2}-\frac{bnx^3}{9e}-\frac{bdx\log(cx^n)}{e^2}+\frac{x^3(a+b\log(cx^n))}{3e} \\
&\quad+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{5/2}}-\frac{ibd^{3/2}n\,\mathrm{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2e^{5/2}}+\frac{ibd^{3/2}n\,\mathrm{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2e^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.25

$$\int \frac{x^4(a+b\log(cx^n))}{d+ex^2}\,dx$$
$$= \frac{-18ad\sqrt{e}x+18bd\sqrt{e}nx-2be^{3/2}nx^3-18bd\sqrt{e}x\log(cx^n)+6e^{3/2}x^3(a+b\log(cx^n))+9\sqrt{-d}d(a+b\log}{}$$

```
[In] Integrate[(x^4*(a + b*Log[c*x^n]))/(d + e*x^2),x]
```

```
[Out] (-18*a*d*Sqrt[e]*x + 18*b*d*Sqrt[e]*n*x - 2*b*e^(3/2)*n*x^3 - 18*b*d*Sqrt[e
]*x*Log[c*x^n] + 6*e^(3/2)*x^3*(a + b*Log[c*x^n]) + 9*Sqrt[-d]*d*(a + b*Log
[c*x^n])*Log[1 + (Sqrt[e]*x)/Sqrt[-d]] + 9*(-d)^(3/2)*(a + b*Log[c*x^n])*Lo
g[1 + (d*Sqrt[e]*x)/(-d)^(3/2)] + 9*b*(-d)^(3/2)*n*PolyLog[2, (Sqrt[e]*x)/S
qrt[-d]] - 9*b*(-d)^(3/2)*n*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)])/(18*e^(5/
2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.49 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.19

method	result
risch	$\frac{b\ln(x^n)x^3}{3e}-\frac{b\ln(x^n)dx}{e^2}-\frac{b\,d^2\arctan\left(\frac{xe}{\sqrt{de}}\right)n\ln(x)}{e^2\sqrt{de}}+\frac{b\,d^2\arctan\left(\frac{xe}{\sqrt{de}}\right)\ln(x^n)}{e^2\sqrt{de}}-\frac{bn\,x^3}{9e}+\frac{bdnx}{e^2}+\frac{bn\,d^2\ln(x)\ln\left(\frac{-ex+\sqrt{-}}{\sqrt{-de}}\right.}{2e^2\sqrt{-de}}$

```
[In] int(x^4*(a+b*ln(c*x^n))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*ln(x^n)/e*x^3-b*ln(x^n)/e^2*d*x-b*d^2/e^2/(d*e)^(1/2)*arctan(x*e/(d*e
)^(1/2))*n*ln(x)+b*d^2/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*ln(x^n)-1/9*
b*n*x^3/e+b*d*n*x/e^2+1/2*b*n*d^2/e^2*ln(x)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1
/2))/(-d*e)^(1/2))-1/2*b*n*d^2/e^2*ln(x)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))
/(-d*e)^(1/2))+1/2*b*n*d^2/e^2/(-d*e)^(1/2)*dilog((-e*x+(-d*e)^(1/2))/(-d*e
)^(1/2))-1/2*b*n*d^2/e^2/(-d*e)^(1/2)*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)
)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csg
n(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n
)^3+b*ln(c)+a)*(1/e^2*(1/3*e*x^3-d*x)+d^2/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^
(1/2)))
```

Fricas [F]

$$\int \frac{x^4(a+b\log(cx^n))}{d+ex^2}\,dx = \int \frac{(b\log(cx^n)+a)x^4}{ex^2+d}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*x^4*log(c*x^n) + a*x^4)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\log(cx^n))}{d+ex^2}\,dx = \int \frac{x^4(a+b\log(cx^n))}{d+ex^2}\,dx$$

```
[In] integrate(x**4*(a+b*ln(c*x**n))/(e*x**2+d),x)
[Out] Integral(x**4*(a + b*log(c*x**n))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\log(cx^n))}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^4(a+b\log(cx^n))}{d+ex^2}\,dx = \int \frac{(b\log(cx^n)+a)x^4}{ex^2+d}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^4/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(cx^n))}{d+ex^2}\,dx = \int \frac{x^4\,(a+b\,\ln(c\,x^n))}{e\,x^2+d}\,dx$$

```
[In] int((x^4*(a + b*log(c*x^n)))/(d + e*x^2),x)
```

```
[Out] int((x^4*(a + b*log(c*x^n)))/(d + e*x^2), x)
```

3.217 $\int \frac{x^2(a+b\log(cx^n))}{d+ex^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 132

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex^2}\,dx = \frac{ax}{e}-\frac{bnx}{e}+\frac{bx\log(cx^n)}{e}-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{3/2}} + \frac{ib\sqrt{d}n\operatorname{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2e^{3/2}}-\frac{ib\sqrt{d}n\operatorname{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2e^{3/2}}$$

```
[Out] a*x/e-b*n*x/e+b*x*ln(c*x^n)/e-arctan(x*e^(1/2)/d^(1/2))*(a+b*ln(c*x^n))*d^(
1/2)/e^(3/2)+1/2*I*b*n*polylog(2,-I*x*e^(1/2)/d^(1/2))*d^(1/2)/e^(3/2)-1/2*
I*b*n*polylog(2,I*x*e^(1/2)/d^(1/2))*d^(1/2)/e^(3/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {327, 211, 2393, 2332, 2361, 12, 4940, 2438}

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex^2}\,dx = -\frac{\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{3/2}}+\frac{ax}{e}+\frac{bx\log(cx^n)}{e} + \frac{ib\sqrt{d}n\operatorname{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2e^{3/2}}-\frac{ib\sqrt{d}n\operatorname{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2e^{3/2}}-\frac{bnx}{e}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/(d + e*x^2),x]
```

```
[Out] (a*x)/e - (b*n*x)/e + (b*x*Log[c*x^n])/e - (Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]*(a + b*Log[c*x^n]))/e^(3/2) + ((I/2)*b*Sqrt[d]*n*PolyLog[2, ((-I)*Sqrt
```

```
[e]*x)/Sqrt[d]])/e^(3/2) - ((I/2)*b*Sqrt[d]*n*PolyLog[2, (I*Sqrt[e]*x)/Sqrt
[d]])/e^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log(cx^n)}{e} - \frac{d(a+b\log(cx^n))}{e(d+ex^2)}\right) dx$$

$$= \frac{\int (a+b\log(cx^n))\,dx}{e} - \frac{d\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx}{e}$$

$$= \frac{ax}{e} - \frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{3/2}} + \frac{b\int \log(cx^n)\,dx}{e} + \frac{(bdn)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}\,dx}{e}$$

$$= \frac{ax}{e} - \frac{bnx}{e} + \frac{bx\log(cx^n)}{e} - \frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{3/2}} + \frac{\left(b\sqrt{d}n\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{e^{3/2}}$$

$$= \frac{ax}{e} - \frac{bnx}{e} + \frac{bx\log(cx^n)}{e} - \frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{3/2}} + \frac{\left(ib\sqrt{d}n\right)\int \frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2e^{3/2}} - \frac{\left(ib\sqrt{d}n\right)\int \frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2e^{3/2}}$$

$$= \frac{ax}{e} - \frac{bnx}{e} + \frac{bx\log(cx^n)}{e} - \frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{3/2}} + \frac{ib\sqrt{d}n\text{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2e^{3/2}} - \frac{ib\sqrt{d}n\text{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2e^{3/2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.29

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex^2}\,dx = \frac{2a\sqrt{e}x - 2b\sqrt{e}nx + 2b\sqrt{e}x\log(cx^n) - \sqrt{-d}(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right) + \sqrt{-d}(a+b\log(cx^n))\log\left(1\right.}{2e^{3/2}}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*x^2),x]
```

```
[Out] (2*a*Sqrt[e]*x - 2*b*Sqrt[e]*n*x + 2*b*Sqrt[e]*x*Log[c*x^n] - Sqrt[-d]*(a +
 b*Log[c*x^n])*Log[1 + (Sqrt[e]*x)/Sqrt[-d]] + Sqrt[-d]*(a + b*Log[c*x^n])*
Log[1 + (d*Sqrt[e]*x)/(-d)^(3/2)] + b*Sqrt[-d]*n*PolyLog[2, (Sqrt[e]*x)/Sqr
t[-d]] - b*Sqrt[-d]*n*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)])/(2*e^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.41 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.40

method	result
risch	$\frac{b\ln(x^n)x}{e}+\frac{bd\arctan\left(\frac{xe}{\sqrt{de}}\right)n\ln(x)}{e\sqrt{de}}-\frac{bd\arctan\left(\frac{xe}{\sqrt{de}}\right)\ln(x^n)}{e\sqrt{de}}-\frac{bnx}{e}-\frac{bnd\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2e\sqrt{-de}}+\frac{bnd\ln(x)\ln\left(\frac{ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2e\sqrt{-de}}$

```
[In] int(x^2*(a+b*ln(c*x^n))/(e*x^2+d),x,method=_RETURNVERBOSE)
[Out] b*ln(x^n)/e*x+b*d/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*n*ln(x)-b*d/e/(d*e)
^(1/2)*arctan(x*e/(d*e)^(1/2))*ln(x^n)-b*n*x/e-1/2*b*n*d/e*ln(x)/(-d*e)^(1/
2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*b*n*d/e*ln(x)/(-d*e)^(1/2)*ln((
e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*n*d/e/(-d*e)^(1/2)*dilog((-e*x+(-d*e)
^(1/2))/(-d*e)^(1/2))+1/2*b*n*d/e/(-d*e)^(1/2)*dilog((e*x+(-d*e)^(1/2))/(-d
*e)^(1/2))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn
(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csg
n(I*c*x^n)^3+b*ln(c)+a)*(x/e-d/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))
```

Fricas [F]

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex^2}\,dx=\int \frac{(b\log(cx^n)+a)x^2}{ex^2+d}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*x^2*log(c*x^n) + a*x^2)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex^2}\,dx=\int \frac{x^2(a+b\log(cx^n))}{d+ex^2}\,dx$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(e*x**2+d),x)
[Out] Integral(x**2*(a + b*log(c*x**n))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex^2}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{ex^2+d}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex^2}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,x^n))}{e\,x^2+d}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e*x^2),x)
```

```
[Out] int((x^2*(a + b*log(c*x^n)))/(d + e*x^2), x)
```

3.218 $\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 105

$$\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx = \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{e}} - \frac{ibn\,\text{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{e}} + \frac{ibn\,\text{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{e}}$$

```
[Out] arctan(x*e^(1/2)/d^(1/2))*(a+b*ln(c*x^n))/d^(1/2)/e^(1/2)-1/2*I*b*n*polylog
(2,-I*x*e^(1/2)/d^(1/2))/d^(1/2)/e^(1/2)+1/2*I*b*n*polylog(2,I*x*e^(1/2)/d^
(1/2))/d^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {211, 2361, 12, 4940, 2438}

$$\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx = \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{e}} - \frac{ibn\,\text{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{e}} + \frac{ibn\,\text{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{e}}$$

```
[In] Int[(a + b*Log[c*x^n])/(d + e*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/(Sqrt[d]*Sqrt[e]) - ((I/2)
*b*n*PolyLog[2, ((-I)*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]) + ((I/2)*b*n*P
olyLog[2, (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{e}} - (bn)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}\,dx\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{e}} - \frac{(bn)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{\sqrt{d}\sqrt{e}}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{e}} - \frac{(ibn)\int\frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2\sqrt{d}\sqrt{e}} + \frac{(ibn)\int\frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2\sqrt{d}\sqrt{e}}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{e}} - \frac{ibn\text{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{e}} + \frac{ibn\text{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{e}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02

$$\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx = \frac{-\left((a+b\log(cx^n))\left(\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)-\log\left(1+\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)\right)\right)+bn\,\text{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)-bn\,\text{PolyLog}\left(2,\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{2\sqrt{-d}\sqrt{e}}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*x^2),x]
```

```
[Out] (-((a + b*Log[c*x^n])*(Log[1 + (Sqrt[e]*x)/Sqrt[-d]] - Log[1 + (d*Sqrt[e]*x
)/(-d)^(3/2)])) + b*n*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]] - b*n*PolyLog[2, (d*
Sqrt[e]*x)/(-d)^(3/2)])/(2*Sqrt[-d]*Sqrt[e])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.50

method	result
risch	$-\frac{b\arctan\left(\frac{xe}{\sqrt{de}}\right)n\ln(x)}{\sqrt{de}}+\frac{b\arctan\left(\frac{xe}{\sqrt{de}}\right)\ln(x^n)}{\sqrt{de}}+\frac{bn\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2\sqrt{-de}}-\frac{bn\ln(x)\ln\left(\frac{ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2\sqrt{-de}}+\frac{bn\,\text{dilog}\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2\sqrt{-de}}$

```
[In] int((a+b*ln(c*x^n))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -b/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*n*ln(x)+b/(d*e)^(1/2)*arctan(x*e/(d*
e)^(1/2))*ln(x^n)+1/2*b*n*ln(x)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^
(1/2))-1/2*b*n*ln(x)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*b
*n/(-d*e)^(1/2)*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*n/(-d*e)^(1/2
)*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)
*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*
csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)/(d*e)^(1/2)*arctan(x*
e/(d*e)^(1/2))
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx = \int \frac{b\log(cx^n)+a}{ex^2+d}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx = \int \frac{a+b\log(cx^n)}{d+ex^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/(e*x**2+d),x)
[Out] Integral((a + b*log(c*x**n))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx = \int \frac{b\log(cx^n)+a}{ex^2+d}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{d+ex^2}\,dx=\int \frac{a+b\,\ln\left(c\,x^n\right)}{e\,x^2+d}\,dx$$

```
[In] int((a + b*log(c*x^n))/(d + e*x^2),x)
[Out] int((a + b*log(c*x^n))/(d + e*x^2), x)
```

3.219 $\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 134

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)}\,dx=-\frac{bn}{dx}-\frac{a+b\log(cx^n)}{dx}-\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}}+\frac{ib\sqrt{e}n\operatorname{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}}-\frac{ib\sqrt{e}n\operatorname{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}}$$

```
[Out] -b*n/d/x+(-a-b*ln(c*x^n))/d/x-arctan(x*e^(1/2)/d^(1/2))*(a+b*ln(c*x^n))*e^(
1/2)/d^(3/2)+1/2*I*b*n*polylog(2,-I*x*e^(1/2)/d^(1/2))*e^(1/2)/d^(3/2)-1/2*
I*b*n*polylog(2,I*x*e^(1/2)/d^(1/2))*e^(1/2)/d^(3/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2380, 2341, 211, 2361, 12, 4940, 2438}

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)}\,dx=-\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}}-\frac{a+b\log(cx^n)}{dx}+\frac{ib\sqrt{e}n\operatorname{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}}-\frac{ib\sqrt{e}n\operatorname{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}}-\frac{bn}{dx}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^2*(d + e*x^2)),x]
```

```
[Out] -((b*n)/(d*x)) - (a + b*Log[c*x^n])/(d*x) - (Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqr
t[d]]*(a + b*Log[c*x^n]))/d^(3/2) + ((I/2)*b*Sqrt[e]*n*PolyLog[2, ((-I)*Sqr
```

```
t[e]*x)/Sqrt[d]])/d^(3/2) - ((I/2)*b*Sqrt[e]*n*PolyLog[2, (I*Sqrt[e]*x)/Sqr
t[d]])/d^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d} - \frac{e\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx}{d}$$

$$= -\frac{bn}{dx} - \frac{a+b\log(cx^n)}{dx} - \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} + \frac{(ben)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}dx}{d}$$

$$= -\frac{bn}{dx} - \frac{a+b\log(cx^n)}{dx} - \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} + \frac{(b\sqrt{e}n)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{d^{3/2}}$$

$$= -\frac{bn}{dx} - \frac{a+b\log(cx^n)}{dx} - \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} + \frac{(ib\sqrt{e}n)\int \frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{2d^{3/2}} - \frac{(ib\sqrt{e}n)\int \frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{2d^{3/2}}$$

$$= -\frac{bn}{dx} - \frac{a+b\log(cx^n)}{dx} - \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{3/2}} + \frac{ib\sqrt{e}n\text{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}} - \frac{ib\sqrt{e}n\text{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.29

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)}dx = \frac{d\left(-2b(-d)^{3/2}n + 2\sqrt{-d}d(a+b\log(cx^n)) - d\sqrt{e}x(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right) + d\sqrt{e}x(a+b\log(cx^n))\right.}{2(-d)^{7/2}x}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*x^2)),x]
```

```
[Out] (d*(-2*b*(-d)^(3/2)*n + 2*Sqrt[-d]*d*(a + b*Log[c*x^n]) - d*Sqrt[e]*x*(a +
b*Log[c*x^n])*Log[1 + (Sqrt[e]*x)/Sqrt[-d]] + d*Sqrt[e]*x*(a + b*Log[c*x^n]
)*Log[1 + (d*Sqrt[e]*x)/(-d)^(3/2)] + b*d*Sqrt[e]*n*x*PolyLog[2, (Sqrt[e]*x
)/Sqrt[-d]] - b*d*Sqrt[e]*n*x*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)]))/(2*(-d
)^(7/2)*x)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.43

method	result
risch	$\frac{be\arctan\left(\frac{xe}{\sqrt{de}}\right)n\ln(x)}{d\sqrt{de}} - \frac{be\arctan\left(\frac{xe}{\sqrt{de}}\right)\ln(x^n)}{d\sqrt{de}} - \frac{b\ln(x^n)}{dx} - \frac{bne\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2d\sqrt{-de}} + \frac{bne\ln(x)\ln\left(\frac{ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2d\sqrt{-de}} - \frac{bne\,\text{dil}}{}$

```
[In] int((a+b*ln(c*x^n))/x^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] b*e/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*n*ln(x)-b*e/d/(d*e)^(1/2)*arctan(
x*e/(d*e)^(1/2))*ln(x^n)-b*ln(x^n)/d/x-1/2*b*n*e/d*ln(x)/(-d*e)^(1/2)*ln((-
e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*b*n*e/d*ln(x)/(-d*e)^(1/2)*ln((e*x+(-d*
e)^(1/2))/(-d*e)^(1/2))-1/2*b*n*e/d/(-d*e)^(1/2)*dilog((-e*x+(-d*e)^(1/2))/
(-d*e)^(1/2))+1/2*b*n*e/d/(-d*e)^(1/2)*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2
))-b*n/d/x+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn
(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csg
n(I*c*x^n)^3+b*ln(c)+a)*(-e/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-1/d/x)
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^2\right)}\,dx=\int \frac{b\log\left(cx^n\right)+a}{(ex^2+d)x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e*x^4 + d*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^2\right)}\,dx=\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^2\right)}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*log(c*x**n))/(x**2*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^2\right)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)}\,dx = \int \frac{b\log(cx^n)+a}{(ex^2+d)x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^2\,(e\,x^2+d)}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*x^2)),x)
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*x^2)), x)
```

3.220 $\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 165

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)}\,dx = -\frac{bn}{9dx^3}+\frac{ben}{d^2x}-\frac{a+b\log(cx^n)}{3dx^3}+\frac{e(a+b\log(cx^n))}{d^2x}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{5/2}}-\frac{ibe^{3/2}n\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{ibe^{3/2}n\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}}$$

```
[Out] -1/9*b*n/d/x^3+b*e*n/d^2/x+1/3*(-a-b*ln(c*x^n))/d/x^3+e*(a+b*ln(c*x^n))/d^2
/x+e^(3/2)*arctan(x*e^(1/2)/d^(1/2))*(a+b*ln(c*x^n))/d^(5/2)-1/2*I*b*e^(3/2
)*n*polylog(2,-I*x*e^(1/2)/d^(1/2))/d^(5/2)+1/2*I*b*e^(3/2)*n*polylog(2,I*x
*e^(1/2)/d^(1/2))/d^(5/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2380, 2341, 211, 2361, 12, 4940, 2438}

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)}\,dx = \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{5/2}}+\frac{e(a+b\log(cx^n))}{d^2x}-\frac{a+b\log(cx^n)}{3dx^3}-\frac{ibe^{3/2}n\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{ibe^{3/2}n\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{ben}{d^2x}-\frac{bn}{9dx^3}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^4*(d + e*x^2)),x]
```

```
[Out] -1/9*(b*n)/(d*x^3) + (b*e*n)/(d^2*x) - (a + b*Log[c*x^n])/(3*d*x^3) + (e*(a
 + b*Log[c*x^n]))/(d^2*x) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(a + b*Log
[c*x^n]))/d^(5/2) - ((I/2)*b*e^(3/2)*n*PolyLog[2, ((-I)*Sqrt[e]*x)/Sqrt[d]]
)/d^(5/2) + ((I/2)*b*e^(3/2)*n*PolyLog[2, (I*Sqrt[e]*x)/Sqrt[d]])/d^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{a+b\log(cx^n)}{x^4}\,dx}{d} - \frac{e\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)}\,dx}{d}$$

$$= -\frac{bn}{9dx^3} - \frac{a+b\log(cx^n)}{3dx^3} - \frac{e\int \frac{a+b\log(cx^n)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx}{d^2}$$

$$= -\frac{bn}{9dx^3} + \frac{ben}{d^2x} - \frac{a+b\log(cx^n)}{3dx^3} + \frac{e(a+b\log(cx^n))}{d^2x} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{5/2}} - \frac{(be^2n)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}\,dx}{d^2}$$

$$= -\frac{bn}{9dx^3} + \frac{ben}{d^2x} - \frac{a+b\log(cx^n)}{3dx^3} + \frac{e(a+b\log(cx^n))}{d^2x} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{5/2}} - \frac{(be^{3/2}n)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{d^{5/2}}$$

$$= -\frac{bn}{9dx^3} + \frac{ben}{d^2x} - \frac{a+b\log(cx^n)}{3dx^3} + \frac{e(a+b\log(cx^n))}{d^2x} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{5/2}} - \frac{(ibe^{3/2}n)\int \frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2d^{5/2}} + \frac{(ibe^{3/2}n)\int \frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2d^{5/2}}$$

$$= -\frac{bn}{9dx^3} + \frac{ben}{d^2x} - \frac{a+b\log(cx^n)}{3dx^3} + \frac{e(a+b\log(cx^n))}{d^2x} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{d^{5/2}} - \frac{ibe^{3/2}n\text{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}} + \frac{ibe^{3/2}n\text{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.28

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)}\,dx = \frac{1}{18}\left(-\frac{2bn}{dx^3} + \frac{18ben}{d^2x} - \frac{6(a+b\log(cx^n))}{dx^3} + \frac{18e(a+b\log(cx^n))}{d^2x} - \frac{9e^{3/2}(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{(-d)^{5/2}} + \frac{9e^{3/2}(a+b\log(cx^n))\log\left(1+\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{(-d)^{5/2}} + \frac{9be^{3/2}n\,\text{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{(-d)^{5/2}} - \frac{9be^{3/2}n\,\text{PolyLog}\left(2,\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{(-d)^{5/2}}\right)$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^4*(d + e*x^2)),x]
```

```
[Out] ((-2*b*n)/(d*x^3) + (18*b*e*n)/(d^2*x) - (6*(a + b*Log[c*x^n]))/(d*x^3) + (
18*e*(a + b*Log[c*x^n]))/(d^2*x) - (9*e^(3/2)*(a + b*Log[c*x^n])*Log[1 + (S
qrt[e]*x)/Sqrt[-d]])/(-d)^(5/2) + (9*e^(3/2)*(a + b*Log[c*x^n])*Log[1 + (d*
Sqrt[e]*x)/(-d)^(3/2)])/(-d)^(5/2) + (9*b*e^(3/2)*n*PolyLog[2, (Sqrt[e]*x)/
Sqrt[-d]])/(-d)^(5/2) - (9*b*e^(3/2)*n*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)]
)/(-d)^(5/2))/18
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.24

method	result
risch	$-\frac{b\,e^{2}\arctan\left(\frac{xe}{\sqrt{de}}\right)n\ln\left(x\right)}{d^{2}\sqrt{de}}+\frac{b\,e^{2}\arctan\left(\frac{xe}{\sqrt{de}}\right)\ln\left(x^{n}\right)}{d^{2}\sqrt{de}}-\frac{b\ln\left(x^{n}\right)}{3d\,x^{3}}+\frac{b\ln\left(x^{n}\right)e}{d^{2}x}+\frac{bn\,e^{2}\ln\left(x\right)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2d^{2}\sqrt{-de}}-\frac{bn\,e^{2}\ln\left(x\right)\ln\left(\right.}{2d^{2}\sqrt{-}}$

```
[In] int((a+b*ln(c*x^n))/x^4/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -b*e^2/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*n*ln(x)+b*e^2/d^2/(d*e)^(1/2
)*arctan(x*e/(d*e)^(1/2))*ln(x^n)-1/3*b*ln(x^n)/d/x^3+b*ln(x^n)*e/d^2/x+1/2
*b*n*e^2/d^2*ln(x)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*
n*e^2/d^2*ln(x)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*b*n*e^
2/d^2/(-d*e)^(1/2)*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*n*e^2/d^2/
(-d*e)^(1/2)*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/9*b*n/d/x^3+b*e*n/d^2
/x+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*cs
gn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^
n)^3+b*ln(c)+a)*(e^2/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-1/3/d/x^3+e/d^
2/x)
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{4}\left(d+ex^{2}\right)}\,dx=\int \frac{b\log\left(cx^{n}\right)+a}{(ex^{2}+d)x^{4}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e*x^6 + d*x^4), x)
```

Sympy [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^4\left(d+ex^2\right)}\,dx=\int \frac{a+b\log\left(cx^n\right)}{x^4\left(d+ex^2\right)}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**4/(e*x**2+d),x)
[Out] Integral((a + b*log(c*x**n))/(x**4*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(cx^n\right)}{x^4\left(d+ex^2\right)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^4\left(d+ex^2\right)}\,dx=\int \frac{b\log\left(cx^n\right)+a}{(ex^2+d)x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x^4\left(d+ex^2\right)}\,dx=\int \frac{a+b\,\ln\left(c\,x^n\right)}{x^4\,\left(e\,x^2+d\right)}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^4*(d + e*x^2)),x)
[Out] int((a + b*log(c*x^n))/(x^4*(d + e*x^2)), x)
```

3.221 $\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = -\frac{bnx^2}{4e^2}+\frac{x^2(a+b\log(cx^n))}{2e^2}+\frac{dx^2(a+b\log(cx^n))}{2e^2(d+ex^2)}-\frac{bdn\log(d+ex^2)}{4e^3}-\frac{d(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{e^3}-\frac{bdn\,\mathrm{PolyLog}\left(2,-\frac{ex^2}{d}\right)}{2e^3}$$

```
[Out] -1/4*b*n*x^2/e^2+1/2*x^2*(a+b*ln(c*x^n))/e^2+1/2*d*x^2*(a+b*ln(c*x^n))/e^2/
(e*x^2+d)-1/4*b*d*n*ln(e*x^2+d)/e^3-d*(a+b*ln(c*x^n))*ln(1+e*x^2/d)/e^3-1/2
*b*d*n*polylog(2,-e*x^2/d)/e^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {272, 45, 2393, 2341, 2373, 266, 2375, 2438}

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = -\frac{d\log\left(\frac{ex^2}{d}+1\right)(a+b\log(cx^n))}{e^3}+\frac{dx^2(a+b\log(cx^n))}{2e^2(d+ex^2)}+\frac{x^2(a+b\log(cx^n))}{2e^2}-\frac{bdn\,\mathrm{PolyLog}\left(2,-\frac{ex^2}{d}\right)}{2e^3}-\frac{bdn\log(d+ex^2)}{4e^3}-\frac{bnx^2}{4e^2}$$

```
[In] Int[(x^5*(a + b*Log[c*x^n]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(b*n*x^2)/e^2 + (x^2*(a + b*Log[c*x^n]))/(2*e^2) + (d*x^2*(a + b*Log[c
*x^n]))/(2*e^2*(d + e*x^2)) - (b*d*n*Log[d + e*x^2])/(4*e^3) - (d*(a + b*Lo
g[c*x^n])*Log[1 + (e*x^2)/d])/e^3 - (b*d*n*PolyLog[2, -((e*x^2)/d)])/(2*e^3
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x(a+b\log(cx^n))}{e^2}+\frac{d^2x(a+b\log(cx^n))}{e^2(d+ex^2)^2}-\frac{2dx(a+b\log(cx^n))}{e^2(d+ex^2)}\right)dx\\
&= \frac{\int x(a+b\log(cx^n))\,dx}{e^2}-\frac{(2d)\int \frac{x(a+b\log(cx^n))}{d+ex^2}\,dx}{e^2}+\frac{d^2\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^2}\,dx}{e^2}\\
&= -\frac{bnx^2}{4e^2}+\frac{x^2(a+b\log(cx^n))}{2e^2}+\frac{dx^2(a+b\log(cx^n))}{2e^2(d+ex^2)}\\
&\quad -\frac{d(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{e^3}+\frac{(bdn)\int \frac{\log\left(1+\frac{ex^2}{d}\right)}{x}\,dx}{e^3}-\frac{(bdn)\int \frac{x}{d+ex^2}\,dx}{2e^2}\\
&= -\frac{bnx^2}{4e^2}+\frac{x^2(a+b\log(cx^n))}{2e^2}+\frac{dx^2(a+b\log(cx^n))}{2e^2(d+ex^2)}-\frac{bdn\log(d+ex^2)}{4e^3}\\
&\quad -\frac{d(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{e^3}-\frac{bdn\text{Li}_2\left(-\frac{ex^2}{d}\right)}{2e^3}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.22

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^2}\,dx$$

$$= \frac{2ex^2(a-bn\log(x)+b\log(cx^n))-\frac{2d^2(a-bn\log(x)+b\log(cx^n))}{d+ex^2}-4d(a-bn\log(x)+b\log(cx^n))\log(d+ex^2)+}{}$$

```
[In] Integrate[(x^5*(a + b*Log[c*x^n]))/(d + e*x^2)^2,x]
```

[Out] (2*e*x^2*(a - b*n*Log[x] + b*Log[c*x^n]) - (2*d^2*(a - b*n*Log[x] + b*Log[c *x^n]))/(d + e*x^2) - 4*d*(a - b*n*Log[x] + b*Log[c*x^n])*Log[d + e*x^2] + b*n*((d*Sqrt[e]*x*Log[x])/((-I)*Sqrt[d] + Sqrt[e]*x) + (d*Sqrt[e]*x*Log[x]) /(I*Sqrt[d] + Sqrt[e]*x) + e*x^2*(-1 + 2*Log[x]) - d*Log[I*Sqrt[d] - Sqrt[e]*x] - d*Log[I*Sqrt[d] + Sqrt[e]*x] - 4*d*(Log[x]*Log[1 + (I*Sqrt[e]*x)/Sqr t[d]] + PolyLog[2, ((-I)*Sqrt[e]*x)/Sqrt[d]]) - 4*d*(Log[x]*Log[1 - (I*Sqrt [e]*x)/Sqrt[d]] + PolyLog[2, (I*Sqrt[e]*x)/Sqrt[d]])))/(4*e^3)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.54 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.72

method	result
risch	$-\frac{b\ln(x^n)d^2}{2e^3(e\,x^2+d)}-\frac{b\ln(x^n)d\ln(e\,x^2+d)}{e^3}+\frac{b\ln(x^n)x^2}{2e^2}-\frac{bn\,x^2}{4e^2}-\frac{bdn\ln(e\,x^2+d)}{4e^3}+\frac{bnd\ln(x)}{2e^3}+\frac{bnd\ln(x)\ln(e\,x^2+d)}{e^3}-$

[In] int(x^5*(a+b*ln(c*x^n))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)

[Out] -1/2*b*ln(x^n)*d^2/e^3/(e*x^2+d)-b*ln(x^n)*d/e^3*ln(e*x^2+d)+1/2*b*ln(x^n)/ e^2*x^2-1/4*b*n*x^2/e^2-1/4*b*d*n*ln(e*x^2+d)/e^3+1/2*b*n/e^3*d*ln(x)+b*n*d /e^3*ln(x)*ln(e*x^2+d)-b*n*d/e^3*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2)) -b*n*d/e^3*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-b*n*d/e^3*dilog((-e*x+ (-d*e)^(1/2))/(-d*e)^(1/2))-b*n*d/e^3*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csg n(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-1/2*d/e^2*(d/e/(e*x^2+d)+2/e*ln(e*x^2+d))+1/2*x^2/e^2)

Fricas [F]

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^2}\,dx=\int \frac{(b\log(cx^n)+a)x^5}{(ex^2+d)^2}\,dx$$

[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b*x^5*log(c*x^n) + a*x^5)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)

Sympy [A] (verification not implemented)

Time = 44.05 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.45

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^2}\,dx$$

$$= \frac{ad^2\left(\begin{cases}\frac{x^2}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x^2} & \text{otherwise}\end{cases}\right)}{2e^2} - \frac{ad\left(\begin{cases}\frac{x^2}{d} & \text{for } e=0\\ \frac{\log(d+ex^2)}{e} & \text{otherwise}\end{cases}\right)}{e^2} + \frac{ax^2}{2e^2}$$

$$- \frac{bd^2n\left(\begin{cases}\frac{x^2}{2d^2} & \text{for } e=0\\ -\frac{\log(x)}{de} + \frac{\log\left(\frac{d}{e}+x^2\right)}{2de} & \text{otherwise}\end{cases}\right)}{2e^2} + \frac{bd^2\left(\begin{cases}\frac{x^2}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x^2} & \text{otherwise}\end{cases}\right)\log(cx^n)}{2e^2}$$

$$+ \frac{bdn\left(\begin{cases}\frac{x^2}{2d} \\ \frac{\begin{cases}-\frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(d)\log(x) - \frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right) - \frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(d) + G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(d) - \frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{otherwise}\end{cases}}{e}\end{cases}\right)}{e^2}$$

$$- \frac{bd\left(\begin{cases}\frac{x^2}{d} & \text{for } e=0\\ \frac{\log(d+ex^2)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{e^2} - \frac{bnx^2}{4e^2} + \frac{bx^2\log(cx^n)}{2e^2}$$

```
[In] integrate(x**5*(a+b*ln(c*x**n))/(e*x**2+d)**2,x)
```

```
[Out] a*d**2*Piecewise((x**2/d**2, Eq(e, 0)), (-1/(d*e + e**2*x**2), True))/(2*e*
*2) - a*d*Piecewise((x**2/d, Eq(e, 0)), (log(d + e*x**2)/e, True))/e**2 + a
*x**2/(2*e**2) - b*d**2*n*Piecewise((x**2/(2*d**2), Eq(e, 0)), (-log(x)/(d*
e) + log(d/e + x**2)/(2*d*e), True))/(2*e**2) + b*d**2*Piecewise((x**2/d**2
, Eq(e, 0)), (-1/(d*e + e**2*x**2), True))*log(c*x**n)/(2*e**2) + b*d*n*Pie
cewise((x**2/(2*d), Eq(e, 0)), (Piecewise((-polylog(2, e*x**2*exp_polar(I*p
i)/d)/2, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x**2
*exp_polar(I*pi)/d)/2, Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x**2*e
xp_polar(I*pi)/d)/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x
)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x**
```

```
2*exp_polar(I*pi)/d)/2, True))/e, True))/e**2 - b*d*Piecewise((x**2/d, Eq(e
, 0)), (log(d + e*x**2)/e, True))*log(c*x**n)/e**2 - b*n*x**2/(4*e**2) + b*
x**2*log(c*x**n)/(2*e**2)
```

Maxima [F]

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \int \frac{(b\log(cx^n)+a)x^5}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] -1/2*a*(d^2/(e^4*x^2 + d*e^3) - x^2/e^2 + 2*d*log(e*x^2 + d)/e^3) + b*integ
rate((x^5*log(c) + x^5*log(x^n))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Giac [F]

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \int \frac{(b\log(cx^n)+a)x^5}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^5/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \int \frac{x^5\,(a+b\,\ln(c\,x^n))}{(e\,x^2+d)^2}\,dx$$

```
[In] int((x^5*(a + b*log(c*x^n)))/(d + e*x^2)^2,x)
[Out] int((x^5*(a + b*log(c*x^n)))/(d + e*x^2)^2, x)
```

3.222 $\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^2} dx = -\frac{x^2(a+b\log(cx^n))}{2e(d+ex^2)} + \frac{bn\log(d+ex^2)}{4e^2} + \frac{(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{2e^2} + \frac{bn\,\text{PolyLog}\left(2,-\frac{ex^2}{d}\right)}{4e^2}$$

```
[Out] -1/2*x^2*(a+b*ln(c*x^n))/e/(e*x^2+d)+1/4*b*n*ln(e*x^2+d)/e^2+1/2*(a+b*ln(c*
x^n))*ln(1+e*x^2/d)/e^2+1/4*b*n*polylog(2,-e*x^2/d)/e^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {272, 45, 2393, 2373, 266, 2375, 2438}

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^2} dx = \frac{\log\left(\frac{ex^2}{d}+1\right)(a+b\log(cx^n))}{2e^2} - \frac{x^2(a+b\log(cx^n))}{2e(d+ex^2)} + \frac{bn\,\text{PolyLog}\left(2,-\frac{ex^2}{d}\right)}{4e^2} + \frac{bn\log(d+ex^2)}{4e^2}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(x^2*(a + b*Log[c*x^n]))/(e*(d + e*x^2)) + (b*n*Log[d + e*x^2])/(4*e^2
) + ((a + b*Log[c*x^n])*Log[1 + (e*x^2)/d])/(2*e^2) + (b*n*PolyLog[2, -((e*
x^2)/d)])/(4*e^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{dx(a+b\log(cx^n))}{e(d+ex^2)^2} + \frac{x(a+b\log(cx^n))}{e(d+ex^2)}\right) dx$$

$$= \frac{\int \frac{x(a+b\log(cx^n))}{d+ex^2} dx}{e} - \frac{d\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^2} dx}{e}$$

$$= -\frac{x^2(a+b\log(cx^n))}{2e(d+ex^2)} + \frac{(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{2e^2} - \frac{(bn)\int \frac{\log\left(1+\frac{ex^2}{d}\right)}{x} dx}{2e^2} + \frac{(bn)\int \frac{x}{d+ex^2} dx}{2e}$$

$$= -\frac{x^2(a+b\log(cx^n))}{2e(d+ex^2)} + \frac{bn\log(d+ex^2)}{4e^2} + \frac{(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{2e^2} + \frac{bn\text{Li}_2\left(-\frac{ex^2}{d}\right)}{4e^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 321, normalized size of antiderivative = 3.38

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^2} dx$$

$$= \frac{\frac{2d(a-bn\log(x)+b\log(cx^n))}{d+ex^2} + 2(a-bn\log(x)+b\log(cx^n))\log(d+ex^2) + \frac{bn\left(-2ex^2\log(x)+d\log\left(i\sqrt{d}-\sqrt{e}x\right)+ex^2\log\left(i\sqrt{d}\right.\right.}{}}{}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/(d + e*x^2)^2,x]
```

```
[Out] ((2*d*(a - b*n*Log[x] + b*Log[c*x^n]))/(d + e*x^2) + 2*(a - b*n*Log[x] + b*
Log[c*x^n])*Log[d + e*x^2] + (b*n*(-2*e*x^2*Log[x] + d*Log[I*Sqrt[d] - Sqrt
[e]*x] + e*x^2*Log[I*Sqrt[d] - Sqrt[e]*x] + d*Log[I*Sqrt[d] + Sqrt[e]*x] +
e*x^2*Log[I*Sqrt[d] + Sqrt[e]*x] + 2*d*Log[x]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]
] + 2*e*x^2*Log[x]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]] + 2*d*Log[x]*Log[1 + (I*S
qrt[e]*x)/Sqrt[d]] + 2*e*x^2*Log[x]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]] + 2*(d +
 e*x^2)*PolyLog[2, ((-I)*Sqrt[e]*x)/Sqrt[d]] + 2*(d + e*x^2)*PolyLog[2, (I*
Sqrt[e]*x)/Sqrt[d]]))/(d + e*x^2))/(4*e^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.47 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.21

method	result
risch	$\frac{b\ln(x^n)d}{2e^2(e\,x^2+d)}+\frac{b\ln(x^n)\ln(e\,x^2+d)}{2e^2}+\frac{bn\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2e^2}+\frac{bn\ln(x)\ln\left(\frac{ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2e^2}-\frac{bn\ln(x)\ln(e\,x^2+d)}{2e^2}+\frac{bn\,\mathrm{dilog}\left(\right.}{2}$

```
[In] int(x^3*(a+b*ln(c*x^n))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*ln(x^n)*d/e^2/(e*x^2+d)+1/2*b*ln(x^n)/e^2*ln(e*x^2+d)+1/2*b*n/e^2*ln(
x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*b*n/e^2*ln(x)*ln((e*x+(-d*e)^(1
/2))/(-d*e)^(1/2))-1/2*b*n/e^2*ln(x)*ln(e*x^2+d)+1/2*b*n/e^2*dilog((-e*x+(-
d*e)^(1/2))/(-d*e)^(1/2))+1/2*b*n/e^2*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)
)+1/4*b*n*ln(e*x^2+d)/e^2-1/2*b*n/e^2*ln(x)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x
^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^
n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/2*d/e^2/(e*x^2+
d)+1/2/e^2*ln(e*x^2+d))
```

Fricas [F]

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^2}\,dx=\int \frac{(b\log(cx^n)+a)x^3}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*log(c*x^n) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^2}\,dx=\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^2}\,dx$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**3*(a + b*log(c*x**n))/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^3(a + b \log(cx^n))}{(d + ex^2)^2} \, dx = \int \frac{(b \log(cx^n) + a)x^3}{(ex^2 + d)^2} \, dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate((x^3*log(c)
+ x^3*log(x^n))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Giac [F]

$$\int \frac{x^3(a + b \log(cx^n))}{(d + ex^2)^2} \, dx = \int \frac{(b \log(cx^n) + a)x^3}{(ex^2 + d)^2} \, dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^3/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b \log(cx^n))}{(d + ex^2)^2} \, dx = \int \frac{x^3 \, (a + b \, \ln(c \, x^n))}{(e \, x^2 + d)^2} \, dx$$

```
[In] int((x^3*(a + b*log(c*x^n)))/(d + e*x^2)^2,x)
```

```
[Out] int((x^3*(a + b*log(c*x^n)))/(d + e*x^2)^2, x)
```

3.223 $\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^2} dx = \frac{x^2(a+b\log(cx^n))}{2d(d+ex^2)} - \frac{bn\log(d+ex^2)}{4de}$$

```
[Out] 1/2*x^2*(a+b*ln(c*x^n))/d/(e*x^2+d)-1/4*b*n*ln(e*x^2+d)/d/e
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2373, 266}

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^2} dx = \frac{x^2(a+b\log(cx^n))}{2d(d+ex^2)} - \frac{bn\log(d+ex^2)}{4de}$$

```
[In] Int[(x*(a + b*Log[c*x^n]))/(d + e*x^2)^2,x]
```

```
[Out] (x^2*(a + b*Log[c*x^n]))/(2*d*(d + e*x^2)) - (b*n*Log[d + e*x^2])/(4*d*e)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
```

```
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^2(a+b\log(cx^n))}{2d(d+ex^2)} - \frac{(bn)\int \frac{x}{d+ex^2}\,dx}{2d}\\ &= \frac{x^2(a+b\log(cx^n))}{2d(d+ex^2)} - \frac{bn\log(d+ex^2)}{4de}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.48

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = -\frac{2ad-2bn(d+ex^2)\log(x)+2bd\log(cx^n)+bdn\log(d+ex^2)+benx^2\log(d+ex^2)}{4de(d+ex^2)}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e*x^2)^2,x]
[Out] -1/4*(2*a*d - 2*b*n*(d + e*x^2)*Log[x] + 2*b*d*Log[c*x^n] + b*d*n*Log[d + e
*x^2] + b*e*n*x^2*Log[d + e*x^2])/(d*e*(d + e*x^2))
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.48

method	result
parallelrisch	$\frac{-\ln(e\,x^2+d)x^2be\,n^2+2x^2\ln(c\,x^n)ben-\ln(e\,x^2+d)bd\,n^2-2adn}{4den(e\,x^2+d)}$
risch	$-\frac{b\ln(x^n)}{2e(e\,x^2+d)} - \frac{-i\pi bd\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+i\pi bd\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+i\pi bd\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-i\pi bd\,\mathrm{csgn}(ic}{4(e\,x^2+d)ed}$

```
[In] int(x*(a+b*ln(c*x^n))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*(-ln(e*x^2+d)*x^2*b*e*n^2+2*x^2*ln(c*x^n)*b*e*n-ln(e*x^2+d)*b*d*n^2-2*a
*d*n)/d/e/n/(e*x^2+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \frac{2\,benx^2\log(x) - 2\,bd\log(c) - 2\,ad - (benx^2 + bdn)\log(ex^2+d)}{4\,(de^2x^2 + d^2e)}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*e*n*x^2*log(x) - 2*b*d*log(c) - 2*a*d - (b*e*n*x^2 + b*d*n)*log(e*
x^2 + d))/(d*e^2*x^2 + d^2*e)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(39) = 78.

Time = 23.12 (sec) , antiderivative size = 292, normalized size of antiderivative = 5.84

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^2}\,dx$$

$$= \begin{cases} \tilde{\infty}\left(-\frac{a}{2x^2} - \frac{bn}{4x^2} - \frac{b\log(cx^n)}{2x^2}\right) \\ \frac{\frac{ax^2}{2} - \frac{bnx^2}{4} + \frac{bx^2\log(cx^n)}{2}}{d^2} \\ \frac{-\frac{a}{2x^2} - \frac{bn}{4x^2} - \frac{b\log(cx^n)}{2x^2}}{e^2} \\ -\frac{2ad}{4d^2e+4de^2x^2} - \frac{bdn\log\left(x-\sqrt{-\frac{d}{e}}\right)}{4d^2e+4de^2x^2} - \frac{bdn\log\left(x+\sqrt{-\frac{d}{e}}\right)}{4d^2e+4de^2x^2} - \frac{benx^2\log\left(x-\sqrt{-\frac{d}{e}}\right)}{4d^2e+4de^2x^2} - \frac{benx^2\log\left(x+\sqrt{-\frac{d}{e}}\right)}{4d^2e+4de^2x^2} + \frac{2bex^2\log(cx^n)}{4d^2e+4de^2x^2} \end{cases}$$

```
[In] integrate(x*(a+b*ln(c*x**n))/(e*x**2+d)**2,x)
```

```
[Out] Piecewise((zoo*(-a/(2*x**2) - b*n/(4*x**2) - b*log(c*x**n)/(2*x**2)), Eq(d,
 0) & Eq(e, 0)), ((a*x**2/2 - b*n*x**2/4 + b*x**2*log(c*x**n)/2)/d**2, Eq(e
, 0)), ((-a/(2*x**2) - b*n/(4*x**2) - b*log(c*x**n)/(2*x**2))/e**2, Eq(d, 0
)), (-2*a*d/(4*d**2*e + 4*d*e**2*x**2) - b*d*n*log(x - sqrt(-d/e))/(4*d**2*
e + 4*d*e**2*x**2) - b*d*n*log(x + sqrt(-d/e))/(4*d**2*e + 4*d*e**2*x**2) -
 b*e*n*x**2*log(x - sqrt(-d/e))/(4*d**2*e + 4*d*e**2*x**2) - b*e*n*x**2*log
(x + sqrt(-d/e))/(4*d**2*e + 4*d*e**2*x**2) + 2*b*e*x**2*log(c*x**n)/(4*d**
2*e + 4*d*e**2*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = -\frac{1}{4}bn\left(\frac{\log(ex^2+d)}{de}-\frac{\log(x^2)}{de}\right)-\frac{b\log(cx^n)}{2(e^2x^2+de)}-\frac{a}{2(e^2x^2+de)}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] -1/4*b*n*(log(e*x^2 + d)/(d*e) - log(x^2)/(d*e)) - 1/2*b*log(c*x^n)/(e^2*x^
2 + d*e) - 1/2*a/(e^2*x^2 + d*e)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = -\frac{bn\log(x)}{2(e^2x^2+de)}-\frac{bn\log(ex^2+d)}{4de}+\frac{bn\log(x)}{2de}-\frac{b\log(c)+a}{2(e^2x^2+de)}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="giac")
[Out] -1/2*b*n*log(x)/(e^2*x^2 + d*e) - 1/4*b*n*log(e*x^2 + d)/(d*e) + 1/2*b*n*lo
g(x)/(d*e) - 1/2*(b*log(c) + a)/(e^2*x^2 + d*e)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.46

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \frac{b\,n\,\ln(x)}{2\,d\,e}-\frac{b\,\ln(c\,x^n)}{2\,(e^2\,x^2+d\,e)}-\frac{b\,n\,\ln(e\,x^2+d)}{4\,d\,e}-\frac{a}{2\,e^2\,x^2+2\,d\,e}$$

```
[In] int((x*(a + b*log(c*x^n)))/(d + e*x^2)^2,x)
[Out] (b*n*log(x))/(2*d*e) - (b*log(c*x^n))/(2*(d*e + e^2*x^2)) - (b*n*log(d + e*
x^2))/(4*d*e) - a/(2*d*e + 2*e^2*x^2)
```

3.224 $\int \frac{a+b\log(cx^n)}{x(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^2}\,dx = \frac{a+b\log(cx^n)}{2d(d+ex^2)} - \frac{\log\left(1+\frac{d}{ex^2}\right)(2a-bn+2b\log(cx^n))}{4d^2} + \frac{bn\,\text{PolyLog}\left(2,-\frac{d}{ex^2}\right)}{4d^2}$$

```
[Out] 1/2*(a+b*ln(c*x^n))/d/(e*x^2+d)-1/4*ln(1+d/e/x^2)*(2*a-b*n+2*b*ln(c*x^n))/d
^2+1/4*b*n*polylog(2,-d/e/x^2)/d^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2385, 2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^2}\,dx = -\frac{\log\left(\frac{d}{ex^2}+1\right)(2a+2b\log(cx^n)-bn)}{4d^2} + \frac{a+b\log(cx^n)}{2d(d+ex^2)} + \frac{bn\,\text{PolyLog}\left(2,-\frac{d}{ex^2}\right)}{4d^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x^2)^2),x]
```

```
[Out] (a + b*Log[c*x^n])/(2*d*(d + e*x^2)) - (Log[1 + d/(e*x^2)]*(2*a - b*n + 2*b
*Log[c*x^n]))/(4*d^2) + (b*n*PolyLog[2, -(d/(e*x^2))])/(4*d^2)
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
```

```
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2385

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^2)^(q_.), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(q + 1)*((a +
 b*Log[c*x^n])/(2*d*f*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(f*x)^m*(d
+ e*x^2)^(q + 1)*(a*(m + 2*q + 3) + b*n + b*(m + 2*q + 3)*Log[c*x^n]), x],
x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a+b\log(cx^n)}{2d(d+ex^2)}-\frac{\int\frac{-2a+bn-2b\log(cx^n)}{x(d+ex^2)}\,dx}{2d}\\
&= \frac{a+b\log(cx^n)}{2d(d+ex^2)}-\frac{\log\left(1+\frac{d}{ex^2}\right)(2a-bn+2b\log(cx^n))}{4d^2}+\frac{(bn)\int\frac{\log\left(1+\frac{d}{ex^2}\right)}{x}\,dx}{2d^2}\\
&= \frac{a+b\log(cx^n)}{2d(d+ex^2)}-\frac{\log\left(1+\frac{d}{ex^2}\right)(2a-bn+2b\log(cx^n))}{4d^2}+\frac{bn\text{Li}_2\left(-\frac{d}{ex^2}\right)}{4d^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.40

$$\int\frac{a+b\log(cx^n)}{x(d+ex^2)^2}\,dx=\frac{a-bn\log(x)+b\log(cx^n)}{2d^2+2dex^2}$$
$$+\frac{\log(x)(a-bn\log(x)+b\log(cx^n))}{d^2}-\frac{(a-bn\log(x)+b\log(cx^n))\log(d+ex^2)}{2d^2}$$
$$+\frac{bn\left(\frac{\sqrt{e}x\log(x)}{i\sqrt{d}-\sqrt{e}x}-\frac{\sqrt{e}x\log(x)}{i\sqrt{d}+\sqrt{e}x}+2\log^2(x)+\log\left(i\sqrt{d}-\sqrt{e}x\right)+\log\left(i\sqrt{d}+\sqrt{e}x\right)-2\left(\log(x)\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)\right.}{4d^2}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x^2)^2),x]
```

```
[Out] (a - b*n*Log[x] + b*Log[c*x^n])/(2*d^2 + 2*d*e*x^2) + (Log[x]*(a - b*n*Log[
x] + b*Log[c*x^n]))/d^2 - ((a - b*n*Log[x] + b*Log[c*x^n])*Log[d + e*x^2])/
```

(2*d^2) + (b*n*((Sqrt[e]*x*Log[x])/(I*Sqrt[d] - Sqrt[e]*x) - (Sqrt[e]*x*Log[x])/(I*Sqrt[d] + Sqrt[e]*x) + 2*Log[x]^2 + Log[I*Sqrt[d] - Sqrt[e]*x] + Log[I*Sqrt[d] + Sqrt[e]*x] - 2*(Log[x]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]] + PolyLog[2, ((-I)*Sqrt[e]*x)/Sqrt[d]]) - 2*(Log[x]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]] + PolyLog[2, (I*Sqrt[e]*x)/Sqrt[d]])))/(4*d^2)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.59 (sec) , antiderivative size = 338, normalized size of antiderivative = 4.12

method	result
risch	$\frac{b\ln(x^n)}{2d(e\,x^2+d)}-\frac{b\ln(x^n)\ln(e\,x^2+d)}{2d^2}+\frac{b\ln(x^n)\ln(x)}{d^2}+\frac{bn\ln(e\,x^2+d)}{4d^2}-\frac{bn\ln(x)}{2d^2}-\frac{bn\ln(x)^2}{2d^2}+\frac{bn\ln(x)\ln(e\,x^2+d)}{2d^2}-$ $bn\ln$

[In] int((a+b*ln(c*x^n))/x/(e*x^2+d)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*b*ln(x^n)/d/(e*x^2+d)-1/2*b*ln(x^n)/d^2*ln(e*x^2+d)+b*ln(x^n)/d^2*ln(x)+1/4*b*n/d^2*ln(e*x^2+d)-1/2*b*n/d^2*ln(x)-1/2*b*n/d^2*ln(x)^2+1/2*b*n/d^2*ln(x)*ln(e*x^2+d)-1/2*b*n/d^2*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*n/d^2*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*n/d^2*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*n/d^2*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-1/2*e/d^2*(-d/e/(e*x^2+d)+1/e*ln(e*x^2+d))+1/d^2*ln(x))

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^2}\,dx=\int \frac{b\log(cx^n)+a}{(ex^2+d)^2x}\,dx$$

[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b*log(c*x^n) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x\left(d+ex^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*x**n))/x/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x\left(d+ex^2\right)^2}\,dx = \int \frac{b\log\left(cx^n\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + b*integrate
((log(c) + log(x^n))/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x\left(d+ex^2\right)^2}\,dx = \int \frac{b\log\left(cx^n\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x\left(d+ex^2\right)^2}\,dx = \int \frac{a+b\,\ln\left(c\,x^n\right)}{x\,{\left(e\,x^2+d\right)}^2}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*log(c*x^n))/(x*(d + e*x^2)^2), x)
```

3.225 $\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^2}\,dx = -\frac{bn}{2d^2x^2}+\frac{a+b\log(cx^n)}{2dx^2(d+ex^2)}-\frac{4a-bn+4b\log(cx^n)}{4d^2x^2}+\frac{e\log\left(1+\frac{d}{ex^2}\right)(4a-bn+4b\log(cx^n))}{4d^3}-\frac{ben\operatorname{PolyLog}\left(2,-\frac{d}{ex^2}\right)}{2d^3}$$

```
[Out] -1/2*b*n/d^2/x^2+1/2*(a+b*ln(c*x^n))/d/x^2/(e*x^2+d)+1/4*(-4*a+b*n-4*b*ln(c
*x^n))/d^2/x^2+1/4*e*ln(1+d/e/x^2)*(4*a-b*n+4*b*ln(c*x^n))/d^3-1/2*b*e*n*po
lylog(2,-d/e/x^2)/d^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2385, 2380, 2341, 2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^2}\,dx = \frac{e\log\left(\frac{d}{ex^2}+1\right)(4a+4b\log(cx^n)-bn)}{4d^3}-\frac{4a+4b\log(cx^n)-bn}{4d^2x^2}+\frac{a+b\log(cx^n)}{2dx^2(d+ex^2)}-\frac{ben\operatorname{PolyLog}\left(2,-\frac{d}{ex^2}\right)}{2d^3}-\frac{bn}{2d^2x^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^3*(d + e*x^2)^2),x]
```

```
[Out] -1/2*(b*n)/(d^2*x^2) + (a + b*Log[c*x^n])/(2*d*x^2*(d + e*x^2)) - (4*a - b*
n + 4*b*Log[c*x^n])/(4*d^2*x^2) + (e*Log[1 + d/(e*x^2)]*(4*a - b*n + 4*b*Lo
g[c*x^n]))/(4*d^3) - (b*e*n*PolyLog[2, -(d/(e*x^2))])/(2*d^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2385

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^2)^(q_.), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(q + 1)*((a +
 b*Log[c*x^n])/(2*d*f*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(f*x)^m*(d
+ e*x^2)^(q + 1)*(a*(m + 2*q + 3) + b*n + b*(m + 2*q + 3)*Log[c*x^n]), x],
x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a+b\log\left(cx^n\right)}{2dx^2\left(d+ex^2\right)}-\frac{\int \frac{-4a+bn-4b\log(cx^n)}{x^3(d+ex^2)}\,dx}{2d}\\
&= \frac{a+b\log\left(cx^n\right)}{2dx^2\left(d+ex^2\right)}-\frac{\int \frac{-4a+bn-4b\log(cx^n)}{x^3}\,dx}{2d^2}+\frac{e\int \frac{-4a+bn-4b\log(cx^n)}{x(d+ex^2)}\,dx}{2d^2}\\
&= -\frac{bn}{2d^2x^2}+\frac{a+b\log\left(cx^n\right)}{2dx^2\left(d+ex^2\right)}-\frac{4a-bn+4b\log\left(cx^n\right)}{4d^2x^2}\\
&\quad+\frac{e\log\left(1+\frac{d}{ex^2}\right)\left(4a-bn+4b\log\left(cx^n\right)\right)}{4d^3}-\frac{(ben)\int \frac{\log\left(1+\frac{d}{ex^2}\right)}{x}\,dx}{d^3}
\end{aligned}
$$

$$= -\frac{bn}{2d^2x^2} + \frac{a+b\log(cx^n)}{2dx^2(d+ex^2)} - \frac{4a-bn+4b\log(cx^n)}{4d^2x^2} + \frac{e\log\left(1+\frac{d}{ex^2}\right)(4a-bn+4b\log(cx^n))}{4d^3} - \frac{benLi_2\left(-\frac{d}{ex^2}\right)}{2d^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.65

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^2}\,dx$$

$$= \frac{-\frac{2d(a-bn\log(x)+b\log(cx^n))}{x^2} - \frac{2de(a-bn\log(x)+b\log(cx^n))}{d+ex^2} - 8e\log(x)(a-bn\log(x)+b\log(cx^n)) + 4e(a-bn\log(}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*(d + e*x^2)^2),x]
```

```
[Out] ((-2*d*(a - b*n*Log[x] + b*Log[c*x^n]))/x^2 - (2*d*e*(a - b*n*Log[x] + b*Lo
g[c*x^n]))/(d + e*x^2) - 8*e*Log[x]*(a - b*n*Log[x] + b*Log[c*x^n]) + 4*e*(
a - b*n*Log[x] + b*Log[c*x^n])*Log[d + e*x^2] + b*n*((e^(3/2)*x*Log[x])/((-
I)*Sqrt[d] + Sqrt[e]*x) - 4*e*Log[x]^2 - (d + 2*d*Log[x])/x^2 - e*Log[I*Sqr
t[d] - Sqrt[e]*x] + ((-I)*e^(3/2)*x*Log[x] + e*(-Sqrt[d] + I*Sqrt[e]*x)*Log
[I*Sqrt[d] + Sqrt[e]*x])/(Sqrt[d] - I*Sqrt[e]*x) + 4*e*(Log[x]*Log[1 + (I*S
qrt[e]*x)/Sqrt[d]] + PolyLog[2, ((-I)*Sqrt[e]*x)/Sqrt[d]]) + 4*e*(Log[x]*Lo
g[1 - (I*Sqrt[e]*x)/Sqrt[d]] + PolyLog[2, (I*Sqrt[e]*x)/Sqrt[d]])))/(4*d^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.69 (sec) , antiderivative size = 380, normalized size of antiderivative = 3.02

method	result
risch	$-\frac{b\ln(x^n)e}{2d^2(e\,x^2+d)} + \frac{b\ln(x^n)e\ln(e\,x^2+d)}{d^3} - \frac{b\ln(x^n)}{2d^2x^2} - \frac{2b\ln(x^n)e\ln(x)}{d^3} + \frac{bne\ln(x)^2}{d^3} - \frac{bne\ln(x)\ln(e\,x^2+d)}{d^3} + \frac{bne\ln(x)\ln\left(\right.}{d^3}$

```
[In] int((a+b*ln(c*x^n))/x^3/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*ln(x^n)*e/d^2/(e*x^2+d)+b*ln(x^n)*e/d^3*ln(e*x^2+d)-1/2*b*ln(x^n)/d^
2/x^2-2*b*ln(x^n)/d^3*e*ln(x)+b*n/d^3*e*ln(x)^2-b*n/d^3*e*ln(x)*ln(e*x^2+d)
+b*n/d^3*e*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+b*n/d^3*e*ln(x)*ln((e
*x+(-d*e)^(1/2))/(-d*e)^(1/2))+b*n/d^3*e*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(
```

```
1/2))+b*n/d^3*e*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/4*b*n*e/d^3*ln(e*x
^2+d)-1/4*b*n/d^2/x^2+1/2*b*n/d^3*e*ln(x)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n
)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)
*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/2*e^2/d^3*(-d/e/(
e*x^2+d)+2/e*ln(e*x^2+d))-1/2/d^2/x^2-2/d^3*e*ln(x))
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^2\right)^2}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex^2+d\right)^2x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^3), x)
```

Sympy [A] (verification not implemented)

Time = 157.73 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.89

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^2}\,dx = \frac{ae^2\left(\begin{cases}\frac{x^2}{2d^2} & \text{for } e=0\\ -\frac{1}{2de+2e^2x^2} & \text{otherwise}\end{cases}\right)}{d^2} - \frac{a}{2d^2x^2} - \frac{2ae\log(x)}{d^3} + \frac{ae\log(d+ex^2)}{d^3} - \frac{be^2n\left(\begin{cases}\frac{x^2}{2d^2} & \text{for } e=0\\ -\frac{\log(x)}{de}+\frac{\log\left(\frac{d}{e}+x^2\right)}{2de} & \text{otherwise}\end{cases}\right)}{2d^2} + \frac{be^2\left(\begin{cases}\frac{x^2}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x^2} & \text{otherwise}\end{cases}\right)\log(cx^n)}{2d^2} - \frac{bn}{4d^2x^2} - \frac{b\log(cx^n)}{2d^2x^2} - \frac{be^2n\left(\begin{cases}\frac{x^2}{2d} \\ \frac{\begin{cases}-\frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1\wedge|x|<1\\ \log(d)\log(x)-\frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right)-\frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(d)-\frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{otherwise}\end{cases}}{e}\end{cases}\right)}{d^3} + \frac{be^2\left(\begin{cases}\frac{x^2}{d} & \text{for } e=0\\ \frac{\log(d+ex^2)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^3} + \frac{ben\log(x^2)^2}{4d^3} - \frac{be\log(x^2)\log(cx^n)}{d^3}$$

```
[In] integrate((a+b*ln(c*x**n))/x**3/(e*x**2+d)**2,x)
```

```
[Out] a*e**2*Piecewise((x**2/(2*d**2), Eq(e, 0)), (-1/(2*d*e + 2*e**2*x**2), True
))/d**2 - a/(2*d**2*x**2) - 2*a*e*log(x)/d**3 + a*e*log(d + e*x**2)/d**3 -
b*e**2*n*Piecewise((x**2/(2*d**2), Eq(e, 0)), (-log(x)/(d*e) + log(d/e + x*
*2)/(2*d*e), True))/(2*d**2) + b*e**2*Piecewise((x**2/d**2, Eq(e, 0)), (-1/
(d*e + e**2*x**2), True))*log(c*x**n)/(2*d**2) - b*n/(4*d**2*x**2) - b*log(
c*x**n)/(2*d**2*x**2) - b*e**2*n*Piecewise((x**2/(2*d), Eq(e, 0)), (Piecewi
se((-polylog(2, e*x**2*exp_polar(I*pi)/d)/2, (Abs(x) < 1) & (1/Abs(x) < 1))
, (log(d)*log(x) - polylog(2, e*x**2*exp_polar(I*pi)/d)/2, Abs(x) < 1), (-l
og(d)*log(1/x) - polylog(2, e*x**2*exp_polar(I*pi)/d)/2, 1/Abs(x) < 1), (-m
```

```
eijerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (
0, 0)), x)*log(d) - polylog(2, e*x**2*exp_polar(I*pi)/d)/2, True))/e, True)
)/d**3 + b*e**2*Piecewise((x**2/d, Eq(e, 0)), (log(d + e*x**2)/e, True))*lo
g(c*x**n)/d**3 + b*e*n*log(x**2)**2/(4*d**3) - b*e*log(x**2)*log(c*x**n)/d*
*3
```

Maxima [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^2\right)^2}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex^2+d\right)^2x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*((2*e*x^2 + d)/(d^2*e*x^4 + d^3*x^2) - 2*e*log(e*x^2 + d)/d^3 + 4*e*
log(x)/d^3) + b*integrate((log(c) + log(x^n))/(e^2*x^7 + 2*d*e*x^5 + d^2*x^
3), x)
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^2\right)^2}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex^2+d\right)^2x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\,\ln\left(c\,x^n\right)}{x^3\,\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^3*(d + e*x^2)^2), x)
```

3.226 $\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 191

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \frac{ax}{e^2}-\frac{bnx}{e^2}-\frac{b\sqrt{d}n\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{5/2}}+\frac{bx\log(cx^n)}{e^2}+\frac{dx(a+b\log(cx^n))}{2e^2(d+ex^2)}-\frac{3\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{5/2}}+\frac{3ib\sqrt{d}n\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4e^{5/2}}-\frac{3ib\sqrt{d}n\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4e^{5/2}}$$

```
[Out] a*x/e^2-b*n*x/e^2+b*x*ln(c*x^n)/e^2+1/2*d*x*(a+b*ln(c*x^n))/e^2/(e*x^2+d)-1
/2*b*n*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(5/2)-3/2*arctan(x*e^(1/2)/d^(1/
2))*(a+b*ln(c*x^n))*d^(1/2)/e^(5/2)+3/4*I*b*n*polylog(2,-I*x*e^(1/2)/d^(1/2
))*d^(1/2)/e^(5/2)-3/4*I*b*n*polylog(2,I*x*e^(1/2)/d^(1/2))*d^(1/2)/e^(5/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {294, 327, 211, 2393, 2332, 2360, 2361, 12, 4940, 2438}

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = -\frac{3\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{5/2}} + \frac{dx(a+b\log(cx^n))}{2e^2(d+ex^2)} + \frac{ax}{e^2} - \frac{b\sqrt{d}n\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{5/2}} + \frac{bx\log(cx^n)}{e^2} + \frac{3ib\sqrt{d}n\,\text{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4e^{5/2}} - \frac{3ib\sqrt{d}n\,\text{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4e^{5/2}} - \frac{bnx}{e^2}$$

```
[In] Int[(x^4*(a + b*Log[c*x^n]))/(d + e*x^2)^2,x]
```

```
[Out] (a*x)/e^2 - (b*n*x)/e^2 - (b*Sqrt[d]*n*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(5
/2)) + (b*x*Log[c*x^n])/e^2 + (d*x*(a + b*Log[c*x^n]))/(2*e^2*(d + e*x^2))
- (3*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/(2*e^(5/2)) +
(((3*I)/4)*b*Sqrt[d]*n*PolyLog[2, ((-I)*Sqrt[e]*x)/Sqrt[d]])/e^(5/2) - (((3
*I)/4)*b*Sqrt[d]*n*PolyLog[2, (I*Sqrt[e]*x)/Sqrt[d]])/e^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2360

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Sym
bol] :> Simp[(-x)*(d + e*x^2)^(q + 1)*((a + b*Log[c*x^n])/(2*d*(q + 1))), x
] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*Log[c*x^n
]), x], x] + Dist[b*(n/(2*d*(q + 1))), Int[(d + e*x^2)^(q + 1), x], x]) /;
FreeQ[{a, b, c, d, e, n}, x] && LtQ[q, -1]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a + b\log\left(cx^n\right)}{e^2} + \frac{d^2\left(a + b\log\left(cx^n\right)\right)}{e^2\left(d + ex^2\right)^2} - \frac{2d\left(a + b\log\left(cx^n\right)\right)}{e^2\left(d + ex^2\right)} \right) dx$$

$$=\frac{\int(a+b\log(cx^n))\,dx}{e^2}-\frac{(2d)\int\frac{a+b\log(cx^n)}{d+ex^2}\,dx}{e^2}+\frac{d^2\int\frac{a+b\log(cx^n)}{(d+ex^2)^2}\,dx}{e^2}$$

$$=\frac{ax}{e^2}+\frac{dx(a+b\log(cx^n))}{2e^2(d+ex^2)}-\frac{2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{5/2}}+\frac{b\int\log(cx^n)\,dx}{e^2}+\frac{d\int\frac{a+b\log(cx^n)}{d+ex^2}\,dx}{2e^2}-\frac{(bdn)\int\frac{1}{d+ex^2}\,dx}{2e^2}+\frac{(2bdn)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}\,dx}{e^2}$$

$$=\frac{ax}{e^2}-\frac{bnx}{e^2}-\frac{b\sqrt{d}n\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{5/2}}+\frac{bx\log(cx^n)}{e^2}+\frac{dx(a+b\log(cx^n))}{2e^2(d+ex^2)}-\frac{3\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{5/2}}+\frac{\left(2b\sqrt{d}n\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{e^{5/2}}-\frac{(bdn)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}\,dx}{2e^2}$$

$$=\frac{ax}{e^2}-\frac{bnx}{e^2}-\frac{b\sqrt{d}n\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{5/2}}+\frac{bx\log(cx^n)}{e^2}+\frac{dx(a+b\log(cx^n))}{2e^2(d+ex^2)}-\frac{3\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{5/2}}+\frac{\left(ib\sqrt{d}n\right)\int\frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{e^{5/2}}-\frac{\left(ib\sqrt{d}n\right)\int\frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{e^{5/2}}-\frac{\left(b\sqrt{d}n\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2e^{5/2}}$$

$$=\frac{ax}{e^2}-\frac{bnx}{e^2}-\frac{b\sqrt{d}n\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{5/2}}+\frac{bx\log(cx^n)}{e^2}+\frac{dx(a+b\log(cx^n))}{2e^2(d+ex^2)}-\frac{3\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{5/2}}+\frac{ib\sqrt{d}n\text{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}}-\frac{ib\sqrt{d}n\text{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}}-\frac{\left(ib\sqrt{d}n\right)\int\frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{4e^{5/2}}+\frac{\left(ib\sqrt{d}n\right)\int\frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{4e^{5/2}}$$

$$=\frac{ax}{e^2}-\frac{bnx}{e^2}-\frac{b\sqrt{d}n\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{5/2}}+\frac{bx\log(cx^n)}{e^2}+\frac{dx(a+b\log(cx^n))}{2e^2(d+ex^2)}-\frac{3\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{5/2}}+\frac{3ib\sqrt{d}n\text{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4e^{5/2}}-\frac{3ib\sqrt{d}n\text{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4e^{5/2}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.55

$$\int \frac{x^4(a + b\log(cx^n))}{(d + ex^2)^2}\,dx$$
$$= \frac{4a\sqrt{e}x - 4b\sqrt{e}nx + 4b\sqrt{e}x\log(cx^n) - \frac{d(a+b\log(cx^n))}{\sqrt{-d}-\sqrt{e}x} + \frac{d(a+b\log(cx^n))}{\sqrt{-d}+\sqrt{e}x} + \frac{bdn(\log(x)-\log(\sqrt{-d}-\sqrt{e}x))}{\sqrt{-d}} + b\sqrt{-d}n(\mathrm{lo}}{}$$

```
[In] Integrate[(x^4*(a + b*Log[c*x^n]))/(d + e*x^2)^2,x]
```

```
[Out] (4*a*Sqrt[e]*x - 4*b*Sqrt[e]*n*x + 4*b*Sqrt[e]*x*Log[c*x^n] - (d*(a + b*Log
[c*x^n]))/(Sqrt[-d] - Sqrt[e]*x) + (d*(a + b*Log[c*x^n]))/(Sqrt[-d] + Sqrt[
e]*x) + (b*d*n*(Log[x] - Log[Sqrt[-d] - Sqrt[e]*x]))/Sqrt[-d] + b*Sqrt[-d]*
n*(Log[x] - Log[Sqrt[-d] + Sqrt[e]*x]) - 3*Sqrt[-d]*(a + b*Log[c*x^n])*Log[
1 + (Sqrt[e]*x)/Sqrt[-d]] + 3*Sqrt[-d]*(a + b*Log[c*x^n])*Log[1 + (d*Sqrt[e
]*x)/(-d)^(3/2)] + 3*b*Sqrt[-d]*n*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]] - 3*b*Sq
rt[-d]*n*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)])/(4*e^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.93

method	result
risch	$\frac{b\ln(x^n)dx}{2e^2(e\,x^2+d)} + \frac{3bd\arctan\left(\frac{xe}{\sqrt{de}}\right)n\ln(x)}{2e^2\sqrt{de}} - \frac{3bd\arctan\left(\frac{xe}{\sqrt{de}}\right)\ln(x^n)}{2e^2\sqrt{de}} + \frac{b\ln(x^n)x}{e^2} - \frac{bnx}{e^2} - \frac{bnd\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{e^2\sqrt{-de}} + \frac{bnd}{}$

```
[In] int(x^4*(a+b*ln(c*x^n))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*ln(x^n)*d/e^2*x/(e*x^2+d)+3/2*b*d/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1
/2))*n*ln(x)-3/2*b*d/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*ln(x^n)+b*ln(x
^n)/e^2*x-b*n*x/e^2-b*n*d/e^2*ln(x)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d
*e)^(1/2))+b*n*d/e^2*ln(x)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))
-3/4*b*n*d/e^2/(-d*e)^(1/2)*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+3/4*b*n
*d/e^2/(-d*e)^(1/2)*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*n*d/e^2/(d
*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/4*b*n*d/e*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*
ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2-1/4*b*n*d/e*ln(x)/(e*x^2+d)/(-d*e)
^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2+1/4*b*n*d^2/e^2*ln(x)/(e*x^2
+d)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/4*b*n*d^2/e^2*ln(x)
/(e*x^2+d)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+(-1/2*I*b*Pi*cs
gn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*
```

```
I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-
d/e^2*(-1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))+x/e^2)
```

Fricas [F]

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \int \frac{(b\log(cx^n)+a)x^4}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^4*log(c*x^n) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^2}\,dx$$

```
[In] integrate(x**4*(a+b*ln(c*x**n))/(e*x**2+d)**2,x)
[Out] Integral(x**4*(a + b*log(c*x**n))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \int \frac{(b\log(cx^n)+a)x^4}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^4/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \int \frac{x^4\,(a+b\,\ln(c\,x^n))}{(e\,x^2+d)^2}\,dx$$

```
[In] int((x^4*(a + b*log(c*x^n)))/(d + e*x^2)^2,x)
[Out] int((x^4*(a + b*log(c*x^n)))/(d + e*x^2)^2, x)
```

3.227 $\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 164

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \frac{bn\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}} - \frac{x(a+b\log(cx^n))}{2e(d+ex^2)} + \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}e^{3/2}} - \frac{ibn\operatorname{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{d}e^{3/2}} + \frac{ibn\operatorname{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{d}e^{3/2}}$$

```
[Out] -1/2*x*(a+b*ln(c*x^n))/e/(e*x^2+d)+1/2*b*n*arctan(x*e^(1/2)/d^(1/2))/e^(3/2
)/d^(1/2)+1/2*arctan(x*e^(1/2)/d^(1/2))*(a+b*ln(c*x^n))/e^(3/2)/d^(1/2)-1/4
*I*b*n*polylog(2,-I*x*e^(1/2)/d^(1/2))/e^(3/2)/d^(1/2)+1/4*I*b*n*polylog(2,
I*x*e^(1/2)/d^(1/2))/e^(3/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used

= {294, 211, 2393, 2360, 2361, 12, 4940, 2438}

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}e^{3/2}} - \frac{x(a+b\log(cx^n))}{2e(d+ex^2)} + \frac{bn\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}} - \frac{ibn\,\text{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{d}e^{3/2}} + \frac{ibn\,\text{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{d}e^{3/2}}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/(d + e*x^2)^2,x]
```

```
[Out] (b*n*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[d]*e^(3/2)) - (x*(a + b*Log[c*x^n
]))/(2*e*(d + e*x^2)) + (ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/(2
*Sqrt[d]*e^(3/2)) - ((I/4)*b*n*PolyLog[2, ((-I)*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[
d]*e^(3/2)) + ((I/4)*b*n*PolyLog[2, (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(3/2
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2360

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Sym
bol] :> Simp[(-x)*(d + e*x^2)^(q + 1)*((a + b*Log[c*x^n])/(2*d*(q + 1))), x
] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*Log[c*x^n
]), x], x] + Dist[b*(n/(2*d*(q + 1))), Int[(d + e*x^2)^(q + 1), x], x]) /;
FreeQ[{a, b, c, d, e, n}, x] && LtQ[q, -1]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{d(a+b\log(cx^n))}{e(d+ex^2)^2}+\frac{a+b\log(cx^n)}{e(d+ex^2)}\right)dx$$

$$=\frac{\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx}{e}-\frac{d\int \frac{a+b\log(cx^n)}{(d+ex^2)^2}\,dx}{e}$$

$$=-\frac{x(a+b\log(cx^n))}{2e(d+ex^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}e^{3/2}}-\frac{\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx}{2e}+\frac{(bn)\int \frac{1}{d+ex^2}\,dx}{2e}-\frac{(bn)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}\,dx}{e}$$

$$=\frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}}-\frac{x(a+b\log(cx^n))}{2e(d+ex^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}e^{3/2}}-\frac{(bn)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{\sqrt{d}e^{3/2}}+\frac{(bn)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}\,dx}{2e}$$

$$= \frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}} - \frac{x(a+b\log(cx^n))}{2e(d+ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}e^{3/2}} - \frac{(ibn)\int\frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{2\sqrt{d}e^{3/2}} + \frac{(ibn)\int\frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{2\sqrt{d}e^{3/2}} + \frac{(bn)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{2\sqrt{d}e^{3/2}}$$

$$= \frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}} - \frac{x(a+b\log(cx^n))}{2e(d+ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}e^{3/2}} - \frac{ibn\text{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}} + \frac{ibn\text{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}} + \frac{(ibn)\int\frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{4\sqrt{d}e^{3/2}} - \frac{(ibn)\int\frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{4\sqrt{d}e^{3/2}}$$

$$= \frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}} - \frac{x(a+b\log(cx^n))}{2e(d+ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}e^{3/2}} - \frac{ibn\text{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{d}e^{3/2}} + \frac{ibn\text{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{d}e^{3/2}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.57

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^2}\,dx$$

$$= \frac{\frac{a+b\log(cx^n)}{\sqrt{-d}-\sqrt{e}x} - \frac{a+b\log(cx^n)}{\sqrt{-d}+\sqrt{e}x} + \frac{bdn(\log(x)-\log(\sqrt{-d}-\sqrt{e}x))}{(-d)^{3/2}} + \frac{bn(\log(x)-\log(\sqrt{-d}+\sqrt{e}x))}{\sqrt{-d}} + \frac{d(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{(-d)^{3/2}} + \frac{(a+b}{}}{4e^{3/2}}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*x^2)^2,x]
```

```
[Out] ((a + b*Log[c*x^n])/(Sqrt[-d] - Sqrt[e]*x) - (a + b*Log[c*x^n])/(Sqrt[-d] +
 Sqrt[e]*x) + (b*d*n*(Log[x] - Log[Sqrt[-d] - Sqrt[e]*x]))/(-d)^(3/2) + (b*
n*(Log[x] - Log[Sqrt[-d] + Sqrt[e]*x]))/Sqrt[-d] + (d*(a + b*Log[c*x^n])*Lo
g[1 + (Sqrt[e]*x)/Sqrt[-d]])/(-d)^(3/2) + ((a + b*Log[c*x^n])*Log[1 + (d*Sq
rt[e]*x)/(-d)^(3/2)])/Sqrt[-d] + (b*n*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]])/Sqr
t[-d] + (b*d*n*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)])/(-d)^(3/2))/(4*e^(3/2)
)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.49 (sec) , antiderivative size = 516, normalized size of antiderivative = 3.15

method	result
risch	$-\frac{b\ln(x^n)x}{2e(e\,x^2+d)}-\frac{b\arctan\left(\frac{xe}{\sqrt{de}}\right)n\ln(x)}{2e\sqrt{de}}+\frac{b\arctan\left(\frac{xe}{\sqrt{de}}\right)\ln(x^n)}{2e\sqrt{de}}+\frac{bn\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2e\sqrt{-de}}-\frac{bn\ln(x)\ln\left(\frac{ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2e\sqrt{-de}}+\frac{bn\,\mathrm{dil}}{}$

```
[In] int(x^2*(a+b*ln(c*x^n))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*ln(x^n)/e*x/(e*x^2+d)-1/2*b/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*n*
ln(x)+1/2*b/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*ln(x^n)+1/2*b*n/e*ln(x)/(
-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*n/e*ln(x)/(-d*e)^(1/
2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/4*b*n/e/(-d*e)^(1/2)*dilog((-e*x+(
-d*e)^(1/2))/(-d*e)^(1/2))-1/4*b*n/e/(-d*e)^(1/2)*dilog((e*x+(-d*e)^(1/2))/
(-d*e)^(1/2))+1/2*b*n/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-1/4*b*n*ln(x)/(
e*x^2+d)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2+1/4*b*n*ln(x
)/(e*x^2+d)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2-1/4*b*n*d/
e*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/4*b*n
*d/e*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+(-1/2
*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x
^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*l
n(c)+a)*(-1/2/e*x/(e*x^2+d)+1/2/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))
```

Fricas [F]

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^2}\,dx=\int \frac{(b\log(cx^n)+a)x^2}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*log(c*x^n) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^2}\,dx=\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^2}\,dx$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**2*(a + b*log(c*x**n))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,x^n))}{(e\,x^2+d)^2}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e*x^2)^2,x)
```

```
[Out] int((x^2*(a + b*log(c*x^n)))/(d + e*x^2)^2, x)
```

3.228 $\int \frac{a+b\log(cx^n)}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 164

$$\int \frac{a+b\log(cx^n)}{(d+ex^2)^2}\,dx = -\frac{bn\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}+\frac{x(a+b\log(cx^n))}{2d(d+ex^2)}+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}\sqrt{e}}-\frac{ibn\operatorname{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{3/2}\sqrt{e}}+\frac{ibn\operatorname{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{3/2}\sqrt{e}}$$

```
[Out] 1/2*x*(a+b*ln(c*x^n))/d/(e*x^2+d)-1/2*b*n*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)
/e^(1/2)+1/2*arctan(x*e^(1/2)/d^(1/2))*(a+b*ln(c*x^n))/d^(3/2)/e^(1/2)-1/4*
I*b*n*polylog(2,-I*x*e^(1/2)/d^(1/2))/d^(3/2)/e^(1/2)+1/4*I*b*n*polylog(2,I
*x*e^(1/2)/d^(1/2))/d^(3/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2360, 211, 2361, 12, 4940, 2438}

$$\int \frac{a+b\log(cx^n)}{(d+ex^2)^2}\,dx=\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}\sqrt{e}}+\frac{x(a+b\log(cx^n))}{2d(d+ex^2)}-\frac{bn\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}-\frac{ibn\operatorname{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{3/2}\sqrt{e}}+\frac{ibn\operatorname{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{3/2}\sqrt{e}}$$

```
[In] Int[(a + b*Log[c*x^n])/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(b*n*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e]) + (x*(a + b*Log[c*
x^n]))/(2*d*(d + e*x^2)) + (ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))
/(2*d^(3/2)*Sqrt[e]) - ((I/4)*b*n*PolyLog[2, ((-I)*Sqrt[e]*x)/Sqrt[d]])/(d^
(3/2)*Sqrt[e]) + ((I/4)*b*n*PolyLog[2, (I*Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqr
t[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2360

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Sym
bol] :> Simp[(-x)*(d + e*x^2)^(q + 1)*((a + b*Log[c*x^n])/(2*d*(q + 1))), x
] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*Log[c*x^n
]), x], x] + Dist[b*(n/(2*d*(q + 1))), Int[(d + e*x^2)^(q + 1), x], x]) /;
FreeQ[{a, b, c, d, e, n}, x] && LtQ[q, -1]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \frac{x(a+b\log(cx^n))}{2d(d+ex^2)} + \frac{\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx}{2d} - \frac{(bn)\int \frac{1}{d+ex^2}\,dx}{2d}$$

$$=-\frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}+\frac{x(a+b\log(cx^n))}{2d(d+ex^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}\sqrt{e}}-\frac{(bn)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}dx}{2d}$$

$$=-\frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}+\frac{x(a+b\log(cx^n))}{2d(d+ex^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}\sqrt{e}}-\frac{(bn)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{2d^{3/2}\sqrt{e}}$$

$$=-\frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}+\frac{x(a+b\log(cx^n))}{2d(d+ex^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}\sqrt{e}}-\frac{(ibn)\int\frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{4d^{3/2}\sqrt{e}}+\frac{(ibn)\int\frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{4d^{3/2}\sqrt{e}}$$

$$=-\frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}+\frac{x(a+b\log(cx^n))}{2d(d+ex^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}\sqrt{e}}-\frac{ibn\text{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{3/2}\sqrt{e}}+\frac{ibn\text{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{3/2}\sqrt{e}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.76

$$\int\frac{a+b\log(cx^n)}{(d+ex^2)^2}dx=\frac{1}{4}\left(\frac{a+b\log(cx^n)}{d\left(\sqrt{-d}\sqrt{e}+ex\right)}+\frac{a+b\log(cx^n)}{(-d)^{3/2}\sqrt{e}+dex}+\frac{bdn\left(\log(x)-\log\left(\sqrt{-d}-\sqrt{e}x\right)\right)}{(-d)^{5/2}\sqrt{e}}+\frac{bn\left(\log(x)-\log\left(\sqrt{-d}+\sqrt{e}x\right)\right)}{(-d)^{3/2}\sqrt{e}}+\frac{(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{(-d)^{3/2}\sqrt{e}}+\frac{d(a+b\log(cx^n))\log\left(1+\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{(-d)^{5/2}\sqrt{e}}+\frac{bdn\,\text{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{(-d)^{5/2}\sqrt{e}}+\frac{bn\,\text{PolyLog}\left(2,\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{(-d)^{3/2}\sqrt{e}}\right)$$

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*x^2)^2,x]
```

```
[Out] ((a + b*Log[c*x^n])/(d*(Sqrt[-d]*Sqrt[e] + e*x)) + (a + b*Log[c*x^n])/((-d)
^(3/2)*Sqrt[e] + d*e*x) + (b*d*n*(Log[x] - Log[Sqrt[-d] - Sqrt[e]*x]))/((-d
)^(5/2)*Sqrt[e]) + (b*n*(Log[x] - Log[Sqrt[-d] + Sqrt[e]*x]))/((-d)^(3/2)*S
qrt[e]) + ((a + b*Log[c*x^n])*Log[1 + (Sqrt[e]*x)/Sqrt[-d]])/((-d)^(3/2)*Sq
rt[e]) + (d*(a + b*Log[c*x^n])*Log[1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/((-d)^(5/
```

```
2)*Sqrt[e]) + (b*d*n*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]])/((-d)^(5/2)*Sqrt[e])
 + (b*n*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)])/((-d)^(3/2)*Sqrt[e]))/4
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.74

method	result
risch	$\frac{bx\ln(x^n)}{2d(e\,x^2+d)} - \frac{b\arctan\left(\frac{xe}{\sqrt{de}}\right)n\ln(x)}{2d\sqrt{de}} + \frac{b\arctan\left(\frac{xe}{\sqrt{de}}\right)\ln(x^n)}{2d\sqrt{de}} - \frac{bn\arctan\left(\frac{xe}{\sqrt{de}}\right)}{2d\sqrt{de}} + \frac{bn\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)x^2e}{4d(e\,x^2+d)\sqrt{-de}} - \frac{bn\ln(x)}{4d(e}$

```
[In] int((a+b*ln(c*x^n))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*x/d/(e*x^2+d)*ln(x^n)-1/2*b/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*n*l
n(x)+1/2*b/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*ln(x^n)-1/2*b*n/d/(d*e)^(1
/2)*arctan(x*e/(d*e)^(1/2))+1/4*b*n*ln(x)/d/(e*x^2+d)/(-d*e)^(1/2)*ln((-e*x
+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2*e-1/4*b*n*ln(x)/d/(e*x^2+d)/(-d*e)^(1/2)*l
n((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2*e+1/4*b*n*ln(x)/(e*x^2+d)/(-d*e)^(1/
2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/4*b*n*ln(x)/(e*x^2+d)/(-d*e)^(1/2
)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/4*b*n/(-d*e)^(1/2)/d*dilog((-e*x+(-
d*e)^(1/2))/(-d*e)^(1/2))-1/4*b*n/(-d*e)^(1/2)/d*dilog((e*x+(-d*e)^(1/2))/(
-d*e)^(1/2))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*cs
gn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*c
sgn(I*c*x^n)^3+b*ln(c)+a)*(1/2*x/d/(e*x^2+d)+1/2/d/(d*e)^(1/2)*arctan(x*e/(
d*e)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^n\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{b\log\left(cx^n\right)+a}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a + b\log\left(cx^n\right)}{\left(d + ex^2\right)^2}\, dx = \int \frac{a + b\log\left(cx^n\right)}{\left(d + ex^2\right)^2}\, dx$$

```
[In] integrate((a+b*ln(c*x**n))/(e*x**2+d)**2,x)
[Out] Integral((a + b*log(c*x**n))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\log\left(cx^n\right)}{\left(d + ex^2\right)^2}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + b\log\left(cx^n\right)}{\left(d + ex^2\right)^2}\, dx = \int \frac{b\log\left(cx^n\right) + a}{\left(ex^2 + d\right)^2}\, dx$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(cx^n\right)}{\left(d + ex^2\right)^2}\, dx = \int \frac{a + b\,\ln\left(c\,x^n\right)}{\left(e\,x^2 + d\right)^2}\, dx$$

```
[In] int((a + b*log(c*x^n))/(d + e*x^2)^2,x)
[Out] int((a + b*log(c*x^n))/(d + e*x^2)^2, x)
```

3.229 $\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 183

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^2}\,dx = -\frac{3bn}{2d^2x}+\frac{a+b\log(cx^n)}{2dx(d+ex^2)}-\frac{3a-bn+3b\log(cx^n)}{2d^2x} -\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3a-bn+3b\log(cx^n))}{2d^{5/2}} +\frac{3ib\sqrt{e}n\operatorname{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}}-\frac{3ib\sqrt{e}n\operatorname{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}}$$

```
[Out] -3/2*b*n/d^2/x+1/2*(a+b*ln(c*x^n))/d/x/(e*x^2+d)+1/2*(-3*a+b*n-3*b*ln(c*x^n
))/d^2/x-1/2*arctan(x*e^(1/2)/d^(1/2))*(3*a-b*n+3*b*ln(c*x^n))*e^(1/2)/d^(5
/2)+3/4*I*b*n*polylog(2,-I*x*e^(1/2)/d^(1/2))*e^(1/2)/d^(5/2)-3/4*I*b*n*pol
ylog(2,I*x*e^(1/2)/d^(1/2))*e^(1/2)/d^(5/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2385, 2380, 2341, 211, 2361, 12, 4940, 2438}

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^2}\,dx = -\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3a+3b\log(cx^n)-bn)}{2d^{5/2}} -\frac{3a+3b\log(cx^n)-bn}{2d^2x}+\frac{a+b\log(cx^n)}{2dx(d+ex^2)} +\frac{3ib\sqrt{e}n\operatorname{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}}-\frac{3ib\sqrt{e}n\operatorname{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}}-\frac{3bn}{2d^2x}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] (-3*b*n)/(2*d^2*x) + (a + b*Log[c*x^n])/(2*d*x*(d + e*x^2)) - (3*a - b*n +
3*b*Log[c*x^n])/(2*d^2*x) - (Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(3*a - b*n
 + 3*b*Log[c*x^n]))/(2*d^(5/2)) + (((3*I)/4)*b*Sqrt[e]*n*PolyLog[2, ((-I)*S
qrt[e]*x)/Sqrt[d]])/d^(5/2) - (((3*I)/4)*b*Sqrt[e]*n*PolyLog[2, (I*Sqrt[e]*
x)/Sqrt[d]])/d^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2385

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^2)^(q_.), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(q + 1)*((a +
 b*Log[c*x^n])/(2*d*f*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(f*x)^m*(d
+ e*x^2)^(q + 1)*(a*(m + 2*q + 3) + b*n + b*(m + 2*q + 3)*Log[c*x^n]), x],
x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a+b\log(cx^n)}{2dx(d+ex^2)} - \frac{\int \frac{-3a+bn-3b\log(cx^n)}{x^2(d+ex^2)}\,dx}{2d} \\
&= \frac{a+b\log(cx^n)}{2dx(d+ex^2)} - \frac{\int \frac{-3a+bn-3b\log(cx^n)}{x^2}\,dx}{2d^2} + \frac{e\int \frac{-3a+bn-3b\log(cx^n)}{d+ex^2}\,dx}{2d^2} \\
&= -\frac{3bn}{2d^2x} + \frac{a+b\log(cx^n)}{2dx(d+ex^2)} - \frac{3a-bn+3b\log(cx^n)}{2d^2x} \\
&\quad - \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3a-bn+3b\log(cx^n))}{2d^{5/2}} + \frac{(3ben)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}\,dx}{2d^2} \\
&= -\frac{3bn}{2d^2x} + \frac{a+b\log(cx^n)}{2dx(d+ex^2)} - \frac{3a-bn+3b\log(cx^n)}{2d^2x} \\
&\quad - \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3a-bn+3b\log(cx^n))}{2d^{5/2}} + \frac{\left(3b\sqrt{e}n\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2d^{5/2}} \\
&= -\frac{3bn}{2d^2x} + \frac{a+b\log(cx^n)}{2dx(d+ex^2)} - \frac{3a-bn+3b\log(cx^n)}{2d^2x} \\
&\quad - \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3a-bn+3b\log(cx^n))}{2d^{5/2}} \\
&\quad + \frac{\left(3ib\sqrt{e}n\right)\int \frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{4d^{5/2}} - \frac{\left(3ib\sqrt{e}n\right)\int \frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{4d^{5/2}} \\
&= -\frac{3bn}{2d^2x} + \frac{a+b\log(cx^n)}{2dx(d+ex^2)} - \frac{3a-bn+3b\log(cx^n)}{2d^2x} \\
&\quad - \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3a-bn+3b\log(cx^n))}{2d^{5/2}} \\
&\quad + \frac{3ib\sqrt{e}n\text{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}} - \frac{3ib\sqrt{e}n\text{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.79

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^2}\,dx = \frac{1}{4}\left(-\frac{4bn}{d^2x}-\frac{4(a+b\log(cx^n))}{d^2x}+\frac{\sqrt{e}(a+b\log(cx^n))}{d^2(\sqrt{-d}-\sqrt{e}x)} - \frac{\sqrt{e}(a+b\log(cx^n))}{d^2(\sqrt{-d}+\sqrt{e}x)}+\frac{b\sqrt{e}n(-\log(x)+\log(\sqrt{-d}-\sqrt{e}x))}{(-d)^{5/2}} + \frac{b\sqrt{e}n(\log(x)-\log(\sqrt{-d}+\sqrt{e}x))}{(-d)^{5/2}} + \frac{3\sqrt{e}(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{(-d)^{5/2}} - \frac{3\sqrt{e}(a+b\log(cx^n))\log\left(1+\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{(-d)^{5/2}} - \frac{3b\sqrt{e}n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{(-d)^{5/2}}+\frac{3b\sqrt{e}n\,\mathrm{PolyLog}\left(2,\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{(-d)^{5/2}}\right)$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] ((-4*b*n)/(d^2*x) - (4*(a + b*Log[c*x^n]))/(d^2*x) + (Sqrt[e]*(a + b*Log[c*
x^n]))/(d^2*(Sqrt[-d] - Sqrt[e]*x)) - (Sqrt[e]*(a + b*Log[c*x^n]))/(d^2*(Sq
rt[-d] + Sqrt[e]*x)) + (b*Sqrt[e]*n*(-Log[x] + Log[Sqrt[-d] - Sqrt[e]*x]))/
(-d)^(5/2) + (b*Sqrt[e]*n*(Log[x] - Log[Sqrt[-d] + Sqrt[e]*x]))/(-d)^(5/2)
+ (3*Sqrt[e]*(a + b*Log[c*x^n])*Log[1 + (Sqrt[e]*x)/Sqrt[-d]])/(-d)^(5/2) -
 (3*Sqrt[e]*(a + b*Log[c*x^n])*Log[1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(-d)^(5/2
) - (3*b*Sqrt[e]*n*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]])/(-d)^(5/2) + (3*b*Sqrt
[e]*n*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)])/(-d)^(5/2))/4
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.60 (sec) , antiderivative size = 568, normalized size of antiderivative = 3.10

method	result
risch	$-\frac{b\ln(x^n)ex}{2d^2(e\,x^2+d)}+\frac{3be\arctan\left(\frac{xe}{\sqrt{de}}\right)n\ln(x)}{2d^2\sqrt{de}}-\frac{3be\arctan\left(\frac{xe}{\sqrt{de}}\right)\ln(x^n)}{2d^2\sqrt{de}}-\frac{b\ln(x^n)}{d^2x}-\frac{bne\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{2d^2\sqrt{-de}}+\frac{bne\ln(x)\ln(}{2d^2\sqrt{-}}$

[In] int((a+b*ln(c*x^n))/x^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)

[Out] -1/2*b*ln(x^n)/d^2*e*x/(e*x^2+d)+3/2*b*e/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*n*ln(x)-3/2*b*e/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*ln(x^n)-b*ln(x^n)/d^2/x-1/2*b*n*e/d^2*ln(x)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*b*n*e/d^2*ln(x)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-3/4*b*n*e/d^2/(-d*e)^(1/2)*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+3/4*b*n*e/d^2/(-d*e)^(1/2)*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*b*n*e/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-1/4*b*n*e^2/d^2*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2+1/4*b*n*e^2/d^2*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2-1/4*b*n*e/d*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/4*b*n*e/d*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-b*n/d^2/x+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-e/d^2*(1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))-1/d^2/x)

Fricas [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^2\right)^2}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex^2+d\right)^2x^2}\,dx$$

[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b*log(c*x^n) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)

Sympy [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^2\right)^2}\,dx$$

[In] integrate((a+b*ln(c*x**n))/x**2/(e*x**2+d)**2,x)

[Out] Integral((a + b*log(c*x**n))/(x**2*(d + e*x**2)**2), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^2}\,dx = \int \frac{b\log(cx^n)+a}{(ex^2+d)^2x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^2}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^2\,(e\,x^2+d)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*x^2)^2), x)
```

3.230 $\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 224

$$\begin{aligned}\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^2}\,dx = & -\frac{5bn}{18d^2x^3}+\frac{5ben}{2d^3x}+\frac{a+b\log(cx^n)}{2dx^3(d+ex^2)} \\ & -\frac{5a-bn+5b\log(cx^n)}{6d^2x^3}+\frac{e(5a-bn+5b\log(cx^n))}{2d^3x} \\ & +\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(5a-bn+5b\log(cx^n))}{2d^{7/2}} \\ & -\frac{5ibe^{3/2}n\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{7/2}}+\frac{5ibe^{3/2}n\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{7/2}}\end{aligned}$$

```
[Out] -5/18*b*n/d^2/x^3+5/2*b*e*n/d^3/x+1/2*(a+b*ln(c*x^n))/d/x^3/(e*x^2+d)+1/6*(
-5*a+b*n-5*b*ln(c*x^n))/d^2/x^3+1/2*e*(5*a-b*n+5*b*ln(c*x^n))/d^3/x+1/2*e^(
3/2)*arctan(x*e^(1/2)/d^(1/2))*(5*a-b*n+5*b*ln(c*x^n))/d^(7/2)-5/4*I*b*e^(3
/2)*n*polylog(2,-I*x*e^(1/2)/d^(1/2))/d^(7/2)+5/4*I*b*e^(3/2)*n*polylog(2,I
*x*e^(1/2)/d^(1/2))/d^(7/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used

$= \{2385, 2380, 2341, 211, 2361, 12, 4940, 2438\}$

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^2}\,dx = \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(5a+5b\log(cx^n)-bn)}{2d^{7/2}} + \frac{e(5a+5b\log(cx^n)-bn)}{2d^3x} - \frac{5a+5b\log(cx^n)-bn}{6d^2x^3} + \frac{a+b\log(cx^n)}{2dx^3(d+ex^2)} - \frac{5ibe^{3/2}n\,\text{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{7/2}} + \frac{5ibe^{3/2}n\,\text{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{7/2}} + \frac{5ben}{2d^3x} - \frac{5bn}{18d^2x^3}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^4*(d + e*x^2)^2),x]
```

```
[Out] (-5*b*n)/(18*d^2*x^3) + (5*b*e*n)/(2*d^3*x) + (a + b*Log[c*x^n])/(2*d*x^3*(
d + e*x^2)) - (5*a - b*n + 5*b*Log[c*x^n])/(6*d^2*x^3) + (e*(5*a - b*n + 5*
b*Log[c*x^n]))/(2*d^3*x) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(5*a - b*n
+ 5*b*Log[c*x^n]))/(2*d^(7/2)) - (((5*I)/4)*b*e^(3/2)*n*PolyLog[2, ((-I)*Sq
rt[e]*x)/Sqrt[d]])/d^(7/2) + (((5*I)/4)*b*e^(3/2)*n*PolyLog[2, (I*Sqrt[e]*x
)/Sqrt[d]])/d^(7/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
```

```
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2385

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^2)^(q_.), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(q + 1)*((a +
 b*Log[c*x^n])/(2*d*f*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(f*x)^m*(d
+ e*x^2)^(q + 1)*(a*(m + 2*q + 3) + b*n + b*(m + 2*q + 3)*Log[c*x^n]), x],
x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a+b\log(cx^n)}{2dx^3(d+ex^2)} - \frac{\int \frac{-5a+bn-5b\log(cx^n)}{x^4(d+ex^2)}\,dx}{2d} \\
&= \frac{a+b\log(cx^n)}{2dx^3(d+ex^2)} - \frac{\int \frac{-5a+bn-5b\log(cx^n)}{x^4}\,dx}{2d^2} + \frac{e\int \frac{-5a+bn-5b\log(cx^n)}{x^2(d+ex^2)}\,dx}{2d^2} \\
&= -\frac{5bn}{18d^2x^3} + \frac{a+b\log(cx^n)}{2dx^3(d+ex^2)} - \frac{5a-bn+5b\log(cx^n)}{6d^2x^3} \\
&\quad + \frac{e\int \frac{-5a+bn-5b\log(cx^n)}{x^2}\,dx}{2d^3} - \frac{e^2\int \frac{-5a+bn-5b\log(cx^n)}{d+ex^2}\,dx}{2d^3} \\
&= -\frac{5bn}{18d^2x^3} + \frac{5ben}{2d^3x} + \frac{a+b\log(cx^n)}{2dx^3(d+ex^2)} \\
&\quad - \frac{5a-bn+5b\log(cx^n)}{6d^2x^3} + \frac{e(5a-bn+5b\log(cx^n))}{2d^3x} \\
&\quad + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(5a-bn+5b\log(cx^n))}{2d^{7/2}} - \frac{(5be^2n)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}\,dx}{2d^3}
\end{aligned}$$

$$= -\frac{5bn}{18d^2x^3} + \frac{5ben}{2d^3x} + \frac{a+b\log(cx^n)}{2dx^3(d+ex^2)} - \frac{5a-bn+5b\log(cx^n)}{6d^2x^3} + \frac{e(5a-bn+5b\log(cx^n))}{2d^3x} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(5a-bn+5b\log(cx^n))}{2d^{7/2}} - \frac{\left(5be^{3/2}n\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{2d^{7/2}}$$

$$= -\frac{5bn}{18d^2x^3} + \frac{5ben}{2d^3x} + \frac{a+b\log(cx^n)}{2dx^3(d+ex^2)} - \frac{5a-bn+5b\log(cx^n)}{6d^2x^3} + \frac{e(5a-bn+5b\log(cx^n))}{2d^3x} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(5a-bn+5b\log(cx^n))}{2d^{7/2}} - \frac{\left(5ibe^{3/2}n\right)\int \frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{4d^{7/2}} + \frac{\left(5ibe^{3/2}n\right)\int \frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{4d^{7/2}}$$

$$= -\frac{5bn}{18d^2x^3} + \frac{5ben}{2d^3x} + \frac{a+b\log(cx^n)}{2dx^3(d+ex^2)} - \frac{5a-bn+5b\log(cx^n)}{6d^2x^3} + \frac{e(5a-bn+5b\log(cx^n))}{2d^3x} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(5a-bn+5b\log(cx^n))}{2d^{7/2}} - \frac{5ibe^{3/2}n\text{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{7/2}} + \frac{5ibe^{3/2}n\text{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{7/2}}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.61

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^2}dx = \frac{1}{36}\left(-\frac{4bn}{d^2x^3} + \frac{72ben}{d^3x} - \frac{12(a+b\log(cx^n))}{d^2x^3} + \frac{72e(a+b\log(cx^n))}{d^3x} - \frac{9e^{3/2}(a+b\log(cx^n))}{d^3\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{9e^{3/2}(a+b\log(cx^n))}{d^3\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{9be^{3/2}n\left(\log(x)-\log\left(\sqrt{-d}-\sqrt{e}x\right)\right)}{(-d)^{7/2}} + \frac{9be^{3/2}n\left(\log(x)-\log\left(\sqrt{-d}+\sqrt{e}x\right)\right)}{(-d)^{7/2}} + \frac{45e^{3/2}(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{(-d)^{7/2}} - \frac{45e^{3/2}(a+b\log(cx^n))\log\left(1+\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{(-d)^{7/2}} - \frac{45be^{3/2}n\,\text{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{(-d)^{7/2}} + \frac{45be^{3/2}n\,\text{PolyLog}\left(2,\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{(-d)^{7/2}}\right)$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^4*(d + e*x^2)^2),x]
```

```
[Out] ((-4*b*n)/(d^2*x^3) + (72*b*e*n)/(d^3*x) - (12*(a + b*Log[c*x^n]))/(d^2*x^3
) + (72*e*(a + b*Log[c*x^n]))/(d^3*x) - (9*e^(3/2)*(a + b*Log[c*x^n]))/(d^3
*(Sqrt[-d] - Sqrt[e]*x)) + (9*e^(3/2)*(a + b*Log[c*x^n]))/(d^3*(Sqrt[-d] +
Sqrt[e]*x)) - (9*b*e^(3/2)*n*(Log[x] - Log[Sqrt[-d] - Sqrt[e]*x]))/(-d)^(7/
2) + (9*b*e^(3/2)*n*(Log[x] - Log[Sqrt[-d] + Sqrt[e]*x]))/(-d)^(7/2) + (45*
e^(3/2)*(a + b*Log[c*x^n])*Log[1 + (Sqrt[e]*x)/Sqrt[-d]])/(-d)^(7/2) - (45*
e^(3/2)*(a + b*Log[c*x^n])*Log[1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(-d)^(7/2) -
(45*b*e^(3/2)*n*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]])/(-d)^(7/2) + (45*b*e^(3/2
)*n*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)])/(-d)^(7/2))/36
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.86 (sec) , antiderivative size = 622, normalized size of antiderivative = 2.78

method	result
risch	$\frac{b\ln(x^n)e^2x}{2d^3(e\,x^2+d)} - \frac{5b\,e^2\arctan\left(\frac{xe}{\sqrt{de}}\right)n\ln(x)}{2d^3\sqrt{de}} + \frac{5b\,e^2\arctan\left(\frac{xe}{\sqrt{de}}\right)\ln(x^n)}{2d^3\sqrt{de}} - \frac{b\ln(x^n)}{3d^2x^3} + \frac{2b\ln(x^n)e}{d^3x} + \frac{bn\,e^2\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right.}{d^3\sqrt{-de}}$

```
[In] int((a+b*ln(c*x^n))/x^4/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*ln(x^n)*e^2/d^3*x/(e*x^2+d)-5/2*b*e^2/d^3/(d*e)^(1/2)*arctan(x*e/(d*e
)^(1/2))*n*ln(x)+5/2*b*e^2/d^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*ln(x^n)-
1/3*b*ln(x^n)/d^2/x^3+2*b*ln(x^n)/d^3*e/x+b*n*e^2/d^3*ln(x)/(-d*e)^(1/2)*ln
((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-b*n*e^2/d^3*ln(x)/(-d*e)^(1/2)*ln((e*x+(
-d*e)^(1/2))/(-d*e)^(1/2))+5/4*b*n*e^2/d^3/(-d*e)^(1/2)*dilog((-e*x+(-d*e)^
(1/2))/(-d*e)^(1/2))-5/4*b*n*e^2/d^3/(-d*e)^(1/2)*dilog((e*x+(-d*e)^(1/2))/
(-d*e)^(1/2))-1/9*b*n/d^2/x^3-1/2*b*n*e^2/d^3/(d*e)^(1/2)*arctan(x*e/(d*e)^
(1/2))+1/4*b*n*e^3/d^3*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/
(-d*e)^(1/2))*x^2-1/4*b*n*e^3/d^3*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*ln((e*x+(-d*
e)^(1/2))/(-d*e)^(1/2))*x^2+1/4*b*n*e^2/d^2*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*ln
((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/4*b*n*e^2/d^2*ln(x)/(e*x^2+d)/(-d*e)^(
1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+2*b*e*n/d^3/x+(-1/2*I*b*Pi*csgn(I*
c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*P
i*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(e^2/d^
3*(1/2*x/(e*x^2+d)+5/2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))-1/3/d^2/x^3+2/d
^3*e/x)
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^2}\,dx=\int \frac{b\log(cx^n)+a}{(ex^2+d)^2x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(e^2*x^8 + 2*d*e*x^6 + d^2*x^4), x)
```

Sympy [F]

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^2}\,dx=\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**4/(e*x**2+d)**2,x)
[Out] Integral((a + b*log(c*x**n))/(x**4*(d + e*x**2)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^2}\,dx=\int \frac{b\log(cx^n)+a}{(ex^2+d)^2x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x^4\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\,\ln\left(c\,x^n\right)}{x^4\,{\left(e\,x^2+d\right)}^2}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^4*(d + e*x^2)^2),x)
[Out] int((a + b*log(c*x^n))/(x^4*(d + e*x^2)^2), x)
```

3.231 $\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 152

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^3}\,dx=\frac{bdn}{8e^3(d+ex^2)}+\frac{bn\log(x)}{4e^3}-\frac{d^2(a+b\log(cx^n))}{4e^3(d+ex^2)^2}-\frac{x^2(a+b\log(cx^n))}{e^2(d+ex^2)}+\frac{3bn\log(d+ex^2)}{8e^3}+\frac{(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{2e^3}+\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{ex^2}{d}\right)}{4e^3}$$

```
[Out] 1/8*b*d*n/e^3/(e*x^2+d)+1/4*b*n*ln(x)/e^3-1/4*d^2*(a+b*ln(c*x^n))/e^3/(e*x^
2+d)^2-x^2*(a+b*ln(c*x^n))/e^2/(e*x^2+d)+3/8*b*n*ln(e*x^2+d)/e^3+1/2*(a+b*l
n(c*x^n))*ln(1+e*x^2/d)/e^3+1/4*b*n*polylog(2,-e*x^2/d)/e^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {272, 45, 2393, 2376, 46, 2373, 266, 2375, 2438}

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^3}\,dx=-\frac{d^2(a+b\log(cx^n))}{4e^3(d+ex^2)^2}+\frac{\log\left(\frac{ex^2}{d}+1\right)(a+b\log(cx^n))}{2e^3}-\frac{x^2(a+b\log(cx^n))}{e^2(d+ex^2)}+\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{ex^2}{d}\right)}{4e^3}+\frac{bdn}{8e^3(d+ex^2)}+\frac{3bn\log(d+ex^2)}{8e^3}+\frac{bn\log(x)}{4e^3}$$

```
[In] Int[(x^5*(a + b*Log[c*x^n]))/(d + e*x^2)^3,x]
```

```
[Out] (b*d*n)/(8*e^3*(d + e*x^2)) + (b*n*Log[x])/(4*e^3) - (d^2*(a + b*Log[c*x^n]
))/(4*e^3*(d + e*x^2)^2) - (x^2*(a + b*Log[c*x^n]))/(e^2*(d + e*x^2)) + (3*
b*n*Log[d + e*x^2])/(8*e^3) + ((a + b*Log[c*x^n])*Log[1 + (e*x^2)/d])/(2*e^
3) + (b*n*PolyLog[2, -((e*x^2)/d)])/(4*e^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2x(a+b\log(cx^n))}{e^2(d+ex^2)^3} - \frac{2dx(a+b\log(cx^n))}{e^2(d+ex^2)^2} + \frac{x(a+b\log(cx^n))}{e^2(d+ex^2)}\right)dx$$

$$= \frac{\int \frac{x(a+b\log(cx^n))}{d+ex^2}\,dx}{e^2} - \frac{(2d)\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^2}\,dx}{e^2} + \frac{d^2\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^3}\,dx}{e^2}$$

$$= -\frac{d^2(a+b\log(cx^n))}{4e^3(d+ex^2)^2} - \frac{x^2(a+b\log(cx^n))}{e^2(d+ex^2)} + \frac{(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{2e^3} - \frac{(bn)\int \frac{\log\left(1+\frac{ex^2}{d}\right)}{x}\,dx}{2e^3} + \frac{(bd^2n)\int \frac{1}{x(d+ex^2)^2}\,dx}{4e^3} + \frac{(bn)\int \frac{x}{d+ex^2}\,dx}{e^2}$$

$$= -\frac{d^2(a+b\log(cx^n))}{4e^3(d+ex^2)^2} - \frac{x^2(a+b\log(cx^n))}{e^2(d+ex^2)} + \frac{bn\log(d+ex^2)}{2e^3} + \frac{(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{2e^3} + \frac{bn\text{Li}_2\left(-\frac{ex^2}{d}\right)}{4e^3} + \frac{(bd^2n)\,\text{Subst}\left(\int \frac{1}{x(d+ex)^2}\,dx, x, x^2\right)}{8e^3}$$

$$= -\frac{d^2(a+b\log(cx^n))}{4e^3(d+ex^2)^2} - \frac{x^2(a+b\log(cx^n))}{e^2(d+ex^2)} + \frac{bn\log(d+ex^2)}{2e^3} + \frac{(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{2e^3} + \frac{bn\text{Li}_2\left(-\frac{ex^2}{d}\right)}{4e^3} + \frac{(bd^2n)\,\text{Subst}\left(\int\left(\frac{1}{d^2x} - \frac{e}{d(d+ex)^2} - \frac{e}{d^2(d+ex)}\right)dx, x, x^2\right)}{8e^3}$$

$$= \frac{bdn}{8e^3(d+ex^2)} + \frac{bn\log(x)}{4e^3} - \frac{d^2(a+b\log(cx^n))}{4e^3(d+ex^2)^2} - \frac{x^2(a+b\log(cx^n))}{e^2(d+ex^2)} + \frac{3bn\log(d+ex^2)}{8e^3} + \frac{(a+b\log(cx^n))\log\left(1+\frac{ex^2}{d}\right)}{2e^3} + \frac{bn\text{Li}_2\left(-\frac{ex^2}{d}\right)}{4e^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.28

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^3}\,dx$$

$$= \frac{-2d^2(a-bn\log(x)+b\log(cx^n)) + 8d(d+ex^2)(a-bn\log(x)+b\log(cx^n)) + 4(d+ex^2)^2(a-bn\log(x)}{}$$

```
[In] Integrate[(x^5*(a + b*Log[c*x^n]))/(d + e*x^2)^3,x]
```

```
[Out] (-2*d^2*(a - b*n*Log[x] + b*Log[c*x^n]) + 8*d*(d + e*x^2)*(a - b*n*Log[x] +
 b*Log[c*x^n]) + 4*(d + e*x^2)^2*(a - b*n*Log[x] + b*Log[c*x^n])*Log[d + e*
x^2] + b*n*(d^2 + d*e*x^2 - 4*d*e*x^2*Log[x] - 6*e^2*x^4*Log[x] + 3*d^2*Log
[I*Sqrt[d] - Sqrt[e]*x] + 6*d*e*x^2*Log[I*Sqrt[d] - Sqrt[e]*x] + 3*e^2*x^4*
Log[I*Sqrt[d] - Sqrt[e]*x] + 3*d^2*Log[I*Sqrt[d] + Sqrt[e]*x] + 6*d*e*x^2*L
og[I*Sqrt[d] + Sqrt[e]*x] + 3*e^2*x^4*Log[I*Sqrt[d] + Sqrt[e]*x] + 4*d^2*Lo
g[x]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]] + 8*d*e*x^2*Log[x]*Log[1 - (I*Sqrt[e]*x
)/Sqrt[d]] + 4*e^2*x^4*Log[x]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]] + 4*d^2*Log[x]
*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]] + 8*d*e*x^2*Log[x]*Log[1 + (I*Sqrt[e]*x)/Sq
rt[d]] + 4*e^2*x^4*Log[x]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]] + 4*(d + e*x^2)^2*
PolyLog[2, ((-I)*Sqrt[e]*x)/Sqrt[d]] + 4*(d + e*x^2)^2*PolyLog[2, (I*Sqrt[e
]*x)/Sqrt[d]]))/(8*e^3*(d + e*x^2)^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.70 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.36

method	result
risch	$\frac{b\ln(x^n)\ln(e\,x^2+d)}{2e^3}+\frac{b\ln(x^n)d}{e^3(e\,x^2+d)}-\frac{b\ln(x^n)d^2}{4e^3(e\,x^2+d)^2}+\frac{3bn\ln(e\,x^2+d)}{8e^3}+\frac{bdn}{8e^3(e\,x^2+d)}-\frac{3bn\ln(x)}{4e^3}-\frac{bn\ln(x)\ln(e\,x^2+d)}{2e^3}+$

```
[In] int(x^5*(a+b*ln(c*x^n))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*ln(x^n)/e^3*ln(e*x^2+d)+b*ln(x^n)*d/e^3/(e*x^2+d)-1/4*b*ln(x^n)*d^2/e
^3/(e*x^2+d)^2+3/8*b*n*ln(e*x^2+d)/e^3+1/8*b*d*n/e^3/(e*x^2+d)-3/4*b*n*ln(x
)/e^3-1/2*b*n/e^3*ln(x)*ln(e*x^2+d)+1/2*b*n/e^3*ln(x)*ln((-e*x+(-d*e)^(1/2)
)/(-d*e)^(1/2))+1/2*b*n/e^3*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*b
*n/e^3*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*b*n/e^3*dilog((e*x+(-d*e
)^(1/2))/(-d*e)^(1/2))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2
*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/
2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/2/e^3*ln(e*x^2+d)+d/e^3/(e*x^2+d)-1/
4*d^2/e^3/(e*x^2+d)^2)
```

Fricas [F]

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^3}\,dx=\int \frac{(b\log(cx^n)+a)x^5}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^5*log(c*x^n) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 +
d^3), x)
```

Sympy [A] (verification not implemented)

Time = 63.33 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.65

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^3}\,dx$$

$$= \frac{ad^2\left(\begin{cases}\frac{x^2}{d^3} & \text{for } e=0\\ -\frac{1}{2e(d+ex^2)^2} & \text{otherwise}\end{cases}\right)}{2e^2} - \frac{ad\left(\begin{cases}\frac{x^2}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x^2} & \text{otherwise}\end{cases}\right)}{e^2}$$

$$+ \frac{a\left(\begin{cases}\frac{x^2}{d} & \text{for } e=0\\ \frac{\log(d+ex^2)}{e} & \text{otherwise}\end{cases}\right)}{2e^2} - \frac{bd^2n\left(\begin{cases}\frac{x^2}{2d^3} & \text{for } e=0\\ -\frac{1}{4d^2e+4de^2x^2} - \frac{\log(x)}{2d^2e} + \frac{\log\left(\frac{d}{e}+x^2\right)}{4d^2e} & \text{otherwise}\end{cases}\right)}{2e^2}$$

$$+ \frac{bd^2\left(\begin{cases}\frac{x^2}{d^3} & \text{for } e=0\\ -\frac{1}{2e(d+ex^2)^2} & \text{otherwise}\end{cases}\right)\log(cx^n)}{2e^2}$$

$$+ \frac{bdn\left(\begin{cases}\frac{x^2}{2d^2} & \text{for } e=0\\ -\frac{\log(x)}{de} + \frac{\log\left(\frac{d}{e}+x^2\right)}{2de} & \text{otherwise}\end{cases}\right)}{e^2} - \frac{bd\left(\begin{cases}\frac{x^2}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x^2} & \text{otherwise}\end{cases}\right)\log(cx^n)}{e^2}$$

$$- \frac{bn\left(\begin{cases}\frac{x^2}{2d} \\ \frac{\begin{cases}-\frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(d)\log(x) - \frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right) - \frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|\,x\right)\log(d) + G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|\,x\right)\log(d) - \frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{otherwise}\end{cases}}{e}\end{cases}\right)}{2e^2}$$

$$+ \frac{b\left(\begin{cases}\frac{x^2}{d} & \text{for } e=0\\ \frac{\log(d+ex^2)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{2e^2}$$

```
[In] integrate(x**5*(a+b*ln(c*x**n))/(e*x**2+d)**3,x)
```

```
[Out] a*d**2*Piecewise((x**2/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x**2)**2), True))/(
2*e**2) - a*d*Piecewise((x**2/d**2, Eq(e, 0)), (-1/(d*e + e**2*x**2), True)
```

```
)/e**2 + a*Piecewise((x**2/d, Eq(e, 0)), (log(d + e*x**2)/e, True))/(2*e**2
) - b*d**2*n*Piecewise((x**2/(2*d**3), Eq(e, 0)), (-1/(4*d**2*e + 4*d*e**2*
x**2) - log(x)/(2*d**2*e) + log(d/e + x**2)/(4*d**2*e), True))/(2*e**2) + b
*d**2*Piecewise((x**2/d**3, Eq(e, 0)), (-1/(2*e*(d + e*x**2)**2), True))*lo
g(c*x**n)/(2*e**2) + b*d*n*Piecewise((x**2/(2*d**2), Eq(e, 0)), (-log(x)/(d
*e) + log(d/e + x**2)/(2*d*e), True))/e**2 - b*d*Piecewise((x**2/d**2, Eq(e
, 0)), (-1/(d*e + e**2*x**2), True))*log(c*x**n)/e**2 - b*n*Piecewise((x**2
/(2*d), Eq(e, 0)), (Piecewise((-polylog(2, e*x**2*exp_polar(I*pi)/d)/2, (Ab
s(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x**2*exp_polar(I
*pi)/d)/2, Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x**2*exp_polar(I*p
i)/d)/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + m
eijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x**2*exp_polar(
I*pi)/d)/2, True))/e, True))/(2*e**2) + b*Piecewise((x**2/d, Eq(e, 0)), (lo
g(d + e*x**2)/e, True))*log(c*x**n)/(2*e**2)
```

Maxima [F]

$$\int \frac{x^5(a+b\log{(cx^n)})}{(d+ex^2)^3}\,dx = \int \frac{(b\log{(cx^n)}+a)x^5}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
+ d)/e^3) + b*integrate((x^5*log(c) + x^5*log(x^n))/(e^3*x^6 + 3*d*e^2*x^4
+ 3*d^2*e*x^2 + d^3), x)
```

Giac [F]

$$\int \frac{x^5(a+b\log{(cx^n)})}{(d+ex^2)^3}\,dx = \int \frac{(b\log{(cx^n)}+a)x^5}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^5/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = \int \frac{x^5\,(a+b\,\ln(c\,x^n))}{(e\,x^2+d)^3}\,dx$$

```
[In] int((x^5*(a + b*log(c*x^n)))/(d + e*x^2)^3,x)
[Out] int((x^5*(a + b*log(c*x^n)))/(d + e*x^2)^3, x)
```

3.232 $\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^3} dx = -\frac{bn}{8e^2(d+ex^2)} + \frac{x^4(a+b\log(cx^n))}{4d(d+ex^2)^2} - \frac{bn\log(d+ex^2)}{8de^2}$$

```
[Out] -1/8*b*n/e^2/(e*x^2+d)+1/4*x^4*(a+b*ln(c*x^n))/d/(e*x^2+d)^2-1/8*b*n*ln(e*x
^2+d)/d/e^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2373, 272, 45}

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^3} dx = \frac{x^4(a+b\log(cx^n))}{4d(d+ex^2)^2} - \frac{bn}{8e^2(d+ex^2)} - \frac{bn\log(d+ex^2)}{8de^2}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n]))/(d + e*x^2)^3,x]
```

```
[Out] -1/8*(b*n)/(e^2*(d + e*x^2)) + (x^4*(a + b*Log[c*x^n]))/(4*d*(d + e*x^2)^2)
 - (b*n*Log[d + e*x^2])/(8*d*e^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^4(a+b\log(cx^n))}{4d(d+ex^2)^2} - \frac{(bn)\int \frac{x^3}{(d+ex^2)^2}\,dx}{4d} \\
&= \frac{x^4(a+b\log(cx^n))}{4d(d+ex^2)^2} - \frac{(bn)\text{Subst}\left(\int \frac{x}{(d+ex)^2}\,dx, x, x^2\right)}{8d} \\
&= \frac{x^4(a+b\log(cx^n))}{4d(d+ex^2)^2} - \frac{(bn)\text{Subst}\left(\int \left(-\frac{d}{e(d+ex)^2}+\frac{1}{e(d+ex)}\right)dx, x, x^2\right)}{8d} \\
&= -\frac{bn}{8e^2(d+ex^2)} + \frac{x^4(a+b\log(cx^n))}{4d(d+ex^2)^2} - \frac{bn\log(d+ex^2)}{8de^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.90

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = -\frac{2ad^2+bd^2n+4adex^2+bdenx^2-2bn(d+ex^2)^2\log(x)+2bd(d+2ex^2)\log(cx^n)+bd^2n\log(d+ex^2)}{8de^2(d+ex^2)^2}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/(d + e*x^2)^3,x]
```

```
[Out] -1/8*(2*a*d^2 + b*d^2*n + 4*a*d*e*x^2 + b*d*e*n*x^2 - 2*b*n*(d + e*x^2)^2*L
og[x] + 2*b*d*(d + 2*e*x^2)*Log[c*x^n] + b*d^2*n*Log[d + e*x^2] + 2*b*d*e*n
*x^2*Log[d + e*x^2] + b*e^2*n*x^4*Log[d + e*x^2])/(d*e^2*(d + e*x^2)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(62) = 124$.

Time = 0.62 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.91

method	result
parallelrisch	$\frac{-\ln\left(e\,x^{2}+d\right)x^{4}b\,e^{2}n^{2}+2x^{4}\ln\left(c\,x^{n}\right)b\,e^{2}n-2\ln\left(e\,x^{2}+d\right)x^{2}bde\,n^{2}-x^{2}bde\,n^{2}-\ln\left(e\,x^{2}+d\right)b\,d^{2}n^{2}-4x^{2}aden-b\,d^{2}n^{2}-2a\,d^{2}n}{8nd\,e^{2}\left(e\,x^{2}+d\right)^{2}}$
risch	$-\frac{b\left(2e\,x^{2}+d\right)\ln\left(x^{n}\right)}{4\left(e\,x^{2}+d\right)^{2}e^{2}}-\frac{-2i\pi bde\,x^{2}\,\mathrm{csgn}\left(ic\,x^{n}\right)^{3}-2i\pi bde\,x^{2}\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(ic\,x^{n}\right)+i\pi b\,d^{2}\,\mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(ic\,x^{n}\right)^{2}-i\pi b}{}$

```
[In] int(x^3*(a+b*ln(c*x^n))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-ln(e*x^2+d)*x^4*b*e^2*n^2+2*x^4*ln(c*x^n)*b*e^2*n-2*ln(e*x^2+d)*x^2*b
*d*e*n^2-x^2*b*d*e*n^2-ln(e*x^2+d)*b*d^2*n^2-4*x^2*a*d*e*n-b*d^2*n^2-2*a*d^
2*n)/n/d/e^2/(e*x^2+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(62) = 124$.

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.85

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = \frac{2\,be^2nx^4\log(x) - bd^2n - 2\,ad^2 - (bden + 4\,ade)x^2 - (be^2nx^4 + 2\,bdenx^2 + bd^2n)\log(ex^2+d) - 2\,(2\,bdex}{8\,(de^4x^4 + 2\,d^2e^3x^2 + d^3e^2)}$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*b*e^2*n*x^4*log(x) - b*d^2*n - 2*a*d^2 - (b*d*e*n + 4*a*d*e)*x^2 - (
b*e^2*n*x^4 + 2*b*d*e*n*x^2 + b*d^2*n)*log(e*x^2 + d) - 2*(2*b*d*e*x^2 + b*
d^2)*log(c))/(d*e^4*x^4 + 2*d^2*e^3*x^2 + d^3*e^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(58) = 116$.

Time = 167.21 (sec) , antiderivative size = 612, normalized size of antiderivative = 9.00

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^3}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-\frac{a}{2x^2}-\frac{bn}{4x^2}-\frac{b\log(cx^n)}{2x^2}\right) \\ \frac{\frac{ax^4}{4}-\frac{bnx^4}{16}+\frac{bx^4\log(cx^n)}{4}}{d^3} \\ \frac{-\frac{a}{2x^2}-\frac{bn}{4x^2}-\frac{b\log(cx^n)}{2x^2}}{e^3} \\ -\frac{2ad^2}{8d^3e^2+16d^2e^3x^2+8de^4x^4}-\frac{4adex^2}{8d^3e^2+16d^2e^3x^2+8de^4x^4}-\frac{bd^2n\log\left(x-\sqrt{-\frac{d}{e}}\right)}{8d^3e^2+16d^2e^3x^2+8de^4x^4}-\frac{bd^2n\log\left(x+\sqrt{-\frac{d}{e}}\right)}{8d^3e^2+16d^2e^3x^2+8de^4x^4}-\frac{bd^2n}{8d^3e^2+16d^2e^3x^2} \end{cases}$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))/(e*x**2+d)**3,x)
```

```
[Out] Piecewise((zoo*(-a/(2*x**2) - b*n/(4*x**2) - b*log(c*x**n)/(2*x**2)), Eq(d,
 0) & Eq(e, 0)), ((a*x**4/4 - b*n*x**4/16 + b*x**4*log(c*x**n)/4)/d**3, Eq(
e, 0)), ((-a/(2*x**2) - b*n/(4*x**2) - b*log(c*x**n)/(2*x**2))/e**3, Eq(d,
0)), (-2*a*d**2/(8*d**3*e**2 + 16*d**2*e**3*x**2 + 8*d*e**4*x**4) - 4*a*d*e
*x**2/(8*d**3*e**2 + 16*d**2*e**3*x**2 + 8*d*e**4*x**4) - b*d**2*n*log(x -
sqrt(-d/e))/(8*d**3*e**2 + 16*d**2*e**3*x**2 + 8*d*e**4*x**4) - b*d**2*n*lo
g(x + sqrt(-d/e))/(8*d**3*e**2 + 16*d**2*e**3*x**2 + 8*d*e**4*x**4) - b*d**
2*n/(8*d**3*e**2 + 16*d**2*e**3*x**2 + 8*d*e**4*x**4) - 2*b*d*e*n*x**2*log(
x - sqrt(-d/e))/(8*d**3*e**2 + 16*d**2*e**3*x**2 + 8*d*e**4*x**4) - 2*b*d*e
*n*x**2*log(x + sqrt(-d/e))/(8*d**3*e**2 + 16*d**2*e**3*x**2 + 8*d*e**4*x**
4) - b*d*e*n*x**2/(8*d**3*e**2 + 16*d**2*e**3*x**2 + 8*d*e**4*x**4) - b*e**
2*n*x**4*log(x - sqrt(-d/e))/(8*d**3*e**2 + 16*d**2*e**3*x**2 + 8*d*e**4*x*
*4) - b*e**2*n*x**4*log(x + sqrt(-d/e))/(8*d**3*e**2 + 16*d**2*e**3*x**2 +
8*d*e**4*x**4) + 2*b*e**2*x**4*log(c*x**n)/(8*d**3*e**2 + 16*d**2*e**3*x**2
 + 8*d*e**4*x**4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(62) = 124$.

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.88

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = -\frac{1}{8}\,bn\left(\frac{1}{e^3x^2+de^2}+\frac{\log(ex^2+d)}{de^2}-\frac{\log(x^2)}{de^2}\right) - \frac{(2\,ex^2+d)b\log(cx^n)}{4\,(e^4x^4+2\,de^3x^2+d^2e^2)} - \frac{(2\,ex^2+d)a}{4\,(e^4x^4+2\,de^3x^2+d^2e^2)}$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*b*n*(1/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/(d*e^2) - log(x^2)/(d*e^2))
- 1/4*(2*e*x^2 + d)*b*log(c*x^n)/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2) - 1/4*(2
*e*x^2 + d)*a/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(62) = 124$.

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.06

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = -\frac{(2\,benx^2+bdn)\log(x)}{4\,(e^4x^4+2\,de^3x^2+d^2e^2)} - \frac{benx^2+4\,bex^2\log(c)+4\,aex^2+bdn+2\,bd\log(c)+2\,ad}{8\,(e^4x^4+2\,de^3x^2+d^2e^2)} - \frac{bn\log(ex^2+d)}{8\,de^2} + \frac{bn\log(x)}{4\,de^2}$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*e*n*x^2 + b*d*n)*log(x)/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2) - 1/8*(
b*e*n*x^2 + 4*b*e*x^2*log(c) + 4*a*e*x^2 + b*d*n + 2*b*d*log(c) + 2*a*d)/(e
^4*x^4 + 2*d*e^3*x^2 + d^2*e^2) - 1/8*b*n*log(e*x^2 + d)/(d*e^2) + 1/4*b*n*
log(x)/(d*e^2)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.90

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = \frac{b\,n\,\ln(x)}{4\,d\,e^2} - \frac{\ln(c\,x^n)\,\left(\frac{b\,x^2}{2\,e}+\frac{b\,d}{4\,e^2}\right)}{d^2+2\,d\,e\,x^2+e^2\,x^4} - \frac{b\,n\,\ln(e\,x^2+d)}{8\,d\,e^2} - \frac{\left(2\,a\,e+\frac{b\,e\,n}{2}\right)\,x^2+a\,d+\frac{b\,d\,n}{2}}{4\,d^2\,e^2+8\,d\,e^3\,x^2+4\,e^4\,x^4}$$

```
[In] int((x^3*(a + b*log(c*x^n)))/(d + e*x^2)^3,x)
```

```
[Out] (b*n*log(x))/(4*d*e^2) - (log(c*x^n)*((b*x^2)/(2*e) + (b*d)/(4*e^2)))/(d^2
+ e^2*x^4 + 2*d*e*x^2) - (b*n*log(d + e*x^2))/(8*d*e^2) - (a*d + x^2*(2*a*e
 + (b*e*n)/2) + (b*d*n)/2)/(4*d^2*e^2 + 4*e^4*x^4 + 8*d*e^3*x^2)
```

3.233 $\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = \frac{bn}{8de(d+ex^2)} + \frac{bn\log(x)}{4d^2e} - \frac{a+b\log(cx^n)}{4e(d+ex^2)^2} - \frac{bn\log(d+ex^2)}{8d^2e}$$

```
[Out] 1/8*b*n/d/e/(e*x^2+d)+1/4*b*n*ln(x)/d^2/e+1/4*(-a-b*ln(c*x^n))/e/(e*x^2+d)^
2-1/8*b*n*ln(e*x^2+d)/d^2/e
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2376, 272, 46}

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = -\frac{a+b\log(cx^n)}{4e(d+ex^2)^2} - \frac{bn\log(d+ex^2)}{8d^2e} + \frac{bn\log(x)}{4d^2e} + \frac{bn}{8de(d+ex^2)}$$

```
[In] Int[(x*(a + b*Log[c*x^n]))/(d + e*x^2)^3,x]
```

```
[Out] (b*n)/(8*d*e*(d + e*x^2)) + (b*n*Log[x])/(4*d^2*e) - (a + b*Log[c*x^n])/(4*
e*(d + e*x^2)^2) - (b*n*Log[d + e*x^2])/(8*d^2*e)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\log(cx^n)}{4e(d+ex^2)^2} + \frac{(bn)\int \frac{1}{x(d+ex^2)^2}\,dx}{4e}$$

$$= -\frac{a+b\log(cx^n)}{4e(d+ex^2)^2} + \frac{(bn)\text{Subst}\left(\int \frac{1}{x(d+ex)^2}\,dx, x, x^2\right)}{8e}$$

$$= -\frac{a+b\log(cx^n)}{4e(d+ex^2)^2} + \frac{(bn)\text{Subst}\left(\int \left(\frac{1}{d^2x} - \frac{e}{d(d+ex)^2} - \frac{e}{d^2(d+ex)}\right)\,dx, x, x^2\right)}{8e}$$

$$= \frac{bn}{8de(d+ex^2)} + \frac{bn\log(x)}{4d^2e} - \frac{a+b\log(cx^n)}{4e(d+ex^2)^2} - \frac{bn\log(d+ex^2)}{8d^2e}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.35

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = \frac{bn}{8de(d+ex^2)} + \frac{bn\log(x)}{4d^2e} - \frac{bn\log(x)}{4e(d+ex^2)^2} + \frac{-a-b(-n\log(x)+\log(cx^n))}{4e(d+ex^2)^2} - \frac{bn\log(d+ex^2)}{8d^2e}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e*x^2)^3,x]
```

```
[Out] (b*n)/(8*d*e*(d + e*x^2)) + (b*n*Log[x])/(4*d^2*e) - (b*n*Log[x])/(4*e*(d +
 e*x^2)^2) + (-a - b*(-(n*Log[x]) + Log[c*x^n]))/(4*e*(d + e*x^2)^2) - (b*n
*Log[d + e*x^2])/(8*d^2*e)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.74

method	result
parallelrisch	$\frac{-\ln(e\,x^2+d)x^4 b\,e^3 n^2+2x^4\ln(c\,x^n)b\,e^3 n-2\ln(e\,x^2+d)x^2 bd\,e^2 n^2+4x^2\ln(c\,x^n)bd\,e^2 n+x^2 bd\,e^2 n^2-\ln(e\,x^2+d)b\,d^2 e\,n^2+b\,d^2 e}{8n\,d^2 e^2(e\,x^2+d)^2}$
risch	$-\frac{b\ln(x^n)}{4e(e\,x^2+d)^2}-\frac{\ln(e\,x^2+d)b\,e^2 n\,x^4-2\ln(x)b\,e^2 n\,x^4-i\pi b\,d^2\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)+i\pi b\,d^2\operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^n)^2+i}{}$

[In] int(x*(a+b*ln(c*x^n))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)

[Out] 1/8*(-ln(e*x^2+d)*x^4*b*e^3*n^2+2*x^4*ln(c*x^n)*b*e^3*n-2*ln(e*x^2+d)*x^2*b*d*e^2*n^2+4*x^2*ln(c*x^n)*b*d*e^2*n+x^2*b*d*e^2*n^2-ln(e*x^2+d)*b*d^2*e*n^2+b*d^2*e*n^2-2*a*d^2*e*n)/n/d^2/e^2/(e*x^2+d)^2

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.44

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^3}\,dx$$
$$=\frac{bdenx^2+bd^2n-2\,bd^2\log(c)-2\,ad^2-(be^2nx^4+2\,bdenx^2+bd^2n)\log(ex^2+d)+2\,(be^2nx^4+2\,bdenx^2)}{8\,(d^2e^3x^4+2\,d^3e^2x^2+d^4e)}$$

[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="fricas")

[Out] 1/8*(b*d*e*n*x^2 + b*d^2*n - 2*b*d^2*log(c) - 2*a*d^2 - (b*e^2*n*x^4 + 2*b*d*e*n*x^2 + b*d^2*n)*log(e*x^2 + d) + 2*(b*e^2*n*x^4 + 2*b*d*e*n*x^2)*log(x))/(d^2*e^3*x^4 + 2*d^3*e^2*x^2 + d^4*e)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. 2(71) = 142.

Time = 169.10 (sec) , antiderivative size = 619, normalized size of antiderivative = 7.55

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^3}\,dx$$
$$=\begin{cases} \tilde{\infty}\left(-\frac{a}{4x^4}-\frac{bn}{16x^4}-\frac{b\log(cx^n)}{4x^4}\right) \\ \frac{\frac{ax^2}{2}-\frac{bnx^2}{4}+\frac{bx^2\log(cx^n)}{2}}{d^3} \\ \frac{-\frac{a}{4x^4}-\frac{bn}{16x^4}-\frac{b\log(cx^n)}{4x^4}}{e^3} \\ -\frac{2ad^2}{8d^4e+16d^3e^2x^2+8d^2e^3x^4}-\frac{bd^2n\log\left(x-\sqrt{-\frac{d}{e}}\right)}{8d^4e+16d^3e^2x^2+8d^2e^3x^4}-\frac{bd^2n\log\left(x+\sqrt{-\frac{d}{e}}\right)}{8d^4e+16d^3e^2x^2+8d^2e^3x^4}+\frac{bd^2n}{8d^4e+16d^3e^2x^2+8d^2e^3x^4}-\frac{2bdenx^2\log\left(x\right.}{8d^4e+16d^3e^2x^2}\end{cases}$$

```
[In] integrate(x*(a+b*ln(c*x**n))/(e*x**2+d)**3,x)
```

```
[Out] Piecewise((zoo*(-a/(4*x**4) - b*n/(16*x**4) - b*log(c*x**n)/(4*x**4)), Eq(d
, 0) & Eq(e, 0)), ((a*x**2/2 - b*n*x**2/4 + b*x**2*log(c*x**n)/2)/d**3, Eq(
e, 0)), ((-a/(4*x**4) - b*n/(16*x**4) - b*log(c*x**n)/(4*x**4))/e**3, Eq(d,
 0)), (-2*a*d**2/(8*d**4*e + 16*d**3*e**2*x**2 + 8*d**2*e**3*x**4) - b*d**2
*n*log(x - sqrt(-d/e))/(8*d**4*e + 16*d**3*e**2*x**2 + 8*d**2*e**3*x**4) -
b*d**2*n*log(x + sqrt(-d/e))/(8*d**4*e + 16*d**3*e**2*x**2 + 8*d**2*e**3*x*
*4) + b*d**2*n/(8*d**4*e + 16*d**3*e**2*x**2 + 8*d**2*e**3*x**4) - 2*b*d*e*
n*x**2*log(x - sqrt(-d/e))/(8*d**4*e + 16*d**3*e**2*x**2 + 8*d**2*e**3*x**4
) - 2*b*d*e*n*x**2*log(x + sqrt(-d/e))/(8*d**4*e + 16*d**3*e**2*x**2 + 8*d*
*2*e**3*x**4) + b*d*e*n*x**2/(8*d**4*e + 16*d**3*e**2*x**2 + 8*d**2*e**3*x*
*4) + 4*b*d*e*x**2*log(c*x**n)/(8*d**4*e + 16*d**3*e**2*x**2 + 8*d**2*e**3*
x**4) - b*e**2*n*x**4*log(x - sqrt(-d/e))/(8*d**4*e + 16*d**3*e**2*x**2 + 8
*d**2*e**3*x**4) - b*e**2*n*x**4*log(x + sqrt(-d/e))/(8*d**4*e + 16*d**3*e*
*2*x**2 + 8*d**2*e**3*x**4) + 2*b*e**2*x**4*log(c*x**n)/(8*d**4*e + 16*d**3
*e**2*x**2 + 8*d**2*e**3*x**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.33

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = \frac{1}{8}\,bn\left(\frac{1}{de^2x^2+d^2e}-\frac{\log(ex^2+d)}{d^2e}+\frac{\log(x^2)}{d^2e}\right) - \frac{b\log(cx^n)}{4\,(e^3x^4+2\,de^2x^2+d^2e)} - \frac{a}{4\,(e^3x^4+2\,de^2x^2+d^2e)}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*b*n*(1/(d*e^2*x^2 + d^2*e) - log(e*x^2 + d)/(d^2*e) + log(x^2)/(d^2*e))
 - 1/4*b*log(c*x^n)/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e) - 1/4*a/(e^3*x^4 + 2*d*
e^2*x^2 + d^2*e)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.41

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = -\frac{bdn\log(x)}{4\,(de^3x^4+2\,d^2e^2x^2+d^3e)} + \frac{benx^2+bdn-2\,bd\log(c)-2\,ad}{8\,(de^3x^4+2\,d^2e^2x^2+d^3e)} - \frac{bn\log(ex^2+d)}{8\,d^2e} + \frac{bn\log(x)}{4\,d^2e}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/4*b*d*n*log(x)/(d*e^3*x^4 + 2*d^2*e^2*x^2 + d^3*e) + 1/8*(b*e*n*x^2 + b*
d*n - 2*b*d*log(c) - 2*a*d)/(d*e^3*x^4 + 2*d^2*e^2*x^2 + d^3*e) - 1/8*b*n*l
og(e*x^2 + d)/(d^2*e) + 1/4*b*n*log(x)/(d^2*e)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.33

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = \frac{\frac{b\,n}{2}-a+\frac{b\,e\,n\,x^2}{2\,d}}{4\,d^2\,e+8\,d\,e^2\,x^2+4\,e^3\,x^4}-\frac{b\,\ln\left(c\,x^n\right)}{4\,e\,\left(d^2+2\,d\,e\,x^2+e^2\,x^4\right)}-\frac{b\,n\,\ln\left(e\,x^2+d\right)}{8\,d^2\,e}+\frac{b\,n\,\ln\left(x\right)}{4\,d^2\,e}$$

```
[In] int((x*(a + b*log(c*x^n)))/(d + e*x^2)^3,x)
```

```
[Out] ((b*n)/2 - a + (b*e*n*x^2)/(2*d))/(4*d^2*e + 4*e^3*x^4 + 8*d*e^2*x^2) - (b*
log(c*x^n))/(4*e*(d^2 + e^2*x^4 + 2*d*e*x^2)) - (b*n*log(d + e*x^2))/(8*d^2
*e) + (b*n*log(x))/(4*d^2*e)
```

3.234 $\int \frac{a+b\log(cx^n)}{x(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^3}\,dx = \frac{a+b\log(cx^n)}{4d(d+ex^2)^2} - \frac{\log\left(1+\frac{d}{ex^2}\right)(4a-3bn+4b\log(cx^n))}{8d^3} + \frac{4a-bn+4b\log(cx^n)}{8d^2(d+ex^2)} + \frac{bn\,\mathrm{PolyLog}\left(2,-\frac{d}{ex^2}\right)}{4d^3}$$

```
[Out] 1/4*(a+b*ln(c*x^n))/d/(e*x^2+d)^2-1/8*ln(1+d/e/x^2)*(4*a-3*b*n+4*b*ln(c*x^n
))/d^3+1/8*(4*a-b*n+4*b*ln(c*x^n))/d^2/(e*x^2+d)+1/4*b*n*polylog(2,-d/e/x^2
)/d^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2385, 2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^3}\,dx = -\frac{\log\left(\frac{d}{ex^2}+1\right)(4a+4b\log(cx^n)-3bn)}{8d^3} + \frac{4a+4b\log(cx^n)-bn}{8d^2(d+ex^2)} + \frac{a+b\log(cx^n)}{4d(d+ex^2)^2} + \frac{bn\,\mathrm{PolyLog}\left(2,-\frac{d}{ex^2}\right)}{4d^3}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x^2)^3),x]
```

```
[Out] (a + b*Log[c*x^n])/(4*d*(d + e*x^2)^2) - (Log[1 + d/(e*x^2)]*(4*a - 3*b*n +
 4*b*Log[c*x^n]))/(8*d^3) + (4*a - b*n + 4*b*Log[c*x^n])/(8*d^2*(d + e*x^2)
) + (b*n*PolyLog[2, -(d/(e*x^2))])/(4*d^3)
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2385

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^2)^(q_.), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(q + 1)*((a +
 b*Log[c*x^n])/(2*d*f*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(f*x)^m*(d
+ e*x^2)^(q + 1)*(a*(m + 2*q + 3) + b*n + b*(m + 2*q + 3)*Log[c*x^n]), x],
x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a+b\log\left(cx^n\right)}{4d\left(d+ex^2\right)^2}-\frac{\int\frac{-4a+bn-4b\log(cx^n)}{x(d+ex^2)^2}\,dx}{4d}\\
&= \frac{a+b\log\left(cx^n\right)}{4d\left(d+ex^2\right)^2}+\frac{4a-bn+4b\log\left(cx^n\right)}{8d^2\left(d+ex^2\right)}+\frac{\int\frac{-4bn-2(-4a+bn)+8b\log(cx^n)}{x(d+ex^2)}\,dx}{8d^2}\\
&= \frac{a+b\log\left(cx^n\right)}{4d\left(d+ex^2\right)^2}-\frac{\log\left(1+\frac{d}{ex^2}\right)\left(4a-3bn+4b\log\left(cx^n\right)\right)}{8d^3}\\
&\quad+\frac{4a-bn+4b\log\left(cx^n\right)}{8d^2\left(d+ex^2\right)}+\frac{(bn)\int\frac{\log\left(1+\frac{d}{ex^2}\right)}{x}\,dx}{2d^3}\\
&= \frac{a+b\log\left(cx^n\right)}{4d\left(d+ex^2\right)^2}-\frac{\log\left(1+\frac{d}{ex^2}\right)\left(4a-3bn+4b\log\left(cx^n\right)\right)}{8d^3}\\
&\quad+\frac{4a-bn+4b\log\left(cx^n\right)}{8d^2\left(d+ex^2\right)}+\frac{bn\text{Li}_2\left(-\frac{d}{ex^2}\right)}{4d^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.44

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^3}\,dx$$

$$= \frac{\frac{4d^2(a-bn\log(x)+b\log(cx^n))}{(d+ex^2)^2}+\frac{8d(a-bn\log(x)+b\log(cx^n))}{d+ex^2}+16\log(x)\left(a-bn\log(x)+b\log\left(cx^n\right)\right)-8(a-bn\log(x)+\cdots}{\cdots}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x^2)^3),x]
```

```
[Out] ((4*d^2*(a - b*n*Log[x] + b*Log[c*x^n]))/(d + e*x^2)^2 + (8*d*(a - b*n*Log[
x] + b*Log[c*x^n]))/(d + e*x^2) + 16*Log[x]*(a - b*n*Log[x] + b*Log[c*x^n])
 - 8*(a - b*n*Log[x] + b*Log[c*x^n])*Log[d + e*x^2] - b*n*(d/(d - I*Sqrt[d]
*Sqrt[e]*x) + d/(d + I*Sqrt[d]*Sqrt[e]*x) + 2*Log[x] - (d*Log[x])/(Sqrt[d]
- I*Sqrt[e]*x)^2 - (d*Log[x])/(Sqrt[d] + I*Sqrt[e]*x)^2 + (5*Sqrt[e]*x*Log[
x])/((-I)*Sqrt[d] + Sqrt[e]*x) + (5*Sqrt[e]*x*Log[x])/(I*Sqrt[d] + Sqrt[e]*
x) - 8*Log[x]^2 - 6*Log[I*Sqrt[d] - Sqrt[e]*x] - 6*Log[I*Sqrt[d] + Sqrt[e]*
x] + 8*Log[x]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]] + 8*Log[x]*Log[1 + (I*Sqrt[e]*
x)/Sqrt[d]] + 8*PolyLog[2, ((-I)*Sqrt[e]*x)/Sqrt[d]] + 8*PolyLog[2, (I*Sqrt
[e]*x)/Sqrt[d]]))/(16*d^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.07 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.39

method	result
risch	$-\frac{b\ln(x^n)\ln(e\,x^2+d)}{2d^3}+\frac{b\ln(x^n)}{2d^2(e\,x^2+d)}+\frac{b\ln(x^n)}{4d(e\,x^2+d)^2}+\frac{b\ln(x^n)\ln(x)}{d^3}-\frac{bn\ln(x)^2}{2d^3}+\frac{bn\ln(x)\ln(e\,x^2+d)}{2d^3}-\frac{bn\ln(x)\ln\left(\frac{-ex+\cdots}{\sqrt{\cdots}}\right)}{2d^3}\cdots$

```
[In] int((a+b*ln(c*x^n))/x/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*ln(x^n)/d^3*ln(e*x^2+d)+1/2*b*ln(x^n)/d^2/(e*x^2+d)+1/4*b*ln(x^n)/d/
(e*x^2+d)^2+b*ln(x^n)/d^3*ln(x)-1/2*b*n/d^3*ln(x)^2+1/2*b*n/d^3*ln(x)*ln(e*
x^2+d)-1/2*b*n/d^3*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*n/d^3*l
n(x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*n/d^3*dilog((-e*x+(-d*e)^(1/
2))/(-d*e)^(1/2))-1/2*b*n/d^3*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+3/8*b*
n/d^3*ln(e*x^2+d)-1/8*b*n/d^2/(e*x^2+d)-3/4*b*n*ln(x)/d^3+(-1/2*I*b*Pi*csgn
(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*
```

```
b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-1/
2*e/d^3*(1/e*ln(e*x^2+d)-d/e/(e*x^2+d)-1/2*d^2/e/(e*x^2+d)^2)+1/d^3*ln(x))
```

Fricas [F]

$$\int \frac{a + b\log\left(cx^n\right)}{x\left(d + ex^2\right)^3}\,dx = \int \frac{b\log\left(cx^n\right) + a}{\left(ex^2 + d\right)^3 x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x),
x)
```

Sympy [A] (verification not implemented)

Time = 119.75 (sec) , antiderivative size = 403, normalized size of antiderivative = 3.50

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^3}\,dx = -\frac{ae\left(\begin{cases}\frac{x^2}{2d^3} & \text{for } e=0\\ -\frac{1}{4e(d+ex^2)^2} & \text{otherwise}\end{cases}\right)}{d} - \frac{ae\left(\begin{cases}\frac{x^2}{2d^2} & \text{for } e=0\\ -\frac{1}{2de+2e^2x^2} & \text{otherwise}\end{cases}\right)}{d^2}$$
$$+\frac{a\log(x)}{d^3} - \frac{a\log(d+ex^2)}{2d^3} + \frac{be^2n\left(\begin{cases}-\frac{1}{2e^3x^2} & \text{for } d=0\\ -\frac{1}{4de^2+4e^3x^2} - \frac{\log(d+ex^2)}{4de^2} & \text{otherwise}\end{cases}\right)}{2d^2}$$
$$-\frac{be^2\left(\begin{cases}\frac{1}{e^3x^2} & \text{for } d=0\\ -\frac{1}{2d\left(\frac{d}{x^2}+e\right)^2} & \text{otherwise}\end{cases}\right)\log(cx^n)}{2d^2}$$
$$-\frac{ben\left(\begin{cases}-\frac{1}{2e^2x^2} & \text{for } d=0\\ -\frac{\log(d+ex^2)}{2de} & \text{otherwise}\end{cases}\right)}{d^2} + \frac{be\left(\begin{cases}\frac{1}{e^2x^2} & \text{for } d=0\\ -\frac{1}{\frac{d^2}{x^2}+de} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^2}$$
$$+\frac{bn\left(\begin{cases}-\frac{1}{2ex^2} & \text{for } \\ \frac{\begin{cases}\frac{\mathrm{Li}_2\left(\frac{de^{i\pi}}{ex^2}\right)}{2} & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(e)\log(x)+\frac{\mathrm{Li}_2\left(\frac{de^{i\pi}}{ex^2}\right)}{2} & \text{for } |x|<1\\ -\log(e)\log\left(\frac{1}{x}\right)+\frac{\mathrm{Li}_2\left(\frac{de^{i\pi}}{ex^2}\right)}{2} & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(e)+\frac{\mathrm{Li}_2\left(\frac{de^{i\pi}}{ex^2}\right)}{2} & \text{otherwise}\end{cases}}{d} & \text{oth}\end{cases}\right)}{2d^2}$$
$$-\frac{b\left(\begin{cases}\frac{1}{ex^2} & \text{for } d=0\\ \frac{\log\left(\frac{d}{x^2}+e\right)}{d} & \text{otherwise}\end{cases}\right)\log(cx^n)}{2d^2}$$

```
[In] integrate((a+b*ln(c*x**n))/x/(e*x**2+d)**3,x)
```

```
[Out] -a*e*Piecewise((x**2/(2*d**3), Eq(e, 0)), (-1/(4*e*(d + e*x**2)**2), True))
/d - a*e*Piecewise((x**2/(2*d**2), Eq(e, 0)), (-1/(2*d*e + 2*e**2*x**2), Tr
ue))/d**2 + a*log(x)/d**3 - a*log(d + e*x**2)/(2*d**3) + b*e**2*n*Piecewise
((-1/(2*e**3*x**2), Eq(d, 0)), (-1/(4*d*e**2 + 4*e**3*x**2) - log(d + e*x**
```

```
2)/(4*d*e**2), True))/(2*d**2) - b*e**2*Piecewise((1/(e**3*x**2), Eq(d, 0))
, (-1/(2*d*(d/x**2 + e)**2), True))*log(c*x**n)/(2*d**2) - b*e*n*Piecewise(
(-1/(2*e**2*x**2), Eq(d, 0)), (-log(d + e*x**2)/(2*d*e), True))/d**2 + b*e*
Piecewise((1/(e**2*x**2), Eq(d, 0)), (-1/(d**2/x**2 + d*e), True))*log(c*x*
*n)/d**2 + b*n*Piecewise((-1/(2*e*x**2), Eq(d, 0)), (Piecewise((polylog(2,
d*exp_polar(I*pi)/(e*x**2))/2, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(e)*log(
x) + polylog(2, d*exp_polar(I*pi)/(e*x**2))/2, Abs(x) < 1), (-log(e)*log(1/
x) + polylog(2, d*exp_polar(I*pi)/(e*x**2))/2, 1/Abs(x) < 1), (-meijerg(((()
, (1, 1)), ((0, 0), ()), x)*log(e) + meijerg(((1, 1), ()), ((), (0, 0)), x)
*log(e) + polylog(2, d*exp_polar(I*pi)/(e*x**2))/2, True))/d, True))/(2*d**
2) - b*Piecewise((1/(e*x**2), Eq(d, 0)), (log(d/x**2 + e)/d, True))*log(c*x
**n)/(2*d**2)
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^3}\,dx=\int \frac{b\log(cx^n)+a}{(ex^2+d)^3x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d)
/d^3 + 4*log(x)/d^3) + b*integrate((log(c) + log(x^n))/(e^3*x^7 + 3*d*e^2*x
^5 + 3*d^2*e*x^3 + d^3*x), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^3}\,dx=\int \frac{b\log(cx^n)+a}{(ex^2+d)^3x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^3}\,dx=\int \frac{a+b\,\ln(c\,x^n)}{x\,(e\,x^2+d)^3}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x^2)^3),x)
```

```
[Out] int((a + b*log(c*x^n))/(x*(d + e*x^2)^3), x)
```

3.235 $\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 162

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^3}\,dx = -\frac{3bn}{4d^3x^2}+\frac{a+b\log(cx^n)}{4dx^2(d+ex^2)^2}+\frac{6a-bn+6b\log(cx^n)}{8d^2x^2(d+ex^2)} - \frac{12a-5bn+12b\log(cx^n)}{8d^3x^2} + \frac{e\log\left(1+\frac{d}{ex^2}\right)(12a-5bn+12b\log(cx^n))}{8d^4} - \frac{3ben\,\text{PolyLog}\left(2,-\frac{d}{ex^2}\right)}{4d^4}$$

```
[Out] -3/4*b*n/d^3/x^2+1/4*(a+b*ln(c*x^n))/d/x^2/(e*x^2+d)^2+1/8*(6*a-b*n+6*b*ln(
c*x^n))/d^2/x^2/(e*x^2+d)+1/8*(-12*a+5*b*n-12*b*ln(c*x^n))/d^3/x^2+1/8*e*ln
(1+d/e/x^2)*(12*a-5*b*n+12*b*ln(c*x^n))/d^4-3/4*b*e*n*polylog(2,-d/e/x^2)/d
^4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2385, 2380, 2341, 2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^3}\,dx = \frac{e\log\left(\frac{d}{ex^2}+1\right)(12a+12b\log(cx^n)-5bn)}{8d^4} - \frac{12a+12b\log(cx^n)-5bn}{8d^3x^2}+\frac{6a+6b\log(cx^n)-bn}{8d^2x^2(d+ex^2)} + \frac{a+b\log(cx^n)}{4dx^2(d+ex^2)^2}-\frac{3ben\,\text{PolyLog}\left(2,-\frac{d}{ex^2}\right)}{4d^4}-\frac{3bn}{4d^3x^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^3*(d + e*x^2)^3),x]
```

```
[Out] (-3*b*n)/(4*d^3*x^2) + (a + b*Log[c*x^n])/(4*d*x^2*(d + e*x^2)^2) + (6*a -
b*n + 6*b*Log[c*x^n])/(8*d^2*x^2*(d + e*x^2)) - (12*a - 5*b*n + 12*b*Log[c*
x^n])/(8*d^3*x^2) + (e*Log[1 + d/(e*x^2)]*(12*a - 5*b*n + 12*b*Log[c*x^n]))
/(8*d^4) - (3*b*e*n*PolyLog[2, -(d/(e*x^2))])/(4*d^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2385

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^2)^(q_.), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(q + 1)*((a +
 b*Log[c*x^n])/(2*d*f*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(f*x)^m*(d
+ e*x^2)^(q + 1)*(a*(m + 2*q + 3) + b*n + b*(m + 2*q + 3)*Log[c*x^n]), x],
x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \frac{a+b\log(cx^n)}{4dx^2(d+ex^2)^2} - \frac{\int \frac{-6a+bn-6b\log(cx^n)}{x^3(d+ex^2)^2}\,dx}{4d}$$

$$= \frac{a+b\log(cx^n)}{4dx^2(d+ex^2)^2} + \frac{6a-bn+6b\log(cx^n)}{8d^2x^2(d+ex^2)} + \frac{\int \frac{-6bn-4(-6a+bn)+24b\log(cx^n)}{x^3(d+ex^2)}\,dx}{8d^2}$$

$$
\begin{aligned}
&= \frac{a+b\log(cx^n)}{4dx^2(d+ex^2)^2} + \frac{6a-bn+6b\log(cx^n)}{8d^2x^2(d+ex^2)} \\
&\quad + \frac{\int \frac{-6bn-4(-6a+bn)+24b\log(cx^n)}{x^3}\,dx}{8d^3} - \frac{e\int \frac{-6bn-4(-6a+bn)+24b\log(cx^n)}{x(d+ex^2)}\,dx}{8d^3} \\
&= -\frac{3bn}{4d^3x^2} + \frac{a+b\log(cx^n)}{4dx^2(d+ex^2)^2} + \frac{6a-bn+6b\log(cx^n)}{8d^2x^2(d+ex^2)} - \frac{12a-5bn+12b\log(cx^n)}{8d^3x^2} \\
&\quad + \frac{e\log\left(1+\frac{d}{ex^2}\right)(12a-5bn+12b\log(cx^n))}{8d^4} - \frac{(3ben)\int \frac{\log\left(1+\frac{d}{ex^2}\right)}{x}\,dx}{2d^4} \\
&= -\frac{3bn}{4d^3x^2} + \frac{a+b\log(cx^n)}{4dx^2(d+ex^2)^2} + \frac{6a-bn+6b\log(cx^n)}{8d^2x^2(d+ex^2)} - \frac{12a-5bn+12b\log(cx^n)}{8d^3x^2} \\
&\quad + \frac{e\log\left(1+\frac{d}{ex^2}\right)(12a-5bn+12b\log(cx^n))}{8d^4} - \frac{3ben\operatorname{Li}_2\left(-\frac{d}{ex^2}\right)}{4d^4}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 507, normalized size of antiderivative = 3.13

$$
\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^3}\,dx
$$

$$
= \frac{-\frac{8d(a-bn\log(x)+b\log(cx^n))}{x^2} - \frac{4d^2e(a-bn\log(x)+b\log(cx^n))}{(d+ex^2)^2} - \frac{16de(a-bn\log(x)+b\log(cx^n))}{d+ex^2} - 48e\log(x)\left(a-bn\log(x)+b\log\right.}{}
$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*(d + e*x^2)^3),x]
```

```
[Out] ((-8*d*(a - b*n*Log[x] + b*Log[c*x^n]))/x^2 - (4*d^2*e*(a - b*n*Log[x] + b*
Log[c*x^n]))/(d + e*x^2)^2 - (16*d*e*(a - b*n*Log[x] + b*Log[c*x^n]))/(d +
e*x^2) - 48*e*Log[x]*(a - b*n*Log[x] + b*Log[c*x^n]) + 24*e*(a - b*n*Log[x]
 + b*Log[c*x^n])*Log[d + e*x^2] + b*n*((9*e^(3/2)*x*Log[x])/((-I)*Sqrt[d] +
 Sqrt[e]*x) - 24*e*Log[x]^2 - (4*d*(1 + 2*Log[x]))/x^2 + e*(d/(d + I*Sqrt[d
]*Sqrt[e]*x) + Log[x] - (d*Log[x])/(Sqrt[d] + I*Sqrt[e]*x)^2 - Log[I*Sqrt[d
] - Sqrt[e]*x]) - 9*e*Log[I*Sqrt[d] - Sqrt[e]*x] + e*(d/(d - I*Sqrt[d]*Sqrt
[e]*x) + Log[x] - (d*Log[x])/(Sqrt[d] - I*Sqrt[e]*x)^2 - Log[I*Sqrt[d] + Sq
rt[e]*x]) + ((-9*I)*e^(3/2)*x*Log[x] + (9*I)*e*(I*Sqrt[d] + Sqrt[e]*x)*Log[
I*Sqrt[d] + Sqrt[e]*x])/(Sqrt[d] - I*Sqrt[e]*x) + 24*e*(Log[x]*Log[1 + (I*S
qrt[e]*x)/Sqrt[d]] + PolyLog[2, ((-I)*Sqrt[e]*x)/Sqrt[d]]) + 24*e*(Log[x]*L
og[1 - (I*Sqrt[e]*x)/Sqrt[d]] + PolyLog[2, (I*Sqrt[e]*x)/Sqrt[d]])))/(16*d^
4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.18 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.72

method	result
risch	$-\frac{b\ln(x^n)e}{4d^2(e\,x^2+d)^2}+\frac{3b\ln(x^n)e\ln(e\,x^2+d)}{2d^4}-\frac{b\ln(x^n)e}{d^3(e\,x^2+d)}-\frac{b\ln(x^n)}{2d^3x^2}-\frac{3b\ln(x^n)e\ln(x)}{d^4}+\frac{3bne\ln(x)^2}{2d^4}-\frac{3bne\ln(x)\ln(e\,x^2+}{2d^4}$

```
[In] int((a+b*ln(c*x^n))/x^3/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b*ln(x^n)*e/d^2/(e*x^2+d)^2+3/2*b*ln(x^n)*e/d^4*ln(e*x^2+d)-b*ln(x^n)*
e/d^3/(e*x^2+d)-1/2*b*ln(x^n)/d^3/x^2-3*b*ln(x^n)/d^4*e*ln(x)+3/2*b*n/d^4*e
*ln(x)^2-3/2*b*n/d^4*e*ln(x)*ln(e*x^2+d)+3/2*b*n/d^4*e*ln(x)*ln((-e*x+(-d*e
)^(1/2))/(-d*e)^(1/2))+3/2*b*n/d^4*e*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/
2))+3/2*b*n/d^4*e*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+3/2*b*n/d^4*e*dil
og((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-5/8*b*n*e/d^4*ln(e*x^2+d)+1/8*b*n*e/d^3
/(e*x^2+d)-1/4*b*n/d^3/x^2+5/4*b*e*n*ln(x)/d^4+(-1/2*I*b*Pi*csgn(I*c)*csgn(
I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I
*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/2*e^2/d^4*(-
1/2*d^2/e/(e*x^2+d)^2+3/e*ln(e*x^2+d)-2*d/e/(e*x^2+d))-1/2/d^3/x^2-3/d^4*e*
ln(x))
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^3}\,dx=\int \frac{b\log(cx^n)+a}{(ex^2+d)^3x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^3)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^3}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*x**n))/x**3/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^2\right)^3}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex^2+d\right)^3x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*((6*e^2*x^4 + 9*d*e*x^2 + 2*d^2)/(d^3*e^2*x^6 + 2*d^4*e*x^4 + d^5*x^
2) - 6*e*log(e*x^2 + d)/d^4 + 12*e*log(x)/d^4) + b*integrate((log(c) + log(
x^n))/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^3), x)
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^2\right)^3}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex^2+d\right)^3x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)^3*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^2\right)^3}\,dx=\int \frac{a+b\,\ln\left(c\,x^n\right)}{x^3\,\left(e\,x^2+d\right)^3}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e*x^2)^3),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^3*(d + e*x^2)^3), x)
```

3.236 $\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 211

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = -\frac{bnx}{8e^2(d+ex^2)} + \frac{bn\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{5/2}} + \frac{dx(a+b\log(cx^n))}{4e^2(d+ex^2)^2} - \frac{5x(a+b\log(cx^n))}{8e^2(d+ex^2)} + \frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8\sqrt{d}e^{5/2}} - \frac{3ibn\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16\sqrt{d}e^{5/2}} + \frac{3ibn\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16\sqrt{d}e^{5/2}}$$

```
[Out] -1/8*b*n*x/e^2/(e*x^2+d)+1/4*d*x*(a+b*ln(c*x^n))/e^2/(e*x^2+d)^2-5/8*x*(a+b
*ln(c*x^n))/e^2/(e*x^2+d)+1/2*b*n*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)/d^(1/2)
+3/8*arctan(x*e^(1/2)/d^(1/2))*(a+b*ln(c*x^n))/e^(5/2)/d^(1/2)-3/16*I*b*n*p
olylog(2,-I*x*e^(1/2)/d^(1/2))/e^(5/2)/d^(1/2)+3/16*I*b*n*polylog(2,I*x*e^(
1/2)/d^(1/2))/e^(5/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used

$= \{294, 211, 2393, 2360, 2361, 12, 4940, 2438, 205\}$

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = \frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8\sqrt{d}e^{5/2}} - \frac{5x(a+b\log(cx^n))}{8e^2(d+ex^2)} + \frac{dx(a+b\log(cx^n))}{4e^2(d+ex^2)^2} + \frac{bn\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{5/2}} - \frac{3ibn\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16\sqrt{d}e^{5/2}} + \frac{3ibn\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16\sqrt{d}e^{5/2}} - \frac{bnx}{8e^2(d+ex^2)}$$

```
[In] Int[(x^4*(a + b*Log[c*x^n]))/(d + e*x^2)^3,x]
```

```
[Out] -1/8*(b*n*x)/(e^2*(d + e*x^2)) + (b*n*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[
d]*e^(5/2)) + (d*x*(a + b*Log[c*x^n]))/(4*e^2*(d + e*x^2)^2) - (5*x*(a + b*
Log[c*x^n]))/(8*e^2*(d + e*x^2)) + (3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(a + b*Lo
g[c*x^n]))/(8*Sqrt[d]*e^(5/2)) - (((3*I)/16)*b*n*PolyLog[2, ((-I)*Sqrt[e]*x
)/Sqrt[d]])/(Sqrt[d]*e^(5/2)) + (((3*I)/16)*b*n*PolyLog[2, (I*Sqrt[e]*x)/Sq
rt[d]])/(Sqrt[d]*e^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2360

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Sym
bol] :> Simp[(-x)*(d + e*x^2)^(q + 1)*((a + b*Log[c*x^n])/(2*d*(q + 1))), x
] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*Log[c*x^n
]), x], x] + Dist[b*(n/(2*d*(q + 1))), Int[(d + e*x^2)^(q + 1), x], x]) /;
FreeQ[{a, b, c, d, e, n}, x] && LtQ[q, -1]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2(a+b\log(cx^n))}{e^2(d+ex^2)^3} - \frac{2d(a+b\log(cx^n))}{e^2(d+ex^2)^2} + \frac{a+b\log(cx^n)}{e^2(d+ex^2)}\right) dx$$

$$= \frac{\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx}{e^2} - \frac{(2d)\int \frac{a+b\log(cx^n)}{(d+ex^2)^2}\,dx}{e^2} + \frac{d^2\int \frac{a+b\log(cx^n)}{(d+ex^2)^3}\,dx}{e^2}$$

$$=\frac{dx(a+b\log(cx^n))}{4e^2(d+ex^2)^2}-\frac{x(a+b\log(cx^n))}{e^2(d+ex^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}e^{5/2}}-\frac{\int\frac{a+b\log(cx^n)}{d+ex^2}\,dx}{e^2}$$
$$+\frac{(3d)\int\frac{a+b\log(cx^n)}{(d+ex^2)^2}\,dx}{4e^2}+\frac{(bn)\int\frac{1}{d+ex^2}\,dx}{e^2}-\frac{(bn)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}\,dx}{e^2}-\frac{(bdn)\int\frac{1}{(d+ex^2)^2}\,dx}{4e^2}$$

$$=-\frac{bnx}{8e^2(d+ex^2)}+\frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}+\frac{dx(a+b\log(cx^n))}{4e^2(d+ex^2)^2}$$
$$-\frac{5x(a+b\log(cx^n))}{8e^2(d+ex^2)}+\frac{3\int\frac{a+b\log(cx^n)}{d+ex^2}\,dx}{8e^2}-\frac{(bn)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{\sqrt{d}e^{5/2}}$$
$$-\frac{(bn)\int\frac{1}{d+ex^2}\,dx}{8e^2}-\frac{(3bn)\int\frac{1}{d+ex^2}\,dx}{8e^2}+\frac{(bn)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}\,dx}{e^2}$$

$$=-\frac{bnx}{8e^2(d+ex^2)}+\frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{5/2}}+\frac{dx(a+b\log(cx^n))}{4e^2(d+ex^2)^2}-\frac{5x(a+b\log(cx^n))}{8e^2(d+ex^2)}$$
$$+\frac{3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8\sqrt{d}e^{5/2}}-\frac{(ibn)\int\frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2\sqrt{d}e^{5/2}}$$
$$+\frac{(ibn)\int\frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2\sqrt{d}e^{5/2}}+\frac{(bn)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{\sqrt{d}e^{5/2}}-\frac{(3bn)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}\,dx}{8e^2}$$

$$=-\frac{bnx}{8e^2(d+ex^2)}+\frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{5/2}}+\frac{dx(a+b\log(cx^n))}{4e^2(d+ex^2)^2}-\frac{5x(a+b\log(cx^n))}{8e^2(d+ex^2)}$$
$$+\frac{3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8\sqrt{d}e^{5/2}}-\frac{ibn\operatorname{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{5/2}}+\frac{ibn\operatorname{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{5/2}}$$
$$+\frac{(ibn)\int\frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2\sqrt{d}e^{5/2}}-\frac{(ibn)\int\frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2\sqrt{d}e^{5/2}}-\frac{(3bn)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{8\sqrt{d}e^{5/2}}$$

$$=-\frac{bnx}{8e^2(d+ex^2)}+\frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{5/2}}+\frac{dx(a+b\log(cx^n))}{4e^2(d+ex^2)^2}-\frac{5x(a+b\log(cx^n))}{8e^2(d+ex^2)}$$
$$+\frac{3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8\sqrt{d}e^{5/2}}-\frac{(3ibn)\int\frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{16\sqrt{d}e^{5/2}}+\frac{(3ibn)\int\frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{16\sqrt{d}e^{5/2}}$$

$$=-\frac{bnx}{8e^2(d+ex^2)}+\frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{5/2}}+\frac{dx(a+b\log(cx^n))}{4e^2(d+ex^2)^2}-\frac{5x(a+b\log(cx^n))}{8e^2(d+ex^2)}$$
$$+\frac{3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8\sqrt{d}e^{5/2}}-\frac{3ibn\operatorname{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16\sqrt{d}e^{5/2}}+\frac{3ibn\operatorname{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16\sqrt{d}e^{5/2}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 495 vs. 2(211) = 422.

Time = 0.77 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.35

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^3}\,dx$$

$$= \frac{-\frac{\sqrt{-d}(a+b\log(cx^n))}{\left(\sqrt{-d}-\sqrt{e}x\right)^2} + \frac{5(a+b\log(cx^n))}{\sqrt{-d}-\sqrt{e}x} + \frac{\sqrt{-d}(a+b\log(cx^n))}{\left(\sqrt{-d}+\sqrt{e}x\right)^2} - \frac{5(a+b\log(cx^n))}{\sqrt{-d}+\sqrt{e}x} - \frac{5bn\left(\log(x)-\log\left(\sqrt{-d}-\sqrt{e}x\right)\right)}{\sqrt{-d}} + \frac{5bn(\log(x)-\log}{\sqrt{-}}}{}$$

```
[In] Integrate[(x^4*(a + b*Log[c*x^n]))/(d + e*x^2)^3,x]
```

```
[Out] (-((Sqrt[-d]*(a + b*Log[c*x^n]))/(Sqrt[-d] - Sqrt[e]*x)^2) + (5*(a + b*Log[
c*x^n]))/(Sqrt[-d] - Sqrt[e]*x) + (Sqrt[-d]*(a + b*Log[c*x^n]))/(Sqrt[-d] +
 Sqrt[e]*x)^2 - (5*(a + b*Log[c*x^n]))/(Sqrt[-d] + Sqrt[e]*x) - (5*b*n*(Log
[x] - Log[Sqrt[-d] - Sqrt[e]*x]))/Sqrt[-d] + (5*b*n*(Log[x] - Log[Sqrt[-d]
+ Sqrt[e]*x]))/Sqrt[-d] - (b*n*(d + (d - Sqrt[-d]*Sqrt[e]*x)*Log[x] + (-d +
 Sqrt[-d]*Sqrt[e]*x)*Log[Sqrt[-d] + Sqrt[e]*x]))/(d*(Sqrt[-d] + Sqrt[e]*x))
 - (3*(a + b*Log[c*x^n])*Log[1 + (Sqrt[e]*x)/Sqrt[-d]])/Sqrt[-d] + (b*n*(d
+ (d + Sqrt[-d]*Sqrt[e]*x)*Log[x] - (d + Sqrt[-d]*Sqrt[e]*x)*Log[(-d)^(3/2)
 + d*Sqrt[e]*x]))/(d*(Sqrt[-d] - Sqrt[e]*x)) + (3*(a + b*Log[c*x^n])*Log[1
+ (d*Sqrt[e]*x)/(-d)^(3/2)])/Sqrt[-d] + (3*b*n*PolyLog[2, (Sqrt[e]*x)/Sqrt[
-d]])/Sqrt[-d] - (3*b*n*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)])/Sqrt[-d])/(16
*e^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.69 (sec) , antiderivative size = 900, normalized size of antiderivative = 4.27

method	result	size
risch	Expression too large to display	900

```
[In] int(x^4*(a+b*ln(c*x^n))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*b*n*d/e*ln(x)/(e*x^2+d)^2/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1
/2))*x^2-3/8*b*n*d/e*ln(x)/(e*x^2+d)^2/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(
-d*e)^(1/2))*x^2+1/2*b*n/e^2*ln(x)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*
e)^(1/2))-1/2*b*n/e^2*ln(x)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)
)+b*n/e*ln(x)/(e*x^2+d)^2*x^3+3/16*b*n*ln(x)/(e*x^2+d)^2/(-d*e)^(1/2)*ln((-
e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^4-b*n/e^2*ln(x)*x/(e*x^2+d)-3/16*b*n*ln(x
)/(e*x^2+d)^2/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^4+b*n*d/e^
2*ln(x)/(e*x^2+d)^2*x-3/8*b/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*n*ln(x)
```

```
-3/8*b*ln(x^n)*d/e^2/(e*x^2+d)^2*x+3/16*b*n*d^2/e^2*ln(x)/(e*x^2+d)^2/(-d*e
)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+3/16*b*n/e^2/(-d*e)^(1/2)*dilo
g((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-3/16*b*n/e^2/(-d*e)^(1/2)*dilog((e*x+(-
d*e)^(1/2))/(-d*e)^(1/2))-1/8*b*n*x/e^2/(e*x^2+d)+1/2*b*n/e^2/(d*e)^(1/2)*a
rctan(x*e/(d*e)^(1/2))-5/8*b/(e*x^2+d)^2/e*x^3*ln(x^n)+3/8*b/e^2/(d*e)^(1/2
)*arctan(x*e/(d*e)^(1/2))*ln(x^n)-3/16*b*n*d^2/e^2*ln(x)/(e*x^2+d)^2/(-d*e)
^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*b*n/e*ln(x)/(e*x^2+d)/(-d*e)
^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2+1/2*b*n/e*ln(x)/(e*x^2+d)/(
-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2-1/2*b*n*d/e^2*ln(x)/(e*
x^2+d)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/2*b*n*d/e^2*ln(x
)/(e*x^2+d)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+(-1/2*I*b*Pi*c
sgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2
*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(
(-5/8/e*x^3-3/8*d/e^2*x)/(e*x^2+d)^2+3/8/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(
1/2)))
```

Fricas [F]

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^3}\,dx=\int \frac{(b\log(cx^n)+a)x^4}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*x^4*log(c*x^n) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 +
d^3), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^3}\,dx=\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^3}\,dx$$

```
[In] integrate(x**4*(a+b*ln(c*x**n))/(e*x**2+d)**3,x)
[Out] Integral(x**4*(a + b*log(c*x**n))/(d + e*x**2)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = \int \frac{(b\log(cx^n)+a)x^4}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^4/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = \int \frac{x^4\,(a+b\,\ln(c\,x^n))}{(e\,x^2+d)^3}\,dx$$

```
[In] int((x^4*(a + b*log(c*x^n)))/(d + e*x^2)^3,x)
```

```
[Out] int((x^4*(a + b*log(c*x^n)))/(d + e*x^2)^3, x)
```

3.237 $\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 187

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = \frac{bnx}{8de(d+ex^2)} - \frac{x(a+b\log(cx^n))}{4e(d+ex^2)^2} + \frac{x(a+b\log(cx^n))}{8de(d+ex^2)} + \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8d^{3/2}e^{3/2}} - \frac{ibn\,\text{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{3/2}e^{3/2}} + \frac{ibn\,\text{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{3/2}e^{3/2}}$$

```
[Out] 1/8*b*n*x/d/e/(e*x^2+d)-1/4*x*(a+b*ln(c*x^n))/e/(e*x^2+d)^2+1/8*x*(a+b*ln(c
*x^n))/d/e/(e*x^2+d)+1/8*arctan(x*e^(1/2)/d^(1/2))*(a+b*ln(c*x^n))/d^(3/2)/
e^(3/2)-1/16*I*b*n*polylog(2,-I*x*e^(1/2)/d^(1/2))/d^(3/2)/e^(3/2)+1/16*I*b
*n*polylog(2,I*x*e^(1/2)/d^(1/2))/d^(3/2)/e^(3/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {294, 205, 211, 2393, 2360, 2361, 12, 4940, 2438}

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8d^{3/2}e^{3/2}} + \frac{x(a+b\log(cx^n))}{8de(d+ex^2)} - \frac{x(a+b\log(cx^n))}{4e(d+ex^2)^2} - \frac{ibn\,\text{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{3/2}e^{3/2}} + \frac{ibn\,\text{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{3/2}e^{3/2}} + \frac{bnx}{8de(d+ex^2)}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/(d + e*x^2)^3,x]
```

```
[Out] (b*n*x)/(8*d*e*(d + e*x^2)) - (x*(a + b*Log[c*x^n]))/(4*e*(d + e*x^2)^2) +
(x*(a + b*Log[c*x^n]))/(8*d*e*(d + e*x^2)) + (ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(
a + b*Log[c*x^n]))/(8*d^(3/2)*e^(3/2)) - ((I/16)*b*n*PolyLog[2, ((-I)*Sqrt[
e]*x)/Sqrt[d]])/(d^(3/2)*e^(3/2)) + ((I/16)*b*n*PolyLog[2, (I*Sqrt[e]*x)/Sq
rt[d]])/(d^(3/2)*e^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2360

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Sym
bol] :> Simp[(-x)*(d + e*x^2)^(q + 1)*((a + b*Log[c*x^n])/(2*d*(q + 1))), x
] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*Log[c*x^n
]), x], x] + Dist[b*(n/(2*d*(q + 1))), Int[(d + e*x^2)^(q + 1), x], x]) /;
FreeQ[{a, b, c, d, e, n}, x] && LtQ[q, -1]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{d(a+b\log(cx^n))}{e(d+ex^2)^3}+\frac{a+b\log(cx^n)}{e(d+ex^2)^2}\right)dx\\
&= \frac{\int \frac{a+b\log(cx^n)}{(d+ex^2)^2}\,dx}{e}-\frac{d\int \frac{a+b\log(cx^n)}{(d+ex^2)^3}\,dx}{e}\\
&= -\frac{x(a+b\log(cx^n))}{4e(d+ex^2)^2}+\frac{x(a+b\log(cx^n))}{2de(d+ex^2)}-\frac{3\int \frac{a+b\log(cx^n)}{(d+ex^2)^2}\,dx}{4e}\\
&\quad+\frac{\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx}{2de}+\frac{(bn)\int \frac{1}{(d+ex^2)^2}\,dx}{4e}-\frac{(bn)\int \frac{1}{d+ex^2}\,dx}{2de}\\
&= \frac{bnx}{8de(d+ex^2)}-\frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{3/2}}-\frac{x(a+b\log(cx^n))}{4e(d+ex^2)^2}\\
&\quad+\frac{x(a+b\log(cx^n))}{8de(d+ex^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}e^{3/2}}-\frac{3\int \frac{a+b\log(cx^n)}{d+ex^2}\,dx}{8de}\\
&\quad+\frac{(bn)\int \frac{1}{d+ex^2}\,dx}{8de}+\frac{(3bn)\int \frac{1}{d+ex^2}\,dx}{8de}-\frac{(bn)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}\,dx}{2de}\\
&= \frac{bnx}{8de(d+ex^2)}-\frac{x(a+b\log(cx^n))}{4e(d+ex^2)^2}+\frac{x(a+b\log(cx^n))}{8de(d+ex^2)}\\
&\quad+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8d^{3/2}e^{3/2}}-\frac{(bn)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2d^{3/2}e^{3/2}}+\frac{(3bn)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}\,dx}{8de}
\end{aligned}
$$

$$= \frac{bnx}{8de(d+ex^2)} - \frac{x(a+b\log(cx^n))}{4e(d+ex^2)^2} + \frac{x(a+b\log(cx^n))}{8de(d+ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8d^{3/2}e^{3/2}} - \frac{(ibn)\int\frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{4d^{3/2}e^{3/2}} + \frac{(ibn)\int\frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{4d^{3/2}e^{3/2}} + \frac{(3bn)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{8d^{3/2}e^{3/2}}$$

$$= \frac{bnx}{8de(d+ex^2)} - \frac{x(a+b\log(cx^n))}{4e(d+ex^2)^2} + \frac{x(a+b\log(cx^n))}{8de(d+ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8d^{3/2}e^{3/2}} - \frac{ibn\text{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{3/2}e^{3/2}} + \frac{ibn\text{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4d^{3/2}e^{3/2}} + \frac{(3ibn)\int\frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{16d^{3/2}e^{3/2}} - \frac{(3ibn)\int\frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{16d^{3/2}e^{3/2}}$$

$$= \frac{bnx}{8de(d+ex^2)} - \frac{x(a+b\log(cx^n))}{4e(d+ex^2)^2} + \frac{x(a+b\log(cx^n))}{8de(d+ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8d^{3/2}e^{3/2}} - \frac{ibn\text{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{3/2}e^{3/2}} + \frac{ibn\text{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{3/2}e^{3/2}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 497 vs. 2(187) = 374.

Time = 0.61 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.66

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^3}\,dx$$

$$= \frac{\frac{d(a+b\log(cx^n))}{(-d)^{3/2}(\sqrt{-d}-\sqrt{e}x)^2} + \frac{a+b\log(cx^n)}{\sqrt{-d}(\sqrt{-d}+\sqrt{e}x)^2} - \frac{a+b\log(cx^n)}{\sqrt{-d}d-d\sqrt{e}x} + \frac{a+b\log(cx^n)}{\sqrt{-d}d+d\sqrt{e}x} + \frac{bdn(\log(x)-\log(\sqrt{-d}-\sqrt{e}x))}{(-d)^{5/2}} + \frac{bn(\log(x)-\log(\sqrt{-d}-}{(-d)^{3/2}}}{}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*x^2)^3,x]
```

```
[Out] ((d*(a + b*Log[c*x^n]))/((-d)^(3/2)*(Sqrt[-d] - Sqrt[e]*x)^2) + (a + b*Log[
c*x^n])/(Sqrt[-d]*(Sqrt[-d] + Sqrt[e]*x)^2) - (a + b*Log[c*x^n])/(Sqrt[-d]*
d - d*Sqrt[e]*x) + (a + b*Log[c*x^n])/(Sqrt[-d]*d + d*Sqrt[e]*x) + (b*d*n*(
Log[x] - Log[Sqrt[-d] - Sqrt[e]*x]))/(-d)^(5/2) + (b*n*(Log[x] - Log[Sqrt[-
d] + Sqrt[e]*x]))/(-d)^(3/2) + (b*n*(d + (d - Sqrt[-d]*Sqrt[e]*x)*Log[x] +
(-d + Sqrt[-d]*Sqrt[e]*x)*Log[Sqrt[-d] + Sqrt[e]*x]))/(d^2*(Sqrt[-d] + Sqrt
[e]*x)) + ((a + b*Log[c*x^n])*Log[1 + (Sqrt[e]*x)/Sqrt[-d]])/(-d)^(3/2) - (
b*n*(d + (d + Sqrt[-d]*Sqrt[e]*x)*Log[x] - (d + Sqrt[-d]*Sqrt[e]*x)*Log[(-d
)^(3/2) + d*Sqrt[e]*x]))/(d^2*(Sqrt[-d] - Sqrt[e]*x)) + (d*(a + b*Log[c*x^n
])*Log[1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(-d)^(5/2) + (b*d*n*PolyLog[2, (Sqrt[
e]*x)/Sqrt[-d]])/(-d)^(5/2) + (b*n*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)])/(-
d)^(3/2))/(16*e^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.65 (sec) , antiderivative size = 826, normalized size of antiderivative = 4.42

method	result
risch	$-\frac{bn\ln(x)x^3}{2d(e\,x^2+d)^2}+\frac{b\,x^3\ln(x^n)}{8(e\,x^2+d)^2d}-\frac{bn\ln(x)x}{2e(e\,x^2+d)^2}-\frac{bx\ln(x^n)}{8(e\,x^2+d)^2e}-\frac{b\arctan\left(\frac{xe}{\sqrt{de}}\right)n\ln(x)}{8ed\sqrt{de}}+\frac{b\arctan\left(\frac{xe}{\sqrt{de}}\right)\ln(x^n)}{8ed\sqrt{de}}+\frac{bnx}{8de(e\,x^2+}$

```
[In] int(x^2*(a+b*ln(c*x^n))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*n/d*ln(x)/(e*x^2+d)^2*x^3+1/8*b/(e*x^2+d)^2/d*x^3*ln(x^n)-1/2*b*n/e*
ln(x)/(e*x^2+d)^2*x-1/8*b/(e*x^2+d)^2*x/e*ln(x^n)-1/8*b/e/d/(d*e)^(1/2)*arc
tan(x*e/(d*e)^(1/2))*n*ln(x)+1/8*b/e/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*
ln(x^n)+1/8*b*n*x/d/e/(e*x^2+d)-3/16*b*n/d*e*ln(x)/(e*x^2+d)^2/(-d*e)^(1/2)
*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^4+3/16*b*n/d*e*ln(x)/(e*x^2+d)^2/(-
d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^4-3/8*b*n*ln(x)/(e*x^2+d)^
2/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2+3/8*b*n*ln(x)/(e*x^
2+d)^2/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2-3/16*b*n*d/e*ln
(x)/(e*x^2+d)^2/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+3/16*b*n*
d/e*ln(x)/(e*x^2+d)^2/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/16
*b*n/d/e/(-d*e)^(1/2)*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/16*b*n/d/e/
(-d*e)^(1/2)*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/4*b*n*ln(x)/d/(e*x^2+
d)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2-1/4*b*n*ln(x)/d/(e
*x^2+d)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2+1/2*b*n/e*ln(x
)/d/(e*x^2+d)*x+1/4*b*n/e*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2
))/(-d*e)^(1/2))-1/4*b*n/e*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2
))/(-d*e)^(1/2))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*P
i*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*
Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*((1/8/d*x^3-1/8*x/e)/(e*x^2+d)^2+1/8/e/d/(d*e
)^(1/2)*arctan(x*e/(d*e)^(1/2)))
```

Fricas [F]

$$\int \frac{x^2(a+b\log{(cx^n)})}{{(d+ex^2)}^3}\,dx=\int \frac{(b\log{(cx^n)}+a)x^2}{{(ex^2+d)}^3}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*log(c*x^n) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 +
d^3), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = \int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^3}\,dx$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(e*x**2+d)**3,x)
[Out] Integral(x**2*(a + b*log(c*x**n))/(d + e*x**2)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^2/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^3}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,x^n))}{(e\,x^2+d)^3}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e*x^2)^3,x)
[Out] int((x^2*(a + b*log(c*x^n)))/(d + e*x^2)^3, x)
```

3.238 $\int \frac{a+b\log(cx^n)}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 210

$$\int \frac{a+b\log(cx^n)}{(d+ex^2)^3}\,dx = -\frac{bnx}{8d^2(d+ex^2)} - \frac{bn\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}} + \frac{x(a+b\log(cx^n))}{4d(d+ex^2)^2} + \frac{3x(a+b\log(cx^n))}{8d^2(d+ex^2)} + \frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8d^{5/2}\sqrt{e}} - \frac{3ibn\operatorname{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{5/2}\sqrt{e}} + \frac{3ibn\operatorname{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{5/2}\sqrt{e}}$$

```
[Out] -1/8*b*n*x/d^2/(e*x^2+d)+1/4*x*(a+b*ln(c*x^n))/d/(e*x^2+d)^2+3/8*x*(a+b*ln(
c*x^n))/d^2/(e*x^2+d)-1/2*b*n*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(1/2)+3/8
*arctan(x*e^(1/2)/d^(1/2))*(a+b*ln(c*x^n))/d^(5/2)/e^(1/2)-3/16*I*b*n*polyl
og(2,-I*x*e^(1/2)/d^(1/2))/d^(5/2)/e^(1/2)+3/16*I*b*n*polylog(2,I*x*e^(1/2)
/d^(1/2))/d^(5/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used

= {2360, 211, 2361, 12, 4940, 2438, 205}

$$\int \frac{a+b\log(cx^n)}{(d+ex^2)^3}\,dx = \frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8d^{5/2}\sqrt{e}} + \frac{3x(a+b\log(cx^n))}{8d^2(d+ex^2)} + \frac{x(a+b\log(cx^n))}{4d(d+ex^2)^2} - \frac{bn\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}} - \frac{3ibn\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{5/2}\sqrt{e}} + \frac{3ibn\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{5/2}\sqrt{e}} - \frac{bnx}{8d^2(d+ex^2)}$$

```
[In] Int[(a + b*Log[c*x^n])/(d + e*x^2)^3,x]
```

```
[Out] -1/8*(b*n*x)/(d^2*(d + e*x^2)) - (b*n*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/
2)*Sqrt[e]) + (x*(a + b*Log[c*x^n]))/(4*d*(d + e*x^2)^2) + (3*x*(a + b*Log[
c*x^n]))/(8*d^2*(d + e*x^2)) + (3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(a + b*Log[c*
x^n]))/(8*d^(5/2)*Sqrt[e]) - (((3*I)/16)*b*n*PolyLog[2, ((-I)*Sqrt[e]*x)/Sq
rt[d]])/(d^(5/2)*Sqrt[e]) + (((3*I)/16)*b*n*PolyLog[2, (I*Sqrt[e]*x)/Sqrt[d
]])/(d^(5/2)*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2360

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Sym
bol] :> Simp[(-x)*(d + e*x^2)^(q + 1)*((a + b*Log[c*x^n])/(2*d*(q + 1))), x
] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*Log[c*x^n
]), x], x] + Dist[b*(n/(2*d*(q + 1))), Int[(d + e*x^2)^(q + 1), x], x]) /;
FreeQ[{a, b, c, d, e, n}, x] && LtQ[q, -1]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a+b\log(cx^n))}{4d(d+ex^2)^2}+\frac{3\int\frac{a+b\log(cx^n)}{(d+ex^2)^2}\,dx}{4d}-\frac{(bn)\int\frac{1}{(d+ex^2)^2}\,dx}{4d}\\
&= -\frac{bnx}{8d^2(d+ex^2)}+\frac{x(a+b\log(cx^n))}{4d(d+ex^2)^2}+\frac{3x(a+b\log(cx^n))}{8d^2(d+ex^2)}\\
&\quad+\frac{3\int\frac{a+b\log(cx^n)}{d+ex^2}\,dx}{8d^2}-\frac{(bn)\int\frac{1}{d+ex^2}\,dx}{8d^2}-\frac{(3bn)\int\frac{1}{d+ex^2}\,dx}{8d^2}\\
&= -\frac{bnx}{8d^2(d+ex^2)}-\frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}}+\frac{x(a+b\log(cx^n))}{4d(d+ex^2)^2}+\frac{3x(a+b\log(cx^n))}{8d^2(d+ex^2)}\\
&\quad+\frac{3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8d^{5/2}\sqrt{e}}-\frac{(3bn)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}\,dx}{8d^2}\\
&= -\frac{bnx}{8d^2(d+ex^2)}-\frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}}+\frac{x(a+b\log(cx^n))}{4d(d+ex^2)^2}+\frac{3x(a+b\log(cx^n))}{8d^2(d+ex^2)}\\
&\quad+\frac{3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8d^{5/2}\sqrt{e}}-\frac{(3bn)\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{8d^{5/2}\sqrt{e}}
\end{aligned}$$

$$= -\frac{bnx}{8d^2(d+ex^2)} - \frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}} + \frac{x(a+b\log(cx^n))}{4d(d+ex^2)^2} + \frac{3x(a+b\log(cx^n))}{8d^2(d+ex^2)} + \frac{3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8d^{5/2}\sqrt{e}} - \frac{(3ibn)\int\frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{16d^{5/2}\sqrt{e}} + \frac{(3ibn)\int\frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{16d^{5/2}\sqrt{e}}$$

$$= -\frac{bnx}{8d^2(d+ex^2)} - \frac{bn\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}} + \frac{x(a+b\log(cx^n))}{4d(d+ex^2)^2} + \frac{3x(a+b\log(cx^n))}{8d^2(d+ex^2)} + \frac{3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8d^{5/2}\sqrt{e}} - \frac{3ibn\text{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{5/2}\sqrt{e}} + \frac{3ibn\text{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{5/2}\sqrt{e}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 544 vs. $2(210) = 420$.

Time = 0.59 (sec) , antiderivative size = 544, normalized size of antiderivative = 2.59

$$\int \frac{a+b\log(cx^n)}{(d+ex^2)^3}\,dx$$

$$= \frac{1}{16}\left(\frac{d(a+b\log(cx^n))}{(-d)^{5/2}\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)^2} + \frac{a+b\log(cx^n)}{(-d)^{3/2}\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)^2} + \frac{3(a+b\log(cx^n))}{(-d)^{5/2}\sqrt{e}+d^2ex} + \frac{3(a+b\log(cx^n))}{(-d)^{3/2}d\sqrt{e}+d^2ex} + \frac{3bn\left(\log(x)-\log\left(\sqrt{-d}-\sqrt{e}x\right)\right)}{(-d)^{5/2}\sqrt{e}} - \frac{3bn\left(\log(x)-\log\left(\sqrt{-d}+\sqrt{e}x\right)\right)}{(-d)^{5/2}\sqrt{e}} - \frac{bn\left(d+\left(d-\sqrt{-d}\sqrt{e}x\right)\log(x)+\left(-d+\sqrt{-d}\sqrt{e}x\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)\right)}{d^3\left(\sqrt{-d}\sqrt{e}+ex\right)} - \frac{3(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{(-d)^{5/2}\sqrt{e}} - \frac{bn\left(d+\left(d+\sqrt{-d}\sqrt{e}x\right)\log(x)-\left(d+\sqrt{-d}\sqrt{e}x\right)\log\left((-d)^{3/2}+d\sqrt{e}x\right)\right)}{(-d)^{7/2}\sqrt{e}+d^3ex} + \frac{3(a+b\log(cx^n))\log\left(1+\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{(-d)^{5/2}\sqrt{e}} + \frac{3bn\,\text{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{(-d)^{5/2}\sqrt{e}} - \frac{3bn\,\text{PolyLog}\left(2,\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{(-d)^{5/2}\sqrt{e}}\right)$$

[In] Integrate[(a + b*Log[c*x^n])/(d + e*x^2)^3,x]

[Out] ((d*(a + b*Log[c*x^n]))/((-d)^(5/2)*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)^2) + (a + b*Log[c*x^n])/((-d)^(3/2)*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)^2) + (3*(a + b*Log[c*x^n]))/((-d)^(5/2)*Sqrt[e] + d^2*e*x) + (3*(a + b*Log[c*x^n]))/((-d)^(3/2)*d*Sqrt[e] + d^2*e*x) + (3*b*n*(Log[x] - Log[Sqrt[-d] - Sqrt[e]*x]))/((-d)^(5/2)*Sqrt[e]) - (3*b*n*(Log[x] - Log[Sqrt[-d] + Sqrt[e]*x]))/((-d)^(5/2)*Sqrt[e]) - (b*n*(d + (d - Sqrt[-d]*Sqrt[e]*x)*Log[x] + (-d + Sqrt[-d]*Sqrt[e]*x)*Log[Sqrt[-d] + Sqrt[e]*x]))/(d^3*(Sqrt[-d]*Sqrt[e] + e*x)) - (3*(a + b*Log[c*x^n])*Log[1 + (Sqrt[e]*x)/Sqrt[-d]])/((-d)^(5/2)*Sqrt[e]) - (b*n*(d + (d + Sqrt[-d]*Sqrt[e]*x)*Log[x] - (d + Sqrt[-d]*Sqrt[e]*x)*Log[(-d)^(3/2) + d*Sqrt[e]*x]))/((-d)^(7/2)*Sqrt[e] + d^3*e*x) + (3*(a + b*Log[c*x^n])*Log[1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/((-d)^(5/2)*Sqrt[e]) + (3*b*n*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]])/((-d)^(5/2)*Sqrt[e]) - (3*b*n*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)])/((-d)^(5/2)*Sqrt[e]))/16

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.63 (sec) , antiderivative size = 664, normalized size of antiderivative = 3.16

method	result
risch	$\frac{3bn\ln(x)x}{8d(e\,x^2+d)^2}+\frac{bx\ln(x^n)}{4d(e\,x^2+d)^2}-\frac{3bxn\ln(x)}{8d^2(e\,x^2+d)}+\frac{3bx\ln(x^n)}{8d^2(e\,x^2+d)}-\frac{3b\arctan\left(\frac{xe}{\sqrt{de}}\right)n\ln(x)}{8d^2\sqrt{de}}+\frac{3b\arctan\left(\frac{xe}{\sqrt{de}}\right)\ln(x^n)}{8d^2\sqrt{de}}-\frac{bnx}{8d^2(e\,x^2+}$

[In] int((a+b*ln(c*x^n))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)

[Out] 3/8*b*n*ln(x)/d/(e*x^2+d)^2*x+1/4*b*x/d/(e*x^2+d)^2*ln(x^n)-3/8*b/d^2*x/(e*x^2+d)*n*ln(x)+3/8*b/d^2*x/(e*x^2+d)*ln(x^n)-3/8*b/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*n*ln(x)+3/8*b/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*ln(x^n)-1/8*b*n*x/d^2/(e*x^2+d)-1/2*b*n/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+3/16*b*n*ln(x)/d^2/(e*x^2+d)^2/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^4*e^2-3/16*b*n*ln(x)/d^2/(e*x^2+d)^2/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^4*e^2+3/8*b*n*ln(x)/d^2/(e*x^2+d)^2*x^3*e+3/8*b*n*ln(x)/d/(e*x^2+d)^2/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2*e-3/8*b*n*ln(x)/d/(e*x^2+d)^2/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2*e+3/16*b*n*ln(x)/(e*x^2+d)^2/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-3/16*b*n*ln(x)/(e*x^2+d)^2/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+3/16*b*n/(-d*e)^(1/2)/d^2*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-3/16*b*n/(-d*e)^(1/2)/d^2*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/4*x/d/(e*x^2+d)^2+3/4/d*(1/2*x/d/(e*x^2+d)+1/2/d/(d*e)^(1/2)*arctan(x*

```
e/(d*e)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{\left(d+ex^{2}\right)^{3}}\,dx=\int \frac{b\log\left(cx^{n}\right)+a}{\left(ex^{2}+d\right)^{3}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{\left(d+ex^{2}\right)^{3}}\,dx=\int \frac{a+b\log\left(cx^{n}\right)}{\left(d+ex^{2}\right)^{3}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/(e*x**2+d)**3,x)
[Out] Integral((a + b*log(c*x**n))/(d + e*x**2)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(cx^{n}\right)}{\left(d+ex^{2}\right)^{3}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + b\log(cx^n)}{(d + ex^2)^3} \, dx = \int \frac{b\log(cx^n) + a}{(ex^2 + d)^3} \, dx$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{(d + ex^2)^3} \, dx = \int \frac{a + b\,\ln(c\,x^n)}{(e\,x^2 + d)^3} \, dx$$

```
[In] int((a + b*log(c*x^n))/(d + e*x^2)^3,x)
```

```
[Out] int((a + b*log(c*x^n))/(d + e*x^2)^3, x)
```

3.239 $\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 219

$$\begin{aligned}\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^3}\,dx = & -\frac{15bn}{8d^3x}+\frac{a+b\log(cx^n)}{4dx(d+ex^2)^2}+\frac{5a-bn+5b\log(cx^n)}{8d^2x(d+ex^2)} \\ & -\frac{15a-8bn+15b\log(cx^n)}{8d^3x} \\ & -\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(15a-8bn+15b\log(cx^n))}{8d^{7/2}} \\ & +\frac{15ib\sqrt{e}n\,\text{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}}-\frac{15ib\sqrt{e}n\,\text{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}}\end{aligned}$$

```
[Out] -15/8*b*n/d^3/x+1/4*(a+b*ln(c*x^n))/d/x/(e*x^2+d)^2+1/8*(5*a-b*n+5*b*ln(c*x
^n))/d^2/x/(e*x^2+d)+1/8*(-15*a+8*b*n-15*b*ln(c*x^n))/d^3/x-1/8*arctan(x*e^
(1/2)/d^(1/2))*(15*a-8*b*n+15*b*ln(c*x^n))*e^(1/2)/d^(7/2)+15/16*I*b*n*poly
log(2,-I*x*e^(1/2)/d^(1/2))*e^(1/2)/d^(7/2)-15/16*I*b*n*polylog(2,I*x*e^(1/
2)/d^(1/2))*e^(1/2)/d^(7/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used

$= \{2385, 2380, 2341, 211, 2361, 12, 4940, 2438\}$

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^3}\,dx = -\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(15a+15b\log(cx^n)-8bn)}{8d^{7/2}} - \frac{15a+15b\log(cx^n)-8bn}{8d^3x} + \frac{5a+5b\log(cx^n)-bn}{8d^2x(d+ex^2)} + \frac{a+b\log(cx^n)}{4dx(d+ex^2)^2} + \frac{15ib\sqrt{e}n\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}} - \frac{15ib\sqrt{e}n\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}} - \frac{15bn}{8d^3x}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^2*(d + e*x^2)^3),x]
```

```
[Out] (-15*b*n)/(8*d^3*x) + (a + b*Log[c*x^n])/(4*d*x*(d + e*x^2)^2) + (5*a - b*n
 + 5*b*Log[c*x^n])/(8*d^2*x*(d + e*x^2)) - (15*a - 8*b*n + 15*b*Log[c*x^n])
/(8*d^3*x) - (Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(15*a - 8*b*n + 15*b*Log[
c*x^n]))/(8*d^(7/2)) + (((15*I)/16)*b*Sqrt[e]*n*PolyLog[2, ((-I)*Sqrt[e]*x)
/Sqrt[d]])/d^(7/2) - (((15*I)/16)*b*Sqrt[e]*n*PolyLog[2, (I*Sqrt[e]*x)/Sqrt
[d]])/d^(7/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
```

```
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2385

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^2)^(q_.), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(q + 1)*((a +
 b*Log[c*x^n])/(2*d*f*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(f*x)^m*(d
+ e*x^2)^(q + 1)*(a*(m + 2*q + 3) + b*n + b*(m + 2*q + 3)*Log[c*x^n]), x],
x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a+b\log(cx^n)}{4dx(d+ex^2)^2} - \frac{\int \frac{-5a+bn-5b\log(cx^n)}{x^2(d+ex^2)^2}\,dx}{4d} \\
&= \frac{a+b\log(cx^n)}{4dx(d+ex^2)^2} + \frac{5a-bn+5b\log(cx^n)}{8d^2x(d+ex^2)} + \frac{\int \frac{-5bn-3(-5a+bn)+15b\log(cx^n)}{x^2(d+ex^2)}\,dx}{8d^2} \\
&= \frac{a+b\log(cx^n)}{4dx(d+ex^2)^2} + \frac{5a-bn+5b\log(cx^n)}{8d^2x(d+ex^2)} \\
&\quad + \frac{\int \frac{-5bn-3(-5a+bn)+15b\log(cx^n)}{x^2}\,dx}{8d^3} - \frac{e\int \frac{-5bn-3(-5a+bn)+15b\log(cx^n)}{d+ex^2}\,dx}{8d^3} \\
&= -\frac{15bn}{8d^3x} + \frac{a+b\log(cx^n)}{4dx(d+ex^2)^2} + \frac{5a-bn+5b\log(cx^n)}{8d^2x(d+ex^2)} - \frac{15a-8bn+15b\log(cx^n)}{8d^3x} \\
&\quad - \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(15a-8bn+15b\log(cx^n))}{8d^{7/2}} + \frac{(15ben)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}x}\,dx}{8d^3} \\
&= -\frac{15bn}{8d^3x} + \frac{a+b\log(cx^n)}{4dx(d+ex^2)^2} + \frac{5a-bn+5b\log(cx^n)}{8d^2x(d+ex^2)} - \frac{15a-8bn+15b\log(cx^n)}{8d^3x} \\
&\quad - \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(15a-8bn+15b\log(cx^n))}{8d^{7/2}} + \frac{(15b\sqrt{e}n)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{8d^{7/2}}
\end{aligned}$$

$$=-\frac{15bn}{8d^3x}+\frac{a+b\log(cx^n)}{4dx(d+ex^2)^2}+\frac{5a-bn+5b\log(cx^n)}{8d^2x(d+ex^2)}-\frac{15a-8bn+15b\log(cx^n)}{8d^3x}-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(15a-8bn+15b\log(cx^n))}{8d^{7/2}}+\frac{(15ib\sqrt{e}n)\int\frac{\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{16d^{7/2}}-\frac{(15ib\sqrt{e}n)\int\frac{\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{16d^{7/2}}$$

$$=-\frac{15bn}{8d^3x}+\frac{a+b\log(cx^n)}{4dx(d+ex^2)^2}+\frac{5a-bn+5b\log(cx^n)}{8d^2x(d+ex^2)}-\frac{15a-8bn+15b\log(cx^n)}{8d^3x}-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(15a-8bn+15b\log(cx^n))}{8d^{7/2}}+\frac{15ib\sqrt{e}n\mathrm{Li}_2\left(-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}}-\frac{15ib\sqrt{e}n\mathrm{Li}_2\left(\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 552 vs. $2(219) = 438$.

Time = 0.94 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.52

$$\int\frac{a+b\log(cx^n)}{x^2(d+ex^2)^3}dx=\frac{1}{16}\left(-\frac{16bn}{d^3x}-\frac{16(a+b\log(cx^n))}{d^3x}+\frac{d\sqrt{e}(a+b\log(cx^n))}{(-d)^{7/2}\left(\sqrt{-d}-\sqrt{e}x\right)^2}+\frac{7\sqrt{e}(a+b\log(cx^n))}{d^3\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{e}(a+b\log(cx^n))}{(-d)^{5/2}\left(\sqrt{-d}+\sqrt{e}x\right)^2}-\frac{7\sqrt{e}(a+b\log(cx^n))}{d^3\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{7b\sqrt{e}n\left(\log(x)-\log\left(\sqrt{-d}-\sqrt{e}x\right)\right)}{(-d)^{7/2}}-\frac{7b\sqrt{e}n\left(\log(x)-\log\left(\sqrt{-d}+\sqrt{e}x\right)\right)}{(-d)^{7/2}}+\frac{bd\sqrt{e}n\left(\frac{1}{\sqrt{-d}(\sqrt{-d}+\sqrt{e}x)}-\frac{\log(x)}{d}+\frac{\log(\sqrt{-d}+\sqrt{e}x)}{d}\right)}{(-d)^{7/2}}-\frac{15\sqrt{e}(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{(-d)^{7/2}}+\frac{b\sqrt{e}n\left(\frac{1}{\sqrt{-d}(\sqrt{-d}-\sqrt{e}x)}-\frac{\log(x)}{d}+\frac{\log((-d)^{3/2}+d\sqrt{e}x)}{d}\right)}{(-d)^{5/2}}+\frac{15\sqrt{e}(a+b\log(cx^n))\log\left(1+\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{(-d)^{7/2}}+\frac{15b\sqrt{e}n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{(-d)^{7/2}}-\frac{15b\sqrt{e}n\,\mathrm{PolyLog}\left(2,\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{(-d)^{7/2}}\right)$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*x^2)^3),x]
```

```
[Out] ((-16*b*n)/(d^3*x) - (16*(a + b*Log[c*x^n]))/(d^3*x) + (d*Sqrt[e]*(a + b*Lo
g[c*x^n]))/((-d)^(7/2)*(Sqrt[-d] - Sqrt[e]*x)^2) + (7*Sqrt[e]*(a + b*Log[c*
x^n]))/(d^3*(Sqrt[-d] - Sqrt[e]*x)) + (Sqrt[e]*(a + b*Log[c*x^n]))/((-d)^(5
/2)*(Sqrt[-d] + Sqrt[e]*x)^2) - (7*Sqrt[e]*(a + b*Log[c*x^n]))/(d^3*(Sqrt[-
d] + Sqrt[e]*x)) + (7*b*Sqrt[e]*n*(Log[x] - Log[Sqrt[-d] - Sqrt[e]*x]))/(-d
)^(7/2) - (7*b*Sqrt[e]*n*(Log[x] - Log[Sqrt[-d] + Sqrt[e]*x]))/(-d)^(7/2) +
 (b*d*Sqrt[e]*n*(1/(Sqrt[-d]*(Sqrt[-d] + Sqrt[e]*x)) - Log[x]/d + Log[Sqrt[
-d] + Sqrt[e]*x]/d))/(-d)^(7/2) - (15*Sqrt[e]*(a + b*Log[c*x^n])*Log[1 + (S
qrt[e]*x)/Sqrt[-d]])/(-d)^(7/2) + (b*Sqrt[e]*n*(1/(Sqrt[-d]*(Sqrt[-d] - Sqr
t[e]*x)) - Log[x]/d + Log[(-d)^(3/2) + d*Sqrt[e]*x]/d))/(-d)^(5/2) + (15*Sq
rt[e]*(a + b*Log[c*x^n])*Log[1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(-d)^(7/2) + (1
5*b*Sqrt[e]*n*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]])/(-d)^(7/2) - (15*b*Sqrt[e]*
n*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)])/(-d)^(7/2))/16
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.00 (sec) , antiderivative size = 964, normalized size of antiderivative = 4.40

method	result	size
risch	Expression too large to display	964

```
[In] int((a+b*ln(c*x^n))/x^2/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b*ln(x^n)/d^3/x+15/16*b*n/d^3*e/(-d*e)^(1/2)*dilog((e*x+(-d*e)^(1/2))/(-d*
e)^(1/2))+b*n*e/d^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-7/8*b/d^3*e^2/(e*x^
2+d)^2*x^3*ln(x^n)-15/16*b*n/d^3*e/(-d*e)^(1/2)*dilog((-e*x+(-d*e)^(1/2))/(
-d*e)^(1/2))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*cs
gn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*c
sgn(I*c*x^n)^3+b*ln(c)+a)*(-1/d^3*e*((7/8*e*x^3+9/8*d*x)/(e*x^2+d)^2+15/8/(
d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))-1/d^3/x)+1/8*b*n/d^3*e*x/(e*x^2+d)-1/4*
b*n*e/d^2*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))
+1/4*b*n*e/d^2*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1
/2))-3/16*b*n*e/d*ln(x)/(e*x^2+d)^2/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d
*e)^(1/2))+3/16*b*n*e/d*ln(x)/(e*x^2+d)^2/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2)
)/(-d*e)^(1/2))-b*n/d^3/x+15/8*b*e/d^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*
n*ln(x)+1/2*b*n*e^2/d^3*ln(x)/(e*x^2+d)^2*x^3+1/2*b*n*e/d^2*ln(x)/(e*x^2+d)
^2*x-1/2*b*n/d^3*e*ln(x)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+
1/2*b*n/d^3*e*ln(x)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-3/16*b
*n*e^3/d^3*ln(x)/(e*x^2+d)^2/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/
2))*x^4+3/16*b*n*e^3/d^3*ln(x)/(e*x^2+d)^2/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2
))/(-d*e)^(1/2))*x^4-3/8*b*n*e^2/d^2*ln(x)/(e*x^2+d)^2/(-d*e)^(1/2)*ln((-e*
x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2+3/8*b*n*e^2/d^2*ln(x)/(e*x^2+d)^2/(-d*e)^
```

```
(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2-1/4*b*n*e^2/d^3*ln(x)/(e*x^2+
d)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2+1/4*b*n*e^2/d^3*ln
(x)/(e*x^2+d)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2-9/8*b*ln
(x^n)/d^2/(e*x^2+d)^2*e*x-1/2*b*n/d^3*ln(x)*e*x/(e*x^2+d)-15/8*b*e/d^3/(d*e
)^(1/2)*arctan(x*e/(d*e)^(1/2))*ln(x^n)
```

Fricas [F]

$$\int \frac{a + b\log\left(cx^n\right)}{x^2\left(d + ex^2\right)^3}\,dx = \int \frac{b\log\left(cx^n\right) + a}{\left(ex^2 + d\right)^3 x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(e^3*x^8 + 3*d*e^2*x^6 + 3*d^2*e*x^4 + d^3*x^2)
, x)
```

Sympy [F]

$$\int \frac{a + b\log\left(cx^n\right)}{x^2\left(d + ex^2\right)^3}\,dx = \int \frac{a + b\log\left(cx^n\right)}{x^2\left(d + ex^2\right)^3}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**2/(e*x**2+d)**3,x)
[Out] Integral((a + b*log(c*x**n))/(x**2*(d + e*x**2)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\log\left(cx^n\right)}{x^2\left(d + ex^2\right)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d)^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^3}\,dx = \int \frac{b\log(cx^n)+a}{(ex^2+d)^3x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^3}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^2\,(e\,x^2+d)^3}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*x^2)^3),x)
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*x^2)^3), x)
```

3.240 $\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 260

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^3}\,dx = -\frac{35bn}{72d^3x^3}+\frac{35ben}{8d^4x}+\frac{a+b\log(cx^n)}{4dx^3(d+ex^2)^2}+\frac{7a-bn+7b\log(cx^n)}{8d^2x^3(d+ex^2)} - \frac{35a-12bn+35b\log(cx^n)}{24d^3x^3}+\frac{e(35a-12bn+35b\log(cx^n))}{8d^4x} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(35a-12bn+35b\log(cx^n))}{8d^{9/2}} - \frac{35ibe^{3/2}n\,\text{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{9/2}}+\frac{35ibe^{3/2}n\,\text{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{9/2}}$$

```
[Out] -35/72*b*n/d^3/x^3+35/8*b*e*n/d^4/x+1/4*(a+b*ln(c*x^n))/d/x^3/(e*x^2+d)^2+1
/8*(7*a-b*n+7*b*ln(c*x^n))/d^2/x^3/(e*x^2+d)+1/24*(-35*a+12*b*n-35*b*ln(c*x
^n))/d^3/x^3+1/8*e*(35*a-12*b*n+35*b*ln(c*x^n))/d^4/x+1/8*e^(3/2)*arctan(x*
e^(1/2)/d^(1/2))*(35*a-12*b*n+35*b*ln(c*x^n))/d^(9/2)-35/16*I*b*e^(3/2)*n*p
olylog(2,-I*x*e^(1/2)/d^(1/2))/d^(9/2)+35/16*I*b*e^(3/2)*n*polylog(2,I*x*e^
(1/2)/d^(1/2))/d^(9/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used

$= \{2385, 2380, 2341, 211, 2361, 12, 4940, 2438\}$

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^3}\,dx = \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(35a+35b\log(cx^n)-12bn)}{8d^{9/2}} + \frac{e(35a+35b\log(cx^n)-12bn)}{8d^4x} - \frac{35a+35b\log(cx^n)-12bn}{24d^3x^3} + \frac{7a+7b\log(cx^n)-bn}{8d^2x^3(d+ex^2)} + \frac{a+b\log(cx^n)}{4dx^3(d+ex^2)^2} - \frac{35ibe^{3/2}n\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{9/2}} + \frac{35ibe^{3/2}n\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{16d^{9/2}} + \frac{35ben}{8d^4x} - \frac{35bn}{72d^3x^3}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^4*(d + e*x^2)^3),x]
```

```
[Out] (-35*b*n)/(72*d^3*x^3) + (35*b*e*n)/(8*d^4*x) + (a + b*Log[c*x^n])/(4*d*x^3
*(d + e*x^2)^2) + (7*a - b*n + 7*b*Log[c*x^n])/(8*d^2*x^3*(d + e*x^2)) - (3
5*a - 12*b*n + 35*b*Log[c*x^n])/(24*d^3*x^3) + (e*(35*a - 12*b*n + 35*b*Log
[c*x^n]))/(8*d^4*x) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(35*a - 12*b*n +
 35*b*Log[c*x^n]))/(8*d^(9/2)) - (((35*I)/16)*b*e^(3/2)*n*PolyLog[2, ((-I)*
Sqrt[e]*x)/Sqrt[d]])/d^(9/2) + (((35*I)/16)*b*e^(3/2)*n*PolyLog[2, (I*Sqrt[
e]*x)/Sqrt[d]])/d^(9/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2385

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^2)^(q_.), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(q + 1)*((a +
 b*Log[c*x^n])/(2*d*f*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(f*x)^m*(d
+ e*x^2)^(q + 1)*(a*(m + 2*q + 3) + b*n + b*(m + 2*q + 3)*Log[c*x^n]), x],
x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \frac{a+b\log(cx^n)}{4dx^3(d+ex^2)^2} - \frac{\int \frac{-7a+bn-7b\log(cx^n)}{x^4(d+ex^2)^2}\,dx}{4d}$$

$$= \frac{a+b\log(cx^n)}{4dx^3(d+ex^2)^2} + \frac{7a-bn+7b\log(cx^n)}{8d^2x^3(d+ex^2)} + \frac{\int \frac{-7bn-5(-7a+bn)+35b\log(cx^n)}{x^4(d+ex^2)}\,dx}{8d^2}$$

$$= \frac{a+b\log(cx^n)}{4dx^3(d+ex^2)^2} + \frac{7a-bn+7b\log(cx^n)}{8d^2x^3(d+ex^2)} + \frac{\int \frac{-7bn-5(-7a+bn)+35b\log(cx^n)}{x^4}\,dx}{8d^3} - \frac{e\int \frac{-7bn-5(-7a+bn)+35b\log(cx^n)}{x^2(d+ex^2)}\,dx}{8d^3}$$

$$= -\frac{35bn}{72d^3x^3} + \frac{a+b\log(cx^n)}{4dx^3(d+ex^2)^2} + \frac{7a-bn+7b\log(cx^n)}{8d^2x^3(d+ex^2)} - \frac{35a-12bn+35b\log(cx^n)}{24d^3x^3} - \frac{e\int \frac{-7bn-5(-7a+bn)+35b\log(cx^n)}{x^2}\,dx}{8d^4} + \frac{e^2\int \frac{-7bn-5(-7a+bn)+35b\log(cx^n)}{d+ex^2}\,dx}{8d^4}$$

$$= -\frac{35bn}{72d^3x^3} + \frac{35ben}{8d^4x} + \frac{a+b\log(cx^n)}{4dx^3(d+ex^2)^2} + \frac{7a-bn+7b\log(cx^n)}{8d^2x^3(d+ex^2)} - \frac{35a-12bn+35b\log(cx^n)}{24d^3x^3} + \frac{e(35a-12bn+35b\log(cx^n))}{8d^4x} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(35a-12bn+35b\log(cx^n))}{8d^{9/2}} - \frac{(35be^2n)\int\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{ex}}\,dx}{8d^4}$$

$$= -\frac{35bn}{72d^3x^3} + \frac{35ben}{8d^4x} + \frac{a+b\log(cx^n)}{4dx^3(d+ex^2)^2} + \frac{7a-bn+7b\log(cx^n)}{8d^2x^3(d+ex^2)} - \frac{35a-12bn+35b\log(cx^n)}{24d^3x^3} + \frac{e(35a-12bn+35b\log(cx^n))}{8d^4x} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(35a-12bn+35b\log(cx^n))}{8d^{9/2}} - \frac{\left(35be^{3/2}n\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{x}\,dx}{8d^{9/2}}$$

$$= -\frac{35bn}{72d^3x^3} + \frac{35ben}{8d^4x} + \frac{a+b\log(cx^n)}{4dx^3(d+ex^2)^2} + \frac{7a-bn+7b\log(cx^n)}{8d^2x^3(d+ex^2)} - \frac{35a-12bn+35b\log(cx^n)}{24d^3x^3} + \frac{e(35a-12bn+35b\log(cx^n))}{8d^4x} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(35a-12bn+35b\log(cx^n))}{8d^{9/2}} - \frac{\left(35ibe^{3/2}n\right)\int\frac{\log\left(1-\frac{i\sqrt{ex}}{\sqrt{d}}\right)}{x}\,dx}{16d^{9/2}} + \frac{\left(35ibe^{3/2}n\right)\int\frac{\log\left(1+\frac{i\sqrt{ex}}{\sqrt{d}}\right)}{x}\,dx}{16d^{9/2}}$$

$$= -\frac{35bn}{72d^3x^3} + \frac{35ben}{8d^4x} + \frac{a+b\log(cx^n)}{4dx^3(d+ex^2)^2} + \frac{7a-bn+7b\log(cx^n)}{8d^2x^3(d+ex^2)} - \frac{35a-12bn+35b\log(cx^n)}{24d^3x^3} + \frac{e(35a-12bn+35b\log(cx^n))}{8d^4x} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(35a-12bn+35b\log(cx^n))}{8d^{9/2}} - \frac{35ibe^{3/2}n\mathrm{Li}_2\left(-\frac{i\sqrt{ex}}{\sqrt{d}}\right)}{16d^{9/2}} + \frac{35ibe^{3/2}n\mathrm{Li}_2\left(\frac{i\sqrt{ex}}{\sqrt{d}}\right)}{16d^{9/2}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 584 vs. 2(260) = 520.

Time = 1.03 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.25

$$\int \frac{a+b\log\left(cx^n\right)}{x^4\left(d+ex^2\right)^3}\,dx=\frac{1}{144}\left(-\frac{16bn}{d^3x^3}+\frac{432ben}{d^4x}-\frac{48(a+b\log\left(cx^n\right))}{d^3x^3}+\frac{432e(a+b\log\left(cx^n\right))}{d^4x}-\frac{9e^{3/2}(a+b\log\left(cx^n\right))}{(-d)^{7/2}\left(\sqrt{-d}-\sqrt{e}x\right)^2}-\frac{99e^{3/2}(a+b\log\left(cx^n\right))}{d^4\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{9e^{3/2}(a+b\log\left(cx^n\right))}{(-d)^{7/2}\left(\sqrt{-d}+\sqrt{e}x\right)^2}+\frac{99e^{3/2}(a+b\log\left(cx^n\right))}{d^4\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{99be^{3/2}n\left(\log(x)-\log\left(\sqrt{-d}-\sqrt{e}x\right)\right)}{(-d)^{9/2}}-\frac{99be^{3/2}n\left(\log(x)-\log\left(\sqrt{-d}+\sqrt{e}x\right)\right)}{(-d)^{9/2}}-\frac{9be^{3/2}n\left(\frac{1}{\sqrt{-d}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{\log(x)}{d}+\frac{\log\left(\sqrt{-d}+\sqrt{e}x\right)}{d}\right)}{(-d)^{7/2}}-\frac{315e^{3/2}(a+b\log\left(cx^n\right))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{(-d)^{9/2}}+\frac{9be^{3/2}n\left(\frac{1}{\sqrt{-d}\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{\log(x)}{d}+\frac{\log\left((-d)^{3/2}+d\sqrt{e}x\right)}{d}\right)}{(-d)^{7/2}}+\frac{315e^{3/2}(a+b\log\left(cx^n\right))\log\left(1+\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{(-d)^{9/2}}+\frac{315be^{3/2}n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{(-d)^{9/2}}-\frac{315be^{3/2}n\,\mathrm{PolyLog}\left(2,\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{(-d)^{9/2}}\right)$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^4*(d + e*x^2)^3),x]
```

```
[Out] ((-16*b*n)/(d^3*x^3) + (432*b*e*n)/(d^4*x) - (48*(a + b*Log[c*x^n]))/(d^3*x
^3) + (432*e*(a + b*Log[c*x^n]))/(d^4*x) - (9*e^(3/2)*(a + b*Log[c*x^n]))/(
(-d)^(7/2)*(Sqrt[-d] - Sqrt[e]*x)^2) - (99*e^(3/2)*(a + b*Log[c*x^n]))/(d^4
*(Sqrt[-d] - Sqrt[e]*x)) + (9*e^(3/2)*(a + b*Log[c*x^n]))/((-d)^(7/2)*(Sqrt
[-d] + Sqrt[e]*x)^2) + (99*e^(3/2)*(a + b*Log[c*x^n]))/(d^4*(Sqrt[-d] + Sqr
t[e]*x)) + (99*b*e^(3/2)*n*(Log[x] - Log[Sqrt[-d] - Sqrt[e]*x]))/(-d)^(9/2)
 - (99*b*e^(3/2)*n*(Log[x] - Log[Sqrt[-d] + Sqrt[e]*x]))/(-d)^(9/2) - (9*b*
e^(3/2)*n*(1/(Sqrt[-d]*(Sqrt[-d] + Sqrt[e]*x)) - Log[x]/d + Log[Sqrt[-d] +
Sqrt[e]*x]/d))/(-d)^(7/2) - (315*e^(3/2)*(a + b*Log[c*x^n])*Log[1 + (Sqrt[e
```

```
]*x)/Sqrt[-d]])/(-d)^(9/2) + (9*b*e^(3/2)*n*(1/(Sqrt[-d]*(Sqrt[-d] - Sqrt[e
]*x)) - Log[x]/d + Log[(-d)^(3/2) + d*Sqrt[e]*x]/d))/(-d)^(7/2) + (315*e^(3
/2)*(a + b*Log[c*x^n])*Log[1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(-d)^(9/2) + (315
*b*e^(3/2)*n*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]])/(-d)^(9/2) - (315*b*e^(3/2)*
n*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)])/(-d)^(9/2))/144
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.39 (sec) , antiderivative size = 1029, normalized size of antiderivative = 3.96

method	result	size
risch	Expression too large to display	1029

```
[In] int((a+b*ln(c*x^n))/x^4/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*n*e^2/d^3*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^
(1/2))-1/2*b*n*e^2/d^3*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(
-d*e)^(1/2))+3/16*b*n*e^2/d^2*ln(x)/(e*x^2+d)^2/(-d*e)^(1/2)*ln((-e*x+(-d*e
)^(1/2))/(-d*e)^(1/2))-3/16*b*n*e^2/d^2*ln(x)/(e*x^2+d)^2/(-d*e)^(1/2)*ln((
e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c
*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*
x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/d^4*e^2*((11/8*e*x^3+13/8*d
*x)/(e*x^2+d)^2+35/8/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))-1/3/d^3/x^3+3/d^4
*e/x)+3*b*e*n/d^4/x+b*n/d^4*e^2*ln(x)*x/(e*x^2+d)+35/8*b*e^2/d^4/(d*e)^(1/2
)*arctan(x*e/(d*e)^(1/2))*ln(x^n)+35/16*b*n/d^4*e^2/(-d*e)^(1/2)*dilog((-e*
x+(-d*e)^(1/2))/(-d*e)^(1/2))-35/16*b*n/d^4*e^2/(-d*e)^(1/2)*dilog((e*x+(-d
*e)^(1/2))/(-d*e)^(1/2))-3/2*b*n*e^2/d^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)
)+11/8*b/d^4*e^3/(e*x^2+d)^2*x^3*ln(x^n)+3/8*b*n*e^3/d^3*ln(x)/(e*x^2+d)^2/
(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2-3/8*b*n*e^3/d^3*ln(x)
/(e*x^2+d)^2/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2+1/2*b*n*e
^3/d^4*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^
2+3/16*b*n*e^4/d^4*ln(x)/(e*x^2+d)^2/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-
d*e)^(1/2))*x^4-3/16*b*n*e^4/d^4*ln(x)/(e*x^2+d)^2/(-d*e)^(1/2)*ln((e*x+(-d
*e)^(1/2))/(-d*e)^(1/2))*x^4-1/2*b*n*e^3/d^4*ln(x)/(e*x^2+d)/(-d*e)^(1/2)*l
n((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))*x^2-1/3*b*ln(x^n)/d^3/x^3-1/8*b*n*e^2/d^
4*x/(e*x^2+d)+13/8*b*ln(x^n)*e^2/d^3/(e*x^2+d)^2*x-b*n*e^2/d^3*ln(x)/(e*x^2
+d)^2*x-3/2*b*n/d^4*e^2*ln(x)/(-d*e)^(1/2)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/
2))+3/2*b*n/d^4*e^2*ln(x)/(-d*e)^(1/2)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))
-35/8*b*e^2/d^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*n*ln(x)-b*n*e^3/d^4*ln(
x)/(e*x^2+d)^2*x^3+3*b*ln(x^n)/d^4*e/x-1/9*b*n/d^3/x^3
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^4\left(d+ex^2\right)^3}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex^2+d\right)^3x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(e^3*x^10 + 3*d*e^2*x^8 + 3*d^2*e*x^6 + d^3*x^4
), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x^4\left(d+ex^2\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*x**n))/x**4/(e*x**2+d)**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(cx^n\right)}{x^4\left(d+ex^2\right)^3}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d)^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^4\left(d+ex^2\right)^3}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex^2+d\right)^3x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)^3*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^3}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^4\,(e\,x^2+d)^3}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^4*(d + e*x^2)^3),x)
[Out] int((a + b*log(c*x^n))/(x^4*(d + e*x^2)^3), x)
```

3.241 $\int \frac{x \log(cx^2)}{1-cx^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 17

$$\int \frac{x \log(cx^2)}{1-cx^2} dx = \frac{\text{PolyLog}(2, 1-cx^2)}{2c}$$

```
[Out] 1/2*polylog(2,-c*x^2+1)/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2374, 2352}

$$\int \frac{x \log(cx^2)}{1-cx^2} dx = \frac{\text{PolyLog}(2, 1-cx^2)}{2c}$$

```
[In] Int[(x*Log[c*x^2])/(1 - c*x^2),x]
```

```
[Out] PolyLog[2, 1 - c*x^2]/(2*c)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2374

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[f^m/n, Subst[Int[(d + e*x)^q*(a +
b*Log[c*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x]
&& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && EqQ[r, n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\log(cx)}{1-cx}\,dx, x, x^2\right)\\
&= \frac{\text{Li}_2(1-cx^2)}{2c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x \log(cx^2)}{1-cx^2}\,dx = \frac{\text{PolyLog}(2, 1-cx^2)}{2c}$$

```
[In] Integrate[(x*Log[c*x^2])/(1 - c*x^2),x]
[Out] PolyLog[2, 1 - c*x^2]/(2*c)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\text{dilog}(c\,x^2)}{2c}$	12
default	$\frac{\text{dilog}(c\,x^2)}{2c}$	12
risch	$\frac{\text{dilog}(c\,x^2)}{2c}$	12
parts	$-\frac{\ln(c\,x^2)\ln(c\,x^2-1)}{2c}+\frac{\ln(x)\ln(c\,x^2-1)-2c\left(\frac{\ln(x)(\ln(1-\sqrt{c}\,x)+\ln(1+\sqrt{c}\,x))}{2c}+\frac{\text{dilog}(1-\sqrt{c}\,x)+\text{dilog}(1+\sqrt{c}\,x)}{2c}\right)}{c}$	89

```
[In] int(x*ln(c*x^2)/(-c*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2/c*dilog(c*x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{x \log(cx^2)}{1 - cx^2} \, dx = \frac{\text{Li}_2(-cx^2 + 1)}{2c}$$

```
[In] integrate(x*log(c*x^2)/(-c*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*dilog(-c*x^2 + 1)/c
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 5.53

$$\int \frac{x \log(cx^2)}{1 - cx^2} \, dx = \frac{\begin{cases} -\frac{\text{Li}_2(cx^2)}{2} & \text{for } \frac{1}{|x|} < 1 \wedge |x| < 1 \\ i\pi \log(x) - \frac{\text{Li}_2(cx^2)}{2} & \text{for } |x| < 1 \\ -i\pi \log\left(\frac{1}{x}\right) - \frac{\text{Li}_2(cx^2)}{2} & \text{for } \frac{1}{|x|} < 1 \\ -i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| x\right) + i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| x\right) - \frac{\text{Li}_2(cx^2)}{2} & \text{otherwise} \end{cases}}{c} - \frac{\log(cx^2)\log(cx^2 - 1)}{2c}$$

```
[In] integrate(x*ln(c*x**2)/(-c*x**2+1),x)
```

```
[Out] Piecewise((-polylog(2, c*x**2)/2, (Abs(x) < 1) & (1/Abs(x) < 1)), (I*pi*log
(x) - polylog(2, c*x**2)/2, Abs(x) < 1), (-I*pi*log(1/x) - polylog(2, c*x**
2)/2, 1/Abs(x) < 1), (-I*pi*meijerg(((), (1, 1)), ((0, 0), ()), x) + I*pi*m
eijerg(((1, 1), ()), ((), (0, 0)), x) - polylog(2, c*x**2)/2, True))/c - lo
g(c*x**2)*log(c*x**2 - 1)/(2*c)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(14) = 28.

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 4.47

$$\int \frac{x \log (cx^2)}{1-cx^2}\,dx = -\frac{\log (cx^2-1)\log (cx^2)}{2\,c} + \frac{\log (cx^2-1)\log (x)}{c} + \frac{\log (cx^2-1)\log (cx^2) - 2\,\log (cx^2-1)\log (x) + \mathrm{Li}_2(-cx^2+1)}{2\,c}$$

```
[In] integrate(x*log(c*x^2)/(-c*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*log(c*x^2 - 1)*log(c*x^2)/c + log(c*x^2 - 1)*log(x)/c + 1/2*(log(c*x^2
 - 1)*log(c*x^2) - 2*log(c*x^2 - 1)*log(x) + dilog(-c*x^2 + 1))/c
```

Giac [F]

$$\int \frac{x \log (cx^2)}{1-cx^2}\,dx = \int -\frac{x \log (cx^2)}{cx^2-1}\,dx$$

```
[In] integrate(x*log(c*x^2)/(-c*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x*log(c*x^2)/(c*x^2 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{x \log (cx^2)}{1-cx^2}\,dx = \frac{\mathrm{Li}_2(c\,x^2)}{2\,c}$$

```
[In] int(-(x*log(c*x^2))/(c*x^2 - 1),x)
```

```
[Out] dilog(c*x^2)/(2*c)
```

3.242 $\int \frac{x \log\left(\frac{x^2}{c}\right)}{c-x^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 16

$$\int \frac{x \log\left(\frac{x^2}{c}\right)}{c-x^2} dx = \frac{1}{2}\operatorname{PolyLog}\left(2, 1 - \frac{x^2}{c}\right)$$

```
[Out] 1/2*polylog(2,1-x^2/c)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2374, 2352}

$$\int \frac{x \log\left(\frac{x^2}{c}\right)}{c-x^2} dx = \frac{1}{2}\operatorname{PolyLog}\left(2, 1 - \frac{x^2}{c}\right)$$

```
[In] Int[(x*Log[x^2/c])/(c - x^2),x]
```

```
[Out] PolyLog[2, 1 - x^2/c]/2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2374

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[f^m/n, Subst[Int[(d + e*x)^q*(a +
```

```
b*Log[c*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x]
&& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && EqQ[r, n]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(\frac{x}{c}\right)}{c-x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Li}_2\left(1-\frac{x^2}{c}\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x\log\left(\frac{x^2}{c}\right)}{c-x^2}\,dx = \frac{1}{2}\,\text{PolyLog}\left(2,\frac{c-x^2}{c}\right)$$

```
[In] Integrate[(x*Log[x^2/c])/(c - x^2),x]
[Out] PolyLog[2, (c - x^2)/c]/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.31

method	result
default	$\frac{\left(\sum_{\_\alpha=\text{RootOf}\left(\_Z^2-c\right)}\left(-\ln(x-\_\alpha)\ln\left(\frac{x^2}{c}\right)+2\,\text{dilog}\left(\frac{x}{\_\alpha}\right)+2\ln(x-\_\alpha)\ln\left(\frac{x}{\_\alpha}\right)\right)\right)}{2}$
risch	$\frac{\left(\sum_{\_\alpha=\text{RootOf}\left(\_Z^2-c\right)}\left(-\ln(x-\_\alpha)\ln\left(\frac{x^2}{c}\right)+2\,\text{dilog}\left(\frac{x}{\_\alpha}\right)+2\ln(x-\_\alpha)\ln\left(\frac{x}{\_\alpha}\right)\right)\right)}{2}$
parts	$-\frac{\ln\left(\frac{x^2}{c}\right)\ln(-x^2+c)}{2}+\ln(x)\ln(-x^2+c)-\ln(x)\ln\left(\frac{\sqrt{c}-x}{\sqrt{c}}\right)-\ln(x)\ln\left(\frac{\sqrt{c}+x}{\sqrt{c}}\right)-\text{dilog}\left(\frac{\sqrt{c}-x}{\sqrt{c}}\right)-\text{d}$

```
[In] int(x*ln(x^2/c)/(-x^2+c),x,method=_RETURNVERBOSE)
[Out] 1/2*sum(-ln(x-_alpha)*ln(x^2/c)+2*dilog(x/_alpha)+2*ln(x-_alpha)*ln(x/_alph
a),_alpha=RootOf(_Z^2-c))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{x\log\left(\frac{x^2}{c}\right)}{c-x^2}\,dx = \frac{1}{2}\,\mathrm{Li}_2\left(-\frac{x^2}{c}+1\right)$$

```
[In] integrate(x*log(x^2/c)/(-x^2+c),x, algorithm="fricas")
[Out] 1/2*dilog(-x^2/c + 1)
```

Sympy [A] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 117, normalized size of antiderivative = 7.31

$$\int \frac{x\log\left(\frac{x^2}{c}\right)}{c-x^2}\,dx = \begin{cases} -\frac{\mathrm{Li}_2\left(\frac{x^2}{c}\right)}{2} \\ \log(c)\log(x)+i\pi\log(x)-\frac{\mathrm{Li}_2\left(\frac{x^2}{c}\right)}{2} \\ -\log(c)\log\left(\frac{1}{x}\right)-i\pi\log\left(\frac{1}{x}\right)-\frac{\mathrm{Li}_2\left(\frac{x^2}{c}\right)}{2} \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(c)-i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)+G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(c)+i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right) \end{cases} - \frac{\log\left(\frac{x^2}{c}\right)\log\left(-c+x^2\right)}{2}$$

```
[In] integrate(x*ln(x**2/c)/(-x**2+c),x)
[Out] Piecewise((-polylog(2, x**2/c)/2, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(c)*l
og(x) + I*pi*log(x) - polylog(2, x**2/c)/2, Abs(x) < 1), (-log(c)*log(1/x)
- I*pi*log(1/x) - polylog(2, x**2/c)/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1
)), ((0, 0), ()), x)*log(c) - I*pi*meijerg(((), (1, 1)), ((0, 0), ()), x) +
 meijerg(((1, 1), ()), ((), (0, 0)), x)*log(c) + I*pi*meijerg(((1, 1), ()),
 ((), (0, 0)), x) - polylog(2, x**2/c)/2, True)) - log(x**2/c)*log(-c + x**
2)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(13) = 26.

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.62

$$\int \frac{x\log\left(\frac{x^2}{c}\right)}{c-x^2}\,dx = -\frac{1}{2}\log\left(x^2-c\right)\log\left(\frac{x^2}{c}\right) + \frac{1}{2}\log\left(x^2-c\right)\log\left(\frac{x^2-c}{c}+1\right) + \frac{1}{2}\,\mathrm{Li}_2\left(-\frac{x^2-c}{c}\right)$$

```
[In] integrate(x*log(x^2/c)/(-x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*log(x^2 - c)*log(x^2/c) + 1/2*log(x^2 - c)*log((x^2 - c)/c + 1) + 1/2*
dilog(-(x^2 - c)/c)
```

Giac [F]

$$\int \frac{x\log\left(\frac{x^2}{c}\right)}{c-x^2}\,dx = \int -\frac{x\log\left(\frac{x^2}{c}\right)}{x^2-c}\,dx$$

```
[In] integrate(x*log(x^2/c)/(-x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-x*log(x^2/c)/(x^2 - c), x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x\log\left(\frac{x^2}{c}\right)}{c-x^2}\,dx = \frac{\mathrm{Li}_2\left(\frac{x^2}{c}\right)}{2}$$

```
[In] int((x*log(x^2/c))/(c - x^2),x)
```

```
[Out] dilog(x^2/c)/2
```

3.243 $\int \frac{\log(x)}{1-x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{\log(x)}{1-x^2} \, dx = \operatorname{arctanh}(x)\log(x) + \frac{\operatorname{PolyLog}(2,-x)}{2} - \frac{\operatorname{PolyLog}(2,x)}{2}$$

```
[Out] arctanh(x)*ln(x)+1/2*polylog(2,-x)-1/2*polylog(2,x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {212, 2361, 6031}

$$\int \frac{\log(x)}{1-x^2} \, dx = \operatorname{arctanh}(x)\log(x) + \frac{\operatorname{PolyLog}(2,-x)}{2} - \frac{\operatorname{PolyLog}(2,x)}{2}$$

```
[In] Int[Log[x]/(1 - x^2),x]
```

```
[Out] ArcTanh[x]*Log[x] + PolyLog[2, -x]/2 - PolyLog[2, x]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
```

```
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \tanh^{-1}(x)\log(x) - \int \frac{\tanh^{-1}(x)}{x}\,dx \\ &= \tanh^{-1}(x)\log(x) + \frac{\text{Li}_2(-x)}{2} - \frac{\text{Li}_2(x)}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{\log(x)}{1-x^2}\,dx = \frac{1}{2}\log(x)\log(1+x) + \frac{\text{PolyLog}(2,1-x)}{2} + \frac{\text{PolyLog}(2,-x)}{2}$$

```
[In] Integrate[Log[x]/(1 - x^2),x]
[Out] (Log[x]*Log[1 + x])/2 + PolyLog[2, 1 - x]/2 + PolyLog[2, -x]/2
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\text{dilog}(x)}{2}+\frac{\text{dilog}(x+1)}{2}+\frac{\ln(x)\ln(x+1)}{2}$	20
risch	$\frac{\text{dilog}(x)}{2}+\frac{\text{dilog}(x+1)}{2}+\frac{\ln(x)\ln(x+1)}{2}$	20
parts	$\frac{\text{dilog}(x)}{2}+\frac{\text{dilog}(x+1)}{2}+\frac{\ln(x)\ln(x+1)}{2}$	20
meijerg	$\left(\frac{\ln(x)\Phi\left(x^2,1,\frac{1}{2}\right)}{2}-\frac{\Phi\left(x^2,2,\frac{1}{2}\right)}{4}\right)x$	22

```
[In] int(ln(x)/(-x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*dilog(x)+1/2*dilog(x+1)+1/2*ln(x)*ln(x+1)
```

Fricas [F]

$$\int \frac{\log(x)}{1-x^2}\,dx = \int -\frac{\log(x)}{x^2-1}\,dx$$

```
[In] integrate(log(x)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-log(x)/(x^2 - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.44 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.86

$$\int \frac{\log(x)}{1-x^2}\,dx = \frac{\begin{cases} -\operatorname{Li}_2(x) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1 \\ i\pi\log(x)-\operatorname{Li}_2(x) & \text{for } |x|<1 \\ -i\pi\log\left(\frac{1}{x}\right)-\operatorname{Li}_2(x) & \text{for } \frac{1}{|x|}<1 \\ -i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)+i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)-\operatorname{Li}_2(x) & \text{otherwise} \end{cases}}{2} - \frac{\log(x)\log(x-1)}{2} + \frac{\log(x)\log(x+1)}{2} + \frac{\operatorname{Li}_2\left(xe^{i\pi}\right)}{2}$$

```
[In] integrate(ln(x)/(-x**2+1),x)
```

```
[Out] Piecewise((-polylog(2, x), (Abs(x) < 1) & (1/Abs(x) < 1)), (I*pi*log(x) - p
olylog(2, x), Abs(x) < 1), (-I*pi*log(1/x) - polylog(2, x), 1/Abs(x) < 1),
(-I*pi*meijerg(((), (1, 1)), ((0, 0), ()), x) + I*pi*meijerg(((1, 1), ()),
((), (0, 0)), x) - polylog(2, x), True))/2 - log(x)*log(x - 1)/2 + log(x)*l
og(x + 1)/2 + polylog(2, x*exp_polar(I*pi))/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(16) = 32$.

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{\log(x)}{1-x^2}\,dx = -\frac{1}{2}\log(-x)\log(x+1) + \frac{1}{2}(\log(x+1) - \log(x-1))\log(x) + \frac{1}{2}\log(x-1)\log(x) - \frac{1}{2}\,\mathrm{Li}_2(x+1) + \frac{1}{2}\,\mathrm{Li}_2(-x+1)$$

```
[In] integrate(log(x)/(-x^2+1),x, algorithm="maxima")
[Out] -1/2*log(-x)*log(x + 1) + 1/2*(log(x + 1) - log(x - 1))*log(x) + 1/2*log(x
- 1)*log(x) - 1/2*dilog(x + 1) + 1/2*dilog(-x + 1)
```

Giac [F]

$$\int \frac{\log(x)}{1-x^2}\,dx = \int -\frac{\log(x)}{x^2-1}\,dx$$

```
[In] integrate(log(x)/(-x^2+1),x, algorithm="giac")
[Out] integrate(-log(x)/(x^2 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\log(x)}{1-x^2}\,dx = \mathrm{atanh}(x)\,\ln(x) + \frac{\mathrm{polylog}(2,-x)}{2} - \frac{\mathrm{polylog}(2,x)}{2}$$

```
[In] int(-log(x)/(x^2 - 1),x)
[Out] atanh(x)*log(x) + polylog(2, -x)/2 - polylog(2, x)/2
```

3.244 $\int \frac{\log(x)}{1+x^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 32

$$\int \frac{\log(x)}{1+x^2} \, dx = \arctan(x)\log(x) - \frac{1}{2}i \operatorname{PolyLog}(2,-ix) + \frac{1}{2}i \operatorname{PolyLog}(2,ix)$$

```
[Out] arctan(x)*ln(x)-1/2*I*polylog(2,-I*x)+1/2*I*polylog(2,I*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {209, 2361, 4940, 2438}

$$\int \frac{\log(x)}{1+x^2} \, dx = \arctan(x)\log(x) - \frac{1}{2}i \operatorname{PolyLog}(2,-ix) + \frac{1}{2}i \operatorname{PolyLog}(2,ix)$$

```
[In] Int[Log[x]/(1 + x^2),x]
```

```
[Out] ArcTan[x]*Log[x] - (I/2)*PolyLog[2, (-I)*x] + (I/2)*PolyLog[2, I*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
```

```
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \tan^{-1}(x)\log(x) - \int \frac{\tan^{-1}(x)}{x}\,dx \\
&= \tan^{-1}(x)\log(x) - \frac{1}{2}i\int \frac{\log(1-ix)}{x}\,dx + \frac{1}{2}i\int \frac{\log(1+ix)}{x}\,dx \\
&= \tan^{-1}(x)\log(x) - \frac{1}{2}i\text{Li}_2(-ix) + \frac{1}{2}i\text{Li}_2(ix)
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 65 vs. 2(32) = 64.

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.03

$$
\begin{aligned}
\int \frac{\log(x)}{1+x^2}\,dx = &-\frac{1}{2}i\log(-i(i-x))\log(x) + \frac{1}{2}i\log(x)\log(-i(i+x)) \\
&- \frac{1}{2}i\,\text{PolyLog}(2,-ix) + \frac{1}{2}i\,\text{PolyLog}(2,ix)
\end{aligned}
$$

```
[In] Integrate[Log[x]/(1 + x^2),x]
```

```
[Out] (-1/2*I)*Log[(-I)*(I - x)]*Log[x] + (I/2)*Log[x]*Log[(-I)*(I + x)] - (I/2)*
PolyLog[2, (-I)*x] + (I/2)*PolyLog[2, I*x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result	size
meijerg	$\left(\frac{\ln(x)\Phi\left(-x^{2},1,\frac{1}{2}\right)}{2}-\frac{\Phi\left(-x^{2},2,\frac{1}{2}\right)}{4}\right)x$	26
default	$-\frac{i\ln(x)\ln(ix+1)}{2}+\frac{i\ln(x)\ln(-ix+1)}{2}-\frac{i\,\mathrm{dilog}(ix+1)}{2}+\frac{i\,\mathrm{dilog}(-ix+1)}{2}$	46
risch	$-\frac{i\ln(x)\ln(ix+1)}{2}+\frac{i\ln(x)\ln(-ix+1)}{2}-\frac{i\,\mathrm{dilog}(ix+1)}{2}+\frac{i\,\mathrm{dilog}(-ix+1)}{2}$	46
parts	$-\frac{i\ln(x)\ln(ix+1)}{2}+\frac{i\ln(x)\ln(-ix+1)}{2}-\frac{i\,\mathrm{dilog}(ix+1)}{2}+\frac{i\,\mathrm{dilog}(-ix+1)}{2}$	46

```
[In] int(ln(x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] (1/2*ln(x)*LerchPhi(-x^2,1,1/2)-1/4*LerchPhi(-x^2,2,1/2))*x
```

Fricas [F]

$$\int \frac{\log(x)}{1+x^2}\,dx=\int \frac{\log\left(x\right)}{x^2+1}\,dx$$

```
[In] integrate(log(x)/(x^2+1),x, algorithm="fricas")
[Out] integral(log(x)/(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{\log(x)}{1+x^2}\,dx=\int \frac{\log\left(x\right)}{x^2+1}\,dx$$

```
[In] integrate(ln(x)/(x**2+1),x)
[Out] Integral(log(x)/(x**2 + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{\log(x)}{1+x^2}\,dx = \frac{1}{4}\,\pi\log\left(x^2+1\right) + \frac{1}{2}i\,\mathrm{Li}_2(i\,x+1) - \frac{1}{2}i\,\mathrm{Li}_2(-i\,x+1)$$

```
[In] integrate(log(x)/(x^2+1),x, algorithm="maxima")
[Out] 1/4*pi*log(x^2 + 1) + 1/2*I*dilog(I*x + 1) - 1/2*I*dilog(-I*x + 1)
```

Giac [F]

$$\int \frac{\log(x)}{1+x^2}\,dx = \int \frac{\log\left(x\right)}{x^2+1}\,dx$$

```
[In] integrate(log(x)/(x^2+1),x, algorithm="giac")
[Out] integrate(log(x)/(x^2 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{\log(x)}{1+x^2}\,dx = \mathrm{atan}(x)\,\ln\left(x\right) - \frac{\mathrm{polylog}(2,-x\,1\mathrm{i})\,1\mathrm{i}}{2} + \frac{\mathrm{polylog}(2,x\,1\mathrm{i})\,1\mathrm{i}}{2}$$

```
[In] int(log(x)/(x^2 + 1),x)
[Out] atan(x)*log(x) - (polylog(2, -x*1i)*1i)/2 + (polylog(2, x*1i)*1i)/2
```

3.245 $\int \frac{a+b\log(cx)}{1-ex^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 62

$$\int \frac{a+b\log(cx)}{1-ex^2}\,dx=\frac{\operatorname{arctanh}\left(\sqrt{e}x\right)(a+b\log(cx))}{\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,-\sqrt{e}x\right)}{2\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,\sqrt{e}x\right)}{2\sqrt{e}}$$

```
[Out] arctanh(x*e^(1/2))*(a+b*ln(c*x))/e^(1/2)+1/2*b*polylog(2,-x*e^(1/2))/e^(1/2
)-1/2*b*polylog(2,x*e^(1/2))/e^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {212, 2361, 12, 6031}

$$\int \frac{a+b\log(cx)}{1-ex^2}\,dx=\frac{\operatorname{arctanh}\left(\sqrt{e}x\right)(a+b\log(cx))}{\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,-\sqrt{e}x\right)}{2\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,\sqrt{e}x\right)}{2\sqrt{e}}$$

```
[In] Int[(a + b*Log[c*x])/(1 - e*x^2),x]
```

```
[Out] (ArcTanh[Sqrt[e]*x]*(a + b*Log[c*x]))/Sqrt[e] + (b*PolyLog[2, -(Sqrt[e]*x)]
)/(2*Sqrt[e]) - (b*PolyLog[2, Sqrt[e]*x])/(2*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\tanh^{-1}\left(\sqrt{e}x\right)(a+b\log(cx))}{\sqrt{e}}-b\int\frac{\tanh^{-1}\left(\sqrt{e}x\right)}{\sqrt{e}x}\,dx\\ &= \frac{\tanh^{-1}\left(\sqrt{e}x\right)(a+b\log(cx))}{\sqrt{e}}-\frac{b\int\frac{\tanh^{-1}\left(\sqrt{e}x\right)}{x}\,dx}{\sqrt{e}}\\ &= \frac{\tanh^{-1}\left(\sqrt{e}x\right)(a+b\log(cx))}{\sqrt{e}}+\frac{b\text{Li}_2\left(-\sqrt{e}x\right)}{2\sqrt{e}}-\frac{b\text{Li}_2\left(\sqrt{e}x\right)}{2\sqrt{e}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int\frac{a+b\log(cx)}{1-ex^2}\,dx$$
$$=\frac{-\left((a+b\log(cx))\left(\log\left(1-\sqrt{e}x\right)-\log\left(1+\sqrt{e}x\right)\right)\right)+b\,\text{PolyLog}\left(2,-\sqrt{e}x\right)-b\,\text{PolyLog}\left(2,\sqrt{e}x\right)}{2\sqrt{e}}$$

```
[In] Integrate[(a + b*Log[c*x])/(1 - e*x^2),x]
```

```
[Out] (-((a + b*Log[c*x])*(Log[1 - Sqrt[e]*x] - Log[1 + Sqrt[e]*x])) + b*PolyLog[
2, -(Sqrt[e]*x)] - b*PolyLog[2, Sqrt[e]*x])/(2*Sqrt[e])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.62 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

method	result	s
meijerg	$\frac{a\,\operatorname{arctanh}\left(x\sqrt{e}\right)}{\sqrt{e}}+\left(\frac{b\ln\left(x\right)\Phi\left(e\,x^{2},1,\frac{1}{2}\right)}{2}-\frac{b\Phi\left(e\,x^{2},2,\frac{1}{2}\right)}{4}\right)x+\frac{b\ln\left(c\right)\operatorname{arctanh}\left(x\sqrt{e}\right)}{\sqrt{e}}$	5
parts	$\frac{a\,\operatorname{arctanh}\left(x\sqrt{e}\right)}{\sqrt{e}}-bc\left(\frac{\ln\left(xc\right)\left(\ln\left(-\frac{\sqrt{e}\,xc-c}{c}\right)-\ln\left(\frac{\sqrt{e}\,xc+c}{c}\right)\right)}{2\sqrt{e}\,c}+\frac{\operatorname{dilog}\left(-\frac{\sqrt{e}\,xc-c}{c}\right)-\operatorname{dilog}\left(\frac{\sqrt{e}\,xc+c}{c}\right)}{2\sqrt{e}\,c}\right)$	1
risch	$\frac{a\,\operatorname{arctanh}\left(x\sqrt{e}\right)}{\sqrt{e}}-\frac{b\ln\left(xc\right)\ln\left(-\frac{\sqrt{e}\,xc-c}{c}\right)}{2\sqrt{e}}+\frac{b\ln\left(xc\right)\ln\left(\frac{\sqrt{e}\,xc+c}{c}\right)}{2\sqrt{e}}-\frac{b\operatorname{dilog}\left(-\frac{\sqrt{e}\,xc-c}{c}\right)}{2\sqrt{e}}+\frac{b\operatorname{dilog}\left(\frac{\sqrt{e}\,xc+c}{c}\right)}{2\sqrt{e}}$	1
derivativedivides	$\frac{\frac{ca\,\operatorname{arctanh}\left(x\sqrt{e}\right)}{\sqrt{e}}+c^{2}b\left(-\frac{\ln\left(xc\right)\left(\ln\left(-\frac{\sqrt{e}\,xc-c}{c}\right)-\ln\left(\frac{\sqrt{e}\,xc+c}{c}\right)\right)}{2\sqrt{e}\,c}-\frac{\operatorname{dilog}\left(-\frac{\sqrt{e}\,xc-c}{c}\right)-\operatorname{dilog}\left(\frac{\sqrt{e}\,xc+c}{c}\right)}{2\sqrt{e}\,c}\right)}{c}$	1
default	$\frac{\frac{ca\,\operatorname{arctanh}\left(x\sqrt{e}\right)}{\sqrt{e}}+c^{2}b\left(-\frac{\ln\left(xc\right)\left(\ln\left(-\frac{\sqrt{e}\,xc-c}{c}\right)-\ln\left(\frac{\sqrt{e}\,xc+c}{c}\right)\right)}{2\sqrt{e}\,c}-\frac{\operatorname{dilog}\left(-\frac{\sqrt{e}\,xc-c}{c}\right)-\operatorname{dilog}\left(\frac{\sqrt{e}\,xc+c}{c}\right)}{2\sqrt{e}\,c}\right)}{c}$	1

```
[In] int((a+b*ln(x*c))/(-e*x^2+1),x,method=_RETURNVERBOSE)
[Out] a/e^(1/2)*arctanh(x*e^(1/2))+(1/2*b*ln(x)*LerchPhi(e*x^2,1,1/2)-1/4*b*Lerch
Phi(e*x^2,2,1/2))*x+b*ln(c)/e^(1/2)*arctanh(x*e^(1/2))
```

Fricas [F]

$$\int \frac{a+b\log(cx)}{1-ex^2}\,dx=\int -\frac{b\log\left(cx\right)+a}{ex^2-1}\,dx$$

```
[In] integrate((a+b*log(c*x))/(-e*x^2+1),x, algorithm="fricas")
[Out] integral(-(b*log(c*x) + a)/(e*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{a+b\log(cx)}{1-ex^2}\,dx=-\int \frac{a}{ex^2-1}\,dx-\int \frac{b\log\left(cx\right)}{ex^2-1}\,dx$$

```
[In] integrate((a+b*ln(c*x))/(-e*x**2+1),x)
[Out] -Integral(a/(e*x**2 - 1), x) - Integral(b*log(c*x)/(e*x**2 - 1), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log(cx)}{1-ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x))/(-e*x^2+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log(cx)}{1-ex^2}\,dx = \int -\frac{b\log(cx)+a}{ex^2-1}\,dx$$

```
[In] integrate((a+b*log(c*x))/(-e*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(b*log(c*x) + a)/(e*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx)}{1-ex^2}\,dx = \int -\frac{a+b\ln(c\,x)}{e\,x^2-1}\,dx$$

```
[In] int(-(a + b*log(c*x))/(e*x^2 - 1),x)
```

```
[Out] int(-(a + b*log(c*x))/(e*x^2 - 1), x)
```

3.246 $\int \frac{a+b\log(cx^n)}{1-ex^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \frac{a+b\log(cx^n)}{1-ex^2}\,dx = \frac{\operatorname{arctanh}\left(\sqrt{e}x\right)(a+b\log(cx^n))}{\sqrt{e}} + \frac{bn\operatorname{PolyLog}\left(2,-\sqrt{e}x\right)}{2\sqrt{e}} - \frac{bn\operatorname{PolyLog}\left(2,\sqrt{e}x\right)}{2\sqrt{e}}$$

```
[Out] arctanh(x*e^(1/2))*(a+b*ln(c*x^n))/e^(1/2)+1/2*b*n*polylog(2,-x*e^(1/2))/e^
(1/2)-1/2*b*n*polylog(2,x*e^(1/2))/e^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {212, 2361, 12, 6031}

$$\int \frac{a+b\log(cx^n)}{1-ex^2}\,dx = \frac{\operatorname{arctanh}\left(\sqrt{e}x\right)(a+b\log(cx^n))}{\sqrt{e}} + \frac{bn\operatorname{PolyLog}\left(2,-\sqrt{e}x\right)}{2\sqrt{e}} - \frac{bn\operatorname{PolyLog}\left(2,\sqrt{e}x\right)}{2\sqrt{e}}$$

```
[In] Int[(a + b*Log[c*x^n])/(1 - e*x^2),x]
```

```
[Out] (ArcTanh[Sqrt[e]*x]*(a + b*Log[c*x^n]))/Sqrt[e] + (b*n*PolyLog[2, -(Sqrt[e]
*x)])/(2*Sqrt[e]) - (b*n*PolyLog[2, Sqrt[e]*x])/(2*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\tanh^{-1}\left(\sqrt{e}x\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{e}}-(bn)\int\frac{\tanh^{-1}\left(\sqrt{e}x\right)}{\sqrt{e}x}\,dx\\ &= \frac{\tanh^{-1}\left(\sqrt{e}x\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{e}}-\frac{(bn)\int\frac{\tanh^{-1}\left(\sqrt{e}x\right)}{x}\,dx}{\sqrt{e}}\\ &= \frac{\tanh^{-1}\left(\sqrt{e}x\right)\left(a+b\log\left(cx^n\right)\right)}{\sqrt{e}}+\frac{bn\text{Li}_2\left(-\sqrt{e}x\right)}{2\sqrt{e}}-\frac{bn\text{Li}_2\left(\sqrt{e}x\right)}{2\sqrt{e}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int\frac{a+b\log\left(cx^n\right)}{1-ex^2}\,dx = \frac{-\left(\left(a+b\log\left(cx^n\right)\right)\left(\log\left(1-\sqrt{e}x\right)-\log\left(1+\sqrt{e}x\right)\right)\right)+bn\,\text{PolyLog}\left(2,-\sqrt{e}x\right)-bn\,\text{PolyLog}\left(2,\sqrt{e}x\right)}{2\sqrt{e}}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(1 - e*x^2),x]
```

```
[Out] (-((a + b*Log[c*x^n])*(Log[1 - Sqrt[e]*x] - Log[1 + Sqrt[e]*x])) + b*n*Poly
Log[2, -(Sqrt[e]*x)] - b*n*PolyLog[2, Sqrt[e]*x])/(2*Sqrt[e])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.68 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

method	result
meijerg	$\frac{a\ \operatorname{arctanh}\left(x\sqrt{e}\right)}{\sqrt{e}}+\frac{b\ln\left(c\right)\operatorname{arctanh}\left(x\sqrt{e}\right)}{\sqrt{e}}+\left(\frac{bn\ln\left(x\right)\Phi\left(e\,x^{2},1,\frac{1}{2}\right)}{2}-\frac{bn\Phi\left(e\,x^{2},2,\frac{1}{2}\right)}{4}\right)x$
risch	$-\frac{\left(\frac{ib\pi\ \operatorname{csgn}\left(ic\right)\operatorname{csgn}\left(ix^{n}\right)\operatorname{csgn}\left(ic\,x^{n}\right)}{2}-\frac{ib\pi\ \operatorname{csgn}\left(ic\right)\operatorname{csgn}\left(ic\,x^{n}\right)^{2}}{2}-\frac{ib\pi\ \operatorname{csgn}\left(ix^{n}\right)\operatorname{csgn}\left(ic\,x^{n}\right)^{2}}{2}+\frac{ib\pi\operatorname{csgn}\left(ic\,x^{n}\right)^{3}}{2}-b\ln\left(c\right)-a\right)\operatorname{arctanh}\left(x\sqrt{e}\right)}{\sqrt{e}}$

[In] int((a+b*ln(c*x^n))/(-e*x^2+1),x,method=_RETURNVERBOSE)

[Out] a/e^(1/2)*arctanh(x*e^(1/2))+b*ln(c)/e^(1/2)*arctanh(x*e^(1/2))+(1/2*b*n*ln(x)*LerchPhi(e*x^2,1,1/2)-1/4*b*n*LerchPhi(e*x^2,2,1/2))*x

Fricas [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{1-ex^{2}}\,dx=\int -\frac{b\log\left(cx^{n}\right)+a}{ex^{2}-1}\,dx$$

[In] integrate((a+b*log(c*x^n))/(-e*x^2+1),x, algorithm="fricas")

[Out] integral(-(b*log(c*x^n) + a)/(e*x^2 - 1), x)

Sympy [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{1-ex^{2}}\,dx=-\int \frac{a}{ex^{2}-1}\,dx-\int \frac{b\log\left(cx^{n}\right)}{ex^{2}-1}\,dx$$

[In] integrate((a+b*ln(c*x**n))/(-e*x**2+1),x)

[Out] -Integral(a/(e*x**2 - 1), x) - Integral(b*log(c*x**n)/(e*x**2 - 1), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log(cx^n)}{1-ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/(-e*x^2+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{1-ex^2}\,dx = \int -\frac{b\log(cx^n)+a}{ex^2-1}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(-e*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(b*log(c*x^n) + a)/(e*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{1-ex^2}\,dx = \int -\frac{a+b\,\ln(c\,x^n)}{e\,x^2-1}\,dx$$

```
[In] int(-(a + b*log(c*x^n))/(e*x^2 - 1),x)
```

```
[Out] int(-(a + b*log(c*x^n))/(e*x^2 - 1), x)
```

3.247 $\int \frac{(a+b\log(cx^n))^2}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 509

$$\begin{aligned}
\int \frac{(a+b\log(cx^n))^2}{(d+ex^2)^2}\,dx &= \frac{x(a+b\log(cx^n))^2}{4(-d)^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{x(a+b\log(cx^n))^2}{4(-d)^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)} \\
&\quad+\frac{bn(a+b\log(cx^n))\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} \\
&\quad-\frac{(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&\quad-\frac{bn(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} \\
&\quad+\frac{(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b^2n^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} \\
&\quad+\frac{bn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} \\
&\quad+\frac{b^2n^2\operatorname{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}-\frac{bn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} \\
&\quad-\frac{b^2n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}+\frac{b^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}
\end{aligned}$$

```
[Out] 1/2*b*n*(a+b*ln(c*x^n))*ln(1-x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)-1/4*(
a+b*ln(c*x^n))^2*ln(1-x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)-1/2*b*n*(a+b
*ln(c*x^n))*ln(1+x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*ln(c*x^n
```

```
))^2*ln(1+x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)-1/2*b^2*n^2*polylog(2,-x
*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)+1/2*b*n*(a+b*ln(c*x^n))*polylog(2,-
x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)+1/2*b^2*n^2*polylog(2,x*e^(1/2)/(-
d)^(1/2))/(-d)^(3/2)/e^(1/2)-1/2*b*n*(a+b*ln(c*x^n))*polylog(2,x*e^(1/2)/(-
d)^(1/2))/(-d)^(3/2)/e^(1/2)-1/2*b^2*n^2*polylog(3,-x*e^(1/2)/(-d)^(1/2))/(
-d)^(3/2)/e^(1/2)+1/2*b^2*n^2*polylog(3,x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^
(1/2)+1/4*x*(a+b*ln(c*x^n))^2/(-d)^(3/2)/((-d)^(1/2)-x*e^(1/2))+1/4*x*(a+b*
ln(c*x^n))^2/(-d)^(3/2)/((-d)^(1/2)+x*e^(1/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2367, 2355, 2354, 2438, 2421, 6724}

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex^2)^2}\,dx = \frac{bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))}{2(-d)^{3/2}\sqrt{e}} - \frac{bn\operatorname{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))}{2(-d)^{3/2}\sqrt{e}} + \frac{bn\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))}{2(-d)^{3/2}\sqrt{e}} - \frac{bn\log\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)(a+b\log(cx^n))}{2(-d)^{3/2}\sqrt{e}} + \frac{x(a+b\log(cx^n))^2}{4(-d)^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{x(a+b\log(cx^n))^2}{4(-d)^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))^2}{4(-d)^{3/2}\sqrt{e}} + \frac{\log\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)(a+b\log(cx^n))^2}{4(-d)^{3/2}\sqrt{e}} - \frac{b^2n^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{b^2n^2\operatorname{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{b^2n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{b^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(d + e*x^2)^2,x]
```

```
[Out] (x*(a + b*Log[c*x^n])^2)/(4*(-d)^(3/2)*(Sqrt[-d] - Sqrt[e]*x)) + (x*(a + b*
Log[c*x^n])^2)/(4*(-d)^(3/2)*(Sqrt[-d] + Sqrt[e]*x)) + (b*n*(a + b*Log[c*x^
n])*Log[1 - (Sqrt[e]*x)/Sqrt[-d]])/(2*(-d)^(3/2)*Sqrt[e]) - ((a + b*Log[c*x
^n])^2*Log[1 - (Sqrt[e]*x)/Sqrt[-d]])/(4*(-d)^(3/2)*Sqrt[e]) - (b*n*(a + b*
```

```
Log[c*x^n])*Log[1 + (Sqrt[e]*x)/Sqrt[-d]])/(2*(-d)^(3/2)*Sqrt[e]) + ((a + b
*Log[c*x^n])^2*Log[1 + (Sqrt[e]*x)/Sqrt[-d]])/(4*(-d)^(3/2)*Sqrt[e]) - (b^2
*n^2*PolyLog[2, -((Sqrt[e]*x)/Sqrt[-d])])/(2*(-d)^(3/2)*Sqrt[e]) + (b*n*(a
+ b*Log[c*x^n])*PolyLog[2, -((Sqrt[e]*x)/Sqrt[-d])])/(2*(-d)^(3/2)*Sqrt[e])
 + (b^2*n^2*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]])/(2*(-d)^(3/2)*Sqrt[e]) - (b*n
*(a + b*Log[c*x^n])*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]])/(2*(-d)^(3/2)*Sqrt[e]
) - (b^2*n^2*PolyLog[3, -((Sqrt[e]*x)/Sqrt[-d])])/(2*(-d)^(3/2)*Sqrt[e]) +
(b^2*n^2*PolyLog[3, (Sqrt[e]*x)/Sqrt[-d]])/(2*(-d)^(3/2)*Sqrt[e])
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{e(a+b\log(cx^n))^2}{4d\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{e(a+b\log(cx^n))^2}{4d\left(\sqrt{-d}\sqrt{e}+ex\right)^2}-\frac{e(a+b\log(cx^n))^2}{2d\left(-de-e^2x^2\right)}\right)dx\\
&= -\frac{e\int \frac{(a+b\log(cx^n))^2}{\left(\sqrt{-d}\sqrt{e}-ex\right)^2}\,dx}{4d}-\frac{e\int \frac{(a+b\log(cx^n))^2}{\left(\sqrt{-d}\sqrt{e}+ex\right)^2}\,dx}{4d}-\frac{e\int \frac{(a+b\log(cx^n))^2}{-de-e^2x^2}\,dx}{2d}\\
&= \frac{x(a+b\log(cx^n))^2}{4(-d)^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{x(a+b\log(cx^n))^2}{4(-d)^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}\\
&\quad-\frac{e\int \left(-\frac{\sqrt{-d}(a+b\log(cx^n))^2}{2de\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{\sqrt{-d}(a+b\log(cx^n))^2}{2de\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{2d}\\
&\quad-\frac{\left(b\sqrt{e}n\right)\int \frac{a+b\log(cx^n)}{\sqrt{-d}\sqrt{e}-ex}\,dx}{2(-d)^{3/2}}-\frac{\left(b\sqrt{e}n\right)\int \frac{a+b\log(cx^n)}{\sqrt{-d}\sqrt{e}+ex}\,dx}{2(-d)^{3/2}}\\
&= \frac{x(a+b\log(cx^n))^2}{4(-d)^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{x(a+b\log(cx^n))^2}{4(-d)^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}\\
&\quad+\frac{bn(a+b\log(cx^n))\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}-\frac{bn(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{\int \frac{(a+b\log(cx^n))^2}{\sqrt{-d}-\sqrt{e}x}\,dx}{4(-d)^{3/2}}+\frac{\int \frac{(a+b\log(cx^n))^2}{\sqrt{-d}+\sqrt{e}x}\,dx}{4(-d)^{3/2}}\\
&\quad-\frac{\left(b^2n^2\right)\int \frac{\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{x}\,dx}{2(-d)^{3/2}\sqrt{e}}+\frac{\left(b^2n^2\right)\int \frac{\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{x}\,dx}{2(-d)^{3/2}\sqrt{e}}\\
&= \frac{x(a+b\log(cx^n))^2}{4(-d)^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{x(a+b\log(cx^n))^2}{4(-d)^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}\\
&\quad+\frac{bn(a+b\log(cx^n))\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}-\frac{(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad-\frac{bn(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}+\frac{(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad-\frac{b^2n^2\text{Li}_2\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}+\frac{b^2n^2\text{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{(bn)\int \frac{(a+b\log(cx^n))\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{x}\,dx}{2(-d)^{3/2}\sqrt{e}}-\frac{(bn)\int \frac{(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{x}\,dx}{2(-d)^{3/2}\sqrt{e}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{x(a+b\log(cx^n))^2}{4(-d)^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{x(a+b\log(cx^n))^2}{4(-d)^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)} \\
&+ \frac{bn(a+b\log(cx^n))\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&- \frac{bn(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&- \frac{b^2n^2\operatorname{Li}_2\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{b^2n^2\operatorname{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} \\
&- \frac{bn(a+b\log(cx^n))\operatorname{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{(b^2n^2)\int\frac{\operatorname{Li}_2\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{x}dx}{2(-d)^{3/2}\sqrt{e}} + \frac{(b^2n^2)\int\frac{\operatorname{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{x}dx}{2(-d)^{3/2}\sqrt{e}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x(a+b\log(cx^n))^2}{4(-d)^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{x(a+b\log(cx^n))^2}{4(-d)^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)} \\
&+ \frac{bn(a+b\log(cx^n))\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&- \frac{bn(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&- \frac{b^2n^2\operatorname{Li}_2\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{b^2n^2\operatorname{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} \\
&- \frac{bn(a+b\log(cx^n))\operatorname{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{b^2n^2\operatorname{Li}_3\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{b^2n^2\operatorname{Li}_3\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex^2)^2}\,dx$$

$$= \frac{-\frac{(a+b\log(cx^n))^2}{d(\sqrt{-d}-\sqrt{e}x)} + \frac{(a+b\log(cx^n))^2}{d(\sqrt{-d}+\sqrt{e}x)} - \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{(-d)^{3/2}} + \frac{(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{(-d)^{3/2}} + \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{d\cdots}{(-\cdots}\right.}{(-d)^{3/2}}}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(d + e*x^2)^2,x]
```

```
[Out] (-((a + b*Log[c*x^n])^2/(d*(Sqrt[-d] - Sqrt[e]*x))) + (a + b*Log[c*x^n])^2/
(d*(Sqrt[-d] + Sqrt[e]*x)) - (2*b*n*(a + b*Log[c*x^n])*Log[1 + (Sqrt[e]*x)/
Sqrt[-d]])/(-d)^(3/2) + ((a + b*Log[c*x^n])^2*Log[1 + (Sqrt[e]*x)/Sqrt[-d]]
)/(-d)^(3/2) + (2*b*n*(a + b*Log[c*x^n])*Log[1 + (d*Sqrt[e]*x)/(-d)^(3/2)])
```

```
/(-d)^(3/2) + (d*(a + b*Log[c*x^n])^2*Log[1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(-
d)^(5/2) + (2*b^2*n^2*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]])/(-d)^(3/2) - (2*b*n
*(a + b*Log[c*x^n])*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]])/(-d)^(3/2) - (2*b^2*n
^2*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)])/(-d)^(3/2) + (2*b*n*(a + b*Log[c*x
^n])*PolyLog[2, (d*Sqrt[e]*x)/(-d)^(3/2)])/(-d)^(3/2) + (2*b^2*n^2*PolyLog[
3, (Sqrt[e]*x)/Sqrt[-d]])/(-d)^(3/2) - (2*b^2*n^2*PolyLog[3, (d*Sqrt[e]*x)/
(-d)^(3/2)])/(-d)^(3/2))/(4*Sqrt[e])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}}{\left(e\,x^{2}+d\right)^{2}}dx$$

```
[In] int((a+b*ln(c*x^n))^2/(e*x^2+d)^2,x)
[Out] int((a+b*ln(c*x^n))^2/(e*x^2+d)^2,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex^2)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex^2+d)^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e^2*x^4 + 2*d*e*x^2 +
 d^2), x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex^2)^2}\,dx = \int \frac{(a+b\log(cx^n))^2}{(d+ex^2)^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**2/(e*x**2+d)**2,x)
[Out] Integral((a + b*log(c*x**n))**2/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex^2)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex^2+d)^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex^2)^2}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{(e\,x^2+d)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(d + e*x^2)^2,x)
```

```
[Out] int((a + b*log(c*x^n))^2/(d + e*x^2)^2, x)
```

3.248 $\int \frac{(a+b\log(cx^n))^3}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 711

$$\int \frac{(a+b\log(cx^n))^3}{(d+ex^2)^2}\,dx = \frac{x(a+b\log(cx^n))^3}{4(-d)^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{x(a+b\log(cx^n))^3}{4(-d)^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{3bn(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{(a+b\log(cx^n))^3\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{3bn(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{(a+b\log(cx^n))^3\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{3b^2n^2(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}+\frac{3bn(a+b\log(cx^n))^2\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{3b^2n^2(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}-\frac{3bn(a+b\log(cx^n))^2\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{3b^3n^3\,\mathrm{PolyLog}\left(3,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}-\frac{3b^2n^2(a+b\log(cx^n))\,\mathrm{PolyLog}\left(3,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}-\frac{3b^3n^3\,\mathrm{PolyLog}\left(3,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}+\frac{3b^2n^2(a+b\log(cx^n))\,\mathrm{PolyLog}\left(3,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}+\frac{3b^3n^3\,\mathrm{PolyLog}\left(4,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}-\frac{3b^3n^3\,\mathrm{PolyLog}\left(4,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

```
[Out] 3/4*b*n*(a+b*ln(c*x^n))^2*ln(1-x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)-1/4
*(a+b*ln(c*x^n))^3*ln(1-x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)-3/4*b*n*(a
```

```
+b*ln(c*x^n))^2*ln(1+x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*ln(c
*x^n))^3*ln(1+x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)-3/2*b^2*n^2*(a+b*ln(
c*x^n))*polylog(2,-x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)+3/4*b*n*(a+b*ln
(c*x^n))^2*polylog(2,-x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)+3/2*b^2*n^2*
(a+b*ln(c*x^n))*polylog(2,x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)-3/4*b*n*
(a+b*ln(c*x^n))^2*polylog(2,x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)+3/2*b^
3*n^3*polylog(3,-x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)-3/2*b^2*n^2*(a+b*
ln(c*x^n))*polylog(3,-x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)-3/2*b^3*n^3*
polylog(3,x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)+3/2*b^2*n^2*(a+b*ln(c*x^
n))*polylog(3,x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)+3/2*b^3*n^3*polylog(
4,-x*e^(1/2)/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)-3/2*b^3*n^3*polylog(4,x*e^(1/2)
/(-d)^(1/2))/(-d)^(3/2)/e^(1/2)+1/4*x*(a+b*ln(c*x^n))^3/(-d)^(3/2)/((-d)^(1
/2)-x*e^(1/2))+1/4*x*(a+b*ln(c*x^n))^3/(-d)^(3/2)/((-d)^(1/2)+x*e^(1/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used

$= \{2367, 2355, 2354, 2421, 6724, 2430\}$

$$\int \frac{(a+b\log(cx^n))^3}{(d+ex^2)^2}\,dx = -\frac{3b^2n^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))}{2(-d)^{3/2}\sqrt{e}} + \frac{3b^2n^2\operatorname{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))}{2(-d)^{3/2}\sqrt{e}} - \frac{3b^2n^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))}{2(-d)^{3/2}\sqrt{e}} + \frac{3b^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))}{2(-d)^{3/2}\sqrt{e}} + \frac{3bn\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))^2}{4(-d)^{3/2}\sqrt{e}} - \frac{3bn\operatorname{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))^2}{4(-d)^{3/2}\sqrt{e}} + \frac{3bn\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))^2}{4(-d)^{3/2}\sqrt{e}} - \frac{3bn\log\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)(a+b\log(cx^n))^2}{4(-d)^{3/2}\sqrt{e}} + \frac{x(a+b\log(cx^n))^3}{4(-d)^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{x(a+b\log(cx^n))^3}{4(-d)^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)(a+b\log(cx^n))^3}{4(-d)^{3/2}\sqrt{e}} + \frac{\log\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)(a+b\log(cx^n))^3}{4(-d)^{3/2}\sqrt{e}} + \frac{3b^3n^3\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{3b^3n^3\operatorname{PolyLog}\left(3,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{3b^3n^3\operatorname{PolyLog}\left(4,-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{3b^3n^3\operatorname{PolyLog}\left(4,\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

```
[In] Int[(a + b*Log[c*x^n])^3/(d + e*x^2)^2,x]
```

```
[Out] (x*(a + b*Log[c*x^n])^3)/(4*(-d)^(3/2)*(Sqrt[-d] - Sqrt[e]*x)) + (x*(a + b*
Log[c*x^n])^3)/(4*(-d)^(3/2)*(Sqrt[-d] + Sqrt[e]*x)) + (3*b*n*(a + b*Log[c*
x^n])^2*Log[1 - (Sqrt[e]*x)/Sqrt[-d]])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*Log
[c*x^n])^3*Log[1 - (Sqrt[e]*x)/Sqrt[-d]])/(4*(-d)^(3/2)*Sqrt[e]) - (3*b*n*(
a + b*Log[c*x^n])^2*Log[1 + (Sqrt[e]*x)/Sqrt[-d]])/(4*(-d)^(3/2)*Sqrt[e]) +
 ((a + b*Log[c*x^n])^3*Log[1 + (Sqrt[e]*x)/Sqrt[-d]])/(4*(-d)^(3/2)*Sqrt[e]
```

```
) - (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((Sqrt[e]*x)/Sqrt[-d])])/(2*(
-d)^(3/2)*Sqrt[e]) + (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[2, -((Sqrt[e]*x)/S
qrt[-d])])/(4*(-d)^(3/2)*Sqrt[e]) + (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2
, (Sqrt[e]*x)/Sqrt[-d]])/(2*(-d)^(3/2)*Sqrt[e]) - (3*b*n*(a + b*Log[c*x^n])
^2*PolyLog[2, (Sqrt[e]*x)/Sqrt[-d]])/(4*(-d)^(3/2)*Sqrt[e]) + (3*b^3*n^3*Po
lyLog[3, -((Sqrt[e]*x)/Sqrt[-d])])/(2*(-d)^(3/2)*Sqrt[e]) - (3*b^2*n^2*(a +
 b*Log[c*x^n])*PolyLog[3, -((Sqrt[e]*x)/Sqrt[-d])])/(2*(-d)^(3/2)*Sqrt[e])
- (3*b^3*n^3*PolyLog[3, (Sqrt[e]*x)/Sqrt[-d]])/(2*(-d)^(3/2)*Sqrt[e]) + (3*
b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, (Sqrt[e]*x)/Sqrt[-d]])/(2*(-d)^(3/2)*
Sqrt[e]) + (3*b^3*n^3*PolyLog[4, -((Sqrt[e]*x)/Sqrt[-d])])/(2*(-d)^(3/2)*Sq
rt[e]) - (3*b^3*n^3*PolyLog[4, (Sqrt[e]*x)/Sqrt[-d]])/(2*(-d)^(3/2)*Sqrt[e]
)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
```

```
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{e(a+b\log(cx^n))^3}{4d\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{e(a+b\log(cx^n))^3}{4d\left(\sqrt{-d}\sqrt{e}+ex\right)^2}-\frac{e(a+b\log(cx^n))^3}{2d\left(-de-e^2x^2\right)}\right)dx$$

$$= -\frac{e\int \frac{(a+b\log(cx^n))^3}{\left(\sqrt{-d}\sqrt{e}-ex\right)^2}dx}{4d}-\frac{e\int \frac{(a+b\log(cx^n))^3}{\left(\sqrt{-d}\sqrt{e}+ex\right)^2}dx}{4d}-\frac{e\int \frac{(a+b\log(cx^n))^3}{-de-e^2x^2}dx}{2d}$$

$$= \frac{x(a+b\log(cx^n))^3}{4(-d)^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{x(a+b\log(cx^n))^3}{4(-d)^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{e\int \left(-\frac{\sqrt{-d}(a+b\log(cx^n))^3}{2de\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{\sqrt{-d}(a+b\log(cx^n))^3}{2de\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{2d} - \frac{\left(3b\sqrt{e}n\right)\int \frac{(a+b\log(cx^n))^2}{\sqrt{-d}\sqrt{e}-ex}dx}{4(-d)^{3/2}}-\frac{\left(3b\sqrt{e}n\right)\int \frac{(a+b\log(cx^n))^2}{\sqrt{-d}\sqrt{e}+ex}dx}{4(-d)^{3/2}}$$

$$= \frac{x(a+b\log(cx^n))^3}{4(-d)^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{x(a+b\log(cx^n))^3}{4(-d)^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{3bn(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{3bn(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\int \frac{(a+b\log(cx^n))^3}{\sqrt{-d}-\sqrt{e}x}dx}{4(-d)^{3/2}}+\frac{\int \frac{(a+b\log(cx^n))^3}{\sqrt{-d}+\sqrt{e}x}dx}{4(-d)^{3/2}} - \frac{\left(3b^2n^2\right)\int \frac{(a+b\log(cx^n))\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{x}dx}{2(-d)^{3/2}\sqrt{e}}+\frac{\left(3b^2n^2\right)\int \frac{(a+b\log(cx^n))\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{x}dx}{2(-d)^{3/2}\sqrt{e}}$$

$$= \frac{x(a+b\log(cx^n))^3}{4(-d)^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{x(a+b\log(cx^n))^3}{4(-d)^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}$$
$$+ \frac{3bn(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{(a+b\log(cx^n))^3\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$- \frac{3bn(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{(a+b\log(cx^n))^3\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$- \frac{3b^2n^2(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{3b^2n^2(a+b\log(cx^n))\operatorname{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{(3bn)\int\frac{(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{x}dx}{4(-d)^{3/2}\sqrt{e}} - \frac{(3bn)\int\frac{(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{x}dx}{4(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{(3b^3n^3)\int\frac{\operatorname{Li}_2\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{x}dx}{2(-d)^{3/2}\sqrt{e}} - \frac{(3b^3n^3)\int\frac{\operatorname{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{x}dx}{2(-d)^{3/2}\sqrt{e}}$$

$$= \frac{x(a+b\log(cx^n))^3}{4(-d)^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{x(a+b\log(cx^n))^3}{4(-d)^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}$$
$$+ \frac{3bn(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{(a+b\log(cx^n))^3\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$- \frac{3bn(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{(a+b\log(cx^n))^3\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$- \frac{3b^2n^2(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_2\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{3b^2n^2(a+b\log(cx^n))\operatorname{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{3b^3n^3\operatorname{Li}_3\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{3b^3n^3\operatorname{Li}_3\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}$$
$$- \frac{(3b^2n^2)\int\frac{(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{x}dx}{2(-d)^{3/2}\sqrt{e}} + \frac{(3b^2n^2)\int\frac{(a+b\log(cx^n))\operatorname{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{x}dx}{2(-d)^{3/2}\sqrt{e}}$$

$$= \frac{x(a+b\log(cx^n))^3}{4(-d)^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{x(a+b\log(cx^n))^3}{4(-d)^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}$$
$$+ \frac{3bn(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{(a+b\log(cx^n))^3\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$- \frac{3bn(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{(a+b\log(cx^n))^3\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$- \frac{3b^2n^2(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_2\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{3b^2n^2(a+b\log(cx^n))\operatorname{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{3b^3n^3\operatorname{Li}_3\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{3b^2n^2(a+b\log(cx^n))\operatorname{Li}_3\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}$$
$$- \frac{3b^3n^3\operatorname{Li}_3\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{3b^2n^2(a+b\log(cx^n))\operatorname{Li}_3\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{(3b^3n^3)\int\frac{\operatorname{Li}_3\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{x}\,dx}{2(-d)^{3/2}\sqrt{e}} - \frac{(3b^3n^3)\int\frac{\operatorname{Li}_3\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{x}\,dx}{2(-d)^{3/2}\sqrt{e}}$$

$$= \frac{x(a+b\log(cx^n))^3}{4(-d)^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{x(a+b\log(cx^n))^3}{4(-d)^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}$$
$$+ \frac{3bn(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{(a+b\log(cx^n))^3\log\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$- \frac{3bn(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{(a+b\log(cx^n))^3\log\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$- \frac{3b^2n^2(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_2\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{3b^2n^2(a+b\log(cx^n))\operatorname{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{3b^3n^3\operatorname{Li}_3\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{3b^2n^2(a+b\log(cx^n))\operatorname{Li}_3\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{3b^3n^3\operatorname{Li}_3\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{3b^2n^2(a+b\log(cx^n))\operatorname{Li}_3\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{3b^3n^3\operatorname{Li}_4\left(-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{3b^3n^3\operatorname{Li}_4\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.91 (sec) , antiderivative size = 1073, normalized size of antiderivative = 1.51

$$\int \frac{(a+b\log(cx^n))^3}{(d+ex^2)^2}\,dx$$

$$= \frac{\frac{2\sqrt{d}x(a-bn\log(x)+b\log(cx^n))^3}{d+ex^2} + \frac{2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a-bn\log(x)+b\log(cx^n))^3}{\sqrt{e}} + 3bn(a-bn\log(x)+b\log(cx^n))^2\left(\frac{\sqrt{e}x\log(x)+\cdots}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(a + b*Log[c*x^n])^3/(d + e*x^2)^2,x]
```

```
[Out] ((2*Sqrt[d]*x*(a - b*n*Log[x] + b*Log[c*x^n])^3)/(d + e*x^2) + (2*ArcTan[(S
qrt[e]*x)/Sqrt[d]]*(a - b*n*Log[x] + b*Log[c*x^n])^3)/Sqrt[e] + 3*b*n*(a -
b*n*Log[x] + b*Log[c*x^n])^2*((Sqrt[e]*x*Log[x] + I*(Sqrt[d] + I*Sqrt[e]*x)
*Log[I*Sqrt[d] - Sqrt[e]*x])/(Sqrt[d]*Sqrt[e] + I*e*x) + (Sqrt[e]*x*Log[x]
+ ((-I)*Sqrt[d] - Sqrt[e]*x)*Log[I*Sqrt[d] + Sqrt[e]*x])/(Sqrt[d]*Sqrt[e] -
 I*e*x) - (I*(Log[x]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]] + PolyLog[2, ((-I)*Sqrt
[e]*x)/Sqrt[d]]))/Sqrt[e] + (I*(Log[x]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]] + Pol
yLog[2, (I*Sqrt[e]*x)/Sqrt[d]]))/Sqrt[e]) + 3*b^2*n^2*(a - b*n*Log[x] + b*L
og[c*x^n])*((Log[x]*(Sqrt[e]*x*Log[x] + (2*I)*(Sqrt[d] + I*Sqrt[e]*x)*Log[1
 + (I*Sqrt[e]*x)/Sqrt[d]]) + (2*I)*(Sqrt[d] + I*Sqrt[e]*x)*PolyLog[2, ((-I)
*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e] + I*e*x) + (Log[x]*(Sqrt[e]*x*Log[x]
 - (2*I)*(Sqrt[d] - I*Sqrt[e]*x)*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]]) - 2*(I*Sqr
t[d] + Sqrt[e]*x)*PolyLog[2, (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e] - I*e
*x) - (I*(Log[x]^2*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]] + 2*Log[x]*PolyLog[2, ((-
I)*Sqrt[e]*x)/Sqrt[d]] - 2*PolyLog[3, ((-I)*Sqrt[e]*x)/Sqrt[d]]))/Sqrt[e] +
 (I*(Log[x]^2*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]] + 2*Log[x]*PolyLog[2, (I*Sqrt[
e]*x)/Sqrt[d]] - 2*PolyLog[3, (I*Sqrt[e]*x)/Sqrt[d]]))/Sqrt[e]) + (I*b^3*n^
3*(-Log[x]^3 + (Sqrt[d]*Log[x]^3)/(Sqrt[d] + I*Sqrt[e]*x) + (Sqrt[e]*x*Log[
x]^3)/(I*Sqrt[d] + Sqrt[e]*x) - 3*Log[x]^2*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]] +
 Log[x]^3*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]] + 3*Log[x]^2*Log[1 + (I*Sqrt[e]*x)
/Sqrt[d]] - Log[x]^3*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]] - 3*(-2 + Log[x])*Log[x
]*PolyLog[2, ((-I)*Sqrt[e]*x)/Sqrt[d]] + 3*(-2 + Log[x])*Log[x]*PolyLog[2,
(I*Sqrt[e]*x)/Sqrt[d]] - 6*PolyLog[3, ((-I)*Sqrt[e]*x)/Sqrt[d]] + 6*Log[x]*
PolyLog[3, ((-I)*Sqrt[e]*x)/Sqrt[d]] + 6*PolyLog[3, (I*Sqrt[e]*x)/Sqrt[d]]
- 6*Log[x]*PolyLog[3, (I*Sqrt[e]*x)/Sqrt[d]] - 6*PolyLog[4, ((-I)*Sqrt[e]*x
)/Sqrt[d]] + 6*PolyLog[4, (I*Sqrt[e]*x)/Sqrt[d]]))/Sqrt[e])/(4*d^(3/2))
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))^3}{(e\,x^2+d)^2}dx$$

[In] int((a+b*ln(c*x^n))^3/(e*x^2+d)^2,x)

[Out] int((a+b*ln(c*x^n))^3/(e*x^2+d)^2,x)

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^3}{(d+ex^2)^2}\,dx = \int \frac{(b\log(cx^n)+a)^3}{(ex^2+d)^2}\,dx$$

[In] integrate((a+b*log(c*x^n))^3/(e*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^3}{(d+ex^2)^2}\,dx = \int \frac{(a+b\log(cx^n))^3}{(d+ex^2)^2}\,dx$$

[In] integrate((a+b*ln(c*x**n))**3/(e*x**2+d)**2,x)

[Out] Integral((a + b*log(c*x**n))**3/(d + e*x**2)**2, x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))^3}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

[In] integrate((a+b*log(c*x^n))^3/(e*x^2+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3}{(d+ex^2)^2}\,dx = \int \frac{(b\log(cx^n)+a)^3}{(ex^2+d)^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^3/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^3/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3}{(d+ex^2)^2}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^3}{(e\,x^2+d)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))^3/(d + e*x^2)^2,x)
[Out] int((a + b*log(c*x^n))^3/(d + e*x^2)^2, x)
```

3.249 $\int \frac{1}{\left(d+ex^2\right)^2(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^2\,(a+b\log(cx^n))}\,dx=\operatorname{Int}\left(\frac{1}{(d+ex^2)^2\,(a+b\log(cx^n))},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*ln(c*x^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2\,(a+b\log(cx^n))}\,dx=\int \frac{1}{(d+ex^2)^2\,(a+b\log(cx^n))}\,dx$$

```
[In] Int[1/((d + e*x^2)^2*(a + b*Log[c*x^n])),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*Log[c*x^n])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(d+ex^2)^2\,(a+b\log(cx^n))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))} dx = \int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))} dx$$

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*Log[c*x^n])),x]
[Out] Integrate[1/((d + e*x^2)^2*(a + b*Log[c*x^n])), x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)^2 (a+b\ln(c\,x^n))} dx$$

```
[In] int(1/(e*x^2+d)^2/(a+b*ln(c*x^n)),x)
[Out] int(1/(e*x^2+d)^2/(a+b*ln(c*x^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.50

$$\int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))} dx = \int \frac{1}{(ex^2+d)^2(b\log(cx^n)+a)} dx$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(1/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*
d^2)*log(c*x^n)), x)
```

Sympy [N/A]

Not integrable

Time = 77.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))} dx = \int \frac{1}{(a+b\log(cx^n))(d+ex^2)^2} dx$$

```
[In] integrate(1/(e*x**2+d)**2/(a+b*ln(c*x**n)),x)
[Out] Integral(1/((a + b*log(c*x**n))*(d + e*x**2)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))} dx = \int \frac{1}{(ex^2+d)^2 (b\log(cx^n)+a)} dx$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)^2*(b*log(c*x^n) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))} dx = \int \frac{1}{(ex^2+d)^2 (b\log(cx^n)+a)} dx$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^2*(b*log(c*x^n) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))} dx = \int \frac{1}{(e\,x^2+d)^2\ (a+b\,\ln(c\,x^n))} dx$$

```
[In] int(1/((d + e*x^2)^2*(a + b*log(c*x^n))),x)
[Out] int(1/((d + e*x^2)^2*(a + b*log(c*x^n))), x)
```

3.250 $\int \frac{1}{(d+ex^2)^2(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))^2}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^2 (a+b\log(cx^n))^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*ln(c*x^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))^2}\,dx = \int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))^2}\,dx$$

```
[In] Int[1/((d + e*x^2)^2*(a + b*Log[c*x^n])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*Log[c*x^n])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))^2} \, dx = \int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))^2} \, dx$$

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*Log[c*x^n])^2),x]
[Out] Integrate[1/((d + e*x^2)^2*(a + b*Log[c*x^n])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)^2 (a+b\ln(c\,x^n))^2}dx$$

```
[In] int(1/(e*x^2+d)^2/(a+b*ln(c*x^n))^2,x)
[Out] int(1/(e*x^2+d)^2/(a+b*ln(c*x^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.64

$$\int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))^2} \, dx = \int \frac{1}{(ex^2+d)^2(b\log(cx^n)+a)^2} \, dx$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*log(c*x^n))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*
e*x^2 + b^2*d^2)*log(c*x^n)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)*l
og(c*x^n)), x)
```

Sympy [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.05

$$\int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))^2}\,dx = \int \frac{1}{(a+b\log(cx^n))^2 (d+ex^2)^2}\,dx$$

```
[In] integrate(1/(e*x**2+d)**2/(a+b*ln(c*x**n))**2,x)
[Out] integrate(1/(e*x**2+d)**2/(a+b*ln(c*x**n))**2,x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 261, normalized size of antiderivative = 11.86

$$\int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))^2}\,dx = \int \frac{1}{(ex^2+d)^2(b\log(cx^n)+a)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*log(c*x^n))^2,x, algorithm="maxima")
[Out] -x/(b^2*d^2*n*log(c) + a*b*d^2*n + (b^2*e^2*n*log(c) + a*b*e^2*n)*x^4 + 2*(
b^2*d*e*n*log(c) + a*b*d*e*n)*x^2 + (b^2*e^2*n*x^4 + 2*b^2*d*e*n*x^2 + b^2*
d^2*n)*log(x^n)) - integrate((3*e*x^2 - d)/((b^2*e^3*n*log(c) + a*b*e^3*n)*
x^6 + b^2*d^3*n*log(c) + a*b*d^3*n + 3*(b^2*d*e^2*n*log(c) + a*b*d*e^2*n)*x
^4 + 3*(b^2*d^2*e*n*log(c) + a*b*d^2*e*n)*x^2 + (b^2*e^3*n*x^6 + 3*b^2*d*e^
2*n*x^4 + 3*b^2*d^2*e*n*x^2 + b^2*d^3*n)*log(x^n)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))^2}\,dx = \int \frac{1}{(ex^2+d)^2(b\log(cx^n)+a)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^2/(a+b*log(c*x^n))^2,x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^2*(b*log(c*x^n) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^2 (a+b\log(cx^n))^2} \, dx = \int \frac{1}{(e\,x^2+d)^2 \left(a+b\,\ln\left(c\,x^n\right)\right)^2} \, dx$$

```
[In] int(1/((d + e*x^2)^2*(a + b*log(c*x^n))^2),x)
[Out] int(1/((d + e*x^2)^2*(a + b*log(c*x^n))^2), x)
```

3.251 $\int x^5\sqrt{d+ex^2}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 208

$$\int x^5\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = -\frac{8bd^3n\sqrt{d+ex^2}}{105e^3}-\frac{8bd^2n(d+ex^2)^{3/2}}{315e^3}+\frac{9bdn(d+ex^2)^{5/2}}{175e^3}-\frac{bn(d+ex^2)^{7/2}}{49e^3}+\frac{8bd^{7/2}n\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{105e^3}+\frac{d^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3}-\frac{2d(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3}+\frac{(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^3}$$

```
[Out] -8/315*b*d^2*n*(e*x^2+d)^(3/2)/e^3+9/175*b*d*n*(e*x^2+d)^(5/2)/e^3-1/49*b*n
*(e*x^2+d)^(7/2)/e^3+8/105*b*d^(7/2)*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))/e^3
+1/3*d^2*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/e^3-2/5*d*(e*x^2+d)^(5/2)*(a+b*ln(
c*x^n))/e^3+1/7*(e*x^2+d)^(7/2)*(a+b*ln(c*x^n))/e^3-8/105*b*d^3*n*(e*x^2+d)
^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {272, 45, 2392, 12, 1265, 911, 1275, 214}

$$\int x^5\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \frac{d^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3} - \frac{2d(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3} + \frac{(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^3} + \frac{8bd^{7/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{105e^3} - \frac{8bd^3n\sqrt{d+ex^2}}{105e^3} - \frac{8bd^2n(d+ex^2)^{3/2}}{315e^3} + \frac{9bdn(d+ex^2)^{5/2}}{175e^3} - \frac{bn(d+ex^2)^{7/2}}{49e^3}$$

```
[In] Int[x^5*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-8*b*d^3*n*Sqrt[d + e*x^2])/(105*e^3) - (8*b*d^2*n*(d + e*x^2)^(3/2))/(315
*e^3) + (9*b*d*n*(d + e*x^2)^(5/2))/(175*e^3) - (b*n*(d + e*x^2)^(7/2))/(49
*e^3) + (8*b*d^(7/2)*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(105*e^3) + (d^2*(
d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/(3*e^3) - (2*d*(d + e*x^2)^(5/2)*(a +
b*Log[c*x^n]))/(5*e^3) + ((d + e*x^2)^(7/2)*(a + b*Log[c*x^n]))/(7*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \frac{d^2(d+ex^2)^{3/2}\,(a+b\log(cx^n))}{3e^3} - \frac{2d(d+ex^2)^{5/2}\,(a+b\log(cx^n))}{5e^3} + \frac{(d+ex^2)^{7/2}\,(a+b\log(cx^n))}{7e^3} - (bn)\int \frac{(d+ex^2)^{3/2}\,(8d^2-12dex^2+15e^2x^4)}{105e^3x}\,dx$$

$$= \frac{d^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3} - \frac{2d(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3} + \frac{(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^3} - \frac{(bn)\int \frac{(d+ex^2)^{3/2}(8d^2-12dex^2+15e^2x^4)}{x}\,dx}{105e^3}$$

$$= \frac{d^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3} - \frac{2d(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3} + \frac{(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^3} - \frac{(bn)\text{Subst}\left(\int \frac{(d+ex)^{3/2}(8d^2-12dex+15e^2x^2)}{x}\,dx, x, x^2\right)}{210e^3}$$

$$= \frac{d^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3} - \frac{2d(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3} + \frac{(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^3} - \frac{(bn)\text{Subst}\left(\int \frac{x^4(35d^2-42dx^2+15x^4)}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{105e^4}$$

$$= \frac{d^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3} - \frac{2d(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3} + \frac{(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^3} - \frac{(bn)\text{Subst}\left(\int \left(8d^3e+8d^2ex^2-27dex^4+15ex^6+\frac{8d^4}{-\frac{d}{e}+\frac{x^2}{e}}\right)dx, x, \sqrt{d+ex^2}\right)}{105e^4}$$

$$= -\frac{8bd^3n\sqrt{d+ex^2}}{105e^3} - \frac{8bd^2n(d+ex^2)^{3/2}}{315e^3} + \frac{9bdn(d+ex^2)^{5/2}}{175e^3} - \frac{bn(d+ex^2)^{7/2}}{49e^3} + \frac{d^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3} - \frac{2d(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3} + \frac{(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^3} - \frac{(8bd^4n)\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{105e^4}$$

$$= -\frac{8bd^3n\sqrt{d+ex^2}}{105e^3} - \frac{8bd^2n(d+ex^2)^{3/2}}{315e^3} + \frac{9bdn(d+ex^2)^{5/2}}{175e^3} - \frac{bn(d+ex^2)^{7/2}}{49e^3} + \frac{8bd^{7/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{105e^3} + \frac{d^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3} - \frac{2d(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3} + \frac{(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^3}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.21

$$\int x^5\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = -\frac{8bd^{7/2}n\log(x)}{105e^3} + \frac{bn\sqrt{d+ex^2}(8d^3-4d^2ex^2+3de^2x^4+15e^3x^6)\log(x)}{105e^3} + \sqrt{d+ex^2}\left(\frac{1}{49}x^6(7a-bn+7b(-n\log(x)+\log(cx^n))) + \frac{dx^4(35a-12bn+35b(-n\log(x)+\log(cx^n)))}{1225e} + \frac{2d^3(420a-389bn+420b(-n\log(x)+\log(cx^n)))}{11025e^3} - \frac{d^2x^2(420a-179bn+420b(-n\log(x)+\log(cx^n)))}{11025e^2}\right) + \frac{8bd^{7/2}n\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)}{105e^3}$$

```
[In] Integrate[x^5*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-8*b*d^(7/2)*n*Log[x])/(105*e^3) + (b*n*Sqrt[d + e*x^2]*(8*d^3 - 4*d^2*e*x
^2 + 3*d*e^2*x^4 + 15*e^3*x^6)*Log[x])/(105*e^3) + Sqrt[d + e*x^2]*((x^6*(7
*a - b*n + 7*b*(-(n*Log[x]) + Log[c*x^n])))/49 + (d*x^4*(35*a - 12*b*n + 35
*b*(-(n*Log[x]) + Log[c*x^n])))/(1225*e) + (2*d^3*(420*a - 389*b*n + 420*b*
(-(n*Log[x]) + Log[c*x^n])))/(11025*e^3) - (d^2*x^2*(420*a - 179*b*n + 420*
b*(-(n*Log[x]) + Log[c*x^n])))/(11025*e^2)) + (8*b*d^(7/2)*n*Log[d + Sqrt[d
]*Sqrt[d + e*x^2]])/(105*e^3)
```

Maple [F]

$$\int x^5(a+b\ln(c\,x^n))\sqrt{e\,x^2+d}dx$$

```
[In] int(x^5*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.99

$$\int x^5\sqrt{d+ex^2}(a+b\log(cx^n))\,dx$$
$$=\left[\frac{420\,bd^{\frac{7}{2}}n\log\left(-\frac{ex^2+2\sqrt{ex^2+d}\sqrt{d}+2\,d}{x^2}\right)-(225\,(be^3n-7\,ae^3)x^6+778\,bd^3n+9\,(12\,bde^2n-35\,ade^2)x^4-840\right.$$
$$-\frac{840\,b\sqrt{-d}d^3n\arctan\left(\frac{\sqrt{-d}}{\sqrt{ex^2+d}}\right)+(225\,(be^3n-7\,ae^3)x^6+778\,bd^3n+9\,(12\,bde^2n-35\,ade^2)x^4-840\,a$$

```
[In] integrate(x^5*(a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] [1/11025*(420*b*d^(7/2)*n*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^
2) - (225*(b*e^3*n - 7*a*e^3)*x^6 + 778*b*d^3*n + 9*(12*b*d*e^2*n - 35*a*d*
e^2)*x^4 - 840*a*d^3 - (179*b*d^2*e*n - 420*a*d^2*e)*x^2 - 105*(15*b*e^3*x^
6 + 3*b*d*e^2*x^4 - 4*b*d^2*e*x^2 + 8*b*d^3)*log(c) - 105*(15*b*e^3*n*x^6 +
 3*b*d*e^2*n*x^4 - 4*b*d^2*e*n*x^2 + 8*b*d^3*n)*log(x))*sqrt(e*x^2 + d))/e^
3, -1/11025*(840*b*sqrt(-d)*d^3*n*arctan(sqrt(-d)/sqrt(e*x^2 + d)) + (225*(
b*e^3*n - 7*a*e^3)*x^6 + 778*b*d^3*n + 9*(12*b*d*e^2*n - 35*a*d*e^2)*x^4 -
840*a*d^3 - (179*b*d^2*e*n - 420*a*d^2*e)*x^2 - 105*(15*b*e^3*x^6 + 3*b*d*e
^2*x^4 - 4*b*d^2*e*x^2 + 8*b*d^3)*log(c) - 105*(15*b*e^3*n*x^6 + 3*b*d*e^2*
n*x^4 - 4*b*d^2*e*n*x^2 + 8*b*d^3*n)*log(x))*sqrt(e*x^2 + d))/e^3]
```

Sympy [A] (verification not implemented)

Time = 20.89 (sec) , antiderivative size = 490, normalized size of antiderivative = 2.36

$$\int x^5\sqrt{d+ex^2}(a+b\log(cx^n))\,dx$$
$$=a\left(\begin{cases}\frac{8d^3\sqrt{d+ex^2}}{105e^3}-\frac{4d^2x^2\sqrt{d+ex^2}}{105e^2}+\frac{dx^4\sqrt{d+ex^2}}{35e}+\frac{x^6\sqrt{d+ex^2}}{7} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^6}{6} & \text{otherwise}\end{cases}\right)$$
$$-bn\left(\begin{cases}-\frac{8d^{\frac{7}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{105e^3}+\frac{8d^4}{105e^{\frac{7}{2}}x\sqrt{\frac{d}{ex^2}+1}}+\frac{8d^3x}{105e^{\frac{5}{2}}\sqrt{\frac{d}{ex^2}+1}}-\frac{4d^2\left(\begin{cases}\frac{d\sqrt{d+ex^2}}{3e}+\frac{x^2\sqrt{d+ex^2}}{3} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^2}{2} & \text{otherwise}\end{cases}\right)}{105e^2}+\frac{d\left(\left(\begin{cases}-\right.\right.}{} \\ \frac{\sqrt{d}x^6}{36}\end{cases}\right.$$
$$+b\left(\begin{cases}\frac{8d^3\sqrt{d+ex^2}}{105e^3}-\frac{4d^2x^2\sqrt{d+ex^2}}{105e^2}+\frac{dx^4\sqrt{d+ex^2}}{35e}+\frac{x^6\sqrt{d+ex^2}}{7} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^6}{6} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate(x**5*(a+b*ln(c*x**n))*(e*x**2+d)**(1/2),x)
```

```
[Out] a*Piecewise((8*d**3*sqrt(d + e*x**2)/(105*e**3) - 4*d**2*x**2*sqrt(d + e*x*
*2)/(105*e**2) + d*x**4*sqrt(d + e*x**2)/(35*e) + x**6*sqrt(d + e*x**2)/7,
Ne(e, 0)), (sqrt(d)*x**6/6, True)) - b*n*Piecewise((-8*d**(7/2)*asinh(sqrt(
d)/(sqrt(e)*x))/(105*e**3) + 8*d**4/(105*e**(7/2)*x*sqrt(d/(e*x**2) + 1)) +
 8*d**3*x/(105*e**(5/2)*sqrt(d/(e*x**2) + 1)) - 4*d**2*Piecewise((d*sqrt(d
+ e*x**2)/(3*e) + x**2*sqrt(d + e*x**2)/3, Ne(e, 0)), (sqrt(d)*x**2/2, True
))/(105*e**2) + d*Piecewise((-2*d**2*sqrt(d + e*x**2)/(15*e**2) + d*x**2*sq
rt(d + e*x**2)/(15*e) + x**4*sqrt(d + e*x**2)/5, Ne(e, 0)), (sqrt(d)*x**4/4
, True))/(35*e) + Piecewise((8*d**3*sqrt(d + e*x**2)/(105*e**3) - 4*d**2*x*
*2*sqrt(d + e*x**2)/(105*e**2) + d*x**4*sqrt(d + e*x**2)/(35*e) + x**6*sqrt
(d + e*x**2)/7, Ne(e, 0)), (sqrt(d)*x**6/6, True))/7, (e > -oo) & (e < oo)
& Ne(e, 0)), (sqrt(d)*x**6/36, True)) + b*Piecewise((8*d**3*sqrt(d + e*x**2
)/(105*e**3) - 4*d**2*x**2*sqrt(d + e*x**2)/(105*e**2) + d*x**4*sqrt(d + e*
x**2)/(35*e) + x**6*sqrt(d + e*x**2)/7, Ne(e, 0)), (sqrt(d)*x**6/6, True))*
log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int x^5\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.38

$$\begin{aligned}\int x^5\sqrt{d+ex^2}(a+b\log(cx^n))\,dx &= \frac{1}{7}\sqrt{ex^2+d}bx^6\log(c) + \frac{1}{7}\sqrt{ex^2+d}ax^6 \\ &+ \frac{\sqrt{ex^2+d}bdx^4\log(c)}{35\,e} + \frac{\sqrt{ex^2+d}adx^4}{35\,e} - \frac{4\sqrt{ex^2+d}bd^2x^2\log(c)}{105\,e^2} - \frac{4\sqrt{ex^2+d}ad^2x^2}{105\,e^2} \\ &+ \frac{1}{11025}bn\left(\frac{105\left(15\,(ex^2+d)^{\frac{7}{2}} - 42\,(ex^2+d)^{\frac{5}{2}}d + 35\,(ex^2+d)^{\frac{3}{2}}d^2\right)\log(x)}{e^3} - \frac{\frac{840\,d^4\arctan\left(\frac{\sqrt{ex^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}} + 225\,(\right. \\ &+ \frac{8\sqrt{ex^2+d}bd^3\log(c)}{105\,e^3} + \frac{8\sqrt{ex^2+d}ad^3}{105\,e^3}\end{aligned}$$

```
[In] integrate(x^5*(a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7*sqrt(e*x^2 + d)*b*x^6*log(c) + 1/7*sqrt(e*x^2 + d)*a*x^6 + 1/35*sqrt(e*
x^2 + d)*b*d*x^4*log(c)/e + 1/35*sqrt(e*x^2 + d)*a*d*x^4/e - 4/105*sqrt(e*x
^2 + d)*b*d^2*x^2*log(c)/e^2 - 4/105*sqrt(e*x^2 + d)*a*d^2*x^2/e^2 + 1/1102
5*b*n*(105*(15*(e*x^2 + d)^(7/2) - 42*(e*x^2 + d)^(5/2)*d + 35*(e*x^2 + d)^
(3/2)*d^2)*log(x)/e^3 - (840*d^4*arctan(sqrt(e*x^2 + d)/sqrt(-d))/sqrt(-d)
+ 225*(e*x^2 + d)^(7/2) - 567*(e*x^2 + d)^(5/2)*d + 280*(e*x^2 + d)^(3/2)*d
^2 + 840*sqrt(e*x^2 + d)*d^3)/e^3) + 8/105*sqrt(e*x^2 + d)*b*d^3*log(c)/e^3
 + 8/105*sqrt(e*x^2 + d)*a*d^3/e^3
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \int x^5\,\sqrt{e\,x^2+d}\,(a+b\,\ln(c\,x^n))\,dx$$

```
[In] int(x^5*(d + e*x^2)^(1/2)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^5*(d + e*x^2)^(1/2)*(a + b*log(c*x^n)), x)
```

3.252 $\int x^3\sqrt{d+ex^2}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 154

$$\int x^3\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \frac{2bd^2n\sqrt{d+ex^2}}{15e^2}+\frac{2bdn(d+ex^2)^{3/2}}{45e^2}-\frac{bn(d+ex^2)^{5/2}}{25e^2}-\frac{2bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{15e^2}-\frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2}+\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^2}$$

```
[Out] 2/45*b*d*n*(e*x^2+d)^(3/2)/e^2-1/25*b*n*(e*x^2+d)^(5/2)/e^2-2/15*b*d^(5/2)*
n*arctanh((e*x^2+d)^(1/2)/d^(1/2))/e^2-1/3*d*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n)
)/e^2+1/5*(e*x^2+d)^(5/2)*(a+b*ln(c*x^n))/e^2+2/15*b*d^2*n*(e*x^2+d)^(1/2)/
e^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used

$= \{272, 45, 2392, 12, 457, 81, 52, 65, 214\}$

$$\int x^3\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = -\frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^2} - \frac{2bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{15e^2} + \frac{2bd^2n\sqrt{d+ex^2}}{15e^2} + \frac{2bdn(d+ex^2)^{3/2}}{45e^2} - \frac{bn(d+ex^2)^{5/2}}{25e^2}$$

```
[In] Int[x^3*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]),x]
```

```
[Out] (2*b*d^2*n*Sqrt[d + e*x^2])/(15*e^2) + (2*b*d*n*(d + e*x^2)^(3/2))/(45*e^2)
 - (b*n*(d + e*x^2)^(5/2))/(25*e^2) - (2*b*d^(5/2)*n*ArcTanh[Sqrt[d + e*x^2
]/Sqrt[d]])/(15*e^2) - (d*(d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/(3*e^2) + (
(d + e*x^2)^(5/2)*(a + b*Log[c*x^n]))/(5*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2}+\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^2}\\
&\quad-(bn)\int\frac{(d+ex^2)^{3/2}(-2d+3ex^2)}{15e^2x}\,dx
\end{aligned}
$$

$$= -\frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^2} - \frac{(bn)\int\frac{(d+ex^2)^{3/2}(-2d+3ex^2)}{x}\,dx}{15e^2}$$

$$= -\frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^2} - \frac{(bn)\text{Subst}\left(\int\frac{(d+ex)^{3/2}(-2d+3ex)}{x}\,dx, x, x^2\right)}{30e^2}$$

$$= -\frac{bn(d+ex^2)^{5/2}}{25e^2} - \frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^2} + \frac{(bdn)\text{Subst}\left(\int\frac{(d+ex)^{3/2}}{x}\,dx, x, x^2\right)}{15e^2}$$

$$= \frac{2bdn(d+ex^2)^{3/2}}{45e^2} - \frac{bn(d+ex^2)^{5/2}}{25e^2} - \frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^2} + \frac{(bd^2n)\,\text{Subst}\left(\int\frac{\sqrt{d+ex}}{x}\,dx, x, x^2\right)}{15e^2}$$

$$= \frac{2bd^2n\sqrt{d+ex^2}}{15e^2} + \frac{2bdn(d+ex^2)^{3/2}}{45e^2} - \frac{bn(d+ex^2)^{5/2}}{25e^2} - \frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^2} + \frac{(bd^3n)\,\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)}{15e^2}$$

$$= \frac{2bd^2n\sqrt{d+ex^2}}{15e^2} + \frac{2bdn(d+ex^2)^{3/2}}{45e^2} - \frac{bn(d+ex^2)^{5/2}}{25e^2} - \frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^2} + \frac{(2bd^3n)\,\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{15e^3}$$

$$= \frac{2bd^2n\sqrt{d+ex^2}}{15e^2} + \frac{2bdn(d+ex^2)^{3/2}}{45e^2} - \frac{bn(d+ex^2)^{5/2}}{25e^2} - \frac{2bd^{5/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{15e^2} - \frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^2}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.32

$$\int x^3\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \frac{2bd^{5/2}n\log(x)}{15e^2} - \frac{bn\sqrt{d+ex^2}(2d^2-dex^2-3e^2x^4)\log(x)}{15e^2} + \sqrt{d+ex^2}\left(\frac{1}{25}x^4(5a-bn+5b(-n\log(x)+\log(cx^n))) + \frac{dx^2(15a-8bn+15b(-n\log(x)+\log(cx^n)))}{225e} - \frac{d^2(30a-31bn+30b(-n\log(x)+\log(cx^n)))}{225e^2}\right) - \frac{2bd^{5/2}n\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)}{15e^2}$$

```
[In] Integrate[x^3*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]),x]
```

```
[Out] (2*b*d^(5/2)*n*Log[x])/(15*e^2) - (b*n*Sqrt[d + e*x^2]*(2*d^2 - d*e*x^2 - 3
*e^2*x^4)*Log[x])/(15*e^2) + Sqrt[d + e*x^2]*((x^4*(5*a - b*n + 5*b*(-(n*Lo
g[x]) + Log[c*x^n])))/25 + (d*x^2*(15*a - 8*b*n + 15*b*(-(n*Log[x]) + Log[c
*x^n])))/(225*e) - (d^2*(30*a - 31*b*n + 30*b*(-(n*Log[x]) + Log[c*x^n])))/
(225*e^2)) - (2*b*d^(5/2)*n*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/(15*e^2)
```

Maple [F]

$$\int x^3(a+b\ln(c\,x^n))\,\sqrt{e\,x^2+d}dx$$

```
[In] int(x^3*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.01

$$\int x^3\sqrt{d+ex^2}(a+b\log(cx^n))\,dx$$

$$= \left[\frac{15\,bd^{\frac{5}{2}}n\log\left(-\frac{ex^2-2\,\sqrt{ex^2+d}\sqrt{d}+2\,d}{x^2}\right) - (9\,(be^2n-5\,ae^2)x^4 - 31\,bd^2n + 30\,ad^2 + (8\,bden-15\,ade)x^2 - 15\,(3}{225\,e^2}\right.$$

```
[In] integrate(x^3*(a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/225*(15*b*d^(5/2)*n*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2)
- (9*(b*e^2*n - 5*a*e^2)*x^4 - 31*b*d^2*n + 30*a*d^2 + (8*b*d*e*n - 15*a*d*
e)*x^2 - 15*(3*b*e^2*x^4 + b*d*e*x^2 - 2*b*d^2)*log(c) - 15*(3*b*e^2*n*x^4
+ b*d*e*n*x^2 - 2*b*d^2*n)*log(x))*sqrt(e*x^2 + d))/e^2, 1/225*(30*b*sqrt(-
d)*d^2*n*arctan(sqrt(-d)/sqrt(e*x^2 + d)) - (9*(b*e^2*n - 5*a*e^2)*x^4 - 31
*b*d^2*n + 30*a*d^2 + (8*b*d*e*n - 15*a*d*e)*x^2 - 15*(3*b*e^2*x^4 + b*d*e*
x^2 - 2*b*d^2)*log(c) - 15*(3*b*e^2*n*x^4 + b*d*e*n*x^2 - 2*b*d^2*n)*log(x)
)*sqrt(e*x^2 + d))/e^2]
```

Sympy [A] (verification not implemented)

Time = 14.27 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.23

$$\int x^3\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = a\left(\begin{cases} -\frac{2d^2\sqrt{d+ex^2}}{15e^2}+\frac{dx^2\sqrt{d+ex^2}}{15e}+\frac{x^4\sqrt{d+ex^2}}{5} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^4}{4} & \text{otherwise}\end{cases}\right)$$
$$-bn\left(\begin{cases} \frac{2d^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{15e^2}-\frac{2d^3}{15e^{\frac{5}{2}}x\sqrt{\frac{d}{ex^2}+1}}-\frac{2d^2x}{15e^{\frac{3}{2}}\sqrt{\frac{d}{ex^2}+1}}+\frac{d\left(\begin{cases}\frac{d\sqrt{d+ex^2}}{3e}+\frac{x^2\sqrt{d+ex^2}}{3} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^2}{2} & \text{otherwise}\end{cases}\right)}{15e}+\frac{\begin{cases}-\frac{2d^2\sqrt{d+e}}{15e^2}\\ \frac{\sqrt{d}x^4}{4}\end{cases}}{} \\ \frac{\sqrt{d}x^4}{16} \end{cases}\right)$$
$$+b\left(\begin{cases} -\frac{2d^2\sqrt{d+ex^2}}{15e^2}+\frac{dx^2\sqrt{d+ex^2}}{15e}+\frac{x^4\sqrt{d+ex^2}}{5} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^4}{4} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))*(e*x**2+d)**(1/2),x)
```

```
[Out] a*Piecewise((-2*d**2*sqrt(d + e*x**2)/(15*e**2) + d*x**2*sqrt(d + e*x**2)/(
15*e) + x**4*sqrt(d + e*x**2)/5, Ne(e, 0)), (sqrt(d)*x**4/4, True)) - b*n*P
iecewise((2*d**(5/2)*asinh(sqrt(d)/(sqrt(e)*x))/(15*e**2) - 2*d**3/(15*e**(
5/2)*x*sqrt(d/(e*x**2) + 1)) - 2*d**2*x/(15*e**(3/2)*sqrt(d/(e*x**2) + 1))
+ d*Piecewise((d*sqrt(d + e*x**2)/(3*e) + x**2*sqrt(d + e*x**2)/3, Ne(e, 0)
), (sqrt(d)*x**2/2, True))/(15*e) + Piecewise((-2*d**2*sqrt(d + e*x**2)/(15
*e**2) + d*x**2*sqrt(d + e*x**2)/(15*e) + x**4*sqrt(d + e*x**2)/5, Ne(e, 0)
), (sqrt(d)*x**4/4, True))/5, (e > -oo) & (e < oo) & Ne(e, 0)), (sqrt(d)*x*
*4/16, True)) + b*Piecewise((-2*d**2*sqrt(d + e*x**2)/(15*e**2) + d*x**2*sq
rt(d + e*x**2)/(15*e) + x**4*sqrt(d + e*x**2)/5, Ne(e, 0)), (sqrt(d)*x**4/4
, True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.40

$$\int x^3\sqrt{d+ex^2}(a+b\log(cx^n))\,dx$$
$$= \frac{1}{5}\sqrt{ex^2+d}bx^4\log(c) + \frac{1}{5}\sqrt{ex^2+d}ax^4 + \frac{\sqrt{ex^2+d}bdx^2\log(c)}{15\,e} + \frac{\sqrt{ex^2+d}adx^2}{15\,e}$$
$$+ \frac{1}{225}bn\left(\frac{15\left(3\,(ex^2+d)^{\frac{5}{2}} - 5\,(ex^2+d)^{\frac{3}{2}}d\right)\log(x)}{e^2} + \frac{\frac{30\,d^3\arctan\left(\frac{\sqrt{ex^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}} - 9\,(ex^2+d)^{\frac{5}{2}} + 10\,(ex^2+d)^{\frac{3}{2}}d}{e^2}\right.$$
$$- \frac{2\sqrt{ex^2+d}bd^2\log(c)}{15\,e^2} - \frac{2\sqrt{ex^2+d}ad^2}{15\,e^2}$$

```
[In] integrate(x^3*(a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(e*x^2 + d)*b*x^4*log(c) + 1/5*sqrt(e*x^2 + d)*a*x^4 + 1/15*sqrt(e*
x^2 + d)*b*d*x^2*log(c)/e + 1/15*sqrt(e*x^2 + d)*a*d*x^2/e + 1/225*b*n*(15*
(3*(e*x^2 + d)^(5/2) - 5*(e*x^2 + d)^(3/2)*d)*log(x)/e^2 + (30*d^3*arctan(s
qrt(e*x^2 + d)/sqrt(-d))/sqrt(-d) - 9*(e*x^2 + d)^(5/2) + 10*(e*x^2 + d)^(3
/2)*d + 30*sqrt(e*x^2 + d)*d^2)/e^2) - 2/15*sqrt(e*x^2 + d)*b*d^2*log(c)/e^
2 - 2/15*sqrt(e*x^2 + d)*a*d^2/e^2
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d+ex^2}(a+b\log(cx^n))\,dx=\int x^3\,\sqrt{e\,x^2+d}\,(a+b\,\ln(c\,x^n))\,dx$$

```
[In] int(x^3*(d + e*x^2)^(1/2)*(a + b*log(c*x^n)),x)
[Out] int(x^3*(d + e*x^2)^(1/2)*(a + b*log(c*x^n)), x)
```

3.253 $\int x\sqrt{d+ex^2}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int x\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = -\frac{bdn\sqrt{d+ex^2}}{3e}-\frac{bn(d+ex^2)^{3/2}}{9e}+\frac{bd^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3e}+\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e}$$

```
[Out] -1/9*b*n*(e*x^2+d)^(3/2)/e+1/3*b*d^(3/2)*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))
/e+1/3*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/e-1/3*b*d*n*(e*x^2+d)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2376, 272, 52, 65, 214}

$$\int x\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e}+\frac{bd^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3e}-\frac{bdn\sqrt{d+ex^2}}{3e}-\frac{bn(d+ex^2)^{3/2}}{9e}$$

```
[In] Int[x*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/3*(b*d*n*Sqrt[d + e*x^2])/e - (b*n*(d + e*x^2)^(3/2))/(9*e) + (b*d^(3/2)
*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(3*e) + ((d + e*x^2)^(3/2)*(a + b*Log[
c*x^n]))/(3*e)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e} - \frac{(bn)\int \frac{(d+ex^2)^{3/2}}{x}\,dx}{3e}$$

$$= \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e} - \frac{(bn)\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x}\,dx, x, x^2\right)}{6e}$$

$$= -\frac{bn(d+ex^2)^{3/2}}{9e} + \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e} - \frac{(bdn)\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x}\,dx, x, x^2\right)}{6e}$$

$$= -\frac{bdn\sqrt{d+ex^2}}{3e} - \frac{bn(d+ex^2)^{3/2}}{9e} + \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e} - \frac{(bd^2n)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)}{6e}$$

$$= -\frac{bdn\sqrt{d+ex^2}}{3e} - \frac{bn(d+ex^2)^{3/2}}{9e} + \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e} - \frac{(bd^2n)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{3e^2}$$

$$= -\frac{bdn\sqrt{d+ex^2}}{3e} - \frac{bn(d+ex^2)^{3/2}}{9e} + \frac{bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3e} + \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.33

$$\int x\sqrt{d+ex^2}(a+b\log(cx^n))\,dx$$
$$= \frac{3ad\sqrt{d+ex^2} - 4bdn\sqrt{d+ex^2} + 3aex^2\sqrt{d+ex^2} - benx^2\sqrt{d+ex^2} - 3bd^{3/2}n\log(x) + 3b(d+ex^2)^{3/2}\log(c}{9e}$$

```
[In] Integrate[x*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]),x]
```

```
[Out] (3*a*d*Sqrt[d + e*x^2] - 4*b*d*n*Sqrt[d + e*x^2] + 3*a*e*x^2*Sqrt[d + e*x^2
] - b*e*n*x^2*Sqrt[d + e*x^2] - 3*b*d^(3/2)*n*Log[x] + 3*b*(d + e*x^2)^(3/2
)*Log[c*x^n] + 3*b*d^(3/2)*n*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/(9*e)
```

Maple [F]

$$\int x(a+b\ln(c\,x^n))\sqrt{e\,x^2+d}dx$$

```
[In] int(x*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.98

$$\int x\sqrt{d+ex^2}(a+b\log(cx^n))\,dx$$
$$=\left[\frac{3\,bd^{\frac{3}{2}}n\log\left(-\frac{ex^2+2\,\sqrt{ex^2+d}\sqrt{d}+2\,d}{x^2}\right)-2\,(4\,bdn+(ben-3\,ae)x^2-3\,ad-3\,(bex^2+bd)\log(c)-3\,(benx^2+}{18\,e}\right.$$
$$-\frac{3\,b\sqrt{-d}dn\arctan\left(\frac{\sqrt{-d}}{\sqrt{ex^2+d}}\right)+(4\,bdn+(ben-3\,ae)x^2-3\,ad-3\,(bex^2+bd)\log(c)-3\,(benx^2+bdn)}{9\,e}$$

```
[In] integrate(x*(a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/18*(3*b*d^(3/2)*n*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) -
2*(4*b*d*n + (b*e*n - 3*a*e)*x^2 - 3*a*d - 3*(b*e*x^2 + b*d)*log(c) - 3*(b*
e*n*x^2 + b*d*n)*log(x))*sqrt(e*x^2 + d))/e, -1/9*(3*b*sqrt(-d)*d*n*arctan(
sqrt(-d)/sqrt(e*x^2 + d)) + (4*b*d*n + (b*e*n - 3*a*e)*x^2 - 3*a*d - 3*(b*e
*x^2 + b*d)*log(c) - 3*(b*e*n*x^2 + b*d*n)*log(x))*sqrt(e*x^2 + d))/e]
```

Sympy [A] (verification not implemented)

Time = 11.60 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.14

$$\int x\sqrt{d+ex^2}(a+b\log(cx^n))\,dx=a\left(\begin{cases}\frac{d\sqrt{d+ex^2}}{3e}+\frac{x^2\sqrt{d+ex^2}}{3} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^2}{2} & \text{otherwise}\end{cases}\right)$$
$$-bn\left(\begin{cases}-\frac{d^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{3e}+\frac{d^2}{3e^{\frac{3}{2}}x\sqrt{\frac{d}{ex^2}+1}}+\frac{dx}{3\sqrt{e}\sqrt{\frac{d}{ex^2}+1}}+\frac{\begin{cases}\frac{d\sqrt{d+ex^2}}{3e}+\frac{x^2\sqrt{d+ex^2}}{3} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^2}{2} & \text{otherwise}\end{cases}}{3} & \text{for } e>-\infty\wedge e\\ \frac{\sqrt{d}x^2}{4} & \text{otherwise}\end{cases}\right.$$
$$+b\left(\begin{cases}\frac{d\sqrt{d+ex^2}}{3e}+\frac{x^2\sqrt{d+ex^2}}{3} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^2}{2} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate(x*(a+b*ln(c*x**n))*(e*x**2+d)**(1/2),x)
```

```
[Out] a*Piecewise((d*sqrt(d + e*x**2)/(3*e) + x**2*sqrt(d + e*x**2)/3, Ne(e, 0)),
 (sqrt(d)*x**2/2, True)) - b*n*Piecewise((-d**(3/2)*asinh(sqrt(d)/(sqrt(e)*
```

```
x))/(3*e) + d**2/(3*e**(3/2)*x*sqrt(d/(e*x**2) + 1)) + d*x/(3*sqrt(e)*sqrt(
d/(e*x**2) + 1)) + Piecewise((d*sqrt(d + e*x**2)/(3*e) + x**2*sqrt(d + e*x*
*2)/3, Ne(e, 0)), (sqrt(d)*x**2/2, True))/3, (e > -oo) & (e < oo) & Ne(e, 0
)), (sqrt(d)*x**2/4, True)) + b*Piecewise((d*sqrt(d + e*x**2)/(3*e) + x**2*
sqrt(d + e*x**2)/3, Ne(e, 0)), (sqrt(d)*x**2/2, True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.38

$$\begin{aligned}
&\int x\sqrt{d+ex^2}(a+b\log(cx^n))\,dx \\
&= \frac{1}{3}\sqrt{ex^2+d}bx^2\log(c) + \frac{1}{3}\sqrt{ex^2+d}ax^2 \\
&\quad + \frac{1}{9}\left(\frac{3(ex^2+d)^{\frac{3}{2}}\log(x)}{e} - \frac{\frac{3d^2\arctan\left(\frac{\sqrt{ex^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}} + (ex^2+d)^{\frac{3}{2}} + 3\sqrt{ex^2+d}d}{e}\right)bn \\
&\quad + \frac{\sqrt{ex^2+d}bd\log(c)}{3e} + \frac{\sqrt{ex^2+d}ad}{3e}
\end{aligned}$$

```
[In] integrate(x*(a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(e*x^2 + d)*b*x^2*log(c) + 1/3*sqrt(e*x^2 + d)*a*x^2 + 1/9*(3*(e*x^
2 + d)^(3/2)*log(x)/e - (3*d^2*arctan(sqrt(e*x^2 + d)/sqrt(-d))/sqrt(-d) +
(e*x^2 + d)^(3/2) + 3*sqrt(e*x^2 + d)*d)/e)*b*n + 1/3*sqrt(e*x^2 + d)*b*d*l
og(c)/e + 1/3*sqrt(e*x^2 + d)*a*d/e
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+ex^2}(a+b\log(cx^n))\,dx=\int x\,\sqrt{e\,x^2+d}\,(a+b\,\ln(c\,x^n))\,dx$$

```
[In] int(x*(d + e*x^2)^(1/2)*(a + b*log(c*x^n)),x)
[Out] int(x*(d + e*x^2)^(1/2)*(a + b*log(c*x^n)), x)
```

3.254 $\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 220

$$\begin{aligned}\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x}\,dx = &-bn\sqrt{d+ex^2}+b\sqrt{d}n\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\\&+\frac{1}{2}b\sqrt{d}n\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2+\left(\sqrt{d+ex^2}\right.\\&\left.-\sqrt{d}\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+b\log(cx^n))\\&-b\sqrt{d}n\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)\\&-\frac{1}{2}b\sqrt{d}n\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)\end{aligned}$$

```
[Out] b*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))*d^(1/2)+1/2*b*n*arctanh((e*x^2+d)^(1/2
)/d^(1/2))^2*d^(1/2)-b*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(
1/2)-(e*x^2+d)^(1/2)))*d^(1/2)-1/2*b*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-(e*x^
2+d)^(1/2)))*d^(1/2)-b*n*(e*x^2+d)^(1/2)+(a+b*ln(c*x^n))*(-arctanh((e*x^2+d
)^(1/2)/d^(1/2))*d^(1/2)+(e*x^2+d)^(1/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {272, 52, 65, 214, 2390, 6131, 6055, 2449, 2352}

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x}\,dx = \left(\sqrt{d+ex^2}-\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+b\log(cx^n)) + \frac{1}{2}b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2 + b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) - b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right) - \frac{1}{2}b\sqrt{d}n\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{ex^2+d}}\right) - bn\sqrt{d+ex^2}$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] -(b*n*Sqrt[d + e*x^2]) + b*Sqrt[d]*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] + (b*
Sqrt[d]*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]]^2)/2 + (Sqrt[d + e*x^2] - Sqrt[d
]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])*(a + b*Log[c*x^n]) - b*Sqrt[d]*n*ArcTan
h[Sqrt[d + e*x^2]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])] - (
b*Sqrt[d]*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \left(\sqrt{d+ex^2} - \sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+b\log(cx^n)) - (bn)\int\left(\frac{\sqrt{d+ex^2}}{x} - \frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{x}\right)dx$$

$$= \left(\sqrt{d+ex^2} - \sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+b\log(cx^n)) - (bn)\int\frac{\sqrt{d+ex^2}}{x}\,dx + \left(b\sqrt{d}n\right)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{x}\,dx$$

$$= \left(\sqrt{d+ex^2} - \sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+b\log(cx^n)) - \frac{1}{2}(bn)\text{Subst}\left(\int\frac{\sqrt{d+ex}}{x}\,dx, x, x^2\right) + \frac{1}{2}\left(b\sqrt{d}n\right)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx, x, x^2\right)$$

$$= -bn\sqrt{d+ex^2} + \left(\sqrt{d+ex^2} - \sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+b\log(cx^n)) + \left(b\sqrt{d}n\right)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx, x, \sqrt{d+ex^2}\right) - \frac{1}{2}(bdn)\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)$$

$$= -bn\sqrt{d+ex^2} + \frac{1}{2}b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2 + \left(\sqrt{d+ex^2} - \sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+b\log(cx^n)) - (bn)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx, x, \sqrt{d+ex^2}\right) - \frac{(bdn)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= -bn\sqrt{d+ex^2} + b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) + \frac{1}{2}b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2 + \left(\sqrt{d+ex^2} - \sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+b\log(cx^n)) - b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right) + (bn)\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}\,dx, x, \sqrt{d+ex^2}\right)$$

$$= -bn\sqrt{d+ex^2} + b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) + \frac{1}{2}b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2 + \left(\sqrt{d+ex^2} - \sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+b\log(cx^n)) - b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right) - \left(b\sqrt{d}n\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{d+ex^2}}{\sqrt{d}}}\right)$$

$$= -bn\sqrt{d+ex^2} + b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) + \frac{1}{2}b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2 + \left(\sqrt{d+ex^2} - \sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+b\log(cx^n)) - b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right) - \frac{1}{2}b\sqrt{d}n\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex^2}}{\sqrt{d}}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{bn\sqrt{d+ex^2}\left(-{}_3F_2\left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2};\frac{1}{2},\frac{1}{2};-\frac{d}{ex^2}\right)+\sqrt{1+\frac{d}{ex^2}}\log(x)-\frac{\sqrt{d}\text{arcsinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)\log(x)}{\sqrt{e}x}\right)}{\sqrt{1+\frac{d}{ex^2}}}$$
$$+\sqrt{d+ex^2}(a-bn\log(x)+b\log(cx^n))+\sqrt{d}\log(x)\,(a-bn\log(x)+b\log(cx^n))$$
$$-\sqrt{d}(a-bn\log(x)+b\log(cx^n))\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] (b*n*Sqrt[d + e*x^2]*(-HypergeometricPFQ[{-1/2, -1/2, -1/2}, {1/2, 1/2}, -(
d/(e*x^2))] + Sqrt[1 + d/(e*x^2)]*Log[x] - (Sqrt[d]*ArcSinh[Sqrt[d]/(Sqrt[e
]*x)]*Log[x])/(Sqrt[e]*x)))/Sqrt[1 + d/(e*x^2)] + Sqrt[d + e*x^2]*(a - b*n*
Log[x] + b*Log[c*x^n]) + Sqrt[d]*Log[x]*(a - b*n*Log[x] + b*Log[c*x^n]) - S
qrt[d]*(a - b*n*Log[x] + b*Log[c*x^n])*Log[d + Sqrt[d]*Sqrt[d + e*x^2]]
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))\sqrt{e\,x^2+d}}{x}dx$$

```
[In] int((a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/x,x)
```

```
[Out] int((a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/x,x)
```

Fricas [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\log(cx^n)+a)}{x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral((sqrt(e*x^2 + d)*b*log(c*x^n) + sqrt(e*x^2 + d)*a)/x, x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x}\,dx = \int \frac{(a+b\log(cx^n))\sqrt{d+ex^2}}{x}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))*(e*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral((a + b*log(c*x**n))*sqrt(d + e*x**2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\log(cx^n)+a)}{x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*log(c*x^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{e\,x^2+d}\,(a+b\,\ln(c\,x^n))}{x}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*log(c*x^n)))/x, x)
```

3.255 $\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 252

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^3}\,dx = -\frac{bn\sqrt{d+ex^2}}{4x^2} - \frac{ben\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{4\sqrt{d}} + \frac{ben\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4\sqrt{d}} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2x^2} - \frac{e\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}} - \frac{ben\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2\sqrt{d}} - \frac{ben\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{4\sqrt{d}}$$

```
[Out] -1/4*b*e*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(1/2)+1/4*b*e*n*arctanh((e*x^
2+d)^(1/2)/d^(1/2))^2/d^(1/2)-1/2*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))*(a+b*l
n(c*x^n))/d^(1/2)-1/2*b*e*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(
d^(1/2)-(e*x^2+d)^(1/2)))/d^(1/2)-1/4*b*e*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-
(e*x^2+d)^(1/2)))/d^(1/2)-1/4*b*n*(e*x^2+d)^(1/2)/x^2-1/2*(a+b*ln(c*x^n))*(
e*x^2+d)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {272, 43, 65, 214, 2392, 12, 14, 6131, 6055, 2449, 2352}

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^3}\,dx = -\frac{e\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2x^2} + \frac{ben\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4\sqrt{d}} - \frac{ben\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{4\sqrt{d}} - \frac{ben\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2\sqrt{d}} - \frac{ben\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{ex^2+d}}\right)}{4\sqrt{d}} - \frac{bn\sqrt{d+ex^2}}{4x^2}$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/4*(b*n*Sqrt[d + e*x^2])/x^2 - (b*e*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(
4*Sqrt[d]) + (b*e*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]]^2)/(4*Sqrt[d]) - (Sqrt
[d + e*x^2]*(a + b*Log[c*x^n]))/(2*x^2) - (e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d
]]*(a + b*Log[c*x^n]))/(2*Sqrt[d]) - (b*e*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]
]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/(2*Sqrt[d]) - (b*e*n*PolyLo
g[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/(4*Sqrt[d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2x^2} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}} \\
&\quad -(bn)\int \frac{-\sqrt{d+ex^2} - \frac{ex^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}}{2x^3}\,dx \\
&= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2x^2} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}} \\
&\quad -\frac{1}{2}(bn)\int \frac{-\sqrt{d+ex^2} - \frac{ex^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}}{x^3}\,dx \\
&= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2x^2} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}} \\
&\quad -\frac{1}{2}(bn)\int \left(-\frac{\sqrt{d+ex^2}}{x^3} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}x}\right)dx \\
&= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2x^2} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}} \\
&\quad +\frac{1}{2}(bn)\int \frac{\sqrt{d+ex^2}}{x^3}\,dx + \frac{(ben)\int \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{x}\,dx}{2\sqrt{d}} \\
&= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2x^2} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}} \\
&\quad +\frac{1}{4}(bn)\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x^2}\,dx, x, x^2\right) + \frac{(ben)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx, x, x^2\right)}{4\sqrt{d}}
\end{aligned}$$

$$= -\frac{bn\sqrt{d+ex^2}}{4x^2} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2x^2} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}}$$
$$+ \frac{1}{8}(ben)\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right) + \frac{(ben)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx, x, \sqrt{d+ex^2}\right)}{2\sqrt{d}}$$

$$= -\frac{bn\sqrt{d+ex^2}}{4x^2} + \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4\sqrt{d}} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2x^2}$$
$$- \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}} + \frac{1}{4}(bn)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)$$
$$- \frac{(ben)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx, x, \sqrt{d+ex^2}\right)}{2d}$$

$$= -\frac{bn\sqrt{d+ex^2}}{4x^2} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{4\sqrt{d}} + \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4\sqrt{d}}$$
$$- \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2x^2} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}}$$
$$- \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2\sqrt{d}}$$
$$+ \frac{(ben)\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}\,dx, x, \sqrt{d+ex^2}\right)}{2d}$$

$$= -\frac{bn\sqrt{d+ex^2}}{4x^2} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{4\sqrt{d}} + \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4\sqrt{d}}$$
$$- \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2x^2} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}}$$
$$- \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2\sqrt{d}} - \frac{(ben)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{d+ex^2}}{\sqrt{d}}}\right)}{2\sqrt{d}}$$

$$= -\frac{bn\sqrt{d+ex^2}}{4x^2} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{4\sqrt{d}} + \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4\sqrt{d}} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2x^2} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{d}} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2\sqrt{d}} - \frac{ben\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex^2}}{\sqrt{d}}}\right)}{4\sqrt{d}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.30 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^3}\,dx$$
$$= \frac{-2b\sqrt{d}n\sqrt{d+ex^2}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{d}{ex^2}\right) - b\sqrt{e}nx\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)(1+2\log(x)) + \sqrt{1+\frac{d}{ex^2}}\left(-2a\right.}{}$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] (-2*b*Sqrt[d]*n*Sqrt[d + e*x^2]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/
2}, -(d/(e*x^2))] - b*Sqrt[e]*n*x*Sqrt[d + e*x^2]*ArcSinh[Sqrt[d]/(Sqrt[e]*
x)]*(1 + 2*Log[x]) + Sqrt[1 + d/(e*x^2)]*(-2*a*Sqrt[d]*Sqrt[d + e*x^2] - b*
Sqrt[d]*n*Sqrt[d + e*x^2] - 2*b*e*n*x^2*Log[x]^2 - 2*a*e*x^2*Log[d + Sqrt[d
]*Sqrt[d + e*x^2]] + 2*e*x^2*Log[x]*(a + b*Log[c*x^n] + b*n*Log[d + Sqrt[d]
*Sqrt[d + e*x^2]]) - 2*b*Log[c*x^n]*(Sqrt[d]*Sqrt[d + e*x^2] + e*x^2*Log[d
+ Sqrt[d]*Sqrt[d + e*x^2]])))/(4*Sqrt[d]*Sqrt[1 + d/(e*x^2)]*x^2)
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))\sqrt{e\,x^2+d}}{x^3}dx$$

```
[In] int((a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/x^3,x)
```

```
[Out] int((a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/x^3,x)
```

Fricas [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\sqrt{ex^2+d}(b\log(cx^n)+a)}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
[Out] integral((sqrt(e*x^2 + d)*b*log(c*x^n) + sqrt(e*x^2 + d)*a)/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^3}\,dx = \int \frac{(a+b\log(cx^n))\sqrt{d+ex^2}}{x^3}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))*(e*x**2+d)**(1/2)/x**3,x)
[Out] Integral((a + b*log(c*x**n))*sqrt(d + e*x**2)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\sqrt{ex^2+d}(b\log(cx^n)+a)}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x^3,x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*log(c*x^n) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\sqrt{e\,x^2+d}\,(a+b\,\ln(c\,x^n))}{x^3}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*log(c*x^n)))/x^3,x)
[Out] int(((d + e*x^2)^(1/2)*(a + b*log(c*x^n)))/x^3, x)
```

3.256 $\int x^4\sqrt{d+ex^2}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 469

$$\int x^4\sqrt{d+ex^2}(a+b\log(cx^n))\,dx$$
$$= \frac{7bd^2nx\sqrt{d+ex^2}}{192e^2}-\frac{5bdnx^3\sqrt{d+ex^2}}{288e}-\frac{1}{36}bnx^5\sqrt{d+ex^2}+\frac{5bd^{5/2}n\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{192e^{5/2}\sqrt{1+\frac{ex^2}{d}}}$$
$$+\frac{bd^{5/2}n\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{32e^{5/2}\sqrt{1+\frac{ex^2}{d}}}-\frac{bd^{5/2}n\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}}$$
$$-\frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e^2}+\frac{dx^3\sqrt{d+ex^2}(a+b\log(cx^n))}{24e}$$
$$+\frac{1}{6}x^5\sqrt{d+ex^2}(a+b\log(cx^n))+\frac{d^{5/2}\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}}-\frac{bd^{5/2}n\sqrt{d+ex^2}\operatorname{PolyLog}}{32e^{5/2}\sqrt{1+}}$$

```
[Out] 7/192*b*d^2*n*x*(e*x^2+d)^(1/2)/e^2-5/288*b*d*n*x^3*(e*x^2+d)^(1/2)/e-1/36*
b*n*x^5*(e*x^2+d)^(1/2)-1/16*d^2*x*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/e^2+1/24
*d*x^3*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/e+1/6*x^5*(a+b*ln(c*x^n))*(e*x^2+d)^
(1/2)+5/192*b*d^(5/2)*n*arcsinh(x*e^(1/2)/d^(1/2))*(e*x^2+d)^(1/2)/e^(5/2)/
(1+e*x^2/d)^(1/2)+1/32*b*d^(5/2)*n*arcsinh(x*e^(1/2)/d^(1/2))^2*(e*x^2+d)^(
1/2)/e^(5/2)/(1+e*x^2/d)^(1/2)-1/16*b*d^(5/2)*n*arcsinh(x*e^(1/2)/d^(1/2))*
ln(1-(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*(e*x^2+d)^(1/2)/e^(5/2)/(1+e*
x^2/d)^(1/2)+1/16*d^(5/2)*arcsinh(x*e^(1/2)/d^(1/2))*(a+b*ln(c*x^n))*(e*x^2
+d)^(1/2)/e^(5/2)/(1+e*x^2/d)^(1/2)-1/32*b*d^(5/2)*n*polylog(2,(x*e^(1/2)/d
^(1/2)+(1+e*x^2/d)^(1/2))^2)*(e*x^2+d)^(1/2)/e^(5/2)/(1+e*x^2/d)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {2386, 285, 327, 221, 2392, 12, 14, 201, 5775, 3797, 2221, 2317, 2438}

$$\int x^4\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \frac{d^{5/2}\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{16e^{5/2}\sqrt{\frac{ex^2}{d}+1}} - \frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e^2} + \frac{1}{6}x^5\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{dx^3\sqrt{d+ex^2}(a+b\log(cx^n))}{24e} - \frac{bd^{5/2}n\sqrt{d+ex^2}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{32e^{5/2}\sqrt{\frac{ex^2}{d}+1}} + \frac{bd^{5/2}n\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{32e^{5/2}\sqrt{\frac{ex^2}{d}+1}} + \frac{5bd^{5/2}n\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{192e^{5/2}\sqrt{\frac{ex^2}{d}+1}} - \frac{bd^{5/2}n\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{16e^{5/2}\sqrt{\frac{ex^2}{d}+1}} + \frac{7bd^2nx\sqrt{d+ex^2}}{192e^2} - \frac{1}{36}bnx^5\sqrt{d+ex^2} - \frac{5bdnx^3\sqrt{d+ex^2}}{288e}$$

```
[In] Int[x^4*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]),x]
```

```
[Out] (7*b*d^2*n*x*Sqrt[d + e*x^2])/(192*e^2) - (5*b*d*n*x^3*Sqrt[d + e*x^2])/(28
8*e) - (b*n*x^5*Sqrt[d + e*x^2])/36 + (5*b*d^(5/2)*n*Sqrt[d + e*x^2]*ArcSin
h[(Sqrt[e]*x)/Sqrt[d]])/(192*e^(5/2)*Sqrt[1 + (e*x^2)/d]) + (b*d^(5/2)*n*Sq
rt[d + e*x^2]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)/(32*e^(5/2)*Sqrt[1 + (e*x^2)/
d]) - (b*d^(5/2)*n*Sqrt[d + e*x^2]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[1 - E^(
2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(16*e^(5/2)*Sqrt[1 + (e*x^2)/d]) - (d^2*x
*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/(16*e^2) + (d*x^3*Sqrt[d + e*x^2]*(a +
 b*Log[c*x^n]))/(24*e) + (x^5*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/6 + (d^(5
```

```
/2)*Sqrt[d + e*x^2]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/(16*e^
(5/2)*Sqrt[1 + (e*x^2)/d]) - (b*d^(5/2)*n*Sqrt[d + e*x^2]*PolyLog[2, E^(2*A
rcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(32*e^(5/2)*Sqrt[1 + (e*x^2)/d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2386

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^
(q_), x_Symbol] :> Dist[d^IntPart[q]*((d + e*x^2)^FracPart[q]/(1 + (e/d)*x^
2)^FracPart[q]), Int[x^m*(1 + (e/d)*x^2)^q*(a + b*Log[c*x^n]), x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && IntegerQ[m/2] && IntegerQ[q - 1/2] &&  !(LtQ[
m + 2*q, -2] || GtQ[d, 0])
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d+ex^2}\int x^4\sqrt{1+\frac{ex^2}{d}}(a+b\log(cx^n))\,dx}{\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e^2} + \frac{dx^3\sqrt{d+ex^2}(a+b\log(cx^n))}{24e} + \frac{1}{6}x^5\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bn\sqrt{d+ex^2}\right)\int\frac{\sqrt{e}x\sqrt{1+\frac{ex^2}{d}}(-3d^2+2dex^2+8e^2x^4)+3d^{5/2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{48e^{5/2}x}\,dx}{\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e^2} + \frac{dx^3\sqrt{d+ex^2}(a+b\log(cx^n))}{24e} + \frac{1}{6}x^5\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bn\sqrt{d+ex^2}\right)\int\frac{\sqrt{e}x\sqrt{1+\frac{ex^2}{d}}(-3d^2+2dex^2+8e^2x^4)+3d^{5/2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{48e^{5/2}\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e^2} + \frac{dx^3\sqrt{d+ex^2}(a+b\log(cx^n))}{24e} + \frac{1}{6}x^5\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bn\sqrt{d+ex^2}\right)\int\left(-3d^2\sqrt{e}\sqrt{1+\frac{ex^2}{d}}+2de^{3/2}x^2\sqrt{1+\frac{ex^2}{d}}+8e^{5/2}x^4\sqrt{1+\frac{ex^2}{d}}+\frac{3d^{5/2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\right.}{48e^{5/2}\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e^2} + \frac{dx^3\sqrt{d+ex^2}(a+b\log(cx^n))}{24e} + \frac{1}{6}x^5\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bn\sqrt{d+ex^2}\right)\int x^4\sqrt{1+\frac{ex^2}{d}}\,dx}{6\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bd^{5/2}n\sqrt{d+ex^2}\right)\int \frac{\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(bd^2n\sqrt{d+ex^2}\right)\int \sqrt{1+\frac{ex^2}{d}}\,dx}{16e^2\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bdn\sqrt{d+ex^2}\right)\int x^2\sqrt{1+\frac{ex^2}{d}}\,dx}{24e\sqrt{1+\frac{ex^2}{d}}}$$

$$= \frac{bd^2nx\sqrt{d+ex^2}}{32e^2} - \frac{bdnx^3\sqrt{d+ex^2}}{96e} - \frac{1}{36}bnx^5\sqrt{d+ex^2} - \frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e^2} + \frac{dx^3\sqrt{d+ex^2}(a+b\log(cx^n))}{24e} + \frac{1}{6}x^5\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bn\sqrt{d+ex^2}\right)\int \frac{x^4}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{36\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bd^{5/2}n\sqrt{d+ex^2}\right)\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(bd^2n\sqrt{d+ex^2}\right)\int \frac{1}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{32e^2\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bdn\sqrt{d+ex^2}\right)\int \frac{x^2}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{96e\sqrt{1+\frac{ex^2}{d}}}$$

$$= \frac{5bd^2nx\sqrt{d+ex^2}}{192e^2} - \frac{5bdnx^3\sqrt{d+ex^2}}{288e} - \frac{1}{36}bnx^5\sqrt{d+ex^2} + \frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{32e^{5/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{32e^{5/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e^2} + \frac{dx^3\sqrt{d+ex^2}(a+b\log(cx^n))}{24e} + \frac{1}{6}x^5\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(bd^{5/2}n\sqrt{d+ex^2}\right)\text{S}}{}$$

$$=\frac{7bd^2nx\sqrt{d+ex^2}}{192e^2}-\frac{5bdnx^3\sqrt{d+ex^2}}{288e}-\frac{1}{36}bnx^5\sqrt{d+ex^2}$$
$$+\frac{7bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{192e^{5/2}\sqrt{1+\frac{ex^2}{d}}}+\frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{32e^{5/2}\sqrt{1+\frac{ex^2}{d}}}$$
$$-\frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}}$$
$$-\frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e^2}+\frac{dx^3\sqrt{d+ex^2}(a+b\log(cx^n))}{24e}$$
$$+\frac{1}{6}x^5\sqrt{d+ex^2}(a+b\log(cx^n))+\frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}}+\frac{(bd^{5/2}n\sqrt{d+ex^2})}{}$$

$$=\frac{7bd^2nx\sqrt{d+ex^2}}{192e^2}-\frac{5bdnx^3\sqrt{d+ex^2}}{288e}-\frac{1}{36}bnx^5\sqrt{d+ex^2}$$
$$+\frac{5bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{192e^{5/2}\sqrt{1+\frac{ex^2}{d}}}+\frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{32e^{5/2}\sqrt{1+\frac{ex^2}{d}}}$$
$$-\frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}}$$
$$-\frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e^2}+\frac{dx^3\sqrt{d+ex^2}(a+b\log(cx^n))}{24e}$$
$$+\frac{1}{6}x^5\sqrt{d+ex^2}(a+b\log(cx^n))+\frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}}+\frac{(bd^{5/2}n\sqrt{d+ex^2})}{}$$

$$= \frac{7bd^2nx\sqrt{d+ex^2}}{192e^2} - \frac{5bdnx^3\sqrt{d+ex^2}}{288e} - \frac{1}{36}bnx^5\sqrt{d+ex^2} + \frac{5bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{192e^{5/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{32e^{5/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e^2} + \frac{dx^3\sqrt{d+ex^2}(a+b\log(cx^n))}{24e} + \frac{1}{6}x^5\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{bd^{5/2}n\sqrt{d+ex^2}\mathrm{Li}_2}{32e^{5/2}\sqrt{1}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.38 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.59

$$\int x^4\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \frac{-48be^{5/2}nx^5\sqrt{d+ex^2}\,{}_3F_2\left(-\frac{1}{2},\frac{5}{2},\frac{5}{2};\frac{7}{2},\frac{7}{2};-\frac{ex^2}{d}\right) + 75bd^{5/2}n\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x) + 25\sqrt{1+\frac{ex^2}{d}}(a}{}$$

```
[In] Integrate[x^4*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-48*b*e^(5/2)*n*x^5*Sqrt[d + e*x^2]*HypergeometricPFQ[{-1/2, 5/2, 5/2}, {7
/2, 7/2}, -((e*x^2)/d)] + 75*b*d^(5/2)*n*Sqrt[d + e*x^2]*ArcSinh[(Sqrt[e]*x
)/Sqrt[d]]*Log[x] + 25*Sqrt[1 + (e*x^2)/d]*(a*Sqrt[e]*x*Sqrt[d + e*x^2]*(-3
*d^2 + 2*d*e*x^2 + 8*e^2*x^4) + 3*d^3*(a - b*n*Log[x])*Log[e*x + Sqrt[e]*Sq
rt[d + e*x^2]] + b*Log[c*x^n]*(Sqrt[e]*x*Sqrt[d + e*x^2]*(-3*d^2 + 2*d*e*x^
2 + 8*e^2*x^4) + 3*d^3*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])))/(1200*e^(5/2)*
Sqrt[1 + (e*x^2)/d])
```

Maple [F]

$$\int x^{4}(a+b\ln(c\,x^{n}))\sqrt{e\,x^{2}+d}dx$$

```
[In] int(x^4*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2),x)
[Out] int(x^4*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int x^{4}\sqrt{d+ex^{2}}(a+b\log(cx^{n}))\,dx=\int\sqrt{ex^{2}+d}(b\log(cx^{n})+a)x^{4}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*b*x^4*log(c*x^n) + sqrt(e*x^2 + d)*a*x^4, x)
```

Sympy [F]

$$\int x^{4}\sqrt{d+ex^{2}}(a+b\log(cx^{n}))\,dx=\int x^{4}(a+b\log(cx^{n}))\sqrt{d+ex^{2}}\,dx$$

```
[In] integrate(x**4*(a+b*ln(c*x**n))*(e*x**2+d)**(1/2),x)
[Out] Integral(x**4*(a + b*log(c*x**n))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{4}\sqrt{d+ex^{2}}(a+b\log(cx^{n}))\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^4\sqrt{d+ex^2}(a+b\log(cx^n))\,dx=\int\sqrt{ex^2+d}(b\log(cx^n)+a)x^4\,dx$$

```
[In] integrate(x^4*(a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*log(c*x^n) + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{d+ex^2}(a+b\log(cx^n))\,dx=\int x^4\sqrt{e\,x^2+d}\,(a+b\,\ln(c\,x^n))\,dx$$

```
[In] int(x^4*(d + e*x^2)^(1/2)*(a + b*log(c*x^n)),x)
[Out] int(x^4*(d + e*x^2)^(1/2)*(a + b*log(c*x^n)), x)
```

3.257 $\int x^2\sqrt{d+ex^2}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 409

$$\int x^2\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = -\frac{3bdnx\sqrt{d+ex^2}}{32e} - \frac{1}{16}bnx^3\sqrt{d+ex^2} - \frac{bd^{3/2}n\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{32e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{bd^{3/2}n\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{bd^{3/2}n\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{8e^{3/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{dx\sqrt{d+ex^2}(a+b\log(cx^n))}{8e} + \frac{1}{4}x^3\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{d^{3/2}\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8e^{3/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{bd^{3/2}n\sqrt{d+ex^2}\,\operatorname{PolyLog}}{16e^{3/2}\sqrt{1+}}$$

```
[Out] -3/32*b*d*n*x*(e*x^2+d)^(1/2)/e-1/16*b*n*x^3*(e*x^2+d)^(1/2)+1/8*d*x*(a+b*l
n(c*x^n))*(e*x^2+d)^(1/2)/e+1/4*x^3*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)-1/32*b*
d^(3/2)*n*arcsinh(x*e^(1/2)/d^(1/2))*(e*x^2+d)^(1/2)/e^(3/2)/(1+e*x^2/d)^(1
/2)-1/16*b*d^(3/2)*n*arcsinh(x*e^(1/2)/d^(1/2))^2*(e*x^2+d)^(1/2)/e^(3/2)/(
1+e*x^2/d)^(1/2)+1/8*b*d^(3/2)*n*arcsinh(x*e^(1/2)/d^(1/2))*ln(1-(x*e^(1/2)
/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*(e*x^2+d)^(1/2)/e^(3/2)/(1+e*x^2/d)^(1/2)-1/
8*d^(3/2)*arcsinh(x*e^(1/2)/d^(1/2))*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/e^(3/2
)/(1+e*x^2/d)^(1/2)+1/16*b*d^(3/2)*n*polylog(2,(x*e^(1/2)/d^(1/2)+(1+e*x^2/
d)^(1/2))^2)*(e*x^2+d)^(1/2)/e^(3/2)/(1+e*x^2/d)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2386, 285, 327, 221, 2392, 396, 201, 5775, 3797, 2221, 2317, 2438}

$$\int x^2\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = -\frac{d^{3/2}\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8e^{3/2}\sqrt{\frac{ex^2}{d}+1}} + \frac{dx\sqrt{d+ex^2}(a+b\log(cx^n))}{8e} + \frac{1}{4}x^3\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{bd^{3/2}n\sqrt{d+ex^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{16e^{3/2}\sqrt{\frac{ex^2}{d}+1}} - \frac{bd^{3/2}n\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{16e^{3/2}\sqrt{\frac{ex^2}{d}+1}} - \frac{bd^{3/2}n\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{32e^{3/2}\sqrt{\frac{ex^2}{d}+1}} + \frac{bd^{3/2}n\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{8e^{3/2}\sqrt{\frac{ex^2}{d}+1}} - \frac{bnx(d+ex^2)^{3/2}}{16e} - \frac{bdnx\sqrt{d+ex^2}}{32e}$$

```
[In] Int[x^2*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/32*(b*d*n*x*Sqrt[d + e*x^2])/e - (b*n*x*(d + e*x^2)^(3/2))/(16*e) - (b*d
^(3/2)*n*Sqrt[d + e*x^2]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])/(32*e^(3/2)*Sqrt[1 +
 (e*x^2)/d]) - (b*d^(3/2)*n*Sqrt[d + e*x^2]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)
/(16*e^(3/2)*Sqrt[1 + (e*x^2)/d]) + (b*d^(3/2)*n*Sqrt[d + e*x^2]*ArcSinh[(S
qrt[e]*x)/Sqrt[d]]*Log[1 - E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(8*e^(3/2)*
Sqrt[1 + (e*x^2)/d]) + (d*x*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/(8*e) + (x^
3*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/4 - (d^(3/2)*Sqrt[d + e*x^2]*ArcSinh[
(Sqrt[e]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/(8*e^(3/2)*Sqrt[1 + (e*x^2)/d]) +
(b*d^(3/2)*n*Sqrt[d + e*x^2]*PolyLog[2, E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])]
)/(16*e^(3/2)*Sqrt[1 + (e*x^2)/d])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2386

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^
(q_), x_Symbol] :> Dist[d^IntPart[q]*((d + e*x^2)^FracPart[q]/(1 + (e/d)*x^
2)^FracPart[q]), Int[x^m*(1 + (e/d)*x^2)^q*(a + b*Log[c*x^n]), x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && IntegerQ[m/2] && IntegerQ[q - 1/2] &&  !(LtQ[
m + 2*q, -2] || GtQ[d, 0])
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d+ex^2}\int x^2\sqrt{1+\frac{ex^2}{d}}(a+b\log(cx^n))\,dx}{\sqrt{1+\frac{ex^2}{d}}}$$

$$= \frac{dx\sqrt{d+ex^2}(a+b\log(cx^n))}{8e} + \frac{1}{4}x^3\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{d^{3/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bn\sqrt{d+ex^2}\right)\int\left(\frac{(d+2ex^2)\sqrt{1+\frac{ex^2}{d}}}{8e} - \frac{d^{3/2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8e^{3/2}x}\right)dx}{\sqrt{1+\frac{ex^2}{d}}}$$

$$= \frac{dx\sqrt{d+ex^2}(a+b\log(cx^n))}{8e} + \frac{1}{4}x^3\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{d^{3/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8e^{3/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(bd^{3/2}n\sqrt{d+ex^2}\right)\int\frac{\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{8e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bn\sqrt{d+ex^2}\right)\int(d+2ex^2)\sqrt{1+\frac{ex^2}{d}}dx}{8e\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{bnx(d+ex^2)^{3/2}}{16e} + \frac{dx\sqrt{d+ex^2}(a+b\log(cx^n))}{8e} + \frac{1}{4}x^3\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{d^{3/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8e^{3/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(bd^{3/2}n\sqrt{d+ex^2}\right)\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{8e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bdn\sqrt{d+ex^2}\right)\int\sqrt{1+\frac{ex^2}{d}}dx}{16e\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{bdnx\sqrt{d+ex^2}}{32e} - \frac{bnx(d+ex^2)^{3/2}}{16e} - \frac{bd^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{dx\sqrt{d+ex^2}(a+b\log(cx^n))}{8e} + \frac{1}{4}x^3\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{d^{3/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bd^{3/2}n\sqrt{d+ex^2}\right)}{}$$

$$= -\frac{bdnx\sqrt{d+ex^2}}{32e} - \frac{bnx(d+ex^2)^{3/2}}{16e}$$
$$- \frac{bd^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{32e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{bd^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{bd^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{8e^{3/2}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{dx\sqrt{d+ex^2}(a+b\log(cx^n))}{8e}$$
$$+ \frac{1}{4}x^3\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{d^{3/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bd^{3/2}n\sqrt{d+ex^2}\right) \mathrm{S}}{}$$

$$= -\frac{bdnx\sqrt{d+ex^2}}{32e} - \frac{bnx(d+ex^2)^{3/2}}{16e}$$
$$- \frac{bd^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{32e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{bd^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{bd^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{8e^{3/2}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{dx\sqrt{d+ex^2}(a+b\log(cx^n))}{8e}$$
$$+ \frac{1}{4}x^3\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{d^{3/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bd^{3/2}n\sqrt{d+ex^2}\right) \mathrm{S}}{}$$

$$= -\frac{bdnx\sqrt{d+ex^2}}{32e} - \frac{bnx(d+ex^2)^{3/2}}{16e} - \frac{bd^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{32e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{bd^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{bd^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{8e^{3/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{dx\sqrt{d+ex^2}(a+b\log(cx^n))}{8e} + \frac{1}{4}x^3\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{d^{3/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8e^{3/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{bd^{3/2}n\sqrt{d+ex^2}\mathrm{Li}}{16e^{3/2}\sqrt{}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.27 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.61

$$\int x^2\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \frac{-8be^{3/2}nx^3\sqrt{d+ex^2}\,{}_3F_2\left(-\frac{1}{2},\frac{3}{2},\frac{3}{2};\frac{5}{2},\frac{5}{2};-\frac{ex^2}{d}\right) - 9bd^{3/2}n\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x) + 9\sqrt{1+\frac{ex^2}{d}}(a\sqrt{}}{}$$

```
[In] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-8*b*e^(3/2)*n*x^3*Sqrt[d + e*x^2]*HypergeometricPFQ[{-1/2, 3/2, 3/2}, {5/
2, 5/2}, -((e*x^2)/d)] - 9*b*d^(3/2)*n*Sqrt[d + e*x^2]*ArcSinh[(Sqrt[e]*x)/
Sqrt[d]]*Log[x] + 9*Sqrt[1 + (e*x^2)/d]*(a*Sqrt[e]*x*Sqrt[d + e*x^2]*(d + 2
*e*x^2) + d^2*(-a + b*n*Log[x])*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]] + b*Log[
c*x^n]*(Sqrt[e]*x*Sqrt[d + e*x^2]*(d + 2*e*x^2) - d^2*Log[e*x + Sqrt[e]*Sqr
t[d + e*x^2]])))/(72*e^(3/2)*Sqrt[1 + (e*x^2)/d])
```

Maple [F]

$$\int x^2(a+b\ln(c\,x^n))\,\sqrt{e\,x^2+d}dx$$

```
[In] int(x^2*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int x^2\sqrt{d+ex^2}(a+b\log{(cx^n)})\;dx=\int\sqrt{ex^2+d}(b\log{(cx^n)}+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*b*x^2*log(c*x^n) + sqrt(e*x^2 + d)*a*x^2, x)
```

Sympy [F]

$$\int x^2\sqrt{d+ex^2}(a+b\log{(cx^n)})\;dx=\int x^2(a+b\log{(cx^n)})\,\sqrt{d+ex^2}\,dx$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*log(c*x**n))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+ex^2}(a+b\log{(cx^n)})\;dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^2\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \int \sqrt{ex^2+d}(b\log(cx^n)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*log(c*x^n) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \int x^2\,\sqrt{e\,x^2+d}\,(a+b\,\ln(c\,x^n))\,dx$$

```
[In] int(x^2*(d + e*x^2)^(1/2)*(a + b*log(c*x^n)),x)
[Out] int(x^2*(d + e*x^2)^(1/2)*(a + b*log(c*x^n)), x)
```

3.258 $\int \sqrt{d+ex^2}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 330

$$\int \sqrt{d+ex^2}(a+b\log(cx^n))\,dx = -\frac{1}{4}bnx\sqrt{d+ex^2}+\frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4\sqrt{e}\sqrt{d+ex^2}} - \frac{bdn\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{e}} - \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\sqrt{d+ex^2}} + \frac{1}{2}x\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{e}\sqrt{d+ex^2}} - \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{4\sqrt{e}\sqrt{d+ex^2}}$$

```
[Out] -1/4*b*d*n*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(1/2)-1/4*b*n*x*(e*x^2+d)^(
1/2)+1/2*x*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)+1/4*b*d^(3/2)*n*arcsinh(x*e^(1/2
)/d^(1/2))^2*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)-1/2*b*d^(3/2)*n*arcs
inh(x*e^(1/2)/d^(1/2))*ln(1-(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*(1+e*x
^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)+1/2*d^(3/2)*arcsinh(x*e^(1/2)/d^(1/2))*
(a+b*ln(c*x^n))*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)-1/4*b*d^(3/2)*n*p
olylog(2,(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*(1+e*x^2/d)^(1/2)/e^(1/2)
/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2358, 201, 223, 212, 2364, 2362, 5775, 3797, 2221, 2317, 2438}

$$\int \sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \frac{d^{3/2}\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{e}\sqrt{d+ex^2}} + \frac{1}{2}x\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{bd^{3/2}n\sqrt{\frac{ex^2}{d}+1}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{4\sqrt{e}\sqrt{d+ex^2}} + \frac{bd^{3/2}n\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4\sqrt{e}\sqrt{d+ex^2}} - \frac{bd^{3/2}n\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\sqrt{d+ex^2}} - \frac{bdn\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{e}} - \frac{1}{4}bnx\sqrt{d+ex^2}$$

```
[In] Int[Sqrt[d + e*x^2]*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/4*(b*n*x*Sqrt[d + e*x^2]) + (b*d^(3/2)*n*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sq
rt[e]*x)/Sqrt[d]]^2)/(4*Sqrt[e]*Sqrt[d + e*x^2]) - (b*d*n*ArcTanh[(Sqrt[e]*
x)/Sqrt[d + e*x^2]])/(4*Sqrt[e]) - (b*d^(3/2)*n*Sqrt[1 + (e*x^2)/d]*ArcSinh
[(Sqrt[e]*x)/Sqrt[d]]*Log[1 - E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(2*Sqrt[
e]*Sqrt[d + e*x^2]) + (x*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/2 + (d^(3/2)*S
qrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/(2*Sqrt
[e]*Sqrt[d + e*x^2]) - (b*d^(3/2)*n*Sqrt[1 + (e*x^2)/d]*PolyLog[2, E^(2*Arc
Sinh[(Sqrt[e]*x)/Sqrt[d]])])/(4*Sqrt[e]*Sqrt[d + e*x^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2358

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Sy
mbol] :> Simp[x*(d + e*x^2)^q*((a + b*Log[c*x^n])/(2*q + 1)), x] + (-Dist[b
*(n/(2*q + 1)), Int[(d + e*x^2)^q, x], x] + Dist[2*d*(q/(2*q + 1)), Int[(d
+ e*x^2)^(q - 1)*(a + b*Log[c*x^n]), x], x]) /; FreeQ[{a, b, c, d, e, n}, x
] && GtQ[q, 0]
```

Rule 2362

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[e, 2]), x]
 - Dist[b*(n/Rt[e, 2]), Int[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 2364

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Dist[Sqrt[1 + (e/d)*x^2]/Sqrt[d + e*x^2], Int[(a + b*Log[c*x^n])/Sqr
t[1 + (e/d)*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] &&  !GtQ[d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{1}{2}d\int\frac{a+b\log(cx^n)}{\sqrt{d+ex^2}}\,dx - \frac{1}{2}(bn)\int\sqrt{d+ex^2}\,dx \\
&= -\frac{1}{4}bnx\sqrt{d+ex^2} + \frac{1}{2}x\sqrt{d+ex^2}(a+b\log(cx^n)) \\
&\quad - \frac{1}{4}(bdn)\int\frac{1}{\sqrt{d+ex^2}}\,dx + \frac{\left(d\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{a+b\log(cx^n)}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{2\sqrt{d+ex^2}} \\
&= -\frac{1}{4}bnx\sqrt{d+ex^2} + \frac{1}{2}x\sqrt{d+ex^2}(a+b\log(cx^n)) \\
&\quad + \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{e}\sqrt{d+ex^2}} \\
&\quad - \frac{1}{4}(bdn)\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right) \\
&\quad - \frac{\left(bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2\sqrt{e}\sqrt{d+ex^2}} \\
&= -\frac{1}{4}bnx\sqrt{d+ex^2} - \frac{bdn\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{e}} + \frac{1}{2}x\sqrt{d+ex^2}(a+b\log(cx^n)) \\
&\quad + \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{e}\sqrt{d+ex^2}} \\
&\quad - \frac{\left(bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{2\sqrt{e}\sqrt{d+ex^2}}
\end{aligned}$$

$$= -\frac{1}{4}bnx\sqrt{d+ex^2} + \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4\sqrt{e}\sqrt{d+ex^2}} - \frac{bdn\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{e}}$$
$$+ \frac{1}{2}x\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{e}\sqrt{d+ex^2}}$$
$$+ \frac{\left(bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{\sqrt{e}\sqrt{d+ex^2}}$$

$$= -\frac{1}{4}bnx\sqrt{d+ex^2} + \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4\sqrt{e}\sqrt{d+ex^2}} - \frac{bdn\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{e}}$$
$$- \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\sqrt{d+ex^2}}$$
$$+ \frac{1}{2}x\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{e}\sqrt{d+ex^2}}$$
$$+ \frac{\left(bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{2\sqrt{e}\sqrt{d+ex^2}}$$

$$= -\frac{1}{4}bnx\sqrt{d+ex^2} + \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4\sqrt{e}\sqrt{d+ex^2}} - \frac{bdn\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{e}}$$
$$- \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\sqrt{d+ex^2}}$$
$$+ \frac{1}{2}x\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{e}\sqrt{d+ex^2}}$$
$$+ \frac{\left(bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{4\sqrt{e}\sqrt{d+ex^2}}$$

$$= -\frac{1}{4}bnx\sqrt{d+ex^2} + \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4\sqrt{e}\sqrt{d+ex^2}} - \frac{bdn\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{e}} - \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\sqrt{d+ex^2}} + \frac{1}{2}x\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{e}\sqrt{d+ex^2}} - \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\mathrm{Li}_2\left(e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{4\sqrt{e}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.23 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.72

$$\int \sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \frac{-2b\sqrt{e}nx\sqrt{d+ex^2}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{ex^2}{d}\right) + b\sqrt{d}n\sqrt{d+ex^2}\mathrm{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(-1+2\log(x)) + \sqrt{1+\frac{ex^2}{d}}(\cdots}{}$$

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-2*b*Sqrt[e]*n*x*Sqrt[d + e*x^2]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2,
3/2}, -((e*x^2)/d)] + b*Sqrt[d]*n*Sqrt[d + e*x^2]*ArcSinh[(Sqrt[e]*x)/Sqrt[
d]]*(-1 + 2*Log[x]) + Sqrt[1 + (e*x^2)/d]*(Sqrt[e]*(2*a - b*n)*x*Sqrt[d + e
*x^2] + 2*d*(a - b*n*Log[x])*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]] + 2*b*Log[c
*x^n]*(Sqrt[e]*x*Sqrt[d + e*x^2] + d*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])))/
(4*Sqrt[e]*Sqrt[1 + (e*x^2)/d])
```

Maple [F]

$$\int (a+b\ln(c\,x^n))\sqrt{e\,x^2+d}\,dx$$

```
[In] int((a+b*ln(c*x^n))*(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*ln(c*x^n))*(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \int \sqrt{ex^2+d}(b\log(cx^n)+a)\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*b*log(c*x^n) + sqrt(e*x^2 + d)*a, x)
```

Sympy [F]

$$\int \sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \int (a+b\log(cx^n))\sqrt{d+ex^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))*(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*log(c*x**n))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \int \sqrt{ex^2+d}(b\log(cx^n)+a)\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex^2}(a+b\log(cx^n))\,dx = \int \sqrt{e\,x^2+d}\,(a+b\,\ln(c\,x^n))\,dx$$

```
[In] int((d + e*x^2)^(1/2)*(a + b*log(c*x^n)),x)
[Out] int((d + e*x^2)^(1/2)*(a + b*log(c*x^n)), x)
```

3.259 $\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 345

$$
\begin{aligned}
\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^2}\,dx = & -\frac{bn\sqrt{d+ex^2}}{x}+\frac{b\sqrt{e}n\sqrt{d+ex^2}\,\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} \\
& +\frac{b\sqrt{e}n\sqrt{d+ex^2}\,\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} \\
& -\frac{b\sqrt{e}n\sqrt{d+ex^2}\,\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} \\
& -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x} \\
& +\frac{\sqrt{e}\sqrt{d+ex^2}\,\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} \\
& -\frac{b\sqrt{e}n\sqrt{d+ex^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}
\end{aligned}
$$

```
[Out] -b*n*(e*x^2+d)^(1/2)/x-(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/x+b*n*arcsinh(x*e^(1
/2)/d^(1/2))*e^(1/2)*(e*x^2+d)^(1/2)/d^(1/2)/(1+e*x^2/d)^(1/2)+1/2*b*n*arcs
inh(x*e^(1/2)/d^(1/2))^2*e^(1/2)*(e*x^2+d)^(1/2)/d^(1/2)/(1+e*x^2/d)^(1/2)-
b*n*arcsinh(x*e^(1/2)/d^(1/2))*ln(1-(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2
)*e^(1/2)*(e*x^2+d)^(1/2)/d^(1/2)/(1+e*x^2/d)^(1/2)+arcsinh(x*e^(1/2)/d^(1/
2))*(a+b*ln(c*x^n))*e^(1/2)*(e*x^2+d)^(1/2)/d^(1/2)/(1+e*x^2/d)^(1/2)-1/2*b
```

```
*n*polylog(2,(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*e^(1/2)*(e*x^2+d)^(1/
2)/d^(1/2)/(1+e*x^2/d)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2386, 283, 221, 2392, 14, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^2}\,dx = \frac{\sqrt{e}\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x} - \frac{b\sqrt{e}n\sqrt{d+ex^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{d}\sqrt{\frac{ex^2}{d}+1}} + \frac{b\sqrt{e}n\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d}\sqrt{\frac{ex^2}{d}+1}} + \frac{b\sqrt{e}n\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}} - \frac{b\sqrt{e}n\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}} - \frac{bn\sqrt{d+ex^2}}{x}$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -((b*n*Sqrt[d + e*x^2])/x) + (b*Sqrt[e]*n*Sqrt[d + e*x^2]*ArcSinh[(Sqrt[e]*
x)/Sqrt[d]])/(Sqrt[d]*Sqrt[1 + (e*x^2)/d]) + (b*Sqrt[e]*n*Sqrt[d + e*x^2]*A
rcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)/(2*Sqrt[d]*Sqrt[1 + (e*x^2)/d]) - (b*Sqrt[e]
*n*Sqrt[d + e*x^2]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[1 - E^(2*ArcSinh[(Sqrt[
e]*x)/Sqrt[d]])])/(Sqrt[d]*Sqrt[1 + (e*x^2)/d]) - (Sqrt[d + e*x^2]*(a + b*L
og[c*x^n]))/x + (Sqrt[e]*Sqrt[d + e*x^2]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*(a +
b*Log[c*x^n]))/(Sqrt[d]*Sqrt[1 + (e*x^2)/d]) - (b*Sqrt[e]*n*Sqrt[d + e*x^2]
*PolyLog[2, E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(2*Sqrt[d]*Sqrt[1 + (e*x^2
)/d])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2386

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^
(q_), x_Symbol] :> Dist[d^IntPart[q]*((d + e*x^2)^FracPart[q]/(1 + (e/d)*x^
2)^FracPart[q]), Int[x^m*(1 + (e/d)*x^2)^q*(a + b*Log[c*x^n]), x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && IntegerQ[m/2] && IntegerQ[q - 1/2] &&  !(LtQ[
m + 2*q, -2] || GtQ[d, 0])
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
```

```
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d+ex^2}\int \frac{\sqrt{1+\frac{ex^2}{d}}(a+b\log(cx^n))}{x^2}\,dx}{\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x} + \frac{\sqrt{e}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bn\sqrt{d+ex^2}\right)\int \frac{-\sqrt{1+\frac{ex^2}{d}}+\frac{\sqrt{e}x\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}}{x^2}\,dx}{\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x} + \frac{\sqrt{e}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bn\sqrt{d+ex^2}\right)\int \left(-\frac{\sqrt{1+\frac{ex^2}{d}}}{x^2}+\frac{\sqrt{e}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}x}\right)dx}{\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x} + \frac{\sqrt{e}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{\left(bn\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{x^2}\,dx}{\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(b\sqrt{e}n\sqrt{d+ex^2}\right)\int\frac{\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{bn\sqrt{d+ex^2}}{x} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x} + \frac{\sqrt{e}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$
$$- \frac{\left(b\sqrt{e}n\sqrt{d+ex^2}\right)\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(ben\sqrt{d+ex^2}\right)\int\frac{1}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{d\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{bn\sqrt{d+ex^2}}{x} + \frac{b\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} + \frac{b\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$
$$- \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x} + \frac{\sqrt{e}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{\left(2b\sqrt{e}n\sqrt{d+ex^2}\right)\text{Subst}\left(\int\frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{bn\sqrt{d+ex^2}}{x} + \frac{b\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} + \frac{b\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$
$$- \frac{b\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$
$$- \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x} + \frac{\sqrt{e}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{\left(b\sqrt{e}n\sqrt{d+ex^2}\right)\text{Subst}\left(\int\log\left(1-e^{2x}\right)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{bn\sqrt{d+ex^2}}{x} + \frac{b\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} + \frac{b\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} - \frac{b\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x} + \frac{\sqrt{e}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(b\sqrt{e}n\sqrt{d+ex^2}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{bn\sqrt{d+ex^2}}{x} + \frac{b\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} + \frac{b\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} - \frac{b\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x} + \frac{\sqrt{e}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} - \frac{b\sqrt{e}n\sqrt{d+ex^2}\text{Li}_2\left(e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.33 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^2}\,dx = \frac{bn\sqrt{d+ex^2}\left(-{}_3F_2\left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2};\frac{1}{2},\frac{1}{2};-\frac{ex^2}{d}\right) - \sqrt{1+\frac{ex^2}{d}}\log(x) + \frac{\sqrt{e}x\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{d}}\right)}{x\sqrt{1+\frac{ex^2}{d}}} - \frac{\sqrt{d+ex^2}(a-bn\log(x)+b\log(cx^n))}{x} + \sqrt{e}(a-bn\log(x)+b\log(cx^n))\log\left(ex+\sqrt{e}\sqrt{d+ex^2}\right)$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] (b*n*Sqrt[d + e*x^2]*(-HypergeometricPFQ[{-1/2, -1/2, -1/2}, {1/2, 1/2}, -(
(e*x^2)/d)] - Sqrt[1 + (e*x^2)/d]*Log[x] + (Sqrt[e]*x*ArcSinh[(Sqrt[e]*x)/S
qrt[d]]*Log[x])/Sqrt[d]))/(x*Sqrt[1 + (e*x^2)/d]) - (Sqrt[d + e*x^2]*(a - b
*n*Log[x] + b*Log[c*x^n]))/x + Sqrt[e]*(a - b*n*Log[x] + b*Log[c*x^n])*Log[
e*x + Sqrt[e]*Sqrt[d + e*x^2]]
```

Maple [F]

$$\int \frac{\left(a +b \ln \left(c \,x^{n}\right)\right) \sqrt{e \,x^{2}+d}}{x^{2}}d x$$

```
[In] int((a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/x^2,x)
[Out] int((a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^2}\,dx=\int \frac{\sqrt{ex^2+d}(b\log(cx^n)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral((sqrt(e*x^2 + d)*b*log(c*x^n) + sqrt(e*x^2 + d)*a)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^2}\,dx=\int \frac{(a+b\log(cx^n))\sqrt{d+ex^2}}{x^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))*(e*x**2+d)**(1/2)/x**2,x)
[Out] Integral((a + b*log(c*x**n))*sqrt(d + e*x**2)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\log(cx^n)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*log(c*x^n) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^2}\,dx = \int \frac{\sqrt{e\,x^2+d}\,(a+b\,\ln(c\,x^n))}{x^2}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*log(c*x^n)))/x^2,x)
[Out] int(((d + e*x^2)^(1/2)*(a + b*log(c*x^n)))/x^2, x)
```

3.260 $\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^4}\,dx = -\frac{ben\sqrt{d+ex^2}}{3dx}-\frac{bn(d+ex^2)^{3/2}}{9dx^3}+\frac{be^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d}-\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3dx^3}$$

```
[Out] -1/9*b*n*(e*x^2+d)^(3/2)/d/x^3+1/3*b*e^(3/2)*n*arctanh(x*e^(1/2)/(e*x^2+d)^
(1/2))/d-1/3*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/d/x^3-1/3*b*e*n*(e*x^2+d)^(1/2
)/d/x
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2373, 283, 223, 212}

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^4}\,dx = -\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3dx^3}+\frac{be^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d}-\frac{ben\sqrt{d+ex^2}}{3dx}-\frac{bn(d+ex^2)^{3/2}}{9dx^3}$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] -1/3*(b*e*n*Sqrt[d + e*x^2])/(d*x) - (b*n*(d + e*x^2)^(3/2))/(9*d*x^3) + (b
*e^(3/2)*n*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(3*d) - ((d + e*x^2)^(3/2)
*(a + b*Log[c*x^n]))/(3*d*x^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3dx^3}+\frac{(bn)\int\frac{(d+ex^2)^{3/2}}{x^4}\,dx}{3d}\\
&= -\frac{bn(d+ex^2)^{3/2}}{9dx^3}-\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3dx^3}+\frac{(ben)\int\frac{\sqrt{d+ex^2}}{x^2}\,dx}{3d}\\
&= -\frac{ben\sqrt{d+ex^2}}{3dx}-\frac{bn(d+ex^2)^{3/2}}{9dx^3}-\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3dx^3}+\frac{(be^2n)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{3d}\\
&= -\frac{ben\sqrt{d+ex^2}}{3dx}-\frac{bn(d+ex^2)^{3/2}}{9dx^3}-\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3dx^3}\\
&\quad+\frac{(be^2n)\,\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{3d}\\
&= -\frac{ben\sqrt{d+ex^2}}{3dx}-\frac{bn(d+ex^2)^{3/2}}{9dx^3}+\frac{be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d}-\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3dx^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^4}\,dx = -\frac{\sqrt{d+ex^2}(3a(d+ex^2)+bn(d+4ex^2))+3b(d+ex^2)^{3/2}\log(cx^n)-3be^{3/2}nx^3\log\left(ex+\sqrt{e}\sqrt{d+ex^2}\right)}{9dx^3}$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] -1/9*(Sqrt[d + e*x^2]*(3*a*(d + e*x^2) + b*n*(d + 4*e*x^2)) + 3*b*(d + e*x^
2)^(3/2)*Log[c*x^n] - 3*b*e^(3/2)*n*x^3*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])
/(d*x^3)
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))\sqrt{e\,x^2+d}}{x^4}dx$$

```
[In] int((a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/x^4,x)
```

```
[Out] int((a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^4}\,dx = \left[\frac{3\,be^{\frac{3}{2}}nx^3\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-2\,(bdn+(4\,ben+3\,ae)x^2+3\,ad+3\,(bex^2+bd)\log(c)+}{18\,dx^3}\right. - \frac{3\,b\sqrt{-e}enx^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+(bdn+(4\,ben+3\,ae)x^2+3\,ad+3\,(bex^2+bd)\log(c)+3\,(benx^2+bdn)}{9\,dx^3}$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/18*(3*b*e^(3/2)*n*x^3*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) -
2*(b*d*n + (4*b*e*n + 3*a*e)*x^2 + 3*a*d + 3*(b*e*x^2 + b*d)*log(c) + 3*(b*
e*n*x^2 + b*d*n)*log(x))*sqrt(e*x^2 + d))/(d*x^3), -1/9*(3*b*sqrt(-e)*e*n*x
^3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (b*d*n + (4*b*e*n + 3*a*e)*x^2 + 3*
a*d + 3*(b*e*x^2 + b*d)*log(c) + 3*(b*e*n*x^2 + b*d*n)*log(x))*sqrt(e*x^2 +
 d))/(d*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^4}\,dx = \int \frac{(a+b\log(cx^n))\sqrt{d+ex^2}}{x^4}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))*(e*x**2+d)**(1/2)/x**4,x)
[Out] Integral((a + b*log(c*x**n))*sqrt(d + e*x**2)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^4}\,dx = \int \frac{\sqrt{ex^2+d}(b\log(cx^n)+a)}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*log(c*x^n) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^4}\,dx = \int \frac{\sqrt{e\,x^2+d}\,(a+b\,\ln(c\,x^n))}{x^4}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*log(c*x^n)))/x^4,x)
[Out] int(((d + e*x^2)^(1/2)*(a + b*log(c*x^n)))/x^4, x)
```

3.261 $\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^6}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 170

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^6}\,dx = \frac{2be^2n\sqrt{d+ex^2}}{15d^2x}+\frac{2ben(d+ex^2)^{3/2}}{45d^2x^3} - \frac{bn(d+ex^2)^{5/2}}{25d^2x^5}-\frac{2be^{5/2}n\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{15d^2} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\log(cx^n))}{15d^2x^3}$$

```
[Out] 2/45*b*e*n*(e*x^2+d)^(3/2)/d^2/x^3-1/25*b*n*(e*x^2+d)^(5/2)/d^2/x^5-2/15*b*
e^(5/2)*n*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/d^2-1/5*(e*x^2+d)^(3/2)*(a+b*l
n(c*x^n))/d/x^5+2/15*e*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/d^2/x^3+2/15*b*e^2*n
*(e*x^2+d)^(1/2)/d^2/x
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used

$= \{277, 270, 2392, 12, 462, 283, 223, 212\}$

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^6}\,dx = \frac{2e(d+ex^2)^{3/2}(a+b\log(cx^n))}{15d^2x^3} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{5dx^5} - \frac{2be^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{15d^2} + \frac{2be^2n\sqrt{d+ex^2}}{15d^2x} - \frac{bn(d+ex^2)^{5/2}}{25d^2x^5} + \frac{2ben(d+ex^2)^{3/2}}{45d^2x^3}$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] (2*b*e^2*n*Sqrt[d + e*x^2])/(15*d^2*x) + (2*b*e*n*(d + e*x^2)^(3/2))/(45*d^
2*x^3) - (b*n*(d + e*x^2)^(5/2))/(25*d^2*x^5) - (2*b*e^(5/2)*n*ArcTanh[(Sqr
t[e]*x)/Sqrt[d + e*x^2]])/(15*d^2) - ((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))
/(5*d*x^5) + (2*e*(d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/(15*d^2*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{5dx^5}+\frac{2e(d+ex^2)^{3/2}(a+b\log(cx^n))}{15d^2x^3} \\
&\quad -(bn)\int\frac{(d+ex^2)^{3/2}(-3d+2ex^2)}{15d^2x^6}\,dx \\
&= -\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{5dx^5}+\frac{2e(d+ex^2)^{3/2}(a+b\log(cx^n))}{15d^2x^3} \\
&\quad -\frac{(bn)\int\frac{(d+ex^2)^{3/2}(-3d+2ex^2)}{x^6}\,dx}{15d^2} \\
&= -\frac{bn(d+ex^2)^{5/2}}{25d^2x^5}-\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{5dx^5} \\
&\quad +\frac{2e(d+ex^2)^{3/2}(a+b\log(cx^n))}{15d^2x^3}-\frac{(2ben)\int\frac{(d+ex^2)^{3/2}}{x^4}\,dx}{15d^2} \\
&= \frac{2ben(d+ex^2)^{3/2}}{45d^2x^3}-\frac{bn(d+ex^2)^{5/2}}{25d^2x^5}-\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{5dx^5} \\
&\quad +\frac{2e(d+ex^2)^{3/2}(a+b\log(cx^n))}{15d^2x^3}-\frac{(2be^2n)\int\frac{\sqrt{d+ex^2}}{x^2}\,dx}{15d^2}
\end{aligned}$$

$$= \frac{2be^2n\sqrt{d+ex^2}}{15d^2x} + \frac{2ben(d+ex^2)^{3/2}}{45d^2x^3} - \frac{bn(d+ex^2)^{5/2}}{25d^2x^5} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\log(cx^n))}{15d^2x^3} - \frac{(2be^3n)\int \frac{1}{\sqrt{d+ex^2}}dx}{15d^2}$$

$$= \frac{2be^2n\sqrt{d+ex^2}}{15d^2x} + \frac{2ben(d+ex^2)^{3/2}}{45d^2x^3} - \frac{bn(d+ex^2)^{5/2}}{25d^2x^5} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\log(cx^n))}{15d^2x^3} - \frac{(2be^3n)\,\text{Subst}\left(\int \frac{1}{1-ex^2}dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{15d^2}$$

$$= \frac{2be^2n\sqrt{d+ex^2}}{15d^2x} + \frac{2ben(d+ex^2)^{3/2}}{45d^2x^3} - \frac{bn(d+ex^2)^{5/2}}{25d^2x^5} - \frac{2be^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{15d^2} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\log(cx^n))}{15d^2x^3}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^6}dx = -\frac{\sqrt{d+ex^2}(bn(9d^2+8dex^2-31e^2x^4)+15a(3d^2+dex^2-2e^2x^4))+15b\sqrt{d+ex^2}(3d^2+dex^2-2e^2x^4)}{225d^2x^5}$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] -1/225*(Sqrt[d + e*x^2]*(b*n*(9*d^2 + 8*d*e*x^2 - 31*e^2*x^4) + 15*a*(3*d^2
 + d*e*x^2 - 2*e^2*x^4)) + 15*b*Sqrt[d + e*x^2]*(3*d^2 + d*e*x^2 - 2*e^2*x^
4)*Log[c*x^n] + 30*b*e^(5/2)*n*x^5*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(d^2
*x^5)
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))\sqrt{e\,x^2+d}}{x^6}dx$$

```
[In] int((a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/x^6,x)
```

```
[Out] int((a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/x^6,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.90

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^6}\,dx$$
$$= \left[\frac{15\,be^{\frac{5}{2}}nx^5\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+((31\,be^2n+30\,ae^2)x^4-9\,bd^2n-45\,ad^2-(8\,bden+15\,a}{225\,d^2x^5}\right.$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/225*(15*b*e^(5/2)*n*x^5*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d)
+ ((31*b*e^2*n + 30*a*e^2)*x^4 - 9*b*d^2*n - 45*a*d^2 - (8*b*d*e*n + 15*a*d
*e)*x^2 + 15*(2*b*e^2*x^4 - b*d*e*x^2 - 3*b*d^2)*log(c) + 15*(2*b*e^2*n*x^4
 - b*d*e*n*x^2 - 3*b*d^2*n)*log(x))*sqrt(e*x^2 + d))/(d^2*x^5), 1/225*(30*b
*sqrt(-e)*e^2*n*x^5*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + ((31*b*e^2*n + 30*
a*e^2)*x^4 - 9*b*d^2*n - 45*a*d^2 - (8*b*d*e*n + 15*a*d*e)*x^2 + 15*(2*b*e^
2*x^4 - b*d*e*x^2 - 3*b*d^2)*log(c) + 15*(2*b*e^2*n*x^4 - b*d*e*n*x^2 - 3*b
*d^2*n)*log(x))*sqrt(e*x^2 + d))/(d^2*x^5)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^6}\,dx = \int \frac{(a+b\log(cx^n))\,\sqrt{d+ex^2}}{x^6}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))*(e*x**2+d)**(1/2)/x**6,x)
```

```
[Out] Integral((a + b*log(c*x**n))*sqrt(d + e*x**2)/x**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^6}\,dx = \int \frac{\sqrt{ex^2+d}(b\log(cx^n)+a)}{x^6}\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x^6,x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*log(c*x^n) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^6}\,dx = \int \frac{\sqrt{e\,x^2+d}\,(a+b\,\ln(c\,x^n))}{x^6}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*log(c*x^n)))/x^6,x)
[Out] int(((d + e*x^2)^(1/2)*(a + b*log(c*x^n)))/x^6, x)
```

3.262 $\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^8}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 230

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^8}\,dx = -\frac{8be^3n\sqrt{d+ex^2}}{105d^3x}-\frac{8be^2n(d+ex^2)^{3/2}}{315d^3x^3}-\frac{bn(d+ex^2)^{5/2}}{49d^2x^7}+\frac{38ben(d+ex^2)^{5/2}}{1225d^3x^5}+\frac{8be^{7/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{105d^3}-\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{7dx^7}+\frac{4e(d+ex^2)^{3/2}(a+b\log(cx^n))}{35d^2x^5}-\frac{8e^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{105d^3x^3}$$

```
[Out] -1/49*b*n*(e*x^2+d)^(3/2)/d/x^7+13/1225*b*e*n*(e*x^2+d)^(3/2)/d^2/x^5+62/11
025*b*e^2*n*(e*x^2+d)^(3/2)/d^3/x^3+8/105*b*e^(7/2)*n*arctanh(x*e^(1/2)/(e*
x^2+d)^(1/2))/d^3-1/7*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/d/x^7+4/35*e*(e*x^2+d
)^(3/2)*(a+b*ln(c*x^n))/d^2/x^5-8/105*e^2*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/d
^3/x^3-8/105*b*e^3*n*(e*x^2+d)^(1/2)/d^3/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {277, 270, 2392, 12, 1279, 462, 283, 223, 212}

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^8}\,dx = -\frac{8e^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{105d^3x^3} + \frac{4e(d+ex^2)^{3/2}(a+b\log(cx^n))}{35d^2x^5} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{7dx^7} + \frac{8be^{7/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{105d^3} - \frac{8be^3n\sqrt{d+ex^2}}{105d^3x} - \frac{8be^2n(d+ex^2)^{3/2}}{315d^3x^3} + \frac{38ben(d+ex^2)^{5/2}}{1225d^3x^5} - \frac{bn(d+ex^2)^{5/2}}{49d^2x^7}$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/x^8,x]
```

```
[Out] (-8*b*e^3*n*Sqrt[d + e*x^2])/(105*d^3*x) - (8*b*e^2*n*(d + e*x^2)^(3/2))/(3
15*d^3*x^3) - (b*n*(d + e*x^2)^(5/2))/(49*d^2*x^7) + (38*b*e*n*(d + e*x^2)^
(5/2))/(1225*d^3*x^5) + (8*b*e^(7/2)*n*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
)/(105*d^3) - ((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/(7*d*x^7) + (4*e*(d +
e*x^2)^(3/2)*(a + b*Log[c*x^n]))/(35*d^2*x^5) - (8*e^2*(d + e*x^2)^(3/2)*(a
 + b*Log[c*x^n]))/(105*d^3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = -\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{7dx^7} + \frac{4e(d+ex^2)^{3/2}(a+b\log(cx^n))}{35d^2x^5} - \frac{8e^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{105d^3x^3} - (bn)\int \frac{(d+ex^2)^{3/2}(-15d^2+12dex^2-8e^2x^4)}{105d^3x^8}\,dx$$

$$= -\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{7dx^7} + \frac{4e(d+ex^2)^{3/2}(a+b\log(cx^n))}{35d^2x^5} - \frac{8e^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{105d^3x^3} - \frac{(bn)\int \frac{(d+ex^2)^{3/2}(-15d^2+12dex^2-8e^2x^4)}{x^8}\,dx}{105d^3}$$

$$= -\frac{bn(d+ex^2)^{5/2}}{49d^2x^7} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{7dx^7} + \frac{4e(d+ex^2)^{3/2}(a+b\log(cx^n))}{35d^2x^5} - \frac{8e^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{105d^3x^3} + \frac{(bn)\int \frac{(d+ex^2)^{3/2}(-114d^2e+56de^2x^2)}{x^6}\,dx}{735d^4}$$

$$= -\frac{bn(d+ex^2)^{5/2}}{49d^2x^7} + \frac{38ben(d+ex^2)^{5/2}}{1225d^3x^5} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{7dx^7} + \frac{4e(d+ex^2)^{3/2}(a+b\log(cx^n))}{35d^2x^5} - \frac{8e^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{105d^3x^3} + \frac{(8be^2n)\int \frac{(d+ex^2)^{3/2}}{x^4}\,dx}{105d^3}$$

$$= -\frac{8be^2n(d+ex^2)^{3/2}}{315d^3x^3} - \frac{bn(d+ex^2)^{5/2}}{49d^2x^7} + \frac{38ben(d+ex^2)^{5/2}}{1225d^3x^5} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{7dx^7} + \frac{4e(d+ex^2)^{3/2}(a+b\log(cx^n))}{35d^2x^5} - \frac{8e^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{105d^3x^3} + \frac{(8be^3n)\int \frac{\sqrt{d+ex^2}}{x^2}\,dx}{105d^3}$$

$$= -\frac{8be^3n\sqrt{d+ex^2}}{105d^3x} - \frac{8be^2n(d+ex^2)^{3/2}}{315d^3x^3} - \frac{bn(d+ex^2)^{5/2}}{49d^2x^7} + \frac{38ben(d+ex^2)^{5/2}}{1225d^3x^5} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{7dx^7} + \frac{4e(d+ex^2)^{3/2}(a+b\log(cx^n))}{35d^2x^5} - \frac{8e^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{105d^3x^3} + \frac{(8be^4n)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{105d^3}$$

$$= -\frac{8be^3n\sqrt{d+ex^2}}{105d^3x} - \frac{8be^2n(d+ex^2)^{3/2}}{315d^3x^3} - \frac{bn(d+ex^2)^{5/2}}{49d^2x^7} + \frac{38ben(d+ex^2)^{5/2}}{1225d^3x^5} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{7dx^7} + \frac{4e(d+ex^2)^{3/2}(a+b\log(cx^n))}{35d^2x^5} - \frac{8e^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{105d^3x^3} + \frac{(8be^4n)\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{105d^3}$$

$$= -\frac{8be^3n\sqrt{d+ex^2}}{105d^3x} - \frac{8be^2n(d+ex^2)^{3/2}}{315d^3x^3} - \frac{bn(d+ex^2)^{5/2}}{49d^2x^7} + \frac{38ben(d+ex^2)^{5/2}}{1225d^3x^5} + \frac{8be^{7/2}n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{105d^3} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{7dx^7} + \frac{4e(d+ex^2)^{3/2}(a+b\log(cx^n))}{35d^2x^5} - \frac{8e^2(d+ex^2)^{3/2}(a+b\log(cx^n))}{105d^3x^3}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^8}\,dx = -\frac{\sqrt{d+ex^2}(105a(15d^3+3d^2ex^2-4de^2x^4+8e^3x^6)+bn(225d^3+108d^2ex^2-179de^2x^4+778e^3x^6))+105}{11025d^3x^7}$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/x^8,x]
```

```
[Out] -1/11025*(Sqrt[d + e*x^2]*(105*a*(15*d^3 + 3*d^2*e*x^2 - 4*d*e^2*x^4 + 8*e^
3*x^6) + b*n*(225*d^3 + 108*d^2*e*x^2 - 179*d*e^2*x^4 + 778*e^3*x^6)) + 105
*b*Sqrt[d + e*x^2]*(15*d^3 + 3*d^2*e*x^2 - 4*d*e^2*x^4 + 8*e^3*x^6)*Log[c*x
^n] - 840*b*e^(7/2)*n*x^7*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(d^3*x^7)
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))\sqrt{e\,x^2+d}}{x^8}dx$$

```
[In] int((a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/x^8,x)
```

```
[Out] int((a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/x^8,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^8}\,dx = \left[\frac{420\,be^{\frac{7}{2}}nx^7\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-(2\,(389\,be^3n+420\,ae^3)x^6+225\,bd^3n-(179\,bde^2n+42}{}\right.$$

$$-\frac{840\,b\sqrt{-e}e^3nx^7\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+(2\,(389\,be^3n+420\,ae^3)x^6+225\,bd^3n-(179\,bde^2n+420\,ade^2)x^4+1}{}$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/11025*(420*b*e^(7/2)*n*x^7*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x -
d) - (2*(389*b*e^3*n + 420*a*e^3)*x^6 + 225*b*d^3*n - (179*b*d*e^2*n + 420*
a*d*e^2)*x^4 + 1575*a*d^3 + 9*(12*b*d^2*e*n + 35*a*d^2*e)*x^2 + 105*(8*b*e^
3*x^6 - 4*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + 15*b*d^3)*log(c) + 105*(8*b*e^3*n*x
^6 - 4*b*d*e^2*n*x^4 + 3*b*d^2*e*n*x^2 + 15*b*d^3*n)*log(x))*sqrt(e*x^2 + d
))/(d^3*x^7), -1/11025*(840*b*sqrt(-e)*e^3*n*x^7*arctan(sqrt(-e)*x/sqrt(e*x
^2 + d)) + (2*(389*b*e^3*n + 420*a*e^3)*x^6 + 225*b*d^3*n - (179*b*d*e^2*n
+ 420*a*d*e^2)*x^4 + 1575*a*d^3 + 9*(12*b*d^2*e*n + 35*a*d^2*e)*x^2 + 105*(
8*b*e^3*x^6 - 4*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + 15*b*d^3)*log(c) + 105*(8*b*e
^3*n*x^6 - 4*b*d*e^2*n*x^4 + 3*b*d^2*e*n*x^2 + 15*b*d^3*n)*log(x))*sqrt(e*x
^2 + d))/(d^3*x^7)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^8}\,dx = \int \frac{(a+b\log(cx^n))\sqrt{d+ex^2}}{x^8}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))*(e*x**2+d)**(1/2)/x**8,x)
```

```
[Out] Integral((a + b*log(c*x**n))*sqrt(d + e*x**2)/x**8, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^8}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^8}\,dx = \int \frac{\sqrt{ex^2+d}(b\log(cx^n)+a)}{x^8}\,dx$$

```
[In] integrate((a+b*log(c*x^n))*(e*x^2+d)^(1/2)/x^8,x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*log(c*x^n) + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{x^8}\,dx = \int \frac{\sqrt{e\,x^2+d}\,(a+b\,\ln(c\,x^n))}{x^8}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*log(c*x^n)))/x^8,x)
[Out] int(((d + e*x^2)^(1/2)*(a + b*log(c*x^n)))/x^8, x)
```

3.263 $\int x^5(d+ex^2)^{3/2}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 231

$$\int x^5\left(d+ex^2\right)^{3/2}(a+b\log(cx^n))\,dx=-\frac{8bd^4n\sqrt{d+ex^2}}{315e^3}-\frac{8bd^3n(d+ex^2)^{3/2}}{945e^3}-\frac{8bd^2n(d+ex^2)^{5/2}}{1575e^3}+\frac{11bdn(d+ex^2)^{7/2}}{441e^3}-\frac{bn(d+ex^2)^{9/2}}{81e^3}+\frac{8bd^{9/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{315e^3}+\frac{d^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3}-\frac{2d(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^3}+\frac{(d+ex^2)^{9/2}(a+b\log(cx^n))}{9e^3}$$

```
[Out] -8/945*b*d^3*n*(e*x^2+d)^(3/2)/e^3-8/1575*b*d^2*n*(e*x^2+d)^(5/2)/e^3+11/44
1*b*d*n*(e*x^2+d)^(7/2)/e^3-1/81*b*n*(e*x^2+d)^(9/2)/e^3+8/315*b*d^(9/2)*n*
arctanh((e*x^2+d)^(1/2)/d^(1/2))/e^3+1/5*d^2*(e*x^2+d)^(5/2)*(a+b*ln(c*x^n)
)/e^3-2/7*d*(e*x^2+d)^(7/2)*(a+b*ln(c*x^n))/e^3+1/9*(e*x^2+d)^(9/2)*(a+b*ln
(c*x^n))/e^3-8/315*b*d^4*n*(e*x^2+d)^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used

$= \{272, 45, 2392, 12, 1265, 911, 1275, 214\}$

$$\int x^5 (d+ex^2)^{3/2} (a+b\log(cx^n))\,dx = \frac{d^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3} - \frac{2d(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^3} + \frac{(d+ex^2)^{9/2}(a+b\log(cx^n))}{9e^3} + \frac{8bd^{9/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{315e^3} - \frac{8bd^4n\sqrt{d+ex^2}}{315e^3} - \frac{8bd^3n(d+ex^2)^{3/2}}{945e^3} - \frac{8bd^2n(d+ex^2)^{5/2}}{1575e^3} + \frac{11bdn(d+ex^2)^{7/2}}{441e^3} - \frac{bn(d+ex^2)^{9/2}}{81e^3}$$

```
[In] Int[x^5*(d + e*x^2)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (-8*b*d^4*n*Sqrt[d + e*x^2])/(315*e^3) - (8*b*d^3*n*(d + e*x^2)^(3/2))/(945
*e^3) - (8*b*d^2*n*(d + e*x^2)^(5/2))/(1575*e^3) + (11*b*d*n*(d + e*x^2)^(7
/2))/(441*e^3) - (b*n*(d + e*x^2)^(9/2))/(81*e^3) + (8*b*d^(9/2)*n*ArcTanh[
Sqrt[d + e*x^2]/Sqrt[d]])/(315*e^3) + (d^2*(d + e*x^2)^(5/2)*(a + b*Log[c*x
^n]))/(5*e^3) - (2*d*(d + e*x^2)^(7/2)*(a + b*Log[c*x^n]))/(7*e^3) + ((d +
e*x^2)^(9/2)*(a + b*Log[c*x^n]))/(9*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3} - \frac{2d(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^3} \\
&\quad + \frac{(d+ex^2)^{9/2}(a+b\log(cx^n))}{9e^3} - (bn)\int \frac{(d+ex^2)^{5/2}(8d^2-20dex^2+35e^2x^4)}{315e^3x}\,dx \\
&= \frac{d^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3} - \frac{2d(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^3} \\
&\quad + \frac{(d+ex^2)^{9/2}(a+b\log(cx^n))}{9e^3} - \frac{(bn)\int \frac{(d+ex^2)^{5/2}(8d^2-20dex^2+35e^2x^4)}{x}\,dx}{315e^3} \\
&= \frac{d^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3} - \frac{2d(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^3} \\
&\quad + \frac{(d+ex^2)^{9/2}(a+b\log(cx^n))}{9e^3} - \frac{(bn)\text{Subst}\left(\int \frac{(d+ex)^{5/2}(8d^2-20dex+35e^2x^2)}{x}\,dx, x, x^2\right)}{630e^3}
\end{aligned}$$

$$= \frac{d^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3} - \frac{2d(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^3} + \frac{(d+ex^2)^{9/2}(a+b\log(cx^n))}{9e^3} - \frac{(bn)\text{Subst}\left(\int \frac{x^6(63d^2-90dx^2+35x^4)}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{315e^4}$$

$$= \frac{d^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3} - \frac{2d(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^3} + \frac{(d+ex^2)^{9/2}(a+b\log(cx^n))}{9e^3} - \frac{(bn)\text{Subst}\left(\int \left(8d^4e+8d^3ex^2+8d^2ex^4-55dex^6+35ex^8+\frac{8d^5}{-\frac{d}{e}+\frac{x^2}{e}}\right)dx, x, \sqrt{d+ex^2}\right)}{315e^4}$$

$$= -\frac{8bd^4n\sqrt{d+ex^2}}{315e^3} - \frac{8bd^3n(d+ex^2)^{3/2}}{945e^3} - \frac{8bd^2n(d+ex^2)^{5/2}}{1575e^3} + \frac{11bdn(d+ex^2)^{7/2}}{441e^3} - \frac{bn(d+ex^2)^{9/2}}{81e^3} + \frac{d^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3} - \frac{2d(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^3} + \frac{(d+ex^2)^{9/2}(a+b\log(cx^n))}{9e^3} - \frac{(8bd^5n)\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{315e^4}$$

$$= -\frac{8bd^4n\sqrt{d+ex^2}}{315e^3} - \frac{8bd^3n(d+ex^2)^{3/2}}{945e^3} - \frac{8bd^2n(d+ex^2)^{5/2}}{1575e^3} + \frac{11bdn(d+ex^2)^{7/2}}{441e^3} - \frac{bn(d+ex^2)^{9/2}}{81e^3} + \frac{8bd^{9/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{315e^3} + \frac{d^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3} - \frac{2d(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^3} + \frac{(d+ex^2)^{9/2}(a+b\log(cx^n))}{9e^3}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.11

$$\int x^5(d+ex^2)^{3/2}(a+b\log(cx^n))\,dx = \frac{-2520bd^{9/2}n\log(x)+315bn(d+ex^2)^{5/2}(8d^2-20dex^2+35e^2x^4)\log(x)+\sqrt{d+ex^2}(1225}{}$$

```
[In] Integrate[x^5*(d + e*x^2)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (-2520*b*d^(9/2)*n*Log[x] + 315*b*n*(d + e*x^2)^(5/2)*(8*d^2 - 20*d*e*x^2 +
 35*e^2*x^4)*Log[x] + Sqrt[d + e*x^2]*(1225*e^4*x^8*(9*a - b*n - 9*b*n*Log[
```

```
x] + 9*b*Log[c*x^n]) + 3*d^2*e^2*x^4*(315*a - 143*b*n + 315*b*(-(n*Log[x])
+ Log[c*x^n])) + 25*d*e^3*x^6*(630*a - 97*b*n + 630*b*(-(n*Log[x]) + Log[c*
x^n])) + 2*d^4*(1260*a - 1307*b*n + 1260*b*(-(n*Log[x]) + Log[c*x^n])) - d^
3*e*x^2*(1260*a - 677*b*n + 1260*b*(-(n*Log[x]) + Log[c*x^n]))) + 2520*b*d^
(9/2)*n*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/(99225*e^3)
```

Maple [F]

$$\int x^{5}\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\ln\left(c\,x^{n}\right)\right)dx$$

```
[In] int(x^5*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n)),x)
```

```
[Out] int(x^5*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 514, normalized size of antiderivative = 2.23

$$\int x^5(d+ex^2)^{3/2}(a+b\log(cx^n))\,dx = \left[\frac{1260\,bd^{\frac{9}{2}}n\log\left(-\frac{ex^2+2\sqrt{ex^2+d}\sqrt{d}+2\,d}{x^2}\right)-(1225\,(be^4n-9\,ae^4)x^8+25\,(97\,bde^3n-630\,ade\ldots}{\ldots}\right.$$

$$-\frac{2520\,b\sqrt{-d}d^4n\arctan\left(\frac{\sqrt{-d}}{\sqrt{ex^2+d}}\right)+(1225\,(be^4n-9\,ae^4)x^8+25\,(97\,bde^3n-630\,ade^3)x^6+2614\,bd^4n-252\ldots}{\ldots}$$

```
[In] integrate(x^5*(e*x^2+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] [1/99225*(1260*b*d^(9/2)*n*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x
^2) - (1225*(b*e^4*n - 9*a*e^4)*x^8 + 25*(97*b*d*e^3*n - 630*a*d*e^3)*x^6 +
 2614*b*d^4*n - 2520*a*d^4 + 3*(143*b*d^2*e^2*n - 315*a*d^2*e^2)*x^4 - (677
*b*d^3*e*n - 1260*a*d^3*e)*x^2 - 315*(35*b*e^4*x^8 + 50*b*d*e^3*x^6 + 3*b*d
^2*e^2*x^4 - 4*b*d^3*e*x^2 + 8*b*d^4)*log(c) - 315*(35*b*e^4*n*x^8 + 50*b*d
*e^3*n*x^6 + 3*b*d^2*e^2*n*x^4 - 4*b*d^3*e*n*x^2 + 8*b*d^4*n)*log(x))*sqrt(
e*x^2 + d))/e^3, -1/99225*(2520*b*sqrt(-d)*d^4*n*arctan(sqrt(-d)/sqrt(e*x^2
 + d)) + (1225*(b*e^4*n - 9*a*e^4)*x^8 + 25*(97*b*d*e^3*n - 630*a*d*e^3)*x^
6 + 2614*b*d^4*n - 2520*a*d^4 + 3*(143*b*d^2*e^2*n - 315*a*d^2*e^2)*x^4 - (
677*b*d^3*e*n - 1260*a*d^3*e)*x^2 - 315*(35*b*e^4*x^8 + 50*b*d*e^3*x^6 + 3*
b*d^2*e^2*x^4 - 4*b*d^3*e*x^2 + 8*b*d^4)*log(c) - 315*(35*b*e^4*n*x^8 + 50*
b*d*e^3*n*x^6 + 3*b*d^2*e^2*n*x^4 - 4*b*d^3*e*n*x^2 + 8*b*d^4*n)*log(x))*sq
rt(e*x^2 + d))/e^3]
```

Sympy [A] (verification not implemented)

Time = 95.55 (sec) , antiderivative size = 1161, normalized size of antiderivative = 5.03

$$\int x^5 (d + ex^2)^{3/2} (a + b\log (cx^n)) \, dx = \text{Too large to display}$$

```
[In] integrate(x**5*(e*x**2+d)**(3/2)*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d*Piecewise((8*d**3*sqrt(d + e*x**2)/(105*e**3) - 4*d**2*x**2*sqrt(d + e*
x**2)/(105*e**2) + d*x**4*sqrt(d + e*x**2)/(35*e) + x**6*sqrt(d + e*x**2)/7
, Ne(e, 0)), (sqrt(d)*x**6/6, True)) + a*e*Piecewise((-16*d**4*sqrt(d + e*x
**2)/(315*e**4) + 8*d**3*x**2*sqrt(d + e*x**2)/(315*e**3) - 2*d**2*x**4*sqr
t(d + e*x**2)/(105*e**2) + d*x**6*sqrt(d + e*x**2)/(63*e) + x**8*sqrt(d + e
*x**2)/9, Ne(e, 0)), (sqrt(d)*x**8/8, True)) - b*d*n*Piecewise((-8*d**(7/2)
*asinh(sqrt(d)/(sqrt(e)*x))/(105*e**3) + 8*d**4/(105*e**(7/2)*x*sqrt(d/(e*x
**2) + 1)) + 8*d**3*x/(105*e**(5/2)*sqrt(d/(e*x**2) + 1)) - 4*d**2*Piecewis
e((d*sqrt(d + e*x**2)/(3*e) + x**2*sqrt(d + e*x**2)/3, Ne(e, 0)), (sqrt(d)*
x**2/2, True))/(105*e**2) + d*Piecewise((-2*d**2*sqrt(d + e*x**2)/(15*e**2)
 + d*x**2*sqrt(d + e*x**2)/(15*e) + x**4*sqrt(d + e*x**2)/5, Ne(e, 0)), (sq
rt(d)*x**4/4, True))/(35*e) + Piecewise((8*d**3*sqrt(d + e*x**2)/(105*e**3)
 - 4*d**2*x**2*sqrt(d + e*x**2)/(105*e**2) + d*x**4*sqrt(d + e*x**2)/(35*e)
 + x**6*sqrt(d + e*x**2)/7, Ne(e, 0)), (sqrt(d)*x**6/6, True))/7, (e > -oo)
 & (e < oo) & Ne(e, 0)), (sqrt(d)*x**6/36, True)) + b*d*Piecewise((8*d**3*s
qrt(d + e*x**2)/(105*e**3) - 4*d**2*x**2*sqrt(d + e*x**2)/(105*e**2) + d*x*
*4*sqrt(d + e*x**2)/(35*e) + x**6*sqrt(d + e*x**2)/7, Ne(e, 0)), (sqrt(d)*x
**6/6, True))*log(c*x**n) - b*e*n*Piecewise((16*d**(9/2)*asinh(sqrt(d)/(sqr
t(e)*x))/(315*e**4) - 16*d**5/(315*e**(9/2)*x*sqrt(d/(e*x**2) + 1)) - 16*d*
*4*x/(315*e**(7/2)*sqrt(d/(e*x**2) + 1)) + 8*d**3*Piecewise((d*sqrt(d + e*x
**2)/(3*e) + x**2*sqrt(d + e*x**2)/3, Ne(e, 0)), (sqrt(d)*x**2/2, True))/(3
15*e**3) - 2*d**2*Piecewise((-2*d**2*sqrt(d + e*x**2)/(15*e**2) + d*x**2*sq
rt(d + e*x**2)/(15*e) + x**4*sqrt(d + e*x**2)/5, Ne(e, 0)), (sqrt(d)*x**4/4
, True))/(105*e**2) + d*Piecewise((8*d**3*sqrt(d + e*x**2)/(105*e**3) - 4*d
**2*x**2*sqrt(d + e*x**2)/(105*e**2) + d*x**4*sqrt(d + e*x**2)/(35*e) + x**
6*sqrt(d + e*x**2)/7, Ne(e, 0)), (sqrt(d)*x**6/6, True))/(63*e) + Piecewise
((-16*d**4*sqrt(d + e*x**2)/(315*e**4) + 8*d**3*x**2*sqrt(d + e*x**2)/(315*
e**3) - 2*d**2*x**4*sqrt(d + e*x**2)/(105*e**2) + d*x**6*sqrt(d + e*x**2)/(
63*e) + x**8*sqrt(d + e*x**2)/9, Ne(e, 0)), (sqrt(d)*x**8/8, True))/9, (e >
 -oo) & (e < oo) & Ne(e, 0)), (sqrt(d)*x**8/64, True)) + b*e*Piecewise((-16
*d**4*sqrt(d + e*x**2)/(315*e**4) + 8*d**3*x**2*sqrt(d + e*x**2)/(315*e**3)
 - 2*d**2*x**4*sqrt(d + e*x**2)/(105*e**2) + d*x**6*sqrt(d + e*x**2)/(63*e)
 + x**8*sqrt(d + e*x**2)/9, Ne(e, 0)), (sqrt(d)*x**8/8, True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int x^5 (d + ex^2)^{3/2} (a + b \log (cx^n)) \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(e*x^2+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^5 (d + ex^2)^{3/2} (a + b \log (cx^n)) \, dx = \int (ex^2 + d)^{\frac{3}{2}} (b \log (cx^n) + a) x^5 \, dx$$

```
[In] integrate(x^5*(e*x^2+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*log(c*x^n) + a)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 (d + ex^2)^{3/2} (a + b \log (cx^n)) \, dx = \int x^5 \left(e\,x^2 + d\right)^{3/2} \left(a + b\,\ln\left(c\,x^n\right)\right) dx$$

```
[In] int(x^5*(d + e*x^2)^(3/2)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^5*(d + e*x^2)^(3/2)*(a + b*log(c*x^n)), x)
```

3.264 $\int x^3(d+ex^2)^{3/2} (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 177

$$\int x^3\left(d+ex^2\right)^{3/2}(a+b\log(cx^n))\,dx = \frac{2bd^3n\sqrt{d+ex^2}}{35e^2}+\frac{2bd^2n(d+ex^2)^{3/2}}{105e^2} + \frac{2bdn(d+ex^2)^{5/2}}{175e^2}-\frac{bn(d+ex^2)^{7/2}}{49e^2}-\frac{2bd^{7/2}n\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{35e^2} - \frac{d(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^2}+\frac{(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^2}$$

```
[Out] 2/105*b*d^2*n*(e*x^2+d)^(3/2)/e^2+2/175*b*d*n*(e*x^2+d)^(5/2)/e^2-1/49*b*n*
(e*x^2+d)^(7/2)/e^2-2/35*b*d^(7/2)*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))/e^2-1
/5*d*(e*x^2+d)^(5/2)*(a+b*ln(c*x^n))/e^2+1/7*(e*x^2+d)^(7/2)*(a+b*ln(c*x^n)
)/e^2+2/35*b*d^3*n*(e*x^2+d)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {272, 45, 2392, 12, 457, 81, 52, 65, 214}

$$\int x^3\left(d+ex^2\right)^{3/2}(a+b\log(cx^n))\,dx = -\frac{d(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^2}-\frac{2bd^{7/2}n\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{35e^2} + \frac{2bd^3n\sqrt{d+ex^2}}{35e^2}+\frac{2bd^2n(d+ex^2)^{3/2}}{105e^2}+\frac{2bdn(d+ex^2)^{5/2}}{175e^2}-\frac{bn(d+ex^2)^{7/2}}{49e^2}$$

```
[In] Int[x^3*(d + e*x^2)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (2*b*d^3*n*Sqrt[d + e*x^2])/(35*e^2) + (2*b*d^2*n*(d + e*x^2)^(3/2))/(105*e
^2) + (2*b*d*n*(d + e*x^2)^(5/2))/(175*e^2) - (b*n*(d + e*x^2)^(7/2))/(49*e
^2) - (2*b*d^(7/2)*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(35*e^2) - (d*(d + e
*x^2)^(5/2)*(a + b*Log[c*x^n]))/(5*e^2) + ((d + e*x^2)^(7/2)*(a + b*Log[c*x
^n]))/(7*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(d+ex^2)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^2}+\frac{(d+ex^2)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^2}\\
&\quad-(bn)\int\frac{(d+ex^2)^{5/2}\left(-2d+5ex^2\right)}{35e^2x}\,dx\\
&= -\frac{d(d+ex^2)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^2}\\
&\quad+\frac{(d+ex^2)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^2}-\frac{(bn)\int\frac{(d+ex^2)^{5/2}\left(-2d+5ex^2\right)}{x}\,dx}{35e^2}\\
&= -\frac{d(d+ex^2)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^2}+\frac{(d+ex^2)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^2}\\
&\quad-\frac{(bn)\text{Subst}\left(\int\frac{(d+ex)^{5/2}(-2d+5ex)}{x}\,dx,x,x^2\right)}{70e^2}\\
&= -\frac{bn(d+ex^2)^{7/2}}{49e^2}-\frac{d(d+ex^2)^{5/2}\left(a+b\log\left(cx^n\right)\right)}{5e^2}\\
&\quad+\frac{(d+ex^2)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{7e^2}+\frac{(bdn)\text{Subst}\left(\int\frac{(d+ex)^{5/2}}{x}\,dx,x,x^2\right)}{35e^2}
\end{aligned}$$

$$= \frac{2bdn(d+ex^2)^{5/2}}{175e^2} - \frac{bn(d+ex^2)^{7/2}}{49e^2} - \frac{d(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^2} + \frac{(bd^2n)\operatorname{Subst}\left(\int \frac{(d+ex)^{3/2}}{x}\,dx, x, x^2\right)}{35e^2}$$

$$= \frac{2bd^2n(d+ex^2)^{3/2}}{105e^2} + \frac{2bdn(d+ex^2)^{5/2}}{175e^2} - \frac{bn(d+ex^2)^{7/2}}{49e^2} - \frac{d(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^2} + \frac{(bd^3n)\operatorname{Subst}\left(\int \frac{\sqrt{d+ex}}{x}\,dx, x, x^2\right)}{35e^2}$$

$$= \frac{2bd^3n\sqrt{d+ex^2}}{35e^2} + \frac{2bd^2n(d+ex^2)^{3/2}}{105e^2} + \frac{2bdn(d+ex^2)^{5/2}}{175e^2} - \frac{bn(d+ex^2)^{7/2}}{49e^2} - \frac{d(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^2} + \frac{(bd^4n)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)}{35e^2}$$

$$= \frac{2bd^3n\sqrt{d+ex^2}}{35e^2} + \frac{2bd^2n(d+ex^2)^{3/2}}{105e^2} + \frac{2bdn(d+ex^2)^{5/2}}{175e^2} - \frac{bn(d+ex^2)^{7/2}}{49e^2} - \frac{d(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^2} + \frac{(2bd^4n)\operatorname{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{35e^3}$$

$$= \frac{2bd^3n\sqrt{d+ex^2}}{35e^2} + \frac{2bd^2n(d+ex^2)^{3/2}}{105e^2} + \frac{2bdn(d+ex^2)^{5/2}}{175e^2} - \frac{bn(d+ex^2)^{7/2}}{49e^2} - \frac{2bd^{7/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{35e^2} - \frac{d(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\log(cx^n))}{7e^2}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.28

$$\int x^3 \left(d+ex^2\right)^{3/2} (a+b\log(cx^n))\ dx = \frac{2bd^{7/2}n\log(x)}{35e^2} - \frac{bn(2d-5ex^2)\left(d+ex^2\right)^{5/2}\log(x)}{35e^2}$$
$$+\sqrt{d+ex^2}\left(\frac{1}{49}ex^6(7a-bn+7b(-n\log(x)+\log(cx^n)))+\frac{d^2x^2(105a-71bn+105b(-n\log(x)+\log(cx^n)))}{3675e}-\right.$$

```
[In] Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (2*b*d^(7/2)*n*Log[x])/(35*e^2) - (b*n*(2*d - 5*e*x^2)*(d + e*x^2)^(5/2)*Lo
g[x])/(35*e^2) + Sqrt[d + e*x^2]*((e*x^6*(7*a - b*n + 7*b*(-(n*Log[x]) + Lo
g[c*x^n])))/49 + (d^2*x^2*(105*a - 71*b*n + 105*b*(-(n*Log[x]) + Log[c*x^n]
)))/(3675*e) - (d^3*(210*a - 247*b*n + 210*b*(-(n*Log[x]) + Log[c*x^n])))/(
3675*e^2) + (d*x^4*(280*a - 61*b*n + 280*b*(-(n*Log[x]) + Log[c*x^n])))/122
5) - (2*b*d^(7/2)*n*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/(35*e^2)
```

Maple [F]

$$\int x^3 \left(e\,x^2+d\right)^{\frac{3}{2}} (a+b\ln(c\,x^n))\,dx$$

```
[In] int(x^3*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n)),x)
```

```
[Out] int(x^3*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.31

$$\int x^3 \left(d+ex^2\right)^{3/2} (a+b\log(cx^n))\ dx = \left[\frac{105\,bd^{\frac{7}{2}}n\log\left(-\frac{ex^2-2\,\sqrt{ex^2+d}\sqrt{d}+2\,d}{x^2}\right) - (75\,(be^3n-7\,ae^3)x^6 - 247\,bd^3n + 3\,(61\,bde^2n - 28}{}\right.$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] [1/3675*(105*b*d^(7/2)*n*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2
) - (75*(b*e^3*n - 7*a*e^3)*x^6 - 247*b*d^3*n + 3*(61*b*d*e^2*n - 280*a*d*e
^2)*x^4 + 210*a*d^3 + (71*b*d^2*e*n - 105*a*d^2*e)*x^2 - 105*(5*b*e^3*x^6 +
 8*b*d*e^2*x^4 + b*d^2*e*x^2 - 2*b*d^3)*log(c) - 105*(5*b*e^3*n*x^6 + 8*b*d
```

```
*e^2*n*x^4 + b*d^2*e*n*x^2 - 2*b*d^3*n)*log(x))*sqrt(e*x^2 + d))/e^2, 1/367
5*(210*b*sqrt(-d)*d^3*n*arctan(sqrt(-d)/sqrt(e*x^2 + d)) - (75*(b*e^3*n - 7
*a*e^3)*x^6 - 247*b*d^3*n + 3*(61*b*d*e^2*n - 280*a*d*e^2)*x^4 + 210*a*d^3
+ (71*b*d^2*e*n - 105*a*d^2*e)*x^2 - 105*(5*b*e^3*x^6 + 8*b*d*e^2*x^4 + b*d
^2*e*x^2 - 2*b*d^3)*log(c) - 105*(5*b*e^3*n*x^6 + 8*b*d*e^2*n*x^4 + b*d^2*e
*n*x^2 - 2*b*d^3*n)*log(x))*sqrt(e*x^2 + d))/e^2]
```

Sympy [A] (verification not implemented)

Time = 48.94 (sec) , antiderivative size = 845, normalized size of antiderivative = 4.77

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\log\left(cx^n\right)\right) \, dx = \text{Too large to display}$$

```
[In] integrate(x**3*(e*x**2+d)**(3/2)*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d*Piecewise((-2*d**2*sqrt(d + e*x**2)/(15*e**2) + d*x**2*sqrt(d + e*x**2)
/(15*e) + x**4*sqrt(d + e*x**2)/5, Ne(e, 0)), (sqrt(d)*x**4/4, True)) + a*e
*Piecewise((8*d**3*sqrt(d + e*x**2)/(105*e**3) - 4*d**2*x**2*sqrt(d + e*x**
2)/(105*e**2) + d*x**4*sqrt(d + e*x**2)/(35*e) + x**6*sqrt(d + e*x**2)/7, N
e(e, 0)), (sqrt(d)*x**6/6, True)) - b*d*n*Piecewise((2*d**(5/2)*asinh(sqrt(
d)/(sqrt(e)*x))/(15*e**2) - 2*d**3/(15*e**(5/2)*x*sqrt(d/(e*x**2) + 1)) - 2
*d**2*x/(15*e**(3/2)*sqrt(d/(e*x**2) + 1)) + d*Piecewise((d*sqrt(d + e*x**2
)/(3*e) + x**2*sqrt(d + e*x**2)/3, Ne(e, 0)), (sqrt(d)*x**2/2, True))/(15*e
) + Piecewise((-2*d**2*sqrt(d + e*x**2)/(15*e**2) + d*x**2*sqrt(d + e*x**2)
/(15*e) + x**4*sqrt(d + e*x**2)/5, Ne(e, 0)), (sqrt(d)*x**4/4, True))/5, (e
 > -oo) & (e < oo) & Ne(e, 0)), (sqrt(d)*x**4/16, True)) + b*d*Piecewise((-
2*d**2*sqrt(d + e*x**2)/(15*e**2) + d*x**2*sqrt(d + e*x**2)/(15*e) + x**4*s
qrt(d + e*x**2)/5, Ne(e, 0)), (sqrt(d)*x**4/4, True))*log(c*x**n) - b*e*n*P
iecewise((-8*d**(7/2)*asinh(sqrt(d)/(sqrt(e)*x))/(105*e**3) + 8*d**4/(105*e
**(7/2)*x*sqrt(d/(e*x**2) + 1)) + 8*d**3*x/(105*e**(5/2)*sqrt(d/(e*x**2) +
1)) - 4*d**2*Piecewise((d*sqrt(d + e*x**2)/(3*e) + x**2*sqrt(d + e*x**2)/3,
 Ne(e, 0)), (sqrt(d)*x**2/2, True))/(105*e**2) + d*Piecewise((-2*d**2*sqrt(
d + e*x**2)/(15*e**2) + d*x**2*sqrt(d + e*x**2)/(15*e) + x**4*sqrt(d + e*x*
*2)/5, Ne(e, 0)), (sqrt(d)*x**4/4, True))/(35*e) + Piecewise((8*d**3*sqrt(d
 + e*x**2)/(105*e**3) - 4*d**2*x**2*sqrt(d + e*x**2)/(105*e**2) + d*x**4*sq
rt(d + e*x**2)/(35*e) + x**6*sqrt(d + e*x**2)/7, Ne(e, 0)), (sqrt(d)*x**6/6
, True))/7, (e > -oo) & (e < oo) & Ne(e, 0)), (sqrt(d)*x**6/36, True)) + b*
e*Piecewise((8*d**3*sqrt(d + e*x**2)/(105*e**3) - 4*d**2*x**2*sqrt(d + e*x*
*2)/(105*e**2) + d*x**4*sqrt(d + e*x**2)/(35*e) + x**6*sqrt(d + e*x**2)/7,
Ne(e, 0)), (sqrt(d)*x**6/6, True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3 (d + ex^2)^{3/2} (a + b \log (cx^n)) \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^3 (d + ex^2)^{3/2} (a + b \log (cx^n)) \, dx = \int (ex^2 + d)^{\frac{3}{2}} (b \log (cx^n) + a) x^3 \, dx$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*log(c*x^n) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d + ex^2)^{3/2} (a + b \log (cx^n)) \, dx = \int x^3 \left(e \, x^2 + d\right)^{3/2} (a + b \, \ln (c \, x^n)) \, dx$$

```
[In] int(x^3*(d + e*x^2)^(3/2)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^3*(d + e*x^2)^(3/2)*(a + b*log(c*x^n)), x)
```

3.265 $\int x(d+ex^2)^{3/2}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 125

$$\int x(d+ex^2)^{3/2}(a+b\log(cx^n))\,dx = -\frac{bd^2n\sqrt{d+ex^2}}{5e}-\frac{bdn(d+ex^2)^{3/2}}{15e}-\frac{bn(d+ex^2)^{5/2}}{25e}+\frac{bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{5e}+\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e}$$

```
[Out] -1/15*b*d*n*(e*x^2+d)^(3/2)/e-1/25*b*n*(e*x^2+d)^(5/2)/e+1/5*b*d^(5/2)*n*ar
ctanh((e*x^2+d)^(1/2)/d^(1/2))/e+1/5*(e*x^2+d)^(5/2)*(a+b*ln(c*x^n))/e-1/5*
b*d^2*n*(e*x^2+d)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2376, 272, 52, 65, 214}

$$\int x(d+ex^2)^{3/2}(a+b\log(cx^n))\,dx = \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e}+\frac{bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{5e}-\frac{bd^2n\sqrt{d+ex^2}}{5e}-\frac{bdn(d+ex^2)^{3/2}}{15e}-\frac{bn(d+ex^2)^{5/2}}{25e}$$

```
[In] Int[x*(d + e*x^2)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/5*(b*d^2*n*Sqrt[d + e*x^2])/e - (b*d*n*(d + e*x^2)^(3/2))/(15*e) - (b*n*
(d + e*x^2)^(5/2))/(25*e) + (b*d^(5/2)*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/
(5*e) + ((d + e*x^2)^(5/2)*(a + b*Log[c*x^n]))/(5*e)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e} - \frac{(bn)\int \frac{(d+ex^2)^{5/2}}{x}\,dx}{5e}$$

$$= \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e} - \frac{(bn)\text{Subst}\left(\int \frac{(d+ex)^{5/2}}{x}\,dx, x, x^2\right)}{10e}$$

$$= -\frac{bn(d+ex^2)^{5/2}}{25e} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e} - \frac{(bdn)\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x}\,dx, x, x^2\right)}{10e}$$

$$= -\frac{bdn(d+ex^2)^{3/2}}{15e} - \frac{bn(d+ex^2)^{5/2}}{25e} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e} - \frac{(bd^2n)\,\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x}\,dx, x, x^2\right)}{10e}$$

$$= -\frac{bd^2n\sqrt{d+ex^2}}{5e} - \frac{bdn(d+ex^2)^{3/2}}{15e} - \frac{bn(d+ex^2)^{5/2}}{25e} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e} - \frac{(bd^3n)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)}{10e}$$

$$= -\frac{bd^2n\sqrt{d+ex^2}}{5e} - \frac{bdn(d+ex^2)^{3/2}}{15e} - \frac{bn(d+ex^2)^{5/2}}{25e} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e} - \frac{(bd^3n)\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{5e^2}$$

$$= -\frac{bd^2n\sqrt{d+ex^2}}{5e} - \frac{bdn(d+ex^2)^{3/2}}{15e} - \frac{bn(d+ex^2)^{5/2}}{25e} + \frac{bd^{5/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{5e} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.45

$$\int x\left(d+ex^2\right)^{3/2}(a+b\log(cx^n))\,dx = -\frac{bd^{5/2}n\log(x)}{5e} + \frac{bn(d+ex^2)^{5/2}\log(x)}{5e} + \sqrt{d+ex^2}\left(\frac{1}{25}ex^4(5a-bn+5b(-n\log(x)+\log(cx^n))) + \frac{d^2(15a-23bn+15b(-n\log(x)+\log(cx^n)))}{75e} + \frac{1}{75}\right.$$

```
[In] Integrate[x*(d + e*x^2)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/5*(b*d^(5/2)*n*Log[x])/e + (b*n*(d + e*x^2)^(5/2)*Log[x])/(5*e) + Sqrt[d
 + e*x^2]*((e*x^4*(5*a - b*n + 5*b*(-(n*Log[x]) + Log[c*x^n])))/25 + (d^2*(
15*a - 23*b*n + 15*b*(-(n*Log[x]) + Log[c*x^n])))/(75*e) + (d*x^2*(30*a - 1
1*b*n + 30*b*(-(n*Log[x]) + Log[c*x^n])))/75) + (b*d^(5/2)*n*Log[d + Sqrt[d
]*Sqrt[d + e*x^2]])/(5*e)
```

Maple [F]

$$\int x\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\ln\left(c\,x^{n}\right)\right)dx$$

```
[In] int(x*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n)),x)
```

```
[Out] int(x*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.43

$$\int x(d+ex^2)^{3/2}(a+b\log(cx^n))\,dx = \left[\frac{15\,bd^{\frac{5}{2}}n\log\left(-\frac{ex^2+2\sqrt{ex^2+d}\sqrt{d}+2\,d}{x^2}\right)-2\,(3\,(be^2n-5\,ae^2)x^4+23\,bd^2n-15\,ad^2+(11\,bden\right.$$

$$-\frac{15\,b\sqrt{-d}d^2n\arctan\left(\frac{\sqrt{-d}}{\sqrt{ex^2+d}}\right)+(3\,(be^2n-5\,ae^2)x^4+23\,bd^2n-15\,ad^2+(11\,bden-30\,ade)x^2-15\,(be^2x^4}{75\,e}$$

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] [1/150*(15*b*d^(5/2)*n*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2)
- 2*(3*(b*e^2*n - 5*a*e^2)*x^4 + 23*b*d^2*n - 15*a*d^2 + (11*b*d*e*n - 30*a
*d*e)*x^2 - 15*(b*e^2*x^4 + 2*b*d*e*x^2 + b*d^2)*log(c) - 15*(b*e^2*n*x^4 +
 2*b*d*e*n*x^2 + b*d^2*n)*log(x))*sqrt(e*x^2 + d))/e, -1/75*(15*b*sqrt(-d)*
d^2*n*arctan(sqrt(-d)/sqrt(e*x^2 + d)) + (3*(b*e^2*n - 5*a*e^2)*x^4 + 23*b*
d^2*n - 15*a*d^2 + (11*b*d*e*n - 30*a*d*e)*x^2 - 15*(b*e^2*x^4 + 2*b*d*e*x^
2 + b*d^2)*log(c) - 15*(b*e^2*n*x^4 + 2*b*d*e*n*x^2 + b*d^2*n)*log(x))*sqrt
(e*x^2 + d))/e]
```

Sympy [A] (verification not implemented)

Time = 30.64 (sec) , antiderivative size = 573, normalized size of antiderivative = 4.58

$$\int x(d+ex^2)^{3/2}(a+b\log(cx^n))\,dx = ad\left(\begin{cases}\frac{d\sqrt{d+ex^2}}{3e}+\frac{x^2\sqrt{d+ex^2}}{3} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ae\left(\begin{cases}-\frac{2d^2\sqrt{d+ex^2}}{15e^2}+\frac{dx^2\sqrt{d+ex^2}}{15e}+\frac{x^4\sqrt{d+ex^2}}{5} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^4}{4} & \text{otherwise}\end{cases}\right)$$
$$-bdn\left(\begin{cases}-\frac{d^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{3e}+\frac{d^2}{3e^{\frac{3}{2}}x\sqrt{\frac{d}{ex^2}+1}}+\frac{dx}{3\sqrt{e}\sqrt{\frac{d}{ex^2}+1}}+\frac{\begin{cases}\frac{d\sqrt{d+ex^2}}{3e}+\frac{x^2\sqrt{d+ex^2}}{3} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^2}{2} & \text{otherwise}\end{cases}}{3} & \text{for } e>-\infty\wedge e<\\ \frac{\sqrt{d}x^2}{4} & \text{otherwise}\end{cases}\right.$$
$$+bd\left(\begin{cases}\frac{d\sqrt{d+ex^2}}{3e}+\frac{x^2\sqrt{d+ex^2}}{3} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^2}{2} & \text{otherwise}\end{cases}\right)\log(cx^n)$$
$$-ben\left(\begin{cases}\frac{2d^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{15e^2}-\frac{2d^3}{15e^{\frac{5}{2}}x\sqrt{\frac{d}{ex^2}+1}}-\frac{2d^2x}{15e^{\frac{3}{2}}\sqrt{\frac{d}{ex^2}+1}}+\frac{d\left(\begin{cases}\frac{d\sqrt{d+ex^2}}{3e}+\frac{x^2\sqrt{d+ex^2}}{3} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^2}{2} & \text{otherwise}\end{cases}\right)}{15e}+\frac{\begin{cases}-\frac{2d^2\sqrt{d+ex^2}}{15e^2} \\ \frac{\sqrt{d}x^4}{4}\end{cases}}{} \\ \frac{\sqrt{d}x^4}{16}\end{cases}\right.$$
$$+be\left(\begin{cases}-\frac{2d^2\sqrt{d+ex^2}}{15e^2}+\frac{dx^2\sqrt{d+ex^2}}{15e}+\frac{x^4\sqrt{d+ex^2}}{5} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^4}{4} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate(x*(e*x**2+d)**(3/2)*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d*Piecewise((d*sqrt(d + e*x**2)/(3*e) + x**2*sqrt(d + e*x**2)/3, Ne(e, 0)
), (sqrt(d)*x**2/2, True)) + a*e*Piecewise((-2*d**2*sqrt(d + e*x**2)/(15*e*
*2) + d*x**2*sqrt(d + e*x**2)/(15*e) + x**4*sqrt(d + e*x**2)/5, Ne(e, 0)),
(sqrt(d)*x**4/4, True)) - b*d*n*Piecewise((-d**(3/2)*asinh(sqrt(d)/(sqrt(e)
*x))/(3*e) + d**2/(3*e**(3/2)*x*sqrt(d/(e*x**2) + 1)) + d*x/(3*sqrt(e)*sqrt
(d/(e*x**2) + 1)) + Piecewise((d*sqrt(d + e*x**2)/(3*e) + x**2*sqrt(d + e*x
**2)/3, Ne(e, 0)), (sqrt(d)*x**2/2, True))/3, (e > -oo) & (e < oo) & Ne(e,
0)), (sqrt(d)*x**2/4, True)) + b*d*Piecewise((d*sqrt(d + e*x**2)/(3*e) + x*
*2*sqrt(d + e*x**2)/3, Ne(e, 0)), (sqrt(d)*x**2/2, True))*log(c*x**n) - b*e
*n*Piecewise((2*d**(5/2)*asinh(sqrt(d)/(sqrt(e)*x))/(15*e**2) - 2*d**3/(15*
e**(5/2)*x*sqrt(d/(e*x**2) + 1)) - 2*d**2*x/(15*e**(3/2)*sqrt(d/(e*x**2) +
1)) + d*Piecewise((d*sqrt(d + e*x**2)/(3*e) + x**2*sqrt(d + e*x**2)/3, Ne(e
```

```
, 0)), (sqrt(d)*x**2/2, True))/(15*e) + Piecewise((-2*d**2*sqrt(d + e*x**2)
/(15*e**2) + d*x**2*sqrt(d + e*x**2)/(15*e) + x**4*sqrt(d + e*x**2)/5, Ne(e
, 0)), (sqrt(d)*x**4/4, True))/5, (e > -oo) & (e < oo) & Ne(e, 0)), (sqrt(d
)*x**4/16, True)) + b*e*Piecewise((-2*d**2*sqrt(d + e*x**2)/(15*e**2) + d*x
**2*sqrt(d + e*x**2)/(15*e) + x**4*sqrt(d + e*x**2)/5, Ne(e, 0)), (sqrt(d)*
x**4/4, True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int x(d+ex^2)^{3/2}(a+b\log(cx^n))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x(d+ex^2)^{3/2}(a+b\log(cx^n))\,dx = \int (ex^2+d)^{\frac{3}{2}}(b\log(cx^n)+a)x\,dx$$

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*log(c*x^n) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)^{3/2}(a+b\log(cx^n))\,dx = \int x\,(e\,x^2+d)^{3/2}\,(a+b\,\ln(c\,x^n))\,dx$$

```
[In] int(x*(d + e*x^2)^(3/2)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x*(d + e*x^2)^(3/2)*(a + b*log(c*x^n)), x)
```

3.266 $\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 260

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x}\,dx = -\frac{4}{3}bdn\sqrt{d+ex^2}-\frac{1}{9}bn(d+ex^2)^{3/2} +\frac{4}{3}bd^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+\frac{1}{2}bd^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2+\frac{1}{3}\left(3d\sqrt{d+ex^2}+(d+ex^2)^{3/2}-3d^{3/2}\operatorname{arcta}\right.$$

```
[Out] -1/9*b*n*(e*x^2+d)^(3/2)+4/3*b*d^(3/2)*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))+1
/2*b*d^(3/2)*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))^2-b*d^(3/2)*n*arctanh((e*x^
2+d)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)-(e*x^2+d)^(1/2)))-1/2*b*d^(3/2)*n
*polylog(2,1-2*d^(1/2)/(d^(1/2)-(e*x^2+d)^(1/2)))-4/3*b*d*n*(e*x^2+d)^(1/2)
+1/3*(a+b*ln(c*x^n))*((e*x^2+d)^(3/2)-3*d^(3/2)*arctanh((e*x^2+d)^(1/2)/d^(
1/2))+3*d*(e*x^2+d)^(1/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {272, 52, 65, 214, 2390, 6131, 6055, 2449, 2352}

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x}\,dx = \frac{1}{3}\left(-3d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+3d\sqrt{d+ex^2}+(d+ex^2)^{3/2}\right)(a+b\log$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] (-4*b*d*n*Sqrt[d + e*x^2])/3 - (b*n*(d + e*x^2)^(3/2))/9 + (4*b*d^(3/2)*n*A
rcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/3 + (b*d^(3/2)*n*ArcTanh[Sqrt[d + e*x^2]/S
```

```
qrt[d]]^2)/2 + ((3*d*Sqrt[d + e*x^2] + (d + e*x^2)^(3/2) - 3*d^(3/2)*ArcTan
h[Sqrt[d + e*x^2]/Sqrt[d]])*(a + b*Log[c*x^n]))/3 - b*d^(3/2)*n*ArcTanh[Sqr
t[d + e*x^2]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])] - (b*d^(
3/2)*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\left(3d\sqrt{d+ex^2}+\left(d+ex^2\right)^{3/2}-3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int\left(\frac{d\sqrt{d+ex^2}}{x}+\frac{\left(d+ex^2\right)^{3/2}}{3x}-\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{x}\right)dx \\
&= \frac{1}{3}\left(3d\sqrt{d+ex^2}+\left(d+ex^2\right)^{3/2}-3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{3}(bn)\int\frac{\left(d+ex^2\right)^{3/2}}{x}\,dx-(bdn)\int\frac{\sqrt{d+ex^2}}{x}\,dx+\left(bd^{3/2}n\right)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{x}\,dx \\
&= \frac{1}{3}\left(3d\sqrt{d+ex^2}+\left(d+ex^2\right)^{3/2}-3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{6}(bn)\text{Subst}\left(\int\frac{(d+ex)^{3/2}}{x}\,dx,x,x^2\right)-\frac{1}{2}(bdn)\text{Subst}\left(\int\frac{\sqrt{d+ex}}{x}\,dx,x,x^2\right)+\frac{1}{2}\left(bd^{3/2}n\right)\text{Subs} \\
&= -bdn\sqrt{d+ex^2}-\frac{1}{9}bn\left(d+ex^2\right)^{3/2}+\frac{1}{3}\left(3d\sqrt{d+ex^2}+\left(d+ex^2\right)^{3/2}\right. \\
&\quad \left.-3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+b\log(cx^n))-\frac{1}{6}(bdn)\text{Subst}\left(\int\frac{\sqrt{d+ex}}{x}\,dx,x,x^2\right)+\left(bd^{3/2}n\right)
\end{aligned}$$

$$= -\frac{4}{3}bdn\sqrt{d+ex^2} - \frac{1}{9}bn\left(d+ex^2\right)^{3/2} + \frac{1}{2}bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2$$
$$+\frac{1}{3}\left(3d\sqrt{d+ex^2}+\left(d+ex^2\right)^{3/2}-3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+b\log(cx^n))-(bdn)\text{Subst}\left(\int \frac{\tan}{}\right.$$

$$= -\frac{4}{3}bdn\sqrt{d+ex^2} - \frac{1}{9}bn\left(d+ex^2\right)^{3/2} + bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)$$
$$+\frac{1}{2}bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2+\frac{1}{3}\left(3d\sqrt{d+ex^2}+\left(d+ex^2\right)^{3/2}-3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+$$

$$= -\frac{4}{3}bdn\sqrt{d+ex^2} - \frac{1}{9}bn\left(d+ex^2\right)^{3/2} + \frac{4}{3}bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)$$
$$+\frac{1}{2}bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2+\frac{1}{3}\left(3d\sqrt{d+ex^2}+\left(d+ex^2\right)^{3/2}-3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+$$

$$= -\frac{4}{3}bdn\sqrt{d+ex^2} - \frac{1}{9}bn\left(d+ex^2\right)^{3/2} + \frac{4}{3}bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)$$
$$+\frac{1}{2}bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2+\frac{1}{3}\left(3d\sqrt{d+ex^2}+\left(d+ex^2\right)^{3/2}-3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)(a+$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.50 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.16

$$\int \frac{\left(d+ex^2\right)^{3/2}(a+b\log(cx^n))}{x}\,dx = \frac{benx^2\sqrt{d+ex^2}\left(-\frac{1}{4}\,{}_3F_2\left(-\frac{1}{2},1,1;2,2;-\frac{ex^2}{d}\right)+\frac{d\left(-1+\left(1+\frac{ex^2}{d}\right)^{3/2}\right)\log(x)}{3ex^2}\right)}{\sqrt{1+\frac{ex^2}{d}}}$$
$$+\frac{bdn\sqrt{d+ex^2}\left(-\,{}_3F_2\left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2};\frac{1}{2},\frac{1}{2};-\frac{d}{ex^2}\right)+\sqrt{1+\frac{d}{ex^2}}\log(x)-\frac{\sqrt{d}\,\text{arcsinh}\left(\frac{\sqrt{d}}{\sqrt{ex}}\right)\log(x)}{\sqrt{ex}}\right)}{\sqrt{1+\frac{d}{ex^2}}}$$
$$+\frac{1}{3}\sqrt{d+ex^2}\left(4d+ex^2\right)(a-bn\log(x)+b\log(cx^n))$$
$$+d^{3/2}\log(x)(a-bn\log(x)+b\log(cx^n))-d^{3/2}(a-bn\log(x)+b\log(cx^n))\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] (b*e*n*x^2*Sqrt[d + e*x^2]*(-1/4*HypergeometricPFQ[{-1/2, 1, 1}, {2, 2}, -(
(e*x^2)/d)] + (d*(-1 + (1 + (e*x^2)/d)^(3/2))*Log[x])/(3*e*x^2)))/Sqrt[1 +
(e*x^2)/d] + (b*d*n*Sqrt[d + e*x^2]*(-HypergeometricPFQ[{-1/2, -1/2, -1/2},
 {1/2, 1/2}, -(d/(e*x^2))] + Sqrt[1 + d/(e*x^2)]*Log[x] - (Sqrt[d]*ArcSinh[
Sqrt[d]/(Sqrt[e]*x)]*Log[x])/(Sqrt[e]*x)))/Sqrt[1 + d/(e*x^2)] + (Sqrt[d +
e*x^2]*(4*d + e*x^2)*(a - b*n*Log[x] + b*Log[c*x^n]))/3 + d^(3/2)*Log[x]*(a
 - b*n*Log[x] + b*Log[c*x^n]) - d^(3/2)*(a - b*n*Log[x] + b*Log[c*x^n])*Log
[d + Sqrt[d]*Sqrt[d + e*x^2]]
```

Maple [F]

$$\int \frac{(e\,x^{2}+d)^{\frac{3}{2}}\left(a+b\ln\left(c\,x^{n}\right)\right)}{x}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/x,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/x,x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\log\left(cx^n\right)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] integral(((b*e*x^2 + b*d)*sqrt(e*x^2 + d)*log(c*x^n) + (a*e*x^2 + a*d)*sqrt
(e*x^2 + d))/x, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx=\int \frac{\left(a+b\log\left(cx^n\right)\right)(d+ex^2)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*ln(c*x**n))/x,x)
```

```
[Out] Integral((a + b*log(c*x**n))*(d + e*x**2)**(3/2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\log(cx^n)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*log(c*x^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,(a+b\,\ln(c\,x^n))}{x}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int(((d + e*x^2)^(3/2)*(a + b*log(c*x^n)))/x, x)
```

3.267 $\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 295

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^3} dx = -ben\sqrt{d+ex^2} - \frac{bdn\sqrt{d+ex^2}}{4x^2} + \frac{3}{4}b\sqrt{d}en\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) + \frac{3}{4}b\sqrt{d}en\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2 + \frac{3}{2}e\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{2x^2} - \frac{3}{2}\sqrt{d}e\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n)) - \frac{3}{2}b\sqrt{d}en\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right) - \frac{3}{4}b\sqrt{d}en\,\text{PolyLog}\left(2, 1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)$$

```
[Out] -1/2*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/x^2+3/4*b*e*n*arctanh((e*x^2+d)^(1/2)/
d^(1/2))*d^(1/2)+3/4*b*e*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))^2*d^(1/2)-3/2*e
*arctanh((e*x^2+d)^(1/2)/d^(1/2))*(a+b*ln(c*x^n))*d^(1/2)-3/2*b*e*n*arctanh
((e*x^2+d)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)-(e*x^2+d)^(1/2)))*d^(1/2)-3
/4*b*e*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-(e*x^2+d)^(1/2)))*d^(1/2)-b*e*n*(e*
x^2+d)^(1/2)-1/4*b*d*n*(e*x^2+d)^(1/2)/x^2+3/2*e*(a+b*ln(c*x^n))*(e*x^2+d)^
(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {272, 43, 52, 65, 214, 2392, 12, 14, 6131, 6055, 2449, 2352}

$$\begin{aligned}
&\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^3}\,dx = -\frac{3}{2}\sqrt{d}e\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))\\
&-\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{2x^2}+\frac{3}{2}e\sqrt{d+ex^2}(a+b\log(cx^n))\\
&+\frac{3}{4}b\sqrt{d}en\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2+\frac{3}{4}b\sqrt{d}en\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\\
&-\frac{3}{2}b\sqrt{d}en\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)\\
&-\frac{3}{4}b\sqrt{d}en\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{ex^2+d}}\right)-ben\sqrt{d+ex^2}-\frac{bdn\sqrt{d+ex^2}}{4x^2}
\end{aligned}$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -(b*e*n*Sqrt[d + e*x^2]) - (b*d*n*Sqrt[d + e*x^2])/(4*x^2) + (3*b*Sqrt[d]*e
*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/4 + (3*b*Sqrt[d]*e*n*ArcTanh[Sqrt[d +
e*x^2]/Sqrt[d]]^2)/4 + (3*e*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/2 - ((d + e
*x^2)^(3/2)*(a + b*Log[c*x^n]))/(2*x^2) - (3*Sqrt[d]*e*ArcTanh[Sqrt[d + e*x
^2]/Sqrt[d]]*(a + b*Log[c*x^n]))/2 - (3*b*Sqrt[d]*e*n*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/2 - (3*b*Sqrt[d]*
e*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3}{2}e\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{2x^2} \\
&\quad - \frac{3}{2}\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n)) \\
&\quad - (bn)\int\frac{-\left((d-2ex^2)\sqrt{d+ex^2}\right)-3\sqrt{d}ex^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2x^3}\,dx \\
&= \frac{3}{2}e\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{2x^2} \\
&\quad - \frac{3}{2}\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n)) \\
&\quad - \frac{1}{2}(bn)\int\frac{-\left((d-2ex^2)\sqrt{d+ex^2}\right)-3\sqrt{d}ex^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{x^3}\,dx \\
&= \frac{3}{2}e\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{2x^2} \\
&\quad - \frac{3}{2}\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n)) \\
&\quad - \frac{1}{2}(bn)\int\left(-\frac{d\sqrt{d+ex^2}}{x^3}+\frac{2e\sqrt{d+ex^2}}{x}-\frac{3\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{x}\right)dx
\end{aligned}$$

$$= \frac{3}{2}e\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{2x^2} - \frac{3}{2}\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n)) + \frac{1}{2}(bdn)\int\frac{\sqrt{d+ex^2}}{x^3}\,dx - (ben)\int\frac{\sqrt{d+ex^2}}{x}\,dx + \frac{1}{2}\left(3b\sqrt{d}en\right)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{x}\,dx$$

$$= \frac{3}{2}e\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{2x^2} - \frac{3}{2}\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n)) + \frac{1}{4}(bdn)\text{Subst}\left(\int\frac{\sqrt{d+ex}}{x^2}\,dx, x, x^2\right) - \frac{1}{2}(ben)\text{Subst}\left(\int\frac{\sqrt{d+ex}}{x}\,dx, x, x^2\right) + \frac{1}{4}\left(3b\sqrt{d}en\right)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx, x, x^2\right)$$

$$= -ben\sqrt{d+ex^2} - \frac{bdn\sqrt{d+ex^2}}{4x^2} + \frac{3}{2}e\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{2x^2} - \frac{3}{2}\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n)) + \frac{1}{2}\left(3b\sqrt{d}en\right)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx, x, \sqrt{d+ex^2}\right) + \frac{1}{8}(bden)\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right) - \frac{1}{2}(bden)\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)$$

$$= -ben\sqrt{d+ex^2} - \frac{bdn\sqrt{d+ex^2}}{4x^2} + \frac{3}{4}b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2 + \frac{3}{2}e\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{2x^2} - \frac{3}{2}\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n)) + \frac{1}{4}(bdn)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right) - (bdn)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right) - \frac{1}{2}(3ben)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx, x, \sqrt{d+ex^2}\right)$$

$$= -ben\sqrt{d+ex^2} - \frac{bdn\sqrt{d+ex^2}}{4x^2} + \frac{3}{4}b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) + \frac{3}{4}b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2 + \frac{3}{2}e\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{2x^2} - \frac{3}{2}\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n)) - \frac{3}{2}b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right) + \frac{1}{2}(3ben)\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}\,dx, x, \sqrt{d+ex^2}\right)$$

$$= -ben\sqrt{d+ex^2} - \frac{bdn\sqrt{d+ex^2}}{4x^2} + \frac{3}{4}b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) + \frac{3}{4}b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2 + \frac{3}{2}e\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{2x^2} - \frac{3}{2}\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n)) - \frac{3}{2}b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right) - \frac{1}{2}\left(3b\sqrt{d}en\right)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{d+ex^2}}{\sqrt{d}}}\right)$$

$$
\begin{aligned}
&= -ben\sqrt{d+ex^2} - \frac{bdn\sqrt{d+ex^2}}{4x^2} + \frac{3}{4}b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) \\
&\quad + \frac{3}{4}b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2 + \frac{3}{2}e\sqrt{d+ex^2}(a+b\log(cx^n)) \\
&\quad - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{2x^2} - \frac{3}{2}\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n)) \\
&\quad - \frac{3}{2}b\sqrt{d}en\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right) \\
&\quad - \frac{3}{4}b\sqrt{d}en\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex^2}}{\sqrt{d}}}\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.59 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.18

$$
\begin{aligned}
&\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^3}\,dx = \frac{ben\sqrt{d+ex^2}\left(-{}_3F_2\left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2};\frac{1}{2},\frac{1}{2};-\frac{d}{ex^2}\right)+\sqrt{1+\frac{d}{ex^2}}\log(x)-\frac{\sqrt{d}\ldots}{\ldots}\right.}{\sqrt{1+\frac{d}{ex^2}}} \\
&- \frac{b\sqrt{d}n\sqrt{d+ex^2}\left(2\sqrt{d}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{d}{ex^2}\right)+\left(\sqrt{d}\sqrt{1+\frac{d}{ex^2}}+\sqrt{e}x\text{arcsinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)\right)(1+2\log(x))\right)}{4\sqrt{1+\frac{d}{ex^2}}x^2} \\
&- \frac{(d-2ex^2)\sqrt{d+ex^2}(a-bn\log(x)+b\log(cx^n))}{2x^2} + \frac{3}{2}\sqrt{d}e\log(x)(a-bn\log(x)+b\log(cx^n)) \\
&- \frac{3}{2}\sqrt{d}e(a-bn\log(x)+b\log(cx^n))\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)
\end{aligned}
$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] (b*e*n*Sqrt[d + e*x^2]*(-HypergeometricPFQ[{-1/2, -1/2, -1/2}, {1/2, 1/2},
-(d/(e*x^2))] + Sqrt[1 + d/(e*x^2)]*Log[x] - (Sqrt[d]*ArcSinh[Sqrt[d]/(Sqrt
[e]*x)]*Log[x])/(Sqrt[e]*x)))/Sqrt[1 + d/(e*x^2)] - (b*Sqrt[d]*n*Sqrt[d + e
*x^2]*(2*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, -(d/(e*x^2)
)] + (Sqrt[d]*Sqrt[1 + d/(e*x^2)] + Sqrt[e]*x*ArcSinh[Sqrt[d]/(Sqrt[e]*x)])
*(1 + 2*Log[x])))/(4*Sqrt[1 + d/(e*x^2)]*x^2) - ((d - 2*e*x^2)*Sqrt[d + e*x
^2]*(a - b*n*Log[x] + b*Log[c*x^n]))/(2*x^2) + (3*Sqrt[d]*e*Log[x]*(a - b*n
*Log[x] + b*Log[c*x^n]))/2 - (3*Sqrt[d]*e*(a - b*n*Log[x] + b*Log[c*x^n])*L
og[d + Sqrt[d]*Sqrt[d + e*x^2]])/2
```

Maple [F]

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\ln\left(c\,x^{n}\right)\right)}{x^{3}}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/x^3,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/x^3,x)
```

Fricas [F]

$$\int \frac{\left(d+ex^{2}\right)^{3/2} \left(a+b\log\left(cx^{n}\right)\right)}{x^{3}}\,dx = \int \frac{\left(ex^{2}+d\right)^{\frac{3}{2}} \left(b\log\left(cx^{n}\right)+a\right)}{x^{3}}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^3,x, algorithm="fricas")
[Out] integral(((b*e*x^2 + b*d)*sqrt(e*x^2 + d)*log(c*x^n) + (a*e*x^2 + a*d)*sqrt
(e*x^2 + d))/x^3, x)
```

Sympy [F]

$$\int \frac{\left(d+ex^{2}\right)^{3/2} \left(a+b\log\left(cx^{n}\right)\right)}{x^{3}}\,dx = \int \frac{\left(a+b\log\left(cx^{n}\right)\right)\left(d+ex^{2}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*ln(c*x**n))/x**3,x)
[Out] Integral((a + b*log(c*x**n))*(d + e*x**2)**(3/2)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex^{2}\right)^{3/2} \left(a+b\log\left(cx^{n}\right)\right)}{x^{3}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\log(cx^n))}{x^3}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\log(cx^n)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^3,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*log(c*x^n) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\log(cx^n))}{x^3}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,(a+b\,\ln(c\,x^n))}{x^3}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*log(c*x^n)))/x^3,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*log(c*x^n)))/x^3, x)
```

3.268 $\int x^2(d+ex^2)^{3/2}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 464

$$\int x^2\left(d+ex^2\right)^{3/2}(a+b\log(cx^n))\,dx=-\frac{11bd^2nx\sqrt{d+ex^2}}{192e}-\frac{23}{288}bdnx^3\sqrt{d+ex^2}-\frac{1}{36}benx^5\sqrt{d+ex^2}-\frac{bd^{5/2}n\sqrt{d+ex^2}\,\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{192e^{3/2}\sqrt{1+\frac{ex^2}{d}}}-\frac{bd^{5/2}n\sqrt{d+ex^2}\,\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{32e^{3/2}\sqrt{1+\frac{ex^2}{d}}}+\frac{bd^{5/2}n\sqrt{d+ex^2}\,\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}}+\frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e}+\frac{1}{8}dx^3\sqrt{d+ex^2}(a+b\log(cx^n))+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}(a+b\log(cx^n))-\frac{d^{5/2}\sqrt{d+ex^2}\,\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}}$$

```
[Out] 1/6*x^3*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n))-11/192*b*d^2*n*x*(e*x^2+d)^(1/2)/e-
23/288*b*d*n*x^3*(e*x^2+d)^(1/2)-1/36*b*e*n*x^5*(e*x^2+d)^(1/2)+1/16*d^2*x*
(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/e+1/8*d*x^3*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)
-1/192*b*d^(5/2)*n*arcsinh(x*e^(1/2)/d^(1/2))*(e*x^2+d)^(1/2)/e^(3/2)/(1+e*
x^2/d)^(1/2)-1/32*b*d^(5/2)*n*arcsinh(x*e^(1/2)/d^(1/2))^2*(e*x^2+d)^(1/2)/
e^(3/2)/(1+e*x^2/d)^(1/2)+1/16*b*d^(5/2)*n*arcsinh(x*e^(1/2)/d^(1/2))*ln(1-
(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*(e*x^2+d)^(1/2)/e^(3/2)/(1+e*x^2/d
)^(1/2)-1/16*d^(5/2)*arcsinh(x*e^(1/2)/d^(1/2))*(a+b*ln(c*x^n))*(e*x^2+d)^(
1/2)/e^(3/2)/(1+e*x^2/d)^(1/2)+1/32*b*d^(5/2)*n*polylog(2,(x*e^(1/2)/d^(1/2
)+(1+e*x^2/d)^(1/2))^2)*(e*x^2+d)^(1/2)/e^(3/2)/(1+e*x^2/d)^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {2386, 285, 327, 221, 2392, 12, 14, 201, 5775, 3797, 2221, 2317, 2438}

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\log\left(cx^n\right)\right)\,dx =$$

$$-\frac{d^{5/2}\sqrt{d+ex^2}\,\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(a+b\log\left(cx^n\right)\right)}{16e^{3/2}\sqrt{\frac{ex^2}{d}+1}}+\frac{d^2x\sqrt{d+ex^2}\left(a+b\log\left(cx^n\right)\right)}{16e}$$

$$+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\left(a+b\log\left(cx^n\right)\right)+\frac{1}{8}dx^3\sqrt{d+ex^2}\left(a+b\log\left(cx^n\right)\right)+\frac{bd^{5/2}n\sqrt{d+ex^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right.}\right.}{32e^{3/2}\sqrt{\frac{ex^2}{d}+1}}$$

```
[In] Int[x^2*(d + e*x^2)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (-11*b*d^2*n*x*Sqrt[d + e*x^2])/(192*e) - (23*b*d*n*x^3*Sqrt[d + e*x^2])/28
8 - (b*e*n*x^5*Sqrt[d + e*x^2])/36 - (b*d^(5/2)*n*Sqrt[d + e*x^2]*ArcSinh[(
Sqrt[e]*x)/Sqrt[d]])/(192*e^(3/2)*Sqrt[1 + (e*x^2)/d]) - (b*d^(5/2)*n*Sqrt[
d + e*x^2]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)/(32*e^(3/2)*Sqrt[1 + (e*x^2)/d])
 + (b*d^(5/2)*n*Sqrt[d + e*x^2]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[1 - E^(2*A
rcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(16*e^(3/2)*Sqrt[1 + (e*x^2)/d]) + (d^2*x*Sq
rt[d + e*x^2]*(a + b*Log[c*x^n]))/(16*e) + (d*x^3*Sqrt[d + e*x^2]*(a + b*Lo
g[c*x^n]))/8 + (x^3*(d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/6 - (d^(5/2)*Sqrt
[d + e*x^2]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/(16*e^(3/2)*Sq
rt[1 + (e*x^2)/d]) + (b*d^(5/2)*n*Sqrt[d + e*x^2]*PolyLog[2, E^(2*ArcSinh[(
Sqrt[e]*x)/Sqrt[d]])])/(32*e^(3/2)*Sqrt[1 + (e*x^2)/d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2386

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^
(q_), x_Symbol] :> Dist[d^IntPart[q]*((d + e*x^2)^FracPart[q]/(1 + (e/d)*x^
2)^FracPart[q]), Int[x^m*(1 + (e/d)*x^2)^q*(a + b*Log[c*x^n]), x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && IntegerQ[m/2] && IntegerQ[q - 1/2] &&  !(LtQ[
m + 2*q, -2] || GtQ[d, 0])
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(d\sqrt{d+ex^2}\right)\int x^2\left(1+\frac{ex^2}{d}\right)^{3/2}(a+b\log(cx^n))\,dx}{\sqrt{1+\frac{ex^2}{d}}}$$

$$= \frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e}+\frac{1}{8}dx^3\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{1}{6}x^3\left(d+ex^2\right)^{3/2}(a+b\log(cx^n)) - \frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bdn\sqrt{d+ex^2}\right)\int\frac{\sqrt{e}x\sqrt{1+\frac{ex^2}{d}}(3d^2+14dex^2+8e^2x^4)-3d^{5/2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{48de^{3/2}x}\,dx}{\sqrt{1+\frac{ex^2}{d}}}$$

$$= \frac{d^2 x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e} + \frac{1}{8}dx^3\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{1}{6}x^3\left(d+ex^2\right)^{3/2}(a+b\log(cx^n)) - \frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bn\sqrt{d+ex^2}\right)\int \frac{\sqrt{e}x\sqrt{1+\frac{ex^2}{d}}(3d^2+14dex^2+8e^2x^4)-3d^{5/2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{48e^{3/2}\sqrt{1+\frac{ex^2}{d}}}$$

$$= \frac{d^2 x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e} + \frac{1}{8}dx^3\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{1}{6}x^3\left(d+ex^2\right)^{3/2}(a+b\log(cx^n)) - \frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bn\sqrt{d+ex^2}\right)\int \left(3d^2\sqrt{e}\sqrt{1+\frac{ex^2}{d}} + 14de^{3/2}x^2\sqrt{1+\frac{ex^2}{d}} + 8e^{5/2}x^4\sqrt{1+\frac{ex^2}{d}} - \frac{3d^{5/2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\right)}{48e^{3/2}\sqrt{1+\frac{ex^2}{d}}}$$

$$= \frac{d^2 x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e} + \frac{1}{8}dx^3\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{1}{6}x^3\left(d+ex^2\right)^{3/2}(a+b\log(cx^n)) - \frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(7bdn\sqrt{d+ex^2}\right)\int x^2\sqrt{1+\frac{ex^2}{d}}\,dx}{24\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(bd^{5/2}n\sqrt{d+ex^2}\right)\int \frac{\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bd^2n\sqrt{d+ex^2}\right)\int\sqrt{1+\frac{ex^2}{d}}\,dx}{16e\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(ben\sqrt{d+ex^2}\right)\int x^4\sqrt{1+\frac{ex^2}{d}}\,dx}{6\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{bd^2nx\sqrt{d+ex^2}}{32e} - \frac{7}{96}bdnx^3\sqrt{d+ex^2} - \frac{1}{36}benx^5\sqrt{d+ex^2} + \frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e} + \frac{1}{8}dx^3\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{1}{6}x^3\left(d+ex^2\right)^{3/2}(a+b\log(cx^n)) - \frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(7bdn\sqrt{d+ex^2}\right)\int\frac{x^2}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{96\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(bd^{5/2}n\sqrt{d+ex^2}\right)\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bd^2n\sqrt{d+ex^2}\right)\int\frac{1}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{32e\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(ben\sqrt{d+ex^2}\right)\int\frac{x^4}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{36\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{13bd^2nx\sqrt{d+ex^2}}{192e} - \frac{23}{288}bdnx^3\sqrt{d+ex^2} - \frac{1}{36}benx^5\sqrt{d+ex^2} - \frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{32e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{32e^{3/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e} + \frac{1}{8}dx^3\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{1}{6}x^3\left(d+ex^2\right)^{3/2}(a+b\log(cx^n)) - \frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{11bd^2nx\sqrt{d+ex^2}}{192e} - \frac{23}{288}bdnx^3\sqrt{d+ex^2} - \frac{1}{36}benx^5\sqrt{d+ex^2} + \frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{192e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{32e^{3/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e} + \frac{1}{8}dx^3\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{1}{6}x^3\left(d+ex^2\right)^{3/2}(a+b\log(cx^n)) - \frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{11bd^2nx\sqrt{d+ex^2}}{192e} - \frac{23}{288}bdnx^3\sqrt{d+ex^2} - \frac{1}{36}benx^5\sqrt{d+ex^2} - \frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{192e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{32e^{3/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e} + \frac{1}{8}dx^3\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{1}{6}x^3(d+ex^2)^{3/2}(a+b\log(cx^n)) - \frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{11bd^2nx\sqrt{d+ex^2}}{192e} - \frac{23}{288}bdnx^3\sqrt{d+ex^2} - \frac{1}{36}benx^5\sqrt{d+ex^2} - \frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{192e^{3/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{32e^{3/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{bd^{5/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{d^2x\sqrt{d+ex^2}(a+b\log(cx^n))}{16e} + \frac{1}{8}dx^3\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{1}{6}x^3(d+ex^2)^{3/2}(a+b\log(cx^n)) - \frac{d^{5/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+}{16e^{3/2}\sqrt{1+\frac{ex^2}{d}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.63 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.71

$$\int x^2(d+ex^2)^{3/2}(a+b\log(cx^n))\,dx = \frac{-400bde^{3/2}nx^3\sqrt{d+ex^2}\,{}_3F_2\left(-\frac{1}{2},\frac{3}{2},\frac{3}{2};\frac{5}{2},\frac{5}{2};-\frac{ex^2}{d}\right) - 144be^{5/2}nx^5\sqrt{d+ex^2}\,{}_3F_2\left(-\frac{1}{2},\frac{5}{2},\frac{5}{2};\frac{7}{2}\right.}{}$$

```
[In] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (-400*b*d*e^(3/2)*n*x^3*Sqrt[d + e*x^2]*HypergeometricPFQ[{-1/2, 3/2, 3/2},
 {5/2, 5/2}, -((e*x^2)/d)] - 144*b*e^(5/2)*n*x^5*Sqrt[d + e*x^2]*Hypergeome
tricPFQ[{-1/2, 5/2, 5/2}, {7/2, 7/2}, -((e*x^2)/d)] - 75*(3*b*d^(5/2)*n*Sqr
t[d + e*x^2]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[x] + Sqrt[1 + (e*x^2)/d]*(-(a
*Sqrt[e]*x*Sqrt[d + e*x^2]*(3*d^2 + 14*d*e*x^2 + 8*e^2*x^4)) + 3*d^3*(a - b
*n*Log[x])*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]] - b*Log[c*x^n]*(Sqrt[e]*x*Sqr
t[d + e*x^2]*(3*d^2 + 14*d*e*x^2 + 8*e^2*x^4) - 3*d^3*Log[e*x + Sqrt[e]*Sqr
t[d + e*x^2]]))))/(3600*e^(3/2)*Sqrt[1 + (e*x^2)/d])
```

Maple [F]

$$\int x^{2} \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a +b \ln \left(c\,x^{n}\right)\right)dx$$

```
[In] int(x^2*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n)),x)
```

```
[Out] int(x^2*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\log \left(cx^n\right)\right)\,dx = \int \left(ex^{2}+d\right)^{\frac{3}{2}} (b\log \left(cx^{n}\right)+a)x^{2}\,dx$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] integral((b*e*x^4 + b*d*x^2)*sqrt(e*x^2 + d)*log(c*x^n) + (a*e*x^4 + a*d*x^
2)*sqrt(e*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\log \left(cx^n\right)\right)\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(e*x**2+d)**(3/2)*(a+b*ln(c*x**n)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 (d+ex^2)^{3/2} (a+b\log(cx^n)) \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^2 (d+ex^2)^{3/2} (a+b\log(cx^n)) \, dx = \int (ex^2+d)^{\frac{3}{2}} (b\log(cx^n)+a)x^2 \, dx$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*log(c*x^n) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d+ex^2)^{3/2} (a+b\log(cx^n)) \, dx = \int x^2 \left(e\,x^2+d\right)^{3/2} \left(a+b\,\ln\left(c\,x^n\right)\right) \, dx$$

```
[In] int(x^2*(d + e*x^2)^(3/2)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^2*(d + e*x^2)^(3/2)*(a + b*log(c*x^n)), x)
```

3.269 $\int (d+ex^2)^{3/2} (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 378

$$\int (d+ex^2)^{3/2} (a+b\log(cx^n))\,dx = -\frac{9}{32}bdnx\sqrt{d+ex^2} - \frac{1}{16}bnx(d+ex^2)^{3/2} + \frac{3bd^{5/2}n\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{16\sqrt{e}\sqrt{d+ex^2}} - \frac{9bd^2n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32\sqrt{e}} - \frac{3bd^{5/2}n\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{8\sqrt{e}\sqrt{d+ex^2}} + \frac{3}{8}dx\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{1}{4}x(d+ex^2)^{3/2}(a+b\log(cx^n)) + \frac{3d^{5/2}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx}{8\sqrt{e}\sqrt{d+ex^2}}$$

```
[Out] -1/16*b*n*x*(e*x^2+d)^(3/2)+1/4*x*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n))-9/32*b*d^
2*n*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(1/2)-9/32*b*d*n*x*(e*x^2+d)^(1/2)
+3/8*d*x*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)+3/16*b*d^(5/2)*n*arcsinh(x*e^(1/2)
/d^(1/2))^2*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)-3/8*b*d^(5/2)*n*arcsi
nh(x*e^(1/2)/d^(1/2))*ln(1-(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*(1+e*x^
2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)+3/8*d^(5/2)*arcsinh(x*e^(1/2)/d^(1/2))*(
a+b*ln(c*x^n))*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)-3/16*b*d^(5/2)*n*p
olylog(2,(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*(1+e*x^2/d)^(1/2)/e^(1/2)
/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2358, 201, 223, 212, 2364, 2362, 5775, 3797, 2221, 2317, 2438}

$$\int \left(d+ex^2\right)^{3/2} (a+b\log(cx^n))\,dx = \frac{3d^{5/2}\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8\sqrt{e}\sqrt{d+ex^2}}$$
$$+\frac{3}{8}dx\sqrt{d+ex^2}(a+b\log(cx^n))$$
$$+\frac{1}{4}x\left(d+ex^2\right)^{3/2}(a+b\log(cx^n))-\frac{3bd^{5/2}n\sqrt{\frac{ex^2}{d}+1}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{16\sqrt{e}\sqrt{d+ex^2}}+\frac{3bd^{5/2}n\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{16\sqrt{e}\sqrt{d+ex^2}}$$

```
[In] Int[(d + e*x^2)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (-9*b*d*n*x*Sqrt[d + e*x^2])/32 - (b*n*x*(d + e*x^2)^(3/2))/16 + (3*b*d^(5/
2)*n*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)/(16*Sqrt[e]*Sqrt[d
 + e*x^2]) - (9*b*d^2*n*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(32*Sqrt[e])
- (3*b*d^(5/2)*n*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[1 - E
^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(8*Sqrt[e]*Sqrt[d + e*x^2]) + (3*d*x*Sq
rt[d + e*x^2]*(a + b*Log[c*x^n]))/8 + (x*(d + e*x^2)^(3/2)*(a + b*Log[c*x^n
]))/4 + (3*d^(5/2)*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*(a + b*
Log[c*x^n]))/(8*Sqrt[e]*Sqrt[d + e*x^2]) - (3*b*d^(5/2)*n*Sqrt[1 + (e*x^2)/
d]*PolyLog[2, E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(16*Sqrt[e]*Sqrt[d + e*x
^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2358

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Sy
mbol] :> Simp[x*(d + e*x^2)^q*((a + b*Log[c*x^n])/(2*q + 1)), x] + (-Dist[b
*(n/(2*q + 1)), Int[(d + e*x^2)^q, x], x] + Dist[2*d*(q/(2*q + 1)), Int[(d
+ e*x^2)^(q - 1)*(a + b*Log[c*x^n]), x], x]) /; FreeQ[{a, b, c, d, e, n}, x
] && GtQ[q, 0]
```

Rule 2362

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[e, 2]), x]
 - Dist[b*(n/Rt[e, 2]), Int[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 2364

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Dist[Sqrt[1 + (e/d)*x^2]/Sqrt[d + e*x^2], Int[(a + b*Log[c*x^n])/Sqr
t[1 + (e/d)*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] &&  !GtQ[d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
```

```
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}x\left(d+ex^2\right)^{3/2}\left(a+b\log\left(cx^n\right)\right)+\frac{1}{4}(3d)\int\sqrt{d+ex^2}\left(a+b\log\left(cx^n\right)\right)\,dx-\frac{1}{4}(bn)\int\left(d+ex^2\right)^{3/2}\,dx$$

$$=-\frac{1}{16}bnx\left(d+ex^2\right)^{3/2}+\frac{3}{8}dx\sqrt{d+ex^2}\left(a+b\log\left(cx^n\right)\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\left(a+b\log\left(cx^n\right)\right)+\frac{1}{8}\left(3d^2\right)\int\frac{a+b\log\left(cx^n\right)}{\sqrt{d+ex^2}}\,dx-\frac{1}{16}(3bdn)\int\sqrt{d+ex^2}\,dx-\frac{1}{8}(3bdn)\int\sqrt{d+ex^2}\,dx$$

$$=-\frac{9}{32}bdnx\sqrt{d+ex^2}-\frac{1}{16}bnx\left(d+ex^2\right)^{3/2}+\frac{3}{8}dx\sqrt{d+ex^2}\left(a+b\log\left(cx^n\right)\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\left(a+b\log\left(cx^n\right)\right)-\frac{1}{32}\left(3bd^2n\right)\int\frac{1}{\sqrt{d+ex^2}}\,dx-\frac{1}{16}\left(3bd^2n\right)\int\frac{1}{\sqrt{d+ex^2}}\,dx+\frac{\left(3d^2\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{a+b\log(cx^n)}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{8\sqrt{d+ex^2}}$$

$$=-\frac{9}{32}bdnx\sqrt{d+ex^2}-\frac{1}{16}bnx\left(d+ex^2\right)^{3/2}+\frac{3}{8}dx\sqrt{d+ex^2}\left(a+b\log\left(cx^n\right)\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\left(a+b\log\left(cx^n\right)\right)+\frac{3d^{5/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(a+b\log\left(cx^n\right)\right)}{8\sqrt{e}\sqrt{d+ex^2}}-\frac{1}{32}\left(3bd^2n\right)\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)-\frac{1}{16}\left(3bd^2n\right)\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)$$

$$=-\frac{9}{32}bdnx\sqrt{d+ex^2}-\frac{1}{16}bnx\left(d+ex^2\right)^{3/2}-\frac{9bd^2n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32\sqrt{e}}+\frac{3}{8}dx\sqrt{d+ex^2}\left(a+b\log\left(cx^n\right)\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\left(a+b\log\left(cx^n\right)\right)+\frac{3d^{5/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(a+b\log\left(cx^n\right)\right)}{8\sqrt{e}\sqrt{d+ex^2}}-\frac{\left(3bd^{5/2}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int x\coth(x)\,dx,x,\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{8\sqrt{e}\sqrt{d+ex^2}}$$

$$= -\frac{9}{32}bdnx\sqrt{d+ex^2} - \frac{1}{16}bnx\left(d+ex^2\right)^{3/2} + \frac{3bd^{5/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{16\sqrt{e}\sqrt{d+ex^2}}$$
$$- \frac{9bd^2n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32\sqrt{e}} + \frac{3}{8}dx\sqrt{d+ex^2}(a+b\log(cx^n))$$
$$+ \frac{1}{4}x\left(d+ex^2\right)^{3/2}(a+b\log(cx^n)) + \frac{3d^{5/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{8\sqrt{e}\sqrt{d+ex^2}} + \frac{\left(3bd^{5/2}n\sqrt{1+\right.}{}$$

$$= -\frac{9}{32}bdnx\sqrt{d+ex^2} - \frac{1}{16}bnx\left(d+ex^2\right)^{3/2} + \frac{3bd^{5/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{16\sqrt{e}\sqrt{d+ex^2}}$$
$$- \frac{9bd^2n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32\sqrt{e}} - \frac{3bd^{5/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{8\sqrt{e}\sqrt{d+ex^2}}$$
$$+ \frac{3}{8}dx\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{1}{4}x\left(d+ex^2\right)^{3/2}(a+b\log(cx^n)) + \frac{3d^{5/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a}{8\sqrt{e}\sqrt{d+ex^2}}$$

$$= -\frac{9}{32}bdnx\sqrt{d+ex^2} - \frac{1}{16}bnx\left(d+ex^2\right)^{3/2} + \frac{3bd^{5/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{16\sqrt{e}\sqrt{d+ex^2}}$$
$$- \frac{9bd^2n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32\sqrt{e}} - \frac{3bd^{5/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{8\sqrt{e}\sqrt{d+ex^2}}$$
$$+ \frac{3}{8}dx\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{1}{4}x\left(d+ex^2\right)^{3/2}(a+b\log(cx^n)) + \frac{3d^{5/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a}{8\sqrt{e}\sqrt{d+ex^2}}$$

$$= -\frac{9}{32}bdnx\sqrt{d+ex^2} - \frac{1}{16}bnx\left(d+ex^2\right)^{3/2} + \frac{3bd^{5/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{16\sqrt{e}\sqrt{d+ex^2}}$$
$$- \frac{9bd^2n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32\sqrt{e}} - \frac{3bd^{5/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{8\sqrt{e}\sqrt{d+ex^2}}$$
$$+ \frac{3}{8}dx\sqrt{d+ex^2}(a+b\log(cx^n)) + \frac{1}{4}x\left(d+ex^2\right)^{3/2}(a+b\log(cx^n)) + \frac{3d^{5/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a}{8\sqrt{e}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.55 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.83

$$\int (d+ex^2)^{3/2} (a +b\log(cx^n))\,dx = \frac{-8be^{3/2}nx^3\sqrt{d+ex^2}\,{}_3F_2\left(-\frac{1}{2},\frac{3}{2},\frac{3}{2};\frac{5}{2},\frac{5}{2};-\frac{ex^2}{d}\right)+9\left(-4bd\sqrt{e}nx\sqrt{d+ex^2}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\right.\right.}{}$$

```
[In] Integrate[(d + e*x^2)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (-8*b*e^(3/2)*n*x^3*Sqrt[d + e*x^2]*HypergeometricPFQ[{-1/2, 3/2, 3/2}, {5/
2, 5/2}, -((e*x^2)/d)] + 9*(-4*b*d*Sqrt[e]*n*x*Sqrt[d + e*x^2]*Hypergeometr
icPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, -((e*x^2)/d)] + b*d^(3/2)*n*Sqrt[d + e*x
^2]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*(-2 + 3*Log[x]) + Sqrt[1 + (e*x^2)/d]*(Sqr
t[e]*x*Sqrt[d + e*x^2]*(5*a*d - 2*b*d*n + 2*a*e*x^2) + 3*d^2*(a - b*n*Log[x
])*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]] + b*Log[c*x^n]*(Sqrt[e]*x*Sqrt[d + e*
x^2]*(5*d + 2*e*x^2) + 3*d^2*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]]))))/(72*Sqr
t[e]*Sqrt[1 + (e*x^2)/d])
```

Maple [F]

$$\int \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a +b \ln \left(c\,x^{n}\right)\right)dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*ln(c*x^n)),x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int (d+ex^2)^{3/2} (a+b\log(cx^n))\,dx = \int (ex^2+d)^{\frac{3}{2}}(b\log(cx^n)+a)\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] integral((b*e*x^2 + b*d)*sqrt(e*x^2 + d)*log(c*x^n) + (a*e*x^2 + a*d)*sqrt(
e*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{3/2} (a+b\log\left(cx^n\right))\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*ln(c*x**n)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{3/2} (a+b\log\left(cx^n\right))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \left(d+ex^2\right)^{3/2} (a+b\log\left(cx^n\right))\,dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\log\left(cx^n\right)+a)\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{3/2} (a+b\log\left(cx^n\right))\,dx = \int \left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\ln\left(c\,x^n\right)\right) \,dx$$

```
[In] int((d + e*x^2)^(3/2)*(a + b*log(c*x^n)),x)
```

```
[Out] int((d + e*x^2)^(3/2)*(a + b*log(c*x^n)), x)
```

3.270 $\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 400

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^2} dx = -\frac{bdn\sqrt{d+ex^2}}{x} - \frac{1}{4}benx\sqrt{d+ex^2} + \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{1+\frac{ex^2}{d}}} + \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4\sqrt{1+\frac{ex^2}{d}}} - \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{1+\frac{ex^2}{d}}} + \frac{3}{2}ex\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x} + \frac{3\sqrt{d}\sqrt{e}\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{1+\frac{ex^2}{d}}} - \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{4\sqrt{1+\frac{ex^2}{d}}}$$

```
[Out] -(e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/x-b*d*n*(e*x^2+d)^(1/2)/x-1/4*b*e*n*x*(e*x
^2+d)^(1/2)+3/2*e*x*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)+3/4*b*n*arcsinh(x*e^(1/
2)/d^(1/2))*d^(1/2)*e^(1/2)*(e*x^2+d)^(1/2)/(1+e*x^2/d)^(1/2)+3/4*b*n*arcsi
nh(x*e^(1/2)/d^(1/2))^2*d^(1/2)*e^(1/2)*(e*x^2+d)^(1/2)/(1+e*x^2/d)^(1/2)-3
/2*b*n*arcsinh(x*e^(1/2)/d^(1/2))*ln(1-(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2)
)^2)*d^(1/2)*e^(1/2)*(e*x^2+d)^(1/2)/(1+e*x^2/d)^(1/2)+3/2*arcsinh(x*e^(1/2
```

```
)/d^(1/2))*(a+b*ln(c*x^n))*d^(1/2)*e^(1/2)*(e*x^2+d)^(1/2)/(1+e*x^2/d)^(1/2
)-3/4*b*n*polylog(2,(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*e^(1/2
)*(e*x^2+d)^(1/2)/(1+e*x^2/d)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2386, 283, 201, 221, 2392, 12, 14, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^2}\,dx = \frac{3\sqrt{d}\sqrt{e}\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{\frac{ex^2}{d}+1}} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x} + \frac{3}{2}ex\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{4\sqrt{\frac{ex^2}{d}+1}} + \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4\sqrt{\frac{ex^2}{d}+1}} + \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{\frac{ex^2}{d}+1}} - \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{\frac{ex^2}{d}+1}} - \frac{1}{4}benx\sqrt{d+ex^2} - \frac{bdn\sqrt{d+ex^2}}{x}$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -((b*d*n*Sqrt[d + e*x^2])/x) - (b*e*n*x*Sqrt[d + e*x^2])/4 + (3*b*Sqrt[d]*S
qrt[e]*n*Sqrt[d + e*x^2]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])/(4*Sqrt[1 + (e*x^2)/
d]) + (3*b*Sqrt[d]*Sqrt[e]*n*Sqrt[d + e*x^2]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2
)/(4*Sqrt[1 + (e*x^2)/d]) - (3*b*Sqrt[d]*Sqrt[e]*n*Sqrt[d + e*x^2]*ArcSinh[
(Sqrt[e]*x)/Sqrt[d]]*Log[1 - E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(2*Sqrt[1
 + (e*x^2)/d]) + (3*e*x*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/2 - ((d + e*x^2
)^(3/2)*(a + b*Log[c*x^n]))/x + (3*Sqrt[d]*Sqrt[e]*Sqrt[d + e*x^2]*ArcSinh[
(Sqrt[e]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/(2*Sqrt[1 + (e*x^2)/d]) - (3*b*Sqr
t[d]*Sqrt[e]*n*Sqrt[d + e*x^2]*PolyLog[2, E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]
)])/(4*Sqrt[1 + (e*x^2)/d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2386

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^
(q_), x_Symbol] :> Dist[d^IntPart[q]*((d + e*x^2)^FracPart[q]/(1 + (e/d)*x^
2)^FracPart[q]), Int[x^m*(1 + (e/d)*x^2)^q*(a + b*Log[c*x^n]), x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && IntegerQ[m/2] && IntegerQ[q - 1/2] && !(LtQ[
m + 2*q, -2] || GtQ[d, 0])
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(d\sqrt{d+ex^2}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^{3/2}(a+b\log(cx^n))}{x^2}\,dx}{\sqrt{1+\frac{ex^2}{d}}}$$

$$
\begin{aligned}
&= \frac{3}{2}ex\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x} \\
&\quad + \frac{3\sqrt{d}\sqrt{e}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{1+\frac{ex^2}{d}}} \\
&\quad - \frac{\left(bdn\sqrt{d+ex^2}\right)\int \frac{(-2d+ex^2)\sqrt{1+\frac{ex^2}{d}}+3\sqrt{d}\sqrt{e}x\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2dx^2}\,dx}{\sqrt{1+\frac{ex^2}{d}}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3}{2}ex\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x} \\
&\quad + \frac{3\sqrt{d}\sqrt{e}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{1+\frac{ex^2}{d}}} \\
&\quad - \frac{\left(bn\sqrt{d+ex^2}\right)\int \frac{(-2d+ex^2)\sqrt{1+\frac{ex^2}{d}}+3\sqrt{d}\sqrt{e}x\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x^2}\,dx}{2\sqrt{1+\frac{ex^2}{d}}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3}{2}ex\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x} \\
&\quad + \frac{3\sqrt{d}\sqrt{e}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{1+\frac{ex^2}{d}}} \\
&\quad - \frac{\left(bn\sqrt{d+ex^2}\right)\int \left(e\sqrt{1+\frac{ex^2}{d}} - \frac{2d\sqrt{1+\frac{ex^2}{d}}}{x^2} + \frac{3\sqrt{d}\sqrt{e}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\right)dx}{2\sqrt{1+\frac{ex^2}{d}}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3}{2}ex\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x} \\
&\quad + \frac{3\sqrt{d}\sqrt{e}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(bdn\sqrt{d+ex^2}\right)\int \frac{\sqrt{1+\frac{ex^2}{d}}}{x^2}\,dx}{\sqrt{1+\frac{ex^2}{d}}} \\
&\quad - \frac{\left(3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\right)\int \frac{\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(ben\sqrt{d+ex^2}\right)\int \sqrt{1+\frac{ex^2}{d}}\,dx}{2\sqrt{1+\frac{ex^2}{d}}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bdn\sqrt{d+ex^2}}{x} - \frac{1}{4}benx\sqrt{d+ex^2} + \frac{3}{2}ex\sqrt{d+ex^2}(a+b\log(cx^n)) \\
&\quad - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x} + \frac{3\sqrt{d}\sqrt{e}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{1+\frac{ex^2}{d}}} \\
&\quad - \frac{\left(3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\right)\mathrm{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{2\sqrt{1+\frac{ex^2}{d}}} \\
&\quad - \frac{\left(ben\sqrt{d+ex^2}\right)\int \frac{1}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{4\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(ben\sqrt{d+ex^2}\right)\int \frac{1}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{\sqrt{1+\frac{ex^2}{d}}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bdn\sqrt{d+ex^2}}{x} - \frac{1}{4}benx\sqrt{d+ex^2} + \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{1+\frac{ex^2}{d}}} \\
&\quad + \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4\sqrt{1+\frac{ex^2}{d}}} + \frac{3}{2}ex\sqrt{d+ex^2}(a+b\log(cx^n)) \\
&\quad - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x} + \frac{3\sqrt{d}\sqrt{e}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{1+\frac{ex^2}{d}}} \\
&\quad + \frac{\left(3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\right)\mathrm{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{\sqrt{1+\frac{ex^2}{d}}}
\end{aligned}
$$

$$= -\frac{bdn\sqrt{d+ex^2}}{x} - \frac{1}{4}benx\sqrt{d+ex^2} + \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4\sqrt{1+\frac{ex^2}{d}}}$$
$$- \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{3}{2}ex\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x}$$
$$+ \frac{3\sqrt{d}\sqrt{e}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{\left(3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\right)\text{Subst}\left(\int\log\left(1-e^{2x}\right)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{2\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{bdn\sqrt{d+ex^2}}{x} - \frac{1}{4}benx\sqrt{d+ex^2} + \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4\sqrt{1+\frac{ex^2}{d}}}$$
$$- \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{3}{2}ex\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x}$$
$$+ \frac{3\sqrt{d}\sqrt{e}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{\left(3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{4\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{bdn\sqrt{d+ex^2}}{x} - \frac{1}{4}benx\sqrt{d+ex^2} + \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4\sqrt{1+\frac{ex^2}{d}}}$$
$$- \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{3}{2}ex\sqrt{d+ex^2}(a+b\log(cx^n)) - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x}$$
$$+ \frac{3\sqrt{d}\sqrt{e}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2\sqrt{1+\frac{ex^2}{d}}}$$
$$- \frac{3b\sqrt{d}\sqrt{e}n\sqrt{d+ex^2}\operatorname{Li}_2\left(e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{4\sqrt{1+\frac{ex^2}{d}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.63 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^2}\,dx =$$
$$-\frac{b\sqrt{d}n\sqrt{d+ex^2}\left(\sqrt{d}\,{}_3F_2\left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2};\frac{1}{2},\frac{1}{2};-\frac{ex^2}{d}\right)+\left(\sqrt{d}\sqrt{1+\frac{ex^2}{d}}-\sqrt{e}x\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)\log(x)\right)}{x\sqrt{1+\frac{ex^2}{d}}}$$
$$+\frac{b\sqrt{e}n\sqrt{d+ex^2}\left(-2\sqrt{e}x\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{ex^2}{d}\right)+\left(\sqrt{e}x\sqrt{1+\frac{ex^2}{d}}+\sqrt{d}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)(-1+2\log(x))\right)}{4\sqrt{1+\frac{ex^2}{d}}}$$
$$-\frac{(2d-ex^2)\sqrt{d+ex^2}(a-bn\log(x)+b\log(cx^n))}{2x}$$
$$+\frac{3}{2}d\sqrt{e}(a-bn\log(x)+b\log(cx^n))\log\left(ex+\sqrt{e}\sqrt{d+ex^2}\right)$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -((b*Sqrt[d]*n*Sqrt[d + e*x^2]*(Sqrt[d]*HypergeometricPFQ[{-1/2, -1/2, -1/2
}, {1/2, 1/2}, -((e*x^2)/d)] + (Sqrt[d]*Sqrt[1 + (e*x^2)/d] - Sqrt[e]*x*Arc
```

```
Sinh[(Sqrt[e]*x)/Sqrt[d]])*Log[x]))/(x*Sqrt[1 + (e*x^2)/d])) + (b*Sqrt[e]*n
*Sqrt[d + e*x^2]*(-2*Sqrt[e]*x*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2
}, -((e*x^2)/d)] + (Sqrt[e]*x*Sqrt[1 + (e*x^2)/d] + Sqrt[d]*ArcSinh[(Sqrt[e
]*x)/Sqrt[d]])*(-1 + 2*Log[x])))/(4*Sqrt[1 + (e*x^2)/d]) - ((2*d - e*x^2)*S
qrt[d + e*x^2]*(a - b*n*Log[x] + b*Log[c*x^n]))/(2*x) + (3*d*Sqrt[e]*(a - b
*n*Log[x] + b*Log[c*x^n])*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/2
```

Maple [F]

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\ln\left(c\,x^{n}\right)\right)}{x^{2}}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/x^2,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/x^2,x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\log\left(cx^n\right)+a)}{x^2}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
[Out] integral(((b*e*x^2 + b*d)*sqrt(e*x^2 + d)*log(c*x^n) + (a*e*x^2 + a*d)*sqrt
(e*x^2 + d))/x^2, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\left(a+b\log\left(cx^n\right)\right)(d+ex^2)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*ln(c*x**n))/x**2,x)
[Out] Integral((a + b*log(c*x**n))*(d + e*x**2)**(3/2)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\log(cx^n)+a)}{x^2}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*log(c*x^n) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^2}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,(a+b\,\ln(c\,x^n))}{x^2}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int(((d + e*x^2)^(3/2)*(a + b*log(c*x^n)))/x^2, x)
```

3.271 $\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 400

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^4}\,dx = -\frac{4ben\sqrt{d+ex^2}}{3x} - \frac{bn(d+ex^2)^{3/2}}{9x^3} + \frac{4be^{3/2}n\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} + \frac{be^{3/2}n\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} - \frac{be^{3/2}n\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} - \frac{e\sqrt{d+ex^2}(a+b\log(cx^n))}{x} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3x^3} + \frac{e^{3/2}\sqrt{d+ex^2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} - \frac{be^{3/2}n\sqrt{d+ex^2}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

```
[Out] -1/9*b*n*(e*x^2+d)^(3/2)/x^3-1/3*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/x^3-4/3*b*
e*n*(e*x^2+d)^(1/2)/x-e*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/x+4/3*b*e^(3/2)*n*a
rcsinh(x*e^(1/2)/d^(1/2))*(e*x^2+d)^(1/2)/d^(1/2)/(1+e*x^2/d)^(1/2)+1/2*b*e
^(3/2)*n*arcsinh(x*e^(1/2)/d^(1/2))^2*(e*x^2+d)^(1/2)/d^(1/2)/(1+e*x^2/d)^(
1/2)-b*e^(3/2)*n*arcsinh(x*e^(1/2)/d^(1/2))*ln(1-(x*e^(1/2)/d^(1/2)+(1+e*x^
2/d)^(1/2))^2)*(e*x^2+d)^(1/2)/d^(1/2)/(1+e*x^2/d)^(1/2)+e^(3/2)*arcsinh(x*
```

```
e^(1/2)/d^(1/2))*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/d^(1/2)/(1+e*x^2/d)^(1/2)-
1/2*b*e^(3/2)*n*polylog(2,(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*(e*x^2+d
)^(1/2)/d^(1/2)/(1+e*x^2/d)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2386, 283, 221, 2392, 462, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^4}\,dx = \frac{e^{3/2}\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}} - \frac{e\sqrt{d+ex^2}(a+b\log(cx^n))}{x} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3x^3} - \frac{be^{3/2}n\sqrt{d+ex^2}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{d}\sqrt{\frac{ex^2}{d}+1}} + \frac{be^{3/2}n\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d}\sqrt{\frac{ex^2}{d}+1}} + \frac{4be^{3/2}n\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3\sqrt{d}\sqrt{\frac{ex^2}{d}+1}} - \frac{be^{3/2}n\sqrt{d+ex^2}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}} - \frac{4ben\sqrt{d+ex^2}}{3x} - \frac{bn(d+ex^2)^{3/2}}{9x^3}$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] (-4*b*e*n*Sqrt[d + e*x^2])/(3*x) - (b*n*(d + e*x^2)^(3/2))/(9*x^3) + (4*b*e
^(3/2)*n*Sqrt[d + e*x^2]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])/(3*Sqrt[d]*Sqrt[1 +
(e*x^2)/d]) + (b*e^(3/2)*n*Sqrt[d + e*x^2]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)/
(2*Sqrt[d]*Sqrt[1 + (e*x^2)/d]) - (b*e^(3/2)*n*Sqrt[d + e*x^2]*ArcSinh[(Sqr
t[e]*x)/Sqrt[d]]*Log[1 - E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(Sqrt[d]*Sqrt
[1 + (e*x^2)/d]) - (e*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/x - ((d + e*x^2)^
(3/2)*(a + b*Log[c*x^n]))/(3*x^3) + (e^(3/2)*Sqrt[d + e*x^2]*ArcSinh[(Sqrt[
e]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/(Sqrt[d]*Sqrt[1 + (e*x^2)/d]) - (b*e^(3/
2)*n*Sqrt[d + e*x^2]*PolyLog[2, E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(2*Sqr
t[d]*Sqrt[1 + (e*x^2)/d])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2386

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^
(q_), x_Symbol] :> Dist[d^IntPart[q]*((d + e*x^2)^FracPart[q]/(1 + (e/d)*x^
2)^FracPart[q]), Int[x^m*(1 + (e/d)*x^2)^q*(a + b*Log[c*x^n]), x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && IntegerQ[m/2] && IntegerQ[q - 1/2] &&  !(LtQ[
m + 2*q, -2] || GtQ[d, 0])
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
```

```
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(d\sqrt{d+ex^2}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^{3/2}(a+b\log(cx^n))}{x^4}\,dx}{\sqrt{1+\frac{ex^2}{d}}} \\
&= -\frac{e\sqrt{d+ex^2}(a+b\log(cx^n))}{x} - \frac{\left(d+ex^2\right)^{3/2}(a+b\log(cx^n))}{3x^3} \\
&\quad + \frac{e^{3/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad - \frac{\left(bdn\sqrt{d+ex^2}\right)\int\left(-\frac{(d+4ex^2)\sqrt{1+\frac{ex^2}{d}}}{3dx^4} + \frac{e^{3/2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}x}\right)dx}{\sqrt{1+\frac{ex^2}{d}}}
\end{aligned}$$

$$= -\frac{e\sqrt{d+ex^2}(a+b\log(cx^n))}{x} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3x^3} + \frac{e^{3/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(bn\sqrt{d+ex^2}\right)\int\frac{(d+4ex^2)\sqrt{1+\frac{ex^2}{d}}}{x^4}\,dx}{3\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(be^{3/2}n\sqrt{d+ex^2}\right)\int\frac{\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{bn(d+ex^2)^{3/2}}{9x^3} - \frac{e\sqrt{d+ex^2}(a+b\log(cx^n))}{x} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3x^3} + \frac{e^{3/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(4ben\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{x^2}\,dx}{3\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(be^{3/2}n\sqrt{d+ex^2}\right)\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{4ben\sqrt{d+ex^2}}{3x} - \frac{bn(d+ex^2)^{3/2}}{9x^3} + \frac{be^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} - \frac{e\sqrt{d+ex^2}(a+b\log(cx^n))}{x} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3x^3} + \frac{e^{3/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(2be^{3/2}n\sqrt{d+ex^2}\right)\text{Subst}\left(\int\frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(4be^2n\sqrt{d+ex^2}\right)\int\frac{1}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{3d\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{4ben\sqrt{d+ex^2}}{3x} - \frac{bn(d+ex^2)^{3/2}}{9x^3} + \frac{4be^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{be^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$
$$- \frac{be^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$
$$- \frac{e\sqrt{d+ex^2}(a+b\log(cx^n))}{x} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3x^3}$$
$$+ \frac{e^{3/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{\left(be^{3/2}n\sqrt{d+ex^2}\right)\text{Subst}\left(\int\log\left(1-e^{2x}\right)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

$$= -\frac{4ben\sqrt{d+ex^2}}{3x} - \frac{bn(d+ex^2)^{3/2}}{9x^3} + \frac{4be^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{be^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$
$$- \frac{be^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$
$$- \frac{e\sqrt{d+ex^2}(a+b\log(cx^n))}{x} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3x^3}$$
$$+ \frac{e^{3/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{\left(be^{3/2}n\sqrt{d+ex^2}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

$$
\begin{aligned}
&= -\frac{4ben\sqrt{d+ex^2}}{3x} - \frac{bn(d+ex^2)^{3/2}}{9x^3} \\
&\quad + \frac{4be^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} + \frac{be^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad - \frac{be^{3/2}n\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad - \frac{e\sqrt{d+ex^2}(a+b\log(cx^n))}{x} - \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3x^3} \\
&\quad + \frac{e^{3/2}\sqrt{d+ex^2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad - \frac{be^{3/2}n\sqrt{d+ex^2}\operatorname{Li}_2\left(e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.50 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.67

$$
\begin{aligned}
&\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^4}\,dx = \frac{bdn\sqrt{d+ex^2}\left(-\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},-\frac{ex^2}{d}\right)-3\left(1+\frac{ex^2}{d}\right.\right.}{9x^3\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{ben\sqrt{d+ex^2}\left(-{}_3F_2\left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2};\frac{1}{2},\frac{1}{2};-\frac{ex^2}{d}\right)-\sqrt{1+\frac{ex^2}{d}}\log(x)+\frac{\sqrt{e}x\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{d}}\right)}{x\sqrt{1+\frac{ex^2}{d}}} \\
&- \frac{\sqrt{d+ex^2}(d+4ex^2)(a-bn\log(x)+b\log(cx^n))}{3x^3} \\
&+ e^{3/2}(a-bn\log(x)+b\log(cx^n))\log\left(ex+\sqrt{e}\sqrt{d+ex^2}\right)
\end{aligned}
$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] (b*d*n*Sqrt[d + e*x^2]*(-Hypergeometric2F1[-3/2, -3/2, -1/2, -((e*x^2)/d)]
- 3*(1 + (e*x^2)/d)^(3/2)*Log[x]))/(9*x^3*Sqrt[1 + (e*x^2)/d]) + (b*e*n*Sqr
t[d + e*x^2]*(-HypergeometricPFQ[{-1/2, -1/2, -1/2}, {1/2, 1/2}, -((e*x^2)/
d)] - Sqrt[1 + (e*x^2)/d]*Log[x] + (Sqrt[e]*x*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*
Log[x])/Sqrt[d]))/(x*Sqrt[1 + (e*x^2)/d]) - (Sqrt[d + e*x^2]*(d + 4*e*x^2)*
(a - b*n*Log[x] + b*Log[c*x^n]))/(3*x^3) + e^(3/2)*(a - b*n*Log[x] + b*Log[
c*x^n])*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]]
```

Maple [F]

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\ln\left(c\,x^{n}\right)\right)}{x^{4}}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/x^4,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/x^4,x)
```

Fricas [F]

$$\int \frac{\left(d+ex^{2}\right)^{3/2}\left(a+b\log\left(cx^{n}\right)\right)}{x^{4}}\,dx = \int \frac{\left(ex^{2}+d\right)^{\frac{3}{2}}\left(b\log\left(cx^{n}\right)+a\right)}{x^{4}}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^4,x, algorithm="fricas")
[Out] integral(((b*e*x^2 + b*d)*sqrt(e*x^2 + d)*log(c*x^n) + (a*e*x^2 + a*d)*sqrt
(e*x^2 + d))/x^4, x)
```

Sympy [F]

$$\int \frac{\left(d+ex^{2}\right)^{3/2}\left(a+b\log\left(cx^{n}\right)\right)}{x^{4}}\,dx = \int \frac{\left(a+b\log\left(cx^{n}\right)\right)\left(d+ex^{2}\right)^{\frac{3}{2}}}{x^{4}}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*ln(c*x**n))/x**4,x)
[Out] Integral((a + b*log(c*x**n))*(d + e*x**2)**(3/2)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex^{2}\right)^{3/2}\left(a+b\log\left(cx^{n}\right)\right)}{x^{4}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^4,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\log(cx^n))}{x^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\log(cx^n)+a)}{x^4}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^4,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*log(c*x^n) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\log(cx^n))}{x^4}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,(a+b\,\ln(c\,x^n))}{x^4}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*log(c*x^n)))/x^4,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*log(c*x^n)))/x^4, x)
```

3.272 $\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^6}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^6}\,dx = -\frac{be^2n\sqrt{d+ex^2}}{5dx} - \frac{ben(d+ex^2)^{3/2}}{15dx^3} - \frac{bn(d+ex^2)^{5/2}}{25dx^5} + \frac{be^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5d} - \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5dx^5}$$

```
[Out] -1/15*b*e*n*(e*x^2+d)^(3/2)/d/x^3-1/25*b*n*(e*x^2+d)^(5/2)/d/x^5+1/5*b*e^(5
/2)*n*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/d-1/5*(e*x^2+d)^(5/2)*(a+b*ln(c*x^
n))/d/x^5-1/5*b*e^2*n*(e*x^2+d)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2373, 283, 223, 212}

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^6}\,dx = -\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5dx^5} + \frac{be^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5d} - \frac{be^2n\sqrt{d+ex^2}}{5dx} - \frac{bn(d+ex^2)^{5/2}}{25dx^5} - \frac{ben(d+ex^2)^{3/2}}{15dx^3}$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] -1/5*(b*e^2*n*Sqrt[d + e*x^2])/(d*x) - (b*e*n*(d + e*x^2)^(3/2))/(15*d*x^3)
 - (b*n*(d + e*x^2)^(5/2))/(25*d*x^5) + (b*e^(5/2)*n*ArcTanh[(Sqrt[e]*x)/Sq
rt[d + e*x^2]])/(5*d) - ((d + e*x^2)^(5/2)*(a + b*Log[c*x^n]))/(5*d*x^5)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d+ex^2)^{5/2}\,(a+b\log(cx^n))}{5dx^5}+\frac{(bn)\int\frac{(d+ex^2)^{5/2}}{x^6}\,dx}{5d}\\
&= -\frac{bn(d+ex^2)^{5/2}}{25dx^5}-\frac{(d+ex^2)^{5/2}\,(a+b\log(cx^n))}{5dx^5}+\frac{(ben)\int\frac{(d+ex^2)^{3/2}}{x^4}\,dx}{5d}\\
&= -\frac{ben(d+ex^2)^{3/2}}{15dx^3}-\frac{bn(d+ex^2)^{5/2}}{25dx^5}-\frac{(d+ex^2)^{5/2}\,(a+b\log(cx^n))}{5dx^5}+\frac{(be^2n)\int\frac{\sqrt{d+ex^2}}{x^2}\,dx}{5d}\\
&= -\frac{be^2n\sqrt{d+ex^2}}{5dx}-\frac{ben(d+ex^2)^{3/2}}{15dx^3}-\frac{bn(d+ex^2)^{5/2}}{25dx^5}\\
&\quad-\frac{(d+ex^2)^{5/2}\,(a+b\log(cx^n))}{5dx^5}+\frac{(be^3n)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{5d}\\
&= -\frac{be^2n\sqrt{d+ex^2}}{5dx}-\frac{ben(d+ex^2)^{3/2}}{15dx^3}-\frac{bn(d+ex^2)^{5/2}}{25dx^5}\\
&\quad-\frac{(d+ex^2)^{5/2}\,(a+b\log(cx^n))}{5dx^5}+\frac{(be^3n)\,\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{5d}
\end{aligned}$$

$$= -\frac{be^2n\sqrt{d+ex^2}}{5dx} - \frac{ben(d+ex^2)^{3/2}}{15dx^3} - \frac{bn(d+ex^2)^{5/2}}{25dx^5} + \frac{be^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5d} - \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5dx^5}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^6}\,dx = -\frac{\sqrt{d+ex^2}\left(15a(d+ex^2)^2+bn(3d^2+11dex^2+23e^2x^4)\right)+15b(d+ex^2)^{5/2}\log(cx^n)-15be^{5/2}nx^5\log(ex}{75dx^5}$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] -1/75*(Sqrt[d + e*x^2]*(15*a*(d + e*x^2)^2 + b*n*(3*d^2 + 11*d*e*x^2 + 23*e
^2*x^4)) + 15*b*(d + e*x^2)^(5/2)*Log[c*x^n] - 15*b*e^(5/2)*n*x^5*Log[e*x +
 Sqrt[e]*Sqrt[d + e*x^2]])/(d*x^5)
```

Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}(a+b\ln(c\,x^n))}{x^6}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/x^6,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/x^6,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.28

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^6}\,dx = \left[\frac{15\,be^{\frac{5}{2}}nx^5\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-2\left((23\,be^2n+15\,ae^2\right.}{}\right.$$

$$-\frac{15\,b\sqrt{-e}e^2nx^5\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+((23\,be^2n+15\,ae^2)x^4+3\,bd^2n+15\,ad^2+(11\,bden+30\,ade)x^2+15\,(b}{75\,dx^5}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^6,x, algorithm="fricas")
```

```
[Out] [1/150*(15*b*e^(5/2)*n*x^5*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d)
- 2*((23*b*e^2*n + 15*a*e^2)*x^4 + 3*b*d^2*n + 15*a*d^2 + (11*b*d*e*n + 30*
a*d*e)*x^2 + 15*(b*e^2*x^4 + 2*b*d*e*x^2 + b*d^2)*log(c) + 15*(b*e^2*n*x^4
+ 2*b*d*e*n*x^2 + b*d^2*n)*log(x))*sqrt(e*x^2 + d))/(d*x^5), -1/75*(15*b*sq
rt(-e)*e^2*n*x^5*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + ((23*b*e^2*n + 15*a*e
^2)*x^4 + 3*b*d^2*n + 15*a*d^2 + (11*b*d*e*n + 30*a*d*e)*x^2 + 15*(b*e^2*x^
4 + 2*b*d*e*x^2 + b*d^2)*log(c) + 15*(b*e^2*n*x^4 + 2*b*d*e*n*x^2 + b*d^2*n
)*log(x))*sqrt(e*x^2 + d))/(d*x^5)]
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^6}\,dx = \int \frac{(a+b\log(cx^n))(d+ex^2)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*ln(c*x**n))/x**6,x)
```

```
[Out] Integral((a + b*log(c*x**n))*(d + e*x**2)**(3/2)/x**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^6}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\log(cx^n)+a)}{x^6}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*log(c*x^n) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^6}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,(a+b\,\ln(c\,x^n))}{x^6}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*log(c*x^n)))/x^6,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*log(c*x^n)))/x^6, x)
```

3.273 $\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^8}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 196

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^8}\,dx = \frac{2be^3n\sqrt{d+ex^2}}{35d^2x}+\frac{2be^2n(d+ex^2)^{3/2}}{105d^2x^3}+\frac{2ben(d+ex^2)^{5/2}}{175d^2x^5}-\frac{bn(d+ex^2)^{7/2}}{49d^2x^7}-\frac{2be^{7/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{35d^2}-\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{7dx^7}+\frac{2e(d+ex^2)^{5/2}(a+b\log(cx^n))}{35d^2x^5}$$

```
[Out] 2/105*b*e^2*n*(e*x^2+d)^(3/2)/d^2/x^3+2/175*b*e*n*(e*x^2+d)^(5/2)/d^2/x^5-1
/49*b*n*(e*x^2+d)^(7/2)/d^2/x^7-2/35*b*e^(7/2)*n*arctanh(x*e^(1/2)/(e*x^2+d
)^(1/2))/d^2-1/7*(e*x^2+d)^(5/2)*(a+b*ln(c*x^n))/d/x^7+2/35*e*(e*x^2+d)^(5/
2)*(a+b*ln(c*x^n))/d^2/x^5+2/35*b*e^3*n*(e*x^2+d)^(1/2)/d^2/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {277, 270, 2392, 12, 462, 283, 223, 212}

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^8}\,dx = \frac{2e(d+ex^2)^{5/2}(a+b\log(cx^n))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{7dx^7}-\frac{2be^{7/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{35d^2}+\frac{2be^3n\sqrt{d+ex^2}}{35d^2x}+\frac{2be^2n(d+ex^2)^{3/2}}{105d^2x^3}-\frac{bn(d+ex^2)^{7/2}}{49d^2x^7}+\frac{2ben(d+ex^2)^{5/2}}{175d^2x^5}$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/x^8,x]
```

```
[Out] (2*b*e^3*n*Sqrt[d + e*x^2])/(35*d^2*x) + (2*b*e^2*n*(d + e*x^2)^(3/2))/(105
*d^2*x^3) + (2*b*e*n*(d + e*x^2)^(5/2))/(175*d^2*x^5) - (b*n*(d + e*x^2)^(7
/2))/(49*d^2*x^7) - (2*b*e^(7/2)*n*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(3
5*d^2) - ((d + e*x^2)^(5/2)*(a + b*Log[c*x^n]))/(7*d*x^7) + (2*e*(d + e*x^2
)^(5/2)*(a + b*Log[c*x^n]))/(35*d^2*x^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{7dx^7}+\frac{2e(d+ex^2)^{5/2}(a+b\log(cx^n))}{35d^2x^5} \\
&\quad -(bn)\int\frac{(d+ex^2)^{5/2}(-5d+2ex^2)}{35d^2x^8}\,dx \\
&= -\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{7dx^7}+\frac{2e(d+ex^2)^{5/2}(a+b\log(cx^n))}{35d^2x^5} \\
&\quad -\frac{(bn)\int\frac{(d+ex^2)^{5/2}(-5d+2ex^2)}{x^8}\,dx}{35d^2} \\
&= -\frac{bn(d+ex^2)^{7/2}}{49d^2x^7}-\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{7dx^7} \\
&\quad +\frac{2e(d+ex^2)^{5/2}(a+b\log(cx^n))}{35d^2x^5}-\frac{(2ben)\int\frac{(d+ex^2)^{5/2}}{x^6}\,dx}{35d^2} \\
&= \frac{2ben(d+ex^2)^{5/2}}{175d^2x^5}-\frac{bn(d+ex^2)^{7/2}}{49d^2x^7}-\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{7dx^7} \\
&\quad +\frac{2e(d+ex^2)^{5/2}(a+b\log(cx^n))}{35d^2x^5}-\frac{(2be^2n)\int\frac{(d+ex^2)^{3/2}}{x^4}\,dx}{35d^2} \\
&= \frac{2be^2n(d+ex^2)^{3/2}}{105d^2x^3}+\frac{2ben(d+ex^2)^{5/2}}{175d^2x^5} \\
&\quad -\frac{bn(d+ex^2)^{7/2}}{49d^2x^7}-\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{7dx^7} \\
&\quad +\frac{2e(d+ex^2)^{5/2}(a+b\log(cx^n))}{35d^2x^5}-\frac{(2be^3n)\int\frac{\sqrt{d+ex^2}}{x^2}\,dx}{35d^2}
\end{aligned}$$

$$= \frac{2be^3n\sqrt{d+ex^2}}{35d^2x} + \frac{2be^2n(d+ex^2)^{3/2}}{105d^2x^3} + \frac{2ben(d+ex^2)^{5/2}}{175d^2x^5} - \frac{bn(d+ex^2)^{7/2}}{49d^2x^7} - \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\log(cx^n))}{35d^2x^5} - \frac{(2be^4n)\int\frac{1}{\sqrt{d+ex^2}}dx}{35d^2}$$

$$= \frac{2be^3n\sqrt{d+ex^2}}{35d^2x} + \frac{2be^2n(d+ex^2)^{3/2}}{105d^2x^3} + \frac{2ben(d+ex^2)^{5/2}}{175d^2x^5} - \frac{bn(d+ex^2)^{7/2}}{49d^2x^7} - \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\log(cx^n))}{35d^2x^5} - \frac{(2be^4n)\,\text{Subst}\left(\int\frac{1}{1-ex^2}dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{35d^2}$$

$$= \frac{2be^3n\sqrt{d+ex^2}}{35d^2x} + \frac{2be^2n(d+ex^2)^{3/2}}{105d^2x^3} + \frac{2ben(d+ex^2)^{5/2}}{175d^2x^5} - \frac{bn(d+ex^2)^{7/2}}{49d^2x^7} - \frac{2be^{7/2}n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{35d^2} - \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\log(cx^n))}{35d^2x^5}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.74

$$\int\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^8}dx = -\frac{\sqrt{d+ex^2}\Big(105a(5d-2ex^2)(d+ex^2)^2+bn(75d^3+183d^2ex^2+71de^2x^4-247e^3x^6)\Big)+105b(5d-2ex^2)(}{3675d^2x^7}$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/x^8,x]
[Out] -1/3675*(Sqrt[d + e*x^2]*(105*a*(5*d - 2*e*x^2)*(d + e*x^2)^2 + b*n*(75*d^3
 + 183*d^2*e*x^2 + 71*d*e^2*x^4 - 247*e^3*x^6)) + 105*b*(5*d - 2*e*x^2)*(d
+ e*x^2)^(5/2)*Log[c*x^n] + 210*b*e^(7/2)*n*x^7*Log[e*x + Sqrt[e]*Sqrt[d +
e*x^2]])/(d^2*x^7)
```

Maple [F]

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\ln\left(c\,x^{n}\right)\right)}{x^{8}}dx$$

[In] int((e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/x^8,x)

[Out] int((e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/x^8,x)

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.16

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\log(cx^n))}{x^8}\,dx = \left[\frac{105\,be^{\frac{7}{2}}nx^7\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+\left((247\,be^3n+210\,ae^3\right.}{}\right.$$

[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^8,x, algorithm="fricas")

[Out] [1/3675*(105*b*e^(7/2)*n*x^7*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d
) + ((247*b*e^3*n + 210*a*e^3)*x^6 - 75*b*d^3*n - (71*b*d*e^2*n + 105*a*d*e
^2)*x^4 - 525*a*d^3 - 3*(61*b*d^2*e*n + 280*a*d^2*e)*x^2 + 105*(2*b*e^3*x^6
 - b*d*e^2*x^4 - 8*b*d^2*e*x^2 - 5*b*d^3)*log(c) + 105*(2*b*e^3*n*x^6 - b*d
*e^2*n*x^4 - 8*b*d^2*e*n*x^2 - 5*b*d^3*n)*log(x))*sqrt(e*x^2 + d))/(d^2*x^7
), 1/3675*(210*b*sqrt(-e)*e^3*n*x^7*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + ((
247*b*e^3*n + 210*a*e^3)*x^6 - 75*b*d^3*n - (71*b*d*e^2*n + 105*a*d*e^2)*x^
4 - 525*a*d^3 - 3*(61*b*d^2*e*n + 280*a*d^2*e)*x^2 + 105*(2*b*e^3*x^6 - b*d
*e^2*x^4 - 8*b*d^2*e*x^2 - 5*b*d^3)*log(c) + 105*(2*b*e^3*n*x^6 - b*d*e^2*n
*x^4 - 8*b*d^2*e*n*x^2 - 5*b*d^3*n)*log(x))*sqrt(e*x^2 + d))/(d^2*x^7)]

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\log(cx^n))}{x^8}\,dx = \int \frac{(a+b\log(cx^n))\,(d+ex^2)^{\frac{3}{2}}}{x^8}\,dx$$

[In] integrate((e*x**2+d)**(3/2)*(a+b*ln(c*x**n))/x**8,x)

[Out] Integral((a + b*log(c*x**n))*(d + e*x**2)**(3/2)/x**8, x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^8}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^8}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\log(cx^n)+a)}{x^8}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*log(c*x^n) + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^8}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,(a+b\,\ln(c\,x^n))}{x^8}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*log(c*x^n)))/x^8,x)
```

```
[Out] int(((d + e*x^2)^(3/2)*(a + b*log(c*x^n)))/x^8, x)
```

3.274 $\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^{10}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 256

$$\begin{aligned}\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^{10}}\,dx = &-\frac{8be^4n\sqrt{d+ex^2}}{315d^3x}-\frac{8be^3n(d+ex^2)^{3/2}}{945d^3x^3}\\&-\frac{8be^2n(d+ex^2)^{5/2}}{1575d^3x^5}-\frac{bn(d+ex^2)^{7/2}}{81d^2x^9}+\frac{50ben(d+ex^2)^{7/2}}{3969d^3x^7}\\&+\frac{8be^{9/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{315d^3}-\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{9dx^9}\\&+\frac{4e(d+ex^2)^{5/2}(a+b\log(cx^n))}{63d^2x^7}-\frac{8e^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{315d^3x^5}\end{aligned}$$

```
[Out] -8/945*b*e^3*n*(e*x^2+d)^(3/2)/d^3/x^3-8/1575*b*e^2*n*(e*x^2+d)^(5/2)/d^3/x
^5-1/81*b*n*(e*x^2+d)^(7/2)/d^2/x^9+50/3969*b*e*n*(e*x^2+d)^(7/2)/d^3/x^7+8
/315*b*e^(9/2)*n*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/d^3-1/9*(e*x^2+d)^(5/2)
*(a+b*ln(c*x^n))/d/x^9+4/63*e*(e*x^2+d)^(5/2)*(a+b*ln(c*x^n))/d^2/x^7-8/315
*e^2*(e*x^2+d)^(5/2)*(a+b*ln(c*x^n))/d^3/x^5-8/315*b*e^4*n*(e*x^2+d)^(1/2)/
d^3/x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used

= {277, 270, 2392, 12, 1279, 462, 283, 223, 212}

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^{10}}\,dx = -\frac{8e^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{315d^3x^5} + \frac{4e(d+ex^2)^{5/2}(a+b\log(cx^n))}{63d^2x^7} - \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{9dx^9} + \frac{8be^{9/2}n\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{315d^3} - \frac{8be^4n\sqrt{d+ex^2}}{315d^3x} - \frac{8be^3n(d+ex^2)^{3/2}}{945d^3x^3} - \frac{8be^2n(d+ex^2)^{5/2}}{1575d^3x^5} + \frac{50ben(d+ex^2)^{7/2}}{3969d^3x^7} - \frac{bn(d+ex^2)^{7/2}}{81d^2x^9}$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/x^10,x]
```

```
[Out] (-8*b*e^4*n*Sqrt[d + e*x^2])/(315*d^3*x) - (8*b*e^3*n*(d + e*x^2)^(3/2))/(9
45*d^3*x^3) - (8*b*e^2*n*(d + e*x^2)^(5/2))/(1575*d^3*x^5) - (b*n*(d + e*x^
2)^(7/2))/(81*d^2*x^9) + (50*b*e*n*(d + e*x^2)^(7/2))/(3969*d^3*x^7) + (8*b
*e^(9/2)*n*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(315*d^3) - ((d + e*x^2)^(
5/2)*(a + b*Log[c*x^n]))/(9*d*x^9) + (4*e*(d + e*x^2)^(5/2)*(a + b*Log[c*x^
n]))/(63*d^2*x^7) - (8*e^2*(d + e*x^2)^(5/2)*(a + b*Log[c*x^n]))/(315*d^3*x
^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
```

```
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{9dx^9}+\frac{4e(d+ex^2)^{5/2}(a+b\log(cx^n))}{63d^2x^7} \\
&\quad -\frac{8e^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{315d^3x^5} \\
&\quad -(bn)\int\frac{(d+ex^2)^{5/2}(-35d^2+20dex^2-8e^2x^4)}{315d^3x^{10}}\,dx \\
&= -\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{9dx^9}+\frac{4e(d+ex^2)^{5/2}(a+b\log(cx^n))}{63d^2x^7} \\
&\quad -\frac{8e^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{315d^3x^5}-\frac{(bn)\int\frac{(d+ex^2)^{5/2}(-35d^2+20dex^2-8e^2x^4)}{x^{10}}\,dx}{315d^3} \\
&= -\frac{bn(d+ex^2)^{7/2}}{81d^2x^9}-\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{9dx^9}+\frac{4e(d+ex^2)^{5/2}(a+b\log(cx^n))}{63d^2x^7} \\
&\quad -\frac{8e^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{315d^3x^5}+\frac{(bn)\int\frac{(d+ex^2)^{5/2}(-250d^2e+72de^2x^2)}{x^8}\,dx}{2835d^4} \\
&= -\frac{bn(d+ex^2)^{7/2}}{81d^2x^9}+\frac{50ben(d+ex^2)^{7/2}}{3969d^3x^7} \\
&\quad -\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{9dx^9}+\frac{4e(d+ex^2)^{5/2}(a+b\log(cx^n))}{63d^2x^7} \\
&\quad -\frac{8e^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{315d^3x^5}+\frac{(8be^2n)\int\frac{(d+ex^2)^{5/2}}{x^6}\,dx}{315d^3} \\
&= -\frac{8be^2n(d+ex^2)^{5/2}}{1575d^3x^5}-\frac{bn(d+ex^2)^{7/2}}{81d^2x^9}+\frac{50ben(d+ex^2)^{7/2}}{3969d^3x^7} \\
&\quad -\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{9dx^9}+\frac{4e(d+ex^2)^{5/2}(a+b\log(cx^n))}{63d^2x^7} \\
&\quad -\frac{8e^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{315d^3x^5}+\frac{(8be^3n)\int\frac{(d+ex^2)^{3/2}}{x^4}\,dx}{315d^3} \\
&= -\frac{8be^3n(d+ex^2)^{3/2}}{945d^3x^3}-\frac{8be^2n(d+ex^2)^{5/2}}{1575d^3x^5}-\frac{bn(d+ex^2)^{7/2}}{81d^2x^9}+\frac{50ben(d+ex^2)^{7/2}}{3969d^3x^7} \\
&\quad -\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{9dx^9}+\frac{4e(d+ex^2)^{5/2}(a+b\log(cx^n))}{63d^2x^7} \\
&\quad -\frac{8e^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{315d^3x^5}+\frac{(8be^4n)\int\frac{\sqrt{d+ex^2}}{x^2}\,dx}{315d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{8be^4n\sqrt{d+ex^2}}{315d^3x} - \frac{8be^3n(d+ex^2)^{3/2}}{945d^3x^3} - \frac{8be^2n(d+ex^2)^{5/2}}{1575d^3x^5} \\
&\quad - \frac{bn(d+ex^2)^{7/2}}{81d^2x^9} + \frac{50ben(d+ex^2)^{7/2}}{3969d^3x^7} \\
&\quad - \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{9dx^9} + \frac{4e(d+ex^2)^{5/2}(a+b\log(cx^n))}{63d^2x^7} \\
&\quad - \frac{8e^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{315d^3x^5} + \frac{(8be^5n)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{315d^3} \\
&= -\frac{8be^4n\sqrt{d+ex^2}}{315d^3x} - \frac{8be^3n(d+ex^2)^{3/2}}{945d^3x^3} - \frac{8be^2n(d+ex^2)^{5/2}}{1575d^3x^5} \\
&\quad - \frac{bn(d+ex^2)^{7/2}}{81d^2x^9} + \frac{50ben(d+ex^2)^{7/2}}{3969d^3x^7} - \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{9dx^9} \\
&\quad + \frac{4e(d+ex^2)^{5/2}(a+b\log(cx^n))}{63d^2x^7} - \frac{8e^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{315d^3x^5} \\
&\quad + \frac{(8be^5n)\,\mathrm{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{315d^3} \\
&= -\frac{8be^4n\sqrt{d+ex^2}}{315d^3x} - \frac{8be^3n(d+ex^2)^{3/2}}{945d^3x^3} - \frac{8be^2n(d+ex^2)^{5/2}}{1575d^3x^5} - \frac{bn(d+ex^2)^{7/2}}{81d^2x^9} \\
&\quad + \frac{50ben(d+ex^2)^{7/2}}{3969d^3x^7} + \frac{8be^{9/2}n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{315d^3} - \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{9dx^9} \\
&\quad + \frac{4e(d+ex^2)^{5/2}(a+b\log(cx^n))}{63d^2x^7} - \frac{8e^2(d+ex^2)^{5/2}(a+b\log(cx^n))}{315d^3x^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.70

$$
\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^{10}}\,dx =
$$

$$
-\frac{\sqrt{d+ex^2}\Big(315a(d+ex^2)^2(35d^2-20dex^2+8e^2x^4)+bn(1225d^4+2425d^3ex^2+429d^2e^2x^4-677de^3x^6+2\cdots}{99225\cdots}
$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/x^10,x]
```

```
[Out] -1/99225*(Sqrt[d + e*x^2]*(315*a*(d + e*x^2)^2*(35*d^2 - 20*d*e*x^2 + 8*e^2
*x^4) + b*n*(1225*d^4 + 2425*d^3*e*x^2 + 429*d^2*e^2*x^4 - 677*d*e^3*x^6 +
2614*e^4*x^8)) + 315*b*(d + e*x^2)^(5/2)*(35*d^2 - 20*d*e*x^2 + 8*e^2*x^4)*
Log[c*x^n] - 2520*b*e^(9/2)*n*x^9*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(d^3*
x^9)
```

Maple [F]

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\ln\left(c\,x^{n}\right)\right)}{x^{10}}dx$$

[In] int((e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/x^10,x)

[Out] int((e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/x^10,x)

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.05

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^{10}}\,dx = \left[\frac{1260\,be^{\frac{9}{2}}nx^9\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-(2\,(1307\,be^4n+1\right.$$

$$-\frac{2520\,b\sqrt{-e}e^4nx^9\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+(2\,(1307\,be^4n+1260\,ae^4)x^8-(677\,bde^3n+1260\,ade^3)x^6+1225\,bd^4r}{}$$

[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^10,x, algorithm="fricas")

[Out] [1/99225*(1260*b*e^(9/2)*n*x^9*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) - (2*(1307*b*e^4*n + 1260*a*e^4)*x^8 - (677*b*d*e^3*n + 1260*a*d*e^3)*x^6 + 1225*b*d^4*n + 11025*a*d^4 + 3*(143*b*d^2*e^2*n + 315*a*d^2*e^2)*x^4 + 25*(97*b*d^3*e*n + 630*a*d^3*e)*x^2 + 315*(8*b*e^4*x^8 - 4*b*d*e^3*x^6 + 3*b*d^2*e^2*x^4 + 50*b*d^3*e*x^2 + 35*b*d^4)*log(c) + 315*(8*b*e^4*n*x^8 - 4*b*d*e^3*n*x^6 + 3*b*d^2*e^2*n*x^4 + 50*b*d^3*e*n*x^2 + 35*b*d^4*n)*log(x))*sqrt(e*x^2 + d))/(d^3*x^9), -1/99225*(2520*b*sqrt(-e)*e^4*n*x^9*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (2*(1307*b*e^4*n + 1260*a*e^4)*x^8 - (677*b*d*e^3*n + 1260*a*d*e^3)*x^6 + 1225*b*d^4*n + 11025*a*d^4 + 3*(143*b*d^2*e^2*n + 315*a*d^2*e^2)*x^4 + 25*(97*b*d^3*e*n + 630*a*d^3*e)*x^2 + 315*(8*b*e^4*x^8 - 4*b*d*e^3*x^6 + 3*b*d^2*e^2*x^4 + 50*b*d^3*e*x^2 + 35*b*d^4)*log(c) + 315*(8*b*e^4*n*x^8 - 4*b*d*e^3*n*x^6 + 3*b*d^2*e^2*n*x^4 + 50*b*d^3*e*n*x^2 + 35*b*d^4*n)*log(x))*sqrt(e*x^2 + d))/(d^3*x^9)]

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^{10}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*ln(c*x**n))/x**10,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^{10}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^{10}}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\log(cx^n)+a)}{x^{10}}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*log(c*x^n))/x^10,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*log(c*x^n) + a)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{x^{10}}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,(a+b\,\ln(c\,x^n))}{x^{10}}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*log(c*x^n)))/x^10,x)
```

```
[Out] int(((d + e*x^2)^(3/2)*(a + b*log(c*x^n)))/x^10, x)
```

3.275 $\int x\sqrt{4+x^2}\log(x)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int x\sqrt{4+x^2}\log(x)\,dx=-\frac{4}{3}\sqrt{4+x^2}-\frac{1}{9}\left(4+x^2\right)^{3/2}+\frac{8}{3}\text{arctanh}\left(\frac{\sqrt{4+x^2}}{2}\right)+\frac{1}{3}\left(4+x^2\right)^{3/2}\log(x)$$

```
[Out] -1/9*(x^2+4)^(3/2)+8/3*arctanh(1/2*(x^2+4)^(1/2))+1/3*(x^2+4)^(3/2)*ln(x)-4
/3*(x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2376, 272, 52, 65, 213}

$$\int x\sqrt{4+x^2}\log(x)\,dx=\frac{8}{3}\text{arctanh}\left(\frac{\sqrt{x^2+4}}{2}\right)-\frac{1}{9}\left(x^2+4\right)^{3/2}-\frac{4\sqrt{x^2+4}}{3}+\frac{1}{3}\left(x^2+4\right)^{3/2}\log(x)$$

```
[In] Int[x*Sqrt[4 + x^2]*Log[x],x]
```

```
[Out] (-4*Sqrt[4 + x^2])/3 - (4 + x^2)^(3/2)/9 + (8*ArcTanh[Sqrt[4 + x^2]/2])/3 +
 ((4 + x^2)^(3/2)*Log[x])/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\left(4+x^2\right)^{3/2}\log(x)-\frac{1}{3}\int\frac{\left(4+x^2\right)^{3/2}}{x}\,dx\\
&= \frac{1}{3}\left(4+x^2\right)^{3/2}\log(x)-\frac{1}{6}\text{Subst}\left(\int\frac{(4+x)^{3/2}}{x}\,dx,x,x^2\right)\\
&= -\frac{1}{9}\left(4+x^2\right)^{3/2}+\frac{1}{3}\left(4+x^2\right)^{3/2}\log(x)-\frac{2}{3}\text{Subst}\left(\int\frac{\sqrt{4+x}}{x}\,dx,x,x^2\right)\\
&= -\frac{4}{3}\sqrt{4+x^2}-\frac{1}{9}\left(4+x^2\right)^{3/2}+\frac{1}{3}\left(4+x^2\right)^{3/2}\log(x)-\frac{8}{3}\text{Subst}\left(\int\frac{1}{x\sqrt{4+x}}\,dx,x,x^2\right)
\end{aligned}$$

$$= -\frac{4}{3}\sqrt{4+x^2} - \frac{1}{9}\left(4+x^2\right)^{3/2} + \frac{1}{3}\left(4+x^2\right)^{3/2}\log(x) - \frac{16}{3}\text{Subst}\left(\int \frac{1}{-4+x^2}\,dx, x, \sqrt{4+x^2}\right)$$

$$= -\frac{4}{3}\sqrt{4+x^2} - \frac{1}{9}\left(4+x^2\right)^{3/2} + \frac{8}{3}\tanh^{-1}\left(\frac{\sqrt{4+x^2}}{2}\right) + \frac{1}{3}\left(4+x^2\right)^{3/2}\log(x)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int x\sqrt{4+x^2}\log(x)\,dx = \frac{1}{3}\left(-\frac{1}{3}\sqrt{4+x^2}\left(16+x^2\right) - 8\log(x) + \left(4+x^2\right)^{3/2}\log(x) + 8\log\left(2+\sqrt{4+x^2}\right)\right)$$

```
[In] Integrate[x*Sqrt[4 + x^2]*Log[x],x]
[Out] (-1/3*(Sqrt[4 + x^2]*(16 + x^2)) - 8*Log[x] + (4 + x^2)^(3/2)*Log[x] + 8*Lo
g[2 + Sqrt[4 + x^2]])/3
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.25

method	result	size
meijerg	$\left(-\frac{2\sqrt{1+\frac{x^2}{4}}}{9}+\frac{2\ln(x)\sqrt{1+\frac{x^2}{4}}}{3}\right)x^2+\frac{32}{9}-\frac{32\sqrt{1+\frac{x^2}{4}}}{9}+\ln\left(x\right)\left(-\frac{8}{3}+\frac{8\sqrt{1+\frac{x^2}{4}}}{3}\right)+\frac{8\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^2}{4}}}{2}\right)}{3}$	75

```
[In] int(x*ln(x)*(x^2+4)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-2/9*(1+1/4*x^2)^(1/2)+2/3*ln(x)*(1+1/4*x^2)^(1/2))*x^2+32/9-32/9*(1+1/4*x
^2)^(1/2)+ln(x)*(-8/3+8/3*(1+1/4*x^2)^(1/2))+8/3*ln(1/2+1/2*(1+1/4*x^2)^(1/
2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int x\sqrt{4+x^2}\log(x)\,dx = -\frac{1}{9}\left(x^2-3\left(x^2+4\right)\log\left(x\right)+16\right)\sqrt{x^2+4} + \frac{8}{3}\log\left(-x+\sqrt{x^2+4}+2\right) - \frac{8}{3}\log\left(-x+\sqrt{x^2+4}-2\right)$$

```
[In] integrate(x*log(x)*(x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(x^2 - 3*(x^2 + 4)*log(x) + 16)*sqrt(x^2 + 4) + 8/3*log(-x + sqrt(x^2
+ 4) + 2) - 8/3*log(-x + sqrt(x^2 + 4) - 2)
```

Sympy [A] (verification not implemented)

Time = 7.76 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int x\sqrt{4+x^2}\log(x)\,dx = \left(\frac{x^2}{3}+\frac{4}{3}\right)\sqrt{x^2+4}\log\left(x\right) - \frac{\left(x^2+4\right)^{\frac{3}{2}}}{9} - \frac{4\sqrt{x^2+4}}{3} - \frac{4\log\left(\sqrt{x^2+4}-2\right)}{3} + \frac{4\log\left(\sqrt{x^2+4}+2\right)}{3}$$

```
[In] integrate(x*ln(x)*(x**2+4)**(1/2),x)
```

```
[Out] (x**2/3 + 4/3)*sqrt(x**2 + 4)*log(x) - (x**2 + 4)**(3/2)/9 - 4*sqrt(x**2 +
4)/3 - 4*log(sqrt(x**2 + 4) - 2)/3 + 4*log(sqrt(x**2 + 4) + 2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

$$\int x\sqrt{4+x^2}\log(x)\,dx = \frac{1}{3}\left(x^2+4\right)^{\frac{3}{2}}\log\left(x\right) - \frac{1}{9}\left(x^2+4\right)^{\frac{3}{2}} - \frac{4}{3}\sqrt{x^2+4} + \frac{8}{3}\,\text{arsinh}\left(\frac{2}{|x|}\right)$$

```
[In] integrate(x*log(x)*(x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 + 4)^(3/2)*log(x) - 1/9*(x^2 + 4)^(3/2) - 4/3*sqrt(x^2 + 4) + 8/3*
arcsinh(2/abs(x))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int x\sqrt{4+x^2}\log(x)\,dx = \frac{1}{3}\left(x^2+4\right)^{\frac{3}{2}}\log\left(x\right) - \frac{1}{9}\left(x^2+4\right)^{\frac{3}{2}} - \frac{4}{3}\sqrt{x^2+4} + \frac{4}{3}\log\left(\sqrt{x^2+4}+2\right) - \frac{4}{3}\log\left(\sqrt{x^2+4}-2\right)$$

```
[In] integrate(x*log(x)*(x^2+4)^(1/2),x, algorithm="giac")
[Out] 1/3*(x^2 + 4)^(3/2)*log(x) - 1/9*(x^2 + 4)^(3/2) - 4/3*sqrt(x^2 + 4) + 4/3*
log(sqrt(x^2 + 4) + 2) - 4/3*log(sqrt(x^2 + 4) - 2)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{4+x^2}\log(x)\,dx = \int x\,\ln\left(x\right)\,\sqrt{x^2+4}\,dx$$

```
[In] int(x*log(x)*(x^2 + 4)^(1/2),x)
[Out] int(x*log(x)*(x^2 + 4)^(1/2), x)
```

3.276 $\int \frac{x^5(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 182

$$\int \frac{x^5(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = -\frac{8bd^2n\sqrt{d+ex^2}}{15e^3}+\frac{7bdn(d+ex^2)^{3/2}}{45e^3}-\frac{bn(d+ex^2)^{5/2}}{25e^3}+\frac{8bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{15e^3}+\frac{d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3}-\frac{2d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3}+\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3}$$

```
[Out] 7/45*b*d*n*(e*x^2+d)^(3/2)/e^3-1/25*b*n*(e*x^2+d)^(5/2)/e^3+8/15*b*d^(5/2)*
n*arctanh((e*x^2+d)^(1/2)/d^(1/2))/e^3-2/3*d*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n)
)/e^3+1/5*(e*x^2+d)^(5/2)*(a+b*ln(c*x^n))/e^3-8/15*b*d^2*n*(e*x^2+d)^(1/2)/
e^3+d^2*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {272, 45, 2392, 12, 1265, 911, 1275, 214}

$$\int \frac{x^5(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \frac{d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3}-\frac{2d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3}+\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3}+\frac{8bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{15e^3}-\frac{8bd^2n\sqrt{d+ex^2}}{15e^3}+\frac{7bdn(d+ex^2)^{3/2}}{45e^3}-\frac{bn(d+ex^2)^{5/2}}{25e^3}$$

```
[In] Int[(x^5*(a + b*Log[c*x^n]))/Sqrt[d + e*x^2],x]
```

```
[Out] (-8*b*d^2*n*Sqrt[d + e*x^2])/(15*e^3) + (7*b*d*n*(d + e*x^2)^(3/2))/(45*e^3
) - (b*n*(d + e*x^2)^(5/2))/(25*e^3) + (8*b*d^(5/2)*n*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]])/(15*e^3) + (d^2*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/e^3 - (2*d
*(d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*
Log[c*x^n]))/(5*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3}-\frac{2d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3}\\
&\quad+\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3}-(bn)\int\frac{\sqrt{d+ex^2}(8d^2-4dex^2+3e^2x^4)}{15e^3x}\,dx\\
&=\frac{d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3}-\frac{2d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3}\\
&\quad+\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3}-\frac{(bn)\int\frac{\sqrt{d+ex^2}(8d^2-4dex^2+3e^2x^4)}{x}\,dx}{15e^3}\\
&=\frac{d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3}-\frac{2d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3}\\
&\quad+\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3}-\frac{(bn)\text{Subst}\left(\int\frac{\sqrt{d+ex}(8d^2-4dex+3e^2x^2)}{x}\,dx,x,x^2\right)}{30e^3}\\
&=\frac{d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3}-\frac{2d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3}\\
&\quad+\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3}-\frac{(bn)\text{Subst}\left(\int\frac{x^2(15d^2-10dx^2+3x^4)}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{15e^4}\\
&=\frac{d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3}-\frac{2d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3}\\
&\quad+\frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3}\\
&\quad-\frac{(bn)\text{Subst}\left(\int\left(8d^2e-7dex^2+3ex^4+\frac{8d^3}{-\frac{d}{e}+\frac{x^2}{e}}\right)dx,x,\sqrt{d+ex^2}\right)}{15e^4}
\end{aligned}$$

$$= -\frac{8bd^2n\sqrt{d+ex^2}}{15e^3} + \frac{7bdn(d+ex^2)^{3/2}}{45e^3} - \frac{bn(d+ex^2)^{5/2}}{25e^3} + \frac{d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3} - \frac{2d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3} - \frac{(8bd^3n)\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{15e^4}$$

$$= -\frac{8bd^2n\sqrt{d+ex^2}}{15e^3} + \frac{7bdn(d+ex^2)^{3/2}}{45e^3} - \frac{bn(d+ex^2)^{5/2}}{25e^3} + \frac{8bd^{5/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{15e^3} + \frac{d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3} - \frac{2d(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^3}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.12

$$\int \frac{x^5(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \frac{120ad^2\sqrt{d+ex^2} - 94bd^2n\sqrt{d+ex^2} - 60adex^2\sqrt{d+ex^2} + 17bdenx^2\sqrt{d+ex^2} + 45ae^2x^4\sqrt{d+ex^2} - 9be}{}$$

```
[In] Integrate[(x^5*(a + b*Log[c*x^n]))/Sqrt[d + e*x^2],x]
```

```
[Out] (120*a*d^2*Sqrt[d + e*x^2] - 94*b*d^2*n*Sqrt[d + e*x^2] - 60*a*d*e*x^2*Sqrt
[d + e*x^2] + 17*b*d*e*n*x^2*Sqrt[d + e*x^2] + 45*a*e^2*x^4*Sqrt[d + e*x^2]
 - 9*b*e^2*n*x^4*Sqrt[d + e*x^2] - 120*b*d^(5/2)*n*Log[x] + 15*b*Sqrt[d + e
*x^2]*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4)*Log[c*x^n] + 120*b*d^(5/2)*n*Log[d +
Sqrt[d]*Sqrt[d + e*x^2]])/(225*e^3)
```

Maple [F]

$$\int \frac{x^5(a+b\ln(c\,x^n))}{\sqrt{e\,x^2+d}}dx$$

```
[In] int(x^5*(a+b*ln(c*x^n))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*ln(c*x^n))/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.73

$$\int \frac{x^5(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx$$
$$= \left[\frac{60\,bd^{\frac{5}{2}}n\log\left(-\frac{ex^2+2\sqrt{ex^2+d}\sqrt{d}+2\,d}{x^2}\right) - (9\,(be^2n-5\,ae^2)x^4 + 94\,bd^2n - 120\,ad^2 - (17\,bden-60\,ade)x^2 - 15}{225\,e^3}\right.$$
$$-\frac{120\,b\sqrt{-d}d^2n\arctan\left(\frac{\sqrt{-d}}{\sqrt{ex^2+d}}\right) + (9\,(be^2n-5\,ae^2)x^4 + 94\,bd^2n - 120\,ad^2 - (17\,bden-60\,ade)x^2 - 15\,(}{225\,e^3}$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/225*(60*b*d^(5/2)*n*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2)
- (9*(b*e^2*n - 5*a*e^2)*x^4 + 94*b*d^2*n - 120*a*d^2 - (17*b*d*e*n - 60*a*
d*e)*x^2 - 15*(3*b*e^2*x^4 - 4*b*d*e*x^2 + 8*b*d^2)*log(c) - 15*(3*b*e^2*n*
x^4 - 4*b*d*e*n*x^2 + 8*b*d^2*n)*log(x))*sqrt(e*x^2 + d))/e^3, -1/225*(120*
b*sqrt(-d)*d^2*n*arctan(sqrt(-d)/sqrt(e*x^2 + d)) + (9*(b*e^2*n - 5*a*e^2)*
x^4 + 94*b*d^2*n - 120*a*d^2 - (17*b*d*e*n - 60*a*d*e)*x^2 - 15*(3*b*e^2*x^
4 - 4*b*d*e*x^2 + 8*b*d^2)*log(c) - 15*(3*b*e^2*n*x^4 - 4*b*d*e*n*x^2 + 8*b
*d^2*n)*log(x))*sqrt(e*x^2 + d))/e^3]
```

Sympy [A] (verification not implemented)

Time = 17.14 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.97

$$\int \frac{x^5(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = a\left(\begin{cases}\frac{8d^2\sqrt{d+ex^2}}{15e^3} - \frac{4dx^2\sqrt{d+ex^2}}{15e^2} + \frac{x^4\sqrt{d+ex^2}}{5e} & \text{for } e\neq 0\\ \frac{x^6}{6\sqrt{d}} & \text{otherwise}\end{cases}\right)$$
$$-bn\left(\begin{cases}-\frac{8d^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{15e^3} + \frac{8d^3}{15e^{\frac{7}{2}}x\sqrt{\frac{d}{ex^2}+1}} + \frac{8d^2x}{15e^{\frac{5}{2}}\sqrt{\frac{d}{ex^2}+1}} - \frac{4d\left(\begin{cases}\frac{d\sqrt{d+ex^2}}{3e} + \frac{x^2\sqrt{d+ex^2}}{3} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^2}{2} & \text{otherwise}\end{cases}\right)}{15e^2} + \frac{\begin{cases}-\frac{2d^2\sqrt{d+}}{15e^2}\\ \frac{\sqrt{d}x^4}{4}\end{cases}}{} \\ \frac{x^6}{36\sqrt{d}}\end{cases}\right.$$
$$+b\left(\begin{cases}\frac{8d^2\sqrt{d+ex^2}}{15e^3} - \frac{4dx^2\sqrt{d+ex^2}}{15e^2} + \frac{x^4\sqrt{d+ex^2}}{5e} & \text{for } e\neq 0\\ \frac{x^6}{6\sqrt{d}} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate(x**5*(a+b*ln(c*x**n))/(e*x**2+d)**(1/2),x)
```

```
[Out] a*Piecewise((8*d**2*sqrt(d + e*x**2)/(15*e**3) - 4*d*x**2*sqrt(d + e*x**2)/
(15*e**2) + x**4*sqrt(d + e*x**2)/(5*e), Ne(e, 0)), (x**6/(6*sqrt(d)), True
)) - b*n*Piecewise((-8*d**(5/2)*asinh(sqrt(d)/(sqrt(e)*x))/(15*e**3) + 8*d*
*3/(15*e**(7/2)*x*sqrt(d/(e*x**2) + 1)) + 8*d**2*x/(15*e**(5/2)*sqrt(d/(e*x
**2) + 1)) - 4*d*Piecewise((d*sqrt(d + e*x**2)/(3*e) + x**2*sqrt(d + e*x**2
)/3, Ne(e, 0)), (sqrt(d)*x**2/2, True))/(15*e**2) + Piecewise((-2*d**2*sqrt
(d + e*x**2)/(15*e**2) + d*x**2*sqrt(d + e*x**2)/(15*e) + x**4*sqrt(d + e*x
**2)/5, Ne(e, 0)), (sqrt(d)*x**4/4, True))/(5*e), (e > -oo) & (e < oo) & Ne
(e, 0)), (x**6/(36*sqrt(d)), True)) + b*Piecewise((8*d**2*sqrt(d + e*x**2)/
(15*e**3) - 4*d*x**2*sqrt(d + e*x**2)/(15*e**2) + x**4*sqrt(d + e*x**2)/(5*
e), Ne(e, 0)), (x**6/(6*sqrt(d)), True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^5(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\log(cx^n)+a)x^5}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^5/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^5\,(a+b\,\ln(c\,x^n))}{\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((x^5*(a + b*log(c*x^n)))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x^5*(a + b*log(c*x^n)))/(d + e*x^2)^(1/2), x)
```

3.277 $\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 129

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \frac{2bdn\sqrt{d+ex^2}}{3e^2}-\frac{bn(d+ex^2)^{3/2}}{9e^2}-\frac{2bd^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^2}+\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2}$$

```
[Out] -1/9*b*n*(e*x^2+d)^(3/2)/e^2-2/3*b*d^(3/2)*n*arctanh((e*x^2+d)^(1/2)/d^(1/2
))/e^2+1/3*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/e^2+2/3*b*d*n*(e*x^2+d)^(1/2)/e^
2-d*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {272, 45, 2392, 12, 457, 81, 52, 65, 214}

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = -\frac{d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^2}+\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2}-\frac{2bd^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3e^2}+\frac{2bdn\sqrt{d+ex^2}}{3e^2}-\frac{bn(d+ex^2)^{3/2}}{9e^2}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n]))/Sqrt[d + e*x^2],x]
```

```
[Out] (2*b*d*n*Sqrt[d + e*x^2])/(3*e^2) - (b*n*(d + e*x^2)^(3/2))/(9*e^2) - (2*b*
d^(3/2)*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(3*e^2) - (d*Sqrt[d + e*x^2]*(a
 + b*Log[c*x^n]))/e^2 + ((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/(3*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^2}+\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2} \\
&\quad -(bn)\int\frac{(-2d+ex^2)\sqrt{d+ex^2}}{3e^2x}\,dx \\
&= -\frac{d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^2}+\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2}-\frac{(bn)\int\frac{(-2d+ex^2)\sqrt{d+ex^2}}{x}\,dx}{3e^2} \\
&= -\frac{d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^2}+\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2} \\
&\quad -\frac{(bn)\text{Subst}\left(\int\frac{(-2d+ex)\sqrt{d+ex}}{x}\,dx,x,x^2\right)}{6e^2} \\
&= -\frac{bn(d+ex^2)^{3/2}}{9e^2}-\frac{d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^2} \\
&\quad +\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2}+\frac{(bdn)\text{Subst}\left(\int\frac{\sqrt{d+ex}}{x}\,dx,x,x^2\right)}{3e^2} \\
&= \frac{2bdn\sqrt{d+ex^2}}{3e^2}-\frac{bn(d+ex^2)^{3/2}}{9e^2}-\frac{d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^2} \\
&\quad +\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2}+\frac{(bd^2n)\,\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{3e^2}
\end{aligned}
$$

$$= \frac{2bdn\sqrt{d+ex^2}}{3e^2} - \frac{bn(d+ex^2)^{3/2}}{9e^2} - \frac{d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2} + \frac{(2bd^2n)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{3e^3}$$

$$= \frac{2bdn\sqrt{d+ex^2}}{3e^2} - \frac{bn(d+ex^2)^{3/2}}{9e^2} - \frac{2bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3e^2} - \frac{d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^2}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{-6ad\sqrt{d+ex^2} + 5bdn\sqrt{d+ex^2} + 3aex^2\sqrt{d+ex^2} - benx^2\sqrt{d+ex^2} + 6bd^{3/2}n\log(x) + 3b(-2d+ex^2)}{9e^2}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/Sqrt[d + e*x^2],x]
```

```
[Out] (-6*a*d*Sqrt[d + e*x^2] + 5*b*d*n*Sqrt[d + e*x^2] + 3*a*e*x^2*Sqrt[d + e*x^
2] - b*e*n*x^2*Sqrt[d + e*x^2] + 6*b*d^(3/2)*n*Log[x] + 3*b*(-2*d + e*x^2)*
Sqrt[d + e*x^2]*Log[c*x^n] - 6*b*d^(3/2)*n*Log[d + Sqrt[d]*Sqrt[d + e*x^2]]
)/(9*e^2)
```

Maple [F]

$$\int \frac{x^3(a+b\ln(c\,x^n))}{\sqrt{e\,x^2+d}}dx$$

```
[In] int(x^3*(a+b*ln(c*x^n))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*ln(c*x^n))/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.60

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \left[\frac{3\,bd^{\frac{3}{2}}n\log\left(-\frac{ex^2-2\,\sqrt{ex^2+d}\sqrt{d}+2\,d}{x^2}\right)+(5\,bdn-(ben-3\,ae)x^2-6\,ad+3\,(bex^2-2\,bd)\log(c)+3\,(benx^2-2}{9\,e^2}\right.$$

[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] [1/9*(3*b*d^(3/2)*n*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + (5*b*d*n - (b*e*n - 3*a*e)*x^2 - 6*a*d + 3*(b*e*x^2 - 2*b*d)*log(c) + 3*(b*e*n*x^2 - 2*b*d*n)*log(x))*sqrt(e*x^2 + d))/e^2, 1/9*(6*b*sqrt(-d)*d*n*arctan(sqrt(-d)/sqrt(e*x^2 + d)) + (5*b*d*n - (b*e*n - 3*a*e)*x^2 - 6*a*d + 3*(b*e*x^2 - 2*b*d)*log(c) + 3*(b*e*n*x^2 - 2*b*d*n)*log(x))*sqrt(e*x^2 + d))/e^2]

Sympy [A] (verification not implemented)

Time = 12.28 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.83

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = a\left(\begin{cases}-\frac{2d\sqrt{d+ex^2}}{3e^2}+\frac{x^2\sqrt{d+ex^2}}{3e} & \text{for } e\neq 0\\ \frac{x^4}{4\sqrt{d}} & \text{otherwise}\end{cases}\right) - bn\left(\begin{cases}\frac{2d^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{3e^2}-\frac{2d^2}{3e^{\frac{5}{2}}x\sqrt{\frac{d}{ex^2}+1}}-\frac{2dx}{3e^{\frac{3}{2}}\sqrt{\frac{d}{ex^2}+1}}+\frac{\begin{cases}\frac{d\sqrt{d+ex^2}}{3e}+\frac{x^2\sqrt{d+ex^2}}{3} & \text{for } e\neq 0\\ \frac{\sqrt{d}x^2}{2} & \text{otherwise}\end{cases}}{3e} & \text{for } e>-\infty\wedge e<\\ \frac{x^4}{16\sqrt{d}} & \text{otherwise}\end{cases}\right) + b\left(\begin{cases}-\frac{2d\sqrt{d+ex^2}}{3e^2}+\frac{x^2\sqrt{d+ex^2}}{3e} & \text{for } e\neq 0\\ \frac{x^4}{4\sqrt{d}} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

[In] integrate(x**3*(a+b*ln(c*x**n))/(e*x**2+d)**(1/2),x)

[Out] a*Piecewise((-2*d*sqrt(d + e*x**2)/(3*e**2) + x**2*sqrt(d + e*x**2)/(3*e), Ne(e, 0)), (x**4/(4*sqrt(d)), True)) - b*n*Piecewise((2*d**(3/2)*asinh(sqrt(d)/(sqrt(e)*x))/(3*e**2) - 2*d**2/(3*e**(5/2)*x*sqrt(d/(e*x**2) + 1)) - 2*d*x/(3*e**(3/2)*sqrt(d/(e*x**2) + 1)) + Piecewise((d*sqrt(d + e*x**2)/(3*e)

```
 + x**2*sqrt(d + e*x**2)/3, Ne(e, 0)), (sqrt(d)*x**2/2, True))/(3*e), (e >
-oo) & (e < oo) & Ne(e, 0)), (x**4/(16*sqrt(d)), True)) + b*Piecewise((-2*d
*sqrt(d + e*x**2)/(3*e**2) + x**2*sqrt(d + e*x**2)/(3*e), Ne(e, 0)), (x**4/
(4*sqrt(d)), True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\log(cx^n))}{\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^3(a + b\log(cx^n))}{\sqrt{d + ex^2}}\,dx = \int \frac{(b\log(cx^n) + a)x^3}{\sqrt{ex^2 + d}}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^3/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\log(cx^n))}{\sqrt{d + ex^2}}\,dx = \int \frac{x^3\,(a + b\,\ln(c\,x^n))}{\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n)))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*log(c*x^n)))/(d + e*x^2)^(1/2), x)
```

3.278 $\int \frac{x(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = -\frac{bn\sqrt{d+ex^2}}{e}+\frac{b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{e}+\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e}$$

```
[Out] b*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))*d^(1/2)/e-b*n*(e*x^2+d)^(1/2)/e+(a+b*l
n(c*x^n))*(e*x^2+d)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2376, 272, 52, 65, 214}

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e}+\frac{b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{e}-\frac{bn\sqrt{d+ex^2}}{e}$$

```
[In] Int[(x*(a + b*Log[c*x^n]))/Sqrt[d + e*x^2],x]
```

```
[Out] -((b*n*Sqrt[d + e*x^2])/e) + (b*Sqrt[d]*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])
/e + (Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/e
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e} - \frac{(bn)\int \frac{\sqrt{d+ex^2}}{x}\,dx}{e} \\
&= \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e} - \frac{(bn)\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x}\,dx, x, x^2\right)}{2e} \\
&= -\frac{bn\sqrt{d+ex^2}}{e} + \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e} - \frac{(bdn)\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)}{2e} \\
&= -\frac{bn\sqrt{d+ex^2}}{e} + \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e} - \frac{(bdn)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{e^2}
\end{aligned}$$

$$= -\frac{bn\sqrt{d+ex^2}}{e} + \frac{b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{e} + \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \frac{a\sqrt{d+ex^2} - bn\sqrt{d+ex^2} - b\sqrt{d}n\log(x) + b\sqrt{d+ex^2}\log(cx^n) + b\sqrt{d}n\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)}{e}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/Sqrt[d + e*x^2],x]
[Out] (a*Sqrt[d + e*x^2] - b*n*Sqrt[d + e*x^2] - b*Sqrt[d]*n*Log[x] + b*Sqrt[d +
e*x^2]*Log[c*x^n] + b*Sqrt[d]*n*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/e
```

Maple [F]

$$\int \frac{x(a+b\ln(c\,x^n))}{\sqrt{e\,x^2+d}}dx$$

```
[In] int(x*(a+b*ln(c*x^n))/(e*x^2+d)^(1/2),x)
[Out] int(x*(a+b*ln(c*x^n))/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.70

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \left[\frac{b\sqrt{d}n\log\left(-\frac{ex^2+2\sqrt{ex^2+d}\sqrt{d}+2\,d}{x^2}\right) + 2\sqrt{ex^2+d}(bn\log(x) - bn + b\log(c) + a)}{2\,e}, -\frac{b\sqrt{-d}n\arctan\left(\frac{\sqrt{-d}}{\sqrt{ex^2+d}}\right) - \sqrt{ex^2+d}(bn\log(x) - bn + b\log(c) + a)}{e}\right]$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] [1/2*(b*sqrt(d)*n*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 2*s
qrt(e*x^2 + d)*(b*n*log(x) - b*n + b*log(c) + a))/e, -(b*sqrt(-d)*n*arctan(
sqrt(-d)/sqrt(e*x^2 + d)) - sqrt(e*x^2 + d)*(b*n*log(x) - b*n + b*log(c) +
a))/e]
```

Sympy [A] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx$$
$$= a\left(\begin{cases} \frac{\sqrt{d+ex^2}}{e} & \text{for } e\neq 0 \\ \frac{x^2}{2\sqrt{d}} & \text{otherwise} \end{cases}\right)$$
$$- bn\left(\begin{cases} -\frac{\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{e} + \frac{d}{e^{\frac{3}{2}}x\sqrt{\frac{d}{ex^2}+1}} + \frac{x}{\sqrt{e}\sqrt{\frac{d}{ex^2}+1}} & \text{for } e>-\infty \wedge e<\infty \wedge e\neq 0 \\ \frac{x^2}{4\sqrt{d}} & \text{otherwise} \end{cases}\right)$$
$$+ b\left(\begin{cases} \frac{\sqrt{d+ex^2}}{e} & \text{for } e\neq 0 \\ \frac{x^2}{2\sqrt{d}} & \text{otherwise} \end{cases}\right)\log(cx^n)$$

```
[In] integrate(x*(a+b*ln(c*x**n))/(e*x**2+d)**(1/2),x)
```

```
[Out] a*Piecewise((sqrt(d + e*x**2)/e, Ne(e, 0)), (x**2/(2*sqrt(d)), True)) - b*n
*Piecewise((-sqrt(d)*asinh(sqrt(d)/(sqrt(e)*x))/e + d/(e**(3/2)*x*sqrt(d/(e
*x**2) + 1)) + x/(sqrt(e)*sqrt(d/(e*x**2) + 1)), (e > -oo) & (e < oo) & Ne(
e, 0)), (x**2/(4*sqrt(d)), True)) + b*Piecewise((sqrt(d + e*x**2)/e, Ne(e,
0)), (x**2/(2*sqrt(d)), True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\log(cx^n)+a)x}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \int \frac{x\,(a+b\,\ln(c\,x^n))}{\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((x*(a + b*log(c*x^n)))/(d + e*x^2)^(1/2),x)
[Out] int((x*(a + b*log(c*x^n)))/(d + e*x^2)^(1/2), x)
```

3.279 $\int \frac{a+b\log(cx^n)}{x\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 166

$$\int \frac{a+b\log(cx^n)}{x\sqrt{d+ex^2}}\,dx = \frac{bn\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2\sqrt{d}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} - \frac{bn\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{\sqrt{d}} - \frac{bn\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2\sqrt{d}}$$

```
[Out] 1/2*b*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))^2/d^(1/2)-arctanh((e*x^2+d)^(1/2)/
d^(1/2))*(a+b*ln(c*x^n))/d^(1/2)-b*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))*ln(2*
d^(1/2)/(d^(1/2)-(e*x^2+d)^(1/2)))/d^(1/2)-1/2*b*n*polylog(2,1-2*d^(1/2)/(d
^(1/2)-(e*x^2+d)^(1/2)))/d^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {272, 65, 214, 2390, 12, 6131, 6055, 2449, 2352}

$$\int \frac{a+b\log(cx^n)}{x\sqrt{d+ex^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}}+\frac{bn\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2\sqrt{d}} - \frac{bn\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{\sqrt{d}} - \frac{bn\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{ex^2+d}}\right)}{2\sqrt{d}}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*Sqrt[d + e*x^2]),x]
```

```
[Out] (b*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]]^2)/(2*Sqrt[d]) - (ArcTanh[Sqrt[d + e*
x^2]/Sqrt[d]]*(a + b*Log[c*x^n]))/Sqrt[d] - (b*n*ArcTanh[Sqrt[d + e*x^2]/Sq
rt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/Sqrt[d] - (b*n*PolyLog
[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/(2*Sqrt[d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} + (bn)\int \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}x}\,dx$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} + \frac{(bn)\int \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{x}\,dx}{\sqrt{d}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} + \frac{(bn)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx, x, x^2\right)}{2\sqrt{d}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} + \frac{(bn)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx, x, \sqrt{d+ex^2}\right)}{\sqrt{d}}$$

$$= \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2\sqrt{d}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} - \frac{(bn)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}dx, x, \sqrt{d+ex^2}\right)}{d}$$

$$= \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2\sqrt{d}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} - \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{\sqrt{d}} + \frac{(bn)\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}dx, x, \sqrt{d+ex^2}\right)}{d}$$

$$= \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2\sqrt{d}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} - \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{\sqrt{d}} - \frac{(bn)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}dx, x, \frac{1}{1-\frac{\sqrt{d+ex^2}}{\sqrt{d}}}\right)}{\sqrt{d}}$$

$$= \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2\sqrt{d}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}} - \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{\sqrt{d}} - \frac{bn\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex^2}}{\sqrt{d}}}\right)}{2\sqrt{d}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.98

$$\int \frac{a+b\log(cx^n)}{x\sqrt{d+ex^2}}dx = \frac{bn\sqrt{1+\frac{d}{ex^2}}\left(-{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{d}{ex^2}\right) - \frac{\sqrt{ex}\text{arcsinh}\left(\frac{\sqrt{d}}{\sqrt{ex}}\right)\log(x)}{\sqrt{d}}\right)}{\sqrt{d+ex^2}} - \frac{\log(x)(-a-b(-n\log(x)+\log(cx^n)))}{\sqrt{d}} + \frac{(-a-b(-n\log(x)+\log(cx^n)))\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)}{\sqrt{d}}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*Sqrt[d + e*x^2]),x]
```

```
[Out] (b*n*Sqrt[1 + d/(e*x^2)]*(-HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, -
(d/(e*x^2))] - (Sqrt[e]*x*ArcSinh[Sqrt[d]/(Sqrt[e]*x)]*Log[x])/Sqrt[d]))/Sq
rt[d + e*x^2] - (Log[x]*(-a - b*(-(n*Log[x]) + Log[c*x^n])))/Sqrt[d] + ((-a
 - b*(-(n*Log[x]) + Log[c*x^n]))*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/Sqrt[d]
```

Maple [F]

$$\int \frac{a+b\ln\left(c\,x^{n}\right)}{x\sqrt{e\,x^{2}+d}}dx$$

```
[In] int((a+b*ln(c*x^n))/x/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/x/(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x\sqrt{d+ex^{2}}}\,dx=\int \frac{b\log\left(cx^{n}\right)+a}{\sqrt{ex^{2}+d}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(e*x^2 + d)*b*log(c*x^n) + sqrt(e*x^2 + d)*a)/(e*x^3 + d*x),
x)
```

Sympy [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x\sqrt{d+ex^{2}}}\,dx=\int \frac{a+b\log\left(cx^{n}\right)}{x\sqrt{d+ex^{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*log(c*x**n))/(x*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(cx^n\right)}{x\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x\sqrt{d+ex^2}}\,dx = \int \frac{b\log\left(cx^n\right)+a}{\sqrt{ex^2+d}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/(sqrt(e*x^2 + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\ln\left(c\,x^n\right)}{x\,\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x*(d + e*x^2)^(1/2)), x)
```

3.280 $\int \frac{a+b\log(cx^n)}{x^3\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 258

$$\int \frac{a+b\log(cx^n)}{x^3\sqrt{d+ex^2}}\,dx = -\frac{bn\sqrt{d+ex^2}}{4dx^2} - \frac{ben\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{4d^{3/2}} - \frac{ben\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{3/2}} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2dx^2} + \frac{e\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}} + \frac{ben\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2d^{3/2}} + \frac{ben\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{4d^{3/2}}$$

```
[Out] -1/4*b*e*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)-1/4*b*e*n*arctanh((e*x^
2+d)^(1/2)/d^(1/2))^2/d^(3/2)+1/2*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))*(a+b*l
n(c*x^n))/d^(3/2)+1/2*b*e*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(
d^(1/2)-(e*x^2+d)^(1/2)))/d^(3/2)+1/4*b*e*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-
(e*x^2+d)^(1/2)))/d^(3/2)-1/4*b*n*(e*x^2+d)^(1/2)/d/x^2-1/2*(a+b*ln(c*x^n))
*(e*x^2+d)^(1/2)/d/x^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {272, 44, 65, 214, 2392, 12, 14, 43, 6131, 6055, 2449, 2352}

$$\int \frac{a+b\log(cx^n)}{x^3\sqrt{d+ex^2}}\,dx = \frac{e\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2dx^2} - \frac{ben\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{3/2}} - \frac{ben\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{4d^{3/2}} + \frac{ben\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2d^{3/2}} + \frac{ben\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{ex^2+d}}\right)}{4d^{3/2}} - \frac{bn\sqrt{d+ex^2}}{4dx^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^3*Sqrt[d + e*x^2]),x]
```

```
[Out] -1/4*(b*n*Sqrt[d + e*x^2])/(d*x^2) - (b*e*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]
])/(4*d^(3/2)) - (b*e*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]]^2)/(4*d^(3/2)) - (
Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/(2*d*x^2) + (e*ArcTanh[Sqrt[d + e*x^2]/
Sqrt[d]]*(a + b*Log[c*x^n]))/(2*d^(3/2)) + (b*e*n*ArcTanh[Sqrt[d + e*x^2]/S
qrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/(2*d^(3/2)) + (b*e*n*
PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/(4*d^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2dx^2}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}} \\
&\quad-(bn)\int\frac{-\frac{\sqrt{d+ex^2}}{d}+\frac{ex^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}}{2x^3}\,dx \\
&= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2dx^2}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}} \\
&\quad-\frac{1}{2}(bn)\int\frac{-\frac{\sqrt{d+ex^2}}{d}+\frac{ex^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}}{x^3}\,dx \\
&= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2dx^2}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}} \\
&\quad-\frac{1}{2}(bn)\int\left(-\frac{\sqrt{d+ex^2}}{dx^3}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}x}\right)dx \\
&= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2dx^2}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}} \\
&\quad+\frac{(bn)\int\frac{\sqrt{d+ex^2}}{x^3}\,dx}{2d}-\frac{(ben)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{x}\,dx}{2d^{3/2}} \\
&= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2dx^2}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}} \\
&\quad+\frac{(bn)\text{Subst}\left(\int\frac{\sqrt{d+ex}}{x^2}\,dx,x,x^2\right)}{4d}-\frac{(ben)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx,x,x^2\right)}{4d^{3/2}}
\end{aligned}
$$

$$= -\frac{bn\sqrt{d+ex^2}}{4dx^2} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2dx^2} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}} - \frac{(ben)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}dx, x, \sqrt{d+ex^2}\right)}{2d^{3/2}} + \frac{(ben)\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}dx, x, x^2\right)}{8d}$$

$$= -\frac{bn\sqrt{d+ex^2}}{4dx^2} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{3/2}} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2dx^2} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}} + \frac{(bn)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}dx, x, \sqrt{d+ex^2}\right)}{4d} + \frac{(ben)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}dx, x, \sqrt{d+ex^2}\right)}{2d^2}$$

$$= -\frac{bn\sqrt{d+ex^2}}{4dx^2} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{4d^{3/2}} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{3/2}} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2dx^2} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}} + \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2d^{3/2}} - \frac{(ben)\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}dx, x, \sqrt{d+ex^2}\right)}{2d^2}$$

$$= -\frac{bn\sqrt{d+ex^2}}{4dx^2} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{4d^{3/2}} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{3/2}} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2dx^2} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}} + \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2d^{3/2}} + \frac{(ben)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}dx, x, \frac{1}{1-\frac{\sqrt{d+ex^2}}{\sqrt{d}}}\right)}{2d^{3/2}}$$

$$= -\frac{bn\sqrt{d+ex^2}}{4dx^2} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{4d^{3/2}} - \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{3/2}} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{2dx^2} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{3/2}} + \frac{ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2d^{3/2}} + \frac{ben\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex^2}}{\sqrt{d}}}\right)}{4d^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.67 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.89

$$\int \frac{a+b\log(cx^n)}{x^3\sqrt{d+ex^2}}\,dx$$
$$= \frac{\frac{bn\sqrt{1+\frac{d}{ex^2}}\left(2d^{3/2}\,{}_3F_2\left(\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{5}{2},\frac{5}{2};-\frac{d}{ex^2}\right)+9ex^2\left(-\sqrt{d}\sqrt{1+\frac{d}{ex^2}}+\sqrt{e}x\operatorname{arcsinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)\right)(1+2\log(x))\right)}{x^2\sqrt{d+ex^2}}-\frac{18\sqrt{d}\sqrt{d+ex^2}(a-bn\log(x)+b\log(cx}{x^2}}{36d^{3/}}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*Sqrt[d + e*x^2]),x]
```

```
[Out] ((b*n*Sqrt[1 + d/(e*x^2)]*(2*d^(3/2)*HypergeometricPFQ[{3/2, 3/2, 3/2}, {5/
2, 5/2}, -(d/(e*x^2))] + 9*e*x^2*(-(Sqrt[d]*Sqrt[1 + d/(e*x^2)]) + Sqrt[e]*
x*ArcSinh[Sqrt[d]/(Sqrt[e]*x)])*(1 + 2*Log[x])))/(x^2*Sqrt[d + e*x^2]) - (1
8*Sqrt[d]*Sqrt[d + e*x^2]*(a - b*n*Log[x] + b*Log[c*x^n]))/x^2 - 18*e*Log[x
]*(a - b*n*Log[x] + b*Log[c*x^n]) + 18*e*(a - b*n*Log[x] + b*Log[c*x^n])*Lo
g[d + Sqrt[d]*Sqrt[d + e*x^2]])/(36*d^(3/2))
```

Maple [F]

$$\int \frac{a+b\ln(c\,x^n)}{x^3\sqrt{e\,x^2+d}}dx$$

```
[In] int((a+b*ln(c*x^n))/x^3/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/x^3/(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{b\log(cx^n)+a}{\sqrt{ex^2+d}x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(e*x^2 + d)*b*log(c*x^n) + sqrt(e*x^2 + d)*a)/(e*x^5 + d*x^3)
, x)
```

Sympy [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\log\left(cx^n\right)}{x^3\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**3/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*log(c*x**n))/(x**3*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{b\log\left(cx^n\right)+a}{\sqrt{ex^2+d}x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/(sqrt(e*x^2 + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\ln\left(c\,x^n\right)}{x^3\,\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e*x^2)^(1/2)),x)
[Out] int((a + b*log(c*x^n))/(x^3*(d + e*x^2)^(1/2)), x)
```

3.281 $\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 359

$$\begin{aligned}\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx =& -\frac{bnx\sqrt{d+ex^2}}{4e} - \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4e^{3/2}\sqrt{d+ex^2}} \\ & - \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4e^{3/2}\sqrt{d+ex^2}} \\ & + \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{3/2}\sqrt{d+ex^2}} \\ & + \frac{x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e} \\ & - \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{3/2}\sqrt{d+ex^2}} \\ & + \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{4e^{3/2}\sqrt{d+ex^2}}\end{aligned}$$

```
[Out] -1/4*b*n*x*(e*x^2+d)^(1/2)/e+1/2*x*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/e-1/4*b*
d^(3/2)*n*arcsinh(x*e^(1/2)/d^(1/2))*(1+e*x^2/d)^(1/2)/e^(3/2)/(e*x^2+d)^(1
/2)-1/4*b*d^(3/2)*n*arcsinh(x*e^(1/2)/d^(1/2))^2*(1+e*x^2/d)^(1/2)/e^(3/2)/
(e*x^2+d)^(1/2)+1/2*b*d^(3/2)*n*arcsinh(x*e^(1/2)/d^(1/2))*ln(1-(x*e^(1/2)/
d^(1/2)+(1+e*x^2/d)^(1/2))^2)*(1+e*x^2/d)^(1/2)/e^(3/2)/(e*x^2+d)^(1/2)-1/2
*d^(3/2)*arcsinh(x*e^(1/2)/d^(1/2))*(a+b*ln(c*x^n))*(1+e*x^2/d)^(1/2)/e^(3/
2)/(e*x^2+d)^(1/2)+1/4*b*d^(3/2)*n*polylog(2,(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)
^(1/2))^2)*(1+e*x^2/d)^(1/2)/e^(3/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2386, 327, 221, 2392, 12, 14, 201, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = -\frac{d^{3/2}\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{3/2}\sqrt{d+ex^2}} + \frac{x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e} + \frac{bd^{3/2}n\sqrt{\frac{ex^2}{d}+1}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{4e^{3/2}\sqrt{d+ex^2}} - \frac{bd^{3/2}n\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4e^{3/2}\sqrt{d+ex^2}} - \frac{bd^{3/2}n\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4e^{3/2}\sqrt{d+ex^2}} + \frac{bd^{3/2}n\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{3/2}\sqrt{d+ex^2}} - \frac{bnx\sqrt{d+ex^2}}{4e}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/Sqrt[d + e*x^2],x]
```

```
[Out] -1/4*(b*n*x*Sqrt[d + e*x^2])/e - (b*d^(3/2)*n*Sqrt[1 + (e*x^2)/d]*ArcSinh[(
Sqrt[e]*x)/Sqrt[d]])/(4*e^(3/2)*Sqrt[d + e*x^2]) - (b*d^(3/2)*n*Sqrt[1 + (e
*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)/(4*e^(3/2)*Sqrt[d + e*x^2]) + (b*d
^(3/2)*n*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[1 - E^(2*ArcS
inh[(Sqrt[e]*x)/Sqrt[d]])])/(2*e^(3/2)*Sqrt[d + e*x^2]) + (x*Sqrt[d + e*x^2
]*(a + b*Log[c*x^n]))/(2*e) - (d^(3/2)*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]
*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/(2*e^(3/2)*Sqrt[d + e*x^2]) + (b*d^(3/2)*n
*Sqrt[1 + (e*x^2)/d]*PolyLog[2, E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(4*e^(
3/2)*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2386

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^
(q_), x_Symbol] :> Dist[d^IntPart[q]*((d + e*x^2)^FracPart[q]/(1 + (e/d)*x^
2)^FracPart[q]), Int[x^m*(1 + (e/d)*x^2)^q*(a + b*Log[c*x^n]), x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && IntegerQ[m/2] && IntegerQ[q - 1/2] &&  !(LtQ[
m + 2*q, -2] || GtQ[d, 0])
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{ex^2}{d}}\int \frac{x^2(a+b\log(cx^n))}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{\sqrt{d+ex^2}}$$

$$= \frac{x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e} - \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{3/2}\sqrt{d+ex^2}} - \frac{\left(bn\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{\frac{dx\sqrt{1+\frac{ex^2}{d}}}{e}-\frac{d^{3/2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}}}{2x}\,dx}{\sqrt{d+ex^2}}$$

$$= \frac{x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e} - \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{3/2}\sqrt{d+ex^2}} - \frac{\left(bn\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{\frac{dx\sqrt{1+\frac{ex^2}{d}}}{e}-\frac{d^{3/2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}}}{x}\,dx}{2\sqrt{d+ex^2}}$$

$$= \frac{x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e} - \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{3/2}\sqrt{d+ex^2}} - \frac{\left(bn\sqrt{1+\frac{ex^2}{d}}\right)\int\left(\frac{d\sqrt{1+\frac{ex^2}{d}}}{e} - \frac{d^{3/2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}x}\right)dx}{2\sqrt{d+ex^2}}$$

$$= \frac{x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e} - \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{3/2}\sqrt{d+ex^2}} + \frac{\left(bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{2e^{3/2}\sqrt{d+ex^2}} - \frac{\left(bdn\sqrt{1+\frac{ex^2}{d}}\right)\int\sqrt{1+\frac{ex^2}{d}}\,dx}{2e\sqrt{d+ex^2}}$$

$$= -\frac{bnx\sqrt{d+ex^2}}{4e} + \frac{x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e} - \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{3/2}\sqrt{d+ex^2}} + \frac{\left(bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\right)\operatorname{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{2e^{3/2}\sqrt{d+ex^2}} - \frac{\left(bdn\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1+\frac{ex^2}{d}}}dx}{4e\sqrt{d+ex^2}}$$

$$= -\frac{bnx\sqrt{d+ex^2}}{4e} - \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4e^{3/2}\sqrt{d+ex^2}} - \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4e^{3/2}\sqrt{d+ex^2}} + \frac{x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e} - \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{3/2}\sqrt{d+ex^2}} - \frac{\left(bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\right)\operatorname{Subst}\left(\int\frac{e^{2x}x}{1-e^{2x}}dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{e^{3/2}\sqrt{d+ex^2}}$$

$$= -\frac{bnx\sqrt{d+ex^2}}{4e} - \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4e^{3/2}\sqrt{d+ex^2}} - \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4e^{3/2}\sqrt{d+ex^2}} + \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{3/2}\sqrt{d+ex^2}} + \frac{x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e} - \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{3/2}\sqrt{d+ex^2}} - \frac{\left(bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int\log\left(1-e^{2x}\right)\,dx,x,\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{2e^{3/2}\sqrt{d+ex^2}}$$

$$= -\frac{bnx\sqrt{d+ex^2}}{4e} - \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4e^{3/2}\sqrt{d+ex^2}} - \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4e^{3/2}\sqrt{d+ex^2}} + \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{3/2}\sqrt{d+ex^2}} + \frac{x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e} - \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{3/2}\sqrt{d+ex^2}} - \frac{\left(bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{4e^{3/2}\sqrt{d+ex^2}}$$

$$= -\frac{bnx\sqrt{d+ex^2}}{4e} - \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4e^{3/2}\sqrt{d+ex^2}} - \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4e^{3/2}\sqrt{d+ex^2}} + \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{3/2}\sqrt{d+ex^2}} + \frac{x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e} - \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{3/2}\sqrt{d+ex^2}} + \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\text{Li}_2\left(e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{4e^{3/2}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.51 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.57

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{\frac{bn\sqrt{1+\frac{ex^2}{d}}\left(2e^2x^3\,{}_3F_2\left(\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{5}{2},\frac{5}{2};-\frac{ex^2}{d}\right)+9d\sqrt{e}\left(\sqrt{e}x\sqrt{1+\frac{ex^2}{d}}-\sqrt{d}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)(-1+2\log(x))\right)}{\sqrt{d+ex^2}}+18ex\sqrt{d+ex^2}(a-bn\log}{36e^2}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/Sqrt[d + e*x^2],x]
```

```
[Out] ((b*n*Sqrt[1 + (e*x^2)/d]*(2*e^2*x^3*HypergeometricPFQ[{3/2, 3/2, 3/2}, {5/
2, 5/2}, -((e*x^2)/d)] + 9*d*Sqrt[e]*(Sqrt[e]*x*Sqrt[1 + (e*x^2)/d] - Sqrt[
d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])*(-1 + 2*Log[x])))/Sqrt[d + e*x^2] + 18*e*x
*Sqrt[d + e*x^2]*(a - b*n*Log[x] + b*Log[c*x^n]) - 18*d*Sqrt[e]*(a - b*n*Lo
g[x] + b*Log[c*x^n])*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(36*e^2)
```

Maple [F]

$$\int \frac{x^2(a+b\ln(c\,x^n))}{\sqrt{e\,x^2+d}}dx$$

```
[In] int(x^2*(a+b*ln(c*x^n))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*ln(c*x^n))/(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(e*x^2 + d)*b*x^2*log(c*x^n) + sqrt(e*x^2 + d)*a*x^2)/(e*x^2
+ d), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(e*x**2+d)**(1/2),x)
[Out] Integral(x**2*(a + b*log(c*x**n))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^2/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,x^n))}{\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e*x^2)^(1/2),x)
[Out] int((x^2*(a + b*log(c*x^n)))/(d + e*x^2)^(1/2), x)
```

3.282 $\int \frac{a+b\log(cx^n)}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 250

$$\int \frac{a+b\log(cx^n)}{\sqrt{d+ex^2}}\,dx = \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}} - \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{e}\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{e}\sqrt{d+ex^2}} - \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\sqrt{d+ex^2}}$$

```
[Out] 1/2*b*n*arcsinh(x*e^(1/2)/d^(1/2))^2*d^(1/2)*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x
^2+d)^(1/2)-b*n*arcsinh(x*e^(1/2)/d^(1/2))*ln(1-(x*e^(1/2)/d^(1/2)+(1+e*x^2
/d)^(1/2))^2)*d^(1/2)*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)+arcsinh(x*e
^(1/2)/d^(1/2))*(a+b*ln(c*x^n))*d^(1/2)*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)
^(1/2)-1/2*b*n*polylog(2,(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(
1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2364, 2362, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{a+b\log(cx^n)}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{e}\sqrt{d+ex^2}} - \frac{b\sqrt{d}n\sqrt{\frac{ex^2}{d}+1}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\sqrt{d+ex^2}} + \frac{b\sqrt{d}n\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}} - \frac{b\sqrt{d}n\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{e}\sqrt{d+ex^2}}$$

```
[In] Int[(a + b*Log[c*x^n])/Sqrt[d + e*x^2],x]
```

```
[Out] (b*Sqrt[d]*n*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)/(2*Sqrt[e]
*Sqrt[d + e*x^2]) - (b*Sqrt[d]*n*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sq
rt[d]]*Log[1 - E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(Sqrt[e]*Sqrt[d + e*x^2
]) + (Sqrt[d]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*(a + b*Log[c
*x^n]))/(Sqrt[e]*Sqrt[d + e*x^2]) - (b*Sqrt[d]*n*Sqrt[1 + (e*x^2)/d]*PolyLo
g[2, E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(2*Sqrt[e]*Sqrt[d + e*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2362

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[e, 2]), x]
```

```
 - Dist[b*(n/Rt[e, 2]), Int[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 2364

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Dist[Sqrt[1 + (e/d)*x^2]/Sqrt[d + e*x^2], Int[(a + b*Log[c*x^n])/Sqr
t[1 + (e/d)*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] &&  !GtQ[d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{ex^2}{d}}\int \frac{a+b\log(cx^n)}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{\sqrt{d+ex^2}}$$

$$= \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{e}\sqrt{d+ex^2}} - \frac{\left(b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{\sqrt{e}\sqrt{d+ex^2}}$$

$$= \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{e}\sqrt{d+ex^2}} - \frac{\left(b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{\sqrt{e}\sqrt{d+ex^2}}$$

$$=\frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}}+\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{e}\sqrt{d+ex^2}}+\frac{\left(2b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int\frac{e^{2x}x}{1-e^{2x}}\,dx,x,\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{\sqrt{e}\sqrt{d+ex^2}}$$

$$=\frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}}-\frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{e}\sqrt{d+ex^2}}+\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{e}\sqrt{d+ex^2}}+\frac{\left(b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int\log\left(1-e^{2x}\right)\,dx,x,\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{\sqrt{e}\sqrt{d+ex^2}}$$

$$=\frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}}-\frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{e}\sqrt{d+ex^2}}+\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{e}\sqrt{d+ex^2}}+\frac{\left(b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\sqrt{d+ex^2}}$$

$$=\frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}}-\frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{e}\sqrt{d+ex^2}}+\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{e}\sqrt{d+ex^2}}-\frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\text{Li}_2\left(e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\sqrt{d+ex^2}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.74

$$\int \frac{a+b\log(cx^n)}{\sqrt{d+ex^2}}\,dx = \frac{(a-bn\log(x)+b\log(cx^n))\log\left(ex+\sqrt{e}\sqrt{d+ex^2}\right)}{\sqrt{e}} + \frac{bn\sqrt{1+\frac{ex^2}{d}}\left(-\text{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)^2-2\text{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)\log\left(1-e^{-2\text{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)}\right)+2\log(x)\log\left(\sqrt{\frac{e}{d}}x+\sqrt{1 ...}\right.\right.}{2\sqrt{\frac{e}{d}}\sqrt{d+ex^2}}$$

```
[In] Integrate[(a + b*Log[c*x^n])/Sqrt[d + e*x^2],x]
```

```
[Out] ((a - b*n*Log[x] + b*Log[c*x^n])*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/Sqrt[e
] + (b*n*Sqrt[1 + (e*x^2)/d]*(-ArcSinh[Sqrt[e/d]*x]^2 - 2*ArcSinh[Sqrt[e/d]
*x]*Log[1 - E^(-2*ArcSinh[Sqrt[e/d]*x])] + 2*Log[x]*Log[Sqrt[e/d]*x + Sqrt[
1 + (e*x^2)/d]] + PolyLog[2, E^(-2*ArcSinh[Sqrt[e/d]*x])]))/(2*Sqrt[e/d]*Sq
rt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\ln(c\,x^n)}{\sqrt{e\,x^2+d}}dx$$

```
[In] int((a+b*ln(c*x^n))/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{\sqrt{d+ex^2}}\,dx = \int \frac{b\log(cx^n)+a}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(e*x^2 + d)*b*log(c*x^n) + sqrt(e*x^2 + d)*a)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{a + b\log(cx^n)}{\sqrt{d + ex^2}}\,dx = \int \frac{a + b\log(cx^n)}{\sqrt{d + ex^2}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*log(c*x**n))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\log(cx^n)}{\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + b\log(cx^n)}{\sqrt{d + ex^2}}\,dx = \int \frac{b\log(cx^n) + a}{\sqrt{ex^2 + d}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\ln(c\,x^n)}{\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(d + e*x^2)^(1/2),x)
[Out] int((a + b*log(c*x^n))/(d + e*x^2)^(1/2), x)
```

3.283 $\int \frac{a+b\log(cx^n)}{x^2\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 81

$$\int \frac{a+b\log(cx^n)}{x^2\sqrt{d+ex^2}}\,dx=-\frac{bn\sqrt{d+ex^2}}{dx}+\frac{b\sqrt{e}n\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{d}-\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{dx}$$

```
[Out] b*n*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^(1/2)/d-b*n*(e*x^2+d)^(1/2)/d/x-(a
+b*ln(c*x^n))*(e*x^2+d)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2373, 283, 223, 212}

$$\int \frac{a+b\log(cx^n)}{x^2\sqrt{d+ex^2}}\,dx=-\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{dx}+\frac{b\sqrt{e}n\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{d}-\frac{bn\sqrt{d+ex^2}}{dx}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] -((b*n*Sqrt[d + e*x^2])/(d*x)) + (b*Sqrt[e]*n*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
e*x^2]])/d - (Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/(d*x)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{dx}+\frac{(bn)\int\frac{\sqrt{d+ex^2}}{x^2}\,dx}{d}\\
&= -\frac{bn\sqrt{d+ex^2}}{dx}-\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{dx}+\frac{(ben)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{d}\\
&= -\frac{bn\sqrt{d+ex^2}}{dx}-\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{dx}+\frac{(ben)\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{d}\\
&= -\frac{bn\sqrt{d+ex^2}}{dx}+\frac{b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{d}-\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{dx}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$
\int\frac{a+b\log(cx^n)}{x^2\sqrt{d+ex^2}}\,dx = \frac{-\left((a+bn)\sqrt{d+ex^2}\right)-b\sqrt{d+ex^2}\log(cx^n)+b\sqrt{e}nx\log\left(ex+\sqrt{e}\sqrt{d+ex^2}\right)}{dx}
$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] (-((a + b*n)*Sqrt[d + e*x^2]) - b*Sqrt[d + e*x^2]*Log[c*x^n] + b*Sqrt[e]*n*
x*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(d*x)
```

Maple [F]

$$\int \frac{a+b\ln\left(c\,x^{n}\right)}{x^{2}\sqrt{e\,x^{2}+d}}dx$$

```
[In] int((a+b*ln(c*x^n))/x^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/x^2/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.57

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{2}\sqrt{d+ex^{2}}}\,dx = \left[\frac{b\sqrt{e}nx\log\left(-2\,ex^{2}-2\,\sqrt{ex^{2}+d}\sqrt{e}x-d\right)-2\,\sqrt{ex^{2}+d}(bn\log\left(x\right)+bn+b\log\left(c\right)+a)}{2\,dx}, -\frac{b\sqrt{-e}nx\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^{2}+d}}\right)+\sqrt{ex^{2}+d}(bn\log\left(x\right)+bn+b\log\left(c\right)+a)}{dx}\right]$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(b*sqrt(e)*n*x*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) - 2*sqr
t(e*x^2 + d)*(b*n*log(x) + b*n + b*log(c) + a))/(d*x), -(b*sqrt(-e)*n*x*arc
tan(sqrt(-e)*x/sqrt(e*x^2 + d)) + sqrt(e*x^2 + d)*(b*n*log(x) + b*n + b*log
(c) + a))/(d*x)]
```

Sympy [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{2}\sqrt{d+ex^{2}}}\,dx = \int \frac{a+b\log\left(cx^{n}\right)}{x^{2}\sqrt{d+ex^{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*log(c*x**n))/(x**2*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\log\left(cx^{n}\right)}{x^{2}\sqrt{d + ex^{2}}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + b\log\left(cx^{n}\right)}{x^{2}\sqrt{d + ex^{2}}}\,dx = \int \frac{b\log\left(cx^{n}\right) + a}{\sqrt{ex^{2} + dx^{2}}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/(sqrt(e*x^2 + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(cx^{n}\right)}{x^{2}\sqrt{d + ex^{2}}}\,dx = \int \frac{a + b\,\ln\left(c\,x^{n}\right)}{x^{2}\,\sqrt{e\,x^{2} + d}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*x^2)^(1/2)), x)
```

3.284 $\int \frac{a+b\log(cx^n)}{x^4\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 144

$$\int \frac{a+b\log(cx^n)}{x^4\sqrt{d+ex^2}}\,dx = \frac{2ben\sqrt{d+ex^2}}{3d^2x} - \frac{bn(d+ex^2)^{3/2}}{9d^2x^3} - \frac{2be^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d^2} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{3dx^3} + \frac{2e\sqrt{d+ex^2}(a+b\log(cx^n))}{3d^2x}$$

```
[Out] -1/9*b*n*(e*x^2+d)^(3/2)/d^2/x^3-2/3*b*e^(3/2)*n*arctanh(x*e^(1/2)/(e*x^2+d
)^(1/2))/d^2+2/3*b*e*n*(e*x^2+d)^(1/2)/d^2/x-1/3*(a+b*ln(c*x^n))*(e*x^2+d)^
(1/2)/d/x^3+2/3*e*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/d^2/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {277, 270, 2392, 12, 462, 283, 223, 212}

$$\int \frac{a+b\log(cx^n)}{x^4\sqrt{d+ex^2}}\,dx = \frac{2e\sqrt{d+ex^2}(a+b\log(cx^n))}{3d^2x} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{3dx^3} - \frac{2be^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d^2} + \frac{2ben\sqrt{d+ex^2}}{3d^2x} - \frac{bn(d+ex^2)^{3/2}}{9d^2x^3}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] (2*b*e*n*Sqrt[d + e*x^2])/(3*d^2*x) - (b*n*(d + e*x^2)^(3/2))/(9*d^2*x^3) -
 (2*b*e^(3/2)*n*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(3*d^2) - (Sqrt[d + e
*x^2]*(a + b*Log[c*x^n]))/(3*d*x^3) + (2*e*Sqrt[d + e*x^2]*(a + b*Log[c*x^n
]))/(3*d^2*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{3dx^3}+\frac{2e\sqrt{d+ex^2}(a+b\log(cx^n))}{3d^2x} \\
&\quad -(bn)\int \frac{\sqrt{d+ex^2}(-d+2ex^2)}{3d^2x^4}\,dx \\
&= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{3dx^3}+\frac{2e\sqrt{d+ex^2}(a+b\log(cx^n))}{3d^2x}-\frac{(bn)\int \frac{\sqrt{d+ex^2}(-d+2ex^2)}{x^4}\,dx}{3d^2} \\
&= -\frac{bn(d+ex^2)^{3/2}}{9d^2x^3}-\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{3dx^3} \\
&\quad +\frac{2e\sqrt{d+ex^2}(a+b\log(cx^n))}{3d^2x}-\frac{(2ben)\int \frac{\sqrt{d+ex^2}}{x^2}\,dx}{3d^2} \\
&= \frac{2ben\sqrt{d+ex^2}}{3d^2x}-\frac{bn(d+ex^2)^{3/2}}{9d^2x^3}-\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{3dx^3} \\
&\quad +\frac{2e\sqrt{d+ex^2}(a+b\log(cx^n))}{3d^2x}-\frac{(2be^2n)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{3d^2} \\
&= \frac{2ben\sqrt{d+ex^2}}{3d^2x}-\frac{bn(d+ex^2)^{3/2}}{9d^2x^3}-\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{3dx^3} \\
&\quad +\frac{2e\sqrt{d+ex^2}(a+b\log(cx^n))}{3d^2x}-\frac{(2be^2n)\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{3d^2} \\
&= \frac{2ben\sqrt{d+ex^2}}{3d^2x}-\frac{bn(d+ex^2)^{3/2}}{9d^2x^3}-\frac{2be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d^2} \\
&\quad -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{3dx^3}+\frac{2e\sqrt{d+ex^2}(a+b\log(cx^n))}{3d^2x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

$$\int \frac{a+b\log(cx^n)}{x^4\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}(-3ad-bdn+6aex^2+5benx^2)-3b(d-2ex^2)\sqrt{d+ex^2}\log(cx^n)-6be^{3/2}nx^3\log\left(ex+\sqrt{e}\sqrt{\right.}}{9d^2x^3}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-3*a*d - b*d*n + 6*a*e*x^2 + 5*b*e*n*x^2) - 3*b*(d - 2*e*
x^2)*Sqrt[d + e*x^2]*Log[c*x^n] - 6*b*e^(3/2)*n*x^3*Log[e*x + Sqrt[e]*Sqrt[
d + e*x^2]])/(9*d^2*x^3)
```

Maple [F]

$$\int \frac{a+b\ln(c\,x^n)}{x^4\sqrt{e\,x^2+d}}dx$$

```
[In] int((a+b*ln(c*x^n))/x^4/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/x^4/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.55

$$\int \frac{a+b\log(cx^n)}{x^4\sqrt{d+ex^2}}\,dx = \left[\frac{3\,be^{\frac{3}{2}}nx^3\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-(bdn-(5\,ben+6\,ae)x^2+3\,ad-3\,(2\,bex^2-bd)\log(c)-}{9\,d^2x^3}\right.$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/9*(3*b*e^(3/2)*n*x^3*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) - (
b*d*n - (5*b*e*n + 6*a*e)*x^2 + 3*a*d - 3*(2*b*e*x^2 - b*d)*log(c) - 3*(2*b
*e*n*x^2 - b*d*n)*log(x))*sqrt(e*x^2 + d))/(d^2*x^3), 1/9*(6*b*sqrt(-e)*e*n
*x^3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (b*d*n - (5*b*e*n + 6*a*e)*x^2 +
3*a*d - 3*(2*b*e*x^2 - b*d)*log(c) - 3*(2*b*e*n*x^2 - b*d*n)*log(x))*sqrt(e
*x^2 + d))/(d^2*x^3)]
```

Sympy [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{4}\sqrt{d+ex^{2}}}\,dx=\int \frac{a+b\log\left(cx^{n}\right)}{x^{4}\sqrt{d+ex^{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**4/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*log(c*x**n))/(x**4*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{4}\sqrt{d+ex^{2}}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{4}\sqrt{d+ex^{2}}}\,dx=\int \frac{b\log\left(cx^{n}\right)+a}{\sqrt{ex^{2}+d}x^{4}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{4}\sqrt{d+ex^{2}}}\,dx=\int \frac{a+b\,\ln\left(c\,x^{n}\right)}{x^{4}\,\sqrt{e\,x^{2}+d}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^4*(d + e*x^2)^(1/2)),x)
[Out] int((a + b*log(c*x^n))/(x^4*(d + e*x^2)^(1/2)), x)
```

3.285 $\int \frac{a+b\log(cx^n)}{x^6\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 204

$$\int \frac{a+b\log(cx^n)}{x^6\sqrt{d+ex^2}}\,dx = -\frac{8be^2n\sqrt{d+ex^2}}{15d^3x}-\frac{bn(d+ex^2)^{3/2}}{25d^2x^5}+\frac{26ben(d+ex^2)^{3/2}}{225d^3x^3}+\frac{8be^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{15d^3}-\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{5dx^5}+\frac{4e\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^2x^3}-\frac{8e^2\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^3x}$$

```
[Out] -1/25*b*n*(e*x^2+d)^(3/2)/d^2/x^5+26/225*b*e*n*(e*x^2+d)^(3/2)/d^3/x^3+8/15
*b*e^(5/2)*n*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/d^3-8/15*b*e^2*n*(e*x^2+d)^
(1/2)/d^3/x-1/5*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/d/x^5+4/15*e*(a+b*ln(c*x^n)
)*(e*x^2+d)^(1/2)/d^2/x^3-8/15*e^2*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/d^3/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {277, 270, 2392, 12, 1279, 462, 283, 223, 212}

$$\int \frac{a+b\log(cx^n)}{x^6\sqrt{d+ex^2}}\,dx = -\frac{8e^2\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^3x}+\frac{4e\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^2x^3}-\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{5dx^5}+\frac{8be^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{15d^3}-\frac{8be^2n\sqrt{d+ex^2}}{15d^3x}+\frac{26ben(d+ex^2)^{3/2}}{225d^3x^3}-\frac{bn(d+ex^2)^{3/2}}{25d^2x^5}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^6*Sqrt[d + e*x^2]),x]
```

```
[Out] (-8*b*e^2*n*Sqrt[d + e*x^2])/(15*d^3*x) - (b*n*(d + e*x^2)^(3/2))/(25*d^2*x
^5) + (26*b*e*n*(d + e*x^2)^(3/2))/(225*d^3*x^3) + (8*b*e^(5/2)*n*ArcTanh[(
Sqrt[e]*x)/Sqrt[d + e*x^2]])/(15*d^3) - (Sqrt[d + e*x^2]*(a + b*Log[c*x^n])
)/(5*d*x^5) + (4*e*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/(15*d^2*x^3) - (8*e^
2*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/(15*d^3*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{5dx^5}+\frac{4e\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^2x^3} \\
&\quad -\frac{8e^2\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^3x}-(bn)\int\frac{\sqrt{d+ex^2}(-3d^2+4dex^2-8e^2x^4)}{15d^3x^6}\,dx \\
&= -\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{5dx^5}+\frac{4e\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^2x^3} \\
&\quad -\frac{8e^2\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^3x}-\frac{(bn)\int\frac{\sqrt{d+ex^2}(-3d^2+4dex^2-8e^2x^4)}{x^6}\,dx}{15d^3} \\
&= -\frac{bn(d+ex^2)^{3/2}}{25d^2x^5}-\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{5dx^5}+\frac{4e\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^2x^3} \\
&\quad -\frac{8e^2\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^3x}+\frac{(bn)\int\frac{\sqrt{d+ex^2}(-26d^2e+40de^2x^2)}{x^4}\,dx}{75d^4}
\end{aligned}$$

$$= -\frac{bn(d+ex^2)^{3/2}}{25d^2x^5} + \frac{26ben(d+ex^2)^{3/2}}{225d^3x^3} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{5dx^5} + \frac{4e\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^2x^3} - \frac{8e^2\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^3x} + \frac{(8be^2n)\int\frac{\sqrt{d+ex^2}}{x^2}\,dx}{15d^3}$$

$$= -\frac{8be^2n\sqrt{d+ex^2}}{15d^3x} - \frac{bn(d+ex^2)^{3/2}}{25d^2x^5} + \frac{26ben(d+ex^2)^{3/2}}{225d^3x^3} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{5dx^5} + \frac{4e\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^2x^3} - \frac{8e^2\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^3x} + \frac{(8be^3n)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{15d^3}$$

$$= -\frac{8be^2n\sqrt{d+ex^2}}{15d^3x} - \frac{bn(d+ex^2)^{3/2}}{25d^2x^5} + \frac{26ben(d+ex^2)^{3/2}}{225d^3x^3} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{5dx^5} + \frac{4e\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^2x^3} - \frac{8e^2\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^3x} + \frac{(8be^3n)\,\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{15d^3}$$

$$= -\frac{8be^2n\sqrt{d+ex^2}}{15d^3x} - \frac{bn(d+ex^2)^{3/2}}{25d^2x^5} + \frac{26ben(d+ex^2)^{3/2}}{225d^3x^3} + \frac{8be^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{15d^3} - \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{5dx^5} + \frac{4e\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^2x^3} - \frac{8e^2\sqrt{d+ex^2}(a+b\log(cx^n))}{15d^3x}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.72

$$\int\frac{a+b\log(cx^n)}{x^6\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}(15a(3d^2-4dex^2+8e^2x^4)+bn(9d^2-17dex^2+94e^2x^4))+15b\sqrt{d+ex^2}(3d^2-4dex^2+8e^2x^4)\cdots}{225d^3x^5}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^6*Sqrt[d + e*x^2]),x]
```

```
[Out] -1/225*(Sqrt[d + e*x^2]*(15*a*(3*d^2 - 4*d*e*x^2 + 8*e^2*x^4) + b*n*(9*d^2
- 17*d*e*x^2 + 94*e^2*x^4)) + 15*b*Sqrt[d + e*x^2]*(3*d^2 - 4*d*e*x^2 + 8*e
^2*x^4)*Log[c*x^n] - 120*b*e^(5/2)*n*x^5*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]]
)/(d^3*x^5)
```

Maple [F]

$$\int \frac{a+b\ln\left(c\,x^{n}\right)}{x^{6}\sqrt{e\,x^{2}+d}}dx$$

[In] int((a+b*ln(c*x^n))/x^6/(e*x^2+d)^(1/2),x)

[Out] int((a+b*ln(c*x^n))/x^6/(e*x^2+d)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.60

$$\int \frac{a+b\log(cx^n)}{x^6\sqrt{d+ex^2}}\,dx = \left[\frac{60\,be^{\frac{5}{2}}nx^5\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-(2\,(47\,be^2n+60\,ae^2)x^4+9\,bd^2n+45\,ad^2-(17\,bden+\cdots}{225\,d\cdots}\right.$$

$$-\frac{120\,b\sqrt{-e}e^2nx^5\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+(2\,(47\,be^2n+60\,ae^2)x^4+9\,bd^2n+45\,ad^2-(17\,bden+60\,ade)x^2+\cdots}{225\,d^3x^5}$$

[In] integrate((a+b*log(c*x^n))/x^6/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] [1/225*(60*b*e^(5/2)*n*x^5*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) - (2*(47*b*e^2*n + 60*a*e^2)*x^4 + 9*b*d^2*n + 45*a*d^2 - (17*b*d*e*n + 60*a*d*e)*x^2 + 15*(8*b*e^2*x^4 - 4*b*d*e*x^2 + 3*b*d^2)*log(c) + 15*(8*b*e^2*n*x^4 - 4*b*d*e*n*x^2 + 3*b*d^2*n)*log(x))*sqrt(e*x^2 + d))/(d^3*x^5), -1/225*(120*b*sqrt(-e)*e^2*n*x^5*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (2*(47*b*e^2*n + 60*a*e^2)*x^4 + 9*b*d^2*n + 45*a*d^2 - (17*b*d*e*n + 60*a*d*e)*x^2 + 15*(8*b*e^2*x^4 - 4*b*d*e*x^2 + 3*b*d^2)*log(c) + 15*(8*b*e^2*n*x^4 - 4*b*d*e*n*x^2 + 3*b*d^2*n)*log(x))*sqrt(e*x^2 + d))/(d^3*x^5)]

Sympy [F]

$$\int \frac{a+b\log(cx^n)}{x^6\sqrt{d+ex^2}}\,dx = \int \frac{a+b\log(cx^n)}{x^6\sqrt{d+ex^2}}\,dx$$

[In] integrate((a+b*ln(c*x**n))/x**6/(e*x**2+d)**(1/2),x)

[Out] Integral((a + b*log(c*x**n))/(x**6*sqrt(d + e*x**2)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{6}\sqrt{d+ex^{2}}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x^6/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{6}\sqrt{d+ex^{2}}}\,dx = \int \frac{b\log\left(cx^{n}\right)+a}{\sqrt{ex^{2}+dx^{6}}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^6/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/(sqrt(e*x^2 + d)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{6}\sqrt{d+ex^{2}}}\,dx = \int \frac{a+b\,\ln\left(c\,x^{n}\right)}{x^{6}\,\sqrt{e\,x^{2}+d}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^6*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^6*(d + e*x^2)^(1/2)), x)
```

3.286 $\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{3/2}} dx$

Optimal result

Integrand size = 25, antiderivative size = 209

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{3/2}} dx = -\frac{11bd^2n\sqrt{d+ex^2}}{5e^4} + \frac{4bdn(d+ex^2)^{3/2}}{15e^4} - \frac{bn(d+ex^2)^{5/2}}{25e^4} + \frac{16bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{5e^4} + \frac{d^3(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}} + \frac{3d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4} - \frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{e^4} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^4}$$

```
[Out] 4/15*b*d*n*(e*x^2+d)^(3/2)/e^4-1/25*b*n*(e*x^2+d)^(5/2)/e^4+16/5*b*d^(5/2)*
n*arctanh((e*x^2+d)^(1/2)/d^(1/2))/e^4-d*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/e^
4+1/5*(e*x^2+d)^(5/2)*(a+b*ln(c*x^n))/e^4+d^3*(a+b*ln(c*x^n))/e^4/(e*x^2+d)
^(1/2)-11/5*b*d^2*n*(e*x^2+d)^(1/2)/e^4+3*d^2*(a+b*ln(c*x^n))*(e*x^2+d)^(1/
2)/e^4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {272, 45, 2392, 12, 1813, 1634, 65, 214}

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{3/2}} dx = \frac{d^3(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}} + \frac{3d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4} - \frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{e^4} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^4} + \frac{16bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{5e^4} - \frac{11bd^2n\sqrt{d+ex^2}}{5e^4} + \frac{4bdn(d+ex^2)^{3/2}}{15e^4} - \frac{bn(d+ex^2)^{5/2}}{25e^4}$$

```
[In] Int[(x^7*(a + b*Log[c*x^n]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (-11*b*d^2*n*Sqrt[d + e*x^2])/(5*e^4) + (4*b*d*n*(d + e*x^2)^(3/2))/(15*e^4
) - (b*n*(d + e*x^2)^(5/2))/(25*e^4) + (16*b*d^(5/2)*n*ArcTanh[Sqrt[d + e*x
^2]/Sqrt[d]])/(5*e^4) + (d^3*(a + b*Log[c*x^n]))/(e^4*Sqrt[d + e*x^2]) + (3
*d^2*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/e^4 - (d*(d + e*x^2)^(3/2)*(a + b*
Log[c*x^n]))/e^4 + ((d + e*x^2)^(5/2)*(a + b*Log[c*x^n]))/(5*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \frac{d^3(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}} + \frac{3d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4} - \frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{e^4} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^4} - (bn)\int \frac{16d^3+8d^2ex^2-2de^2x^4+e^3x^6}{5e^4x\sqrt{d+ex^2}}\,dx$$

$$= \frac{d^3(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}} + \frac{3d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4} - \frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{e^4} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^4} - \frac{(bn)\int \frac{16d^3+8d^2ex^2-2de^2x^4+e^3x^6}{x\sqrt{d+ex^2}}\,dx}{5e^4}$$

$$= \frac{d^3(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}} + \frac{3d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4} - \frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{e^4} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^4} - \frac{(bn)\text{Subst}\left(\int \frac{16d^3+8d^2ex-2de^2x^2+e^3x^3}{x\sqrt{d+ex}}\,dx, x, x^2\right)}{10e^4}$$

$$= \frac{d^3(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}} + \frac{3d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4} - \frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{e^4} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^4} - \frac{(bn)\text{Subst}\left(\int \left(\frac{11d^2e}{\sqrt{d+ex}} + \frac{16d^3}{x\sqrt{d+ex}} - 4de\sqrt{d+ex} + e(d+ex)^{3/2}\right) dx, x, x^2\right)}{10e^4}$$

$$= -\frac{11bd^2n\sqrt{d+ex^2}}{5e^4} + \frac{4bdn(d+ex^2)^{3/2}}{15e^4} - \frac{bn(d+ex^2)^{5/2}}{25e^4} + \frac{d^3(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}} + \frac{3d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4} - \frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{e^4} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^4} - \frac{(8bd^3n)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)}{5e^4}$$

$$= -\frac{11bd^2n\sqrt{d+ex^2}}{5e^4} + \frac{4bdn(d+ex^2)^{3/2}}{15e^4} - \frac{bn(d+ex^2)^{5/2}}{25e^4} + \frac{d^3(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}} + \frac{3d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4} - \frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{e^4} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^4} - \frac{(16bd^3n)\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{5e^5}$$

$$= -\frac{11bd^2n\sqrt{d+ex^2}}{5e^4} + \frac{4bdn(d+ex^2)^{3/2}}{15e^4} - \frac{bn(d+ex^2)^{5/2}}{25e^4} + \frac{16bd^{5/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{5e^4} + \frac{d^3(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}} + \frac{3d^2\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4} - \frac{d(d+ex^2)^{3/2}(a+b\log(cx^n))}{e^4} + \frac{(d+ex^2)^{5/2}(a+b\log(cx^n))}{5e^4}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.93

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \frac{240ad^3 - 148bd^3n + 120ad^2ex^2 - 134bd^2enx^2 - 30ade^2x^4 + 11bde^2nx^4 + 15ae^3x^6}{}$$

```
[In] Integrate[(x^7*(a + b*Log[c*x^n]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (240*a*d^3 - 148*b*d^3*n + 120*a*d^2*e*x^2 - 134*b*d^2*e*n*x^2 - 30*a*d*e^2
*x^4 + 11*b*d*e^2*n*x^4 + 15*a*e^3*x^6 - 3*b*e^3*n*x^6 - 240*b*d^(5/2)*n*Sq
rt[d + e*x^2]*Log[x] + 15*b*(16*d^3 + 8*d^2*e*x^2 - 2*d*e^2*x^4 + e^3*x^6)*
Log[c*x^n] + 240*b*d^(5/2)*n*Sqrt[d + e*x^2]*Log[d + Sqrt[d]*Sqrt[d + e*x^2
]])/(75*e^4*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^7(a+b\ln(c\,x^n))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^7*(a+b*ln(c*x^n))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^7*(a+b*ln(c*x^n))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.21

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \left[\frac{120\,(bd^2enx^2+bd^3n)\sqrt{d}\log\left(-\frac{ex^2+2\sqrt{ex^2+d}\sqrt{d}+2\,d}{x^2}\right)-(3\,(be^3n-5\,ae^3)x^6+148\right.$$

$$-\frac{240\,(bd^2enx^2+bd^3n)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}}{\sqrt{ex^2+d}}\right)+(3\,(be^3n-5\,ae^3)x^6+148\,bd^3n-(11\,bde^2n-30\,ade^2)x^4-}{}$$

```
[In] integrate(x^7*(a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/75*(120*(b*d^2*e*n*x^2 + b*d^3*n)*sqrt(d)*log(-(e*x^2 + 2*sqrt(e*x^2 + d
)*sqrt(d) + 2*d)/x^2) - (3*(b*e^3*n - 5*a*e^3)*x^6 + 148*b*d^3*n - (11*b*d*
e^2*n - 30*a*d*e^2)*x^4 - 240*a*d^3 + 2*(67*b*d^2*e*n - 60*a*d^2*e)*x^2 - 1
5*(b*e^3*x^6 - 2*b*d*e^2*x^4 + 8*b*d^2*e*x^2 + 16*b*d^3)*log(c) - 15*(b*e^3
*n*x^6 - 2*b*d*e^2*n*x^4 + 8*b*d^2*e*n*x^2 + 16*b*d^3*n)*log(x))*sqrt(e*x^2
 + d))/(e^5*x^2 + d*e^4), -1/75*(240*(b*d^2*e*n*x^2 + b*d^3*n)*sqrt(-d)*arc
tan(sqrt(-d)/sqrt(e*x^2 + d)) + (3*(b*e^3*n - 5*a*e^3)*x^6 + 148*b*d^3*n -
(11*b*d*e^2*n - 30*a*d*e^2)*x^4 - 240*a*d^3 + 2*(67*b*d^2*e*n - 60*a*d^2*e)
*x^2 - 15*(b*e^3*x^6 - 2*b*d*e^2*x^4 + 8*b*d^2*e*x^2 + 16*b*d^3)*log(c) - 1
5*(b*e^3*n*x^6 - 2*b*d*e^2*n*x^4 + 8*b*d^2*e*n*x^2 + 16*b*d^3*n)*log(x))*sq
rt(e*x^2 + d))/(e^5*x^2 + d*e^4)]
```

Sympy [A] (verification not implemented)

Time = 44.81 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.79

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = a\left(\begin{cases}\frac{d^3}{e^4\sqrt{d+ex^2}}+\frac{3d^2\sqrt{d+ex^2}}{e^4}-\frac{d(d+ex^2)^{\frac{3}{2}}}{e^4}+\frac{(d+ex^2)^{\frac{5}{2}}}{5e^4} & \text{for } e\neq 0\\ \frac{x^8}{8d^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$

$$-bn\left(\begin{cases}-\frac{77d^{\frac{5}{2}}\sqrt{1+\frac{ex^2}{d}}}{75e^4}-\frac{2d^{\frac{5}{2}}\log\left(\frac{ex^2}{d}\right)}{5e^4}+\frac{4d^{\frac{5}{2}}\log\left(\sqrt{1+\frac{ex^2}{d}}+1\right)}{5e^4}-\frac{4d^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{e^4}-\frac{14d^{\frac{3}{2}}x^2\sqrt{1+\frac{ex^2}{d}}}{75e^3}+\frac{\sqrt{d}x^4\sqrt{1+\frac{ex^2}{d}}}{25e^2}+\cdots\\ \frac{x^8}{64d^{\frac{3}{2}}}\end{cases}\right.$$

$$+b\left(\begin{cases}\frac{d^3}{e^4\sqrt{d+ex^2}}+\frac{3d^2\sqrt{d+ex^2}}{e^4}-\frac{d(d+ex^2)^{\frac{3}{2}}}{e^4}+\frac{(d+ex^2)^{\frac{5}{2}}}{5e^4} & \text{for } e\neq 0\\ \frac{x^8}{8d^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate(x**7*(a+b*ln(c*x**n))/(e*x**2+d)**(3/2),x)
```

```
[Out] a*Piecewise((d**3/(e**4*sqrt(d + e*x**2)) + 3*d**2*sqrt(d + e*x**2)/e**4 -
d*(d + e*x**2)**(3/2)/e**4 + (d + e*x**2)**(5/2)/(5*e**4), Ne(e, 0)), (x**8
/(8*d**(3/2)), True)) - b*n*Piecewise((-77*d**(5/2)*sqrt(1 + e*x**2/d)/(75*
e**4) - 2*d**(5/2)*log(e*x**2/d)/(5*e**4) + 4*d**(5/2)*log(sqrt(1 + e*x**2/
d) + 1)/(5*e**4) - 4*d**(5/2)*asinh(sqrt(d)/(sqrt(e)*x))/e**4 - 14*d**(3/2)
*x**2*sqrt(1 + e*x**2/d)/(75*e**3) + sqrt(d)*x**4*sqrt(1 + e*x**2/d)/(25*e*
*2) + 3*d**3/(e**(9/2)*x*sqrt(d/(e*x**2) + 1)) + 3*d**2*x/(e**(7/2)*sqrt(d/
(e*x**2) + 1)), (e > -oo) & (e < oo) & Ne(e, 0)), (x**8/(64*d**(3/2)), True
)) + b*Piecewise((d**3/(e**4*sqrt(d + e*x**2)) + 3*d**2*sqrt(d + e*x**2)/e*
*4 - d*(d + e*x**2)**(3/2)/e**4 + (d + e*x**2)**(5/2)/(5*e**4), Ne(e, 0)),
(x**8/(8*d**(3/2)), True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^7*(a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\log(cx^n)+a)x^7}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^7*(a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^7/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^7\,(a+b\,\ln(c\,x^n))}{(e\,x^2+d)^{3/2}}\,dx$$

```
[In] int((x^7*(a + b*log(c*x^n)))/(d + e*x^2)^(3/2),x)
[Out] int((x^7*(a + b*log(c*x^n)))/(d + e*x^2)^(3/2), x)
```

3.287 $\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^{3/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 158

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^{3/2}} \, dx = \frac{5bdn\sqrt{d+ex^2}}{3e^3} - \frac{bn(d+ex^2)^{3/2}}{9e^3} - \frac{8bd^{3/2}n\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3e^3} - \frac{d^2(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3}$$

```
[Out] -1/9*b*n*(e*x^2+d)^(3/2)/e^3-8/3*b*d^(3/2)*n*arctanh((e*x^2+d)^(1/2)/d^(1/2
))/e^3+1/3*(e*x^2+d)^(3/2)*(a+b*ln(c*x^n))/e^3-d^2*(a+b*ln(c*x^n))/e^3/(e*x
^2+d)^(1/2)+5/3*b*d*n*(e*x^2+d)^(1/2)/e^3-2*d*(a+b*ln(c*x^n))*(e*x^2+d)^(1/
2)/e^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {272, 45, 2392, 12, 1265, 911, 1167, 214}

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^{3/2}} \, dx = -\frac{d^2(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3} - \frac{8bd^{3/2}n\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3e^3} + \frac{5bdn\sqrt{d+ex^2}}{3e^3} - \frac{bn(d+ex^2)^{3/2}}{9e^3}$$

```
[In] Int[(x^5*(a + b*Log[c*x^n]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (5*b*d*n*Sqrt[d + e*x^2])/(3*e^3) - (b*n*(d + e*x^2)^(3/2))/(9*e^3) - (8*b*
d^(3/2)*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(3*e^3) - (d^2*(a + b*Log[c*x^n
]))/(e^3*Sqrt[d + e*x^2]) - (2*d*Sqrt[d + e*x^2]*(a + b*Log[c*x^n]))/e^3 +
((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/(3*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3} \\
&\quad + \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3} - (bn)\int \frac{-8d^2-4dex^2+e^2x^4}{3e^3x\sqrt{d+ex^2}}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3} \\
&\quad + \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3} - \frac{(bn)\int \frac{-8d^2-4dex^2+e^2x^4}{x\sqrt{d+ex^2}}\,dx}{3e^3} \\
&= -\frac{d^2(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3} \\
&\quad + \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3} - \frac{(bn)\text{Subst}\left(\int \frac{-8d^2-4dex+e^2x^2}{x\sqrt{d+ex}}\,dx, x, x^2\right)}{6e^3} \\
&= -\frac{d^2(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3} \\
&\quad + \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3} - \frac{(bn)\text{Subst}\left(\int \frac{-3d^2-6dx^2+x^4}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{3e^4} \\
&= -\frac{d^2(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3} \\
&\quad - \frac{(bn)\text{Subst}\left(\int \left(-5de+ex^2-\frac{8d^2}{-\frac{d}{e}+\frac{x^2}{e}}\right)dx, x, \sqrt{d+ex^2}\right)}{3e^4}
\end{aligned}$$

$$= \frac{5bdn\sqrt{d+ex^2}}{3e^3} - \frac{bn(d+ex^2)^{3/2}}{9e^3} - \frac{d^2(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3} + \frac{(8bd^2n)\operatorname{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{3e^4}$$

$$= \frac{5bdn\sqrt{d+ex^2}}{3e^3} - \frac{bn(d+ex^2)^{3/2}}{9e^3} - \frac{8bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3e^3} - \frac{d^2(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^3}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \frac{-24ad^2+14bd^2n-12adex^2+13bdenx^2+3ae^2x^4-be^2nx^4+24bd^{3/2}n\sqrt{d+ex^2}\ldots}{9e\ldots}$$

```
[In] Integrate[(x^5*(a + b*Log[c*x^n]))/(d + e*x^2)^(3/2),x]
[Out] (-24*a*d^2 + 14*b*d^2*n - 12*a*d*e*x^2 + 13*b*d*e*n*x^2 + 3*a*e^2*x^4 - b*e
^2*n*x^4 + 24*b*d^(3/2)*n*Sqrt[d + e*x^2]*Log[x] - 3*b*(8*d^2 + 4*d*e*x^2 -
 e^2*x^4)*Log[c*x^n] - 24*b*d^(3/2)*n*Sqrt[d + e*x^2]*Log[d + Sqrt[d]*Sqrt[
d + e*x^2]])/(9*e^3*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^5(a+b\ln(c\,x^n))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^5*(a+b*ln(c*x^n))/(e*x^2+d)^(3/2),x)
[Out] int(x^5*(a+b*ln(c*x^n))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.25

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \left[\frac{12\,(bdenx^2+bd^2n)\sqrt{d}\log\left(-\frac{ex^2-2\,\sqrt{ex^2+d}\sqrt{d}+2\,d}{x^2}\right) - ((be^2n-3\,ae^2)x^4-14\,bd^2\ldots}{\ldots}\right.$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/9*(12*(b*d*e*n*x^2 + b*d^2*n)*sqrt(d)*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sq
rt(d) + 2*d)/x^2) - ((b*e^2*n - 3*a*e^2)*x^4 - 14*b*d^2*n + 24*a*d^2 - (13*
b*d*e*n - 12*a*d*e)*x^2 - 3*(b*e^2*x^4 - 4*b*d*e*x^2 - 8*b*d^2)*log(c) - 3*
(b*e^2*n*x^4 - 4*b*d*e*n*x^2 - 8*b*d^2*n)*log(x))*sqrt(e*x^2 + d))/(e^4*x^2
 + d*e^3), 1/9*(24*(b*d*e*n*x^2 + b*d^2*n)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*
x^2 + d)) - ((b*e^2*n - 3*a*e^2)*x^4 - 14*b*d^2*n + 24*a*d^2 - (13*b*d*e*n
- 12*a*d*e)*x^2 - 3*(b*e^2*x^4 - 4*b*d*e*x^2 - 8*b*d^2)*log(c) - 3*(b*e^2*n
*x^4 - 4*b*d*e*n*x^2 - 8*b*d^2*n)*log(x))*sqrt(e*x^2 + d))/(e^4*x^2 + d*e^3
)]
```

Sympy [A] (verification not implemented)

Time = 37.36 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.95

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = a\left(\begin{cases} -\frac{d^2}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}}{e^3} + \frac{(d+ex^2)^{\frac{3}{2}}}{3e^3} & \text{for } e \neq 0 \\ \frac{x^6}{6d^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) - bn\left(\begin{cases} \frac{4d^{\frac{3}{2}}\sqrt{1+\frac{ex^2}{d}}}{9e^3} + \frac{d^{\frac{3}{2}}\log\left(\frac{ex^2}{d}\right)}{6e^3} - \frac{d^{\frac{3}{2}}\log\left(\sqrt{1+\frac{ex^2}{d}}+1\right)}{3e^3} + \frac{3d^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{e^3} + \frac{\sqrt{d}x^2\sqrt{1+\frac{ex^2}{d}}}{9e^2} - \frac{2d^2}{e^{\frac{7}{2}}x\sqrt{\frac{d}{ex^2}+1}} - \frac{2dx}{e^{\frac{5}{2}}\sqrt{\frac{d}{ex^2}+1}} \\ \frac{x^6}{36d^{\frac{3}{2}}} \end{cases}\right) + b\left(\begin{cases} -\frac{d^2}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}}{e^3} + \frac{(d+ex^2)^{\frac{3}{2}}}{3e^3} & \text{for } e \neq 0 \\ \frac{x^6}{6d^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)\log(cx^n)$$

```
[In] integrate(x**5*(a+b*ln(c*x**n))/(e*x**2+d)**(3/2),x)
```

```
[Out] a*Piecewise((-d**2/(e**3*sqrt(d + e*x**2)) - 2*d*sqrt(d + e*x**2)/e**3 + (d
 + e*x**2)**(3/2)/(3*e**3), Ne(e, 0)), (x**6/(6*d**(3/2)), True)) - b*n*Pie
cewise((4*d**(3/2)*sqrt(1 + e*x**2/d)/(9*e**3) + d**(3/2)*log(e*x**2/d)/(6*
e**3) - d**(3/2)*log(sqrt(1 + e*x**2/d) + 1)/(3*e**3) + 3*d**(3/2)*asinh(sq
rt(d)/(sqrt(e)*x))/e**3 + sqrt(d)*x**2*sqrt(1 + e*x**2/d)/(9*e**2) - 2*d**2
/(e**(7/2)*x*sqrt(d/(e*x**2) + 1)) - 2*d*x/(e**(5/2)*sqrt(d/(e*x**2) + 1)),
 (e > -oo) & (e < oo) & Ne(e, 0)), (x**6/(36*d**(3/2)), True)) + b*Piecewis
e((-d**2/(e**3*sqrt(d + e*x**2)) - 2*d*sqrt(d + e*x**2)/e**3 + (d + e*x**2)
**(3/2)/(3*e**3), Ne(e, 0)), (x**6/(6*d**(3/2)), True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b\log{(cx^n)})}{(d + ex^2)^{3/2}}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^5(a + b\log{(cx^n)})}{(d + ex^2)^{3/2}}\, dx = \int \frac{(b\log{(cx^n)} + a)x^5}{(ex^2 + d)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^5/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b\log{(cx^n)})}{(d + ex^2)^{3/2}}\, dx = \int \frac{x^5\,\left(a + b\,\ln{(c\,x^n)}\right)}{{\left(e\,x^2 + d\right)}^{3/2}}\, dx$$

```
[In] int((x^5*(a + b*log(c*x^n)))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((x^5*(a + b*log(c*x^n)))/(d + e*x^2)^(3/2), x)
```

3.288 $\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 100

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = -\frac{bn\sqrt{d+ex^2}}{e^2}+\frac{2b\sqrt{d}n\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{e^2}+\frac{d(a+b\log(cx^n))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e^2}$$

```
[Out] 2*b*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))*d^(1/2)/e^2+d*(a+b*ln(c*x^n))/e^2/(e
*x^2+d)^(1/2)-b*n*(e*x^2+d)^(1/2)/e^2+(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {272, 45, 2392, 12, 457, 81, 65, 214}

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e^2}+\frac{d(a+b\log(cx^n))}{e^2\sqrt{d+ex^2}}+\frac{2b\sqrt{d}n\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{e^2}-\frac{bn\sqrt{d+ex^2}}{e^2}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((b*n*Sqrt[d + e*x^2])/e^2) + (2*b*Sqrt[d]*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[
d]])/e^2 + (d*(a + b*Log[c*x^n]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*
(a + b*Log[c*x^n]))/e^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(a+b\log(cx^n))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e^2}-(bn)\int\frac{2d+ex^2}{e^2x\sqrt{d+ex^2}}\,dx\\
&= \frac{d(a+b\log(cx^n))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e^2}-\frac{(bn)\int\frac{2d+ex^2}{x\sqrt{d+ex^2}}\,dx}{e^2}\\
&= \frac{d(a+b\log(cx^n))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e^2}-\frac{(bn)\text{Subst}\left(\int\frac{2d+ex}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{2e^2}\\
&= -\frac{bn\sqrt{d+ex^2}}{e^2}+\frac{d(a+b\log(cx^n))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e^2}\\
&\quad-\frac{(bdn)\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{e^2}\\
&= -\frac{bn\sqrt{d+ex^2}}{e^2}+\frac{d(a+b\log(cx^n))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e^2}\\
&\quad-\frac{(2bdn)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{e^3}\\
&= -\frac{bn\sqrt{d+ex^2}}{e^2}+\frac{2b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{e^2}+\frac{d(a+b\log(cx^n))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.18

$$\int\frac{x^3(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx=\frac{2ad-bdn+aex^2-benx^2-2b\sqrt{d}n\sqrt{d+ex^2}\log(x)+b(2d+ex^2)\log(cx^n)+2b}{e^2\sqrt{d+ex^2}}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/(d + e*x^2)^(3/2),x]
[Out] (2*a*d - b*d*n + a*e*x^2 - b*e*n*x^2 - 2*b*Sqrt[d]*n*Sqrt[d + e*x^2]*Log[x]
 + b*(2*d + e*x^2)*Log[c*x^n] + 2*b*Sqrt[d]*n*Sqrt[d + e*x^2]*Log[d + Sqrt[
d]*Sqrt[d + e*x^2]])/(e^2*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^3(a+b\ln(c\,x^n))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^3*(a+b*ln(c*x^n))/(e*x^2+d)^(3/2),x)
[Out] int(x^3*(a+b*ln(c*x^n))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.45

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \left[\frac{(benx^2+bdn)\sqrt{d}\log\left(-\frac{ex^2+2\sqrt{ex^2+d}\sqrt{d}+2d}{x^2}\right)-(bdn+(ben-ae)x^2-2\,ad-(b}{e^3x^2+de^2}\right.$$
$$-\frac{2\,(benx^2+bdn)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}}{\sqrt{ex^2+d}}\right)+(bdn+(ben-ae)x^2-2\,ad-(bex^2+2\,bd)\log(c)-(benx^2+2\,b}{e^3x^2+de^2}$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] [((b*e*n*x^2 + b*d*n)*sqrt(d)*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d
)/x^2) - (b*d*n + (b*e*n - a*e)*x^2 - 2*a*d - (b*e*x^2 + 2*b*d)*log(c) - (b
*e*n*x^2 + 2*b*d*n)*log(x))*sqrt(e*x^2 + d))/(e^3*x^2 + d*e^2), -(2*(b*e*n*
x^2 + b*d*n)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x^2 + d)) + (b*d*n + (b*e*n -
a*e)*x^2 - 2*a*d - (b*e*x^2 + 2*b*d)*log(c) - (b*e*n*x^2 + 2*b*d*n)*log(x))
*sqrt(e*x^2 + d))/(e^3*x^2 + d*e^2)]
```

Sympy [A] (verification not implemented)

Time = 24.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.67

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = a\left(\begin{cases}\frac{d}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}}{e^2} & \text{for}\ e\neq 0\\ \frac{x^4}{4d^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$
$$-bn\left(\begin{cases}-\frac{2\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{e^2}+\frac{d}{e^{\frac{5}{2}}x\sqrt{\frac{d}{ex^2}+1}}+\frac{x}{e^{\frac{3}{2}}\sqrt{\frac{d}{ex^2}+1}} & \text{for}\ e>-\infty\wedge e<\infty\wedge e\neq 0\\ \frac{x^4}{16d^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$
$$+b\left(\begin{cases}\frac{d}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}}{e^2} & \text{for}\ e\neq 0\\ \frac{x^4}{4d^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))/(e*x**2+d)**(3/2),x)
```

```
[Out] a*Piecewise((d/(e**2*sqrt(d + e*x**2)) + sqrt(d + e*x**2)/e**2, Ne(e, 0)),
(x**4/(4*d**(3/2)), True)) - b*n*Piecewise((-2*sqrt(d)*asinh(sqrt(d)/(sqrt(
e)*x))/e**2 + d/(e**(5/2)*x*sqrt(d/(e*x**2) + 1)) + x/(e**(3/2)*sqrt(d/(e*x
**2) + 1)), (e > -oo) & (e < oo) & Ne(e, 0)), (x**4/(16*d**(3/2)), True)) +
 b*Piecewise((d/(e**2*sqrt(d + e*x**2)) + sqrt(d + e*x**2)/e**2, Ne(e, 0)),
 (x**4/(4*d**(3/2)), True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\log{(cx^n)})}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^3(a+b\log{(cx^n)})}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\log{(cx^n)}+a)x^3}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log{(cx^n)})}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^3\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n)))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*log(c*x^n)))/(d + e*x^2)^(3/2), x)
```

3.289 $\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = -\frac{bn\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}e} - \frac{a+b\log(cx^n)}{e\sqrt{d+ex^2}}$$

```
[Out] -b*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))/e/d^(1/2)+(-a-b*ln(c*x^n))/e/(e*x^2+d
)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2376, 272, 65, 214}

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = -\frac{a+b\log(cx^n)}{e\sqrt{d+ex^2}} - \frac{bn\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}e}$$

```
[In] Int[(x*(a + b*Log[c*x^n]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((b*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(Sqrt[d]*e)) - (a + b*Log[c*x^n])/
(e*Sqrt[d + e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\log\left(cx^n\right)}{e\sqrt{d+ex^2}}+\frac{(bn)\int\frac{1}{x\sqrt{d+ex^2}}\,dx}{e}\\
&= -\frac{a+b\log\left(cx^n\right)}{e\sqrt{d+ex^2}}+\frac{(bn)\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{2e}\\
&= -\frac{a+b\log\left(cx^n\right)}{e\sqrt{d+ex^2}}+\frac{(bn)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{e^2}\\
&= -\frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}e}-\frac{a+b\log\left(cx^n\right)}{e\sqrt{d+ex^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.35

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = -\frac{\frac{a}{\sqrt{d+ex^2}}-\frac{bn\log(x)}{\sqrt{d}}+\frac{b\log(cx^n)}{\sqrt{d+ex^2}}+\frac{bn\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)}{\sqrt{d}}}{e}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e*x^2)^(3/2),x]
[Out] -((a/Sqrt[d + e*x^2] - (b*n*Log[x])/Sqrt[d] + (b*Log[c*x^n])/Sqrt[d + e*x^2
] + (b*n*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/Sqrt[d])/e)
```

Maple [F]

$$\int \frac{x(a+b\ln(c\,x^n))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x*(a+b*ln(c*x^n))/(e*x^2+d)^(3/2),x)
[Out] int(x*(a+b*ln(c*x^n))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.96

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \left[\frac{(benx^2+bdn)\sqrt{d}\log\left(-\frac{ex^2-2\sqrt{ex^2+d}\sqrt{d}+2d}{x^2}\right)-2\left(bdn\log(x)+bd\log(c)+ad\right)\sqrt{}}{2\left(de^2x^2+d^2e\right)}\right.$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] [1/2*((b*e*n*x^2 + b*d*n)*sqrt(d)*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) +
 2*d)/x^2) - 2*(b*d*n*log(x) + b*d*log(c) + a*d)*sqrt(e*x^2 + d))/(d*e^2*x^
2 + d^2*e), ((b*e*n*x^2 + b*d*n)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x^2 + d))
- (b*d*n*log(x) + b*d*log(c) + a*d)*sqrt(e*x^2 + d))/(d*e^2*x^2 + d^2*e)]
```

Sympy [A] (verification not implemented)

Time = 4.63 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.65

$$\int \frac{x(a + b\log(cx^n))}{(d + ex^2)^{3/2}}\,dx = a\left(\begin{cases} -\frac{1}{e\sqrt{d+ex^2}} & \text{for } e \neq 0 \\ \frac{x^2}{2d^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) - bn\left(\begin{cases} \frac{x^2}{4d^{\frac{3}{2}}} & \text{for } e = 0 \\ \frac{\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{\sqrt{de}} & \text{otherwise} \end{cases}\right) + b\left(\begin{cases} \frac{x^2}{2d^{\frac{3}{2}}} & \text{for } e = 0 \\ -\frac{1}{e\sqrt{d+ex^2}} & \text{otherwise} \end{cases}\right)\log(cx^n)$$

```
[In] integrate(x*(a+b*ln(c*x**n))/(e*x**2+d)**(3/2),x)
```

```
[Out] a*Piecewise((-1/(e*sqrt(d + e*x**2)), Ne(e, 0)), (x**2/(2*d**(3/2)), True))
 - b*n*Piecewise((x**2/(4*d**(3/2)), Eq(e, 0)), (asinh(sqrt(d)/(sqrt(e)*x))
/(sqrt(d)*e), True)) + b*Piecewise((x**2/(2*d**(3/2)), Eq(e, 0)), (-1/(e*sq
rt(d + e*x**2)), True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a + b\log(cx^n))}{(d + ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x(a + b\log(cx^n))}{(d + ex^2)^{3/2}}\,dx = \int \frac{(b\log(cx^n) + a)x}{(ex^2 + d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x\,(a+b\,\ln(c\,x^n))}{(e\,x^2+d)^{3/2}}\,dx$$

```
[In] int((x*(a + b*log(c*x^n)))/(d + e*x^2)^(3/2),x)
[Out] int((x*(a + b*log(c*x^n)))/(d + e*x^2)^(3/2), x)
```

3.290 $\int \frac{a+b\log(cx^n)}{x(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 209

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^{3/2}}\,dx = \frac{bn\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{bn\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2d^{3/2}}$$
$$+\left(\frac{1}{d\sqrt{d+ex^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)(a+b\log(cx^n)) - \frac{bn\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{d^{3/2}} - \frac{bn\,\text{PolyLog}\left(2\right.}{2}$$

```
[Out] b*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)+1/2*b*n*arctanh((e*x^2+d)^(1/2
)/d^(1/2))^2/d^(3/2)-b*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(
1/2)-(e*x^2+d)^(1/2)))/d^(3/2)-1/2*b*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-(e*x^
2+d)^(1/2)))/d^(3/2)+(a+b*ln(c*x^n))*(-arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(
3/2)+1/d/(e*x^2+d)^(1/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {272, 53, 65, 214, 2390, 6131, 6055, 2449, 2352}

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^{3/2}}\,dx = \left(\frac{1}{d\sqrt{d+ex^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)(a+b\log(cx^n))$$
$$+\frac{bn\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2d^{3/2}}+\frac{bn\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}$$
$$-\frac{bn\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{d^{3/2}}-\frac{bn\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{ex^2+d}}\right)}{2d^{3/2}}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x^2)^(3/2)),x]
```

```
[Out] (b*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/d^(3/2) + (b*n*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]]^2)/(2*d^(3/2)) + (1/(d*Sqrt[d + e*x^2]) - ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]]/d^(3/2))*(a + b*Log[c*x^n]) - (b*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt
[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/d^(3/2) - (b*n*PolyLog[2
, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/(2*d^(3/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\frac{1}{d\sqrt{d+ex^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int\left(\frac{1}{dx\sqrt{d+ex^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}x}\right)dx \\
&= \left(\frac{1}{d\sqrt{d+ex^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)(a+b\log(cx^n)) \\
&\quad +\frac{(bn)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{x}\,dx}{d^{3/2}}-\frac{(bn)\int\frac{1}{x\sqrt{d+ex^2}}\,dx}{d} \\
&= \left(\frac{1}{d\sqrt{d+ex^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)(a+b\log(cx^n)) \\
&\quad +\frac{(bn)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx,x,x^2\right)}{2d^{3/2}}-\frac{(bn)\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{2d}
\end{aligned}$$

$$= \left(\frac{1}{d\sqrt{d+ex^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)(a+b\log(cx^n)) + \frac{(bn)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx, x, \sqrt{d+ex^2}\right)}{d^{3/2}} - \frac{(bn)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{de}$$

$$= \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2d^{3/2}} + \left(\frac{1}{d\sqrt{d+ex^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)(a+b\log(cx^n)) - \frac{(bn)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx, x, \sqrt{d+ex^2}\right)}{d^2}$$

$$= \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2d^{3/2}} + \left(\frac{1}{d\sqrt{d+ex^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)(a+b\log(cx^n)) - \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{d^{3/2}} + \frac{(bn}{}$$

$$= \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2d^{3/2}} + \left(\frac{1}{d\sqrt{d+ex^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)(a+b\log(cx^n)) - \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{d^{3/2}} - \frac{(bn}{}$$

$$= \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2d^{3/2}} + \left(\frac{1}{d\sqrt{d+ex^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)(a+b\log(cx^n)) - \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{d^{3/2}} - \frac{bn}{}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.26 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.15

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^{3/2}}\,dx = \frac{-bd^{3/2}n\sqrt{1+\frac{d}{ex^2}}\,{}_3F_2\left(\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{5}{2},\frac{5}{2};-\frac{d}{ex^2}\right)+9ex^2\left(-b\sqrt{e}n\sqrt{1+\frac{d}{ex^2}}x\operatorname{arcsinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)\log\right.}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x^2)^(3/2)),x]
```

```
[Out] (-(b*d^(3/2)*n*Sqrt[1 + d/(e*x^2)]*HypergeometricPFQ[{3/2, 3/2, 3/2}, {5/2,
 5/2}, -(d/(e*x^2))]) + 9*e*x^2*(-(b*Sqrt[e]*n*Sqrt[1 + d/(e*x^2)]*x*ArcSin
h[Sqrt[d]/(Sqrt[e]*x)]*Log[x]) - b*n*Sqrt[d + e*x^2]*Log[x]^2 + Sqrt[d + e*
x^2]*Log[x]*(a + b*Log[c*x^n] + b*n*Log[d + Sqrt[d]*Sqrt[d + e*x^2]]) + (a
+ b*Log[c*x^n])*(Sqrt[d] - Sqrt[d + e*x^2]*Log[d + Sqrt[d]*Sqrt[d + e*x^2]]
)))/(9*d^(3/2)*e*x^2*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\ln(c\,x^n)}{x\,(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*ln(c*x^n))/x/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/x/(e*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^{3/2}}\,dx = \int \frac{b\log(cx^n)+a}{(ex^2+d)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(e*x^2 + d)*b*log(c*x^n) + sqrt(e*x^2 + d)*a)/(e^2*x^5 + 2*d*
e*x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{a+b\log (cx^n)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\log (cx^n)}{x\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b*ln(c*x**n))/x/(e*x**2+d)**(3/2),x)

[Out] Integral((a + b*log(c*x**n))/(x*(d + e*x**2)**(3/2)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log (cx^n)}{x\left(d+ex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e

Giac [F]

$$\int \frac{a+b\log (cx^n)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\log (cx^n)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x}\,dx$$

[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)^(3/2)*x), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log (cx^n)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\ln \left(c\,x^n\right)}{x\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

[In] int((a + b*log(c*x^n))/(x*(d + e*x^2)^(3/2)),x)

[Out] int((a + b*log(c*x^n))/(x*(d + e*x^2)^(3/2)), x)

3.291 $\int \frac{a+b\log(cx^n)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 287

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^{3/2}}\,dx = -\frac{bn\sqrt{d+ex^2}}{4d^2x^2}-\frac{5ben\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{4d^{5/2}} -\frac{3ben\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{5/2}}-\frac{3e(a+b\log(cx^n))}{2d^2\sqrt{d+ex^2}} -\frac{a+b\log(cx^n)}{2dx^2\sqrt{d+ex^2}}+\frac{3e\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{5/2}} +\frac{3ben\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2d^{5/2}}+\frac{3ben\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{4d^{5/2}}$$

```
[Out] -5/4*b*e*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(5/2)-3/4*b*e*n*arctanh((e*x^
2+d)^(1/2)/d^(1/2))^2/d^(5/2)+3/2*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))*(a+b*l
n(c*x^n))/d^(5/2)+3/2*b*e*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(
d^(1/2)-(e*x^2+d)^(1/2)))/d^(5/2)+3/4*b*e*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-
(e*x^2+d)^(1/2)))/d^(5/2)-3/2*e*(a+b*ln(c*x^n))/d^2/(e*x^2+d)^(1/2)+1/2*(-a
-b*ln(c*x^n))/d/x^2/(e*x^2+d)^(1/2)-1/4*b*n*(e*x^2+d)^(1/2)/d^2/x^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {272, 44, 53, 65, 214, 2392, 457, 79, 6131, 6055, 2449, 2352}

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^{3/2}}\,dx = \frac{3e\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{5/2}} - \frac{3e(a+b\log(cx^n))}{2d^2\sqrt{d+ex^2}} - \frac{a+b\log(cx^n)}{2dx^2\sqrt{d+ex^2}} - \frac{3ben\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{5/2}} - \frac{5ben\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{4d^{5/2}} + \frac{3ben\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2d^{5/2}} + \frac{3ben\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{ex^2+d}}\right)}{4d^{5/2}} - \frac{bn\sqrt{d+ex^2}}{4d^2x^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^3*(d + e*x^2)^(3/2)),x]
```

```
[Out] -1/4*(b*n*Sqrt[d + e*x^2])/(d^2*x^2) - (5*b*e*n*ArcTanh[Sqrt[d + e*x^2]/Sqr
t[d]])/(4*d^(5/2)) - (3*b*e*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]]^2)/(4*d^(5/2
)) - (3*e*(a + b*Log[c*x^n]))/(2*d^2*Sqrt[d + e*x^2]) - (a + b*Log[c*x^n])/
(2*d*x^2*Sqrt[d + e*x^2]) + (3*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]]*(a + b*Lo
g[c*x^n]))/(2*d^(5/2)) + (3*b*e*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]]*Log[(2*S
qrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/(2*d^(5/2)) + (3*b*e*n*PolyLog[2, 1 -
 (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/(4*d^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
```

```
IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3e(a+b\log(cx^n))}{2d^2\sqrt{d+ex^2}}-\frac{a+b\log(cx^n)}{2dx^2\sqrt{d+ex^2}}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{5/2}} \\
&\quad -(bn)\int\left(-\frac{d+3ex^2}{2d^2x^3\sqrt{d+ex^2}}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{5/2}x}\right)dx \\
&= -\frac{3e(a+b\log(cx^n))}{2d^2\sqrt{d+ex^2}}-\frac{a+b\log(cx^n)}{2dx^2\sqrt{d+ex^2}}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{5/2}} \\
&\quad +\frac{(bn)\int\frac{d+3ex^2}{x^3\sqrt{d+ex^2}}\,dx}{2d^2}-\frac{(3ben)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{x}\,dx}{2d^{5/2}} \\
&= -\frac{3e(a+b\log(cx^n))}{2d^2\sqrt{d+ex^2}}-\frac{a+b\log(cx^n)}{2dx^2\sqrt{d+ex^2}}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{5/2}} \\
&\quad +\frac{(bn)\text{Subst}\left(\int\frac{d+3ex}{x^2\sqrt{d+ex}}\,dx,x,x^2\right)}{4d^2}-\frac{(3ben)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx,x,x^2\right)}{4d^{5/2}}
\end{aligned}$$

$$=-\frac{bn\sqrt{d+ex^2}}{4d^2x^2}-\frac{3e(a+b\log(cx^n))}{2d^2\sqrt{d+ex^2}}-\frac{a+b\log(cx^n)}{2dx^2\sqrt{d+ex^2}}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{5/2}}-\frac{(3ben)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx,x,\sqrt{d+ex^2}\right)}{2d^{5/2}}+\frac{(5ben)\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{8d^2}$$

$$=-\frac{bn\sqrt{d+ex^2}}{4d^2x^2}-\frac{3ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{5/2}}-\frac{3e(a+b\log(cx^n))}{2d^2\sqrt{d+ex^2}}-\frac{a+b\log(cx^n)}{2dx^2\sqrt{d+ex^2}}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{5/2}}+\frac{(5bn)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{4d^2}+\frac{(3ben)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx,x,\sqrt{d+ex^2}\right)}{2d^3}$$

$$=-\frac{bn\sqrt{d+ex^2}}{4d^2x^2}-\frac{5ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{4d^{5/2}}-\frac{3ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{5/2}}-\frac{3e(a+b\log(cx^n))}{2d^2\sqrt{d+ex^2}}-\frac{a+b\log(cx^n)}{2dx^2\sqrt{d+ex^2}}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{5/2}}+\frac{3ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2d^{5/2}}-\frac{(3ben)\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}\,dx,x,\sqrt{d+ex^2}\right)}{2d^3}$$

$$=-\frac{bn\sqrt{d+ex^2}}{4d^2x^2}-\frac{5ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{4d^{5/2}}-\frac{3ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{5/2}}-\frac{3e(a+b\log(cx^n))}{2d^2\sqrt{d+ex^2}}-\frac{a+b\log(cx^n)}{2dx^2\sqrt{d+ex^2}}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{5/2}}+\frac{3ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2d^{5/2}}+\frac{(3ben)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-\frac{\sqrt{d+ex^2}}{\sqrt{d}}}\right)}{2d^{5/2}}$$

$$= -\frac{bn\sqrt{d+ex^2}}{4d^2x^2} - \frac{5ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{4d^{5/2}} - \frac{3ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{5/2}} - \frac{3e(a+b\log(cx^n))}{2d^2\sqrt{d+ex^2}} - \frac{a+b\log(cx^n)}{2dx^2\sqrt{d+ex^2}} + \frac{3e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{5/2}} + \frac{3ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2d^{5/2}} + \frac{3ben\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex^2}}{\sqrt{d}}}\right)}{4d^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.76

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^{3/2}}\,dx = \frac{3bd^{5/2}n\sqrt{1+\frac{d}{ex^2}}\,{}_3F_2\left(\frac{5}{2},\frac{5}{2},\frac{5}{2};\frac{7}{2},\frac{7}{2};-\frac{d}{ex^2}\right) - 5bd^{5/2}n\sqrt{1+\frac{d}{ex^2}}\,\text{Hypergeometric2F1}\left(\frac{3}{2}\right.}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*(d + e*x^2)^(3/2)),x]
```

```
[Out] (3*b*d^(5/2)*n*Sqrt[1 + d/(e*x^2)]*HypergeometricPFQ[{5/2, 5/2, 5/2}, {7/2,
 7/2}, -(d/(e*x^2))] - 5*b*d^(5/2)*n*Sqrt[1 + d/(e*x^2)]*Hypergeometric2F1[
3/2, 5/2, 7/2, -(d/(e*x^2))]*(1 + 2*Log[x]) - 25*e*x^2*(a - b*n*Log[x] + b*
Log[c*x^n])*(Sqrt[d]*(d + 3*e*x^2) + 3*e*x^2*Sqrt[d + e*x^2]*Log[x] - 3*e*x
^2*Sqrt[d + e*x^2]*Log[d + Sqrt[d]*Sqrt[d + e*x^2]]))/(50*d^(5/2)*e*x^4*Sqr
t[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\ln(c\,x^n)}{x^3\,(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*ln(c*x^n))/x^3/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/x^3/(e*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^{3/2}}\,dx = \int \frac{b\log(cx^n)+a}{(ex^2+d)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(e*x^2 + d)*b*log(c*x^n) + sqrt(e*x^2 + d)*a)/(e^2*x^7 + 2*d*
e*x^5 + d^2*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**3/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*log(c*x**n))/(x**3*(d + e*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^{3/2}}\,dx = \int \frac{b\log(cx^n)+a}{(ex^2+d)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^3\,(e\,x^2+d)^{3/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*log(c*x^n))/(x^3*(d + e*x^2)^(3/2)), x)
```

3.292 $\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 328

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\sqrt{d+ex^2}} + \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2e^{3/2}\sqrt{d+ex^2}} - \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{e^{3/2}\sqrt{d+ex^2}} - \frac{x(a+b\log(cx^n))}{e\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{3/2}\sqrt{d+ex^2}} - \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{3/2}\sqrt{d+ex^2}}$$

```
[Out] -x*(a+b*ln(c*x^n))/e/(e*x^2+d)^(1/2)+b*n*arcsinh(x*e^(1/2)/d^(1/2))*d^(1/2)
*(1+e*x^2/d)^(1/2)/e^(3/2)/(e*x^2+d)^(1/2)+1/2*b*n*arcsinh(x*e^(1/2)/d^(1/2
))^2*d^(1/2)*(1+e*x^2/d)^(1/2)/e^(3/2)/(e*x^2+d)^(1/2)-b*n*arcsinh(x*e^(1/2
)/d^(1/2))*ln(1-(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(1+e*x^2/d
)^(1/2)/e^(3/2)/(e*x^2+d)^(1/2)+arcsinh(x*e^(1/2)/d^(1/2))*(a+b*ln(c*x^n))*
d^(1/2)*(1+e*x^2/d)^(1/2)/e^(3/2)/(e*x^2+d)^(1/2)-1/2*b*n*polylog(2,(x*e^(1
/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(1+e*x^2/d)^(1/2)/e^(3/2)/(e*x^2+
d)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2386, 294, 221, 2392, 14, 21, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \frac{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{3/2}\sqrt{d+ex^2}} - \frac{x(a+b\log(cx^n))}{e\sqrt{d+ex^2}} - \frac{b\sqrt{d}n\sqrt{\frac{ex^2}{d}+1}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{3/2}\sqrt{d+ex^2}} + \frac{b\sqrt{d}n\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2e^{3/2}\sqrt{d+ex^2}} + \frac{b\sqrt{d}n\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\sqrt{d+ex^2}} - \frac{b\sqrt{d}n\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{e^{3/2}\sqrt{d+ex^2}}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (b*Sqrt[d]*n*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*Sqr
t[d + e*x^2]) + (b*Sqrt[d]*n*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d
]]^2)/(2*e^(3/2)*Sqrt[d + e*x^2]) - (b*Sqrt[d]*n*Sqrt[1 + (e*x^2)/d]*ArcSin
h[(Sqrt[e]*x)/Sqrt[d]]*Log[1 - E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(e^(3/2
)*Sqrt[d + e*x^2]) - (x*(a + b*Log[c*x^n]))/(e*Sqrt[d + e*x^2]) + (Sqrt[d]*
Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/(e^(3/
2)*Sqrt[d + e*x^2]) - (b*Sqrt[d]*n*Sqrt[1 + (e*x^2)/d]*PolyLog[2, E^(2*ArcS
inh[(Sqrt[e]*x)/Sqrt[d]])])/(2*e^(3/2)*Sqrt[d + e*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2386

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^
(q_), x_Symbol] :> Dist[d^IntPart[q]*((d + e*x^2)^FracPart[q]/(1 + (e/d)*x^
2)^FracPart[q]), Int[x^m*(1 + (e/d)*x^2)^q*(a + b*Log[c*x^n]), x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && IntegerQ[m/2] && IntegerQ[q - 1/2] &&  !(LtQ[
m + 2*q, -2] || GtQ[d, 0])
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{ex^2}{d}}\int \frac{x^2(a+b\log(cx^n))}{\left(1+\frac{ex^2}{d}\right)^{3/2}}\,dx}{d\sqrt{d+ex^2}}$$

$$= -\frac{x(a+b\log(cx^n))}{e\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{3/2}\sqrt{d+ex^2}} - \frac{\left(bn\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{-\frac{dx}{e\sqrt{1+\frac{ex^2}{d}}}+\frac{d^{3/2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}}}{x}\,dx}{d\sqrt{d+ex^2}}$$

$$= -\frac{x(a+b\log(cx^n))}{e\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{3/2}\sqrt{d+ex^2}} - \frac{\left(bn\sqrt{1+\frac{ex^2}{d}}\right)\int\left(\frac{d^2\sqrt{1+\frac{ex^2}{d}}}{e(-d-ex^2)}+\frac{d^{3/2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}x}\right)dx}{d\sqrt{d+ex^2}}$$

$$= -\frac{x(a+b\log(cx^n))}{e\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{3/2}\sqrt{d+ex^2}} - \frac{\left(b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{e^{3/2}\sqrt{d+ex^2}} - \frac{\left(bdn\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{\sqrt{1+\frac{ex^2}{d}}}{-d-ex^2}\,dx}{e\sqrt{d+ex^2}}$$

$$=-\frac{x(a+b\log(cx^n))}{e\sqrt{d+ex^2}}+\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{3/2}\sqrt{d+ex^2}}-\frac{\left(b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int x\coth(x)\,dx,x,\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{e^{3/2}\sqrt{d+ex^2}}+\frac{\left(bn\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{e\sqrt{d+ex^2}}$$

$$=\frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\sqrt{d+ex^2}}+\frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2e^{3/2}\sqrt{d+ex^2}}-\frac{x(a+b\log(cx^n))}{e\sqrt{d+ex^2}}+\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{3/2}\sqrt{d+ex^2}}+\frac{\left(2b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int\frac{e^{2x}x}{1-e^{2x}}\,dx,x,\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{e^{3/2}\sqrt{d+ex^2}}$$

$$=\frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\sqrt{d+ex^2}}+\frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2e^{3/2}\sqrt{d+ex^2}}-\frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{e^{3/2}\sqrt{d+ex^2}}-\frac{x(a+b\log(cx^n))}{e\sqrt{d+ex^2}}+\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{3/2}\sqrt{d+ex^2}}+\frac{\left(b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int\log\left(1-e^{2x}\right)\,dx,x,\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{e^{3/2}\sqrt{d+ex^2}}$$

$$=\frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\sqrt{d+ex^2}}+\frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2e^{3/2}\sqrt{d+ex^2}}-\frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{e^{3/2}\sqrt{d+ex^2}}-\frac{x(a+b\log(cx^n))}{e\sqrt{d+ex^2}}+\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{3/2}\sqrt{d+ex^2}}+\frac{\left(b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{3/2}\sqrt{d+ex^2}}$$

$$= \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\sqrt{d+ex^2}} + \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2e^{3/2}\sqrt{d+ex^2}} - \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{e^{3/2}\sqrt{d+ex^2}} - \frac{x(a+b\log(cx^n))}{e\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{3/2}\sqrt{d+ex^2}} - \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\operatorname{Li}_2\left(e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{3/2}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.32 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.66

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = -\frac{bn\sqrt{1+\frac{ex^2}{d}}\left(e^{3/2}x^3(d+ex^2)\,{}_3F_2\left(\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{5}{2},\frac{5}{2};-\frac{ex^2}{d}\right)+9d^2\sqrt{e}x\sqrt{1+\frac{ex^2}{d}}\log(x)-9d^{3/2}(d+ex^2)\operatorname{arcsinh}}{9de^{3/2}(d+ex^2)^{3/2}} - \frac{x(a-bn\log(x)+b\log(cx^n))}{e\sqrt{d+ex^2}} + \frac{(a-bn\log(x)+b\log(cx^n))\log\left(ex+\sqrt{e}\sqrt{d+ex^2}\right)}{e^{3/2}}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -1/9*(b*n*Sqrt[1 + (e*x^2)/d]*(e^(3/2)*x^3*(d + e*x^2)*HypergeometricPFQ[{3
/2, 3/2, 3/2}, {5/2, 5/2}, -((e*x^2)/d)] + 9*d^2*Sqrt[e]*x*Sqrt[1 + (e*x^2)
/d]*Log[x] - 9*d^(3/2)*(d + e*x^2)*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[x]))/(d
*e^(3/2)*(d + e*x^2)^(3/2)) - (x*(a - b*n*Log[x] + b*Log[c*x^n]))/(e*Sqrt[d
 + e*x^2]) + ((a - b*n*Log[x] + b*Log[c*x^n])*Log[e*x + Sqrt[e]*Sqrt[d + e*
x^2]])/e^(3/2)
```

Maple [F]

$$\int \frac{x^2(a+b\ln(c\,x^n))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^2*(a+b*ln(c*x^n))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^2*(a+b*ln(c*x^n))/(e*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral((sqrt(e*x^2 + d)*b*x^2*log(c*x^n) + sqrt(e*x^2 + d)*a*x^2)/(e^2*x^
4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(e*x**2+d)**(3/2),x)
[Out] Integral(x**2*(a + b*log(c*x**n))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^2/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,x^n))}{(e\,x^2+d)^{3/2}}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e*x^2)^(3/2),x)
[Out] int((x^2*(a + b*log(c*x^n)))/(d + e*x^2)^(3/2), x)
```

3.293 $\int \frac{a+b\log(cx^n)}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{a+b\log(cx^n)}{(d+ex^2)^{3/2}}\,dx = -\frac{bn\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{d\sqrt{e}} + \frac{x(a+b\log(cx^n))}{d\sqrt{d+ex^2}}$$

```
[Out] -b*n*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/d/e^(1/2)+x*(a+b*ln(c*x^n))/d/(e*x^
2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2351, 223, 212}

$$\int \frac{a+b\log(cx^n)}{(d+ex^2)^{3/2}}\,dx = \frac{x(a+b\log(cx^n))}{d\sqrt{d+ex^2}} - \frac{bn\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

```
[In] Int[(a + b*Log[c*x^n])/(d + e*x^2)^(3/2),x]
```

```
[Out] -((b*n*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(d*Sqrt[e])) + (x*(a + b*Log[c
*x^n]))/(d*Sqrt[d + e*x^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x(a+b\log(cx^n))}{d\sqrt{d+ex^2}} - \frac{(bn)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{d}\\ &= \frac{x(a+b\log(cx^n))}{d\sqrt{d+ex^2}} - \frac{(bn)\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{d}\\ &= -\frac{bn\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{d\sqrt{e}} + \frac{x(a+b\log(cx^n))}{d\sqrt{d+ex^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int\frac{a+b\log(cx^n)}{(d+ex^2)^{3/2}}\,dx = \frac{\frac{ax}{\sqrt{d+ex^2}} + \frac{bx\log(cx^n)}{\sqrt{d+ex^2}} - \frac{bn\log\left(ex+\sqrt{e}\sqrt{d+ex^2}\right)}{\sqrt{e}}}{d}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*x^2)^(3/2),x]
[Out] ((a*x)/Sqrt[d + e*x^2] + (b*x*Log[c*x^n])/Sqrt[d + e*x^2] - (b*n*Log[e*x +
Sqrt[e]*Sqrt[d + e*x^2]])/Sqrt[e])/d
```

Maple [F]

$$\int\frac{a+b\ln(c\,x^n)}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*ln(c*x^n))/(e*x^2+d)^(3/2),x)
[Out] int((a+b*ln(c*x^n))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.97

$$\int \frac{a + b\log(cx^n)}{(d + ex^2)^{3/2}}\,dx = \left[\frac{(benx^2 + bdn)\sqrt{e}\log\left(-2\,ex^2 + 2\,\sqrt{ex^2 + d}\sqrt{e}x - d\right) + 2\,(benx\log(x) + bex\log(c)\right.$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*e*n*x^2 + b*d*n)*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*
x - d) + 2*(b*e*n*x*log(x) + b*e*x*log(c) + a*e*x)*sqrt(e*x^2 + d))/(d*e^2*
x^2 + d^2*e), ((b*e*n*x^2 + b*d*n)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 +
d)) + (b*e*n*x*log(x) + b*e*x*log(c) + a*e*x)*sqrt(e*x^2 + d))/(d*e^2*x^2 +
 d^2*e)]
```

Sympy [F]

$$\int \frac{a + b\log(cx^n)}{(d + ex^2)^{3/2}}\,dx = \int \frac{a + b\log(cx^n)}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*log(c*x**n))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\log(cx^n)}{(d + ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{(d+ex^2)^{3/2}}\,dx = \int \frac{b\log(cx^n)+a}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{(e\,x^2+d)^{3/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(d + e*x^2)^(3/2),x)
[Out] int((a + b*log(c*x^n))/(d + e*x^2)^(3/2), x)
```

3.294 $\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 110

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^{3/2}}\,dx = -\frac{bn\sqrt{d+ex^2}}{d^2x} + \frac{2b\sqrt{e}n\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{d^2} - \frac{a+b\log(cx^n)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\log(cx^n))}{d^2\sqrt{d+ex^2}}$$

```
[Out] 2*b*n*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^(1/2)/d^2+(-a-b*ln(c*x^n))/d/x/(
e*x^2+d)^(1/2)-2*e*x*(a+b*ln(c*x^n))/d^2/(e*x^2+d)^(1/2)-b*n*(e*x^2+d)^(1/2
)/d^2/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {277, 197, 2392, 12, 462, 223, 212}

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^{3/2}}\,dx = -\frac{2ex(a+b\log(cx^n))}{d^2\sqrt{d+ex^2}} - \frac{a+b\log(cx^n)}{dx\sqrt{d+ex^2}} + \frac{2b\sqrt{e}n\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{d^2} - \frac{bn\sqrt{d+ex^2}}{d^2x}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] -((b*n*Sqrt[d + e*x^2])/(d^2*x)) + (2*b*Sqrt[e]*n*ArcTanh[(Sqrt[e]*x)/Sqrt[
d + e*x^2]])/d^2 - (a + b*Log[c*x^n])/(d*x*Sqrt[d + e*x^2]) - (2*e*x*(a + b
*Log[c*x^n]))/(d^2*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = -\frac{a+b\log(cx^n)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\log(cx^n))}{d^2\sqrt{d+ex^2}} - (bn)\int \frac{-d-2ex^2}{d^2x^2\sqrt{d+ex^2}}\,dx$$

$$= -\frac{a+b\log(cx^n)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\log(cx^n))}{d^2\sqrt{d+ex^2}} - \frac{(bn)\int \frac{-d-2ex^2}{x^2\sqrt{d+ex^2}}\,dx}{d^2}$$

$$= -\frac{bn\sqrt{d+ex^2}}{d^2x} - \frac{a+b\log(cx^n)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\log(cx^n))}{d^2\sqrt{d+ex^2}} + \frac{(2ben)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{d^2}$$

$$= -\frac{bn\sqrt{d+ex^2}}{d^2x} - \frac{a+b\log(cx^n)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\log(cx^n))}{d^2\sqrt{d+ex^2}} + \frac{(2ben)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{d^2}$$

$$= -\frac{bn\sqrt{d+ex^2}}{d^2x} + \frac{2b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{d^2} - \frac{a+b\log(cx^n)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\log(cx^n))}{d^2\sqrt{d+ex^2}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^{3/2}}\,dx = \frac{-ad-bdn-2aex^2-benx^2-b(d+2ex^2)\log(cx^n)+2b\sqrt{e}nx\sqrt{d+ex^2}\log\left(ex+\sqrt{\cdots}\right.}{d^2x\sqrt{d+ex^2}}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] (-(a*d) - b*d*n - 2*a*e*x^2 - b*e*n*x^2 - b*(d + 2*e*x^2)*Log[c*x^n] + 2*b*
Sqrt[e]*n*x*Sqrt[d + e*x^2]*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(d^2*x*Sqrt
[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\ln(c\,x^n)}{x^2\,(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*ln(c*x^n))/x^2/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/x^2/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.19

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^{3/2}}\,dx = \left[\frac{(benx^3+bdnx)\sqrt{e}\log\left(-2\,ex^2-2\sqrt{ex^2+d}\sqrt{e}x-d\right)-(bdn+(ben+2\,ae)x^2+}{d^2ex^3+d^3x}\right.$$

$$-\frac{2\,(benx^3+bdnx)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+(bdn+(ben+2\,ae)x^2+ad+(2\,bex^2+bd)\log(c)+(2\,benx^2+}{d^2ex^3+d^3x}$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*e*n*x^3 + b*d*n*x)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x
- d) - (b*d*n + (b*e*n + 2*a*e)*x^2 + a*d + (2*b*e*x^2 + b*d)*log(c) + (2*b
*e*n*x^2 + b*d*n)*log(x))*sqrt(e*x^2 + d))/(d^2*e*x^3 + d^3*x), -(2*(b*e*n*
x^3 + b*d*n*x)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (b*d*n + (b*e*
n + 2*a*e)*x^2 + a*d + (2*b*e*x^2 + b*d)*log(c) + (2*b*e*n*x^2 + b*d*n)*log
(x))*sqrt(e*x^2 + d))/(d^2*e*x^3 + d^3*x)]
```

Sympy [F]

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*log(c*x**n))/(x**2*(d + e*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\ln\left(c\,x^n\right)}{x^2\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*x^2)^(3/2)), x)
```

3.295 $\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 176

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^{3/2}}\,dx = -\frac{bn\sqrt{d+ex^2}}{9d^2x^3}+\frac{14ben\sqrt{d+ex^2}}{9d^3x}-\frac{8be^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d^3} -\frac{a+b\log(cx^n)}{3dx^3\sqrt{d+ex^2}}+\frac{4e(a+b\log(cx^n))}{3d^2x\sqrt{d+ex^2}}+\frac{8e^2x(a+b\log(cx^n))}{3d^3\sqrt{d+ex^2}}$$

```
[Out] -8/3*b*e^(3/2)*n*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/d^3+1/3*(-a-b*ln(c*x^n)
)/d/x^3/(e*x^2+d)^(1/2)+4/3*e*(a+b*ln(c*x^n))/d^2/x/(e*x^2+d)^(1/2)+8/3*e^2
*x*(a+b*ln(c*x^n))/d^3/(e*x^2+d)^(1/2)-1/9*b*n*(e*x^2+d)^(1/2)/d^2/x^3+14/9
*b*e*n*(e*x^2+d)^(1/2)/d^3/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {277, 197, 2392, 12, 1279, 462, 223, 212}

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^{3/2}}\,dx = \frac{8e^2x(a+b\log(cx^n))}{3d^3\sqrt{d+ex^2}}+\frac{4e(a+b\log(cx^n))}{3d^2x\sqrt{d+ex^2}} -\frac{a+b\log(cx^n)}{3dx^3\sqrt{d+ex^2}}-\frac{8be^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d^3}+\frac{14ben\sqrt{d+ex^2}}{9d^3x}-\frac{bn\sqrt{d+ex^2}}{9d^2x^3}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^4*(d + e*x^2)^(3/2)),x]
```

```
[Out] -1/9*(b*n*Sqrt[d + e*x^2])/(d^2*x^3) + (14*b*e*n*Sqrt[d + e*x^2])/(9*d^3*x)
 - (8*b*e^(3/2)*n*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(3*d^3) - (a + b*Lo
```

```
g[c*x^n])/(3*d*x^3*Sqrt[d + e*x^2]) + (4*e*(a + b*Log[c*x^n]))/(3*d^2*x*Sqr
t[d + e*x^2]) + (8*e^2*x*(a + b*Log[c*x^n]))/(3*d^3*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
```

```
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\log(cx^n)}{3dx^3\sqrt{d+ex^2}}+\frac{4e(a+b\log(cx^n))}{3d^2x\sqrt{d+ex^2}} \\
&\quad+\frac{8e^2x(a+b\log(cx^n))}{3d^3\sqrt{d+ex^2}}-(bn)\int\frac{-d^2+4dex^2+8e^2x^4}{3d^3x^4\sqrt{d+ex^2}}\,dx \\
&= -\frac{a+b\log(cx^n)}{3dx^3\sqrt{d+ex^2}}+\frac{4e(a+b\log(cx^n))}{3d^2x\sqrt{d+ex^2}}+\frac{8e^2x(a+b\log(cx^n))}{3d^3\sqrt{d+ex^2}}-\frac{(bn)\int\frac{-d^2+4dex^2+8e^2x^4}{x^4\sqrt{d+ex^2}}\,dx}{3d^3} \\
&= -\frac{bn\sqrt{d+ex^2}}{9d^2x^3}-\frac{a+b\log(cx^n)}{3dx^3\sqrt{d+ex^2}}+\frac{4e(a+b\log(cx^n))}{3d^2x\sqrt{d+ex^2}} \\
&\quad+\frac{8e^2x(a+b\log(cx^n))}{3d^3\sqrt{d+ex^2}}+\frac{(bn)\int\frac{-14d^2e-24de^2x^2}{x^2\sqrt{d+ex^2}}\,dx}{9d^4} \\
&= -\frac{bn\sqrt{d+ex^2}}{9d^2x^3}+\frac{14ben\sqrt{d+ex^2}}{9d^3x}-\frac{a+b\log(cx^n)}{3dx^3\sqrt{d+ex^2}} \\
&\quad+\frac{4e(a+b\log(cx^n))}{3d^2x\sqrt{d+ex^2}}+\frac{8e^2x(a+b\log(cx^n))}{3d^3\sqrt{d+ex^2}}-\frac{(8be^2n)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{3d^3} \\
&= -\frac{bn\sqrt{d+ex^2}}{9d^2x^3}+\frac{14ben\sqrt{d+ex^2}}{9d^3x}-\frac{a+b\log(cx^n)}{3dx^3\sqrt{d+ex^2}}+\frac{4e(a+b\log(cx^n))}{3d^2x\sqrt{d+ex^2}} \\
&\quad+\frac{8e^2x(a+b\log(cx^n))}{3d^3\sqrt{d+ex^2}}-\frac{(8be^2n)\,\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{3d^3} \\
&= -\frac{bn\sqrt{d+ex^2}}{9d^2x^3}+\frac{14ben\sqrt{d+ex^2}}{9d^3x}-\frac{8be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d^3} \\
&\quad-\frac{a+b\log(cx^n)}{3dx^3\sqrt{d+ex^2}}+\frac{4e(a+b\log(cx^n))}{3d^2x\sqrt{d+ex^2}}+\frac{8e^2x(a+b\log(cx^n))}{3d^3\sqrt{d+ex^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.82

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^{3/2}}\,dx = \frac{-3ad^2-bd^2n+12adex^2+13bdenx^2+24ae^2x^4+14be^2nx^4-3b(d^2-4dex^2-8e^2x^{}}{9d^3x^3\sqrt{d+ex^2}}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^4*(d + e*x^2)^(3/2)),x]
```

```
[Out] (-3*a*d^2 - b*d^2*n + 12*a*d*e*x^2 + 13*b*d*e*n*x^2 + 24*a*e^2*x^4 + 14*b*e
^2*n*x^4 - 3*b*(d^2 - 4*d*e*x^2 - 8*e^2*x^4)*Log[c*x^n] - 24*b*e^(3/2)*n*x^
3*Sqrt[d + e*x^2]*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(9*d^3*x^3*Sqrt[d + e
*x^2])
```

Maple [F]

$$\int \frac{a+b\ln(c\,x^n)}{x^4\,(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*ln(c*x^n))/x^4/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/x^4/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.10

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^{3/2}}\,dx = \left[\frac{12\,(be^2nx^5+bdenx^3)\sqrt{e}\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+(2\,(7\,be^2n+12\,ae^2)x}{}\right.$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/9*(12*(b*e^2*n*x^5 + b*d*e*n*x^3)*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 +
d)*sqrt(e)*x - d) + (2*(7*b*e^2*n + 12*a*e^2)*x^4 - b*d^2*n - 3*a*d^2 + (13
*b*d*e*n + 12*a*d*e)*x^2 + 3*(8*b*e^2*x^4 + 4*b*d*e*x^2 - b*d^2)*log(c) + 3
*(8*b*e^2*n*x^4 + 4*b*d*e*n*x^2 - b*d^2*n)*log(x))*sqrt(e*x^2 + d))/(d^3*e*
x^5 + d^4*x^3), 1/9*(24*(b*e^2*n*x^5 + b*d*e*n*x^3)*sqrt(-e)*arctan(sqrt(-e
)*x/sqrt(e*x^2 + d)) + (2*(7*b*e^2*n + 12*a*e^2)*x^4 - b*d^2*n - 3*a*d^2 +
(13*b*d*e*n + 12*a*d*e)*x^2 + 3*(8*b*e^2*x^4 + 4*b*d*e*x^2 - b*d^2)*log(c)
+ 3*(8*b*e^2*n*x^4 + 4*b*d*e*n*x^2 - b*d^2*n)*log(x))*sqrt(e*x^2 + d))/(d^3
*e*x^5 + d^4*x^3)]
```

Sympy [F]

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**4/(e*x**2+d)**(3/2),x)
[Out] Integral((a + b*log(c*x**n))/(x**4*(d + e*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^{3/2}}\,dx = \int \frac{b\log(cx^n)+a}{(ex^2+d)^{\frac{3}{2}}x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^4\,(e\,x^2+d)^{3/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^4*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*log(c*x^n))/(x^4*(d + e*x^2)^(3/2)), x)
```

3.296 $\int \frac{a+b\log(cx^n)}{x^6(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 236

$$\int \frac{a+b\log(cx^n)}{x^6(d+ex^2)^{3/2}}\,dx = -\frac{bn\sqrt{d+ex^2}}{25d^2x^5}+\frac{14ben\sqrt{d+ex^2}}{75d^3x^3} - \frac{148be^2n\sqrt{d+ex^2}}{75d^4x}+\frac{16be^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5d^4}-\frac{a+b\log(cx^n)}{5dx^5\sqrt{d+ex^2}} + \frac{2e(a+b\log(cx^n))}{5d^2x^3\sqrt{d+ex^2}}-\frac{8e^2(a+b\log(cx^n))}{5d^3x\sqrt{d+ex^2}}-\frac{16e^3x(a+b\log(cx^n))}{5d^4\sqrt{d+ex^2}}$$

```
[Out] 16/5*b*e^(5/2)*n*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/d^4+1/5*(-a-b*ln(c*x^n)
)/d/x^5/(e*x^2+d)^(1/2)+2/5*e*(a+b*ln(c*x^n))/d^2/x^3/(e*x^2+d)^(1/2)-8/5*e
^2*(a+b*ln(c*x^n))/d^3/x/(e*x^2+d)^(1/2)-16/5*e^3*x*(a+b*ln(c*x^n))/d^4/(e*
x^2+d)^(1/2)-1/25*b*n*(e*x^2+d)^(1/2)/d^2/x^5+14/75*b*e*n*(e*x^2+d)^(1/2)/d
^3/x^3-148/75*b*e^2*n*(e*x^2+d)^(1/2)/d^4/x
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {277, 197, 2392, 12, 1821, 1599, 1279, 462, 223, 212}

$$\int \frac{a+b\log(cx^n)}{x^6(d+ex^2)^{3/2}}\,dx = -\frac{16e^3x(a+b\log(cx^n))}{5d^4\sqrt{d+ex^2}}-\frac{8e^2(a+b\log(cx^n))}{5d^3x\sqrt{d+ex^2}} + \frac{2e(a+b\log(cx^n))}{5d^2x^3\sqrt{d+ex^2}}-\frac{a+b\log(cx^n)}{5dx^5\sqrt{d+ex^2}}+\frac{16be^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5d^4} - \frac{148be^2n\sqrt{d+ex^2}}{75d^4x}+\frac{14ben\sqrt{d+ex^2}}{75d^3x^3}-\frac{bn\sqrt{d+ex^2}}{25d^2x^5}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^6*(d + e*x^2)^(3/2)),x]
```

```
[Out] -1/25*(b*n*Sqrt[d + e*x^2])/(d^2*x^5) + (14*b*e*n*Sqrt[d + e*x^2])/(75*d^3*
x^3) - (148*b*e^2*n*Sqrt[d + e*x^2])/(75*d^4*x) + (16*b*e^(5/2)*n*ArcTanh[(
Sqrt[e]*x)/Sqrt[d + e*x^2]])/(5*d^4) - (a + b*Log[c*x^n])/(5*d*x^5*Sqrt[d +
 e*x^2]) + (2*e*(a + b*Log[c*x^n]))/(5*d^2*x^3*Sqrt[d + e*x^2]) - (8*e^2*(a
 + b*Log[c*x^n]))/(5*d^3*x*Sqrt[d + e*x^2]) - (16*e^3*x*(a + b*Log[c*x^n]))
/(5*d^4*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\log(cx^n)}{5dx^5\sqrt{d+ex^2}}+\frac{2e(a+b\log(cx^n))}{5d^2x^3\sqrt{d+ex^2}}-\frac{8e^2(a+b\log(cx^n))}{5d^3x\sqrt{d+ex^2}} \\
&\quad -\frac{16e^3x(a+b\log(cx^n))}{5d^4\sqrt{d+ex^2}}-(bn)\int\frac{-d^3+2d^2ex^2-8de^2x^4-16e^3x^6}{5d^4x^6\sqrt{d+ex^2}}\,dx \\
&= -\frac{a+b\log(cx^n)}{5dx^5\sqrt{d+ex^2}}+\frac{2e(a+b\log(cx^n))}{5d^2x^3\sqrt{d+ex^2}}-\frac{8e^2(a+b\log(cx^n))}{5d^3x\sqrt{d+ex^2}} \\
&\quad -\frac{16e^3x(a+b\log(cx^n))}{5d^4\sqrt{d+ex^2}}-\frac{(bn)\int\frac{-d^3+2d^2ex^2-8de^2x^4-16e^3x^6}{x^6\sqrt{d+ex^2}}\,dx}{5d^4}
\end{aligned}$$

$$= -\frac{bn\sqrt{d+ex^2}}{25d^2x^5} - \frac{a+b\log(cx^n)}{5dx^5\sqrt{d+ex^2}} + \frac{2e(a+b\log(cx^n))}{5d^2x^3\sqrt{d+ex^2}} - \frac{8e^2(a+b\log(cx^n))}{5d^3x\sqrt{d+ex^2}} - \frac{16e^3x(a+b\log(cx^n))}{5d^4\sqrt{d+ex^2}} + \frac{(bn)\int \frac{-14d^3ex+40d^2e^2x^3+80de^3x^5}{x^5\sqrt{d+ex^2}}\,dx}{25d^5}$$

$$= -\frac{bn\sqrt{d+ex^2}}{25d^2x^5} - \frac{a+b\log(cx^n)}{5dx^5\sqrt{d+ex^2}} + \frac{2e(a+b\log(cx^n))}{5d^2x^3\sqrt{d+ex^2}} - \frac{8e^2(a+b\log(cx^n))}{5d^3x\sqrt{d+ex^2}} - \frac{16e^3x(a+b\log(cx^n))}{5d^4\sqrt{d+ex^2}} + \frac{(bn)\int \frac{-14d^3e+40d^2e^2x^2+80de^3x^4}{x^4\sqrt{d+ex^2}}\,dx}{25d^5}$$

$$= -\frac{bn\sqrt{d+ex^2}}{25d^2x^5} + \frac{14ben\sqrt{d+ex^2}}{75d^3x^3} - \frac{a+b\log(cx^n)}{5dx^5\sqrt{d+ex^2}} + \frac{2e(a+b\log(cx^n))}{5d^2x^3\sqrt{d+ex^2}} - \frac{8e^2(a+b\log(cx^n))}{5d^3x\sqrt{d+ex^2}} - \frac{16e^3x(a+b\log(cx^n))}{5d^4\sqrt{d+ex^2}} - \frac{(bn)\int \frac{-148d^3e^2-240d^2e^3x^2}{x^2\sqrt{d+ex^2}}\,dx}{75d^6}$$

$$= -\frac{bn\sqrt{d+ex^2}}{25d^2x^5} + \frac{14ben\sqrt{d+ex^2}}{75d^3x^3} - \frac{148be^2n\sqrt{d+ex^2}}{75d^4x} - \frac{a+b\log(cx^n)}{5dx^5\sqrt{d+ex^2}} + \frac{2e(a+b\log(cx^n))}{5d^2x^3\sqrt{d+ex^2}} - \frac{8e^2(a+b\log(cx^n))}{5d^3x\sqrt{d+ex^2}} - \frac{16e^3x(a+b\log(cx^n))}{5d^4\sqrt{d+ex^2}} + \frac{(16be^3n)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{5d^4}$$

$$= -\frac{bn\sqrt{d+ex^2}}{25d^2x^5} + \frac{14ben\sqrt{d+ex^2}}{75d^3x^3} - \frac{148be^2n\sqrt{d+ex^2}}{75d^4x} - \frac{a+b\log(cx^n)}{5dx^5\sqrt{d+ex^2}} + \frac{2e(a+b\log(cx^n))}{5d^2x^3\sqrt{d+ex^2}} - \frac{8e^2(a+b\log(cx^n))}{5d^3x\sqrt{d+ex^2}} - \frac{16e^3x(a+b\log(cx^n))}{5d^4\sqrt{d+ex^2}} + \frac{(16be^3n)\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{5d^4}$$

$$= -\frac{bn\sqrt{d+ex^2}}{25d^2x^5} + \frac{14ben\sqrt{d+ex^2}}{75d^3x^3} - \frac{148be^2n\sqrt{d+ex^2}}{75d^4x} + \frac{16be^{5/2}n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5d^4} - \frac{a+b\log(cx^n)}{5dx^5\sqrt{d+ex^2}} + \frac{2e(a+b\log(cx^n))}{5d^2x^3\sqrt{d+ex^2}} - \frac{8e^2(a+b\log(cx^n))}{5d^3x\sqrt{d+ex^2}} - \frac{16e^3x(a+b\log(cx^n))}{5d^4\sqrt{d+ex^2}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.76

$$\int \frac{a+b\log(cx^n)}{x^6(d+ex^2)^{3/2}}\,dx = \frac{-15ad^3-3bd^3n+30ad^2ex^2+11bd^2enx^2-120ade^2x^4-134bde^2nx^4-240ae^3x^6-}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^6*(d + e*x^2)^(3/2)),x]
```

```
[Out] (-15*a*d^3 - 3*b*d^3*n + 30*a*d^2*e*x^2 + 11*b*d^2*e*n*x^2 - 120*a*d*e^2*x^
4 - 134*b*d*e^2*n*x^4 - 240*a*e^3*x^6 - 148*b*e^3*n*x^6 - 15*b*(d^3 - 2*d^2
*e*x^2 + 8*d*e^2*x^4 + 16*e^3*x^6)*Log[c*x^n] + 240*b*e^(5/2)*n*x^5*Sqrt[d
+ e*x^2]*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(75*d^4*x^5*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\,x^{n}\right)}{x^{6}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*ln(c*x^n))/x^6/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/x^6/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.00

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{6}\left(d+ex^{2}\right)^{3/2}}\,dx = \left[\frac{120\left(be^{3}nx^{7}+bde^{2}nx^{5}\right)\sqrt{e}\log\left(-2\,ex^{2}-2\sqrt{ex^{2}+d}\sqrt{e}x-d\right)-\left(4\left(37\,be^{3}n+60\,ae\right.\right.}{}\right.$$

$$-\frac{240\left(be^{3}nx^{7}+bde^{2}nx^{5}\right)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^{2}+d}}\right)+\left(4\left(37\,be^{3}n+60\,ae^{3}\right)x^{6}+3\,bd^{3}n+2\left(67\,bde^{2}n+60\,ade^{2}\right)x^{4}\right.}{}$$

```
[In] integrate((a+b*log(c*x^n))/x^6/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/75*(120*(b*e^3*n*x^7 + b*d*e^2*n*x^5)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^
2 + d)*sqrt(e)*x - d) - (4*(37*b*e^3*n + 60*a*e^3)*x^6 + 3*b*d^3*n + 2*(67*
b*d*e^2*n + 60*a*d*e^2)*x^4 + 15*a*d^3 - (11*b*d^2*e*n + 30*a*d^2*e)*x^2 +
15*(16*b*e^3*x^6 + 8*b*d*e^2*x^4 - 2*b*d^2*e*x^2 + b*d^3)*log(c) + 15*(16*b
*e^3*n*x^6 + 8*b*d*e^2*n*x^4 - 2*b*d^2*e*n*x^2 + b*d^3*n)*log(x))*sqrt(e*x^
2 + d))/(d^4*e*x^7 + d^5*x^5), -1/75*(240*(b*e^3*n*x^7 + b*d*e^2*n*x^5)*sqr
t(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (4*(37*b*e^3*n + 60*a*e^3)*x^6 +
 3*b*d^3*n + 2*(67*b*d*e^2*n + 60*a*d*e^2)*x^4 + 15*a*d^3 - (11*b*d^2*e*n +
 30*a*d^2*e)*x^2 + 15*(16*b*e^3*x^6 + 8*b*d*e^2*x^4 - 2*b*d^2*e*x^2 + b*d^3
)*log(c) + 15*(16*b*e^3*n*x^6 + 8*b*d*e^2*n*x^4 - 2*b*d^2*e*n*x^2 + b*d^3*n
)*log(x))*sqrt(e*x^2 + d))/(d^4*e*x^7 + d^5*x^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{6}\left(d+ex^{2}\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*x**n))/x**6/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{6}\left(d+ex^{2}\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x^6/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{6}\left(d+ex^{2}\right)^{3/2}}\,dx=\int \frac{b\log\left(cx^{n}\right)+a}{\left(ex^{2}+d\right)^{\frac{3}{2}}x^{6}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^6/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)^(3/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{6}\left(d+ex^{2}\right)^{3/2}}\,dx=\int \frac{a+b\,\ln\left(c\,x^{n}\right)}{x^{6}\,\left(e\,x^{2}+d\right)^{3/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^6*(d + e*x^2)^(3/2)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^6*(d + e*x^2)^(3/2)), x)
```

3.297 $\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 212

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = -\frac{bd^2n}{3e^4\sqrt{d+ex^2}}+\frac{8bdn\sqrt{d+ex^2}}{3e^4}-\frac{bn(d+ex^2)^{3/2}}{9e^4} -\frac{16bd^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3e^4}+\frac{d^3(a+b\log(cx^n))}{3e^4(d+ex^2)^{3/2}}-\frac{3d^2(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}} -\frac{3d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4}+\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^4}$$

```
[Out] -1/9*b*n*(e*x^2+d)^(3/2)/e^4-16/3*b*d^(3/2)*n*arctanh((e*x^2+d)^(1/2)/d^(1/
2))/e^4+1/3*d^3*(a+b*ln(c*x^n))/e^4/(e*x^2+d)^(3/2)+1/3*(e*x^2+d)^(3/2)*(a+
b*ln(c*x^n))/e^4-1/3*b*d^2*n/e^4/(e*x^2+d)^(1/2)-3*d^2*(a+b*ln(c*x^n))/e^4/
(e*x^2+d)^(1/2)+8/3*b*d*n*(e*x^2+d)^(1/2)/e^4-3*d*(a+b*ln(c*x^n))*(e*x^2+d)
^(1/2)/e^4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {272, 45, 2392, 12, 1813, 1633, 65, 214}

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \frac{d^3(a+b\log(cx^n))}{3e^4(d+ex^2)^{3/2}}-\frac{3d^2(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}} -\frac{3d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4}+\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^4} -\frac{16bd^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3e^4}-\frac{bd^2n}{3e^4\sqrt{d+ex^2}}+\frac{8bdn\sqrt{d+ex^2}}{3e^4}-\frac{bn(d+ex^2)^{3/2}}{9e^4}$$

```
[In] Int[(x^7*(a + b*Log[c*x^n]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*d^2*n)/(e^4*Sqrt[d + e*x^2]) + (8*b*d*n*Sqrt[d + e*x^2])/(3*e^4) -
(b*n*(d + e*x^2)^(3/2))/(9*e^4) - (16*b*d^(3/2)*n*ArcTanh[Sqrt[d + e*x^2]/S
qrt[d]])/(3*e^4) + (d^3*(a + b*Log[c*x^n]))/(3*e^4*(d + e*x^2)^(3/2)) - (3*
d^2*(a + b*Log[c*x^n]))/(e^4*Sqrt[d + e*x^2]) - (3*d*Sqrt[d + e*x^2]*(a + b
*Log[c*x^n]))/e^4 + ((d + e*x^2)^(3/2)*(a + b*Log[c*x^n]))/(3*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1633

```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_)), x_Symbol] :> I
nt[ExpandIntegrand[1/Sqrt[c + d*x], Px*((c + d*x)^(n + 1/2)/(a + b*x)), x],
 x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0] && GtQ
[Expon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^3(a+b\log(cx^n))}{3e^4(d+ex^2)^{3/2}}-\frac{3d^2(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}}-\frac{3d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4}\\
&\quad+\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^4}-(bn)\int\frac{-16d^3-24d^2ex^2-6de^2x^4+e^3x^6}{3e^4x(d+ex^2)^{3/2}}\,dx\\
&=\frac{d^3(a+b\log(cx^n))}{3e^4(d+ex^2)^{3/2}}-\frac{3d^2(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}}-\frac{3d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4}\\
&\quad+\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^4}-\frac{(bn)\int\frac{-16d^3-24d^2ex^2-6de^2x^4+e^3x^6}{x(d+ex^2)^{3/2}}\,dx}{3e^4}\\
&=\frac{d^3(a+b\log(cx^n))}{3e^4(d+ex^2)^{3/2}}-\frac{3d^2(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}}-\frac{3d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4}\\
&\quad+\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^4}-\frac{(bn)\text{Subst}\left(\int\frac{-16d^3-24d^2ex-6de^2x^2+e^3x^3}{x(d+ex)^{3/2}}\,dx,x,x^2\right)}{6e^4}\\
&=\frac{d^3(a+b\log(cx^n))}{3e^4(d+ex^2)^{3/2}}-\frac{3d^2(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}}\\
&\quad-\frac{3d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4}+\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^4}\\
&\quad-\frac{(bn)\text{Subst}\left(\int\left(-\frac{d^2e}{(d+ex)^{3/2}}-\frac{7de}{\sqrt{d+ex}}-\frac{16d^2}{x\sqrt{d+ex}}+\frac{e^2x}{\sqrt{d+ex}}\right)dx,x,x^2\right)}{6e^4}\\
&=-\frac{bd^2n}{3e^4\sqrt{d+ex^2}}+\frac{7bdn\sqrt{d+ex^2}}{3e^4}+\frac{d^3(a+b\log(cx^n))}{3e^4(d+ex^2)^{3/2}}-\frac{3d^2(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}}\\
&\quad-\frac{3d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4}+\frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^4}\\
&\quad+\frac{(8bd^2n)\,\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{3e^4}-\frac{(bn)\text{Subst}\left(\int\frac{x}{\sqrt{d+ex}}\,dx,x,x^2\right)}{6e^2}
\end{aligned}$$

$$= -\frac{bd^2n}{3e^4\sqrt{d+ex^2}} + \frac{7bdn\sqrt{d+ex^2}}{3e^4} + \frac{d^3(a+b\log(cx^n))}{3e^4(d+ex^2)^{3/2}} - \frac{3d^2(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}} - \frac{3d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4} + \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^4} + \frac{(16bd^2n)\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{3e^5} - \frac{(bn)\text{Subst}\left(\int \left(-\frac{d}{e\sqrt{d+ex}}+\frac{\sqrt{d+ex}}{e}\right)\,dx, x, x^2\right)}{6e^2}$$

$$= -\frac{bd^2n}{3e^4\sqrt{d+ex^2}} + \frac{8bdn\sqrt{d+ex^2}}{3e^4} - \frac{bn(d+ex^2)^{3/2}}{9e^4} - \frac{16bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3e^4} + \frac{d^3(a+b\log(cx^n))}{3e^4(d+ex^2)^{3/2}} - \frac{3d^2(a+b\log(cx^n))}{e^4\sqrt{d+ex^2}} - \frac{3d\sqrt{d+ex^2}(a+b\log(cx^n))}{e^4} + \frac{(d+ex^2)^{3/2}(a+b\log(cx^n))}{3e^4}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.13

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \frac{-48ad^3+20bd^3n-72ad^2ex^2+42bd^2enx^2-18ade^2x^4+21bde^2nx^4+3ae^3x^6-}{}$$

```
[In] Integrate[(x^7*(a + b*Log[c*x^n]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (-48*a*d^3 + 20*b*d^3*n - 72*a*d^2*e*x^2 + 42*b*d^2*e*n*x^2 - 18*a*d*e^2*x^
4 + 21*b*d*e^2*n*x^4 + 3*a*e^3*x^6 - b*e^3*n*x^6 + 48*b*d^(3/2)*n*(d + e*x^
2)^(3/2)*Log[x] - 3*b*(16*d^3 + 24*d^2*e*x^2 + 6*d*e^2*x^4 - e^3*x^6)*Log[c
*x^n] - 48*b*d^(5/2)*n*Sqrt[d + e*x^2]*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] - 4
8*b*d^(3/2)*e*n*x^2*Sqrt[d + e*x^2]*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/(9*e^
4*(d + e*x^2)^(3/2))
```

Maple [F]

$$\int \frac{x^7(a+b\ln(c\,x^n))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^7*(a+b*ln(c*x^n))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^7*(a+b*ln(c*x^n))/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 504, normalized size of antiderivative = 2.38

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \left[\frac{24\,(bde^2nx^4+2\,bd^2enx^2+bd^3n)\sqrt{d}\log\left(-\frac{ex^2-2\,\sqrt{ex^2+d}\sqrt{d}+2\,d}{x^2}\right)-((be^3n-3\,ae^3)\cdots}{\cdots}\right.$$

```
[In] integrate(x^7*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/9*(24*(b*d*e^2*n*x^4 + 2*b*d^2*e*n*x^2 + b*d^3*n)*sqrt(d)*log(-(e*x^2 -
2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) - ((b*e^3*n - 3*a*e^3)*x^6 - 20*b*d^3
*n - 3*(7*b*d*e^2*n - 6*a*d*e^2)*x^4 + 48*a*d^3 - 6*(7*b*d^2*e*n - 12*a*d^2
*e)*x^2 - 3*(b*e^3*x^6 - 6*b*d*e^2*x^4 - 24*b*d^2*e*x^2 - 16*b*d^3)*log(c)
- 3*(b*e^3*n*x^6 - 6*b*d*e^2*n*x^4 - 24*b*d^2*e*n*x^2 - 16*b*d^3*n)*log(x))
*sqrt(e*x^2 + d))/(e^6*x^4 + 2*d*e^5*x^2 + d^2*e^4), 1/9*(48*(b*d*e^2*n*x^4
 + 2*b*d^2*e*n*x^2 + b*d^3*n)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x^2 + d)) - (
(b*e^3*n - 3*a*e^3)*x^6 - 20*b*d^3*n - 3*(7*b*d*e^2*n - 6*a*d*e^2)*x^4 + 48
*a*d^3 - 6*(7*b*d^2*e*n - 12*a*d^2*e)*x^2 - 3*(b*e^3*x^6 - 6*b*d*e^2*x^4 -
24*b*d^2*e*x^2 - 16*b*d^3)*log(c) - 3*(b*e^3*n*x^6 - 6*b*d*e^2*n*x^4 - 24*b
*d^2*e*n*x^2 - 16*b*d^3*n)*log(x))*sqrt(e*x^2 + d))/(e^6*x^4 + 2*d*e^5*x^2
+ d^2*e^4)]
```

Sympy [A] (verification not implemented)

Time = 104.75 (sec) , antiderivative size = 556, normalized size of antiderivative = 2.62

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = a\left(\begin{cases}\frac{d^3}{3e^4(d+ex^2)^{\frac{3}{2}}}-\frac{3d^2}{e^4\sqrt{d+ex^2}}-\frac{3d\sqrt{d+ex^2}}{e^4}+\frac{(d+ex^2)^{\frac{3}{2}}}{3e^4} & \text{for } e\neq 0\\ \frac{x^8}{8d^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)$$
$$-bn\left(\begin{cases}\frac{4d^{\frac{3}{2}}\sqrt{1+\frac{ex^2}{d}}}{9e^4}+\frac{d^{\frac{3}{2}}\log\left(\frac{ex^2}{d}\right)}{6e^4}-\frac{d^{\frac{3}{2}}\log\left(\sqrt{1+\frac{ex^2}{d}}+1\right)}{3e^4}+\frac{6d^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{ex}}\right)}{e^4}+\frac{\sqrt{d}x^2\sqrt{1+\frac{ex^2}{d}}}{9e^3}+\frac{2d^6\sqrt{1+\frac{ex^2}{d}}}{6d^{\frac{9}{2}}e^4+6d^{\frac{7}{2}}e^5x^2}+\frac{d^6\log(\cdots}{6d^{\frac{9}{2}}e^4+6d\cdots}\cdots\\ \frac{x^8}{64d^{\frac{5}{2}}}\end{cases}\right.$$
$$+b\left(\begin{cases}\frac{d^3}{3e^4(d+ex^2)^{\frac{3}{2}}}-\frac{3d^2}{e^4\sqrt{d+ex^2}}-\frac{3d\sqrt{d+ex^2}}{e^4}+\frac{(d+ex^2)^{\frac{3}{2}}}{3e^4} & \text{for } e\neq 0\\ \frac{x^8}{8d^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate(x**7*(a+b*ln(c*x**n))/(e*x**2+d)**(5/2),x)
```

```
[Out] a*Piecewise((d**3/(3*e**4*(d + e*x**2)**(3/2)) - 3*d**2/(e**4*sqrt(d + e*x*
*2)) - 3*d*sqrt(d + e*x**2)/e**4 + (d + e*x**2)**(3/2)/(3*e**4), Ne(e, 0)),
 (x**8/(8*d**(5/2)), True)) - b*n*Piecewise((4*d**(3/2)*sqrt(1 + e*x**2/d)/
(9*e**4) + d**(3/2)*log(e*x**2/d)/(6*e**4) - d**(3/2)*log(sqrt(1 + e*x**2/d
) + 1)/(3*e**4) + 6*d**(3/2)*asinh(sqrt(d)/(sqrt(e)*x))/e**4 + sqrt(d)*x**2
*sqrt(1 + e*x**2/d)/(9*e**3) + 2*d**6*sqrt(1 + e*x**2/d)/(6*d**(9/2)*e**4 +
 6*d**(7/2)*e**5*x**2) + d**6*log(e*x**2/d)/(6*d**(9/2)*e**4 + 6*d**(7/2)*e
**5*x**2) - 2*d**6*log(sqrt(1 + e*x**2/d) + 1)/(6*d**(9/2)*e**4 + 6*d**(7/2
)*e**5*x**2) + d**5*x**2*log(e*x**2/d)/(6*d**(9/2)*e**3 + 6*d**(7/2)*e**4*x
**2) - 2*d**5*x**2*log(sqrt(1 + e*x**2/d) + 1)/(6*d**(9/2)*e**3 + 6*d**(7/2
)*e**4*x**2) - 3*d**2/(e**(9/2)*x*sqrt(d/(e*x**2) + 1)) - 3*d*x/(e**(7/2)*s
qrt(d/(e*x**2) + 1)), (e > -oo) & (e < oo) & Ne(e, 0)), (x**8/(64*d**(5/2))
, True)) + b*Piecewise((d**3/(3*e**4*(d + e*x**2)**(3/2)) - 3*d**2/(e**4*sq
rt(d + e*x**2)) - 3*d*sqrt(d + e*x**2)/e**4 + (d + e*x**2)**(3/2)/(3*e**4),
 Ne(e, 0)), (x**8/(8*d**(5/2)), True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^7*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\log(cx^n)+a)x^7}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^7*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^7/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^7\,(a+b\,\ln(c\,x^n))}{(e\,x^2+d)^{5/2}}\,dx$$

```
[In] int((x^7*(a + b*log(c*x^n)))/(d + e*x^2)^(5/2),x)
[Out] int((x^7*(a + b*log(c*x^n)))/(d + e*x^2)^(5/2), x)
```

3.298 $\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 155

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \frac{bdn}{3e^3\sqrt{d+ex^2}} - \frac{bn\sqrt{d+ex^2}}{e^3} + \frac{8b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3e^3} - \frac{d^2(a+b\log(cx^n))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3}$$

```
[Out] -1/3*d^2*(a+b*ln(c*x^n))/e^3/(e*x^2+d)^(3/2)+8/3*b*n*arctanh((e*x^2+d)^(1/2
)/d^(1/2))*d^(1/2)/e^3+1/3*b*d*n/e^3/(e*x^2+d)^(1/2)+2*d*(a+b*ln(c*x^n))/e^
3/(e*x^2+d)^(1/2)-b*n*(e*x^2+d)^(1/2)/e^3+(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/e
^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {272, 45, 2392, 12, 1265, 911, 1275, 212}

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = -\frac{d^2(a+b\log(cx^n))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3} + \frac{8b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3e^3} + \frac{bdn}{3e^3\sqrt{d+ex^2}} - \frac{bn\sqrt{d+ex^2}}{e^3}$$

```
[In] Int[(x^5*(a + b*Log[c*x^n]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*d*n)/(3*e^3*Sqrt[d + e*x^2]) - (b*n*Sqrt[d + e*x^2])/e^3 + (8*b*Sqrt[d]*
n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(3*e^3) - (d^2*(a + b*Log[c*x^n]))/(3*e
```

```
^3*(d + e*x^2)^(3/2)) + (2*d*(a + b*Log[c*x^n]))/(e^3*Sqrt[d + e*x^2]) + (S
qrt[d + e*x^2]*(a + b*Log[c*x^n]))/e^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{3e^3(d+ex^2)^{3/2}}+\frac{2d(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}} \\
&\quad+\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3}-(bn)\int\frac{8d^2+12dex^2+3e^2x^4}{3e^3x(d+ex^2)^{3/2}}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{3e^3(d+ex^2)^{3/2}}+\frac{2d(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}} \\
&\quad+\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3}-\frac{(bn)\int\frac{8d^2+12dex^2+3e^2x^4}{x(d+ex^2)^{3/2}}\,dx}{3e^3} \\
&= -\frac{d^2(a+b\log(cx^n))}{3e^3(d+ex^2)^{3/2}}+\frac{2d(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}} \\
&\quad+\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3}-\frac{(bn)\text{Subst}\left(\int\frac{8d^2+12dex+3e^2x^2}{x(d+ex)^{3/2}}\,dx,x,x^2\right)}{6e^3} \\
&= -\frac{d^2(a+b\log(cx^n))}{3e^3(d+ex^2)^{3/2}}+\frac{2d(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3} \\
&\quad-\frac{(bn)\text{Subst}\left(\int\frac{-d^2+6dx^2+3x^4}{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)}\,dx,x,\sqrt{d+ex^2}\right)}{3e^4} \\
&= -\frac{d^2(a+b\log(cx^n))}{3e^3(d+ex^2)^{3/2}}+\frac{2d(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3} \\
&\quad-\frac{(bn)\text{Subst}\left(\int\left(3e+\frac{de}{x^2}-\frac{8de}{d-x^2}\right)dx,x,\sqrt{d+ex^2}\right)}{3e^4} \\
&= \frac{bdn}{3e^3\sqrt{d+ex^2}}-\frac{bn\sqrt{d+ex^2}}{e^3}-\frac{d^2(a+b\log(cx^n))}{3e^3(d+ex^2)^{3/2}}+\frac{2d(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}} \\
&\quad+\frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3}+\frac{(8bdn)\text{Subst}\left(\int\frac{1}{d-x^2}\,dx,x,\sqrt{d+ex^2}\right)}{3e^3}
\end{aligned}
$$

$$= \frac{bdn}{3e^3\sqrt{d+ex^2}} - \frac{bn\sqrt{d+ex^2}}{e^3} + \frac{8b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3e^3} - \frac{d^2(a+b\log(cx^n))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\log(cx^n))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\log(cx^n))}{e^3}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.32

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = -\frac{8b\sqrt{d}n\log(x)}{3e^3} + \frac{bn(8d^2+12dex^2+3e^2x^4)\log(x)}{3e^3(d+ex^2)^{3/2}} + \sqrt{d+ex^2}\left(-\frac{d^2(a+b(-n\log(x)+\log(cx^n)))}{3e^3(d+ex^2)^2} + \frac{a-bn+b(-n\log(x)+\log(cx^n))}{e^3} + \frac{d(6a+bn+6b(-n\log$$
$$+ \frac{8b\sqrt{d}n\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)}{3e^3}$$

```
[In] Integrate[(x^5*(a + b*Log[c*x^n]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (-8*b*Sqrt[d]*n*Log[x])/(3*e^3) + (b*n*(8*d^2 + 12*d*e*x^2 + 3*e^2*x^4)*Log
[x])/(3*e^3*(d + e*x^2)^(3/2)) + Sqrt[d + e*x^2]*(-1/3*(d^2*(a + b*(-(n*Log
[x]) + Log[c*x^n])))/(e^3*(d + e*x^2)^2) + (a - b*n + b*(-(n*Log[x]) + Log[
c*x^n]))/e^3 + (d*(6*a + b*n + 6*b*(-(n*Log[x]) + Log[c*x^n])))/(3*e^3*(d +
 e*x^2))) + (8*b*Sqrt[d]*n*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/(3*e^3)
```

Maple [F]

$$\int \frac{x^5(a+b\ln(c\,x^n))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^5*(a+b*ln(c*x^n))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^5*(a+b*ln(c*x^n))/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.59

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \left[\frac{4(be^2nx^4+2bdenx^2+bd^2n)\sqrt{d}\log\left(-\frac{ex^2+2\sqrt{ex^2+d}\sqrt{d}+2d}{x^2}\right) - (3(be^2n-ae^2)x^4}{} \right.$$
$$-\frac{8(be^2nx^4+2bdenx^2+bd^2n)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}}{\sqrt{ex^2+d}}\right) + (3(be^2n-ae^2)x^4+2bd^2n-8ad^2+(5bden-12ad}{3(e^5x^4+2de^4x^2+}$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(4*(b*e^2*n*x^4 + 2*b*d*e*n*x^2 + b*d^2*n)*sqrt(d)*log(-(e*x^2 + 2*sqr
t(e*x^2 + d)*sqrt(d) + 2*d)/x^2) - (3*(b*e^2*n - a*e^2)*x^4 + 2*b*d^2*n - 8
*a*d^2 + (5*b*d*e*n - 12*a*d*e)*x^2 - (3*b*e^2*x^4 + 12*b*d*e*x^2 + 8*b*d^2
)*log(c) - (3*b*e^2*n*x^4 + 12*b*d*e*n*x^2 + 8*b*d^2*n)*log(x))*sqrt(e*x^2
+ d))/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3), -1/3*(8*(b*e^2*n*x^4 + 2*b*d*e*n*x
^2 + b*d^2*n)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x^2 + d)) + (3*(b*e^2*n - a*e
^2)*x^4 + 2*b*d^2*n - 8*a*d^2 + (5*b*d*e*n - 12*a*d*e)*x^2 - (3*b*e^2*x^4 +
 12*b*d*e*x^2 + 8*b*d^2)*log(c) - (3*b*e^2*n*x^4 + 12*b*d*e*n*x^2 + 8*b*d^2
*n)*log(x))*sqrt(e*x^2 + d))/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3)]
```

Sympy [A] (verification not implemented)

Time = 65.07 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.68

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = a\left(\begin{cases} -\frac{d^2}{3e^3(d+ex^2)^{\frac{3}{2}}} + \frac{2d}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}}{e^3} & \text{for } e\neq 0 \\ \frac{x^6}{6d^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right) - bn\left(\begin{cases} -\frac{3\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{e^3} - \frac{2d^5\sqrt{1+\frac{ex^2}{d}}}{6d^{\frac{9}{2}}e^3+6d^{\frac{7}{2}}e^4x^2} - \frac{d^5\log\left(\frac{ex^2}{d}\right)}{6d^{\frac{9}{2}}e^3+6d^{\frac{7}{2}}e^4x^2} + \frac{2d^5\log\left(\sqrt{1+\frac{ex^2}{d}}+1\right)}{6d^{\frac{9}{2}}e^3+6d^{\frac{7}{2}}e^4x^2} - \frac{d^4x^2\log\left(\frac{ex^2}{d}\right)}{6d^{\frac{9}{2}}e^2+6d^{\frac{7}{2}}e^3x^2} + \frac{2d^4x^2\log\left(\sqrt{1+\right.}{6d^{\frac{9}{2}}e^2+6d^{\frac{7}{2}}} \\ \frac{x^6}{36d^{\frac{5}{2}}} \end{cases}\right) + b\left(\begin{cases} -\frac{d^2}{3e^3(d+ex^2)^{\frac{3}{2}}} + \frac{2d}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}}{e^3} & \text{for } e\neq 0 \\ \frac{x^6}{6d^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right)\log(cx^n)$$

```
[In] integrate(x**5*(a+b*ln(c*x**n))/(e*x**2+d)**(5/2),x)
```

```
[Out] a*Piecewise((-d**2/(3*e**3*(d + e*x**2)**(3/2)) + 2*d/(e**3*sqrt(d + e*x**2
)) + sqrt(d + e*x**2)/e**3, Ne(e, 0)), (x**6/(6*d**(5/2)), True)) - b*n*Pie
cewise((-3*sqrt(d)*asinh(sqrt(d)/(sqrt(e)*x))/e**3 - 2*d**5*sqrt(1 + e*x**2
/d)/(6*d**(9/2)*e**3 + 6*d**(7/2)*e**4*x**2) - d**5*log(e*x**2/d)/(6*d**(9/
2)*e**3 + 6*d**(7/2)*e**4*x**2) + 2*d**5*log(sqrt(1 + e*x**2/d) + 1)/(6*d**
(9/2)*e**3 + 6*d**(7/2)*e**4*x**2) - d**4*x**2*log(e*x**2/d)/(6*d**(9/2)*e*
*2 + 6*d**(7/2)*e**3*x**2) + 2*d**4*x**2*log(sqrt(1 + e*x**2/d) + 1)/(6*d**
(9/2)*e**2 + 6*d**(7/2)*e**3*x**2) + d/(e**(7/2)*x*sqrt(d/(e*x**2) + 1)) +
x/(e**(5/2)*sqrt(d/(e*x**2) + 1)), (e > -oo) & (e < oo) & Ne(e, 0)), (x**6/
(36*d**(5/2)), True)) + b*Piecewise((-d**2/(3*e**3*(d + e*x**2)**(3/2)) + 2
*d/(e**3*sqrt(d + e*x**2)) + sqrt(d + e*x**2)/e**3, Ne(e, 0)), (x**6/(6*d**
(5/2)), True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\log(cx^n)+a)x^5}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^5*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^5/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^5\,(a+b\,\ln(c\,x^n))}{(e\,x^2+d)^{5/2}}\,dx$$

```
[In] int((x^5*(a + b*log(c*x^n)))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^5*(a + b*log(c*x^n)))/(d + e*x^2)^(5/2), x)
```

3.299 $\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 108

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = -\frac{bn}{3e^2\sqrt{d+ex^2}} - \frac{2bn\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3\sqrt{d}e^2} + \frac{d(a+b\log(cx^n))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\log(cx^n)}{e^2\sqrt{d+ex^2}}$$

```
[Out] 1/3*d*(a+b*ln(c*x^n))/e^2/(e*x^2+d)^(3/2)-2/3*b*n*arctanh((e*x^2+d)^(1/2)/d
^(1/2))/e^2/d^(1/2)-1/3*b*n/e^2/(e*x^2+d)^(1/2)+(-a-b*ln(c*x^n))/e^2/(e*x^2
+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {272, 45, 2392, 12, 457, 79, 65, 214}

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = -\frac{a+b\log(cx^n)}{e^2\sqrt{d+ex^2}} + \frac{d(a+b\log(cx^n))}{3e^2(d+ex^2)^{3/2}} - \frac{2bn\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3\sqrt{d}e^2} - \frac{bn}{3e^2\sqrt{d+ex^2}}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*n)/(e^2*Sqrt[d + e*x^2]) - (2*b*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])
/(3*Sqrt[d]*e^2) + (d*(a + b*Log[c*x^n]))/(3*e^2*(d + e*x^2)^(3/2)) - (a +
b*Log[c*x^n])/(e^2*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(a+b\log(cx^n))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\log(cx^n)}{e^2\sqrt{d+ex^2}} - (bn)\int \frac{-2d-3ex^2}{3e^2x(d+ex^2)^{3/2}}\,dx \\
&= \frac{d(a+b\log(cx^n))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\log(cx^n)}{e^2\sqrt{d+ex^2}} - \frac{(bn)\int \frac{-2d-3ex^2}{x(d+ex^2)^{3/2}}\,dx}{3e^2} \\
&= \frac{d(a+b\log(cx^n))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\log(cx^n)}{e^2\sqrt{d+ex^2}} - \frac{(bn)\text{Subst}\left(\int \frac{-2d-3ex}{x(d+ex)^{3/2}}\,dx, x, x^2\right)}{6e^2} \\
&= -\frac{bn}{3e^2\sqrt{d+ex^2}} + \frac{d(a+b\log(cx^n))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\log(cx^n)}{e^2\sqrt{d+ex^2}} + \frac{(bn)\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)}{3e^2} \\
&= -\frac{bn}{3e^2\sqrt{d+ex^2}} + \frac{d(a+b\log(cx^n))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\log(cx^n)}{e^2\sqrt{d+ex^2}} \\
&\quad + \frac{(2bn)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{3e^3} \\
&= -\frac{bn}{3e^2\sqrt{d+ex^2}} - \frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3\sqrt{d}e^2} + \frac{d(a+b\log(cx^n))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\log(cx^n)}{e^2\sqrt{d+ex^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.27

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \frac{\frac{2bn\log(x)}{\sqrt{d}} - \frac{bn(2d+3ex^2)\log(x)}{(d+ex^2)^{3/2}} + \frac{d(a-bn\log(x)+b\log(cx^n))-(d+ex^2)(3a+bn-3bn\log(x)+3b\log(cx^n)}{(d+ex^2)^{3/2}}}{3e^2}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/(d + e*x^2)^(5/2),x]
```

```
[Out] ((2*b*n*Log[x])/Sqrt[d] - (b*n*(2*d + 3*e*x^2)*Log[x])/(d + e*x^2)^(3/2) +
(d*(a - b*n*Log[x] + b*Log[c*x^n]) - (d + e*x^2)*(3*a + b*n - 3*b*n*Log[x]
+ 3*b*Log[c*x^n]))/(d + e*x^2)^(3/2) - (2*b*n*Log[d + Sqrt[d]*Sqrt[d + e*x^
2]])/Sqrt[d])/(3*e^2)
```

Maple [F]

$$\int \frac{x^3(a+b\ln(c\,x^n))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^3*(a+b*ln(c*x^n))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^3*(a+b*ln(c*x^n))/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 325, normalized size of antiderivative = 3.01

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \left[\frac{(be^2nx^4+2\,bdenx^2+bd^2n)\sqrt{d}\log\left(-\frac{ex^2-2\,\sqrt{ex^2+d}\sqrt{d}+2\,d}{x^2}\right)-(bd^2n+2\,ad^2+(bde\cdots}{3\,(de^4x^4+2\,d\cdots}\right.$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*((b*e^2*n*x^4 + 2*b*d*e*n*x^2 + b*d^2*n)*sqrt(d)*log(-(e*x^2 - 2*sqrt(
e*x^2 + d)*sqrt(d) + 2*d)/x^2) - (b*d^2*n + 2*a*d^2 + (b*d*e*n + 3*a*d*e)*x
^2 + (3*b*d*e*x^2 + 2*b*d^2)*log(c) + (3*b*d*e*n*x^2 + 2*b*d^2*n)*log(x))*s
qrt(e*x^2 + d))/(d*e^4*x^4 + 2*d^2*e^3*x^2 + d^3*e^2), 1/3*(2*(b*e^2*n*x^4
+ 2*b*d*e*n*x^2 + b*d^2*n)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x^2 + d)) - (b*d
^2*n + 2*a*d^2 + (b*d*e*n + 3*a*d*e)*x^2 + (3*b*d*e*x^2 + 2*b*d^2)*log(c) +
 (3*b*d*e*n*x^2 + 2*b*d^2*n)*log(x))*sqrt(e*x^2 + d))/(d*e^4*x^4 + 2*d^2*e^
3*x^2 + d^3*e^2)]
```

Sympy [A] (verification not implemented)

Time = 28.71 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.12

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = a\left(\begin{cases}\frac{d}{3e^2(d+ex^2)^{\frac{3}{2}}}-\frac{1}{e^2\sqrt{d+ex^2}} & \text{for } e\neq 0\\ \frac{x^4}{4d^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)$$
$$-bn\left(\begin{cases}\frac{2d^4\sqrt{1+\frac{ex^2}{d}}}{6d^{\frac{9}{2}}e^2+6d^{\frac{7}{2}}e^3x^2}+\frac{d^4\log\left(\frac{ex^2}{d}\right)}{6d^{\frac{9}{2}}e^2+6d^{\frac{7}{2}}e^3x^2}-\frac{2d^4\log\left(\sqrt{1+\frac{ex^2}{d}}+1\right)}{6d^{\frac{9}{2}}e^2+6d^{\frac{7}{2}}e^3x^2}+\frac{d^3x^2\log\left(\frac{ex^2}{d}\right)}{6d^{\frac{9}{2}}e+6d^{\frac{7}{2}}e^2x^2}-\frac{2d^3x^2\log\left(\sqrt{1+\frac{ex^2}{d}}+1\right)}{6d^{\frac{9}{2}}e+6d^{\frac{7}{2}}e^2x^2}+\frac{\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{\sqrt{d}e^2}\\ \frac{x^4}{16d^{\frac{5}{2}}}\end{cases}\right.$$
$$+b\left(\begin{cases}\frac{d}{3e^2(d+ex^2)^{\frac{3}{2}}}-\frac{1}{e^2\sqrt{d+ex^2}} & \text{for } e\neq 0\\ \frac{x^4}{4d^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))/(e*x**2+d)**(5/2),x)
```

```
[Out] a*Piecewise((d/(3*e**2*(d + e*x**2)**(3/2)) - 1/(e**2*sqrt(d + e*x**2)), Ne
(e, 0)), (x**4/(4*d**(5/2)), True)) - b*n*Piecewise((2*d**4*sqrt(1 + e*x**2
/d)/(6*d**(9/2)*e**2 + 6*d**(7/2)*e**3*x**2) + d**4*log(e*x**2/d)/(6*d**(9/
2)*e**2 + 6*d**(7/2)*e**3*x**2) - 2*d**4*log(sqrt(1 + e*x**2/d) + 1)/(6*d**
(9/2)*e**2 + 6*d**(7/2)*e**3*x**2) + d**3*x**2*log(e*x**2/d)/(6*d**(9/2)*e
+ 6*d**(7/2)*e**2*x**2) - 2*d**3*x**2*log(sqrt(1 + e*x**2/d) + 1)/(6*d**(9/
2)*e + 6*d**(7/2)*e**2*x**2) + asinh(sqrt(d)/(sqrt(e)*x))/(sqrt(d)*e**2), (
e > -oo) & (e < oo) & Ne(e, 0)), (x**4/(16*d**(5/2)), True)) + b*Piecewise(
(d/(3*e**2*(d + e*x**2)**(3/2)) - 1/(e**2*sqrt(d + e*x**2)), Ne(e, 0)), (x*
*4/(4*d**(5/2)), True))*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.27

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \frac{1}{3}bn\left(\frac{\log\left(\frac{\sqrt{ex^2+d}-\sqrt{d}}{\sqrt{ex^2+d}+\sqrt{d}}\right)}{\sqrt{d}e^2}-\frac{1}{\sqrt{ex^2+d}e^2}\right)$$
$$-\frac{1}{3}b\left(\frac{3x^2}{(ex^2+d)^{\frac{3}{2}}e}+\frac{2d}{(ex^2+d)^{\frac{3}{2}}e^2}\right)\log(cx^n)-\frac{1}{3}a\left(\frac{3x^2}{(ex^2+d)^{\frac{3}{2}}e}+\frac{2d}{(ex^2+d)^{\frac{3}{2}}e^2}\right)$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*n*(log((sqrt(e*x^2 + d) - sqrt(d))/(sqrt(e*x^2 + d) + sqrt(d)))/(sqrt
(d)*e^2) - 1/(sqrt(e*x^2 + d)*e^2)) - 1/3*b*(3*x^2/((e*x^2 + d)^(3/2)*e) +
2*d/((e*x^2 + d)^(3/2)*e^2))*log(c*x^n) - 1/3*a*(3*x^2/((e*x^2 + d)^(3/2)*e
) + 2*d/((e*x^2 + d)^(3/2)*e^2))
```

Giac [F]

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^3\,(a+b\,\ln(c\,x^n))}{(e\,x^2+d)^{5/2}}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n)))/(d + e*x^2)^(5/2),x)
[Out] int((x^3*(a + b*log(c*x^n)))/(d + e*x^2)^(5/2), x)
```

3.300 $\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx=\frac{bn}{3de\sqrt{d+ex^2}}-\frac{bn\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3d^{3/2}e}-\frac{a+b\log(cx^n)}{3e(d+ex^2)^{3/2}}$$

```
[Out] -1/3*b*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)/e+1/3*(-a-b*ln(c*x^n))/e/
(e*x^2+d)^(3/2)+1/3*b*n/d/e/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2376, 272, 53, 65, 214}

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx=-\frac{a+b\log(cx^n)}{3e(d+ex^2)^{3/2}}-\frac{bn\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3d^{3/2}e}+\frac{bn}{3de\sqrt{d+ex^2}}$$

```
[In] Int[(x*(a + b*Log[c*x^n]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*n)/(3*d*e*Sqrt[d + e*x^2]) - (b*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(3*d
^(3/2)*e) - (a + b*Log[c*x^n])/(3*e*(d + e*x^2)^(3/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\log\left(cx^n\right)}{3e\left(d+ex^2\right)^{3/2}}+\frac{(bn)\int\frac{1}{x(d+ex^2)^{3/2}}\,dx}{3e}\\
&= -\frac{a+b\log\left(cx^n\right)}{3e\left(d+ex^2\right)^{3/2}}+\frac{(bn)\text{Subst}\left(\int\frac{1}{x(d+ex)^{3/2}}\,dx,x,x^2\right)}{6e}\\
&= \frac{bn}{3de\sqrt{d+ex^2}}-\frac{a+b\log\left(cx^n\right)}{3e\left(d+ex^2\right)^{3/2}}+\frac{(bn)\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{6de}\\
&= \frac{bn}{3de\sqrt{d+ex^2}}-\frac{a+b\log\left(cx^n\right)}{3e\left(d+ex^2\right)^{3/2}}+\frac{(bn)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{3de^2}
\end{aligned}$$

$$= \frac{bn}{3de\sqrt{d+ex^2}} - \frac{bn \tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3d^{3/2}e} - \frac{a+b\log\left(cx^n\right)}{3e\left(d+ex^2\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

$$\int \frac{x(a+b\log\left(cx^n\right))}{\left(d+ex^2\right)^{5/2}}\,dx = -\frac{\frac{a}{(d+ex^2)^{3/2}} - \frac{bn}{d\sqrt{d+ex^2}} - \frac{bn\log(x)}{d^{3/2}} + \frac{b\log(cx^n)}{(d+ex^2)^{3/2}} + \frac{bn\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)}{d^{3/2}}}{3e}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(a/(d + e*x^2)^(3/2) - (b*n)/(d*Sqrt[d + e*x^2]) - (b*n*Log[x])/d^(3/2
) + (b*Log[c*x^n])/(d + e*x^2)^(3/2) + (b*n*Log[d + Sqrt[d]*Sqrt[d + e*x^2]
])/d^(3/2))/e
```

Maple [F]

$$\int \frac{x(a+b\ln\left(c\,x^n\right))}{\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

```
[In] int(x*(a+b*ln(c*x^n))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x*(a+b*ln(c*x^n))/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.18

$$\int \frac{x(a+b\log\left(cx^n\right))}{\left(d+ex^2\right)^{5/2}}\,dx = \left[\frac{(be^2nx^4+2\,bdenx^2+bd^2n)\sqrt{d}\log\left(-\frac{ex^2-2\,\sqrt{ex^2+d}\sqrt{d}+2\,d}{x^2}\right)+2\,(bdenx^2-bd^2n\log}{6\,(d^2e^3x^4+2\,d^3e^2x^2+d^4e)}\right.$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*((b*e^2*n*x^4 + 2*b*d*e*n*x^2 + b*d^2*n)*sqrt(d)*log(-(e*x^2 - 2*sqrt(
e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 2*(b*d*e*n*x^2 - b*d^2*n*log(x) + b*d^2*n
- b*d^2*log(c) - a*d^2)*sqrt(e*x^2 + d))/(d^2*e^3*x^4 + 2*d^3*e^2*x^2 + d^4
*e), 1/3*((b*e^2*n*x^4 + 2*b*d*e*n*x^2 + b*d^2*n)*sqrt(-d)*arctan(sqrt(-d)/
sqrt(e*x^2 + d)) + (b*d*e*n*x^2 - b*d^2*n*log(x) + b*d^2*n - b*d^2*log(c) -
 a*d^2)*sqrt(e*x^2 + d))/(d^2*e^3*x^4 + 2*d^3*e^2*x^2 + d^4*e)]
```

Sympy [A] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.24

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = a\left(\begin{cases} -\frac{1}{3e(d+ex^2)^{\frac{3}{2}}} & \text{for } e\neq 0\\ \frac{x^2}{2d^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right) - bn\left(\begin{cases} -\frac{2d^3\sqrt{1+\frac{ex^2}{d}}}{6d^{\frac{9}{2}}e+6d^{\frac{7}{2}}e^2x^2} - \frac{d^3\log\left(\frac{ex^2}{d}\right)}{6d^{\frac{9}{2}}e+6d^{\frac{7}{2}}e^2x^2} + \frac{2d^3\log\left(\sqrt{1+\frac{ex^2}{d}}+1\right)}{6d^{\frac{9}{2}}e+6d^{\frac{7}{2}}e^2x^2} - \frac{d^2x^2\log\left(\frac{ex^2}{d}\right)}{6d^{\frac{9}{2}}+6d^{\frac{7}{2}}ex^2} + \frac{2d^2x^2\log\left(\sqrt{1+\frac{ex^2}{d}}+1\right)}{6d^{\frac{9}{2}}+6d^{\frac{7}{2}}ex^2} & \text{for } e>-\infty\\ \frac{x^2}{4d^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right) + b\left(\begin{cases} -\frac{1}{3e(d+ex^2)^{\frac{3}{2}}} & \text{for } e\neq 0\\ \frac{x^2}{2d^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate(x*(a+b*ln(c*x**n))/(e*x**2+d)**(5/2),x)
```

```
[Out] a*Piecewise((-1/(3*e*(d + e*x**2)**(3/2)), Ne(e, 0)), (x**2/(2*d**(5/2)), T
rue)) - b*n*Piecewise((-2*d**3*sqrt(1 + e*x**2/d)/(6*d**(9/2)*e + 6*d**(7/2
)*e**2*x**2) - d**3*log(e*x**2/d)/(6*d**(9/2)*e + 6*d**(7/2)*e**2*x**2) + 2
*d**3*log(sqrt(1 + e*x**2/d) + 1)/(6*d**(9/2)*e + 6*d**(7/2)*e**2*x**2) - d
**2*x**2*log(e*x**2/d)/(6*d**(9/2) + 6*d**(7/2)*e*x**2) + 2*d**2*x**2*log(s
qrt(1 + e*x**2/d) + 1)/(6*d**(9/2) + 6*d**(7/2)*e*x**2), (e > -oo) & (e < o
o) & Ne(e, 0)), (x**2/(4*d**(5/2)), True)) + b*Piecewise((-1/(3*e*(d + e*x*
*2)**(3/2)), Ne(e, 0)), (x**2/(2*d**(5/2)), True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x(a + b\log(cx^n))}{(d + ex^2)^{5/2}}\,dx = \int \frac{(b\log(cx^n) + a)x}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\log(cx^n))}{(d + ex^2)^{5/2}}\,dx = \int \frac{x\,(a + b\,\ln(c\,x^n))}{(e\,x^2 + d)^{5/2}}\,dx$$

```
[In] int((x*(a + b*log(c*x^n)))/(d + e*x^2)^(5/2),x)
[Out] int((x*(a + b*log(c*x^n)))/(d + e*x^2)^(5/2), x)
```

3.301 $\int \frac{a+b\log(cx^n)}{x\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 251

$$\int \frac{a+b\log(cx^n)}{x\left(d+ex^2\right)^{5/2}}\,dx=-\frac{bn}{3d^2\sqrt{d+ex^2}}+\frac{4bn\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{bn\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2d^{5/2}}+\frac{1}{3}\left(\frac{1}{d\left(d+ex^2\right)^{3/2}}+\frac{3}{d^2\sqrt{d+ex^2}}-\frac{3\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}}\right)(a+b\log(cx^n))-\frac{bn\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+e}}\right.}{d^{5/2}}$$

```
[Out] 4/3*b*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(5/2)+1/2*b*n*arctanh((e*x^2+d)^
(1/2)/d^(1/2))^2/d^(5/2)-b*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))*ln(2*d^(1/2)/
(d^(1/2)-(e*x^2+d)^(1/2)))/d^(5/2)-1/2*b*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-(
e*x^2+d)^(1/2)))/d^(5/2)+1/3*(a+b*ln(c*x^n))*(1/d/(e*x^2+d)^(3/2)-3*arctanh
((e*x^2+d)^(1/2)/d^(1/2))/d^(5/2)+3/d^2/(e*x^2+d)^(1/2))-1/3*b*n/d^2/(e*x^2
+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {272, 53, 65, 214, 2390, 6131, 6055, 2449, 2352}

$$\int \frac{a+b\log(cx^n)}{x\left(d+ex^2\right)^{5/2}}\,dx=\frac{1}{3}\left(-\frac{3\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}}+\frac{3}{d^2\sqrt{d+ex^2}}+\frac{1}{d\left(d+ex^2\right)^{3/2}}\right)(a+b\log(cx^n))+\frac{bn\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2d^{5/2}}+\frac{4bn\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{bn\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\right.}{d^{5/2}}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x^2)^(5/2)),x]
```

```
[Out] -1/3*(b*n)/(d^2*Sqrt[d + e*x^2]) + (4*b*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])
/(3*d^(5/2)) + (b*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]]^2)/(2*d^(5/2)) + ((1/(
d*(d + e*x^2)^(3/2)) + 3/(d^2*Sqrt[d + e*x^2]) - (3*ArcTanh[Sqrt[d + e*x^2]
/Sqrt[d]])/d^(5/2))*(a + b*Log[c*x^n]))/3 - (b*n*ArcTanh[Sqrt[d + e*x^2]/Sq
rt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/d^(5/2) - (b*n*PolyLog
[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/(2*d^(5/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
```

```
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\left(\frac{1}{d\,(d+ex^2)^{3/2}}+\frac{3}{d^2\sqrt{d+ex^2}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}}\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int\left(\frac{1}{3dx\,(d+ex^2)^{3/2}}+\frac{1}{d^2x\sqrt{d+ex^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}x}\right)dx \\
&= \frac{1}{3}\left(\frac{1}{d\,(d+ex^2)^{3/2}}+\frac{3}{d^2\sqrt{d+ex^2}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}}\right)(a+b\log(cx^n)) \\
&\quad +\frac{(bn)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{x}\,dx}{d^{5/2}}-\frac{(bn)\int\frac{1}{x\sqrt{d+ex^2}}\,dx}{d^2}-\frac{(bn)\int\frac{1}{x(d+ex^2)^{3/2}}\,dx}{3d}
\end{aligned}$$

$$= \frac{1}{3}\left(\frac{1}{d\left(d+ex^2\right)^{3/2}} + \frac{3}{d^2\sqrt{d+ex^2}} - \frac{3\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}}\right)(a+b\log(cx^n)) + \frac{(bn)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx, x, x^2\right)}{2d^{5/2}} - \frac{(bn)\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)}{2d^2} - \frac{(bn)\text{Subst}\left(\int \frac{1}{x(d+ex)^{3/2}}\,dx, x, x^2\right)}{6d}$$

$$= -\frac{bn}{3d^2\sqrt{d+ex^2}} + \frac{1}{3}\left(\frac{1}{d\left(d+ex^2\right)^{3/2}} + \frac{3}{d^2\sqrt{d+ex^2}} - \frac{3\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}}\right)(a+b\log(cx^n)) + \frac{(bn)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx, x, \sqrt{d+ex^2}\right)}{d^{5/2}} - \frac{(bn)\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)}{6d^2} - \frac{(bn)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{d^2 e}$$

$$= -\frac{bn}{3d^2\sqrt{d+ex^2}} + \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2d^{5/2}} + \frac{1}{3}\left(\frac{1}{d\left(d+ex^2\right)^{3/2}} + \frac{3}{d^2\sqrt{d+ex^2}} - \frac{3\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}}\right)(a+b\log(cx^n)) - \frac{(bn)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\ldots\right)}{d^{3\ldots}}$$

$$= -\frac{bn}{3d^2\sqrt{d+ex^2}} + \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2d^{5/2}} + \frac{1}{3}\left(\frac{1}{d\left(d+ex^2\right)^{3/2}} + \frac{3}{d^2\sqrt{d+ex^2}} - \frac{3\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}}\right)(a+b\log(cx^n)) - \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\text{lo}\ldots}{d^{5/2}}$$

$$= -\frac{bn}{3d^2\sqrt{d+ex^2}} + \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2d^{5/2}} + \frac{1}{3}\left(\frac{1}{d\left(d+ex^2\right)^{3/2}} + \frac{3}{d^2\sqrt{d+ex^2}} - \frac{3\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}}\right)(a+b\log(cx^n)) - \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\text{lo}\ldots}{d^{5/2}}$$

$$= -\frac{bn}{3d^2\sqrt{d+ex^2}} + \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{2d^{5/2}}$$

$$+\frac{1}{3}\left(\frac{1}{d(d+ex^2)^{3/2}}+\frac{3}{d^2\sqrt{d+ex^2}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}}\right)(a+b\log(cx^n))-\frac{bn\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log}{d^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.31 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.09

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^{5/2}}\,dx = \frac{bn\sqrt{1+\frac{d}{ex^2}}\left(-3d^{5/2}(d+ex^2)^2\ {}_3F_2\left(\frac{5}{2},\frac{5}{2},\frac{5}{2};\frac{7}{2},\frac{7}{2};-\frac{d}{ex^2}\right)+25\sqrt{d}e^3\sqrt{1+\frac{d}{ex^2}}x^6(4d+3e\right.}{75d^{5/2}e^2x^4(d+ex^2)^{5/2}}$$

$$+\frac{(4d+3ex^2)(a-bn\log(x)+b\log(cx^n))}{3d^2(d+ex^2)^{3/2}}+\frac{\log(x)(a-bn\log(x)+b\log(cx^n))}{d^{5/2}}$$

$$-\frac{(a-bn\log(x)+b\log(cx^n))\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)}{d^{5/2}}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x^2)^(5/2)),x]
```

```
[Out] (b*n*Sqrt[1 + d/(e*x^2)]*(-3*d^(5/2)*(d + e*x^2)^2*HypergeometricPFQ[{5/2,
5/2, 5/2}, {7/2, 7/2}, -(d/(e*x^2))] + 25*Sqrt[d]*e^3*Sqrt[1 + d/(e*x^2)]*x
^6*(4*d + 3*e*x^2)*Log[x] - 75*e^(5/2)*x^5*(d + e*x^2)^2*ArcSinh[Sqrt[d]/(S
qrt[e]*x)]*Log[x]))/(75*d^(5/2)*e^2*x^4*(d + e*x^2)^(5/2)) + ((4*d + 3*e*x^
2)*(a - b*n*Log[x] + b*Log[c*x^n]))/(3*d^2*(d + e*x^2)^(3/2)) + (Log[x]*(a
- b*n*Log[x] + b*Log[c*x^n]))/d^(5/2) - ((a - b*n*Log[x] + b*Log[c*x^n])*Lo
g[d + Sqrt[d]*Sqrt[d + e*x^2]])/d^(5/2)
```

Maple [F]

$$\int \frac{a+b\ln(c\,x^n)}{x\,(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*ln(c*x^n))/x/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/x/(e*x^2+d)^(5/2),x)
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^{5/2}}\,dx=\int \frac{b\log(cx^n)+a}{(ex^2+d)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral((sqrt(e*x^2 + d)*b*log(c*x^n) + sqrt(e*x^2 + d)*a)/(e^3*x^7 + 3*d*
e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*x**n))/x/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^{5/2}}\,dx=\int \frac{b\log(cx^n)+a}{(ex^2+d)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x\,(e\,x^2+d)^{5/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*log(c*x^n))/(x*(d + e*x^2)^(5/2)), x)
```

3.302 $\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 337

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^{5/2}}\,dx = \frac{ben}{3d^3\sqrt{d+ex^2}}-\frac{bn\sqrt{d+ex^2}}{4d^3x^2}-\frac{31ben\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{12d^{7/2}}-\frac{5ben\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{7/2}}-\frac{5e(a+b\log(cx^n))}{6d^2(d+ex^2)^{3/2}}-\frac{a+b\log(cx^n)}{2dx^2(d+ex^2)^{3/2}}-\frac{5e(a+b\log(cx^n))}{2d^3\sqrt{d+ex^2}}+\frac{5e\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{7/2}}+\frac{5ben\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2d^{7/2}}+\frac{5ben\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{4d^{7/2}}$$

```
[Out] -31/12*b*e*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(7/2)-5/4*b*e*n*arctanh((e*
x^2+d)^(1/2)/d^(1/2))^2/d^(7/2)-5/6*e*(a+b*ln(c*x^n))/d^2/(e*x^2+d)^(3/2)+1
/2*(-a-b*ln(c*x^n))/d/x^2/(e*x^2+d)^(3/2)+5/2*e*arctanh((e*x^2+d)^(1/2)/d^(
1/2))*(a+b*ln(c*x^n))/d^(7/2)+5/2*b*e*n*arctanh((e*x^2+d)^(1/2)/d^(1/2))*ln
(2*d^(1/2)/(d^(1/2)-(e*x^2+d)^(1/2)))/d^(7/2)+5/4*b*e*n*polylog(2,1-2*d^(1/
2)/(d^(1/2)-(e*x^2+d)^(1/2)))/d^(7/2)+1/3*b*e*n/d^3/(e*x^2+d)^(1/2)-5/2*e*(
a+b*ln(c*x^n))/d^3/(e*x^2+d)^(1/2)-1/4*b*n*(e*x^2+d)^(1/2)/d^3/x^2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {272, 44, 53, 65, 214, 2392, 1265, 911, 1273, 464, 6131, 6055, 2449, 2352}

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^{5/2}}\,dx = \frac{5e\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{7/2}} - \frac{5e(a+b\log(cx^n))}{2d^3\sqrt{d+ex^2}} - \frac{5e(a+b\log(cx^n))}{6d^2(d+ex^2)^{3/2}} - \frac{a+b\log(cx^n)}{2dx^2(d+ex^2)^{3/2}} - \frac{5ben\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{7/2}} - \frac{31ben\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{12d^{7/2}} + \frac{5ben\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2d^{7/2}} + \frac{5ben\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{ex^2+d}}\right)}{4d^{7/2}} - \frac{bn\sqrt{d+ex^2}}{4d^3x^2} + \frac{ben}{3d^3\sqrt{d+ex^2}}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^3*(d + e*x^2)^(5/2)),x]
```

```
[Out] (b*e*n)/(3*d^3*Sqrt[d + e*x^2]) - (b*n*Sqrt[d + e*x^2])/(4*d^3*x^2) - (31*b
*e*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(12*d^(7/2)) - (5*b*e*n*ArcTanh[Sqrt
[d + e*x^2]/Sqrt[d]]^2)/(4*d^(7/2)) - (5*e*(a + b*Log[c*x^n]))/(6*d^2*(d +
e*x^2)^(3/2)) - (a + b*Log[c*x^n])/(2*d*x^2*(d + e*x^2)^(3/2)) - (5*e*(a +
b*Log[c*x^n]))/(2*d^3*Sqrt[d + e*x^2]) + (5*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[
d]]*(a + b*Log[c*x^n]))/(2*d^(7/2)) + (5*b*e*n*ArcTanh[Sqrt[d + e*x^2]/Sqrt
[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/(2*d^(7/2)) + (5*b*e*n*P
olyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^2])])/(4*d^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\frac{5e(a+b\log(cx^n))}{6d^2(d+ex^2)^{3/2}} - \frac{a+b\log(cx^n)}{2dx^2(d+ex^2)^{3/2}} - \frac{5e(a+b\log(cx^n))}{2d^3\sqrt{d+ex^2}} + \frac{5e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{7/2}} - (bn)\int\left(-\frac{3d^2+20dex^2+15e^2x^4}{6d^3x^3(d+ex^2)^{3/2}} + \frac{5e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{7/2}x}\right)dx$$

$$= -\frac{5e(a+b\log(cx^n))}{6d^2(d+ex^2)^{3/2}} - \frac{a+b\log(cx^n)}{2dx^2(d+ex^2)^{3/2}} - \frac{5e(a+b\log(cx^n))}{2d^3\sqrt{d+ex^2}} + \frac{5e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{7/2}} + \frac{(bn)\int\frac{3d^2+20dex^2+15e^2x^4}{x^3(d+ex^2)^{3/2}}dx}{6d^3} - \frac{(5ben)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{x}dx}{2d^{7/2}}$$

$$= -\frac{5e(a+b\log(cx^n))}{6d^2(d+ex^2)^{3/2}} - \frac{a+b\log(cx^n)}{2dx^2(d+ex^2)^{3/2}} - \frac{5e(a+b\log(cx^n))}{2d^3\sqrt{d+ex^2}} + \frac{5e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{7/2}} + \frac{(bn)\text{Subst}\left(\int\frac{3d^2+20dex+15e^2x^2}{x^2(d+ex)^{3/2}}dx, x, x^2\right)}{12d^3} - \frac{(5ben)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}dx, x, x^2\right)}{4d^{7/2}}$$

$$= -\frac{5e(a+b\log(cx^n))}{6d^2(d+ex^2)^{3/2}} - \frac{a+b\log(cx^n)}{2dx^2(d+ex^2)^{3/2}} - \frac{5e(a+b\log(cx^n))}{2d^3\sqrt{d+ex^2}} + \frac{5e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{7/2}} + \frac{(bn)\text{Subst}\left(\int\frac{-2d^2-10dx^2+15x^4}{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2}dx, x, \sqrt{d+ex^2}\right)}{6d^3e} - \frac{(5ben)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}dx, x, \sqrt{d+ex^2}\right)}{2d^{7/2}}$$

$$= -\frac{bn\sqrt{d+ex^2}}{4d^3x^2} - \frac{5ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{7/2}} - \frac{5e(a+b\log(cx^n))}{6d^2(d+ex^2)^{3/2}} - \frac{a+b\log(cx^n)}{2dx^2(d+ex^2)^{3/2}} - \frac{5e(a+b\log(cx^n))}{2d^3\sqrt{d+ex^2}} + \frac{5e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{7/2}} + \frac{(5ben)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx, x, \sqrt{d+ex^2}\right)}{2d^4} - \frac{(be^3n)\,\text{Subst}\left(\int \frac{-\frac{4d^3}{e^3}-\frac{27d^2x^2}{e^3}}{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{12d^5}$$

$$= \frac{ben}{3d^3\sqrt{d+ex^2}} - \frac{bn\sqrt{d+ex^2}}{4d^3x^2} - \frac{5ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{7/2}} - \frac{5e(a+b\log(cx^n))}{6d^2(d+ex^2)^{3/2}} - \frac{a+b\log(cx^n)}{2dx^2(d+ex^2)^{3/2}} - \frac{5e(a+b\log(cx^n))}{2d^3\sqrt{d+ex^2}} + \frac{5e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{7/2}} + \frac{5ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2d^{7/2}} + \frac{(31bn)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{12d^3} - \frac{(5ben)\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}\,dx, x, \sqrt{d+ex^2}\right)}{2d^4}$$

$$= \frac{ben}{3d^3\sqrt{d+ex^2}} - \frac{bn\sqrt{d+ex^2}}{4d^3x^2} - \frac{31ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{12d^{7/2}} - \frac{5ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{7/2}} - \frac{5e(a+b\log(cx^n))}{6d^2(d+ex^2)^{3/2}} - \frac{a+b\log(cx^n)}{2dx^2(d+ex^2)^{3/2}} - \frac{5e(a+b\log(cx^n))}{2d^3\sqrt{d+ex^2}} + \frac{5e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{7/2}} + \frac{5ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2d^{7/2}} + \frac{(5ben)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{d+ex^2}}{\sqrt{d}}}\right)}{2d^{7/2}}$$

$$= \frac{ben}{3d^3\sqrt{d+ex^2}} - \frac{bn\sqrt{d+ex^2}}{4d^3x^2} - \frac{31ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{12d^{7/2}} - \frac{5ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)^2}{4d^{7/2}} - \frac{5e(a+b\log(cx^n))}{6d^2(d+ex^2)^{3/2}} - \frac{a+b\log(cx^n)}{2dx^2(d+ex^2)^{3/2}} - \frac{5e(a+b\log(cx^n))}{2d^3\sqrt{d+ex^2}} + \frac{5e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(a+b\log(cx^n))}{2d^{7/2}} + \frac{5ben\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^2}}\right)}{2d^{7/2}} + \frac{5ben\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex^2}}{\sqrt{d}}}\right)}{4d^{7/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.19 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.67

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^2)^{5/2}}\,dx = \frac{bn\sqrt{1+\frac{d}{ex^2}}\left(5\,{}_3F_2\left(\frac{7}{2},\frac{7}{2},\frac{7}{2};\frac{9}{2},\frac{9}{2};-\frac{d}{ex^2}\right) - 7\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{7}{2},\frac{9}{2},-\frac{d}{ex^2}\right)(1+2}{98e^2x^6\sqrt{d+ex^2}} - \frac{(3d^2+20dex^2+15e^2x^4)(a-bn\log(x)+b\log(cx^n))}{6d^3x^2(d+ex^2)^{3/2}} - \frac{5e\log(x)(a-bn\log(x)+b\log(cx^n))}{2d^{7/2}} + \frac{5e(a-bn\log(x)+b\log(cx^n))\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)}{2d^{7/2}}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*(d + e*x^2)^(5/2)),x]
```

```
[Out] (b*n*Sqrt[1 + d/(e*x^2)]*(5*HypergeometricPFQ[{7/2, 7/2, 7/2}, {9/2, 9/2},
-(d/(e*x^2))] - 7*Hypergeometric2F1[5/2, 7/2, 9/2, -(d/(e*x^2))]*(1 + 2*Log
[x])))/(98*e^2*x^6*Sqrt[d + e*x^2]) - ((3*d^2 + 20*d*e*x^2 + 15*e^2*x^4)*(a
 - b*n*Log[x] + b*Log[c*x^n]))/(6*d^3*x^2*(d + e*x^2)^(3/2)) - (5*e*Log[x]*
(a - b*n*Log[x] + b*Log[c*x^n]))/(2*d^(7/2)) + (5*e*(a - b*n*Log[x] + b*Log
[c*x^n])*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/(2*d^(7/2))
```

Maple [F]

$$\int \frac{a + b \ln \left(c \,x^{n}\right)}{x^{3} \left(e \,x^{2}+d \right)^{\frac{5}{2}}}d x$$

```
[In] int((a+b*ln(c*x^n))/x^3/(e*x^2+d)^(5/2),x)
[Out] int((a+b*ln(c*x^n))/x^3/(e*x^2+d)^(5/2),x)
```

Fricas [F]

$$\int \frac{a + b \log \left(cx^{n}\right)}{x^{3} \left(d + ex^{2}\right)^{5/2}}\, dx = \int \frac{b \log \left(cx^{n}\right) + a}{{\left(ex^{2} + d\right)}^{\frac{5}{2}} x^{3}}\, dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral((sqrt(e*x^2 + d)*b*log(c*x^n) + sqrt(e*x^2 + d)*a)/(e^3*x^9 + 3*d*
e^2*x^7 + 3*d^2*e*x^5 + d^3*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \log \left(cx^{n}\right)}{x^{3} \left(d + ex^{2}\right)^{5/2}}\, dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*x**n))/x**3/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \log \left(cx^{n}\right)}{x^{3} \left(d + ex^{2}\right)^{5/2}}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + b\log(cx^n)}{x^3 (d + ex^2)^{5/2}} \, dx = \int \frac{b\log(cx^n) + a}{(ex^2 + d)^{\frac{5}{2}} x^3} \, dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)^(5/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{x^3 (d + ex^2)^{5/2}} \, dx = \int \frac{a + b\,\ln(c\,x^n)}{x^3\,(e\,x^2 + d)^{5/2}} \, dx$$

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*log(c*x^n))/(x^3*(d + e*x^2)^(5/2)), x)
```

3.303 $\int \frac{x^6(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 443

$$\begin{aligned}\int \frac{x^6(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx &= \frac{bdnx}{3e^3\sqrt{d+ex^2}}-\frac{bnx\sqrt{d+ex^2}}{4e^3}\\ &-\frac{31bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{12e^{7/2}\sqrt{d+ex^2}}-\frac{5bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4e^{7/2}\sqrt{d+ex^2}}\\ &+\frac{5bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{7/2}\sqrt{d+ex^2}}\\ &-\frac{x^5(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}}-\frac{5x^3(a+b\log(cx^n))}{3e^2\sqrt{d+ex^2}}+\frac{5x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e^3}\\ &-\frac{5d^{3/2}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{7/2}\sqrt{d+ex^2}}\\ &+\frac{5bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{4e^{7/2}\sqrt{d+ex^2}}\end{aligned}$$

```
[Out] -1/3*x^5*(a+b*ln(c*x^n))/e/(e*x^2+d)^(3/2)+5/6*b*d*n*x/e^3/(e*x^2+d)^(1/2)+
1/2*b*n*x^3/e^2/(e*x^2+d)^(1/2)-5/3*x^3*(a+b*ln(c*x^n))/e^2/(e*x^2+d)^(1/2)
-3/4*b*n*x*(e*x^2+d)^(1/2)/e^3+5/2*x*(a+b*ln(c*x^n))*(e*x^2+d)^(1/2)/e^3-31
/12*b*d^(3/2)*n*arcsinh(x*e^(1/2)/d^(1/2))*(1+e*x^2/d)^(1/2)/e^(7/2)/(e*x^2
+d)^(1/2)-5/4*b*d^(3/2)*n*arcsinh(x*e^(1/2)/d^(1/2))^2*(1+e*x^2/d)^(1/2)/e^
(7/2)/(e*x^2+d)^(1/2)-5*b*d^(3/2)*n*arcsinh(x*e^(1/2)/d^(1/2))*arctanh((x*e
^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*(1+e*x^2/d)^(1/2)/e^(7/2)/(e*x^2+d)^(1
/2)+5/2*b*d^(3/2)*n*arcsinh(x*e^(1/2)/d^(1/2))*ln(1+(x*e^(1/2)/d^(1/2)+(1+e
```

```
*x^2/d)^(1/2))^2)*(1+e*x^2/d)^(1/2)/e^(7/2)/(e*x^2+d)^(1/2)-5/2*d^(3/2)*arc
sinh(x*e^(1/2)/d^(1/2))*(a+b*ln(c*x^n))*(1+e*x^2/d)^(1/2)/e^(7/2)/(e*x^2+d)
^(1/2)+5/4*b*d^(3/2)*n*polylog(2,(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*(
1+e*x^2/d)^(1/2)/e^(7/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {2386, 294, 327, 221, 2392, 21, 1171, 396, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{x^6(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = -\frac{5d^{3/2}\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{7/2}\sqrt{d+ex^2}} + \frac{5x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e^3} - \frac{5x^3(a+b\log(cx^n))}{3e^2\sqrt{d+ex^2}} - \frac{x^5(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}} + \frac{5bd^{3/2}n\sqrt{\frac{ex^2}{d}+1}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{4e^{7/2}\sqrt{d+ex^2}} - \frac{5bd^{3/2}n\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4e^{7/2}\sqrt{d+ex^2}} - \frac{31bd^{3/2}n\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{12e^{7/2}\sqrt{d+ex^2}} + \frac{5bd^{3/2}n\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{7/2}\sqrt{d+ex^2}} - \frac{bnx\sqrt{d+ex^2}}{4e^3} + \frac{bdnx}{3e^3\sqrt{d+ex^2}}$$

```
[In] Int[(x^6*(a + b*Log[c*x^n]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*d*n*x)/(3*e^3*Sqrt[d + e*x^2]) - (b*n*x*Sqrt[d + e*x^2])/(4*e^3) - (31*b
*d^(3/2)*n*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])/(12*e^(7/2)*Sq
rt[d + e*x^2]) - (5*b*d^(3/2)*n*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqr
t[d]]^2)/(4*e^(7/2)*Sqrt[d + e*x^2]) + (5*b*d^(3/2)*n*Sqrt[1 + (e*x^2)/d]*A
rcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[1 - E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(2
*e^(7/2)*Sqrt[d + e*x^2]) - (x^5*(a + b*Log[c*x^n]))/(3*e*(d + e*x^2)^(3/2)
) - (5*x^3*(a + b*Log[c*x^n]))/(3*e^2*Sqrt[d + e*x^2]) + (5*x*Sqrt[d + e*x^
2]*(a + b*Log[c*x^n]))/(2*e^3) - (5*d^(3/2)*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sq
rt[e]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/(2*e^(7/2)*Sqrt[d + e*x^2]) + (5*b*d^
(3/2)*n*Sqrt[1 + (e*x^2)/d]*PolyLog[2, E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])
/(4*e^(7/2)*Sqrt[d + e*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2386

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^
(q_), x_Symbol] :> Dist[d^IntPart[q]*((d + e*x^2)^FracPart[q]/(1 + (e/d)*x^
2)^FracPart[q]), Int[x^m*(1 + (e/d)*x^2)^q*(a + b*Log[c*x^n]), x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && IntegerQ[m/2] && IntegerQ[q - 1/2] &&  !(LtQ[
m + 2*q, -2] || GtQ[d, 0])
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{ex^2}{d}}\int \frac{x^6(a+b\log(cx^n))}{\left(1+\frac{ex^2}{d}\right)^{5/2}}\,dx}{d^2\sqrt{d+ex^2}}$$

$$= -\frac{x^5(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}} - \frac{5x^3(a+b\log(cx^n))}{3e^2\sqrt{d+ex^2}} + \frac{5x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e^3} - \frac{5d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{7/2}\sqrt{d+ex^2}} - \frac{\left(bn\sqrt{1+\frac{ex^2}{d}}\right)\int\left(\frac{d^3\sqrt{1+\frac{ex^2}{d}}(15d^2+20dex^2+3e^2x^4)}{6e^3(d+ex^2)^2} - \frac{5d^{7/2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{7/2}x}\right)dx}{d^2\sqrt{d+ex^2}}$$

$$= -\frac{x^5(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}} - \frac{5x^3(a+b\log(cx^n))}{3e^2\sqrt{d+ex^2}} + \frac{5x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e^3} - \frac{5d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{7/2}\sqrt{d+ex^2}} + \frac{\left(5bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{2e^{7/2}\sqrt{d+ex^2}} - \frac{\left(bdn\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}(15d^2+20dex^2+3e^2x^4)}{(d+ex^2)^2}\,dx}{6e^3\sqrt{d+ex^2}}$$

$$= -\frac{x^5(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}} - \frac{5x^3(a+b\log(cx^n))}{3e^2\sqrt{d+ex^2}} + \frac{5x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e^3} - \frac{5d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{7/2}\sqrt{d+ex^2}} + \frac{\left(5bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{2e^{7/2}\sqrt{d+ex^2}} - \frac{\left(bn\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{15d^2+20dex^2+3e^2x^4}{\left(1+\frac{ex^2}{d}\right)^{3/2}}\,dx}{6de^3\sqrt{d+ex^2}}$$

$$= \frac{bdnx}{3e^3\sqrt{d+ex^2}} - \frac{5bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4e^{7/2}\sqrt{d+ex^2}} - \frac{x^5(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}} - \frac{5x^3(a+b\log(cx^n))}{3e^2\sqrt{d+ex^2}} + \frac{5x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e^3} - \frac{5d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{7/2}\sqrt{d+ex^2}} - \frac{\left(5bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int\frac{e^{2x}x}{1-e^{2x}}\,dx,x,\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{e^{7/2}\sqrt{d+ex^2}} + \frac{\left(bn\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{-17d^2-3dex^2}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{6de^3\sqrt{d+ex^2}}$$

$$= \frac{bdnx}{3e^3\sqrt{d+ex^2}} - \frac{bnx\sqrt{d+ex^2}}{4e^3} - \frac{5bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4e^{7/2}\sqrt{d+ex^2}} + \frac{5bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{7/2}\sqrt{d+ex^2}} - \frac{x^5(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}} - \frac{5x^3(a+b\log(cx^n))}{3e^2\sqrt{d+ex^2}} + \frac{5x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e^3} - \frac{5d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{7/2}\sqrt{d+ex^2}} - \frac{\left(5bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int\log\left(1-e^{2x}\right)\,dx,x,\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{2e^{7/2}\sqrt{d+ex^2}} - \frac{\left(31bdn\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{12e^3\sqrt{d+ex^2}}$$

$$= \frac{bdnx}{3e^3\sqrt{d+ex^2}} - \frac{bnx\sqrt{d+ex^2}}{4e^3} - \frac{31bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{12e^{7/2}\sqrt{d+ex^2}} - \frac{5bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4e^{7/2}\sqrt{d+ex^2}} + \frac{5bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{7/2}\sqrt{d+ex^2}} - \frac{x^5(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}} - \frac{5x^3(a+b\log(cx^n))}{3e^2\sqrt{d+ex^2}} + \frac{5x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e^3} - \frac{5d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{7/2}\sqrt{d+ex^2}} - \frac{\left(5bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{4e^{7/2}\sqrt{d+ex^2}}$$

$$= \frac{bdnx}{3e^3\sqrt{d+ex^2}} - \frac{bnx\sqrt{d+ex^2}}{4e^3} - \frac{31bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{12e^{7/2}\sqrt{d+ex^2}} - \frac{5bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{4e^{7/2}\sqrt{d+ex^2}} + \frac{5bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{7/2}\sqrt{d+ex^2}} - \frac{x^5(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}} - \frac{5x^3(a+b\log(cx^n))}{3e^2\sqrt{d+ex^2}} + \frac{5x\sqrt{d+ex^2}(a+b\log(cx^n))}{2e^3} - \frac{5d^{3/2}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{2e^{7/2}\sqrt{d+ex^2}} + \frac{5bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\text{Li}_2\left(e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{4e^{7/2}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.45

$$\int \frac{x^6(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \frac{bnx^7\sqrt{1+\frac{ex^2}{d}}\left(5\,{}_3F_2\left(\frac{7}{2},\frac{7}{2},\frac{7}{2};\frac{9}{2},\frac{9}{2};-\frac{ex^2}{d}\right)+7\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{7}{2},\frac{9}{2},-\frac{ex^2}{d}\right.\right.}{98d^2\sqrt{d+ex^2}}$$
$$+\frac{x(15d^2+20dex^2+3e^2x^4)\,(a-bn\log(x)+b\log(cx^n))}{6e^3\,(d+ex^2)^{3/2}}$$
$$-\frac{5d(a-bn\log(x)+b\log(cx^n))\log\left(ex+\sqrt{e}\sqrt{d+ex^2}\right)}{2e^{7/2}}$$

```
[In] Integrate[(x^6*(a + b*Log[c*x^n]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*n*x^7*Sqrt[1 + (e*x^2)/d]*(5*HypergeometricPFQ[{7/2, 7/2, 7/2}, {9/2, 9/
2}, -((e*x^2)/d)] + 7*Hypergeometric2F1[5/2, 7/2, 9/2, -((e*x^2)/d)]*(-1 +
2*Log[x])))/(98*d^2*Sqrt[d + e*x^2]) + (x*(15*d^2 + 20*d*e*x^2 + 3*e^2*x^4)
*(a - b*n*Log[x] + b*Log[c*x^n]))/(6*e^3*(d + e*x^2)^(3/2)) - (5*d*(a - b*n
*Log[x] + b*Log[c*x^n])*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(2*e^(7/2))
```

Maple [F]

$$\int \frac{x^6(a+b\ln(c\,x^n))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^6*(a+b*ln(c*x^n))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^6*(a+b*ln(c*x^n))/(e*x^2+d)^(5/2),x)
```

Fricas [F]

$$\int \frac{x^6(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\log(cx^n)+a)x^6}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^6*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(e*x^2 + d)*b*x^6*log(c*x^n) + sqrt(e*x^2 + d)*a*x^6)/(e^3*x^
6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**6*(a+b*ln(c*x**n))/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^6*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^6(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\log(cx^n)+a)x^6}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^6*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^6/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^6\,(a+b\,\ln(c\,x^n))}{(e\,x^2+d)^{5/2}}\,dx$$

```
[In] int((x^6*(a + b*log(c*x^n)))/(d + e*x^2)^(5/2),x)
[Out] int((x^6*(a + b*log(c*x^n)))/(d + e*x^2)^(5/2), x)
```

3.304 $\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 383

$$
\begin{aligned}
\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx &= -\frac{bnx}{3e^2\sqrt{d+ex^2}} \\
&+ \frac{4b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\,\mathrm{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{5/2}\sqrt{d+ex^2}} + \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\,\mathrm{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2e^{5/2}\sqrt{d+ex^2}} \\
&- \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\,\mathrm{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\mathrm{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{e^{5/2}\sqrt{d+ex^2}} - \frac{x^3(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}} \\
&- \frac{x(a+b\log(cx^n))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\,\mathrm{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{5/2}\sqrt{d+ex^2}} \\
&- \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\,\mathrm{PolyLog}\left(2,e^{2\mathrm{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{5/2}\sqrt{d+ex^2}}
\end{aligned}
$$

```
[Out] -1/3*x^3*(a+b*ln(c*x^n))/e/(e*x^2+d)^(3/2)-1/3*b*n*x/e^2/(e*x^2+d)^(1/2)-x*
(a+b*ln(c*x^n))/e^2/(e*x^2+d)^(1/2)+4/3*b*n*arcsinh(x*e^(1/2)/d^(1/2))*d^(1
/2)*(1+e*x^2/d)^(1/2)/e^(5/2)/(e*x^2+d)^(1/2)+1/2*b*n*arcsinh(x*e^(1/2)/d^(
1/2))^2*d^(1/2)*(1+e*x^2/d)^(1/2)/e^(5/2)/(e*x^2+d)^(1/2)-b*n*arcsinh(x*e^(
1/2)/d^(1/2))*ln(1-(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(1+e*x^
2/d)^(1/2)/e^(5/2)/(e*x^2+d)^(1/2)+arcsinh(x*e^(1/2)/d^(1/2))*(a+b*ln(c*x^n
))*d^(1/2)*(1+e*x^2/d)^(1/2)/e^(5/2)/(e*x^2+d)^(1/2)-1/2*b*n*polylog(2,(x*e
^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(1+e*x^2/d)^(1/2)/e^(5/2)/(e*x
^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2386, 294, 221, 2392, 21, 393, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \frac{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{5/2}\sqrt{d+ex^2}} - \frac{x(a+b\log(cx^n))}{e^2\sqrt{d+ex^2}} - \frac{x^3(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}} - \frac{b\sqrt{d}n\sqrt{\frac{ex^2}{d}+1}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{5/2}\sqrt{d+ex^2}} + \frac{b\sqrt{d}n\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2e^{5/2}\sqrt{d+ex^2}} + \frac{4b\sqrt{d}n\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{5/2}\sqrt{d+ex^2}} - \frac{b\sqrt{d}n\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{e^{5/2}\sqrt{d+ex^2}} - \frac{bnx}{3e^2\sqrt{d+ex^2}}$$

```
[In] Int[(x^4*(a + b*Log[c*x^n]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*n*x)/(e^2*Sqrt[d + e*x^2]) + (4*b*Sqrt[d]*n*Sqrt[1 + (e*x^2)/d]*Arc
Sinh[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(5/2)*Sqrt[d + e*x^2]) + (b*Sqrt[d]*n*Sqrt[
1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)/(2*e^(5/2)*Sqrt[d + e*x^2])
- (b*Sqrt[d]*n*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[1 - E^(
2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(e^(5/2)*Sqrt[d + e*x^2]) - (x^3*(a + b*L
og[c*x^n]))/(3*e*(d + e*x^2)^(3/2)) - (x*(a + b*Log[c*x^n]))/(e^2*Sqrt[d +
e*x^2]) + (Sqrt[d]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*(a + b*
Log[c*x^n]))/(e^(5/2)*Sqrt[d + e*x^2]) - (b*Sqrt[d]*n*Sqrt[1 + (e*x^2)/d]*P
olyLog[2, E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(2*e^(5/2)*Sqrt[d + e*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2386

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^
(q_), x_Symbol] :> Dist[d^IntPart[q]*((d + e*x^2)^FracPart[q]/(1 + (e/d)*x^
2)^FracPart[q]), Int[x^m*(1 + (e/d)*x^2)^q*(a + b*Log[c*x^n]), x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && IntegerQ[m/2] && IntegerQ[q - 1/2] &&  !(LtQ[
m + 2*q, -2] || GtQ[d, 0])
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{ex^2}{d}}\int \frac{x^4(a+b\log(cx^n))}{\left(1+\frac{ex^2}{d}\right)^{5/2}}\,dx}{d^2\sqrt{d+ex^2}}$$

$$= -\frac{x^3(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}} - \frac{x(a+b\log(cx^n))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{5/2}\sqrt{d+ex^2}} - \frac{\left(bn\sqrt{1+\frac{ex^2}{d}}\right)\int\left(-\frac{d^3(3d+4ex^2)\sqrt{1+\frac{ex^2}{d}}}{3e^2(d+ex^2)^2}+\frac{d^{5/2}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}x}\right)dx}{d^2\sqrt{d+ex^2}}$$

$$= -\frac{x^3(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}} - \frac{x(a+b\log(cx^n))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{5/2}\sqrt{d+ex^2}} - \frac{\left(b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{e^{5/2}\sqrt{d+ex^2}} + \frac{\left(bdn\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{(3d+4ex^2)\sqrt{1+\frac{ex^2}{d}}}{(d+ex^2)^2}\,dx}{3e^2\sqrt{d+ex^2}}$$

$$= -\frac{x^3(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}} - \frac{x(a+b\log(cx^n))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{5/2}\sqrt{d+ex^2}} - \frac{\left(b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{e^{5/2}\sqrt{d+ex^2}} + \frac{\left(bn\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{3d+4ex^2}{\left(1+\frac{ex^2}{d}\right)^{3/2}}\,dx}{3de^2\sqrt{d+ex^2}}$$

$$= -\frac{bnx}{3e^2\sqrt{d+ex^2}} + \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2e^{5/2}\sqrt{d+ex^2}} - \frac{x^3(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}} - \frac{x(a+b\log(cx^n))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{5/2}\sqrt{d+ex^2}} + \frac{\left(2b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int\frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{e^{5/2}\sqrt{d+ex^2}} + \frac{\left(4bn\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{3e^2\sqrt{d+ex^2}}$$

$$= -\frac{bnx}{3e^2\sqrt{d+ex^2}} + \frac{4b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{5/2}\sqrt{d+ex^2}} + \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2e^{5/2}\sqrt{d+ex^2}} - \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{e^{5/2}\sqrt{d+ex^2}} - \frac{x^3(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}} - \frac{x(a+b\log(cx^n))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{5/2}\sqrt{d+ex^2}} + \frac{\left(b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int\log\left(1-e^{2x}\right)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{e^{5/2}\sqrt{d+ex^2}}$$

$$= -\frac{bnx}{3e^2\sqrt{d+ex^2}} + \frac{4b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{5/2}\sqrt{d+ex^2}} + \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2e^{5/2}\sqrt{d+ex^2}} - \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{e^{5/2}\sqrt{d+ex^2}} - \frac{x^3(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}} - \frac{x(a+b\log(cx^n))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{5/2}\sqrt{d+ex^2}} + \frac{\left(b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{5/2}\sqrt{d+ex^2}}$$

$$= -\frac{bnx}{3e^2\sqrt{d+ex^2}} + \frac{4b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{5/2}\sqrt{d+ex^2}} + \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2e^{5/2}\sqrt{d+ex^2}} - \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{e^{5/2}\sqrt{d+ex^2}} - \frac{x^3(a+b\log(cx^n))}{3e(d+ex^2)^{3/2}} - \frac{x(a+b\log(cx^n))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\log(cx^n))}{e^{5/2}\sqrt{d+ex^2}} - \frac{b\sqrt{d}n\sqrt{1+\frac{ex^2}{d}}\text{Li}_2\left(e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2e^{5/2}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.59 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.64

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = -\frac{bn\sqrt{1+\frac{ex^2}{d}}\left(3e^{5/2}x^5(d+ex^2)^2\,{}_3F_2\left(\frac{5}{2},\frac{5}{2},\frac{5}{2};\frac{7}{2},\frac{7}{2};-\frac{ex^2}{d}\right)+25d^3\sqrt{e}x(3d+4ex^2)\sqrt{1+\frac{ex^2}{d}}\log(x)-75d^{5/2}(d}{75d^2e^{5/2}(d+ex^2)^{5/2}} - \frac{x(3d+4ex^2)(a-bn\log(x)+b\log(cx^n))}{3e^2(d+ex^2)^{3/2}} + \frac{(a-bn\log(x)+b\log(cx^n))\log\left(ex+\sqrt{e}\sqrt{d+ex^2}\right)}{e^{5/2}}$$

```
[In] Integrate[(x^4*(a + b*Log[c*x^n]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/75*(b*n*Sqrt[1 + (e*x^2)/d]*(3*e^(5/2)*x^5*(d + e*x^2)^2*HypergeometricP
FQ[{5/2, 5/2, 5/2}, {7/2, 7/2}, -((e*x^2)/d)] + 25*d^3*Sqrt[e]*x*(3*d + 4*e
*x^2)*Sqrt[1 + (e*x^2)/d]*Log[x] - 75*d^(5/2)*(d + e*x^2)^2*ArcSinh[(Sqrt[e
]*x)/Sqrt[d]]*Log[x]))/(d^2*e^(5/2)*(d + e*x^2)^(5/2)) - (x*(3*d + 4*e*x^2)
*(a - b*n*Log[x] + b*Log[c*x^n]))/(3*e^2*(d + e*x^2)^(3/2)) + ((a - b*n*Log
[x] + b*Log[c*x^n])*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/e^(5/2)
```

Maple [F]

$$\int \frac{x^4 (a + b \ln (c\, x^n))}{(e\, x^2 + d)^{\frac{5}{2}}} dx$$

```
[In] int(x^4*(a+b*ln(c*x^n))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^4*(a+b*ln(c*x^n))/(e*x^2+d)^(5/2),x)
```

Fricas [F]

$$\int \frac{x^4 (a + b \log (cx^n))}{(d + ex^2)^{5/2}} \, dx = \int \frac{(b \log (cx^n) + a) x^4}{(ex^2 + d)^{\frac{5}{2}}} \, dx$$

```
[In] integrate(x^4*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(e*x^2 + d)*b*x^4*log(c*x^n) + sqrt(e*x^2 + d)*a*x^4)/(e^3*x^
6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{x^4 (a + b \log (cx^n))}{(d + ex^2)^{5/2}} \, dx = \int \frac{x^4 (a + b \log (cx^n))}{(d + ex^2)^{\frac{5}{2}}} \, dx$$

```
[In] integrate(x**4*(a+b*ln(c*x**n))/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*log(c*x**n))/(d + e*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\log(cx^n)+a)x^4}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^4/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^4\,(a+b\,\ln(c\,x^n))}{(e\,x^2+d)^{5/2}}\,dx$$

```
[In] int((x^4*(a + b*log(c*x^n)))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^4*(a + b*log(c*x^n)))/(d + e*x^2)^(5/2), x)
```

3.305 $\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \frac{bnx}{3de\sqrt{d+ex^2}} - \frac{bn\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3de^{3/2}} + \frac{x^3(a+b\log(cx^n))}{3d(d+ex^2)^{3/2}}$$

```
[Out] -1/3*b*n*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/d/e^(3/2)+1/3*x^3*(a+b*ln(c*x^n
))/d/(e*x^2+d)^(3/2)+1/3*b*n*x/d/e/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2373, 294, 223, 212}

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \frac{x^3(a+b\log(cx^n))}{3d(d+ex^2)^{3/2}} - \frac{bn\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3de^{3/2}} + \frac{bnx}{3de\sqrt{d+ex^2}}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*n*x)/(3*d*e*Sqrt[d + e*x^2]) - (b*n*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
)/(3*d*e^(3/2)) + (x^3*(a + b*Log[c*x^n]))/(3*d*(d + e*x^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^3(a+b\log(cx^n))}{3d(d+ex^2)^{3/2}} - \frac{(bn)\int\frac{x^2}{(d+ex^2)^{3/2}}\,dx}{3d}\\
&= \frac{bnx}{3de\sqrt{d+ex^2}} + \frac{x^3(a+b\log(cx^n))}{3d(d+ex^2)^{3/2}} - \frac{(bn)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{3de}\\
&= \frac{bnx}{3de\sqrt{d+ex^2}} + \frac{x^3(a+b\log(cx^n))}{3d(d+ex^2)^{3/2}} - \frac{(bn)\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{3de}\\
&= \frac{bnx}{3de\sqrt{d+ex^2}} - \frac{bn\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3de^{3/2}} + \frac{x^3(a+b\log(cx^n))}{3d(d+ex^2)^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.13

$$\int\frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \frac{\sqrt{e}x(aex^2+bn(d+ex^2)) + be^{3/2}x^3\log(cx^n) - bn(d+ex^2)^{3/2}\log\left(ex+\sqrt{e}\sqrt{d+\right.}{3de^{3/2}(d+ex^2)^{3/2}}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (Sqrt[e]*x*(a*e*x^2 + b*n*(d + e*x^2)) + b*e^(3/2)*x^3*Log[c*x^n] - b*n*(d
+ e*x^2)^(3/2)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(3*d*e^(3/2)*(d + e*x^2)
^(3/2))
```

Maple [F]

$$\int \frac{x^{2}\left(a+b\ln\left(c\,x^{n}\right)\right)}{\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

```
[In] int(x^2*(a+b*ln(c*x^n))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^2*(a+b*ln(c*x^n))/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 277, normalized size of antiderivative = 3.11

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \left[\frac{(be^2nx^4+2\,bdenx^2+bd^2n)\sqrt{e}\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\,(be^2nx^3}{6\,(de^4x^4+2\,d^2e^3x^2+d}\right.$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*((b*e^2*n*x^4 + 2*b*d*e*n*x^2 + b*d^2*n)*sqrt(e)*log(-2*e*x^2 + 2*sqrt
(e*x^2 + d)*sqrt(e)*x - d) + 2*(b*e^2*n*x^3*log(x) + b*e^2*x^3*log(c) + b*d
*e*n*x + (b*e^2*n + a*e^2)*x^3)*sqrt(e*x^2 + d))/(d*e^4*x^4 + 2*d^2*e^3*x^2
 + d^3*e^2), 1/3*((b*e^2*n*x^4 + 2*b*d*e*n*x^2 + b*d^2*n)*sqrt(-e)*arctan(s
qrt(-e)*x/sqrt(e*x^2 + d)) + (b*e^2*n*x^3*log(x) + b*e^2*x^3*log(c) + b*d*e
*n*x + (b*e^2*n + a*e^2)*x^3)*sqrt(e*x^2 + d))/(d*e^4*x^4 + 2*d^2*e^3*x^2 +
 d^3*e^2)]
```

Sympy [F]

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^{2}\left(a+b\log\left(cx^{n}\right)\right)}{\left(d+ex^{2}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*log(c*x**n))/(d + e*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(x/((e*x^2 + d)^(3/2)*e) - x/(sqrt(e*x^2 + d)*d*e)) + b*integrate((x
^2*log(c) + x^2*log(x^n))/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,x^n))}{(e\,x^2+d)^{5/2}}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*log(c*x^n)))/(d + e*x^2)^(5/2), x)
```

3.306 $\int \frac{a+b\log(cx^n)}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{a+b\log(cx^n)}{(d+ex^2)^{5/2}}\,dx = -\frac{bnx}{3d^2\sqrt{d+ex^2}} - \frac{2bn\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}} + \frac{x(a+b\log(cx^n))}{3d(d+ex^2)^{3/2}} + \frac{2x(a+b\log(cx^n))}{3d^2\sqrt{d+ex^2}}$$

```
[Out] 1/3*x*(a+b*ln(c*x^n))/d/(e*x^2+d)^(3/2)-2/3*b*n*arctanh(x*e^(1/2)/(e*x^2+d)
^(1/2))/d^2/e^(1/2)-1/3*b*n*x/d^2/(e*x^2+d)^(1/2)+2/3*x*(a+b*ln(c*x^n))/d^2
/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2360, 2351, 223, 212, 197}

$$\int \frac{a+b\log(cx^n)}{(d+ex^2)^{5/2}}\,dx = \frac{2x(a+b\log(cx^n))}{3d^2\sqrt{d+ex^2}} + \frac{x(a+b\log(cx^n))}{3d(d+ex^2)^{3/2}} - \frac{2bn\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}} - \frac{bnx}{3d^2\sqrt{d+ex^2}}$$

```
[In] Int[(a + b*Log[c*x^n])/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*n*x)/(d^2*Sqrt[d + e*x^2]) - (2*b*n*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*
x^2]])/(3*d^2*Sqrt[e]) + (x*(a + b*Log[c*x^n]))/(3*d*(d + e*x^2)^(3/2)) + (
2*x*(a + b*Log[c*x^n]))/(3*d^2*Sqrt[d + e*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2360

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Sym
bol] :> Simp[(-x)*(d + e*x^2)^(q + 1)*((a + b*Log[c*x^n])/(2*d*(q + 1))), x
] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*Log[c*x^n
]), x], x] + Dist[b*(n/(2*d*(q + 1))), Int[(d + e*x^2)^(q + 1), x], x]) /;
FreeQ[{a, b, c, d, e, n}, x] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a+b\log(cx^n))}{3d(d+ex^2)^{3/2}}+\frac{2\int\frac{a+b\log(cx^n)}{(d+ex^2)^{3/2}}\,dx}{3d}-\frac{(bn)\int\frac{1}{(d+ex^2)^{3/2}}\,dx}{3d}\\
&= -\frac{bnx}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\log(cx^n))}{3d(d+ex^2)^{3/2}}+\frac{2x(a+b\log(cx^n))}{3d^2\sqrt{d+ex^2}}-\frac{(2bn)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{3d^2}\\
&= -\frac{bnx}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\log(cx^n))}{3d(d+ex^2)^{3/2}}+\frac{2x(a+b\log(cx^n))}{3d^2\sqrt{d+ex^2}}\\
&\quad-\frac{(2bn)\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{3d^2}\\
&= -\frac{bnx}{3d^2\sqrt{d+ex^2}}-\frac{2bn\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}}+\frac{x(a+b\log(cx^n))}{3d(d+ex^2)^{3/2}}+\frac{2x(a+b\log(cx^n))}{3d^2\sqrt{d+ex^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03

$$\int \frac{a+b\log(cx^n)}{(d+ex^2)^{5/2}}\,dx = \frac{\sqrt{e}x(-bn(d+ex^2)+a(3d+2ex^2))+b\sqrt{e}x(3d+2ex^2)\log(cx^n)-2bn(d+ex^2)^{3/2}}{3d^2\sqrt{e}\,(d+ex^2)^{3/2}}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*x^2)^(5/2),x]
```

```
[Out] (Sqrt[e]*x*(-(b*n*(d + e*x^2)) + a*(3*d + 2*e*x^2)) + b*Sqrt[e]*x*(3*d + 2*
e*x^2)*Log[c*x^n] - 2*b*n*(d + e*x^2)^(3/2)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^
2]])/(3*d^2*Sqrt[e]*(d + e*x^2)^(3/2))
```

Maple [F]

$$\int \frac{a+b\ln(c\,x^n)}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*ln(c*x^n))/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.98

$$\int \frac{a+b\log(cx^n)}{(d+ex^2)^{5/2}}\,dx = \left[\frac{(be^2nx^4+2\,bdenx^2+bd^2n)\sqrt{e}\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-((be^2n-2\,ae}{3\,(d^2e^3x^4}\right.$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*((b*e^2*n*x^4 + 2*b*d*e*n*x^2 + b*d^2*n)*sqrt(e)*log(-2*e*x^2 + 2*sqrt
(e*x^2 + d)*sqrt(e)*x - d) - ((b*e^2*n - 2*a*e^2)*x^3 + (b*d*e*n - 3*a*d*e)
*x - (2*b*e^2*x^3 + 3*b*d*e*x)*log(c) - (2*b*e^2*n*x^3 + 3*b*d*e*n*x)*log(x
))*sqrt(e*x^2 + d))/(d^2*e^3*x^4 + 2*d^3*e^2*x^2 + d^4*e), 1/3*(2*(b*e^2*n*
x^4 + 2*b*d*e*n*x^2 + b*d^2*n)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))
- ((b*e^2*n - 2*a*e^2)*x^3 + (b*d*e*n - 3*a*d*e)*x - (2*b*e^2*x^3 + 3*b*d*e
*x)*log(c) - (2*b*e^2*n*x^3 + 3*b*d*e*n*x)*log(x))*sqrt(e*x^2 + d))/(d^2*e^
3*x^4 + 2*d^3*e^2*x^2 + d^4*e)]
```

Sympy [F]

$$\int \frac{a+b\log(cx^n)}{(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\log(cx^n)}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/(e*x**2+d)**(5/2),x)
[Out] Integral((a + b*log(c*x**n))/(d + e*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{(d+ex^2)^{5/2}}\,dx = \int \frac{b\log(cx^n)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] 1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate((
log(c) + log(x^n))/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{(d+ex^2)^{5/2}}\,dx = \int \frac{b\log(cx^n)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{(e\,x^2+d)^{5/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(d + e*x^2)^(5/2),x)
[Out] int((a + b*log(c*x^n))/(d + e*x^2)^(5/2), x)
```

3.307 $\int \frac{a+b\log(cx^n)}{x^2\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 166

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^2\right)^{5/2}}\,dx=-\frac{bn}{d^2x\sqrt{d+ex^2}}-\frac{2benx}{3d^3\sqrt{d+ex^2}}+\frac{8b\sqrt{e}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d^3}-\frac{a+b\log\left(cx^n\right)}{dx\left(d+ex^2\right)^{3/2}}-\frac{4ex(a+b\log\left(cx^n\right))}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{8ex(a+b\log\left(cx^n\right))}{3d^3\sqrt{d+ex^2}}$$

```
[Out] (-a-b*ln(c*x^n))/d/x/(e*x^2+d)^(3/2)-4/3*e*x*(a+b*ln(c*x^n))/d^2/(e*x^2+d)^
(3/2)+8/3*b*n*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^(1/2)/d^3-b*n/d^2/x/(e*x
^2+d)^(1/2)-2/3*b*e*n*x/d^3/(e*x^2+d)^(1/2)-8/3*e*x*(a+b*ln(c*x^n))/d^3/(e*
x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {277, 198, 197, 2392, 12, 1279, 393, 223, 212}

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^2\right)^{5/2}}\,dx=-\frac{8ex(a+b\log\left(cx^n\right))}{3d^3\sqrt{d+ex^2}}-\frac{4ex(a+b\log\left(cx^n\right))}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\log\left(cx^n\right)}{dx\left(d+ex^2\right)^{3/2}}+\frac{8b\sqrt{e}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d^3}-\frac{2benx}{3d^3\sqrt{d+ex^2}}-\frac{bn}{d^2x\sqrt{d+ex^2}}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^2*(d + e*x^2)^(5/2)),x]
```

```
[Out] -((b*n)/(d^2*x*Sqrt[d + e*x^2])) - (2*b*e*n*x)/(3*d^3*Sqrt[d + e*x^2]) + (8
*b*Sqrt[e]*n*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(3*d^3) - (a + b*Log[c*x
```

```
^n])/(d*x*(d + e*x^2)^(3/2)) - (4*e*x*(a + b*Log[c*x^n]))/(3*d^2*(d + e*x^2
)^(3/2)) - (8*e*x*(a + b*Log[c*x^n]))/(3*d^3*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\log(cx^n)}{dx(d+ex^2)^{3/2}}-\frac{4ex(a+b\log(cx^n))}{3d^2(d+ex^2)^{3/2}} \\
&\quad -\frac{8ex(a+b\log(cx^n))}{3d^3\sqrt{d+ex^2}}-(bn)\int\frac{-3d^2-12dex^2-8e^2x^4}{3d^3x^2(d+ex^2)^{3/2}}\,dx \\
&= -\frac{a+b\log(cx^n)}{dx(d+ex^2)^{3/2}}-\frac{4ex(a+b\log(cx^n))}{3d^2(d+ex^2)^{3/2}}-\frac{8ex(a+b\log(cx^n))}{3d^3\sqrt{d+ex^2}}-\frac{(bn)\int\frac{-3d^2-12dex^2-8e^2x^4}{x^2(d+ex^2)^{3/2}}\,dx}{3d^3} \\
&= -\frac{bn}{d^2x\sqrt{d+ex^2}}-\frac{a+b\log(cx^n)}{dx(d+ex^2)^{3/2}}-\frac{4ex(a+b\log(cx^n))}{3d^2(d+ex^2)^{3/2}} \\
&\quad -\frac{8ex(a+b\log(cx^n))}{3d^3\sqrt{d+ex^2}}+\frac{(bn)\int\frac{6d^2e+8de^2x^2}{(d+ex^2)^{3/2}}\,dx}{3d^4} \\
&= -\frac{bn}{d^2x\sqrt{d+ex^2}}-\frac{2benx}{3d^3\sqrt{d+ex^2}}-\frac{a+b\log(cx^n)}{dx(d+ex^2)^{3/2}} \\
&\quad -\frac{4ex(a+b\log(cx^n))}{3d^2(d+ex^2)^{3/2}}-\frac{8ex(a+b\log(cx^n))}{3d^3\sqrt{d+ex^2}}+\frac{(8ben)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{3d^3} \\
&= -\frac{bn}{d^2x\sqrt{d+ex^2}}-\frac{2benx}{3d^3\sqrt{d+ex^2}}-\frac{a+b\log(cx^n)}{dx(d+ex^2)^{3/2}}-\frac{4ex(a+b\log(cx^n))}{3d^2(d+ex^2)^{3/2}} \\
&\quad -\frac{8ex(a+b\log(cx^n))}{3d^3\sqrt{d+ex^2}}+\frac{(8ben)\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{3d^3}
\end{aligned}$$

$$= -\frac{bn}{d^2x\sqrt{d+ex^2}} - \frac{2benx}{3d^3\sqrt{d+ex^2}} + \frac{8b\sqrt{e}n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d^3} - \frac{a+b\log(cx^n)}{dx(d+ex^2)^{3/2}} - \frac{4ex(a+b\log(cx^n))}{3d^2(d+ex^2)^{3/2}} - \frac{8ex(a+b\log(cx^n))}{3d^3\sqrt{d+ex^2}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^{5/2}}\,dx = \frac{-3ad^2-3bd^2n-12adex^2-5bdenx^2-8ae^2x^4-2be^2nx^4-b(3d^2+12dex^2+8e^2x^4)}{3d^3x(d+ex^2)^{3/2}}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*x^2)^(5/2)),x]
```

```
[Out] (-3*a*d^2 - 3*b*d^2*n - 12*a*d*e*x^2 - 5*b*d*e*n*x^2 - 8*a*e^2*x^4 - 2*b*e^
2*n*x^4 - b*(3*d^2 + 12*d*e*x^2 + 8*e^2*x^4)*Log[c*x^n] + 8*b*Sqrt[e]*n*x*(
d + e*x^2)^(3/2)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(3*d^3*x*(d + e*x^2)^(
3/2))
```

Maple [F]

$$\int \frac{a+b\ln(c\,x^n)}{x^2\,(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*ln(c*x^n))/x^2/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/x^2/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.40

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^2)^{5/2}}\,dx = \left[\frac{4\,(be^2nx^5+2\,bdenx^3+bd^2nx)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-(2\,(be^2n+4\right.$$

$$-\frac{8\,(be^2nx^5+2\,bdenx^3+bd^2nx)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+(2\,(be^2n+4\,ae^2)x^4+3\,bd^2n+3\,ad^2+(5\,bden+12}{3\,(d^3e^2x^5+2\,d^4ex}$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(4*(b*e^2*n*x^5 + 2*b*d*e*n*x^3 + b*d^2*n*x)*sqrt(e)*log(-2*e*x^2 - 2*
sqrt(e*x^2 + d)*sqrt(e)*x - d) - (2*(b*e^2*n + 4*a*e^2)*x^4 + 3*b*d^2*n + 3
```

```
*a*d^2 + (5*b*d*e*n + 12*a*d*e)*x^2 + (8*b*e^2*x^4 + 12*b*d*e*x^2 + 3*b*d^2
)*log(c) + (8*b*e^2*n*x^4 + 12*b*d*e*n*x^2 + 3*b*d^2*n)*log(x))*sqrt(e*x^2
+ d))/(d^3*e^2*x^5 + 2*d^4*e*x^3 + d^5*x), -1/3*(8*(b*e^2*n*x^5 + 2*b*d*e*n
*x^3 + b*d^2*n*x)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (2*(b*e^2*n
 + 4*a*e^2)*x^4 + 3*b*d^2*n + 3*a*d^2 + (5*b*d*e*n + 12*a*d*e)*x^2 + (8*b*e
^2*x^4 + 12*b*d*e*x^2 + 3*b*d^2)*log(c) + (8*b*e^2*n*x^4 + 12*b*d*e*n*x^2 +
 3*b*d^2*n)*log(x))*sqrt(e*x^2 + d))/(d^3*e^2*x^5 + 2*d^4*e*x^3 + d^5*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^2\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*x**n))/x**2/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^2\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^2\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\ln\left(c\,x^n\right)}{x^2\left(e\,x^2+d\right)^{5/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*x^2)^(5/2)), x)
```

3.308 $\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 230

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^{5/2}}\,dx = -\frac{be^2nx}{3d^4\sqrt{d+ex^2}}-\frac{bn\sqrt{d+ex^2}}{9d^3x^3}+\frac{23ben\sqrt{d+ex^2}}{9d^4x}-\frac{16be^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d^4}-\frac{a+b\log(cx^n)}{3dx^3(d+ex^2)^{3/2}}+\frac{2e(a+b\log(cx^n))}{d^2x(d+ex^2)^{3/2}}+\frac{8e^2x(a+b\log(cx^n))}{3d^3(d+ex^2)^{3/2}}+\frac{16e^2x(a+b\log(cx^n))}{3d^4\sqrt{d+ex^2}}$$

```
[Out] -16/3*b*e^(3/2)*n*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/d^4+1/3*(-a-b*ln(c*x^n
))/d/x^3/(e*x^2+d)^(3/2)+2*e*(a+b*ln(c*x^n))/d^2/x/(e*x^2+d)^(3/2)+8/3*e^2*
x*(a+b*ln(c*x^n))/d^3/(e*x^2+d)^(3/2)-1/3*b*e^2*n*x/d^4/(e*x^2+d)^(1/2)+16/
3*e^2*x*(a+b*ln(c*x^n))/d^4/(e*x^2+d)^(1/2)-1/9*b*n*(e*x^2+d)^(1/2)/d^3/x^3
+23/9*b*e*n*(e*x^2+d)^(1/2)/d^4/x
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {277, 198, 197, 2392, 12, 1819, 1279, 462, 223, 212}

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^{5/2}}\,dx = \frac{16e^2x(a+b\log(cx^n))}{3d^4\sqrt{d+ex^2}}+\frac{8e^2x(a+b\log(cx^n))}{3d^3(d+ex^2)^{3/2}}+\frac{2e(a+b\log(cx^n))}{d^2x(d+ex^2)^{3/2}}-\frac{a+b\log(cx^n)}{3dx^3(d+ex^2)^{3/2}}-\frac{16be^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d^4}-\frac{be^2nx}{3d^4\sqrt{d+ex^2}}+\frac{23ben\sqrt{d+ex^2}}{9d^4x}-\frac{bn\sqrt{d+ex^2}}{9d^3x^3}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^4*(d + e*x^2)^(5/2)),x]
```

```
[Out] -1/3*(b*e^2*n*x)/(d^4*Sqrt[d + e*x^2]) - (b*n*Sqrt[d + e*x^2])/(9*d^3*x^3)
+ (23*b*e*n*Sqrt[d + e*x^2])/(9*d^4*x) - (16*b*e^(3/2)*n*ArcTanh[(Sqrt[e]*x
)/Sqrt[d + e*x^2]])/(3*d^4) - (a + b*Log[c*x^n])/(3*d*x^3*(d + e*x^2)^(3/2)
) + (2*e*(a + b*Log[c*x^n]))/(d^2*x*(d + e*x^2)^(3/2)) + (8*e^2*x*(a + b*Lo
g[c*x^n]))/(3*d^3*(d + e*x^2)^(3/2)) + (16*e^2*x*(a + b*Log[c*x^n]))/(3*d^4
*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a+b\log(cx^n)}{3dx^3(d+ex^2)^{3/2}}+\frac{2e(a+b\log(cx^n))}{d^2x(d+ex^2)^{3/2}}+\frac{8e^2x(a+b\log(cx^n))}{3d^3(d+ex^2)^{3/2}}\\&\quad+\frac{16e^2x(a+b\log(cx^n))}{3d^4\sqrt{d+ex^2}}-(bn)\int\frac{-d^3+6d^2ex^2+24de^2x^4+16e^3x^6}{3d^4x^4(d+ex^2)^{3/2}}\,dx\\&= -\frac{a+b\log(cx^n)}{3dx^3(d+ex^2)^{3/2}}+\frac{2e(a+b\log(cx^n))}{d^2x(d+ex^2)^{3/2}}+\frac{8e^2x(a+b\log(cx^n))}{3d^3(d+ex^2)^{3/2}}\\&\quad+\frac{16e^2x(a+b\log(cx^n))}{3d^4\sqrt{d+ex^2}}-\frac{(bn)\int\frac{-d^3+6d^2ex^2+24de^2x^4+16e^3x^6}{x^4(d+ex^2)^{3/2}}\,dx}{3d^4}\end{aligned}$$

$$= -\frac{be^2nx}{3d^4\sqrt{d+ex^2}} - \frac{a+b\log(cx^n)}{3dx^3(d+ex^2)^{3/2}} + \frac{2e(a+b\log(cx^n))}{d^2x(d+ex^2)^{3/2}} + \frac{8e^2x(a+b\log(cx^n))}{3d^3(d+ex^2)^{3/2}} + \frac{16e^2x(a+b\log(cx^n))}{3d^4\sqrt{d+ex^2}} + \frac{(bn)\int \frac{d^3-7d^2ex^2-16de^2x^4}{x^4\sqrt{d+ex^2}}\,dx}{3d^5}$$

$$= -\frac{be^2nx}{3d^4\sqrt{d+ex^2}} - \frac{bn\sqrt{d+ex^2}}{9d^3x^3} - \frac{a+b\log(cx^n)}{3dx^3(d+ex^2)^{3/2}} + \frac{2e(a+b\log(cx^n))}{d^2x(d+ex^2)^{3/2}} + \frac{8e^2x(a+b\log(cx^n))}{3d^3(d+ex^2)^{3/2}} + \frac{16e^2x(a+b\log(cx^n))}{3d^4\sqrt{d+ex^2}} - \frac{(bn)\int \frac{23d^3e+48d^2e^2x^2}{x^2\sqrt{d+ex^2}}\,dx}{9d^6}$$

$$= -\frac{be^2nx}{3d^4\sqrt{d+ex^2}} - \frac{bn\sqrt{d+ex^2}}{9d^3x^3} + \frac{23ben\sqrt{d+ex^2}}{9d^4x} - \frac{a+b\log(cx^n)}{3dx^3(d+ex^2)^{3/2}} + \frac{2e(a+b\log(cx^n))}{d^2x(d+ex^2)^{3/2}} + \frac{8e^2x(a+b\log(cx^n))}{3d^3(d+ex^2)^{3/2}} + \frac{16e^2x(a+b\log(cx^n))}{3d^4\sqrt{d+ex^2}} - \frac{(16be^2n)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{3d^4}$$

$$= -\frac{be^2nx}{3d^4\sqrt{d+ex^2}} - \frac{bn\sqrt{d+ex^2}}{9d^3x^3} + \frac{23ben\sqrt{d+ex^2}}{9d^4x} - \frac{a+b\log(cx^n)}{3dx^3(d+ex^2)^{3/2}} + \frac{2e(a+b\log(cx^n))}{d^2x(d+ex^2)^{3/2}} + \frac{8e^2x(a+b\log(cx^n))}{3d^3(d+ex^2)^{3/2}} + \frac{16e^2x(a+b\log(cx^n))}{3d^4\sqrt{d+ex^2}} - \frac{(16be^2n)\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{3d^4}$$

$$= -\frac{be^2nx}{3d^4\sqrt{d+ex^2}} - \frac{bn\sqrt{d+ex^2}}{9d^3x^3} + \frac{23ben\sqrt{d+ex^2}}{9d^4x} - \frac{16be^{3/2}n\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3d^4} - \frac{a+b\log(cx^n)}{3dx^3(d+ex^2)^{3/2}} + \frac{2e(a+b\log(cx^n))}{d^2x(d+ex^2)^{3/2}} + \frac{8e^2x(a+b\log(cx^n))}{3d^3(d+ex^2)^{3/2}} + \frac{16e^2x(a+b\log(cx^n))}{3d^4\sqrt{d+ex^2}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.79

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^{5/2}}\,dx = \frac{-3ad^3 - bd^3n + 18ad^2ex^2 + 21bd^2enx^2 + 72ade^2x^4 + 42bde^2nx^4 + 48ae^3x^6 + 20be^3n}{9}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^4*(d + e*x^2)^(5/2)),x]
```

```
[Out] (-3*a*d^3 - b*d^3*n + 18*a*d^2*e*x^2 + 21*b*d^2*e*n*x^2 + 72*a*d*e^2*x^4 +
42*b*d*e^2*n*x^4 + 48*a*e^3*x^6 + 20*b*e^3*n*x^6 + 3*b*(-d^3 + 6*d^2*e*x^2
+ 24*d*e^2*x^4 + 16*e^3*x^6)*Log[c*x^n] - 48*b*e^(3/2)*n*x^3*(d + e*x^2)^(3
/2)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(9*d^4*x^3*(d + e*x^2)^(3/2))
```

Maple [F]

$$\int \frac{a+b\ln \left(c\,x^{n}\right)}{x^{4} \left(e\,x^{2}+d \right)^{\frac{5}{2}}}d x$$

```
[In] int((a+b*ln(c*x^n))/x^4/(e*x^2+d)^(5/2),x)
[Out] int((a+b*ln(c*x^n))/x^4/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 520, normalized size of antiderivative = 2.26

$$\int \frac{a+b\log\left(cx^n\right)}{x^4\left(d+ex^2\right)^{5/2}}\,dx = \left[\frac{24\left(be^3nx^7+2\,bde^2nx^5+bd^2enx^3\right)\sqrt{e}\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+(4\,(5\,b$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] [1/9*(24*(b*e^3*n*x^7 + 2*b*d*e^2*n*x^5 + b*d^2*e*n*x^3)*sqrt(e)*log(-2*e*x
^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + (4*(5*b*e^3*n + 12*a*e^3)*x^6 - b*d
^3*n + 6*(7*b*d*e^2*n + 12*a*d*e^2)*x^4 - 3*a*d^3 + 3*(7*b*d^2*e*n + 6*a*d^
2*e)*x^2 + 3*(16*b*e^3*x^6 + 24*b*d*e^2*x^4 + 6*b*d^2*e*x^2 - b*d^3)*log(c)
 + 3*(16*b*e^3*n*x^6 + 24*b*d*e^2*n*x^4 + 6*b*d^2*e*n*x^2 - b*d^3*n)*log(x)
)*sqrt(e*x^2 + d))/(d^4*e^2*x^7 + 2*d^5*e*x^5 + d^6*x^3), 1/9*(48*(b*e^3*n*
x^7 + 2*b*d*e^2*n*x^5 + b*d^2*e*n*x^3)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^
2 + d)) + (4*(5*b*e^3*n + 12*a*e^3)*x^6 - b*d^3*n + 6*(7*b*d*e^2*n + 12*a*d
*e^2)*x^4 - 3*a*d^3 + 3*(7*b*d^2*e*n + 6*a*d^2*e)*x^2 + 3*(16*b*e^3*x^6 + 2
4*b*d*e^2*x^4 + 6*b*d^2*e*x^2 - b*d^3)*log(c) + 3*(16*b*e^3*n*x^6 + 24*b*d*
e^2*n*x^4 + 6*b*d^2*e*n*x^2 - b*d^3*n)*log(x))*sqrt(e*x^2 + d))/(d^4*e^2*x^
7 + 2*d^5*e*x^5 + d^6*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x^4\left(d+ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*x**n))/x**4/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^{5/2}}\,dx = \int \frac{b\log(cx^n)+a}{(ex^2+d)^{\frac{5}{2}}x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x^2 + d)^(5/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^4(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^4\,(e\,x^2+d)^{5/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^4*(d + e*x^2)^(5/2)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^4*(d + e*x^2)^(5/2)), x)
```

3.309 $\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 251

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{2bd^2n(d^2-e^2x^2)}{3e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn(d^2-e^2x^2)^2}{9e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{2bd^4n\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{3e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{d^2(d^2-e^2x^2)\left(a+b\log\left(cx^n\right)\right)}{e^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{(d^2-e^2x^2)^2\left(a+b\log\left(cx^n\right)\right)}{3e^4\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] 2/3*b*d^2*n*(-e^2*x^2+d^2)/e^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/9*b*n*(-e^2*x
^2+d^2)^2/e^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-d^2*(-e^2*x^2+d^2)*(a+b*ln(c*x^n
))/e^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2)+1/3*(-e^2*x^2+d^2)^2*(a+b*ln(c*x^n))/e^
4/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-2/3*b*d^4*n*arctanh((1-e^2*x^2/d^2)^(1/2))*(
1-e^2*x^2/d^2)^(1/2)/e^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used

$= \{2387, 272, 45, 2392, 12, 457, 81, 52, 65, 214\}$

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{d^2(d^2-e^2x^2)(a+b\log(cx^n))}{e^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{(d^2-e^2x^2)^2(a+b\log(cx^n))}{3e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{2bd^4n\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{3e^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{2bd^2n(d^2-e^2x^2)}{3e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn(d^2-e^2x^2)^2}{9e^4\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n]))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (2*b*d^2*n*(d^2 - e^2*x^2))/(3*e^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - (b*n*(d^2
 - e^2*x^2)^2)/(9*e^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - (2*b*d^4*n*Sqrt[1 - (e
^2*x^2)/d^2]*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]])/(3*e^4*Sqrt[d - e*x]*Sqrt[d
+ e*x]) - (d^2*(d^2 - e^2*x^2)*(a + b*Log[c*x^n]))/(e^4*Sqrt[d - e*x]*Sqrt[
d + e*x]) + ((d^2 - e^2*x^2)^2*(a + b*Log[c*x^n]))/(3*e^4*Sqrt[d - e*x]*Sqr
t[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2387

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d1_) + (e1_.)*(x_))^
(q_)*((d2_) + (e2_.)*(x_))^(q_), x_Symbol] :> Dist[(d1 + e1*x)^q*((d2 + e2*
x)^q/(1 + e1*(e2/(d1*d2))*x^2)^q), Int[x^m*(1 + e1*(e2/(d1*d2))*x^2)^q*(a +
 b*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*
e1 + d1*e2, 0] && IntegerQ[m] && IntegerQ[q - 1/2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int\frac{x^3(a+b\log(cx^n))}{\sqrt{1-\frac{e^2x^2}{d^2}}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{d^2(d^2-e^2x^2)\,(a+b\log(cx^n))}{e^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{(d^2-e^2x^2)^2\,(a+b\log(cx^n))}{3e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(bn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int\frac{d^2(-2d^2-e^2x^2)\sqrt{1-\frac{e^2x^2}{d^2}}}{3e^4x}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{d^2(d^2-e^2x^2)\,(a+b\log(cx^n))}{e^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{(d^2-e^2x^2)^2\,(a+b\log(cx^n))}{3e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(bd^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int\frac{(-2d^2-e^2x^2)\sqrt{1-\frac{e^2x^2}{d^2}}}{x}\,dx}{3e^4\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{d^2(d^2-e^2x^2)\,(a+b\log(cx^n))}{e^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{(d^2-e^2x^2)^2\,(a+b\log(cx^n))}{3e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(bd^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int\frac{(-2d^2-e^2x)\sqrt{1-\frac{e^2x}{d^2}}}{x}\,dx,x,x^2\right)}{6e^4\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{bn(d^2-e^2x^2)^2}{9e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{d^2(d^2-e^2x^2)\,(a+b\log(cx^n))}{e^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{(d^2-e^2x^2)^2\,(a+b\log(cx^n))}{3e^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(bd^4n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{e^2x}{d^2}}}{x}\,dx,x,x^2\right)}{3e^4\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{2bd^2n(d^2-e^2x^2)}{3e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn(d^2-e^2x^2)^2}{9e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{d^2(d^2-e^2x^2)\,(a+b\log(cx^n))}{e^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{(d^2-e^2x^2)^2\,(a+b\log(cx^n))}{3e^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(bd^4n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{e^2x}{d^2}}}\,dx,x,x^2\right)}{3e^4\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{2bd^2n(d^2-e^2x^2)}{3e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn(d^2-e^2x^2)^2}{9e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{d^2(d^2-e^2x^2)\,(a+b\log(cx^n))}{e^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{(d^2-e^2x^2)^2\,(a+b\log(cx^n))}{3e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(2bd^6n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int\frac{1}{\frac{d^2}{e^2}-\frac{d^2x^2}{e^2}}\,dx,x,\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{3e^6\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{2bd^2n(d^2-e^2x^2)}{3e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn(d^2-e^2x^2)^2}{9e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{2bd^4n\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{3e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{d^2(d^2-e^2x^2)(a+b\log(cx^n))}{e^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{(d^2-e^2x^2)^2(a+b\log(cx^n))}{3e^4\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.65

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{-6bd^3n\log(x)+3bn\sqrt{d-ex}\sqrt{d+ex}(2d^2+e^2x^2)\log(x)+\sqrt{d-ex}\sqrt{d+ex}(e^2x^2(3a-bn-3bn\log(x}{9e^4}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/9*(-6*b*d^3*n*Log[x] + 3*b*n*Sqrt[d - e*x]*Sqrt[d + e*x]*(2*d^2 + e^2*x^
2)*Log[x] + Sqrt[d - e*x]*Sqrt[d + e*x]*(e^2*x^2*(3*a - b*n - 3*b*n*Log[x]
+ 3*b*Log[c*x^n]) + d^2*(6*a - 5*b*n - 6*b*n*Log[x] + 6*b*Log[c*x^n])) + 6*
b*d^3*n*Log[d + Sqrt[d - e*x]*Sqrt[d + e*x]])/e^4
```

Maple [F]

$$\int \frac{x^3(a+b\ln(c\,x^n))}{\sqrt{-ex+d}\,\sqrt{ex+d}}dx$$

```
[In] int(x^3*(a+b*ln(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*ln(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.50

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{6\,bd^3n\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)+(5\,bd^2n-6\,ad^2+(be^2n-3\,ae^2)x^2-3\,(be^2x^2+2\,bd^2)\log(c)-3\,(be^2nx^2+}{9\,e^4}$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/9*(6*b*d^3*n*log((sqrt(e*x + d)*sqrt(-e*x + d) - d)/x) + (5*b*d^2*n - 6*a
*d^2 + (b*e^2*n - 3*a*e^2)*x^2 - 3*(b*e^2*x^2 + 2*b*d^2)*log(c) - 3*(b*e^2*
n*x^2 + 2*b*d^2*n)*log(x))*sqrt(e*x + d)*sqrt(-e*x + d))/e^4
```

Sympy [F]

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx=\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*log(c*x**n))/(sqrt(d - e*x)*sqrt(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx=&\\ &-\frac{1}{9}bn\left(\frac{3\,d^3\log\left(d+\sqrt{-e^2x^2+d^2}\right)}{e^4}-\frac{3\,d^3\log\left(-d+\sqrt{-e^2x^2+d^2}\right)}{e^4}-\frac{6\sqrt{-e^2x^2+d^2}d^2-\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{e^4}\right.\\ &-\frac{1}{3}b\left(\frac{\sqrt{-e^2x^2+d^2}x^2}{e^2}+\frac{2\sqrt{-e^2x^2+d^2}d^2}{e^4}\right)\log\left(cx^n\right)\\ &-\frac{1}{3}a\left(\frac{\sqrt{-e^2x^2+d^2}x^2}{e^2}+\frac{2\sqrt{-e^2x^2+d^2}d^2}{e^4}\right)\end{aligned}$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] -1/9*b*n*(3*d^3*log(d + sqrt(-e^2*x^2 + d^2))/e^4 - 3*d^3*log(-d + sqrt(-e^
2*x^2 + d^2))/e^4 - (6*sqrt(-e^2*x^2 + d^2)*d^2 - (-e^2*x^2 + d^2)^(3/2))/e
^4) - 1/3*b*(sqrt(-e^2*x^2 + d^2)*x^2/e^2 + 2*sqrt(-e^2*x^2 + d^2)*d^2/e^4)
*log(c*x^n) - 1/3*a*(sqrt(-e^2*x^2 + d^2)*x^2/e^2 + 2*sqrt(-e^2*x^2 + d^2)*
d^2/e^4)
```

Giac [F]

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{\sqrt{ex+d}\sqrt{-ex+d}}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^3/(sqrt(e*x + d)*sqrt(-e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{x^3\,(a+b\,\ln(c\,x^n))}{\sqrt{d+e\,x}\,\sqrt{d-e\,x}}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n)))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] int((x^3*(a + b*log(c*x^n)))/((d + e*x)^(1/2)*(d - e*x)^(1/2)), x)
```

3.310 $\int \frac{x(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 148

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{bn(d^2-e^2x^2)}{e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{bd^2n\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)\,(a+b\log(cx^n))}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] b*n*(-e^2*x^2+d^2)/e^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-(-e^2*x^2+d^2)*(a+b*ln(
c*x^n))/e^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-b*d^2*n*arctanh((1-e^2*x^2/d^2)^(1
/2))*(1-e^2*x^2/d^2)^(1/2)/e^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2387, 2376, 272, 52, 65, 214}

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(d^2-e^2x^2)\,(a+b\log(cx^n))}{e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{bd^2n\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{bn(d^2-e^2x^2)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(x*(a + b*Log[c*x^n]))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (b*n*(d^2 - e^2*x^2))/(e^2*Sqrt[d - e*x]*Sqrt[d + e*x]) - (b*d^2*n*Sqrt[1 -
 (e^2*x^2)/d^2]*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]])/(e^2*Sqrt[d - e*x]*Sqrt[d
```

```
 + e*x]) - ((d^2 - e^2*x^2)*(a + b*Log[c*x^n]))/(e^2*Sqrt[d - e*x]*Sqrt[d +
 e*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rule 2387

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d1_) + (e1_.)*(x_))^
(q_)*((d2_) + (e2_.)*(x_))^(q_), x_Symbol] :> Dist[(d1 + e1*x)^q*((d2 + e2*
x)^q/(1 + e1*(e2/(d1*d2))*x^2)^q), Int[x^m*(1 + e1*(e2/(d1*d2))*x^2)^q*(a +
 b*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*
e1 + d1*e2, 0] && IntegerQ[m] && IntegerQ[q - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{e^2x^2}{d^2}} \int \frac{x(a+b\log(cx^n))}{\sqrt{1-\frac{e^2x^2}{d^2}}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{(d^2-e^2x^2)(a+b\log(cx^n))}{e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(bd^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{\sqrt{1-\frac{e^2x^2}{d^2}}}{x}\,dx}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{(d^2-e^2x^2)(a+b\log(cx^n))}{e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(bd^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{e^2x}{d^2}}}{x}\,dx, x, x^2\right)}{2e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{bn(d^2-e^2x^2)}{e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)(a+b\log(cx^n))}{e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(bd^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{e^2x}{d^2}}}\,dx, x, x^2\right)}{2e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{bn(d^2-e^2x^2)}{e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)(a+b\log(cx^n))}{e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(bd^4n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{d^2x^2}{e^2}}\,dx, x, \sqrt{1-\frac{e^2x^2}{d^2}}\right)}{e^4\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{bn(d^2-e^2x^2)}{e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{bd^2n\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)(a+b\log(cx^n))}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.76

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{bdn\log(x)}{e^2} - \frac{bn\sqrt{d-ex}\sqrt{d+ex}\log(x)}{e^2} - \frac{\sqrt{d-ex}\sqrt{d+ex}(a-bn+b(-n\log(x)+\log(cx^n)))}{e^2} - \frac{bdn\log\left(d+\sqrt{d-ex}\sqrt{d+ex}\right)}{e^2}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (b*d*n*Log[x])/e^2 - (b*n*Sqrt[d - e*x]*Sqrt[d + e*x]*Log[x])/e^2 - (Sqrt[d
 - e*x]*Sqrt[d + e*x]*(a - b*n + b*(-(n*Log[x]) + Log[c*x^n])))/e^2 - (b*d*
n*Log[d + Sqrt[d - e*x]*Sqrt[d + e*x]])/e^2
```

Maple [F]

$$\int \frac{x(a+b\ln(c\,x^{n}))}{\sqrt{-ex+d}\,\sqrt{ex+d}}dx$$

[In] int(x*(a+b*ln(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)

[Out] int(x*(a+b*ln(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.45

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{bdn\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)-(bn\log(x)-bn+b\log(c)+a)\sqrt{ex+d}\sqrt{-ex+d}}{e^2}$$

[In] integrate(x*(a+b*log(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] (b*d*n*log((sqrt(e*x + d)*sqrt(-e*x + d) - d)/x) - (b*n*log(x) - b*n + b*log(c) + a)*sqrt(e*x + d)*sqrt(-e*x + d))/e^2

Sympy [F]

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{x(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

[In] integrate(x*(a+b*ln(c*x**n))/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)

[Out] Integral(x*(a + b*log(c*x**n))/(sqrt(d - e*x)*sqrt(d + e*x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.71

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\left(d\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)-\sqrt{-e^2x^2+d^2}\right)bn}{e^2} - \frac{\sqrt{-e^2x^2+d^2}b\log(cx^n)}{e^2} - \frac{\sqrt{-e^2x^2+d^2}a}{e^2}$$

```
[In] integrate(x*(a+b*log(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] -(d*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - sqrt(-e^2*x^2 + d
^2))*b*n/e^2 - sqrt(-e^2*x^2 + d^2)*b*log(c*x^n)/e^2 - sqrt(-e^2*x^2 + d^2)
*a/e^2
```

Giac [F]

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx=\int \frac{(b\log(cx^n)+a)x}{\sqrt{ex+d}\sqrt{-ex+d}}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*log(c*x^n) + a)*x/(sqrt(e*x + d)*sqrt(-e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx=\int \frac{x\,(a+b\,\ln(c\,x^n))}{\sqrt{d+e\,x}\,\sqrt{d-e\,x}}\,dx$$

```
[In] int((x*(a + b*log(c*x^n)))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] int((x*(a + b*log(c*x^n)))/((d + e*x)^(1/2)*(d - e*x)^(1/2)), x)
```

3.311 $\int \frac{a+b\log(cx^n)}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 301

$$\int \frac{a+b\log(cx^n)}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{bn\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)^2}{2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)\log\left(\frac{2}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{PolyLog}\left(2,-\frac{1+\sqrt{1-\frac{e^2x^2}{d^2}}}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] 1/2*b*n*arctanh((1-e^2*x^2/d^2)^(1/2))^2*(1-e^2*x^2/d^2)^(1/2)/(-e*x+d)^(1/
2)/(e*x+d)^(1/2)-arctanh((1-e^2*x^2/d^2)^(1/2))*(a+b*ln(c*x^n))*(1-e^2*x^2/
d^2)^(1/2)/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-b*n*arctanh((1-e^2*x^2/d^2)^(1/2))*
ln(2/(1-(1-e^2*x^2/d^2)^(1/2)))*(1-e^2*x^2/d^2)^(1/2)/(-e*x+d)^(1/2)/(e*x+d
)^(1/2)-1/2*b*n*polylog(2,(-1-(1-e^2*x^2/d^2)^(1/2))/(1-(1-e^2*x^2/d^2)^(1/
2)))*(1-e^2*x^2/d^2)^(1/2)/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2387, 272, 65, 214, 2390, 6131, 6055, 2449, 2352}

$$\int \frac{a+b\log(cx^n)}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}} + \frac{bn\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)^2}{2\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)\log\left(\frac{2}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1-\frac{e^2x^2}{d^2}}+1}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (b*n*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]]^2)/(2*Sqrt[d
- e*x]*Sqrt[d + e*x]) - (Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqrt[1 - (e^2*x^2)
/d^2]]*(a + b*Log[c*x^n]))/(Sqrt[d - e*x]*Sqrt[d + e*x]) - (b*n*Sqrt[1 - (e
^2*x^2)/d^2]*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]]*Log[2/(1 - Sqrt[1 - (e^2*x^2)
/d^2])])/(Sqrt[d - e*x]*Sqrt[d + e*x]) - (b*n*Sqrt[1 - (e^2*x^2)/d^2]*PolyL
og[2, -((1 + Sqrt[1 - (e^2*x^2)/d^2])/(1 - Sqrt[1 - (e^2*x^2)/d^2]))])/(2*S
qrt[d - e*x]*Sqrt[d + e*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2387

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d1_) + (e1_.)*(x_))^
(q_)*((d2_) + (e2_.)*(x_))^(q_), x_Symbol] :> Dist[(d1 + e1*x)^q*((d2 + e2*
x)^q/(1 + e1*(e2/(d1*d2))*x^2)^q), Int[x^m*(1 + e1*(e2/(d1*d2))*x^2)^q*(a +
 b*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*
e1 + d1*e2, 0] && IntegerQ[m] && IntegerQ[q - 1/2]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{a+b\log(cx^n)}{x\sqrt{1-\frac{e^2x^2}{d^2}}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(bn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{x}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(bn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\sqrt{1-\frac{e^2x}{d^2}}\right)}{x}\,dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(bn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int \frac{x\tanh^{-1}(x)}{-1+x^2}\,dx, x, \sqrt{1-\frac{e^2x^2}{d^2}}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{bn\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)^2}{2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(bn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{1-x}\,dx, x, \sqrt{1-\frac{e^2x^2}{d^2}}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{bn\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)^2}{2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)\log\left(\frac{2}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(bn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-x}\right)}{1-x^2}\,dx, x, \sqrt{1-\frac{e^2x^2}{d^2}}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{bn\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)^2}{2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)\log\left(\frac{2}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(bn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{bn\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)^2}{2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)\log\left(\frac{2}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Li}_2\left(1-\frac{2}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.03

$$\int \frac{a+b\log(cx^n)}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\log(x)\left(a-bn\log(x)+b\log(cx^n)\right)}{d} - \frac{\left(a-bn\log(x)+b\log(cx^n)\right)\log\left(d+\sqrt{d-ex}\sqrt{d+ex}\right)}{d} + \frac{bn\sqrt{-d^2+e^2x^2}\left(-\frac{4\operatorname{arctanh}\left(\frac{\sqrt{-d^2+e^2x^2}}{\sqrt{-d^2}}\right)\left(2\log(x)-\log\left(\frac{e^2x^2}{d^2}\right)\right)}{\sqrt{-d^2}} + \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\left(\log^2\left(\frac{e^2x^2}{d^2}\right)-4\log\left(\frac{e^2x^2}{d^2}\right)\log\left(\frac{1}{2}\left(1+\sqrt{1-\frac{e^2x^2}{d^2}}\right.\right.\right.}{\sqrt{-}}\right.}{8\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (Log[x]*(a - b*n*Log[x] + b*Log[c*x^n]))/d - ((a - b*n*Log[x] + b*Log[c*x^n
])*Log[d + Sqrt[d - e*x]*Sqrt[d + e*x]])/d + (b*n*Sqrt[-d^2 + e^2*x^2]*((-4
*ArcTanh[Sqrt[-d^2 + e^2*x^2]/Sqrt[-d^2]]*(2*Log[x] - Log[(e^2*x^2)/d^2]))/
Sqrt[-d^2] + (Sqrt[1 - (e^2*x^2)/d^2]*(Log[(e^2*x^2)/d^2]^2 - 4*Log[(e^2*x^
2)/d^2]*Log[(1 + Sqrt[1 - (e^2*x^2)/d^2])/2] + 2*Log[(1 + Sqrt[1 - (e^2*x^2
)/d^2])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[1 - (e^2*x^2)/d^2]/2]))/Sqrt[-d^2 +
e^2*x^2]))/(8*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Maple [F]

$$\int \frac{a+b\ln(c\,x^n)}{x\sqrt{-ex+d}\,\sqrt{ex+d}}dx$$

```
[In] int((a+b*ln(c*x^n))/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{b\log(cx^n)+a}{\sqrt{ex+d}\sqrt{-ex+d}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-(sqrt(e*x + d)*sqrt(-e*x + d)*b*log(c*x^n) + sqrt(e*x + d)*sqrt(-
e*x + d)*a)/(e^2*x^3 - d^2*x), x)
```

Sympy [F]

$$\int \frac{a + b\log(cx^n)}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{a + b\log(cx^n)}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
[Out] Integral((a + b*log(c*x**n))/(x*sqrt(d - e*x)*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{a + b\log(cx^n)}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{b\log(cx^n) + a}{\sqrt{ex+d}\sqrt{-ex+d}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="max
ima")
[Out] b*integrate((log(c) + log(x^n))/(sqrt(e*x + d)*sqrt(-e*x + d)*x), x) - a*lo
g(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d
```

Giac [F]

$$\int \frac{a + b\log(cx^n)}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{b\log(cx^n) + a}{\sqrt{ex+d}\sqrt{-ex+d}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gia
c")
[Out] integrate((b*log(c*x^n) + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{a + b\ln(c\,x^n)}{x\,\sqrt{d+e\,x}\,\sqrt{d-e\,x}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
[Out] int((a + b*log(c*x^n))/(x*(d + e*x)^(1/2)*(d - e*x)^(1/2)), x)
```

3.312 $\int \frac{a+b\log(cx^n)}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 489

$$\begin{aligned}\int \frac{a+b\log(cx^n)}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx =& -\frac{bn(d^2-e^2x^2)}{4d^2x^2\sqrt{d-ex}\sqrt{d+ex}} \\ &+\frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{4d^2\sqrt{d-ex}\sqrt{d+ex}} \\ &+\frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)^2}{4d^2\sqrt{d-ex}\sqrt{d+ex}} \\ &-\frac{(d^2-e^2x^2)(a+b\log(cx^n))}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} \\ &-\frac{e^2\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)(a+b\log(cx^n))}{2d^2\sqrt{d-ex}\sqrt{d+ex}} \\ &-\frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)\log\left(\frac{2}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{2d^2\sqrt{d-ex}\sqrt{d+ex}} \\ &-\frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{PolyLog}\left(2,-\frac{1+\sqrt{1-\frac{e^2x^2}{d^2}}}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{4d^2\sqrt{d-ex}\sqrt{d+ex}}\end{aligned}$$

```
[Out] -1/4*b*n*(-e^2*x^2+d^2)/d^2/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/2*(-e^2*x^2+
d^2)*(a+b*ln(c*x^n))/d^2/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2)+1/4*b*e^2*n*arcta
nh((1-e^2*x^2/d^2)^(1/2))*(1-e^2*x^2/d^2)^(1/2)/d^2/(-e*x+d)^(1/2)/(e*x+d)^
(1/2)+1/4*b*e^2*n*arctanh((1-e^2*x^2/d^2)^(1/2))^2*(1-e^2*x^2/d^2)^(1/2)/d^
2/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/2*e^2*arctanh((1-e^2*x^2/d^2)^(1/2))*(a+b*
```

```
ln(c*x^n))*(1-e^2*x^2/d^2)^(1/2)/d^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/2*b*e^2
*n*arctanh((1-e^2*x^2/d^2)^(1/2))*ln(2/(1-(1-e^2*x^2/d^2)^(1/2)))*(1-e^2*x^
2/d^2)^(1/2)/d^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/4*b*e^2*n*polylog(2,(-1-(1-
e^2*x^2/d^2)^(1/2))/(1-(1-e^2*x^2/d^2)^(1/2)))*(1-e^2*x^2/d^2)^(1/2)/d^2/(-
e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2387, 272, 44, 65, 214, 2392, 43, 6131, 6055, 2449, 2352}

$$\int \frac{a+b\log(cx^n)}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{e^2\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)(a+b\log(cx^n))}{2d^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)(a+b\log(cx^n))}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)^2}{4d^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{4d^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{arctanh}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)\log\left(\frac{2}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{2d^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1-\frac{e^2x^2}{d^2}}+1}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{4d^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn(d^2-e^2x^2)}{4d^2x^2\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^3*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/4*(b*n*(d^2 - e^2*x^2))/(d^2*x^2*Sqrt[d - e*x]*Sqrt[d + e*x]) + (b*e^2*n
*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]])/(4*d^2*Sqrt[d -
e*x]*Sqrt[d + e*x]) + (b*e^2*n*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqrt[1 - (e^
2*x^2)/d^2]]^2)/(4*d^2*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((d^2 - e^2*x^2)*(a +
 b*Log[c*x^n]))/(2*d^2*x^2*Sqrt[d - e*x]*Sqrt[d + e*x]) - (e^2*Sqrt[1 - (e^
2*x^2)/d^2]*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]]*(a + b*Log[c*x^n]))/(2*d^2*Sqr
t[d - e*x]*Sqrt[d + e*x]) - (b*e^2*n*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqrt[1
 - (e^2*x^2)/d^2]]*Log[2/(1 - Sqrt[1 - (e^2*x^2)/d^2])])/(2*d^2*Sqrt[d - e*
x]*Sqrt[d + e*x]) - (b*e^2*n*Sqrt[1 - (e^2*x^2)/d^2]*PolyLog[2, -((1 + Sqrt
```

```
[1 - (e^2*x^2)/d^2])/(1 - Sqrt[1 - (e^2*x^2)/d^2]))])/(4*d^2*Sqrt[d - e*x]*
Sqrt[d + e*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2387

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d1_) + (e1_.)*(x_))^
(q_)*((d2_) + (e2_.)*(x_))^(q_), x_Symbol] :> Dist[(d1 + e1*x)^q*((d2 + e2*
x)^q/(1 + e1*(e2/(d1*d2))*x^2)^q), Int[x^m*(1 + e1*(e2/(d1*d2))*x^2)^q*(a +
```

```
 b*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*
e1 + d1*e2, 0] && IntegerQ[m] && IntegerQ[q - 1/2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{a+b\log(cx^n)}{x^3\sqrt{1-\frac{e^2x^2}{d^2}}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{(d^2-e^2x^2)\,(a+b\log(cx^n))}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{e^2\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)(a+b\log(cx^n))}{2d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$-\frac{\left(bn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int\left(-\frac{\sqrt{1-\frac{e^2x^2}{d^2}}}{2x^3}-\frac{e^2\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{2d^2x}\right)dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{(d^2-e^2x^2)(a+b\log(cx^n))}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{e^2\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)(a+b\log(cx^n))}{2d^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(bn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int\frac{\sqrt{1-\frac{e^2x^2}{d^2}}}{x^3}\,dx}{2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int\frac{\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{x}\,dx}{2d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{(d^2-e^2x^2)(a+b\log(cx^n))}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{e^2\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)(a+b\log(cx^n))}{2d^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(bn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{e^2x}{d^2}}}{x^2}\,dx,x,x^2\right)}{4\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\sqrt{1-\frac{e^2x}{d^2}}\right)}{x}\,dx,x,x^2\right)}{4d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{bn(d^2-e^2x^2)}{4d^2x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)(a+b\log(cx^n))}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{e^2\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)(a+b\log(cx^n))}{2d^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{e^2x}{d^2}}}\,dx,x,x^2\right)}{8d^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int\frac{x\tanh^{-1}(x)}{-1+x^2}\,dx,x,\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{2d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{bn(d^2-e^2x^2)}{4d^2x^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)^2}{4d^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)\left(a+b\log\left(cx^n\right)\right)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{e^2\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)\left(a+b\log\left(cx^n\right)\right)}{2d^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(bn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int\frac{1}{\frac{d^2}{e^2}-\frac{d^2x^2}{e^2}}\,dx, x, \sqrt{1-\frac{e^2x^2}{d^2}}\right)}{4\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int\frac{\tanh^{-1}(x)}{1-x}\,dx, x, \sqrt{1-\frac{e^2x^2}{d^2}}\right)}{2d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{bn(d^2-e^2x^2)}{4d^2x^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{4d^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)^2}{4d^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)\left(a+b\log\left(cx^n\right)\right)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{e^2\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)\left(a+b\log\left(cx^n\right)\right)}{2d^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)\log\left(\frac{2}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{2d^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-x}\right)}{1-x^2}\,dx, x, \sqrt{1-\frac{e^2x^2}{d^2}}\right)}{2d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{bn(d^2-e^2x^2)}{4d^2x^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{4d^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)^2}{4d^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)(a+b\log(cx^n))}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{e^2\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)(a+b\log(cx^n))}{2d^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)\log\left(\frac{2}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{2d^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{2d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{bn(d^2-e^2x^2)}{4d^2x^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{4d^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)^2}{4d^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)(a+b\log(cx^n))}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{e^2\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)(a+b\log(cx^n))}{2d^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)\log\left(\frac{2}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{2d^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\text{Li}_2\left(1-\frac{2}{1-\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{4d^2\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.64 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.52

$$\int\frac{a+b\log(cx^n)}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

$$= \frac{\frac{bn(-d^2+e^2x^2)\left(2d^3\,{}_3F_2\left(\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{5}{2},\frac{5}{2};\frac{d^2}{e^2x^2}\right)+9e^2x^2\left(d\sqrt{1-\frac{d^2}{e^2x^2}}-ex\arcsin\left(\frac{d}{ex}\right)\right)(1+2\log(x))\right)}{e^2\sqrt{1-\frac{d^2}{e^2x^2}}x^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{18d\sqrt{d-ex}\sqrt{d+ex}(a-bn\log(x)+b\log(cx^n))}{x^2}}{36}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] ((b*n*(-d^2 + e^2*x^2)*(2*d^3*HypergeometricPFQ[{3/2, 3/2, 3/2}, {5/2, 5/2}
, d^2/(e^2*x^2)] + 9*e^2*x^2*(d*Sqrt[1 - d^2/(e^2*x^2)] - e*x*ArcSin[d/(e*x
)])*(1 + 2*Log[x])))/(e^2*Sqrt[1 - d^2/(e^2*x^2)]*x^4*Sqrt[d - e*x]*Sqrt[d
+ e*x]) - (18*d*Sqrt[d - e*x]*Sqrt[d + e*x]*(a - b*n*Log[x] + b*Log[c*x^n])
)/x^2 + 18*e^2*Log[x]*(a - b*n*Log[x] + b*Log[c*x^n]) - 18*e^2*(a - b*n*Log
[x] + b*Log[c*x^n])*Log[d + Sqrt[d - e*x]*Sqrt[d + e*x]])/(36*d^3)
```

Maple [F]

$$\int \frac{a+b\ln\left(c\,x^{n}\right)}{x^{3}\sqrt{-ex+d}\,\sqrt{ex+d}}dx$$

```
[In] int((a+b*ln(c*x^n))/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{3}\sqrt{d-ex}\sqrt{d+ex}}\,dx=\int \frac{b\log\left(cx^{n}\right)+a}{\sqrt{ex+d}\sqrt{-ex+d}x^{3}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-(sqrt(e*x + d)*sqrt(-e*x + d)*b*log(c*x^n) + sqrt(e*x + d)*sqrt(-
e*x + d)*a)/(e^2*x^5 - d^2*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^{n}\right)}{x^{3}\sqrt{d-ex}\sqrt{d+ex}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*x**n))/x**3/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{b\log(cx^n)+a}{\sqrt{ex+d}\sqrt{-ex+d}x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] -1/2*a*(e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 + sqrt(
-e^2*x^2 + d^2)/(d^2*x^2)) + b*integrate((log(c) + log(x^n))/(sqrt(e*x + d)
*sqrt(-e*x + d)*x^3), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{b\log(cx^n)+a}{\sqrt{ex+d}\sqrt{-ex+d}x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*log(c*x^n) + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^3\,\sqrt{d+e\,x}\,\sqrt{d-e\,x}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^3*(d + e*x)^(1/2)*(d - e*x)^(1/2)), x)
```

3.313 $\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 406

$$\begin{aligned}\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx =\ & \frac{bnx(d^2-e^2x^2)}{4e^2\sqrt{d-ex}\sqrt{d+ex}}+\frac{bd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)}{4e^3\sqrt{d-ex}\sqrt{d+ex}} \\ & +\frac{ibd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)^2}{4e^3\sqrt{d-ex}\sqrt{d+ex}} \\ & -\frac{bd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)\log\left(1-e^{2i\arcsin\left(\frac{ex}{d}\right)}\right)}{2e^3\sqrt{d-ex}\sqrt{d+ex}} \\ & -\frac{x(d^2-e^2x^2)(a+b\log(cx^n))}{2e^2\sqrt{d-ex}\sqrt{d+ex}} \\ & +\frac{d^3\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)(a+b\log(cx^n))}{2e^3\sqrt{d-ex}\sqrt{d+ex}} \\ & +\frac{ibd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{PolyLog}\left(2,e^{2i\arcsin\left(\frac{ex}{d}\right)}\right)}{4e^3\sqrt{d-ex}\sqrt{d+ex}}\end{aligned}$$

```
[Out] 1/4*b*n*x*(-e^2*x^2+d^2)/e^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/2*x*(-e^2*x^2+d
^2)*(a+b*ln(c*x^n))/e^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2)+1/4*b*d^3*n*arcsin(e*x
/d)*(1-e^2*x^2/d^2)^(1/2)/e^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)+1/4*I*b*d^3*n*ar
csin(e*x/d)^2*(1-e^2*x^2/d^2)^(1/2)/e^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/2*b*
d^3*n*arcsin(e*x/d)*ln(1-(I*e*x/d+(1-e^2*x^2/d^2)^(1/2))^2)*(1-e^2*x^2/d^2)
^(1/2)/e^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)+1/2*d^3*arcsin(e*x/d)*(a+b*ln(c*x^n
))*(1-e^2*x^2/d^2)^(1/2)/e^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)+1/4*I*b*d^3*n*pol
ylog(2,(I*e*x/d+(1-e^2*x^2/d^2)^(1/2))^2)*(1-e^2*x^2/d^2)^(1/2)/e^3/(-e*x+d
)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2387, 327, 222, 2392, 12, 14, 201, 4721, 3798, 2221, 2317, 2438}

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{d^3\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)(a+b\log(cx^n))}{2e^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{x(d^2-e^2x^2)(a+b\log(cx^n))}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{ibd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{PolyLog}\left(2,e^{2i\arcsin\left(\frac{ex}{d}\right)}\right)}{4e^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{ibd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)^2}{4e^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{bd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)}{4e^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{bd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)\log\left(1-e^{2i\arcsin\left(\frac{ex}{d}\right)}\right)}{2e^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{bnx(d^2-e^2x^2)}{4e^2\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (b*n*x*(d^2 - e^2*x^2))/(4*e^2*Sqrt[d - e*x]*Sqrt[d + e*x]) + (b*d^3*n*Sqrt
[1 - (e^2*x^2)/d^2]*ArcSin[(e*x)/d])/(4*e^3*Sqrt[d - e*x]*Sqrt[d + e*x]) +
((I/4)*b*d^3*n*Sqrt[1 - (e^2*x^2)/d^2]*ArcSin[(e*x)/d]^2)/(e^3*Sqrt[d - e*x
]*Sqrt[d + e*x]) - (b*d^3*n*Sqrt[1 - (e^2*x^2)/d^2]*ArcSin[(e*x)/d]*Log[1 -
 E^((2*I)*ArcSin[(e*x)/d])])/(2*e^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - (x*(d^2
- e^2*x^2)*(a + b*Log[c*x^n]))/(2*e^2*Sqrt[d - e*x]*Sqrt[d + e*x]) + (d^3*S
qrt[1 - (e^2*x^2)/d^2]*ArcSin[(e*x)/d]*(a + b*Log[c*x^n]))/(2*e^3*Sqrt[d -
e*x]*Sqrt[d + e*x]) + ((I/4)*b*d^3*n*Sqrt[1 - (e^2*x^2)/d^2]*PolyLog[2, E^(
(2*I)*ArcSin[(e*x)/d])])/(e^3*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2387

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d1_) + (e1_.)*(x_))^
(q_)*((d2_) + (e2_.)*(x_))^(q_), x_Symbol] :> Dist[(d1 + e1*x)^q*((d2 + e2*
x)^q/(1 + e1*(e2/(d1*d2))*x^2)^q), Int[x^m*(1 + e1*(e2/(d1*d2))*x^2)^q*(a +
 b*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*
e1 + d1*e2, 0] && IntegerQ[m] && IntegerQ[q - 1/2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
```

```
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{x^2(a+b\log(cx^n))}{\sqrt{1-\frac{e^2x^2}{d^2}}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{x(d^2-e^2x^2)\left(a+b\log\left(cx^n\right)\right)}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{d^3\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)\left(a+b\log\left(cx^n\right)\right)}{2e^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(bn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{d^2\left(-ex\sqrt{\frac{d^2-e^2x^2}{d^2}}+d\sin^{-1}\left(\frac{ex}{d}\right)\right)}{2e^3x}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{x(d^2-e^2x^2)\left(a+b\log\left(cx^n\right)\right)}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{d^3\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)\left(a+b\log\left(cx^n\right)\right)}{2e^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(bd^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{-ex\sqrt{\frac{d^2-e^2x^2}{d^2}}+d\sin^{-1}\left(\frac{ex}{d}\right)}{x}\,dx}{2e^3\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{x(d^2-e^2x^2)\left(a+b\log\left(cx^n\right)\right)}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{d^3\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)\left(a+b\log\left(cx^n\right)\right)}{2e^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(bd^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \left(-e\sqrt{1-\frac{e^2x^2}{d^2}}+\frac{d\sin^{-1}\left(\frac{ex}{d}\right)}{x}\right)dx}{2e^3\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{x(d^2-e^2x^2)\,(a+b\log(cx^n))}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{d^3\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)(a+b\log(cx^n))}{2e^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(bd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int\frac{\sin^{-1}\left(\frac{ex}{d}\right)}{x}\,dx}{2e^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(bd^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int\sqrt{1-\frac{e^2x^2}{d^2}}\,dx}{2e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{bnx(d^2-e^2x^2)}{4e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{x(d^2-e^2x^2)\,(a+b\log(cx^n))}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{d^3\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)(a+b\log(cx^n))}{2e^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(bd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int x\cot(x)\,dx, x, \sin^{-1}\left(\frac{ex}{d}\right)\right)}{2e^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(bd^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int\frac{1}{\sqrt{1-\frac{e^2x^2}{d^2}}}\,dx}{4e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{bnx(d^2-e^2x^2)}{4e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{bd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)}{4e^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{ibd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)^2}{4e^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{x(d^2-e^2x^2)\,(a+b\log(cx^n))}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{d^3\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)(a+b\log(cx^n))}{2e^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(ibd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int\frac{e^{2ix}x}{1-e^{2ix}}\,dx, x, \sin^{-1}\left(\frac{ex}{d}\right)\right)}{e^3\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{bnx(d^2-e^2x^2)}{4e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{bd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)}{4e^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{ibd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)^2}{4e^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{bd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)\log\left(1-e^{2i\sin^{-1}\left(\frac{ex}{d}\right)}\right)}{2e^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{x(d^2-e^2x^2)\,(a+b\log(cx^n))}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{d^3\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)(a+b\log(cx^n))}{2e^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(bd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int\log\left(1-e^{2ix}\right)\,dx, x, \sin^{-1}\left(\frac{ex}{d}\right)\right)}{2e^3\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{bnx(d^2-e^2x^2)}{4e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{bd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)}{4e^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{ibd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)^2}{4e^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{bd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)\log\left(1-e^{2i\sin^{-1}\left(\frac{ex}{d}\right)}\right)}{2e^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{x(d^2-e^2x^2)(a+b\log(cx^n))}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{d^3\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)(a+b\log(cx^n))}{2e^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(ibd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2i\sin^{-1}\left(\frac{ex}{d}\right)}\right)}{4e^3\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{bnx(d^2-e^2x^2)}{4e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{bd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)}{4e^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{ibd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)^2}{4e^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{bd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)\log\left(1-e^{2i\sin^{-1}\left(\frac{ex}{d}\right)}\right)}{2e^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{x(d^2-e^2x^2)(a+b\log(cx^n))}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{d^3\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)(a+b\log(cx^n))}{2e^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{ibd^3n\sqrt{1-\frac{e^2x^2}{d^2}}\text{Li}_2\left(e^{2i\sin^{-1}\left(\frac{ex}{d}\right)}\right)}{4e^3\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.78

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

$$= \frac{-2ex\sqrt{d-ex}\sqrt{d+ex}(a-bn\log(x)+b\log(cx^n)) + 2d^2\arctan\left(\frac{ex}{\sqrt{d-ex}\sqrt{d+ex}}\right)(a-bn\log(x)+b\log(cx^n))}{}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-2*e*x*Sqrt[d - e*x]*Sqrt[d + e*x]*(a - b*n*Log[x] + b*Log[c*x^n]) + 2*d^2
*ArcTan[(e*x)/(Sqrt[d - e*x]*Sqrt[d + e*x])]*(a - b*n*Log[x] + b*Log[c*x^n]
) + (b*n*(d^3*Sqrt[1 - (e^2*x^2)/d^2]*ArcSin[(e*x)/d] + e*x*(-d^2 + e^2*x^2
)*(-1 + 2*Log[x]) + (e^3*Sqrt[1 - (e^2*x^2)/d^2]*(ArcSinh[Sqrt[-(e^2/d^2)]*
x]^2 + 2*ArcSinh[Sqrt[-(e^2/d^2)]*x]*Log[1 - E^(-2*ArcSinh[Sqrt[-(e^2/d^2)]
*x])] - 2*Log[x]*Log[Sqrt[-(e^2/d^2)]*x + Sqrt[1 - (e^2*x^2)/d^2]] - PolyLo
g[2, E^(-2*ArcSinh[Sqrt[-(e^2/d^2)]*x])]))/(-(e^2/d^2))^(3/2)))/(Sqrt[d - e
*x]*Sqrt[d + e*x]))/(4*e^3)
```

Maple [F]

$$\int \frac{x^2(a+b\ln(c\,x^n))}{\sqrt{-ex+d}\,\sqrt{ex+d}}dx$$

[In] int(x^2*(a+b*ln(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)

[Out] int(x^2*(a+b*ln(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)

Fricas [F]

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{\sqrt{ex+d}\sqrt{-ex+d}}\,dx$$

[In] integrate(x^2*(a+b*log(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] integral(-(sqrt(e*x + d)*sqrt(-e*x + d)*b*x^2*log(c*x^n) + sqrt(e*x + d)*sqrt(-e*x + d)*a*x^2)/(e^2*x^2 - d^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

[In] integrate(x**2*(a+b*ln(c*x**n))/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{\sqrt{ex+d}\sqrt{-ex+d}}\,dx$$

[In] integrate(x^2*(a+b*log(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")

[Out] 1/2*a*(d^2*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - sqrt(-e^2*x^2 + d^2)*x/e^2) + b*integrate((x^2*log(c) + x^2*log(x^n))/(sqrt(e*x + d)*sqrt(-e*x + d)), x)

Giac [F]

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{\sqrt{ex+d}\sqrt{-ex+d}}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2/(sqrt(e*x + d)*sqrt(-e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(cx^n))}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,x^n))}{\sqrt{d+e\,x}\,\sqrt{d-e\,x}}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n)))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] int((x^2*(a + b*log(c*x^n)))/((d + e*x)^(1/2)*(d - e*x)^(1/2)), x)
```

3.314 $\int \frac{a+b\log(cx^n)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 248

$$\int \frac{a+b\log(cx^n)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{ibdn\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)^2}{2e\sqrt{d-ex}\sqrt{d+ex}} - \frac{bdn\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)\log\left(1-e^{2i\arcsin\left(\frac{ex}{d}\right)}\right)}{e\sqrt{d-ex}\sqrt{d+ex}} + \frac{d\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)(a+b\log(cx^n))}{e\sqrt{d-ex}\sqrt{d+ex}} + \frac{ibdn\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{PolyLog}\left(2,e^{2i\arcsin\left(\frac{ex}{d}\right)}\right)}{2e\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] 1/2*I*b*d*n*arcsin(e*x/d)^2*(1-e^2*x^2/d^2)^(1/2)/e/(-e*x+d)^(1/2)/(e*x+d)^
(1/2)-b*d*n*arcsin(e*x/d)*ln(1-(I*e*x/d+(1-e^2*x^2/d^2)^(1/2))^2)*(1-e^2*x^
2/d^2)^(1/2)/e/(-e*x+d)^(1/2)/(e*x+d)^(1/2)+d*arcsin(e*x/d)*(a+b*ln(c*x^n))
*(1-e^2*x^2/d^2)^(1/2)/e/(-e*x+d)^(1/2)/(e*x+d)^(1/2)+1/2*I*b*d*n*polylog(2
,(I*e*x/d+(1-e^2*x^2/d^2)^(1/2))^2)*(1-e^2*x^2/d^2)^(1/2)/e/(-e*x+d)^(1/2)/
(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2365, 2363, 4721, 3798, 2221, 2317, 2438}

$$\int \frac{a+b\log(cx^n)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{d\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)(a+b\log(cx^n))}{e\sqrt{d-ex}\sqrt{d+ex}} + \frac{ibdn\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{PolyLog}\left(2,e^{2i\arcsin\left(\frac{ex}{d}\right)}\right)}{2e\sqrt{d-ex}\sqrt{d+ex}} + \frac{ibdn\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)^2}{2e\sqrt{d-ex}\sqrt{d+ex}} - \frac{bdn\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)\log\left(1-e^{2i\arcsin\left(\frac{ex}{d}\right)}\right)}{e\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(a + b*Log[c*x^n])/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] ((I/2)*b*d*n*Sqrt[1 - (e^2*x^2)/d^2]*ArcSin[(e*x)/d]^2)/(e*Sqrt[d - e*x]*Sq
rt[d + e*x]) - (b*d*n*Sqrt[1 - (e^2*x^2)/d^2]*ArcSin[(e*x)/d]*Log[1 - E^((2
*I)*ArcSin[(e*x)/d])])/(e*Sqrt[d - e*x]*Sqrt[d + e*x]) + (d*Sqrt[1 - (e^2*x
^2)/d^2]*ArcSin[(e*x)/d]*(a + b*Log[c*x^n]))/(e*Sqrt[d - e*x]*Sqrt[d + e*x]
) + ((I/2)*b*d*n*Sqrt[1 - (e^2*x^2)/d^2]*PolyLog[2, E^((2*I)*ArcSin[(e*x)/d
])])/(e*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
```

```
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2365

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + e1*(e2/(d1*d2))*x^2]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + e1*(e2/(d1*d2
))*x^2], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{a+b\log(cx^n)}{\sqrt{1-\frac{e^2x^2}{d^2}}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}\\
&= \frac{d\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)(a+b\log(cx^n))}{e\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(bdn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{\sin^{-1}\left(\frac{ex}{d}\right)}{x}\,dx}{e\sqrt{d-ex}\sqrt{d+ex}}\\
&= \frac{d\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)(a+b\log(cx^n))}{e\sqrt{d-ex}\sqrt{d+ex}}\\
&\quad-\frac{\left(bdn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int x\cot(x)\,dx,x,\sin^{-1}\left(\frac{ex}{d}\right)\right)}{e\sqrt{d-ex}\sqrt{d+ex}}
\end{aligned}$$

$$= \frac{ibdn\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)^2}{2e\sqrt{d-ex}\sqrt{d+ex}} + \frac{d\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)(a+b\log(cx^n))}{e\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(2ibdn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx,x,\sin^{-1}\left(\frac{ex}{d}\right)\right)}{e\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{ibdn\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)^2}{2e\sqrt{d-ex}\sqrt{d+ex}} - \frac{bdn\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)\log\left(1-e^{2i\sin^{-1}\left(\frac{ex}{d}\right)}\right)}{e\sqrt{d-ex}\sqrt{d+ex}} + \frac{d\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)(a+b\log(cx^n))}{e\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(bdn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int \log\left(1-e^{2ix}\right)\,dx,x,\sin^{-1}\left(\frac{ex}{d}\right)\right)}{e\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{ibdn\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)^2}{2e\sqrt{d-ex}\sqrt{d+ex}} - \frac{bdn\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)\log\left(1-e^{2i\sin^{-1}\left(\frac{ex}{d}\right)}\right)}{e\sqrt{d-ex}\sqrt{d+ex}} + \frac{d\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)(a+b\log(cx^n))}{e\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(ibdn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2i\sin^{-1}\left(\frac{ex}{d}\right)}\right)}{2e\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{ibdn\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)^2}{2e\sqrt{d-ex}\sqrt{d+ex}} - \frac{bdn\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)\log\left(1-e^{2i\sin^{-1}\left(\frac{ex}{d}\right)}\right)}{e\sqrt{d-ex}\sqrt{d+ex}} + \frac{d\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)(a+b\log(cx^n))}{e\sqrt{d-ex}\sqrt{d+ex}} + \frac{ibdn\sqrt{1-\frac{e^2x^2}{d^2}}\text{Li}_2\left(e^{2i\sin^{-1}\left(\frac{ex}{d}\right)}\right)}{2e\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.88

$$\int \frac{a+b\log(cx^n)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\arctan\left(\frac{ex}{\sqrt{d-ex}\sqrt{d+ex}}\right)(a-bn\log(x)+b\log(cx^n))}{e} - \frac{bn\sqrt{1-\frac{e^2x^2}{d^2}}\left(\text{arcsinh}\left(\sqrt{-\frac{e^2}{d^2}}x\right)^2+2\text{arcsinh}\left(\sqrt{-\frac{e^2}{d^2}}x\right)\log\left(1-e^{-2\text{arcsinh}\left(\sqrt{-\frac{e^2}{d^2}}x\right)}\right)-2\log(x)\log\left(\sqrt{\right.\right.}{2\sqrt{-\frac{e^2}{d^2}}\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

[Out] (ArcTan[(e*x)/(Sqrt[d - e*x]*Sqrt[d + e*x])]*(a - b*n*Log[x] + b*Log[c*x^n]
))/e - (b*n*Sqrt[1 - (e^2*x^2)/d^2]*(ArcSinh[Sqrt[-(e^2/d^2)]*x]^2 + 2*ArcS
inh[Sqrt[-(e^2/d^2)]*x]*Log[1 - E^(-2*ArcSinh[Sqrt[-(e^2/d^2)]*x])] - 2*Log
[x]*Log[Sqrt[-(e^2/d^2)]*x + Sqrt[1 - (e^2*x^2)/d^2]] - PolyLog[2, E^(-2*Ar
cSinh[Sqrt[-(e^2/d^2)]*x])]))/(2*Sqrt[-(e^2/d^2)]*Sqrt[d - e*x]*Sqrt[d + e*
x])

Maple [F]

$$\int \frac{a+b\ln\left(c\,x^{n}\right)}{\sqrt{-ex+d}\,\sqrt{ex+d}}dx$$

[In] int((a+b*ln(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)

[Out] int((a+b*ln(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)

Fricas [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{b\log\left(cx^{n}\right)+a}{\sqrt{ex+d}\sqrt{-ex+d}}\,dx$$

[In] integrate((a+b*log(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="frica
s")

[Out] integral(-(sqrt(e*x + d)*sqrt(-e*x + d)*b*log(c*x^n) + sqrt(e*x + d)*sqrt(-
e*x + d)*a)/(e^2*x^2 - d^2), x)

Sympy [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{a+b\log\left(cx^{n}\right)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

[In] integrate((a+b*ln(c*x**n))/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)

[Out] Integral((a + b*log(c*x**n))/(sqrt(d - e*x)*sqrt(d + e*x)), x)

Maxima [F]

$$\int \frac{a + b\log (cx^n)}{\sqrt{d - ex}\sqrt{d + ex}}\,dx = \int \frac{b\log (cx^n) + a}{\sqrt{ex + d}\sqrt{-ex + d}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] b*integrate((log(c) + log(x^n))/(sqrt(e*x + d)*sqrt(-e*x + d)), x) + a*arcs
in(e^2*x/(d*sqrt(e^2)))/sqrt(e^2)
```

Giac [F]

$$\int \frac{a + b\log (cx^n)}{\sqrt{d - ex}\sqrt{d + ex}}\,dx = \int \frac{b\log (cx^n) + a}{\sqrt{ex + d}\sqrt{-ex + d}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*log(c*x^n) + a)/(sqrt(e*x + d)*sqrt(-e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log (cx^n)}{\sqrt{d - ex}\sqrt{d + ex}}\,dx = \int \frac{a + b\,\ln (c\,x^n)}{\sqrt{d + e\,x}\,\sqrt{d - e\,x}}\,dx$$

```
[In] int((a + b*log(c*x^n))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] int((a + b*log(c*x^n))/((d + e*x)^(1/2)*(d - e*x)^(1/2)), x)
```

3.315 $\int \frac{a+b\log(cx^n)}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 142

$$\int \frac{a+b\log(cx^n)}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{bn(d^2-e^2x^2)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} - \frac{ben\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)}{d\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)\,(a+b\log(cx^n))}{d^2x\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -b*n*(-e^2*x^2+d^2)/d^2/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-(-e^2*x^2+d^2)*(a+b*
ln(c*x^n))/d^2/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-b*e*n*arcsin(e*x/d)*(1-e^2*x^
2/d^2)^(1/2)/d/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2387, 2373, 283, 222}

$$\int \frac{a+b\log(cx^n)}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(d^2-e^2x^2)\,(a+b\log(cx^n))}{d^2x\sqrt{d-ex}\sqrt{d+ex}} - \frac{ben\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)}{d\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn(d^2-e^2x^2)}{d^2x\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((b*n*(d^2 - e^2*x^2))/(d^2*x*Sqrt[d - e*x]*Sqrt[d + e*x])) - (b*e*n*Sqrt[
1 - (e^2*x^2)/d^2]*ArcSin[(e*x)/d])/(d*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((d^2
 - e^2*x^2)*(a + b*Log[c*x^n]))/(d^2*x*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2387

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d1_) + (e1_.)*(x_))^
(q_)*((d2_) + (e2_.)*(x_))^(q_), x_Symbol] :> Dist[(d1 + e1*x)^q*((d2 + e2*
x)^q/(1 + e1*(e2/(d1*d2))*x^2)^q), Int[x^m*(1 + e1*(e2/(d1*d2))*x^2)^q*(a +
 b*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*
e1 + d1*e2, 0] && IntegerQ[m] && IntegerQ[q - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{a+b\log(cx^n)}{x^2\sqrt{1-\frac{e^2x^2}{d^2}}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{(d^2-e^2x^2)\,(a+b\log(cx^n))}{d^2x\sqrt{d-ex}\sqrt{d+ex}}+\frac{\left(bn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{\sqrt{1-\frac{e^2x^2}{d^2}}}{x^2}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{bn(d^2-e^2x^2)}{d^2x\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)\,(a+b\log(cx^n))}{d^2x\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{1}{\sqrt{1-\frac{e^2x^2}{d^2}}}\,dx}{d^2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{bn(d^2-e^2x^2)}{d^2x\sqrt{d-ex}\sqrt{d+ex}}-\frac{ben\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)}{d\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)\,(a+b\log(cx^n))}{d^2x\sqrt{d-ex}\sqrt{d+ex}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.49

$$\int \frac{a+b\log(cx^n)}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{benx\arctan\left(\frac{ex}{\sqrt{d-ex}\sqrt{d+ex}}\right)+\sqrt{d-ex}\sqrt{d+ex}(a+bn+b\log(cx^n))}{d^2x}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((b*e*n*x*ArcTan[(e*x)/(Sqrt[d - e*x]*Sqrt[d + e*x])] + Sqrt[d - e*x]*Sqrt
[d + e*x]*(a + b*n + b*Log[c*x^n]))/(d^2*x))
```

Maple [F]

$$\int \frac{a+b\ln(c\,x^n)}{x^2\sqrt{-ex+d}\,\sqrt{ex+d}}dx$$

```
[In] int((a+b*ln(c*x^n))/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.51

$$\int \frac{a+b\log(cx^n)}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{2\,benx\arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right)-(bn\log(x)+bn+b\log(c)+a)\sqrt{ex+d}\sqrt{-ex+d}}{d^2x}$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] (2*b*e*n*x*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)) - (b*n*log(x) +
 b*n + b*log(c) + a)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^2*x)
```

Sympy [F]

$$\int \frac{a+b\log(cx^n)}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{a+b\log(cx^n)}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**2/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*log(c*x**n))/(x**2*sqrt(d - e*x)*sqrt(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.72

$$\int \frac{a+b\log(cx^n)}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\left(\frac{e^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}}+\frac{\sqrt{-e^2x^2+d^2}}{x}\right)bn}{d^2} - \frac{\sqrt{-e^2x^2+d^2}b\log(cx^n)}{d^2x} - \frac{\sqrt{-e^2x^2+d^2}a}{d^2x}$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] -(e^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + sqrt(-e^2*x^2 + d^2)/x)*b*n/d
^2 - sqrt(-e^2*x^2 + d^2)*b*log(c*x^n)/(d^2*x) - sqrt(-e^2*x^2 + d^2)*a/(d^
2*x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{b\log(cx^n)+a}{\sqrt{ex+d}\sqrt{-ex+d}x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*log(c*x^n) + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^2\,\sqrt{d+e\,x}\,\sqrt{d-e\,x}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*x)^(1/2)*(d - e*x)^(1/2)), x)
```

3.316 $\int \frac{a+b\log(cx^n)}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 252

$$\int \frac{a+b\log(cx^n)}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{2be^2n(d^2-e^2x^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn(d^2-e^2x^2)^2}{9d^4x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{2be^3n\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)}{3d^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)\,(a+b\log(cx^n))}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{2e^2(d^2-e^2x^2)\,(a+b\log(cx^n))}{3d^4x\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -2/3*b*e^2*n*(-e^2*x^2+d^2)/d^4/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/9*b*n*(-e^
2*x^2+d^2)^2/d^4/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/3*(-e^2*x^2+d^2)*(a+b*l
n(c*x^n))/d^2/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-2/3*e^2*(-e^2*x^2+d^2)*(a+b*
ln(c*x^n))/d^4/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-2/3*b*e^3*n*arcsin(e*x/d)*(1-
e^2*x^2/d^2)^(1/2)/d^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2387, 277, 270, 2392, 12, 462, 283, 222}

$$\int \frac{a+b\log(cx^n)}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(d^2-e^2x^2)\,(a+b\log(cx^n))}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{2e^2(d^2-e^2x^2)\,(a+b\log(cx^n))}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} - \frac{2be^3n\sqrt{1-\frac{e^2x^2}{d^2}}\arcsin\left(\frac{ex}{d}\right)}{3d^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{2be^2n(d^2-e^2x^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn(d^2-e^2x^2)^2}{9d^4x^3\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(a + b*Log[c*x^n])/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-2*b*e^2*n*(d^2 - e^2*x^2))/(3*d^4*x*Sqrt[d - e*x]*Sqrt[d + e*x]) - (b*n*(
d^2 - e^2*x^2)^2)/(9*d^4*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - (2*b*e^3*n*Sqrt
[1 - (e^2*x^2)/d^2]*ArcSin[(e*x)/d])/(3*d^3*Sqrt[d - e*x]*Sqrt[d + e*x]) -
((d^2 - e^2*x^2)*(a + b*Log[c*x^n]))/(3*d^2*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]
) - (2*e^2*(d^2 - e^2*x^2)*(a + b*Log[c*x^n]))/(3*d^4*x*Sqrt[d - e*x]*Sqrt[
d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
```

```
Q[m + n, -1]))
```

Rule 2387

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d1_) + (e1_.)*(x_))^
(q_)*((d2_) + (e2_.)*(x_))^(q_), x_Symbol] :> Dist[(d1 + e1*x)^q*((d2 + e2*
x)^q/(1 + e1*(e2/(d1*d2))*x^2)^q), Int[x^m*(1 + e1*(e2/(d1*d2))*x^2)^q*(a +
 b*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*
e1 + d1*e2, 0] && IntegerQ[m] && IntegerQ[q - 1/2]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int\frac{a+b\log(cx^n)}{x^4\sqrt{1-\frac{e^2x^2}{d^2}}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}} \\
&= -\frac{(d^2-e^2x^2)\,(a+b\log{(cx^n)})}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{2e^2(d^2-e^2x^2)\,(a+b\log{(cx^n)})}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} \\
&\quad -\frac{\left(bn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int\frac{(-d^2-2e^2x^2)\sqrt{1-\frac{e^2x^2}{d^2}}}{3d^2x^4}\,dx}{\sqrt{d-ex}\sqrt{d+ex}} \\
&= -\frac{(d^2-e^2x^2)\,(a+b\log{(cx^n)})}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{2e^2(d^2-e^2x^2)\,(a+b\log{(cx^n)})}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} \\
&\quad -\frac{\left(bn\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int\frac{(-d^2-2e^2x^2)\sqrt{1-\frac{e^2x^2}{d^2}}}{x^4}\,dx}{3d^2\sqrt{d-ex}\sqrt{d+ex}} \\
&= -\frac{bn(d^2-e^2x^2)^2}{9d^4x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)\,(a+b\log{(cx^n)})}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} \\
&\quad -\frac{2e^2(d^2-e^2x^2)\,(a+b\log{(cx^n)})}{3d^4x\sqrt{d-ex}\sqrt{d+ex}}+\frac{\left(2be^2n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int\frac{\sqrt{1-\frac{e^2x^2}{d^2}}}{x^2}\,dx}{3d^2\sqrt{d-ex}\sqrt{d+ex}}
\end{aligned}
$$

$$= -\frac{2be^2n(d^2-e^2x^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn(d^2-e^2x^2)^2}{9d^4x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)(a+b\log(cx^n))}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{2e^2(d^2-e^2x^2)(a+b\log(cx^n))}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(2be^4n\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int\frac{1}{\sqrt{1-\frac{e^2x^2}{d^2}}}\,dx}{3d^4\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{2be^2n(d^2-e^2x^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} - \frac{bn(d^2-e^2x^2)^2}{9d^4x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{2be^3n\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)}{3d^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)(a+b\log(cx^n))}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{2e^2(d^2-e^2x^2)(a+b\log(cx^n))}{3d^4x\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.46

$$\int \frac{a+b\log(cx^n)}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{6be^3nx^3\arctan\left(\frac{ex}{\sqrt{d-ex}\sqrt{d+ex}}\right) + \sqrt{d-ex}\sqrt{d+ex}(3a(d^2+2e^2x^2) + bn(d^2+5e^2x^2) + 3b(d^2+2e^2x^2)\text{lo}}{9d^4x^3}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/9*(6*b*e^3*n*x^3*ArcTan[(e*x)/(Sqrt[d - e*x]*Sqrt[d + e*x])] + Sqrt[d -
e*x]*Sqrt[d + e*x]*(3*a*(d^2 + 2*e^2*x^2) + b*n*(d^2 + 5*e^2*x^2) + 3*b*(d^
2 + 2*e^2*x^2)*Log[c*x^n]))/(d^4*x^3)
```

Maple [F]

$$\int \frac{a+b\ln(c\,x^n)}{x^4\sqrt{-ex+d}\,\sqrt{ex+d}}dx$$

```
[In] int((a+b*ln(c*x^n))/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] int((a+b*ln(c*x^n))/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.54

$$\int \frac{a+b\log(cx^n)}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{12\,be^3nx^3\arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right) - (bd^2n + 3\,ad^2 + (5\,be^2n + 6\,ae^2)x^2 + 3\,(2\,be^2x^2 + bd^2)\log(c) + 3\,(2}{9\,d^4x^3}$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/9*(12*b*e^3*n*x^3*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)) - (b*d
^2*n + 3*a*d^2 + (5*b*e^2*n + 6*a*e^2)*x^2 + 3*(2*b*e^2*x^2 + b*d^2)*log(c)
 + 3*(2*b*e^2*n*x^2 + b*d^2*n)*log(x))*sqrt(e*x + d)*sqrt(-e*x + d))/(d^4*x
^3)
```

Sympy [F]

$$\int \frac{a+b\log(cx^n)}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{a+b\log(cx^n)}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**4/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*log(c*x**n))/(x**4*sqrt(d - e*x)*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{b\log(cx^n)+a}{\sqrt{ex+d}\sqrt{-ex+d}x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] -1/3*a*(2*sqrt(-e^2*x^2 + d^2)*e^2/(d^4*x) + sqrt(-e^2*x^2 + d^2)/(d^2*x^3)
) + b*integrate((log(c) + log(x^n))/(sqrt(e*x + d)*sqrt(-e*x + d)*x^4), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{b\log(cx^n)+a}{\sqrt{ex+d}\sqrt{-ex+d}x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*log(c*x^n) + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^4\,\sqrt{d+e\,x}\,\sqrt{d-e\,x}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^4*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
[Out] int((a + b*log(c*x^n))/(x^4*(d + e*x)^(1/2)*(d - e*x)^(1/2)), x)
```

3.317 $\int \frac{x \log(x)}{\sqrt{-1+x^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{x \log(x)}{\sqrt{-1+x^2}} \, dx = -\sqrt{-1+x^2} + \arctan\left(\sqrt{-1+x^2}\right) + \sqrt{-1+x^2} \log(x)$$

```
[Out] arctan((x^2-1)^(1/2))-(x^2-1)^(1/2)+ln(x)*(x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2376, 272, 52, 65, 209}

$$\int \frac{x \log(x)}{\sqrt{-1+x^2}} \, dx = \arctan\left(\sqrt{x^2-1}\right) - \sqrt{x^2-1} + \sqrt{x^2-1} \log(x)$$

```
[In] Int[(x*Log[x])/Sqrt[-1 + x^2],x]
[Out] -Sqrt[-1 + x^2] + ArcTan[Sqrt[-1 + x^2]] + Sqrt[-1 + x^2]*Log[x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \sqrt{-1+x^2}\log(x) - \int \frac{\sqrt{-1+x^2}}{x}\,dx \\
&= \sqrt{-1+x^2}\log(x) - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, x^2\right) \\
&= -\sqrt{-1+x^2} + \sqrt{-1+x^2}\log(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x}\,dx, x, x^2\right) \\
&= -\sqrt{-1+x^2} + \sqrt{-1+x^2}\log(x) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^2}\right) \\
&= -\sqrt{-1+x^2} + \tan^{-1}\left(\sqrt{-1+x^2}\right) + \sqrt{-1+x^2}\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{x\log(x)}{\sqrt{-1+x^2}}\,dx = -\arctan\left(\frac{1}{\sqrt{-1+x^2}}\right)+\sqrt{-1+x^2}(-1+\log(x))$$

```
[In] Integrate[(x*Log[x])/Sqrt[-1 + x^2],x]
[Out] -ArcTan[1/Sqrt[-1 + x^2]] + Sqrt[-1 + x^2]*(-1 + Log[x])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.50

method	result
meijerg	$-\frac{\sqrt{-\operatorname{signum}(x^2-1)}\left(2-2\sqrt{-x^2+1}\right)}{4\sqrt{\operatorname{signum}(x^2-1)}}+\frac{\sqrt{-\operatorname{signum}(x^2-1)}\,\ln(x)\left(2-2\sqrt{-x^2+1}\right)}{2\sqrt{\operatorname{signum}(x^2-1)}}+\frac{\sqrt{-\operatorname{signum}(x^2-1)}\left(-16+16\sqrt{-x^2+1}-32\ln\right.}{32\sqrt{\operatorname{signum}(x^2-1)}}$

```
[In] int(x*ln(x)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4/signum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*(2-2*(-x^2+1)^(1/2))+1/2/si
gnum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*ln(x)*(2-2*(-x^2+1)^(1/2))+1/32/si
gnum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*(-16+16*(-x^2+1)^(1/2)-32*ln(1/2+1
/2*(-x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{x\log(x)}{\sqrt{-1+x^2}}\,dx = \sqrt{x^2-1}(\log(x)-1)+2\arctan\left(-x+\sqrt{x^2-1}\right)$$

```
[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^2 - 1)*(log(x) - 1) + 2*arctan(-x + sqrt(x^2 - 1))
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{x\log(x)}{\sqrt{-1+x^2}}\,dx = \sqrt{x^2-1}\log(x) - \begin{cases} \sqrt{x^2-1} - \operatorname{acos}\left(\frac{1}{x}\right) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

```
[In] integrate(x*ln(x)/(x**2-1)**(1/2),x)
[Out] sqrt(x**2 - 1)*log(x) - Piecewise((sqrt(x**2 - 1) - acos(1/x), (x > -1) & (
x < 1)))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{x\log(x)}{\sqrt{-1+x^2}}\,dx = \sqrt{x^2-1}\log(x) - \sqrt{x^2-1} - \arcsin\left(\frac{1}{|x|}\right)$$

```
[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 - 1)*log(x) - sqrt(x^2 - 1) - arcsin(1/abs(x))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{x\log(x)}{\sqrt{-1+x^2}}\,dx = \sqrt{x^2-1}\log(x) - \sqrt{x^2-1} + \arctan\left(\sqrt{x^2-1}\right)$$

```
[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 - 1)*log(x) - sqrt(x^2 - 1) + arctan(sqrt(x^2 - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\log(x)}{\sqrt{-1+x^2}}\,dx = \int \frac{x\,\ln(x)}{\sqrt{x^2-1}}\,dx$$

```
[In] int((x*log(x))/(x^2 - 1)^(1/2),x)
[Out] int((x*log(x))/(x^2 - 1)^(1/2), x)
```

3.318 $\int (fx)^m \left(d+ex^2\right)^3 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 211

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\log(cx^n))\,dx = -\frac{bd^3n(fx)^{1+m}}{f(1+m)^2}-\frac{3bd^2en(fx)^{3+m}}{f^3(3+m)^2}-\frac{3bde^2n(fx)^{5+m}}{f^5(5+m)^2}-\frac{be^3n(fx)^{7+m}}{f^7(7+m)^2}+\frac{d^3(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)}+\frac{3d^2e(fx)^{3+m}(a+b\log(cx^n))}{f^3(3+m)}+\frac{3de^2(fx)^{5+m}(a+b\log(cx^n))}{f^5(5+m)}+\frac{e^3(fx)^{7+m}(a+b\log(cx^n))}{f^7(7+m)}$$

```
[Out] -b*d^3*n*(f*x)^(1+m)/f/(1+m)^2-3*b*d^2*e*n*(f*x)^(3+m)/f^3/(3+m)^2-3*b*d*e^
2*n*(f*x)^(5+m)/f^5/(5+m)^2-b*e^3*n*(f*x)^(7+m)/f^7/(7+m)^2+d^3*(f*x)^(1+m)
*(a+b*ln(c*x^n))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(a+b*ln(c*x^n))/f^3/(3+m)+3*d*
e^2*(f*x)^(5+m)*(a+b*ln(c*x^n))/f^5/(5+m)+e^3*(f*x)^(7+m)*(a+b*ln(c*x^n))/f
^7/(7+m)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used

= {276, 2392, 14}

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\log(cx^n))\,dx = \frac{d^3(fx)^{m+1}(a+b\log(cx^n))}{f(m+1)} + \frac{3d^2e(fx)^{m+3}(a+b\log(cx^n))}{f^3(m+3)} + \frac{3de^2(fx)^{m+5}(a+b\log(cx^n))}{f^5(m+5)} + \frac{e^3(fx)^{m+7}(a+b\log(cx^n))}{f^7(m+7)} - \frac{bd^3n(fx)^{m+1}}{f(m+1)^2} - \frac{3bd^2en(fx)^{m+3}}{f^3(m+3)^2} - \frac{3bde^2n(fx)^{m+5}}{f^5(m+5)^2} - \frac{be^3n(fx)^{m+7}}{f^7(m+7)^2}$$

```
[In] Int[(f*x)^m*(d + e*x^2)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -((b*d^3*n*(f*x)^(1 + m))/(f*(1 + m)^2)) - (3*b*d^2*e*n*(f*x)^(3 + m))/(f^3
*(3 + m)^2) - (3*b*d*e^2*n*(f*x)^(5 + m))/(f^5*(5 + m)^2) - (b*e^3*n*(f*x)^
(7 + m))/(f^7*(7 + m)^2) + (d^3*(f*x)^(1 + m)*(a + b*Log[c*x^n]))/(f*(1 + m
)) + (3*d^2*e*(f*x)^(3 + m)*(a + b*Log[c*x^n]))/(f^3*(3 + m)) + (3*d*e^2*(f
*x)^(5 + m)*(a + b*Log[c*x^n]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*L
og[c*x^n]))/(f^7*(7 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{d^3(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)}+\frac{3d^2e(fx)^{3+m}(a+b\log(cx^n))}{f^3(3+m)} \\
&\quad+\frac{3de^2(fx)^{5+m}(a+b\log(cx^n))}{f^5(5+m)}+\frac{e^3(fx)^{7+m}(a+b\log(cx^n))}{f^7(7+m)} \\
&\quad-(bn)\int(fx)^m\left(\frac{d^3}{1+m}+\frac{3d^2ex^2}{3+m}+\frac{3de^2x^4}{5+m}+\frac{e^3x^6}{7+m}\right)dx \\
&= \frac{d^3(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)}+\frac{3d^2e(fx)^{3+m}(a+b\log(cx^n))}{f^3(3+m)} \\
&\quad+\frac{3de^2(fx)^{5+m}(a+b\log(cx^n))}{f^5(5+m)}+\frac{e^3(fx)^{7+m}(a+b\log(cx^n))}{f^7(7+m)} \\
&\quad-(bn)\int\left(\frac{d^3(fx)^m}{1+m}+\frac{3d^2e(fx)^{2+m}}{f^2(3+m)}+\frac{3de^2(fx)^{4+m}}{f^4(5+m)}+\frac{e^3(fx)^{6+m}}{f^6(7+m)}\right)dx \\
&= -\frac{bd^3n(fx)^{1+m}}{f(1+m)^2}-\frac{3bd^2en(fx)^{3+m}}{f^3(3+m)^2}-\frac{3bde^2n(fx)^{5+m}}{f^5(5+m)^2}-\frac{be^3n(fx)^{7+m}}{f^7(7+m)^2} \\
&\quad+\frac{d^3(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)}+\frac{3d^2e(fx)^{3+m}(a+b\log(cx^n))}{f^3(3+m)} \\
&\quad+\frac{3de^2(fx)^{5+m}(a+b\log(cx^n))}{f^5(5+m)}+\frac{e^3(fx)^{7+m}(a+b\log(cx^n))}{f^7(7+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.74

$$
\int(fx)^m\left(d+ex^2\right)^3(a+b\log(cx^n))\,dx = x(fx)^m\left(-\frac{bd^3n}{(1+m)^2}-\frac{3bd^2enx^2}{(3+m)^2}-\frac{3bde^2nx^4}{(5+m)^2}-\frac{be^3nx^6}{(7+m)^2}+\frac{d^3(a+b\log(cx^n))}{1+m}+\frac{3d^2ex^2(a+b\log(cx^n))}{3+m}+\frac{3de^2x^4(a+b\log(cx^n))}{5+m}+\frac{e^3x^6(a+b\log(cx^n))}{7+m}\right)
$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] x*(f*x)^m*(-((b*d^3*n)/(1 + m)^2) - (3*b*d^2*e*n*x^2)/(3 + m)^2 - (3*b*d*e^
2*n*x^4)/(5 + m)^2 - (b*e^3*n*x^6)/(7 + m)^2 + (d^3*(a + b*Log[c*x^n]))/(1
+ m) + (3*d^2*e*x^2*(a + b*Log[c*x^n]))/(3 + m) + (3*d*e^2*x^4*(a + b*Log[c
*x^n]))/(5 + m) + (e^3*x^6*(a + b*Log[c*x^n]))/(7 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1760 vs. 2(211) = 422.

Time = 27.45 (sec) , antiderivative size = 1761, normalized size of antiderivative = 8.35

method	result	size
parallelrisch	Expression too large to display	1761
risch	Expression too large to display	5073

```
[In] int((f*x)^m*(e*x^2+d)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -(-1575*e^3*b*ln(c*x^n)*(f*x)^m*x^7-x^7*(f*x)^m*a*e^3*m^7-25*x^7*(f*x)^m*a*
e^3*m^6-253*x^7*(f*x)^m*a*e^3*m^5-1333*x^7*(f*x)^m*a*e^3*m^4-3907*x^7*(f*x)
^m*a*e^3*m^3-6283*x^7*(f*x)^m*a*e^3*m^2-5055*x^7*(f*x)^m*a*e^3*m+225*x^7*(f
*x)^m*b*e^3*n-6615*x^5*(f*x)^m*a*d*e^2-11025*x^3*(f*x)^m*a*d^2*e-11025*x*(f
*x)^m*a*d^3-1575*x^7*(f*x)^m*a*e^3-3*x^5*(f*x)^m*ln(c*x^n)*b*d*e^2*m^7-81*x
^5*(f*x)^m*ln(c*x^n)*b*d*e^2*m^6+3*x^5*(f*x)^m*b*d*e^2*m^6*n-879*x^5*(f*x)^
m*ln(c*x^n)*b*d*e^2*m^5+66*x^5*(f*x)^m*b*d*e^2*m^5*n-3*x^3*(f*x)^m*ln(c*x^n
)*b*d^2*e*m^7-4917*x^5*(f*x)^m*ln(c*x^n)*b*d*e^2*m^4+549*x^5*(f*x)^m*b*d*e^
2*m^4*n-87*x^3*(f*x)^m*ln(c*x^n)*b*d^2*e*m^6+3*x^3*(f*x)^m*b*d^2*e*m^6*n-15
129*x^5*(f*x)^m*ln(c*x^n)*b*d*e^2*m^3+2172*x^5*(f*x)^m*b*d*e^2*m^3*n-1023*x
^3*(f*x)^m*ln(c*x^n)*b*d^2*e*m^5+78*x^3*(f*x)^m*b*d^2*e*m^5*n-25251*x^5*(f*
x)^m*ln(c*x^n)*b*d*e^2*m^2+4269*x^5*(f*x)^m*b*d*e^2*m^2*n-6243*x^3*(f*x)^m*
ln(c*x^n)*b*d^2*e*m^4+789*x^3*(f*x)^m*b*d^2*e*m^4*n-20853*x^5*(f*x)^m*ln(c*
x^n)*b*d*e^2*m+3906*x^5*(f*x)^m*b*d*e^2*m*n-20985*x^3*(f*x)^m*ln(c*x^n)*b*d
^2*e*m^3+3876*x^3*(f*x)^m*b*d^2*e*m^3*n-37941*x^3*(f*x)^m*ln(c*x^n)*b*d^2*e
*m^2+9357*x^3*(f*x)^m*b*d^2*e*m^2*n-33285*x^3*(f*x)^m*ln(c*x^n)*b*d^2*e*m+9
870*x^3*(f*x)^m*b*d^2*e*m*n+11025*x*(f*x)^m*b*d^3*n-11025*b*d^3*ln(c*x^n)*(
f*x)^m*x-10531*x*(f*x)^m*ln(c*x^n)*b*d^3*m^3+2340*x*(f*x)^m*b*d^3*m^3*n-231
01*x*(f*x)^m*ln(c*x^n)*b*d^3*m^2+8191*x*(f*x)^m*b*d^3*m^2*n-x*(f*x)^m*a*d^3
*m^7-31*x*(f*x)^m*a*d^3*m^6-397*x*(f*x)^m*a*d^3*m^5-2707*x*(f*x)^m*a*d^3*m^
4-10531*x*(f*x)^m*a*d^3*m^3-23101*x*(f*x)^m*a*d^3*m^2-25935*x*(f*x)^m*a*d^3
*m-25935*x*(f*x)^m*ln(c*x^n)*b*d^3*m+14910*x*(f*x)^m*b*d^3*m*n-x*(f*x)^m*ln
(c*x^n)*b*d^3*m^7-31*x*(f*x)^m*ln(c*x^n)*b*d^3*m^6+x*(f*x)^m*b*d^3*m^6*n-39
7*x*(f*x)^m*ln(c*x^n)*b*d^3*m^5+30*x*(f*x)^m*b*d^3*m^5*n-2707*x*(f*x)^m*ln(
c*x^n)*b*d^3*m^4+367*x*(f*x)^m*b*d^3*m^4*n-25251*x^5*(f*x)^m*a*d*e^2*m^2-62
43*x^3*(f*x)^m*a*d^2*e*m^4-20853*x^5*(f*x)^m*a*d*e^2*m+1323*x^5*(f*x)^m*b*d
*e^2*n-20985*x^3*(f*x)^m*a*d^2*e*m^3-37941*x^3*(f*x)^m*a*d^2*e*m^2-33285*x^
3*(f*x)^m*a*d^2*e*m+3675*x^3*(f*x)^m*b*d^2*e*n-x^7*(f*x)^m*ln(c*x^n)*b*e^3*
m^7-11025*e*d^2*b*ln(c*x^n)*(f*x)^m*x^3-6615*e^2*d*b*ln(c*x^n)*(f*x)^m*x^5-
25*x^7*(f*x)^m*ln(c*x^n)*b*e^3*m^6+x^7*(f*x)^m*b*e^3*m^6*n-253*x^7*(f*x)^m*
ln(c*x^n)*b*e^3*m^5+18*x^7*(f*x)^m*b*e^3*m^5*n-1333*x^7*(f*x)^m*ln(c*x^n)*b
*e^3*m^4+127*x^7*(f*x)^m*b*e^3*m^4*n-3*x^5*(f*x)^m*a*d*e^2*m^7-3907*x^7*(f*
x)^m*ln(c*x^n)*b*e^3*m^3+444*x^7*(f*x)^m*b*e^3*m^3*n-81*x^5*(f*x)^m*a*d*e^2
*m^6-6283*x^7*(f*x)^m*ln(c*x^n)*b*e^3*m^2+799*x^7*(f*x)^m*b*e^3*m^2*n-879*x
```

```
^5*(f*x)^m*a*d*e^2*m^5-3*x^3*(f*x)^m*a*d^2*e*m^7-5055*x^7*(f*x)^m*ln(c*x^n)
*b*e^3*m+690*x^7*(f*x)^m*b*e^3*m*n-4917*x^5*(f*x)^m*a*d*e^2*m^4-87*x^3*(f*x
)^m*a*d^2*e*m^6-15129*x^5*(f*x)^m*a*d*e^2*m^3-1023*x^3*(f*x)^m*a*d^2*e*m^5)
/(m^2+14*m+49)/(m^2+10*m+25)/(m^2+6*m+9)/(1+m)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1222 vs. $2(211) = 422$.

Time = 0.30 (sec) , antiderivative size = 1222, normalized size of antiderivative = 5.79

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\log(cx^n)) \, dx = \text{Too large to display}$$

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] ((a*e^3*m^7 + 25*a*e^3*m^6 + 253*a*e^3*m^5 + 1333*a*e^3*m^4 + 3907*a*e^3*m^
3 + 6283*a*e^3*m^2 + 5055*a*e^3*m + 1575*a*e^3 - (b*e^3*m^6 + 18*b*e^3*m^5
+ 127*b*e^3*m^4 + 444*b*e^3*m^3 + 799*b*e^3*m^2 + 690*b*e^3*m + 225*b*e^3)*
n)*x^7 + 3*(a*d*e^2*m^7 + 27*a*d*e^2*m^6 + 293*a*d*e^2*m^5 + 1639*a*d*e^2*m
^4 + 5043*a*d*e^2*m^3 + 8417*a*d*e^2*m^2 + 6951*a*d*e^2*m + 2205*a*d*e^2 -
(b*d*e^2*m^6 + 22*b*d*e^2*m^5 + 183*b*d*e^2*m^4 + 724*b*d*e^2*m^3 + 1423*b*
d*e^2*m^2 + 1302*b*d*e^2*m + 441*b*d*e^2)*n)*x^5 + 3*(a*d^2*e*m^7 + 29*a*d^
2*e*m^6 + 341*a*d^2*e*m^5 + 2081*a*d^2*e*m^4 + 6995*a*d^2*e*m^3 + 12647*a*d
^2*e*m^2 + 11095*a*d^2*e*m + 3675*a*d^2*e - (b*d^2*e*m^6 + 26*b*d^2*e*m^5 +
 263*b*d^2*e*m^4 + 1292*b*d^2*e*m^3 + 3119*b*d^2*e*m^2 + 3290*b*d^2*e*m + 1
225*b*d^2*e)*n)*x^3 + (a*d^3*m^7 + 31*a*d^3*m^6 + 397*a*d^3*m^5 + 2707*a*d^
3*m^4 + 10531*a*d^3*m^3 + 23101*a*d^3*m^2 + 25935*a*d^3*m + 11025*a*d^3 - (
b*d^3*m^6 + 30*b*d^3*m^5 + 367*b*d^3*m^4 + 2340*b*d^3*m^3 + 8191*b*d^3*m^2
+ 14910*b*d^3*m + 11025*b*d^3)*n)*x + ((b*e^3*m^7 + 25*b*e^3*m^6 + 253*b*e^
3*m^5 + 1333*b*e^3*m^4 + 3907*b*e^3*m^3 + 6283*b*e^3*m^2 + 5055*b*e^3*m + 1
575*b*e^3)*x^7 + 3*(b*d*e^2*m^7 + 27*b*d*e^2*m^6 + 293*b*d*e^2*m^5 + 1639*b
*d*e^2*m^4 + 5043*b*d*e^2*m^3 + 8417*b*d*e^2*m^2 + 6951*b*d*e^2*m + 2205*b*
d*e^2)*x^5 + 3*(b*d^2*e*m^7 + 29*b*d^2*e*m^6 + 341*b*d^2*e*m^5 + 2081*b*d^2
*e*m^4 + 6995*b*d^2*e*m^3 + 12647*b*d^2*e*m^2 + 11095*b*d^2*e*m + 3675*b*d^
2*e)*x^3 + (b*d^3*m^7 + 31*b*d^3*m^6 + 397*b*d^3*m^5 + 2707*b*d^3*m^4 + 105
31*b*d^3*m^3 + 23101*b*d^3*m^2 + 25935*b*d^3*m + 11025*b*d^3)*x)*log(c) + (
(b*e^3*m^7 + 25*b*e^3*m^6 + 253*b*e^3*m^5 + 1333*b*e^3*m^4 + 3907*b*e^3*m^3
 + 6283*b*e^3*m^2 + 5055*b*e^3*m + 1575*b*e^3)*n*x^7 + 3*(b*d*e^2*m^7 + 27*
b*d*e^2*m^6 + 293*b*d*e^2*m^5 + 1639*b*d*e^2*m^4 + 5043*b*d*e^2*m^3 + 8417*
b*d*e^2*m^2 + 6951*b*d*e^2*m + 2205*b*d*e^2)*n*x^5 + 3*(b*d^2*e*m^7 + 29*b*
d^2*e*m^6 + 341*b*d^2*e*m^5 + 2081*b*d^2*e*m^4 + 6995*b*d^2*e*m^3 + 12647*b
*d^2*e*m^2 + 11095*b*d^2*e*m + 3675*b*d^2*e)*n*x^3 + (b*d^3*m^7 + 31*b*d^3*
m^6 + 397*b*d^3*m^5 + 2707*b*d^3*m^4 + 10531*b*d^3*m^3 + 23101*b*d^3*m^2 +
25935*b*d^3*m + 11025*b*d^3)*n*x)*log(x))*e^(m*log(f) + m*log(x))/(m^8 + 32
*m^7 + 428*m^6 + 3104*m^5 + 13238*m^4 + 33632*m^3 + 49036*m^2 + 36960*m + 1
1025)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6217 vs. 2(206) = 412.

Time = 13.51 (sec) , antiderivative size = 6217, normalized size of antiderivative = 29.46

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\log(cx^n))\,dx = \text{Too large to display}$$

```
[In] integrate((f*x)**m*(e*x**2+d)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise(((-a*d**3/(6*x**6) - 3*a*d**2*e/(4*x**4) - 3*a*d*e**2/(2*x**2) +
a*e**3*log(x) + b*d**3*(-n/(36*x**6) - log(c*x**n)/(6*x**6)) + 3*b*d**2*e*(
-n/(16*x**4) - log(c*x**n)/(4*x**4)) + 3*b*d*e**2*(-n/(4*x**2) - log(c*x**n
)/(2*x**2)) - b*e**3*Piecewise((-log(c)*log(x), Eq(n, 0)), (-log(c*x**n)**2
/(2*n), True)))/f**7, Eq(m, -7)), ((-a*d**3/(4*x**4) - 3*a*d**2*e/(2*x**2)
+ 3*a*d*e**2*log(c*x**n)/n + a*e**3*x**2/2 - b*d**3*n/(16*x**4) - b*d**3*lo
g(c*x**n)/(4*x**4) - 3*b*d**2*e*n/(4*x**2) - 3*b*d**2*e*log(c*x**n)/(2*x**2
) + 3*b*d*e**2*log(c*x**n)**2/(2*n) - b*e**3*n*x**2/4 + b*e**3*x**2*log(c*x
**n)/2)/f**5, Eq(m, -5)), ((-a*d**3/(2*x**2) + 3*a*d**2*e*log(c*x**n)/n + 3
*a*d*e**2*x**2/2 + a*e**3*x**4/4 - b*d**3*n/(4*x**2) - b*d**3*log(c*x**n)/(
2*x**2) + 3*b*d**2*e*log(c*x**n)**2/(2*n) - 3*b*d*e**2*n*x**2/4 + 3*b*d*e**
2*x**2*log(c*x**n)/2 - b*e**3*n*x**4/16 + b*e**3*x**4*log(c*x**n)/4)/f**3,
Eq(m, -3)), ((a*d**3*log(c*x**n)/n + 3*a*d**2*e*x**2/2 + 3*a*d*e**2*x**4/4
+ a*e**3*x**6/6 + b*d**3*log(c*x**n)**2/(2*n) - 3*b*d**2*e*n*x**2/4 + 3*b*d
**2*e*x**2*log(c*x**n)/2 - 3*b*d*e**2*n*x**4/16 + 3*b*d*e**2*x**4*log(c*x**
n)/4 - b*e**3*n*x**6/36 + b*e**3*x**6*log(c*x**n)/6)/f, Eq(m, -1)), (a*d**3
*m**7*x*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 3363
2*m**3 + 49036*m**2 + 36960*m + 11025) + 31*a*d**3*m**6*x*(f*x)**m/(m**8 +
32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 369
60*m + 11025) + 397*a*d**3*m**5*x*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 310
4*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 2707*a*d
**3*m**4*x*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 3
3632*m**3 + 49036*m**2 + 36960*m + 11025) + 10531*a*d**3*m**3*x*(f*x)**m/(m
**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2
 + 36960*m + 11025) + 23101*a*d**3*m**2*x*(f*x)**m/(m**8 + 32*m**7 + 428*m*
*6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) +
25935*a*d**3*m*x*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m*
*4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 11025*a*d**3*x*(f*x)**m/(
m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**
2 + 36960*m + 11025) + 3*a*d**2*e*m**7*x**3*(f*x)**m/(m**8 + 32*m**7 + 428*
m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025)
+ 87*a*d**2*e*m**6*x**3*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 1
3238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 1023*a*d**2*e*m**5
*x**3*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*
m**3 + 49036*m**2 + 36960*m + 11025) + 6243*a*d**2*e*m**4*x**3*(f*x)**m/(m*
```

*8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2
+ 36960*m + 11025) + 20985*a*d**2*e*m**3*x**3*(f*x)**m/(m**8 + 32*m**7 + 42
8*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025
) + 37941*a*d**2*e*m**2*x**3*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**
5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 33285*a*d**2*
e*m*x**3*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 336
32*m**3 + 49036*m**2 + 36960*m + 11025) + 11025*a*d**2*e*x**3*(f*x)**m/(m**
8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 +
 36960*m + 11025) + 3*a*d*e**2*m**7*x**5*(f*x)**m/(m**8 + 32*m**7 + 428*m**
6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 8
1*a*d*e**2*m**6*x**5*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 1323
8*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 879*a*d*e**2*m**5*x**
5*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3
 + 49036*m**2 + 36960*m + 11025) + 4917*a*d*e**2*m**4*x**5*(f*x)**m/(m**8 +
 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36
960*m + 11025) + 15129*a*d*e**2*m**3*x**5*(f*x)**m/(m**8 + 32*m**7 + 428*m*
*6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) +
25251*a*d*e**2*m**2*x**5*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 +
13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 20853*a*d*e**2*m*
x**5*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m
3 + 49036*m2 + 36960*m + 11025) + 6615*a*d*e**2*x**5*(f*x)**m/(m**8 + 3
2*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 3696
0*m + 11025) + a*e**3*m**7*x**7*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*
m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 25*a*e**3*
m**6*x**7*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33
632*m**3 + 49036*m**2 + 36960*m + 11025) + 253*a*e**3*m**5*x**7*(f*x)**m/(m
8 + 32*m7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2
 + 36960*m + 11025) + 1333*a*e**3*m**4*x**7*(f*x)**m/(m**8 + 32*m**7 + 428*
m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025)
+ 3907*a*e**3*m**3*x**7*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 1
3238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 6283*a*e**3*m**2*x
7*(f*x)m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m*
*3 + 49036*m**2 + 36960*m + 11025) + 5055*a*e**3*m*x**7*(f*x)**m/(m**8 + 32
*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960
*m + 11025) + 1575*a*e**3*x**7*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m
5 + 13238*m4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + b*d**3*m**7
x(f*x)**m*log(c*x**n)/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4
 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) - b*d**3*m**6*n*x*(f*x)**m/(m
8 + 32*m7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2
 + 36960*m + 11025) + 31*b*d**3*m**6*x*(f*x)**m*log(c*x**n)/(m**8 + 32*m**7
 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m +
11025) - 30*b*d**3*m**5*n*x*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5
 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 397*b*d**3*m**
5*x*(f*x)**m*log(c*x**n)/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**
4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) - 367*b*d**3*m**4*n*x*(f*x)*

```
*m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036
*m**2 + 36960*m + 11025) + 2707*b*d**3*m**4*x*(f*x)**m*log(c*x**n)/(m**8 +
32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 369
60*m + 11025) - 2340*b*d**3*m**3*n*x*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 +
3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 10531
*b*d**3*m**3*x*(f*x)**m*log(c*x**n)/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5
+ 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) - 8191*b*d**3*m**
2*n*x*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*
m**3 + 49036*m**2 + 36960*m + 11025) + 23101*b*d**3*m**2*x*(f*x)**m*log(c*x
**n)/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 490
36*m**2 + 36960*m + 11025) - 14910*b*d**3*m*n*x*(f*x)**m/(m**8 + 32*m**7 +
428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 110
25) + 25935*b*d**3*m*x*(f*x)**m*log(c*x**n)/(m**8 + 32*m**7 + 428*m**6 + 31
04*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) - 11025*b
*d**3*n*x*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33
632*m**3 + 49036*m**2 + 36960*m + 11025) + 11025*b*d**3*x*(f*x)**m*log(c*x*
*n)/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 4903
6*m**2 + 36960*m + 11025) + 3*b*d**2*e*m**7*x**3*(f*x)**m*log(c*x**n)/(m**8
 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 +
36960*m + 11025) - 3*b*d**2*e*m**6*n*x**3*(f*x)**m/(m**8 + 32*m**7 + 428*m*
*6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) +
87*b*d**2*e*m**6*x**3*(f*x)**m*log(c*x**n)/(m**8 + 32*m**7 + 428*m**6 + 310
4*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) - 78*b*d**
2*e*m**5*n*x**3*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**
4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 1023*b*d**2*e*m**5*x**3*(f
*x)**m*log(c*x**n)/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33
632*m**3 + 49036*m**2 + 36960*m + 11025) - 789*b*d**2*e*m**4*n*x**3*(f*x)**
m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*
m**2 + 36960*m + 11025) + 6243*b*d**2*e*m**4*x**3*(f*x)**m*log(c*x**n)/(m**
8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 +
 36960*m + 11025) - 3876*b*d**2*e*m**3*n*x**3*(f*x)**m/(m**8 + 32*m**7 + 42
8*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025
) + 20985*b*d**2*e*m**3*x**3*(f*x)**m*log(c*x**n)/(m**8 + 32*m**7 + 428*m**
6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) - 9
357*b*d**2*e*m**2*n*x**3*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 +
13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 37941*b*d**2*e*m*
*2*x**3*(f*x)**m*log(c*x**n)/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238
*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) - 9870*b*d**2*e*m*n*x**3
*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3
+ 49036*m**2 + 36960*m + 11025) + 33285*b*d**2*e*m*x**3*(f*x)**m*log(c*x**n
)/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*
m**2 + 36960*m + 11025) - 3675*b*d**2*e*n*x**3*(f*x)**m/(m**8 + 32*m**7 + 4
28*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 1102
5) + 11025*b*d**2*e*x**3*(f*x)**m*log(c*x**n)/(m**8 + 32*m**7 + 428*m**6 +
3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 3*b*d
```

```
*e**2*m**7*x**5*(f*x)**m*log(c*x**n)/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5
 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) - 3*b*d*e**2*m**
6*n*x**5*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 336
32*m**3 + 49036*m**2 + 36960*m + 11025) + 81*b*d*e**2*m**6*x**5*(f*x)**m*lo
g(c*x**n)/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3
+ 49036*m**2 + 36960*m + 11025) - 66*b*d*e**2*m**5*n*x**5*(f*x)**m/(m**8 +
32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 369
60*m + 11025) + 879*b*d*e**2*m**5*x**5*(f*x)**m*log(c*x**n)/(m**8 + 32*m**7
 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m +
11025) - 549*b*d*e**2*m**4*n*x**5*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 310
4*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 4917*b*d
*e**2*m**4*x**5*(f*x)**m*log(c*x**n)/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5
 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) - 2172*b*d*e**2*
m**3*n*x**5*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 +
33632*m**3 + 49036*m**2 + 36960*m + 11025) + 15129*b*d*e**2*m**3*x**5*(f*x)
**m*log(c*x**n)/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632
*m**3 + 49036*m**2 + 36960*m + 11025) - 4269*b*d*e**2*m**2*n*x**5*(f*x)**m/
(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m*
*2 + 36960*m + 11025) + 25251*b*d*e**2*m**2*x**5*(f*x)**m*log(c*x**n)/(m**8
 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 +
36960*m + 11025) - 3906*b*d*e**2*m*n*x**5*(f*x)**m/(m**8 + 32*m**7 + 428*m*
*6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) +
20853*b*d*e**2*m*x**5*(f*x)**m*log(c*x**n)/(m**8 + 32*m**7 + 428*m**6 + 310
4*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) - 1323*b*d
*e**2*n*x**5*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 +
 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 6615*b*d*e**2*x**5*(f*x)**m*l
og(c*x**n)/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3
 + 49036*m**2 + 36960*m + 11025) + b*e**3*m**7*x**7*(f*x)**m*log(c*x**n)/(m
**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2
 + 36960*m + 11025) - b*e**3*m**6*n*x**7*(f*x)**m/(m**8 + 32*m**7 + 428*m**
6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 2
5*b*e**3*m**6*x**7*(f*x)**m*log(c*x**n)/(m**8 + 32*m**7 + 428*m**6 + 3104*m
**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) - 18*b*e**3*m
**5*n*x**7*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 3
3632*m**3 + 49036*m**2 + 36960*m + 11025) + 253*b*e**3*m**5*x**7*(f*x)**m*l
og(c*x**n)/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3
 + 49036*m**2 + 36960*m + 11025) - 127*b*e**3*m**4*n*x**7*(f*x)**m/(m**8 +
32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 369
60*m + 11025) + 1333*b*e**3*m**4*x**7*(f*x)**m*log(c*x**n)/(m**8 + 32*m**7
+ 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 1
1025) - 444*b*e**3*m**3*n*x**7*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m
**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 3907*b*e**3
*m**3*x**7*(f*x)**m*log(c*x**n)/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13
238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) - 799*b*e**3*m**2*n*x
**7*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m*
```

```
*3 + 49036*m**2 + 36960*m + 11025) + 6283*b*e**3*m**2*x**7*(f*x)**m*log(c*x
**n)/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 490
36*m**2 + 36960*m + 11025) - 690*b*e**3*m*n*x**7*(f*x)**m/(m**8 + 32*m**7 +
 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11
025) + 5055*b*e**3*m*x**7*(f*x)**m*log(c*x**n)/(m**8 + 32*m**7 + 428*m**6 +
 3104*m**5 + 13238*m**4 + 33632*m**3 + 49036*m**2 + 36960*m + 11025) - 225*
b*e**3*n*x**7*(f*x)**m/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4
+ 33632*m**3 + 49036*m**2 + 36960*m + 11025) + 1575*b*e**3*x**7*(f*x)**m*lo
g(c*x**n)/(m**8 + 32*m**7 + 428*m**6 + 3104*m**5 + 13238*m**4 + 33632*m**3
+ 49036*m**2 + 36960*m + 11025), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.28

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\log(cx^n))\,dx = \frac{be^3f^mx^7x^m\log(cx^n)}{m+7} + \frac{ae^3f^mx^7x^m}{m+7} - \frac{be^3f^mnx^7x^m}{(m+7)^2} + \frac{3\,bde^2f^mx^5x^m\log(cx^n)}{m+5} + \frac{3\,ade^2f^mx^5x^m}{m+5} - \frac{3\,bde^2f^mnx^5x^m}{(m+5)^2} + \frac{3\,bd^2ef^mx^3x^m\log(cx^n)}{m+3} + \frac{3\,ad^2ef^mx^3x^m}{m+3} - \frac{3\,bd^2ef^mnx^3x^m}{(m+3)^2} - \frac{bd^3f^mnxx^m}{(m+1)^2} + \frac{(fx)^{m+1}\,bd^3\log(cx^n)}{f(m+1)} + \frac{(fx)^{m+1}\,ad^3}{f(m+1)}$$

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] b*e^3*f^m*x^7*x^m*log(c*x^n)/(m + 7) + a*e^3*f^m*x^7*x^m/(m + 7) - b*e^3*f^
m*n*x^7*x^m/(m + 7)^2 + 3*b*d*e^2*f^m*x^5*x^m*log(c*x^n)/(m + 5) + 3*a*d*e^
2*f^m*x^5*x^m/(m + 5) - 3*b*d*e^2*f^m*n*x^5*x^m/(m + 5)^2 + 3*b*d^2*e*f^m*x
^3*x^m*log(c*x^n)/(m + 3) + 3*a*d^2*e*f^m*x^3*x^m/(m + 3) - 3*b*d^2*e*f^m*n
*x^3*x^m/(m + 3)^2 - b*d^3*f^m*n*x*x^m/(m + 1)^2 + (f*x)^(m + 1)*b*d^3*log(
c*x^n)/(f*(m + 1)) + (f*x)^(m + 1)*a*d^3/(f*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(211) = 422$.

Time = 0.38 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.64

$$\int (fx)^m (d+ex^2)^3 (a+b\log(cx^n))\,dx = \frac{be^3f^6f^mx^7x^m\log(c)}{f^6m+7f^6} + \frac{ae^3f^6f^mx^7x^m}{f^6m+7f^6} + \frac{3bde^2f^4f^mx^5x^m\log(c)}{f^4m+5f^4} + \frac{be^3f^mmnx^7x^m\log(x)}{m^2+14m+49} + \frac{3ade^2f^4f^mx^5x^m}{f^4m+5f^4} + \frac{7be^3f^mnx^7x^m\log(x)}{m^2+14m+49} - \frac{be^3f^mnx^7x^m}{m^2+14m+49} + \frac{3bde^2f^mmnx^5x^m\log(x)}{m^2+10m+25} + \frac{15bde^2f^mnx^5x^m\log(x)}{m^2+10m+25} - \frac{3bde^2f^mnx^5x^m}{m^2+10m+25} + \frac{3bd^2ef^2f^mx^3x^m\log(c)}{f^2m+3f^2} + \frac{3bd^2ef^mmnx^3x^m\log(x)}{m^2+6m+9} + \frac{3ad^2ef^2f^mx^3x^m}{f^2m+3f^2} + \frac{9bd^2ef^mnx^3x^m\log(x)}{m^2+6m+9} - \frac{3bd^2ef^mnx^3x^m}{m^2+6m+9} + \frac{bd^3f^mmnxx^m\log(x)}{m^2+2m+1} + \frac{bd^3f^mnxx^m\log(x)}{m^2+2m+1} - \frac{bd^3f^mnxx^m}{m^2+2m+1} + \frac{(fx)^m bd^3x\log(c)}{m+1} + \frac{(fx)^m ad^3x}{m+1}$$

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*e^3*f^6*f^m*x^7*x^m*log(c)/(f^6*m + 7*f^6) + a*e^3*f^6*f^m*x^7*x^m/(f^6*m
 + 7*f^6) + 3*b*d*e^2*f^4*f^m*x^5*x^m*log(c)/(f^4*m + 5*f^4) + b*e^3*f^m*m*
n*x^7*x^m*log(x)/(m^2 + 14*m + 49) + 3*a*d*e^2*f^4*f^m*x^5*x^m/(f^4*m + 5*f
^4) + 7*b*e^3*f^m*n*x^7*x^m*log(x)/(m^2 + 14*m + 49) - b*e^3*f^m*n*x^7*x^m/
(m^2 + 14*m + 49) + 3*b*d*e^2*f^m*m*n*x^5*x^m*log(x)/(m^2 + 10*m + 25) + 15
*b*d*e^2*f^m*n*x^5*x^m*log(x)/(m^2 + 10*m + 25) - 3*b*d*e^2*f^m*n*x^5*x^m/(
m^2 + 10*m + 25) + 3*b*d^2*e*f^2*f^m*x^3*x^m*log(c)/(f^2*m + 3*f^2) + 3*b*d
^2*e*f^m*m*n*x^3*x^m*log(x)/(m^2 + 6*m + 9) + 3*a*d^2*e*f^2*f^m*x^3*x^m/(f^
2*m + 3*f^2) + 9*b*d^2*e*f^m*n*x^3*x^m*log(x)/(m^2 + 6*m + 9) - 3*b*d^2*e*f
^m*n*x^3*x^m/(m^2 + 6*m + 9) + b*d^3*f^m*m*n*x*x^m*log(x)/(m^2 + 2*m + 1) +
 b*d^3*f^m*n*x*x^m*log(x)/(m^2 + 2*m + 1) - b*d^3*f^m*n*x*x^m/(m^2 + 2*m +
1) + (f*x)^m*b*d^3*x*log(c)/(m + 1) + (f*x)^m*a*d^3*x/(m + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\log{(cx^n)})\,dx = \int (f\,x)^m \left(e\,x^2+d\right)^3 (a+b\,\ln{(c\,x^n)})\,dx$$

```
[In] int((f*x)^m*(d + e*x^2)^3*(a + b*log(c*x^n)),x)
[Out] int((f*x)^m*(d + e*x^2)^3*(a + b*log(c*x^n)), x)
```

3.319 $\int (fx)^m (d+ex^2)^2 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int (fx)^m (d+ex^2)^2 (a+b\log(cx^n))\,dx = -\frac{bd^2n(fx)^{1+m}}{f(1+m)^2} - \frac{2bden(fx)^{3+m}}{f^3(3+m)^2} - \frac{be^2n(fx)^{5+m}}{f^5(5+m)^2} + \frac{d^2(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)} + \frac{2de(fx)^{3+m}(a+b\log(cx^n))}{f^3(3+m)} + \frac{e^2(fx)^{5+m}(a+b\log(cx^n))}{f^5(5+m)}$$

```
[Out] -b*d^2*n*(f*x)^(1+m)/f/(1+m)^2-2*b*d*e*n*(f*x)^(3+m)/f^3/(3+m)^2-b*e^2*n*(f
*x)^(5+m)/f^5/(5+m)^2+d^2*(f*x)^(1+m)*(a+b*ln(c*x^n))/f/(1+m)+2*d*e*(f*x)^(
3+m)*(a+b*ln(c*x^n))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*ln(c*x^n))/f^5/(5+m)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {276, 2392, 12, 14}

$$\int (fx)^m (d+ex^2)^2 (a+b\log(cx^n))\,dx = \frac{d^2(fx)^{m+1}(a+b\log(cx^n))}{f(m+1)} + \frac{2de(fx)^{m+3}(a+b\log(cx^n))}{f^3(m+3)} + \frac{e^2(fx)^{m+5}(a+b\log(cx^n))}{f^5(m+5)} - \frac{bd^2n(fx)^{m+1}}{f(m+1)^2} - \frac{2bden(fx)^{m+3}}{f^3(m+3)^2} - \frac{be^2n(fx)^{m+5}}{f^5(m+5)^2}$$

```
[In] Int[(f*x)^m*(d + e*x^2)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -((b*d^2*n*(f*x)^(1 + m))/(f*(1 + m)^2)) - (2*b*d*e*n*(f*x)^(3 + m))/(f^3*(
3 + m)^2) - (b*e^2*n*(f*x)^(5 + m))/(f^5*(5 + m)^2) + (d^2*(f*x)^(1 + m)*(a
 + b*Log[c*x^n]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)*(a + b*Log[c*x^n]))/(f
^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*Log[c*x^n]))/(f^5*(5 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

integral

$$= \frac{d^2(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)}+\frac{2de(fx)^{3+m}\left(a+b\log\left(cx^n\right)\right)}{f^3(3+m)}+\frac{e^2(fx)^{5+m}\left(a+b\log\left(cx^n\right)\right)}{f^5(5+m)} -(bn)\int\frac{(fx)^m\left(d^2(15+8m+m^2)+2de(5+6m+m^2)\,x^2+e^2(3+4m+m^2)\,x^4\right)}{(1+m)(3+m)(5+m)}\,dx$$

$$= \frac{d^2(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)}+\frac{2de(fx)^{3+m}\left(a+b\log\left(cx^n\right)\right)}{f^3(3+m)}+\frac{e^2(fx)^{5+m}\left(a+b\log\left(cx^n\right)\right)}{f^5(5+m)} -\frac{(bn)\int(fx)^m\left(d^2(15+8m+m^2)+2de(5+6m+m^2)\,x^2+e^2(3+4m+m^2)\,x^4\right)\,dx}{15+23m+9m^2+m^3}$$

$$= \frac{d^2(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)} + \frac{2de(fx)^{3+m}(a+b\log(cx^n))}{f^3(3+m)} + \frac{e^2(fx)^{5+m}(a+b\log(cx^n))}{f^5(5+m)} - \frac{(bn)\int\left(d^2(3+m)(5+m)(fx)^m + \frac{2de(1+m)(5+m)(fx)^{2+m}}{f^2} + \frac{e^2(1+m)(3+m)(fx)^{4+m}}{f^4}\right)dx}{15+23m+9m^2+m^3}$$

$$= -\frac{bd^2n(fx)^{1+m}}{f(1+m)^2} - \frac{2bden(fx)^{3+m}}{f^3(3+m)^2} - \frac{be^2n(fx)^{5+m}}{f^5(5+m)^2} + \frac{d^2(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)} + \frac{2de(fx)^{3+m}(a+b\log(cx^n))}{f^3(3+m)} + \frac{e^2(fx)^{5+m}(a+b\log(cx^n))}{f^5(5+m)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.73

$$\int (fx)^m\left(d+ex^2\right)^2(a+b\log(cx^n))\,dx = x(fx)^m\left(-\frac{bd^2n}{(1+m)^2} - \frac{2bdenx^2}{(3+m)^2} - \frac{be^2nx^4}{(5+m)^2} + \frac{d^2(a+b\log(cx^n))}{1+m} + \frac{2dex^2(a+b\log(cx^n))}{3+m} + \frac{e^2x^4(a+b\log(cx^n))}{5+m}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] x*(f*x)^m*(-((b*d^2*n)/(1 + m)^2) - (2*b*d*e*n*x^2)/(3 + m)^2 - (b*e^2*n*x^
4)/(5 + m)^2 + (d^2*(a + b*Log[c*x^n]))/(1 + m) + (2*d*e*x^2*(a + b*Log[c*x
^n]))/(3 + m) + (e^2*x^4*(a + b*Log[c*x^n]))/(5 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. 2(153) = 306.

Time = 5.72 (sec) , antiderivative size = 916, normalized size of antiderivative = 5.99

method	result
parallelrisch	$-\frac{-225x(fx)^m a\,d^2-150x^3(fx)^m ade-x^5(fx)^m a\,e^2m^5-13x^5(fx)^m a\,e^2m^4-62x^5(fx)^m a\,e^2m^3-134x^5(fx)^m a\,e^2m^2-129x^5(f}{}$
risch	Expression too large to display

```
[In] int((f*x)^m*(e*x^2+d)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -(-225*x*(f*x)^m*a*d^2-150*x^3*(f*x)^m*a*d*e-x^5*(f*x)^m*a*e^2*m^5-13*x^5*(
f*x)^m*a*e^2*m^4-62*x^5*(f*x)^m*a*e^2*m^3-134*x^5*(f*x)^m*a*e^2*m^2-129*x^5
*(f*x)^m*a*e^2*m+9*x^5*(f*x)^m*b*e^2*n-45*e^2*b*ln(c*x^n)*(f*x)^m*x^5-225*x
*(f*x)^m*ln(c*x^n)*b*d^2-x*(f*x)^m*a*d^2*m^5-17*x*(f*x)^m*a*d^2*m^4-110*x*(
f*x)^m*a*d^2*m^3-334*x*(f*x)^m*a*d^2*m^2-465*x*(f*x)^m*a*d^2*m+225*x*(f*x)^
```

```
m*b*d^2*n-2*x^3*(f*x)^m*ln(c*x^n)*b*d*e*m^5-30*x^3*(f*x)^m*ln(c*x^n)*b*d*e*
m^4+2*x^3*(f*x)^m*b*d*e*m^4*n-164*x^3*(f*x)^m*ln(c*x^n)*b*d*e*m^3+24*x^3*(f
*x)^m*b*d*e*m^3*n-396*x^3*(f*x)^m*ln(c*x^n)*b*d*e*m^2+92*x^3*(f*x)^m*b*d*e*
m^2*n-410*x^3*(f*x)^m*ln(c*x^n)*b*d*e*m+120*x^3*(f*x)^m*b*d*e*m*n-x*(f*x)^m
*ln(c*x^n)*b*d^2*m^5-17*x*(f*x)^m*ln(c*x^n)*b*d^2*m^4+x*(f*x)^m*b*d^2*m^4*n
-110*x*(f*x)^m*ln(c*x^n)*b*d^2*m^3+16*x*(f*x)^m*b*d^2*m^3*n-334*x*(f*x)^m*l
n(c*x^n)*b*d^2*m^2+94*x*(f*x)^m*b*d^2*m^2*n-465*x*(f*x)^m*ln(c*x^n)*b*d^2*m
+240*x*(f*x)^m*b*d^2*m*n-45*x^5*(f*x)^m*a*e^2-13*x^5*(f*x)^m*ln(c*x^n)*b*e^
2*m^4+x^5*(f*x)^m*b*e^2*m^4*n-62*x^5*(f*x)^m*ln(c*x^n)*b*e^2*m^3+8*x^5*(f*x
)^m*b*e^2*m^3*n-134*x^5*(f*x)^m*ln(c*x^n)*b*e^2*m^2+22*x^5*(f*x)^m*b*e^2*m^
2*n-150*b*d*e*ln(c*x^n)*(f*x)^m*x^3-2*x^3*(f*x)^m*a*d*e*m^5-129*x^5*(f*x)^m
*ln(c*x^n)*b*e^2*m+24*x^5*(f*x)^m*b*e^2*m*n-30*x^3*(f*x)^m*a*d*e*m^4-164*x^
3*(f*x)^m*a*d*e*m^3-396*x^3*(f*x)^m*a*d*e*m^2-410*x^3*(f*x)^m*a*d*e*m+50*x^
3*(f*x)^m*b*d*e*n-x^5*(f*x)^m*ln(c*x^n)*b*e^2*m^5)/(m^2+10*m+25)/(3+m)^2/(1
+m)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(153) = 306$.

Time = 0.30 (sec) , antiderivative size = 633, normalized size of antiderivative = 4.14

$$\int (fx)^m \left(d+ex^2\right)^2 (a+b\log(cx^n))\,dx = \frac{((ae^2m^5+13\,ae^2m^4+62\,ae^2m^3+134\,ae^2m^2+129\,ae^2m+45\,ae^2-(be^2m^4+8\,be^2m^3+22\,be^2m^2+24\,b}{}$$

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] ((a*e^2*m^5 + 13*a*e^2*m^4 + 62*a*e^2*m^3 + 134*a*e^2*m^2 + 129*a*e^2*m + 4
5*a*e^2 - (b*e^2*m^4 + 8*b*e^2*m^3 + 22*b*e^2*m^2 + 24*b*e^2*m + 9*b*e^2)*n
)*x^5 + 2*(a*d*e*m^5 + 15*a*d*e*m^4 + 82*a*d*e*m^3 + 198*a*d*e*m^2 + 205*a*
d*e*m + 75*a*d*e - (b*d*e*m^4 + 12*b*d*e*m^3 + 46*b*d*e*m^2 + 60*b*d*e*m +
25*b*d*e)*n)*x^3 + (a*d^2*m^5 + 17*a*d^2*m^4 + 110*a*d^2*m^3 + 334*a*d^2*m^
2 + 465*a*d^2*m + 225*a*d^2 - (b*d^2*m^4 + 16*b*d^2*m^3 + 94*b*d^2*m^2 + 24
0*b*d^2*m + 225*b*d^2)*n)*x + ((b*e^2*m^5 + 13*b*e^2*m^4 + 62*b*e^2*m^3 + 1
34*b*e^2*m^2 + 129*b*e^2*m + 45*b*e^2)*x^5 + 2*(b*d*e*m^5 + 15*b*d*e*m^4 +
82*b*d*e*m^3 + 198*b*d*e*m^2 + 205*b*d*e*m + 75*b*d*e)*x^3 + (b*d^2*m^5 + 1
7*b*d^2*m^4 + 110*b*d^2*m^3 + 334*b*d^2*m^2 + 465*b*d^2*m + 225*b*d^2)*x)*l
og(c) + ((b*e^2*m^5 + 13*b*e^2*m^4 + 62*b*e^2*m^3 + 134*b*e^2*m^2 + 129*b*e
^2*m + 45*b*e^2)*n*x^5 + 2*(b*d*e*m^5 + 15*b*d*e*m^4 + 82*b*d*e*m^3 + 198*b
*d*e*m^2 + 205*b*d*e*m + 75*b*d*e)*n*x^3 + (b*d^2*m^5 + 17*b*d^2*m^4 + 110*
b*d^2*m^3 + 334*b*d^2*m^2 + 465*b*d^2*m + 225*b*d^2)*n*x)*log(x))*e^(m*log(
f) + m*log(x))/(m^6 + 18*m^5 + 127*m^4 + 444*m^3 + 799*m^2 + 690*m + 225)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2820 vs. 2(146) = 292.

Time = 5.45 (sec) , antiderivative size = 2820, normalized size of antiderivative = 18.43

$$\int (fx)^m \left(d+ex^2\right)^2 (a+b\log(cx^n))\,dx = \text{Too large to display}$$

```
[In] integrate((f*x)**m*(e*x**2+d)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise(((-a*d**2/(4*x**4) - a*d*e/x**2 + a*e**2*log(x) + b*d**2*(-n/(16*
x**4) - log(c*x**n)/(4*x**4)) + 2*b*d*e*(-n/(4*x**2) - log(c*x**n)/(2*x**2)
) - b*e**2*Piecewise((-log(c)*log(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), Tr
ue)))/f**5, Eq(m, -5)), ((-a*d**2/(2*x**2) + 2*a*d*e*log(c*x**n)/n + a*e**2
*x**2/2 - b*d**2*n/(4*x**2) - b*d**2*log(c*x**n)/(2*x**2) + b*d*e*log(c*x**
n)**2/n - b*e**2*n*x**2/4 + b*e**2*x**2*log(c*x**n)/2)/f**3, Eq(m, -3)), ((
a*d**2*log(c*x**n)/n + a*d*e*x**2 + a*e**2*x**4/4 + b*d**2*log(c*x**n)**2/(
2*n) - b*d*e*n*x**2/2 + b*d*e*x**2*log(c*x**n) - b*e**2*n*x**4/16 + b*e**2*
x**4*log(c*x**n)/4)/f, Eq(m, -1)), (a*d**2*m**5*x*(f*x)**m/(m**6 + 18*m**5
+ 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) + 17*a*d**2*m**4*x*(f*x)**m
/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) + 110*a*d*
*2*m**3*x*(f*x)**m/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m
 + 225) + 334*a*d**2*m**2*x*(f*x)**m/(m**6 + 18*m**5 + 127*m**4 + 444*m**3
+ 799*m**2 + 690*m + 225) + 465*a*d**2*m*x*(f*x)**m/(m**6 + 18*m**5 + 127*m
**4 + 444*m**3 + 799*m**2 + 690*m + 225) + 225*a*d**2*x*(f*x)**m/(m**6 + 18
*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) + 2*a*d*e*m**5*x**3*(
f*x)**m/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) + 3
0*a*d*e*m**4*x**3*(f*x)**m/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2
 + 690*m + 225) + 164*a*d*e*m**3*x**3*(f*x)**m/(m**6 + 18*m**5 + 127*m**4 +
 444*m**3 + 799*m**2 + 690*m + 225) + 396*a*d*e*m**2*x**3*(f*x)**m/(m**6 +
18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) + 410*a*d*e*m*x**3*
(f*x)**m/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) +
150*a*d*e*x**3*(f*x)**m/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 +
690*m + 225) + a*e**2*m**5*x**5*(f*x)**m/(m**6 + 18*m**5 + 127*m**4 + 444*m
**3 + 799*m**2 + 690*m + 225) + 13*a*e**2*m**4*x**5*(f*x)**m/(m**6 + 18*m**
5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) + 62*a*e**2*m**3*x**5*(f*
x)**m/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) + 134
*a*e**2*m**2*x**5*(f*x)**m/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2
 + 690*m + 225) + 129*a*e**2*m*x**5*(f*x)**m/(m**6 + 18*m**5 + 127*m**4 + 4
44*m**3 + 799*m**2 + 690*m + 225) + 45*a*e**2*x**5*(f*x)**m/(m**6 + 18*m**5
 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) + b*d**2*m**5*x*(f*x)**m*l
og(c*x**n)/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225)
- b*d**2*m**4*n*x*(f*x)**m/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2
 + 690*m + 225) + 17*b*d**2*m**4*x*(f*x)**m*log(c*x**n)/(m**6 + 18*m**5 + 1
27*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) - 16*b*d**2*m**3*n*x*(f*x)**m/
```

```
(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) + 110*b*d**
2*m**3*x*(f*x)**m*log(c*x**n)/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m
**2 + 690*m + 225) - 94*b*d**2*m**2*n*x*(f*x)**m/(m**6 + 18*m**5 + 127*m**4
 + 444*m**3 + 799*m**2 + 690*m + 225) + 334*b*d**2*m**2*x*(f*x)**m*log(c*x*
*n)/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) - 240*b
*d**2*m*n*x*(f*x)**m/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690
*m + 225) + 465*b*d**2*m*x*(f*x)**m*log(c*x**n)/(m**6 + 18*m**5 + 127*m**4
+ 444*m**3 + 799*m**2 + 690*m + 225) - 225*b*d**2*n*x*(f*x)**m/(m**6 + 18*m
**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) + 225*b*d**2*x*(f*x)**m
*log(c*x**n)/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225
) + 2*b*d*e*m**5*x**3*(f*x)**m*log(c*x**n)/(m**6 + 18*m**5 + 127*m**4 + 444
*m**3 + 799*m**2 + 690*m + 225) - 2*b*d*e*m**4*n*x**3*(f*x)**m/(m**6 + 18*m
**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) + 30*b*d*e*m**4*x**3*(f
*x)**m*log(c*x**n)/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m
 + 225) - 24*b*d*e*m**3*n*x**3*(f*x)**m/(m**6 + 18*m**5 + 127*m**4 + 444*m*
*3 + 799*m**2 + 690*m + 225) + 164*b*d*e*m**3*x**3*(f*x)**m*log(c*x**n)/(m*
*6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) - 92*b*d*e*m**
2*n*x**3*(f*x)**m/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m
+ 225) + 396*b*d*e*m**2*x**3*(f*x)**m*log(c*x**n)/(m**6 + 18*m**5 + 127*m**
4 + 444*m**3 + 799*m**2 + 690*m + 225) - 120*b*d*e*m*n*x**3*(f*x)**m/(m**6
+ 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) + 410*b*d*e*m*x**
3*(f*x)**m*log(c*x**n)/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 6
90*m + 225) - 50*b*d*e*n*x**3*(f*x)**m/(m**6 + 18*m**5 + 127*m**4 + 444*m**
3 + 799*m**2 + 690*m + 225) + 150*b*d*e*x**3*(f*x)**m*log(c*x**n)/(m**6 + 1
8*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) + b*e**2*m**5*x**5*(
f*x)**m*log(c*x**n)/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*
m + 225) - b*e**2*m**4*n*x**5*(f*x)**m/(m**6 + 18*m**5 + 127*m**4 + 444*m**
3 + 799*m**2 + 690*m + 225) + 13*b*e**2*m**4*x**5*(f*x)**m*log(c*x**n)/(m**
6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) - 8*b*e**2*m**3
*n*x**5*(f*x)**m/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m +
 225) + 62*b*e**2*m**3*x**5*(f*x)**m*log(c*x**n)/(m**6 + 18*m**5 + 127*m**4
 + 444*m**3 + 799*m**2 + 690*m + 225) - 22*b*e**2*m**2*n*x**5*(f*x)**m/(m**
6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) + 134*b*e**2*m*
*2*x**5*(f*x)**m*log(c*x**n)/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m*
*2 + 690*m + 225) - 24*b*e**2*m*n*x**5*(f*x)**m/(m**6 + 18*m**5 + 127*m**4
+ 444*m**3 + 799*m**2 + 690*m + 225) + 129*b*e**2*m*x**5*(f*x)**m*log(c*x**
n)/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m + 225) - 9*b*e*
*2*n*x**5*(f*x)**m/(m**6 + 18*m**5 + 127*m**4 + 444*m**3 + 799*m**2 + 690*m
 + 225) + 45*b*e**2*x**5*(f*x)**m*log(c*x**n)/(m**6 + 18*m**5 + 127*m**4 +
444*m**3 + 799*m**2 + 690*m + 225), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.27

$$\int (fx)^m \left(d+ex^2\right)^2 (a+b\log(cx^n))\, dx = \frac{be^2f^mx^5x^m\log(cx^n)}{m+5} + \frac{ae^2f^mx^5x^m}{m+5} - \frac{be^2f^mnx^5x^m}{(m+5)^2} + \frac{2\,bdef^mx^3x^m\log(cx^n)}{m+3} + \frac{2\,adef^mx^3x^m}{m+3} - \frac{2\,bdef^mnx^3x^m}{(m+3)^2} - \frac{bd^2f^mnxx^m}{(m+1)^2} + \frac{(fx)^{m+1}bd^2\log(cx^n)}{f(m+1)} + \frac{(fx)^{m+1}ad^2}{f(m+1)}$$

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] b*e^2*f^m*x^5*x^m*log(c*x^n)/(m + 5) + a*e^2*f^m*x^5*x^m/(m + 5) - b*e^2*f^
m*n*x^5*x^m/(m + 5)^2 + 2*b*d*e*f^m*x^3*x^m*log(c*x^n)/(m + 3) + 2*a*d*e*f^
m*x^3*x^m/(m + 3) - 2*b*d*e*f^m*n*x^3*x^m/(m + 3)^2 - b*d^2*f^m*n*x*x^m/(m
+ 1)^2 + (f*x)^(m + 1)*b*d^2*log(c*x^n)/(f*(m + 1)) + (f*x)^(m + 1)*a*d^2/(
f*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(153) = 306.

Time = 0.37 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.59

$$\int (fx)^m \left(d+ex^2\right)^2 (a+b\log(cx^n))\, dx = \frac{be^2f^4f^mx^5x^m\log(c)}{f^4m+5\,f^4} + \frac{ae^2f^4f^mx^5x^m}{f^4m+5\,f^4} + \frac{be^2f^mmnx^5x^m\log(x)}{m^2+10\,m+25} + \frac{5\,be^2f^mnx^5x^m\log(x)}{m^2+10\,m+25} - \frac{be^2f^mnx^5x^m}{m^2+10\,m+25} + \frac{2\,bdef^2f^mx^3x^m\log(c)}{f^2m+3\,f^2} + \frac{2\,bdef^mmnx^3x^m\log(x)}{m^2+6\,m+9} + \frac{2\,adef^2f^mx^3x^m}{f^2m+3\,f^2} + \frac{6\,bdef^mnx^3x^m\log(x)}{m^2+6\,m+9} - \frac{2\,bdef^mnx^3x^m}{m^2+6\,m+9} + \frac{bd^2f^mmnxx^m\log(x)}{m^2+2\,m+1} + \frac{bd^2f^mnxx^m\log(x)}{m^2+2\,m+1} - \frac{bd^2f^mnxx^m}{m^2+2\,m+1} + \frac{(fx)^m\,bd^2x\log(c)}{m+1} + \frac{(fx)^m\,ad^2x}{m+1}$$

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*e^2*f^4*f^m*x^5*x^m*log(c)/(f^4*m + 5*f^4) + a*e^2*f^4*f^m*x^5*x^m/(f^4*m
 + 5*f^4) + b*e^2*f^m*m*n*x^5*x^m*log(x)/(m^2 + 10*m + 25) + 5*b*e^2*f^m*n*
x^5*x^m*log(x)/(m^2 + 10*m + 25) - b*e^2*f^m*n*x^5*x^m/(m^2 + 10*m + 25) +
2*b*d*e*f^2*f^m*x^3*x^m*log(c)/(f^2*m + 3*f^2) + 2*b*d*e*f^m*m*n*x^3*x^m*lo
g(x)/(m^2 + 6*m + 9) + 2*a*d*e*f^2*f^m*x^3*x^m/(f^2*m + 3*f^2) + 6*b*d*e*f^
m*n*x^3*x^m*log(x)/(m^2 + 6*m + 9) - 2*b*d*e*f^m*n*x^3*x^m/(m^2 + 6*m + 9)
+ b*d^2*f^m*m*n*x*x^m*log(x)/(m^2 + 2*m + 1) + b*d^2*f^m*n*x*x^m*log(x)/(m^
2 + 2*m + 1) - b*d^2*f^m*n*x*x^m/(m^2 + 2*m + 1) + (f*x)^m*b*d^2*x*log(c)/(
m + 1) + (f*x)^m*a*d^2*x/(m + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^2 (a+b\log\left(cx^n\right))\,dx=\int (f\,x)^m\,\left(e\,x^2+d\right)^2\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

```
[In] int((f*x)^m*(d + e*x^2)^2*(a + b*log(c*x^n)),x)
```

```
[Out] int((f*x)^m*(d + e*x^2)^2*(a + b*log(c*x^n)), x)
```

3.320 $\int (fx)^m (d+ex^2) (a+b\log(cx^n)) \, dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int (fx)^m (d+ex^2) (a+b\log(cx^n)) \, dx = -\frac{bdn(fx)^{1+m}}{f(1+m)^2} - \frac{ben(fx)^{3+m}}{f^3(3+m)^2} + \frac{d(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)} + \frac{e(fx)^{3+m}(a+b\log(cx^n))}{f^3(3+m)}$$

```
[Out] -b*d*n*(f*x)^(1+m)/f/(1+m)^2-b*e*n*(f*x)^(3+m)/f^3/(3+m)^2+d*(f*x)^(1+m)*(a
+b*ln(c*x^n))/f/(1+m)+e*(f*x)^(3+m)*(a+b*ln(c*x^n))/f^3/(3+m)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {14, 2392}

$$\int (fx)^m (d+ex^2) (a+b\log(cx^n)) \, dx = \frac{d(fx)^{m+1}(a+b\log(cx^n))}{f(m+1)} + \frac{e(fx)^{m+3}(a+b\log(cx^n))}{f^3(m+3)} - \frac{bdn(fx)^{m+1}}{f(m+1)^2} - \frac{ben(fx)^{m+3}}{f^3(m+3)^2}$$

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*Log[c*x^n]),x]
```

```
[Out] -((b*d*n*(f*x)^(1 + m))/(f*(1 + m)^2)) - (b*e*n*(f*x)^(3 + m))/(f^3*(3 + m)
^2) + (d*(f*x)^(1 + m)*(a + b*Log[c*x^n]))/(f*(1 + m)) + (e*(f*x)^(3 + m)*(
a + b*Log[c*x^n]))/(f^3*(3 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{d(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\log\left(cx^n\right)\right)}{f^3(3+m)} \\
&\quad -(bn)\int(fx)^m\left(\frac{d}{1+m}+\frac{ex^2}{3+m}\right)dx \\
&= \frac{d(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\log\left(cx^n\right)\right)}{f^3(3+m)} \\
&\quad -(bn)\int\left(\frac{d(fx)^m}{1+m}+\frac{e(fx)^{2+m}}{f^2(3+m)}\right)dx \\
&= -\frac{bdn(fx)^{1+m}}{f(1+m)^2}-\frac{ben(fx)^{3+m}}{f^3(3+m)^2}+\frac{d(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\log\left(cx^n\right)\right)}{f^3(3+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$
\int(fx)^m\left(d+ex^2\right)\left(a+b\log\left(cx^n\right)\right)dx=x(fx)^m\left(-\frac{bdn}{(1+m)^2}-\frac{benx^2}{(3+m)^2}+\frac{d\left(a+b\log\left(cx^n\right)\right)}{1+m}+\frac{ex^2\left(a+b\log\left(cx^n\right)\right)}{3+m}\right)
$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*Log[c*x^n]),x]
[Out] x*(f*x)^m*(-((b*d*n)/(1 + m)^2) - (b*e*n*x^2)/(3 + m)^2 + (d*(a + b*Log[c*x
^n]))/(1 + m) + (e*x^2*(a + b*Log[c*x^n]))/(3 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(95) = 190$.

Time = 0.90 (sec) , antiderivative size = 343, normalized size of antiderivative = 3.61

method	result
parallelrisch	$-\frac{-3x^3(fx)^m ae-9x(fx)^m ad-x(fx)^m ad\,m^3-7x(fx)^m ad\,m^2-15x(fx)^m adm+9x(fx)^m bdn-9x(fx)^m \ln(c\,x^n)bd-x^3(fx)^m a}{}$
risch	Expression too large to display

```
[In] int((f*x)^m*(e*x^2+d)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -(-3*x^3*(f*x)^m*a*e-9*x*(f*x)^m*a*d-x*(f*x)^m*a*d*m^3-7*x*(f*x)^m*a*d*m^2-
15*x*(f*x)^m*a*d*m+9*x*(f*x)^m*b*d*n-9*x*(f*x)^m*ln(c*x^n)*b*d-x^3*(f*x)^m*
a*e*m^3-5*x^3*(f*x)^m*a*e*m^2-7*x^3*(f*x)^m*a*e*m+x^3*(f*x)^m*b*e*n-3*x^3*(
f*x)^m*ln(c*x^n)*b*e-x*(f*x)^m*ln(c*x^n)*b*d*m^3-7*x*(f*x)^m*ln(c*x^n)*b*d*
m^2+x*(f*x)^m*b*d*m^2*n-15*x*(f*x)^m*ln(c*x^n)*b*d*m+6*x*(f*x)^m*b*d*m*n-x^
3*(f*x)^m*ln(c*x^n)*b*e*m^3-5*x^3*(f*x)^m*ln(c*x^n)*b*e*m^2+x^3*(f*x)^m*b*e
*m^2*n-7*x^3*(f*x)^m*ln(c*x^n)*b*e*m+2*x^3*(f*x)^m*b*e*m*n)/(3+m)^2/(1+m)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(95) = 190$.

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.47

$$\int (fx)^m \left(d+ex^2\right) (a+b\log(cx^n))\,dx$$
$$=\frac{((aem^3+5\,aem^2+7\,aem+3\,ae-(bem^2+2\,bem+be)n)x^3+(adm^3+7\,adm^2+15\,adm+9\,ad-(bdm^2}{}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] ((a*e*m^3 + 5*a*e*m^2 + 7*a*e*m + 3*a*e - (b*e*m^2 + 2*b*e*m + b*e)*n)*x^3
+ (a*d*m^3 + 7*a*d*m^2 + 15*a*d*m + 9*a*d - (b*d*m^2 + 6*b*d*m + 9*b*d)*n)*
x + ((b*e*m^3 + 5*b*e*m^2 + 7*b*e*m + 3*b*e)*x^3 + (b*d*m^3 + 7*b*d*m^2 + 1
5*b*d*m + 9*b*d)*x)*log(c) + ((b*e*m^3 + 5*b*e*m^2 + 7*b*e*m + 3*b*e)*n*x^3
 + (b*d*m^3 + 7*b*d*m^2 + 15*b*d*m + 9*b*d)*n*x)*log(x))*e^(m*log(f) + m*lo
g(x))/(m^4 + 8*m^3 + 22*m^2 + 24*m + 9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(87) = 174$.

Time = 2.71 (sec) , antiderivative size = 920, normalized size of antiderivative = 9.68

$$\int (fx)^m (d+ex^2) (a+b\log(cx^n))\, dx$$
$$= \begin{cases} \frac{-\frac{ad}{2x^2}+ae\log(x)+bd\left(-\frac{n}{4x^2}-\frac{\log(cx^n)}{2x^2}\right)-be\left(\begin{cases}-\log(c)\log(x) & \text{for } n=0\\ -\frac{\log(cx^n)^2}{2n} & \text{otherwise}\end{cases}\right)}{f^3} \\ \frac{\frac{ad\log(cx^n)}{n}+\frac{aex^2}{2}+\frac{bd\log(cx^n)^2}{2n}-\frac{benx^2}{4}+\frac{bex^2\log(cx^n)}{2}}{f} \\ \frac{adm^3x(fx)^m}{m^4+8m^3+22m^2+24m+9}+\frac{7adm^2x(fx)^m}{m^4+8m^3+22m^2+24m+9}+\frac{15admx(fx)^m}{m^4+8m^3+22m^2+24m+9}+\frac{9adx(fx)^m}{m^4+8m^3+22m^2+24m+9}+\frac{aem^3x^3(fx)}{m^4+8m^3+22m^2+} \end{cases}$$

```
[In] integrate((f*x)**m*(e*x**2+d)*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise(((-a*d/(2*x**2) + a*e*log(x) + b*d*(-n/(4*x**2) - log(c*x**n)/(2*
x**2)) - b*e*Piecewise((-log(c)*log(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n),
True)))/f**3, Eq(m, -3)), ((a*d*log(c*x**n)/n + a*e*x**2/2 + b*d*log(c*x**n
)**2/(2*n) - b*e*n*x**2/4 + b*e*x**2*log(c*x**n)/2)/f, Eq(m, -1)), (a*d*m**
3*x*(f*x)**m/(m**4 + 8*m**3 + 22*m**2 + 24*m + 9) + 7*a*d*m**2*x*(f*x)**m/(
m**4 + 8*m**3 + 22*m**2 + 24*m + 9) + 15*a*d*m*x*(f*x)**m/(m**4 + 8*m**3 +
22*m**2 + 24*m + 9) + 9*a*d*x*(f*x)**m/(m**4 + 8*m**3 + 22*m**2 + 24*m + 9)
 + a*e*m**3*x**3*(f*x)**m/(m**4 + 8*m**3 + 22*m**2 + 24*m + 9) + 5*a*e*m**2
*x**3*(f*x)**m/(m**4 + 8*m**3 + 22*m**2 + 24*m + 9) + 7*a*e*m*x**3*(f*x)**m
/(m**4 + 8*m**3 + 22*m**2 + 24*m + 9) + 3*a*e*x**3*(f*x)**m/(m**4 + 8*m**3
+ 22*m**2 + 24*m + 9) + b*d*m**3*x*(f*x)**m*log(c*x**n)/(m**4 + 8*m**3 + 22
*m**2 + 24*m + 9) - b*d*m**2*n*x*(f*x)**m/(m**4 + 8*m**3 + 22*m**2 + 24*m +
 9) + 7*b*d*m**2*x*(f*x)**m*log(c*x**n)/(m**4 + 8*m**3 + 22*m**2 + 24*m + 9
) - 6*b*d*m*n*x*(f*x)**m/(m**4 + 8*m**3 + 22*m**2 + 24*m + 9) + 15*b*d*m*x*
(f*x)**m*log(c*x**n)/(m**4 + 8*m**3 + 22*m**2 + 24*m + 9) - 9*b*d*n*x*(f*x)
**m/(m**4 + 8*m**3 + 22*m**2 + 24*m + 9) + 9*b*d*x*(f*x)**m*log(c*x**n)/(m*
*4 + 8*m**3 + 22*m**2 + 24*m + 9) + b*e*m**3*x**3*(f*x)**m*log(c*x**n)/(m**
4 + 8*m**3 + 22*m**2 + 24*m + 9) - b*e*m**2*n*x**3*(f*x)**m/(m**4 + 8*m**3
+ 22*m**2 + 24*m + 9) + 5*b*e*m**2*x**3*(f*x)**m*log(c*x**n)/(m**4 + 8*m**3
 + 22*m**2 + 24*m + 9) - 2*b*e*m*n*x**3*(f*x)**m/(m**4 + 8*m**3 + 22*m**2 +
 24*m + 9) + 7*b*e*m*x**3*(f*x)**m*log(c*x**n)/(m**4 + 8*m**3 + 22*m**2 + 2
4*m + 9) - b*e*n*x**3*(f*x)**m/(m**4 + 8*m**3 + 22*m**2 + 24*m + 9) + 3*b*e
*x**3*(f*x)**m*log(c*x**n)/(m**4 + 8*m**3 + 22*m**2 + 24*m + 9), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25

$$\int (fx)^m \left(d+ex^2\right) (a+b\log(cx^n)) \, dx = \frac{bef^m x^3 x^m \log(cx^n)}{m+3} + \frac{aef^m x^3 x^m}{m+3} - \frac{bef^m n x^3 x^m}{(m+3)^2} - \frac{bdf^m n x x^m}{(m+1)^2} + \frac{(fx)^{m+1} bd \log(cx^n)}{f(m+1)} + \frac{(fx)^{m+1} ad}{f(m+1)}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] b*e*f^m*x^3*x^m*log(c*x^n)/(m + 3) + a*e*f^m*x^3*x^m/(m + 3) - b*e*f^m*n*x^
3*x^m/(m + 3)^2 - b*d*f^m*n*x*x^m/(m + 1)^2 + (f*x)^(m + 1)*b*d*log(c*x^n)/
(f*(m + 1)) + (f*x)^(m + 1)*a*d/(f*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(95) = 190.

Time = 0.34 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.46

$$\int (fx)^m \left(d+ex^2\right) (a+b\log(cx^n)) \, dx = \frac{bef^2 f^m x^3 x^m \log(c)}{f^2 m + 3f^2} + \frac{bef^m m n x^3 x^m \log(x)}{m^2 + 6m + 9} + \frac{aef^2 f^m x^3 x^m}{f^2 m + 3f^2} + \frac{3bef^m n x^3 x^m \log(x)}{m^2 + 6m + 9} - \frac{bef^m n x^3 x^m}{m^2 + 6m + 9} + \frac{bdf^m m n x x^m \log(x)}{m^2 + 2m + 1} + \frac{bdf^m n x x^m \log(x)}{m^2 + 2m + 1} - \frac{bdf^m n x x^m}{m^2 + 2m + 1} + \frac{(fx)^m bdx \log(c)}{m+1} + \frac{(fx)^m adx}{m+1}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*e*f^2*f^m*x^3*x^m*log(c)/(f^2*m + 3*f^2) + b*e*f^m*m*n*x^3*x^m*log(x)/(m^
2 + 6*m + 9) + a*e*f^2*f^m*x^3*x^m/(f^2*m + 3*f^2) + 3*b*e*f^m*n*x^3*x^m*lo
g(x)/(m^2 + 6*m + 9) - b*e*f^m*n*x^3*x^m/(m^2 + 6*m + 9) + b*d*f^m*m*n*x*x^
m*log(x)/(m^2 + 2*m + 1) + b*d*f^m*n*x*x^m*log(x)/(m^2 + 2*m + 1) - b*d*f^m
*n*x*x^m/(m^2 + 2*m + 1) + (f*x)^m*b*d*x*log(c)/(m + 1) + (f*x)^m*a*d*x/(m
+ 1)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right) (a+b\log \left(cx^n\right)) \, dx = \int {\left(f\,x\right)}^m\,\left(e\,x^2+d\right)\,\left(a+b\,\ln \left(c\,x^n\right)\right) \,d x$$

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*log(c*x^n)),x)
[Out] int((f*x)^m*(d + e*x^2)*(a + b*log(c*x^n)), x)
```

3.321 $\int (fx)^m (a + b\log (cx^n))\ dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int (fx)^m (a + b\log (cx^n))\ dx = -\frac{bn(fx)^{1+m}}{f(1+m)^2} + \frac{(fx)^{1+m}(a + b\log (cx^n))}{f(1+m)}$$

```
[Out] -b*n*(f*x)^(1+m)/f/(1+m)^2+(f*x)^(1+m)*(a+b*ln(c*x^n))/f/(1+m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2341}

$$\int (fx)^m (a + b\log (cx^n))\ dx = \frac{(fx)^{m+1}(a + b\log (cx^n))}{f(m+1)} - \frac{bn(fx)^{m+1}}{f(m+1)^2}$$

```
[In] Int[(f*x)^m*(a + b*Log[c*x^n]),x]
```

```
[Out] -((b*n*(f*x)^(1 + m))/(f*(1 + m)^2)) + ((f*x)^(1 + m)*(a + b*Log[c*x^n]))/(
f*(1 + m))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{bn(fx)^{1+m}}{f(1+m)^2} + \frac{(fx)^{1+m}(a + b\log (cx^n))}{f(1+m)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int (fx)^m (a+b\log(cx^n))\,dx=\frac{x(fx)^m(a+am-bn+b(1+m)\log(cx^n))}{(1+m)^2}$$

```
[In] Integrate[(f*x)^m*(a + b*Log[c*x^n]),x]
[Out] (x*(f*x)^m*(a + a*m - b*n + b*(1 + m)*Log[c*x^n]))/(1 + m)^2
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

method	result
parallelrisch	$-\frac{-x(fx)^m\ln(cx^n)bm-x(fx)^m\ln(cx^n)b-x(fx)^m am+x(fx)^m bn-x(fx)^m a}{(1+m)^2}$
risch	$\frac{bx\,x^m f^m \mathrm{e}^{\frac{i\,\mathrm{csgn}(ifx)\pi m(\mathrm{csgn}(ifx)-\mathrm{csgn}(ix))(-\mathrm{csgn}(ifx)+\mathrm{csgn}(if))}{2}}\ln(x^n)}{1+m}-\frac{\left(i\pi b\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)m-i\pi b\,\mathrm{csgn}(i\right.}{}$

```
[In] int((f*x)^m*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] -(-x*(f*x)^m*ln(c*x^n)*b*m-x*(f*x)^m*ln(c*x^n)*b-x*(f*x)^m*a*m+x*(f*x)^m*b*
n-x*(f*x)^m*a)/(1+m)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int (fx)^m (a+b\log(cx^n))\,dx=\frac{((bm+b)nx\log(x)+(bm+b)x\log(c)+(am-bn+a)x)e^{(m\log(f)+m\log(x))}}{m^2+2\,m+1}$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] ((b*m + b)*n*x*log(x) + (b*m + b)*x*log(c) + (a*m - b*n + a)*x)*e^(m*log(f)
 + m*log(x))/(m^2 + 2*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(37) = 74$.

Time = 2.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.07

$$\int (fx)^m (a + b\log(cx^n))\,dx = \begin{cases} \frac{amx(fx)^m}{m^2+2m+1} + \frac{ax(fx)^m}{m^2+2m+1} + \frac{bmx(fx)^m\log(cx^n)}{m^2+2m+1} - \frac{bnx(fx)^m}{m^2+2m+1} + \frac{bx(fx)^m\log(cx^n)}{m^2+2m+1} & \text{for } m \neq -1 \\ \frac{\begin{cases} a\log(x) & \text{for } b = 0 \\ -(-a - b\log(c))\log(x) & \text{for } n = 0 \\ \frac{(-a-b\log(cx^n))^2}{2bn} & \text{otherwise} \end{cases}}{f} & \text{otherwise} \end{cases}$$

```
[In] integrate((f*x)**m*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((a*m*x*(f*x)**m/(m**2 + 2*m + 1) + a*x*(f*x)**m/(m**2 + 2*m + 1)
+ b*m*x*(f*x)**m*log(c*x**n)/(m**2 + 2*m + 1) - b*n*x*(f*x)**m/(m**2 + 2*m
+ 1) + b*x*(f*x)**m*log(c*x**n)/(m**2 + 2*m + 1), Ne(m, -1)), (Piecewise((a
*log(x), Eq(b, 0)), (-(-a - b*log(c))*log(x), Eq(n, 0)), ((-a - b*log(c*x**
n))**2/(2*b*n), True))/f, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int (fx)^m (a + b\log(cx^n))\,dx = -\frac{bf^m nxx^m}{(m+1)^2} + \frac{(fx)^{m+1} b\log(cx^n)}{f(m+1)} + \frac{(fx)^{m+1} a}{f(m+1)}$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -b*f^m*n*x*x^m/(m + 1)^2 + (f*x)^(m + 1)*b*log(c*x^n)/(f*(m + 1)) + (f*x)^(
m + 1)*a/(f*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(46) = 92$.

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int (fx)^m (a + b\log(cx^n))\,dx = \frac{bf^m mnxx^m\log(x)}{m^2+2m+1} + \frac{bf^m nxx^m\log(x)}{m^2+2m+1} - \frac{bf^m nxx^m}{m^2+2m+1} + \frac{(fx)^m bx\log(c)}{m+1} + \frac{(fx)^m ax}{m+1}$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*f^m*m*n*x*x^m*log(x)/(m^2 + 2*m + 1) + b*f^m*n*x*x^m*log(x)/(m^2 + 2*m +
1) - b*f^m*n*x*x^m/(m^2 + 2*m + 1) + (f*x)^m*b*x*log(c)/(m + 1) + (f*x)^m*a
*x/(m + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (a + b\log(cx^n))\,dx = \int (f\,x)^m\,(a + b\,\ln(c\,x^n))\,dx$$

```
[In] int((f*x)^m*(a + b*log(c*x^n)),x)
```

```
[Out] int((f*x)^m*(a + b*log(c*x^n)), x)
```

3.322 $\int \frac{(fx)^m(a+b\log(cx^n))}{d+ex^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m\,(a+b\log\left(cx^n\right))}{d+ex^2}\,dx=\operatorname{Int}\left(\frac{(fx)^m\,(a+b\log\left(cx^n\right))}{d+ex^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*ln(c*x^n))/(e*x^2+d),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\,(a+b\log\left(cx^n\right))}{d+ex^2}\,dx=\int \frac{(fx)^m\,(a+b\log\left(cx^n\right))}{d+ex^2}\,dx$$

```
[In] Int[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x^2),x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{(fx)^m\,(a+b\log\left(cx^n\right))}{d+ex^2}\,dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 108 vs. $2(28) = 56$.

Time = 0.70 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.32

$$\int \frac{(fx)^m (a + b\log(cx^n))}{d + ex^2} dx = \frac{x(fx)^m \left(-bn\,{}_3F_2\left(1, \frac{1}{2} + \frac{m}{2}, \frac{1}{2} + \frac{m}{2}; \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}; -\frac{ex^2}{d}\right) + (1+m)\,\text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, -\right.\right.}{d(1+m)^2}$$

```
[In] Integrate[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x^2),x]
```

```
[Out] (x*(f*x)^m*(-(b*n*HypergeometricPFQ[{1, 1/2 + m/2, 1/2 + m/2}, {3/2 + m/2,
3/2 + m/2}, -((e*x^2)/d)]) + (1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m
)/2, -((e*x^2)/d)]*(a + b*Log[c*x^n])))/(d*(1 + m)^2)
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b\ln(c\,x^n))}{e\,x^2 + d} dx$$

```
[In] int((f*x)^m*(a+b*ln(c*x^n))/(e*x^2+d),x)
```

```
[Out] int((f*x)^m*(a+b*ln(c*x^n))/(e*x^2+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{(fx)^m (a + b\log(cx^n))}{d + ex^2} dx = \int \frac{(b\log(cx^n) + a)(fx)^m}{ex^2 + d} dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(((f*x)^m*b*log(c*x^n) + (f*x)^m*a)/(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 7.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(fx)^m (a + b \log(cx^n))}{d + ex^2} \, dx = \int \frac{(fx)^m (a + b \log(cx^n))}{d + ex^2} \, dx$$

```
[In] integrate((f*x)**m*(a+b*ln(c*x**n))/(e*x**2+d),x)
[Out] Integral((f*x)**m*(a + b*log(c*x**n))/(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m (a + b \log(cx^n))}{d + ex^2} \, dx = \int \frac{(b \log(cx^n) + a)(fx)^m}{ex^2 + d} \, dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*(f*x)^m/(e*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m (a + b \log(cx^n))}{d + ex^2} \, dx = \int \frac{(b \log(cx^n) + a)(fx)^m}{ex^2 + d} \, dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*(f*x)^m/(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m (a+b\log(cx^n))}{d+ex^2} \, dx = \int \frac{(f\,x)^m\,(a+b\,\ln(c\,x^n))}{e\,x^2+d} \, dx$$

```
[In] int(((f*x)^m*(a + b*log(c*x^n)))/(d + e*x^2),x)
[Out] int(((f*x)^m*(a + b*log(c*x^n)))/(d + e*x^2), x)
```

3.323 $\int \frac{(fx)^m(a+b\log(cx^n))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m\,(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \text{Int}\left(\frac{(fx)^m\,(a+b\log(cx^n))}{(d+ex^2)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*ln(c*x^n))/(e*x^2+d)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\,(a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \int \frac{(fx)^m\,(a+b\log(cx^n))}{(d+ex^2)^2}\,dx$$

```
[In] Int[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x^2)^2,x]
[Out] Defer[Int][((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x^2)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(fx)^m\,(a+b\log(cx^n))}{(d+ex^2)^2}\,dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 108 vs. $2(28) = 56$.

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.32

$$\int \frac{(fx)^m (a + b\log(cx^n))}{(d + ex^2)^2} dx$$
$$= \frac{x(fx)^m \left(-bn\,{}_3F_2\left(2, \frac{1}{2} + \frac{m}{2}, \frac{1}{2} + \frac{m}{2}; \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}; -\frac{ex^2}{d}\right) + (1+m)\,\text{Hypergeometric2F1}\left(2, \frac{1+m}{2}, \frac{3+m}{2}, -\right.\right.}{d^2(1+m)^2}$$

```
[In] Integrate[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x^2)^2,x]
```

```
[Out] (x*(f*x)^m*(-(b*n*HypergeometricPFQ[{2, 1/2 + m/2, 1/2 + m/2}, {3/2 + m/2,
3/2 + m/2}, -((e*x^2)/d)]) + (1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m
)/2, -((e*x^2)/d)]*(a + b*Log[c*x^n])))/(d^2*(1 + m)^2)
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b\ln(c\,x^n))}{(e\,x^2 + d)^2} dx$$

```
[In] int((f*x)^m*(a+b*ln(c*x^n))/(e*x^2+d)^2,x)
```

```
[Out] int((f*x)^m*(a+b*ln(c*x^n))/(e*x^2+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{(fx)^m (a + b\log(cx^n))}{(d + ex^2)^2} dx = \int \frac{(b\log(cx^n) + a)(fx)^m}{(ex^2 + d)^2} dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral(((f*x)^m*b*log(c*x^n) + (f*x)^m*a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 162.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m (a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \int \frac{(fx)^m (a+b\log(cx^n))}{(d+ex^2)^2}\,dx$$

```
[In] integrate((f*x)**m*(a+b*ln(c*x**n))/(e*x**2+d)**2,x)
[Out] Integral((f*x)**m*(a + b*log(c*x**n))/(d + e*x**2)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m (a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \int \frac{(b\log(cx^n)+a)(fx)^m}{(ex^2+d)^2}\,dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m (a+b\log(cx^n))}{(d+ex^2)^2}\,dx = \int \frac{(b\log(cx^n)+a)(fx)^m}{(ex^2+d)^2}\,dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m (a+b\log(cx^n))}{(d+ex^2)^2} dx = \int \frac{(f\,x)^m \left(a+b\,\ln\left(c\,x^n\right)\right)}{\left(e\,x^2+d\right)^2} dx$$

```
[In] int(((f*x)^m*(a + b*log(c*x^n)))/(d + e*x^2)^2,x)
[Out] int(((f*x)^m*(a + b*log(c*x^n)))/(d + e*x^2)^2, x)
```

3.324 $\int \frac{(a+b\log(cx^n))^3}{(d+ex^3)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 1198

$$
\begin{aligned}
\int \frac{(a+b\log(cx^n))^3}{(d+ex^3)^2}\,dx &= \frac{x(a+b\log(cx^n))^3}{9d^{5/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} - \frac{\sqrt[3]{-1}x(a+b\log(cx^n))^3}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)} \\
&\quad + \frac{x(a+b\log(cx^n))^3}{9d^{5/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)} \\
&\quad - \frac{bn(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{3d^{5/3}\sqrt[3]{e}} \\
&\quad + \frac{2(a+b\log(cx^n))^3\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}} \\
&\quad + \frac{3\sqrt[3]{-1}bn(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}} \\
&\quad - \frac{2i\sqrt{3}(a+b\log(cx^n))^3\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}} \\
&\quad + \frac{\sqrt[3]{-1}bn(a+b\log(cx^n))^2\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{3d^{5/3}\sqrt[3]{e}} \\
&\quad + \frac{2(a+b\log(cx^n))^3\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}} \\
&\quad - \frac{2b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{3d^{5/3}\sqrt[3]{e}} \\
&\quad + \frac{2bn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{3d^{5/3}\sqrt[3]{e}} \\
&\quad + \frac{6\sqrt[3]{-1}b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}} \\
&\quad - \frac{6i\sqrt{3}bn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}} \\
&\quad + \frac{2\sqrt[3]{-1}b^2n^2(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{3d^{5/3}\sqrt[3]{e}} \\
&\quad + \frac{6bn(a+b\log(cx^n))^2\operatorname{PolyLog}\left(2,-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}}
\end{aligned}
$$

```
[Out] 1/9*x*(a+b*ln(c*x^n))^3/d^(5/3)/(d^(1/3)+e^(1/3)*x)-(-1)^(1/3)*x*(a+b*ln(c*
x^n))^3/(1+(-1)^(1/3))^4/d^(5/3)/((-1)^(2/3)*d^(1/3)+e^(1/3)*x)+1/9*x*(a+b*
ln(c*x^n))^3/d^(5/3)/(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)-1/3*b*n*(a+b*ln(c*x^n))
^2*ln(1+e^(1/3)*x/d^(1/3))/d^(5/3)/e^(1/3)+2/9*(a+b*ln(c*x^n))^3*ln(1+e^(1/
3)*x/d^(1/3))/d^(5/3)/e^(1/3)+3*(-1)^(1/3)*b*n*(a+b*ln(c*x^n))^2*ln(1-(-1)^
(1/3)*e^(1/3)*x/d^(1/3))/(1+(-1)^(1/3))^4/d^(5/3)/e^(1/3)+1/3*(-1)^(1/3)*b*
n*(a+b*ln(c*x^n))^2*ln(1+(-1)^(2/3)*e^(1/3)*x/d^(1/3))/d^(5/3)/e^(1/3)-2/3*
b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-e^(1/3)*x/d^(1/3))/d^(5/3)/e^(1/3)+2/3*b
*n*(a+b*ln(c*x^n))^2*polylog(2,-e^(1/3)*x/d^(1/3))/d^(5/3)/e^(1/3)+6*(-1)^(
1/3)*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,(-1)^(1/3)*e^(1/3)*x/d^(1/3))/(1+(-1
)^(1/3))^4/d^(5/3)/e^(1/3)+2/3*(-1)^(1/3)*b^2*n^2*(a+b*ln(c*x^n))*polylog(2
,-(-1)^(2/3)*e^(1/3)*x/d^(1/3))/d^(5/3)/e^(1/3)+2/3*b^3*n^3*polylog(3,-e^(1
/3)*x/d^(1/3))/d^(5/3)/e^(1/3)-4/3*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-e^(1/
3)*x/d^(1/3))/d^(5/3)/e^(1/3)-6*(-1)^(1/3)*b^3*n^3*polylog(3,(-1)^(1/3)*e^(
1/3)*x/d^(1/3))/(1+(-1)^(1/3))^4/d^(5/3)/e^(1/3)-2/3*(-1)^(1/3)*b^3*n^3*pol
ylog(3,-(-1)^(2/3)*e^(1/3)*x/d^(1/3))/d^(5/3)/e^(1/3)+4/3*b^3*n^3*polylog(4
,-e^(1/3)*x/d^(1/3))/d^(5/3)/e^(1/3)-4/9*(a+b*ln(c*x^n))^3*ln(1-1/2*e^(1/3)
*x*(1-I*3^(1/2))/d^(1/3))/d^(5/3)/e^(1/3)/(1-I*3^(1/2))-4/3*b*n*(a+b*ln(c*x
^n))^2*polylog(2,1/2*e^(1/3)*x*(1-I*3^(1/2))/d^(1/3))/d^(5/3)/e^(1/3)/(1-I*
3^(1/2))+8/3*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,1/2*e^(1/3)*x*(1-I*3^(1/2))/
d^(1/3))/d^(5/3)/e^(1/3)/(1-I*3^(1/2))-8/3*b^3*n^3*polylog(4,1/2*e^(1/3)*x*
(1-I*3^(1/2))/d^(1/3))/d^(5/3)/e^(1/3)/(1-I*3^(1/2))-4/9*(a+b*ln(c*x^n))^3*
ln(1-1/2*e^(1/3)*x*(1+I*3^(1/2))/d^(1/3))/d^(5/3)/e^(1/3)/(1+I*3^(1/2))-4/3
*b*n*(a+b*ln(c*x^n))^2*polylog(2,1/2*e^(1/3)*x*(1+I*3^(1/2))/d^(1/3))/d^(5/
3)/e^(1/3)/(1+I*3^(1/2))+8/3*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,1/2*e^(1/3)*
x*(1+I*3^(1/2))/d^(1/3))/d^(5/3)/e^(1/3)/(1+I*3^(1/2))-8/3*b^3*n^3*polylog(
4,1/2*e^(1/3)*x*(1+I*3^(1/2))/d^(1/3))/d^(5/3)/e^(1/3)/(1+I*3^(1/2))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 1198, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used

$= \{2367, 2355, 2354, 2421, 6724, 2430\}$

$$\int \frac{(a+b\log(cx^n))^3}{(d+ex^3)^2}\,dx = \frac{2b^3\,\mathrm{PolyLog}\left(3,-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^3}{3d^{5/3}\sqrt[3]{e}}$$

$$-\frac{6\sqrt[3]{-1}b^3\,\mathrm{PolyLog}\left(3,\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^3}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}}$$

$$-\frac{2\sqrt[3]{-1}b^3\,\mathrm{PolyLog}\left(3,-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^3}{3d^{5/3}\sqrt[3]{e}}$$

$$+\frac{4b^3\,\mathrm{PolyLog}\left(4,-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^3}{3d^{5/3}\sqrt[3]{e}}$$

$$-\frac{12i\sqrt{3}b^3\,\mathrm{PolyLog}\left(4,\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^3}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$

$$+\frac{12b^3\,\mathrm{PolyLog}\left(4,-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^3}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}}$$

$$-\frac{2b^2(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^2}{3d^{5/3}\sqrt[3]{e}}$$

$$+\frac{6\sqrt[3]{-1}b^2(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^2}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}}$$

$$+\frac{2\sqrt[3]{-1}b^2(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^2}{3d^{5/3}\sqrt[3]{e}}$$

$$-\frac{4b^2(a+b\log(cx^n))\,\mathrm{PolyLog}\left(3,-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^2}{3d^{5/3}\sqrt[3]{e}}$$

$$+\frac{12i\sqrt{3}b^2(a+b\log(cx^n))\,\mathrm{PolyLog}\left(3,\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^2}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$

$$-\frac{12b^2(a+b\log(cx^n))\,\mathrm{PolyLog}\left(3,-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^2}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}}$$

$$-\frac{b(a+b\log(cx^n))^2\log\left(\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}+1\right)n}{3d^{5/3}\sqrt[3]{e}}$$

$$+\frac{3\sqrt[3]{-1}b(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}}$$

```
[In] Int[(a + b*Log[c*x^n])^3/(d + e*x^3)^2,x]
```

```
[Out] (x*(a + b*Log[c*x^n])^3)/(9*d^(5/3)*(d^(1/3) + e^(1/3)*x)) - ((-1)^(1/3)*x*
(a + b*Log[c*x^n])^3)/((1 + (-1)^(1/3))^4*d^(5/3)*((-1)^(2/3)*d^(1/3) + e^(
1/3)*x)) + (x*(a + b*Log[c*x^n])^3)/(9*d^(5/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3
)*x)) - (b*n*(a + b*Log[c*x^n])^2*Log[1 + (e^(1/3)*x)/d^(1/3)])/(3*d^(5/3)*
e^(1/3)) + (2*(a + b*Log[c*x^n])^3*Log[1 + (e^(1/3)*x)/d^(1/3)])/(9*d^(5/3)
*e^(1/3)) + (3*(-1)^(1/3)*b*n*(a + b*Log[c*x^n])^2*Log[1 - ((-1)^(1/3)*e^(1
/3)*x)/d^(1/3)])/((1 + (-1)^(1/3))^4*d^(5/3)*e^(1/3)) - ((2*I)*Sqrt[3]*(a +
 b*Log[c*x^n])^3*Log[1 - ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)])/((1 + (-1)^(1/3))
^5*d^(5/3)*e^(1/3)) + ((-1)^(1/3)*b*n*(a + b*Log[c*x^n])^2*Log[1 + ((-1)^(2
/3)*e^(1/3)*x)/d^(1/3)])/(3*d^(5/3)*e^(1/3)) + (2*(a + b*Log[c*x^n])^3*Log[
1 + ((-1)^(2/3)*e^(1/3)*x)/d^(1/3)])/((1 + (-1)^(1/3))^4*d^(5/3)*e^(1/3)) -
 (2*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((e^(1/3)*x)/d^(1/3))])/(3*d^(5/
3)*e^(1/3)) + (2*b*n*(a + b*Log[c*x^n])^2*PolyLog[2, -((e^(1/3)*x)/d^(1/3))
])/(3*d^(5/3)*e^(1/3)) + (6*(-1)^(1/3)*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2
, ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)])/((1 + (-1)^(1/3))^4*d^(5/3)*e^(1/3)) - (
(6*I)*Sqrt[3]*b*n*(a + b*Log[c*x^n])^2*PolyLog[2, ((-1)^(1/3)*e^(1/3)*x)/d^
(1/3)])/((1 + (-1)^(1/3))^5*d^(5/3)*e^(1/3)) + (2*(-1)^(1/3)*b^2*n^2*(a + b
*Log[c*x^n])*PolyLog[2, -(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))])/(3*d^(5/3)*e^(1
/3)) + (6*b*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(((-1)^(2/3)*e^(1/3)*x)/d^(1
/3))])/((1 + (-1)^(1/3))^4*d^(5/3)*e^(1/3)) + (2*b^3*n^3*PolyLog[3, -((e^(1
/3)*x)/d^(1/3))])/(3*d^(5/3)*e^(1/3)) - (4*b^2*n^2*(a + b*Log[c*x^n])*PolyL
og[3, -((e^(1/3)*x)/d^(1/3))])/(3*d^(5/3)*e^(1/3)) - (6*(-1)^(1/3)*b^3*n^3*
PolyLog[3, ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)])/((1 + (-1)^(1/3))^4*d^(5/3)*e^(
1/3)) + ((12*I)*Sqrt[3]*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, ((-1)^(1/3)*e
^(1/3)*x)/d^(1/3)])/((1 + (-1)^(1/3))^5*d^(5/3)*e^(1/3)) - (2*(-1)^(1/3)*b^
3*n^3*PolyLog[3, -(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))])/(3*d^(5/3)*e^(1/3)) -
(12*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))
])/((1 + (-1)^(1/3))^4*d^(5/3)*e^(1/3)) + (4*b^3*n^3*PolyLog[4, -((e^(1/3)*
x)/d^(1/3))])/(3*d^(5/3)*e^(1/3)) - ((12*I)*Sqrt[3]*b^3*n^3*PolyLog[4, ((-1
)^(1/3)*e^(1/3)*x)/d^(1/3)])/((1 + (-1)^(1/3))^5*d^(5/3)*e^(1/3)) + (12*b^3
*n^3*PolyLog[4, -(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))])/((1 + (-1)^(1/3))^4*d^(
5/3)*e^(1/3))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
```

```
, p}, x] && GtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a+b\log(cx^n))^3}{9d^{4/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}+\frac{2(a+b\log(cx^n))^3}{9d^{5/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}+\frac{(-1)^{2/3}(a+b\log(cx^n))^3}{\left(1+\sqrt[3]{-1}\right)^4 d^{4/3}\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)^2}-\frac{2(-1)^{5/6}\sqrt{3}(a+b\log(cx^n))^3}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)}+\frac{(a+b\log(cx^n))^3}{\left(-1+\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4 d^{4/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)^2}+\frac{2(-1)^{2/3}(a+b\log(cx^n))^3}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}\right)dx$$

$$= \frac{2\int \frac{(a+b\log(cx^n))^3}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{9d^{5/3}} + \frac{2\int \frac{(a+b\log(cx^n))^3}{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}\,dx}{9d^{5/3}} - \frac{\left(2(-1)^{5/6}\sqrt{3}\right)\int \frac{(a+b\log(cx^n))^3}{-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x}\,dx}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}}$$

$$+ \frac{\int \frac{(a+b\log(cx^n))^3}{\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}\,dx}{9d^{4/3}} + \frac{\int \frac{(a+b\log(cx^n))^3}{\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)^2}\,dx}{9d^{4/3}} + \frac{\int \frac{(a+b\log(cx^n))^3}{\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)^2}\,dx}{9d^{4/3}}$$

$$= \frac{x(a+b\log(cx^n))^3}{9d^{5/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{(-1)^{2/3}x(a+b\log(cx^n))^3}{9d^{5/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}$$

$$+ \frac{x(a+b\log(cx^n))^3}{9d^{5/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)} + \frac{2(a+b\log(cx^n))^3\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

$$- \frac{2i\sqrt{3}(a+b\log(cx^n))^3\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$

$$- \frac{2\sqrt[3]{-1}(a+b\log(cx^n))^3\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

$$- \frac{(bn)\int \frac{(a+b\log(cx^n))^2}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{3d^{5/3}} + \frac{(bn)\int \frac{(a+b\log(cx^n))^2}{-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x}\,dx}{3d^{5/3}}$$

$$- \frac{(bn)\int \frac{(a+b\log(cx^n))^2}{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}\,dx}{3d^{5/3}} - \frac{(2bn)\int \frac{(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}\,dx}{3d^{5/3}\sqrt[3]{e}}$$

$$+ \frac{\left(2\sqrt[3]{-1}bn\right)\int \frac{(a+b\log(cx^n))^2\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}\,dx}{3d^{5/3}\sqrt[3]{e}}$$

$$+ \frac{\left(6i\sqrt{3}bn\right)\int \frac{(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}\,dx}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$

$$= \frac{x(a+b\log(cx^n))^3}{9d^{5/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{(-1)^{2/3}x(a+b\log(cx^n))^3}{9d^{5/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{x(a+b\log(cx^n))^3}{9d^{5/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}$$
$$- \frac{bn(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{3d^{5/3}\sqrt[3]{e}} + \frac{2(a+b\log(cx^n))^3\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$
$$- \frac{(-1)^{2/3}bn(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{3d^{5/3}\sqrt[3]{e}}$$
$$- \frac{2i\sqrt{3}(a+b\log(cx^n))^3\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$
$$+ \frac{\sqrt[3]{-1}bn(a+b\log(cx^n))^2\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{3d^{5/3}\sqrt[3]{e}}$$
$$- \frac{2\sqrt[3]{-1}(a+b\log(cx^n))^3\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$
$$+ \frac{2bn(a+b\log(cx^n))^2\operatorname{Li}_2\left(-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{3d^{5/3}\sqrt[3]{e}} - \frac{6i\sqrt{3}bn(a+b\log(cx^n))^2\operatorname{Li}_2\left(\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$
$$- \frac{2\sqrt[3]{-1}bn(a+b\log(cx^n))^2\operatorname{Li}_2\left(-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{3d^{5/3}\sqrt[3]{e}}$$
$$+ \frac{(2b^2n^2)\int\frac{(a+b\log(cx^n))\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}dx}{3d^{5/3}\sqrt[3]{e}} - \frac{(4b^2n^2)\int\frac{(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}dx}{3d^{5/3}\sqrt[3]{e}}$$
$$- \frac{\left(2\sqrt[3]{-1}b^2n^2\right)\int\frac{(a+b\log(cx^n))\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}dx}{3d^{5/3}\sqrt[3]{e}}$$
$$+ \frac{\left(4\sqrt[3]{-1}b^2n^2\right)\int\frac{(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}dx}{3d^{5/3}\sqrt[3]{e}}$$
$$+ \frac{\left(2(-1)^{2/3}b^2n^2\right)\int\frac{(a+b\log(cx^n))\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}dx}{3d^{5/3}\sqrt[3]{e}}$$
$$+ \frac{\left(12i\sqrt{3}b^2n^2\right)\int\frac{(a+b\log(cx^n))\operatorname{Li}_2\left(\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}dx}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$

= Too large to display

Mathematica [A] (verified)

Time = 7.31 (sec) , antiderivative size = 2215, normalized size of antiderivative = 1.85

$$\int \frac{(a+b\log(cx^n))^3}{(d+ex^3)^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Log[c*x^n])^3/(d + e*x^3)^2,x]
```

```
[Out] (x*(a + b*(-(n*Log[x]) + Log[c*x^n]))^3)/(3*d*(d + e*x^3)) + (2*ArcTan[(-d^
(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))]*(a + b*(-(n*Log[x]) + Log[c*x^n]))^
3)/(3*Sqrt[3]*d^(5/3)*e^(1/3)) + (2*(a + b*(-(n*Log[x]) + Log[c*x^n]))^3*Lo
g[d^(1/3) + e^(1/3)*x])/(9*d^(5/3)*e^(1/3)) - ((a + b*(-(n*Log[x]) + Log[c*
x^n]))^3*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(9*d^(5/3)*e^(1/3)
) + 3*b*n*(a + b*(-(n*Log[x]) + Log[c*x^n]))^2*(-1/3*((-1 + (-1)^(1/3))*((-
((-1)^(1/3)/d^(1/3)) - ((-1)^(2/3)*d^(1/3) + e^(1/3)*x)^(-1))*Log[x] + ((-1
)^(1/3)*Log[-((-1)^(2/3)*d^(1/3)) - e^(1/3)*x])/d^(1/3)))/((1 + (-1)^(1/3))
^2*d^(4/3)*e^(1/3)) + ((-1)^(1/3)*((d^(-1/3) - (d^(1/3) + e^(1/3)*x)^(-1))*
Log[x] - Log[d^(1/3) + e^(1/3)*x]/d^(1/3)))/(3*(1 + (-1)^(1/3))^2*d^(4/3)*e
^(1/3)) - (Log[x]/(e^(1/3)*((-1)^(1/3)*d^(1/3) - e^(1/3)*x)) - (-(((-1)^(2/
3)*Log[x])/d^(1/3)) + ((-1)^(2/3)*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/d^(1
/3))/e^(1/3))/(3*(1 + (-1)^(1/3))^2*d^(4/3)) + (2*(-1)^(1/3)*(Log[x]*Log[1
+ (e^(1/3)*x)/d^(1/3)] + PolyLog[2, -((e^(1/3)*x)/d^(1/3))]))/(3*(1 + (-1)^
(1/3))^2*d^(5/3)*e^(1/3)) - (2*(Log[x]*Log[1 - ((-1)^(1/3)*e^(1/3)*x)/d^(1/
3)] + PolyLog[2, ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)]))/(3*(1 + (-1)^(1/3))^2*d^
(5/3)*e^(1/3)) - (2*(-1 + (-1)^(1/3))*(Log[x]*Log[1 + ((-1)^(2/3)*e^(1/3)*x
)/d^(1/3)] + PolyLog[2, -(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))]))/(3*(1 + (-1)^(
1/3))^2*d^(5/3)*e^(1/3))) + 3*b^2*n^2*(a + b*(-(n*Log[x]) + Log[c*x^n]))*((
(-1)^(1/3)*(Log[x]*((e^(1/3)*x*Log[x])/(d^(1/3) + e^(1/3)*x) - 2*Log[1 + (e
^(1/3)*x)/d^(1/3)]) - 2*PolyLog[2, -((e^(1/3)*x)/d^(1/3))]))/(3*(1 + (-1)^(
1/3))^2*d^(5/3)*e^(1/3)) - ((-1 + (-1)^(1/3))*(Log[x]*((-((-1)^(1/3)/d^(1/3
)) - ((-1)^(2/3)*d^(1/3) + e^(1/3)*x)^(-1))*Log[x] + (2*(-1)^(1/3)*Log[1 -
((-1)^(1/3)*e^(1/3)*x)/d^(1/3)])/d^(1/3)) + (2*(-1)^(1/3)*PolyLog[2, ((-1)^
(1/3)*e^(1/3)*x)/d^(1/3)])/d^(1/3)))/(3*(1 + (-1)^(1/3))^2*d^(4/3)*e^(1/3))
 - (Log[x]*((-1)^(2/3)*e^(1/3)*x*Log[x] - 2*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x
)*Log[1 + ((-1)^(2/3)*e^(1/3)*x)/d^(1/3)]) - 2*(d^(1/3) + (-1)^(2/3)*e^(1/3
)*x)*PolyLog[2, -(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))])/(3*(1 + (-1)^(1/3))^2*d
^(4/3)*(-((-1)^(1/3)*d^(2/3)*e^(1/3)) + d^(1/3)*e^(2/3)*x)) + (2*(-1)^(1/3)
*(Log[x]^2*Log[1 + (e^(1/3)*x)/d^(1/3)] + 2*Log[x]*PolyLog[2, -((e^(1/3)*x)
/d^(1/3))] - 2*PolyLog[3, -((e^(1/3)*x)/d^(1/3))]))/(3*(1 + (-1)^(1/3))^2*d
^(5/3)*e^(1/3)) - (2*(Log[x]^2*Log[1 - ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)] + 2*
Log[x]*PolyLog[2, ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)] - 2*PolyLog[3, ((-1)^(1/3
)*e^(1/3)*x)/d^(1/3)]))/(3*(1 + (-1)^(1/3))^2*d^(5/3)*e^(1/3)) - (2*(-1 + (
```

```
-1)^(1/3))*(Log[x]^2*Log[1 + ((-1)^(2/3)*e^(1/3)*x)/d^(1/3)] + 2*Log[x]*Pol
yLog[2, -(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))] - 2*PolyLog[3, -(((-1)^(2/3)*e^(
1/3)*x)/d^(1/3))]))/(3*(1 + (-1)^(1/3))^2*d^(5/3)*e^(1/3))) + b^3*n^3*(((-1
)^(1/3)*(Log[x]^2*((d^(-1/3) - (d^(1/3) + e^(1/3)*x)^(-1))*Log[x] - (3*Log[
1 + (e^(1/3)*x)/d^(1/3)])/d^(1/3)) - (6*Log[x]*PolyLog[2, -((e^(1/3)*x)/d^(
1/3))])/d^(1/3) + (6*PolyLog[3, -((e^(1/3)*x)/d^(1/3))])/d^(1/3)))/(3*(1 +
(-1)^(1/3))^2*d^(4/3)*e^(1/3)) - ((-1 + (-1)^(1/3))*(-(((-1)^(1/3)*Log[x]^3
)/d^(1/3)) - Log[x]^3/((-1)^(2/3)*d^(1/3) + e^(1/3)*x) + (3*(-1)^(1/3)*Log[
x]^2*Log[1 - ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)])/d^(1/3) + (6*(-1)^(1/3)*(Log[
x]*PolyLog[2, ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)] - PolyLog[3, ((-1)^(1/3)*e^(1
/3)*x)/d^(1/3)]))/d^(1/3)))/(3*(1 + (-1)^(1/3))^2*d^(4/3)*e^(1/3)) - (((-1)
^(2/3)*Log[x]^3)/d^(1/3) + Log[x]^3/((-1)^(1/3)*d^(1/3) - e^(1/3)*x) - (3*(
-1)^(2/3)*Log[x]^2*Log[1 + ((-1)^(2/3)*e^(1/3)*x)/d^(1/3)])/d^(1/3) - (6*(-
1)^(2/3)*(Log[x]*PolyLog[2, -(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))] - PolyLog[3,
 -(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))]))/d^(1/3))/(3*(1 + (-1)^(1/3))^2*d^(4/3
)*e^(1/3)) + (2*(-1)^(1/3)*(Log[x]^3*Log[1 + (e^(1/3)*x)/d^(1/3)] + 3*Log[x
]^2*PolyLog[2, -((e^(1/3)*x)/d^(1/3))] - 6*Log[x]*PolyLog[3, -((e^(1/3)*x)/
d^(1/3))] + 6*PolyLog[4, -((e^(1/3)*x)/d^(1/3))]))/(3*(1 + (-1)^(1/3))^2*d^
(5/3)*e^(1/3)) - (2*(Log[x]^3*Log[1 - ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)] + 3*L
og[x]^2*PolyLog[2, ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)] - 6*Log[x]*PolyLog[3, ((
-1)^(1/3)*e^(1/3)*x)/d^(1/3)] + 6*PolyLog[4, ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)
]))/(3*(1 + (-1)^(1/3))^2*d^(5/3)*e^(1/3)) - (2*(-1 + (-1)^(1/3))*(Log[x]^3
*Log[1 + ((-1)^(2/3)*e^(1/3)*x)/d^(1/3)] + 3*Log[x]^2*PolyLog[2, -(((-1)^(2
/3)*e^(1/3)*x)/d^(1/3))] - 6*Log[x]*PolyLog[3, -(((-1)^(2/3)*e^(1/3)*x)/d^(
1/3))] + 6*PolyLog[4, -(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))]))/(3*(1 + (-1)^(1/
3))^2*d^(5/3)*e^(1/3)))
```

Maple [F]

$$\int \frac{(a+b\ln (c\,x^{n}))^{3}}{(e\,x^{3}+d)^{2}}dx$$

```
[In] int((a+b*ln(c*x^n))^3/(e*x^3+d)^2,x)
```

```
[Out] int((a+b*ln(c*x^n))^3/(e*x^3+d)^2,x)
```

Fricas [F]

$$\int \frac{(a+b\log (cx^{n}))^{3}}{(d+ex^{3})^{2}}\,dx=\int \frac{(b\log (cx^{n})+a)^{3}}{(ex^{3}+d)^{2}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^3/(e*x^3+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)/(e^2*x^6 + 2*d*e*x^3 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3}{(d+ex^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*x**n))**3/(e*x**3+d)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))^3}{(d+ex^3)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))^3/(e*x^3+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^3}{(d+ex^3)^2}\,dx = \int \frac{(b\log(cx^n)+a)^3}{(ex^3+d)^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^3/(e*x^3+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3/(e*x^3 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^3}{(d+ex^3)^2}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^3}{(e\,x^3+d)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))^3/(d + e*x^3)^2,x)
```

```
[Out] int((a + b*log(c*x^n))^3/(d + e*x^3)^2, x)
```

3.325 $\int \frac{(a+b\log(cx^n))^2}{(d+ex^3)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 860

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex^3)^2}\,dx = \frac{x(a+b\log(cx^n))^2}{9d^{5/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} - \frac{\sqrt[3]{-1}x(a+b\log(cx^n))^2}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}$$
$$+ \frac{x(a+b\log(cx^n))^2}{9d^{5/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}$$
$$- \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$
$$+ \frac{2(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$
$$+ \frac{2\sqrt[3]{-1}bn(a+b\log(cx^n))\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}}$$
$$- \frac{2i\sqrt{3}(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$
$$+ \frac{2\sqrt[3]{-1}bn(a+b\log(cx^n))\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$
$$+ \frac{2(a+b\log(cx^n))^2\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}}$$
$$- \frac{2b^2n^2\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$
$$+ \frac{4bn(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$
$$+ \frac{2\sqrt[3]{-1}b^2n^2\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}}$$
$$- \frac{4i\sqrt{3}bn(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$
$$+ \frac{2\sqrt[3]{-1}b^2n^2\,\mathrm{PolyLog}\left(2,-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$
$$+ \frac{4bn(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}}$$

```
[Out] 1/9*x*(a+b*ln(c*x^n))^2/d^(5/3)/(d^(1/3)+e^(1/3)*x)-(-1)^(1/3)*x*(a+b*ln(c*
x^n))^2/(1+(-1)^(1/3))^4/d^(5/3)/((-1)^(2/3)*d^(1/3)+e^(1/3)*x)+1/9*x*(a+b*
ln(c*x^n))^2/d^(5/3)/(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)-2/9*b*n*(a+b*ln(c*x^n))
*ln(1+e^(1/3)*x/d^(1/3))/d^(5/3)/e^(1/3)+2/9*(a+b*ln(c*x^n))^2*ln(1+e^(1/3)
*x/d^(1/3))/d^(5/3)/e^(1/3)+2*(-1)^(1/3)*b*n*(a+b*ln(c*x^n))*ln(1-(-1)^(1/3
)*e^(1/3)*x/d^(1/3))/(1+(-1)^(1/3))^4/d^(5/3)/e^(1/3)+2/9*(-1)^(1/3)*b*n*(a
+b*ln(c*x^n))*ln(1+(-1)^(2/3)*e^(1/3)*x/d^(1/3))/d^(5/3)/e^(1/3)-2/9*b^2*n^
2*polylog(2,-e^(1/3)*x/d^(1/3))/d^(5/3)/e^(1/3)+4/9*b*n*(a+b*ln(c*x^n))*pol
ylog(2,-e^(1/3)*x/d^(1/3))/d^(5/3)/e^(1/3)+2*(-1)^(1/3)*b^2*n^2*polylog(2,(
-1)^(1/3)*e^(1/3)*x/d^(1/3))/(1+(-1)^(1/3))^4/d^(5/3)/e^(1/3)+2/9*(-1)^(1/3
)*b^2*n^2*polylog(2,-(-1)^(2/3)*e^(1/3)*x/d^(1/3))/d^(5/3)/e^(1/3)-4/9*b^2*
n^2*polylog(3,-e^(1/3)*x/d^(1/3))/d^(5/3)/e^(1/3)-4/9*(a+b*ln(c*x^n))^2*ln(
1-1/2*e^(1/3)*x*(1-I*3^(1/2))/d^(1/3))/d^(5/3)/e^(1/3)/(1-I*3^(1/2))-8/9*b*
n*(a+b*ln(c*x^n))*polylog(2,1/2*e^(1/3)*x*(1-I*3^(1/2))/d^(1/3))/d^(5/3)/e^
(1/3)/(1-I*3^(1/2))+8/9*b^2*n^2*polylog(3,1/2*e^(1/3)*x*(1-I*3^(1/2))/d^(1/
3))/d^(5/3)/e^(1/3)/(1-I*3^(1/2))-4/9*(a+b*ln(c*x^n))^2*ln(1-1/2*e^(1/3)*x*
(1+I*3^(1/2))/d^(1/3))/d^(5/3)/e^(1/3)/(1+I*3^(1/2))-8/9*b*n*(a+b*ln(c*x^n)
)*polylog(2,1/2*e^(1/3)*x*(1+I*3^(1/2))/d^(1/3))/d^(5/3)/e^(1/3)/(1+I*3^(1/
2))+8/9*b^2*n^2*polylog(3,1/2*e^(1/3)*x*(1+I*3^(1/2))/d^(1/3))/d^(5/3)/e^(1
/3)/(1+I*3^(1/2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 860, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used

$= \{2367, 2355, 2354, 2438, 2421, 6724\}$

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex^3)^2}\,dx = -\frac{2b^2\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^2}{9d^{5/3}\sqrt[3]{e}} + \frac{2\sqrt[3]{-1}b^2\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^2}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}} + \frac{2\sqrt[3]{-1}b^2\,\mathrm{PolyLog}\left(2,-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^2}{9d^{5/3}\sqrt[3]{e}} - \frac{4b^2\,\mathrm{PolyLog}\left(3,-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^2}{9d^{5/3}\sqrt[3]{e}} + \frac{4i\sqrt{3}b^2\,\mathrm{PolyLog}\left(3,\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^2}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}} - \frac{4b^2\,\mathrm{PolyLog}\left(3,-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n^2}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}} - \frac{2b(a+b\log(cx^n))\log\left(\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}+1\right)n}{9d^{5/3}\sqrt[3]{e}} + \frac{2\sqrt[3]{-1}b(a+b\log(cx^n))\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}} + \frac{2\sqrt[3]{-1}b(a+b\log(cx^n))\log\left(\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}+1\right)n}{9d^{5/3}\sqrt[3]{e}} + \frac{4b(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n}{9d^{5/3}\sqrt[3]{e}} - \frac{4i\sqrt{3}b(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}} + \frac{4b(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)n}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}} + \frac{x(a+b\log(cx^n))^2}{9d^{5/3}\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)} - \frac{\sqrt[3]{-1}x(a+b\log(cx^n))^2}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\left(\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}\right)} + \frac{x(a+b\log(cx^n))^2}{9d^{5/3}\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)} + \frac{2(a+b\log(cx^n))^2\log\left(\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}+1\right)}{9d^{5/3}\sqrt[3]{e}}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(d + e*x^3)^2,x]
```

```
[Out] (x*(a + b*Log[c*x^n])^2)/(9*d^(5/3)*(d^(1/3) + e^(1/3)*x)) - ((-1)^(1/3)*x*
(a + b*Log[c*x^n])^2)/((1 + (-1)^(1/3))^4*d^(5/3)*((-1)^(2/3)*d^(1/3) + e^(
1/3)*x)) + (x*(a + b*Log[c*x^n])^2)/(9*d^(5/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3
)*x)) - (2*b*n*(a + b*Log[c*x^n])*Log[1 + (e^(1/3)*x)/d^(1/3)])/(9*d^(5/3)*
e^(1/3)) + (2*(a + b*Log[c*x^n])^2*Log[1 + (e^(1/3)*x)/d^(1/3)])/(9*d^(5/3)
*e^(1/3)) + (2*(-1)^(1/3)*b*n*(a + b*Log[c*x^n])*Log[1 - ((-1)^(1/3)*e^(1/3
)*x)/d^(1/3)])/((1 + (-1)^(1/3))^4*d^(5/3)*e^(1/3)) - ((2*I)*Sqrt[3]*(a + b
*Log[c*x^n])^2*Log[1 - ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)])/((1 + (-1)^(1/3))^5
*d^(5/3)*e^(1/3)) + (2*(-1)^(1/3)*b*n*(a + b*Log[c*x^n])*Log[1 + ((-1)^(2/3
)*e^(1/3)*x)/d^(1/3)])/(9*d^(5/3)*e^(1/3)) + (2*(a + b*Log[c*x^n])^2*Log[1
+ ((-1)^(2/3)*e^(1/3)*x)/d^(1/3)])/((1 + (-1)^(1/3))^4*d^(5/3)*e^(1/3)) - (
2*b^2*n^2*PolyLog[2, -((e^(1/3)*x)/d^(1/3))])/(9*d^(5/3)*e^(1/3)) + (4*b*n*
(a + b*Log[c*x^n])*PolyLog[2, -((e^(1/3)*x)/d^(1/3))])/(9*d^(5/3)*e^(1/3))
+ (2*(-1)^(1/3)*b^2*n^2*PolyLog[2, ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)])/((1 + (
-1)^(1/3))^4*d^(5/3)*e^(1/3)) - ((4*I)*Sqrt[3]*b*n*(a + b*Log[c*x^n])*PolyL
og[2, ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)])/((1 + (-1)^(1/3))^5*d^(5/3)*e^(1/3))
 + (2*(-1)^(1/3)*b^2*n^2*PolyLog[2, -(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))])/(9*
d^(5/3)*e^(1/3)) + (4*b*n*(a + b*Log[c*x^n])*PolyLog[2, -(((-1)^(2/3)*e^(1/
3)*x)/d^(1/3))])/((1 + (-1)^(1/3))^4*d^(5/3)*e^(1/3)) - (4*b^2*n^2*PolyLog[
3, -((e^(1/3)*x)/d^(1/3))])/(9*d^(5/3)*e^(1/3)) + ((4*I)*Sqrt[3]*b^2*n^2*Po
lyLog[3, ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)])/((1 + (-1)^(1/3))^5*d^(5/3)*e^(1/
3)) - (4*b^2*n^2*PolyLog[3, -(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))])/((1 + (-1)^
(1/3))^4*d^(5/3)*e^(1/3))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a+b\log(cx^n))^2}{9d^{4/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)^2} + \frac{2(a+b\log(cx^n))^2}{9d^{5/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{(-1)^{2/3}(a+b\log(cx^n))^2}{\left(1+\sqrt[3]{-1}\right)^4 d^{4/3}\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)^2} - \frac{2(-1)^{5/6}\sqrt{3}(a+b\log(cx^n))^2}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)} + \frac{(a+b\log(cx^n))^2}{\left(-1+\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4 d^{4/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)^2} + \frac{2(-1)^{2/3}(a+b\log(cx^n))^2}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)} \right) dx$$

$$= \frac{2\int \frac{(a+b\log(cx^n))^2}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{9d^{5/3}} + \frac{2\int \frac{(a+b\log(cx^n))^2}{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}\,dx}{9d^{5/3}} - \frac{\left(2(-1)^{5/6}\sqrt{3}\right)\int \frac{(a+b\log(cx^n))^2}{-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x}\,dx}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}} + \frac{\int \frac{(a+b\log(cx^n))^2}{\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}\,dx}{9d^{4/3}} + \frac{\int \frac{(a+b\log(cx^n))^2}{\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)^2}\,dx}{9d^{4/3}} + \frac{\int \frac{(a+b\log(cx^n))^2}{\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)^2}\,dx}{9d^{4/3}}$$

$$= \frac{x(a+b\log(cx^n))^2}{9d^{5/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{(-1)^{2/3}x(a+b\log(cx^n))^2}{9d^{5/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}$$

$$+ \frac{x(a+b\log(cx^n))^2}{9d^{5/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)} + \frac{2(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

$$- \frac{2i\sqrt{3}(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$

$$- \frac{2\sqrt[3]{-1}(a+b\log(cx^n))^2\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

$$- \frac{(2bn)\int\frac{a+b\log(cx^n)}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{9d^{5/3}} + \frac{(2bn)\int\frac{a+b\log(cx^n)}{-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x}\,dx}{9d^{5/3}}$$

$$- \frac{(2bn)\int\frac{a+b\log(cx^n)}{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}\,dx}{9d^{5/3}} - \frac{(4bn)\int\frac{(a+b\log(cx^n))\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}\,dx}{9d^{5/3}\sqrt[3]{e}}$$

$$+ \frac{\left(4\sqrt[3]{-1}bn\right)\int\frac{(a+b\log(cx^n))\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}\,dx}{9d^{5/3}\sqrt[3]{e}}$$

$$+ \frac{\left(4i\sqrt{3}bn\right)\int\frac{(a+b\log(cx^n))\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}\,dx}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$

$$= \frac{x(a+b\log(cx^n))^2}{9d^{5/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{(-1)^{2/3}x(a+b\log(cx^n))^2}{9d^{5/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{x(a+b\log(cx^n))^2}{9d^{5/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}$$

$$- \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}} + \frac{2(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

$$- \frac{2(-1)^{2/3}bn(a+b\log(cx^n))\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

$$- \frac{2i\sqrt{3}(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$

$$+ \frac{2\sqrt[3]{-1}bn(a+b\log(cx^n))\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

$$- \frac{2\sqrt[3]{-1}(a+b\log(cx^n))^2\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

$$+ \frac{4bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}} - \frac{4i\sqrt{3}bn(a+b\log(cx^n))\operatorname{Li}_2\left(\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$

$$- \frac{4\sqrt[3]{-1}bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

$$+ \frac{(2b^2n^2)\int\frac{\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}dx}{9d^{5/3}\sqrt[3]{e}} - \frac{(4b^2n^2)\int\frac{\operatorname{Li}_2\left(-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}dx}{9d^{5/3}\sqrt[3]{e}}$$

$$- \frac{\left(2\sqrt[3]{-1}b^2n^2\right)\int\frac{\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}dx}{9d^{5/3}\sqrt[3]{e}} + \frac{\left(4\sqrt[3]{-1}b^2n^2\right)\int\frac{\operatorname{Li}_2\left(-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}dx}{9d^{5/3}\sqrt[3]{e}}$$

$$+ \frac{\left(2(-1)^{2/3}b^2n^2\right)\int\frac{\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}dx}{9d^{5/3}\sqrt[3]{e}} + \frac{\left(4i\sqrt{3}b^2n^2\right)\int\frac{\operatorname{Li}_2\left(\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}dx}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$

$$= \frac{x(a+b\log(cx^n))^2}{9d^{5/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{(-1)^{2/3}x(a+b\log(cx^n))^2}{9d^{5/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{x(a+b\log(cx^n))^2}{9d^{5/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}$$

$$- \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}} + \frac{2(a+b\log(cx^n))^2\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

$$- \frac{2(-1)^{2/3}bn(a+b\log(cx^n))\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

$$- \frac{2i\sqrt{3}(a+b\log(cx^n))^2\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$

$$+ \frac{2\sqrt[3]{-1}bn(a+b\log(cx^n))\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

$$- \frac{2\sqrt[3]{-1}(a+b\log(cx^n))^2\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}} - \frac{2b^2n^2\operatorname{Li}_2\left(-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

$$+ \frac{4bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}} - \frac{2(-1)^{2/3}b^2n^2\operatorname{Li}_2\left(\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

$$- \frac{4i\sqrt{3}bn(a+b\log(cx^n))\operatorname{Li}_2\left(\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}} + \frac{2\sqrt[3]{-1}b^2n^2\operatorname{Li}_2\left(-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

$$- \frac{4\sqrt[3]{-1}bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}} - \frac{4b^2n^2\operatorname{Li}_3\left(-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

$$+ \frac{4i\sqrt{3}b^2n^2\operatorname{Li}_3\left(\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}} + \frac{4\sqrt[3]{-1}b^2n^2\operatorname{Li}_3\left(-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

Mathematica [A] (verified)

Time = 2.72 (sec) , antiderivative size = 1180, normalized size of antiderivative = 1.37

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex^3)^2}\,dx$$

$$= \frac{\frac{3d^{2/3}x(a-bn\log(x)+b\log(cx^n))^2}{d+ex^3} + \frac{2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)(a-bn\log(x)+b\log(cx^n))^2}{\sqrt[3]{e}} + \frac{2(a-bn\log(x)+b\log(cx^n))^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right.}{\sqrt[3]{e}}}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(d + e*x^3)^2,x]
```

```
[Out] ((3*d^(2/3)*x*(a - b*n*Log[x] + b*Log[c*x^n])^2)/(d + e*x^3) + (2*Sqrt[3]*A
rcTan[(-d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))]*(a - b*n*Log[x] + b*Log[c
*x^n])^2)/e^(1/3) + (2*(a - b*n*Log[x] + b*Log[c*x^n])^2*Log[d^(1/3) + e^(1
/3)*x])/e^(1/3) - ((a - b*n*Log[x] + b*Log[c*x^n])^2*Log[d^(2/3) - d^(1/3)*
e^(1/3)*x + e^(2/3)*x^2])/e^(1/3) + (6*b*n*(a - b*n*Log[x] + b*Log[c*x^n])*
(((-1 + (-1)^(1/3))*((-1)^(1/3)*e^(1/3)*x*Log[x] + (d^(1/3) - (-1)^(1/3)*e^
(1/3)*x)*Log[-((-1)^(2/3)*d^(1/3)) - e^(1/3)*x]))/((-1)^(2/3)*d^(1/3)*e^(1/
3) + e^(2/3)*x) + (-1)^(1/3)*((x*Log[x])/(d^(1/3) + e^(1/3)*x) - Log[d^(1/3
) + e^(1/3)*x]/e^(1/3)) + (-((-1)^(2/3)*e^(1/3)*x*Log[x]) + (d^(1/3) + (-1)
^(2/3)*e^(1/3)*x)*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/(-((-1)^(1/3)*d^(1/3
)*e^(1/3)) + e^(2/3)*x) + (2*(-1)^(1/3)*(Log[x]*Log[1 + (e^(1/3)*x)/d^(1/3)
] + PolyLog[2, -((e^(1/3)*x)/d^(1/3))]))/e^(1/3) - (2*(Log[x]*Log[1 - ((-1)
^(1/3)*e^(1/3)*x)/d^(1/3)] + PolyLog[2, ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)]))/e
^(1/3) - (2*(-1 + (-1)^(1/3))*(Log[x]*Log[1 + ((-1)^(2/3)*e^(1/3)*x)/d^(1/3
)] + PolyLog[2, -(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))]))/e^(1/3)))/(1 + (-1)^(1
/3))^2 + (3*b^2*n^2*(((-1)^(1/3)*(Log[x]*((e^(1/3)*x*Log[x])/(d^(1/3) + e^(
1/3)*x) - 2*Log[1 + (e^(1/3)*x)/d^(1/3)]) - 2*PolyLog[2, -((e^(1/3)*x)/d^(1
/3))]))/e^(1/3) - ((-1 + (-1)^(1/3))*d^(1/3)*(Log[x]*((-((-1)^(1/3)/d^(1/3)
) - ((-1)^(2/3)*d^(1/3) + e^(1/3)*x)^(-1))*Log[x] + (2*(-1)^(1/3)*Log[1 - (
(-1)^(1/3)*e^(1/3)*x)/d^(1/3)])/d^(1/3)) + (2*(-1)^(1/3)*PolyLog[2, ((-1)^(
1/3)*e^(1/3)*x)/d^(1/3)])/d^(1/3)))/e^(1/3) + (Log[x]*((-1)^(2/3)*e^(1/3)*x
*Log[x] - 2*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)*Log[1 + ((-1)^(2/3)*e^(1/3)*x)
/d^(1/3)]) - 2*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)*PolyLog[2, -(((-1)^(2/3)*e^
(1/3)*x)/d^(1/3))])/((-1)^(1/3)*d^(1/3)*e^(1/3) - e^(2/3)*x) + (2*(-1)^(1/3
)*(Log[x]^2*Log[1 + (e^(1/3)*x)/d^(1/3)] + 2*Log[x]*PolyLog[2, -((e^(1/3)*x
)/d^(1/3))] - 2*PolyLog[3, -((e^(1/3)*x)/d^(1/3))]))/e^(1/3) - (2*(Log[x]^2
```

```
*Log[1 - ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)] + 2*Log[x]*PolyLog[2, ((-1)^(1/3)*
e^(1/3)*x)/d^(1/3)] - 2*PolyLog[3, ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)]))/e^(1/3
) - (2*(-1 + (-1)^(1/3))*(Log[x]^2*Log[1 + ((-1)^(2/3)*e^(1/3)*x)/d^(1/3)]
+ 2*Log[x]*PolyLog[2, -(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))] - 2*PolyLog[3, -((
(-1)^(2/3)*e^(1/3)*x)/d^(1/3))]))/e^(1/3)))/(1 + (-1)^(1/3))^2)/(9*d^(5/3))
```

Maple [F]

$$\int \frac{\left(a +b \ln \left(c \,x^{n}\right)\right)^{2}}{\left(e \,x^{3}+d \right)^{2}}d x$$

```
[In] int((a+b*ln(c*x^n))^2/(e*x^3+d)^2,x)
```

```
[Out] int((a+b*ln(c*x^n))^2/(e*x^3+d)^2,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex^3)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex^3+d)^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/(e*x^3+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)/(e^2*x^6 + 2*d*e*x^3 +
 d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*x**n))**2/(e*x**3+d)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex^3)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))^2/(e*x^3+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex^3)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex^3+d)^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/(e*x^3+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2/(e*x^3 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{(d+ex^3)^2}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{(e\,x^3+d)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(d + e*x^3)^2,x)
```

```
[Out] int((a + b*log(c*x^n))^2/(d + e*x^3)^2, x)
```

3.326 $\int \frac{a+b\log(cx^n)}{(d+ex^3)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 520

$$\begin{aligned}
\int \frac{a+b\log(cx^n)}{(d+ex^3)^2}\,dx &= \frac{x(a+b\log(cx^n))}{9d^{5/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} - \frac{\sqrt[3]{-1}x(a+b\log(cx^n))}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)} \\
&\quad + \frac{x(a+b\log(cx^n))}{9d^{5/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)} + \frac{\sqrt[3]{-1}bn\log\left(-(-1)^{2/3}\sqrt[3]{d}-\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}} \\
&\quad - \frac{bn\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{5/3}\sqrt[3]{e}} + \frac{\sqrt[3]{-1}bn\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{9d^{5/3}\sqrt[3]{e}} \\
&\quad + \frac{2(a+b\log(cx^n))\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}} \\
&\quad - \frac{2i\sqrt{3}(a+b\log(cx^n))\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}} \\
&\quad + \frac{2(a+b\log(cx^n))\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}} \\
&\quad + \frac{2bn\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}} - \frac{2i\sqrt{3}bn\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}} \\
&\quad + \frac{2bn\,\mathrm{PolyLog}\left(2,-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}}
\end{aligned}$$

```
[Out] 1/9*x*(a+b*ln(c*x^n))/d^(5/3)/(d^(1/3)+e^(1/3)*x)-(-1)^(1/3)*x*(a+b*ln(c*x^
n))/(1+(-1)^(1/3))^4/d^(5/3)/((-1)^(2/3)*d^(1/3)+e^(1/3)*x)+1/9*x*(a+b*ln(c
*x^n))/d^(5/3)/(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)+(-1)^(1/3)*b*n*ln(-(-1)^(2/3)
*d^(1/3)-e^(1/3)*x)/(1+(-1)^(1/3))^4/d^(5/3)/e^(1/3)-1/9*b*n*ln(d^(1/3)+e^(
1/3)*x)/d^(5/3)/e^(1/3)+1/9*(-1)^(1/3)*b*n*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)
/d^(5/3)/e^(1/3)+2/9*(a+b*ln(c*x^n))*ln(1+e^(1/3)*x/d^(1/3))/d^(5/3)/e^(1/3
)+2/9*b*n*polylog(2,-e^(1/3)*x/d^(1/3))/d^(5/3)/e^(1/3)-4/9*(a+b*ln(c*x^n))
*ln(1-1/2*e^(1/3)*x*(1-I*3^(1/2))/d^(1/3))/d^(5/3)/e^(1/3)/(1-I*3^(1/2))-4/
9*b*n*polylog(2,1/2*e^(1/3)*x*(1-I*3^(1/2))/d^(1/3))/d^(5/3)/e^(1/3)/(1-I*3
^(1/2))-4/9*(a+b*ln(c*x^n))*ln(1-1/2*e^(1/3)*x*(1+I*3^(1/2))/d^(1/3))/d^(5/
3)/e^(1/3)/(1+I*3^(1/2))-4/9*b*n*polylog(2,1/2*e^(1/3)*x*(1+I*3^(1/2))/d^(1
/3))/d^(5/3)/e^(1/3)/(1+I*3^(1/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules

used = {205, 206, 31, 648, 631, 210, 642, 2367, 2351, 2354, 2438}

$$\int \frac{a+b\log(cx^n)}{(d+ex^3)^2}\,dx = \frac{2\log\left(\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}+1\right)(a+b\log(cx^n))}{9d^{5/3}\sqrt[3]{e}} - \frac{2i\sqrt{3}\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)(a+b\log(cx^n))}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}} + \frac{2\log\left(\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}+1\right)(a+b\log(cx^n))}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}} + \frac{x(a+b\log(cx^n))}{9d^{5/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} - \frac{\sqrt[3]{-1}x(a+b\log(cx^n))}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{x(a+b\log(cx^n))}{9d^{5/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)} + \frac{2bn\,\text{PolyLog}\left(2,-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}} - \frac{2i\sqrt{3}bn\,\text{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}} + \frac{2bn\,\text{PolyLog}\left(2,-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}} + \frac{\sqrt[3]{-1}bn\log\left(-(-1)^{2/3}\sqrt[3]{d}-\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\sqrt[3]{e}} - \frac{bn\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{5/3}\sqrt[3]{e}} + \frac{\sqrt[3]{-1}bn\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{9d^{5/3}\sqrt[3]{e}}$$

```
[In] Int[(a + b*Log[c*x^n])/(d + e*x^3)^2,x]
```

```
[Out] (x*(a + b*Log[c*x^n]))/(9*d^(5/3)*(d^(1/3) + e^(1/3)*x)) - ((-1)^(1/3)*x*(a
 + b*Log[c*x^n]))/((1 + (-1)^(1/3))^4*d^(5/3)*((-1)^(2/3)*d^(1/3) + e^(1/3)
*x)) + (x*(a + b*Log[c*x^n]))/(9*d^(5/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))
+ ((-1)^(1/3)*b*n*Log[-((-1)^(2/3)*d^(1/3)) - e^(1/3)*x])/((1 + (-1)^(1/3))
^4*d^(5/3)*e^(1/3)) - (b*n*Log[d^(1/3) + e^(1/3)*x])/(9*d^(5/3)*e^(1/3)) +
((-1)^(1/3)*b*n*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/(9*d^(5/3)*e^(1/3)) +
(2*(a + b*Log[c*x^n])*Log[1 + (e^(1/3)*x)/d^(1/3)])/(9*d^(5/3)*e^(1/3)) - (
(2*I)*Sqrt[3]*(a + b*Log[c*x^n])*Log[1 - ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)])/(
(1 + (-1)^(1/3))^5*d^(5/3)*e^(1/3)) + (2*(a + b*Log[c*x^n])*Log[1 + ((-1)^(
2/3)*e^(1/3)*x)/d^(1/3)])/((1 + (-1)^(1/3))^4*d^(5/3)*e^(1/3)) + (2*b*n*Pol
yLog[2, -((e^(1/3)*x)/d^(1/3))])/(9*d^(5/3)*e^(1/3)) - ((2*I)*Sqrt[3]*b*n*P
olyLog[2, ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)])/((1 + (-1)^(1/3))^5*d^(5/3)*e^(1
/3)) + (2*b*n*PolyLog[2, -(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))])/((1 + (-1)^(1/
3))^4*d^(5/3)*e^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log(cx^n)}{9d^{4/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)^2} + \frac{2(a+b\log(cx^n))}{9d^{5/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{(-1)^{2/3}(a+b\log(cx^n))}{\left(1+\sqrt[3]{-1}\right)^4 d^{4/3}\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)^2} - \frac{2(-1)^{5/6}\sqrt{3}(a+b\log(cx^n))}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)} + \frac{a+b\log(cx^n)}{\left(-1+\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4 d^{4/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)^2} + \frac{2(-1)^{2/3}(a+b\log(cx^n))}{\left(1+\sqrt[3]{-1}\right)^4 d^{5/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)} \right) dx$$

$$= \frac{2\int \frac{a+b\log(cx^n)}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{9d^{5/3}} + \frac{2\int \frac{a+b\log(cx^n)}{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}\,dx}{9d^{5/3}} - \frac{\left(2(-1)^{5/6}\sqrt{3}\right)\int \frac{a+b\log(cx^n)}{-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x}\,dx}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}}$$
$$+ \frac{\int \frac{a+b\log(cx^n)}{\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}\,dx}{9d^{4/3}} + \frac{\int \frac{a+b\log(cx^n)}{\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)^2}\,dx}{9d^{4/3}} + \frac{\int \frac{a+b\log(cx^n)}{\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)^2}\,dx}{9d^{4/3}}$$

$$= \frac{x(a+b\log(cx^n))}{9d^{5/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{(-1)^{2/3}x(a+b\log(cx^n))}{9d^{5/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{x(a+b\log(cx^n))}{9d^{5/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}$$
$$+ \frac{2(a+b\log(cx^n))\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}} - \frac{2i\sqrt{3}(a+b\log(cx^n))\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$
$$- \frac{2\sqrt[3]{-1}(a+b\log(cx^n))\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}} - \frac{(bn)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{9d^{5/3}}$$
$$+ \frac{(bn)\int \frac{1}{-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x}\,dx}{9d^{5/3}} - \frac{(bn)\int \frac{1}{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}\,dx}{9d^{5/3}} - \frac{(2bn)\int \frac{\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}\,dx}{9d^{5/3}\sqrt[3]{e}}$$
$$+ \frac{\left(2\sqrt[3]{-1}bn\right)\int \frac{\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}\,dx}{9d^{5/3}\sqrt[3]{e}} + \frac{\left(2i\sqrt{3}bn\right)\int \frac{\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{x}\,dx}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$

$$= \frac{x(a+b\log(cx^n))}{9d^{5/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{(-1)^{2/3}x(a+b\log(cx^n))}{9d^{5/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}$$
$$+ \frac{x(a+b\log(cx^n))}{9d^{5/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)} - \frac{(-1)^{2/3}bn\log\left(-(-1)^{2/3}\sqrt[3]{d}-\sqrt[3]{e}x\right)}{9d^{5/3}\sqrt[3]{e}}$$
$$- \frac{bn\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{5/3}\sqrt[3]{e}} + \frac{\sqrt[3]{-1}bn\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{9d^{5/3}\sqrt[3]{e}}$$
$$+ \frac{2(a+b\log(cx^n))\log\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}} - \frac{2i\sqrt{3}(a+b\log(cx^n))\log\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}}$$
$$- \frac{2\sqrt[3]{-1}(a+b\log(cx^n))\log\left(1+\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}} + \frac{2bn\text{Li}_2\left(-\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$
$$- \frac{2i\sqrt{3}bn\text{Li}_2\left(\frac{\sqrt[3]{-1}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^5 d^{5/3}\sqrt[3]{e}} - \frac{2\sqrt[3]{-1}bn\text{Li}_2\left(-\frac{(-1)^{2/3}\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{5/3}\sqrt[3]{e}}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.10

$$\int \frac{a+b\log(cx^n)}{(d+ex^3)^2}\,dx$$

$$= \frac{\frac{3d^{2/3}x(a-bn\log(x)+b\log(cx^n))}{d+ex^3} - \frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)(a-bn\log(x)+b\log(cx^n))}{\sqrt[3]{e}} + \frac{2(a-bn\log(x)+b\log(cx^n))\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt[3]{e}} - }{}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*x^3)^2,x]
```

```
[Out] ((3*d^(2/3)*x*(a - b*n*Log[x] + b*Log[c*x^n]))/(d + e*x^3) - (2*Sqrt[3]*Arc
Tan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]]*(a - b*n*Log[x] + b*Log[c*x^n]))/e
^(1/3) + (2*(a - b*n*Log[x] + b*Log[c*x^n])*Log[d^(1/3) + e^(1/3)*x])/e^(1/
3) - ((a - b*n*Log[x] + b*Log[c*x^n])*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(
2/3)*x^2])/e^(1/3) + (3*b*n*(((-1 + (-1)^(1/3))*((-1)^(1/3)*e^(1/3)*x*Log[x
] + (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)*Log[-((-1)^(2/3)*d^(1/3)) - e^(1/3)*x]
))/((-1)^(2/3)*d^(1/3)*e^(1/3) + e^(2/3)*x) + (-1)^(1/3)*((x*Log[x])/(d^(1/
3) + e^(1/3)*x) - Log[d^(1/3) + e^(1/3)*x]/e^(1/3)) + (-((-1)^(2/3)*e^(1/3)
*x*Log[x]) + (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)*Log[d^(1/3) + (-1)^(2/3)*e^(1
/3)*x])/(-((-1)^(1/3)*d^(1/3)*e^(1/3)) + e^(2/3)*x) + (2*(-1)^(1/3)*(Log[x]
*Log[1 + (e^(1/3)*x)/d^(1/3)] + PolyLog[2, -((e^(1/3)*x)/d^(1/3))]))/e^(1/3
) - (2*(Log[x]*Log[1 - ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)] + PolyLog[2, ((-1)^(
1/3)*e^(1/3)*x)/d^(1/3)]))/e^(1/3) - (2*(-1 + (-1)^(1/3))*(Log[x]*Log[1 + (
(-1)^(2/3)*e^(1/3)*x)/d^(1/3)] + PolyLog[2, -(((-1)^(2/3)*e^(1/3)*x)/d^(1/3
))]))/e^(1/3)))/(1 + (-1)^(1/3))^2)/(9*d^(5/3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.13 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.15

method	result
risch	$\frac{bx\ln(x^n)}{3d(e\,x^3+d)}-\frac{2b\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)n\ln(x)}{9e\left(\frac{d}{e}\right)^{\frac{2}{3}}d}+\frac{2b\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)\ln(x^n)}{9e\left(\frac{d}{e}\right)^{\frac{2}{3}}d}+\frac{b\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)n\ln(x)}{9e\left(\frac{d}{e}\right)^{\frac{2}{3}}d}-\frac{b\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\right.\right.}{9e\left(\frac{d}{e}\right)^{\frac{2}{3}}}$

```
[In] int((a+b*ln(c*x^n))/(e*x^3+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*x/d/(e*x^3+d)*ln(x^n)-2/9*b/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))/d*n*ln(x)
+2/9*b/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))/d*ln(x^n)+1/9*b/e/(d/e)^(2/3)*ln(x^2
-(d/e)^(1/3)*x+(d/e)^(2/3))/d*n*ln(x)-1/9*b/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3
)*x+(d/e)^(2/3))/d*ln(x^n)-2/9*b/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(d/e)^(1/3)*x-1))/d*n*ln(x)+2/9*b/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(d/e)^(1/3)*x-1))/d*ln(x^n)-1/9*b*n/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))/d+
1/18*b*n/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))/d-1/9*b*n/e/(d/e)^
(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))/d+2/9*b*n/e/d*sum(1/_
R2^2*(ln(x)*ln((_R2-x)/_R2)+dilog((_R2-x)/_R2)),_R2=RootOf(_Z^3*e+d))+(-1/2
*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x
^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*l
n(c)+a)*(1/3*x/d/(e*x^3+d)+2/3/d*(1/3/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))-1/6/e
/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))+1/3/e/(d/e)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))))
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^n\right)}{\left(d+ex^3\right)^2}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex^3+d\right)^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^3+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e^2*x^6 + 2*d*e*x^3 + d^2), x)
```

Sympy [F]

$$\int \frac{a+b\log(cx^n)}{(d+ex^3)^2}\,dx=\int \frac{a+b\log(cx^n)}{(d+ex^3)^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/(e*x**3+d)**2,x)
[Out] Integral((a + b*log(c*x**n))/(d + e*x**3)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log(cx^n)}{(d+ex^3)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^3+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{(d+ex^3)^2}\,dx=\int \frac{b\log(cx^n)+a}{(ex^3+d)^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(e*x^3+d)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/(e*x^3 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{(d+ex^3)^2}\,dx=\int \frac{a+b\,\ln(c\,x^n)}{(e\,x^3+d)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))/(d + e*x^3)^2,x)
[Out] int((a + b*log(c*x^n))/(d + e*x^3)^2, x)
```

3.327 $\int \frac{1}{(d+ex^3)^2(a+b\log(cx^n))} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^3)^2 (a+b\log(cx^n))} dx = \text{Int}\left(\frac{1}{(d+ex^3)^2 (a+b\log(cx^n))}, x\right)$$

```
[Out] Unintegrable(1/(e*x^3+d)^2/(a+b*ln(c*x^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex^3)^2 (a+b\log(cx^n))} dx = \int \frac{1}{(d+ex^3)^2 (a+b\log(cx^n))} dx$$

```
[In] Int[1/((d + e*x^3)^2*(a + b*Log[c*x^n])),x]
```

```
[Out] Defer[Int][1/((d + e*x^3)^2*(a + b*Log[c*x^n])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(d+ex^3)^2 (a+b\log(cx^n))} dx$$

Mathematica [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^3)^2\,(a+b\log{(cx^n)})}\,dx=\int \frac{1}{(d+ex^3)^2\,(a+b\log{(cx^n)})}\,dx$$

```
[In] Integrate[1/((d + e*x^3)^2*(a + b*Log[c*x^n])),x]
[Out] Integrate[1/((d + e*x^3)^2*(a + b*Log[c*x^n])), x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(e\,x^{3}+d\right)^{2}\left(a+b\ln\left(c\,x^{n}\right)\right)}dx$$

```
[In] int(1/(e*x^3+d)^2/(a+b*ln(c*x^n)),x)
[Out] int(1/(e*x^3+d)^2/(a+b*ln(c*x^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.50

$$\int \frac{1}{(d+ex^3)^2\,(a+b\log{(cx^n)})}\,dx=\int \frac{1}{(ex^3+d)^2(b\log{(cx^n)}+a)}\,dx$$

```
[In] integrate(1/(e*x^3+d)^2/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(1/(a*e^2*x^6 + 2*a*d*e*x^3 + a*d^2 + (b*e^2*x^6 + 2*b*d*e*x^3 + b*
d^2)*log(c*x^n)), x)
```

Sympy [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.05

$$\int \frac{1}{(d+ex^3)^2 (a+b\log(cx^n))} dx = \int \frac{1}{(a+b\log(cx^n))(d+ex^3)^2} dx$$

```
[In] integrate(1/(e*x**3+d)**2/(a+b*ln(c*x**n)),x)
[Out] integrate(1/(e*x**3+d)**2/(a+b*ln(c*x**n)),x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^3)^2 (a+b\log(cx^n))} dx = \int \frac{1}{(ex^3+d)^2(b\log(cx^n)+a)} dx$$

```
[In] integrate(1/(e*x^3+d)^2/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(1/((e*x^3 + d)^2*(b*log(c*x^n) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^3)^2 (a+b\log(cx^n))} dx = \int \frac{1}{(ex^3+d)^2(b\log(cx^n)+a)} dx$$

```
[In] integrate(1/(e*x^3+d)^2/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(1/((e*x^3 + d)^2*(b*log(c*x^n) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^3)^2 (a+b\log(cx^n))} \, dx = \int \frac{1}{(e\,x^3+d)^2 \, (a+b\,\ln(c\,x^n))} \, dx$$

```
[In] int(1/((d + e*x^3)^2*(a + b*log(c*x^n))),x)
[Out] int(1/((d + e*x^3)^2*(a + b*log(c*x^n))), x)
```

3.328 $\int \frac{1}{(d+ex^3)^2(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^3)^2(a+b\log(cx^n))^2}\,dx=\operatorname{Int}\left(\frac{1}{(d+ex^3)^2(a+b\log(cx^n))^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^3+d)^2/(a+b*ln(c*x^n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^3)^2(a+b\log(cx^n))^2}\,dx=\int \frac{1}{(d+ex^3)^2(a+b\log(cx^n))^2}\,dx$$

```
[In] Int[1/((d + e*x^3)^2*(a + b*Log[c*x^n])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x^3)^2*(a + b*Log[c*x^n])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(d+ex^3)^2(a+b\log(cx^n))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 17.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^3)^2 (a+b\log(cx^n))^2} dx = \int \frac{1}{(d+ex^3)^2 (a+b\log(cx^n))^2} dx$$

```
[In] Integrate[1/((d + e*x^3)^2*(a + b*Log[c*x^n])^2),x]
[Out] Integrate[1/((d + e*x^3)^2*(a + b*Log[c*x^n])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^3+d)^2 (a+b\ln(c\,x^n))^2}dx$$

```
[In] int(1/(e*x^3+d)^2/(a+b*ln(c*x^n))^2,x)
[Out] int(1/(e*x^3+d)^2/(a+b*ln(c*x^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.64

$$\int \frac{1}{(d+ex^3)^2 (a+b\log(cx^n))^2} dx = \int \frac{1}{(ex^3+d)^2(b\log(cx^n)+a)^2} dx$$

```
[In] integrate(1/(e*x^3+d)^2/(a+b*log(c*x^n))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*e^2*x^6 + 2*a^2*d*e*x^3 + a^2*d^2 + (b^2*e^2*x^6 + 2*b^2*d*
e*x^3 + b^2*d^2)*log(c*x^n)^2 + 2*(a*b*e^2*x^6 + 2*a*b*d*e*x^3 + a*b*d^2)*l
og(c*x^n)), x)
```

Sympy [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.05

$$\int \frac{1}{(d+ex^3)^2 (a+b\log(cx^n))^2} \, dx = \int \frac{1}{(a+b\log(cx^n))^2 (d+ex^3)^2} \, dx$$

```
[In] integrate(1/(e*x**3+d)**2/(a+b*ln(c*x**n))**2,x)
[Out] integrate(1/(e*x**3+d)**2/(a+b*ln(c*x**n))**2,x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 261, normalized size of antiderivative = 11.86

$$\int \frac{1}{(d+ex^3)^2 (a+b\log(cx^n))^2} \, dx = \int \frac{1}{(ex^3+d)^2(b\log(cx^n)+a)^2} \, dx$$

```
[In] integrate(1/(e*x^3+d)^2/(a+b*log(c*x^n))^2,x, algorithm="maxima")
[Out] -x/((b^2*e^2*n*log(c) + a*b*e^2*n)*x^6 + b^2*d^2*n*log(c) + a*b*d^2*n + 2*(
b^2*d*e*n*log(c) + a*b*d*e*n)*x^3 + (b^2*e^2*n*x^6 + 2*b^2*d*e*n*x^3 + b^2*
d^2*n)*log(x^n)) - integrate((5*e*x^3 - d)/((b^2*e^3*n*log(c) + a*b*e^3*n)*
x^9 + 3*(b^2*d*e^2*n*log(c) + a*b*d*e^2*n)*x^6 + b^2*d^3*n*log(c) + a*b*d^3
*n + 3*(b^2*d^2*e*n*log(c) + a*b*d^2*e*n)*x^3 + (b^2*e^3*n*x^9 + 3*b^2*d*e^
2*n*x^6 + 3*b^2*d^2*e*n*x^3 + b^2*d^3*n)*log(x^n)), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^3)^2 (a+b\log(cx^n))^2} \, dx = \int \frac{1}{(ex^3+d)^2(b\log(cx^n)+a)^2} \, dx$$

```
[In] integrate(1/(e*x^3+d)^2/(a+b*log(c*x^n))^2,x, algorithm="giac")
[Out] integrate(1/((e*x^3 + d)^2*(b*log(c*x^n) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^3)^2 (a+b\log(cx^n))^2}\,dx = \int \frac{1}{(e\,x^3+d)^2\,(a+b\,\ln(c\,x^n))^2}\,dx$$

```
[In] int(1/((d + e*x^3)^2*(a + b*log(c*x^n))^2),x)
[Out] int(1/((d + e*x^3)^2*(a + b*log(c*x^n))^2), x)
```

3.329 $\int \frac{x^3(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 185

$$\int \frac{x^3(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = -\frac{ae^3x}{d^4}+\frac{be^3nx}{d^4}-\frac{be^2nx^2}{4d^3}+\frac{benx^3}{9d^2}-\frac{bnx^4}{16d} -\frac{be^3x\log(cx^n)}{d^4}+\frac{e^2x^2(a+b\log(cx^n))}{2d^3} -\frac{ex^3(a+b\log(cx^n))}{3d^2}+\frac{x^4(a+b\log(cx^n))}{4d} +\frac{e^4(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{d^5}+\frac{be^4n\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^5}$$

```
[Out] -a*e^3*x/d^4+b*e^3*n*x/d^4-1/4*b*e^2*n*x^2/d^3+1/9*b*e*n*x^3/d^2-1/16*b*n*x
^4/d-b*e^3*x*ln(c*x^n)/d^4+1/2*e^2*x^2*(a+b*ln(c*x^n))/d^3-1/3*e*x^3*(a+b*l
n(c*x^n))/d^2+1/4*x^4*(a+b*ln(c*x^n))/d+e^4*(a+b*ln(c*x^n))*ln(1+d*x/e)/d^5
+b*e^4*n*polylog(2,-d*x/e)/d^5
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used

$= \{269, 45, 2393, 2332, 2341, 2354, 2438\}$

$$\int \frac{x^3(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = \frac{e^4\log\left(\frac{dx}{e}+1\right)(a+b\log(cx^n))}{d^5} + \frac{e^2x^2(a+b\log(cx^n))}{2d^3} - \frac{ex^3(a+b\log(cx^n))}{3d^2} + \frac{x^4(a+b\log(cx^n))}{4d} - \frac{ae^3x}{d^4} - \frac{be^3x\log(cx^n)}{d^4} + \frac{be^4n\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^5} + \frac{be^3nx}{d^4} - \frac{be^2nx^2}{4d^3} + \frac{benx^3}{9d^2} - \frac{bnx^4}{16d}$$

```
[In] Int[(x^3*(a + b*Log[c*x^n]))/(d + e/x),x]
```

```
[Out] -((a*e^3*x)/d^4) + (b*e^3*n*x)/d^4 - (b*e^2*n*x^2)/(4*d^3) + (b*e*n*x^3)/(9
*d^2) - (b*n*x^4)/(16*d) - (b*e^3*x*Log[c*x^n])/d^4 + (e^2*x^2*(a + b*Log[c
*x^n]))/(2*d^3) - (e*x^3*(a + b*Log[c*x^n]))/(3*d^2) + (x^4*(a + b*Log[c*x^
n]))/(4*d) + (e^4*(a + b*Log[c*x^n])*Log[1 + (d*x)/e])/d^5 + (b*e^4*n*PolyL
og[2, -((d*x)/e)])/d^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
```

```
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{e^3(a+b\log(cx^n))}{d^4}+\frac{e^2x(a+b\log(cx^n))}{d^3}-\frac{ex^2(a+b\log(cx^n))}{d^2}+\frac{x^3(a+b\log(cx^n))}{d}+\frac{e^4(a+b\log(cx^n))}{d^4(e+dx)}\right)dx \\
&= \frac{\int x^3(a+b\log(cx^n))\,dx}{d}-\frac{e\int x^2(a+b\log(cx^n))\,dx}{d^2}+\frac{e^2\int x(a+b\log(cx^n))\,dx}{d^3}-\frac{e^3\int (a+b\log(cx^n))\,dx}{d^4}+\frac{e^4\int \frac{a+b\log(cx^n)}{e+dx}\,dx}{d^4} \\
&= -\frac{ae^3x}{d^4}-\frac{be^2nx^2}{4d^3}+\frac{benx^3}{9d^2}-\frac{bnx^4}{16d}+\frac{e^2x^2(a+b\log(cx^n))}{2d^3}-\frac{ex^3(a+b\log(cx^n))}{3d^2}+\frac{x^4(a+b\log(cx^n))}{4d}+\frac{e^4(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{d^5} \\
&\quad -\frac{(be^3)\int \log(cx^n)\,dx}{d^4}-\frac{(be^4n)\int \frac{\log\left(1+\frac{dx}{e}\right)}{x}\,dx}{d^5} \\
&= -\frac{ae^3x}{d^4}+\frac{be^3nx}{d^4}-\frac{be^2nx^2}{4d^3}+\frac{benx^3}{9d^2}-\frac{bnx^4}{16d}-\frac{be^3x\log(cx^n)}{d^4}+\frac{e^2x^2(a+b\log(cx^n))}{2d^3}-\frac{ex^3(a+b\log(cx^n))}{3d^2}+\frac{x^4(a+b\log(cx^n))}{4d} \\
&\quad +\frac{e^4(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{d^5}+\frac{be^4n\text{Li}_2\left(-\frac{dx}{e}\right)}{d^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.92

$$\int \frac{x^3(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx$$
$$=\frac{-144ade^3x+144bde^3nx-36bd^2e^2nx^2+16bd^3enx^3-9bd^4nx^4-144bde^3x\log(cx^n)+72d^2e^2x^2(a+b\log}{}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/(d + e/x),x]
```

```
[Out] (-144*a*d*e^3*x + 144*b*d*e^3*n*x - 36*b*d^2*e^2*n*x^2 + 16*b*d^3*e*n*x^3 -
 9*b*d^4*n*x^4 - 144*b*d*e^3*x*Log[c*x^n] + 72*d^2*e^2*x^2*(a + b*Log[c*x^n
]) - 48*d^3*e*x^3*(a + b*Log[c*x^n]) + 36*d^4*x^4*(a + b*Log[c*x^n]) + 144*
e^4*(a + b*Log[c*x^n])*Log[1 + (d*x)/e] + 144*b*e^4*n*PolyLog[2, -((d*x)/e)
])/(144*d^5)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.69

method	result
risch	$\frac{b\ln(x^n)x^4}{4d}-\frac{b\ln(x^n)e\,x^3}{3d^2}+\frac{b\ln(x^n)x^2e^2}{2d^3}-\frac{b\ln(x^n)x\,e^3}{d^4}+\frac{b\ln(x^n)e^4\ln(dx+e)}{d^5}-\frac{bn\,e^4\ln(dx+e)\ln\left(-\frac{dx}{e}\right)}{d^5}-\frac{bn\,e^4\,\mathrm{dilog}(}{d^5}$

```
[In] int(x^3*(a+b*ln(c*x^n))/(d+e/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b*ln(x^n)/d*x^4-1/3*b*ln(x^n)/d^2*e*x^3+1/2*b*ln(x^n)/d^3*x^2*e^2-b*ln(
x^n)/d^4*x*e^3+b*ln(x^n)*e^4/d^5*ln(d*x+e)-b*n*e^4/d^5*ln(d*x+e)*ln(-d*x/e)
-b*n*e^4/d^5*dilog(-d*x/e)-1/16*b*n*x^4/d+1/9*b*e*n*x^3/d^2-1/4*b*e^2*n*x^2
/d^3+b*e^3*n*x/d^4+205/144*b*n*e^4/d^5+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*c
sgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*cs
gn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/d^4*(1/4*d^3*x^4-1/3
*e*d^2*x^3+1/2*d*e^2*x^2-x*e^3)+e^4/d^5*ln(d*x+e))
```

Fricas [F]

$$\int \frac{x^3(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{d+\frac{e}{x}}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(d+e/x),x, algorithm="fricas")
[Out] integral((b*x^4*log(c*x^n) + a*x^4)/(d*x + e), x)
```

Sympy [A] (verification not implemented)

Time = 75.86 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.71

$$\int \frac{x^3(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = \frac{ax^4}{4d} - \frac{aex^3}{3d^2} + \frac{ae^2x^2}{2d^3} + \frac{ae^4\left(\begin{cases}\frac{x}{e} & \text{for}\ d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)}{d^4} - \frac{ae^3x}{d^4} - \frac{bnx^4}{16d} + \frac{bx^4\log(cx^n)}{4d} + \frac{benx^3}{9d^2} - \frac{bex^3\log(cx^n)}{3d^2} - \frac{be^2nx^2}{4d^3} + \frac{be^2x^2\log(cx^n)}{2d^3} - \frac{be^4n\left(\begin{cases}\frac{x}{e} \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for}\ \frac{1}{|x|}<1\wedge|x|<1\\ \log(e)\log(x)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for}\ |x|<1\\ -\log(e)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for}\ \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log(e)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{otherwise}\end{cases}}{d}\end{cases}\right)}{d^4} + \frac{be^4\left(\begin{cases}\frac{x}{e} & \text{for}\ d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^4} + \frac{be^3nx}{d^4} - \frac{be^3x\log(cx^n)}{d^4}$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))/(d+e/x),x)
[Out] a*x**4/(4*d) - a*e*x**3/(3*d**2) + a*e**2*x**2/(2*d**3) + a*e**4*Piecewise(
(x/e, Eq(d, 0)), (log(d*x + e)/d, True))/d**4 - a*e**3*x/d**4 - b*n*x**4/(1
6*d) + b*x**4*log(c*x**n)/(4*d) + b*e*n*x**3/(9*d**2) - b*e*x**3*log(c*x**n
)/(3*d**2) - b*e**2*n*x**2/(4*d**3) + b*e**2*x**2*log(c*x**n)/(2*d**3) - b*
e**4*n*Piecewise((x/e, Eq(d, 0)), (Piecewise((-polylog(2, d*x*exp_polar(I*p
i)/e), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(e)*log(x) - polylog(2, d*x*exp_
```

```
polar(I*pi)/e), Abs(x) < 1), (-log(e)*log(1/x) - polylog(2, d*x*exp_polar(I
*pi)/e), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(e) + m
eijerg(((1, 1), ()), ((), (0, 0)), x)*log(e) - polylog(2, d*x*exp_polar(I*p
i)/e), True))/d, True))/d**4 + b*e**4*Piecewise((x/e, Eq(d, 0)), (log(d*x +
 e)/d, True))*log(c*x**n)/d**4 + b*e**3*n*x/d**4 - b*e**3*x*log(c*x**n)/d**
4
```

Maxima [F]

$$\int \frac{x^3(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{d+\frac{e}{x}}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(d+e/x),x, algorithm="maxima")
[Out] 1/12*a*(12*e^4*log(d*x + e)/d^5 + (3*d^3*x^4 - 4*d^2*e*x^3 + 6*d*e^2*x^2 -
12*e^3*x)/d^4) + b*integrate((x^4*log(c) + x^4*log(x^n))/(d*x + e), x)
```

Giac [F]

$$\int \frac{x^3(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{d+\frac{e}{x}}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(d+e/x),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^3/(d + e/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = \int \frac{x^3\,(a+b\,\ln(c\,x^n))}{d+\frac{e}{x}}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n)))/(d + e/x),x)
[Out] int((x^3*(a + b*log(c*x^n)))/(d + e/x), x)
```

3.330 $\int \frac{x^2(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{x^2(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = \frac{ae^2x}{d^3}-\frac{be^2nx}{d^3}+\frac{benx^2}{4d^2}-\frac{bnx^3}{9d}+\frac{be^2x\log(cx^n)}{d^3} - \frac{ex^2(a+b\log(cx^n))}{2d^2}+\frac{x^3(a+b\log(cx^n))}{3d} - \frac{e^3(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{d^4}-\frac{be^3n\operatorname{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^4}$$

```
[Out] a*e^2*x/d^3-b*e^2*n*x/d^3+1/4*b*e*n*x^2/d^2-1/9*b*n*x^3/d+b*e^2*x*ln(c*x^n)
/d^3-1/2*e*x^2*(a+b*ln(c*x^n))/d^2+1/3*x^3*(a+b*ln(c*x^n))/d-e^3*(a+b*ln(c*
x^n))*ln(1+d*x/e)/d^4-b*e^3*n*polylog(2,-d*x/e)/d^4
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {269, 45, 2393, 2332, 2341, 2354, 2438}

$$\int \frac{x^2(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = -\frac{e^3\log\left(\frac{dx}{e}+1\right)(a+b\log(cx^n))}{d^4}-\frac{ex^2(a+b\log(cx^n))}{2d^2} +\frac{x^3(a+b\log(cx^n))}{3d}+\frac{ae^2x}{d^3}+\frac{be^2x\log(cx^n)}{d^3} -\frac{be^3n\operatorname{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^4}-\frac{be^2nx}{d^3}+\frac{benx^2}{4d^2}-\frac{bnx^3}{9d}$$

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/(d + e/x),x]
```

```
[Out] (a*e^2*x)/d^3 - (b*e^2*n*x)/d^3 + (b*e*n*x^2)/(4*d^2) - (b*n*x^3)/(9*d) + (
b*e^2*x*Log[c*x^n])/d^3 - (e*x^2*(a + b*Log[c*x^n]))/(2*d^2) + (x^3*(a + b*
Log[c*x^n]))/(3*d) - (e^3*(a + b*Log[c*x^n])*Log[1 + (d*x)/e])/d^4 - (b*e^3
*n*PolyLog[2, -((d*x)/e)])/d^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^2(a+b\log(cx^n))}{d^3} - \frac{ex(a+b\log(cx^n))}{d^2} + \frac{x^2(a+b\log(cx^n))}{d} - \frac{e^3(a+b\log(cx^n))}{d^3(e+dx)}\right) dx$$

$$= \frac{\int x^2(a+b\log(cx^n))\,dx}{d} - \frac{e\int x(a+b\log(cx^n))\,dx}{d^2} + \frac{e^2\int (a+b\log(cx^n))\,dx}{d^3} - \frac{e^3\int \frac{a+b\log(cx^n)}{e+dx}\,dx}{d^3}$$

$$= \frac{ae^2x}{d^3} + \frac{benx^2}{4d^2} - \frac{bnx^3}{9d} - \frac{ex^2(a+b\log(cx^n))}{2d^2} + \frac{x^3(a+b\log(cx^n))}{3d} - \frac{e^3(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{d^4} + \frac{(be^2)\int \log(cx^n)\,dx}{d^3} + \frac{(be^3n)\int \frac{\log\left(1+\frac{dx}{e}\right)}{x}\,dx}{d^4}$$

$$= \frac{ae^2x}{d^3} - \frac{be^2nx}{d^3} + \frac{benx^2}{4d^2} - \frac{bnx^3}{9d} + \frac{be^2x\log(cx^n)}{d^3} - \frac{ex^2(a+b\log(cx^n))}{2d^2} + \frac{x^3(a+b\log(cx^n))}{3d} - \frac{e^3(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{d^4} - \frac{be^3n\text{Li}_2\left(-\frac{dx}{e}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = \frac{36ade^2x - 36bde^2nx - 18ad^2ex^2 + 9bd^2enx^2 + 12ad^3x^3 - 4bd^3nx^3 - 36ae^3\log\left(1+\frac{dx}{e}\right) + 6b\log(cx^n)\left(dx\right.}{36d^4}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e/x),x]
```

```
[Out] (36*a*d*e^2*x - 36*b*d*e^2*n*x - 18*a*d^2*e*x^2 + 9*b*d^2*e*n*x^2 + 12*a*d^
3*x^3 - 4*b*d^3*n*x^3 - 36*a*e^3*Log[1 + (d*x)/e] + 6*b*Log[c*x^n]*(d*x*(6*
e^2 - 3*d*e*x + 2*d^2*x^2) - 6*e^3*Log[1 + (d*x)/e]) - 36*b*e^3*n*PolyLog[2
, -((d*x)/e)])/(36*d^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.84

method	result
risch	$\frac{b\ln(x^n)x^3}{3d} - \frac{b\ln(x^n)e\,x^2}{2d^2} + \frac{b\ln(x^n)x\,e^2}{d^3} - \frac{b\ln(x^n)e^3\ln(dx+e)}{d^4} - \frac{bn\,x^3}{9d} + \frac{ben\,x^2}{4d^2} - \frac{b\,e^2nx}{d^3} - \frac{49bn\,e^3}{36d^4} + \frac{bn\,e^3\ln(dx+e)}{d^4}$

[In] int(x^2*(a+b*ln(c*x^n))/(d+e/x),x,method=_RETURNVERBOSE)

[Out] 1/3*b*ln(x^n)/d*x^3-1/2*b*ln(x^n)/d^2*e*x^2+b*ln(x^n)/d^3*x*e^2-b*ln(x^n)*e^3/d^4*ln(d*x+e)-1/9*b*n*x^3/d+1/4*b*e*n*x^2/d^2-b*e^2*n*x/d^3-49/36*b*n*e^3/d^4+b*n*e^3/d^4*ln(d*x+e)*ln(-d*x/e)+b*n*e^3/d^4*dilog(-d*x/e)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/d^3*(1/3*d^2*x^3-1/2*d*e*x^2+x*e^2)-e^3/d^4*ln(d*x+e))

Fricas [F]

$$\int \frac{x^2(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{d+\frac{e}{x}}\,dx$$

[In] integrate(x^2*(a+b*log(c*x^n))/(d+e/x),x, algorithm="fricas")

[Out] integral((b*x^3*log(c*x^n) + a*x^3)/(d*x + e), x)

Sympy [A] (verification not implemented)

Time = 80.12 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.80

$$\int \frac{x^2(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = \frac{ax^3}{3d} - \frac{aex^2}{2d^2} - \frac{ae^3\left(\begin{cases}\frac{x}{e} & \text{for } d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)}{d^3} + \frac{ae^2x}{d^3} - \frac{bnx^3}{9d} + \frac{bx^3\log(cx^n)}{3d} + \frac{benx^2}{4d^2} - \frac{bex^2\log(cx^n)}{2d^2} + \frac{be^3n\left(\begin{cases}\frac{x}{e} \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(e)\log(x)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } |x|<1\\ -\log(e)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(e)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{otherwise}\end{cases}}{d}\end{cases}\right)}{d^3} - \frac{be^3\left(\begin{cases}\frac{x}{e} & \text{for } d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^3} - \frac{be^2nx}{d^3} + \frac{be^2x\log(cx^n)}{d^3}$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(d+e/x),x)
```

```
[Out] a*x**3/(3*d) - a*e*x**2/(2*d**2) - a*e**3*Piecewise((x/e, Eq(d, 0)), (log(d
*x + e)/d, True))/d**3 + a*e**2*x/d**3 - b*n*x**3/(9*d) + b*x**3*log(c*x**n
)/(3*d) + b*e*n*x**2/(4*d**2) - b*e*x**2*log(c*x**n)/(2*d**2) + b*e**3*n*Pi
ecewise((x/e, Eq(d, 0)), (Piecewise((-polylog(2, d*x*exp_polar(I*pi)/e), (A
bs(x) < 1) & (1/Abs(x) < 1)), (log(e)*log(x) - polylog(2, d*x*exp_polar(I*p
i)/e), Abs(x) < 1), (-log(e)*log(1/x) - polylog(2, d*x*exp_polar(I*pi)/e),
1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(e) + meijerg(((
1, 1), ()), ((), (0, 0)), x)*log(e) - polylog(2, d*x*exp_polar(I*pi)/e), Tr
ue))/d, True))/d**3 - b*e**3*Piecewise((x/e, Eq(d, 0)), (log(d*x + e)/d, Tr
ue))*log(c*x**n)/d**3 - b*e**2*n*x/d**3 + b*e**2*x*log(c*x**n)/d**3
```

Maxima [F]

$$\int \frac{x^2(a + b\log(cx^n))}{d + \frac{e}{x}}\,dx = \int \frac{(b\log(cx^n) + a)x^2}{d + \frac{e}{x}}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(d+e/x),x, algorithm="maxima")
```

```
[Out] -1/6*a*(6*e^3*log(d*x + e)/d^4 - (2*d^2*x^3 - 3*d*e*x^2 + 6*e^2*x)/d^3) + b
*integrate((x^3*log(c) + x^3*log(x^n))/(d*x + e), x)
```

Giac [F]

$$\int \frac{x^2(a + b\log(cx^n))}{d + \frac{e}{x}}\,dx = \int \frac{(b\log(cx^n) + a)x^2}{d + \frac{e}{x}}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(d+e/x),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2/(d + e/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\log(cx^n))}{d + \frac{e}{x}}\,dx = \int \frac{x^2\,(a + b\,\ln(c\,x^n))}{d + \frac{e}{x}}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e/x),x)
```

```
[Out] int((x^2*(a + b*log(c*x^n)))/(d + e/x), x)
```

3.331 $\int \frac{x(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{x(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = -\frac{aex}{d^2}+\frac{benx}{d^2}-\frac{bnx^2}{4d}-\frac{bex\log(cx^n)}{d^2}+\frac{x^2(a+b\log(cx^n))}{2d}+\frac{e^2(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{d^3}+\frac{be^2n\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^3}$$

```
[Out] -a*e*x/d^2+b*e*n*x/d^2-1/4*b*n*x^2/d-b*e*x*ln(c*x^n)/d^2+1/2*x^2*(a+b*ln(c*
x^n))/d+e^2*(a+b*ln(c*x^n))*ln(1+d*x/e)/d^3+b*e^2*n*polylog(2,-d*x/e)/d^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {269, 45, 2393, 2332, 2341, 2354, 2438}

$$\int \frac{x(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = \frac{e^2\log\left(\frac{dx}{e}+1\right)(a+b\log(cx^n))}{d^3}+\frac{x^2(a+b\log(cx^n))}{2d}-\frac{aex}{d^2}-\frac{bex\log(cx^n)}{d^2}+\frac{be^2n\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^3}+\frac{benx}{d^2}-\frac{bnx^2}{4d}$$

```
[In] Int[(x*(a + b*Log[c*x^n]))/(d + e/x),x]
```

```
[Out] -((a*e*x)/d^2) + (b*e*n*x)/d^2 - (b*n*x^2)/(4*d) - (b*e*x*Log[c*x^n])/d^2 +
 (x^2*(a + b*Log[c*x^n]))/(2*d) + (e^2*(a + b*Log[c*x^n])*Log[1 + (d*x)/e])
/d^3 + (b*e^2*n*PolyLog[2, -((d*x)/e)])/d^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{e(a+b\log(cx^n))}{d^2}+\frac{x(a+b\log(cx^n))}{d}+\frac{e^2(a+b\log(cx^n))}{d^2(e+dx)}\right) dx$$

$$= \frac{\int x(a+b\log(cx^n))\,dx}{d} - \frac{e\int(a+b\log(cx^n))\,dx}{d^2} + \frac{e^2\int\frac{a+b\log(cx^n)}{e+dx}\,dx}{d^2}$$

$$= -\frac{aex}{d^2} - \frac{bnx^2}{4d} + \frac{x^2(a+b\log(cx^n))}{2d} + \frac{e^2(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{d^3} - \frac{(be)\int\log(cx^n)\,dx}{d^2} - \frac{(be^2n)\int\frac{\log\left(1+\frac{dx}{e}\right)}{x}\,dx}{d^3}$$

$$= -\frac{aex}{d^2} + \frac{benx}{d^2} - \frac{bnx^2}{4d} - \frac{bex\log(cx^n)}{d^2} + \frac{x^2(a+b\log(cx^n))}{2d} + \frac{e^2(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{d^3} + \frac{be^2n\mathrm{Li}_2\left(-\frac{dx}{e}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98

$$\int\frac{x(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = \frac{-4adex+4bdenx+2ad^2x^2-bd^2nx^2+4ae^2\log\left(1+\frac{dx}{e}\right)+2b\log(cx^n)\left(dx(-2e+dx)+2e^2\log\left(1+\frac{dx}{e}\right)\right.}{4d^3}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e/x),x]
```

```
[Out] (-4*a*d*e*x + 4*b*d*e*n*x + 2*a*d^2*x^2 - b*d^2*n*x^2 + 4*a*e^2*Log[1 + (d*
x)/e] + 2*b*Log[c*x^n]*(d*x*(-2*e + d*x) + 2*e^2*Log[1 + (d*x)/e]) + 4*b*e^
2*n*PolyLog[2, -((d*x)/e)])/(4*d^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.18

method	result
risch	$\frac{b\ln(x^n)x^2}{2d} - \frac{b\ln(x^n)ex}{d^2} + \frac{b\ln(x^n)e^2\ln(dx+e)}{d^3} - \frac{bn\,e^2\ln(dx+e)\ln\left(-\frac{dx}{e}\right)}{d^3} - \frac{bn\,e^2\operatorname{dilog}\left(-\frac{dx}{e}\right)}{d^3} - \frac{bn\,x^2}{4d} + \frac{benx}{d^2} + \frac{5bn\,e^2}{4d^3}$

```
[In] int(x*(a+b*ln(c*x^n))/(d+e/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*ln(x^n)/d*x^2-b*ln(x^n)/d^2*e*x+b*ln(x^n)*e^2/d^3*ln(d*x+e)-b*n*e^2/d
^3*ln(d*x+e)*ln(-d*x/e)-b*n*e^2/d^3*dilog(-d*x/e)-1/4*b*n*x^2/d+b*e*n*x/d^2
+5/4*b*n*e^2/d^3+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*P
i*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*
Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(1/d^2*(1/2*d*x^2-e*x)+e^2/d^3*ln(d*x+e))
```

Fricas [F]

$$\int \frac{x(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = \int \frac{(b\log(cx^n)+a)x}{d+\frac{e}{x}}\,dx$$

[In] integrate(x*(a+b*log(c*x^n))/(d+e/x),x, algorithm="fricas")

[Out] integral((b*x^2*log(c*x^n) + a*x^2)/(d*x + e), x)

Sympy [A] (verification not implemented)

Time = 53.55 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.04

$$\int \frac{x(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = \frac{ax^2}{2d} + \frac{ae^2\left(\begin{cases}\frac{x}{e} & \text{for } d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)}{d^2} - \frac{aex}{d^2} - \frac{bnx^2}{4d} + \frac{bx^2\log(cx^n)}{2d} - \frac{be^2n\left(\begin{cases}\frac{x}{e} & \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(e)\log(x)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } |x|<1\\ -\log(e)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(e)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{otherwise}\end{cases}}{d} & \end{cases}\right)}{d^2} + \frac{be^2\left(\begin{cases}\frac{x}{e} & \text{for } d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^2} + \frac{benx}{d^2} - \frac{bex\log(cx^n)}{d^2}$$

[In] integrate(x*(a+b*ln(c*x**n))/(d+e/x),x)

[Out] a*x**2/(2*d) + a*e**2*Piecewise((x/e, Eq(d, 0)), (log(d*x + e)/d, True))/d**2 - a*e*x/d**2 - b*n*x**2/(4*d) + b*x**2*log(c*x**n)/(2*d) - b*e**2*n*Piecewise((x/e, Eq(d, 0)), (Piecewise((-polylog(2, d*x*exp_polar(I*pi)/e), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(e)*log(x) - polylog(2, d*x*exp_polar(I*pi)/e), Abs(x) < 1), (-log(e)*log(1/x) - polylog(2, d*x*exp_polar(I*pi)/e), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(e) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(e) - polylog(2, d*x*exp_polar(I*pi)/e), True))/d, True))/d**2 + b*e**2*Piecewise((x/e, Eq(d, 0)), (log(d*x + e)/d, True))*log(c*x**n)/d**2 + b*e*n*x/d**2 - b*e*x*log(c*x**n)/d**2

Maxima [F]

$$\int \frac{x(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = \int \frac{(b\log(cx^n)+a)x}{d+\frac{e}{x}}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(d+e/x),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*e^2*log(d*x + e)/d^3 + (d*x^2 - 2*e*x)/d^2) + b*integrate((x^2*log
(c) + x^2*log(x^n))/(d*x + e), x)
```

Giac [F]

$$\int \frac{x(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = \int \frac{(b\log(cx^n)+a)x}{d+\frac{e}{x}}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(d+e/x),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x/(d + e/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(cx^n))}{d+\frac{e}{x}}\,dx = \int \frac{x\,(a+b\,\ln(c\,x^n))}{d+\frac{e}{x}}\,dx$$

```
[In] int((x*(a + b*log(c*x^n)))/(d + e/x),x)
```

```
[Out] int((x*(a + b*log(c*x^n)))/(d + e/x), x)
```

3.332 $\int \frac{a+b\log(cx^n)}{d+\frac{e}{x}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{a+b\log(cx^n)}{d+\frac{e}{x}}\,dx = \frac{ax}{d}-\frac{bnx}{d}+\frac{bx\log(cx^n)}{d} - \frac{e(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{d^2}-\frac{ben\,\text{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^2}$$

```
[Out] a*x/d-b*n*x/d+b*x*ln(c*x^n)/d-e*(a+b*ln(c*x^n))*ln(1+d*x/e)/d^2-b*e*n*polyl
og(2,-d*x/e)/d^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {199, 45, 2367, 2332, 2354, 2438}

$$\int \frac{a+b\log(cx^n)}{d+\frac{e}{x}}\,dx = -\frac{e\log\left(\frac{dx}{e}+1\right)(a+b\log(cx^n))}{d^2}+\frac{ax}{d} +\frac{bx\log(cx^n)}{d}-\frac{ben\,\text{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^2}-\frac{bnx}{d}$$

```
[In] Int[(a + b*Log[c*x^n])/(d + e/x),x]
```

```
[Out] (a*x)/d - (b*n*x)/d + (b*x*Log[c*x^n])/d - (e*(a + b*Log[c*x^n])*Log[1 + (d
*x)/e])/d^2 - (b*e*n*PolyLog[2, -((d*x)/e)])/d^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a+b\log(cx^n)}{d}-\frac{e(a+b\log(cx^n))}{d(e+dx)}\right)\,dx\\
&=\frac{\int (a+b\log(cx^n))\,dx}{d}-\frac{e\int \frac{a+b\log(cx^n)}{e+dx}\,dx}{d}\\
&=\frac{ax}{d}-\frac{e(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{d^2}+\frac{b\int \log(cx^n)\,dx}{d}+\frac{(ben)\int \frac{\log\left(1+\frac{dx}{e}\right)}{x}\,dx}{d^2}\\
&=\frac{ax}{d}-\frac{bnx}{d}+\frac{bx\log(cx^n)}{d}-\frac{e(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{d^2}-\frac{ben\text{Li}_2\left(-\frac{dx}{e}\right)}{d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{a+b\log(cx^n)}{d+\frac{e}{x}}\,dx = \frac{adx-bdnx-ae\log\left(1+\frac{dx}{e}\right)+b\log(cx^n)\left(dx-e\log\left(1+\frac{dx}{e}\right)\right)-ben\,\text{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^2}$$

[In] Integrate[(a + b*Log[c*x^n])/(d + e/x),x]

[Out] (a*d*x - b*d*n*x - a*e*Log[1 + (d*x)/e] + b*Log[c*x^n]*(d*x - e*Log[1 + (d*x)/e]) - b*e*n*PolyLog[2, -((d*x)/e)])/d^2

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.72

method	result
risch	$\frac{b\ln(x^n)x}{d}-\frac{b\ln(x^n)e\ln(dx+e)}{d^2}-\frac{bnx}{d}-\frac{bne}{d^2}+\frac{bne\ln(dx+e)\ln\left(-\frac{dx}{e}\right)}{d^2}+\frac{bne\,\text{dilog}\left(-\frac{dx}{e}\right)}{d^2}+\left(-\frac{ib\pi\,\text{csgn}(ic)\,\text{csgn}(ix^n)\,\text{cs}}{2}\right.$

[In] int((a+b*ln(c*x^n))/(d+e/x),x,method=_RETURNVERBOSE)

[Out] b*ln(x^n)/d*x-b*ln(x^n)*e/d^2*ln(d*x+e)-b*n*x/d-b*n*e/d^2+b*n*e/d^2*ln(d*x+e)*ln(-d*x/e)+b*n*e/d^2*dilog(-d*x/e)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(x/d-e/d^2*ln(d*x+e))

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{d+\frac{e}{x}}\,dx = \int \frac{b\log(cx^n)+a}{d+\frac{e}{x}}\,dx$$

[In] integrate((a+b*log(c*x^n))/(d+e/x),x, algorithm="fricas")

[Out] integral((b*x*log(c*x^n) + a*x)/(d*x + e), x)

Sympy [A] (verification not implemented)

Time = 42.53 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.36

$$\int \frac{a+b\log(cx^n)}{d+\frac{e}{x}}\,dx = -\frac{ae\left(\begin{cases}\frac{x}{e} & \text{for } d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)}{d} + \frac{ax}{d} + \frac{ben\left(\begin{cases}\frac{x}{e} & \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(e)\log(x)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } |x|<1\\ -\log(e)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(e)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{otherwise}\end{cases}}{d} & \end{cases}\right)}{d} - \frac{be\left(\begin{cases}\frac{x}{e} & \text{for } d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d} - \frac{bnx}{d} + \frac{bx\log(cx^n)}{d}$$

```
[In] integrate((a+b*ln(c*x**n))/(d+e/x),x)
```

```
[Out] -a*e*Piecewise((x/e, Eq(d, 0)), (log(d*x + e)/d, True))/d + a*x/d + b*e*n*P
iecewise((x/e, Eq(d, 0)), (Piecewise((-polylog(2, d*x*exp_polar(I*pi)/e), (
Abs(x) < 1) & (1/Abs(x) < 1)), (log(e)*log(x) - polylog(2, d*x*exp_polar(I*
pi)/e), Abs(x) < 1), (-log(e)*log(1/x) - polylog(2, d*x*exp_polar(I*pi)/e),
 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(e) + meijerg((
(1, 1), ()), ((), (0, 0)), x)*log(e) - polylog(2, d*x*exp_polar(I*pi)/e), T
rue))/d, True))/d - b*e*Piecewise((x/e, Eq(d, 0)), (log(d*x + e)/d, True))*
log(c*x**n)/d - b*n*x/d + b*x*log(c*x**n)/d
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{d+\frac{e}{x}}\,dx = \int \frac{b\log(cx^n)+a}{d+\frac{e}{x}}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(d+e/x),x, algorithm="maxima")
```

```
[Out] a*(x/d - e*log(d*x + e)/d^2) + b*integrate((x*log(c) + x*log(x^n))/(d*x + e
), x)
```

Giac [F]

$$\int \frac{a + b\log(cx^n)}{d + \frac{e}{x}}\, dx = \int \frac{b\log(cx^n) + a}{d + \frac{e}{x}}\, dx$$

```
[In] integrate((a+b*log(c*x^n))/(d+e/x),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/(d + e/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{d + \frac{e}{x}}\, dx = \int \frac{a + b\,\ln(c\,x^n)}{d + \frac{e}{x}}\, dx$$

```
[In] int((a + b*log(c*x^n))/(d + e/x),x)
[Out] int((a + b*log(c*x^n))/(d + e/x), x)
```

3.333 $\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 39

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x}\,dx=\frac{(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{d}+\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{d}$$

```
[Out] (a+b*ln(c*x^n))*ln(1+d*x/e)/d+b*n*polylog(2,-d*x/e)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2370, 2354, 2438}

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x}\,dx=\frac{\log\left(\frac{dx}{e}+1\right)(a+b\log(cx^n))}{d}+\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{d}$$

```
[In] Int[(a + b*Log[c*x^n])/((d + e/x)*x),x]
```

```
[Out] ((a + b*Log[c*x^n])*Log[1 + (d*x)/e])/d + (b*n*PolyLog[2, -((d*x)/e)])/d
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{a+b\log(cx^n)}{e+dx}\,dx \\
&= \frac{(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{d} - \frac{(bn)\int \frac{\log\left(1+\frac{dx}{e}\right)}{x}\,dx}{d} \\
&= \frac{(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{d} + \frac{bn\mathrm{Li}_2\left(-\frac{dx}{e}\right)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$
\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x}\,dx = \frac{(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)+bn\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{d}
$$

```
[In] Integrate[(a + b*Log[c*x^n])/((d + e/x)*x),x]
[Out] ((a + b*Log[c*x^n])*Log[1 + (d*x)/e] + b*n*PolyLog[2, -((d*x)/e)])/d
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.87

method	result
risch	$\frac{b\ln(x^n)\ln(dx+e)}{d} - \frac{bn\ln(dx+e)\ln\left(-\frac{dx}{e}\right)}{d} - \frac{bn\,\mathrm{dilog}\left(-\frac{dx}{e}\right)}{d} + \frac{\left(-\frac{ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)}{2}+\frac{ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2}{2}+\right.}{}$

```
[In] int((a+b*ln(c*x^n))/(d+e/x)/x,x,method=_RETURNVERBOSE)
[Out] b*ln(x^n)*ln(d*x+e)/d-b/d*n*ln(d*x+e)*ln(-d*x/e)-b/d*n*dilog(-d*x/e)+(-1/2*
I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^
```

```
n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln
(c)+a)*ln(d*x+e)/d
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x}\,dx = \int \frac{b\log(cx^n)+a}{\left(d+\frac{e}{x}\right)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(d+e/x)/x,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(d*x + e), x)
```

Sympy [F]

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x}\,dx = \int \frac{a+b\log(cx^n)}{dx+e}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/(d+e/x)/x,x)
[Out] Integral((a + b*log(c*x**n))/(d*x + e), x)
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x}\,dx = \int \frac{b\log(cx^n)+a}{\left(d+\frac{e}{x}\right)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(d+e/x)/x,x, algorithm="maxima")
[Out] b*integrate((log(c) + log(x^n))/(d*x + e), x) + a*log(d*x + e)/d
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x}\,dx = \int \frac{b\log(cx^n)+a}{\left(d+\frac{e}{x}\right)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(d+e/x)/x,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((d + e/x)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x}\,dx=\int \frac{a+b\,\ln(c\,x^n)}{x\,\left(d+\frac{e}{x}\right)}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e/x)),x)
[Out] int((a + b*log(c*x^n))/(x*(d + e/x)), x)
```

3.334 $\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 44

$$\int \frac{a+b\log\left(cx^n\right)}{\left(d+\frac{e}{x}\right)x^2}\,dx=-\frac{\log\left(1+\frac{e}{dx}\right)\left(a+b\log\left(cx^n\right)\right)}{e}+\frac{bn\,\text{PolyLog}\left(2,-\frac{e}{dx}\right)}{e}$$

```
[Out] -ln(1+e/d/x)*(a+b*ln(c*x^n))/e+b*n*polylog(2,-e/d/x)/e
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2375, 2438}

$$\int \frac{a+b\log\left(cx^n\right)}{\left(d+\frac{e}{x}\right)x^2}\,dx=\frac{bn\,\text{PolyLog}\left(2,-\frac{e}{dx}\right)}{e}-\frac{\log\left(\frac{e}{dx}+1\right)\left(a+b\log\left(cx^n\right)\right)}{e}$$

```
[In] Int[(a + b*Log[c*x^n])/((d + e/x)*x^2),x]
```

```
[Out] -((Log[1 + e/(d*x)]*(a + b*Log[c*x^n]))/e) + (b*n*PolyLog[2, -(e/(d*x))])/e
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\log\left(1+\frac{e}{dx}\right)(a+b\log(cx^n))}{e}+\frac{(bn)\int\frac{\log\left(1+\frac{e}{dx}\right)}{x}\,dx}{e}\\ &= -\frac{\log\left(1+\frac{e}{dx}\right)(a+b\log(cx^n))}{e}+\frac{bn\mathrm{Li}_2\left(-\frac{e}{dx}\right)}{e}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.43

$$\int\frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^2}\,dx=\frac{(a+b\log(cx^n))\left(a+b\log(cx^n)-2bn\log\left(1+\frac{dx}{e}\right)\right)}{2ben}-\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{e}$$

```
[In] Integrate[(a + b*Log[c*x^n])/((d + e/x)*x^2),x]
```

```
[Out] ((a + b*Log[c*x^n])*(a + b*Log[c*x^n] - 2*b*n*Log[1 + (d*x)/e]))/(2*b*e*n)
- (b*n*PolyLog[2, -((d*x)/e)])/e
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 4.11

method	result
risch	$-\frac{b\ln(x^n)\ln(dx+e)}{e}+\frac{b\ln(x^n)\ln(x)}{e}-\frac{bn\ln(x)^2}{2e}+\frac{bn\ln(dx+e)\ln\left(-\frac{dx}{e}\right)}{e}+\frac{bn\,\mathrm{dilog}\left(-\frac{dx}{e}\right)}{e}+\left(-\frac{ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{c}}{2}\right.$

```
[In] int((a+b*ln(c*x^n))/(d+e/x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b*ln(x^n)/e*ln(d*x+e)+b*ln(x^n)/e*ln(x)-1/2*b*n/e*ln(x)^2+b*n/e*ln(d*x+e)*
ln(-d*x/e)+b*n/e*dilog(-d*x/e)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*
x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x
^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-1/e*ln(d*x+e)+1/e*ln(x))
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^2}\,dx = \int \frac{b\log(cx^n)+a}{\left(d+\frac{e}{x}\right)x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(d+e/x)/x^2,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(d*x^2 + e*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 3.93

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^2}\,dx = \frac{2ad\left(\begin{cases} -\frac{x}{e}-\frac{1}{2d} & \text{for } d=0 \\ \frac{\log(2dx)}{2d} & \text{otherwise} \end{cases}\right)}{e} - \frac{2ad\left(\begin{cases} \frac{x}{e}+\frac{1}{2d} & \text{for } d=0 \\ \frac{\log(2dx+2e)}{2d} & \text{otherwise} \end{cases}\right)}{e} + bn\left(\begin{cases} -\frac{1}{dx} & \text{fo} \\ \frac{\begin{cases} \mathrm{Li}_2\left(\frac{ee^{i\pi}}{dx}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1 \\ \log(d)\log(x)+\mathrm{Li}_2\left(\frac{ee^{i\pi}}{dx}\right) & \text{for } |x|<1 \\ -\log(d)\log\left(\frac{1}{x}\right)+\mathrm{Li}_2\left(\frac{ee^{i\pi}}{dx}\right) & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(d)+\mathrm{Li}_2\left(\frac{ee^{i\pi}}{dx}\right) & \text{otherwise} \end{cases}}{e} & \text{ot} \end{cases}\right) - b\left(\begin{cases} \frac{1}{dx} & \text{for } e=0 \\ \frac{\log\left(d+\frac{e}{x}\right)}{e} & \text{otherwise} \end{cases}\right)\log(cx^n)$$

```
[In] integrate((a+b*ln(c*x**n))/(d+e/x)/x**2,x)
[Out] 2*a*d*Piecewise((-x/e - 1/(2*d), Eq(d, 0)), (log(2*d*x)/(2*d), True))/e - 2
*a*d*Piecewise((x/e + 1/(2*d), Eq(d, 0)), (log(2*d*x + 2*e)/(2*d), True))/e
 + b*n*Piecewise((-1/(d*x), Eq(e, 0)), (Piecewise((polylog(2, e*exp_polar(I
*pi)/(d*x)), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) + polylog(2, e*
exp_polar(I*pi)/(d*x)), Abs(x) < 1), (-log(d)*log(1/x) + polylog(2, e*exp_p
olar(I*pi)/(d*x)), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*
log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) + polylog(2, e*exp_p
olar(I*pi)/(d*x)), True))/e, True)) - b*Piecewise((1/(d*x), Eq(e, 0)), (log
(d + e/x)/e, True))*log(c*x**n)
```

Maxima [F]

$$\int \frac{a+b\log\left(cx^n\right)}{\left(d+\frac{e}{x}\right)x^2}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(d+\frac{e}{x}\right)x^2}\,dx$$

[In] integrate((a+b*log(c*x^n))/(d+e/x)/x^2,x, algorithm="maxima")

[Out] -a*(log(d*x + e)/e - log(x)/e) + b*integrate((log(c) + log(x^n))/(d*x^2 + e*x), x)

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{\left(d+\frac{e}{x}\right)x^2}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(d+\frac{e}{x}\right)x^2}\,dx$$

[In] integrate((a+b*log(c*x^n))/(d+e/x)/x^2,x, algorithm="giac")

[Out] integrate((b*log(c*x^n) + a)/((d + e/x)*x^2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{\left(d+\frac{e}{x}\right)x^2}\,dx=\int \frac{a+b\,\ln\left(c\,x^n\right)}{x^2\,\left(d+\frac{e}{x}\right)}\,dx$$

[In] int((a + b*log(c*x^n))/(x^2*(d + e/x)),x)

[Out] int((a + b*log(c*x^n))/(x^2*(d + e/x)), x)

3.335 $\int \frac{a+b\log(cx^n)}{(d+\frac{e}{x})x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^3}\,dx = -\frac{bn}{ex}-\frac{a+b\log(cx^n)}{ex}-\frac{d(a+b\log(cx^n))^2}{2be^2n} + \frac{d(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{e^2}+\frac{bdn\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{e^2}$$

```
[Out] -b*n/e/x+(-a-b*ln(c*x^n))/e/x-1/2*d*(a+b*ln(c*x^n))^2/b/e^2/n+d*(a+b*ln(c*x
^n))*ln(1+d*x/e)/e^2+b*d*n*polylog(2,-d*x/e)/e^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {269, 46, 2393, 2341, 2338, 2354, 2438}

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^3}\,dx = -\frac{d(a+b\log(cx^n))^2}{2be^2n}+\frac{d\log\left(\frac{dx}{e}+1\right)(a+b\log(cx^n))}{e^2} - \frac{a+b\log(cx^n)}{ex}+\frac{bdn\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{e^2}-\frac{bn}{ex}$$

```
[In] Int[(a + b*Log[c*x^n])/((d + e/x)*x^3),x]
```

```
[Out] -((b*n)/(e*x)) - (a + b*Log[c*x^n])/(e*x) - (d*(a + b*Log[c*x^n])^2)/(2*b*e
^2*n) + (d*(a + b*Log[c*x^n])*Log[1 + (d*x)/e])/e^2 + (b*d*n*PolyLog[2, -((
d*x)/e)])/e^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log\left(cx^n\right)}{ex^2} - \frac{d(a+b\log\left(cx^n\right))}{e^2x} + \frac{d^2(a+b\log\left(cx^n\right))}{e^2(e+dx)} \right) dx$$

$$= -\frac{d\int \frac{a+b\log(cx^n)}{x}\,dx}{e^2} + \frac{d^2\int \frac{a+b\log(cx^n)}{e+dx}\,dx}{e^2} + \frac{\int \frac{a+b\log(cx^n)}{x^2}\,dx}{e}$$

$$= -\frac{bn}{ex} - \frac{a+b\log(cx^n)}{ex} - \frac{d(a+b\log(cx^n))^2}{2be^2n} + \frac{d(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{e^2} - \frac{(bdn)\int \frac{\log\left(1+\frac{dx}{e}\right)}{x}\,dx}{e^2}$$

$$= -\frac{bn}{ex} - \frac{a+b\log(cx^n)}{ex} - \frac{d(a+b\log(cx^n))^2}{2be^2n} + \frac{d(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{e^2} + \frac{bdn\text{Li}_2\left(-\frac{dx}{e}\right)}{e^2}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^3}\,dx = -\frac{\frac{2ben}{x} + \frac{2e(a+b\log(cx^n))}{x} + \frac{d(a+b\log(cx^n))^2}{bn} - 2d(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right) - 2bdn\,\text{PolyLog}\left(2,-\frac{dx}{e}\right)}{2e^2}$$

```
[In] Integrate[(a + b*Log[c*x^n])/((d + e/x)*x^3),x]
```

```
[Out] -1/2*((2*b*e*n)/x + (2*e*(a + b*Log[c*x^n]))/x + (d*(a + b*Log[c*x^n])^2)/(
b*n) - 2*d*(a + b*Log[c*x^n])*Log[1 + (d*x)/e] - 2*b*d*n*PolyLog[2, -((d*x)
/e)])/e^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.33

method	result
risch	$\frac{b\ln(x^n)d\ln(dx+e)}{e^2} - \frac{b\ln(x^n)}{ex} - \frac{b\ln(x^n)d\ln(x)}{e^2} - \frac{bnd\ln(dx+e)\ln\left(-\frac{dx}{e}\right)}{e^2} - \frac{bnd\,\text{dilog}\left(-\frac{dx}{e}\right)}{e^2} - \frac{bn}{ex} + \frac{bnd\ln(x)^2}{2e^2} + \left(-\frac{ib\pi}{}\right.$

```
[In] int((a+b*ln(c*x^n))/(d+e/x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b*ln(x^n)*d/e^2*ln(d*x+e)-b*ln(x^n)/e/x-b*ln(x^n)*d/e^2*ln(x)-b*n*d/e^2*ln(
d*x+e)*ln(-d*x/e)-b*n*d/e^2*dilog(-d*x/e)-b*n/e/x+1/2*b*n*d/e^2*ln(x)^2+(-1
/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c
*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b
*ln(c)+a)*(d/e^2*ln(d*x+e)-1/e/x-d/e^2*ln(x))
```

Fricas [F]

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^3}\,dx = \int \frac{b\log(cx^n)+a}{\left(d+\frac{e}{x}\right)x^3}\,dx$$

[In] integrate((a+b*log(c*x^n))/(d+e/x)/x^3,x, algorithm="fricas")

[Out] integral((b*log(c*x^n) + a)/(d*x^3 + e*x^2), x)

Sympy [A] (verification not implemented)

Time = 37.15 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.27

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^3}\,dx = \frac{ad^2\left(\begin{cases}\frac{x}{e} & \text{for}\ d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)}{e^2} - \frac{ad\log(x)}{e^2} - \frac{a}{ex} - \frac{bd^2n\left(\begin{cases}\frac{x}{e} \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for}\ \frac{1}{|x|}<1\wedge|x|<1\\ \log(e)\log(x)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for}\ |x|<1\\ -\log(e)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for}\ \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log(e)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{otherwise}\end{cases}}{d}\end{cases}\right)}{e^2} + \frac{bd^2\left(\begin{cases}\frac{x}{e} & \text{for}\ d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)\log(cx^n)}{e^2} + \frac{bdn\log(x)^2}{2e^2} - \frac{bd\log(x)\log(cx^n)}{e^2} - \frac{bn}{ex} - \frac{b\log(cx^n)}{ex}$$

[In] integrate((a+b*ln(c*x**n))/(d+e/x)/x**3,x)

[Out] a*d**2*Piecewise((x/e, Eq(d, 0)), (log(d*x + e)/d, True))/e**2 - a*d*log(x)/e**2 - a/(e*x) - b*d**2*n*Piecewise((x/e, Eq(d, 0)), (Piecewise((-polylog(2, d*x*exp_polar(I*pi)/e), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(e)*log(x) - polylog(2, d*x*exp_polar(I*pi)/e), Abs(x) < 1), (-log(e)*log(1/x) - polylog(2, d*x*exp_polar(I*pi)/e), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0)

```
, ()), x)*log(e) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(e) - polylog(
2, d*x*exp_polar(I*pi)/e), True))/d, True))/e**2 + b*d**2*Piecewise((x/e, E
q(d, 0)), (log(d*x + e)/d, True))*log(c*x**n)/e**2 + b*d*n*log(x)**2/(2*e**
2) - b*d*log(x)*log(c*x**n)/e**2 - b*n/(e*x) - b*log(c*x**n)/(e*x)
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^3}\,dx=\int \frac{b\log(cx^n)+a}{\left(d+\frac{e}{x}\right)x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(d+e/x)/x^3,x, algorithm="maxima")
```

```
[Out] a*(d*log(d*x + e)/e^2 - d*log(x)/e^2 - 1/(e*x)) + b*integrate((log(c) + log
(x^n))/(d*x^3 + e*x^2), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^3}\,dx=\int \frac{b\log(cx^n)+a}{\left(d+\frac{e}{x}\right)x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(d+e/x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((d + e/x)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^3}\,dx=\int \frac{a+b\,\ln(c\,x^n)}{x^3\,\left(d+\frac{e}{x}\right)}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e/x)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^3*(d + e/x)), x)
```

3.336 $\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^4}\,dx = -\frac{bn}{4ex^2}+\frac{bdn}{e^2x}-\frac{a+b\log(cx^n)}{2ex^2}+\frac{d(a+b\log(cx^n))}{e^2x}+\frac{d^2(a+b\log(cx^n))^2}{2be^3n}-\frac{d^2(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{e^3}-\frac{bd^2n\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{e^3}$$

```
[Out] -1/4*b*n/e/x^2+b*d*n/e^2/x+1/2*(-a-b*ln(c*x^n))/e/x^2+d*(a+b*ln(c*x^n))/e^2
/x+1/2*d^2*(a+b*ln(c*x^n))^2/b/e^3/n-d^2*(a+b*ln(c*x^n))*ln(1+d*x/e)/e^3-b*
d^2*n*polylog(2,-d*x/e)/e^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {269, 46, 2393, 2341, 2338, 2354, 2438}

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^4}\,dx = \frac{d^2(a+b\log(cx^n))^2}{2be^3n}-\frac{d^2\log\left(\frac{dx}{e}+1\right)(a+b\log(cx^n))}{e^3}+\frac{d(a+b\log(cx^n))}{e^2x}-\frac{a+b\log(cx^n)}{2ex^2}-\frac{bd^2n\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{e^3}+\frac{bdn}{e^2x}-\frac{bn}{4ex^2}$$

```
[In] Int[(a + b*Log[c*x^n])/((d + e/x)*x^4),x]
```

```
[Out] -1/4*(b*n)/(e*x^2) + (b*d*n)/(e^2*x) - (a + b*Log[c*x^n])/(2*e*x^2) + (d*(a
 + b*Log[c*x^n]))/(e^2*x) + (d^2*(a + b*Log[c*x^n])^2)/(2*b*e^3*n) - (d^2*(
a + b*Log[c*x^n])*Log[1 + (d*x)/e])/e^3 - (b*d^2*n*PolyLog[2, -((d*x)/e)])/
e^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log(cx^n)}{ex^3} - \frac{d(a+b\log(cx^n))}{e^2x^2} + \frac{d^2(a+b\log(cx^n))}{e^3x} - \frac{d^3(a+b\log(cx^n))}{e^3(e+dx)}\right) dx$$

$$= \frac{d^2 \int \frac{a+b\log(cx^n)}{x}\,dx}{e^3} - \frac{d^3 \int \frac{a+b\log(cx^n)}{e+dx}\,dx}{e^3} - \frac{d \int \frac{a+b\log(cx^n)}{x^2}\,dx}{e^2} + \frac{\int \frac{a+b\log(cx^n)}{x^3}\,dx}{e}$$

$$= -\frac{bn}{4ex^2} + \frac{bdn}{e^2x} - \frac{a+b\log(cx^n)}{2ex^2} + \frac{d(a+b\log(cx^n))}{e^2x} + \frac{d^2(a+b\log(cx^n))^2}{2be^3n} - \frac{d^2(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{e^3} + \frac{(bd^2n)\int \frac{\log\left(1+\frac{dx}{e}\right)}{x}\,dx}{e^3}$$

$$= -\frac{bn}{4ex^2} + \frac{bdn}{e^2x} - \frac{a+b\log(cx^n)}{2ex^2} + \frac{d(a+b\log(cx^n))}{e^2x} + \frac{d^2(a+b\log(cx^n))^2}{2be^3n} - \frac{d^2(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right)}{e^3} - \frac{bd^2n\mathrm{Li}_2\left(-\frac{dx}{e}\right)}{e^3}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.92

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^4}\,dx = -\frac{\frac{be^2n}{x^2} - \frac{4bden}{x} + \frac{2e^2(a+b\log(cx^n))}{x^2} - \frac{4de(a+b\log(cx^n))}{x} - \frac{2d^2(a+b\log(cx^n))^2}{bn} + 4d^2(a+b\log(cx^n))\log\left(1+\frac{dx}{e}\right) + 4b}{4e^3}$$

```
[In] Integrate[(a + b*Log[c*x^n])/((d + e/x)*x^4),x]
```

```
[Out] -1/4*((b*e^2*n)/x^2 - (4*b*d*e*n)/x + (2*e^2*(a + b*Log[c*x^n]))/x^2 - (4*d
*e*(a + b*Log[c*x^n]))/x - (2*d^2*(a + b*Log[c*x^n])^2)/(b*n) + 4*d^2*(a +
b*Log[c*x^n])*Log[1 + (d*x)/e] + 4*b*d^2*n*PolyLog[2, -((d*x)/e)])/e^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.96

method	result
risch	$-\frac{b\ln(x^n)d^2\ln(dx+e)}{e^3} - \frac{b\ln(x^n)}{2e\,x^2} + \frac{b\ln(x^n)d^2\ln(x)}{e^3} + \frac{b\ln(x^n)d}{e^2x} + \frac{bdn}{e^2x} - \frac{bn}{4e\,x^2} - \frac{bn\,d^2\ln(x)^2}{2e^3} + \frac{bn\,d^2\ln(dx+e)\ln\left(-\frac{dx}{e}\right.}{e^3}$

```
[In] int((a+b*ln(c*x^n))/(d+e/x)/x^4,x,method=_RETURNVERBOSE)
[Out] -b*ln(x^n)*d^2/e^3*ln(d*x+e)-1/2*b*ln(x^n)/e/x^2+b*ln(x^n)*d^2/e^3*ln(x)+b*
ln(x^n)*d/e^2/x+b*d*n/e^2/x-1/4*b*n/e/x^2-1/2*b*n*d^2/e^3*ln(x)^2+b*n*d^2/e
^3*ln(d*x+e)*ln(-d*x/e)+b*n*d^2/e^3*dilog(-d*x/e)+(-1/2*I*b*Pi*csgn(I*c)*cs
gn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csg
n(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-d^2/e^3*ln
(d*x+e)-1/2/e/x^2+d^2/e^3*ln(x)+d/e^2/x)
```

Fricas [F]

$$\int \frac{a+b\log\left(cx^n\right)}{\left(d+\frac{e}{x}\right)x^4}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(d+\frac{e}{x}\right)x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(d+e/x)/x^4,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(d*x^4 + e*x^3), x)
```

Sympy [A] (verification not implemented)

Time = 42.97 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.96

$$\begin{aligned}\int \frac{a+b\log\left(cx^n\right)}{\left(d+\frac{e}{x}\right)x^4}\,dx=&-\frac{ad^3\left(\begin{cases}\frac{x}{e} & \text{for } d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)}{e^3}+\frac{ad^2\log(x)}{e^3}+\frac{ad}{e^2x}-\frac{a}{2ex^2}\\&+\frac{bd^3n\left(\begin{cases}\frac{x}{e} \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|}<1\wedge|x|<1\\ \log(e)\log(x)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } |x|<1\\ -\log(e)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(e)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{otherwise}\end{cases}}{d}\end{cases}\right)}{e^3}\\&-\frac{bd^3\left(\begin{cases}\frac{x}{e} & \text{for } d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)\log\left(cx^n\right)}{e^3}-\frac{bd^2n\log(x)^2}{2e^3}\\&+\frac{bd^2\log(x)\log\left(cx^n\right)}{e^3}+\frac{bdn}{e^2x}+\frac{bd\log\left(cx^n\right)}{e^2x}-\frac{bn}{4ex^2}-\frac{b\log\left(cx^n\right)}{2ex^2}\end{aligned}$$

```
[In] integrate((a+b*ln(c*x**n))/(d+e/x)/x**4,x)
```

```
[Out] -a*d**3*Piecewise((x/e, Eq(d, 0)), (log(d*x + e)/d, True))/e**3 + a*d**2*lo
g(x)/e**3 + a*d/(e**2*x) - a/(2*e*x**2) + b*d**3*n*Piecewise((x/e, Eq(d, 0)
), (Piecewise((-polylog(2, d*x*exp_polar(I*pi)/e), (Abs(x) < 1) & (1/Abs(x)
 < 1)), (log(e)*log(x) - polylog(2, d*x*exp_polar(I*pi)/e), Abs(x) < 1), (-
log(e)*log(1/x) - polylog(2, d*x*exp_polar(I*pi)/e), 1/Abs(x) < 1), (-meije
rg(((), (1, 1)), ((0, 0), ()), x)*log(e) + meijerg(((1, 1), ()), ((), (0, 0
)), x)*log(e) - polylog(2, d*x*exp_polar(I*pi)/e), True))/d, True))/e**3 -
b*d**3*Piecewise((x/e, Eq(d, 0)), (log(d*x + e)/d, True))*log(c*x**n)/e**3
- b*d**2*n*log(x)**2/(2*e**3) + b*d**2*log(x)*log(c*x**n)/e**3 + b*d*n/(e**
2*x) + b*d*log(c*x**n)/(e**2*x) - b*n/(4*e*x**2) - b*log(c*x**n)/(2*e*x**2)
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^4}\,dx = \int \frac{b\log(cx^n)+a}{\left(d+\frac{e}{x}\right)x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(d+e/x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*d^2*log(d*x + e)/e^3 - 2*d^2*log(x)/e^3 - (2*d*x - e)/(e^2*x^2))
+ b*integrate((log(c) + log(x^n))/(d*x^4 + e*x^3), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^4}\,dx = \int \frac{b\log(cx^n)+a}{\left(d+\frac{e}{x}\right)x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(d+e/x)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((d + e/x)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{\left(d+\frac{e}{x}\right)x^4}\,dx = \int \frac{a+b\,\ln\left(c\,x^n\right)}{x^4\,\left(d+\frac{e}{x}\right)}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^4*(d + e/x)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^4*(d + e/x)), x)
```

3.337 $\int \frac{x^3(a+b\log(cx))}{d+\frac{e}{x}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 170

$$\int \frac{x^3(a+b\log(cx))}{d+\frac{e}{x}}\,dx = -\frac{ae^3x}{d^4}+\frac{be^3x}{d^4}-\frac{be^2x^2}{4d^3}+\frac{bex^3}{9d^2}-\frac{bx^4}{16d}-\frac{be^3x\log(cx)}{d^4}+\frac{e^2x^2(a+b\log(cx))}{2d^3}-\frac{ex^3(a+b\log(cx))}{3d^2}+\frac{x^4(a+b\log(cx))}{4d}+\frac{e^4(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{d^5}+\frac{be^4\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^5}$$

```
[Out] -a*e^3*x/d^4+b*e^3*x/d^4-1/4*b*e^2*x^2/d^3+1/9*b*e*x^3/d^2-1/16*b*x^4/d-b*e
^3*x*ln(c*x)/d^4+1/2*e^2*x^2*(a+b*ln(c*x))/d^3-1/3*e*x^3*(a+b*ln(c*x))/d^2+
1/4*x^4*(a+b*ln(c*x))/d+e^4*(a+b*ln(c*x))*ln(1+d*x/e)/d^5+b*e^4*polylog(2,-
d*x/e)/d^5
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {269, 45, 2393, 2332, 2341, 2354, 2438}

$$\int \frac{x^3(a+b\log(cx))}{d+\frac{e}{x}}\,dx = \frac{e^4\log\left(\frac{dx}{e}+1\right)(a+b\log(cx))}{d^5}+\frac{e^2x^2(a+b\log(cx))}{2d^3}-\frac{ex^3(a+b\log(cx))}{3d^2}+\frac{x^4(a+b\log(cx))}{4d}-\frac{ae^3x}{d^4}-\frac{be^3x\log(cx)}{d^4}+\frac{be^4\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^5}+\frac{be^3x}{d^4}-\frac{be^2x^2}{4d^3}+\frac{bex^3}{9d^2}-\frac{bx^4}{16d}$$

```
[In] Int[(x^3*(a + b*Log[c*x]))/(d + e/x),x]
```

```
[Out] -((a*e^3*x)/d^4) + (b*e^3*x)/d^4 - (b*e^2*x^2)/(4*d^3) + (b*e*x^3)/(9*d^2)
- (b*x^4)/(16*d) - (b*e^3*x*Log[c*x])/d^4 + (e^2*x^2*(a + b*Log[c*x]))/(2*d
^3) - (e*x^3*(a + b*Log[c*x]))/(3*d^2) + (x^4*(a + b*Log[c*x]))/(4*d) + (e^
4*(a + b*Log[c*x])*Log[1 + (d*x)/e])/d^5 + (b*e^4*PolyLog[2, -((d*x)/e)])/d
^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{e^3(a+b\log(cx))}{d^4}+\frac{e^2x(a+b\log(cx))}{d^3}-\frac{ex^2(a+b\log(cx))}{d^2}+\frac{x^3(a+b\log(cx))}{d}+\frac{e^4(a+b\log(cx))}{d^4(e+dx)}\right)dx$$

$$=\frac{\int x^3(a+b\log(cx))\,dx}{d}-\frac{e\int x^2(a+b\log(cx))\,dx}{d^2}+\frac{e^2\int x(a+b\log(cx))\,dx}{d^3}-\frac{e^3\int (a+b\log(cx))\,dx}{d^4}+\frac{e^4\int \frac{a+b\log(cx)}{e+dx}\,dx}{d^4}$$

$$=-\frac{ae^3x}{d^4}-\frac{be^2x^2}{4d^3}+\frac{bex^3}{9d^2}-\frac{bx^4}{16d}+\frac{e^2x^2(a+b\log(cx))}{2d^3}-\frac{ex^3(a+b\log(cx))}{3d^2}+\frac{x^4(a+b\log(cx))}{4d}+\frac{e^4(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{d^5}-\frac{(be^3)\int\log(cx)\,dx}{d^4}-\frac{(be^4)\int\frac{\log\left(1+\frac{dx}{e}\right)}{x}\,dx}{d^5}$$

$$=-\frac{ae^3x}{d^4}+\frac{be^3x}{d^4}-\frac{be^2x^2}{4d^3}+\frac{bex^3}{9d^2}-\frac{bx^4}{16d}-\frac{be^3x\log(cx)}{d^4}+\frac{e^2x^2(a+b\log(cx))}{2d^3}-\frac{ex^3(a+b\log(cx))}{3d^2}+\frac{x^4(a+b\log(cx))}{4d}+\frac{e^4(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{d^5}+\frac{be^4\text{Li}_2\left(-\frac{dx}{e}\right)}{d^5}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int \frac{x^3(a+b\log(cx))}{d+\frac{e}{x}}\,dx = \frac{-144ade^3x+144bde^3x-36bd^2e^2x^2+16bd^3ex^3-9bd^4x^4-144bde^3x\log(cx)+72d^2e^2x^2(a+b\log(cx))-}{1}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x]))/(d + e/x),x]
```

```
[Out] (-144*a*d*e^3*x + 144*b*d*e^3*x - 36*b*d^2*e^2*x^2 + 16*b*d^3*e*x^3 - 9*b*d
^4*x^4 - 144*b*d*e^3*x*Log[c*x] + 72*d^2*e^2*x^2*(a + b*Log[c*x]) - 48*d^3*
e*x^3*(a + b*Log[c*x]) + 36*d^4*x^4*(a + b*Log[c*x]) + 144*e^4*(a + b*Log[c
*x])*Log[1 + (d*x)/e] + 144*b*e^4*PolyLog[2, -((d*x)/e)])/(144*d^5)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.20

method	result
parts	$a\left(\frac{\frac{1}{4}d^3x^4-\frac{1}{3}e\,d^2x^3+\frac{1}{2}d\,e^2x^2-x\,e^3}{d^4}+\frac{e^4\ln(dx+e)}{d^5}\right)+\frac{b\,x^4\ln(xc)}{4d}-\frac{b\,x^4}{16d}-\frac{be\,x^3\ln(xc)}{3d^2}+\frac{be\,x^3}{9d^2}+\frac{b\,e^2x^2\ln(x}{2d^3}$
risch	$\frac{a\,x^4}{4d}-\frac{ae\,x^3}{3d^2}+\frac{a\,e^2x^2}{2d^3}-\frac{a\,e^3x}{d^4}+\frac{a\,e^4\ln(dx+e)}{d^5}+\frac{b\,x^4\ln(xc)}{4d}-\frac{b\,x^4}{16d}-\frac{be\,x^3\ln(xc)}{3d^2}+\frac{be\,x^3}{9d^2}+\frac{b\,e^2x^2\ln(xc}{2d^3}$
derivativedivides	$\frac{a\left(-\frac{c^4e^3x-\frac{1}{2}d\,c^4e^2x^2+\frac{1}{3}e\,c^4x^3d^2-\frac{1}{4}x^4c^4d^3}{d^4}+\frac{c^4e^4\ln(cdx+ce)}{d^5}\right)+b\left(\frac{\frac{x^4c^4\ln(xc)}{4}-\frac{x^4c^4}{16}}{d}-\frac{ce\left(\frac{x^3c^3\ln(xc)}{3}-\frac{x^3c^3}{9}\right)}{d^2}+\frac{c^2e^2\left(x\right.}{}\right.}{c^4}$
default	$\frac{a\left(-\frac{c^4e^3x-\frac{1}{2}d\,c^4e^2x^2+\frac{1}{3}e\,c^4x^3d^2-\frac{1}{4}x^4c^4d^3}{d^4}+\frac{c^4e^4\ln(cdx+ce)}{d^5}\right)+b\left(\frac{\frac{x^4c^4\ln(xc)}{4}-\frac{x^4c^4}{16}}{d}-\frac{ce\left(\frac{x^3c^3\ln(xc)}{3}-\frac{x^3c^3}{9}\right)}{d^2}+\frac{c^2e^2\left(x\right.}{}\right.}{c^4}$

```
[In] int(x^3*(a+b*ln(x*c))/(d+e/x),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/d^4*(1/4*d^3*x^4-1/3*e*d^2*x^3+1/2*d*e^2*x^2-x*e^3)+e^4/d^5*ln(d*x+e))
+1/4*b/d*x^4*ln(x*c)-1/16*b*x^4/d-1/3*b/d^2*e*x^3*ln(x*c)+1/9*b*e*x^3/d^2+1
/2*b/d^3*e^2*x^2*ln(x*c)-1/4*b*e^2*x^2/d^3-b*e^3*x*ln(x*c)/d^4+b*e^3*x/d^4+
b*e^4/d^5*dilog((c*d*x+c*e)/e/c)+b*e^4/d^5*ln(x*c)*ln((c*d*x+c*e)/e/c)
```

Fricas [F]

$$\int \frac{x^3(a+b\log(cx))}{d+\frac{e}{x}}\,dx=\int \frac{(b\log(cx)+a)x^3}{d+\frac{e}{x}}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x))/(d+e/x),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*log(c*x) + a*x^4)/(d*x + e), x)
```

Sympy [A] (verification not implemented)

Time = 82.83 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.76

$$\int \frac{x^3(a+b\log(cx))}{d+\frac{e}{x}}\,dx = \frac{ax^4}{4d} - \frac{aex^3}{3d^2} + \frac{ae^2x^2}{2d^3} + \frac{ae^4\left(\begin{cases}\frac{x}{e} & \text{for } d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)}{d^4} - \frac{ae^3x}{d^4} + \frac{bx^4\log(cx)}{4d} - \frac{bx^4}{16d} - \frac{bex^3\log(cx)}{3d^2} + \frac{bex^3}{9d^2} + \frac{be^2x^2\log(cx)}{2d^3} - \frac{be^2x^2}{4d^3} - \frac{be^4\left(\begin{cases}\frac{x}{e} \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(e)\log(x)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } |x|<1\\ -\log(e)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(e)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{otherwise}\end{cases}}{d}\end{cases}\right)}{d^4} + \frac{be^4\left(\begin{cases}\frac{x}{e} & \text{for } d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)\log(cx)}{d^4} - \frac{be^3x\log(cx)}{d^4} + \frac{be^3x}{d^4}$$

```
[In] integrate(x**3*(a+b*ln(c*x))/(d+e/x),x)
```

```
[Out] a*x**4/(4*d) - a*e*x**3/(3*d**2) + a*e**2*x**2/(2*d**3) + a*e**4*Piecewise(
(x/e, Eq(d, 0)), (log(d*x + e)/d, True))/d**4 - a*e**3*x/d**4 + b*x**4*log(
c*x)/(4*d) - b*x**4/(16*d) - b*e*x**3*log(c*x)/(3*d**2) + b*e*x**3/(9*d**2)
 + b*e**2*x**2*log(c*x)/(2*d**3) - b*e**2*x**2/(4*d**3) - b*e**4*Piecewise(
(x/e, Eq(d, 0)), (Piecewise((-polylog(2, d*x*exp_polar(I*pi)/e), (Abs(x) <
1) & (1/Abs(x) < 1)), (log(e)*log(x) - polylog(2, d*x*exp_polar(I*pi)/e), A
bs(x) < 1), (-log(e)*log(1/x) - polylog(2, d*x*exp_polar(I*pi)/e), 1/Abs(x)
 < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(e) + meijerg(((1, 1), (
)), ((), (0, 0)), x)*log(e) - polylog(2, d*x*exp_polar(I*pi)/e), True))/d,
True))/d**4 + b*e**4*Piecewise((x/e, Eq(d, 0)), (log(d*x + e)/d, True))*log
(c*x)/d**4 - b*e**3*x*log(c*x)/d**4 + b*e**3*x/d**4
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.24

$$\int \frac{x^3(a+b\log(cx))}{d+\frac{e}{x}}\,dx = \frac{\left(\log\left(\frac{dx}{e}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{dx}{e}\right)\right)be^4}{d^5} + \frac{9\left(4\,ad^3+(4\,d^3\log(c)-d^3)b\right)x^4-16\left(3\,ad^2e+(3\,d^2e\log(c)-d^2e)b\right)x^3+36\left(2\,ade^2+(2\,de^2\log(c)-\cdots\right.}{144\,d^4} + \frac{(be^4\log(c)+ae^4)\log(dx+e)}{d^5}$$

```
[In] integrate(x^3*(a+b*log(c*x))/(d+e/x),x, algorithm="maxima")
```

```
[Out] (log(d*x/e + 1)*log(x) + dilog(-d*x/e))*b*e^4/d^5 + 1/144*(9*(4*a*d^3 + (4*
d^3*log(c) - d^3)*b)*x^4 - 16*(3*a*d^2*e + (3*d^2*e*log(c) - d^2*e)*b)*x^3
+ 36*(2*a*d*e^2 + (2*d*e^2*log(c) - d*e^2)*b)*x^2 - 144*(a*e^3 + (e^3*log(c
) - e^3)*b)*x + 12*(3*b*d^3*x^4 - 4*b*d^2*e*x^3 + 6*b*d*e^2*x^2 - 12*b*e^3*
x)*log(x))/d^4 + (b*e^4*log(c) + a*e^4)*log(d*x + e)/d^5
```

Giac [F]

$$\int \frac{x^3(a+b\log(cx))}{d+\frac{e}{x}}\,dx = \int \frac{(b\log(cx)+a)x^3}{d+\frac{e}{x}}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x))/(d+e/x),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x) + a)*x^3/(d + e/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(cx))}{d+\frac{e}{x}}\,dx = \int \frac{x^3\,(a+b\,\ln(c\,x))}{d+\frac{e}{x}}\,dx$$

```
[In] int((x^3*(a + b*log(c*x)))/(d + e/x),x)
```

```
[Out] int((x^3*(a + b*log(c*x)))/(d + e/x), x)
```

3.338 $\int \frac{x^2(a+b\log(cx))}{d+\frac{e}{x}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \frac{x^2(a+b\log(cx))}{d+\frac{e}{x}}\,dx = \frac{ae^2x}{d^3}-\frac{be^2x}{d^3}+\frac{bex^2}{4d^2}-\frac{bx^3}{9d}+\frac{be^2x\log(cx)}{d^3} - \frac{ex^2(a+b\log(cx))}{2d^2}+\frac{x^3(a+b\log(cx))}{3d} - \frac{e^3(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{d^4}-\frac{be^3\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^4}$$

```
[Out] a*e^2*x/d^3-b*e^2*x/d^3+1/4*b*e*x^2/d^2-1/9*b*x^3/d+b*e^2*x*ln(c*x)/d^3-1/2
*e*x^2*(a+b*ln(c*x))/d^2+1/3*x^3*(a+b*ln(c*x))/d-e^3*(a+b*ln(c*x))*ln(1+d*x
/e)/d^4-b*e^3*polylog(2,-d*x/e)/d^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {269, 45, 2393, 2332, 2341, 2354, 2438}

$$\int \frac{x^2(a+b\log(cx))}{d+\frac{e}{x}}\,dx = -\frac{e^3\log\left(\frac{dx}{e}+1\right)(a+b\log(cx))}{d^4}-\frac{ex^2(a+b\log(cx))}{2d^2} +\frac{x^3(a+b\log(cx))}{3d}+\frac{ae^2x}{d^3}+\frac{be^2x\log(cx)}{d^3} -\frac{be^3\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^4}-\frac{be^2x}{d^3}+\frac{bex^2}{4d^2}-\frac{bx^3}{9d}$$

```
[In] Int[(x^2*(a + b*Log[c*x]))/(d + e/x),x]
```

```
[Out] (a*e^2*x)/d^3 - (b*e^2*x)/d^3 + (b*e*x^2)/(4*d^2) - (b*x^3)/(9*d) + (b*e^2*
x*Log[c*x])/d^3 - (e*x^2*(a + b*Log[c*x]))/(2*d^2) + (x^3*(a + b*Log[c*x]))
/(3*d) - (e^3*(a + b*Log[c*x])*Log[1 + (d*x)/e])/d^4 - (b*e^3*PolyLog[2, -(
(d*x)/e)])/d^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^2(a+b\log(cx))}{d^3} - \frac{ex(a+b\log(cx))}{d^2} + \frac{x^2(a+b\log(cx))}{d} - \frac{e^3(a+b\log(cx))}{d^3(e+dx)}\right)dx$$

$$= \frac{\int x^2(a+b\log(cx))\,dx}{d} - \frac{e\int x(a+b\log(cx))\,dx}{d^2} + \frac{e^2\int(a+b\log(cx))\,dx}{d^3} - \frac{e^3\int\frac{a+b\log(cx)}{e+dx}\,dx}{d^3}$$

$$= \frac{ae^2x}{d^3} + \frac{bex^2}{4d^2} - \frac{bx^3}{9d} - \frac{ex^2(a+b\log(cx))}{2d^2} + \frac{x^3(a+b\log(cx))}{3d} - \frac{e^3(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{d^4} + \frac{(be^2)\int\log(cx)\,dx}{d^3} + \frac{(be^3)\int\frac{\log\left(1+\frac{dx}{e}\right)}{x}\,dx}{d^4}$$

$$= \frac{ae^2x}{d^3} - \frac{be^2x}{d^3} + \frac{bex^2}{4d^2} - \frac{bx^3}{9d} + \frac{be^2x\log(cx)}{d^3} - \frac{ex^2(a+b\log(cx))}{2d^2} + \frac{x^3(a+b\log(cx))}{3d} - \frac{e^3(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{d^4} - \frac{be^3\text{Li}_2\left(-\frac{dx}{e}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.92

$$\int\frac{x^2(a+b\log(cx))}{d+\frac{e}{x}}\,dx = \frac{36ade^2x - 36bde^2x + 9bd^2ex^2 - 4bd^3x^3 + 36bde^2x\log(cx) - 18d^2ex^2(a+b\log(cx)) + 12d^3x^3(a+b\log(cx)}{36d^4}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x]))/(d + e/x),x]
```

```
[Out] (36*a*d*e^2*x - 36*b*d*e^2*x + 9*b*d^2*e*x^2 - 4*b*d^3*x^3 + 36*b*d*e^2*x*L
og[c*x] - 18*d^2*e*x^2*(a + b*Log[c*x]) + 12*d^3*x^3*(a + b*Log[c*x]) - 36*
e^3*(a + b*Log[c*x])*Log[1 + (d*x)/e] - 36*b*e^3*PolyLog[2, -((d*x)/e)])/(3
6*d^4)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.24

method	result
risch	$\frac{a\,x^{3}}{3d}-\frac{ae\,x^{2}}{2d^{2}}+\frac{a\,e^{2}x}{d^{3}}-\frac{a\,e^{3}\ln(dx+e)}{d^{4}}+\frac{b\,x^{3}\ln(xc)}{3d}-\frac{b\,x^{3}}{9d}-\frac{be\,x^{2}\ln(xc)}{2d^{2}}+\frac{be\,x^{2}}{4d^{2}}+\frac{b\,e^{2}x\ln(xc)}{d^{3}}-\frac{b\,e^{2}x}{d^{3}}$
parts	$\frac{a\,x^{3}}{3d}-\frac{ae\,x^{2}}{2d^{2}}+\frac{a\,e^{2}x}{d^{3}}-\frac{a\,e^{3}\ln(dx+e)}{d^{4}}+\frac{b\,x^{3}\ln(xc)}{3d}-\frac{b\,x^{3}}{9d}-\frac{be\,x^{2}\ln(xc)}{2d^{2}}+\frac{be\,x^{2}}{4d^{2}}+\frac{b\,e^{2}x\ln(xc)}{d^{3}}-\frac{b\,e^{2}x}{d^{3}}$
derivativedivides	$\frac{\frac{a\,c^{3}e^{2}x}{d^{3}}-\frac{a\,c^{3}e\,x^{2}}{2d^{2}}+\frac{a\,x^{3}c^{3}}{3d}-\frac{a\,c^{3}e^{3}\ln(cdx+ce)}{d^{4}}+b\left(\frac{\frac{x^{3}c^{3}\ln(xc)}{3}-\frac{x^{3}c^{3}}{9}}{d}-\frac{ce\left(\frac{x^{2}c^{2}\ln(xc)}{2}-\frac{x^{2}c^{2}}{4}\right)}{d^{2}}+\frac{c^{2}e^{2}(xc\ln(xc)-xc)}{d^{3}}-\frac{c^{3}e^{3}}{}\right.}{c^{3}}$
default	$\frac{\frac{a\,c^{3}e^{2}x}{d^{3}}-\frac{a\,c^{3}e\,x^{2}}{2d^{2}}+\frac{a\,x^{3}c^{3}}{3d}-\frac{a\,c^{3}e^{3}\ln(cdx+ce)}{d^{4}}+b\left(\frac{\frac{x^{3}c^{3}\ln(xc)}{3}-\frac{x^{3}c^{3}}{9}}{d}-\frac{ce\left(\frac{x^{2}c^{2}\ln(xc)}{2}-\frac{x^{2}c^{2}}{4}\right)}{d^{2}}+\frac{c^{2}e^{2}(xc\ln(xc)-xc)}{d^{3}}-\frac{c^{3}e^{3}(}{}\right.}{c^{3}}$

[In] int(x^2*(a+b*ln(x*c))/(d+e/x),x,method=_RETURNVERBOSE)

[Out] 1/3*a/d*x^3-1/2*a*e/d^2*x^2+a*e^2*x/d^3-a*e^3/d^4*ln(d*x+e)+1/3*b/d*x^3*ln(x*c)-1/9*b*x^3/d-1/2*b/d^2*e*x^2*ln(x*c)+1/4*b*e*x^2/d^2+b*e^2*x*ln(x*c)/d^3-b*e^2*x/d^3-b*e^3/d^4*dilog((c*d*x+c*e)/e/c)-b*e^3/d^4*ln(x*c)*ln((c*d*x+c*e)/e/c)

Fricas [F]

$$\int \frac{x^2(a+b\log(cx))}{d+\frac{e}{x}}\,dx=\int \frac{(b\log(cx)+a)x^2}{d+\frac{e}{x}}\,dx$$

[In] integrate(x^2*(a+b*log(c*x))/(d+e/x),x, algorithm="fricas")

[Out] integral((b*x^3*log(c*x) + a*x^3)/(d*x + e), x)

Sympy [A] (verification not implemented)

Time = 79.52 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.86

$$\int \frac{x^2(a+b\log(cx))}{d+\frac{e}{x}}\,dx = \frac{ax^3}{3d} - \frac{aex^2}{2d^2} - \frac{ae^3\left(\begin{cases}\frac{x}{e} & \text{for } d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)}{d^3} + \frac{ae^2x}{d^3} + \frac{bx^3\log(cx)}{3d} - \frac{bx^3}{9d} - \frac{bex^2\log(cx)}{2d^2} + \frac{bex^2}{4d^2} + \frac{be^3\left(\begin{cases}\frac{x}{e} & \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(e)\log(x)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } |x|<1\\ -\log(e)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(e)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{otherwise}\end{cases}}{d} & \end{cases}\right)}{d^3} - \frac{be^3\left(\begin{cases}\frac{x}{e} & \text{for } d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)\log(cx)}{d^3} + \frac{be^2x\log(cx)}{d^3} - \frac{be^2x}{d^3}$$

```
[In] integrate(x**2*(a+b*ln(c*x))/(d+e/x),x)
```

```
[Out] a*x**3/(3*d) - a*e*x**2/(2*d**2) - a*e**3*Piecewise((x/e, Eq(d, 0)), (log(d
*x + e)/d, True))/d**3 + a*e**2*x/d**3 + b*x**3*log(c*x)/(3*d) - b*x**3/(9*
d) - b*e*x**2*log(c*x)/(2*d**2) + b*e*x**2/(4*d**2) + b*e**3*Piecewise((x/e
, Eq(d, 0)), (Piecewise((-polylog(2, d*x*exp_polar(I*pi)/e), (Abs(x) < 1) &
 (1/Abs(x) < 1)), (log(e)*log(x) - polylog(2, d*x*exp_polar(I*pi)/e), Abs(x
) < 1), (-log(e)*log(1/x) - polylog(2, d*x*exp_polar(I*pi)/e), 1/Abs(x) < 1
), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(e) + meijerg(((1, 1), ()),
((), (0, 0)), x)*log(e) - polylog(2, d*x*exp_polar(I*pi)/e), True))/d, True
))/d**3 - b*e**3*Piecewise((x/e, Eq(d, 0)), (log(d*x + e)/d, True))*log(c*x
)/d**3 + b*e**2*x*log(c*x)/d**3 - b*e**2*x/d**3
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.21

$$\int \frac{x^2(a+b\log(cx))}{d+\frac{e}{x}}\,dx = -\frac{\left(\log\left(\frac{dx}{e}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{dx}{e}\right)\right)be^3}{d^4} + \frac{4\,(3\,ad^2+(3\,d^2\log\left(c\right)-d^2)b)x^3-9\,(2\,ade+(2\,de\log\left(c\right)-de)b)x^2+36\,(ae^2+(e^2\log\left(c\right)-e^2)b)x+6}{36\,d^3} - \frac{(be^3\log\left(c\right)+ae^3)\log\left(dx+e\right)}{d^4}$$

```
[In] integrate(x^2*(a+b*log(c*x))/(d+e/x),x, algorithm="maxima")
```

```
[Out] -(log(d*x/e + 1)*log(x) + dilog(-d*x/e))*b*e^3/d^4 + 1/36*(4*(3*a*d^2 + (3*
d^2*log(c) - d^2)*b)*x^3 - 9*(2*a*d*e + (2*d*e*log(c) - d*e)*b)*x^2 + 36*(a
*e^2 + (e^2*log(c) - e^2)*b)*x + 6*(2*b*d^2*x^3 - 3*b*d*e*x^2 + 6*b*e^2*x)*
log(x))/d^3 - (b*e^3*log(c) + a*e^3)*log(d*x + e)/d^4
```

Giac [F]

$$\int \frac{x^2(a+b\log(cx))}{d+\frac{e}{x}}\,dx = \int \frac{(b\log\left(cx\right)+a)x^2}{d+\frac{e}{x}}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x))/(d+e/x),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x) + a)*x^2/(d + e/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(cx))}{d+\frac{e}{x}}\,dx = \int \frac{x^2\,\left(a+b\,\ln\left(c\,x\right)\right)}{d+\frac{e}{x}}\,dx$$

```
[In] int((x^2*(a + b*log(c*x)))/(d + e/x),x)
```

```
[Out] int((x^2*(a + b*log(c*x)))/(d + e/x), x)
```

3.339 $\int \frac{x(a+b\log(cx))}{d+\frac{e}{x}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 98

$$\int \frac{x(a+b\log(cx))}{d+\frac{e}{x}}\,dx = -\frac{aex}{d^2}+\frac{bex}{d^2}-\frac{bx^2}{4d}-\frac{bex\log(cx)}{d^2}+\frac{x^2(a+b\log(cx))}{2d}+\frac{e^2(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{d^3}+\frac{be^2\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^3}$$

```
[Out] -a*e*x/d^2+b*e*x/d^2-1/4*b*x^2/d-b*e*x*ln(c*x)/d^2+1/2*x^2*(a+b*ln(c*x))/d+
e^2*(a+b*ln(c*x))*ln(1+d*x/e)/d^3+b*e^2*polylog(2,-d*x/e)/d^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {269, 45, 2393, 2332, 2341, 2354, 2438}

$$\int \frac{x(a+b\log(cx))}{d+\frac{e}{x}}\,dx = \frac{e^2\log\left(\frac{dx}{e}+1\right)(a+b\log(cx))}{d^3}+\frac{x^2(a+b\log(cx))}{2d}-\frac{aex}{d^2}-\frac{bex\log(cx)}{d^2}+\frac{be^2\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^3}+\frac{bex}{d^2}-\frac{bx^2}{4d}$$

```
[In] Int[(x*(a + b*Log[c*x]))/(d + e/x),x]
```

```
[Out] -((a*e*x)/d^2) + (b*e*x)/d^2 - (b*x^2)/(4*d) - (b*e*x*Log[c*x])/d^2 + (x^2*
(a + b*Log[c*x]))/(2*d) + (e^2*(a + b*Log[c*x])*Log[1 + (d*x)/e])/d^3 + (b*
e^2*PolyLog[2, -((d*x)/e)])/d^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{e(a+b\log(cx))}{d^2}+\frac{x(a+b\log(cx))}{d}+\frac{e^2(a+b\log(cx))}{d^2(e+dx)}\right)dx$$

$$
\begin{aligned}
&= \frac{\int x(a+b\log(cx))\,dx}{d} - \frac{e\int(a+b\log(cx))\,dx}{d^2} + \frac{e^2\int\frac{a+b\log(cx)}{e+dx}\,dx}{d^2} \\
&= -\frac{aex}{d^2} - \frac{bx^2}{4d} + \frac{x^2(a+b\log(cx))}{2d} + \frac{e^2(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{d^3} \\
&\quad - \frac{(be)\int\log(cx)\,dx}{d^2} - \frac{(be^2)\int\frac{\log\left(1+\frac{dx}{e}\right)}{x}\,dx}{d^3} \\
&= -\frac{aex}{d^2} + \frac{bex}{d^2} - \frac{bx^2}{4d} - \frac{bex\log(cx)}{d^2} + \frac{x^2(a+b\log(cx))}{2d} \\
&\quad + \frac{e^2(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{d^3} + \frac{be^2\mathrm{Li}_2\left(-\frac{dx}{e}\right)}{d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$
\begin{aligned}
\int\frac{x(a+b\log(cx))}{d+\frac{e}{x}}\,dx &= -\frac{aex}{d^2} + \frac{bex}{d^2} - \frac{bx^2}{4d} - \frac{bex\log(cx)}{d^2} + \frac{x^2(a+b\log(cx))}{2d} \\
&\quad + \frac{e^2(a+b\log(cx))\log\left(\frac{e+dx}{e}\right)}{d^3} + \frac{be^2\,\mathrm{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^3}
\end{aligned}
$$

```
[In] Integrate[(x*(a + b*Log[c*x]))/(d + e/x),x]
[Out] -((a*e*x)/d^2) + (b*e*x)/d^2 - (b*x^2)/(4*d) - (b*e*x*Log[c*x])/d^2 + (x^2*
(a + b*Log[c*x]))/(2*d) + (e^2*(a + b*Log[c*x])*Log[(e + d*x)/e])/d^3 + (b*
e^2*PolyLog[2, -((d*x)/e)])/d^3
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.29

method	result
risch	$\frac{a\,x^2}{2d}-\frac{aex}{d^2}+\frac{a\,e^2\ln(dx+e)}{d^3}+\frac{b\,x^2\ln(xc)}{2d}-\frac{b\,x^2}{4d}-\frac{bex\ln(xc)}{d^2}+\frac{bex}{d^2}+\frac{b\,e^2\operatorname{dilog}\left(\frac{cdx+ce}{ec}\right)}{d^3}+\frac{b\,e^2\ln(xc)\ln(}{d^3}$
parts	$\frac{a\,x^2}{2d}-\frac{aex}{d^2}+\frac{a\,e^2\ln(dx+e)}{d^3}+\frac{b\,x^2\ln(xc)}{2d}-\frac{b\,x^2}{4d}-\frac{bex\ln(xc)}{d^2}+\frac{bex}{d^2}+\frac{b\,e^2\operatorname{dilog}\left(\frac{cdx+ce}{ec}\right)}{d^3}+\frac{b\,e^2\ln(xc)\ln(}{d^3}$
derivativedivides	$\frac{-\frac{a\,c^2ex}{d^2}+\frac{a\,x^2c^2}{2d}+\frac{a\,c^2e^2\ln(cdx+ce)}{d^3}+b\left(\frac{\frac{x^2c^2\ln(xc)}{2}-\frac{x^2c^2}{4}}{d}-\frac{ec(xc\ln(xc)-xc)}{d^2}+\frac{c^2e^2\left(\frac{\operatorname{dilog}\left(\frac{cdx+ce}{ec}\right)}{d}+\frac{\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{d}\right)}{d^2}\right.}{c^2}$
default	$\frac{-\frac{a\,c^2ex}{d^2}+\frac{a\,x^2c^2}{2d}+\frac{a\,c^2e^2\ln(cdx+ce)}{d^3}+b\left(\frac{\frac{x^2c^2\ln(xc)}{2}-\frac{x^2c^2}{4}}{d}-\frac{ec(xc\ln(xc)-xc)}{d^2}+\frac{c^2e^2\left(\frac{\operatorname{dilog}\left(\frac{cdx+ce}{ec}\right)}{d}+\frac{\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{d}\right)}{d^2}\right.}{c^2}$

```
[In] int(x*(a+b*ln(x*c))/(d+e/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2/d-a*e*x/d^2+a*e^2/d^3*ln(d*x+e)+1/2*b/d*x^2*ln(x*c)-1/4*b*x^2/d-b
*e*x*ln(x*c)/d^2+b*e*x/d^2+b*e^2/d^3*dilog((c*d*x+c*e)/e/c)+b*e^2/d^3*ln(x*
c)*ln((c*d*x+c*e)/e/c)
```

Fricas [F]

$$\int \frac{x(a+b\log(cx))}{d+\frac{e}{x}}\,dx=\int \frac{(b\log(cx)+a)x}{d+\frac{e}{x}}\,dx$$

```
[In] integrate(x*(a+b*log(c*x))/(d+e/x),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*log(c*x) + a*x^2)/(d*x + e), x)
```

Sympy [A] (verification not implemented)

Time = 52.73 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.11

$$\int \frac{x(a+b\log(cx))}{d+\frac{e}{x}}\,dx = \frac{ax^2}{2d} + \frac{ae^2\left(\begin{cases}\frac{x}{e} & \text{for } d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)}{d^2} - \frac{aex}{d^2} + \frac{bx^2\log(cx)}{2d} - \frac{bx^2}{4d} - \frac{be^2\left(\begin{cases}\frac{x}{e} \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1\\ \log(e)\log(x)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } |x|<1\\ -\log(e)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(e)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{otherwise}\end{cases}}{d}\end{cases}\right)}{d^2} + \frac{be^2\left(\begin{cases}\frac{x}{e} & \text{for } d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)\log(cx)}{d^2} - \frac{bex\log(cx)}{d^2} + \frac{bex}{d^2}$$

```
[In] integrate(x*(a+b*ln(c*x))/(d+e/x),x)
```

```
[Out] a*x**2/(2*d) + a*e**2*Piecewise((x/e, Eq(d, 0)), (log(d*x + e)/d, True))/d*
*2 - a*e*x/d**2 + b*x**2*log(c*x)/(2*d) - b*x**2/(4*d) - b*e**2*Piecewise((
x/e, Eq(d, 0)), (Piecewise((-polylog(2, d*x*exp_polar(I*pi)/e), (Abs(x) < 1
) & (1/Abs(x) < 1)), (log(e)*log(x) - polylog(2, d*x*exp_polar(I*pi)/e), Ab
s(x) < 1), (-log(e)*log(1/x) - polylog(2, d*x*exp_polar(I*pi)/e), 1/Abs(x)
< 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(e) + meijerg(((1, 1), ()
), ((), (0, 0)), x)*log(e) - polylog(2, d*x*exp_polar(I*pi)/e), True))/d, T
rue))/d**2 + b*e**2*Piecewise((x/e, Eq(d, 0)), (log(d*x + e)/d, True))*log(
c*x)/d**2 - b*e*x*log(c*x)/d**2 + b*e*x/d**2
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int \frac{x(a+b\log(cx))}{d+\frac{e}{x}}\,dx = \frac{\left(\log\left(\frac{dx}{e}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{dx}{e}\right)\right)be^2}{d^3} + \frac{((2\,d\log(c)-d)b+2\,ad)x^2-4\,((e\log(c)-e)b+ae)x+2\,(bdx^2-2\,bex)\log(x)}{4\,d^2} + \frac{(be^2\log(c)+ae^2)\log(dx+e)}{d^3}$$

```
[In] integrate(x*(a+b*log(c*x))/(d+e/x),x, algorithm="maxima")
[Out] (log(d*x/e + 1)*log(x) + dilog(-d*x/e))*b*e^2/d^3 + 1/4*(((2*d*log(c) - d)*
b + 2*a*d)*x^2 - 4*((e*log(c) - e)*b + a*e)*x + 2*(b*d*x^2 - 2*b*e*x)*log(x
))/d^2 + (b*e^2*log(c) + a*e^2)*log(d*x + e)/d^3
```

Giac [F]

$$\int \frac{x(a+b\log(cx))}{d+\frac{e}{x}}\,dx = \int \frac{(b\log(cx)+a)x}{d+\frac{e}{x}}\,dx$$

```
[In] integrate(x*(a+b*log(c*x))/(d+e/x),x, algorithm="giac")
[Out] integrate((b*log(c*x) + a)*x/(d + e/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log(cx))}{d+\frac{e}{x}}\,dx = \int \frac{x\,(a+b\,\ln(c\,x))}{d+\frac{e}{x}}\,dx$$

```
[In] int((x*(a + b*log(c*x)))/(d + e/x),x)
[Out] int((x*(a + b*log(c*x)))/(d + e/x), x)
```

3.340 $\int \frac{a+b\log(cx)}{d+\frac{e}{x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int \frac{a+b\log(cx)}{d+\frac{e}{x}}\,dx=\frac{ax}{d}-\frac{bx}{d}+\frac{bx\log(cx)}{d}-\frac{e(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{d^2}-\frac{be\,\text{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^2}$$

```
[Out] a*x/d-b*x/d+b*x*ln(c*x)/d-e*(a+b*ln(c*x))*ln(1+d*x/e)/d^2-b*e*polylog(2,-d*
x/e)/d^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {199, 45, 2367, 2332, 2354, 2438}

$$\int \frac{a+b\log(cx)}{d+\frac{e}{x}}\,dx=-\frac{e\log\left(\frac{dx}{e}+1\right)(a+b\log(cx))}{d^2}+\frac{ax}{d}+\frac{bx\log(cx)}{d}-\frac{be\,\text{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^2}-\frac{bx}{d}$$

```
[In] Int[(a + b*Log[c*x])/(d + e/x),x]
```

```
[Out] (a*x)/d - (b*x)/d + (b*x*Log[c*x])/d - (e*(a + b*Log[c*x])*Log[1 + (d*x)/e]
)/d^2 - (b*e*PolyLog[2, -((d*x)/e)])/d^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a+b\log(cx)}{d}-\frac{e(a+b\log(cx))}{d(e+dx)}\right)dx\\
&= \frac{\int (a+b\log(cx))\,dx}{d}-\frac{e\int \frac{a+b\log(cx)}{e+dx}\,dx}{d}\\
&= \frac{ax}{d}-\frac{e(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{d^2}+\frac{b\int \log(cx)\,dx}{d}+\frac{(be)\int \frac{\log\left(1+\frac{dx}{e}\right)}{x}\,dx}{d^2}\\
&= \frac{ax}{d}-\frac{bx}{d}+\frac{bx\log(cx)}{d}-\frac{e(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{d^2}-\frac{be\text{Li}_2\left(-\frac{dx}{e}\right)}{d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{a+b\log(cx)}{d+\frac{e}{x}}\,dx=\frac{ax}{d}-\frac{bx}{d}+\frac{bx\log(cx)}{d}-\frac{e(a+b\log(cx))\log\left(\frac{e+dx}{e}\right)}{d^2}-\frac{be\,\text{PolyLog}\left(2,-\frac{dx}{e}\right)}{d^2}$$

```
[In] Integrate[(a + b*Log[c*x])/(d + e/x),x]
```

```
[Out] (a*x)/d - (b*x)/d + (b*x*Log[c*x])/d - (e*(a + b*Log[c*x])*Log[(e + d*x)/e]
)/d^2 - (b*e*PolyLog[2, -((d*x)/e)])/d^2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.40

method	result	size
risch	$\frac{ax}{d}-\frac{ae\ln(dx+e)}{d^2}+\frac{bx\ln(xc)}{d}-\frac{bx}{d}-\frac{be\,\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{d^2}-\frac{be\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{d^2}$	88
parts	$\frac{ax}{d}-\frac{ae\ln(dx+e)}{d^2}+\frac{bx\ln(xc)}{d}-\frac{bx}{d}-\frac{be\,\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{d^2}-\frac{be\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{d^2}$	88
derivativedivides	$\frac{\frac{axc}{d}-\frac{aec\ln(cdx+ce)}{d^2}+b\left(\frac{xc\ln(xc)-xc}{d}-\frac{ec\left(\frac{\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{d}+\frac{\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{d}\right)}{d}\right)}{c}$	101
default	$\frac{\frac{axc}{d}-\frac{aec\ln(cdx+ce)}{d^2}+b\left(\frac{xc\ln(xc)-xc}{d}-\frac{ec\left(\frac{\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{d}+\frac{\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{d}\right)}{d}\right)}{c}$	101

```
[In] int((a+b*ln(x*c))/(d+e/x),x,method=_RETURNVERBOSE)
```

```
[Out] a*x/d-a*e/d^2*ln(d*x+e)+b*x*ln(x*c)/d-b*x/d-b*e/d^2*dilog((c*d*x+c*e)/e/c)-
b*e/d^2*ln(x*c)*ln((c*d*x+c*e)/e/c)
```

Fricas [F]

$$\int \frac{a+b\log(cx)}{d+\frac{e}{x}}\,dx=\int \frac{b\log(cx)+a}{d+\frac{e}{x}}\,dx$$

```
[In] integrate((a+b*log(c*x))/(d+e/x),x, algorithm="fricas")
```

```
[Out] integral((b*x*log(c*x) + a*x)/(d*x + e), x)
```

Sympy [A] (verification not implemented)

Time = 43.43 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.48

$$\int \frac{a+b\log(cx)}{d+\frac{e}{x}}\,dx = -\frac{ae\left(\begin{cases}\frac{x}{e} & \text{for}\ d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)}{d}+\frac{ax}{d}+\frac{be\left(\begin{cases}\frac{x}{e}\\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for}\ \frac{1}{|x|}<1\wedge|x|<1\\ \log(e)\log(x)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for}\ |x|<1\\ -\log(e)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for}\ \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log(e)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{otherwise}\end{cases}}{d}\end{cases}\right)}{d}-\frac{be\left(\begin{cases}\frac{x}{e} & \text{for}\ d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)\log(cx)}{d}+\frac{bx\log(cx)}{d}-\frac{bx}{d}$$

```
[In] integrate((a+b*ln(c*x))/(d+e/x),x)
```

```
[Out] -a*e*Piecewise((x/e, Eq(d, 0)), (log(d*x + e)/d, True))/d + a*x/d + b*e*Pie
cewise((x/e, Eq(d, 0)), (Piecewise((-polylog(2, d*x*exp_polar(I*pi)/e), (Ab
s(x) < 1) & (1/Abs(x) < 1)), (log(e)*log(x) - polylog(2, d*x*exp_polar(I*pi
)/e), Abs(x) < 1), (-log(e)*log(1/x) - polylog(2, d*x*exp_polar(I*pi)/e), 1
/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(e) + meijerg(((1
, 1), ()), ((), (0, 0)), x)*log(e) - polylog(2, d*x*exp_polar(I*pi)/e), Tru
e))/d, True))/d - b*e*Piecewise((x/e, Eq(d, 0)), (log(d*x + e)/d, True))*lo
g(c*x)/d + b*x*log(c*x)/d - b*x/d
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

$$\int \frac{a+b\log(cx)}{d+\frac{e}{x}}\,dx = -\frac{\left(\log\left(\frac{dx}{e}+1\right)\log(x)+\operatorname{Li}_2\left(-\frac{dx}{e}\right)\right)be}{d^2}+\frac{bx\log(x)+(b(\log(c)-1)+a)x}{d}-\frac{(be\log(c)+ae)\log(dx+e)}{d^2}$$

```
[In] integrate((a+b*log(c*x))/(d+e/x),x, algorithm="maxima")
[Out] -(log(d*x/e + 1)*log(x) + dilog(-d*x/e))*b*e/d^2 + (b*x*log(x) + (b*(log(c)
 - 1) + a)*x)/d - (b*e*log(c) + a*e)*log(d*x + e)/d^2
```

Giac [F]

$$\int \frac{a+b\log(cx)}{d+\frac{e}{x}}\,dx=\int \frac{b\log(cx)+a}{d+\frac{e}{x}}\,dx$$

```
[In] integrate((a+b*log(c*x))/(d+e/x),x, algorithm="giac")
[Out] integrate((b*log(c*x) + a)/(d + e/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx)}{d+\frac{e}{x}}\,dx=\int \frac{a+b\,\ln(c\,x)}{d+\frac{e}{x}}\,dx$$

```
[In] int((a + b*log(c*x))/(d + e/x),x)
[Out] int((a + b*log(c*x))/(d + e/x), x)
```

3.341 $\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x}\,dx=\frac{(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{d}+\frac{b\,\text{PolyLog}\left(2,-\frac{dx}{e}\right)}{d}$$

```
[Out] (a+b*ln(c*x))*ln(1+d*x/e)/d+b*polylog(2,-d*x/e)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2370, 2354, 2438}

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x}\,dx=\frac{\log\left(\frac{dx}{e}+1\right)(a+b\log(cx))}{d}+\frac{b\,\text{PolyLog}\left(2,-\frac{dx}{e}\right)}{d}$$

```
[In] Int[(a + b*Log[c*x])/((d + e/x)*x),x]
```

```
[Out] ((a + b*Log[c*x])*Log[1 + (d*x)/e])/d + (b*PolyLog[2, -((d*x)/e)])/d
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{a+b\log(cx)}{e+dx}\,dx\\ &= \frac{(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{d}-\frac{b\int \frac{\log\left(1+\frac{dx}{e}\right)}{x}\,dx}{d}\\ &= \frac{(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{d}+\frac{b\text{Li}_2\left(-\frac{dx}{e}\right)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x}\,dx=\frac{(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)+b\,\text{PolyLog}\left(2,-\frac{dx}{e}\right)}{d}$$

```
[In] Integrate[(a + b*Log[c*x])/((d + e/x)*x),x]
[Out] ((a + b*Log[c*x])*Log[1 + (d*x)/e] + b*PolyLog[2, -((d*x)/e)])/d
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

method	result	size
risch	$\frac{a\ln(dx+e)}{d}+\frac{b\,\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{d}+\frac{b\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{d}$	59
parts	$\frac{a\ln(dx+e)}{d}+\frac{b\,\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{d}+\frac{b\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{d}$	59
derivativedivides	$\frac{a\ln(cdx+ce)}{d}+\frac{b\,\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{d}+\frac{b\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{d}$	62
default	$\frac{a\ln(cdx+ce)}{d}+\frac{b\,\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{d}+\frac{b\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{d}$	62

```
[In] int((a+b*ln(x*c))/(d+e/x)/x,x,method=_RETURNVERBOSE)
[Out] a*ln(d*x+e)/d+b*dilog((c*d*x+c*e)/e/c)/d+b*ln(x*c)*ln((c*d*x+c*e)/e/c)/d
```

Fricas [F]

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x}\,dx = \int \frac{b\log\left(cx\right)+a}{\left(d+\frac{e}{x}\right)x}\,dx$$

```
[In] integrate((a+b*log(c*x))/(d+e/x)/x,x, algorithm="fricas")
[Out] integral((b*log(c*x) + a)/(d*x + e), x)
```

Sympy [F]

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x}\,dx = \int \frac{a+b\log\left(cx\right)}{dx+e}\,dx$$

```
[In] integrate((a+b*ln(c*x))/(d+e/x)/x,x)
[Out] Integral((a + b*log(c*x))/(d*x + e), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x}\,dx = \frac{\left(\log\left(\frac{dx}{e}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{dx}{e}\right)\right)b}{d} + \frac{\left(b\log\left(c\right)+a\right)\log\left(dx+e\right)}{d}$$

```
[In] integrate((a+b*log(c*x))/(d+e/x)/x,x, algorithm="maxima")
[Out] (log(d*x/e + 1)*log(x) + dilog(-d*x/e))*b/d + (b*log(c) + a)*log(d*x + e)/d
```

Giac [F]

$$\int \frac{a + b\log(cx)}{\left(d + \frac{e}{x}\right)x}\,dx = \int \frac{b\log{(cx)} + a}{\left(d + \frac{e}{x}\right)x}\,dx$$

[In] integrate((a+b*log(c*x))/(d+e/x)/x,x, algorithm="giac")

[Out] integrate((b*log(c*x) + a)/((d + e/x)*x), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx)}{\left(d + \frac{e}{x}\right)x}\,dx = \int \frac{a + b\,\ln{(c\,x)}}{x\,\left(d + \frac{e}{x}\right)}\,dx$$

[In] int((a + b*log(c*x))/(x*(d + e/x)),x)

[Out] int((a + b*log(c*x))/(x*(d + e/x)), x)

3.342 $\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^2}\,dx=-\frac{\log\left(1+\frac{e}{dx}\right)(a+b\log(cx))}{e}+\frac{b\,\mathrm{PolyLog}\left(2,-\frac{e}{dx}\right)}{e}$$

```
[Out] -ln(1+e/d/x)*(a+b*ln(c*x))/e+b*polylog(2,-e/d/x)/e
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2375, 2438}

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^2}\,dx=\frac{b\,\mathrm{PolyLog}\left(2,-\frac{e}{dx}\right)}{e}-\frac{\log\left(\frac{e}{dx}+1\right)(a+b\log(cx))}{e}$$

```
[In] Int[(a + b*Log[c*x])/((d + e/x)*x^2),x]
[Out] -((Log[1 + e/(d*x)]*(a + b*Log[c*x]))/e) + (b*PolyLog[2, -(e/(d*x))])/e
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\log\left(1+\frac{e}{dx}\right)(a+b\log(cx))}{e}+\frac{b\int\frac{\log\left(1+\frac{e}{dx}\right)}{x}\,dx}{e}\\ &= -\frac{\log\left(1+\frac{e}{dx}\right)(a+b\log(cx))}{e}+\frac{b\text{Li}_2\left(-\frac{e}{dx}\right)}{e}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int\frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^2}\,dx=\frac{(a+b\log(cx))\left(a+b\log(cx)-2b\log\left(1+\frac{dx}{e}\right)\right)-2b^2\,\text{PolyLog}\left(2,-\frac{dx}{e}\right)}{2be}$$

```
[In] Integrate[(a + b*Log[c*x])/((d + e/x)*x^2),x]
[Out] ((a + b*Log[c*x])*(a + b*Log[c*x] - 2*b*Log[1 + (d*x)/e]) - 2*b^2*PolyLog[2
, -((d*x)/e)])/(2*b*e)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.98

method	result	size
risch	$-\frac{a\ln(dx+e)}{e}+\frac{a\ln(x)}{e}-\frac{b\,\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{e}-\frac{b\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{e}+\frac{b\ln(xc)^2}{2e}$	81
parts	$-\frac{a\ln(dx+e)}{e}+\frac{a\ln(x)}{e}-\frac{b\,\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{e}-\frac{b\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{e}+\frac{b\ln(xc)^2}{2e}$	81
derivativedivides	$c\left(\frac{a\ln(xc)}{ec}-\frac{a\ln(cdx+ce)}{ec}+\frac{b\ln(xc)^2}{2ec}-\frac{b\,\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{ec}-\frac{b\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{ec}\right)$	103
default	$c\left(\frac{a\ln(xc)}{ec}-\frac{a\ln(cdx+ce)}{ec}+\frac{b\ln(xc)^2}{2ec}-\frac{b\,\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{ec}-\frac{b\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{ec}\right)$	103

```
[In] int((a+b*ln(x*c))/(d+e/x)/x^2,x,method=_RETURNVERBOSE)
[Out] -a/e*ln(d*x+e)+a/e*ln(x)-b/e*dilog((c*d*x+c*e)/e/c)-b/e*ln(x*c)*ln((c*d*x+c
*e)/e/c)+1/2*b/e*ln(x*c)^2
```

Fricas [F]

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^2}\,dx=\int \frac{b\log\left(cx\right)+a}{\left(d+\frac{e}{x}\right)x^2}\,dx$$

[In] integrate((a+b*log(c*x))/(d+e/x)/x^2,x, algorithm="fricas")

[Out] integral((b*log(c*x) + a)/(d*x^2 + e*x), x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.95 (sec) , antiderivative size = 170, normalized size of antiderivative = 4.15

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^2}\,dx=\frac{2ad\left(\begin{cases}-\frac{x}{e}-\frac{1}{2d} & \text{for } d=0\\ \frac{\log\left(2dx\right)}{2d} & \text{otherwise}\end{cases}\right)}{e}-\frac{2ad\left(\begin{cases}\frac{x}{e}+\frac{1}{2d} & \text{for } d=0\\ \frac{\log\left(2dx+2e\right)}{2d} & \text{otherwise}\end{cases}\right)}{e}$$
$$+b\left(\begin{cases}-\frac{1}{dx} & \text{fo}\\ \frac{\begin{cases}\mathrm{Li}_2\left(\frac{ee^{i\pi}}{dx}\right) & \text{for } \frac{1}{|x|}<1\wedge|x|<1\\ \log\left(d\right)\log\left(x\right)+\mathrm{Li}_2\left(\frac{ee^{i\pi}}{dx}\right) & \text{for } |x|<1\\ -\log\left(d\right)\log\left(\frac{1}{x}\right)+\mathrm{Li}_2\left(\frac{ee^{i\pi}}{dx}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log\left(d\right)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log\left(d\right)+\mathrm{Li}_2\left(\frac{ee^{i\pi}}{dx}\right) & \text{otherwise}\end{cases}}{e} & \text{ot}\end{cases}\right)$$
$$-b\left(\begin{cases}\frac{1}{dx} & \text{for } e=0\\ \frac{\log\left(d+\frac{e}{x}\right)}{e} & \text{otherwise}\end{cases}\right)\log\left(cx\right)$$

[In] integrate((a+b*ln(c*x))/(d+e/x)/x**2,x)

[Out] 2*a*d*Piecewise((-x/e - 1/(2*d), Eq(d, 0)), (log(2*d*x)/(2*d), True))/e - 2*a*d*Piecewise((x/e + 1/(2*d), Eq(d, 0)), (log(2*d*x + 2*e)/(2*d), True))/e + b*Piecewise((-1/(d*x), Eq(e, 0)), (Piecewise((polylog(2, e*exp_polar(I*pi)/(d*x)), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) + polylog(2, e*exp_polar(I*pi)/(d*x)), Abs(x) < 1), (-log(d)*log(1/x) + polylog(2, e*exp_polar(I*pi)/(d*x)), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) + polylog(2, e*exp_polar(I*pi)/(d*x)), True))/e, True)) - b*Piecewise((1/(d*x), Eq(e, 0)), (log(d + e/x)/e, True))*log(c*x)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.63

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^2}\,dx = \frac{b\log\left(x\right)^2}{2\,e} - \frac{\left(\log\left(\frac{dx}{e}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{dx}{e}\right)\right)b}{e} - \frac{\left(b\log\left(c\right)+a\right)\log\left(dx+e\right)}{e} + \frac{\left(b\log\left(c\right)+a\right)\log\left(x\right)}{e}$$

```
[In] integrate((a+b*log(c*x))/(d+e/x)/x^2,x, algorithm="maxima")
[Out] 1/2*b*log(x)^2/e - (log(d*x/e + 1)*log(x) + dilog(-d*x/e))*b/e - (b*log(c)
+ a)*log(d*x + e)/e + (b*log(c) + a)*log(x)/e
```

Giac [F]

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^2}\,dx = \int \frac{b\log\left(cx\right)+a}{\left(d+\frac{e}{x}\right)x^2}\,dx$$

```
[In] integrate((a+b*log(c*x))/(d+e/x)/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*x) + a)/((d + e/x)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^2}\,dx = \int \frac{a+b\,\ln\left(c\,x\right)}{x^2\,\left(d+\frac{e}{x}\right)}\,dx$$

```
[In] int((a + b*log(c*x))/(x^2*(d + e/x)),x)
[Out] int((a + b*log(c*x))/(x^2*(d + e/x)), x)
```

3.343 $\int \frac{a+b\log(cx)}{(d+\frac{e}{x})x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^3}\,dx=-\frac{b}{ex}-\frac{a+b\log(cx)}{ex}-\frac{d(a+b\log(cx))^2}{2be^2}+\frac{d(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{e^2}+\frac{bd\,\text{PolyLog}\left(2,-\frac{dx}{e}\right)}{e^2}$$

```
[Out] -b/e/x+(-a-b*ln(c*x))/e/x-1/2*d*(a+b*ln(c*x))^2/b/e^2+d*(a+b*ln(c*x))*ln(1+
d*x/e)/e^2+b*d*polylog(2,-d*x/e)/e^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {269, 46, 2393, 2341, 2338, 2354, 2438}

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^3}\,dx=-\frac{d(a+b\log(cx))^2}{2be^2}+\frac{d\log\left(\frac{dx}{e}+1\right)(a+b\log(cx))}{e^2}-\frac{a+b\log(cx)}{ex}+\frac{bd\,\text{PolyLog}\left(2,-\frac{dx}{e}\right)}{e^2}-\frac{b}{ex}$$

```
[In] Int[(a + b*Log[c*x])/((d + e/x)*x^3),x]
```

```
[Out] -(b/(e*x)) - (a + b*Log[c*x])/(e*x) - (d*(a + b*Log[c*x])^2)/(2*b*e^2) + (d
*(a + b*Log[c*x])*Log[1 + (d*x)/e])/e^2 + (b*d*PolyLog[2, -((d*x)/e)])/e^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a+b\log(cx)}{ex^2}-\frac{d(a+b\log(cx))}{e^2x}+\frac{d^2(a+b\log(cx))}{e^2(e+dx)}\right)dx\\ &= -\frac{d\int \frac{a+b\log(cx)}{x}\,dx}{e^2}+\frac{d^2\int \frac{a+b\log(cx)}{e+dx}\,dx}{e^2}+\frac{\int \frac{a+b\log(cx)}{x^2}\,dx}{e}\end{aligned}$$

$$= -\frac{b}{ex} - \frac{a+b\log(cx)}{ex} - \frac{d(a+b\log(cx))^2}{2be^2} + \frac{d(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{e^2} - \frac{(bd)\int \frac{\log\left(1+\frac{dx}{e}\right)}{x}\,dx}{e^2}$$

$$= -\frac{b}{ex} - \frac{a+b\log(cx)}{ex} - \frac{d(a+b\log(cx))^2}{2be^2} + \frac{d(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{e^2} + \frac{bd\text{Li}_2\left(-\frac{dx}{e}\right)}{e^2}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^3}\,dx = -\frac{\frac{2be}{x} + \frac{2e(a+b\log(cx))}{x} + \frac{d(a+b\log(cx))^2}{b} - 2d(a+b\log(cx))\log\left(1+\frac{dx}{e}\right) - 2bd\,\text{PolyLog}\left(2,-\frac{dx}{e}\right)}{2e^2}$$

```
[In] Integrate[(a + b*Log[c*x])/((d + e/x)*x^3),x]
```

```
[Out] -1/2*((2*b*e)/x + (2*e*(a + b*Log[c*x]))/x + (d*(a + b*Log[c*x])^2)/b - 2*d
*(a + b*Log[c*x])*Log[1 + (d*x)/e] - 2*b*d*PolyLog[2, -((d*x)/e)])/e^2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.37

method	result
risch	$\frac{ad\ln(dx+e)}{e^2} - \frac{a}{ex} - \frac{ad\ln(x)}{e^2} + \frac{bd\,\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{e^2} + \frac{bd\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{e^2} - \frac{b\ln(xc)}{ex} - \frac{b}{ex} - \frac{bd\ln(xc)^2}{2e^2}$
parts	$a\left(\frac{d\ln(dx+e)}{e^2} - \frac{1}{ex} - \frac{d\ln(x)}{e^2}\right) + \frac{bd\,\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{e^2} + \frac{bd\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{e^2} - \frac{b\ln(xc)}{ex} - \frac{b}{ex} - \frac{bd\ln(xc)^2}{2e^2}$
derivativedivides	$c^2\left(a\left(-\frac{1}{e\,c^2x} - \frac{d\ln(xc)}{e^2c^2} + \frac{d\ln(cdx+ce)}{e^2c^2}\right) + b\left(-\frac{d\ln(xc)^2}{2e^2c^2} + \frac{-\frac{\ln(xc)}{xc} - \frac{1}{xc}}{ec} + \frac{d^2\left(\frac{\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{d} + \frac{\ln(xc)\ln}{}\right.}{e^2c^2}\right.\right.$
default	$c^2\left(a\left(-\frac{1}{e\,c^2x} - \frac{d\ln(xc)}{e^2c^2} + \frac{d\ln(cdx+ce)}{e^2c^2}\right) + b\left(-\frac{d\ln(xc)^2}{2e^2c^2} + \frac{-\frac{\ln(xc)}{xc} - \frac{1}{xc}}{ec} + \frac{d^2\left(\frac{\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{d} + \frac{\ln(xc)\ln}{}\right.}{e^2c^2}\right.\right.$

```
[In] int((a+b*ln(x*c))/(d+e/x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*d/e^2*ln(d*x+e)-a/e/x-a*d/e^2*ln(x)+b/e^2*d*dilog((c*d*x+c*e)/e/c)+b/e^2*
d*ln(x*c)*ln((c*d*x+c*e)/e/c)-b/e*ln(x*c)/x-b/e/x-1/2*b/e^2*d*ln(x*c)^2
```

Fricas [F]

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^3}\,dx = \int \frac{b\log(cx)+a}{\left(d+\frac{e}{x}\right)x^3}\,dx$$

```
[In] integrate((a+b*log(c*x))/(d+e/x)/x^3,x, algorithm="fricas")
[Out] integral((b*log(c*x) + a)/(d*x^3 + e*x^2), x)
```

Sympy [A] (verification not implemented)

Time = 37.73 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.45

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^3}\,dx = \frac{ad^2\left(\begin{cases}\frac{x}{e} & \text{for } d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)}{e^2} - \frac{ad\log(x)}{e^2} - \frac{a}{ex} - \frac{bd^2\left(\begin{cases}\frac{x}{e} & \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|}<1\wedge|x|<1\\ \log(e)\log(x)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } |x|<1\\ -\log(e)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log(e)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{otherwise}\end{cases}}{d} & \end{cases}\right)}{e^2} + \frac{bd^2\left(\begin{cases}\frac{x}{e} & \text{for } d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)\log(cx)}{e^2} + \frac{bd\log(x)^2}{2e^2} - \frac{bd\log(x)\log(cx)}{e^2} - \frac{b\log(cx)}{ex} - \frac{b}{ex}$$

```
[In] integrate((a+b*ln(c*x))/(d+e/x)/x**3,x)
[Out] a*d**2*Piecewise((x/e, Eq(d, 0)), (log(d*x + e)/d, True))/e**2 - a*d*log(x)
/e**2 - a/(e*x) - b*d**2*Piecewise((x/e, Eq(d, 0)), (Piecewise((-polylog(2,
 d*x*exp_polar(I*pi)/e), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(e)*log(x) - p
olylog(2, d*x*exp_polar(I*pi)/e), Abs(x) < 1), (-log(e)*log(1/x) - polylog(
2, d*x*exp_polar(I*pi)/e), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0),
()), x)*log(e) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(e) - polylog(2,
 d*x*exp_polar(I*pi)/e), True))/d, True))/e**2 + b*d**2*Piecewise((x/e, Eq(
d, 0)), (log(d*x + e)/d, True))*log(c*x)/e**2 + b*d*log(x)**2/(2*e**2) - b*
d*log(x)*log(c*x)/e**2 - b*log(c*x)/(e*x) - b/(e*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.14

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^3}\,dx = \frac{\left(\log\left(\frac{dx}{e}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{dx}{e}\right)\right)bd}{e^2} + \frac{(bd\log(c)+ad)\log(dx+e)}{e^2} - \frac{bdx\log(x)^2+2\,(e\log(c)+e)b+2\,ae+2\,(be+(bd\log(c)+ad)x)\log(x)}{2\,e^2x}$$

```
[In] integrate((a+b*log(c*x))/(d+e/x)/x^3,x, algorithm="maxima")
```

```
[Out] (log(d*x/e + 1)*log(x) + dilog(-d*x/e))*b*d/e^2 + (b*d*log(c) + a*d)*log(d*
x + e)/e^2 - 1/2*(b*d*x*log(x)^2 + 2*(e*log(c) + e)*b + 2*a*e + 2*(b*e + (b
*d*log(c) + a*d)*x)*log(x))/(e^2*x)
```

Giac [F]

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^3}\,dx = \int \frac{b\log(cx)+a}{\left(d+\frac{e}{x}\right)x^3}\,dx$$

```
[In] integrate((a+b*log(c*x))/(d+e/x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x) + a)/((d + e/x)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^3}\,dx = \int \frac{a+b\ln(c\,x)}{x^3\left(d+\frac{e}{x}\right)}\,dx$$

```
[In] int((a + b*log(c*x))/(x^3*(d + e/x)),x)
```

```
[Out] int((a + b*log(c*x))/(x^3*(d + e/x)), x)
```

3.344 $\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 121

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^4}\,dx = -\frac{b}{4ex^2}+\frac{bd}{e^2x}-\frac{a+b\log(cx)}{2ex^2}+\frac{d(a+b\log(cx))}{e^2x}+\frac{d^2(a+b\log(cx))^2}{2be^3}-\frac{d^2(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{e^3}-\frac{bd^2\operatorname{PolyLog}\left(2,-\frac{dx}{e}\right)}{e^3}$$

```
[Out] -1/4*b/e/x^2+b*d/e^2/x+1/2*(-a-b*ln(c*x))/e/x^2+d*(a+b*ln(c*x))/e^2/x+1/2*d
^2*(a+b*ln(c*x))^2/b/e^3-d^2*(a+b*ln(c*x))*ln(1+d*x/e)/e^3-b*d^2*polylog(2,
-d*x/e)/e^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {269, 46, 2393, 2341, 2338, 2354, 2438}

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^4}\,dx = \frac{d^2(a+b\log(cx))^2}{2be^3}-\frac{d^2\log\left(\frac{dx}{e}+1\right)(a+b\log(cx))}{e^3}+\frac{d(a+b\log(cx))}{e^2x}-\frac{a+b\log(cx)}{2ex^2}-\frac{bd^2\operatorname{PolyLog}\left(2,-\frac{dx}{e}\right)}{e^3}+\frac{bd}{e^2x}-\frac{b}{4ex^2}$$

```
[In] Int[(a + b*Log[c*x])/((d + e/x)*x^4),x]
```

```
[Out] -1/4*b/(e*x^2) + (b*d)/(e^2*x) - (a + b*Log[c*x])/(2*e*x^2) + (d*(a + b*Log
[c*x]))/(e^2*x) + (d^2*(a + b*Log[c*x])^2)/(2*b*e^3) - (d^2*(a + b*Log[c*x]
)*Log[1 + (d*x)/e])/e^3 - (b*d^2*PolyLog[2, -((d*x)/e)])/e^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a+b\log(cx)}{ex^3} - \frac{d(a+b\log(cx))}{e^2x^2} + \frac{d^2(a+b\log(cx))}{e^3x} - \frac{d^3(a+b\log(cx))}{e^3(e+dx)}\right) dx$$

$$= \frac{d^2 \int \frac{a+b\log(cx)}{x}\,dx}{e^3} - \frac{d^3 \int \frac{a+b\log(cx)}{e+dx}\,dx}{e^3} - \frac{d \int \frac{a+b\log(cx)}{x^2}\,dx}{e^2} + \frac{\int \frac{a+b\log(cx)}{x^3}\,dx}{e}$$

$$= -\frac{b}{4ex^2} + \frac{bd}{e^2x} - \frac{a+b\log(cx)}{2ex^2} + \frac{d(a+b\log(cx))}{e^2x} + \frac{d^2(a+b\log(cx))^2}{2be^3} - \frac{d^2(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{e^3} + \frac{(bd^2)\int \frac{\log\left(1+\frac{dx}{e}\right)}{x}\,dx}{e^3}$$

$$= -\frac{b}{4ex^2} + \frac{bd}{e^2x} - \frac{a+b\log(cx)}{2ex^2} + \frac{d(a+b\log(cx))}{e^2x} + \frac{d^2(a+b\log(cx))^2}{2be^3} - \frac{d^2(a+b\log(cx))\log\left(1+\frac{dx}{e}\right)}{e^3} - \frac{bd^2\mathrm{Li}_2\left(-\frac{dx}{e}\right)}{e^3}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.91

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^4}\,dx = -\frac{\frac{be^2}{x^2} - \frac{4bde}{x} + \frac{2e^2(a+b\log(cx))}{x^2} - \frac{4de(a+b\log(cx))}{x} - \frac{2d^2(a+b\log(cx))^2}{b} + 4d^2(a+b\log(cx))\log\left(1+\frac{dx}{e}\right) + 4bd^2\,\mathrm{PolyI}}{4e^3}$$

```
[In] Integrate[(a + b*Log[c*x])/((d + e/x)*x^4),x]
```

```
[Out] -1/4*((b*e^2)/x^2 - (4*b*d*e)/x + (2*e^2*(a + b*Log[c*x]))/x^2 - (4*d*e*(a
+ b*Log[c*x]))/x - (2*d^2*(a + b*Log[c*x])^2)/b + 4*d^2*(a + b*Log[c*x])*Lo
g[1 + (d*x)/e] + 4*b*d^2*PolyLog[2, -((d*x)/e)])/e^3
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.30

method	result
parts	$a\left(-\frac{d^2\ln(dx+e)}{e^3}-\frac{1}{2e\,x^2}+\frac{d^2\ln(x)}{e^3}+\frac{d}{e^2x}\right)-\frac{b\,d^2\,\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{e^3}-\frac{b\,d^2\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{e^3}+\frac{bd\ln(xc)}{e^2x}+$
risch	$-\frac{a\,d^2\ln(dx+e)}{e^3}-\frac{a}{2e\,x^2}+\frac{a\,d^2\ln(x)}{e^3}+\frac{ad}{e^2x}-\frac{b\,d^2\,\text{dilog}\left(\frac{cdx+ce}{ec}\right)}{e^3}-\frac{b\,d^2\ln(xc)\ln\left(\frac{cdx+ce}{ec}\right)}{e^3}+\frac{bd\ln(xc)}{e^2x}+$
derivativedivides	$c^3\left(a\left(-\frac{1}{2e\,c^3x^2}+\frac{d^2\ln(xc)}{e^3c^3}+\frac{d}{e^2c^3x}-\frac{d^2\ln(cdx+ce)}{e^3c^3}\right)+b\left(\frac{-\frac{\ln(xc)}{2x^2c^2}-\frac{1}{4x^2c^2}}{ec}+\frac{d^2\ln(xc)^2}{2e^3c^3}-\frac{d\left(-\frac{\ln(xc)}{xc}\right.}{e^2c^2}\right.\right.$
default	$c^3\left(a\left(-\frac{1}{2e\,c^3x^2}+\frac{d^2\ln(xc)}{e^3c^3}+\frac{d}{e^2c^3x}-\frac{d^2\ln(cdx+ce)}{e^3c^3}\right)+b\left(\frac{-\frac{\ln(xc)}{2x^2c^2}-\frac{1}{4x^2c^2}}{ec}+\frac{d^2\ln(xc)^2}{2e^3c^3}-\frac{d\left(-\frac{\ln(xc)}{xc}\right.}{e^2c^2}\right.\right.$

```
[In] int((a+b*ln(x*c))/(d+e/x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-d^2/e^3*ln(d*x+e)-1/2/e/x^2+d^2/e^3*ln(x)+d/e^2/x)-b/e^3*d^2*dilog((c*d
*x+c*e)/e/c)-b/e^3*d^2*ln(x*c)*ln((c*d*x+c*e)/e/c)+b/e^2*d*ln(x*c)/x+b*d/e^
2/x+1/2*b/e^3*d^2*ln(x*c)^2-1/2*b/e*ln(x*c)/x^2-1/4*b/e/x^2
```

Fricas [F]

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^4}\,dx=\int \frac{b\log\left(cx\right)+a}{\left(d+\frac{e}{x}\right)x^4}\,dx$$

```
[In] integrate((a+b*log(c*x))/(d+e/x)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x) + a)/(d*x^4 + e*x^3), x)
```

Sympy [A] (verification not implemented)

Time = 42.65 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.08

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^4}\,dx = -\frac{ad^3\left(\begin{cases}\frac{x}{e} & \text{for}\ d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)}{e^3}+\frac{ad^2\log(x)}{e^3}+\frac{ad}{e^2x}-\frac{a}{2ex^2} + \frac{bd^3\left(\begin{cases}\frac{x}{e} & \\ \frac{\begin{cases}-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for}\ \frac{1}{|x|}<1\wedge|x|<1\\ \log(e)\log(x)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for}\ |x|<1\\ -\log(e)\log\left(\frac{1}{x}\right)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for}\ \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log(e)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log(e)-\operatorname{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{otherwise}\end{cases}}{d} & \end{cases}\right)}{e^3} - \frac{bd^3\left(\begin{cases}\frac{x}{e} & \text{for}\ d=0\\ \frac{\log(dx+e)}{d} & \text{otherwise}\end{cases}\right)\log(cx)}{e^3}-\frac{bd^2\log(x)^2}{2e^3} + \frac{bd^2\log(x)\log(cx)}{e^3}+\frac{bd\log(cx)}{e^2x}+\frac{bd}{e^2x}-\frac{b\log(cx)}{2ex^2}-\frac{b}{4ex^2}$$

```
[In] integrate((a+b*ln(c*x))/(d+e/x)/x**4,x)
```

```
[Out] -a*d**3*Piecewise((x/e, Eq(d, 0)), (log(d*x + e)/d, True))/e**3 + a*d**2*lo
g(x)/e**3 + a*d/(e**2*x) - a/(2*e*x**2) + b*d**3*Piecewise((x/e, Eq(d, 0)),
 (Piecewise((-polylog(2, d*x*exp_polar(I*pi)/e), (Abs(x) < 1) & (1/Abs(x) <
 1)), (log(e)*log(x) - polylog(2, d*x*exp_polar(I*pi)/e), Abs(x) < 1), (-lo
g(e)*log(1/x) - polylog(2, d*x*exp_polar(I*pi)/e), 1/Abs(x) < 1), (-meijerg
(((), (1, 1)), ((0, 0), ()), x)*log(e) + meijerg(((1, 1), ()), ((), (0, 0))
, x)*log(e) - polylog(2, d*x*exp_polar(I*pi)/e), True))/d, True))/e**3 - b*
d**3*Piecewise((x/e, Eq(d, 0)), (log(d*x + e)/d, True))*log(c*x)/e**3 - b*d
**2*log(x)**2/(2*e**3) + b*d**2*log(x)*log(c*x)/e**3 + b*d*log(c*x)/(e**2*x
) + b*d/(e**2*x) - b*log(c*x)/(2*e*x**2) - b/(4*e*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.25

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^4}\,dx = -\frac{\left(\log\left(\frac{dx}{e}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{dx}{e}\right)\right)bd^2}{e^3} - \frac{\left(bd^2\log\left(c\right)+ad^2\right)\log\left(dx+e\right)}{e^3} + \frac{2\,bd^2x^2\log\left(x\right)^2 - 2\,ae^2 - \left(2\,e^2\log\left(c\right)+e^2\right)b + 4\left(ade+\left(de\log\left(c\right)+de\right)b\right)x + 2\left(2\,bdex - be^2 + 2\left(bd^2\,\mathrm{l}\right.\right.}{4\,e^3x^2}$$

```
[In] integrate((a+b*log(c*x))/(d+e/x)/x^4,x, algorithm="maxima")
```

```
[Out] -(log(d*x/e + 1)*log(x) + dilog(-d*x/e))*b*d^2/e^3 - (b*d^2*log(c) + a*d^2)
*log(d*x + e)/e^3 + 1/4*(2*b*d^2*x^2*log(x)^2 - 2*a*e^2 - (2*e^2*log(c) + e
^2)*b + 4*(a*d*e + (d*e*log(c) + d*e)*b)*x + 2*(2*b*d*e*x - b*e^2 + 2*(b*d^
2*log(c) + a*d^2)*x^2)*log(x))/(e^3*x^2)
```

Giac [F]

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^4}\,dx = \int \frac{b\log\left(cx\right)+a}{\left(d+\frac{e}{x}\right)x^4}\,dx$$

```
[In] integrate((a+b*log(c*x))/(d+e/x)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x) + a)/((d + e/x)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx)}{\left(d+\frac{e}{x}\right)x^4}\,dx = \int \frac{a+b\,\ln\left(c\,x\right)}{x^4\,\left(d+\frac{e}{x}\right)}\,dx$$

```
[In] int((a + b*log(c*x))/(x^4*(d + e/x)),x)
```

```
[Out] int((a + b*log(c*x))/(x^4*(d + e/x)), x)
```

3.345 $\int \frac{x^{-1+n}\log(ex^n)}{1-ex^n}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 17

$$\int \frac{x^{-1+n}\log(ex^n)}{1-ex^n}\,dx = \frac{\text{PolyLog}(2,1-ex^n)}{en}$$

```
[Out] polylog(2,1-e*x^n)/e/n
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2374, 2352}

$$\int \frac{x^{-1+n}\log(ex^n)}{1-ex^n}\,dx = \frac{\text{PolyLog}(2,1-ex^n)}{en}$$

```
[In] Int[(x^(-1 + n)*Log[e*x^n])/(1 - e*x^n),x]
```

```
[Out] PolyLog[2, 1 - e*x^n]/(e*n)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2374

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[f^m/n, Subst[Int[(d + e*x)^q*(a +
b*Log[c*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x]
&& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && EqQ[r, n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log(ex)}{1-ex}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Li}_2(1-ex^n)}{en}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}\log(ex^n)}{1-ex^n}\,dx = \frac{\text{PolyLog}(2,1-ex^n)}{en}$$

```
[In] Integrate[(x^(-1 + n)*Log[e*x^n])/(1 - e*x^n),x]
[Out] PolyLog[2, 1 - e*x^n]/(e*n)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result
default	$\frac{\operatorname{dilog}(e\,x^n)}{en}$
risch	$-\frac{\ln(1-e\,x^n)\ln(x^n)}{ne}+\frac{\ln(1-e\,x^n)\ln(e\,x^n)}{ne}+\frac{\operatorname{dilog}(e\,x^n)}{en}+\frac{\left(-\frac{i\pi\ \operatorname{csgn}(ie)\operatorname{csgn}(ie\,x^n)^2}{2}+\frac{i\pi\ \operatorname{csgn}(ie)\operatorname{csgn}(ie\,x^n)\operatorname{csgn}(ix^n)}{2}+\frac{i\pi\operatorname{csgn}\cdots}{\cdots}\cdots\right.}{ne}\cdots$
meijerg	$\frac{i(-1)^{\frac{\operatorname{csgn}(ie)}{2}-\frac{\operatorname{csgn}(ix^n)}{2}-\frac{\operatorname{csgn}(ix^n)\operatorname{csgn}(ie)}{2}-\frac{1}{n}-\frac{n-1}{n}}\ln(e)\ln\left(1+ix^n e(-1)^{-\frac{\operatorname{csgn}(ie)}{2}+\frac{\operatorname{csgn}(ix^n)}{2}+\frac{\operatorname{csgn}(ix^n)\operatorname{csgn}(ie)}{2}}\right)}{en}-\frac{i(-1)^{\frac{\operatorname{csgn}(\cdots}{2}\cdots}}{\cdots}$

```
[In] int(x^(n-1)*ln(e*x^n)/(1-e*x^n),x,method=_RETURNVERBOSE)
[Out] 1/e/n*dilog(e*x^n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(16) = 32$.

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \frac{x^{-1+n}\log(ex^n)}{1-ex^n}\,dx = -\frac{n\log(-ex^n+1)\log(x)+\log(ex^n-1)\log(e)+\mathrm{Li}_2(ex^n)}{en}$$

```
[In] integrate(x^(-1+n)*log(e*x^n)/(1-e*x^n),x, algorithm="fricas")
[Out] -(n*log(-e*x^n + 1)*log(x) + log(e*x^n - 1)*log(e) + dilog(e*x^n))/(e*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+n}\log(ex^n)}{1-ex^n}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x**(-1+n)*ln(e*x**n)/(1-e*x**n),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(16) = 32$.

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 3.06

$$\int \frac{x^{-1+n}\log(ex^n)}{1-ex^n}\,dx = -\frac{\log(e)\log\left(\frac{ex^n-1}{e}\right)}{en} - \frac{\log(-ex^n+1)\log(x^n)+\mathrm{Li}_2(ex^n)}{en}$$

```
[In] integrate(x^(-1+n)*log(e*x^n)/(1-e*x^n),x, algorithm="maxima")
[Out] -log(e)*log((e*x^n - 1)/e)/(e*n) - (log(-e*x^n + 1)*log(x^n) + dilog(e*x^n)
)/(e*n)
```

Giac [F]

$$\int \frac{x^{-1+n}\log(ex^n)}{1-ex^n}\,dx = \int -\frac{x^{n-1}\log(ex^n)}{ex^n-1}\,dx$$

```
[In] integrate(x^(-1+n)*log(e*x^n)/(1-e*x^n),x, algorithm="giac")
[Out] integrate(-x^(n - 1)*log(e*x^n)/(e*x^n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x^{-1+n} \log (ex^n)}{1 - ex^n} \, dx = \frac{\mathrm{Li}_2(e\, x^n)}{e\, n}$$

```
[In] int(-(x^(n - 1)*log(e*x^n))/(e*x^n - 1),x)
[Out] dilog(e*x^n)/(e*n)
```

3.346 $\int \frac{x^{-1+n}\log\left(\frac{x^n}{d}\right)}{d-x^n}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{x^{-1+n}\log\left(\frac{x^n}{d}\right)}{d-x^n}\,dx = \frac{\text{PolyLog}\left(2,1-\frac{x^n}{d}\right)}{n}$$

```
[Out] polylog(2,1-x^n/d)/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2374, 2352}

$$\int \frac{x^{-1+n}\log\left(\frac{x^n}{d}\right)}{d-x^n}\,dx = \frac{\text{PolyLog}\left(2,1-\frac{x^n}{d}\right)}{n}$$

```
[In] Int[(x^(-1 + n)*Log[x^n/d])/(d - x^n),x]
```

```
[Out] PolyLog[2, 1 - x^n/d]/n
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2374

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[f^m/n, Subst[Int[(d + e*x)^q*(a +
b*Log[c*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x]
```

```
&& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && EqQ[r, n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\log\left(\frac{x}{d}\right)}{d-x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Li}_2\left(1-\frac{x^n}{d}\right)}{n}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x^{-1+n}\log\left(\frac{x^n}{d}\right)}{d-x^n}\,dx = \frac{\text{PolyLog}\left(2,\frac{d-x^n}{d}\right)}{n}$$

```
[In] Integrate[(x^(-1 + n)*Log[x^n/d])/(d - x^n),x]
[Out] PolyLog[2, (d - x^n)/d]/n
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result
default	$\frac{\text{dilog}\left(\frac{x^n}{d}\right)}{n}$
risch	$-\frac{\ln(x^n)\ln\left(-\frac{-d+x^n}{d}\right)}{n}-\frac{\text{dilog}\left(-\frac{-d+x^n}{d}\right)}{n}+\frac{\left(-\frac{i\pi\ \text{csgn}\left(\frac{i}{d}\right)\text{csgn}\left(\frac{ix^n}{d}\right)^2}{2}+\frac{i\pi\ \text{csgn}\left(\frac{i}{d}\right)\text{csgn}\left(\frac{ix^n}{d}\right)\text{csgn}(ix^n)}{2}+\frac{i\pi\ \text{csgn}\left(\frac{ix^n}{d}\right)^3}{2}-\frac{i\pi\ \text{cs}}{}\right.}{n}$

```
[In] int(x^(n-1)*ln(x^n/d)/(d-x^n),x,method=_RETURNVERBOSE)
[Out] 1/n*dilog(x^n/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(15) = 30$.

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 3.12

$$\int \frac{x^{-1+n} \log\left(\frac{x^n}{d}\right)}{d-x^n}\,dx = -\frac{n\log(x)\log\left(\frac{d-x^n}{d}\right) + \log(-d+x^n)\log\left(\frac{1}{d}\right) + \text{Li}_2\left(-\frac{d-x^n}{d}+1\right)}{n}$$

```
[In] integrate(x^(-1+n)*log(x^n/d)/(d-x^n),x, algorithm="fricas")
[Out] -(n*log(x)*log((d - x^n)/d) + log(-d + x^n)*log(1/d) + dilog(-(d - x^n)/d +
 1))/n
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+n} \log\left(\frac{x^n}{d}\right)}{d-x^n}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x**(-1+n)*ln(x**n/d)/(d-x**n),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(15) = 30$.

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.81

$$\int \frac{x^{-1+n} \log\left(\frac{x^n}{d}\right)}{d-x^n}\,dx = \frac{\log(d)\log(-d+x^n)}{n} - \frac{\log(x^n)\log\left(-\frac{x^n}{d}+1\right) + \text{Li}_2\left(\frac{x^n}{d}\right)}{n}$$

```
[In] integrate(x^(-1+n)*log(x^n/d)/(d-x^n),x, algorithm="maxima")
[Out] log(d)*log(-d + x^n)/n - (log(x^n)*log(-x^n/d + 1) + dilog(x^n/d))/n
```

Giac [F]

$$\int \frac{x^{-1+n}\log\left(\frac{x^n}{d}\right)}{d-x^n}\,dx = \int \frac{x^{n-1}\log\left(\frac{x^n}{d}\right)}{d-x^n}\,dx$$

```
[In] integrate(x^(-1+n)*log(x^n/d)/(d-x^n),x, algorithm="giac")
[Out] integrate(x^(n - 1)*log(x^n/d)/(d - x^n), x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^{-1+n}\log\left(\frac{x^n}{d}\right)}{d-x^n}\,dx = \frac{\mathrm{Li}_2\left(\frac{x^n}{d}\right)}{n}$$

```
[In] int((x^(n - 1)*log(x^n/d))/(d - x^n),x)
[Out] dilog(x^n/d)/n
```

3.347 $\int \frac{x^{-1+n}\log\left(-\frac{ex^n}{d}\right)}{d+ex^n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 20

$$\int \frac{x^{-1+n}\log\left(-\frac{ex^n}{d}\right)}{d+ex^n}\,dx = -\frac{\text{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{en}$$

```
[Out] -polylog(2,1+e*x^n/d)/e/n
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2374, 2352}

$$\int \frac{x^{-1+n}\log\left(-\frac{ex^n}{d}\right)}{d+ex^n}\,dx = -\frac{\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{en}$$

```
[In] Int[(x^(-1 + n)*Log[-((e*x^n)/d)])/(d + e*x^n),x]
```

```
[Out] -(PolyLog[2, 1 + (e*x^n)/d]/(e*n))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2374

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[f^m/n, Subst[Int[(d + e*x)^q*(a +
b*Log[c*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x]
```

```
&& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && EqQ[r, n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex} \, dx, x, x^n\right)}{n}$$
$$= -\frac{\text{Li}_2\left(1 + \frac{ex^n}{d}\right)}{en}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{x^{-1+n} \log\left(-\frac{ex^n}{d}\right)}{d+ex^n} \, dx = -\frac{\text{PolyLog}\left(2, \frac{d+ex^n}{d}\right)}{en}$$

```
[In] Integrate[(x^(-1 + n)*Log[-((e*x^n)/d)])/(d + e*x^n),x]
[Out] -(PolyLog[2, (d + e*x^n)/d]/(e*n))
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result
default	$-\frac{\operatorname{dilog}\left(-\frac{e\,x^{n}}{d}\right)}{ne}$
risch	$\frac{\operatorname{dilog}\left(\frac{d+e\,x^{n}}{d}\right)}{ne}+\frac{\ln\left(x^{n}\right)\ln\left(\frac{d+e\,x^{n}}{d}\right)}{ne}+\frac{\left(\frac{i\pi\ \operatorname{csgn}\left(ie\right)\operatorname{csgn}\left(ie\,x^{n}\right)^{2}}{2}-\frac{i\pi\ \operatorname{csgn}\left(ie\right)\operatorname{csgn}\left(ie\,x^{n}\right)\operatorname{csgn}\left(ix^{n}\right)}{2}-\frac{i\pi\operatorname{csgn}\left(ie\,x^{n}\right)^{3}}{2}+\frac{i\pi\operatorname{csgn}\left(ie\,x^{n}\right)^{2}}{2}\right.}{}$

```
[In] int(x^(n-1)*ln(-e*x^n/d)/(d+e*x^n),x,method=_RETURNVERBOSE)
[Out] -1/n/e*dilog(-e*x^n/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(19) = 38$.

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75

$$\int \frac{x^{-1+n}\log\left(-\frac{ex^n}{d}\right)}{d+ex^n}\,dx = \frac{n\log(x)\log\left(\frac{ex^n+d}{d}\right)+\log(ex^n+d)\log\left(-\frac{e}{d}\right)+\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)}{en}$$

```
[In] integrate(x^(-1+n)*log(-e*x^n/d)/(d+e*x^n),x, algorithm="fricas")
[Out] (n*log(x)*log((e*x^n + d)/d) + log(e*x^n + d)*log(-e/d) + dilog(-(e*x^n + d
)/d + 1))/(e*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+n}\log\left(-\frac{ex^n}{d}\right)}{d+ex^n}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x**(-1+n)*ln(-e*x**n/d)/(d+e*x**n),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(19) = 38$.

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.20

$$\int \frac{x^{-1+n}\log\left(-\frac{ex^n}{d}\right)}{d+ex^n}\,dx = -\frac{(\log(d)-\log(e))\log\left(\frac{ex^n+d}{e}\right)}{en} + \frac{\log\left(\frac{ex^n}{d}+1\right)\log(-x^n)+\mathrm{Li}_2\left(-\frac{ex^n}{d}\right)}{en}$$

```
[In] integrate(x^(-1+n)*log(-e*x^n/d)/(d+e*x^n),x, algorithm="maxima")
[Out] -(log(d) - log(e))*log((e*x^n + d)/e)/(e*n) + (log(e*x^n/d + 1)*log(-x^n) +
 dilog(-e*x^n/d))/(e*n)
```

Giac [F]

$$\int \frac{x^{-1+n}\log\left(-\frac{ex^n}{d}\right)}{d+ex^n}\,dx = \int \frac{x^{n-1}\log\left(-\frac{ex^n}{d}\right)}{ex^n+d}\,dx$$

```
[In] integrate(x^(-1+n)*log(-e*x^n/d)/(d+e*x^n),x, algorithm="giac")
[Out] integrate(x^(n - 1)*log(-e*x^n/d)/(e*x^n + d), x)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x^{-1+n}\log\left(-\frac{ex^n}{d}\right)}{d+ex^n}\,dx = -\frac{\mathrm{Li}_2\left(-\frac{e\,x^n}{d}\right)}{e\,n}$$

```
[In] int((x^(n - 1)*log(-(e*x^n)/d))/(d + e*x^n),x)
[Out] -dilog(-(e*x^n)/d)/(e*n)
```

3.348 $\int \frac{\log\left(\frac{a}{x}\right)}{ax-x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \frac{\log\left(\frac{a}{x}\right)}{ax-x^2}\,dx = \frac{\text{PolyLog}\left(2,1-\frac{a}{x}\right)}{a}$$

```
[Out] polylog(2,1-a/x)/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1607, 2378, 2370, 2352}

$$\int \frac{\log\left(\frac{a}{x}\right)}{ax-x^2}\,dx = \frac{\text{PolyLog}\left(2,1-\frac{a}{x}\right)}{a}$$

```
[In] Int[Log[a/x]/(a*x - x^2),x]
```

```
[Out] PolyLog[2, 1 - a/x]/a
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\log\left(\frac{a}{x}\right)}{(a-x)x}\,dx \\
&= -\text{Subst}\left(\int \frac{\log(ax)}{\left(a-\frac{1}{x}\right)x}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \frac{\log(ax)}{-1+ax}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\text{Li}_2\left(1-\frac{a}{x}\right)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$
\int \frac{\log\left(\frac{a}{x}\right)}{ax-x^2}\,dx = \frac{\text{PolyLog}\left(2,-\frac{a-x}{x}\right)}{a}
$$

```
[In] Integrate[Log[a/x]/(a*x - x^2),x]
[Out] PolyLog[2, -((a - x)/x)]/a
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(\frac{a}{x}\right)}{a}$	11
default	$\frac{\operatorname{dilog}\left(\frac{a}{x}\right)}{a}$	11
risch	$\frac{\operatorname{dilog}\left(\frac{a}{x}\right)}{a}$	11
parts	$\frac{\ln\left(\frac{a}{x}\right)\ln(x)}{a}-\frac{\ln\left(\frac{a}{x}\right)\ln(a-x)}{a}+\frac{\ln(x)^2}{2a}-\frac{\ln(a-x)\ln\left(\frac{x}{a}\right)}{a}-\frac{\operatorname{dilog}\left(\frac{x}{a}\right)}{a}$	68

```
[In] int(ln(a/x)/(a*x-x^2),x,method=_RETURNVERBOSE)
[Out] 1/a*dilog(a/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{\log\left(\frac{a}{x}\right)}{ax-x^2}\,dx=\frac{\operatorname{Li}_2\left(-\frac{a}{x}+1\right)}{a}$$

```
[In] integrate(log(a/x)/(a*x-x^2),x, algorithm="fricas")
[Out] dilog(-a/x + 1)/a
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.88 (sec) , antiderivative size = 82, normalized size of antiderivative = 5.86

$$\int \frac{\log\left(\frac{a}{x}\right)}{ax-x^2}\,dx=-\left(\begin{cases}-\frac{1}{x} & \text{for}\ a=0\\ \frac{\log\left(\frac{a}{x}-1\right)}{a} & \text{otherwise}\end{cases}\right)\log\left(\frac{a}{x}\right) - \begin{cases}\frac{1}{x} & \text{for}\ a=0\\ \frac{\begin{cases}\operatorname{Li}_2\left(\frac{a}{x}\right) & \text{for}\ \frac{1}{|x|}<1\wedge|x|<1\\ i\pi\log\left(x\right)+\operatorname{Li}_2\left(\frac{a}{x}\right) & \text{for}\ |x|<1\\ -i\pi\log\left(\frac{1}{x}\right)+\operatorname{Li}_2\left(\frac{a}{x}\right) & \text{for}\ \frac{1}{|x|}<1\\ -i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)+i\pi G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)+\operatorname{Li}_2\left(\frac{a}{x}\right) & \text{otherwise}\end{cases}}{a} & \text{otherwise}\end{cases}$$

```
[In] integrate(ln(a/x)/(a*x-x**2),x)
```

```
[Out] -Piecewise((-1/x, Eq(a, 0)), (log(a/x - 1)/a, True))*log(a/x) - Piecewise((
1/x, Eq(a, 0)), (Piecewise((polylog(2, a/x), (Abs(x) < 1) & (1/Abs(x) < 1))
, (I*pi*log(x) + polylog(2, a/x), Abs(x) < 1), (-I*pi*log(1/x) + polylog(2,
 a/x), 1/Abs(x) < 1), (-I*pi*meijerg(((), (1, 1)), ((0, 0), ()), x) + I*pi*
meijerg(((1, 1), ()), ((), (0, 0)), x) + polylog(2, a/x), True))/a, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(13) = 26$.

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 5.14

$$\int \frac{\log\left(\frac{a}{x}\right)}{ax-x^2}\,dx = -\left(\frac{\log\left(-a+x\right)}{a}-\frac{\log\left(x\right)}{a}\right)\log\left(\frac{a}{x}\right) - \frac{2\,\log\left(-a+x\right)\log\left(x\right)-\log\left(x\right)^2}{2\,a} + \frac{\log\left(x\right)\log\left(-\frac{x}{a}+1\right)+\mathrm{Li}_2\left(\frac{x}{a}\right)}{a}$$

```
[In] integrate(log(a/x)/(a*x-x^2),x, algorithm="maxima")
```

```
[Out] -(log(-a + x)/a - log(x)/a)*log(a/x) - 1/2*(2*log(-a + x)*log(x) - log(x)^2
)/a + (log(x)*log(-x/a + 1) + dilog(x/a))/a
```

Giac [F]

$$\int \frac{\log\left(\frac{a}{x}\right)}{ax-x^2}\,dx = \int \frac{\log\left(\frac{a}{x}\right)}{ax-x^2}\,dx$$

```
[In] integrate(log(a/x)/(a*x-x^2),x, algorithm="giac")
```

```
[Out] integrate(log(a/x)/(a*x - x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\log\left(\frac{a}{x}\right)}{ax-x^2}\,dx = \frac{\mathrm{Li}_2\left(\frac{a}{x}\right)}{a}$$

```
[In] int(log(a/x)/(a*x - x^2),x)
```

```
[Out] dilog(a/x)/a
```

3.349 $\int \frac{\log\left(\frac{a}{x^2}\right)}{ax-x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 17

$$\int \frac{\log\left(\frac{a}{x^2}\right)}{ax-x^3}\,dx = \frac{\text{PolyLog}\left(2,1-\frac{a}{x^2}\right)}{2a}$$

```
[Out] 1/2*polylog(2,1-a/x^2)/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1607, 2378, 2370, 2352}

$$\int \frac{\log\left(\frac{a}{x^2}\right)}{ax-x^3}\,dx = \frac{\text{PolyLog}\left(2,1-\frac{a}{x^2}\right)}{2a}$$

```
[In] Int[Log[a/x^2]/(a*x - x^3),x]
[Out] PolyLog[2, 1 - a/x^2]/(2*a)
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\log\left(\frac{a}{x^2}\right)}{x\left(a-x^2\right)}\,dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\log(ax)}{\left(a-\frac{1}{x}\right)x}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\log(ax)}{-1+ax}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{\text{Li}_2\left(1-\frac{a}{x^2}\right)}{2a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$
\int \frac{\log\left(\frac{a}{x^2}\right)}{ax-x^3}\,dx = \frac{\text{PolyLog}\left(2,-\frac{a-x^2}{x^2}\right)}{2a}
$$

```
[In] Integrate[Log[a/x^2]/(a*x - x^3),x]
[Out] PolyLog[2, -((a - x^2)/x^2)]/(2*a)
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{\operatorname{dilog}\left(\frac{a}{x^2}\right)}{2a}$
default	$\frac{\operatorname{dilog}\left(\frac{a}{x^2}\right)}{2a}$
risch	$\frac{\operatorname{dilog}\left(\frac{a}{x^2}\right)}{2a}$
parts	$\frac{\ln\left(\frac{a}{x^2}\right)\ln(x)}{a}-\frac{\ln\left(\frac{a}{x^2}\right)\ln(-x^2+a)}{2a}+\frac{\ln(x)^2}{a}-\frac{\ln(x)\ln(-x^2+a)-\ln(x)\ln\left(\frac{\sqrt{a}-x}{\sqrt{a}}\right)-\ln(x)\ln\left(\frac{\sqrt{a}+x}{\sqrt{a}}\right)-\operatorname{dilog}\left(\frac{\sqrt{a}-x}{\sqrt{a}}\right.}{a}$

```
[In] int(ln(1/x^2*a)/(-x^3+a*x),x,method=_RETURNVERBOSE)
[Out] 1/2/a*dilog(1/x^2*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{\log\left(\frac{a}{x^2}\right)}{ax-x^3}\,dx=\frac{\operatorname{Li}_2\left(-\frac{a}{x^2}+1\right)}{2\,a}$$

```
[In] integrate(log(a/x^2)/(-x^3+a*x),x, algorithm="fricas")
[Out] 1/2*dilog(-a/x^2 + 1)/a
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 5.41

$$\int \frac{\log\left(\frac{a}{x^2}\right)}{ax-x^3}\,dx = -\frac{\begin{cases}\frac{\operatorname{Li}_2\left(\frac{a}{x^2}\right)}{2} & \text{for}\ \frac{1}{|x|}<1\wedge|x|<1\\ i\pi\log\left(x\right)+\frac{\operatorname{Li}_2\left(\frac{a}{x^2}\right)}{2} & \text{for}\ |x|<1\\ -i\pi\log\left(\frac{1}{x}\right)+\frac{\operatorname{Li}_2\left(\frac{a}{x^2}\right)}{2} & \text{for}\ \frac{1}{|x|}<1\\ -i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)+i\pi G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)+\frac{\operatorname{Li}_2\left(\frac{a}{x^2}\right)}{2} & \text{otherwise}\end{cases}}{a} - \frac{\log\left(\frac{a}{x^2}\right)\log\left(\frac{a}{x^2}-1\right)}{2a}$$

```
[In] integrate(ln(a/x**2)/(-x**3+a*x),x)
```

```
[Out] -Piecewise((polylog(2, a/x**2)/2, (Abs(x) < 1) & (1/Abs(x) < 1)), (I*pi*log
(x) + polylog(2, a/x**2)/2, Abs(x) < 1), (-I*pi*log(1/x) + polylog(2, a/x**
2)/2, 1/Abs(x) < 1), (-I*pi*meijerg(((), (1, 1)), ((0, 0), ()), x) + I*pi*m
eijerg(((1, 1), ()), ((), (0, 0)), x) + polylog(2, a/x**2)/2, True))/a - lo
g(a/x**2)*log(a/x**2 - 1)/(2*a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 4.76

$$\int \frac{\log\left(\frac{a}{x^2}\right)}{ax-x^3}\,dx = -\frac{1}{2}\left(\frac{\log\left(x^2-a\right)}{a}-\frac{2\,\log\left(x\right)}{a}\right)\log\left(\frac{a}{x^2}\right) - \frac{\log\left(x^2-a\right)\log\left(x\right)-\log\left(x\right)^2}{a} + \frac{2\,\log\left(x\right)\log\left(-\frac{x^2}{a}+1\right)+\mathrm{Li}_2\left(\frac{x^2}{a}\right)}{2\,a}$$

```
[In] integrate(log(a/x^2)/(-x^3+a*x),x, algorithm="maxima")
```

```
[Out] -1/2*(log(x^2 - a)/a - 2*log(x)/a)*log(a/x^2) - (log(x^2 - a)*log(x) - log(
x)^2)/a + 1/2*(2*log(x)*log(-x^2/a + 1) + dilog(x^2/a))/a
```

Giac [F]

$$\int \frac{\log\left(\frac{a}{x^2}\right)}{ax-x^3}\,dx = \int -\frac{\log\left(\frac{a}{x^2}\right)}{x^3-ax}\,dx$$

```
[In] integrate(log(a/x^2)/(-x^3+a*x),x, algorithm="giac")
```

```
[Out] integrate(-log(a/x^2)/(x^3 - a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\log\left(\frac{a}{x^2}\right)}{ax-x^3}\,dx = \frac{\mathrm{Li}_2\left(\frac{a}{x^2}\right)}{2\,a}$$

```
[In] int(log(a/x^2)/(a*x - x^3),x)
```

```
[Out] dilog(a/x^2)/(2*a)
```

3.350 $\int \frac{\log(ax^{1-n})}{ax-x^n}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 26

$$\int \frac{\log(ax^{1-n})}{ax-x^n}\,dx = -\frac{\text{PolyLog}(2,1-ax^{1-n})}{a(1-n)}$$

```
[Out] -polylog(2,1-a*x^(1-n))/a/(1-n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1607, 2374, 2352}

$$\int \frac{\log(ax^{1-n})}{ax-x^n}\,dx = -\frac{\text{PolyLog}(2,1-ax^{1-n})}{a(1-n)}$$

```
[In] Int[Log[a*x^(1 - n)]/(a*x - x^n),x]
[Out] -(PolyLog[2, 1 - a*x^(1 - n)]/(a*(1 - n)))
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2374

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[f^m/n, Subst[Int[(d + e*x)^q*(a +
b*Log[c*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x]
&& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && EqQ[r, n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^{-n}\log\left(ax^{1-n}\right)}{-1+ax^{1-n}}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{\log(ax)}{-1+ax}\,dx, x, x^{1-n}\right)}{1-n}\\ &= -\frac{\text{Li}_2\left(1-ax^{1-n}\right)}{a(1-n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(ax^{1-n}\right)}{ax-x^n}\,dx = \frac{\text{PolyLog}\left(2, 1-ax^{1-n}\right)}{a(-1+n)}$$

```
[In] Integrate[Log[a*x^(1 - n)]/(a*x - x^n),x]
[Out] PolyLog[2, 1 - a*x^(1 - n)]/(a*(-1 + n))
```

Maple [F]

$$\int \frac{\ln\left(a\,x^{-n+1}\right)}{ax-x^n}dx$$

```
[In] int(ln(a*x^(-n+1))/(a*x-x^n),x)
[Out] int(ln(a*x^(-n+1))/(a*x-x^n),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(22) = 44$.

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.42

$$\int \frac{\log(ax^{1-n})}{ax - x^n} dx = \frac{2(n-1)\log(a)\log(x) - (n^2 - 2n + 1)\log(x)^2 + 2(n-1)\log(x)\log\left(\frac{a-x^{n-1}}{a}\right) - 2\log(a)\log(-a + x^{n-1}) + 2\,{\rm Li}_2\left(-\frac{a-x^{n-1}}{a} + 1\right)}{2(an - a)}$$

```
[In] integrate(log(a*x^(1-n))/(a*x-x^n),x, algorithm="fricas")
[Out] 1/2*(2*(n - 1)*log(a)*log(x) - (n^2 - 2*n + 1)*log(x)^2 + 2*(n - 1)*log(x)*
log((a - x^(n - 1))/a) - 2*log(a)*log(-a + x^(n - 1)) + 2*dilog(-(a - x^(n
- 1))/a + 1))/(a*n - a)
```

Sympy [F]

$$\int \frac{\log(ax^{1-n})}{ax - x^n} dx = \int \frac{\log(ax^{1-n})}{ax - x^n} dx$$

```
[In] integrate(ln(a*x**(1-n))/(a*x-x**n),x)
[Out] Integral(log(a*x**(1 - n))/(a*x - x**n), x)
```

Maxima [F]

$$\int \frac{\log(ax^{1-n})}{ax - x^n} dx = \int \frac{\log(ax^{-n+1})}{ax - x^n} dx$$

```
[In] integrate(log(a*x^(1-n))/(a*x-x^n),x, algorithm="maxima")
[Out] integrate(log(a*x^(-n + 1))/(a*x - x^n), x)
```

Giac [F]

$$\int \frac{\log (ax^{1-n})}{ax - x^n}\, dx = \int \frac{\log (ax^{-n+1})}{ax - x^n}\, dx$$

```
[In] integrate(log(a*x^(1-n))/(a*x-x^n),x, algorithm="giac")
[Out] integrate(log(a*x^(-n + 1))/(a*x - x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log (ax^{1-n})}{ax - x^n}\, dx = \int \frac{\ln (a\, x^{1-n})}{a\, x - x^n}\, dx$$

```
[In] int(log(a*x^(1 - n))/(a*x - x^n),x)
[Out] int(log(a*x^(1 - n))/(a*x - x^n), x)
```

3.351 $\int (fx)^{-1+m} (d+ex^m)^3 (a+b\log (cx^n)) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 171

$$\int (fx)^{-1+m} (d+ex^m)^3 (a+b\log (cx^n)) \, dx = -\frac{bd^3nx(fx)^{-1+m}}{m^2} - \frac{3bd^2enx^{1+m}(fx)^{-1+m}}{4m^2} - \frac{bde^2nx^{1+2m}(fx)^{-1+m}}{3m^2} - \frac{be^3nx^{1+3m}(fx)^{-1+m}}{16m^2} - \frac{bd^4nx^{1-m}(fx)^{-1+m}\log(x)}{4em} + \frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^4 (a+b\log (cx^n))}{4em}$$

```
[Out] -b*d^3*n*x*(f*x)^(-1+m)/m^2-3/4*b*d^2*e*n*x^(1+m)*(f*x)^(-1+m)/m^2-1/3*b*d*
e^2*n*x^(1+2*m)*(f*x)^(-1+m)/m^2-1/16*b*e^3*n*x^(1+3*m)*(f*x)^(-1+m)/m^2-1/
4*b*d^4*n*x^(1-m)*(f*x)^(-1+m)*ln(x)/e/m+1/4*x^(1-m)*(f*x)^(-1+m)*(d+e*x^m)
^4*(a+b*ln(c*x^n))/e/m
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used

$= \{2377, 2376, 272, 45\}$

$$\int (fx)^{-1+m} (d+ex^m)^3 (a+b\log(cx^n))\,dx = \frac{x^{1-m}(fx)^{m-1}(d+ex^m)^4(a+b\log(cx^n))}{4em} - \frac{bd^4nx^{1-m}\log(x)(fx)^{m-1}}{4em} - \frac{bd^3nx(fx)^{m-1}}{m^2} - \frac{3bd^2enx^{m+1}(fx)^{m-1}}{4m^2} - \frac{bde^2nx^{2m+1}(fx)^{m-1}}{3m^2} - \frac{be^3nx^{3m+1}(fx)^{m-1}}{16m^2}$$

```
[In] Int[(f*x)^(-1 + m)*(d + e*x^m)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -((b*d^3*n*x*(f*x)^(-1 + m))/m^2) - (3*b*d^2*e*n*x^(1 + m)*(f*x)^(-1 + m))/
(4*m^2) - (b*d*e^2*n*x^(1 + 2*m)*(f*x)^(-1 + m))/(3*m^2) - (b*e^3*n*x^(1 +
3*m)*(f*x)^(-1 + m))/(16*m^2) - (b*d^4*n*x^(1 - m)*(f*x)^(-1 + m)*Log[x])/(
4*e*m) + (x^(1 - m)*(f*x)^(-1 + m)*(d + e*x^m)^4*(a + b*Log[c*x^n]))/(4*e*m
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rule 2377

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(d + e*x^r)^q
*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] &&  !(IntegerQ[m] || GtQ[f, 0])
```

Rubi steps

$$\text{integral} = \left(x^{1-m}(fx)^{-1+m}\right)\int x^{-1+m}(d+ex^m)^3\,(a+b\log(cx^n))\,dx$$

$$= \frac{x^{1-m}(fx)^{-1+m}\,(d+ex^m)^4\,(a+b\log(cx^n))}{4em} - \frac{\left(bnx^{1-m}(fx)^{-1+m}\right)\int\frac{(d+ex^m)^4}{x}\,dx}{4em}$$

$$= \frac{x^{1-m}(fx)^{-1+m}\,(d+ex^m)^4\,(a+b\log(cx^n))}{4em} - \frac{\left(bnx^{1-m}(fx)^{-1+m}\right)\text{Subst}\left(\int\frac{(d+ex)^4}{x}\,dx,x,x^m\right)}{4em^2}$$

$$= \frac{x^{1-m}(fx)^{-1+m}\,(d+ex^m)^4\,(a+b\log(cx^n))}{4em} - \frac{\left(bnx^{1-m}(fx)^{-1+m}\right)\text{Subst}\left(\int\left(4d^3e+\frac{d^4}{x}+6d^2e^2x+4de^3x^2+e^4x^3\right)\,dx,x,x^m\right)}{4em^2}$$

$$= -\frac{bd^3nx(fx)^{-1+m}}{m^2} - \frac{3bd^2enx^{1+m}(fx)^{-1+m}}{4m^2} - \frac{bde^2nx^{1+2m}(fx)^{-1+m}}{3m^2} - \frac{be^3nx^{1+3m}(fx)^{-1+m}}{16m^2} - \frac{bd^4nx^{1-m}(fx)^{-1+m}\log(x)}{4em} + \frac{x^{1-m}(fx)^{-1+m}\,(d+ex^m)^4\,(a+b\log(cx^n))}{4em}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.82

$$\int (fx)^{-1+m}\,(d+ex^m)^3\,(a+b\log(cx^n))\,dx = \frac{(fx)^m\,(12am(4d^3+6d^2ex^m+4de^2x^{2m}+e^3x^{3m})-bn(48d^3+36d^2ex^m+16de^2x^{2m}+3e^3x^{3m})+12bm(4d^3}{48fm^2}$$

```
[In] Integrate[(f*x)^(-1 + m)*(d + e*x^m)^3*(a + b*Log[c*x^n]),x]
[Out] ((f*x)^m*(12*a*m*(4*d^3 + 6*d^2*e*x^m + 4*d*e^2*x^(2*m) + e^3*x^(3*m)) - b*
n*(48*d^3 + 36*d^2*e*x^m + 16*d*e^2*x^(2*m) + 3*e^3*x^(3*m)) + 12*b*m*(4*d^
3 + 6*d^2*e*x^m + 4*d*e^2*x^(2*m) + e^3*x^(3*m))*Log[c*x^n]))/(48*f*m^2)
```

Maple [A] (verified)

Time = 54.99 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.50

method	result
parallelrisch	$-\frac{-12e^3b(fx)^{m-1}\ln(cx^n)x\,x^{3m}m-12x\,x^{3m}(fx)^{m-1}a\,e^3m+3x\,x^{3m}(fx)^{m-1}b\,e^3n-48e^2db(fx)^{m-1}\ln(cx^n)x^{2m}xm-48x\,x^{2m}}{}$
risch	$\frac{b(e^3x^{3m}+4d\,e^2x^{2m}+6d^2e\,x^m+4d^3)\,x\,e^{\frac{(m-1)\left(-i\pi\,\text{csgn}(ifx)^3+i\pi\,\text{csgn}(ifx)^2\,\text{csgn}(if)+i\pi\,\text{csgn}(ifx)^2\,\text{csgn}(ix)-i\pi\,\text{csgn}(ifx)\,\text{csgn}(if)\,\text{csgn}}{2}}}{4m}$

```
[In] int((f*x)^(m-1)*(d+e*x^m)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(-12*e^3*b*(f*x)^(m-1)*ln(c*x^n)*x*(x^m)^3*m-12*x*(x^m)^3*(f*x)^(m-1)
*a*e^3*m+3*x*(x^m)^3*(f*x)^(m-1)*b*e^3*n-48*e^2*d*b*(f*x)^(m-1)*ln(c*x^n)*(
x^m)^2*x*m-48*x*(x^m)^2*(f*x)^(m-1)*a*d*e^2*m+16*x*(x^m)^2*(f*x)^(m-1)*b*d*
e^2*n-72*e*d^2*b*(f*x)^(m-1)*ln(c*x^n)*x^m*x*m-72*x*x^m*(f*x)^(m-1)*a*d^2*e
*m+36*x*x^m*(f*x)^(m-1)*b*d^2*e*n-48*b*d^3*(f*x)^(m-1)*ln(c*x^n)*x*m-48*x*(
f*x)^(m-1)*a*d^3*m+48*x*(f*x)^(m-1)*b*d^3*n)/m^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.13

$$\int (fx)^{-1+m} (d+ex^m)^3 (a+b\log(cx^n)) \, dx = \frac{3\left(4\,be^3mn\log(x)+4\,be^3m\log(c)+4\,ae^3m-be^3n\right)f^{m-1}x^{4\,m}+16\,(3\,bde^2mn\log(x)+3\,bde^2m\log(c)+\cdots}{\cdots}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(4*b*e^3*m*n*log(x) + 4*b*e^3*m*log(c) + 4*a*e^3*m - b*e^3*n)*f^(m
- 1)*x^(4*m) + 16*(3*b*d*e^2*m*n*log(x) + 3*b*d*e^2*m*log(c) + 3*a*d*e^2*m
- b*d*e^2*n)*f^(m - 1)*x^(3*m) + 36*(2*b*d^2*e*m*n*log(x) + 2*b*d^2*e*m*log
(c) + 2*a*d^2*e*m - b*d^2*e*n)*f^(m - 1)*x^(2*m) + 48*(b*d^3*m*n*log(x) + b
*d^3*m*log(c) + a*d^3*m - b*d^3*n)*f^(m - 1)*x^m)/m^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(160) = 320$.

Time = 8.48 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.95

$$\int (fx)^{-1+m} (d+ex^m)^3 (a+b\log(cx^n)) \, dx = \begin{cases} \frac{ad^3x(fx)^{m-1}}{m} + \frac{3ad^2exx^m(fx)^{m-1}}{2m} + \frac{ade^2xx^{2m}(fx)^{m-1}}{m} + \frac{ae^3xx^{3m}(fx)^{m-1}}{4m} + \frac{bd^3x(fx)^{m-1}\log(cx^n)}{m} - \frac{bd^3nx(fx)^{m-1}}{m^2} + \frac{3bd^2\cdots}{\cdots} \\ \frac{(d+e)^3\left(\begin{cases} a\log(x) & \text{for}\ b=0 \\ -(-a-b\log(c))\log(x) & \text{for}\ n=0 \\ \frac{(-a-b\log(cx^n))^2}{2bn} & \text{otherwise} \end{cases}\right)}{f} \end{cases}$$

```
[In] integrate((f*x)**(-1+m)*(d+e*x**m)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((a*d**3*x*(f*x)**(m - 1)/m + 3*a*d**2*e*x*x**m*(f*x)**(m - 1)/(2*
m) + a*d*e**2*x*x**(2*m)*(f*x)**(m - 1)/m + a*e**3*x*x**(3*m)*(f*x)**(m - 1
```

```
)/(4*m) + b*d**3*x*(f*x)**(m - 1)*log(c*x**n)/m - b*d**3*n*x*(f*x)**(m - 1)
/m**2 + 3*b*d**2*e*x*x**m*(f*x)**(m - 1)*log(c*x**n)/(2*m) - 3*b*d**2*e*n*x
*x**m*(f*x)**(m - 1)/(4*m**2) + b*d*e**2*x*x**(2*m)*(f*x)**(m - 1)*log(c*x*
*n)/m - b*d*e**2*n*x*x**(2*m)*(f*x)**(m - 1)/(3*m**2) + b*e**3*x*x**(3*m)*(
f*x)**(m - 1)*log(c*x**n)/(4*m) - b*e**3*n*x*x**(3*m)*(f*x)**(m - 1)/(16*m*
*2), Ne(m, 0)), ((d + e)**3*Piecewise((a*log(x), Eq(b, 0)), (-(-a - b*log(c
))*log(x), Eq(n, 0)), ((-a - b*log(c*x**n))**2/(2*b*n), True))/f, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.48

$$\int (fx)^{-1+m}(d+ex^m)^3(a+b\log(cx^n))\,dx = \frac{be^3f^{m-1}x^{4m}\log(cx^n)}{4m} + \frac{bde^2f^{m-1}x^{3m}\log(cx^n)}{m} + \frac{3bd^2ef^{m-1}x^{2m}\log(cx^n)}{2m} + \frac{ae^3f^{m-1}x^{4m}}{4m} - \frac{be^3f^{m-1}nx^{4m}}{16m^2} + \frac{ade^2f^{m-1}x^{3m}}{m} - \frac{bde^2f^{m-1}nx^{3m}}{3m^2} + \frac{3ad^2ef^{m-1}x^{2m}}{2m} - \frac{3bd^2ef^{m-1}nx^{2m}}{4m^2} - \frac{bd^3f^{m-1}nx^m}{m^2} + \frac{(fx)^m bd^3\log(cx^n)}{fm} + \frac{(fx)^m ad^3}{fm}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*e^3*f^(m - 1)*x^(4*m)*log(c*x^n)/m + b*d*e^2*f^(m - 1)*x^(3*m)*log(c*
x^n)/m + 3/2*b*d^2*e*f^(m - 1)*x^(2*m)*log(c*x^n)/m + 1/4*a*e^3*f^(m - 1)*x
^(4*m)/m - 1/16*b*e^3*f^(m - 1)*n*x^(4*m)/m^2 + a*d*e^2*f^(m - 1)*x^(3*m)/m
 - 1/3*b*d*e^2*f^(m - 1)*n*x^(3*m)/m^2 + 3/2*a*d^2*e*f^(m - 1)*x^(2*m)/m -
3/4*b*d^2*e*f^(m - 1)*n*x^(2*m)/m^2 - b*d^3*f^(m - 1)*n*x^m/m^2 + (f*x)^m*b
*d^3*log(c*x^n)/(f*m) + (f*x)^m*a*d^3/(f*m)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(161) = 322$.

Time = 0.43 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.98

$$\int (fx)^{-1+m}(d+ex^m)^3(a+b\log(cx^n))\,dx = \frac{be^3f^mnx^{4m}\log(x)}{4fm} + \frac{bde^2f^mnx^{3m}\log(x)}{fm} + \frac{3bd^2ef^mnx^{2m}\log(x)}{2fm} + \frac{bd^3f^mnx^m\log(x)}{fm} + \frac{be^3f^mx^{4m}\log(c)}{4fm} + \frac{bde^2f^mx^{3m}\log(c)}{fm} + \frac{3bd^2ef^mx^{2m}\log(c)}{2fm} + \frac{bd^3f^mx^m\log(c)}{fm} + \frac{ae^3f^mx^{4m}}{4fm} - \frac{be^3f^mnx^{4m}}{16fm^2} + \frac{ade^2f^mx^{3m}}{fm} - \frac{bde^2f^mnx^{3m}}{3fm^2} + \frac{3ad^2ef^mx^{2m}}{2fm} - \frac{3bd^2ef^mnx^{2m}}{4fm^2} + \frac{ad^3f^mx^m}{fm} - \frac{bd^3f^mnx^m}{fm^2}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*e^3*f^m*n*x^(4*m)*log(x)/(f*m) + b*d*e^2*f^m*n*x^(3*m)*log(x)/(f*m) +
 3/2*b*d^2*e*f^m*n*x^(2*m)*log(x)/(f*m) + b*d^3*f^m*n*x^m*log(x)/(f*m) + 1/
4*b*e^3*f^m*x^(4*m)*log(c)/(f*m) + b*d*e^2*f^m*x^(3*m)*log(c)/(f*m) + 3/2*b
*d^2*e*f^m*x^(2*m)*log(c)/(f*m) + b*d^3*f^m*x^m*log(c)/(f*m) + 1/4*a*e^3*f^
m*x^(4*m)/(f*m) - 1/16*b*e^3*f^m*n*x^(4*m)/(f*m^2) + a*d*e^2*f^m*x^(3*m)/(f
*m) - 1/3*b*d*e^2*f^m*n*x^(3*m)/(f*m^2) + 3/2*a*d^2*e*f^m*x^(2*m)/(f*m) - 3
/4*b*d^2*e*f^m*n*x^(2*m)/(f*m^2) + a*d^3*f^m*x^m/(f*m) - b*d^3*f^m*n*x^m/(f
*m^2)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+m}(d+ex^m)^3(a+b\log(cx^n))\,dx = \int (f\,x)^{m-1}\,(d+e\,x^m)^3\,(a+b\,\ln(c\,x^n))\,dx$$

```
[In] int((f*x)^(m - 1)*(d + e*x^m)^3*(a + b*log(c*x^n)),x)
```

```
[Out] int((f*x)^(m - 1)*(d + e*x^m)^3*(a + b*log(c*x^n)), x)
```

3.352 $\int (fx)^{-1+m} (d+ex^m)^2 (a+b\log (cx^n)) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 142

$$\int (fx)^{-1+m} (d+ex^m)^2 (a+b\log (cx^n)) \, dx = -\frac{bd^2nx(fx)^{-1+m}}{m^2} - \frac{bdenx^{1+m}(fx)^{-1+m}}{2m^2} - \frac{be^2nx^{1+2m}(fx)^{-1+m}}{9m^2} - \frac{bd^3nx^{1-m}(fx)^{-1+m}\log(x)}{3em} + \frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^3(a+b\log(cx^n))}{3em}$$

```
[Out] -b*d^2*n*x*(f*x)^(-1+m)/m^2-1/2*b*d*e*n*x^(1+m)*(f*x)^(-1+m)/m^2-1/9*b*e^2*
n*x^(1+2*m)*(f*x)^(-1+m)/m^2-1/3*b*d^3*n*x^(1-m)*(f*x)^(-1+m)*ln(x)/e/m+1/3
*x^(1-m)*(f*x)^(-1+m)*(d+e*x^m)^3*(a+b*ln(c*x^n))/e/m
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2377, 2376, 272, 45}

$$\int (fx)^{-1+m} (d+ex^m)^2 (a+b\log (cx^n)) \, dx = \frac{x^{1-m}(fx)^{m-1}(d+ex^m)^3(a+b\log(cx^n))}{3em} - \frac{bd^3nx^{1-m}\log(x)(fx)^{m-1}}{3em} - \frac{bd^2nx(fx)^{m-1}}{m^2} - \frac{bdenx^{m+1}(fx)^{m-1}}{2m^2} - \frac{be^2nx^{2m+1}(fx)^{m-1}}{9m^2}$$

```
[In] Int[(f*x)^(-1 + m)*(d + e*x^m)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -((b*d^2*n*x*(f*x)^(-1 + m))/m^2) - (b*d*e*n*x^(1 + m)*(f*x)^(-1 + m))/(2*m
^2) - (b*e^2*n*x^(1 + 2*m)*(f*x)^(-1 + m))/(9*m^2) - (b*d^3*n*x^(1 - m)*(f*
x)^(-1 + m)*Log[x])/(3*e*m) + (x^(1 - m)*(f*x)^(-1 + m)*(d + e*x^m)^3*(a +
b*Log[c*x^n]))/(3*e*m)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rule 2377

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(d + e*x^r)^q
*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] &&  !(IntegerQ[m] || GtQ[f, 0])
```

Rubi steps

$$\text{integral} = \left(x^{1-m}(fx)^{-1+m}\right)\int x^{-1+m}(d+ex^m)^2\,(a+b\log(cx^n))\,dx$$

$$= \frac{x^{1-m}(fx)^{-1+m}\,(d+ex^m)^3\,(a+b\log(cx^n))}{3em} - \frac{\left(bnx^{1-m}(fx)^{-1+m}\right)\int\frac{(d+ex^m)^3}{x}\,dx}{3em}$$

$$= \frac{x^{1-m}(fx)^{-1+m}\,(d+ex^m)^3\,(a+b\log(cx^n))}{3em} - \frac{\left(bnx^{1-m}(fx)^{-1+m}\right)\text{Subst}\left(\int\frac{(d+ex)^3}{x}\,dx,x,x^m\right)}{3em^2}$$

$$= \frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^3(a+b\log(cx^n))}{3em} - \frac{(bnx^{1-m}(fx)^{-1+m})\,\text{Subst}\left(\int\left(3d^2e+\frac{d^3}{x}+3de^2x+e^3x^2\right)dx,x,x^m\right)}{3em^2}$$

$$= -\frac{bd^2nx(fx)^{-1+m}}{m^2} - \frac{bdenx^{1+m}(fx)^{-1+m}}{2m^2} - \frac{be^2nx^{1+2m}(fx)^{-1+m}}{9m^2} - \frac{bd^3nx^{1-m}(fx)^{-1+m}\log(x)}{3em} + \frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^3(a+b\log(cx^n))}{3em}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.71

$$\int (fx)^{-1+m}(d+ex^m)^2(a+b\log(cx^n))\,dx = \frac{(fx)^m\left(6am(3d^2+3dex^m+e^2x^{2m})-bn(18d^2+9dex^m+2e^2x^{2m})+6bm(3d^2+3dex^m+e^2x^{2m})\log(cx^n)\right)}{18fm^2}$$

```
[In] Integrate[(f*x)^(-1 + m)*(d + e*x^m)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] ((f*x)^m*(6*a*m*(3*d^2 + 3*d*e*x^m + e^2*x^(2*m)) - b*n*(18*d^2 + 9*d*e*x^m
 + 2*e^2*x^(2*m)) + 6*b*m*(3*d^2 + 3*d*e*x^m + e^2*x^(2*m))*Log[c*x^n]))/(1
8*f*m^2)
```

Maple [A] (verified)

Time = 12.18 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.27

method	result
parallelrisch	$-\frac{-6e^2b(fx)^{m-1}\ln(c\,x^n)x^{2m}xm-6x\,x^{2m}(fx)^{m-1}a\,e^2m+2x\,x^{2m}(fx)^{m-1}b\,e^2n-18bde(fx)^{m-1}\ln(c\,x^n)x^mxm-18x\,x^m(fx)^{m}}{18m^2}$
risch	$\frac{b(e^2x^{2m}+3de\,x^m+3d^2)x\,\mathrm{e}^{\frac{(m-1)\left(-i\pi\,\mathrm{csgn}(ifx)^3+i\pi\,\mathrm{csgn}(ifx)^2\,\mathrm{csgn}(if)+i\pi\,\mathrm{csgn}(ifx)^2\,\mathrm{csgn}(ix)-i\pi\,\mathrm{csgn}(ifx)\,\mathrm{csgn}(if)\,\mathrm{csgn}(ix)+2\ln(x)+}{2}}}{3m}$

```
[In] int((f*x)^(m-1)*(d+e*x^m)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*(-6*e^2*b*(f*x)^(m-1)*ln(c*x^n)*(x^m)^2*x*m-6*x*(x^m)^2*(f*x)^(m-1)*a
*e^2*m+2*x*(x^m)^2*(f*x)^(m-1)*b*e^2*n-18*b*d*e*(f*x)^(m-1)*ln(c*x^n)*x^m*x
*m-18*x*x^m*(f*x)^(m-1)*a*d*e*m+9*x*x^m*(f*x)^(m-1)*b*d*e*n-18*b*d^2*(f*x)^
(m-1)*ln(c*x^n)*x*m-18*x*(f*x)^(m-1)*a*d^2*m+18*x*(f*x)^(m-1)*b*d^2*n)/m^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.95

$$\int (fx)^{-1+m} (d+ex^m)^2 (a+b\log(cx^n))\,dx$$
$$= \frac{2\,(3\,be^2mn\log(x)+3\,be^2m\log(c)+3\,ae^2m-be^2n)f^{m-1}x^{3\,m}+9\,(2\,bdemn\log(x)+2\,bdem\log(c)+2\,ad}{18\,m^2}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 1/18*(2*(3*b*e^2*m*n*log(x) + 3*b*e^2*m*log(c) + 3*a*e^2*m - b*e^2*n)*f^(m
- 1)*x^(3*m) + 9*(2*b*d*e*m*n*log(x) + 2*b*d*e*m*log(c) + 2*a*d*e*m - b*d*e
*n)*f^(m - 1)*x^(2*m) + 18*(b*d^2*m*n*log(x) + b*d^2*m*log(c) + a*d^2*m - b
*d^2*n)*f^(m - 1)*x^m)/m^2
```

Sympy [A] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.70

$$\int (fx)^{-1+m} (d+ex^m)^2 (a+b\log(cx^n))\,dx$$
$$= \begin{cases} \frac{ad^2x(fx)^{m-1}}{m} + \frac{adexx^m(fx)^{m-1}}{m} + \frac{ae^2xx^{2m}(fx)^{m-1}}{3m} + \frac{bd^2x(fx)^{m-1}\log(cx^n)}{m} - \frac{bd^2nx(fx)^{m-1}}{m^2} + \frac{bdexx^m(fx)^{m-1}\log(cx^n)}{m} - \\ \frac{(d+e)^2\left(\begin{cases} a\log(x) & \text{for } b=0 \\ -(-a-b\log(c))\log(x) & \text{for } n=0 \\ \frac{(-a-b\log(cx^n))^2}{2bn} & \text{otherwise} \end{cases}\right)}{f} \end{cases}$$

```
[In] integrate((f*x)**(-1+m)*(d+e*x**m)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((a*d**2*x*(f*x)**(m - 1)/m + a*d*e*x*x**m*(f*x)**(m - 1)/m + a*e*
*2*x*x**(2*m)*(f*x)**(m - 1)/(3*m) + b*d**2*x*(f*x)**(m - 1)*log(c*x**n)/m
- b*d**2*n*x*(f*x)**(m - 1)/m**2 + b*d*e*x*x**m*(f*x)**(m - 1)*log(c*x**n)/
m - b*d*e*n*x*x**m*(f*x)**(m - 1)/(2*m**2) + b*e**2*x*x**(2*m)*(f*x)**(m -
1)*log(c*x**n)/(3*m) - b*e**2*n*x*x**(2*m)*(f*x)**(m - 1)/(9*m**2), Ne(m, 0
)), ((d + e)**2*Piecewise((a*log(x), Eq(b, 0)), (-(-a - b*log(c))*log(x), E
q(n, 0)), ((-a - b*log(c*x**n))**2/(2*b*n), True))/f, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.27

$$\int (fx)^{-1+m}(d+ex^m)^2(a+b\log(cx^n))\,dx = \frac{be^2f^{m-1}x^{3m}\log(cx^n)}{3m} + \frac{bdef^{m-1}x^{2m}\log(cx^n)}{m} + \frac{ae^2f^{m-1}x^{3m}}{3m} - \frac{be^2f^{m-1}nx^{3m}}{9m^2} + \frac{adef^{m-1}x^{2m}}{m} - \frac{bdef^{m-1}nx^{2m}}{2m^2} - \frac{bd^2f^{m-1}nx^m}{m^2} + \frac{(fx)^m bd^2\log(cx^n)}{fm} + \frac{(fx)^m ad^2}{fm}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*e^2*f^(m - 1)*x^(3*m)*log(c*x^n)/m + b*d*e*f^(m - 1)*x^(2*m)*log(c*x^
n)/m + 1/3*a*e^2*f^(m - 1)*x^(3*m)/m - 1/9*b*e^2*f^(m - 1)*n*x^(3*m)/m^2 +
a*d*e*f^(m - 1)*x^(2*m)/m - 1/2*b*d*e*f^(m - 1)*n*x^(2*m)/m^2 - b*d^2*f^(m
- 1)*n*x^m/m^2 + (f*x)^m*b*d^2*log(c*x^n)/(f*m) + (f*x)^m*a*d^2/(f*m)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.70

$$\int (fx)^{-1+m}(d+ex^m)^2(a+b\log(cx^n))\,dx = \frac{be^2f^mnx^{3m}\log(x)}{3fm} + \frac{bdef^mnx^{2m}\log(x)}{fm} + \frac{bd^2f^mnx^m\log(x)}{fm} + \frac{be^2f^mx^{3m}\log(c)}{3fm} + \frac{bdef^mx^{2m}\log(c)}{fm} + \frac{bd^2f^mx^m\log(c)}{fm} + \frac{ae^2f^mx^{3m}}{3fm} - \frac{be^2f^mnx^{3m}}{9fm^2} + \frac{adef^mx^{2m}}{fm} - \frac{bdef^mnx^{2m}}{2fm^2} + \frac{ad^2f^mx^m}{fm} - \frac{bd^2f^mnx^m}{fm^2}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*e^2*f^m*n*x^(3*m)*log(x)/(f*m) + b*d*e*f^m*n*x^(2*m)*log(x)/(f*m) + b
*d^2*f^m*n*x^m*log(x)/(f*m) + 1/3*b*e^2*f^m*x^(3*m)*log(c)/(f*m) + b*d*e*f^
m*x^(2*m)*log(c)/(f*m) + b*d^2*f^m*x^m*log(c)/(f*m) + 1/3*a*e^2*f^m*x^(3*m)
/(f*m) - 1/9*b*e^2*f^m*n*x^(3*m)/(f*m^2) + a*d*e*f^m*x^(2*m)/(f*m) - 1/2*b*
d*e*f^m*n*x^(2*m)/(f*m^2) + a*d^2*f^m*x^m/(f*m) - b*d^2*f^m*n*x^m/(f*m^2)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+m} (d+ex^m)^2 (a+b\log(cx^n)) \, dx = \int (f\,x)^{m-1} (d+e\,x^m)^2 (a+b\,\ln(c\,x^n)) \, dx$$

```
[In] int((f*x)^(m - 1)*(d + e*x^m)^2*(a + b*log(c*x^n)),x)
[Out] int((f*x)^(m - 1)*(d + e*x^m)^2*(a + b*log(c*x^n)), x)
```

3.353 $\int (fx)^{-1+m} (d+ex^m) (a+b\log(cx^n)) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 90

$$\int (fx)^{-1+m} (d+ex^m) (a+b\log(cx^n)) \, dx = -\frac{bdn(fx)^m}{fm^2} - \frac{benx^m(fx)^m}{4fm^2} + \frac{d(fx)^m (a+b\log(cx^n))}{fm} + \frac{ex^m(fx)^m (a+b\log(cx^n))}{2fm}$$

```
[Out] -b*d*n*(f*x)^m/f/m^2-1/4*b*e*n*x^m*(f*x)^m/f/m^2+d*(f*x)^m*(a+b*ln(c*x^n))/
f/m+1/2*e*x^m*(f*x)^m*(a+b*ln(c*x^n))/f/m
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.26, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2377, 2376, 272, 45}

$$\int (fx)^{-1+m} (d+ex^m) (a+b\log(cx^n)) \, dx = \frac{x^{1-m}(fx)^{m-1} (d+ex^m)^2 (a+b\log(cx^n))}{2em} - \frac{bd^2nx^{1-m}\log(x)(fx)^{m-1}}{2em} - \frac{bdnx(fx)^{m-1}}{m^2} - \frac{benx^{m+1}(fx)^{m-1}}{4m^2}$$

```
[In] Int[(f*x)^(-1 + m)*(d + e*x^m)*(a + b*Log[c*x^n]),x]
```

```
[Out] -((b*d*n*x*(f*x)^(-1 + m))/m^2) - (b*e*n*x^(1 + m)*(f*x)^(-1 + m))/(4*m^2)
- (b*d^2*n*x^(1 - m)*(f*x)^(-1 + m)*Log[x])/(2*e*m) + (x^(1 - m)*(f*x)^(-1
+ m)*(d + e*x^m)^2*(a + b*Log[c*x^n]))/(2*e*m)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rule 2377

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(d + e*x^r)^q
*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] &&  !(IntegerQ[m] || GtQ[f, 0])
```

Rubi steps

$$\text{integral} = \left(x^{1-m}(fx)^{-1+m}\right)\int x^{-1+m}(d+ex^m)\left(a+b\log\left(cx^n\right)\right)\,dx$$

$$=\frac{x^{1-m}(fx)^{-1+m}\left(d+ex^m\right)^2\left(a+b\log\left(cx^n\right)\right)}{2em}-\frac{\left(bnx^{1-m}(fx)^{-1+m}\right)\int\frac{(d+ex^m)^2}{x}\,dx}{2em}$$

$$=\frac{x^{1-m}(fx)^{-1+m}\left(d+ex^m\right)^2\left(a+b\log\left(cx^n\right)\right)}{2em}-\frac{\left(bnx^{1-m}(fx)^{-1+m}\right)\text{Subst}\left(\int\frac{(d+ex)^2}{x}\,dx,x,x^m\right)}{2em^2}$$

$$=\frac{x^{1-m}(fx)^{-1+m}\left(d+ex^m\right)^2\left(a+b\log\left(cx^n\right)\right)}{2em}-\frac{\left(bnx^{1-m}(fx)^{-1+m}\right)\text{Subst}\left(\int\left(2de+\frac{d^2}{x}+e^2x\right)\,dx,x,x^m\right)}{2em^2}$$

$$= -\frac{bdnx(fx)^{-1+m}}{m^2} - \frac{benx^{1+m}(fx)^{-1+m}}{4m^2} - \frac{bd^2nx^{1-m}(fx)^{-1+m}\log(x)}{2em} + \frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^2(a+b\log(cx^n))}{2em}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.68

$$\int (fx)^{-1+m}(d+ex^m)(a+b\log(cx^n))\,dx = \frac{(fx)^m(2am(2d+ex^m)-bn(4d+ex^m)+2bm(2d+ex^m)\log(cx^n))}{4fm^2}$$

```
[In] Integrate[(f*x)^(-1 + m)*(d + e*x^m)*(a + b*Log[c*x^n]),x]
```

```
[Out] ((f*x)^m*(2*a*m*(2*d + e*x^m) - b*n*(4*d + e*x^m) + 2*b*m*(2*d + e*x^m)*Log
[c*x^n]))/(4*f*m^2)
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

method	result
parallelrisch	$-\frac{-2x\,x^m\ln(c\,x^n)(fx)^{m-1}bem-2x\,x^m(fx)^{m-1}aem+x\,x^m(fx)^{m-1}ben-4x\ln(c\,x^n)(fx)^{m-1}bdm-4x(fx)^{m-1}adm+4x(fx)^{m}}{4m^2}$
risch	$\frac{b(e\,x^m+2d)x\,\mathrm{e}^{\frac{(m-1)\left(-i\pi\,\mathrm{csgn}(ifx)^3+i\pi\,\mathrm{csgn}(ifx)^2\,\mathrm{csgn}(if)+i\pi\,\mathrm{csgn}(ifx)^2\,\mathrm{csgn}(ix)-i\pi\,\mathrm{csgn}(ifx)\,\mathrm{csgn}(if)\,\mathrm{csgn}(ix)+2\ln(x)+2\ln(f)\right)}{2}}\ln(x}{2m}$

```
[In] int((f*x)^(m-1)*(d+e*x^m)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-2*x*x^m*ln(c*x^n)*(f*x)^(m-1)*b*e*m-2*x*x^m*(f*x)^(m-1)*a*e*m+x*x^m*
(f*x)^(m-1)*b*e*n-4*x*ln(c*x^n)*(f*x)^(m-1)*b*d*m-4*x*(f*x)^(m-1)*a*d*m+4*x
*(f*x)^(m-1)*b*d*n)/m^2
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int (fx)^{-1+m}(d+ex^m)(a+b\log(cx^n))\,dx$$
$$= \frac{(2\,bemn\log(x)+2\,bem\log(c)+2\,aem-ben)f^{m-1}x^{2\,m}+4\,(bdmn\log(x)+bdm\log(c)+adm-bdn)f^{m}}{4\,m^2}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b*e*m*n*log(x) + 2*b*e*m*log(c) + 2*a*e*m - b*e*n)*f^(m - 1)*x^(2*m
) + 4*(b*d*m*n*log(x) + b*d*m*log(c) + a*d*m - b*d*n)*f^(m - 1)*x^m)/m^2
```

Sympy [A] (verification not implemented)

Time = 3.01 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.73

$$\int (fx)^{-1+m}(d+ex^m)(a+b\log(cx^n))\,dx$$
$$= \begin{cases} \frac{adx(fx)^{m-1}}{m}+\frac{aexx^m(fx)^{m-1}}{2m}+\frac{bdx(fx)^{m-1}\log(cx^n)}{m}-\frac{bdnx(fx)^{m-1}}{m^2}+\frac{bexx^m(fx)^{m-1}\log(cx^n)}{2m}-\frac{benxx^m(fx)^{m-1}}{4m^2} & \text{for } m \\ \frac{(d+e)\left(\begin{cases} a\log(x) & \text{for } b=0 \\ -(-a-b\log(c))\log(x) & \text{for } n=0 \\ \frac{(-a-b\log(cx^n))^2}{2bn} & \text{otherwise} \end{cases}\right)}{f} & \text{other} \end{cases}$$

```
[In] integrate((f*x)**(-1+m)*(d+e*x**m)*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((a*d*x*(f*x)**(m - 1)/m + a*e*x*x**m*(f*x)**(m - 1)/(2*m) + b*d*x
*(f*x)**(m - 1)*log(c*x**n)/m - b*d*n*x*(f*x)**(m - 1)/m**2 + b*e*x*x**m*(f
*x)**(m - 1)*log(c*x**n)/(2*m) - b*e*n*x*x**m*(f*x)**(m - 1)/(4*m**2), Ne(m
, 0)), ((d + e)*Piecewise((a*log(x), Eq(b, 0)), (-(-a - b*log(c))*log(x), E
q(n, 0)), ((-a - b*log(c*x**n))**2/(2*b*n), True))/f, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.21

$$\int (fx)^{-1+m} (d+ex^m) (a+b\log (cx^n)) \, dx = \frac{bef^{m-1}x^{2\,m}\log (cx^n)}{2\,m} + \frac{aef^{m-1}x^{2\,m}}{2\,m} - \frac{bef^{m-1}nx^{2\,m}}{4\,m^2} - \frac{bdf^{m-1}nx^m}{m^2} + \frac{(fx)^m\,bd\log (cx^n)}{fm} + \frac{(fx)^m\,ad}{fm}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*b*e*f^(m - 1)*x^(2*m)*log(c*x^n)/m + 1/2*a*e*f^(m - 1)*x^(2*m)/m - 1/4*
b*e*f^(m - 1)*n*x^(2*m)/m^2 - b*d*f^(m - 1)*n*x^m/m^2 + (f*x)^m*b*d*log(c*x
^n)/(f*m) + (f*x)^m*a*d/(f*m)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.62

$$\int (fx)^{-1+m} (d+ex^m) (a+b\log (cx^n)) \, dx = \frac{bef^mnx^{2\,m}\log (x)}{2\,fm} + \frac{bdf^mnx^m\log (x)}{fm} + \frac{bef^mx^{2\,m}\log (c)}{2\,fm} + \frac{bdf^mx^m\log (c)}{fm} + \frac{aef^mx^{2\,m}}{2\,fm} - \frac{bef^mnx^{2\,m}}{4\,fm^2} + \frac{adf^mx^m}{fm} - \frac{bdf^mnx^m}{fm^2}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*e*f^m*n*x^(2*m)*log(x)/(f*m) + b*d*f^m*n*x^m*log(x)/(f*m) + 1/2*b*e*f
^m*x^(2*m)*log(c)/(f*m) + b*d*f^m*x^m*log(c)/(f*m) + 1/2*a*e*f^m*x^(2*m)/(f
*m) - 1/4*b*e*f^m*n*x^(2*m)/(f*m^2) + a*d*f^m*x^m/(f*m) - b*d*f^m*n*x^m/(f*
m^2)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+m}(d+ex^m)(a+b\log(cx^n))\,dx = \int (f\,x)^{m-1}\,(d+e\,x^m)\,(a+b\,\ln(c\,x^n))\,dx$$

```
[In] int((f*x)^(m - 1)*(d + e*x^m)*(a + b*log(c*x^n)),x)
[Out] int((f*x)^(m - 1)*(d + e*x^m)*(a + b*log(c*x^n)), x)
```

3.354 $\int (fx)^{-1+m} (a + b\log(cx^n))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 38

$$\int (fx)^{-1+m} (a + b\log(cx^n))\,dx = -\frac{bn(fx)^m}{fm^2} + \frac{(fx)^m (a + b\log(cx^n))}{fm}$$

```
[Out] -b*n*(f*x)^m/f/m^2+(f*x)^m*(a+b*ln(c*x^n))/f/m
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2341}

$$\int (fx)^{-1+m} (a + b\log(cx^n))\,dx = \frac{(fx)^m (a + b\log(cx^n))}{fm} - \frac{bn(fx)^m}{fm^2}$$

```
[In] Int[(f*x)^(-1 + m)*(a + b*Log[c*x^n]),x]
```

```
[Out] -((b*n*(f*x)^m)/(f*m^2)) + ((f*x)^m*(a + b*Log[c*x^n]))/(f*m)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{bn(fx)^m}{fm^2} + \frac{(fx)^m (a + b\log(cx^n))}{fm}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int (fx)^{-1+m}(a+b\log(cx^n))\,dx=\frac{(fx)^m(am-bn+bm\log(cx^n))}{fm^2}$$

```
[In] Integrate[(f*x)^(-1 + m)*(a + b*Log[c*x^n]),x]
[Out] ((f*x)^m*(a*m - b*n + b*m*Log[c*x^n]))/(f*m^2)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

method	result
parallelrisch	$-\frac{-x\ln(c\,x^n)(fx)^{m-1}bm-x(fx)^{m-1}am+x(fx)^{m-1}bn}{m^2}$
risch	$\frac{bx\,\mathrm{e}^{\frac{(m-1)\left(-i\pi\,\mathrm{csgn}(ifx)^3+i\pi\,\mathrm{csgn}(ifx)^2\,\mathrm{csgn}(if)+i\pi\,\mathrm{csgn}(ifx)^2\,\mathrm{csgn}(ix)-i\pi\,\mathrm{csgn}(ifx)\,\mathrm{csgn}(if)\,\mathrm{csgn}(ix)+2\ln(x)+2\ln(f)\right)}{2}}}{m}\ln(x^n)+$

```
[In] int((f*x)^(m-1)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] -(-x*ln(c*x^n)*(f*x)^(m-1)*b*m-x*(f*x)^(m-1)*a*m+x*(f*x)^(m-1)*b*n)/m^2
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int (fx)^{-1+m}(a+b\log(cx^n))\,dx$$
$$=\frac{(bmnx\log(x)+bmx\log(c)+(am-bn)x)e^{((m-1)\log(f)+(m-1)\log(x))}}{m^2}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] (b*m*n*x*log(x) + b*m*x*log(c) + (a*m - b*n)*x)*e^((m - 1)*log(f) + (m - 1)
*log(x))/m^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(31) = 62$.

Time = 2.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int (fx)^{-1+m}\left(a+b\log\left(cx^{n}\right)\right)\,dx=\begin{cases}\frac{ax(fx)^{m-1}}{m}+\frac{bx(fx)^{m-1}\log\left(cx^{n}\right)}{m}-\frac{bnx(fx)^{m-1}}{m^{2}} & \text{for}\ m\neq 0\\ \frac{\begin{cases}a\log\left(x\right) & \text{for}\ b=0\\ -\left(-a-b\log\left(c\right)\right)\log\left(x\right) & \text{for}\ n=0\\ \frac{\left(-a-b\log\left(cx^{n}\right)\right)^{2}}{2bn} & \text{otherwise}\end{cases}}{f} & \text{otherwise}\end{cases}$$

```
[In] integrate((f*x)**(-1+m)*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((a*x*(f*x)**(m - 1)/m + b*x*(f*x)**(m - 1)*log(c*x**n)/m - b*n*x*
(f*x)**(m - 1)/m**2, Ne(m, 0)), (Piecewise((a*log(x), Eq(b, 0)), (-(-a - b*
log(c))*log(x), Eq(n, 0)), ((-a - b*log(c*x**n))**2/(2*b*n), True))/f, True
))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int (fx)^{-1+m}\left(a+b\log\left(cx^{n}\right)\right)\,dx=-\frac{bf^{m-1}nx^{m}}{m^{2}}+\frac{(fx)^{m}\,b\log\left(cx^{n}\right)}{fm}+\frac{(fx)^{m}\,a}{fm}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -b*f^(m - 1)*n*x^m/m^2 + (f*x)^m*b*log(c*x^n)/(f*m) + (f*x)^m*a/(f*m)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.68

$$\int (fx)^{-1+m}\left(a+b\log\left(cx^{n}\right)\right)\,dx=\frac{bf^{m}nx^{m}\log\left(x\right)}{fm}+\frac{bf^{m}x^{m}\log\left(c\right)}{fm}+\frac{af^{m}x^{m}}{fm}-\frac{bf^{m}nx^{m}}{fm^{2}}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*f^m*n*x^m*log(x)/(f*m) + b*f^m*x^m*log(c)/(f*m) + a*f^m*x^m/(f*m) - b*f^m
*n*x^m/(f*m^2)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+m}\,(a+b\log(cx^n))\;dx=\int (f\,x)^{m-1}\,(a+b\,\ln(c\,x^n))\;dx$$

```
[In] int((f*x)^(m - 1)*(a + b*log(c*x^n)),x)
[Out] int((f*x)^(m - 1)*(a + b*log(c*x^n)), x)
```

3.355 $\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{d+ex^m}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 77

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{d+ex^m}\,dx = \frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))\log\left(1+\frac{ex^m}{d}\right)}{em} + \frac{bnx^{1-m}(fx)^{-1+m}\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{em^2}$$

```
[Out] x^(1-m)*(f*x)^(-1+m)*(a+b*ln(c*x^n))*ln(1+e*x^m/d)/e/m+b*n*x^(1-m)*(f*x)^(-
1+m)*polylog(2,-e*x^m/d)/e/m^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2377, 2375, 2438}

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{d+ex^m}\,dx = \frac{x^{1-m}(fx)^{m-1}\log\left(\frac{ex^m}{d}+1\right)(a+b\log(cx^n))}{em} + \frac{bnx^{1-m}(fx)^{m-1}\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{em^2}$$

```
[In] Int[((f*x)^(-1 + m)*(a + b*Log[c*x^n]))/(d + e*x^m),x]
```

```
[Out] (x^(1 - m)*(f*x)^(-1 + m)*(a + b*Log[c*x^n])*Log[1 + (e*x^m)/d])/(e*m) + (b
*n*x^(1 - m)*(f*x)^(-1 + m)*PolyLog[2, -((e*x^m)/d)])/(e*m^2)
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
```

```
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2377

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(d + e*x^r)^q
*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] &&  !(IntegerQ[m] || GtQ[f, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \left(x^{1-m}(fx)^{-1+m}\right)\int \frac{x^{-1+m}(a+b\log(cx^n))}{d+ex^m}\,dx$$

$$= \frac{x^{1-m}(fx)^{-1+m}\left(a+b\log(cx^n)\right)\log\left(1+\frac{ex^m}{d}\right)}{em} - \frac{\left(bnx^{1-m}(fx)^{-1+m}\right)\int \frac{\log\left(1+\frac{ex^m}{d}\right)}{x}\,dx}{em}$$

$$= \frac{x^{1-m}(fx)^{-1+m}\left(a+b\log(cx^n)\right)\log\left(1+\frac{ex^m}{d}\right)}{em} + \frac{bnx^{1-m}(fx)^{-1+m}\text{Li}_2\left(-\frac{ex^m}{d}\right)}{em^2}$$

Mathematica [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.83

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{d+ex^m}\,dx$$
$$= \frac{x^{-m}(fx)^m\left(-bm^2n\log^2(x)+am\log(d-dx^m)+bm\log(cx^n)\log(d-dx^m)-bn\log\left(-\frac{ex^m}{d}\right)\log(d+ex\right.}{efm}$$

```
[In] Integrate[((f*x)^(-1 + m)*(a + b*Log[c*x^n]))/(d + e*x^m),x]
```

```
[Out] ((f*x)^m*(-(b*m^2*n*Log[x]^2) + a*m*Log[d - d*x^m] + b*m*Log[c*x^n]*Log[d -
 d*x^m] - b*n*Log[-((e*x^m)/d)]*Log[d + e*x^m] + m*Log[x]*(a*m + b*m*Log[c*
x^n] - b*n*Log[d - d*x^m] + b*n*Log[d + e*x^m]) - b*n*PolyLog[2, 1 + (e*x^m
)/d]))/(e*f*m^2*x^m)
```

Maple [F]

$$\int \frac{(fx)^{m-1} (a+b\ln(c\,x^n))}{d+e\,x^m}dx$$

```
[In] int((f*x)^(m-1)*(a+b*ln(c*x^n))/(d+e*x^m),x)
```

```
[Out] int((f*x)^(m-1)*(a+b*ln(c*x^n))/(d+e*x^m),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^{-1+m} (a+b\log(cx^n))}{d+ex^m}\,dx = \frac{bf^{m-1}mn\log(x)\log\left(\frac{ex^m+d}{d}\right)+bf^{m-1}n\mathrm{Li}_2\left(-\frac{ex^m+d}{d}+1\right)+(bm\log(c)+am)f^{m-1}\log(ex^m+d)}{em^2}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))/(d+e*x^m),x, algorithm="fricas")
```

```
[Out] (b*f^(m - 1)*m*n*log(x)*log((e*x^m + d)/d) + b*f^(m - 1)*n*dilog(-(e*x^m +
d)/d + 1) + (b*m*log(c) + a*m)*f^(m - 1)*log(e*x^m + d))/(e*m^2)
```

Sympy [F]

$$\int \frac{(fx)^{-1+m} (a+b\log(cx^n))}{d+ex^m}\,dx = \int \frac{(fx)^{m-1} (a+b\log(cx^n))}{d+ex^m}\,dx$$

```
[In] integrate((f*x)**(-1+m)*(a+b*ln(c*x**n))/(d+e*x**m),x)
```

```
[Out] Integral((f*x)**(m - 1)*(a + b*log(c*x**n))/(d + e*x**m), x)
```

Maxima [F]

$$\int \frac{(fx)^{-1+m} (a+b\log(cx^n))}{d+ex^m}\,dx = \int \frac{(b\log(cx^n)+a)(fx)^{m-1}}{ex^m+d}\,dx$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))/(d+e*x^m),x, algorithm="maxima")
```

```
[Out] b*integrate((f^m*x^m*log(c) + f^m*x^m*log(x^n))/(e*f*x*x^m + d*f*x), x) + a
*f^(m - 1)*log((e*x^m + d)/e)/(e*m)
```

Giac [F]

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{d+ex^m}\,dx = \int \frac{(b\log(cx^n)+a)(fx)^{m-1}}{ex^m+d}\,dx$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))/(d+e*x^m),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*(f*x)^(m - 1)/(e*x^m + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{d+ex^m}\,dx = \int \frac{(f\,x)^{m-1}\,(a+b\,\ln(c\,x^n))}{d+e\,x^m}\,dx$$

```
[In] int(((f*x)^(m - 1)*(a + b*log(c*x^n)))/(d + e*x^m),x)
[Out] int(((f*x)^(m - 1)*(a + b*log(c*x^n)))/(d + e*x^m), x)
```

3.356 $\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 69

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^2}\,dx=\frac{(fx)^m(a+b\log(cx^n))}{dfm(d+ex^m)}-\frac{bnx^{-m}(fx)^m\log(d+ex^m)}{defm^2}$$

```
[Out] (f*x)^m*(a+b*ln(c*x^n))/d/f/m/(d+e*x^m)-b*n*(f*x)^m*ln(d+e*x^m)/d/e/f/m^2/(
x^m)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2373, 274, 266}

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^2}\,dx=\frac{(fx)^m(a+b\log(cx^n))}{dfm(d+ex^m)}-\frac{bnx^{-m}(fx)^m\log(d+ex^m)}{defm^2}$$

```
[In] Int[((f*x)^(-1 + m)*(a + b*Log[c*x^n]))/(d + e*x^m)^2,x]
```

```
[Out] ((f*x)^m*(a + b*Log[c*x^n]))/(d*f*m*(d + e*x^m)) - (b*n*(f*x)^m*Log[d + e*x
^m])/(d*e*f*m^2*x^m)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 274

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a + b*x^n)^p, x], x] /;
```

```
FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(fx)^m (a+b\log(cx^n))}{dfm(d+ex^m)} - \frac{(bn)\int \frac{(fx)^{-1+m}}{d+ex^m}\,dx}{dm} \\
&= \frac{(fx)^m (a+b\log(cx^n))}{dfm(d+ex^m)} - \frac{(bnx^{-m}(fx)^m)\int \frac{x^{-1+m}}{d+ex^m}\,dx}{dfm} \\
&= \frac{(fx)^m (a+b\log(cx^n))}{dfm(d+ex^m)} - \frac{bnx^{-m}(fx)^m\log(d+ex^m)}{defm^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^2}\,dx = -\frac{x^{-m}(fx)^m (adm - bmn(d+ex^m)\log(x) + bdm\log(cx^n) + bdn\log(d+ex^m) + benx^m\log(d+ex^m))}{defm^2(d+ex^m)}$$

```
[In] Integrate[((f*x)^(-1 + m)*(a + b*Log[c*x^n]))/(d + e*x^m)^2,x]
```

```
[Out] -(((f*x)^m*(a*d*m - b*m*n*(d + e*x^m)*Log[x] + b*d*m*Log[c*x^n] + b*d*n*Log
[d + e*x^m] + b*e*n*x^m*Log[d + e*x^m]))/(d*e*f*m^2*x^m*(d + e*x^m)))
```

Maple [F]

$$\int \frac{(fx)^{m-1}(a+b\ln(c\,x^n))}{(d+e\,x^m)^2}dx$$

```
[In] int((f*x)^(m-1)*(a+b*ln(c*x^n))/(d+e*x^m)^2,x)
```

```
[Out] int((f*x)^(m-1)*(a+b*ln(c*x^n))/(d+e*x^m)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^2}\,dx = \frac{bef^{m-1}mnx^m\log(x)-(bdm\log(c)+adm)f^{m-1}-(bef^{m-1}nx^m+bdf^{m-1}n)\log(ex^m+d)}{de^2m^2x^m+d^2em^2}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))/(d+e*x^m)^2,x, algorithm="fricas")
```

```
[Out] (b*e*f^(m - 1)*m*n*x^m*log(x) - (b*d*m*log(c) + a*d*m)*f^(m - 1) - (b*e*f^(
m - 1)*n*x^m + b*d*f^(m - 1)*n)*log(e*x^m + d))/(d*e^2*m^2*x^m + d^2*e*m^2)
```

Sympy [F]

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^2}\,dx = \int \frac{(fx)^{m-1}(a+b\log(cx^n))}{(d+ex^m)^2}\,dx$$

```
[In] integrate((f*x)**(-1+m)*(a+b*ln(c*x**n))/(d+e*x**m)**2,x)
```

```
[Out] Integral((f*x)**(m - 1)*(a + b*log(c*x**n))/(d + e*x**m)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.41

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^2}\,dx = bf^mn\left(\frac{\log(x)}{defm}-\frac{\log(ex^m+d)}{defm^2}\right) - \frac{bf^m\log(cx^n)}{e^2fmx^m+defm} - \frac{af^m}{e^2fmx^m+defm}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))/(d+e*x^m)^2,x, algorithm="maxima")
```

```
[Out] b*f^m*n*(log(x)/(d*e*f*m) - log(e*x^m + d)/(d*e*f*m^2)) - b*f^m*log(c*x^n)/
(e^2*f*m*x^m + d*e*f*m) - a*f^m/(e^2*f*m*x^m + d*e*f*m)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(69) = 138$.

Time = 0.35 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.93

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^2}\,dx = \frac{befmmnxx^m\log(x)}{de^2fm^2xx^m+d^2efm^2x} - \frac{bef^mnxx^m\log(ex^m+d)}{de^2fm^2xx^m+d^2efm^2x} - \frac{bdf^mnx\log(ex^m+d)}{de^2fm^2xx^m+d^2efm^2x} - \frac{bdf^mmx\log(c)}{de^2fm^2xx^m+d^2efm^2x} - \frac{adf^mmx}{de^2fm^2xx^m+d^2efm^2x}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))/(d+e*x^m)^2,x, algorithm="giac")
```

```
[Out] b*e*f^m*m*n*x*x^m*log(x)/(d*e^2*f*m^2*x*x^m + d^2*e*f*m^2*x) - b*e*f^m*n*x*
x^m*log(e*x^m + d)/(d*e^2*f*m^2*x*x^m + d^2*e*f*m^2*x) - b*d*f^m*n*x*log(e*
x^m + d)/(d*e^2*f*m^2*x*x^m + d^2*e*f*m^2*x) - b*d*f^m*m*x*log(c)/(d*e^2*f*
m^2*x*x^m + d^2*e*f*m^2*x) - a*d*f^m*m*x/(d*e^2*f*m^2*x*x^m + d^2*e*f*m^2*x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^2}\,dx = \int \frac{(f\,x)^{m-1}\,(a+b\,\ln(c\,x^n))}{(d+e\,x^m)^2}\,dx$$

```
[In] int(((f*x)^(m - 1)*(a + b*log(c*x^n)))/(d + e*x^m)^2,x)
```

```
[Out] int(((f*x)^(m - 1)*(a + b*log(c*x^n)))/(d + e*x^m)^2, x)
```

3.357 $\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 150

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^3}\,dx = \frac{bnx^{1-m}(fx)^{-1+m}}{2dem^2(d+ex^m)} + \frac{bnx^{1-m}(fx)^{-1+m}\log(x)}{2d^2em} - \frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))}{2em(d+ex^m)^2} - \frac{bnx^{1-m}(fx)^{-1+m}\log(d+ex^m)}{2d^2em^2}$$

```
[Out] 1/2*b*n*x^(1-m)*(f*x)^(-1+m)/d/e/m^2/(d+e*x^m)+1/2*b*n*x^(1-m)*(f*x)^(-1+m)
*ln(x)/d^2/e/m-1/2*x^(1-m)*(f*x)^(-1+m)*(a+b*ln(c*x^n))/e/m/(d+e*x^m)^2-1/2
*b*n*x^(1-m)*(f*x)^(-1+m)*ln(d+e*x^m)/d^2/e/m^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2377, 2376, 272, 46}

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^3}\,dx = -\frac{x^{1-m}(fx)^{m-1}(a+b\log(cx^n))}{2em(d+ex^m)^2} - \frac{bnx^{1-m}(fx)^{m-1}\log(d+ex^m)}{2d^2em^2} + \frac{bnx^{1-m}\log(x)(fx)^{m-1}}{2d^2em} + \frac{bnx^{1-m}(fx)^{m-1}}{2dem^2(d+ex^m)}$$

```
[In] Int[((f*x)^(-1 + m)*(a + b*Log[c*x^n]))/(d + e*x^m)^3,x]
```

```
[Out] (b*n*x^(1 - m)*(f*x)^(-1 + m))/(2*d*e*m^2*(d + e*x^m)) + (b*n*x^(1 - m)*(f*
x)^(-1 + m)*Log[x])/(2*d^2*e*m) - (x^(1 - m)*(f*x)^(-1 + m)*(a + b*Log[c*x^
n]))/(2*e*m*(d + e*x^m)^2) - (b*n*x^(1 - m)*(f*x)^(-1 + m)*Log[d + e*x^m])/
(2*d^2*e*m^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rule 2377

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(d + e*x^r)^q
*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] &&  !(IntegerQ[m] || GtQ[f, 0])
```

Rubi steps

$$\text{integral} = \left(x^{1-m}(fx)^{-1+m}\right)\int \frac{x^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^3}\,dx$$

$$= -\frac{x^{1-m}(fx)^{-1+m}\left(a+b\log(cx^n)\right)}{2em\left(d+ex^m\right)^2} + \frac{\left(bnx^{1-m}(fx)^{-1+m}\right)\int \frac{1}{x(d+ex^m)^2}\,dx}{2em}$$

$$= -\frac{x^{1-m}(fx)^{-1+m}\left(a+b\log(cx^n)\right)}{2em\left(d+ex^m\right)^2} + \frac{\left(bnx^{1-m}(fx)^{-1+m}\right)\text{Subst}\left(\int \frac{1}{x(d+ex)^2}\,dx, x, x^m\right)}{2em^2}$$

$$= -\frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))}{2em(d+ex^m)^2} + \frac{(bnx^{1-m}(fx)^{-1+m})\,\text{Subst}\left(\int\left(\frac{1}{d^2x}-\frac{e}{d(d+ex)^2}-\frac{e}{d^2(d+ex)}\right)dx,x,x^m\right)}{2em^2}$$

$$= \frac{bnx^{1-m}(fx)^{-1+m}}{2dem^2(d+ex^m)} + \frac{bnx^{1-m}(fx)^{-1+m}\log(x)}{2d^2em} - \frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))}{2em(d+ex^m)^2} - \frac{bnx^{1-m}(fx)^{-1+m}\log(d+ex^m)}{2d^2em^2}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int\frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^3}\,dx = \frac{x^{-m}(fx)^m\left(-ad^2m+bd^2n+bdenx^m+bmn(d+ex^m)^2\log(x)-bd^2m\log(cx^n)-bd^2n\log(d+ex^m)-2b\right.}{2d^2efm^2(d+ex^m)^2}$$

```
[In] Integrate[((f*x)^(-1 + m)*(a + b*Log[c*x^n]))/(d + e*x^m)^3,x]
[Out] ((f*x)^m*(-(a*d^2*m) + b*d^2*n + b*d*e*n*x^m + b*m*n*(d + e*x^m)^2*Log[x] -
 b*d^2*m*Log[c*x^n] - b*d^2*n*Log[d + e*x^m] - 2*b*d*e*n*x^m*Log[d + e*x^m]
 - b*e^2*n*x^(2*m)*Log[d + e*x^m]))/(2*d^2*e*f*m^2*x^m*(d + e*x^m)^2)
```

Maple [F]

$$\int\frac{(fx)^{m-1}(a+b\ln(cx^n))}{(d+ex^m)^3}dx$$

```
[In] int((f*x)^(m-1)*(a+b*ln(c*x^n))/(d+e*x^m)^3,x)
[Out] int((f*x)^(m-1)*(a+b*ln(c*x^n))/(d+e*x^m)^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.11

$$\int\frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^3}\,dx = \frac{be^2f^{m-1}mnx^{2m}\log(x)+(2bdemn\log(x)+bden)f^{m-1}x^m-(bd^2m\log(c)+ad^2m-bd^2n)f^{m-1}-(be^2f^m}{2(d^2e^3m^2x^{2m}+2d^3e^2m^2x^m+d^4em^2)}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))/(d+e*x^m)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*e^2*f^(m - 1)*m*n*x^(2*m)*log(x) + (2*b*d*e*m*n*log(x) + b*d*e*n)*f^
(m - 1)*x^m - (b*d^2*m*log(c) + a*d^2*m - b*d^2*n)*f^(m - 1) - (b*e^2*f^(m
- 1)*n*x^(2*m) + 2*b*d*e*f^(m - 1)*n*x^m + b*d^2*f^(m - 1)*n)*log(e*x^m + d
))/(d^2*e^3*m^2*x^(2*m) + 2*d^3*e^2*m^2*x^m + d^4*e*m^2)
```

Sympy [F]

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^3}\,dx=\int \frac{(fx)^{m-1}(a+b\log(cx^n))}{(d+ex^m)^3}\,dx$$

```
[In] integrate((f*x)**(-1+m)*(a+b*ln(c*x**n))/(d+e*x**m)**3,x)
```

```
[Out] Integral((f*x)**(m - 1)*(a + b*log(c*x**n))/(d + e*x**m)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.01

$$\begin{aligned}&\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^3}\,dx\\&=\frac{1}{2}bf^mn\left(\frac{1}{(de^2fmx^m+d^2efm)m}+\frac{\log(x)}{d^2efm}-\frac{\log(ex^m+d)}{d^2efm^2}\right)\\&\quad-\frac{bf^m\log(cx^n)}{2(e^3fmx^{2m}+2de^2fmx^m+d^2efm)}-\frac{af^m}{2(e^3fmx^{2m}+2de^2fmx^m+d^2efm)}\end{aligned}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))/(d+e*x^m)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*f^m*n*(1/((d*e^2*f*m*x^m + d^2*e*f*m)*m) + log(x)/(d^2*e*f*m) - log(e
*x^m + d)/(d^2*e*f*m^2)) - 1/2*b*f^m*log(c*x^n)/(e^3*f*m*x^(2*m) + 2*d*e^2*
f*m*x^m + d^2*e*f*m) - 1/2*a*f^m/(e^3*f*m*x^(2*m) + 2*d*e^2*f*m*x^m + d^2*e
*f*m)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(142) = 284$.

Time = 0.36 (sec) , antiderivative size = 633, normalized size of antiderivative = 4.22

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^3}\,dx = \frac{be^2f^mmnx^2x^{2m}\log(x)}{2\left(d^2e^3fm^2x^2x^{2m}+2\,d^3e^2fm^2x^2x^m+d^4efm^2x^2\right)} + \frac{bdef^mmnx^2x^m\log(x)}{d^2e^3fm^2x^2x^{2m}+2\,d^3e^2fm^2x^2x^m+d^4efm^2x^2} - \frac{be^2f^mnx^2x^{2m}\log(ex^m+d)}{2\left(d^2e^3fm^2x^2x^{2m}+2\,d^3e^2fm^2x^2x^m+d^4efm^2x^2\right)} - \frac{bdef^mnx^2x^m\log(ex^m+d)}{d^2e^3fm^2x^2x^{2m}+2\,d^3e^2fm^2x^2x^m+d^4efm^2x^2} + \frac{bdef^mnx^2x^m}{2\left(d^2e^3fm^2x^2x^{2m}+2\,d^3e^2fm^2x^2x^m+d^4efm^2x^2\right)} - \frac{bd^2f^mnx^2\log(ex^m+d)}{2\left(d^2e^3fm^2x^2x^{2m}+2\,d^3e^2fm^2x^2x^m+d^4efm^2x^2\right)} - \frac{bd^2f^mmx^2\log(c)}{2\left(d^2e^3fm^2x^2x^{2m}+2\,d^3e^2fm^2x^2x^m+d^4efm^2x^2\right)} - \frac{ad^2f^mmx^2}{2\left(d^2e^3fm^2x^2x^{2m}+2\,d^3e^2fm^2x^2x^m+d^4efm^2x^2\right)} + \frac{bd^2f^mnx^2}{2\left(d^2e^3fm^2x^2x^{2m}+2\,d^3e^2fm^2x^2x^m+d^4efm^2x^2\right)}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))/(d+e*x^m)^3,x, algorithm="giac")
```

```
[Out] 1/2*b*e^2*f^m*m*n*x^2*x^(2*m)*log(x)/(d^2*e^3*f*m^2*x^2*x^(2*m) + 2*d^3*e^2
*f*m^2*x^2*x^m + d^4*e*f*m^2*x^2) + b*d*e*f^m*m*n*x^2*x^m*log(x)/(d^2*e^3*f
*m^2*x^2*x^(2*m) + 2*d^3*e^2*f*m^2*x^2*x^m + d^4*e*f*m^2*x^2) - 1/2*b*e^2*f
^m*n*x^2*x^(2*m)*log(e*x^m + d)/(d^2*e^3*f*m^2*x^2*x^(2*m) + 2*d^3*e^2*f*m^
2*x^2*x^m + d^4*e*f*m^2*x^2) - b*d*e*f^m*n*x^2*x^m*log(e*x^m + d)/(d^2*e^3*
f*m^2*x^2*x^(2*m) + 2*d^3*e^2*f*m^2*x^2*x^m + d^4*e*f*m^2*x^2) + 1/2*b*d*e*
f^m*n*x^2*x^m/(d^2*e^3*f*m^2*x^2*x^(2*m) + 2*d^3*e^2*f*m^2*x^2*x^m + d^4*e*
f*m^2*x^2) - 1/2*b*d^2*f^m*n*x^2*log(e*x^m + d)/(d^2*e^3*f*m^2*x^2*x^(2*m)
+ 2*d^3*e^2*f*m^2*x^2*x^m + d^4*e*f*m^2*x^2) - 1/2*b*d^2*f^m*m*x^2*log(c)/(
d^2*e^3*f*m^2*x^2*x^(2*m) + 2*d^3*e^2*f*m^2*x^2*x^m + d^4*e*f*m^2*x^2) - 1/
2*a*d^2*f^m*m*x^2/(d^2*e^3*f*m^2*x^2*x^(2*m) + 2*d^3*e^2*f*m^2*x^2*x^m + d^
4*e*f*m^2*x^2) + 1/2*b*d^2*f^m*n*x^2/(d^2*e^3*f*m^2*x^2*x^(2*m) + 2*d^3*e^2
*f*m^2*x^2*x^m + d^4*e*f*m^2*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^3}\,dx = \int \frac{(f\,x)^{m-1}\,(a+b\,\ln(c\,x^n))}{(d+e\,x^m)^3}\,dx$$

```
[In] int(((f*x)^(m - 1)*(a + b*log(c*x^n)))/(d + e*x^m)^3,x)
[Out] int(((f*x)^(m - 1)*(a + b*log(c*x^n)))/(d + e*x^m)^3, x)
```

3.358 $\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 188

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^4}\,dx = \frac{bnx^{1-m}(fx)^{-1+m}}{6dem^2(d+ex^m)^2} + \frac{bnx^{1-m}(fx)^{-1+m}}{3d^2em^2(d+ex^m)} + \frac{bnx^{1-m}(fx)^{-1+m}\log(x)}{3d^3em} - \frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))}{3em(d+ex^m)^3} - \frac{bnx^{1-m}(fx)^{-1+m}\log(d+ex^m)}{3d^3em^2}$$

```
[Out] 1/6*b*n*x^(1-m)*(f*x)^(-1+m)/d/e/m^2/(d+e*x^m)^2+1/3*b*n*x^(1-m)*(f*x)^(-1+
m)/d^2/e/m^2/(d+e*x^m)+1/3*b*n*x^(1-m)*(f*x)^(-1+m)*ln(x)/d^3/e/m-1/3*x^(1-
m)*(f*x)^(-1+m)*(a+b*ln(c*x^n))/e/m/(d+e*x^m)^3-1/3*b*n*x^(1-m)*(f*x)^(-1+m
)*ln(d+e*x^m)/d^3/e/m^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used

$= \{2377, 2376, 272, 46\}$

$$\int \frac{(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)}{\left(d+ex^m\right)^4}\,dx = -\frac{x^{1-m}(fx)^{m-1}\left(a+b\log\left(cx^n\right)\right)}{3em\left(d+ex^m\right)^3} - \frac{bnx^{1-m}(fx)^{m-1}\log\left(d+ex^m\right)}{3d^3em^2} + \frac{bnx^{1-m}\log(x)(fx)^{m-1}}{3d^3em} + \frac{bnx^{1-m}(fx)^{m-1}}{3d^2em^2\left(d+ex^m\right)} + \frac{bnx^{1-m}(fx)^{m-1}}{6dem^2\left(d+ex^m\right)^2}$$

```
[In] Int[((f*x)^(-1 + m)*(a + b*Log[c*x^n]))/(d + e*x^m)^4,x]
```

```
[Out] (b*n*x^(1 - m)*(f*x)^(-1 + m))/(6*d*e*m^2*(d + e*x^m)^2) + (b*n*x^(1 - m)*(
f*x)^(-1 + m))/(3*d^2*e*m^2*(d + e*x^m)) + (b*n*x^(1 - m)*(f*x)^(-1 + m)*Lo
g[x])/(3*d^3*e*m) - (x^(1 - m)*(f*x)^(-1 + m)*(a + b*Log[c*x^n]))/(3*e*m*(d
 + e*x^m)^3) - (b*n*x^(1 - m)*(f*x)^(-1 + m)*Log[d + e*x^m])/(3*d^3*e*m^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rule 2377

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(d + e*x^r)^q
*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] &&  !(IntegerQ[m] || GtQ[f, 0])
```

Rubi steps

$$\text{integral} = \left(x^{1-m}(fx)^{-1+m}\right)\int \frac{x^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^4}\,dx$$

$$= -\frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))}{3em(d+ex^m)^3} + \frac{(bnx^{1-m}(fx)^{-1+m})\int \frac{1}{x(d+ex^m)^3}\,dx}{3em}$$

$$= -\frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))}{3em(d+ex^m)^3} + \frac{(bnx^{1-m}(fx)^{-1+m})\,\text{Subst}\left(\int \frac{1}{x(d+ex)^3}\,dx, x, x^m\right)}{3em^2}$$

$$= -\frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))}{3em(d+ex^m)^3} + \frac{(bnx^{1-m}(fx)^{-1+m})\,\text{Subst}\left(\int \left(\frac{1}{d^3x} - \frac{e}{d(d+ex)^3} - \frac{e}{d^2(d+ex)^2} - \frac{e}{d^3(d+ex)}\right)dx, x, x^m\right)}{3em^2}$$

$$= \frac{bnx^{1-m}(fx)^{-1+m}}{6dem^2(d+ex^m)^2} + \frac{bnx^{1-m}(fx)^{-1+m}}{3d^2em^2(d+ex^m)} + \frac{bnx^{1-m}(fx)^{-1+m}\log(x)}{3d^3em} - \frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))}{3em(d+ex^m)^3} - \frac{bnx^{1-m}(fx)^{-1+m}\log(d+ex^m)}{3d^3em^2}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.95

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^4}\,dx = \frac{x^{-m}(fx)^m\left(-2ad^3m+3bd^3n+5bd^2enx^m+2bde^2nx^{2m}+2bmn(d+ex^m)^3\log(x)-2bd^3m\log(cx^n)-2bd\cdots\right.}{6d^3efm^2(d+\cdots}$$

```
[In] Integrate[((f*x)^(-1 + m)*(a + b*Log[c*x^n]))/(d + e*x^m)^4,x]
```

```
[Out] ((f*x)^m*(-2*a*d^3*m + 3*b*d^3*n + 5*b*d^2*e*n*x^m + 2*b*d*e^2*n*x^(2*m) +
2*b*m*n*(d + e*x^m)^3*Log[x] - 2*b*d^3*m*Log[c*x^n] - 2*b*d^3*n*Log[d + e*x
^m] - 6*b*d^2*e*n*x^m*Log[d + e*x^m] - 6*b*d*e^2*n*x^(2*m)*Log[d + e*x^m] -
 2*b*e^3*n*x^(3*m)*Log[d + e*x^m]))/(6*d^3*e*f*m^2*x^m*(d + e*x^m)^3)
```

Maple [F]

$$\int \frac{(fx)^{m-1} (a + b\ln (c\,x^{n}))}{(d + e\,x^{m})^{4}}dx$$

[In] int((f*x)^(m-1)*(a+b*ln(c*x^n))/(d+e*x^m)^4,x)

[Out] int((f*x)^(m-1)*(a+b*ln(c*x^n))/(d+e*x^m)^4,x)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.29

$$\int \frac{(fx)^{-1+m} (a + b\log (cx^{n}))}{(d + ex^{m})^{4}}\,dx = \frac{2\,be^{3}f^{m-1}mnx^{3\,m}\log (x) + 2\,(3\,bde^{2}mn\log (x) + bde^{2}n)f^{m-1}x^{2\,m} + (6\,bd^{2}emn\log (x) + 5\,bd^{2}en)f^{m-1}x^{m}}{6\,(d^{3}e^{4}m^{2}x^{3\,m} +}$$

[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))/(d+e*x^m)^4,x, algorithm="fricas")

[Out] 1/6*(2*b*e^3*f^(m - 1)*m*n*x^(3*m)*log(x) + 2*(3*b*d*e^2*m*n*log(x) + b*d*e^2*n)*f^(m - 1)*x^(2*m) + (6*b*d^2*e*m*n*log(x) + 5*b*d^2*e*n)*f^(m - 1)*x^m - (2*b*d^3*m*log(c) + 2*a*d^3*m - 3*b*d^3*n)*f^(m - 1) - 2*(b*e^3*f^(m - 1)*n*x^(3*m) + 3*b*d*e^2*f^(m - 1)*n*x^(2*m) + 3*b*d^2*e*f^(m - 1)*n*x^m + b*d^3*f^(m - 1)*n)*log(e*x^m + d))/(d^3*e^4*m^2*x^(3*m) + 3*d^4*e^3*m^2*x^(2*m) + 3*d^5*e^2*m^2*x^m + d^6*e*m^2)

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^{-1+m} (a + b\log (cx^{n}))}{(d + ex^{m})^{4}}\,dx = \text{Timed out}$$

[In] integrate((f*x)**(-1+m)*(a+b*ln(c*x**n))/(d+e*x**m)**4,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.12

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^4}\,dx$$
$$= \frac{1}{6}bf^m n\left(\frac{2\,ex^m+3\,d}{(d^2e^3fmx^{2\,m}+2\,d^3e^2fmx^m+d^4efm)m}+\frac{2\,\log(x)}{d^3efm}-\frac{2\,\log(ex^m+d)}{d^3efm^2}\right)$$
$$-\frac{bf^m\log(cx^n)}{3\,(e^4fmx^{3\,m}+3\,de^3fmx^{2\,m}+3\,d^2e^2fmx^m+d^3efm)}$$
$$-\frac{af^m}{3\,(e^4fmx^{3\,m}+3\,de^3fmx^{2\,m}+3\,d^2e^2fmx^m+d^3efm)}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))/(d+e*x^m)^4,x, algorithm="maxima")
```

```
[Out] 1/6*b*f^m*n*((2*e*x^m + 3*d)/((d^2*e^3*f*m*x^(2*m) + 2*d^3*e^2*f*m*x^m + d^
4*e*f*m)*m) + 2*log(x)/(d^3*e*f*m) - 2*log(e*x^m + d)/(d^3*e*f*m^2)) - 1/3*
b*f^m*log(c*x^n)/(e^4*f*m*x^(3*m) + 3*d*e^3*f*m*x^(2*m) + 3*d^2*e^2*f*m*x^m
 + d^3*e*f*m) - 1/3*a*f^m/(e^4*f*m*x^(3*m) + 3*d*e^3*f*m*x^(2*m) + 3*d^2*e^
2*f*m*x^m + d^3*e*f*m)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1102 vs. 2(178) = 356.

Time = 0.37 (sec) , antiderivative size = 1102, normalized size of antiderivative = 5.86

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^4}\,dx = \frac{be^3f^mmnx^3x^{3m}\log(x)}{3(d^3e^4fm^2x^3x^{3m}+3d^4e^3fm^2x^3x^{2m}+3d^5e^2fm^2x^3x^m+d^6efm^2x^3)} + \frac{bde^2f^mmnx^3x^{2m}\log(x)}{d^3e^4fm^2x^3x^{3m}+3d^4e^3fm^2x^3x^{2m}+3d^5e^2fm^2x^3x^m+d^6efm^2x^3} + \frac{bd^2ef^mmnx^3x^m\log(x)}{d^3e^4fm^2x^3x^{3m}+3d^4e^3fm^2x^3x^{2m}+3d^5e^2fm^2x^3x^m+d^6efm^2x^3} - \frac{be^3f^mnx^3x^{3m}\log(ex^m+d)}{3(d^3e^4fm^2x^3x^{3m}+3d^4e^3fm^2x^3x^{2m}+3d^5e^2fm^2x^3x^m+d^6efm^2x^3)} - \frac{bde^2f^mnx^3x^{2m}\log(ex^m+d)}{d^3e^4fm^2x^3x^{3m}+3d^4e^3fm^2x^3x^{2m}+3d^5e^2fm^2x^3x^m+d^6efm^2x^3} - \frac{bd^2ef^mnx^3x^m\log(ex^m+d)}{d^3e^4fm^2x^3x^{3m}+3d^4e^3fm^2x^3x^{2m}+3d^5e^2fm^2x^3x^m+d^6efm^2x^3} + \frac{bde^2f^mnx^3x^{2m}}{3(d^3e^4fm^2x^3x^{3m}+3d^4e^3fm^2x^3x^{2m}+3d^5e^2fm^2x^3x^m+d^6efm^2x^3)} + \frac{5bd^2ef^mnx^3x^m}{6(d^3e^4fm^2x^3x^{3m}+3d^4e^3fm^2x^3x^{2m}+3d^5e^2fm^2x^3x^m+d^6efm^2x^3)} - \frac{bd^3f^mnx^3\log(ex^m+d)}{3(d^3e^4fm^2x^3x^{3m}+3d^4e^3fm^2x^3x^{2m}+3d^5e^2fm^2x^3x^m+d^6efm^2x^3)} - \frac{bd^3f^mmx^3\log(c)}{3(d^3e^4fm^2x^3x^{3m}+3d^4e^3fm^2x^3x^{2m}+3d^5e^2fm^2x^3x^m+d^6efm^2x^3)} - \frac{ad^3f^mmx^3}{3(d^3e^4fm^2x^3x^{3m}+3d^4e^3fm^2x^3x^{2m}+3d^5e^2fm^2x^3x^m+d^6efm^2x^3)} + \frac{bd^3f^mnx^3}{2(d^3e^4fm^2x^3x^{3m}+3d^4e^3fm^2x^3x^{2m}+3d^5e^2fm^2x^3x^m+d^6efm^2x^3)}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))/(d+e*x^m)^4,x, algorithm="giac")
```

```
[Out] 1/3*b*e^3*f^m*m*n*x^3*x^(3*m)*log(x)/(d^3*e^4*f*m^2*x^3*x^(3*m) + 3*d^4*e^3
*f*m^2*x^3*x^(2*m) + 3*d^5*e^2*f*m^2*x^3*x^m + d^6*e*f*m^2*x^3) + b*d*e^2*f
^m*m*n*x^3*x^(2*m)*log(x)/(d^3*e^4*f*m^2*x^3*x^(3*m) + 3*d^4*e^3*f*m^2*x^3*
x^(2*m) + 3*d^5*e^2*f*m^2*x^3*x^m + d^6*e*f*m^2*x^3) + b*d^2*e*f^m*m*n*x^3*
x^m*log(x)/(d^3*e^4*f*m^2*x^3*x^(3*m) + 3*d^4*e^3*f*m^2*x^3*x^(2*m) + 3*d^5
*e^2*f*m^2*x^3*x^m + d^6*e*f*m^2*x^3) - 1/3*b*e^3*f^m*n*x^3*x^(3*m)*log(e*x
^m + d)/(d^3*e^4*f*m^2*x^3*x^(3*m) + 3*d^4*e^3*f*m^2*x^3*x^(2*m) + 3*d^5*e^
2*f*m^2*x^3*x^m + d^6*e*f*m^2*x^3) - b*d*e^2*f^m*n*x^3*x^(2*m)*log(e*x^m +
d)/(d^3*e^4*f*m^2*x^3*x^(3*m) + 3*d^4*e^3*f*m^2*x^3*x^(2*m) + 3*d^5*e^2*f*m
^2*x^3*x^m + d^6*e*f*m^2*x^3) - b*d^2*e*f^m*n*x^3*x^m*log(e*x^m + d)/(d^3*e
^4*f*m^2*x^3*x^(3*m) + 3*d^4*e^3*f*m^2*x^3*x^(2*m) + 3*d^5*e^2*f*m^2*x^3*x^
```

```
m + d^6*e*f*m^2*x^3) + 1/3*b*d*e^2*f^m*n*x^3*x^(2*m)/(d^3*e^4*f*m^2*x^3*x^(
3*m) + 3*d^4*e^3*f*m^2*x^3*x^(2*m) + 3*d^5*e^2*f*m^2*x^3*x^m + d^6*e*f*m^2*
x^3) + 5/6*b*d^2*e*f^m*n*x^3*x^m/(d^3*e^4*f*m^2*x^3*x^(3*m) + 3*d^4*e^3*f*m
^2*x^3*x^(2*m) + 3*d^5*e^2*f*m^2*x^3*x^m + d^6*e*f*m^2*x^3) - 1/3*b*d^3*f^m
*n*x^3*log(e*x^m + d)/(d^3*e^4*f*m^2*x^3*x^(3*m) + 3*d^4*e^3*f*m^2*x^3*x^(2
*m) + 3*d^5*e^2*f*m^2*x^3*x^m + d^6*e*f*m^2*x^3) - 1/3*b*d^3*f^m*m*x^3*log(
c)/(d^3*e^4*f*m^2*x^3*x^(3*m) + 3*d^4*e^3*f*m^2*x^3*x^(2*m) + 3*d^5*e^2*f*m
^2*x^3*x^m + d^6*e*f*m^2*x^3) - 1/3*a*d^3*f^m*m*x^3/(d^3*e^4*f*m^2*x^3*x^(3
*m) + 3*d^4*e^3*f*m^2*x^3*x^(2*m) + 3*d^5*e^2*f*m^2*x^3*x^m + d^6*e*f*m^2*x
^3) + 1/2*b*d^3*f^m*n*x^3/(d^3*e^4*f*m^2*x^3*x^(3*m) + 3*d^4*e^3*f*m^2*x^3*
x^(2*m) + 3*d^5*e^2*f*m^2*x^3*x^m + d^6*e*f*m^2*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{-1+m}\,(a+b\log{(cx^n)})}{(d+ex^m)^4}\,dx = \int \frac{(f\,x)^{m-1}\,(a+b\,\ln{(c\,x^n)})}{(d+e\,x^m)^4}\,dx$$

```
[In] int(((f*x)^(m - 1)*(a + b*log(c*x^n)))/(d + e*x^m)^4,x)
[Out] int(((f*x)^(m - 1)*(a + b*log(c*x^n)))/(d + e*x^m)^4, x)
```

3.359 $\int (fx)^{-1+m} (d+ex^m)^3 (a+b\log(cx^n))^2 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 372

$$\begin{aligned}\int (fx)^{-1+m} (d+ex^m)^3 (a+b\log(cx^n))^2 \, dx &= \frac{2b^2d^3n^2x(fx)^{-1+m}}{m^3} + \frac{3b^2d^2en^2x^{1+m}(fx)^{-1+m}}{4m^3} \\ &+ \frac{2b^2de^2n^2x^{1+2m}(fx)^{-1+m}}{9m^3} \\ &+ \frac{b^2e^3n^2x^{1+3m}(fx)^{-1+m}}{32m^3} \\ &+ \frac{b^2d^4n^2x^{1-m}(fx)^{-1+m}\log^2(x)}{4em} \\ &- \frac{2bd^3nx(fx)^{-1+m}(a+b\log(cx^n))}{m^2} \\ &- \frac{3bd^2enx^{1+m}(fx)^{-1+m}(a+b\log(cx^n))}{2m^2} \\ &- \frac{2bde^2nx^{1+2m}(fx)^{-1+m}(a+b\log(cx^n))}{3m^2} \\ &- \frac{be^3nx^{1+3m}(fx)^{-1+m}(a+b\log(cx^n))}{8m^2} \\ &- \frac{bd^4nx^{1-m}(fx)^{-1+m}\log(x)(a+b\log(cx^n))}{2em} \\ &+ \frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^4(a+b\log(cx^n))^2}{4em}\end{aligned}$$

```
[Out] 2*b^2*d^3*n^2*x*(f*x)^(-1+m)/m^3+3/4*b^2*d^2*e*n^2*x^(1+m)*(f*x)^(-1+m)/m^3
+2/9*b^2*d*e^2*n^2*x^(1+2*m)*(f*x)^(-1+m)/m^3+1/32*b^2*e^3*n^2*x^(1+3*m)*(f
*x)^(-1+m)/m^3+1/4*b^2*d^4*n^2*x^(1-m)*(f*x)^(-1+m)*ln(x)^2/e/m-2*b*d^3*n*x
*(f*x)^(-1+m)*(a+b*ln(c*x^n))/m^2-3/2*b*d^2*e*n*x^(1+m)*(f*x)^(-1+m)*(a+b*l
n(c*x^n))/m^2-2/3*b*d*e^2*n*x^(1+2*m)*(f*x)^(-1+m)*(a+b*ln(c*x^n))/m^2-1/8*
b*e^3*n*x^(1+3*m)*(f*x)^(-1+m)*(a+b*ln(c*x^n))/m^2-1/2*b*d^4*n*x^(1-m)*(f*x
```

```
)^(-1+m)*ln(x)*(a+b*ln(c*x^n))/e/m+1/4*x^(1-m)*(f*x)^(-1+m)*(d+e*x^m)^4*(a+
b*ln(c*x^n))^2/e/m
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2377, 2376, 272, 45, 2372, 14, 2338}

$$\int (fx)^{-1+m} (d+ex^m)^3 (a+b\log(cx^n))^2 \, dx = -\frac{bd^4nx^{1-m}\log(x)(fx)^{m-1}(a+b\log(cx^n))}{2em} - \frac{2bd^3nx(fx)^{m-1}(a+b\log(cx^n))}{m^2} - \frac{3bd^2enx^{m+1}(fx)^{m-1}(a+b\log(cx^n))}{2m^2} - \frac{2bde^2nx^{2m+1}(fx)^{m-1}(a+b\log(cx^n))}{3m^2} + \frac{x^{1-m}(fx)^{m-1}(d+ex^m)^4(a+b\log(cx^n))^2}{4em} - \frac{be^3nx^{3m+1}(fx)^{m-1}(a+b\log(cx^n))}{8m^2} + \frac{b^2d^4n^2x^{1-m}\log^2(x)(fx)^{m-1}}{4em} + \frac{2b^2d^3n^2x(fx)^{m-1}}{m^3} + \frac{3b^2d^2en^2x^{m+1}(fx)^{m-1}}{4m^3} + \frac{2b^2de^2n^2x^{2m+1}(fx)^{m-1}}{9m^3} + \frac{b^2e^3n^2x^{3m+1}(fx)^{m-1}}{32m^3}$$

```
[In] Int[(f*x)^(-1 + m)*(d + e*x^m)^3*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (2*b^2*d^3*n^2*x*(f*x)^(-1 + m))/m^3 + (3*b^2*d^2*e*n^2*x^(1 + m)*(f*x)^(-1
 + m))/(4*m^3) + (2*b^2*d*e^2*n^2*x^(1 + 2*m)*(f*x)^(-1 + m))/(9*m^3) + (b^
2*e^3*n^2*x^(1 + 3*m)*(f*x)^(-1 + m))/(32*m^3) + (b^2*d^4*n^2*x^(1 - m)*(f*
x)^(-1 + m)*Log[x]^2)/(4*e*m) - (2*b*d^3*n*x*(f*x)^(-1 + m)*(a + b*Log[c*x^
n]))/m^2 - (3*b*d^2*e*n*x^(1 + m)*(f*x)^(-1 + m)*(a + b*Log[c*x^n]))/(2*m^2
) - (2*b*d*e^2*n*x^(1 + 2*m)*(f*x)^(-1 + m)*(a + b*Log[c*x^n]))/(3*m^2) - (
b*e^3*n*x^(1 + 3*m)*(f*x)^(-1 + m)*(a + b*Log[c*x^n]))/(8*m^2) - (b*d^4*n*x
^(1 - m)*(f*x)^(-1 + m)*Log[x]*(a + b*Log[c*x^n]))/(2*e*m) + (x^(1 - m)*(f*
x)^(-1 + m)*(d + e*x^m)^4*(a + b*Log[c*x^n])^2)/(4*e*m)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rule 2377

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(d + e*x^r)^q
*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] &&  !(IntegerQ[m] || GtQ[f, 0])
```

Rubi steps

$$\text{integral} = \left(x^{1-m}(fx)^{-1+m}\right)\int x^{-1+m}(d+ex^m)^3\,(a+b\log(cx^n))^2\,dx$$

$$=\frac{x^{1-m}(fx)^{-1+m}\,(d+ex^m)^4\,(a+b\log(cx^n))^2}{4em}-\frac{\left(bnx^{1-m}(fx)^{-1+m}\right)\int\frac{(d+ex^m)^4(a+b\log(cx^n))}{x}\,dx}{2em}$$

$$= -\frac{2bd^3nx(fx)^{-1+m}(a+b\log(cx^n))}{m^2} - \frac{3bd^2enx^{1+m}(fx)^{-1+m}(a+b\log(cx^n))}{2m^2} - \frac{2bde^2nx^{1+2m}(fx)^{-1+m}(a+b\log(cx^n))}{3m^2} - \frac{be^3nx^{1+3m}(fx)^{-1+m}(a+b\log(cx^n))}{8m^2} - \frac{bd^4nx^{1-m}(fx)^{-1+m}\log(x)(a+b\log(cx^n))}{2em} + \frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^4(a+b\log(cx^n))^2}{4em} + \frac{(b^2n^2x^{1-m}(fx)^{-1+m})\int\left(\frac{ex^{-1+m}(48d^3+36d^2ex^m+16de^2x^{2m}+3e^3x^{3m})}{12m}+\frac{d^4\log(x)}{x}\right)dx}{2em}$$

$$= -\frac{2bd^3nx(fx)^{-1+m}(a+b\log(cx^n))}{m^2} - \frac{3bd^2enx^{1+m}(fx)^{-1+m}(a+b\log(cx^n))}{2m^2} - \frac{2bde^2nx^{1+2m}(fx)^{-1+m}(a+b\log(cx^n))}{3m^2} - \frac{be^3nx^{1+3m}(fx)^{-1+m}(a+b\log(cx^n))}{8m^2} - \frac{bd^4nx^{1-m}(fx)^{-1+m}\log(x)(a+b\log(cx^n))}{2em} + \frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^4(a+b\log(cx^n))^2}{4em} + \frac{(b^2n^2x^{1-m}(fx)^{-1+m})\int x^{-1+m}(48d^3+36d^2ex^m+16de^2x^{2m}+3e^3x^{3m})\,dx}{24m^2} + \frac{(b^2d^4n^2x^{1-m}(fx)^{-1+m})\int\frac{\log(x)}{x}\,dx}{2em}$$

$$= \frac{b^2d^4n^2x^{1-m}(fx)^{-1+m}\log^2(x)}{4em} - \frac{2bd^3nx(fx)^{-1+m}(a+b\log(cx^n))}{m^2} - \frac{3bd^2enx^{1+m}(fx)^{-1+m}(a+b\log(cx^n))}{2m^2} - \frac{2bde^2nx^{1+2m}(fx)^{-1+m}(a+b\log(cx^n))}{3m^2} - \frac{be^3nx^{1+3m}(fx)^{-1+m}(a+b\log(cx^n))}{8m^2} - \frac{bd^4nx^{1-m}(fx)^{-1+m}\log(x)(a+b\log(cx^n))}{2em} + \frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^4(a+b\log(cx^n))^2}{4em} + \frac{(b^2n^2x^{1-m}(fx)^{-1+m})\int(48d^3x^{-1+m}+36d^2ex^{-1+2m}+16de^2x^{-1+3m}+3e^3x^{-1+4m})\,dx}{24m^2}$$

$$= \frac{2b^2d^3n^2x(fx)^{-1+m}}{m^3} + \frac{3b^2d^2en^2x^{1+m}(fx)^{-1+m}}{4m^3} + \frac{2b^2de^2n^2x^{1+2m}(fx)^{-1+m}}{9m^3} + \frac{b^2e^3n^2x^{1+3m}(fx)^{-1+m}}{32m^3} + \frac{b^2d^4n^2x^{1-m}(fx)^{-1+m}\log^2(x)}{4em} - \frac{2bd^3nx(fx)^{-1+m}(a+b\log(cx^n))}{m^2} - \frac{3bd^2enx^{1+m}(fx)^{-1+m}(a+b\log(cx^n))}{2m^2} - \frac{2bde^2nx^{1+2m}(fx)^{-1+m}(a+b\log(cx^n))}{3m^2} - \frac{be^3nx^{1+3m}(fx)^{-1+m}(a+b\log(cx^n))}{8m^2} - \frac{bd^4nx^{1-m}(fx)^{-1+m}\log(x)(a+b\log(cx^n))}{2em} + \frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^4(a+b\log(cx^n))^2}{4em}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.77

$$\int (fx)^{-1+m}(d+ex^m)^3(a+b\log(cx^n))^2\,dx = \frac{(fx)^m(72a^2m^2(4d^3+6d^2ex^m+4de^2x^{2m}+e^3x^{3m})-12abmn(48d^3+36d^2ex^m+16de^2x^{2m}+3e^3x^{3m})+\cdots}{}$$

```
[In] Integrate[(f*x)^(-1 + m)*(d + e*x^m)^3*(a + b*Log[c*x^n])^2,x]
```

```
[Out] ((f*x)^m*(72*a^2*m^2*(4*d^3 + 6*d^2*e*x^m + 4*d*e^2*x^(2*m) + e^3*x^(3*m))
- 12*a*b*m*n*(48*d^3 + 36*d^2*e*x^m + 16*d*e^2*x^(2*m) + 3*e^3*x^(3*m)) + b
^2*n^2*(576*d^3 + 216*d^2*e*x^m + 64*d*e^2*x^(2*m) + 9*e^3*x^(3*m)) + 12*b*
m*(12*a*m*(4*d^3 + 6*d^2*e*x^m + 4*d*e^2*x^(2*m) + e^3*x^(3*m)) - b*n*(48*d
^3 + 36*d^2*e*x^m + 16*d*e^2*x^(2*m) + 3*e^3*x^(3*m)))*Log[c*x^n] + 72*b^2*
m^2*(4*d^3 + 6*d^2*e*x^m + 4*d*e^2*x^(2*m) + e^3*x^(3*m))*Log[c*x^n]^2))/(2
88*f*m^3)
```

Maple [A] (verified)

Time = 205.35 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.66

method	result
parallelrisch	$-\frac{-288b^2d^3(fx)^{m-1}\ln(cx^n)^2xm^2-72x\,x^{3m}(fx)^{m-1}a^2e^3m^2-9x\,x^{3m}(fx)^{m-1}b^2e^3n^2-432b^2d^2e(fx)^{m-1}\ln(cx^n)^2x^mxm^2\cdots}{}$
risch	Expression too large to display

```
[In] int((f*x)^(m-1)*(d+e*x^m)^3*(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/288*(-288*b^2*d^3*(f*x)^(m-1)*ln(c*x^n)^2*x*m^2-72*x*(x^m)^3*(f*x)^(m-1)
*a^2*e^3*m^2-9*x*(x^m)^3*(f*x)^(m-1)*b^2*e^3*n^2-432*b^2*d^2*e*(f*x)^(m-1)*
```

```
ln(c*x^n)^2*x^m*x*m^2-864*x*x^m*ln(c*x^n)*(f*x)^(m-1)*a*b*d^2*e*m^2+432*x*x
^m*ln(c*x^n)*(f*x)^(m-1)*b^2*d^2*e*m*n-576*x*(x^m)^2*ln(c*x^n)*(f*x)^(m-1)*
a*b*d*e^2*m^2+192*x*(x^m)^2*ln(c*x^n)*(f*x)^(m-1)*b^2*d*e^2*m*n+432*x*x^m*(
f*x)^(m-1)*a*b*d^2*e*m*n+192*x*(x^m)^2*(f*x)^(m-1)*a*b*d*e^2*m*n-288*x*(f*x
)^(m-1)*a^2*d^3*m^2-576*x*(f*x)^(m-1)*b^2*d^3*n^2-216*x*x^m*(f*x)^(m-1)*b^2
*d^2*e*n^2-288*x*(x^m)^2*(f*x)^(m-1)*a^2*d*e^2*m^2-64*x*(x^m)^2*(f*x)^(m-1)
*b^2*d*e^2*n^2-72*b^2*e^3*(f*x)^(m-1)*ln(c*x^n)^2*x*(x^m)^3*m^2-576*x*ln(c*
x^n)*(f*x)^(m-1)*a*b*d^3*m^2+576*x*ln(c*x^n)*(f*x)^(m-1)*b^2*d^3*m*n+576*x*
(f*x)^(m-1)*a*b*d^3*m*n-432*x*x^m*(f*x)^(m-1)*a^2*d^2*e*m^2+36*x*(x^m)^3*(f
*x)^(m-1)*a*b*e^3*m*n-144*x*(x^m)^3*ln(c*x^n)*(f*x)^(m-1)*a*b*e^3*m^2+36*x*
(x^m)^3*ln(c*x^n)*(f*x)^(m-1)*b^2*e^3*m*n-288*b^2*d*e^2*(f*x)^(m-1)*ln(c*x^
n)^2*(x^m)^2*x*m^2)/m^3
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.59

$$\int (fx)^{-1+m} (d+ex^m)^3 (a+b\log(cx^n))^2 \, dx = \frac{9 \left(8 b^2 e^3 m^2 n^2 \log(x)^2 + 8 b^2 e^3 m^2 \log(c)^2 + 8 a^2 e^3 m^2 - 4 abe^3 mn + b^2 e^3 n^2 + 4 (4 abe^3 m^2 - b^2 e^3 mn) \log(c)\right.}{}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)^3*(a+b*log(c*x^n))^2,x, algorithm="fricas"
)
```

```
[Out] 1/288*(9*(8*b^2*e^3*m^2*n^2*log(x)^2 + 8*b^2*e^3*m^2*log(c)^2 + 8*a^2*e^3*m
^2 - 4*a*b*e^3*m*n + b^2*e^3*n^2 + 4*(4*a*b*e^3*m^2 - b^2*e^3*m*n)*log(c) +
 4*(4*b^2*e^3*m^2*n*log(c) + 4*a*b*e^3*m^2*n - b^2*e^3*m*n^2)*log(x))*f^(m
- 1)*x^(4*m) + 32*(9*b^2*d*e^2*m^2*n^2*log(x)^2 + 9*b^2*d*e^2*m^2*log(c)^2
+ 9*a^2*d*e^2*m^2 - 6*a*b*d*e^2*m*n + 2*b^2*d*e^2*n^2 + 6*(3*a*b*d*e^2*m^2
- b^2*d*e^2*m*n)*log(c) + 6*(3*b^2*d*e^2*m^2*n*log(c) + 3*a*b*d*e^2*m^2*n -
 b^2*d*e^2*m*n^2)*log(x))*f^(m - 1)*x^(3*m) + 216*(2*b^2*d^2*e*m^2*n^2*log(
x)^2 + 2*b^2*d^2*e*m^2*log(c)^2 + 2*a^2*d^2*e*m^2 - 2*a*b*d^2*e*m*n + b^2*d
^2*e*n^2 + 2*(2*a*b*d^2*e*m^2 - b^2*d^2*e*m*n)*log(c) + 2*(2*b^2*d^2*e*m^2*
n*log(c) + 2*a*b*d^2*e*m^2*n - b^2*d^2*e*m*n^2)*log(x))*f^(m - 1)*x^(2*m) +
 288*(b^2*d^3*m^2*n^2*log(x)^2 + b^2*d^3*m^2*log(c)^2 + a^2*d^3*m^2 - 2*a*b
*d^3*m*n + 2*b^2*d^3*n^2 + 2*(a*b*d^3*m^2 - b^2*d^3*m*n)*log(c) + 2*(b^2*d^
3*m^2*n*log(c) + a*b*d^3*m^2*n - b^2*d^3*m*n^2)*log(x))*f^(m - 1)*x^m)/m^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(364) = 728$.

Time = 38.32 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.04

$$\int (fx)^{-1+m} (d+ex^m)^3 (a+b\log(cx^n))^2 \, dx$$
$$= \begin{cases} \frac{a^2d^3x(fx)^{m-1}}{m} + \frac{3a^2d^2exx^m(fx)^{m-1}}{2m} + \frac{a^2de^2xx^{2m}(fx)^{m-1}}{m} + \frac{a^2e^3xx^{3m}(fx)^{m-1}}{4m} + \frac{2abd^3x(fx)^{m-1}\log(cx^n)}{m} - \frac{2abd^3nx(fx)^{m-1}}{m^2} \\ \frac{(d+e)^3\left(\begin{cases} \frac{a^2\log(cx^n)+ab\log(cx^n)^2+\frac{b^2\log(cx^n)^3}{3}}{n} & \text{for } n \neq 0 \\ (a^2+2ab\log(c)+b^2\log(c)^2)\log(x) & \text{otherwise} \end{cases}\right)}{f} \end{cases}$$

```
[In] integrate((f*x)**(-1+m)*(d+e*x**m)**3*(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((a**2*d**3*x*(f*x)**(m - 1)/m + 3*a**2*d**2*e*x*x**m*(f*x)**(m -
1)/(2*m) + a**2*d*e**2*x*x**(2*m)*(f*x)**(m - 1)/m + a**2*e**3*x*x**(3*m)*(
f*x)**(m - 1)/(4*m) + 2*a*b*d**3*x*(f*x)**(m - 1)*log(c*x**n)/m - 2*a*b*d**
3*n*x*(f*x)**(m - 1)/m**2 + 3*a*b*d**2*e*x*x**m*(f*x)**(m - 1)*log(c*x**n)/
m - 3*a*b*d**2*e*n*x*x**m*(f*x)**(m - 1)/(2*m**2) + 2*a*b*d*e**2*x*x**(2*m)
*(f*x)**(m - 1)*log(c*x**n)/m - 2*a*b*d*e**2*n*x*x**(2*m)*(f*x)**(m - 1)/(3
*m**2) + a*b*e**3*x*x**(3*m)*(f*x)**(m - 1)*log(c*x**n)/(2*m) - a*b*e**3*n*
x*x**(3*m)*(f*x)**(m - 1)/(8*m**2) + b**2*d**3*x*(f*x)**(m - 1)*log(c*x**n)
**2/m - 2*b**2*d**3*n*x*(f*x)**(m - 1)*log(c*x**n)/m**2 + 2*b**2*d**3*n**2*
x*(f*x)**(m - 1)/m**3 + 3*b**2*d**2*e*x*x**m*(f*x)**(m - 1)*log(c*x**n)**2/
(2*m) - 3*b**2*d**2*e*n*x*x**m*(f*x)**(m - 1)*log(c*x**n)/(2*m**2) + 3*b**2
*d**2*e*n**2*x*x**m*(f*x)**(m - 1)/(4*m**3) + b**2*d*e**2*x*x**(2*m)*(f*x)*
*(m - 1)*log(c*x**n)**2/m - 2*b**2*d*e**2*n*x*x**(2*m)*(f*x)**(m - 1)*log(c
*x**n)/(3*m**2) + 2*b**2*d*e**2*n**2*x*x**(2*m)*(f*x)**(m - 1)/(9*m**3) + b
**2*e**3*x*x**(3*m)*(f*x)**(m - 1)*log(c*x**n)**2/(4*m) - b**2*e**3*n*x*x**
(3*m)*(f*x)**(m - 1)*log(c*x**n)/(8*m**2) + b**2*e**3*n**2*x*x**(3*m)*(f*x)
**(m - 1)/(32*m**3), Ne(m, 0)), ((d + e)**3*Piecewise(((a**2*log(c*x**n) +
a*b*log(c*x**n)**2 + b**2*log(c*x**n)**3/3)/n, Ne(n, 0)), ((a**2 + 2*a*b*lo
g(c) + b**2*log(c)**2)*log(x), True))/f, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.55

$$\int (fx)^{-1+m}(d+ex^m)^3(a+b\log(cx^n))^2\,dx$$
$$= \frac{b^2e^3f^{m-1}x^{4m}\log(cx^n)^2}{4m} + \frac{b^2de^2f^{m-1}x^{3m}\log(cx^n)^2}{m} + \frac{3b^2d^2ef^{m-1}x^{2m}\log(cx^n)^2}{2m}$$
$$+ \frac{abe^3f^{m-1}x^{4m}\log(cx^n)}{2m} + \frac{2abde^2f^{m-1}x^{3m}\log(cx^n)}{m}$$
$$+ \frac{3abd^2ef^{m-1}x^{2m}\log(cx^n)}{m} - 2\left(\frac{f^{m-1}nx^m\log(cx^n)}{m^2} - \frac{f^{m-1}n^2x^m}{m^3}\right)b^2d^3$$
$$- \frac{3}{4}\left(\frac{2f^{m-1}nx^{2m}\log(cx^n)}{m^2} - \frac{f^{m-1}n^2x^{2m}}{m^3}\right)b^2d^2e$$
$$- \frac{2}{9}\left(\frac{3f^{m-1}nx^{3m}\log(cx^n)}{m^2} - \frac{f^{m-1}n^2x^{3m}}{m^3}\right)b^2de^2$$
$$- \frac{1}{32}\left(\frac{4f^{m-1}nx^{4m}\log(cx^n)}{m^2} - \frac{f^{m-1}n^2x^{4m}}{m^3}\right)b^2e^3 + \frac{a^2e^3f^{m-1}x^{4m}}{4m} - \frac{abe^3f^{m-1}nx^{4m}}{8m^2}$$
$$+ \frac{a^2de^2f^{m-1}x^{3m}}{m} - \frac{2abde^2f^{m-1}nx^{3m}}{3m^2} + \frac{3a^2d^2ef^{m-1}x^{2m}}{2m} - \frac{3abd^2ef^{m-1}nx^{2m}}{2m^2}$$
$$- \frac{2abd^3f^{m-1}nx^m}{m^2} + \frac{(fx)^m b^2d^3\log(cx^n)^2}{fm} + \frac{2(fx)^m abd^3\log(cx^n)}{fm} + \frac{(fx)^m a^2d^3}{fm}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)^3*(a+b*log(c*x^n))^2,x, algorithm="maxima"
)
```

```
[Out] 1/4*b^2*e^3*f^(m - 1)*x^(4*m)*log(c*x^n)^2/m + b^2*d*e^2*f^(m - 1)*x^(3*m)*
log(c*x^n)^2/m + 3/2*b^2*d^2*e*f^(m - 1)*x^(2*m)*log(c*x^n)^2/m + 1/2*a*b*e
^3*f^(m - 1)*x^(4*m)*log(c*x^n)/m + 2*a*b*d*e^2*f^(m - 1)*x^(3*m)*log(c*x^n
)/m + 3*a*b*d^2*e*f^(m - 1)*x^(2*m)*log(c*x^n)/m - 2*(f^(m - 1)*n*x^m*log(c
*x^n)/m^2 - f^(m - 1)*n^2*x^m/m^3)*b^2*d^3 - 3/4*(2*f^(m - 1)*n*x^(2*m)*log
(c*x^n)/m^2 - f^(m - 1)*n^2*x^(2*m)/m^3)*b^2*d^2*e - 2/9*(3*f^(m - 1)*n*x^(
3*m)*log(c*x^n)/m^2 - f^(m - 1)*n^2*x^(3*m)/m^3)*b^2*d*e^2 - 1/32*(4*f^(m -
 1)*n*x^(4*m)*log(c*x^n)/m^2 - f^(m - 1)*n^2*x^(4*m)/m^3)*b^2*e^3 + 1/4*a^2
*e^3*f^(m - 1)*x^(4*m)/m - 1/8*a*b*e^3*f^(m - 1)*n*x^(4*m)/m^2 + a^2*d*e^2*
f^(m - 1)*x^(3*m)/m - 2/3*a*b*d*e^2*f^(m - 1)*n*x^(3*m)/m^2 + 3/2*a^2*d^2*e
*f^(m - 1)*x^(2*m)/m - 3/2*a*b*d^2*e*f^(m - 1)*n*x^(2*m)/m^2 - 2*a*b*d^3*f^
(m - 1)*n*x^m/m^2 + (f*x)^m*b^2*d^3*log(c*x^n)^2/(f*m) + 2*(f*x)^m*a*b*d^3*
log(c*x^n)/(f*m) + (f*x)^m*a^2*d^3/(f*m)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 995 vs. $2(354) = 708$.

Time = 0.62 (sec) , antiderivative size = 995, normalized size of antiderivative = 2.67

$$\int (fx)^{-1+m}(d+ex^m)^3(a+b\log(cx^n))^2\,dx$$
$$\begin{aligned}
&= \frac{b^2e^3f^mn^2x^{4m}\log(x)^2}{4fm} + \frac{b^2de^2f^mn^2x^{3m}\log(x)^2}{fm} + \frac{3b^2d^2ef^mn^2x^{2m}\log(x)^2}{2fm} \\
&+ \frac{b^2d^3f^mn^2x^m\log(x)^2}{fm} + \frac{b^2e^3f^mnx^{4m}\log(c)\log(x)}{2fm} + \frac{2b^2de^2f^mnx^{3m}\log(c)\log(x)}{fm} \\
&+ \frac{3b^2d^2ef^mnx^{2m}\log(c)\log(x)}{fm} + \frac{2b^2d^3f^mnx^m\log(c)\log(x)}{fm} + \frac{b^2e^3f^mx^{4m}\log(c)^2}{4fm} \\
&+ \frac{b^2de^2f^mx^{3m}\log(c)^2}{fm} + \frac{3b^2d^2ef^mx^{2m}\log(c)^2}{2fm} + \frac{b^2d^3f^mx^m\log(c)^2}{fm} \\
&+ \frac{abe^3f^mnx^{4m}\log(x)}{2fm} - \frac{b^2e^3f^mn^2x^{4m}\log(x)}{8fm^2} + \frac{2abde^2f^mnx^{3m}\log(x)}{fm} \\
&- \frac{2b^2de^2f^mn^2x^{3m}\log(x)}{3fm^2} + \frac{3abd^2ef^mnx^{2m}\log(x)}{fm} - \frac{3b^2d^2ef^mn^2x^{2m}\log(x)}{2fm^2} \\
&+ \frac{2abd^3f^mnx^m\log(x)}{fm} - \frac{2b^2d^3f^mn^2x^m\log(x)}{fm^2} + \frac{abe^3f^mx^{4m}\log(c)}{2fm} \\
&- \frac{b^2e^3f^mnx^{4m}\log(c)}{8fm^2} + \frac{2abde^2f^mx^{3m}\log(c)}{fm} - \frac{2b^2de^2f^mnx^{3m}\log(c)}{3fm^2} \\
&+ \frac{3abd^2ef^mx^{2m}\log(c)}{fm} - \frac{3b^2d^2ef^mnx^{2m}\log(c)}{2fm^2} + \frac{2abd^3f^mx^m\log(c)}{fm} \\
&- \frac{2b^2d^3f^mnx^m\log(c)}{fm^2} + \frac{a^2e^3f^mx^{4m}}{4fm} - \frac{abe^3f^mnx^{4m}}{8fm^2} + \frac{b^2e^3f^mn^2x^{4m}}{32fm^3} \\
&+ \frac{a^2de^2f^mx^{3m}}{fm} - \frac{2abde^2f^mnx^{3m}}{3fm^2} + \frac{2b^2de^2f^mn^2x^{3m}}{9fm^3} + \frac{3a^2d^2ef^mx^{2m}}{2fm} \\
&- \frac{3abd^2ef^mnx^{2m}}{2fm^2} + \frac{3b^2d^2ef^mn^2x^{2m}}{4fm^3} + \frac{a^2d^3f^mx^m}{fm} - \frac{2abd^3f^mnx^m}{fm^2} + \frac{2b^2d^3f^mn^2x^m}{fm^3}
\end{aligned}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)^3*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/4*b^2*e^3*f^m*n^2*x^(4*m)*log(x)^2/(f*m) + b^2*d*e^2*f^m*n^2*x^(3*m)*log(
x)^2/(f*m) + 3/2*b^2*d^2*e*f^m*n^2*x^(2*m)*log(x)^2/(f*m) + b^2*d^3*f^m*n^2
*x^m*log(x)^2/(f*m) + 1/2*b^2*e^3*f^m*n*x^(4*m)*log(c)*log(x)/(f*m) + 2*b^2
*d*e^2*f^m*n*x^(3*m)*log(c)*log(x)/(f*m) + 3*b^2*d^2*e*f^m*n*x^(2*m)*log(c)
*log(x)/(f*m) + 2*b^2*d^3*f^m*n*x^m*log(c)*log(x)/(f*m) + 1/4*b^2*e^3*f^m*x
^(4*m)*log(c)^2/(f*m) + b^2*d*e^2*f^m*x^(3*m)*log(c)^2/(f*m) + 3/2*b^2*d^2*
e*f^m*x^(2*m)*log(c)^2/(f*m) + b^2*d^3*f^m*x^m*log(c)^2/(f*m) + 1/2*a*b*e^3
*f^m*n*x^(4*m)*log(x)/(f*m) - 1/8*b^2*e^3*f^m*n^2*x^(4*m)*log(x)/(f*m^2) +
2*a*b*d*e^2*f^m*n*x^(3*m)*log(x)/(f*m) - 2/3*b^2*d*e^2*f^m*n^2*x^(3*m)*log(
```

```
x)/(f*m^2) + 3*a*b*d^2*e*f^m*n*x^(2*m)*log(x)/(f*m) - 3/2*b^2*d^2*e*f^m*n^2
*x^(2*m)*log(x)/(f*m^2) + 2*a*b*d^3*f^m*n*x^m*log(x)/(f*m) - 2*b^2*d^3*f^m*
n^2*x^m*log(x)/(f*m^2) + 1/2*a*b*e^3*f^m*x^(4*m)*log(c)/(f*m) - 1/8*b^2*e^3
*f^m*n*x^(4*m)*log(c)/(f*m^2) + 2*a*b*d*e^2*f^m*x^(3*m)*log(c)/(f*m) - 2/3*
b^2*d*e^2*f^m*n*x^(3*m)*log(c)/(f*m^2) + 3*a*b*d^2*e*f^m*x^(2*m)*log(c)/(f*
m) - 3/2*b^2*d^2*e*f^m*n*x^(2*m)*log(c)/(f*m^2) + 2*a*b*d^3*f^m*x^m*log(c)/
(f*m) - 2*b^2*d^3*f^m*n*x^m*log(c)/(f*m^2) + 1/4*a^2*e^3*f^m*x^(4*m)/(f*m)
- 1/8*a*b*e^3*f^m*n*x^(4*m)/(f*m^2) + 1/32*b^2*e^3*f^m*n^2*x^(4*m)/(f*m^3)
+ a^2*d*e^2*f^m*x^(3*m)/(f*m) - 2/3*a*b*d*e^2*f^m*n*x^(3*m)/(f*m^2) + 2/9*b
^2*d*e^2*f^m*n^2*x^(3*m)/(f*m^3) + 3/2*a^2*d^2*e*f^m*x^(2*m)/(f*m) - 3/2*a*
b*d^2*e*f^m*n*x^(2*m)/(f*m^2) + 3/4*b^2*d^2*e*f^m*n^2*x^(2*m)/(f*m^3) + a^2
*d^3*f^m*x^m/(f*m) - 2*a*b*d^3*f^m*n*x^m/(f*m^2) + 2*b^2*d^3*f^m*n^2*x^m/(f
*m^3)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+m} (d+ex^m)^3 (a+b\log(cx^n))^2 \, dx = \int (f\,x)^{m-1} (d+e\,x^m)^3 (a+b\,\ln(c\,x^n))^2 \, dx$$

```
[In] int((f*x)^(m - 1)*(d + e*x^m)^3*(a + b*log(c*x^n))^2,x)
[Out] int((f*x)^(m - 1)*(d + e*x^m)^3*(a + b*log(c*x^n))^2, x)
```

3.360 $\int (fx)^{-1+m} (d+ex^m)^2 (a+b\log(cx^n))^2 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 298

$$\begin{aligned}\int (fx)^{-1+m} (d+ex^m)^2 (a+b\log(cx^n))^2 \, dx &= \frac{2b^2d^2n^2x(fx)^{-1+m}}{m^3} + \frac{b^2den^2x^{1+m}(fx)^{-1+m}}{2m^3} + \frac{2b^2e^2n^2x^{1+2m}(fx)^{-1+m}}{27m^3} \\ &+ \frac{b^2d^3n^2x^{1-m}(fx)^{-1+m}\log^2(x)}{3em} - \frac{2bd^2nx(fx)^{-1+m}(a+b\log(cx^n))}{m^2} \\ &- \frac{bdenx^{1+m}(fx)^{-1+m}(a+b\log(cx^n))}{m^2} - \frac{2be^2nx^{1+2m}(fx)^{-1+m}(a+b\log(cx^n))}{9m^2} \\ &- \frac{2bd^3nx^{1-m}(fx)^{-1+m}\log(x)(a+b\log(cx^n))}{3em} \\ &+ \frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^3(a+b\log(cx^n))^2}{3em}\end{aligned}$$

```
[Out] 2*b^2*d^2*n^2*x*(f*x)^(-1+m)/m^3+1/2*b^2*d*e*n^2*x^(1+m)*(f*x)^(-1+m)/m^3+2
/27*b^2*e^2*n^2*x^(1+2*m)*(f*x)^(-1+m)/m^3+1/3*b^2*d^3*n^2*x^(1-m)*(f*x)^(-
1+m)*ln(x)^2/e/m-2*b*d^2*n*x*(f*x)^(-1+m)*(a+b*ln(c*x^n))/m^2-b*d*e*n*x^(1+
m)*(f*x)^(-1+m)*(a+b*ln(c*x^n))/m^2-2/9*b*e^2*n*x^(1+2*m)*(f*x)^(-1+m)*(a+b
*ln(c*x^n))/m^2-2/3*b*d^3*n*x^(1-m)*(f*x)^(-1+m)*ln(x)*(a+b*ln(c*x^n))/e/m+
1/3*x^(1-m)*(f*x)^(-1+m)*(d+e*x^m)^3*(a+b*ln(c*x^n))^2/e/m
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2377, 2376, 272, 45, 2372, 12, 14, 2338}

$$\int (fx)^{-1+m}(d+ex^m)^2(a+b\log(cx^n))^2\,dx = -\frac{2bd^3nx^{1-m}\log(x)(fx)^{m-1}(a+b\log(cx^n))}{3em} - \frac{2bd^2nx(fx)^{m-1}(a+b\log(cx^n))}{m^2} - \frac{bdenx^{m+1}(fx)^{m-1}(a+b\log(cx^n))}{m^2} + \frac{x^{1-m}(fx)^{m-1}(d+ex^m)^3(a+b\log(cx^n))^2}{3em} - \frac{2be^2nx^{2m+1}(fx)^{m-1}(a+b\log(cx^n))}{9m^2} + \frac{b^2d^3n^2x^{1-m}\log^2(x)(fx)^{m-1}}{3em} + \frac{2b^2d^2n^2x(fx)^{m-1}}{m^3} + \frac{b^2den^2x^{m+1}(fx)^{m-1}}{2m^3} + \frac{2b^2e^2n^2x^{2m+1}(fx)^{m-1}}{27m^3}$$

```
[In] Int[(f*x)^(-1 + m)*(d + e*x^m)^2*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (2*b^2*d^2*n^2*x*(f*x)^(-1 + m))/m^3 + (b^2*d*e*n^2*x^(1 + m)*(f*x)^(-1 + m
))/(2*m^3) + (2*b^2*e^2*n^2*x^(1 + 2*m)*(f*x)^(-1 + m))/(27*m^3) + (b^2*d^3
*n^2*x^(1 - m)*(f*x)^(-1 + m)*Log[x]^2)/(3*e*m) - (2*b*d^2*n*x*(f*x)^(-1 +
m)*(a + b*Log[c*x^n]))/m^2 - (b*d*e*n*x^(1 + m)*(f*x)^(-1 + m)*(a + b*Log[c
*x^n]))/m^2 - (2*b*e^2*n*x^(1 + 2*m)*(f*x)^(-1 + m)*(a + b*Log[c*x^n]))/(9*
m^2) - (2*b*d^3*n*x^(1 - m)*(f*x)^(-1 + m)*Log[x]*(a + b*Log[c*x^n]))/(3*e*
m) + (x^(1 - m)*(f*x)^(-1 + m)*(d + e*x^m)^3*(a + b*Log[c*x^n])^2)/(3*e*m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rule 2377

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(d + e*x^r)^q
*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] &&  !(IntegerQ[m] || GtQ[f, 0])
```

Rubi steps

$$\text{integral} = \left(x^{1-m}(fx)^{-1+m}\right)\int x^{-1+m}(d+ex^m)^2\,(a+b\log(cx^n))^2\,dx$$

$$= \frac{x^{1-m}(fx)^{-1+m}\,(d+ex^m)^3\,(a+b\log(cx^n))^2}{3em} - \frac{\left(2bnx^{1-m}(fx)^{-1+m}\right)\int\frac{(d+ex^m)^3(a+b\log(cx^n))}{x}\,dx}{3em}$$

$$=-\frac{2bd^2nx(fx)^{-1+m}(a+b\log(cx^n))}{m^2}-\frac{bdenx^{1+m}(fx)^{-1+m}(a+b\log(cx^n))}{m^2}-\frac{2be^2nx^{1+2m}(fx)^{-1+m}(a+b\log(cx^n))}{9m^2}-\frac{2bd^3nx^{1-m}(fx)^{-1+m}\log(x)(a+b\log(cx^n))}{3em}+\frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^3(a+b\log(cx^n))^2}{3em}+\frac{\left(2b^2n^2x^{1-m}(fx)^{-1+m}\right)\int\frac{ex^m(18d^2+9dex^m+2e^2x^{2m})+6d^3m\log(x)}{6mx}\,dx}{3em}$$

$$=-\frac{2bd^2nx(fx)^{-1+m}(a+b\log(cx^n))}{m^2}-\frac{bdenx^{1+m}(fx)^{-1+m}(a+b\log(cx^n))}{m^2}-\frac{2be^2nx^{1+2m}(fx)^{-1+m}(a+b\log(cx^n))}{9m^2}-\frac{2bd^3nx^{1-m}(fx)^{-1+m}\log(x)(a+b\log(cx^n))}{3em}+\frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^3(a+b\log(cx^n))^2}{3em}+\frac{\left(b^2n^2x^{1-m}(fx)^{-1+m}\right)\int\frac{ex^m(18d^2+9dex^m+2e^2x^{2m})+6d^3m\log(x)}{x}\,dx}{9em^2}$$

$$=-\frac{2bd^2nx(fx)^{-1+m}(a+b\log(cx^n))}{m^2}-\frac{bdenx^{1+m}(fx)^{-1+m}(a+b\log(cx^n))}{m^2}-\frac{2be^2nx^{1+2m}(fx)^{-1+m}(a+b\log(cx^n))}{9m^2}-\frac{2bd^3nx^{1-m}(fx)^{-1+m}\log(x)(a+b\log(cx^n))}{3em}+\frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^3(a+b\log(cx^n))^2}{3em}+\frac{\left(b^2n^2x^{1-m}(fx)^{-1+m}\right)\int\left(18d^2ex^{-1+m}+9de^2x^{-1+2m}+2e^3x^{-1+3m}+\frac{6d^3m\log(x)}{x}\right)dx}{9em^2}$$

$$=\frac{2b^2d^2n^2x(fx)^{-1+m}}{m^3}+\frac{b^2den^2x^{1+m}(fx)^{-1+m}}{2m^3}+\frac{2b^2e^2n^2x^{1+2m}(fx)^{-1+m}}{27m^3}-\frac{2bd^2nx(fx)^{-1+m}(a+b\log(cx^n))}{m^2}-\frac{bdenx^{1+m}(fx)^{-1+m}(a+b\log(cx^n))}{m^2}-\frac{2be^2nx^{1+2m}(fx)^{-1+m}(a+b\log(cx^n))}{9m^2}-\frac{2bd^3nx^{1-m}(fx)^{-1+m}\log(x)(a+b\log(cx^n))}{3em}+\frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^3(a+b\log(cx^n))^2}{3em}+\frac{\left(2b^2d^3n^2x^{1-m}(fx)^{-1+m}\right)\int\frac{\log(x)}{x}\,dx}{3em}$$

$$= \frac{2b^2d^2n^2x(fx)^{-1+m}}{m^3} + \frac{b^2den^2x^{1+m}(fx)^{-1+m}}{2m^3} + \frac{2b^2e^2n^2x^{1+2m}(fx)^{-1+m}}{27m^3} + \frac{b^2d^3n^2x^{1-m}(fx)^{-1+m}\log^2(x)}{3em} - \frac{2bd^2nx(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)}{m^2} - \frac{bdenx^{1+m}(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)}{m^2} - \frac{2be^2nx^{1+2m}(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)}{9m^2} - \frac{2bd^3nx^{1-m}(fx)^{-1+m}\log(x)\left(a+b\log\left(cx^n\right)\right)}{3em} + \frac{x^{1-m}(fx)^{-1+m}\left(d+ex^m\right)^3\left(a+b\log\left(cx^n\right)\right)^2}{3em}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.69

$$\int (fx)^{-1+m}\left(d+ex^m\right)^2\left(a+b\log\left(cx^n\right)\right)^2\,dx$$
$$= \frac{(fx)^m\left(18a^2m^2(3d^2+3dex^m+e^2x^{2m}) - 6abmn(18d^2+9dex^m+2e^2x^{2m}) + b^2n^2(108d^2+27dex^m+4e^2\cdots\right.}{\cdots}$$

```
[In] Integrate[(f*x)^(-1 + m)*(d + e*x^m)^2*(a + b*Log[c*x^n])^2,x]
```

```
[Out] ((f*x)^m*(18*a^2*m^2*(3*d^2 + 3*d*e*x^m + e^2*x^(2*m)) - 6*a*b*m*n*(18*d^2
+ 9*d*e*x^m + 2*e^2*x^(2*m)) + b^2*n^2*(108*d^2 + 27*d*e*x^m + 4*e^2*x^(2*m
)) + 6*b*m*(6*a*m*(3*d^2 + 3*d*e*x^m + e^2*x^(2*m)) - b*n*(18*d^2 + 9*d*e*x
^m + 2*e^2*x^(2*m)))*Log[c*x^n] + 18*b^2*m^2*(3*d^2 + 3*d*e*x^m + e^2*x^(2*
m))*Log[c*x^n]^2))/(54*f*m^3)
```

Maple [A] (verified)

Time = 50.86 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.47

method	result
parallelrisch	$-\frac{-18e^2b^2(fx)^{m-1}\ln(cx^n)^2x^{2m}xm^2-36xx^{2m}\ln(cx^n)(fx)^{m-1}abe^2m^2+12xx^{2m}\ln(cx^n)(fx)^{m-1}b^2e^2mn-54b^2de(fx)^m\cdots}{\cdots}$
risch	Expression too large to display

```
[In] int((f*x)^(m-1)*(d+e*x^m)^2*(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/54*(-18*e^2*b^2*(f*x)^(m-1)*ln(c*x^n)^2*(x^m)^2*x*m^2-36*x*(x^m)^2*ln(c*
x^n)*(f*x)^(m-1)*a*b*e^2*m^2+12*x*(x^m)^2*ln(c*x^n)*(f*x)^(m-1)*b^2*e^2*m*n
-54*b^2*d*e*(f*x)^(m-1)*ln(c*x^n)^2*x^m*x*m^2-18*x*(x^m)^2*(f*x)^(m-1)*a^2*
e^2*m^2+12*x*(x^m)^2*(f*x)^(m-1)*a*b*e^2*m*n-4*x*(x^m)^2*(f*x)^(m-1)*b^2*e^
2*n^2-108*x*x^m*ln(c*x^n)*(f*x)^(m-1)*a*b*d*e*m^2+54*x*x^m*ln(c*x^n)*(f*x)^
(m-1)*b^2*d*e*m*n-54*b^2*d^2*(f*x)^(m-1)*ln(c*x^n)^2*x*m^2-54*x*x^m*(f*x)^(
```

```
m-1)*a^2*d*e*m^2+54*x*x^m*(f*x)^(m-1)*a*b*d*e*m*n-27*x*x^m*(f*x)^(m-1)*b^2*
d*e*n^2-108*x*ln(c*x^n)*(f*x)^(m-1)*a*b*d^2*m^2+108*x*ln(c*x^n)*(f*x)^(m-1)
*b^2*d^2*m*n-54*x*(f*x)^(m-1)*a^2*d^2*m^2+108*x*(f*x)^(m-1)*a*b*d^2*m*n-108
*x*(f*x)^(m-1)*b^2*d^2*n^2)/m^3
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.41

$$\int (fx)^{-1+m}(d+ex^m)^2(a+b\log(cx^n))^2\,dx = \frac{2\left(9\,b^2e^2m^2n^2\log(x)^2+9\,b^2e^2m^2\log(c)^2+9\,a^2e^2m^2-6\,abe^2mn+2\,b^2e^2n^2+6\,(3\,abe^2m^2-b^2e^2mn)\log(\ldots\right.}{}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)^2*(a+b*log(c*x^n))^2,x, algorithm="fricas"
)
```

```
[Out] 1/54*(2*(9*b^2*e^2*m^2*n^2*log(x)^2 + 9*b^2*e^2*m^2*log(c)^2 + 9*a^2*e^2*m^
2 - 6*a*b*e^2*m*n + 2*b^2*e^2*n^2 + 6*(3*a*b*e^2*m^2 - b^2*e^2*m*n)*log(c)
+ 6*(3*b^2*e^2*m^2*n*log(c) + 3*a*b*e^2*m^2*n - b^2*e^2*m*n^2)*log(x))*f^(m
 - 1)*x^(3*m) + 27*(2*b^2*d*e*m^2*n^2*log(x)^2 + 2*b^2*d*e*m^2*log(c)^2 + 2
*a^2*d*e*m^2 - 2*a*b*d*e*m*n + b^2*d*e*n^2 + 2*(2*a*b*d*e*m^2 - b^2*d*e*m*n
)*log(c) + 2*(2*b^2*d*e*m^2*n*log(c) + 2*a*b*d*e*m^2*n - b^2*d*e*m*n^2)*log
(x))*f^(m - 1)*x^(2*m) + 54*(b^2*d^2*m^2*n^2*log(x)^2 + b^2*d^2*m^2*log(c)^
2 + a^2*d^2*m^2 - 2*a*b*d^2*m*n + 2*b^2*d^2*n^2 + 2*(a*b*d^2*m^2 - b^2*d^2*
m*n)*log(c) + 2*(b^2*d^2*m^2*n*log(c) + a*b*d^2*m^2*n - b^2*d^2*m*n^2)*log(
x))*f^(m - 1)*x^m)/m^3
```

Sympy [A] (verification not implemented)

Time = 12.60 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.85

$$\int (fx)^{-1+m}(d+ex^m)^2(a+b\log(cx^n))^2\,dx = \begin{cases} \frac{a^2d^2x(fx)^{m-1}}{m}+\frac{a^2dexx^m(fx)^{m-1}}{m}+\frac{a^2e^2xx^{2m}(fx)^{m-1}}{3m}+\frac{2abd^2x(fx)^{m-1}\log(cx^n)}{m}-\frac{2abd^2nx(fx)^{m-1}}{m^2}+\frac{2abdexx^m(fx)^{m-1}\log(\ldots}{m} \\ \frac{(d+e)^2\left(\begin{cases}\frac{a^2\log(cx^n)+ab\log(cx^n)^2+\frac{b^2\log(cx^n)^3}{3}}{n} & \text{for}\ n\neq 0\\ \left(a^2+2ab\log(c)+b^2\log(c)^2\right)\log(x) & \text{otherwise}\end{cases}\right)}{f} \end{cases}$$

```
[In] integrate((f*x)**(-1+m)*(d+e*x**m)**2*(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x*(f*x)**(m - 1)/m + a**2*d*e*x*x**m*(f*x)**(m - 1)/m
+ a**2*e**2*x*x**(2*m)*(f*x)**(m - 1)/(3*m) + 2*a*b*d**2*x*(f*x)**(m - 1)*l
og(c*x**n)/m - 2*a*b*d**2*n*x*(f*x)**(m - 1)/m**2 + 2*a*b*d*e*x*x**m*(f*x)*
*(m - 1)*log(c*x**n)/m - a*b*d*e*n*x*x**m*(f*x)**(m - 1)/m**2 + 2*a*b*e**2*
x*x**(2*m)*(f*x)**(m - 1)*log(c*x**n)/(3*m) - 2*a*b*e**2*n*x*x**(2*m)*(f*x)
**(m - 1)/(9*m**2) + b**2*d**2*x*(f*x)**(m - 1)*log(c*x**n)**2/m - 2*b**2*d
**2*n*x*(f*x)**(m - 1)*log(c*x**n)/m**2 + 2*b**2*d**2*n**2*x*(f*x)**(m - 1)
/m**3 + b**2*d*e*x*x**m*(f*x)**(m - 1)*log(c*x**n)**2/m - b**2*d*e*n*x*x**m
*(f*x)**(m - 1)*log(c*x**n)/m**2 + b**2*d*e*n**2*x*x**m*(f*x)**(m - 1)/(2*m
**3) + b**2*e**2*x*x**(2*m)*(f*x)**(m - 1)*log(c*x**n)**2/(3*m) - 2*b**2*e*
*2*n*x*x**(2*m)*(f*x)**(m - 1)*log(c*x**n)/(9*m**2) + 2*b**2*e**2*n**2*x*x*
*(2*m)*(f*x)**(m - 1)/(27*m**3), Ne(m, 0)), ((d + e)**2*Piecewise(((a**2*lo
g(c*x**n) + a*b*log(c*x**n)**2 + b**2*log(c*x**n)**3/3)/n, Ne(n, 0)), ((a**
2 + 2*a*b*log(c) + b**2*log(c)**2)*log(x), True))/f, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.40

$$\begin{aligned}
\int (fx)^{-1+m} (d+ex^m)^2 (a+b\log(cx^n))^2 \, dx
&= \frac{b^2e^2f^{m-1}x^{3m}\log(cx^n)^2}{3m} + \frac{b^2def^{m-1}x^{2m}\log(cx^n)^2}{m} + \frac{2abe^2f^{m-1}x^{3m}\log(cx^n)}{3m} \\
&\quad + \frac{2abdef^{m-1}x^{2m}\log(cx^n)}{m} - 2\left(\frac{f^{m-1}nx^m\log(cx^n)}{m^2} - \frac{f^{m-1}n^2x^m}{m^3}\right)b^2d^2 \\
&\quad - \frac{1}{2}\left(\frac{2f^{m-1}nx^{2m}\log(cx^n)}{m^2} - \frac{f^{m-1}n^2x^{2m}}{m^3}\right)b^2de \\
&\quad - \frac{2}{27}\left(\frac{3f^{m-1}nx^{3m}\log(cx^n)}{m^2} - \frac{f^{m-1}n^2x^{3m}}{m^3}\right)b^2e^2 + \frac{a^2e^2f^{m-1}x^{3m}}{3m} \\
&\quad - \frac{2abe^2f^{m-1}nx^{3m}}{9m^2} + \frac{a^2def^{m-1}x^{2m}}{m} - \frac{abdef^{m-1}nx^{2m}}{m^2} - \frac{2abd^2f^{m-1}nx^m}{m^2} \\
&\quad + \frac{(fx)^m b^2d^2\log(cx^n)^2}{fm} + \frac{2(fx)^m abd^2\log(cx^n)}{fm} + \frac{(fx)^m a^2d^2}{fm}
\end{aligned}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)^2*(a+b*log(c*x^n))^2,x, algorithm="maxima"
)
```

```
[Out] 1/3*b^2*e^2*f^(m - 1)*x^(3*m)*log(c*x^n)^2/m + b^2*d*e*f^(m - 1)*x^(2*m)*lo
g(c*x^n)^2/m + 2/3*a*b*e^2*f^(m - 1)*x^(3*m)*log(c*x^n)/m + 2*a*b*d*e*f^(m
- 1)*x^(2*m)*log(c*x^n)/m - 2*(f^(m - 1)*n*x^m*log(c*x^n)/m^2 - f^(m - 1)*n
^2*x^m/m^3)*b^2*d^2 - 1/2*(2*f^(m - 1)*n*x^(2*m)*log(c*x^n)/m^2 - f^(m - 1)
*n^2*x^(2*m)/m^3)*b^2*d*e - 2/27*(3*f^(m - 1)*n*x^(3*m)*log(c*x^n)/m^2 - f^
(m - 1)*n^2*x^(3*m)/m^3)*b^2*e^2 + 1/3*a^2*e^2*f^(m - 1)*x^(3*m)/m - 2/9*a*
```

```
b*e^2*f^(m - 1)*n*x^(3*m)/m^2 + a^2*d*e*f^(m - 1)*x^(2*m)/m - a*b*d*e*f^(m
- 1)*n*x^(2*m)/m^2 - 2*a*b*d^2*f^(m - 1)*n*x^m/m^2 + (f*x)^m*b^2*d^2*log(c*
x^n)^2/(f*m) + 2*(f*x)^m*a*b*d^2*log(c*x^n)/(f*m) + (f*x)^m*a^2*d^2/(f*m)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 715 vs. 2(286) = 572.

Time = 0.52 (sec) , antiderivative size = 715, normalized size of antiderivative = 2.40

$$\begin{aligned}&\int (fx)^{-1+m}(d+ex^m)^2(a+b\log(cx^n))^2\,dx\\
&=\frac{b^2e^2f^mn^2x^{3m}\log(x)^2}{3fm}+\frac{b^2def^mn^2x^{2m}\log(x)^2}{fm}+\frac{b^2d^2f^mn^2x^m\log(x)^2}{fm}\\
&+\frac{2b^2e^2f^mnx^{3m}\log(c)\log(x)}{3fm}+\frac{2b^2def^mnx^{2m}\log(c)\log(x)}{fm}\\
&+\frac{2b^2d^2f^mnx^m\log(c)\log(x)}{fm}+\frac{b^2e^2f^mx^{3m}\log(c)^2}{3fm}+\frac{b^2def^mx^{2m}\log(c)^2}{fm}\\
&+\frac{b^2d^2f^mx^m\log(c)^2}{fm}+\frac{2abe^2f^mnx^{3m}\log(x)}{3fm}-\frac{2b^2e^2f^mn^2x^{3m}\log(x)}{9fm^2}\\
&+\frac{2abdef^mnx^{2m}\log(x)}{fm}-\frac{b^2def^mn^2x^{2m}\log(x)}{fm^2}+\frac{2abd^2f^mnx^m\log(x)}{fm}\\
&-\frac{2b^2d^2f^mn^2x^m\log(x)}{fm^2}+\frac{2abe^2f^mx^{3m}\log(c)}{3fm}-\frac{2b^2e^2f^mnx^{3m}\log(c)}{9fm^2}\\
&+\frac{2abdef^mx^{2m}\log(c)}{fm}-\frac{b^2def^mnx^{2m}\log(c)}{fm^2}+\frac{2abd^2f^mx^m\log(c)}{fm}\\
&-\frac{2b^2d^2f^mnx^m\log(c)}{fm^2}+\frac{a^2e^2f^mx^{3m}}{3fm}-\frac{2abe^2f^mnx^{3m}}{9fm^2}+\frac{2b^2e^2f^mn^2x^{3m}}{27fm^3}+\frac{a^2def^mx^{2m}}{fm}\\
&-\frac{abdef^mnx^{2m}}{fm^2}+\frac{b^2def^mn^2x^{2m}}{2fm^3}+\frac{a^2d^2f^mx^m}{fm}-\frac{2abd^2f^mnx^m}{fm^2}+\frac{2b^2d^2f^mn^2x^m}{fm^3}\end{aligned}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)^2*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*e^2*f^m*n^2*x^(3*m)*log(x)^2/(f*m) + b^2*d*e*f^m*n^2*x^(2*m)*log(x)
^2/(f*m) + b^2*d^2*f^m*n^2*x^m*log(x)^2/(f*m) + 2/3*b^2*e^2*f^m*n*x^(3*m)*l
og(c)*log(x)/(f*m) + 2*b^2*d*e*f^m*n*x^(2*m)*log(c)*log(x)/(f*m) + 2*b^2*d^
2*f^m*n*x^m*log(c)*log(x)/(f*m) + 1/3*b^2*e^2*f^m*x^(3*m)*log(c)^2/(f*m) +
b^2*d*e*f^m*x^(2*m)*log(c)^2/(f*m) + b^2*d^2*f^m*x^m*log(c)^2/(f*m) + 2/3*a
*b*e^2*f^m*n*x^(3*m)*log(x)/(f*m) - 2/9*b^2*e^2*f^m*n^2*x^(3*m)*log(x)/(f*m
^2) + 2*a*b*d*e*f^m*n*x^(2*m)*log(x)/(f*m) - b^2*d*e*f^m*n^2*x^(2*m)*log(x)
/(f*m^2) + 2*a*b*d^2*f^m*n*x^m*log(x)/(f*m) - 2*b^2*d^2*f^m*n^2*x^m*log(x)/
(f*m^2) + 2/3*a*b*e^2*f^m*x^(3*m)*log(c)/(f*m) - 2/9*b^2*e^2*f^m*n*x^(3*m)*
log(c)/(f*m^2) + 2*a*b*d*e*f^m*x^(2*m)*log(c)/(f*m) - b^2*d*e*f^m*n*x^(2*m)
*log(c)/(f*m^2) + 2*a*b*d^2*f^m*x^m*log(c)/(f*m) - 2*b^2*d^2*f^m*n*x^m*log(
```

```
c)/(f*m^2) + 1/3*a^2*e^2*f^m*x^(3*m)/(f*m) - 2/9*a*b*e^2*f^m*n*x^(3*m)/(f*m
^2) + 2/27*b^2*e^2*f^m*n^2*x^(3*m)/(f*m^3) + a^2*d*e*f^m*x^(2*m)/(f*m) - a*
b*d*e*f^m*n*x^(2*m)/(f*m^2) + 1/2*b^2*d*e*f^m*n^2*x^(2*m)/(f*m^3) + a^2*d^2
*f^m*x^m/(f*m) - 2*a*b*d^2*f^m*n*x^m/(f*m^2) + 2*b^2*d^2*f^m*n^2*x^m/(f*m^3
)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+m} (d+ex^m)^2 (a+b\log(cx^n))^2 \, dx = \int (f\,x)^{m-1} (d+e\,x^m)^2 (a+b\,\ln(c\,x^n))^2 \, dx$$

```
[In] int((f*x)^(m - 1)*(d + e*x^m)^2*(a + b*log(c*x^n))^2,x)
[Out] int((f*x)^(m - 1)*(d + e*x^m)^2*(a + b*log(c*x^n))^2, x)
```

3.361 $\int (fx)^{-1+m}(d+ex^m)(a+b\log(cx^n))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 226

$$\begin{aligned}\int (fx)^{-1+m}(d+ex^m)(a+b\log(cx^n))^2\,dx =\ & \frac{2b^2dn^2x(fx)^{-1+m}}{m^3}+\frac{b^2en^2x^{1+m}(fx)^{-1+m}}{4m^3} \\ & +\frac{b^2d^2n^2x^{1-m}(fx)^{-1+m}\log^2(x)}{2em} \\ & -\frac{2bdnx(fx)^{-1+m}(a+b\log(cx^n))}{m^2} \\ & -\frac{benx^{1+m}(fx)^{-1+m}(a+b\log(cx^n))}{2m^2} \\ & -\frac{bd^2nx^{1-m}(fx)^{-1+m}\log(x)(a+b\log(cx^n))}{em} \\ & +\frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^2(a+b\log(cx^n))^2}{2em}\end{aligned}$$

```
[Out] 2*b^2*d*n^2*x*(f*x)^(-1+m)/m^3+1/4*b^2*e*n^2*x^(1+m)*(f*x)^(-1+m)/m^3+1/2*b
^2*d^2*n^2*x^(1-m)*(f*x)^(-1+m)*ln(x)^2/e/m-2*b*d*n*x*(f*x)^(-1+m)*(a+b*ln(
c*x^n))/m^2-1/2*b*e*n*x^(1+m)*(f*x)^(-1+m)*(a+b*ln(c*x^n))/m^2-b*d^2*n*x^(1
-m)*(f*x)^(-1+m)*ln(x)*(a+b*ln(c*x^n))/e/m+1/2*x^(1-m)*(f*x)^(-1+m)*(d+e*x^
m)^2*(a+b*ln(c*x^n))^2/e/m
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2377, 2376, 272, 45, 2372, 12, 14, 2338}

$$\int (fx)^{-1+m}(d+ex^m)(a+b\log(cx^n))^2\,dx = -\frac{bd^2nx^{1-m}\log(x)(fx)^{m-1}(a+b\log(cx^n))}{em} + \frac{x^{1-m}(fx)^{m-1}(d+ex^m)^2(a+b\log(cx^n))^2}{2em} - \frac{2bdnx(fx)^{m-1}(a+b\log(cx^n))}{m^2} - \frac{benx^{m+1}(fx)^{m-1}(a+b\log(cx^n))}{2m^2} + \frac{b^2d^2n^2x^{1-m}\log^2(x)(fx)^{m-1}}{2em} + \frac{2b^2dn^2x(fx)^{m-1}}{m^3} + \frac{b^2en^2x^{m+1}(fx)^{m-1}}{4m^3}$$

```
[In] Int[(f*x)^(-1 + m)*(d + e*x^m)*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (2*b^2*d*n^2*x*(f*x)^(-1 + m))/m^3 + (b^2*e*n^2*x^(1 + m)*(f*x)^(-1 + m))/(
4*m^3) + (b^2*d^2*n^2*x^(1 - m)*(f*x)^(-1 + m)*Log[x]^2)/(2*e*m) - (2*b*d*n
*x*(f*x)^(-1 + m)*(a + b*Log[c*x^n]))/m^2 - (b*e*n*x^(1 + m)*(f*x)^(-1 + m)
*(a + b*Log[c*x^n]))/(2*m^2) - (b*d^2*n*x^(1 - m)*(f*x)^(-1 + m)*Log[x]*(a
+ b*Log[c*x^n]))/(e*m) + (x^(1 - m)*(f*x)^(-1 + m)*(d + e*x^m)^2*(a + b*Log
[c*x^n])^2)/(2*e*m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rule 2377

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(d + e*x^r)^q
*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] &&  !(IntegerQ[m] || GtQ[f, 0])
```

Rubi steps

$$\text{integral} = \left(x^{1-m}(fx)^{-1+m}\right)\int x^{-1+m}(d+ex^m)\left(a+b\log\left(cx^n\right)\right)^2\,dx$$

$$=\frac{x^{1-m}(fx)^{-1+m}\left(d+ex^m\right)^2\left(a+b\log\left(cx^n\right)\right)^2}{2em}-\frac{\left(bnx^{1-m}(fx)^{-1+m}\right)\int\frac{(d+ex^m)^2(a+b\log(cx^n))}{x}\,dx}{em}$$

$$= -\frac{2bdnx(fx)^{-1+m}(a+b\log(cx^n))}{m^2} - \frac{benx^{1+m}(fx)^{-1+m}(a+b\log(cx^n))}{2m^2} - \frac{bd^2nx^{1-m}(fx)^{-1+m}\log(x)(a+b\log(cx^n))}{em} + \frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^2(a+b\log(cx^n))^2}{2em} + \frac{(b^2n^2x^{1-m}(fx)^{-1+m})\int \frac{ex^m(4d+ex^m)+2d^2m\log(x)}{2mx}dx}{em}$$

$$= -\frac{2bdnx(fx)^{-1+m}(a+b\log(cx^n))}{m^2} - \frac{benx^{1+m}(fx)^{-1+m}(a+b\log(cx^n))}{2m^2} - \frac{bd^2nx^{1-m}(fx)^{-1+m}\log(x)(a+b\log(cx^n))}{em} + \frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^2(a+b\log(cx^n))^2}{2em} + \frac{(b^2n^2x^{1-m}(fx)^{-1+m})\int \frac{ex^m(4d+ex^m)+2d^2m\log(x)}{x}dx}{2em^2}$$

$$= -\frac{2bdnx(fx)^{-1+m}(a+b\log(cx^n))}{m^2} - \frac{benx^{1+m}(fx)^{-1+m}(a+b\log(cx^n))}{2m^2} - \frac{bd^2nx^{1-m}(fx)^{-1+m}\log(x)(a+b\log(cx^n))}{em} + \frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^2(a+b\log(cx^n))^2}{2em} + \frac{(b^2n^2x^{1-m}(fx)^{-1+m})\int \left(4dex^{-1+m}+e^2x^{-1+2m}+\frac{2d^2m\log(x)}{x}\right)dx}{2em^2}$$

$$= \frac{2b^2dn^2x(fx)^{-1+m}}{m^3} + \frac{b^2en^2x^{1+m}(fx)^{-1+m}}{4m^3} - \frac{2bdnx(fx)^{-1+m}(a+b\log(cx^n))}{m^2} - \frac{benx^{1+m}(fx)^{-1+m}(a+b\log(cx^n))}{2m^2} - \frac{bd^2nx^{1-m}(fx)^{-1+m}\log(x)(a+b\log(cx^n))}{em} + \frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^2(a+b\log(cx^n))^2}{2em} + \frac{(b^2d^2n^2x^{1-m}(fx)^{-1+m})\int \frac{\log(x)}{x}dx}{em}$$

$$= \frac{2b^2dn^2x(fx)^{-1+m}}{m^3} + \frac{b^2en^2x^{1+m}(fx)^{-1+m}}{4m^3} + \frac{b^2d^2n^2x^{1-m}(fx)^{-1+m}\log^2(x)}{2em} - \frac{2bdnx(fx)^{-1+m}(a+b\log(cx^n))}{m^2} - \frac{benx^{1+m}(fx)^{-1+m}(a+b\log(cx^n))}{2m^2} - \frac{bd^2nx^{1-m}(fx)^{-1+m}\log(x)(a+b\log(cx^n))}{em} + \frac{x^{1-m}(fx)^{-1+m}(d+ex^m)^2(a+b\log(cx^n))^2}{2em}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.55

$$\int (fx)^{-1+m}(d+ex^m)(a+b\log(cx^n))^2\,dx$$
$$=\frac{(fx)^m\left(2a^2m^2(2d+ex^m)-2abmn(4d+ex^m)+b^2n^2(8d+ex^m)-2bm(-2am(2d+ex^m)+bn(4d+ex^m)\right.}{4fm^3}$$

```
[In] Integrate[(f*x)^(-1 + m)*(d + e*x^m)*(a + b*Log[c*x^n])^2,x]
```

```
[Out] ((f*x)^m*(2*a^2*m^2*(2*d + e*x^m) - 2*a*b*m*n*(4*d + e*x^m) + b^2*n^2*(8*d
+ e*x^m) - 2*b*m*(-2*a*m*(2*d + e*x^m) + b*n*(4*d + e*x^m))*Log[c*x^n] + 2*
b^2*m^2*(2*d + e*x^m)*Log[c*x^n]^2))/(4*f*m^3)
```

Maple [A] (verified)

Time = 10.75 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.15

method	result
parallelrisch	$-\frac{-2eb^2(fx)^{m-1}\ln(cx^n)^2x^mxm^2-4xx^m\ln(cx^n)(fx)^{m-1}abem^2+2xx^m\ln(cx^n)(fx)^{m-1}b^2emn-4b^2d(fx)^{m-1}\ln(cx^n)^2x}{}$
risch	Expression too large to display

```
[In] int((f*x)^(m-1)*(d+e*x^m)*(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-2*e*b^2*(f*x)^(m-1)*ln(c*x^n)^2*x^m*x*m^2-4*x*x^m*ln(c*x^n)*(f*x)^(m
-1)*a*b*e*m^2+2*x*x^m*ln(c*x^n)*(f*x)^(m-1)*b^2*e*m*n-4*b^2*d*(f*x)^(m-1)*l
n(c*x^n)^2*x*m^2-2*x*x^m*(f*x)^(m-1)*a^2*e*m^2+2*x*x^m*(f*x)^(m-1)*a*b*e*m*
n-x*x^m*(f*x)^(m-1)*b^2*e*n^2-8*x*ln(c*x^n)*(f*x)^(m-1)*a*b*d*m^2+8*x*ln(c*
x^n)*(f*x)^(m-1)*b^2*d*m*n-4*x*(f*x)^(m-1)*a^2*d*m^2+8*x*(f*x)^(m-1)*a*b*d*
m*n-8*x*(f*x)^(m-1)*b^2*d*n^2)/m^3
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.08

$$\int (fx)^{-1+m}(d+ex^m)(a+b\log(cx^n))^2\,dx$$
$$=\frac{\left(2\,b^2em^2n^2\log(x)^2+2\,b^2em^2\log(c)^2+2\,a^2em^2-2\,abemn+b^2en^2+2\,(2\,abem^2-b^2emn)\log(c)+2\,(2\,b\right.}{}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*b^2*e*m^2*n^2*log(x)^2 + 2*b^2*e*m^2*log(c)^2 + 2*a^2*e*m^2 - 2*a*b
*e*m*n + b^2*e*n^2 + 2*(2*a*b*e*m^2 - b^2*e*m*n)*log(c) + 2*(2*b^2*e*m^2*n*
log(c) + 2*a*b*e*m^2*n - b^2*e*m*n^2)*log(x))*f^(m - 1)*x^(2*m) + 4*(b^2*d*
m^2*n^2*log(x)^2 + b^2*d*m^2*log(c)^2 + a^2*d*m^2 - 2*a*b*d*m*n + 2*b^2*d*n
^2 + 2*(a*b*d*m^2 - b^2*d*m*n)*log(c) + 2*(b^2*d*m^2*n*log(c) + a*b*d*m^2*n
 - b^2*d*m*n^2)*log(x))*f^(m - 1)*x^m)/m^3
```

Sympy [A] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.56

$$\int (fx)^{-1+m}\left(d+ex^m\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx = \begin{cases} \frac{a^2dx(fx)^{m-1}}{m}+\frac{a^2exx^m(fx)^{m-1}}{2m}+\frac{2abdx(fx)^{m-1}\log\left(cx^n\right)}{m}-\frac{2abdnx(fx)^{m-1}}{m^2}+\frac{abexx^m(fx)^{m-1}\log\left(cx^n\right)}{m}-\frac{abenxx^m(fx)^{m-1}}{2m^2} \\ \frac{(d+e)\left(\begin{cases}\frac{a^2\log\left(cx^n\right)+ab\log\left(cx^n\right)^2+\frac{b^2\log\left(cx^n\right)^3}{3}}{n} & \text{for } n\neq 0\\ \left(a^2+2ab\log\left(c\right)+b^2\log\left(c\right)^2\right)\log\left(x\right) & \text{otherwise}\end{cases}\right)}{f} \end{cases}$$

```
[In] integrate((f*x)**(-1+m)*(d+e*x**m)*(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((a**2*d*x*(f*x)**(m - 1)/m + a**2*e*x*x**m*(f*x)**(m - 1)/(2*m) +
 2*a*b*d*x*(f*x)**(m - 1)*log(c*x**n)/m - 2*a*b*d*n*x*(f*x)**(m - 1)/m**2 +
 a*b*e*x*x**m*(f*x)**(m - 1)*log(c*x**n)/m - a*b*e*n*x*x**m*(f*x)**(m - 1)/
(2*m**2) + b**2*d*x*(f*x)**(m - 1)*log(c*x**n)**2/m - 2*b**2*d*n*x*(f*x)**(
m - 1)*log(c*x**n)/m**2 + 2*b**2*d*n**2*x*(f*x)**(m - 1)/m**3 + b**2*e*x*x*
*m*(f*x)**(m - 1)*log(c*x**n)**2/(2*m) - b**2*e*n*x*x**m*(f*x)**(m - 1)*log
(c*x**n)/(2*m**2) + b**2*e*n**2*x*x**m*(f*x)**(m - 1)/(4*m**3), Ne(m, 0)),
((d + e)*Piecewise(((a**2*log(c*x**n) + a*b*log(c*x**n)**2 + b**2*log(c*x**
n)**3/3)/n, Ne(n, 0)), ((a**2 + 2*a*b*log(c) + b**2*log(c)**2)*log(x), True
))/f, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.14

$$\int (fx)^{-1+m}\left(d+ex^m\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx = \frac{b^2ef^{m-1}x^{2m}\log\left(cx^n\right)^2}{2m}+\frac{abef^{m-1}x^{2m}\log\left(cx^n\right)}{m} - 2\left(\frac{f^{m-1}nx^m\log\left(cx^n\right)}{m^2}-\frac{f^{m-1}n^2x^m}{m^3}\right)b^2d - \frac{1}{4}\left(\frac{2f^{m-1}nx^{2m}\log\left(cx^n\right)}{m^2}-\frac{f^{m-1}n^2x^{2m}}{m^3}\right)b^2e+\frac{a^2ef^{m-1}x^{2m}}{2m}-\frac{abef^{m-1}nx^{2m}}{2m^2} - \frac{2abdf^{m-1}nx^m}{m^2}+\frac{(fx)^m b^2d\log\left(cx^n\right)^2}{fm}+\frac{2(fx)^m abd\log\left(cx^n\right)}{fm}+\frac{(fx)^m a^2d}{fm}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*e*f^(m - 1)*x^(2*m)*log(c*x^n)^2/m + a*b*e*f^(m - 1)*x^(2*m)*log(c*
x^n)/m - 2*(f^(m - 1)*n*x^m*log(c*x^n)/m^2 - f^(m - 1)*n^2*x^m/m^3)*b^2*d -
 1/4*(2*f^(m - 1)*n*x^(2*m)*log(c*x^n)/m^2 - f^(m - 1)*n^2*x^(2*m)/m^3)*b^2
*e + 1/2*a^2*e*f^(m - 1)*x^(2*m)/m - 1/2*a*b*e*f^(m - 1)*n*x^(2*m)/m^2 - 2*
a*b*d*f^(m - 1)*n*x^m/m^2 + (f*x)^m*b^2*d*log(c*x^n)^2/(f*m) + 2*(f*x)^m*a*
b*d*log(c*x^n)/(f*m) + (f*x)^m*a^2*d/(f*m)
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.92

$$\int (fx)^{-1+m}(d+ex^m)(a+b\log(cx^n))^2\,dx = \frac{b^2ef^mn^2x^{2m}\log(x)^2}{2fm} + \frac{b^2df^mn^2x^m\log(x)^2}{fm} + \frac{b^2ef^mnx^{2m}\log(c)\log(x)}{fm} + \frac{2b^2df^mnx^m\log(c)\log(x)}{fm} + \frac{b^2ef^mx^{2m}\log(c)^2}{2fm} + \frac{b^2df^mx^m\log(c)^2}{fm} + \frac{abef^mnx^{2m}\log(x)}{fm} - \frac{b^2ef^mn^2x^{2m}\log(x)}{2fm^2} + \frac{2abdf^mnx^m\log(x)}{fm} - \frac{2b^2df^mn^2x^m\log(x)}{fm^2} + \frac{abef^mx^{2m}\log(c)}{fm} - \frac{b^2ef^mnx^{2m}\log(c)}{2fm^2} + \frac{2abdf^mx^m\log(c)}{fm} - \frac{2b^2df^mnx^m\log(c)}{fm^2} + \frac{a^2ef^mx^{2m}}{2fm} - \frac{abef^mnx^{2m}}{2fm^2} + \frac{b^2ef^mn^2x^{2m}}{4fm^3} + \frac{a^2df^mx^m}{fm} - \frac{2abdf^mnx^m}{fm^2} + \frac{2b^2df^mn^2x^m}{fm^3}$$

```
[In] integrate((f*x)^(-1+m)*(d+e*x^m)*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*e*f^m*n^2*x^(2*m)*log(x)^2/(f*m) + b^2*d*f^m*n^2*x^m*log(x)^2/(f*m)
 + b^2*e*f^m*n*x^(2*m)*log(c)*log(x)/(f*m) + 2*b^2*d*f^m*n*x^m*log(c)*log(x
)/(f*m) + 1/2*b^2*e*f^m*x^(2*m)*log(c)^2/(f*m) + b^2*d*f^m*x^m*log(c)^2/(f*
m) + a*b*e*f^m*n*x^(2*m)*log(x)/(f*m) - 1/2*b^2*e*f^m*n^2*x^(2*m)*log(x)/(f
*m^2) + 2*a*b*d*f^m*n*x^m*log(x)/(f*m) - 2*b^2*d*f^m*n^2*x^m*log(x)/(f*m^2)
 + a*b*e*f^m*x^(2*m)*log(c)/(f*m) - 1/2*b^2*e*f^m*n*x^(2*m)*log(c)/(f*m^2)
+ 2*a*b*d*f^m*x^m*log(c)/(f*m) - 2*b^2*d*f^m*n*x^m*log(c)/(f*m^2) + 1/2*a^2
*e*f^m*x^(2*m)/(f*m) - 1/2*a*b*e*f^m*n*x^(2*m)/(f*m^2) + 1/4*b^2*e*f^m*n^2*
x^(2*m)/(f*m^3) + a^2*d*f^m*x^m/(f*m) - 2*a*b*d*f^m*n*x^m/(f*m^2) + 2*b^2*d
*f^m*n^2*x^m/(f*m^3)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+m}\,(d+ex^m)\,(a+b\log(cx^n))^2\,dx=\int (f\,x)^{m-1}\,(d+e\,x^m)\,\left(a+b\,\ln(c\,x^n)\right)^2\,dx$$

```
[In] int((f*x)^(m - 1)*(d + e*x^m)*(a + b*log(c*x^n))^2,x)
[Out] int((f*x)^(m - 1)*(d + e*x^m)*(a + b*log(c*x^n))^2, x)
```

3.362 $\int (fx)^{-1+m} (a + b\log(cx^n))^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int (fx)^{-1+m} (a + b\log(cx^n))^2 \, dx = \frac{2b^2n^2(fx)^m}{fm^3} - \frac{2bn(fx)^m (a + b\log(cx^n))}{fm^2} + \frac{(fx)^m (a + b\log(cx^n))^2}{fm}$$

```
[Out] 2*b^2*n^2*(f*x)^m/f/m^3-2*b*n*(f*x)^m*(a+b*ln(c*x^n))/f/m^2+(f*x)^m*(a+b*ln
(c*x^n))^2/f/m
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2342, 2341}

$$\int (fx)^{-1+m} (a + b\log(cx^n))^2 \, dx = -\frac{2bn(fx)^m (a + b\log(cx^n))}{fm^2} + \frac{(fx)^m (a + b\log(cx^n))^2}{fm} + \frac{2b^2n^2(fx)^m}{fm^3}$$

```
[In] Int[(f*x)^(-1 + m)*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (2*b^2*n^2*(f*x)^m)/(f*m^3) - (2*b*n*(f*x)^m*(a + b*Log[c*x^n]))/(f*m^2) +
((f*x)^m*(a + b*Log[c*x^n])^2)/(f*m)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(fx)^m\,(a+b\log(cx^n))^2}{fm}-\frac{(2bn)\int (fx)^{-1+m}\,(a+b\log(cx^n))\;dx}{m}\\ &= \frac{2b^2n^2(fx)^m}{fm^3}-\frac{2bn(fx)^m\,(a+b\log(cx^n))}{fm^2}+\frac{(fx)^m\,(a+b\log(cx^n))^2}{fm}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int (fx)^{-1+m}\,(a+b\log(cx^n))^2\;dx = \frac{(fx)^m\left(a^2m^2-2abmn+2b^2n^2+2bm(am-bn)\log(cx^n)+b^2m^2\log^2(cx^n)\right)}{fm^3}$$

```
[In] Integrate[(f*x)^(-1 + m)*(a + b*Log[c*x^n])^2,x]
```

```
[Out] ((f*x)^m*(a^2*m^2 - 2*a*b*m*n + 2*b^2*n^2 + 2*b*m*(a*m - b*n)*Log[c*x^n] +
b^2*m^2*Log[c*x^n]^2))/(f*m^3)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.72

method	result
parallelrisch	$-\frac{-x\ln(cx^n)^2(fx)^{m-1}b^2m^2-2x\ln(cx^n)(fx)^{m-1}ab\,m^2+2x\ln(cx^n)(fx)^{m-1}b^2mn-x(fx)^{m-1}a^2m^2+2x(fx)^{m-1}abmn-2x(}{m^3}$
risch	Expression too large to display

```
[In] int((f*x)^(m-1)*(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-x*ln(c*x^n)^2*(f*x)^(m-1)*b^2*m^2-2*x*ln(c*x^n)*(f*x)^(m-1)*a*b*m^2+2*x*
ln(c*x^n)*(f*x)^(m-1)*b^2*m*n-x*(f*x)^(m-1)*a^2*m^2+2*x*(f*x)^(m-1)*a*b*m*n
-2*x*(f*x)^(m-1)*b^2*n^2)/m^3
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.80

$$\int (fx)^{-1+m} (a + b\log(cx^n))^2 \, dx$$
$$= \frac{\left(b^2m^2n^2x\log(x)^2 + b^2m^2x\log(c)^2 + 2\,(abm^2 - b^2mn)x\log(c) + (a^2m^2 - 2\,abmn + 2\,b^2n^2)x + 2\,(b^2m^2n\right.}{m^3}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] (b^2*m^2*n^2*x*log(x)^2 + b^2*m^2*x*log(c)^2 + 2*(a*b*m^2 - b^2*m*n)*x*log(
c) + (a^2*m^2 - 2*a*b*m*n + 2*b^2*n^2)*x + 2*(b^2*m^2*n*x*log(c) + (a*b*m^2
*n - b^2*m*n^2)*x)*log(x))*e^((m - 1)*log(f) + (m - 1)*log(x))/m^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(63) = 126.

Time = 6.77 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.67

$$\int (fx)^{-1+m} (a + b\log(cx^n))^2 \, dx$$
$$= \begin{cases} \frac{a^2x(fx)^{m-1}}{m} + \frac{2abx(fx)^{m-1}\log(cx^n)}{m} - \frac{2abnx(fx)^{m-1}}{m^2} + \frac{b^2x(fx)^{m-1}\log(cx^n)^2}{m} - \frac{2b^2nx(fx)^{m-1}\log(cx^n)}{m^2} + \frac{2b^2n^2x(fx)^{m-1}}{m^3} \\ \frac{\begin{cases} \frac{a^2\log(cx^n) + ab\log(cx^n)^2 + \frac{b^2\log(cx^n)^3}{3}}{n} & \text{for } n \neq 0 \\ \left(a^2 + 2ab\log(c) + b^2\log(c)^2\right)\log(x) & \text{otherwise} \end{cases}}{f} \end{cases}$$

```
[In] integrate((f*x)**(-1+m)*(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((a**2*x*(f*x)**(m - 1)/m + 2*a*b*x*(f*x)**(m - 1)*log(c*x**n)/m -
 2*a*b*n*x*(f*x)**(m - 1)/m**2 + b**2*x*(f*x)**(m - 1)*log(c*x**n)**2/m - 2
*b**2*n*x*(f*x)**(m - 1)*log(c*x**n)/m**2 + 2*b**2*n**2*x*(f*x)**(m - 1)/m*
*3, Ne(m, 0)), (Piecewise(((a**2*log(c*x**n) + a*b*log(c*x**n)**2 + b**2*lo
g(c*x**n)**3/3)/n, Ne(n, 0)), ((a**2 + 2*a*b*log(c) + b**2*log(c)**2)*log(x
), True))/f, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.70

$$\int (fx)^{-1+m}(a+b\log(cx^n))^2\,dx = -2\left(\frac{f^{m-1}nx^m\log(cx^n)}{m^2} - \frac{f^{m-1}n^2x^m}{m^3}\right)b^2 - \frac{2\,abf^{m-1}nx^m}{m^2} + \frac{(fx)^m b^2\log(cx^n)^2}{fm} + \frac{2\,(fx)^m ab\log(cx^n)}{fm} + \frac{(fx)^m a^2}{fm}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] -2*(f^(m - 1)*n*x^m*log(c*x^n)/m^2 - f^(m - 1)*n^2*x^m/m^3)*b^2 - 2*a*b*f^(
m - 1)*n*x^m/m^2 + (f*x)^m*b^2*log(c*x^n)^2/(f*m) + 2*(f*x)^m*a*b*log(c*x^n
)/(f*m) + (f*x)^m*a^2/(f*m)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(69) = 138.

Time = 0.42 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.87

$$\int (fx)^{-1+m}(a+b\log(cx^n))^2\,dx = \frac{b^2f^mn^2x^m\log(x)^2}{fm} + \frac{2\,b^2f^mnx^m\log(c)\log(x)}{fm} + \frac{b^2f^mx^m\log(c)^2}{fm} + \frac{2\,abf^mnx^m\log(x)}{fm} - \frac{2\,b^2f^mn^2x^m\log(x)}{fm^2} + \frac{2\,abf^mx^m\log(c)}{fm} - \frac{2\,b^2f^mnx^m\log(c)}{fm^2} + \frac{a^2f^mx^m}{fm} - \frac{2\,abf^mnx^m}{fm^2} + \frac{2\,b^2f^mn^2x^m}{fm^3}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] b^2*f^m*n^2*x^m*log(x)^2/(f*m) + 2*b^2*f^m*n*x^m*log(c)*log(x)/(f*m) + b^2*
f^m*x^m*log(c)^2/(f*m) + 2*a*b*f^m*n*x^m*log(x)/(f*m) - 2*b^2*f^m*n^2*x^m*l
og(x)/(f*m^2) + 2*a*b*f^m*x^m*log(c)/(f*m) - 2*b^2*f^m*n*x^m*log(c)/(f*m^2)
 + a^2*f^m*x^m/(f*m) - 2*a*b*f^m*n*x^m/(f*m^2) + 2*b^2*f^m*n^2*x^m/(f*m^3)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)^2\,dx=\int {\left(f\,x\right)}^{m-1}\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\,dx$$

```
[In] int((f*x)^(m - 1)*(a + b*log(c*x^n))^2,x)
[Out] int((f*x)^(m - 1)*(a + b*log(c*x^n))^2, x)
```

3.363 $\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{d+ex^m}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 129

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{d+ex^m}\,dx = \frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))^2\log\left(1+\frac{ex^m}{d}\right)}{em} + \frac{2bnx^{1-m}(fx)^{-1+m}(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{em^2} - \frac{2b^2n^2x^{1-m}(fx)^{-1+m}\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{em^3}$$

```
[Out] x^(1-m)*(f*x)^(-1+m)*(a+b*ln(c*x^n))^2*ln(1+e*x^m/d)/e/m+2*b*n*x^(1-m)*(f*x
)^(-1+m)*(a+b*ln(c*x^n))*polylog(2,-e*x^m/d)/e/m^2-2*b^2*n^2*x^(1-m)*(f*x)^
(-1+m)*polylog(3,-e*x^m/d)/e/m^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2377, 2375, 2421, 6724}

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{d+ex^m}\,dx = \frac{2bnx^{1-m}(fx)^{m-1}\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)(a+b\log(cx^n))}{em^2} + \frac{x^{1-m}(fx)^{m-1}\log\left(\frac{ex^m}{d}+1\right)(a+b\log(cx^n))^2}{em} - \frac{2b^2n^2x^{1-m}(fx)^{m-1}\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{em^3}$$

```
[In] Int[((f*x)^(-1 + m)*(a + b*Log[c*x^n])^2)/(d + e*x^m),x]
```

```
[Out] (x^(1 - m)*(f*x)^(-1 + m)*(a + b*Log[c*x^n])^2*Log[1 + (e*x^m)/d])/(e*m) +
(2*b*n*x^(1 - m)*(f*x)^(-1 + m)*(a + b*Log[c*x^n])*PolyLog[2, -((e*x^m)/d)]
)/(e*m^2) - (2*b^2*n^2*x^(1 - m)*(f*x)^(-1 + m)*PolyLog[3, -((e*x^m)/d)])/(
e*m^3)
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2377

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(d + e*x^r)^q
*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] &&  !(IntegerQ[m] || GtQ[f, 0])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(x^{1-m}(fx)^{-1+m}\right)\int \frac{x^{-1+m}(a+b\log(cx^n))^2}{d+ex^m}\,dx\\ &= \frac{x^{1-m}(fx)^{-1+m}\left(a+b\log(cx^n)\right)^2\log\left(1+\frac{ex^m}{d}\right)}{em}\\ &\quad-\frac{\left(2bnx^{1-m}(fx)^{-1+m}\right)\int \frac{(a+b\log(cx^n))\log\left(1+\frac{ex^m}{d}\right)}{x}\,dx}{em}\end{aligned}$$

$$
\begin{aligned}
&= \frac{x^{1-m}(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)^2\log\left(1+\frac{ex^m}{d}\right)}{em} \\
&\quad + \frac{2bnx^{1-m}(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)\operatorname{Li}_2\left(-\frac{ex^m}{d}\right)}{em^2} \\
&\quad - \frac{\left(2b^2n^2x^{1-m}(fx)^{-1+m}\right)\int\frac{\operatorname{Li}_2\left(-\frac{ex^m}{d}\right)}{x}\,dx}{em^2} \\
&= \frac{x^{1-m}(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)^2\log\left(1+\frac{ex^m}{d}\right)}{em} \\
&\quad + \frac{2bnx^{1-m}(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)\operatorname{Li}_2\left(-\frac{ex^m}{d}\right)}{em^2} - \frac{2b^2n^2x^{1-m}(fx)^{-1+m}\operatorname{Li}_3\left(-\frac{ex^m}{d}\right)}{em^3}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 502 vs. $2(129) = 258$.

Time = 0.40 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.89

$$
\int \frac{(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)^2}{d+ex^m}\,dx
= \frac{x^{-m}(fx)^m\left(3a^2m^3\log(x)-6abm^3n\log^2(x)+4b^2m^3n^2\log^3(x)+6abm^3\log(x)\log\left(cx^n\right)-6b^2m^3n\log^2(x)\right.}{}
$$

```
[In] Integrate[((f*x)^(-1 + m)*(a + b*Log[c*x^n])^2)/(d + e*x^m),x]
```

```
[Out] ((f*x)^m*(3*a^2*m^3*Log[x] - 6*a*b*m^3*n*Log[x]^2 + 4*b^2*m^3*n^2*Log[x]^3
+ 6*a*b*m^3*Log[x]*Log[c*x^n] - 6*b^2*m^3*n*Log[x]^2*Log[c*x^n] + 3*b^2*m^3
*Log[x]*Log[c*x^n]^2 + 3*b^2*m^2*n^2*Log[x]^2*Log[1 + d/(e*x^m)] + 3*a^2*m^
2*Log[d - d*x^m] - 6*a*b*m^2*n*Log[x]*Log[d - d*x^m] + 3*b^2*m^2*n^2*Log[x]
^2*Log[d - d*x^m] + 6*a*b*m^2*Log[c*x^n]*Log[d - d*x^m] - 6*b^2*m^2*n*Log[x
]*Log[c*x^n]*Log[d - d*x^m] + 3*b^2*m^2*Log[c*x^n]^2*Log[d - d*x^m] + 6*a*b
*m^2*n*Log[x]*Log[d + e*x^m] - 6*b^2*m^2*n^2*Log[x]^2*Log[d + e*x^m] - 6*a*
b*m*n*Log[-((e*x^m)/d)]*Log[d + e*x^m] + 6*b^2*m*n^2*Log[x]*Log[-((e*x^m)/d
)]*Log[d + e*x^m] + 6*b^2*m^2*n*Log[x]*Log[c*x^n]*Log[d + e*x^m] - 6*b^2*m*
n*Log[-((e*x^m)/d)]*Log[c*x^n]*Log[d + e*x^m] - 6*b^2*m*n^2*Log[x]*PolyLog[
2, -(d/(e*x^m))] - 6*b*m*n*(a - b*n*Log[x] + b*Log[c*x^n])*PolyLog[2, 1 + (
e*x^m)/d] - 6*b^2*n^2*PolyLog[3, -(d/(e*x^m))]))/(3*e*f*m^3*x^m)
```

Maple [F]

$$\int \frac{(fx)^{m-1} (a+b\ln(c\,x^n))^2}{d+e\,x^m}dx$$

```
[In] int((f*x)^(m-1)*(a+b*ln(c*x^n))^2/(d+e*x^m),x)
```

```
[Out] int((f*x)^(m-1)*(a+b*ln(c*x^n))^2/(d+e*x^m),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.38

$$\int \frac{(fx)^{-1+m} (a+b\log(cx^n))^2}{d+ex^m}\,dx = -\frac{2\,b^2 f^{m-1} n^2 \mathrm{polylog}\left(3,-\frac{ex^m}{d}\right) - 2\left(b^2 m n^2 \log(x) + b^2 m n \log(c) + abmn\right) f^{m-1} \mathrm{Li}_2\left(-\frac{ex^m+d}{d}+1\right) - (b^2 m}{}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))^2/(d+e*x^m),x, algorithm="fricas")
```

```
[Out] -(2*b^2*f^(m - 1)*n^2*polylog(3, -e*x^m/d) - 2*(b^2*m*n^2*log(x) + b^2*m*n*
log(c) + a*b*m*n)*f^(m - 1)*dilog(-(e*x^m + d)/d + 1) - (b^2*m^2*log(c)^2 +
 2*a*b*m^2*log(c) + a^2*m^2)*f^(m - 1)*log(e*x^m + d) - (b^2*m^2*n^2*log(x)
^2 + 2*(b^2*m^2*n*log(c) + a*b*m^2*n)*log(x))*f^(m - 1)*log((e*x^m + d)/d))
/(e*m^3)
```

Sympy [F]

$$\int \frac{(fx)^{-1+m} (a+b\log(cx^n))^2}{d+ex^m}\,dx = \int \frac{(fx)^{m-1} (a+b\log(cx^n))^2}{d+ex^m}\,dx$$

```
[In] integrate((f*x)**(-1+m)*(a+b*ln(c*x**n))**2/(d+e*x**m),x)
```

```
[Out] Integral((f*x)**(m - 1)*(a + b*log(c*x**n))**2/(d + e*x**m), x)
```

Maxima [F]

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{d+ex^m}\,dx = \int \frac{(b\log(cx^n)+a)^2(fx)^{m-1}}{ex^m+d}\,dx$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))^2/(d+e*x^m),x, algorithm="maxima")
```

```
[Out] a^2*f^(m - 1)*log((e*x^m + d)/e)/(e*m) + integrate((b^2*f^m*x^m*log(x^n)^2
+ 2*(b^2*f^m*log(c) + a*b*f^m)*x^m*log(x^n) + (b^2*f^m*log(c)^2 + 2*a*b*f^m
*log(c))*x^m)/(e*f*x*x^m + d*f*x), x)
```

Giac [F]

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{d+ex^m}\,dx = \int \frac{(b\log(cx^n)+a)^2(fx)^{m-1}}{ex^m+d}\,dx$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))^2/(d+e*x^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*(f*x)^(m - 1)/(e*x^m + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{d+ex^m}\,dx = \int \frac{(f\,x)^{m-1}\,(a+b\,\ln(c\,x^n))^2}{d+e\,x^m}\,dx$$

```
[In] int(((f*x)^(m - 1)*(a + b*log(c*x^n))^2)/(d + e*x^m),x)
```

```
[Out] int(((f*x)^(m - 1)*(a + b*log(c*x^n))^2)/(d + e*x^m), x)
```

3.364 $\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 138

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^2}\,dx = -\frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))^2}{em(d+ex^m)} - \frac{2bnx^{1-m}(fx)^{-1+m}(a+b\log(cx^n))\log\left(1+\frac{dx^{-m}}{e}\right)}{dem^2} + \frac{2b^2n^2x^{1-m}(fx)^{-1+m}\operatorname{PolyLog}\left(2,-\frac{dx^{-m}}{e}\right)}{dem^3}$$

```
[Out] -x^(1-m)*(f*x)^(-1+m)*(a+b*ln(c*x^n))^2/e/m/(d+e*x^m)-2*b*n*x^(1-m)*(f*x)^(
-1+m)*(a+b*ln(c*x^n))*ln(1+d/e/(x^m))/d/e/m^2+2*b^2*n^2*x^(1-m)*(f*x)^(-1+m
)*polylog(2,-d/e/(x^m))/d/e/m^3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2377, 2376, 2379, 2438}

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^2}\,dx = -\frac{2bnx^{1-m}(fx)^{m-1}\log\left(\frac{dx^{-m}}{e}+1\right)(a+b\log(cx^n))}{dem^2} - \frac{x^{1-m}(fx)^{m-1}(a+b\log(cx^n))^2}{em(d+ex^m)} + \frac{2b^2n^2x^{1-m}(fx)^{m-1}\operatorname{PolyLog}\left(2,-\frac{dx^{-m}}{e}\right)}{dem^3}$$

```
[In] Int[((f*x)^(-1 + m)*(a + b*Log[c*x^n])^2)/(d + e*x^m)^2,x]
```

```
[Out] -((x^(1 - m)*(f*x)^(-1 + m)*(a + b*Log[c*x^n])^2)/(e*m*(d + e*x^m))) - (2*b
*n*x^(1 - m)*(f*x)^(-1 + m)*(a + b*Log[c*x^n])*Log[1 + d/(e*x^m)])/(d*e*m^2
) + (2*b^2*n^2*x^(1 - m)*(f*x)^(-1 + m)*PolyLog[2, -(d/(e*x^m))])/(d*e*m^3)
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rule 2377

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(d + e*x^r)^q
*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] &&  !(IntegerQ[m] || GtQ[f, 0])
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(x^{1-m}(fx)^{-1+m}\right)\int \frac{x^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^2}\,dx\\ &= -\frac{x^{1-m}(fx)^{-1+m}\,(a+b\log(cx^n))^2}{em\,(d+ex^m)}+\frac{(2bnx^{1-m}(fx)^{-1+m})\int \frac{a+b\log(cx^n)}{x(d+ex^m)}\,dx}{em}\end{aligned}$$

$$= -\frac{x^{1-m}(fx)^{-1+m}\left(a+b\log\left(cx^{n}\right)\right)^{2}}{em\left(d+ex^{m}\right)} - \frac{2bnx^{1-m}(fx)^{-1+m}\left(a+b\log\left(cx^{n}\right)\right)\log\left(1+\frac{dx^{-m}}{e}\right)}{dem^{2}} + \frac{\left(2b^{2}n^{2}x^{1-m}(fx)^{-1+m}\right)\int\frac{\log\left(1+\frac{dx^{-m}}{e}\right)}{x}dx}{dem^{2}}$$

$$= -\frac{x^{1-m}(fx)^{-1+m}\left(a+b\log\left(cx^{n}\right)\right)^{2}}{em\left(d+ex^{m}\right)} - \frac{2bnx^{1-m}(fx)^{-1+m}\left(a+b\log\left(cx^{n}\right)\right)\log\left(1+\frac{dx^{-m}}{e}\right)}{dem^{2}} + \frac{2b^{2}n^{2}x^{1-m}(fx)^{-1+m}\operatorname{Li}_{2}\left(-\frac{dx^{-m}}{e}\right)}{dem^{3}}$$

Mathematica [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.14

$$\int\frac{(fx)^{-1+m}\left(a+b\log\left(cx^{n}\right)\right)^{2}}{\left(d+ex^{m}\right)^{2}}dx$$

$$= \frac{x^{-m}(fx)^{m}\left(-\frac{m^{2}(a+b\log(cx^{n}))^{2}}{d+ex^{m}} - \frac{2abmn\log(d-dx^{m})}{d} + \frac{2b^{2}mn(n\log(x)-\log(cx^{n}))\log(d-dx^{m})}{d} + \frac{2b^{2}n^{2}\left(\frac{1}{2}m^{2}\log^{2}(x)+\left(-m\log(\right.\right.}{}\right.}{efm^{3}}$$

```
[In] Integrate[((f*x)^(-1 + m)*(a + b*Log[c*x^n])^2)/(d + e*x^m)^2,x]
```

```
[Out] ((f*x)^m*(-((m^2*(a + b*Log[c*x^n])^2)/(d + e*x^m)) - (2*a*b*m*n*Log[d - d*
x^m])/d + (2*b^2*m*n*(n*Log[x] - Log[c*x^n])*Log[d - d*x^m])/d + (2*b^2*n^2
*((m^2*Log[x]^2)/2 + (-(m*Log[x]) + Log[-((e*x^m)/d)])*Log[d + e*x^m] + Pol
yLog[2, 1 + (e*x^m)/d]))/d))/(e*f*m^3*x^m)
```

Maple [F]

$$\int\frac{(fx)^{m-1}\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}}{\left(d+e\,x^{m}\right)^{2}}dx$$

```
[In] int((f*x)^(m-1)*(a+b*ln(c*x^n))^2/(d+e*x^m)^2,x)
```

```
[Out] int((f*x)^(m-1)*(a+b*ln(c*x^n))^2/(d+e*x^m)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.93

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^2}\,dx$$
$$= \frac{(b^2em^2n^2\log(x)^2+2(b^2em^2n\log(c)+abem^2n)\log(x))f^{m-1}x^m-(b^2dm^2\log(c)^2+2abdm^2\log(c)+a^2d}{}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))^2/(d+e*x^m)^2,x, algorithm="fricas"
)
```

```
[Out] ((b^2*e*m^2*n^2*log(x)^2 + 2*(b^2*e*m^2*n*log(c) + a*b*e*m^2*n)*log(x))*f^(
m - 1)*x^m - (b^2*d*m^2*log(c)^2 + 2*a*b*d*m^2*log(c) + a^2*d*m^2)*f^(m - 1
) - 2*(b^2*e*f^(m - 1)*n^2*x^m + b^2*d*f^(m - 1)*n^2)*dilog(-(e*x^m + d)/d
+ 1) - 2*((b^2*e*m*n*log(c) + a*b*e*m*n)*f^(m - 1)*x^m + (b^2*d*m*n*log(c)
+ a*b*d*m*n)*f^(m - 1))*log(e*x^m + d) - 2*(b^2*e*f^(m - 1)*m*n^2*x^m*log(x
) + b^2*d*f^(m - 1)*m*n^2*log(x))*log((e*x^m + d)/d))/(d*e^2*m^3*x^m + d^2*
e*m^3)
```

Sympy [F]

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^2}\,dx = \int \frac{(fx)^{m-1}(a+b\log(cx^n))^2}{(d+ex^m)^2}\,dx$$

```
[In] integrate((f*x)**(-1+m)*(a+b*ln(c*x**n))**2/(d+e*x**m)**2,x)
```

```
[Out] Integral((f*x)**(m - 1)*(a + b*log(c*x**n))**2/(d + e*x**m)**2, x)
```

Maxima [F]

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2(fx)^{m-1}}{(ex^m+d)^2}\,dx$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))^2/(d+e*x^m)^2,x, algorithm="maxima"
)
```

```
[Out] 2*a*b*f^m*n*(log(x)/(d*e*f*m) - log(e*x^m + d)/(d*e*f*m^2)) - (f^m*log(x^n)
^2/(e^2*f*m*x^m + d*e*f*m) - integrate((e*f^m*m*x^m*log(c)^2 + 2*(d*f^m*n +
 (e*f^m*m*log(c) + e*f^m*n)*x^m)*log(x^n))/(e^3*f*m*x*x^(2*m) + 2*d*e^2*f*m
*x*x^m + d^2*e*f*m*x), x))*b^2 - 2*a*b*f^m*log(c*x^n)/(e^2*f*m*x^m + d*e*f*
m) - a^2*f^m/(e^2*f*m*x^m + d*e*f*m)
```

Giac [F]

$$\int \frac{(fx)^{-1+m} (a + b \log(cx^n))^2}{(d + ex^m)^2} \, dx = \int \frac{(b \log(cx^n) + a)^2 (fx)^{m-1}}{(ex^m + d)^2} \, dx$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))^2/(d+e*x^m)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*(f*x)^(m - 1)/(e*x^m + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{-1+m} (a + b \log(cx^n))^2}{(d + ex^m)^2} \, dx = \int \frac{(f\,x)^{m-1} (a + b\,\ln(c\,x^n))^2}{(d + e\,x^m)^2} \, dx$$

```
[In] int(((f*x)^(m - 1)*(a + b*log(c*x^n))^2)/(d + e*x^m)^2,x)
[Out] int(((f*x)^(m - 1)*(a + b*log(c*x^n))^2)/(d + e*x^m)^2, x)
```

3.365 $\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 214

$$\begin{aligned}\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^3}\,dx =& -\frac{bnx(fx)^{-1+m}(a+b\log(cx^n))}{d^2m^2(d+ex^m)} \\ &- \frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))^2}{2em(d+ex^m)^2} \\ &- \frac{bnx^{1-m}(fx)^{-1+m}(a+b\log(cx^n))\log\left(1+\frac{dx^{-m}}{e}\right)}{d^2em^2} \\ &+ \frac{b^2n^2x^{1-m}(fx)^{-1+m}\log(d+ex^m)}{d^2em^3} \\ &+ \frac{b^2n^2x^{1-m}(fx)^{-1+m}\text{PolyLog}\left(2,-\frac{dx^{-m}}{e}\right)}{d^2em^3}\end{aligned}$$

```
[Out] -b*n*x*(f*x)^(-1+m)*(a+b*ln(c*x^n))/d^2/m^2/(d+e*x^m)-1/2*x^(1-m)*(f*x)^(-1
+m)*(a+b*ln(c*x^n))^2/e/m/(d+e*x^m)^2-b*n*x^(1-m)*(f*x)^(-1+m)*(a+b*ln(c*x^
n))*ln(1+d/e/(x^m))/d^2/e/m^2+b^2*n^2*x^(1-m)*(f*x)^(-1+m)*ln(d+e*x^m)/d^2/
e/m^3+b^2*n^2*x^(1-m)*(f*x)^(-1+m)*polylog(2,-d/e/(x^m))/d^2/e/m^3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2377, 2376, 2391, 2379, 2438, 2373, 266}

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^3}\,dx = -\frac{bnx^{1-m}(fx)^{m-1}\log\left(\frac{dx^{-m}}{e}+1\right)(a+b\log(cx^n))}{d^2em^2} - \frac{bnx(fx)^{m-1}(a+b\log(cx^n))}{d^2m^2(d+ex^m)} - \frac{x^{1-m}(fx)^{m-1}(a+b\log(cx^n))^2}{2em(d+ex^m)^2} + \frac{b^2n^2x^{1-m}(fx)^{m-1}\operatorname{PolyLog}\left(2,-\frac{dx^{-m}}{e}\right)}{d^2em^3} + \frac{b^2n^2x^{1-m}(fx)^{m-1}\log(d+ex^m)}{d^2em^3}$$

```
[In] Int[((f*x)^(-1 + m)*(a + b*Log[c*x^n])^2)/(d + e*x^m)^3,x]
```

```
[Out] -((b*n*x*(f*x)^(-1 + m)*(a + b*Log[c*x^n]))/(d^2*m^2*(d + e*x^m))) - (x^(1
- m)*(f*x)^(-1 + m)*(a + b*Log[c*x^n])^2)/(2*e*m*(d + e*x^m)^2) - (b*n*x^(1
 - m)*(f*x)^(-1 + m)*(a + b*Log[c*x^n])*Log[1 + d/(e*x^m)])/(d^2*e*m^2) + (
b^2*n^2*x^(1 - m)*(f*x)^(-1 + m)*Log[d + e*x^m])/(d^2*e*m^3) + (b^2*n^2*x^(
1 - m)*(f*x)^(-1 + m)*PolyLog[2, -(d/(e*x^m))])/(d^2*e*m^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
```

```
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rule 2377

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(d + e*x^r)^q
*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] &&  !(IntegerQ[m] || GtQ[f, 0])
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^
(q_))/(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x^r)^(q + 1)*((a + b*Log[c*x^
n])^p/x), x], x] - Dist[e/d, Int[x^(r - 1)*(d + e*x^r)^q*(a + b*Log[c*x^n])
^p, x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0] && ILtQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(x^{1-m}(fx)^{-1+m}\right)\int \frac{x^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^3}\,dx\\
&= -\frac{x^{1-m}(fx)^{-1+m}\left(a+b\log(cx^n)\right)^2}{2em\left(d+ex^m\right)^2}+\frac{\left(bnx^{1-m}(fx)^{-1+m}\right)\int \frac{a+b\log(cx^n)}{x(d+ex^m)^2}\,dx}{em}\\
&= -\frac{x^{1-m}(fx)^{-1+m}\left(a+b\log(cx^n)\right)^2}{2em\left(d+ex^m\right)^2}-\frac{\left(bnx^{1-m}(fx)^{-1+m}\right)\int \frac{x^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^2}\,dx}{dm}\\
&\quad+\frac{\left(bnx^{1-m}(fx)^{-1+m}\right)\int \frac{a+b\log(cx^n)}{x(d+ex^m)}\,dx}{dem}
\end{aligned}
$$

$$= -\frac{bnx(fx)^{-1+m}(a+b\log(cx^n))}{d^2m^2(d+ex^m)} - \frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))^2}{2em(d+ex^m)^2} - \frac{bnx^{1-m}(fx)^{-1+m}(a+b\log(cx^n))\log\left(1+\frac{dx^{-m}}{e}\right)}{d^2em^2} + \frac{(b^2n^2x^{1-m}(fx)^{-1+m})\int\frac{x^{-1+m}}{d+ex^m}\,dx}{d^2m^2} + \frac{(b^2n^2x^{1-m}(fx)^{-1+m})\int\frac{\log\left(1+\frac{dx^{-m}}{e}\right)}{x}\,dx}{d^2em^2}$$

$$= -\frac{bnx(fx)^{-1+m}(a+b\log(cx^n))}{d^2m^2(d+ex^m)} - \frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))^2}{2em(d+ex^m)^2} - \frac{bnx^{1-m}(fx)^{-1+m}(a+b\log(cx^n))\log\left(1+\frac{dx^{-m}}{e}\right)}{d^2em^2} + \frac{b^2n^2x^{1-m}(fx)^{-1+m}\log(d+ex^m)}{d^2em^3} + \frac{b^2n^2x^{1-m}(fx)^{-1+m}\text{Li}_2\left(-\frac{dx^{-m}}{e}\right)}{d^2em^3}$$

Mathematica [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.97

$$\int\frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^3}\,dx = \frac{x^{-m}(fx)^m\left(\frac{2bmn(a+b\log(cx^n))}{d(d+ex^m)} - \frac{m^2(a+b\log(cx^n))^2}{(d+ex^m)^2} - \frac{2abmn\log(d-dx^m)}{d^2} + \frac{2b^2n^2\log(d-dx^m)}{d^2} + \frac{2b^2mn(n\log(x)-\log(cx^n))\log}{d^2}\right.}{2efm^3}$$

```
[In] Integrate[((f*x)^(-1 + m)*(a + b*Log[c*x^n])^2)/(d + e*x^m)^3,x]
```

```
[Out] ((f*x)^m*((2*b*m*n*(a + b*Log[c*x^n]))/(d*(d + e*x^m)) - (m^2*(a + b*Log[c*
x^n])^2)/(d + e*x^m)^2 - (2*a*b*m*n*Log[d - d*x^m])/d^2 + (2*b^2*n^2*Log[d
- d*x^m])/d^2 + (2*b^2*m*n*(n*Log[x] - Log[c*x^n])*Log[d - d*x^m])/d^2 + (2
*b^2*n^2*((m^2*Log[x]^2)/2 + (-(m*Log[x]) + Log[-((e*x^m)/d)])*Log[d + e*x^
m] + PolyLog[2, 1 + (e*x^m)/d]))/d^2))/(2*e*f*m^3*x^m)
```

Maple [F]

$$\int\frac{(fx)^{m-1}(a+b\ln(cx^n))^2}{(d+ex^m)^3}dx$$

```
[In] int((f*x)^(m-1)*(a+b*ln(c*x^n))^2/(d+e*x^m)^3,x)
```

```
[Out] int((f*x)^(m-1)*(a+b*ln(c*x^n))^2/(d+e*x^m)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. 2(211) = 422.

Time = 0.30 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.50

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^3}\,dx$$
$$= \frac{\left(b^2e^2m^2n^2\log(x)^2+2\left(b^2e^2m^2n\log(c)+abe^2m^2n-b^2e^2mn^2\right)\log(x)\right)f^{m-1}x^{2m}+2\left(b^2dem^2n^2\log(x)^2+\right.}{}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))^2/(d+e*x^m)^3,x, algorithm="fricas"
)
```

```
[Out] 1/2*((b^2*e^2*m^2*n^2*log(x)^2 + 2*(b^2*e^2*m^2*n*log(c) + a*b*e^2*m^2*n -
b^2*e^2*m*n^2)*log(x))*f^(m - 1)*x^(2*m) + 2*(b^2*d*e*m^2*n^2*log(x)^2 + b^
2*d*e*m*n*log(c) + a*b*d*e*m*n + (2*b^2*d*e*m^2*n*log(c) + 2*a*b*d*e*m^2*n
- b^2*d*e*m*n^2)*log(x))*f^(m - 1)*x^m - (b^2*d^2*m^2*log(c)^2 + a^2*d^2*m^
2 - 2*a*b*d^2*m*n + 2*(a*b*d^2*m^2 - b^2*d^2*m*n)*log(c))*f^(m - 1) - 2*(b^
2*e^2*f^(m - 1)*n^2*x^(2*m) + 2*b^2*d*e*f^(m - 1)*n^2*x^m + b^2*d^2*f^(m -
1)*n^2)*dilog(-(e*x^m + d)/d + 1) - 2*((b^2*e^2*m*n*log(c) + a*b*e^2*m*n -
b^2*e^2*n^2)*f^(m - 1)*x^(2*m) + 2*(b^2*d*e*m*n*log(c) + a*b*d*e*m*n - b^2*
d*e*n^2)*f^(m - 1)*x^m + (b^2*d^2*m*n*log(c) + a*b*d^2*m*n - b^2*d^2*n^2)*f
^(m - 1))*log(e*x^m + d) - 2*(b^2*e^2*f^(m - 1)*m*n^2*x^(2*m)*log(x) + 2*b^
2*d*e*f^(m - 1)*m*n^2*x^m*log(x) + b^2*d^2*f^(m - 1)*m*n^2*log(x))*log((e*x
^m + d)/d))/(d^2*e^3*m^3*x^(2*m) + 2*d^3*e^2*m^3*x^m + d^4*e*m^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^3}\,dx = \text{Timed out}$$

```
[In] integrate((f*x)**(-1+m)*(a+b*ln(c*x**n))**2/(d+e*x**m)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(fx)^{-1+m} (a + b\log(cx^n))^2}{(d + ex^m)^3}\, dx = \int \frac{(b\log(cx^n) + a)^2 (fx)^{m-1}}{(ex^m + d)^3}\, dx$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))^2/(d+e*x^m)^3,x, algorithm="maxima"
)
```

```
[Out] a*b*f^m*n*(1/((d*e^2*f*m*x^m + d^2*e*f*m)*m) + log(x)/(d^2*e*f*m) - log(e*x
^m + d)/(d^2*e*f*m^2)) - 1/2*(f^m*log(x^n)^2/(e^3*f*m*x^(2*m) + 2*d*e^2*f*m
*x^m + d^2*e*f*m) - 2*integrate((e*f^m*m*x^m*log(c)^2 + (d*f^m*n + (2*e*f^m
*m*log(c) + e*f^m*n)*x^m)*log(x^n))/(e^4*f*m*x*x^(3*m) + 3*d*e^3*f*m*x*x^(2
*m) + 3*d^2*e^2*f*m*x*x^m + d^3*e*f*m*x), x))*b^2 - a*b*f^m*log(c*x^n)/(e^3
*f*m*x^(2*m) + 2*d*e^2*f*m*x^m + d^2*e*f*m) - 1/2*a^2*f^m/(e^3*f*m*x^(2*m)
+ 2*d*e^2*f*m*x^m + d^2*e*f*m)
```

Giac [F]

$$\int \frac{(fx)^{-1+m} (a + b\log(cx^n))^2}{(d + ex^m)^3}\, dx = \int \frac{(b\log(cx^n) + a)^2 (fx)^{m-1}}{(ex^m + d)^3}\, dx$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))^2/(d+e*x^m)^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*(f*x)^(m - 1)/(e*x^m + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{-1+m} (a + b\log(cx^n))^2}{(d + ex^m)^3}\, dx = \int \frac{(f\,x)^{m-1}\,(a + b\,\ln(c\,x^n))^2}{(d + e\,x^m)^3}\, dx$$

```
[In] int(((f*x)^(m - 1)*(a + b*log(c*x^n))^2)/(d + e*x^m)^3,x)
```

```
[Out] int(((f*x)^(m - 1)*(a + b*log(c*x^n))^2)/(d + e*x^m)^3, x)
```

3.366 $\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 346

$$\begin{aligned}\int \frac{(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)^2}{\left(d+ex^m\right)^4}\,dx &= -\frac{b^2n^2x^{1-m}(fx)^{-1+m}}{3d^2em^3\left(d+ex^m\right)}-\frac{b^2n^2x^{1-m}(fx)^{-1+m}\log(x)}{3d^3em^2} \\ &+\frac{bnx^{1-m}(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)}{3dem^2\left(d+ex^m\right)^2} \\ &-\frac{2bnx(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)}{3d^3m^2\left(d+ex^m\right)} \\ &-\frac{x^{1-m}(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)^2}{3em\left(d+ex^m\right)^3} \\ &-\frac{2bnx^{1-m}(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)\log\left(1+\frac{dx^{-m}}{e}\right)}{3d^3em^2} \\ &+\frac{b^2n^2x^{1-m}(fx)^{-1+m}\log\left(d+ex^m\right)}{d^3em^3} \\ &+\frac{2b^2n^2x^{1-m}(fx)^{-1+m}\operatorname{PolyLog}\left(2,-\frac{dx^{-m}}{e}\right)}{3d^3em^3}\end{aligned}$$

```
[Out] -1/3*b^2*n^2*x^(1-m)*(f*x)^(-1+m)/d^2/e/m^3/(d+e*x^m)-1/3*b^2*n^2*x^(1-m)*(
f*x)^(-1+m)*ln(x)/d^3/e/m^2+1/3*b*n*x^(1-m)*(f*x)^(-1+m)*(a+b*ln(c*x^n))/d/
e/m^2/(d+e*x^m)^2-2/3*b*n*x*(f*x)^(-1+m)*(a+b*ln(c*x^n))/d^3/m^2/(d+e*x^m)-
1/3*x^(1-m)*(f*x)^(-1+m)*(a+b*ln(c*x^n))^2/e/m/(d+e*x^m)^3-2/3*b*n*x^(1-m)*
(f*x)^(-1+m)*(a+b*ln(c*x^n))*ln(1+d/e/(x^m))/d^3/e/m^2+b^2*n^2*x^(1-m)*(f*x
)^(-1+m)*ln(d+e*x^m)/d^3/e/m^3+2/3*b^2*n^2*x^(1-m)*(f*x)^(-1+m)*polylog(2,-
d/e/(x^m))/d^3/e/m^3
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2377, 2376, 2391, 2379, 2438, 2373, 266, 272, 46}

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^4}\,dx = -\frac{2bnx^{1-m}(fx)^{m-1}\log\left(\frac{dx^{-m}}{e}+1\right)(a+b\log(cx^n))}{3d^3em^2} - \frac{2bnx(fx)^{m-1}(a+b\log(cx^n))}{3d^3m^2(d+ex^m)} + \frac{bnx^{1-m}(fx)^{m-1}(a+b\log(cx^n))}{3dem^2(d+ex^m)^2} - \frac{x^{1-m}(fx)^{m-1}(a+b\log(cx^n))^2}{3em(d+ex^m)^3} + \frac{2b^2n^2x^{1-m}(fx)^{m-1}\operatorname{PolyLog}\left(2,-\frac{dx^{-m}}{e}\right)}{3d^3em^3} + \frac{b^2n^2x^{1-m}(fx)^{m-1}\log(d+ex^m)}{d^3em^3} - \frac{b^2n^2x^{1-m}\log(x)(fx)^{m-1}}{3d^3em^2} - \frac{b^2n^2x^{1-m}(fx)^{m-1}}{3d^2em^3(d+ex^m)}$$

```
[In] Int[((f*x)^(-1 + m)*(a + b*Log[c*x^n])^2)/(d + e*x^m)^4,x]
```

```
[Out] -1/3*(b^2*n^2*x^(1 - m)*(f*x)^(-1 + m))/(d^2*e*m^3*(d + e*x^m)) - (b^2*n^2*
x^(1 - m)*(f*x)^(-1 + m)*Log[x])/(3*d^3*e*m^2) + (b*n*x^(1 - m)*(f*x)^(-1 +
 m)*(a + b*Log[c*x^n]))/(3*d*e*m^2*(d + e*x^m)^2) - (2*b*n*x*(f*x)^(-1 + m)
*(a + b*Log[c*x^n]))/(3*d^3*m^2*(d + e*x^m)) - (x^(1 - m)*(f*x)^(-1 + m)*(a
 + b*Log[c*x^n])^2)/(3*e*m*(d + e*x^m)^3) - (2*b*n*x^(1 - m)*(f*x)^(-1 + m)
*(a + b*Log[c*x^n])*Log[1 + d/(e*x^m)])/(3*d^3*e*m^2) + (b^2*n^2*x^(1 - m)*
(f*x)^(-1 + m)*Log[d + e*x^m])/(d^3*e*m^3) + (2*b^2*n^2*x^(1 - m)*(f*x)^(-1
 + m)*PolyLog[2, -(d/(e*x^m))])/(3*d^3*e*m^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rule 2377

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(d + e*x^r)^q
*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] &&  !(IntegerQ[m] || GtQ[f, 0])
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^
(q_))/(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x^r)^(q + 1)*((a + b*Log[c*x^
n])^p/x), x], x] - Dist[e/d, Int[x^(r - 1)*(d + e*x^r)^q*(a + b*Log[c*x^n])
^p, x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0] && ILtQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \left(x^{1-m}(fx)^{-1+m}\right)\int \frac{x^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^4}\,dx$$

$$= -\frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))^2}{3em(d+ex^m)^3} + \frac{\left(2bnx^{1-m}(fx)^{-1+m}\right)\int \frac{a+b\log(cx^n)}{x(d+ex^m)^3}\,dx}{3em}$$

$$= -\frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))^2}{3em(d+ex^m)^3} - \frac{\left(2bnx^{1-m}(fx)^{-1+m}\right)\int \frac{x^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^3}\,dx}{3dm} + \frac{\left(2bnx^{1-m}(fx)^{-1+m}\right)\int \frac{a+b\log(cx^n)}{x(d+ex^m)^2}\,dx}{3dem}$$

$$= \frac{bnx^{1-m}(fx)^{-1+m}(a+b\log(cx^n))}{3dem^2(d+ex^m)^2} - \frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))^2}{3em(d+ex^m)^3} - \frac{\left(2bnx^{1-m}(fx)^{-1+m}\right)\int \frac{x^{-1+m}(a+b\log(cx^n))}{(d+ex^m)^2}\,dx}{3d^2m} + \frac{\left(2bnx^{1-m}(fx)^{-1+m}\right)\int \frac{a+b\log(cx^n)}{x(d+ex^m)}\,dx}{3d^2em} - \frac{\left(b^2n^2x^{1-m}(fx)^{-1+m}\right)\int \frac{1}{x(d+ex^m)^2}\,dx}{3dem^2}$$

$$= \frac{bnx^{1-m}(fx)^{-1+m}(a+b\log(cx^n))}{3dem^2(d+ex^m)^2} - \frac{2bnx(fx)^{-1+m}(a+b\log(cx^n))}{3d^3m^2(d+ex^m)} - \frac{x^{1-m}(fx)^{-1+m}(a+b\log(cx^n))^2}{3em(d+ex^m)^3} - \frac{2bnx^{1-m}(fx)^{-1+m}(a+b\log(cx^n))\log\left(1+\frac{dx^{-m}}{e}\right)}{3d^3em^2} - \frac{\left(b^2n^2x^{1-m}(fx)^{-1+m}\right)\text{Subst}\left(\int \frac{1}{x(d+ex)^2}\,dx, x, x^m\right)}{3dem^3} + \frac{\left(2b^2n^2x^{1-m}(fx)^{-1+m}\right)\int \frac{x^{-1+m}}{d+ex^m}\,dx}{3d^3m^2} + \frac{\left(2b^2n^2x^{1-m}(fx)^{-1+m}\right)\int \frac{\log\left(1+\frac{dx^{-m}}{e}\right)}{x}\,dx}{3d^3em^2}$$

$$= \frac{bnx^{1-m}(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)}{3dem^2\left(d+ex^m\right)^2} - \frac{2bnx(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)}{3d^3m^2\left(d+ex^m\right)}$$
$$- \frac{x^{1-m}(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)^2}{3em\left(d+ex^m\right)^3}$$
$$- \frac{2bnx^{1-m}(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)\log\left(1+\frac{dx^{-m}}{e}\right)}{3d^3em^2}$$
$$+ \frac{2b^2n^2x^{1-m}(fx)^{-1+m}\log\left(d+ex^m\right)}{3d^3em^3} + \frac{2b^2n^2x^{1-m}(fx)^{-1+m}\text{Li}_2\left(-\frac{dx^{-m}}{e}\right)}{3d^3em^3}$$
$$- \frac{\left(b^2n^2x^{1-m}(fx)^{-1+m}\right)\text{Subst}\left(\int\left(\frac{1}{d^2x}-\frac{e}{d(d+ex)^2}-\frac{e}{d^2(d+ex)}\right)dx,x,x^m\right)}{3dem^3}$$
$$= -\frac{b^2n^2x^{1-m}(fx)^{-1+m}}{3d^2em^3\left(d+ex^m\right)} - \frac{b^2n^2x^{1-m}(fx)^{-1+m}\log(x)}{3d^3em^2}$$
$$+ \frac{bnx^{1-m}(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)}{3dem^2\left(d+ex^m\right)^2}$$
$$- \frac{2bnx(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)}{3d^3m^2\left(d+ex^m\right)} - \frac{x^{1-m}(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)^2}{3em\left(d+ex^m\right)^3}$$
$$- \frac{2bnx^{1-m}(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)\log\left(1+\frac{dx^{-m}}{e}\right)}{3d^3em^2}$$
$$+ \frac{b^2n^2x^{1-m}(fx)^{-1+m}\log\left(d+ex^m\right)}{d^3em^3} + \frac{2b^2n^2x^{1-m}(fx)^{-1+m}\text{Li}_2\left(-\frac{dx^{-m}}{e}\right)}{3d^3em^3}$$

Mathematica [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.69

$$\int \frac{(fx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)^2}{\left(d+ex^m\right)^4}\,dx$$
$$= \frac{x^{-m}(fx)^m\left(\frac{bmn(a+b\log(cx^n))}{d(d+ex^m)^2} - \frac{m^2(a+b\log(cx^n))^2}{(d+ex^m)^3} + \frac{bn(2am-bn+2bm\log(cx^n))}{d^2(d+ex^m)} - \frac{2abmn\log(d-dx^m)}{d^3} + \frac{3b^2n^2\log(d-dx^m)}{d^3} + \cdots\right.}{3efm^3}$$

```
[In] Integrate[((f*x)^(-1 + m)*(a + b*Log[c*x^n])^2)/(d + e*x^m)^4,x]
```

```
[Out] ((f*x)^m*((b*m*n*(a + b*Log[c*x^n]))/(d*(d + e*x^m)^2) - (m^2*(a + b*Log[c*
x^n])^2)/(d + e*x^m)^3 + (b*n*(2*a*m - b*n + 2*b*m*Log[c*x^n]))/(d^2*(d + e
*x^m)) - (2*a*b*m*n*Log[d - d*x^m])/d^3 + (3*b^2*n^2*Log[d - d*x^m])/d^3 +
(2*b^2*m*n*(n*Log[x] - Log[c*x^n])*Log[d - d*x^m])/d^3 + (2*b^2*n^2*((m^2*L
og[x]^2)/2 + (-(m*Log[x]) + Log[-((e*x^m)/d)])*Log[d + e*x^m] + PolyLog[2,
1 + (e*x^m)/d]))/d^3))/(3*e*f*m^3*x^m)
```

Maple [F]

$$\int \frac{(fx)^{m-1} (a + b\ln (c\,x^{n}))^{2}}{(d + e\,x^{m})^{4}}dx$$

```
[In] int((f*x)^(m-1)*(a+b*ln(c*x^n))^2/(d+e*x^m)^4,x)
```

```
[Out] int((f*x)^(m-1)*(a+b*ln(c*x^n))^2/(d+e*x^m)^4,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 810 vs. $2(331) = 662$.

Time = 0.31 (sec) , antiderivative size = 810, normalized size of antiderivative = 2.34

$$\int \frac{(fx)^{-1+m} (a + b\log (cx^{n}))^{2}}{(d + ex^{m})^{4}}\,dx = \frac{\left(b^{2}e^{3}m^{2}n^{2}\log (x)^{2} + (2\,b^{2}e^{3}m^{2}n\log (c) + 2\,abe^{3}m^{2}n - 3\,b^{2}e^{3}mn^{2})\log (x)\right)f^{m-1}x^{3\,m} + \left(3\,b^{2}de^{2}m^{2}n^{2}\log (\right.}{}$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))^2/(d+e*x^m)^4,x, algorithm="fricas"
)
```

```
[Out] 1/3*((b^2*e^3*m^2*n^2*log(x)^2 + (2*b^2*e^3*m^2*n*log(c) + 2*a*b*e^3*m^2*n
- 3*b^2*e^3*m*n^2)*log(x))*f^(m - 1)*x^(3*m) + (3*b^2*d*e^2*m^2*n^2*log(x)^
2 + 2*b^2*d*e^2*m*n*log(c) + 2*a*b*d*e^2*m*n - b^2*d*e^2*n^2 + (6*b^2*d*e^2
*m^2*n*log(c) + 6*a*b*d*e^2*m^2*n - 7*b^2*d*e^2*m*n^2)*log(x))*f^(m - 1)*x^
(2*m) + (3*b^2*d^2*e*m^2*n^2*log(x)^2 + 5*b^2*d^2*e*m*n*log(c) + 5*a*b*d^2*
e*m*n - 2*b^2*d^2*e*n^2 + 2*(3*b^2*d^2*e*m^2*n*log(c) + 3*a*b*d^2*e*m^2*n -
 2*b^2*d^2*e*m*n^2)*log(x))*f^(m - 1)*x^m - (b^2*d^3*m^2*log(c)^2 + a^2*d^3
*m^2 - 3*a*b*d^3*m*n + b^2*d^3*n^2 + (2*a*b*d^3*m^2 - 3*b^2*d^3*m*n)*log(c)
)*f^(m - 1) - 2*(b^2*e^3*f^(m - 1)*n^2*x^(3*m) + 3*b^2*d*e^2*f^(m - 1)*n^2*
x^(2*m) + 3*b^2*d^2*e*f^(m - 1)*n^2*x^m + b^2*d^3*f^(m - 1)*n^2)*dilog(-(e*
x^m + d)/d + 1) - ((2*b^2*e^3*m*n*log(c) + 2*a*b*e^3*m*n - 3*b^2*e^3*n^2)*f
^(m - 1)*x^(3*m) + 3*(2*b^2*d*e^2*m*n*log(c) + 2*a*b*d*e^2*m*n - 3*b^2*d*e^
2*n^2)*f^(m - 1)*x^(2*m) + 3*(2*b^2*d^2*e*m*n*log(c) + 2*a*b*d^2*e*m*n - 3*
b^2*d^2*e*n^2)*f^(m - 1)*x^m + (2*b^2*d^3*m*n*log(c) + 2*a*b*d^3*m*n - 3*b^
2*d^3*n^2)*f^(m - 1))*log(e*x^m + d) - 2*(b^2*e^3*f^(m - 1)*m*n^2*x^(3*m)*l
og(x) + 3*b^2*d*e^2*f^(m - 1)*m*n^2*x^(2*m)*log(x) + 3*b^2*d^2*e*f^(m - 1)*
m*n^2*x^m*log(x) + b^2*d^3*f^(m - 1)*m*n^2*log(x))*log((e*x^m + d)/d))/(d^3
*e^4*m^3*x^(3*m) + 3*d^4*e^3*m^3*x^(2*m) + 3*d^5*e^2*m^3*x^m + d^6*e*m^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^4}\,dx = \text{Timed out}$$

```
[In] integrate((f*x)**(-1+m)*(a+b*ln(c*x**n))**2/(d+e*x**m)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2(fx)^{m-1}}{(ex^m+d)^4}\,dx$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))^2/(d+e*x^m)^4,x, algorithm="maxima"
)
[Out] 1/3*a*b*f^m*n*((2*e*x^m + 3*d)/((d^2*e^3*f*m*x^(2*m) + 2*d^3*e^2*f*m*x^m +
d^4*e*f*m)*m) + 2*log(x)/(d^3*e*f*m) - 2*log(e*x^m + d)/(d^3*e*f*m^2)) - 1/
3*(f^m*log(x^n)^2/(e^4*f*m*x^(3*m) + 3*d*e^3*f*m*x^(2*m) + 3*d^2*e^2*f*m*x^
m + d^3*e*f*m) - 3*integrate(1/3*(3*e*f^m*m*x^m*log(c)^2 + 2*(d*f^m*n + (3*
e*f^m*m*log(c) + e*f^m*n)*x^m)*log(x^n))/(e^5*f*m*x*x^(4*m) + 4*d*e^4*f*m*x
*x^(3*m) + 6*d^2*e^3*f*m*x*x^(2*m) + 4*d^3*e^2*f*m*x*x^m + d^4*e*f*m*x), x)
)*b^2 - 2/3*a*b*f^m*log(c*x^n)/(e^4*f*m*x^(3*m) + 3*d*e^3*f*m*x^(2*m) + 3*d
^2*e^2*f*m*x^m + d^3*e*f*m) - 1/3*a^2*f^m/(e^4*f*m*x^(3*m) + 3*d*e^3*f*m*x^
(2*m) + 3*d^2*e^2*f*m*x^m + d^3*e*f*m)
```

Giac [F]

$$\int \frac{(fx)^{-1+m}(a+b\log(cx^n))^2}{(d+ex^m)^4}\,dx = \int \frac{(b\log(cx^n)+a)^2(fx)^{m-1}}{(ex^m+d)^4}\,dx$$

```
[In] integrate((f*x)^(-1+m)*(a+b*log(c*x^n))^2/(d+e*x^m)^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*(f*x)^(m - 1)/(e*x^m + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{-1+m} (a + b\log{(cx^n)})^2}{(d + ex^m)^4} dx = \int \frac{(f\,x)^{m-1} (a + b\,\ln{(c\,x^n)})^2}{(d + e\,x^m)^4} dx$$

```
[In] int(((f*x)^(m - 1)*(a + b*log(c*x^n))^2)/(d + e*x^m)^4,x)
[Out] int(((f*x)^(m - 1)*(a + b*log(c*x^n))^2)/(d + e*x^m)^4, x)
```

3.367 $\int x^5(d+ex^r)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int x^5(d+ex^r)(a+b\log(cx^n))\,dx=-\frac{1}{36}bdnx^6-\frac{benx^{6+r}}{(6+r)^2}+\frac{1}{6}\left(dx^6+\frac{6ex^{6+r}}{6+r}\right)(a+b\log(cx^n))$$

```
[Out] -1/36*b*d*n*x^6-b*e*n*x^(6+r)/(6+r)^2+1/6*(d*x^6+6*e*x^(6+r)/(6+r))*(a+b*ln
(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {14, 2371, 12}

$$\int x^5(d+ex^r)(a+b\log(cx^n))\,dx=\frac{1}{6}\left(dx^6+\frac{6ex^{r+6}}{r+6}\right)(a+b\log(cx^n))-\frac{1}{36}bdnx^6-\frac{benx^{r+6}}{(r+6)^2}$$

```
[In] Int[x^5*(d + e*x^r)*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/36*(b*d*n*x^6) - (b*e*n*x^(6 + r))/(6 + r)^2 + ((d*x^6 + (6*e*x^(6 + r))
/(6 + r))*(a + b*Log[c*x^n]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{6}\left(dx^6+\frac{6ex^{6+r}}{6+r}\right)(a+b\log(cx^n))-(bn)\int\frac{1}{6}x^5\left(d+\frac{6ex^r}{6+r}\right)dx\\ &= \frac{1}{6}\left(dx^6+\frac{6ex^{6+r}}{6+r}\right)(a+b\log(cx^n))-\frac{1}{6}(bn)\int x^5\left(d+\frac{6ex^r}{6+r}\right)dx\\ &= \frac{1}{6}\left(dx^6+\frac{6ex^{6+r}}{6+r}\right)(a+b\log(cx^n))-\frac{1}{6}(bn)\int\left(dx^5+\frac{6ex^{5+r}}{6+r}\right)dx\\ &= -\frac{1}{36}bdnx^6-\frac{benx^{6+r}}{(6+r)^2}+\frac{1}{6}\left(dx^6+\frac{6ex^{6+r}}{6+r}\right)(a+b\log(cx^n))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int x^5(d+ex^r)(a+b\log(cx^n))\,dx$$
$$=\frac{x^6(6a(6+r)(d(6+r)+6ex^r)-bn(d(6+r)^2+36ex^r)+6b(6+r)(d(6+r)+6ex^r)\log(cx^n))}{36(6+r)^2}$$

```
[In] Integrate[x^5*(d + e*x^r)*(a + b*Log[c*x^n]),x]
[Out] (x^6*(6*a*(6 + r)*(d*(6 + r) + 6*e*x^r) - b*n*(d*(6 + r)^2 + 36*e*x^r) + 6*
b*(6 + r)*(d*(6 + r) + 6*e*x^r)*Log[c*x^n]))/(36*(6 + r)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(55) = 110$.

Time = 4.80 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.95

method	result
parallelrisch	$-\frac{-36x^6x^r\ln(c\,x^n)ber-6x^6\ln(c\,x^n)bd\,r^2+x^6bdn\,r^2-216be\ln(c\,x^n)x^rx^6-36x^6x^raer+36x^6x^rben-72x^6\ln(c\,x^n)bdr-6x^6ad\,r^2}{36(r^2+12r+36)}$
risch	$\frac{b\,x^6(dr+6e\,x^r+6d)\ln(x^n)}{36+6r}-\frac{x^6\left(-216x^rae+36bdn-216ad-36x^raer+36x^rben-108i\pi bd\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-3i\pi bd\,r^2\,\mathrm{csg}\right.}{}$

```
[In] int(x^5*(d+e*x^r)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/36*(-36*x^6*x^r*ln(c*x^n)*b*e*r-6*x^6*ln(c*x^n)*b*d*r^2+x^6*b*d*n*r^2-21
6*b*e*ln(c*x^n)*x^r*x^6-36*x^6*x^r*a*e*r+36*x^6*x^r*b*e*n-72*x^6*ln(c*x^n)*
b*d*r-6*x^6*a*d*r^2+12*x^6*b*d*n*r-216*x^6*x^r*a*e-216*x^6*ln(c*x^n)*b*d-72
*x^6*a*d*r+36*b*d*n*x^6-216*a*d*x^6)/(r^2+12*r+36)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(55) = 110$.

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.69

$$\int x^5(d+ex^r)\,(a+b\log{(cx^n)})\;dx$$
$$=\frac{6\,(bdr^2+12\,bdr+36\,bd)x^6\log{(c)}+6\,(bdnr^2+12\,bdnr+36\,bdn)x^6\log{(x)}-(36\,bdn+(bdn-6\,ad)r^2-2}{36\,(r^2+}$$

```
[In] integrate(x^5*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 1/36*(6*(b*d*r^2 + 12*b*d*r + 36*b*d)*x^6*log(c) + 6*(b*d*n*r^2 + 12*b*d*n*
r + 36*b*d*n)*x^6*log(x) - (36*b*d*n + (b*d*n - 6*a*d)*r^2 - 216*a*d + 12*(
b*d*n - 6*a*d)*r)*x^6 + 36*((b*e*r + 6*b*e)*x^6*log(c) + (b*e*n*r + 6*b*e*n
)*x^6*log(x) - (b*e*n - a*e*r - 6*a*e)*x^6)*x^r)/(r^2 + 12*r + 36)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(51) = 102$.

Time = 8.57 (sec) , antiderivative size = 398, normalized size of antiderivative = 6.75

$$\int x^5(d+ex^r)\,(a+b\log{(cx^n)})\;dx$$
$$=\begin{cases}\frac{6adr^2x^6}{36r^2+432r+1296}+\frac{72adrx^6}{36r^2+432r+1296}+\frac{216adx^6}{36r^2+432r+1296}+\frac{36aerx^6x^r}{36r^2+432r+1296}+\frac{216aex^6x^r}{36r^2+432r+1296}-\frac{bdnr^2x^6}{36r^2+432r+1296}-\frac{12bdn}{36r^2+432}\\ \frac{adx^6}{6}+\frac{ae\log{(cx^n)}}{n}-\frac{bdnx^6}{36}+\frac{bdx^6\log{(cx^n)}}{6}+\frac{be\log{(cx^n)}^2}{2n}\end{cases}$$

```
[In] integrate(x**5*(d+e*x**r)*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((6*a*d*r**2*x**6/(36*r**2 + 432*r + 1296) + 72*a*d*r*x**6/(36*r**
2 + 432*r + 1296) + 216*a*d*x**6/(36*r**2 + 432*r + 1296) + 36*a*e*r*x**6*x
**r/(36*r**2 + 432*r + 1296) + 216*a*e*x**6*x**r/(36*r**2 + 432*r + 1296) -
 b*d*n*r**2*x**6/(36*r**2 + 432*r + 1296) - 12*b*d*n*r*x**6/(36*r**2 + 432*
r + 1296) - 36*b*d*n*x**6/(36*r**2 + 432*r + 1296) + 6*b*d*r**2*x**6*log(c*
x**n)/(36*r**2 + 432*r + 1296) + 72*b*d*r*x**6*log(c*x**n)/(36*r**2 + 432*r
 + 1296) + 216*b*d*x**6*log(c*x**n)/(36*r**2 + 432*r + 1296) - 36*b*e*n*x**
6*x**r/(36*r**2 + 432*r + 1296) + 36*b*e*r*x**6*x**r*log(c*x**n)/(36*r**2 +
 432*r + 1296) + 216*b*e*x**6*x**r*log(c*x**n)/(36*r**2 + 432*r + 1296), Ne
(r, -6)), (a*d*x**6/6 + a*e*log(c*x**n)/n - b*d*n*x**6/36 + b*d*x**6*log(c*
x**n)/6 + b*e*log(c*x**n)**2/(2*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int x^5(d+ex^r)\,(a+b\log{(cx^n)})\;dx = -\frac{1}{36}\,bdnx^6 + \frac{1}{6}\,bdx^6\log{(cx^n)} + \frac{1}{6}\,adx^6 + \frac{bex^{r+6}\log{(cx^n)}}{r+6} - \frac{benx^{r+6}}{(r+6)^2} + \frac{aex^{r+6}}{r+6}$$

```
[In] integrate(x^5*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/36*b*d*n*x^6 + 1/6*b*d*x^6*log(c*x^n) + 1/6*a*d*x^6 + b*e*x^(r + 6)*log(
c*x^n)/(r + 6) - b*e*n*x^(r + 6)/(r + 6)^2 + a*e*x^(r + 6)/(r + 6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(55) = 110.

Time = 0.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.24

$$\int x^5(d+ex^r)\,(a+b\log{(cx^n)})\;dx = \frac{benrx^6x^r\log{(x)}}{r^2+12\,r+36} + \frac{6\,benx^6x^r\log{(x)}}{r^2+12\,r+36} + \frac{1}{6}\,bdnx^6\log{(x)} - \frac{benx^6x^r}{r^2+12\,r+36} - \frac{1}{36}\,bdnx^6 + \frac{bex^6x^r\log{(c)}}{r+6} + \frac{1}{6}\,bdx^6\log{(c)} + \frac{aex^6x^r}{r+6} + \frac{1}{6}\,adx^6$$

```
[In] integrate(x^5*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*e*n*r*x^6*x^r*log(x)/(r^2 + 12*r + 36) + 6*b*e*n*x^6*x^r*log(x)/(r^2 + 12
*r + 36) + 1/6*b*d*n*x^6*log(x) - b*e*n*x^6*x^r/(r^2 + 12*r + 36) - 1/36*b*
d*n*x^6 + b*e*x^6*x^r*log(c)/(r + 6) + 1/6*b*d*x^6*log(c) + a*e*x^6*x^r/(r
+ 6) + 1/6*a*d*x^6
```

Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx=\int x^5\left(d+e\,x^r\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

```
[In] int(x^5*(d + e*x^r)*(a + b*log(c*x^n)),x)
[Out] int(x^5*(d + e*x^r)*(a + b*log(c*x^n)), x)
```

3.368 $\int x^3(d+ex^r)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int x^3(d+ex^r)(a+b\log(cx^n))\,dx=-\frac{1}{16}bdnx^4-\frac{benx^{4+r}}{(4+r)^2}+\frac{1}{4}\left(dx^4+\frac{4ex^{4+r}}{4+r}\right)(a+b\log(cx^n))$$

```
[Out] -1/16*b*d*n*x^4-b*e*n*x^(4+r)/(4+r)^2+1/4*(d*x^4+4*e*x^(4+r)/(4+r))*(a+b*ln
(c*x^n))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {14, 2371, 12}

$$\int x^3(d+ex^r)(a+b\log(cx^n))\,dx=\frac{1}{4}\left(dx^4+\frac{4ex^{r+4}}{r+4}\right)(a+b\log(cx^n))-\frac{1}{16}bdnx^4-\frac{benx^{r+4}}{(r+4)^2}$$

```
[In] Int[x^3*(d + e*x^r)*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/16*(b*d*n*x^4) - (b*e*n*x^(4 + r))/(4 + r)^2 + ((d*x^4 + (4*e*x^(4 + r))
/(4 + r))*(a + b*Log[c*x^n]))/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\left(dx^4+\frac{4ex^{4+r}}{4+r}\right)(a+b\log(cx^n))-(bn)\int\frac{1}{4}x^3\left(d+\frac{4ex^r}{4+r}\right)dx\\&= \frac{1}{4}\left(dx^4+\frac{4ex^{4+r}}{4+r}\right)(a+b\log(cx^n))-\frac{1}{4}(bn)\int x^3\left(d+\frac{4ex^r}{4+r}\right)dx\\&= \frac{1}{4}\left(dx^4+\frac{4ex^{4+r}}{4+r}\right)(a+b\log(cx^n))-\frac{1}{4}(bn)\int\left(dx^3+\frac{4ex^{3+r}}{4+r}\right)dx\\&= -\frac{1}{16}bdnx^4-\frac{benx^{4+r}}{(4+r)^2}+\frac{1}{4}\left(dx^4+\frac{4ex^{4+r}}{4+r}\right)(a+b\log(cx^n))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int x^3(d+ex^r)(a+b\log(cx^n))\,dx = \frac{x^4(4a(4+r)(d(4+r)+4ex^r)-bn(d(4+r)^2+16ex^r)+4b(4+r)(d(4+r)+4ex^r)\log(cx^n))}{16(4+r)^2}$$

```
[In] Integrate[x^3*(d + e*x^r)*(a + b*Log[c*x^n]),x]
[Out] (x^4*(4*a*(4 + r)*(d*(4 + r) + 4*e*x^r) - b*n*(d*(4 + r)^2 + 16*e*x^r) + 4*
b*(4 + r)*(d*(4 + r) + 4*e*x^r)*Log[c*x^n]))/(16*(4 + r)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(55) = 110$.

Time = 1.32 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.86

method	result
parallelrisch	$-\frac{-16x^4x^r \ln(c\,x^n)ber-4x^4 \ln(c\,x^n)bd\,r^2+x^4bdn\,r^2-64x^4x^r \ln(c\,x^n)be-16x^4x^r aer+16x^4x^r ben-32x^4 \ln(c\,x^n)bdr-4x^4ad\,r}{16(4+r)^2}$
risch	$\frac{b\,x^4(dr+4e\,x^r+4d)\ln(x^n)}{16+4r}-\frac{x^4\left(-64x^r ae+16bdn-64ad-16x^r aer+16x^r ben-32i\pi bd\,\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2-2i\pi bd\,r^2\operatorname{csgn}\right.}{}$

```
[In] int(x^3*(d+e*x^r)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(-16*x^4*x^r*ln(c*x^n)*b*e*r-4*x^4*ln(c*x^n)*b*d*r^2+x^4*b*d*n*r^2-64
*x^4*x^r*ln(c*x^n)*b*e-16*x^4*x^r*a*e*r+16*x^4*x^r*b*e*n-32*x^4*ln(c*x^n)*b
*d*r-4*x^4*a*d*r^2+8*x^4*b*d*n*r-64*x^4*x^r*a*e-64*x^4*ln(c*x^n)*b*d-32*x^4
*a*d*r+16*b*d*n*x^4-64*a*d*x^4)/(4+r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(55) = 110$.

Time = 0.31 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.69

$$\int x^3(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{4\,(bdr^2+8\,bdr+16\,bd)x^4\log\left(c\right)+4\,(bdnr^2+8\,bdnr+16\,bdn)x^4\log\left(x\right)-(16\,bdn+(bdn-4\,ad)r^2-64}{16\,(r^2+}$$

```
[In] integrate(x^3*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*(b*d*r^2 + 8*b*d*r + 16*b*d)*x^4*log(c) + 4*(b*d*n*r^2 + 8*b*d*n*r
+ 16*b*d*n)*x^4*log(x) - (16*b*d*n + (b*d*n - 4*a*d)*r^2 - 64*a*d + 8*(b*d*
n - 4*a*d)*r)*x^4 + 16*((b*e*r + 4*b*e)*x^4*log(c) + (b*e*n*r + 4*b*e*n)*x^
4*log(x) - (b*e*n - a*e*r - 4*a*e)*x^4)*x^r)/(r^2 + 8*r + 16)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(51) = 102$.

Time = 2.86 (sec) , antiderivative size = 398, normalized size of antiderivative = 6.75

$$\int x^3(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx = \begin{cases} \frac{4adr^2x^4}{16r^2+128r+256}+\frac{32adrx^4}{16r^2+128r+256}+\frac{64adx^4}{16r^2+128r+256}+\frac{16aerx^4x^r}{16r^2+128r+256}+\frac{64aex^4x^r}{16r^2+128r+256}-\frac{bdnr^2x^4}{16r^2+128r+256}-\frac{8bdnrx^4}{16r^2+128r+25} \\ \frac{adx^4}{4}+\frac{ae\log\left(cx^n\right)}{n}-\frac{bdnx^4}{16}+\frac{bdx^4\log\left(cx^n\right)}{4}+\frac{be\log\left(cx^n\right)^2}{2n} \end{cases}$$

```
[In] integrate(x**3*(d+e*x**r)*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((4*a*d*r**2*x**4/(16*r**2 + 128*r + 256) + 32*a*d*r*x**4/(16*r**2
 + 128*r + 256) + 64*a*d*x**4/(16*r**2 + 128*r + 256) + 16*a*e*r*x**4*x**r/
(16*r**2 + 128*r + 256) + 64*a*e*x**4*x**r/(16*r**2 + 128*r + 256) - b*d*n*
r**2*x**4/(16*r**2 + 128*r + 256) - 8*b*d*n*r*x**4/(16*r**2 + 128*r + 256)
- 16*b*d*n*x**4/(16*r**2 + 128*r + 256) + 4*b*d*r**2*x**4*log(c*x**n)/(16*r
**2 + 128*r + 256) + 32*b*d*r*x**4*log(c*x**n)/(16*r**2 + 128*r + 256) + 64
*b*d*x**4*log(c*x**n)/(16*r**2 + 128*r + 256) - 16*b*e*n*x**4*x**r/(16*r**2
 + 128*r + 256) + 16*b*e*r*x**4*x**r*log(c*x**n)/(16*r**2 + 128*r + 256) +
64*b*e*x**4*x**r*log(c*x**n)/(16*r**2 + 128*r + 256), Ne(r, -4)), (a*d*x**4
/4 + a*e*log(c*x**n)/n - b*d*n*x**4/16 + b*d*x**4*log(c*x**n)/4 + b*e*log(c
*x**n)**2/(2*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int x^3(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{1}{16}\,bdnx^4+\frac{1}{4}\,bdx^4\log\left(cx^n\right)+\frac{1}{4}\,adx^4+\frac{bex^{r+4}\log\left(cx^n\right)}{r+4}-\frac{benx^{r+4}}{(r+4)^2}+\frac{aex^{r+4}}{r+4}$$

```
[In] integrate(x^3*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/16*b*d*n*x^4 + 1/4*b*d*x^4*log(c*x^n) + 1/4*a*d*x^4 + b*e*x^(r + 4)*log(
c*x^n)/(r + 4) - b*e*n*x^(r + 4)/(r + 4)^2 + a*e*x^(r + 4)/(r + 4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(55) = 110.

Time = 0.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.24

$$\int x^3(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{benrx^4x^r\log\left(x\right)}{r^2+8\,r+16}+\frac{4\,benx^4x^r\log\left(x\right)}{r^2+8\,r+16}+\frac{1}{4}\,bdnx^4\log\left(x\right)-\frac{benx^4x^r}{r^2+8\,r+16}-\frac{1}{16}\,bdnx^4+\frac{bex^4x^r\log\left(c\right)}{r+4}+\frac{1}{4}\,bdx^4\log\left(c\right)+\frac{aex^4x^r}{r+4}+\frac{1}{4}\,adx^4$$

```
[In] integrate(x^3*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*e*n*r*x^4*x^r*log(x)/(r^2 + 8*r + 16) + 4*b*e*n*x^4*x^r*log(x)/(r^2 + 8*r
 + 16) + 1/4*b*d*n*x^4*log(x) - b*e*n*x^4*x^r/(r^2 + 8*r + 16) - 1/16*b*d*n
*x^4 + b*e*x^4*x^r*log(c)/(r + 4) + 1/4*b*d*x^4*log(c) + a*e*x^4*x^r/(r + 4
) + 1/4*a*d*x^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx=\int x^3\left(d+e\,x^r\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

```
[In] int(x^3*(d + e*x^r)*(a + b*log(c*x^n)),x)
[Out] int(x^3*(d + e*x^r)*(a + b*log(c*x^n)), x)
```

3.369 $\int x(d+ex^r)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int x(d+ex^r)(a+b\log(cx^n))\,dx=-\frac{1}{4}bdnx^2-\frac{benx^{2+r}}{(2+r)^2}+\frac{1}{2}\left(dx^2+\frac{2ex^{2+r}}{2+r}\right)(a+b\log(cx^n))$$

```
[Out] -1/4*b*d*n*x^2-b*e*n*x^(2+r)/(2+r)^2+1/2*(d*x^2+2*e*x^(2+r)/(2+r))*(a+b*ln(
c*x^n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {14, 2371, 12}

$$\int x(d+ex^r)(a+b\log(cx^n))\,dx=\frac{1}{2}\left(dx^2+\frac{2ex^{r+2}}{r+2}\right)(a+b\log(cx^n))-\frac{1}{4}bdnx^2-\frac{benx^{r+2}}{(r+2)^2}$$

```
[In] Int[x*(d + e*x^r)*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/4*(b*d*n*x^2) - (b*e*n*x^(2 + r))/(2 + r)^2 + ((d*x^2 + (2*e*x^(2 + r))/
(2 + r))*(a + b*Log[c*x^n]))/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\left(dx^2+\frac{2ex^{2+r}}{2+r}\right)(a+b\log(cx^n))-(bn)\int \frac{1}{2}x\left(d+\frac{2ex^r}{2+r}\right)dx\\
&= \frac{1}{2}\left(dx^2+\frac{2ex^{2+r}}{2+r}\right)(a+b\log(cx^n))-\frac{1}{2}(bn)\int x\left(d+\frac{2ex^r}{2+r}\right)dx\\
&= \frac{1}{2}\left(dx^2+\frac{2ex^{2+r}}{2+r}\right)(a+b\log(cx^n))-\frac{1}{2}(bn)\int \left(dx+\frac{2ex^{1+r}}{2+r}\right)dx\\
&= -\frac{1}{4}bdnx^2-\frac{benx^{2+r}}{(2+r)^2}+\frac{1}{2}\left(dx^2+\frac{2ex^{2+r}}{2+r}\right)(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int x(d+ex^r)(a+b\log(cx^n))\,dx = \frac{x^2(2a(2+r)(d(2+r)+2ex^r)-bn(d(2+r)^2+4ex^r)+2b(2+r)(d(2+r)+2ex^r)\log(cx^n))}{4(2+r)^2}$$

```
[In] Integrate[x*(d + e*x^r)*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^2*(2*a*(2 + r)*(d*(2 + r) + 2*e*x^r) - b*n*(d*(2 + r)^2 + 4*e*x^r) + 2*b
*(2 + r)*(d*(2 + r) + 2*e*x^r)*Log[c*x^n]))/(4*(2 + r)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(55) = 110$.

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.95

method	result
parallelrisch	$-\frac{-4x^2x^r\ln(c\,x^n)ber-2x^2\ln(c\,x^n)bd\,r^2+x^2bdn\,r^2-8x^2x^r\ln(c\,x^n)be-4x^2x^raer+4x^2x^rben-8x^2\ln(c\,x^n)bdr-2x^2ad\,r^2+4x^2b}{4(r^2+4r+4)}$
risch	$\frac{b\,x^2(dr+2e\,x^r+2d)\ln(x^n)}{4+2r}-\frac{x^2\left(-8x^rae+4bdn-8ad-4x^raer+4x^rben-4i\pi bd\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-i\pi bd\,r^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(i\right.}{}$

[In] int(x*(d+e*x^r)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)

[Out] -1/4*(-4*x^2*x^r*ln(c*x^n)*b*e*r-2*x^2*ln(c*x^n)*b*d*r^2+x^2*b*d*n*r^2-8*x^2*x^r*ln(c*x^n)*b*e-4*x^2*x^r*a*e*r+4*x^2*x^r*b*e*n-8*x^2*ln(c*x^n)*b*d*r-2*x^2*a*d*r^2+4*x^2*b*d*n*r-8*x^2*x^r*a*e-8*x^2*ln(c*x^n)*b*d-8*x^2*a*d*r+4*b*d*n*x^2-8*a*d*x^2)/(r^2+4*r+4)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(55) = 110$.

Time = 0.32 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.69

$$\int x(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{2\left(bdr^2+4\,bdr+4\,bd\right)x^2\log\left(c\right)+2\left(bdnr^2+4\,bdnr+4\,bdn\right)x^2\log\left(x\right)-\left(4\,bdn+\left(bdn-2\,ad\right)r^2-8\,ad+\right.}{4\left(r^2+4\,r+\right.}$$

[In] integrate(x*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="fricas")

[Out] 1/4*(2*(b*d*r^2 + 4*b*d*r + 4*b*d)*x^2*log(c) + 2*(b*d*n*r^2 + 4*b*d*n*r + 4*b*d*n)*x^2*log(x) - (4*b*d*n + (b*d*n - 2*a*d)*r^2 - 8*a*d + 4*(b*d*n - 2*a*d)*r)*x^2 + 4*((b*e*r + 2*b*e)*x^2*log(c) + (b*e*n*r + 2*b*e*n)*x^2*log(x) - (b*e*n - a*e*r - 2*a*e)*x^2)*x^r)/(r^2 + 4*r + 4)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(51) = 102$.

Time = 0.86 (sec) , antiderivative size = 398, normalized size of antiderivative = 6.75

$$\int x(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\begin{cases}\frac{2adr^2x^2}{4r^2+16r+16}+\frac{8adrx^2}{4r^2+16r+16}+\frac{8adx^2}{4r^2+16r+16}+\frac{4aerx^2x^r}{4r^2+16r+16}+\frac{8aex^2x^r}{4r^2+16r+16}-\frac{bdnr^2x^2}{4r^2+16r+16}-\frac{4bdnrx^2}{4r^2+16r+16}-\frac{4bdnx^2}{4r^2+16r+16}+\frac{2b}{} \\ \frac{adx^2}{2}+\frac{ae\log\left(cx^n\right)}{n}-\frac{bdnx^2}{4}+\frac{bdx^2\log\left(cx^n\right)}{2}+\frac{be\log\left(cx^n\right)^2}{2n}\end{cases}$$

```
[In] integrate(x*(d+e*x**r)*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((2*a*d*r**2*x**2/(4*r**2 + 16*r + 16) + 8*a*d*r*x**2/(4*r**2 + 16
*r + 16) + 8*a*d*x**2/(4*r**2 + 16*r + 16) + 4*a*e*r*x**2*x**r/(4*r**2 + 16
*r + 16) + 8*a*e*x**2*x**r/(4*r**2 + 16*r + 16) - b*d*n*r**2*x**2/(4*r**2 +
 16*r + 16) - 4*b*d*n*r*x**2/(4*r**2 + 16*r + 16) - 4*b*d*n*x**2/(4*r**2 +
16*r + 16) + 2*b*d*r**2*x**2*log(c*x**n)/(4*r**2 + 16*r + 16) + 8*b*d*r*x**
2*log(c*x**n)/(4*r**2 + 16*r + 16) + 8*b*d*x**2*log(c*x**n)/(4*r**2 + 16*r
+ 16) - 4*b*e*n*x**2*x**r/(4*r**2 + 16*r + 16) + 4*b*e*r*x**2*x**r*log(c*x*
*n)/(4*r**2 + 16*r + 16) + 8*b*e*x**2*x**r*log(c*x**n)/(4*r**2 + 16*r + 16)
, Ne(r, -2)), (a*d*x**2/2 + a*e*log(c*x**n)/n - b*d*n*x**2/4 + b*d*x**2*log
(c*x**n)/2 + b*e*log(c*x**n)**2/(2*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int x(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{1}{4}\,bdnx^2 + \frac{1}{2}\,bdx^2\log\left(cx^n\right) + \frac{1}{2}\,adx^2 + \frac{bex^{r+2}\log\left(cx^n\right)}{r+2} - \frac{benx^{r+2}}{(r+2)^2} + \frac{aex^{r+2}}{r+2}$$

```
[In] integrate(x*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*n*x^2 + 1/2*b*d*x^2*log(c*x^n) + 1/2*a*d*x^2 + b*e*x^(r + 2)*log(c
*x^n)/(r + 2) - b*e*n*x^(r + 2)/(r + 2)^2 + a*e*x^(r + 2)/(r + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(55) = 110.

Time = 0.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.24

$$\int x(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{benrx^2x^r\log\left(x\right)}{r^2+4\,r+4} + \frac{2\,benx^2x^r\log\left(x\right)}{r^2+4\,r+4} + \frac{1}{2}\,bdnx^2\log\left(x\right) - \frac{benx^2x^r}{r^2+4\,r+4} - \frac{1}{4}\,bdnx^2 + \frac{bex^2x^r\log\left(c\right)}{r+2} + \frac{1}{2}\,bdx^2\log\left(c\right) + \frac{aex^2x^r}{r+2} + \frac{1}{2}\,adx^2$$

```
[In] integrate(x*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*e*n*r*x^2*x^r*log(x)/(r^2 + 4*r + 4) + 2*b*e*n*x^2*x^r*log(x)/(r^2 + 4*r
+ 4) + 1/2*b*d*n*x^2*log(x) - b*e*n*x^2*x^r/(r^2 + 4*r + 4) - 1/4*b*d*n*x^2
 + b*e*x^2*x^r*log(c)/(r + 2) + 1/2*b*d*x^2*log(c) + a*e*x^2*x^r/(r + 2) +
1/2*a*d*x^2
```

Mupad [F(-1)]

Timed out.

$$\int x(d + ex^r) (a + b \log (cx^n)) \, dx = \int x \left(d + e\, x^r\right) \left(a + b\, \ln \left(c\, x^n\right)\right) \, dx$$

```
[In] int(x*(d + e*x^r)*(a + b*log(c*x^n)),x)
[Out] int(x*(d + e*x^r)*(a + b*log(c*x^n)), x)
```

3.370 $\int \frac{(d+ex^r)(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{(d+ex^r)\,(a+b\log(cx^n))}{x}\,dx=-\frac{benx^r}{r^2}+\frac{ex^r(a+b\log(cx^n))}{r}+\frac{d(a+b\log(cx^n))^2}{2bn}$$

```
[Out] -b*e*n*x^r/r^2+e*x^r*(a+b*ln(c*x^n))/r+1/2*d*(a+b*ln(c*x^n))^2/b/n
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {14, 2393, 2338, 2341}

$$\int \frac{(d+ex^r)\,(a+b\log(cx^n))}{x}\,dx=\frac{d(a+b\log(cx^n))^2}{2bn}+\frac{ex^r(a+b\log(cx^n))}{r}-\frac{benx^r}{r^2}$$

```
[In] Int[((d + e*x^r)*(a + b*Log[c*x^n]))/x,x]
[Out] -((b*e*n*x^r)/r^2) + (e*x^r*(a + b*Log[c*x^n]))/r + (d*(a + b*Log[c*x^n])^2
)/(2*b*n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d(a+b\log(cx^n))}{x}+ex^{-1+r}(a+b\log(cx^n))\right)dx\\
&= d\int \frac{a+b\log(cx^n)}{x}\,dx+e\int x^{-1+r}(a+b\log(cx^n))\,dx\\
&= -\frac{benx^r}{r^2}+\frac{ex^r(a+b\log(cx^n))}{r}+\frac{d(a+b\log(cx^n))^2}{2bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x}\,dx=\frac{e(-bn+ar)x^r}{r^2}+ad\log(x)+\frac{bex^r\log(cx^n)}{r}+\frac{bd\log^2(cx^n)}{2n}$$

```
[In] Integrate[((d + e*x^r)*(a + b*Log[c*x^n]))/x,x]
[Out] (e*(-(b*n) + a*r)*x^r)/r^2 + a*d*Log[x] + (b*e*x^r*Log[c*x^n])/r + (b*d*Log
[c*x^n]^2)/(2*n)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

method	result
parallelrisch	$\frac{2\ln(x)adn\,r^2+2x^r\ln(c\,x^n)bern+bd\ln(c\,x^n)^2r^2+2x^r aenr-2x^r be\,n^2}{2r^2n}$
risch	$\frac{b(dr\ln(x)+e\,x^r)\ln(x^n)}{r}-\frac{i\ln(x)\pi bd\,\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)}{2}+\frac{i\ln(x)\pi bd\,\operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^n)^2}{2}+\frac{i\ln(x)\pi bd\,\operatorname{csgn}(ix}{2}$

```
[In] int((d+e*x^r)*(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*ln(x)*a*d*n*r^2+2*x^r*ln(c*x^n)*b*e*r*n+b*d*ln(c*x^n)^2*r^2+2*x^r*a*
e*n*r-2*x^r*b*e*n^2)/r^2/n
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x}\,dx = \frac{bdnr^2\log(x)^2 + 2\,(benr\log(x) + ber\log(c) - ben + aer)x^r + 2\,(bdr^2\log(c) + adr^2)\log(x)}{2\,r^2}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*n*r^2*log(x)^2 + 2*(b*e*n*r*log(x) + b*e*r*log(c) - b*e*n + a*e*r)
*x^r + 2*(b*d*r^2*log(c) + a*d*r^2)*log(x))/r^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(46) = 92.

Time = 2.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.47

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x}\,dx = \begin{cases} (a+b\log(c))\,(d+e)\log(x) & \text{for}\ n=0 \wedge r=0 \\ (a+b\log(c))\left(d\log(x)+\frac{ex^r}{r}\right) & \text{for}\ n=0 \\ (d+e)\left(\begin{cases} a\log(x) & \text{for}\ b=0 \\ -(-a-b\log(c))\log(x) & \text{for}\ n=0 \\ \frac{(-a-b\log(cx^n))^2}{2bn} & \text{otherwise} \end{cases}\right) & \text{for}\ r=0 \\ \frac{ad\log(cx^n)}{n}+\frac{aex^r}{r}+\frac{bd\log(cx^n)^2}{2n}-\frac{benx^r}{r^2}+\frac{bex^r\log(cx^n)}{r} & \text{otherwise} \end{cases}$$

```
[In] integrate((d+e*x**r)*(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise(((a + b*log(c))*(d + e)*log(x), Eq(n, 0) & Eq(r, 0)), ((a + b*log
(c))*(d*log(x) + e*x**r/r), Eq(n, 0)), ((d + e)*Piecewise((a*log(x), Eq(b,
0)), (-(-a - b*log(c))*log(x), Eq(n, 0)), ((-a - b*log(c*x**n))**2/(2*b*n),
 True)), Eq(r, 0)), (a*d*log(c*x**n)/n + a*e*x**r/r + b*d*log(c*x**n)**2/(2
*n) - b*e*n*x**r/r**2 + b*e*x**r*log(c*x**n)/r, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x}\,dx = \frac{bex^r\log(cx^n)}{r}+\frac{bd\log(cx^n)^2}{2n}+ad\log(x)-\frac{benx^r}{r^2}+\frac{aex^r}{r}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x,x, algorithm="maxima")
[Out] b*e*x^r*log(c*x^n)/r + 1/2*b*d*log(c*x^n)^2/n + a*d*log(x) - b*e*n*x^r/r^2
+ a*e*x^r/r
```

Giac [F]

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x}\,dx = \int \frac{(ex^r+d)(b\log(cx^n)+a)}{x}\,dx$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x,x, algorithm="giac")
[Out] integrate((e*x^r + d)*(b*log(c*x^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x}\,dx = \int \frac{(d+e\,x^r)\,(a+b\,\ln(c\,x^n))}{x}\,dx$$

```
[In] int(((d + e*x^r)*(a + b*log(c*x^n)))/x,x)
[Out] int(((d + e*x^r)*(a + b*log(c*x^n)))/x, x)
```

3.371 $\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^3}\,dx = -\frac{bdn}{4x^2}-\frac{benx^{-2+r}}{(2-r)^2}-\frac{d(a+b\log(cx^n))}{2x^2}-\frac{ex^{-2+r}(a+b\log(cx^n))}{2-r}$$

```
[Out] -1/4*b*d*n/x^2-b*e*n*x^(-2+r)/(2-r)^2-1/2*d*(a+b*ln(c*x^n))/x^2-e*x^(-2+r)*
(a+b*ln(c*x^n))/(2-r)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {14, 2372}

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^3}\,dx = -\frac{d(a+b\log(cx^n))}{2x^2}-\frac{ex^{r-2}(a+b\log(cx^n))}{2-r}-\frac{bdn}{4x^2}-\frac{benx^{r-2}}{(2-r)^2}$$

```
[In] Int[((d + e*x^r)*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/4*(b*d*n)/x^2 - (b*e*n*x^(-2 + r))/(2 - r)^2 - (d*(a + b*Log[c*x^n]))/(2
*x^2) - (e*x^(-2 + r)*(a + b*Log[c*x^n]))/(2 - r)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{d(a+b\log(cx^n))}{2x^2} - \frac{ex^{-2+r}(a+b\log(cx^n))}{2-r} - (bn)\int\left(-\frac{d}{2x^3} + \frac{ex^{-3+r}}{-2+r}\right)dx \\ &= -\frac{bdn}{4x^2} - \frac{benx^{-2+r}}{(2-r)^2} - \frac{d(a+b\log(cx^n))}{2x^2} - \frac{ex^{-2+r}(a+b\log(cx^n))}{2-r}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^3}\,dx = -\frac{2a(-2+r)(d(-2+r)-2ex^r)+bn(d(-2+r)^2+4ex^r)+2b(-2+r)(d(-2+r)-2ex^r)\log(cx^n)}{4(-2+r)^2x^2}$$

```
[In] Integrate[((d + e*x^r)*(a + b*Log[c*x^n]))/x^3,x]
[Out] -1/4*(2*a*(-2 + r)*(d*(-2 + r) - 2*e*x^r) + b*n*(d*(-2 + r)^2 + 4*e*x^r) +
2*b*(-2 + r)*(d*(-2 + r) - 2*e*x^r)*Log[c*x^n])/((-2 + r)^2*x^2)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.90

method	result
parallelrisch	$-\frac{-4x^r\ln(cx^n)ber+2\ln(cx^n)bdr^2+bdnr^2+8x^r\ln(cx^n)be-4x^raer+4x^rben-8\ln(cx^n)bdr+2adr^2-4bdnr+8x^rae+8b\ln(cx^n}{4x^2(r^2-4r+4)}$
risch	$-\frac{b(dr-2ex^r-2d)\ln(x^n)}{2(-2+r)x^2} - \frac{8x^rae+4bdn+8ad-4x^raer+4x^rben+4i\pi bd\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2+i\pi bdr^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(icx^n)^2}{}$

```
[In] int((d+e*x^r)*(a+b*ln(c*x^n))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-4*x^r*ln(c*x^n)*b*e*r+2*ln(c*x^n)*b*d*r^2+b*d*n*r^2+8*x^r*ln(c*x^n)*
b*e-4*x^r*a*e*r+4*x^r*b*e*n-8*ln(c*x^n)*b*d*r+2*a*d*r^2-4*b*d*n*r+8*x^r*a*e
+8*b*ln(c*x^n)*d-8*a*d*r+4*b*d*n+8*a*d)/x^2/(r^2-4*r+4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(62) = 124.

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.97

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^3}\,dx = -\frac{4\,bdn+(bdn+2\,ad)r^2+8\,ad-4\,(bdn+2\,ad)r+4\,(ben-aer+2\,ae-(ber-2\,be)\log(c)-(benr-}{4\,(r^2-4\,r+4)x^2}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b*d*n + (b*d*n + 2*a*d)*r^2 + 8*a*d - 4*(b*d*n + 2*a*d)*r + 4*(b*e*
n - a*e*r + 2*a*e - (b*e*r - 2*b*e)*log(c) - (b*e*n*r - 2*b*e*n)*log(x))*x^
r + 2*(b*d*r^2 - 4*b*d*r + 4*b*d)*log(c) + 2*(b*d*n*r^2 - 4*b*d*n*r + 4*b*d
*n)*log(x))/((r^2 - 4*r + 4)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(63) = 126.

Time = 2.04 (sec) , antiderivative size = 495, normalized size of antiderivative = 6.97

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^3}\,dx$$

$$= \begin{cases} -\frac{2adr^2}{4r^2x^2-16rx^2+16x^2} + \frac{8adr}{4r^2x^2-16rx^2+16x^2} - \frac{8ad}{4r^2x^2-16rx^2+16x^2} + \frac{4aerx^r}{4r^2x^2-16rx^2+16x^2} - \frac{8aex^r}{4r^2x^2-16rx^2+16x^2} - \frac{bdnr^2}{4r^2x^2-16rx^2} \\ -\frac{ad}{2x^2} + ae\log(x) + bd\left(-\frac{n}{4x^2} - \frac{\log(cx^n)}{2x^2}\right) - be\left(\begin{cases} -\log(c)\log(x) & \text{for } n=0 \\ -\frac{\log(cx^n)^2}{2n} & \text{otherwise} \end{cases}\right) \end{cases}$$

```
[In] integrate((d+e*x**r)*(a+b*ln(c*x**n))/x**3,x)
```

```
[Out] Piecewise((-2*a*d*r**2/(4*r**2*x**2 - 16*r*x**2 + 16*x**2) + 8*a*d*r/(4*r**
2*x**2 - 16*r*x**2 + 16*x**2) - 8*a*d/(4*r**2*x**2 - 16*r*x**2 + 16*x**2) +
 4*a*e*r*x**r/(4*r**2*x**2 - 16*r*x**2 + 16*x**2) - 8*a*e*x**r/(4*r**2*x**2
 - 16*r*x**2 + 16*x**2) - b*d*n*r**2/(4*r**2*x**2 - 16*r*x**2 + 16*x**2) +
4*b*d*n*r/(4*r**2*x**2 - 16*r*x**2 + 16*x**2) - 4*b*d*n/(4*r**2*x**2 - 16*r
*x**2 + 16*x**2) - 2*b*d*r**2*log(c*x**n)/(4*r**2*x**2 - 16*r*x**2 + 16*x**
2) + 8*b*d*r*log(c*x**n)/(4*r**2*x**2 - 16*r*x**2 + 16*x**2) - 8*b*d*log(c*
x**n)/(4*r**2*x**2 - 16*r*x**2 + 16*x**2) - 4*b*e*n*x**r/(4*r**2*x**2 - 16*
```

```
r*x**2 + 16*x**2) + 4*b*e*r*x**r*log(c*x**n)/(4*r**2*x**2 - 16*r*x**2 + 16*
x**2) - 8*b*e*x**r*log(c*x**n)/(4*r**2*x**2 - 16*r*x**2 + 16*x**2), Ne(r, 2
)), (-a*d/(2*x**2) + a*e*log(x) + b*d*(-n/(4*x**2) - log(c*x**n)/(2*x**2))
- b*e*Piecewise((-log(c)*log(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), True)),
 True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-3>0)', see `assume?` for more det
ails)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(62) = 124.

Time = 0.34 (sec) , antiderivative size = 389, normalized size of antiderivative = 5.48

$$\begin{aligned}\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^3}\,dx = &-\frac{bdnr^2\log(x)}{2(r^2-4r+4)x^2}+\frac{benrx^r\log(x)}{(r^2-4r+4)x^2}\\&-\frac{bdnr^2}{4(r^2-4r+4)x^2}-\frac{bdr^2\log(c)}{2(r^2-4r+4)x^2}\\&+\frac{berx^r\log(c)}{(r^2-4r+4)x^2}+\frac{2\,bdnr\log(x)}{(r^2-4r+4)x^2}-\frac{2\,benx^r\log(x)}{(r^2-4r+4)x^2}\\&+\frac{bdnr}{(r^2-4r+4)x^2}-\frac{adr^2}{2(r^2-4r+4)x^2}-\frac{benx^r}{(r^2-4r+4)x^2}\\&+\frac{aerx^r}{(r^2-4r+4)x^2}+\frac{2\,bdr\log(c)}{(r^2-4r+4)x^2}-\frac{2\,bex^r\log(c)}{(r^2-4r+4)x^2}\\&-\frac{2\,bdn\log(x)}{(r^2-4r+4)x^2}-\frac{bdn}{(r^2-4r+4)x^2}+\frac{2\,adr}{(r^2-4r+4)x^2}\\&-\frac{2\,aex^r}{(r^2-4r+4)x^2}-\frac{2\,bd\log(c)}{(r^2-4r+4)x^2}-\frac{2\,ad}{(r^2-4r+4)x^2}\end{aligned}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*b*d*n*r^2*log(x)/((r^2 - 4*r + 4)*x^2) + b*e*n*r*x^r*log(x)/((r^2 - 4*
r + 4)*x^2) - 1/4*b*d*n*r^2/((r^2 - 4*r + 4)*x^2) - 1/2*b*d*r^2*log(c)/((r^
```

```
2 - 4*r + 4)*x^2) + b*e*r*x^r*log(c)/((r^2 - 4*r + 4)*x^2) + 2*b*d*n*r*log(
x)/((r^2 - 4*r + 4)*x^2) - 2*b*e*n*x^r*log(x)/((r^2 - 4*r + 4)*x^2) + b*d*n
*r/((r^2 - 4*r + 4)*x^2) - 1/2*a*d*r^2/((r^2 - 4*r + 4)*x^2) - b*e*n*x^r/((
r^2 - 4*r + 4)*x^2) + a*e*r*x^r/((r^2 - 4*r + 4)*x^2) + 2*b*d*r*log(c)/((r^
2 - 4*r + 4)*x^2) - 2*b*e*x^r*log(c)/((r^2 - 4*r + 4)*x^2) - 2*b*d*n*log(x)
/((r^2 - 4*r + 4)*x^2) - b*d*n/((r^2 - 4*r + 4)*x^2) + 2*a*d*r/((r^2 - 4*r
+ 4)*x^2) - 2*a*e*x^r/((r^2 - 4*r + 4)*x^2) - 2*b*d*log(c)/((r^2 - 4*r + 4)
*x^2) - 2*a*d/((r^2 - 4*r + 4)*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^3}\,dx=\int \frac{(d+e\,x^r)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

```
[In] int(((d + e*x^r)*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int(((d + e*x^r)*(a + b*log(c*x^n)))/x^3, x)
```

3.372 $\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^5} dx$

Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^5} dx = -\frac{bdn}{16x^4} - \frac{benx^{-4+r}}{(4-r)^2} - \frac{d(a+b\log(cx^n))}{4x^4} - \frac{ex^{-4+r}(a+b\log(cx^n))}{4-r}$$

```
[Out] -1/16*b*d*n/x^4-b*e*n*x^(-4+r)/(4-r)^2-1/4*d*(a+b*ln(c*x^n))/x^4-e*x^(-4+r)
*(a+b*ln(c*x^n))/(4-r)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {14, 2372}

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^5} dx = -\frac{d(a+b\log(cx^n))}{4x^4} - \frac{ex^{r-4}(a+b\log(cx^n))}{4-r} - \frac{bdn}{16x^4} - \frac{benx^{r-4}}{(4-r)^2}$$

```
[In] Int[((d + e*x^r)*(a + b*Log[c*x^n]))/x^5,x]
```

```
[Out] -1/16*(b*d*n)/x^4 - (b*e*n*x^(-4 + r))/(4 - r)^2 - (d*(a + b*Log[c*x^n]))/(
4*x^4) - (e*x^(-4 + r)*(a + b*Log[c*x^n]))/(4 - r)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{d(a+b\log(cx^n))}{4x^4} - \frac{ex^{-4+r}(a+b\log(cx^n))}{4-r} - (bn)\int\left(-\frac{d}{4x^5}+\frac{ex^{-5+r}}{-4+r}\right)dx\\ &= -\frac{bdn}{16x^4} - \frac{benx^{-4+r}}{(4-r)^2} - \frac{d(a+b\log(cx^n))}{4x^4} - \frac{ex^{-4+r}(a+b\log(cx^n))}{4-r}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int\frac{(d+ex^r)(a+b\log(cx^n))}{x^5}\,dx = -\frac{4a(-4+r)(d(-4+r)-4ex^r)+bn(d(-4+r)^2+16ex^r)+4b(-4+r)(d(-4+r)-4ex^r)\log(cx^n)}{16(-4+r)^2x^4}$$

```
[In] Integrate[((d + e*x^r)*(a + b*Log[c*x^n]))/x^5,x]
[Out] -1/16*(4*a*(-4 + r)*(d*(-4 + r) - 4*e*x^r) + b*n*(d*(-4 + r)^2 + 16*e*x^r)
+ 4*b*(-4 + r)*(d*(-4 + r) - 4*e*x^r)*Log[c*x^n])/((-4 + r)^2*x^4)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.90

method	result
parallelrisch	$-\frac{-16x^r\ln(cx^n)ber+4\ln(cx^n)bdr^2+bdnr^2+64x^r\ln(cx^n)be-16x^raer+16x^rben-32\ln(cx^n)bdr+4adr^2-8bdnr+64x^rae+\cdots}{16x^4(r^2-8r+16)}$
risch	$-\frac{b(dr-4ex^r-4d)\ln(x^n)}{4(-4+r)x^4} - \frac{64x^rae+16bdn+64ad-16x^raer+16x^rben+32i\pi bd\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2+2i\pi bdr^2\,\mathrm{csgn}(ic)\,\mathrm{cs}\cdots}{}$

```
[In] int((d+e*x^r)*(a+b*ln(c*x^n))/x^5,x,method=_RETURNVERBOSE)
```

[Out] -1/16*(-16*x^r*ln(c*x^n)*b*e*r+4*ln(c*x^n)*b*d*r^2+b*d*n*r^2+64*x^r*ln(c*x^n)*b*e-16*x^r*a*e*r+16*x^r*b*e*n-32*ln(c*x^n)*b*d*r+4*a*d*r^2-8*b*d*n*r+64*x^r*a*e+64*b*ln(c*x^n)*d-32*a*d*r+16*b*d*n+64*a*d)/x^4/(r^2-8*r+16)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(62) = 124.

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.97

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^5}\,dx = -\frac{16\,bdn + (bdn+4\,ad)r^2 + 64\,ad - 8\,(bdn+4\,ad)r + 16\,(ben - aer + 4\,ae - (ber - 4\,be)\log(c) - (benr}{16\,(r^2 - 8\,r + 16)}$$

[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x^5,x, algorithm="fricas")

[Out] -1/16*(16*b*d*n + (b*d*n + 4*a*d)*r^2 + 64*a*d - 8*(b*d*n + 4*a*d)*r + 16*(b*e*n - a*e*r + 4*a*e - (b*e*r - 4*b*e)*log(c) - (b*e*n*r - 4*b*e*n)*log(x))*x^r + 4*(b*d*r^2 - 8*b*d*r + 16*b*d)*log(c) + 4*(b*d*n*r^2 - 8*b*d*n*r + 16*b*d*n)*log(x))/((r^2 - 8*r + 16)*x^4)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(63) = 126.

Time = 3.42 (sec) , antiderivative size = 495, normalized size of antiderivative = 6.97

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^5}\,dx = \begin{cases} -\frac{4adr^2}{16r^2x^4-128rx^4+256x^4} + \frac{32adr}{16r^2x^4-128rx^4+256x^4} - \frac{64ad}{16r^2x^4-128rx^4+256x^4} + \frac{16aerx^r}{16r^2x^4-128rx^4+256x^4} - \frac{64aex^r}{16r^2x^4-128rx^4+256x^4} - \\ -\frac{ad}{4x^4} + ae\log(x) + bd\left(-\frac{n}{16x^4} - \frac{\log(cx^n)}{4x^4}\right) - be\left(\begin{cases} -\log(c)\log(x) & \text{for } n=0 \\ -\frac{\log(cx^n)^2}{2n} & \text{otherwise} \end{cases}\right) \end{cases}$$

[In] integrate((d+e*x**r)*(a+b*ln(c*x**n))/x**5,x)

[Out] Piecewise((-4*a*d*r**2/(16*r**2*x**4 - 128*r*x**4 + 256*x**4) + 32*a*d*r/(16*r**2*x**4 - 128*r*x**4 + 256*x**4) - 64*a*d/(16*r**2*x**4 - 128*r*x**4 + 256*x**4) + 16*a*e*r*x**r/(16*r**2*x**4 - 128*r*x**4 + 256*x**4) - 64*a*e*x**r/(16*r**2*x**4 - 128*r*x**4 + 256*x**4) - b*d*n*r**2/(16*r**2*x**4 - 128*r*x**4 + 256*x**4) + 8*b*d*n*r/(16*r**2*x**4 - 128*r*x**4 + 256*x**4) - 16*b*d*n/(16*r**2*x**4 - 128*r*x**4 + 256*x**4) - 4*b*d*r**2*log(c*x**n)/(16*r**2*x**4 - 128*r*x**4 + 256*x**4) + 32*b*d*r*log(c*x**n)/(16*r**2*x**4 - 128*r*x**4 + 256*x**4) - 64*b*d*log(c*x**n)/(16*r**2*x**4 - 128*r*x**4 + 256

```
*x**4) - 16*b*e*n*x**r/(16*r**2*x**4 - 128*r*x**4 + 256*x**4) + 16*b*e*r*x*
*r*log(c*x**n)/(16*r**2*x**4 - 128*r*x**4 + 256*x**4) - 64*b*e*x**r*log(c*x
**n)/(16*r**2*x**4 - 128*r*x**4 + 256*x**4), Ne(r, 4)), (-a*d/(4*x**4) + a*
e*log(x) + b*d*(-n/(16*x**4) - log(c*x**n)/(4*x**4)) - b*e*Piecewise((-log(
c)*log(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), True)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^5}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-5>0)', see `assume?` for more det
ails)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(62) = 124.

Time = 0.34 (sec) , antiderivative size = 390, normalized size of antiderivative = 5.49

$$\begin{aligned}\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^5}\,dx = &-\frac{bdnr^2\log(x)}{4(r^2-8r+16)x^4}+\frac{benrx^r\log(x)}{(r^2-8r+16)x^4}\\&-\frac{bdnr^2}{16(r^2-8r+16)x^4}\\&-\frac{bdr^2\log(c)}{4(r^2-8r+16)x^4}+\frac{berx^r\log(c)}{(r^2-8r+16)x^4}\\&+\frac{2\,bdnr\log(x)}{(r^2-8r+16)x^4}-\frac{4\,benx^r\log(x)}{(r^2-8r+16)x^4}\\&+\frac{bdnr}{2(r^2-8r+16)x^4}-\frac{adr^2}{4(r^2-8r+16)x^4}\\&-\frac{benx^r}{(r^2-8r+16)x^4}+\frac{aerx^r}{(r^2-8r+16)x^4}\\&+\frac{2\,bdr\log(c)}{(r^2-8r+16)x^4}-\frac{4\,bex^r\log(c)}{(r^2-8r+16)x^4}\\&-\frac{4\,bdn\log(x)}{(r^2-8r+16)x^4}-\frac{bdn}{(r^2-8r+16)x^4}\\&+\frac{2\,adr}{(r^2-8r+16)x^4}-\frac{4\,aex^r}{(r^2-8r+16)x^4}\\&-\frac{4\,bd\log(c)}{(r^2-8r+16)x^4}-\frac{4\,ad}{(r^2-8r+16)x^4}\end{aligned}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x^5,x, algorithm="giac")
```

```
[Out] -1/4*b*d*n*r^2*log(x)/((r^2 - 8*r + 16)*x^4) + b*e*n*r*x^r*log(x)/((r^2 - 8
*r + 16)*x^4) - 1/16*b*d*n*r^2/((r^2 - 8*r + 16)*x^4) - 1/4*b*d*r^2*log(c)/
((r^2 - 8*r + 16)*x^4) + b*e*r*x^r*log(c)/((r^2 - 8*r + 16)*x^4) + 2*b*d*n*
r*log(x)/((r^2 - 8*r + 16)*x^4) - 4*b*e*n*x^r*log(x)/((r^2 - 8*r + 16)*x^4)
 + 1/2*b*d*n*r/((r^2 - 8*r + 16)*x^4) - 1/4*a*d*r^2/((r^2 - 8*r + 16)*x^4)
- b*e*n*x^r/((r^2 - 8*r + 16)*x^4) + a*e*r*x^r/((r^2 - 8*r + 16)*x^4) + 2*b
*d*r*log(c)/((r^2 - 8*r + 16)*x^4) - 4*b*e*x^r*log(c)/((r^2 - 8*r + 16)*x^4
) - 4*b*d*n*log(x)/((r^2 - 8*r + 16)*x^4) - b*d*n/((r^2 - 8*r + 16)*x^4) +
2*a*d*r/((r^2 - 8*r + 16)*x^4) - 4*a*e*x^r/((r^2 - 8*r + 16)*x^4) - 4*b*d*l
og(c)/((r^2 - 8*r + 16)*x^4) - 4*a*d/((r^2 - 8*r + 16)*x^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^5}\,dx = \int \frac{(d+e\,x^r)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^5}\,dx$$

```
[In] int(((d + e*x^r)*(a + b*log(c*x^n)))/x^5,x)
```

```
[Out] int(((d + e*x^r)*(a + b*log(c*x^n)))/x^5, x)
```

3.373 $\int x^4(d+ex^r)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int x^4(d+ex^r)(a+b\log(cx^n))\,dx=-\frac{1}{25}bdnx^5-\frac{benx^{5+r}}{(5+r)^2}+\frac{1}{5}\left(dx^5+\frac{5ex^{5+r}}{5+r}\right)(a+b\log(cx^n))$$

```
[Out] -1/25*b*d*n*x^5-b*e*n*x^(5+r)/(5+r)^2+1/5*(d*x^5+5*e*x^(5+r)/(5+r))*(a+b*ln
(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {14, 2371, 12}

$$\int x^4(d+ex^r)(a+b\log(cx^n))\,dx=\frac{1}{5}\left(dx^5+\frac{5ex^{r+5}}{r+5}\right)(a+b\log(cx^n))-\frac{1}{25}bdnx^5-\frac{benx^{r+5}}{(r+5)^2}$$

```
[In] Int[x^4*(d + e*x^r)*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/25*(b*d*n*x^5) - (b*e*n*x^(5 + r))/(5 + r)^2 + ((d*x^5 + (5*e*x^(5 + r))
/(5 + r))*(a + b*Log[c*x^n]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}\left(dx^5+\frac{5ex^{5+r}}{5+r}\right)(a+b\log(cx^n))-(bn)\int\frac{1}{5}x^4\left(d+\frac{5ex^r}{5+r}\right)dx\\
&= \frac{1}{5}\left(dx^5+\frac{5ex^{5+r}}{5+r}\right)(a+b\log(cx^n))-\frac{1}{5}(bn)\int x^4\left(d+\frac{5ex^r}{5+r}\right)dx\\
&= \frac{1}{5}\left(dx^5+\frac{5ex^{5+r}}{5+r}\right)(a+b\log(cx^n))-\frac{1}{5}(bn)\int\left(dx^4+\frac{5ex^{4+r}}{5+r}\right)dx\\
&= -\frac{1}{25}bdnx^5-\frac{benx^{5+r}}{(5+r)^2}+\frac{1}{5}\left(dx^5+\frac{5ex^{5+r}}{5+r}\right)(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int x^4(d+ex^r)(a+b\log(cx^n))\,dx = \frac{x^5(5a(5+r)(d(5+r)+5ex^r)-bn(d(5+r)^2+25ex^r)+5b(5+r)(d(5+r)+5ex^r)\log(cx^n))}{25(5+r)^2}$$

```
[In] Integrate[x^4*(d + e*x^r)*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^5*(5*a*(5 + r)*(d*(5 + r) + 5*e*x^r) - b*n*(d*(5 + r)^2 + 25*e*x^r) + 5*
b*(5 + r)*(d*(5 + r) + 5*e*x^r)*Log[c*x^n]))/(25*(5 + r)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(55) = 110.

Time = 2.62 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.86

method	result
parallelrisch	$-\frac{-25x^5x^r\ln(cx^n)ber-5x^5\ln(cx^n)bdr^2+x^5bdnr^2-125x^5x^r\ln(cx^n)be-25x^5x^raer+25x^5x^rben-50x^5\ln(cx^n)bdr-5x^5ad}{25(5+r)^2}$
risch	$\frac{bx^5(dr+5ex^r+5d)\ln(x^n)}{25+5r}-\frac{x^5\left(-250x^rae+50bdn-250ad-50x^raer+50x^rben-125i\pi bd\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2-5i\pi bdr^2\,c\right.}{}$

```
[In] int(x^4*(d+e*x^r)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/25*(-25*x^5*x^r*ln(c*x^n)*b*e*r-5*x^5*ln(c*x^n)*b*d*r^2+x^5*b*d*n*r^2-12
5*x^5*x^r*ln(c*x^n)*b*e-25*x^5*x^r*a*e*r+25*x^5*x^r*b*e*n-50*x^5*ln(c*x^n)*
b*d*r-5*x^5*a*d*r^2+10*x^5*b*d*n*r-125*x^5*x^r*a*e-125*x^5*b*ln(c*x^n)*d-50
*x^5*a*d*r+25*b*d*n*x^5-125*x^5*a*d)/(5+r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(55) = 110.

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.69

$$\int x^4(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{5\,(bdr^2+10\,bdr+25\,bd)x^5\log\left(c\right)+5\,(bdnr^2+10\,bdnr+25\,bdn)x^5\log\left(x\right)-(25\,bdn+(bdn-5\,ad)r^2-}{25\,(r^2}$$

```
[In] integrate(x^4*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 1/25*(5*(b*d*r^2 + 10*b*d*r + 25*b*d)*x^5*log(c) + 5*(b*d*n*r^2 + 10*b*d*n*
r + 25*b*d*n)*x^5*log(x) - (25*b*d*n + (b*d*n - 5*a*d)*r^2 - 125*a*d + 10*(
b*d*n - 5*a*d)*r)*x^5 + 25*((b*e*r + 5*b*e)*x^5*log(c) + (b*e*n*r + 5*b*e*n
)*x^5*log(x) - (b*e*n - a*e*r - 5*a*e)*x^5)*x^r)/(r^2 + 10*r + 25)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(51) = 102.

Time = 5.03 (sec) , antiderivative size = 398, normalized size of antiderivative = 6.75

$$\int x^4(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\begin{cases}\frac{5adr^2x^5}{25r^2+250r+625}+\frac{50adrx^5}{25r^2+250r+625}+\frac{125adx^5}{25r^2+250r+625}+\frac{25aerx^5x^r}{25r^2+250r+625}+\frac{125aex^5x^r}{25r^2+250r+625}-\frac{bdnr^2x^5}{25r^2+250r+625}-\frac{10bdnrx^5}{25r^2+250r+62} \\ \frac{adx^5}{5}+\frac{ae\log\left(cx^n\right)}{n}-\frac{bdnx^5}{25}+\frac{bdx^5\log\left(cx^n\right)}{5}+\frac{be\log\left(cx^n\right)^2}{2n}\end{cases}$$

```
[In] integrate(x**4*(d+e*x**r)*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((5*a*d*r**2*x**5/(25*r**2 + 250*r + 625) + 50*a*d*r*x**5/(25*r**2
 + 250*r + 625) + 125*a*d*x**5/(25*r**2 + 250*r + 625) + 25*a*e*r*x**5*x**r
/(25*r**2 + 250*r + 625) + 125*a*e*x**5*x**r/(25*r**2 + 250*r + 625) - b*d*
n*r**2*x**5/(25*r**2 + 250*r + 625) - 10*b*d*n*r*x**5/(25*r**2 + 250*r + 62
5) - 25*b*d*n*x**5/(25*r**2 + 250*r + 625) + 5*b*d*r**2*x**5*log(c*x**n)/(2
5*r**2 + 250*r + 625) + 50*b*d*r*x**5*log(c*x**n)/(25*r**2 + 250*r + 625) +
 125*b*d*x**5*log(c*x**n)/(25*r**2 + 250*r + 625) - 25*b*e*n*x**5*x**r/(25*
r**2 + 250*r + 625) + 25*b*e*r*x**5*x**r*log(c*x**n)/(25*r**2 + 250*r + 625
) + 125*b*e*x**5*x**r*log(c*x**n)/(25*r**2 + 250*r + 625), Ne(r, -5)), (a*d
*x**5/5 + a*e*log(c*x**n)/n - b*d*n*x**5/25 + b*d*x**5*log(c*x**n)/5 + b*e*
log(c*x**n)**2/(2*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int x^4(d+ex^r)\,(a+b\log{(cx^n)})\;dx = -\frac{1}{25}\,bdnx^5+\frac{1}{5}\,bdx^5\log{(cx^n)}+\frac{1}{5}\,adx^5+\frac{bex^{r+5}\log{(cx^n)}}{r+5}-\frac{benx^{r+5}}{(r+5)^2}+\frac{aex^{r+5}}{r+5}$$

```
[In] integrate(x^4*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/25*b*d*n*x^5 + 1/5*b*d*x^5*log(c*x^n) + 1/5*a*d*x^5 + b*e*x^(r + 5)*log(
c*x^n)/(r + 5) - b*e*n*x^(r + 5)/(r + 5)^2 + a*e*x^(r + 5)/(r + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(55) = 110.

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.24

$$\int x^4(d+ex^r)\,(a+b\log{(cx^n)})\;dx = \frac{benrx^5x^r\log{(x)}}{r^2+10\,r+25}+\frac{5\,benx^5x^r\log{(x)}}{r^2+10\,r+25}+\frac{1}{5}\,bdnx^5\log{(x)}-\frac{benx^5x^r}{r^2+10\,r+25}-\frac{1}{25}\,bdnx^5+\frac{bex^5x^r\log{(c)}}{r+5}+\frac{1}{5}\,bdx^5\log{(c)}+\frac{aex^5x^r}{r+5}+\frac{1}{5}\,adx^5$$

```
[In] integrate(x^4*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*e*n*r*x^5*x^r*log(x)/(r^2 + 10*r + 25) + 5*b*e*n*x^5*x^r*log(x)/(r^2 + 10
*r + 25) + 1/5*b*d*n*x^5*log(x) - b*e*n*x^5*x^r/(r^2 + 10*r + 25) - 1/25*b*
d*n*x^5 + b*e*x^5*x^r*log(c)/(r + 5) + 1/5*b*d*x^5*log(c) + a*e*x^5*x^r/(r
+ 5) + 1/5*a*d*x^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx=\int x^4\left(d+e\,x^r\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

```
[In] int(x^4*(d + e*x^r)*(a + b*log(c*x^n)),x)
[Out] int(x^4*(d + e*x^r)*(a + b*log(c*x^n)), x)
```

3.374 $\int x^2(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int x^2(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{9}bdnx^3-\frac{benx^{3+r}}{(3+r)^2}+\frac{1}{3}\left(dx^3+\frac{3ex^{3+r}}{3+r}\right)\left(a+b\log\left(cx^n\right)\right)$$

```
[Out] -1/9*b*d*n*x^3-b*e*n*x^(3+r)/(3+r)^2+1/3*(d*x^3+3*e*x^(3+r)/(3+r))*(a+b*ln(
c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {14, 2371, 12}

$$\int x^2(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{1}{3}\left(dx^3+\frac{3ex^{r+3}}{r+3}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{9}bdnx^3-\frac{benx^{r+3}}{(r+3)^2}$$

```
[In] Int[x^2*(d + e*x^r)*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/9*(b*d*n*x^3) - (b*e*n*x^(3 + r))/(3 + r)^2 + ((d*x^3 + (3*e*x^(3 + r))/
(3 + r))*(a + b*Log[c*x^n]))/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\left(dx^3+\frac{3ex^{3+r}}{3+r}\right)(a+b\log(cx^n))-(bn)\int\frac{1}{3}x^2\left(d+\frac{3ex^r}{3+r}\right)\,dx\\ &=\frac{1}{3}\left(dx^3+\frac{3ex^{3+r}}{3+r}\right)(a+b\log(cx^n))-\frac{1}{3}(bn)\int x^2\left(d+\frac{3ex^r}{3+r}\right)\,dx\\ &=\frac{1}{3}\left(dx^3+\frac{3ex^{3+r}}{3+r}\right)(a+b\log(cx^n))-\frac{1}{3}(bn)\int\left(dx^2+\frac{3ex^{2+r}}{3+r}\right)\,dx\\ &=-\frac{1}{9}bdnx^3-\frac{benx^{3+r}}{(3+r)^2}+\frac{1}{3}\left(dx^3+\frac{3ex^{3+r}}{3+r}\right)(a+b\log(cx^n))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int x^2(d+ex^r)\left(a+b\log(cx^n)\right)\,dx$$
$$=\frac{x^3(3a(3+r)\left(d(3+r)+3ex^r\right)-bn(d(3+r)^2+9ex^r)+3b(3+r)\left(d(3+r)+3ex^r\right)\log(cx^n))}{9(3+r)^2}$$

```
[In] Integrate[x^2*(d + e*x^r)*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^3*(3*a*(3 + r)*(d*(3 + r) + 3*e*x^r) - b*n*(d*(3 + r)^2 + 9*e*x^r) + 3*b
*(3 + r)*(d*(3 + r) + 3*e*x^r)*Log[c*x^n]))/(9*(3 + r)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(55) = 110.

Time = 0.70 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.86

method	result
parallelrisch	$-\frac{-9x^3x^r\ln(cx^n)ber-3x^3\ln(cx^n)bdr^2+x^3bdnr^2-27x^3x^r\ln(cx^n)be-9x^3x^raer+9x^3x^rben-18x^3\ln(cx^n)bdr-3x^3adr^2+6x}{9(3+r)^2}$
risch	$\frac{bx^3(dr+3ex^r+3d)\ln(x^n)}{9+3r}-\frac{x^3\left(-54x^rae+18bdn-54ad-18x^raer+18x^rben-27i\pi bd\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2-3i\pi bdr^2\,\mathrm{csgn}(i\right.}{}$

```
[In] int(x^2*(d+e*x^r)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*(-9*x^3*x^r*ln(c*x^n)*b*e*r-3*x^3*ln(c*x^n)*b*d*r^2+x^3*b*d*n*r^2-27*x
^3*x^r*ln(c*x^n)*b*e-9*x^3*x^r*a*e*r+9*x^3*x^r*b*e*n-18*x^3*ln(c*x^n)*b*d*r
-3*x^3*a*d*r^2+6*x^3*b*d*n*r-27*x^3*x^r*a*e-27*x^3*ln(c*x^n)*b*d-18*x^3*a*d
*r+9*b*d*n*x^3-27*x^3*a*d)/(3+r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(55) = 110.

Time = 0.31 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.69

$$\int x^2(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{3\left(bdr^2+6\,bdr+9\,bd\right)x^3\log\left(c\right)+3\left(bdnr^2+6\,bdnr+9\,bdn\right)x^3\log\left(x\right)-\left(9\,bdn+\left(bdn-3\,ad\right)r^2-27\,ad+\right.}{9\left(r^2+6\,r\right.}$$

```
[In] integrate(x^2*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b*d*r^2 + 6*b*d*r + 9*b*d)*x^3*log(c) + 3*(b*d*n*r^2 + 6*b*d*n*r +
9*b*d*n)*x^3*log(x) - (9*b*d*n + (b*d*n - 3*a*d)*r^2 - 27*a*d + 6*(b*d*n -
3*a*d)*r)*x^3 + 9*((b*e*r + 3*b*e)*x^3*log(c) + (b*e*n*r + 3*b*e*n)*x^3*log
(x) - (b*e*n - a*e*r - 3*a*e)*x^3)*x^r)/(r^2 + 6*r + 9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(51) = 102.

Time = 1.51 (sec) , antiderivative size = 398, normalized size of antiderivative = 6.75

$$\int x^2(d+ex^r)\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\begin{cases}\frac{3adr^2x^3}{9r^2+54r+81}+\frac{18adrx^3}{9r^2+54r+81}+\frac{27adx^3}{9r^2+54r+81}+\frac{9aerx^3x^r}{9r^2+54r+81}+\frac{27aex^3x^r}{9r^2+54r+81}-\frac{bdnr^2x^3}{9r^2+54r+81}-\frac{6bdnrx^3}{9r^2+54r+81}-\frac{9bdnx^3}{9r^2+54r+81}+\frac{3b}{}\\ \frac{adx^3}{3}+\frac{ae\log\left(cx^n\right)}{n}-\frac{bdnx^3}{9}+\frac{bdx^3\log\left(cx^n\right)}{3}+\frac{be\log\left(cx^n\right)^2}{2n}\end{cases}$$

```
[In] integrate(x**2*(d+e*x**r)*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((3*a*d*r**2*x**3/(9*r**2 + 54*r + 81) + 18*a*d*r*x**3/(9*r**2 + 5
4*r + 81) + 27*a*d*x**3/(9*r**2 + 54*r + 81) + 9*a*e*r*x**3*x**r/(9*r**2 +
54*r + 81) + 27*a*e*x**3*x**r/(9*r**2 + 54*r + 81) - b*d*n*r**2*x**3/(9*r**
2 + 54*r + 81) - 6*b*d*n*r*x**3/(9*r**2 + 54*r + 81) - 9*b*d*n*x**3/(9*r**2
 + 54*r + 81) + 3*b*d*r**2*x**3*log(c*x**n)/(9*r**2 + 54*r + 81) + 18*b*d*r
*x**3*log(c*x**n)/(9*r**2 + 54*r + 81) + 27*b*d*x**3*log(c*x**n)/(9*r**2 +
54*r + 81) - 9*b*e*n*x**3*x**r/(9*r**2 + 54*r + 81) + 9*b*e*r*x**3*x**r*log
(c*x**n)/(9*r**2 + 54*r + 81) + 27*b*e*x**3*x**r*log(c*x**n)/(9*r**2 + 54*r
 + 81), Ne(r, -3)), (a*d*x**3/3 + a*e*log(c*x**n)/n - b*d*n*x**3/9 + b*d*x*
*3*log(c*x**n)/3 + b*e*log(c*x**n)**2/(2*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int x^2(d+ex^r)\,(a+b\log(cx^n))\ dx = -\frac{1}{9}\,bdnx^3 + \frac{1}{3}\,bdx^3\log(cx^n) + \frac{1}{3}\,adx^3 + \frac{bex^{r+3}\log(cx^n)}{r+3} - \frac{benx^{r+3}}{(r+3)^2} + \frac{aex^{r+3}}{r+3}$$

```
[In] integrate(x^2*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/9*b*d*n*x^3 + 1/3*b*d*x^3*log(c*x^n) + 1/3*a*d*x^3 + b*e*x^(r + 3)*log(c
*x^n)/(r + 3) - b*e*n*x^(r + 3)/(r + 3)^2 + a*e*x^(r + 3)/(r + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(55) = 110.

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.24

$$\int x^2(d+ex^r)\,(a+b\log(cx^n))\ dx = \frac{benrx^3x^r\log(x)}{r^2+6\,r+9} + \frac{3\,benx^3x^r\log(x)}{r^2+6\,r+9} + \frac{1}{3}\,bdnx^3\log(x) - \frac{benx^3x^r}{r^2+6\,r+9} - \frac{1}{9}\,bdnx^3 + \frac{bex^3x^r\log(c)}{r+3} + \frac{1}{3}\,bdx^3\log(c) + \frac{aex^3x^r}{r+3} + \frac{1}{3}\,adx^3$$

```
[In] integrate(x^2*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*e*n*r*x^3*x^r*log(x)/(r^2 + 6*r + 9) + 3*b*e*n*x^3*x^r*log(x)/(r^2 + 6*r
+ 9) + 1/3*b*d*n*x^3*log(x) - b*e*n*x^3*x^r/(r^2 + 6*r + 9) - 1/9*b*d*n*x^3
 + b*e*x^3*x^r*log(c)/(r + 3) + 1/3*b*d*x^3*log(c) + a*e*x^3*x^r/(r + 3) +
1/3*a*d*x^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^r)\,(a+b\log{(cx^n)})\ dx = \int x^2\,(d+e\,x^r)\ (a+b\,\ln{(c\,x^n)})\ dx$$

```
[In] int(x^2*(d + e*x^r)*(a + b*log(c*x^n)),x)
[Out] int(x^2*(d + e*x^r)*(a + b*log(c*x^n)), x)
```

3.375 $\int (d+ex^r)(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 57

$$\int (d+ex^r)(a+b\log(cx^n))\,dx = -bdnx - \frac{benx^{1+r}}{(1+r)^2} + dx(a+b\log(cx^n)) + \frac{ex^{1+r}(a+b\log(cx^n))}{1+r}$$

```
[Out] -b*d*n*x-b*e*n*x^(1+r)/(1+r)^2+d*x*(a+b*ln(c*x^n))+e*x^(1+r)*(a+b*ln(c*x^n)
)/(1+r)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2350, 12}

$$\int (d+ex^r)(a+b\log(cx^n))\,dx = dx(a+b\log(cx^n)) + \frac{ex^{r+1}(a+b\log(cx^n))}{r+1} - bdnx - \frac{benx^{r+1}}{(r+1)^2}$$

```
[In] Int[(d + e*x^r)*(a + b*Log[c*x^n]),x]
```

```
[Out] -(b*d*n*x) - (b*e*n*x^(1 + r))/(1 + r)^2 + d*x*(a + b*Log[c*x^n]) + (e*x^(1
 + r)*(a + b*Log[c*x^n]))/(1 + r)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2350

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Dist[a + b*Log[c*x^n], u
, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= dx(a+b\log(cx^n))+\frac{ex^{1+r}(a+b\log(cx^n))}{1+r}-(bn)\int\frac{d+dr+ex^r}{1+r}\,dx\\&= dx(a+b\log(cx^n))+\frac{ex^{1+r}(a+b\log(cx^n))}{1+r}-\frac{(bn)\int(d+dr+ex^r)\,dx}{1+r}\\&= -bdnx-\frac{benx^{1+r}}{(1+r)^2}+dx(a+b\log(cx^n))+\frac{ex^{1+r}(a+b\log(cx^n))}{1+r}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int(d+ex^r)(a+b\log(cx^n))\,dx=x\left(ad-bdn-\frac{benx^r}{(1+r)^2}+bd\log(cx^n)+\frac{ex^r(a+b\log(cx^n))}{1+r}\right)$$

```
[In] Integrate[(d + e*x^r)*(a + b*Log[c*x^n]),x]
[Out] x*(a*d - b*d*n - (b*e*n*x^r)/(1 + r)^2 + b*d*Log[c*x^n] + (e*x^r*(a + b*Log
[c*x^n]))/(1 + r))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(57) = 114.

Time = 0.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.53

method	result
parallelrisch	$-\frac{-x\,x^r\ln(c\,x^n)ber-x\ln(c\,x^n)bd\,r^2+xbdn\,r^2-x\,x^r\ln(c\,x^n)be-x\,x^r aer+x\,x^r ben-2x\ln(c\,x^n)bdr-xad\,r^2+2xbdnr-x\,x^r ae-x\,\mathrm{l}}{r^2+2r+1}$
risch	$\frac{bx(dr+e\,x^r+d)\ln(x^n)}{1+r}-\frac{x\left(-2x^r ae+2bdn-2ad-2x^r aer+2x^r ben-i\pi bd\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-i\pi bd\,r^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2\right.}{}$

```
[In] int((d+e*x^r)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] -(-x*x^r*ln(c*x^n)*b*e*r-x*ln(c*x^n)*b*d*r^2+x*b*d*n*r^2-x*x^r*ln(c*x^n)*b*
e-x*x^r*a*e*r+x*x^r*b*e*n-2*x*ln(c*x^n)*b*d*r-x*a*d*r^2+2*x*b*d*n*r-x*x^r*a
*e-x*ln(c*x^n)*b*d-2*x*a*d*r+b*d*n*x-x*a*d)/(r^2+2*r+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(57) = 114$.

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.42

$$\int (d+ex^r)(a+b\log(cx^n))\,dx$$
$$= \frac{(bdr^2+2\,bdr+bd)x\log(c)+(bdnr^2+2\,bdnr+bdn)x\log(x)-(bdn+(bdn-ad)r^2-ad+2\,(bdn-ad$$
$$r^2+2\,r+1$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] ((b*d*r^2 + 2*b*d*r + b*d)*x*log(c) + (b*d*n*r^2 + 2*b*d*n*r + b*d*n)*x*log
(x) - (b*d*n + (b*d*n - a*d)*r^2 - a*d + 2*(b*d*n - a*d)*r)*x + ((b*e*r + b
*e)*x*log(c) + (b*e*n*r + b*e*n)*x*log(x) - (b*e*n - a*e*r - a*e)*x)*x^r)/(
r^2 + 2*r + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(54) = 108$.

Time = 0.47 (sec) , antiderivative size = 323, normalized size of antiderivative = 5.67

$$\int (d+ex^r)(a+b\log(cx^n))\,dx$$
$$= \begin{cases} \frac{adr^2x}{r^2+2r+1}+\frac{2adrx}{r^2+2r+1}+\frac{adx}{r^2+2r+1}+\frac{aerxx^r}{r^2+2r+1}+\frac{aexx^r}{r^2+2r+1}-\frac{bdnr^2x}{r^2+2r+1}-\frac{2bdnrx}{r^2+2r+1}-\frac{bdnx}{r^2+2r+1}+\frac{bdr^2x\log(cx^n)}{r^2+2r+1}+\frac{2bdrx\log}{r^2+2r} \\ adx+\frac{ae\log(cx^n)}{n}-bdnx+bdx\log(cx^n)+\frac{be\log(cx^n)^2}{2n} \end{cases}$$

```
[In] integrate((d+e*x**r)*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((a*d*r**2*x/(r**2 + 2*r + 1) + 2*a*d*r*x/(r**2 + 2*r + 1) + a*d*x
/(r**2 + 2*r + 1) + a*e*r*x*x**r/(r**2 + 2*r + 1) + a*e*x*x**r/(r**2 + 2*r
+ 1) - b*d*n*r**2*x/(r**2 + 2*r + 1) - 2*b*d*n*r*x/(r**2 + 2*r + 1) - b*d*n
*x/(r**2 + 2*r + 1) + b*d*r**2*x*log(c*x**n)/(r**2 + 2*r + 1) + 2*b*d*r*x*l
og(c*x**n)/(r**2 + 2*r + 1) + b*d*x*log(c*x**n)/(r**2 + 2*r + 1) - b*e*n*x*
x**r/(r**2 + 2*r + 1) + b*e*r*x*x**r*log(c*x**n)/(r**2 + 2*r + 1) + b*e*x*x
**r*log(c*x**n)/(r**2 + 2*r + 1), Ne(r, -1)), (a*d*x + a*e*log(c*x**n)/n -
b*d*n*x + b*d*x*log(c*x**n) + b*e*log(c*x**n)**2/(2*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int (d+ex^r)(a+b\log(cx^n))\,dx = -bdnx + bdx\log(cx^n) + adx + \frac{bex^{r+1}\log(cx^n)}{r+1} - \frac{benx^{r+1}}{(r+1)^2} + \frac{aex^{r+1}}{r+1}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -b*d*n*x + b*d*x*log(c*x^n) + a*d*x + b*e*x^(r + 1)*log(c*x^n)/(r + 1) - b*
e*n*x^(r + 1)/(r + 1)^2 + a*e*x^(r + 1)/(r + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.93

$$\int (d+ex^r)(a+b\log(cx^n))\,dx = \frac{benrxx^r\log(x)}{r^2+2r+1} + bdnx\log(x) + \frac{benxx^r\log(x)}{r^2+2r+1} - bdnx - \frac{benxx^r}{r^2+2r+1} + bdx\log(c) + \frac{bexx^r\log(c)}{r+1} + adx + \frac{aexx^r}{r+1}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*e*n*r*x*x^r*log(x)/(r^2 + 2*r + 1) + b*d*n*x*log(x) + b*e*n*x*x^r*log(x)/
(r^2 + 2*r + 1) - b*d*n*x - b*e*n*x*x^r/(r^2 + 2*r + 1) + b*d*x*log(c) + b*
e*x*x^r*log(c)/(r + 1) + a*d*x + a*e*x*x^r/(r + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^r)(a+b\log(cx^n))\,dx = \int (d+e\,x^r)\,(a+b\,\ln(c\,x^n))\,dx$$

```
[In] int((d + e*x^r)*(a + b*log(c*x^n)),x)
```

```
[Out] int((d + e*x^r)*(a + b*log(c*x^n)), x)
```

3.376 $\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^2}\,dx = -\frac{bdn}{x} - \frac{benx^{-1+r}}{(1-r)^2} - \frac{d(a+b\log(cx^n))}{x} - \frac{ex^{-1+r}(a+b\log(cx^n))}{1-r}$$

```
[Out] -b*d*n/x-b*e*n*x^(-1+r)/(1-r)^2-d*(a+b*ln(c*x^n))/x-e*x^(-1+r)*(a+b*ln(c*x^
n))/(1-r)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {14, 2372, 12}

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^2}\,dx = -\frac{d(a+b\log(cx^n))}{x} - \frac{ex^{r-1}(a+b\log(cx^n))}{1-r} - \frac{bdn}{x} - \frac{benx^{r-1}}{(1-r)^2}$$

```
[In] Int[((d + e*x^r)*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -((b*d*n)/x) - (b*e*n*x^(-1 + r))/(1 - r)^2 - (d*(a + b*Log[c*x^n]))/x - (e
*x^(-1 + r)*(a + b*Log[c*x^n]))/(1 - r)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{d(a+b\log(cx^n))}{x}-\frac{ex^{-1+r}(a+b\log(cx^n))}{1-r}-(bn)\int\frac{-d+dr-ex^r}{(1-r)x^2}\,dx\\ &= -\frac{d(a+b\log(cx^n))}{x}-\frac{ex^{-1+r}(a+b\log(cx^n))}{1-r}-\frac{(bn)\int\frac{-d+dr-ex^r}{x^2}\,dx}{1-r}\\ &= -\frac{d(a+b\log(cx^n))}{x}-\frac{ex^{-1+r}(a+b\log(cx^n))}{1-r}-\frac{(bn)\int\left(\frac{d(-1+r)}{x^2}-ex^{-2+r}\right)dx}{1-r}\\ &= -\frac{bdn}{x}-\frac{benx^{-1+r}}{(1-r)^2}-\frac{d(a+b\log(cx^n))}{x}-\frac{ex^{-1+r}(a+b\log(cx^n))}{1-r}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int\frac{(d+ex^r)(a+b\log(cx^n))}{x^2}\,dx = -\frac{a(-1+r)(d(-1+r)-ex^r)+bn(d(-1+r)^2+ex^r)+b(-1+r)(d(-1+r)-ex^r)\log(cx^n)}{(-1+r)^2x}$$

```
[In] Integrate[((d + e*x^r)*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -((a*(-1 + r)*(d*(-1 + r) - e*x^r) + b*n*(d*(-1 + r)^2 + e*x^r) + b*(-1 + r
)*(d*(-1 + r) - e*x^r)*Log[c*x^n])/((-1 + r)^2*x))
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.90

method	result
parallelrisch	$-\frac{-x^r \ln(c\,x^n)ber+\ln(c\,x^n)bd\,r^2+bdn\,r^2+x^r \ln(c\,x^n)be-x^r aer+x^r ben-2\ln(c\,x^n)bdr+ad\,r^2-2bdnr+x^r ae+b\ln(c\,x^n)d-2ad}{x(r^2-2r+1)}$
risch	$-\frac{b(dr-e\,x^r-d)\ln(x^n)}{(-1+r)x}-\frac{2x^r ae+2bdn+2ad-2x^r aer+2x^r ben+i\pi bd\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2+i\pi bd\,r^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+}{}$

```
[In] int((d+e*x^r)*(a+b*ln(c*x^n))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-x^r*ln(c*x^n)*b*e*r+ln(c*x^n)*b*d*r^2+b*d*n*r^2+x^r*ln(c*x^n)*b*e-x^r*a*
e*r+x^r*b*e*n-2*ln(c*x^n)*b*d*r+a*d*r^2-2*b*d*n*r+x^r*a*e+b*ln(c*x^n)*d-2*a
*d*r+b*d*n+a*d)/x/(r^2-2*r+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(62) = 124.

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.94

$$\int \frac{(d+ex^r)\,(a+b\log{(cx^n)})}{x^2}\,dx = -\frac{bdn+(bdn+ad)r^2+ad-2\,(bdn+ad)r+(ben-aer+ae-(ber-be)\log{(c)}-(benr-ben)\log{(x)}}{(r^2-2\,r+1)x}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
```

```
[Out] -(b*d*n + (b*d*n + a*d)*r^2 + a*d - 2*(b*d*n + a*d)*r + (b*e*n - a*e*r + a*
e - (b*e*r - b*e)*log(c) - (b*e*n*r - b*e*n)*log(x))*x^r + (b*d*r^2 - 2*b*d
*r + b*d)*log(c) + (b*d*n*r^2 - 2*b*d*n*r + b*d*n)*log(x))/((r^2 - 2*r + 1)
*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(56) = 112.

Time = 2.18 (sec) , antiderivative size = 348, normalized size of antiderivative = 5.19

$$\int \frac{(d+ex^r)\,(a+b\log{(cx^n)})}{x^2}\,dx = \begin{cases} -\frac{adr^2}{r^2x-2rx+x}+\frac{2adr}{r^2x-2rx+x}-\frac{ad}{r^2x-2rx+x}+\frac{aerx^r}{r^2x-2rx+x}-\frac{aex^r}{r^2x-2rx+x}-\frac{bdnr^2}{r^2x-2rx+x}+\frac{2bdnr}{r^2x-2rx+x}-\frac{bdn}{r^2x-2rx+x}-\frac{bdr^2}{r^2x} \\ -\frac{ad}{x}+ae\log{(x)}+bd\left(-\frac{n}{x}-\frac{\log{(cx^n)}}{x}\right)-be\left(\begin{cases} -\log{(c)}\log{(x)} & \text{for}\ n=0 \\ -\frac{\log{(cx^n)}^2}{2n} & \text{otherwise} \end{cases}\right) \end{cases}$$

```
[In] integrate((d+e*x**r)*(a+b*ln(c*x**n))/x**2,x)
```

```
[Out] Piecewise((-a*d*r**2/(r**2*x - 2*r*x + x) + 2*a*d*r/(r**2*x - 2*r*x + x) -
a*d/(r**2*x - 2*r*x + x) + a*e*r*x**r/(r**2*x - 2*r*x + x) - a*e*x**r/(r**2
*x - 2*r*x + x) - b*d*n*r**2/(r**2*x - 2*r*x + x) + 2*b*d*n*r/(r**2*x - 2*r
*x + x) - b*d*n/(r**2*x - 2*r*x + x) - b*d*r**2*log(c*x**n)/(r**2*x - 2*r*x
 + x) + 2*b*d*r*log(c*x**n)/(r**2*x - 2*r*x + x) - b*d*log(c*x**n)/(r**2*x
- 2*r*x + x) - b*e*n*x**r/(r**2*x - 2*r*x + x) + b*e*r*x**r*log(c*x**n)/(r*
*2*x - 2*r*x + x) - b*e*x**r*log(c*x**n)/(r**2*x - 2*r*x + x), Ne(r, 1)), (
-a*d/x + a*e*log(x) + b*d*(-n/x - log(c*x**n)/x) - b*e*Piecewise((-log(c)*l
og(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), True)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-2>0)', see `assume?` for more det
ails)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(62) = 124$.

Time = 0.38 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.78

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^2}\,dx = \frac{benrx^r\log(x)}{(r^2-2r+1)x} + \frac{berx^r\log(c)}{(r^2-2r+1)x} - \frac{bdn\log(x)}{x} - \frac{benx^r\log(x)}{(r^2-2r+1)x} - \frac{bdn}{x} - \frac{benx^r}{(r^2-2r+1)x} + \frac{aerx^r}{(r^2-2r+1)x} - \frac{bd\log(c)}{x} - \frac{bex^r\log(c)}{(r^2-2r+1)x} - \frac{ad}{x} - \frac{aex^r}{(r^2-2r+1)x}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

```
[Out] b*e*n*r*x^r*log(x)/((r^2 - 2*r + 1)*x) + b*e*r*x^r*log(c)/((r^2 - 2*r + 1)*
x) - b*d*n*log(x)/x - b*e*n*x^r*log(x)/((r^2 - 2*r + 1)*x) - b*d*n/x - b*e*
n*x^r/((r^2 - 2*r + 1)*x) + a*e*r*x^r/((r^2 - 2*r + 1)*x) - b*d*log(c)/x -
b*e*x^r*log(c)/((r^2 - 2*r + 1)*x) - a*d/x - a*e*x^r/((r^2 - 2*r + 1)*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\left(d+e\,x^r\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

```
[In] int(((d + e*x^r)*(a + b*log(c*x^n)))/x^2,x)
[Out] int(((d + e*x^r)*(a + b*log(c*x^n)))/x^2, x)
```

3.377 $\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^4}\,dx = -\frac{bdn}{9x^3} - \frac{benx^{-3+r}}{(3-r)^2} - \frac{d(a+b\log(cx^n))}{3x^3} - \frac{ex^{-3+r}(a+b\log(cx^n))}{3-r}$$

```
[Out] -1/9*b*d*n/x^3-b*e*n*x^(-3+r)/(3-r)^2-1/3*d*(a+b*ln(c*x^n))/x^3-e*x^(-3+r)*
(a+b*ln(c*x^n))/(3-r)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {14, 2372}

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^4}\,dx = -\frac{d(a+b\log(cx^n))}{3x^3} - \frac{ex^{r-3}(a+b\log(cx^n))}{3-r} - \frac{bdn}{9x^3} - \frac{benx^{r-3}}{(3-r)^2}$$

```
[In] Int[((d + e*x^r)*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] -1/9*(b*d*n)/x^3 - (b*e*n*x^(-3 + r))/(3 - r)^2 - (d*(a + b*Log[c*x^n]))/(3
*x^3) - (e*x^(-3 + r)*(a + b*Log[c*x^n]))/(3 - r)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{d(a+b\log(cx^n))}{3x^3}-\frac{ex^{-3+r}(a+b\log(cx^n))}{3-r}-(bn)\int\left(-\frac{d}{3x^4}+\frac{ex^{-4+r}}{-3+r}\right)dx\\ &= -\frac{bdn}{9x^3}-\frac{benx^{-3+r}}{(3-r)^2}-\frac{d(a+b\log(cx^n))}{3x^3}-\frac{ex^{-3+r}(a+b\log(cx^n))}{3-r}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int\frac{(d+ex^r)(a+b\log(cx^n))}{x^4}\,dx = -\frac{3a(-3+r)(d(-3+r)-3ex^r)+bn(d(-3+r)^2+9ex^r)+3b(-3+r)(d(-3+r)-3ex^r)\log(cx^n)}{9(-3+r)^2x^3}$$

```
[In] Integrate[((d + e*x^r)*(a + b*Log[c*x^n]))/x^4,x]
[Out] -1/9*(3*a*(-3 + r)*(d*(-3 + r) - 3*e*x^r) + b*n*(d*(-3 + r)^2 + 9*e*x^r) +
3*b*(-3 + r)*(d*(-3 + r) - 3*e*x^r)*Log[c*x^n])/((-3 + r)^2*x^3)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.90

method	result
parallelrisch	$-\frac{-9x^r\ln(cx^n)ber+3\ln(cx^n)bdr^2+bdnr^2+27x^r\ln(cx^n)be-9x^raer+9x^rben-18\ln(cx^n)bdr+3adr^2-6bdnr+27x^rae+27b}{9x^3(r^2-6r+9)}$
risch	$-\frac{b(dr-3ex^r-3d)\ln(x^n)}{3(-3+r)x^3}-\frac{54x^rae+18bdn+54ad-18x^raer+18x^rben+27i\pi bd\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2+3i\pi bdr^2\,\mathrm{csgn}(ic)\,\mathrm{cs}}{}$

```
[In] int((d+e*x^r)*(a+b*ln(c*x^n))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*(-9*x^r*ln(c*x^n)*b*e*r+3*ln(c*x^n)*b*d*r^2+b*d*n*r^2+27*x^r*ln(c*x^n)
*b*e-9*x^r*a*e*r+9*x^r*b*e*n-18*ln(c*x^n)*b*d*r+3*a*d*r^2-6*b*d*n*r+27*x^r*
a*e+27*b*ln(c*x^n)*d-18*a*d*r+9*b*d*n+27*a*d)/x^3/(r^2-6*r+9)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(62) = 124$.

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.97

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^4}\,dx = -\frac{9\,bdn + (bdn + 3\,ad)r^2 + 27\,ad - 6\,(bdn + 3\,ad)r + 9\,(ben - aer + 3\,ae - (ber - 3\,be)\log(c) - (benr - }{9\,(r^2 - 6\,r + 9)x^3}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(9*b*d*n + (b*d*n + 3*a*d)*r^2 + 27*a*d - 6*(b*d*n + 3*a*d)*r + 9*(b*e
*n - a*e*r + 3*a*e - (b*e*r - 3*b*e)*log(c) - (b*e*n*r - 3*b*e*n)*log(x))*x
^r + 3*(b*d*r^2 - 6*b*d*r + 9*b*d)*log(c) + 3*(b*d*n*r^2 - 6*b*d*n*r + 9*b*
d*n)*log(x))/((r^2 - 6*r + 9)*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(63) = 126$.

Time = 2.57 (sec) , antiderivative size = 495, normalized size of antiderivative = 6.97

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^4}\,dx = \begin{cases} -\frac{3adr^2}{9r^2x^3-54rx^3+81x^3} + \frac{18adr}{9r^2x^3-54rx^3+81x^3} - \frac{27ad}{9r^2x^3-54rx^3+81x^3} + \frac{9aerx^r}{9r^2x^3-54rx^3+81x^3} - \frac{27aex^r}{9r^2x^3-54rx^3+81x^3} - \frac{bdnr^2}{9r^2x^3-54rx^3+} \\ -\frac{ad}{3x^3} + ae\log(x) + bd\left(-\frac{n}{9x^3} - \frac{\log(cx^n)}{3x^3}\right) - be\left(\begin{cases} -\log(c)\log(x) & \text{for } n = 0 \\ -\frac{\log(cx^n)^2}{2n} & \text{otherwise} \end{cases}\right) \end{cases}$$

```
[In] integrate((d+e*x**r)*(a+b*ln(c*x**n))/x**4,x)
```

```
[Out] Piecewise((-3*a*d*r**2/(9*r**2*x**3 - 54*r*x**3 + 81*x**3) + 18*a*d*r/(9*r*
*2*x**3 - 54*r*x**3 + 81*x**3) - 27*a*d/(9*r**2*x**3 - 54*r*x**3 + 81*x**3)
 + 9*a*e*r*x**r/(9*r**2*x**3 - 54*r*x**3 + 81*x**3) - 27*a*e*x**r/(9*r**2*x
**3 - 54*r*x**3 + 81*x**3) - b*d*n*r**2/(9*r**2*x**3 - 54*r*x**3 + 81*x**3)
 + 6*b*d*n*r/(9*r**2*x**3 - 54*r*x**3 + 81*x**3) - 9*b*d*n/(9*r**2*x**3 - 5
4*r*x**3 + 81*x**3) - 3*b*d*r**2*log(c*x**n)/(9*r**2*x**3 - 54*r*x**3 + 81*
x**3) + 18*b*d*r*log(c*x**n)/(9*r**2*x**3 - 54*r*x**3 + 81*x**3) - 27*b*d*l
og(c*x**n)/(9*r**2*x**3 - 54*r*x**3 + 81*x**3) - 9*b*e*n*x**r/(9*r**2*x**3
```

```
- 54*r*x**3 + 81*x**3) + 9*b*e*r*x**r*log(c*x**n)/(9*r**2*x**3 - 54*r*x**3
+ 81*x**3) - 27*b*e*x**r*log(c*x**n)/(9*r**2*x**3 - 54*r*x**3 + 81*x**3), N
e(r, 3)), (-a*d/(3*x**3) + a*e*log(x) + b*d*(-n/(9*x**3) - log(c*x**n)/(3*x
**3)) - b*e*Piecewise((-log(c)*log(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), T
rue)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-4>0)', see `assume?` for more det
ails)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(62) = 124$.

Time = 0.35 (sec) , antiderivative size = 390, normalized size of antiderivative = 5.49

$$\begin{aligned}\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^4}\,dx =& -\frac{bdnr^2\log(x)}{3(r^2-6r+9)x^3} + \frac{benrx^r\log(x)}{(r^2-6r+9)x^3} \\ &- \frac{bdnr^2}{9(r^2-6r+9)x^3} - \frac{bdr^2\log(c)}{3(r^2-6r+9)x^3} \\ &+ \frac{berx^r\log(c)}{(r^2-6r+9)x^3} + \frac{2\,bdnr\log(x)}{(r^2-6r+9)x^3} \\ &- \frac{3\,benx^r\log(x)}{(r^2-6r+9)x^3} + \frac{2\,bdnr}{3(r^2-6r+9)x^3} \\ &- \frac{adr^2}{3(r^2-6r+9)x^3} - \frac{benx^r}{(r^2-6r+9)x^3} \\ &+ \frac{aerx^r}{(r^2-6r+9)x^3} + \frac{2\,bdr\log(c)}{(r^2-6r+9)x^3} - \frac{3\,bex^r\log(c)}{(r^2-6r+9)x^3} \\ &- \frac{3\,bdn\log(x)}{(r^2-6r+9)x^3} - \frac{bdn}{(r^2-6r+9)x^3} + \frac{2\,adr}{(r^2-6r+9)x^3} \\ &- \frac{3\,aex^r}{(r^2-6r+9)x^3} - \frac{3\,bd\log(c)}{(r^2-6r+9)x^3} - \frac{3\,ad}{(r^2-6r+9)x^3}\end{aligned}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x^4,x, algorithm="giac")
```

```
[Out] -1/3*b*d*n*r^2*log(x)/((r^2 - 6*r + 9)*x^3) + b*e*n*r*x^r*log(x)/((r^2 - 6*
r + 9)*x^3) - 1/9*b*d*n*r^2/((r^2 - 6*r + 9)*x^3) - 1/3*b*d*r^2*log(c)/((r^
2 - 6*r + 9)*x^3) + b*e*r*x^r*log(c)/((r^2 - 6*r + 9)*x^3) + 2*b*d*n*r*log(
x)/((r^2 - 6*r + 9)*x^3) - 3*b*e*n*x^r*log(x)/((r^2 - 6*r + 9)*x^3) + 2/3*b
*d*n*r/((r^2 - 6*r + 9)*x^3) - 1/3*a*d*r^2/((r^2 - 6*r + 9)*x^3) - b*e*n*x^
r/((r^2 - 6*r + 9)*x^3) + a*e*r*x^r/((r^2 - 6*r + 9)*x^3) + 2*b*d*r*log(c)/
((r^2 - 6*r + 9)*x^3) - 3*b*e*x^r*log(c)/((r^2 - 6*r + 9)*x^3) - 3*b*d*n*lo
g(x)/((r^2 - 6*r + 9)*x^3) - b*d*n/((r^2 - 6*r + 9)*x^3) + 2*a*d*r/((r^2 -
6*r + 9)*x^3) - 3*a*e*x^r/((r^2 - 6*r + 9)*x^3) - 3*b*d*log(c)/((r^2 - 6*r
+ 9)*x^3) - 3*a*d/((r^2 - 6*r + 9)*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)\,(a+b\log{(cx^n)})}{x^4}\,dx = \int \frac{\left(d+e\,x^r\right)\,\left(a+b\,\ln{\left(c\,x^n\right)}\right)}{x^4}\,dx$$

```
[In] int(((d + e*x^r)*(a + b*log(c*x^n)))/x^4,x)
[Out] int(((d + e*x^r)*(a + b*log(c*x^n)))/x^4, x)
```

3.378 $\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^6}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^6}\,dx = -\frac{bdn}{25x^5}-\frac{benx^{-5+r}}{(5-r)^2}-\frac{d(a+b\log(cx^n))}{5x^5} - \frac{ex^{-5+r}(a+b\log(cx^n))}{5-r}$$

```
[Out] -1/25*b*d*n/x^5-b*e*n*x^(-5+r)/(5-r)^2-1/5*d*(a+b*ln(c*x^n))/x^5-e*x^(-5+r)
*(a+b*ln(c*x^n))/(5-r)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {14, 2372}

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x^6}\,dx = -\frac{d(a+b\log(cx^n))}{5x^5} - \frac{ex^{r-5}(a+b\log(cx^n))}{5-r}-\frac{bdn}{25x^5}-\frac{benx^{r-5}}{(5-r)^2}$$

```
[In] Int[((d + e*x^r)*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] -1/25*(b*d*n)/x^5 - (b*e*n*x^(-5 + r))/(5 - r)^2 - (d*(a + b*Log[c*x^n]))/(
5*x^5) - (e*x^(-5 + r)*(a + b*Log[c*x^n]))/(5 - r)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{d(a+b\log(cx^n))}{5x^5}-\frac{ex^{-5+r}(a+b\log(cx^n))}{5-r}-(bn)\int\left(-\frac{d}{5x^6}+\frac{ex^{-6+r}}{-5+r}\right)dx\\ &= -\frac{bdn}{25x^5}-\frac{benx^{-5+r}}{(5-r)^2}-\frac{d(a+b\log(cx^n))}{5x^5}-\frac{ex^{-5+r}(a+b\log(cx^n))}{5-r}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int\frac{(d+ex^r)(a+b\log(cx^n))}{x^6}\,dx = -\frac{5a(-5+r)(d(-5+r)-5ex^r)+bn(d(-5+r)^2+25ex^r)+5b(-5+r)(d(-5+r)-5ex^r)\log(cx^n)}{25(-5+r)^2x^5}$$

```
[In] Integrate[((d + e*x^r)*(a + b*Log[c*x^n]))/x^6,x]
[Out] -1/25*(5*a*(-5 + r)*(d*(-5 + r) - 5*e*x^r) + b*n*(d*(-5 + r)^2 + 25*e*x^r)
+ 5*b*(-5 + r)*(d*(-5 + r) - 5*e*x^r)*Log[c*x^n])/((-5 + r)^2*x^5)
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.90

method	result
parallelrisch	$-\frac{-25x^r\ln(cx^n)ber+5\ln(cx^n)bdr^2+bdnr^2+125x^r\ln(cx^n)be-25x^raer+25x^rben-50\ln(cx^n)bdr+5adr^2-10bdnr+125x^rae-}{25x^5(r^2-10r+25)}$
risch	$-\frac{b(dr-5ex^r-5d)\ln(x^n)}{5(-5+r)x^5}-\frac{250x^rae+50bdn+250ad-50x^raer+50x^rben+125i\pi bd\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2+5i\pi bdr^2\,\mathrm{csgn}(ic)}{}$

```
[In] int((d+e*x^r)*(a+b*ln(c*x^n))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/25*(-25*x^r*ln(c*x^n)*b*e*r+5*ln(c*x^n)*b*d*r^2+b*d*n*r^2+125*x^r*ln(c*x
^n)*b*e-25*x^r*a*e*r+25*x^r*b*e*n-50*ln(c*x^n)*b*d*r+5*a*d*r^2-10*b*d*n*r+1
25*x^r*a*e+125*b*ln(c*x^n)*d-50*a*d*r+25*b*d*n+125*a*d)/x^5/(r^2-10*r+25)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(62) = 124.

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.97

$$\int \frac{(d+ex^r)\,(a+b\log{(cx^n)})}{x^6}\,dx = -\frac{25\,bdn+(bdn+5\,ad)r^2+125\,ad-10\,(bdn+5\,ad)r+25\,(ben-aer+5\,ae-(ber-5\,be)\log{(c)}-(be}{25\,(r^2-10\,r+}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x^6,x, algorithm="fricas")
```

```
[Out] -1/25*(25*b*d*n + (b*d*n + 5*a*d)*r^2 + 125*a*d - 10*(b*d*n + 5*a*d)*r + 25
*(b*e*n - a*e*r + 5*a*e - (b*e*r - 5*b*e)*log(c) - (b*e*n*r - 5*b*e*n)*log(
x))*x^r + 5*(b*d*r^2 - 10*b*d*r + 25*b*d)*log(c) + 5*(b*d*n*r^2 - 10*b*d*n*
r + 25*b*d*n)*log(x))/((r^2 - 10*r + 25)*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(63) = 126.

Time = 5.03 (sec) , antiderivative size = 495, normalized size of antiderivative = 6.97

$$\int \frac{(d+ex^r)\,(a+b\log{(cx^n)})}{x^6}\,dx = \begin{cases} -\frac{5adr^2}{25r^2x^5-250rx^5+625x^5}+\frac{50adr}{25r^2x^5-250rx^5+625x^5}-\frac{125ad}{25r^2x^5-250rx^5+625x^5}+\frac{25aerx^r}{25r^2x^5-250rx^5+625x^5}-\frac{125aex^r}{25r^2x^5-250rx^5+625x^5} \\ -\frac{ad}{5x^5}+ae\log{(x)}+bd\left(-\frac{n}{25x^5}-\frac{\log{(cx^n)}}{5x^5}\right)-be\left(\begin{cases} -\log{(c)}\log{(x)} & \text{for } n=0 \\ -\frac{\log{(cx^n)}^2}{2n} & \text{otherwise} \end{cases}\right) \end{cases}$$

```
[In] integrate((d+e*x**r)*(a+b*ln(c*x**n))/x**6,x)
```

```
[Out] Piecewise((-5*a*d*r**2/(25*r**2*x**5 - 250*r*x**5 + 625*x**5) + 50*a*d*r/(2
5*r**2*x**5 - 250*r*x**5 + 625*x**5) - 125*a*d/(25*r**2*x**5 - 250*r*x**5 +
 625*x**5) + 25*a*e*r*x**r/(25*r**2*x**5 - 250*r*x**5 + 625*x**5) - 125*a*e
*x**r/(25*r**2*x**5 - 250*r*x**5 + 625*x**5) - b*d*n*r**2/(25*r**2*x**5 - 2
50*r*x**5 + 625*x**5) + 10*b*d*n*r/(25*r**2*x**5 - 250*r*x**5 + 625*x**5) -
 25*b*d*n/(25*r**2*x**5 - 250*r*x**5 + 625*x**5) - 5*b*d*r**2*log(c*x**n)/(
25*r**2*x**5 - 250*r*x**5 + 625*x**5) + 50*b*d*r*log(c*x**n)/(25*r**2*x**5
- 250*r*x**5 + 625*x**5) - 125*b*d*log(c*x**n)/(25*r**2*x**5 - 250*r*x**5 +
```

```
625*x**5) - 25*b*e*n*x**r/(25*r**2*x**5 - 250*r*x**5 + 625*x**5) + 25*b*e*
r*x**r*log(c*x**n)/(25*r**2*x**5 - 250*r*x**5 + 625*x**5) - 125*b*e*x**r*lo
g(c*x**n)/(25*r**2*x**5 - 250*r*x**5 + 625*x**5), Ne(r, 5)), (-a*d/(5*x**5)
 + a*e*log(x) + b*d*(-n/(25*x**5) - log(c*x**n)/(5*x**5)) - b*e*Piecewise((
-log(c)*log(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), True)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)\left(a+b\log\left(cx^n\right)\right)}{x^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-6>0)', see `assume?` for more det
ails)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(62) = 124$.

Time = 0.35 (sec) , antiderivative size = 390, normalized size of antiderivative = 5.49

$$\begin{aligned}\int \frac{(d+ex^r)\left(a+b\log\left(cx^n\right)\right)}{x^6}\,dx = &-\frac{bdnr^2\log\left(x\right)}{5\,(r^2-10\,r+25)x^5}+\frac{benrx^r\log\left(x\right)}{(r^2-10\,r+25)x^5}\\&-\frac{bdnr^2}{25\,(r^2-10\,r+25)x^5}\\&-\frac{bdr^2\log\left(c\right)}{5\,(r^2-10\,r+25)x^5}+\frac{berx^r\log\left(c\right)}{(r^2-10\,r+25)x^5}\\&+\frac{2\,bdnr\log\left(x\right)}{(r^2-10\,r+25)x^5}-\frac{5\,benx^r\log\left(x\right)}{(r^2-10\,r+25)x^5}\\&+\frac{2\,bdnr}{5\,(r^2-10\,r+25)x^5}-\frac{adr^2}{5\,(r^2-10\,r+25)x^5}\\&-\frac{benx^r}{(r^2-10\,r+25)x^5}+\frac{aerx^r}{(r^2-10\,r+25)x^5}\\&+\frac{2\,bdr\log\left(c\right)}{(r^2-10\,r+25)x^5}-\frac{5\,bex^r\log\left(c\right)}{(r^2-10\,r+25)x^5}\\&-\frac{5\,bdn\log\left(x\right)}{(r^2-10\,r+25)x^5}-\frac{bdn}{(r^2-10\,r+25)x^5}\\&+\frac{2\,adr}{(r^2-10\,r+25)x^5}-\frac{5\,aex^r}{(r^2-10\,r+25)x^5}\\&-\frac{5\,bd\log\left(c\right)}{(r^2-10\,r+25)x^5}-\frac{5\,ad}{(r^2-10\,r+25)x^5}\end{aligned}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x^6,x, algorithm="giac")
```

```
[Out] -1/5*b*d*n*r^2*log(x)/((r^2 - 10*r + 25)*x^5) + b*e*n*r*x^r*log(x)/((r^2 -
10*r + 25)*x^5) - 1/25*b*d*n*r^2/((r^2 - 10*r + 25)*x^5) - 1/5*b*d*r^2*log(
c)/((r^2 - 10*r + 25)*x^5) + b*e*r*x^r*log(c)/((r^2 - 10*r + 25)*x^5) + 2*b
*d*n*r*log(x)/((r^2 - 10*r + 25)*x^5) - 5*b*e*n*x^r*log(x)/((r^2 - 10*r + 2
5)*x^5) + 2/5*b*d*n*r/((r^2 - 10*r + 25)*x^5) - 1/5*a*d*r^2/((r^2 - 10*r +
25)*x^5) - b*e*n*x^r/((r^2 - 10*r + 25)*x^5) + a*e*r*x^r/((r^2 - 10*r + 25)
*x^5) + 2*b*d*r*log(c)/((r^2 - 10*r + 25)*x^5) - 5*b*e*x^r*log(c)/((r^2 - 1
0*r + 25)*x^5) - 5*b*d*n*log(x)/((r^2 - 10*r + 25)*x^5) - b*d*n/((r^2 - 10*
r + 25)*x^5) + 2*a*d*r/((r^2 - 10*r + 25)*x^5) - 5*a*e*x^r/((r^2 - 10*r + 2
5)*x^5) - 5*b*d*log(c)/((r^2 - 10*r + 25)*x^5) - 5*a*d/((r^2 - 10*r + 25)*x
^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)\,(a+b\log{(cx^n)})}{x^6}\,dx=\int \frac{\left(d+e\,x^r\right)\,\left(a+b\,\ln{\left(c\,x^n\right)}\right)}{x^6}\,dx$$

```
[In] int(((d + e*x^r)*(a + b*log(c*x^n)))/x^6,x)
```

```
[Out] int(((d + e*x^r)*(a + b*log(c*x^n)))/x^6, x)
```

3.379 $\int x^5(d+ex^r)^2 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int x^5(d+ex^r)^2 (a+b\log(cx^n))\,dx = -\frac{1}{36}bd^2nx^6 - \frac{be^2nx^{2(3+r)}}{4(3+r)^2} - \frac{2bdenx^{6+r}}{(6+r)^2} + \frac{1}{6}\left(d^2x^6 + \frac{3e^2x^{2(3+r)}}{3+r} + \frac{12dex^{6+r}}{6+r}\right)(a+b\log(cx^n))$$

```
[Out] -1/36*b*d^2*n*x^6-1/4*b*e^2*n*x^(6+2*r)/(3+r)^2-2*b*d*e*n*x^(6+r)/(6+r)^2+1
/6*(d^2*x^6+3*e^2*x^(6+2*r)/(3+r)+12*d*e*x^(6+r)/(6+r))*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2371, 12, 14}

$$\int x^5(d+ex^r)^2 (a+b\log(cx^n))\,dx = \frac{1}{6}\left(d^2x^6 + \frac{12dex^{r+6}}{r+6} + \frac{3e^2x^{2(r+3)}}{r+3}\right)(a+b\log(cx^n)) - \frac{1}{36}bd^2nx^6 - \frac{2bdenx^{r+6}}{(r+6)^2} - \frac{be^2nx^{2(r+3)}}{4(r+3)^2}$$

```
[In] Int[x^5*(d + e*x^r)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/36*(b*d^2*n*x^6) - (b*e^2*n*x^(2*(3 + r)))/(4*(3 + r)^2) - (2*b*d*e*n*x^
(6 + r))/(6 + r)^2 + ((d^2*x^6 + (3*e^2*x^(2*(3 + r)))/(3 + r) + (12*d*e*x^
(6 + r))/(6 + r))*(a + b*Log[c*x^n]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{6}\left(d^2x^6+\frac{3e^2x^{2(3+r)}}{3+r}+\frac{12dex^{6+r}}{6+r}\right)(a+b\log(cx^n)) \\
&\quad-(bn)\int\frac{1}{6}x^5\left(d^2+\frac{12dex^r}{6+r}+\frac{3e^2x^{2r}}{3+r}\right)dx \\
&= \frac{1}{6}\left(d^2x^6+\frac{3e^2x^{2(3+r)}}{3+r}+\frac{12dex^{6+r}}{6+r}\right)(a+b\log(cx^n))-\frac{1}{6}(bn)\int x^5\left(d^2+\frac{12dex^r}{6+r}+\frac{3e^2x^{2r}}{3+r}\right)dx \\
&= \frac{1}{6}\left(d^2x^6+\frac{3e^2x^{2(3+r)}}{3+r}+\frac{12dex^{6+r}}{6+r}\right)(a+b\log(cx^n)) \\
&\quad-\frac{1}{6}(bn)\int\left(d^2x^5+\frac{12dex^{5+r}}{6+r}+\frac{3e^2x^{5+2r}}{3+r}\right)dx \\
&= -\frac{1}{36}bd^2nx^6-\frac{be^2nx^{2(3+r)}}{4(3+r)^2}-\frac{2bdenx^{6+r}}{(6+r)^2}+\frac{1}{6}\left(d^2x^6+\frac{3e^2x^{2(3+r)}}{3+r}+\frac{12dex^{6+r}}{6+r}\right)(a+b\log(cx^n))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int x^5 (d+ex^r)^2 (a+b\log(cx^n))\,dx = \frac{1}{36}x^6\left(bn\left(-d^2-\frac{72dex^r}{(6+r)^2}-\frac{9e^2x^{2r}}{(3+r)^2}\right)+6a\left(d^2+\frac{12dex^r}{6+r}+\frac{3e^2x^{2r}}{3+r}\right)+6b\left(d^2+\frac{12dex^r}{6+r}+\frac{3e^2x^{2r}}{3+r}\right)\log(cx^n)\right)$$

```
[In] Integrate[x^5*(d + e*x^r)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^6*(b*n*(-d^2 - (72*d*e*x^r)/(6 + r)^2 - (9*e^2*x^(2*r))/(3 + r)^2) + 6*a
*(d^2 + (12*d*e*x^r)/(6 + r) + (3*e^2*x^(2*r))/(3 + r)) + 6*b*(d^2 + (12*d*
e*x^r)/(6 + r) + (3*e^2*x^(2*r))/(3 + r))*Log[c*x^n]))/36
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. 2(97) = 194.

Time = 15.65 (sec) , antiderivative size = 583, normalized size of antiderivative = 5.66

method	result
parallelrisch	$-\frac{-1944a\,d^2x^6-18x^6x^{2r}\ln(c\,x^n)b\,e^2r^3-270x^6x^{2r}\ln(c\,x^n)b\,e^2r^2-1296x^6x^{2r}\ln(c\,x^n)b\,e^2r+18x^6b\,d^2n\,r^3+117x^6b\,d^2n\,r^2+324x}{}$
risch	Expression too large to display

```
[In] int(x^5*(d+e*x^r)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/36*(-1944*a*d^2*x^6-1944*e^2*b*ln(c*x^n)*(x^r)^2*x^6+18*x^6*b*d^2*n*r^3+
117*x^6*b*d^2*n*r^2+324*x^6*b*d^2*n*r-72*x^6*x^r*a*d*e*r^3-864*x^6*x^r*a*d*
e*r^2-3240*x^6*x^r*a*d*e*r+648*x^6*x^r*b*d*e*n-3888*b*d*e*ln(c*x^n)*x^r*x^6
+9*x^6*(x^r)^2*b*e^2*n*r^2+108*x^6*(x^r)^2*b*e^2*n*r-18*x^6*(x^r)^2*ln(c*x^
n)*b*e^2*r^3-270*x^6*(x^r)^2*ln(c*x^n)*b*e^2*r^2-1296*x^6*(x^r)^2*ln(c*x^n)
*b*e^2*r-1944*x^6*(x^r)^2*a*e^2-6*x^6*a*d^2*r^4-108*x^6*a*d^2*r^3-702*x^6*a
*d^2*r^2-1944*x^6*a*d^2*r+72*x^6*x^r*b*d*e*n*r^2-1296*x^6*(x^r)^2*a*e^2*r+3
24*x^6*(x^r)^2*b*e^2*n-6*x^6*ln(c*x^n)*b*d^2*r^4-108*x^6*ln(c*x^n)*b*d^2*r^
3-702*x^6*ln(c*x^n)*b*d^2*r^2-1944*x^6*ln(c*x^n)*b*d^2*r-3888*x^6*x^r*a*d*e
+x^6*b*d^2*n*r^4-18*x^6*(x^r)^2*a*e^2*r^3-270*x^6*(x^r)^2*a*e^2*r^2-72*x^6*
x^r*ln(c*x^n)*b*d*e*r^3-864*x^6*x^r*ln(c*x^n)*b*d*e*r^2-3240*x^6*x^r*ln(c*x
^n)*b*d*e*r+432*x^6*x^r*b*d*e*n*r+324*b*d^2*n*x^6-1944*x^6*ln(c*x^n)*b*d^2)
/(r^2+6*r+9)/(6+r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(97) = 194$.

Time = 0.30 (sec) , antiderivative size = 489, normalized size of antiderivative = 4.75

$$\int x^5(d+ex^r)^2(a+b\log(cx^n))\,dx$$
$$= \frac{6\,(bd^2r^4+18\,bd^2r^3+117\,bd^2r^2+324\,bd^2r+324\,bd^2)x^6\log(c)+6\,(bd^2nr^4+18\,bd^2nr^3+117\,bd^2nr^2+32}{}$$

```
[In] integrate(x^5*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 1/36*(6*(b*d^2*r^4 + 18*b*d^2*r^3 + 117*b*d^2*r^2 + 324*b*d^2*r + 324*b*d^2
)*x^6*log(c) + 6*(b*d^2*n*r^4 + 18*b*d^2*n*r^3 + 117*b*d^2*n*r^2 + 324*b*d^
2*n*r + 324*b*d^2*n)*x^6*log(x) - ((b*d^2*n - 6*a*d^2)*r^4 + 324*b*d^2*n +
18*(b*d^2*n - 6*a*d^2)*r^3 - 1944*a*d^2 + 117*(b*d^2*n - 6*a*d^2)*r^2 + 324
*(b*d^2*n - 6*a*d^2)*r)*x^6 + 9*(2*(b*e^2*r^3 + 15*b*e^2*r^2 + 72*b*e^2*r +
 108*b*e^2)*x^6*log(c) + 2*(b*e^2*n*r^3 + 15*b*e^2*n*r^2 + 72*b*e^2*n*r + 1
08*b*e^2*n)*x^6*log(x) + (2*a*e^2*r^3 - 36*b*e^2*n + 216*a*e^2 - (b*e^2*n -
 30*a*e^2)*r^2 - 12*(b*e^2*n - 12*a*e^2)*r)*x^6)*x^(2*r) + 72*((b*d*e*r^3 +
 12*b*d*e*r^2 + 45*b*d*e*r + 54*b*d*e)*x^6*log(c) + (b*d*e*n*r^3 + 12*b*d*e
*n*r^2 + 45*b*d*e*n*r + 54*b*d*e*n)*x^6*log(x) + (a*d*e*r^3 - 9*b*d*e*n + 5
4*a*d*e - (b*d*e*n - 12*a*d*e)*r^2 - 3*(2*b*d*e*n - 15*a*d*e)*r)*x^6)*x^r)/
(r^4 + 18*r^3 + 117*r^2 + 324*r + 324)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1634 vs. $2(97) = 194$.

Time = 26.36 (sec) , antiderivative size = 1634, normalized size of antiderivative = 15.86

$$\int x^5(d+ex^r)^2(a+b\log(cx^n))\,dx = \text{Too large to display}$$

```
[In] integrate(x**5*(d+e*x**r)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((a*d**2*x**6/6 + 2*a*d*e*log(c*x**n)/n - a*e**2/(6*x**6) - b*d**2
*n*x**6/36 + b*d**2*x**6*log(c*x**n)/6 + b*d*e*log(c*x**n)**2/n - b*e**2*n/
(36*x**6) - b*e**2*log(c*x**n)/(6*x**6), Eq(r, -6)), (a*d**2*x**6/6 + 2*a*d
*e*x**3/3 + a*e**2*log(c*x**n)/n - b*d**2*n*x**6/36 + b*d**2*x**6*log(c*x**
n)/6 - 2*b*d*e*n*x**3/9 + 2*b*d*e*x**3*log(c*x**n)/3 + b*e**2*log(c*x**n)**
2/(2*n), Eq(r, -3)), (6*a*d**2*r**4*x**6/(36*r**4 + 648*r**3 + 4212*r**2 +
11664*r + 11664) + 108*a*d**2*r**3*x**6/(36*r**4 + 648*r**3 + 4212*r**2 + 1
1664*r + 11664) + 702*a*d**2*r**2*x**6/(36*r**4 + 648*r**3 + 4212*r**2 + 11
664*r + 11664) + 1944*a*d**2*r*x**6/(36*r**4 + 648*r**3 + 4212*r**2 + 11664
```

```
*r + 11664) + 1944*a*d**2*x**6/(36*r**4 + 648*r**3 + 4212*r**2 + 11664*r +
11664) + 72*a*d*e*r**3*x**6*x**r/(36*r**4 + 648*r**3 + 4212*r**2 + 11664*r
+ 11664) + 864*a*d*e*r**2*x**6*x**r/(36*r**4 + 648*r**3 + 4212*r**2 + 11664
*r + 11664) + 3240*a*d*e*r*x**6*x**r/(36*r**4 + 648*r**3 + 4212*r**2 + 1166
4*r + 11664) + 3888*a*d*e*x**6*x**r/(36*r**4 + 648*r**3 + 4212*r**2 + 11664
*r + 11664) + 18*a*e**2*r**3*x**6*x**(2*r)/(36*r**4 + 648*r**3 + 4212*r**2
+ 11664*r + 11664) + 270*a*e**2*r**2*x**6*x**(2*r)/(36*r**4 + 648*r**3 + 42
12*r**2 + 11664*r + 11664) + 1296*a*e**2*r*x**6*x**(2*r)/(36*r**4 + 648*r**
3 + 4212*r**2 + 11664*r + 11664) + 1944*a*e**2*x**6*x**(2*r)/(36*r**4 + 648
*r**3 + 4212*r**2 + 11664*r + 11664) - b*d**2*n*r**4*x**6/(36*r**4 + 648*r*
*3 + 4212*r**2 + 11664*r + 11664) - 18*b*d**2*n*r**3*x**6/(36*r**4 + 648*r*
*3 + 4212*r**2 + 11664*r + 11664) - 117*b*d**2*n*r**2*x**6/(36*r**4 + 648*r
**3 + 4212*r**2 + 11664*r + 11664) - 324*b*d**2*n*r*x**6/(36*r**4 + 648*r**
3 + 4212*r**2 + 11664*r + 11664) - 324*b*d**2*n*x**6/(36*r**4 + 648*r**3 +
4212*r**2 + 11664*r + 11664) + 6*b*d**2*r**4*x**6*log(c*x**n)/(36*r**4 + 64
8*r**3 + 4212*r**2 + 11664*r + 11664) + 108*b*d**2*r**3*x**6*log(c*x**n)/(3
6*r**4 + 648*r**3 + 4212*r**2 + 11664*r + 11664) + 702*b*d**2*r**2*x**6*log
(c*x**n)/(36*r**4 + 648*r**3 + 4212*r**2 + 11664*r + 11664) + 1944*b*d**2*r
*x**6*log(c*x**n)/(36*r**4 + 648*r**3 + 4212*r**2 + 11664*r + 11664) + 1944
*b*d**2*x**6*log(c*x**n)/(36*r**4 + 648*r**3 + 4212*r**2 + 11664*r + 11664)
 - 72*b*d*e*n*r**2*x**6*x**r/(36*r**4 + 648*r**3 + 4212*r**2 + 11664*r + 11
664) - 432*b*d*e*n*r*x**6*x**r/(36*r**4 + 648*r**3 + 4212*r**2 + 11664*r +
11664) - 648*b*d*e*n*x**6*x**r/(36*r**4 + 648*r**3 + 4212*r**2 + 11664*r +
11664) + 72*b*d*e*r**3*x**6*x**r*log(c*x**n)/(36*r**4 + 648*r**3 + 4212*r**
2 + 11664*r + 11664) + 864*b*d*e*r**2*x**6*x**r*log(c*x**n)/(36*r**4 + 648*
r**3 + 4212*r**2 + 11664*r + 11664) + 3240*b*d*e*r*x**6*x**r*log(c*x**n)/(3
6*r**4 + 648*r**3 + 4212*r**2 + 11664*r + 11664) + 3888*b*d*e*x**6*x**r*log
(c*x**n)/(36*r**4 + 648*r**3 + 4212*r**2 + 11664*r + 11664) - 9*b*e**2*n*r*
*2*x**6*x**(2*r)/(36*r**4 + 648*r**3 + 4212*r**2 + 11664*r + 11664) - 108*b
*e**2*n*r*x**6*x**(2*r)/(36*r**4 + 648*r**3 + 4212*r**2 + 11664*r + 11664)
- 324*b*e**2*n*x**6*x**(2*r)/(36*r**4 + 648*r**3 + 4212*r**2 + 11664*r + 11
664) + 18*b*e**2*r**3*x**6*x**(2*r)*log(c*x**n)/(36*r**4 + 648*r**3 + 4212*
r**2 + 11664*r + 11664) + 270*b*e**2*r**2*x**6*x**(2*r)*log(c*x**n)/(36*r**
4 + 648*r**3 + 4212*r**2 + 11664*r + 11664) + 1296*b*e**2*r*x**6*x**(2*r)*l
og(c*x**n)/(36*r**4 + 648*r**3 + 4212*r**2 + 11664*r + 11664) + 1944*b*e**2
*x**6*x**(2*r)*log(c*x**n)/(36*r**4 + 648*r**3 + 4212*r**2 + 11664*r + 1166
4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.44

$$\int x^5(d+ex^r)^2\,(a+b\log{(cx^n)})\;dx = -\frac{1}{36}\,bd^2nx^6+\frac{1}{6}\,bd^2x^6\log{(cx^n)}+\frac{1}{6}\,ad^2x^6 + \frac{be^2x^{2\,r+6}\log{(cx^n)}}{2\,(r+3)}+\frac{2\,bdex^{r+6}\log{(cx^n)}}{r+6} - \frac{be^2nx^{2\,r+6}}{4\,(r+3)^2}+\frac{ae^2x^{2\,r+6}}{2\,(r+3)}-\frac{2\,bdenx^{r+6}}{(r+6)^2}+\frac{2\,adex^{r+6}}{r+6}$$

```
[In] integrate(x^5*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/36*b*d^2*n*x^6 + 1/6*b*d^2*x^6*log(c*x^n) + 1/6*a*d^2*x^6 + 1/2*b*e^2*x^
(2*r + 6)*log(c*x^n)/(r + 3) + 2*b*d*e*x^(r + 6)*log(c*x^n)/(r + 6) - 1/4*b
*e^2*n*x^(2*r + 6)/(r + 3)^2 + 1/2*a*e^2*x^(2*r + 6)/(r + 3) - 2*b*d*e*n*x^
(r + 6)/(r + 6)^2 + 2*a*d*e*x^(r + 6)/(r + 6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 744 vs. 2(97) = 194.

Time = 0.37 (sec) , antiderivative size = 744, normalized size of antiderivative = 7.22

$$\int x^5(d+ex^r)^2\,(a+b\log{(cx^n)})\;dx = \frac{18\,be^2nr^3x^6x^{2\,r}\log{(x)}+72\,bdenr^3x^6x^r\log{(x)}+6\,bd^2nr^4x^6\log{(x)}-bd^2nr^4x^6+18\,be^2r^3x^6x^{2\,r}\log{(c)}+\cdots}{}$$

```
[In] integrate(x^5*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/36*(18*b*e^2*n*r^3*x^6*x^(2*r)*log(x) + 72*b*d*e*n*r^3*x^6*x^r*log(x) + 6
*b*d^2*n*r^4*x^6*log(x) - b*d^2*n*r^4*x^6 + 18*b*e^2*r^3*x^6*x^(2*r)*log(c)
 + 72*b*d*e*r^3*x^6*x^r*log(c) + 6*b*d^2*r^4*x^6*log(c) + 270*b*e^2*n*r^2*x
^6*x^(2*r)*log(x) + 864*b*d*e*n*r^2*x^6*x^r*log(x) + 108*b*d^2*n*r^3*x^6*lo
g(x) - 9*b*e^2*n*r^2*x^6*x^(2*r) + 18*a*e^2*r^3*x^6*x^(2*r) - 72*b*d*e*n*r^
2*x^6*x^r + 72*a*d*e*r^3*x^6*x^r - 18*b*d^2*n*r^3*x^6 + 6*a*d^2*r^4*x^6 + 2
70*b*e^2*r^2*x^6*x^(2*r)*log(c) + 864*b*d*e*r^2*x^6*x^r*log(c) + 108*b*d^2*
r^3*x^6*log(c) + 1296*b*e^2*n*r*x^6*x^(2*r)*log(x) + 3240*b*d*e*n*r*x^6*x^r
*log(x) + 702*b*d^2*n*r^2*x^6*log(x) - 108*b*e^2*n*r*x^6*x^(2*r) + 270*a*e^
2*r^2*x^6*x^(2*r) - 432*b*d*e*n*r*x^6*x^r + 864*a*d*e*r^2*x^6*x^r - 117*b*d
^2*n*r^2*x^6 + 108*a*d^2*r^3*x^6 + 1296*b*e^2*r*x^6*x^(2*r)*log(c) + 3240*b
*d*e*r*x^6*x^r*log(c) + 702*b*d^2*r^2*x^6*log(c) + 1944*b*e^2*n*x^6*x^(2*r)
*log(x) + 3888*b*d*e*n*x^6*x^r*log(x) + 1944*b*d^2*n*r*x^6*log(x) - 324*b*e
```

```
^2*n*x^6*x^(2*r) + 1296*a*e^2*r*x^6*x^(2*r) - 648*b*d*e*n*x^6*x^r + 3240*a*
d*e*r*x^6*x^r - 324*b*d^2*n*r*x^6 + 702*a*d^2*r^2*x^6 + 1944*b*e^2*x^6*x^(2
*r)*log(c) + 3888*b*d*e*x^6*x^r*log(c) + 1944*b*d^2*r*x^6*log(c) + 1944*b*d
^2*n*x^6*log(x) + 1944*a*e^2*x^6*x^(2*r) + 3888*a*d*e*x^6*x^r - 324*b*d^2*n
*x^6 + 1944*a*d^2*r*x^6 + 1944*b*d^2*x^6*log(c) + 1944*a*d^2*x^6)/(r^4 + 18
*r^3 + 117*r^2 + 324*r + 324)
```

Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex^r)^2\left(a+b\log\left(cx^n\right)\right)\,dx=\int x^5\left(d+e\,x^r\right)^2\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

```
[In] int(x^5*(d + e*x^r)^2*(a + b*log(c*x^n)),x)
[Out] int(x^5*(d + e*x^r)^2*(a + b*log(c*x^n)), x)
```

3.380 $\int x^3(d+ex^r)^2 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int x^3(d+ex^r)^2 (a+b\log(cx^n))\,dx = -\frac{1}{16}bd^2nx^4 - \frac{be^2nx^{2(2+r)}}{4(2+r)^2} - \frac{2bdenx^{4+r}}{(4+r)^2} + \frac{1}{4}\left(d^2x^4 + \frac{2e^2x^{2(2+r)}}{2+r} + \frac{8dex^{4+r}}{4+r}\right)(a+b\log(cx^n))$$

```
[Out] -1/16*b*d^2*n*x^4-1/4*b*e^2*n*x^(4+2*r)/(2+r)^2-2*b*d*e*n*x^(4+r)/(4+r)^2+1
/4*(d^2*x^4+2*e^2*x^(4+2*r)/(2+r)+8*d*e*x^(4+r)/(4+r))*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2371, 12, 14}

$$\int x^3(d+ex^r)^2 (a+b\log(cx^n))\,dx = \frac{1}{4}\left(d^2x^4 + \frac{8dex^{r+4}}{r+4} + \frac{2e^2x^{2(r+2)}}{r+2}\right)(a+b\log(cx^n)) - \frac{1}{16}bd^2nx^4 - \frac{2bdenx^{r+4}}{(r+4)^2} - \frac{be^2nx^{2(r+2)}}{4(r+2)^2}$$

```
[In] Int[x^3*(d + e*x^r)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/16*(b*d^2*n*x^4) - (b*e^2*n*x^(2*(2 + r)))/(4*(2 + r)^2) - (2*b*d*e*n*x^
(4 + r))/(4 + r)^2 + ((d^2*x^4 + (2*e^2*x^(2*(2 + r)))/(2 + r) + (8*d*e*x^(
4 + r))/(4 + r))*(a + b*Log[c*x^n]))/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\left(d^2x^4+\frac{2e^2x^{2(2+r)}}{2+r}+\frac{8dex^{4+r}}{4+r}\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int \frac{1}{4}x^3\left(d^2+\frac{8dex^r}{4+r}+\frac{2e^2x^{2r}}{2+r}\right)dx \\
&= \frac{1}{4}\left(d^2x^4+\frac{2e^2x^{2(2+r)}}{2+r}+\frac{8dex^{4+r}}{4+r}\right)(a+b\log(cx^n))-\frac{1}{4}(bn)\int x^3\left(d^2+\frac{8dex^r}{4+r}+\frac{2e^2x^{2r}}{2+r}\right)dx \\
&= \frac{1}{4}\left(d^2x^4+\frac{2e^2x^{2(2+r)}}{2+r}+\frac{8dex^{4+r}}{4+r}\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{4}(bn)\int\left(d^2x^3+\frac{8dex^{3+r}}{4+r}+\frac{2e^2x^{3+2r}}{2+r}\right)dx \\
&= -\frac{1}{16}bd^2nx^4-\frac{be^2nx^{2(2+r)}}{4(2+r)^2}-\frac{2bdenx^{4+r}}{(4+r)^2}+\frac{1}{4}\left(d^2x^4+\frac{2e^2x^{2(2+r)}}{2+r}+\frac{8dex^{4+r}}{4+r}\right)(a+b\log(cx^n))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int x^3(d+ex^r)^2\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{1}{16}x^4\left(bn\left(-d^2-\frac{32dex^r}{(4+r)^2}-\frac{4e^2x^{2r}}{(2+r)^2}\right)+4a\left(d^2+\frac{8dex^r}{4+r}+\frac{2e^2x^{2r}}{2+r}\right)+4b\left(d^2+\frac{8dex^r}{4+r}+\frac{2e^2x^{2r}}{2+r}\right)\log\left(cx^n\right)\right)$$

```
[In] Integrate[x^3*(d + e*x^r)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^4*(b*n*(-d^2 - (32*d*e*x^r)/(4 + r)^2 - (4*e^2*x^(2*r))/(2 + r)^2) + 4*a
*(d^2 + (8*d*e*x^r)/(4 + r) + (2*e^2*x^(2*r))/(2 + r)) + 4*b*(d^2 + (8*d*e*
x^r)/(4 + r) + (2*e^2*x^(2*r))/(2 + r))*Log[c*x^n]))/16
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(97) = 194$.

Time = 5.23 (sec) , antiderivative size = 588, normalized size of antiderivative = 5.71

method	result
parallelrisch	$-\frac{-256a\,d^2x^4-256x^4x^{2r}\ln(c\,x^n)b\,e^2r+4x^4x^{2r}b\,e^2n\,r^2+32x^4x^{2r}b\,e^2nr-8x^4x^{2r}\ln(c\,x^n)b\,e^2r^3-80x^4x^{2r}\ln(c\,x^n)b\,e^2r^2+128}{}$
risch	Expression too large to display

```
[In] int(x^3*(d+e*x^r)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(-256*a*d^2*x^4+128*x^4*x^r*b*d*e*n*r-32*x^4*x^r*ln(c*x^n)*b*d*e*r^3-
256*x^4*x^r*ln(c*x^n)*b*d*e*r^2-640*x^4*x^r*ln(c*x^n)*b*d*e*r+32*x^4*x^r*b*
d*e*n*r^2-4*x^4*a*d^2*r^4-48*x^4*a*d^2*r^3-208*x^4*a*d^2*r^2-384*x^4*a*d^2*
r-256*x^4*(x^r)^2*a*e^2-256*x^4*(x^r)^2*ln(c*x^n)*b*e^2*r-32*x^4*x^r*a*d*e*
r^3-256*x^4*x^r*a*d*e*r^2-640*x^4*x^r*a*d*e*r-512*b*d*e*ln(c*x^n)*x^r*x^4+1
28*x^4*x^r*b*d*e*n+4*x^4*(x^r)^2*b*e^2*n*r^2+32*x^4*(x^r)^2*b*e^2*n*r-8*x^4
*(x^r)^2*ln(c*x^n)*b*e^2*r^3-80*x^4*(x^r)^2*ln(c*x^n)*b*e^2*r^2-256*x^4*ln(
c*x^n)*b*d^2-208*x^4*ln(c*x^n)*b*d^2*r^2-384*x^4*ln(c*x^n)*b*d^2*r-512*x^4*
x^r*a*d*e-8*x^4*(x^r)^2*a*e^2*r^3-80*x^4*(x^r)^2*a*e^2*r^2-256*x^4*(x^r)^2*
a*e^2*r+64*x^4*(x^r)^2*b*e^2*n-256*e^2*b*ln(c*x^n)*(x^r)^2*x^4+x^4*b*d^2*n*
r^4+12*x^4*b*d^2*n*r^3+52*x^4*b*d^2*n*r^2+96*x^4*b*d^2*n*r-4*x^4*ln(c*x^n)*
b*d^2*r^4-48*x^4*ln(c*x^n)*b*d^2*r^3+64*b*d^2*n*x^4)/(r^2+4*r+4)/(r^2+8*r+1
6)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(97) = 194$.

Time = 0.31 (sec) , antiderivative size = 488, normalized size of antiderivative = 4.74

$$\int x^3(d+ex^r)^2\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{4\left(bd^2r^4+12\,bd^2r^3+52\,bd^2r^2+96\,bd^2r+64\,bd^2\right)x^4\log\left(c\right)+4\left(bd^2nr^4+12\,bd^2nr^3+52\,bd^2nr^2+96\,bd^2nr\right.}{}$$

```
[In] integrate(x^3*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*(b*d^2*r^4 + 12*b*d^2*r^3 + 52*b*d^2*r^2 + 96*b*d^2*r + 64*b*d^2)*x
^4*log(c) + 4*(b*d^2*n*r^4 + 12*b*d^2*n*r^3 + 52*b*d^2*n*r^2 + 96*b*d^2*n*r
 + 64*b*d^2*n)*x^4*log(x) - ((b*d^2*n - 4*a*d^2)*r^4 + 64*b*d^2*n + 12*(b*d
^2*n - 4*a*d^2)*r^3 - 256*a*d^2 + 52*(b*d^2*n - 4*a*d^2)*r^2 + 96*(b*d^2*n
- 4*a*d^2)*r)*x^4 + 4*(2*(b*e^2*r^3 + 10*b*e^2*r^2 + 32*b*e^2*r + 32*b*e^2)
*x^4*log(c) + 2*(b*e^2*n*r^3 + 10*b*e^2*n*r^2 + 32*b*e^2*n*r + 32*b*e^2*n)*
x^4*log(x) + (2*a*e^2*r^3 - 16*b*e^2*n + 64*a*e^2 - (b*e^2*n - 20*a*e^2)*r^
2 - 8*(b*e^2*n - 8*a*e^2)*r)*x^4)*x^(2*r) + 32*((b*d*e*r^3 + 8*b*d*e*r^2 +
20*b*d*e*r + 16*b*d*e)*x^4*log(c) + (b*d*e*n*r^3 + 8*b*d*e*n*r^2 + 20*b*d*e
*n*r + 16*b*d*e*n)*x^4*log(x) + (a*d*e*r^3 - 4*b*d*e*n + 16*a*d*e - (b*d*e*
n - 8*a*d*e)*r^2 - 4*(b*d*e*n - 5*a*d*e)*r)*x^4)*x^r)/(r^4 + 12*r^3 + 52*r^
2 + 96*r + 64)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1625 vs. $2(97) = 194$.

Time = 7.10 (sec) , antiderivative size = 1625, normalized size of antiderivative = 15.78

$$\int x^3(d+ex^r)^2\left(a+b\log\left(cx^n\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x**3*(d+e*x**r)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((a*d**2*x**4/4 + 2*a*d*e*log(c*x**n)/n - a*e**2/(4*x**4) - b*d**2
*n*x**4/16 + b*d**2*x**4*log(c*x**n)/4 + b*d*e*log(c*x**n)**2/n - b*e**2*n/
(16*x**4) - b*e**2*log(c*x**n)/(4*x**4), Eq(r, -4)), (a*d**2*x**4/4 + a*d*e
*x**2 + a*e**2*log(c*x**n)/n - b*d**2*n*x**4/16 + b*d**2*x**4*log(c*x**n)/4
 - b*d*e*n*x**2/2 + b*d*e*x**2*log(c*x**n) + b*e**2*log(c*x**n)**2/(2*n), E
q(r, -2)), (4*a*d**2*r**4*x**4/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 10
24) + 48*a*d**2*r**3*x**4/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) +
 208*a*d**2*r**2*x**4/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 384
*a*d**2*r*x**4/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 256*a*d**2
```

```
*x**4/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 32*a*d*e*r**3*x**4*
x**r/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 256*a*d*e*r**2*x**4*
x**r/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 640*a*d*e*r*x**4*x**
r/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 512*a*d*e*x**4*x**r/(16
*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 8*a*e**2*r**3*x**4*x**(2*r)/
(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 80*a*e**2*r**2*x**4*x**(2
*r)/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 256*a*e**2*r*x**4*x**
(2*r)/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 256*a*e**2*x**4*x**
(2*r)/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) - b*d**2*n*r**4*x**4/
(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) - 12*b*d**2*n*r**3*x**4/(16
*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) - 52*b*d**2*n*r**2*x**4/(16*r*
*4 + 192*r**3 + 832*r**2 + 1536*r + 1024) - 96*b*d**2*n*r*x**4/(16*r**4 + 1
92*r**3 + 832*r**2 + 1536*r + 1024) - 64*b*d**2*n*x**4/(16*r**4 + 192*r**3
+ 832*r**2 + 1536*r + 1024) + 4*b*d**2*r**4*x**4*log(c*x**n)/(16*r**4 + 192
*r**3 + 832*r**2 + 1536*r + 1024) + 48*b*d**2*r**3*x**4*log(c*x**n)/(16*r**
4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 208*b*d**2*r**2*x**4*log(c*x**n)
/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 384*b*d**2*r*x**4*log(c*
x**n)/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 256*b*d**2*x**4*log
(c*x**n)/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) - 32*b*d*e*n*r**2*
x**4*x**r/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) - 128*b*d*e*n*r*x
**4*x**r/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) - 128*b*d*e*n*x**4
*x**r/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 32*b*d*e*r**3*x**4*
x**r*log(c*x**n)/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 256*b*d*
e*r**2*x**4*x**r*log(c*x**n)/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024
) + 640*b*d*e*r*x**4*x**r*log(c*x**n)/(16*r**4 + 192*r**3 + 832*r**2 + 1536
*r + 1024) + 512*b*d*e*x**4*x**r*log(c*x**n)/(16*r**4 + 192*r**3 + 832*r**2
 + 1536*r + 1024) - 4*b*e**2*n*r**2*x**4*x**(2*r)/(16*r**4 + 192*r**3 + 832
*r**2 + 1536*r + 1024) - 32*b*e**2*n*r*x**4*x**(2*r)/(16*r**4 + 192*r**3 +
832*r**2 + 1536*r + 1024) - 64*b*e**2*n*x**4*x**(2*r)/(16*r**4 + 192*r**3 +
 832*r**2 + 1536*r + 1024) + 8*b*e**2*r**3*x**4*x**(2*r)*log(c*x**n)/(16*r*
*4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 80*b*e**2*r**2*x**4*x**(2*r)*lo
g(c*x**n)/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 256*b*e**2*r*x*
*4*x**(2*r)*log(c*x**n)/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r + 1024) + 2
56*b*e**2*x**4*x**(2*r)*log(c*x**n)/(16*r**4 + 192*r**3 + 832*r**2 + 1536*r
 + 1024), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.44

$$\int x^3(d+ex^r)^2\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{16}\,bd^2nx^4+\frac{1}{4}\,bd^2x^4\log\left(cx^n\right)+\frac{1}{4}\,ad^2x^4+\frac{be^2x^{2\,r+4}\log\left(cx^n\right)}{2\,(r+2)}+\frac{2\,bdex^{r+4}\log\left(cx^n\right)}{r+4}-\frac{be^2nx^{2\,r+4}}{4\,(r+2)^2}+\frac{ae^2x^{2\,r+4}}{2\,(r+2)}-\frac{2\,bdenx^{r+4}}{(r+4)^2}+\frac{2\,adex^{r+4}}{r+4}$$

```
[In] integrate(x^3*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/16*b*d^2*n*x^4 + 1/4*b*d^2*x^4*log(c*x^n) + 1/4*a*d^2*x^4 + 1/2*b*e^2*x^
(2*r + 4)*log(c*x^n)/(r + 2) + 2*b*d*e*x^(r + 4)*log(c*x^n)/(r + 4) - 1/4*b
*e^2*n*x^(2*r + 4)/(r + 2)^2 + 1/2*a*e^2*x^(2*r + 4)/(r + 2) - 2*b*d*e*n*x^
(r + 4)/(r + 4)^2 + 2*a*d*e*x^(r + 4)/(r + 4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 744 vs. 2(97) = 194.

Time = 0.38 (sec) , antiderivative size = 744, normalized size of antiderivative = 7.22

$$\int x^3(d+ex^r)^2\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{8\,be^2nr^3x^4x^{2\,r}\log\left(x\right)+32\,bdenr^3x^4x^r\log\left(x\right)+4\,bd^2nr^4x^4\log\left(x\right)-bd^2nr^4x^4+8\,be^2r^3x^4x^{2\,r}\log\left(c\right)+32\,bc}{}$$

```
[In] integrate(x^3*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/16*(8*b*e^2*n*r^3*x^4*x^(2*r)*log(x) + 32*b*d*e*n*r^3*x^4*x^r*log(x) + 4*
b*d^2*n*r^4*x^4*log(x) - b*d^2*n*r^4*x^4 + 8*b*e^2*r^3*x^4*x^(2*r)*log(c) +
 32*b*d*e*r^3*x^4*x^r*log(c) + 4*b*d^2*r^4*x^4*log(c) + 80*b*e^2*n*r^2*x^4*
x^(2*r)*log(x) + 256*b*d*e*n*r^2*x^4*x^r*log(x) + 48*b*d^2*n*r^3*x^4*log(x)
 - 4*b*e^2*n*r^2*x^4*x^(2*r) + 8*a*e^2*r^3*x^4*x^(2*r) - 32*b*d*e*n*r^2*x^4
*x^r + 32*a*d*e*r^3*x^4*x^r - 12*b*d^2*n*r^3*x^4 + 4*a*d^2*r^4*x^4 + 80*b*e
^2*r^2*x^4*x^(2*r)*log(c) + 256*b*d*e*r^2*x^4*x^r*log(c) + 48*b*d^2*r^3*x^4
*log(c) + 256*b*e^2*n*r*x^4*x^(2*r)*log(x) + 640*b*d*e*n*r*x^4*x^r*log(x) +
 208*b*d^2*n*r^2*x^4*log(x) - 32*b*e^2*n*r*x^4*x^(2*r) + 80*a*e^2*r^2*x^4*x
^(2*r) - 128*b*d*e*n*r*x^4*x^r + 256*a*d*e*r^2*x^4*x^r - 52*b*d^2*n*r^2*x^4
 + 48*a*d^2*r^3*x^4 + 256*b*e^2*r*x^4*x^(2*r)*log(c) + 640*b*d*e*r*x^4*x^r*
log(c) + 208*b*d^2*r^2*x^4*log(c) + 256*b*e^2*n*x^4*x^(2*r)*log(x) + 512*b*
d*e*n*x^4*x^r*log(x) + 384*b*d^2*n*r*x^4*log(x) - 64*b*e^2*n*x^4*x^(2*r) +
```

```
256*a*e^2*r*x^4*x^(2*r) - 128*b*d*e*n*x^4*x^r + 640*a*d*e*r*x^4*x^r - 96*b*
d^2*n*r*x^4 + 208*a*d^2*r^2*x^4 + 256*b*e^2*x^4*x^(2*r)*log(c) + 512*b*d*e*
x^4*x^r*log(c) + 384*b*d^2*r*x^4*log(c) + 256*b*d^2*n*x^4*log(x) + 256*a*e^
2*x^4*x^(2*r) + 512*a*d*e*x^4*x^r - 64*b*d^2*n*x^4 + 384*a*d^2*r*x^4 + 256*
b*d^2*x^4*log(c) + 256*a*d^2*x^4)/(r^4 + 12*r^3 + 52*r^2 + 96*r + 64)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^r)^2\left(a+b\log\left(cx^n\right)\right)\,dx=\int x^3\left(d+e\,x^r\right)^2\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

```
[In] int(x^3*(d + e*x^r)^2*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^3*(d + e*x^r)^2*(a + b*log(c*x^n)), x)
```

3.381 $\int x(d+ex^r)^2 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int x(d+ex^r)^2 (a+b\log(cx^n))\,dx = -\frac{1}{4}bd^2nx^2 - \frac{be^2nx^{2(1+r)}}{4(1+r)^2} - \frac{2bdenx^{2+r}}{(2+r)^2} + \frac{1}{2}\left(d^2x^2 + \frac{e^2x^{2(1+r)}}{1+r} + \frac{4dex^{2+r}}{2+r}\right)(a+b\log(cx^n))$$

```
[Out] -1/4*b*d^2*n*x^2-1/4*b*e^2*n*x^(2+2*r)/(1+r)^2-2*b*d*e*n*x^(2+r)/(2+r)^2+1/
2*(d^2*x^2+e^2*x^(2+2*r)/(1+r)+4*d*e*x^(2+r)/(2+r))*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {276, 2371, 12, 14}

$$\int x(d+ex^r)^2 (a+b\log(cx^n))\,dx = \frac{1}{2}\left(d^2x^2 + \frac{4dex^{r+2}}{r+2} + \frac{e^2x^{2(r+1)}}{r+1}\right)(a+b\log(cx^n)) - \frac{1}{4}bd^2nx^2 - \frac{2bdenx^{r+2}}{(r+2)^2} - \frac{be^2nx^{2(r+1)}}{4(r+1)^2}$$

```
[In] Int[x*(d + e*x^r)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/4*(b*d^2*n*x^2) - (b*e^2*n*x^(2*(1 + r)))/(4*(1 + r)^2) - (2*b*d*e*n*x^(
2 + r))/(2 + r)^2 + ((d^2*x^2 + (e^2*x^(2*(1 + r)))/(1 + r) + (4*d*e*x^(2 +
 r))/(2 + r))*(a + b*Log[c*x^n]))/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\left(d^2x^2+\frac{e^2x^{2(1+r)}}{1+r}+\frac{4dex^{2+r}}{2+r}\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int \frac{1}{2}x\left(d^2+\frac{4dex^r}{2+r}+\frac{e^2x^{2r}}{1+r}\right)dx \\
&= \frac{1}{2}\left(d^2x^2+\frac{e^2x^{2(1+r)}}{1+r}+\frac{4dex^{2+r}}{2+r}\right)(a+b\log(cx^n))-\frac{1}{2}(bn)\int x\left(d^2+\frac{4dex^r}{2+r}+\frac{e^2x^{2r}}{1+r}\right)dx \\
&= \frac{1}{2}\left(d^2x^2+\frac{e^2x^{2(1+r)}}{1+r}+\frac{4dex^{2+r}}{2+r}\right)(a+b\log(cx^n))-\frac{1}{2}(bn)\int \left(d^2x+\frac{4dex^{1+r}}{2+r}+\frac{e^2x^{1+2r}}{1+r}\right)dx \\
&= -\frac{1}{4}bd^2nx^2-\frac{be^2nx^{2(1+r)}}{4(1+r)^2}-\frac{2bdenx^{2+r}}{(2+r)^2}+\frac{1}{2}\left(d^2x^2+\frac{e^2x^{2(1+r)}}{1+r}+\frac{4dex^{2+r}}{2+r}\right)(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.14

$$\int x(d+ex^r)^2 (a+b\log(cx^n))\,dx = \frac{1}{4}x^2\left(bn\left(-d^2-\frac{8dex^r}{(2+r)^2}-\frac{e^2x^{2r}}{(1+r)^2}\right)+2a\left(d^2+\frac{4dex^r}{2+r}+\frac{e^2x^{2r}}{1+r}\right)+2b\left(d^2+\frac{4dex^r}{2+r}+\frac{e^2x^{2r}}{1+r}\right)\log(cx^n)\right)$$

```
[In] Integrate[x*(d + e*x^r)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^2*(b*n*(-d^2 - (8*d*e*x^r)/(2 + r)^2 - (e^2*x^(2*r))/(1 + r)^2) + 2*a*(d
^2 + (4*d*e*x^r)/(2 + r) + (e^2*x^(2*r))/(1 + r)) + 2*b*(d^2 + (4*d*e*x^r)/
(2 + r) + (e^2*x^(2*r))/(1 + r))*Log[c*x^n]))/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(96) = 192.

Time = 1.79 (sec) , antiderivative size = 577, normalized size of antiderivative = 5.66

method	result
parallelrisch	$-\frac{6x^2 b\, d^2 n\, r^3+13x^2 b\, d^2 n\, r^2+12x^2 b\, d^2 n r-16x^2 x^r a d e-2x^2 \ln(c\, x^n) b\, d^2 r^4-12x^2 \ln(c\, x^n) b\, d^2 r^3-26x^2 \ln(c\, x^n) b\, d^2 r^2-24x^2 \ln(c}{}$
risch	Expression too large to display

```
[In] int(x*(d+e*x^r)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(6*x^2*b*d^2*n*r^3+13*x^2*b*d^2*n*r^2+12*x^2*b*d^2*n*r-16*x^2*x^r*a*d*
e-2*x^2*(x^r)^2*a*e^2*r^3-10*x^2*(x^r)^2*a*e^2*r^2-16*x^2*(x^r)^2*a*e^2*r+4
*x^2*(x^r)^2*b*e^2*n-2*x^2*ln(c*x^n)*b*d^2*r^4-12*x^2*ln(c*x^n)*b*d^2*r^3-2
6*x^2*ln(c*x^n)*b*d^2*r^2-24*x^2*ln(c*x^n)*b*d^2*r-8*x^2*(x^r)^2*ln(c*x^n)*
b*e^2-8*x^2*b*ln(c*x^n)*d^2-8*a*d^2*x^2-32*x^2*x^r*a*d*e*r^2-40*x^2*x^r*a*d
*e*r+8*x^2*x^r*b*d*e*n+x^2*(x^r)^2*b*e^2*n*r^2+4*x^2*(x^r)^2*b*e^2*n*r-2*x^
2*(x^r)^2*ln(c*x^n)*b*e^2*r^3-10*x^2*(x^r)^2*ln(c*x^n)*b*e^2*r^2-16*x^2*(x^
r)^2*ln(c*x^n)*b*e^2*r-8*x^2*x^r*a*d*e*r^3+x^2*b*d^2*n*r^4+8*x^2*x^r*b*d*e*
n*r^2+16*x^2*x^r*b*d*e*n*r-8*x^2*x^r*ln(c*x^n)*b*d*e*r^3-32*x^2*x^r*ln(c*x^
n)*b*d*e*r^2-40*x^2*x^r*ln(c*x^n)*b*d*e*r-16*x^2*x^r*ln(c*x^n)*b*d*e-2*x^2*
a*d^2*r^4-12*x^2*a*d^2*r^3-26*x^2*a*d^2*r^2-24*x^2*a*d^2*r-8*x^2*(x^r)^2*a*
e^2+4*b*d^2*n*x^2)/(1+r)^2/(2+r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(96) = 192$.

Time = 0.30 (sec) , antiderivative size = 488, normalized size of antiderivative = 4.78

$$\int x(d+ex^r)^2(a+b\log(cx^n))\,dx = \frac{2\,(bd^2r^4+6\,bd^2r^3+13\,bd^2r^2+12\,bd^2r+4\,bd^2)x^2\log(c)+2\,(bd^2nr^4+6\,bd^2nr^3+13\,bd^2nr^2+12\,bd^2nr+}{}$$

```
[In] integrate(x*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*d^2*r^4 + 6*b*d^2*r^3 + 13*b*d^2*r^2 + 12*b*d^2*r + 4*b*d^2)*x^2*
log(c) + 2*(b*d^2*n*r^4 + 6*b*d^2*n*r^3 + 13*b*d^2*n*r^2 + 12*b*d^2*n*r + 4
*b*d^2*n)*x^2*log(x) - ((b*d^2*n - 2*a*d^2)*r^4 + 4*b*d^2*n + 6*(b*d^2*n -
2*a*d^2)*r^3 - 8*a*d^2 + 13*(b*d^2*n - 2*a*d^2)*r^2 + 12*(b*d^2*n - 2*a*d^2
)*r)*x^2 + (2*(b*e^2*r^3 + 5*b*e^2*r^2 + 8*b*e^2*r + 4*b*e^2)*x^2*log(c) +
2*(b*e^2*n*r^3 + 5*b*e^2*n*r^2 + 8*b*e^2*n*r + 4*b*e^2*n)*x^2*log(x) + (2*a
*e^2*r^3 - 4*b*e^2*n + 8*a*e^2 - (b*e^2*n - 10*a*e^2)*r^2 - 4*(b*e^2*n - 4*
a*e^2)*r)*x^2)*x^(2*r) + 8*((b*d*e*r^3 + 4*b*d*e*r^2 + 5*b*d*e*r + 2*b*d*e)
*x^2*log(c) + (b*d*e*n*r^3 + 4*b*d*e*n*r^2 + 5*b*d*e*n*r + 2*b*d*e*n)*x^2*l
og(x) + (a*d*e*r^3 - b*d*e*n + 2*a*d*e - (b*d*e*n - 4*a*d*e)*r^2 - (2*b*d*e
*n - 5*a*d*e)*r)*x^2)*x^r)/(r^4 + 6*r^3 + 13*r^2 + 12*r + 4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1622 vs. $2(97) = 194$.

Time = 1.73 (sec) , antiderivative size = 1622, normalized size of antiderivative = 15.90

$$\int x(d+ex^r)^2(a+b\log(cx^n))\,dx = \text{Too large to display}$$

```
[In] integrate(x*(d+e*x**r)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((a*d**2*x**2/2 + 2*a*d*e*log(c*x**n)/n - a*e**2/(2*x**2) - b*d**2
*n*x**2/4 + b*d**2*x**2*log(c*x**n)/2 + b*d*e*log(c*x**n)**2/n - b*e**2*n/(
4*x**2) - b*e**2*log(c*x**n)/(2*x**2), Eq(r, -2)), (a*d**2*x**2/2 + 2*a*d*e
*x + a*e**2*log(c*x**n)/n - b*d**2*n*x**2/4 + b*d**2*x**2*log(c*x**n)/2 - 2
*b*d*e*n*x + 2*b*d*e*x*log(c*x**n) + b*e**2*log(c*x**n)**2/(2*n), Eq(r, -1)
), (2*a*d**2*r**4*x**2/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16) + 12*a*d**2
*r**3*x**2/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16) + 26*a*d**2*r**2*x**2/(
4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16) + 24*a*d**2*r*x**2/(4*r**4 + 24*r**
3 + 52*r**2 + 48*r + 16) + 8*a*d**2*x**2/(4*r**4 + 24*r**3 + 52*r**2 + 48*r
 + 16) + 8*a*d*e*r**3*x**2*x**r/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16) +
```

```
32*a*d*e*r**2*x**2*x**r/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16) + 40*a*d*e
*r*x**2*x**r/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16) + 16*a*d*e*x**2*x**r/
(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16) + 2*a*e**2*r**3*x**2*x**(2*r)/(4*r
**4 + 24*r**3 + 52*r**2 + 48*r + 16) + 10*a*e**2*r**2*x**2*x**(2*r)/(4*r**4
 + 24*r**3 + 52*r**2 + 48*r + 16) + 16*a*e**2*r*x**2*x**(2*r)/(4*r**4 + 24*
r**3 + 52*r**2 + 48*r + 16) + 8*a*e**2*x**2*x**(2*r)/(4*r**4 + 24*r**3 + 52
*r**2 + 48*r + 16) - b*d**2*n*r**4*x**2/(4*r**4 + 24*r**3 + 52*r**2 + 48*r
+ 16) - 6*b*d**2*n*r**3*x**2/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16) - 13*
b*d**2*n*r**2*x**2/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16) - 12*b*d**2*n*r
*x**2/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16) - 4*b*d**2*n*x**2/(4*r**4 +
24*r**3 + 52*r**2 + 48*r + 16) + 2*b*d**2*r**4*x**2*log(c*x**n)/(4*r**4 + 2
4*r**3 + 52*r**2 + 48*r + 16) + 12*b*d**2*r**3*x**2*log(c*x**n)/(4*r**4 + 2
4*r**3 + 52*r**2 + 48*r + 16) + 26*b*d**2*r**2*x**2*log(c*x**n)/(4*r**4 + 2
4*r**3 + 52*r**2 + 48*r + 16) + 24*b*d**2*r*x**2*log(c*x**n)/(4*r**4 + 24*r
**3 + 52*r**2 + 48*r + 16) + 8*b*d**2*x**2*log(c*x**n)/(4*r**4 + 24*r**3 +
52*r**2 + 48*r + 16) - 8*b*d*e*n*r**2*x**2*x**r/(4*r**4 + 24*r**3 + 52*r**2
 + 48*r + 16) - 16*b*d*e*n*r*x**2*x**r/(4*r**4 + 24*r**3 + 52*r**2 + 48*r +
 16) - 8*b*d*e*n*x**2*x**r/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16) + 8*b*d
*e*r**3*x**2*x**r*log(c*x**n)/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16) + 32
*b*d*e*r**2*x**2*x**r*log(c*x**n)/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16)
+ 40*b*d*e*r*x**2*x**r*log(c*x**n)/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16)
 + 16*b*d*e*x**2*x**r*log(c*x**n)/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16)
- b*e**2*n*r**2*x**2*x**(2*r)/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16) - 4*
b*e**2*n*r*x**2*x**(2*r)/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16) - 4*b*e**
2*n*x**2*x**(2*r)/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16) + 2*b*e**2*r**3*
x**2*x**(2*r)*log(c*x**n)/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16) + 10*b*e
**2*r**2*x**2*x**(2*r)*log(c*x**n)/(4*r**4 + 24*r**3 + 52*r**2 + 48*r + 16)
 + 16*b*e**2*r*x**2*x**(2*r)*log(c*x**n)/(4*r**4 + 24*r**3 + 52*r**2 + 48*r
 + 16) + 8*b*e**2*x**2*x**(2*r)*log(c*x**n)/(4*r**4 + 24*r**3 + 52*r**2 + 4
8*r + 16), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.45

$$\int x(d+ex^r)^2\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{4}\,bd^2nx^2+\frac{1}{2}\,bd^2x^2\log\left(cx^n\right)+\frac{1}{2}\,ad^2x^2+\frac{be^2x^{2\,r+2}\log\left(cx^n\right)}{2\,(r+1)}+\frac{2\,bdex^{r+2}\log\left(cx^n\right)}{r+2}-\frac{be^2nx^{2\,r+2}}{4\,(r+1)^2}+\frac{ae^2x^{2\,r+2}}{2\,(r+1)}-\frac{2\,bdenx^{r+2}}{(r+2)^2}+\frac{2\,adex^{r+2}}{r+2}$$

```
[In] integrate(x*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d^2*n*x^2 + 1/2*b*d^2*x^2*log(c*x^n) + 1/2*a*d^2*x^2 + 1/2*b*e^2*x^(
2*r + 2)*log(c*x^n)/(r + 1) + 2*b*d*e*x^(r + 2)*log(c*x^n)/(r + 2) - 1/4*b*
e^2*n*x^(2*r + 2)/(r + 1)^2 + 1/2*a*e^2*x^(2*r + 2)/(r + 1) - 2*b*d*e*n*x^(
r + 2)/(r + 2)^2 + 2*a*d*e*x^(r + 2)/(r + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 744 vs. $2(96) = 192$.

Time = 0.31 (sec) , antiderivative size = 744, normalized size of antiderivative = 7.29

$$\int x(d+ex^r)^2\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$= \frac{2\,be^2nr^3x^2x^{2\,r}\log\left(x\right)+8\,bdenr^3x^2x^r\log\left(x\right)+2\,bd^2nr^4x^2\log\left(x\right)-bd^2nr^4x^2+2\,be^2r^3x^2x^{2\,r}\log\left(c\right)+8\,bd}{}$$

```
[In] integrate(x*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/4*(2*b*e^2*n*r^3*x^2*x^(2*r)*log(x) + 8*b*d*e*n*r^3*x^2*x^r*log(x) + 2*b*
d^2*n*r^4*x^2*log(x) - b*d^2*n*r^4*x^2 + 2*b*e^2*r^3*x^2*x^(2*r)*log(c) + 8
*b*d*e*r^3*x^2*x^r*log(c) + 2*b*d^2*r^4*x^2*log(c) + 10*b*e^2*n*r^2*x^2*x^(
2*r)*log(x) + 32*b*d*e*n*r^2*x^2*x^r*log(x) + 12*b*d^2*n*r^3*x^2*log(x) - b
*e^2*n*r^2*x^2*x^(2*r) + 2*a*e^2*r^3*x^2*x^(2*r) - 8*b*d*e*n*r^2*x^2*x^r +
8*a*d*e*r^3*x^2*x^r - 6*b*d^2*n*r^3*x^2 + 2*a*d^2*r^4*x^2 + 10*b*e^2*r^2*x^
2*x^(2*r)*log(c) + 32*b*d*e*r^2*x^2*x^r*log(c) + 12*b*d^2*r^3*x^2*log(c) +
16*b*e^2*n*r*x^2*x^(2*r)*log(x) + 40*b*d*e*n*r*x^2*x^r*log(x) + 26*b*d^2*n*
r^2*x^2*log(x) - 4*b*e^2*n*r*x^2*x^(2*r) + 10*a*e^2*r^2*x^2*x^(2*r) - 16*b*
d*e*n*r*x^2*x^r + 32*a*d*e*r^2*x^2*x^r - 13*b*d^2*n*r^2*x^2 + 12*a*d^2*r^3*
x^2 + 16*b*e^2*r*x^2*x^(2*r)*log(c) + 40*b*d*e*r*x^2*x^r*log(c) + 26*b*d^2*
r^2*x^2*log(c) + 8*b*e^2*n*x^2*x^(2*r)*log(x) + 16*b*d*e*n*x^2*x^r*log(x) +
 24*b*d^2*n*r*x^2*log(x) - 4*b*e^2*n*x^2*x^(2*r) + 16*a*e^2*r*x^2*x^(2*r) -
 8*b*d*e*n*x^2*x^r + 40*a*d*e*r*x^2*x^r - 12*b*d^2*n*r*x^2 + 26*a*d^2*r^2*x
^2 + 8*b*e^2*x^2*x^(2*r)*log(c) + 16*b*d*e*x^2*x^r*log(c) + 24*b*d^2*r*x^2*
log(c) + 8*b*d^2*n*x^2*log(x) + 8*a*e^2*x^2*x^(2*r) + 16*a*d*e*x^2*x^r - 4*
b*d^2*n*x^2 + 24*a*d^2*r*x^2 + 8*b*d^2*x^2*log(c) + 8*a*d^2*x^2)/(r^4 + 6*r
^3 + 13*r^2 + 12*r + 4)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^r)^2 (a+b\log(cx^n)) \, dx = \int x\,(d+e\,x^r)^2\,(a+b\,\ln(c\,x^n)) \, dx$$

```
[In] int(x*(d + e*x^r)^2*(a + b*log(c*x^n)),x)
[Out] int(x*(d + e*x^r)^2*(a + b*log(c*x^n)), x)
```

3.382 $\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x}\,dx = -\frac{2bdenx^r}{r^2}-\frac{be^2nx^{2r}}{4r^2}-\frac{1}{2}bd^2n\log^2(x)+\frac{2dex^r(a+b\log(cx^n))}{r}+\frac{e^2x^{2r}(a+b\log(cx^n))}{2r}+d^2\log(x)(a+b\log(cx^n))$$

```
[Out] -2*b*d*e*n*x^r/r^2-1/4*b*e^2*n*x^(2*r)/r^2-1/2*b*d^2*n*ln(x)^2+2*d*e*x^r*(a
+b*ln(c*x^n))/r+1/2*e^2*x^(2*r)*(a+b*ln(c*x^n))/r+d^2*ln(x)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {272, 45, 2372, 12, 14, 2338}

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x}\,dx = d^2\log(x)(a+b\log(cx^n))+\frac{2dex^r(a+b\log(cx^n))}{r}+\frac{e^2x^{2r}(a+b\log(cx^n))}{2r}-\frac{1}{2}bd^2n\log^2(x)-\frac{2bdenx^r}{r^2}-\frac{be^2nx^{2r}}{4r^2}$$

```
[In] Int[((d + e*x^r)^2*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] (-2*b*d*e*n*x^r)/r^2 - (b*e^2*n*x^(2*r))/(4*r^2) - (b*d^2*n*Log[x]^2)/2 + (
2*d*e*x^r*(a + b*Log[c*x^n]))/r + (e^2*x^(2*r)*(a + b*Log[c*x^n]))/(2*r) +
d^2*Log[x]*(a + b*Log[c*x^n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2dex^r(a+b\log(cx^n))}{r}+\frac{e^2x^{2r}(a+b\log(cx^n))}{2r} \\
&\quad +d^2\log(x)\,(a+b\log(cx^n))-(bn)\int\frac{ex^r(4d+ex^r)+2d^2r\log(x)}{2rx}\,dx \\
&= \frac{2dex^r(a+b\log(cx^n))}{r}+\frac{e^2x^{2r}(a+b\log(cx^n))}{2r} \\
&\quad +d^2\log(x)\,(a+b\log(cx^n))-\frac{(bn)\int\frac{ex^r(4d+ex^r)+2d^2r\log(x)}{x}\,dx}{2r}
\end{aligned}
$$

$$= \frac{2dex^r(a+b\log(cx^n))}{r} + \frac{e^2x^{2r}(a+b\log(cx^n))}{2r} + d^2\log(x)(a+b\log(cx^n)) - \frac{(bn)\int\left(4dex^{-1+r}+e^2x^{-1+2r}+\frac{2d^2r\log(x)}{x}\right)dx}{2r}$$

$$= -\frac{2bdenx^r}{r^2} - \frac{be^2nx^{2r}}{4r^2} + \frac{2dex^r(a+b\log(cx^n))}{r} + \frac{e^2x^{2r}(a+b\log(cx^n))}{2r} + d^2\log(x)(a+b\log(cx^n)) - (bd^2n)\int\frac{\log(x)}{x}dx$$

$$= -\frac{2bdenx^r}{r^2} - \frac{be^2nx^{2r}}{4r^2} - \frac{1}{2}bd^2n\log^2(x) + \frac{2dex^r(a+b\log(cx^n))}{r} + \frac{e^2x^{2r}(a+b\log(cx^n))}{2r} + d^2\log(x)(a+b\log(cx^n))$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int\frac{(d+ex^r)^2(a+b\log(cx^n))}{x}dx = \frac{1}{4}\left(\frac{ex^r(2ar(4d+ex^r)-bn(8d+ex^r))}{r^2} + 4ad^2\log(x) + \frac{2bex^r(4d+ex^r)\log(cx^n)}{r} + \frac{2bd^2\log^2(cx^n)}{n}\right)$$

```
[In] Integrate[((d + e*x^r)^2*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] ((e*x^r*(2*a*r*(4*d + e*x^r) - b*n*(8*d + e*x^r)))/r^2 + 4*a*d^2*Log[x] + (
2*b*e*x^r*(4*d + e*x^r)*Log[c*x^n])/r + (2*b*d^2*Log[c*x^n]^2)/n)/4
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17

method	result
parallelrisch	$\frac{2x^{2r}\ln(cx^n)be^2rn+4\ln(x)ad^2nr^2+2x^{2r}ae^2nr-x^{2r}be^2n^2+8x^r\ln(cx^n)bdern+2bd^2\ln(cx^n)^2r^2+8x^radenr-8x^rbden^2}{4r^2n}$
risch	$\frac{b(2d^2\ln(x)r+e^2x^{2r}+4dex^r)\ln(x^n)}{2r} + \frac{i\pi be^2\operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)^2x^{2r}}{4r} - \frac{i\pi bde\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)x^r}{r} - \frac{i\pi b e}{}$

```
[In] int((d+e*x^r)^2*(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*(x^r)^2*ln(c*x^n)*b*e^2*r*n+4*ln(x)*a*d^2*n*r^2+2*(x^r)^2*a*e^2*n*r-
(x^r)^2*b*e^2*n^2+8*x^r*ln(c*x^n)*b*d*e*r*n+2*b*d^2*ln(c*x^n)^2*r^2+8*x^r*a
*d*e*n*r-8*x^r*b*d*e*n^2)/r^2/n
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x} dx = \frac{2\,bd^2nr^2\log(x)^2 + (2\,be^2nr\log(x) + 2\,be^2r\log(c) - be^2n + 2\,ae^2r)x^{2\,r} + 8\,(bdenr\log(x) + bder\log(c) - b}{4\,r^2}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*d^2*n*r^2*log(x)^2 + (2*b*e^2*n*r*log(x) + 2*b*e^2*r*log(c) - b*e^
2*n + 2*a*e^2*r)*x^(2*r) + 8*(b*d*e*n*r*log(x) + b*d*e*r*log(c) - b*d*e*n +
 a*d*e*r)*x^r + 4*(b*d^2*r^2*log(c) + a*d^2*r^2)*log(x))/r^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(104) = 208.

Time = 2.44 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.08

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x} dx = \begin{cases} (a+b\log(c))\,(d+e)^2\log(x) & \text{for } n=0 \wedge r \\ (a+b\log(c))\left(d^2\log(x) + \frac{2dex^r}{r} + \frac{e^2x^{2r}}{2r}\right) & \text{for } n=0 \\ (d+e)^2\left(\begin{cases} a\log(x) & \text{for } b=0 \\ -(-a-b\log(c))\log(x) & \text{for } n=0 \\ \frac{(-a-b\log(cx^n))^2}{2bn} & \text{otherwise} \end{cases}\right) & \text{for } r=0 \\ \frac{ad^2\log(cx^n)}{n} + \frac{2adex^r}{r} + \frac{ae^2x^{2r}}{2r} + \frac{bd^2\log(cx^n)^2}{2n} - \frac{2bdenx^r}{r^2} + \frac{2bdex^r\log(cx^n)}{r} - \frac{be^2nx^{2r}}{4r^2} + \frac{be^2x^{2r}\log(cx^n)}{2r} & \text{otherwise} \end{cases}$$

```
[In] integrate((d+e*x**r)**2*(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise(((a + b*log(c))*(d + e)**2*log(x), Eq(n, 0) & Eq(r, 0)), ((a + b*
log(c))*(d**2*log(x) + 2*d*e*x**r/r + e**2*x**(2*r)/(2*r)), Eq(n, 0)), ((d
+ e)**2*Piecewise((a*log(x), Eq(b, 0)), (-(-a - b*log(c))*log(x), Eq(n, 0))
, ((-a - b*log(c*x**n))**2/(2*b*n), True)), Eq(r, 0)), (a*d**2*log(c*x**n)/
n + 2*a*d*e*x**r/r + a*e**2*x**(2*r)/(2*r) + b*d**2*log(c*x**n)**2/(2*n) -
2*b*d*e*n*x**r/r**2 + 2*b*d*e*x**r*log(c*x**n)/r - b*e**2*n*x**(2*r)/(4*r**
2) + b*e**2*x**(2*r)*log(c*x**n)/(2*r), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex^r)^2\,(a+b\log(cx^n))}{x}\,dx = \frac{be^2x^{2r}\log(cx^n)}{2r} + \frac{2bdex^r\log(cx^n)}{r} + \frac{bd^2\log(cx^n)^2}{2n} + ad^2\log(x) - \frac{be^2nx^{2r}}{4r^2} + \frac{ae^2x^{2r}}{2r} - \frac{2bdenx^r}{r^2} + \frac{2adex^r}{r}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x,x, algorithm="maxima")
[Out] 1/2*b*e^2*x^(2*r)*log(c*x^n)/r + 2*b*d*e*x^r*log(c*x^n)/r + 1/2*b*d^2*log(c
*x^n)^2/n + a*d^2*log(x) - 1/4*b*e^2*n*x^(2*r)/r^2 + 1/2*a*e^2*x^(2*r)/r -
2*b*d*e*n*x^r/r^2 + 2*a*d*e*x^r/r
```

Giac [F]

$$\int \frac{(d+ex^r)^2\,(a+b\log(cx^n))}{x}\,dx = \int \frac{(ex^r+d)^2(b\log(cx^n)+a)}{x}\,dx$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x,x, algorithm="giac")
[Out] integrate((e*x^r + d)^2*(b*log(c*x^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^2\,(a+b\log(cx^n))}{x}\,dx = \int \frac{(d+e\,x^r)^2\,(a+b\,\ln(c\,x^n))}{x}\,dx$$

```
[In] int(((d + e*x^r)^2*(a + b*log(c*x^n)))/x,x)
[Out] int(((d + e*x^r)^2*(a + b*log(c*x^n)))/x, x)
```

3.383 $\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x^3}\,dx = -\frac{bd^2n}{4x^2}-\frac{be^2nx^{-2(1-r)}}{4(1-r)^2}-\frac{2bdenx^{-2+r}}{(2-r)^2}-\frac{d^2(a+b\log(cx^n))}{2x^2}-\frac{e^2x^{-2(1-r)}(a+b\log(cx^n))}{2(1-r)}-\frac{2dex^{-2+r}(a+b\log(cx^n))}{2-r}$$

```
[Out] -1/4*b*d^2*n/x^2-1/4*b*e^2*n/(1-r)^2/(x^(2-2*r))-2*b*d*e*n*x^(-2+r)/(2-r)^2
-1/2*d^2*(a+b*ln(c*x^n))/x^2-1/2*e^2*(a+b*ln(c*x^n))/(1-r)/(x^(2-2*r))-2*d*
e*x^(-2+r)*(a+b*ln(c*x^n))/(2-r)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2372, 12, 14}

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x^3}\,dx = -\frac{d^2(a+b\log(cx^n))}{2x^2}-\frac{2dex^{r-2}(a+b\log(cx^n))}{2-r}-\frac{e^2x^{-2(1-r)}(a+b\log(cx^n))}{2(1-r)}-\frac{bd^2n}{4x^2}-\frac{2bdenx^{r-2}}{(2-r)^2}-\frac{be^2nx^{-2(1-r)}}{4(1-r)^2}$$

```
[In] Int[((d + e*x^r)^2*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/4*(b*d^2*n)/x^2 - (b*e^2*n)/(4*(1 - r)^2*x^(2*(1 - r))) - (2*b*d*e*n*x^(
-2 + r))/(2 - r)^2 - (d^2*(a + b*Log[c*x^n]))/(2*x^2) - (e^2*(a + b*Log[c*x
```

```
^n]))/(2*(1 - r)*x^(2*(1 - r))) - (2*d*e*x^(-2 + r)*(a + b*Log[c*x^n]))/(2
- r)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{2x^2}-\frac{e^2x^{-2(1-r)}(a+b\log(cx^n))}{2(1-r)}-\frac{2dex^{-2+r}(a+b\log(cx^n))}{2-r} \\
&\quad -(bn)\int\frac{-d^2(2-3r+r^2)+4de(-1+r)x^r+e^2(-2+r)x^{2r}}{2(1-r)(2-r)x^3}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{2x^2}-\frac{e^2x^{-2(1-r)}(a+b\log(cx^n))}{2(1-r)} \\
&\quad -\frac{2dex^{-2+r}(a+b\log(cx^n))}{2-r}-\frac{(bn)\int\frac{-d^2(2-3r+r^2)+4de(-1+r)x^r+e^2(-2+r)x^{2r}}{x^3}\,dx}{2(2-3r+r^2)} \\
&= -\frac{d^2(a+b\log(cx^n))}{2x^2}-\frac{e^2x^{-2(1-r)}(a+b\log(cx^n))}{2(1-r)}-\frac{2dex^{-2+r}(a+b\log(cx^n))}{2-r} \\
&\quad -\frac{(bn)\int\left(-\frac{d^2(-2+r)(-1+r)}{x^3}+4de(-1+r)x^{-3+r}+e^2(-2+r)x^{-3+2r}\right)dx}{2(2-3r+r^2)}
\end{aligned}
$$

$$= -\frac{bd^2n}{4x^2} - \frac{be^2nx^{-2(1-r)}}{4(1-r)^2} - \frac{2bdenx^{-2+r}}{(2-r)^2} - \frac{d^2(a+b\log(cx^n))}{2x^2} - \frac{e^2x^{-2(1-r)}(a+b\log(cx^n))}{2(1-r)} - \frac{2dex^{-2+r}(a+b\log(cx^n))}{2-r}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x^3}\,dx = \frac{bn\left(-d^2-\frac{8dex^r}{(-2+r)^2}-\frac{e^2x^{2r}}{(-1+r)^2}\right)+a\left(-2d^2+\frac{8dex^r}{-2+r}+\frac{2e^2x^{2r}}{-1+r}\right)+2b\left(-d^2+\frac{4dex^r}{-2+r}+\frac{e^2x^{2r}}{-1+r}\right)\log(cx^n)}{4x^2}$$

```
[In] Integrate[((d + e*x^r)^2*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] (b*n*(-d^2 - (8*d*e*x^r)/(-2 + r)^2 - (e^2*x^(2*r))/(-1 + r)^2) + a*(-2*d^2
 + (8*d*e*x^r)/(-2 + r) + (2*e^2*x^(2*r))/(-1 + r)) + 2*b*(-d^2 + (4*d*e*x^
r)/(-2 + r) + (e^2*x^(2*r))/(-1 + r))*Log[c*x^n])/(4*x^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(127) = 254.

Time = 1.15 (sec) , antiderivative size = 474, normalized size of antiderivative = 3.51

method	result
parallelrisch	$-\frac{8b\ln(cx^n)d^2+8bdenx^r+13bd^2nr^2+16dex^ra-12bd^2nr+16dex^rb\ln(cx^n)+2ad^2r^4-12ad^2r^3-8ader^3x^r+4bd^2n+8ad^2+2}{}$
risch	Expression too large to display

```
[In] int((d+e*x^r)^2*(a+b*ln(c*x^n))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^2*(8*b*ln(c*x^n)*d^2-4*b*e^2*n*r*(x^r)^2+8*b*d*e*n*x^r+10*a*e^2*r^2*
(x^r)^2-16*a*e^2*r*(x^r)^2+4*b*e^2*n*(x^r)^2-2*a*e^2*r^3*(x^r)^2+8*e^2*(x^r
)^2*a+13*b*d^2*n*r^2+16*d*e*x^r*a+8*e^2*(x^r)^2*b*ln(c*x^n)-12*b*d^2*n*r+16
*d*e*x^r*b*ln(c*x^n)+2*a*d^2*r^4-12*a*d^2*r^3-8*a*d*e*r^3*x^r+4*b*d^2*n+8*a
*d^2+2*ln(c*x^n)*b*d^2*r^4-12*ln(c*x^n)*b*d^2*r^3+26*ln(c*x^n)*b*d^2*r^2-24
*ln(c*x^n)*b*d^2*r+b*d^2*n*r^4-6*b*d^2*n*r^3-16*b*d*e*n*r*x^r+26*a*d^2*r^2-
24*a*d^2*r-2*(x^r)^2*ln(c*x^n)*b*e^2*r^3+10*(x^r)^2*ln(c*x^n)*b*e^2*r^2-16*
(x^r)^2*ln(c*x^n)*b*e^2*r+8*b*d*e*n*r^2*x^r+32*a*d*e*r^2*x^r-40*a*d*e*r*x^r
+b*e^2*n*r^2*(x^r)^2-8*x^r*ln(c*x^n)*b*d*e*r^3+32*x^r*ln(c*x^n)*b*d*e*r^2-4
0*x^r*ln(c*x^n)*b*d*e*r)/(-1+r)^2/(r^2-4*r+4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(119) = 238$.

Time = 0.30 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.39

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^3}\,dx = -\frac{(bd^2n+2\,ad^2)r^4+4\,bd^2n-6\,(bd^2n+2\,ad^2)r^3+8\,ad^2+13\,(bd^2n+2\,ad^2)r^2-12\,(bd^2n+2\,ad^2)r-(2}{}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*((b*d^2*n + 2*a*d^2)*r^4 + 4*b*d^2*n - 6*(b*d^2*n + 2*a*d^2)*r^3 + 8*a
*d^2 + 13*(b*d^2*n + 2*a*d^2)*r^2 - 12*(b*d^2*n + 2*a*d^2)*r - (2*a*e^2*r^3
 - 4*b*e^2*n - 8*a*e^2 - (b*e^2*n + 10*a*e^2)*r^2 + 4*(b*e^2*n + 4*a*e^2)*r
 + 2*(b*e^2*r^3 - 5*b*e^2*r^2 + 8*b*e^2*r - 4*b*e^2)*log(c) + 2*(b*e^2*n*r^
3 - 5*b*e^2*n*r^2 + 8*b*e^2*n*r - 4*b*e^2*n)*log(x))*x^(2*r) - 8*(a*d*e*r^3
 - b*d*e*n - 2*a*d*e - (b*d*e*n + 4*a*d*e)*r^2 + (2*b*d*e*n + 5*a*d*e)*r +
(b*d*e*r^3 - 4*b*d*e*r^2 + 5*b*d*e*r - 2*b*d*e)*log(c) + (b*d*e*n*r^3 - 4*b
*d*e*n*r^2 + 5*b*d*e*n*r - 2*b*d*e*n)*log(x))*x^r + 2*(b*d^2*r^4 - 6*b*d^2*
r^3 + 13*b*d^2*r^2 - 12*b*d^2*r + 4*b*d^2)*log(c) + 2*(b*d^2*n*r^4 - 6*b*d^
2*n*r^3 + 13*b*d^2*n*r^2 - 12*b*d^2*n*r + 4*b*d^2*n)*log(x))/((r^4 - 6*r^3
+ 13*r^2 - 12*r + 4)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2118 vs. $2(119) = 238$.

Time = 3.30 (sec) , antiderivative size = 2118, normalized size of antiderivative = 15.69

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate((d+e*x**r)**2*(a+b*ln(c*x**n))/x**3,x)
```

```
[Out] Piecewise((-a*d**2/(2*x**2) - 2*a*d*e/x + a*e**2*log(x) + b*d**2*(-n/(4*x**
2) - log(c*x**n)/(2*x**2)) + 2*b*d*e*(-n/x - log(c*x**n)/x) - b*e**2*Piecew
ise((-log(c)*log(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), True)), Eq(r, 1)),
(-a*d**2/(2*x**2) + 2*a*d*e*log(c*x**n)/n + a*e**2*x**2/2 - b*d**2*n/(4*x**
2) - b*d**2*log(c*x**n)/(2*x**2) + b*d*e*log(c*x**n)**2/n - b*e**2*n*x**2/4
 + b*e**2*x**2*log(c*x**n)/2, Eq(r, 2)), (-2*a*d**2*r**4/(4*r**4*x**2 - 24*
r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) + 12*a*d**2*r**3/(4*r**4*x*
*2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) - 26*a*d**2*r**2/(4
*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) + 24*a*d**2
*r/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) - 8*a*
```

```
d**2/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) + 8*
a*d*e*r**3*x**r/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16
*x**2) - 32*a*d*e*r**2*x**r/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48
*r*x**2 + 16*x**2) + 40*a*d*e*r*x**r/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*
x**2 - 48*r*x**2 + 16*x**2) - 16*a*d*e*x**r/(4*r**4*x**2 - 24*r**3*x**2 + 5
2*r**2*x**2 - 48*r*x**2 + 16*x**2) + 2*a*e**2*r**3*x**(2*r)/(4*r**4*x**2 -
24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) - 10*a*e**2*r**2*x**(2*r
)/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) + 16*a*
e**2*r*x**(2*r)/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16
*x**2) - 8*a*e**2*x**(2*r)/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*
r*x**2 + 16*x**2) - b*d**2*n*r**4/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**
2 - 48*r*x**2 + 16*x**2) + 6*b*d**2*n*r**3/(4*r**4*x**2 - 24*r**3*x**2 + 52
*r**2*x**2 - 48*r*x**2 + 16*x**2) - 13*b*d**2*n*r**2/(4*r**4*x**2 - 24*r**3
*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) + 12*b*d**2*n*r/(4*r**4*x**2 -
24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) - 4*b*d**2*n/(4*r**4*x**
2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) - 2*b*d**2*r**4*log(
c*x**n)/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) +
 12*b*d**2*r**3*log(c*x**n)/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48
*r*x**2 + 16*x**2) - 26*b*d**2*r**2*log(c*x**n)/(4*r**4*x**2 - 24*r**3*x**2
 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) + 24*b*d**2*r*log(c*x**n)/(4*r**4*x*
*2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) - 8*b*d**2*log(c*x*
*n)/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) - 8*b
*d*e*n*r**2*x**r/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 1
6*x**2) + 16*b*d*e*n*r*x**r/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48
*r*x**2 + 16*x**2) - 8*b*d*e*n*x**r/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x
**2 - 48*r*x**2 + 16*x**2) + 8*b*d*e*r**3*x**r*log(c*x**n)/(4*r**4*x**2 - 2
4*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) - 32*b*d*e*r**2*x**r*log(
c*x**n)/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) +
 40*b*d*e*r*x**r*log(c*x**n)/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 4
8*r*x**2 + 16*x**2) - 16*b*d*e*x**r*log(c*x**n)/(4*r**4*x**2 - 24*r**3*x**2
 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) - b*e**2*n*r**2*x**(2*r)/(4*r**4*x**
2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) + 4*b*e**2*n*r*x**(2
*r)/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) - 4*b
*e**2*n*x**(2*r)/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 1
6*x**2) + 2*b*e**2*r**3*x**(2*r)*log(c*x**n)/(4*r**4*x**2 - 24*r**3*x**2 +
52*r**2*x**2 - 48*r*x**2 + 16*x**2) - 10*b*e**2*r**2*x**(2*r)*log(c*x**n)/(
4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2) + 16*b*e**
2*r*x**(2*r)*log(c*x**n)/(4*r**4*x**2 - 24*r**3*x**2 + 52*r**2*x**2 - 48*r*
x**2 + 16*x**2) - 8*b*e**2*x**(2*r)*log(c*x**n)/(4*r**4*x**2 - 24*r**3*x**2
 + 52*r**2*x**2 - 48*r*x**2 + 16*x**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^3} dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-3>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^3} dx = \int \frac{(ex^r+d)^2(b\log(cx^n)+a)}{x^3} dx$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^r + d)^2*(b*log(c*x^n) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^3} dx = \int \frac{(d+e\,x^r)^2 (a+b\,\ln(c\,x^n))}{x^3} dx$$

```
[In] int(((d + e*x^r)^2*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int(((d + e*x^r)^2*(a + b*log(c*x^n)))/x^3, x)
```

3.384 $\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x^5}\,dx = -\frac{bd^2n}{16x^4}-\frac{be^2nx^{-2(2-r)}}{4(2-r)^2}-\frac{2bdenx^{-4+r}}{(4-r)^2}-\frac{d^2(a+b\log(cx^n))}{4x^4}-\frac{e^2x^{-2(2-r)}(a+b\log(cx^n))}{2(2-r)}-\frac{2dex^{-4+r}(a+b\log(cx^n))}{4-r}$$

```
[Out] -1/16*b*d^2*n/x^4-1/4*b*e^2*n/(2-r)^2/(x^(4-2*r))-2*b*d*e*n*x^(-4+r)/(4-r)^
2-1/4*d^2*(a+b*ln(c*x^n))/x^4-1/2*e^2*(a+b*ln(c*x^n))/(2-r)/(x^(4-2*r))-2*d
*e*x^(-4+r)*(a+b*ln(c*x^n))/(4-r)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2372, 12, 14}

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x^5}\,dx = -\frac{d^2(a+b\log(cx^n))}{4x^4}-\frac{2dex^{r-4}(a+b\log(cx^n))}{4-r}-\frac{e^2x^{-2(2-r)}(a+b\log(cx^n))}{2(2-r)}-\frac{bd^2n}{16x^4}-\frac{2bdenx^{r-4}}{(4-r)^2}-\frac{be^2nx^{-2(2-r)}}{4(2-r)^2}$$

```
[In] Int[((d + e*x^r)^2*(a + b*Log[c*x^n]))/x^5,x]
```

```
[Out] -1/16*(b*d^2*n)/x^4 - (b*e^2*n)/(4*(2 - r)^2*x^(2*(2 - r))) - (2*b*d*e*n*x^
(-4 + r))/(4 - r)^2 - (d^2*(a + b*Log[c*x^n]))/(4*x^4) - (e^2*(a + b*Log[c*
```

```
x^n]))/(2*(2 - r)*x^(2*(2 - r))) - (2*d*e*x^(-4 + r)*(a + b*Log[c*x^n]))/(4
 - r)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{4x^4}-\frac{e^2x^{-2(2-r)}(a+b\log(cx^n))}{2(2-r)}-\frac{2dex^{-4+r}(a+b\log(cx^n))}{4-r} \\
&\quad -(bn)\int\frac{-d^2(8-6r+r^2)+8de(-2+r)x^r+2e^2(-4+r)x^{2r}}{4(2-r)(4-r)x^5}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{4x^4}-\frac{e^2x^{-2(2-r)}(a+b\log(cx^n))}{2(2-r)} \\
&\quad -\frac{2dex^{-4+r}(a+b\log(cx^n))}{4-r}-\frac{(bn)\int\frac{-d^2(8-6r+r^2)+8de(-2+r)x^r+2e^2(-4+r)x^{2r}}{x^5}\,dx}{4(8-6r+r^2)} \\
&= -\frac{d^2(a+b\log(cx^n))}{4x^4}-\frac{e^2x^{-2(2-r)}(a+b\log(cx^n))}{2(2-r)}-\frac{2dex^{-4+r}(a+b\log(cx^n))}{4-r} \\
&\quad -\frac{(bn)\int\left(-\frac{d^2(-4+r)(-2+r)}{x^5}+8de(-2+r)x^{-5+r}+2e^2(-4+r)x^{-5+2r}\right)dx}{4(8-6r+r^2)}
\end{aligned}$$

$$= -\frac{bd^2n}{16x^4} - \frac{be^2nx^{-2(2-r)}}{4(2-r)^2} - \frac{2bdenx^{-4+r}}{(4-r)^2} - \frac{d^2(a+b\log(cx^n))}{4x^4} - \frac{e^2x^{-2(2-r)}(a+b\log(cx^n))}{2(2-r)} - \frac{2dex^{-4+r}(a+b\log(cx^n))}{4-r}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x^5}\,dx = \frac{bn\left(-d^2-\frac{32dex^r}{(-4+r)^2}-\frac{4e^2x^{2r}}{(-2+r)^2}\right)+a\left(-4d^2+\frac{32dex^r}{-4+r}+\frac{8e^2x^{2r}}{-2+r}\right)+4b\left(-d^2+\frac{8dex^r}{-4+r}+\frac{2e^2x^{2r}}{-2+r}\right)\log(cx^n)}{16x^4}$$

```
[In] Integrate[((d + e*x^r)^2*(a + b*Log[c*x^n]))/x^5,x]
```

```
[Out] (b*n*(-d^2 - (32*d*e*x^r)/(-4 + r)^2 - (4*e^2*x^(2*r))/(-2 + r)^2) + a*(-4*
d^2 + (32*d*e*x^r)/(-4 + r) + (8*e^2*x^(2*r))/(-2 + r)) + 4*b*(-d^2 + (8*d*
e*x^r)/(-4 + r) + (2*e^2*x^(2*r))/(-2 + r))*Log[c*x^n])/(16*x^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(127) = 254.

Time = 1.10 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.52

method	result
parallelrisch	$-\frac{256b\ln(cx^n)d^2+128bdenx^r+52bd^2nr^2+512dex^ra-96bd^2nr+512dex^rb\ln(cx^n)+4ad^2r^4-48ad^2r^3-32ader^3x^r+64bd^2n\cdots}{}$
risch	Expression too large to display

```
[In] int((d+e*x^r)^2*(a+b*ln(c*x^n))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/x^4*(256*b*ln(c*x^n)*d^2-32*b*e^2*n*r*(x^r)^2+128*b*d*e*n*x^r+80*a*e^
2*r^2*(x^r)^2-256*a*e^2*r*(x^r)^2+64*b*e^2*n*(x^r)^2-8*a*e^2*r^3*(x^r)^2+25
6*e^2*(x^r)^2*a+52*b*d^2*n*r^2+512*d*e*x^r*a+256*e^2*(x^r)^2*b*ln(c*x^n)-96
*b*d^2*n*r+512*d*e*x^r*b*ln(c*x^n)+4*a*d^2*r^4-48*a*d^2*r^3-32*a*d*e*r^3*x^
r+64*b*d^2*n+256*a*d^2+4*ln(c*x^n)*b*d^2*r^4-48*ln(c*x^n)*b*d^2*r^3+208*ln(
c*x^n)*b*d^2*r^2-384*ln(c*x^n)*b*d^2*r+b*d^2*n*r^4-12*b*d^2*n*r^3-128*b*d*e
*n*r*x^r+208*a*d^2*r^2-384*a*d^2*r-8*(x^r)^2*ln(c*x^n)*b*e^2*r^3+80*(x^r)^2
*ln(c*x^n)*b*e^2*r^2-256*(x^r)^2*ln(c*x^n)*b*e^2*r+32*b*d*e*n*r^2*x^r+256*a
*d*e*r^2*x^r-640*a*d*e*r*x^r+4*b*e^2*n*r^2*(x^r)^2-32*x^r*ln(c*x^n)*b*d*e*r
^3+256*x^r*ln(c*x^n)*b*d*e*r^2-640*x^r*ln(c*x^n)*b*d*e*r)/(-2+r)^2/(r^2-8*r
+16)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(119) = 238$.

Time = 0.30 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.39

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^5}\,dx = -\frac{(bd^2n+4\,ad^2)r^4+64\,bd^2n-12\,(bd^2n+4\,ad^2)r^3+256\,ad^2+52\,(bd^2n+4\,ad^2)r^2-96\,(bd^2n+4\,ad^2)r}{}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*((b*d^2*n + 4*a*d^2)*r^4 + 64*b*d^2*n - 12*(b*d^2*n + 4*a*d^2)*r^3 +
256*a*d^2 + 52*(b*d^2*n + 4*a*d^2)*r^2 - 96*(b*d^2*n + 4*a*d^2)*r - 4*(2*a*
e^2*r^3 - 16*b*e^2*n - 64*a*e^2 - (b*e^2*n + 20*a*e^2)*r^2 + 8*(b*e^2*n + 8
*a*e^2)*r + 2*(b*e^2*r^3 - 10*b*e^2*r^2 + 32*b*e^2*r - 32*b*e^2)*log(c) + 2
*(b*e^2*n*r^3 - 10*b*e^2*n*r^2 + 32*b*e^2*n*r - 32*b*e^2*n)*log(x))*x^(2*r)
 - 32*(a*d*e*r^3 - 4*b*d*e*n - 16*a*d*e - (b*d*e*n + 8*a*d*e)*r^2 + 4*(b*d*
e*n + 5*a*d*e)*r + (b*d*e*r^3 - 8*b*d*e*r^2 + 20*b*d*e*r - 16*b*d*e)*log(c)
 + (b*d*e*n*r^3 - 8*b*d*e*n*r^2 + 20*b*d*e*n*r - 16*b*d*e*n)*log(x))*x^r +
4*(b*d^2*r^4 - 12*b*d^2*r^3 + 52*b*d^2*r^2 - 96*b*d^2*r + 64*b*d^2)*log(c)
+ 4*(b*d^2*n*r^4 - 12*b*d^2*n*r^3 + 52*b*d^2*n*r^2 - 96*b*d^2*n*r + 64*b*d^
2*n)*log(x))/((r^4 - 12*r^3 + 52*r^2 - 96*r + 64)*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2127 vs. $2(119) = 238$.

Time = 4.82 (sec) , antiderivative size = 2127, normalized size of antiderivative = 15.76

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^5}\,dx = \text{Too large to display}$$

```
[In] integrate((d+e*x**r)**2*(a+b*ln(c*x**n))/x**5,x)
```

```
[Out] Piecewise((-a*d**2/(4*x**4) - a*d*e/x**2 + a*e**2*log(x) + b*d**2*(-n/(16*x
**4) - log(c*x**n)/(4*x**4)) + 2*b*d*e*(-n/(4*x**2) - log(c*x**n)/(2*x**2))
 - b*e**2*Piecewise((-log(c)*log(x), Eq(n, 0)), (-log(c*x**n)**2/(2*n), Tru
e)), Eq(r, 2)), (-a*d**2/(4*x**4) + 2*a*d*e*log(c*x**n)/n + a*e**2*x**4/4 -
 b*d**2*n/(16*x**4) - b*d**2*log(c*x**n)/(4*x**4) + b*d*e*log(c*x**n)**2/n
- b*e**2*n*x**4/16 + b*e**2*x**4*log(c*x**n)/4, Eq(r, 4)), (-4*a*d**2*r**4/
(16*r**4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x**4 + 1024*x**4) +
48*a*d**2*r**3/(16*r**4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x**4
+ 1024*x**4) - 208*a*d**2*r**2/(16*r**4*x**4 - 192*r**3*x**4 + 832*r**2*x**
4 - 1536*r*x**4 + 1024*x**4) + 384*a*d**2*r/(16*r**4*x**4 - 192*r**3*x**4 +
```

```
832*r**2*x**4 - 1536*r*x**4 + 1024*x**4) - 256*a*d**2/(16*r**4*x**4 - 192*
r**3*x**4 + 832*r**2*x**4 - 1536*r*x**4 + 1024*x**4) + 32*a*d*e*r**3*x**r/(
16*r**4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x**4 + 1024*x**4) - 2
56*a*d*e*r**2*x**r/(16*r**4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x
**4 + 1024*x**4) + 640*a*d*e*r*x**r/(16*r**4*x**4 - 192*r**3*x**4 + 832*r**
2*x**4 - 1536*r*x**4 + 1024*x**4) - 512*a*d*e*x**r/(16*r**4*x**4 - 192*r**3
*x**4 + 832*r**2*x**4 - 1536*r*x**4 + 1024*x**4) + 8*a*e**2*r**3*x**(2*r)/(
16*r**4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x**4 + 1024*x**4) - 8
0*a*e**2*r**2*x**(2*r)/(16*r**4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536
*r*x**4 + 1024*x**4) + 256*a*e**2*r*x**(2*r)/(16*r**4*x**4 - 192*r**3*x**4
+ 832*r**2*x**4 - 1536*r*x**4 + 1024*x**4) - 256*a*e**2*x**(2*r)/(16*r**4*x
**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x**4 + 1024*x**4) - b*d**2*n*r
**4/(16*r**4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x**4 + 1024*x**4
) + 12*b*d**2*n*r**3/(16*r**4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r
*x**4 + 1024*x**4) - 52*b*d**2*n*r**2/(16*r**4*x**4 - 192*r**3*x**4 + 832*r
**2*x**4 - 1536*r*x**4 + 1024*x**4) + 96*b*d**2*n*r/(16*r**4*x**4 - 192*r**
3*x**4 + 832*r**2*x**4 - 1536*r*x**4 + 1024*x**4) - 64*b*d**2*n/(16*r**4*x*
*4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x**4 + 1024*x**4) - 4*b*d**2*r*
*4*log(c*x**n)/(16*r**4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x**4
+ 1024*x**4) + 48*b*d**2*r**3*log(c*x**n)/(16*r**4*x**4 - 192*r**3*x**4 + 8
32*r**2*x**4 - 1536*r*x**4 + 1024*x**4) - 208*b*d**2*r**2*log(c*x**n)/(16*r
**4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x**4 + 1024*x**4) + 384*b
*d**2*r*log(c*x**n)/(16*r**4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*
x**4 + 1024*x**4) - 256*b*d**2*log(c*x**n)/(16*r**4*x**4 - 192*r**3*x**4 +
832*r**2*x**4 - 1536*r*x**4 + 1024*x**4) - 32*b*d*e*n*r**2*x**r/(16*r**4*x*
*4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x**4 + 1024*x**4) + 128*b*d*e*n
*r*x**r/(16*r**4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x**4 + 1024*
x**4) - 128*b*d*e*n*x**r/(16*r**4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 15
36*r*x**4 + 1024*x**4) + 32*b*d*e*r**3*x**r*log(c*x**n)/(16*r**4*x**4 - 192
*r**3*x**4 + 832*r**2*x**4 - 1536*r*x**4 + 1024*x**4) - 256*b*d*e*r**2*x**r
*log(c*x**n)/(16*r**4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x**4 +
1024*x**4) + 640*b*d*e*r*x**r*log(c*x**n)/(16*r**4*x**4 - 192*r**3*x**4 + 8
32*r**2*x**4 - 1536*r*x**4 + 1024*x**4) - 512*b*d*e*x**r*log(c*x**n)/(16*r*
*4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x**4 + 1024*x**4) - 4*b*e*
*2*n*r**2*x**(2*r)/(16*r**4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x
**4 + 1024*x**4) + 32*b*e**2*n*r*x**(2*r)/(16*r**4*x**4 - 192*r**3*x**4 + 8
32*r**2*x**4 - 1536*r*x**4 + 1024*x**4) - 64*b*e**2*n*x**(2*r)/(16*r**4*x**
4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x**4 + 1024*x**4) + 8*b*e**2*r**
3*x**(2*r)*log(c*x**n)/(16*r**4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536
*r*x**4 + 1024*x**4) - 80*b*e**2*r**2*x**(2*r)*log(c*x**n)/(16*r**4*x**4 -
192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x**4 + 1024*x**4) + 256*b*e**2*r*x**
(2*r)*log(c*x**n)/(16*r**4*x**4 - 192*r**3*x**4 + 832*r**2*x**4 - 1536*r*x*
*4 + 1024*x**4) - 256*b*e**2*x**(2*r)*log(c*x**n)/(16*r**4*x**4 - 192*r**3*
x**4 + 832*r**2*x**4 - 1536*r*x**4 + 1024*x**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^5}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-5>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^5}\,dx = \int \frac{(ex^r+d)^2(b\log(cx^n)+a)}{x^5}\,dx$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^5,x, algorithm="giac")
```

```
[Out] integrate((e*x^r + d)^2*(b*log(c*x^n) + a)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^5}\,dx = \int \frac{(d+e\,x^r)^2\,(a+b\,\ln(c\,x^n))}{x^5}\,dx$$

```
[In] int(((d + e*x^r)^2*(a + b*log(c*x^n)))/x^5,x)
```

```
[Out] int(((d + e*x^r)^2*(a + b*log(c*x^n)))/x^5, x)
```

3.385 $\int x^4(d+ex^r)^2 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int x^4(d+ex^r)^2 (a+b\log(cx^n))\,dx=-\frac{1}{25}bd^2nx^5-\frac{2bdenx^{5+r}}{(5+r)^2}-\frac{be^2nx^{5+2r}}{(5+2r)^2}+\frac{1}{5}\left(d^2x^5+\frac{10dex^{5+r}}{5+r}+\frac{5e^2x^{5+2r}}{5+2r}\right)(a+b\log(cx^n))$$

```
[Out] -1/25*b*d^2*n*x^5-2*b*d*e*n*x^(5+r)/(5+r)^2-b*e^2*n*x^(5+2*r)/(5+2*r)^2+1/5
*(d^2*x^5+10*d*e*x^(5+r)/(5+r)+5*e^2*x^(5+2*r)/(5+2*r))*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2371, 12, 14}

$$\int x^4(d+ex^r)^2 (a+b\log(cx^n))\,dx=\frac{1}{5}\left(d^2x^5+\frac{10dex^{r+5}}{r+5}+\frac{5e^2x^{2r+5}}{2r+5}\right)(a+b\log(cx^n))-\frac{1}{25}bd^2nx^5-\frac{2bdenx^{r+5}}{(r+5)^2}-\frac{be^2nx^{2r+5}}{(2r+5)^2}$$

```
[In] Int[x^4*(d + e*x^r)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/25*(b*d^2*n*x^5) - (2*b*d*e*n*x^(5 + r))/(5 + r)^2 - (b*e^2*n*x^(5 + 2*r
))/(5 + 2*r)^2 + ((d^2*x^5 + (10*d*e*x^(5 + r))/(5 + r) + (5*e^2*x^(5 + 2*r
))/(5 + 2*r))*(a + b*Log[c*x^n]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}\left(d^2x^5+\frac{10dex^{5+r}}{5+r}+\frac{5e^2x^{5+2r}}{5+2r}\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int \frac{1}{5}x^4\left(d^2+\frac{10dex^r}{5+r}+\frac{5e^2x^{2r}}{5+2r}\right)dx \\
&= \frac{1}{5}\left(d^2x^5+\frac{10dex^{5+r}}{5+r}+\frac{5e^2x^{5+2r}}{5+2r}\right)(a+b\log(cx^n))-\frac{1}{5}(bn)\int x^4\left(d^2+\frac{10dex^r}{5+r}+\frac{5e^2x^{2r}}{5+2r}\right)dx \\
&= \frac{1}{5}\left(d^2x^5+\frac{10dex^{5+r}}{5+r}+\frac{5e^2x^{5+2r}}{5+2r}\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{5}(bn)\int\left(d^2x^4+\frac{5e^2x^{2(2+r)}}{5+2r}+\frac{10dex^{4+r}}{5+r}\right)dx \\
&= -\frac{1}{25}bd^2nx^5-\frac{2bdenx^{5+r}}{(5+r)^2}-\frac{be^2nx^{5+2r}}{(5+2r)^2}+\frac{1}{5}\left(d^2x^5+\frac{10dex^{5+r}}{5+r}+\frac{5e^2x^{5+2r}}{5+2r}\right)(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.18

$$\int x^4(d+ex^r)^2(a+b\log(cx^n))\,dx = \frac{1}{25}x^5\left(bn\left(-d^2-\frac{50dex^r}{(5+r)^2}-\frac{25e^2x^{2r}}{(5+2r)^2}\right)+5a\left(d^2+\frac{10dex^r}{5+r}+\frac{5e^2x^{2r}}{5+2r}\right)+5b\left(d^2+\frac{10dex^r}{5+r}+\frac{5e^2x^{2r}}{5+2r}\right)\log(cx^n)\right)$$

```
[In] Integrate[x^4*(d + e*x^r)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^5*(b*n*(-d^2 - (50*d*e*x^r)/(5 + r)^2 - (25*e^2*x^(2*r))/(5 + 2*r)^2) +
5*a*(d^2 + (10*d*e*x^r)/(5 + r) + (5*e^2*x^(2*r))/(5 + 2*r)) + 5*b*(d^2 + (
10*d*e*x^r)/(5 + r) + (5*e^2*x^(2*r))/(5 + 2*r))*Log[c*x^n]))/25
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. 2(101) = 202.

Time = 9.10 (sec) , antiderivative size = 586, normalized size of antiderivative = 5.58

method	result
parallelrisch	$-\frac{-3125x^5a\,d^2-6250bde\ln(c\,x^n)x^r r\,x^5-200x^5x^r\ln(c\,x^n)bde\,r^3-2000x^5x^r\ln(c\,x^n)bde\,r^2-625x^5x^{2r}a\,e^2r^2+625x^5x^{2r}b\,e^2n-3}{}$
risch	Expression too large to display

```
[In] int(x^4*(d+e*x^r)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/25*(-3125*x^5*a*d^2-2500*e^2*b*ln(c*x^n)*(x^r)^2*x^5*r-6250*b*d*e*ln(c*x
^n)*x^r*r*x^5-625*x^5*(x^r)^2*a*e^2*r^2-200*x^5*x^r*ln(c*x^n)*b*d*e*r^3-200
0*x^5*x^r*ln(c*x^n)*b*d*e*r^2-20*x^5*a*d^2*r^4-300*x^5*a*d^2*r^3-1625*x^5*a
*d^2*r^2-3750*x^5*a*d^2*r+625*x^5*(x^r)^2*b*e^2*n+4*x^5*b*d^2*n*r^4+60*x^5*
b*d^2*n*r^3+325*x^5*b*d^2*n*r^2+750*x^5*b*d^2*n*r-6250*x^5*d*e*x^r*b*ln(c*x
^n)-3125*x^5*e^2*(x^r)^2*a-3125*x^5*b*ln(c*x^n)*d^2-3750*x^5*ln(c*x^n)*b*d^
2*r-6250*x^5*d*e*x^r*a-3125*x^5*e^2*(x^r)^2*b*ln(c*x^n)-2500*x^5*(x^r)^2*a*
e^2*r-50*x^5*(x^r)^2*ln(c*x^n)*b*e^2*r^3-625*x^5*(x^r)^2*ln(c*x^n)*b*e^2*r^
2-50*x^5*(x^r)^2*a*e^2*r^3-20*x^5*ln(c*x^n)*b*d^2*r^4-300*x^5*ln(c*x^n)*b*d
^2*r^3-1625*x^5*ln(c*x^n)*b*d^2*r^2+1250*x^5*x^r*b*d*e*n+25*x^5*(x^r)^2*b*e
^2*n*r^2+250*x^5*(x^r)^2*b*e^2*n*r+200*x^5*x^r*b*d*e*n*r^2+1000*x^5*x^r*b*d
*e*n*r+625*b*d^2*n*x^5-200*x^5*x^r*a*d*e*r^3-2000*x^5*x^r*a*d*e*r^2-6250*x^
5*x^r*a*d*e*r)/(r^2+10*r+25)/(5+2*r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(101) = 202$.

Time = 0.30 (sec) , antiderivative size = 497, normalized size of antiderivative = 4.73

$$\int x^4(d+ex^r)^2\,(a+b\log{(cx^n)})\;dx$$
$$= \frac{5\,(4\,bd^2r^4+60\,bd^2r^3+325\,bd^2r^2+750\,bd^2r+625\,bd^2)x^5\log{(c)}+5\,(4\,bd^2nr^4+60\,bd^2nr^3+325\,bd^2nr^2+\cdots}{}$$

```
[In] integrate(x^4*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 1/25*(5*(4*b*d^2*r^4 + 60*b*d^2*r^3 + 325*b*d^2*r^2 + 750*b*d^2*r + 625*b*d
^2)*x^5*log(c) + 5*(4*b*d^2*n*r^4 + 60*b*d^2*n*r^3 + 325*b*d^2*n*r^2 + 750*
b*d^2*n*r + 625*b*d^2*n)*x^5*log(x) - (4*(b*d^2*n - 5*a*d^2)*r^4 + 625*b*d^
2*n + 60*(b*d^2*n - 5*a*d^2)*r^3 - 3125*a*d^2 + 325*(b*d^2*n - 5*a*d^2)*r^2
 + 750*(b*d^2*n - 5*a*d^2)*r)*x^5 + 25*((2*b*e^2*r^3 + 25*b*e^2*r^2 + 100*b
*e^2*r + 125*b*e^2)*x^5*log(c) + (2*b*e^2*n*r^3 + 25*b*e^2*n*r^2 + 100*b*e^
2*n*r + 125*b*e^2*n)*x^5*log(x) + (2*a*e^2*r^3 - 25*b*e^2*n + 125*a*e^2 - (
b*e^2*n - 25*a*e^2)*r^2 - 10*(b*e^2*n - 10*a*e^2)*r)*x^5)*x^(2*r) + 50*((4*
b*d*e*r^3 + 40*b*d*e*r^2 + 125*b*d*e*r + 125*b*d*e)*x^5*log(c) + (4*b*d*e*n
*r^3 + 40*b*d*e*n*r^2 + 125*b*d*e*n*r + 125*b*d*e*n)*x^5*log(x) + (4*a*d*e*
r^3 - 25*b*d*e*n + 125*a*d*e - 4*(b*d*e*n - 10*a*d*e)*r^2 - 5*(4*b*d*e*n -
25*a*d*e)*r)*x^5)*x^r)/(4*r^4 + 60*r^3 + 325*r^2 + 750*r + 625)
```

Sympy [F(-1)]

Timed out.

$$\int x^4(d+ex^r)^2\,(a+b\log{(cx^n)})\;dx = \text{Timed out}$$

```
[In] integrate(x**4*(d+e*x**r)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.45

$$\int x^4(d+ex^r)^2\,(a+b\log{(cx^n)})\;dx = -\frac{1}{25}\,bd^2nx^5+\frac{1}{5}\,bd^2x^5\log{(cx^n)}+\frac{1}{5}\,ad^2x^5 + \frac{be^2x^{2\,r+5}\log{(cx^n)}}{2\,r+5}+\frac{2\,bdex^{r+5}\log{(cx^n)}}{r+5} - \frac{be^2nx^{2\,r+5}}{(2\,r+5)^2}+\frac{ae^2x^{2\,r+5}}{2\,r+5}-\frac{2\,bdenx^{r+5}}{(r+5)^2}+\frac{2\,adex^{r+5}}{r+5}$$

```
[In] integrate(x^4*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/25*b*d^2*n*x^5 + 1/5*b*d^2*x^5*log(c*x^n) + 1/5*a*d^2*x^5 + b*e^2*x^(2*r
 + 5)*log(c*x^n)/(2*r + 5) + 2*b*d*e*x^(r + 5)*log(c*x^n)/(r + 5) - b*e^2*n
*x^(2*r + 5)/(2*r + 5)^2 + a*e^2*x^(2*r + 5)/(2*r + 5) - 2*b*d*e*n*x^(r + 5
)/(r + 5)^2 + 2*a*d*e*x^(r + 5)/(r + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(101) = 202$.

Time = 0.35 (sec) , antiderivative size = 746, normalized size of antiderivative = 7.10

$$\int x^4(d+ex^r)^2\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$= \frac{50\,be^2nr^3x^5x^{2\,r}\log\left(x\right)+200\,bdenr^3x^5x^r\log\left(x\right)+20\,bd^2nr^4x^5\log\left(x\right)-4\,bd^2nr^4x^5+50\,be^2r^3x^5x^{2\,r}\log\left(c\right)}{}$$

```
[In] integrate(x^4*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/25*(50*b*e^2*n*r^3*x^5*x^(2*r)*log(x) + 200*b*d*e*n*r^3*x^5*x^r*log(x) +
20*b*d^2*n*r^4*x^5*log(x) - 4*b*d^2*n*r^4*x^5 + 50*b*e^2*r^3*x^5*x^(2*r)*lo
g(c) + 200*b*d*e*r^3*x^5*x^r*log(c) + 20*b*d^2*r^4*x^5*log(c) + 625*b*e^2*n
*r^2*x^5*x^(2*r)*log(x) + 2000*b*d*e*n*r^2*x^5*x^r*log(x) + 300*b*d^2*n*r^3
*x^5*log(x) - 25*b*e^2*n*r^2*x^5*x^(2*r) + 50*a*e^2*r^3*x^5*x^(2*r) - 200*b
*d*e*n*r^2*x^5*x^r + 200*a*d*e*r^3*x^5*x^r - 60*b*d^2*n*r^3*x^5 + 20*a*d^2*
r^4*x^5 + 625*b*e^2*r^2*x^5*x^(2*r)*log(c) + 2000*b*d*e*r^2*x^5*x^r*log(c)
+ 300*b*d^2*r^3*x^5*log(c) + 2500*b*e^2*n*r*x^5*x^(2*r)*log(x) + 6250*b*d*e
*n*r*x^5*x^r*log(x) + 1625*b*d^2*n*r^2*x^5*log(x) - 250*b*e^2*n*r*x^5*x^(2*
r) + 625*a*e^2*r^2*x^5*x^(2*r) - 1000*b*d*e*n*r*x^5*x^r + 2000*a*d*e*r^2*x^
5*x^r - 325*b*d^2*n*r^2*x^5 + 300*a*d^2*r^3*x^5 + 2500*b*e^2*r*x^5*x^(2*r)*
log(c) + 6250*b*d*e*r*x^5*x^r*log(c) + 1625*b*d^2*r^2*x^5*log(c) + 3125*b*e
^2*n*x^5*x^(2*r)*log(x) + 6250*b*d*e*n*x^5*x^r*log(x) + 3750*b*d^2*n*r*x^5*
log(x) - 625*b*e^2*n*x^5*x^(2*r) + 2500*a*e^2*r*x^5*x^(2*r) - 1250*b*d*e*n*
x^5*x^r + 6250*a*d*e*r*x^5*x^r - 750*b*d^2*n*r*x^5 + 1625*a*d^2*r^2*x^5 + 3
125*b*e^2*x^5*x^(2*r)*log(c) + 6250*b*d*e*x^5*x^r*log(c) + 3750*b*d^2*r*x^5
*log(c) + 3125*b*d^2*n*x^5*log(x) + 3125*a*e^2*x^5*x^(2*r) + 6250*a*d*e*x^5
*x^r - 625*b*d^2*n*x^5 + 3750*a*d^2*r*x^5 + 3125*b*d^2*x^5*log(c) + 3125*a*
d^2*x^5)/(4*r^4 + 60*r^3 + 325*r^2 + 750*r + 625)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex^r)^2\,(a+b\log{(cx^n)})\;dx=\int x^4\,{\left(d+e\,x^r\right)}^2\,\left(a+b\,\ln\left(c\,x^n\right)\right)\;dx$$

```
[In] int(x^4*(d + e*x^r)^2*(a + b*log(c*x^n)),x)
[Out] int(x^4*(d + e*x^r)^2*(a + b*log(c*x^n)), x)
```

3.386 $\int x^2(d+ex^r)^2 (a+b\log(cx^n))\, dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int x^2(d+ex^r)^2 (a+b\log(cx^n))\, dx = -\frac{1}{9}bd^2nx^3 - \frac{2bdenx^{3+r}}{(3+r)^2} - \frac{be^2nx^{3+2r}}{(3+2r)^2} + \frac{1}{3}\left(d^2x^3 + \frac{6dex^{3+r}}{3+r} + \frac{3e^2x^{3+2r}}{3+2r}\right)(a+b\log(cx^n))$$

```
[Out] -1/9*b*d^2*n*x^3-2*b*d*e*n*x^(3+r)/(3+r)^2-b*e^2*n*x^(3+2*r)/(3+2*r)^2+1/3*
(d^2*x^3+6*d*e*x^(3+r)/(3+r)+3*e^2*x^(3+2*r)/(3+2*r))*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2371, 12, 14}

$$\int x^2(d+ex^r)^2 (a+b\log(cx^n))\, dx = \frac{1}{3}\left(d^2x^3 + \frac{6dex^{r+3}}{r+3} + \frac{3e^2x^{2r+3}}{2r+3}\right)(a+b\log(cx^n)) - \frac{1}{9}bd^2nx^3 - \frac{2bdenx^{r+3}}{(r+3)^2} - \frac{be^2nx^{2r+3}}{(2r+3)^2}$$

```
[In] Int[x^2*(d + e*x^r)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/9*(b*d^2*n*x^3) - (2*b*d*e*n*x^(3 + r))/(3 + r)^2 - (b*e^2*n*x^(3 + 2*r)
)/(3 + 2*r)^2 + ((d^2*x^3 + (6*d*e*x^(3 + r))/(3 + r) + (3*e^2*x^(3 + 2*r))
/(3 + 2*r))*(a + b*Log[c*x^n]))/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\left(d^2x^3+\frac{6dex^{3+r}}{3+r}+\frac{3e^2x^{3+2r}}{3+2r}\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int \frac{1}{3}x^2\left(d^2+\frac{6dex^r}{3+r}+\frac{3e^2x^{2r}}{3+2r}\right)dx \\
&= \frac{1}{3}\left(d^2x^3+\frac{6dex^{3+r}}{3+r}+\frac{3e^2x^{3+2r}}{3+2r}\right)(a+b\log(cx^n))-\frac{1}{3}(bn)\int x^2\left(d^2+\frac{6dex^r}{3+r}+\frac{3e^2x^{2r}}{3+2r}\right)dx \\
&= \frac{1}{3}\left(d^2x^3+\frac{6dex^{3+r}}{3+r}+\frac{3e^2x^{3+2r}}{3+2r}\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{3}(bn)\int\left(d^2x^2+\frac{3e^2x^{2(1+r)}}{3+2r}+\frac{6dex^{2+r}}{3+r}\right)dx \\
&= -\frac{1}{9}bd^2nx^3-\frac{2bdenx^{3+r}}{(3+r)^2}-\frac{be^2nx^{3+2r}}{(3+2r)^2}+\frac{1}{3}\left(d^2x^3+\frac{6dex^{3+r}}{3+r}+\frac{3e^2x^{3+2r}}{3+2r}\right)(a+b\log(cx^n))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.18

$$\int x^2(d+ex^r)^2\,(a+b\log(cx^n))\,dx = \frac{1}{9}x^3\left(bn\left(-d^2-\frac{18dex^r}{(3+r)^2}-\frac{9e^2x^{2r}}{(3+2r)^2}\right)+3a\left(d^2+\frac{6dex^r}{3+r}+\frac{3e^2x^{2r}}{3+2r}\right)+3b\left(d^2+\frac{6dex^r}{3+r}+\frac{3e^2x^{2r}}{3+2r}\right)\log(cx^n)\right)$$

```
[In] Integrate[x^2*(d + e*x^r)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^3*(b*n*(-d^2 - (18*d*e*x^r)/(3 + r)^2 - (9*e^2*x^(2*r))/(3 + 2*r)^2) + 3
*a*(d^2 + (6*d*e*x^r)/(3 + r) + (3*e^2*x^(2*r))/(3 + 2*r)) + 3*b*(d^2 + (6*
d*e*x^r)/(3 + r) + (3*e^2*x^(2*r))/(3 + 2*r))*Log[c*x^n]))/9
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(101) = 202.

Time = 3.03 (sec) , antiderivative size = 581, normalized size of antiderivative = 5.53

method	result
parallelrisch	$-\frac{-72x^3x^r\ln(c\,x^n)bde\,r^3-432x^3x^r\ln(c\,x^n)bde\,r^2-810x^3x^r\ln(c\,x^n)bder-72x^3x^r ade\,r^3-432x^3x^r ade\,r^2-810x^3x^r ader+72x^3}{}$
risch	Expression too large to display

```
[In] int(x^2*(d+e*x^r)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*(-135*x^3*(x^r)^2*ln(c*x^n)*b*e^2*r^2-72*x^3*x^r*ln(c*x^n)*b*d*e*r^3-4
32*x^3*x^r*ln(c*x^n)*b*d*e*r^2-810*x^3*x^r*ln(c*x^n)*b*d*e*r-324*x^3*(x^r)^
2*a*e^2*r-324*x^3*(x^r)^2*ln(c*x^n)*b*e^2*r-72*x^3*x^r*a*d*e*r^3-432*x^3*x^
r*a*d*e*r^2-810*x^3*x^r*a*d*e*r+72*x^3*x^r*b*d*e*n*r^2+216*x^3*x^r*b*d*e*n*
r-486*x^3*d*e*x^r*b*ln(c*x^n)-243*x^3*e^2*(x^r)^2*a-243*x^3*b*ln(c*x^n)*d^2
-135*x^3*(x^r)^2*a*e^2*r^2-108*x^3*a*d^2*r^3-351*x^3*a*d^2*r^2-486*x^3*a*d^
2*r+162*x^3*x^r*b*d*e*n+9*x^3*(x^r)^2*b*e^2*n*r^2+54*x^3*(x^r)^2*b*e^2*n*r-
12*x^3*ln(c*x^n)*b*d^2*r^4-108*x^3*ln(c*x^n)*b*d^2*r^3-243*x^3*e^2*(x^r)^2*
b*ln(c*x^n)-243*a*d^2*x^3-486*x^3*ln(c*x^n)*b*d^2*r-18*x^3*(x^r)^2*a*e^2*r^
3-486*x^3*d*e*x^r*a-18*x^3*(x^r)^2*ln(c*x^n)*b*e^2*r^3-351*x^3*ln(c*x^n)*b*
d^2*r^2-12*x^3*a*d^2*r^4+81*b*d^2*n*x^3+81*x^3*(x^r)^2*b*e^2*n+4*x^3*b*d^2*
n*r^4+36*x^3*b*d^2*n*r^3+117*x^3*b*d^2*n*r^2+162*x^3*b*d^2*n*r)/(3+2*r)^2/(
3+r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(101) = 202$.

Time = 0.28 (sec) , antiderivative size = 497, normalized size of antiderivative = 4.73

$$\int x^2(d+ex^r)^2(a+b\log(cx^n))\,dx$$
$$= \frac{3\,(4\,bd^2r^4+36\,bd^2r^3+117\,bd^2r^2+162\,bd^2r+81\,bd^2)x^3\log(c)+3\,(4\,bd^2nr^4+36\,bd^2nr^3+117\,bd^2nr^2+}{}$$

```
[In] integrate(x^2*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(4*b*d^2*r^4 + 36*b*d^2*r^3 + 117*b*d^2*r^2 + 162*b*d^2*r + 81*b*d^2
)*x^3*log(c) + 3*(4*b*d^2*n*r^4 + 36*b*d^2*n*r^3 + 117*b*d^2*n*r^2 + 162*b*
d^2*n*r + 81*b*d^2*n)*x^3*log(x) - (4*(b*d^2*n - 3*a*d^2)*r^4 + 81*b*d^2*n
+ 36*(b*d^2*n - 3*a*d^2)*r^3 - 243*a*d^2 + 117*(b*d^2*n - 3*a*d^2)*r^2 + 16
2*(b*d^2*n - 3*a*d^2)*r)*x^3 + 9*((2*b*e^2*r^3 + 15*b*e^2*r^2 + 36*b*e^2*r
+ 27*b*e^2)*x^3*log(c) + (2*b*e^2*n*r^3 + 15*b*e^2*n*r^2 + 36*b*e^2*n*r + 2
7*b*e^2*n)*x^3*log(x) + (2*a*e^2*r^3 - 9*b*e^2*n + 27*a*e^2 - (b*e^2*n - 15
*a*e^2)*r^2 - 6*(b*e^2*n - 6*a*e^2)*r)*x^3)*x^(2*r) + 18*((4*b*d*e*r^3 + 24
*b*d*e*r^2 + 45*b*d*e*r + 27*b*d*e)*x^3*log(c) + (4*b*d*e*n*r^3 + 24*b*d*e*
n*r^2 + 45*b*d*e*n*r + 27*b*d*e*n)*x^3*log(x) + (4*a*d*e*r^3 - 9*b*d*e*n +
27*a*d*e - 4*(b*d*e*n - 6*a*d*e)*r^2 - 3*(4*b*d*e*n - 15*a*d*e)*r)*x^3)*x^r
)/(4*r^4 + 36*r^3 + 117*r^2 + 162*r + 81)
```

Sympy [A] (verification not implemented)

Time = 83.39 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.29

$$\int x^2(d+ex^r)^2\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$= \frac{ad^2x^3}{3} + 2ade\left(\begin{cases}\frac{x^3x^r}{r+3} & \text{for}\ r\neq -3\\ x^3x^r\log(x) & \text{otherwise}\end{cases}\right)$$
$$+ ae^2\left(\begin{cases}\frac{x^3x^{2r}}{2r+3} & \text{for}\ r\neq -\frac{3}{2}\\ x^3x^{2r}\log(x) & \text{otherwise}\end{cases}\right) - \frac{bd^2nx^3}{9} + \frac{bd^2x^3\log(cx^n)}{3}$$
$$- 2bden\left(\begin{cases}\frac{\begin{cases}\frac{x^{r+3}}{r+3} & \text{for}\ r\neq -3\\ \log(x) & \text{otherwise}\end{cases}}{r+3} & \text{for}\ r>-\infty\wedge r<\infty\wedge r\neq -3\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ 2bde\left(\begin{cases}\frac{x^{r+3}}{r+3} & \text{for}\ r\neq -3\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$
$$- be^2n\left(\begin{cases}\frac{\begin{cases}\frac{x^{2r+3}}{2r+3} & \text{for}\ r\neq -\frac{3}{2}\\ \log(x) & \text{otherwise}\end{cases}}{2r+3} & \text{for}\ r>-\infty\wedge r<\infty\wedge r\neq -\frac{3}{2}\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ be^2\left(\begin{cases}\frac{x^{2r+3}}{2r+3} & \text{for}\ r\neq -\frac{3}{2}\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate(x**2*(d+e*x**r)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**2*x**3/3 + 2*a*d*e*Piecewise((x**3*x**r/(r + 3), Ne(r, -3)), (x**3*x**
r*log(x), True)) + a*e**2*Piecewise((x**3*x**(2*r)/(2*r + 3), Ne(r, -3/2)),
 (x**3*x**(2*r)*log(x), True)) - b*d**2*n*x**3/9 + b*d**2*x**3*log(c*x**n)/
3 - 2*b*d*e*n*Piecewise((Piecewise((x**(r + 3)/(r + 3), Ne(r, -3)), (log(x)
, True))/(r + 3), (r > -oo) & (r < oo) & Ne(r, -3)), (log(x)**2/2, True)) +
 2*b*d*e*Piecewise((x**(r + 3)/(r + 3), Ne(r, -3)), (log(x), True))*log(c*x
**n) - b*e**2*n*Piecewise((Piecewise((x**(2*r + 3)/(2*r + 3), Ne(r, -3/2)),
 (log(x), True))/(2*r + 3), (r > -oo) & (r < oo) & Ne(r, -3/2)), (log(x)**2
/2, True)) + b*e**2*Piecewise((x**(2*r + 3)/(2*r + 3), Ne(r, -3/2)), (log(x
), True))*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.45

$$\int x^2(d+ex^r)^2\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{1}{9}\,bd^2nx^3+\frac{1}{3}\,bd^2x^3\log\left(cx^n\right)+\frac{1}{3}\,ad^2x^3+\frac{be^2x^{2\,r+3}\log\left(cx^n\right)}{2\,r+3}+\frac{2\,bdex^{r+3}\log\left(cx^n\right)}{r+3}-\frac{be^2nx^{2\,r+3}}{(2\,r+3)^2}+\frac{ae^2x^{2\,r+3}}{2\,r+3}-\frac{2\,bdenx^{r+3}}{(r+3)^2}+\frac{2\,adex^{r+3}}{r+3}$$

```
[In] integrate(x^2*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/9*b*d^2*n*x^3 + 1/3*b*d^2*x^3*log(c*x^n) + 1/3*a*d^2*x^3 + b*e^2*x^(2*r
+ 3)*log(c*x^n)/(2*r + 3) + 2*b*d*e*x^(r + 3)*log(c*x^n)/(r + 3) - b*e^2*n*
x^(2*r + 3)/(2*r + 3)^2 + a*e^2*x^(2*r + 3)/(2*r + 3) - 2*b*d*e*n*x^(r + 3)
/(r + 3)^2 + 2*a*d*e*x^(r + 3)/(r + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. 2(101) = 202.

Time = 0.37 (sec) , antiderivative size = 746, normalized size of antiderivative = 7.10

$$\int x^2(d+ex^r)^2\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{18\,be^2nr^3x^3x^{2\,r}\log\left(x\right)+72\,bdenr^3x^3x^r\log\left(x\right)+12\,bd^2nr^4x^3\log\left(x\right)-4\,bd^2nr^4x^3+18\,be^2r^3x^3x^{2\,r}\log\left(c\right)}{}$$

```
[In] integrate(x^2*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/9*(18*b*e^2*n*r^3*x^3*x^(2*r)*log(x) + 72*b*d*e*n*r^3*x^3*x^r*log(x) + 12
*b*d^2*n*r^4*x^3*log(x) - 4*b*d^2*n*r^4*x^3 + 18*b*e^2*r^3*x^3*x^(2*r)*log(
c) + 72*b*d*e*r^3*x^3*x^r*log(c) + 12*b*d^2*r^4*x^3*log(c) + 135*b*e^2*n*r^
2*x^3*x^(2*r)*log(x) + 432*b*d*e*n*r^2*x^3*x^r*log(x) + 108*b*d^2*n*r^3*x^3
*log(x) - 9*b*e^2*n*r^2*x^3*x^(2*r) + 18*a*e^2*r^3*x^3*x^(2*r) - 72*b*d*e*n
*r^2*x^3*x^r + 72*a*d*e*r^3*x^3*x^r - 36*b*d^2*n*r^3*x^3 + 12*a*d^2*r^4*x^3
 + 135*b*e^2*r^2*x^3*x^(2*r)*log(c) + 432*b*d*e*r^2*x^3*x^r*log(c) + 108*b*
d^2*r^3*x^3*log(c) + 324*b*e^2*n*r*x^3*x^(2*r)*log(x) + 810*b*d*e*n*r*x^3*x
^r*log(x) + 351*b*d^2*n*r^2*x^3*log(x) - 54*b*e^2*n*r*x^3*x^(2*r) + 135*a*e
^2*r^2*x^3*x^(2*r) - 216*b*d*e*n*r*x^3*x^r + 432*a*d*e*r^2*x^3*x^r - 117*b*
d^2*n*r^2*x^3 + 108*a*d^2*r^3*x^3 + 324*b*e^2*r*x^3*x^(2*r)*log(c) + 810*b*
d*e*r*x^3*x^r*log(c) + 351*b*d^2*r^2*x^3*log(c) + 243*b*e^2*n*x^3*x^(2*r)*l
og(x) + 486*b*d*e*n*x^3*x^r*log(x) + 486*b*d^2*n*r*x^3*log(x) - 81*b*e^2*n*
```

```
x^3*x^(2*r) + 324*a*e^2*r*x^3*x^(2*r) - 162*b*d*e*n*x^3*x^r + 810*a*d*e*r*x
^3*x^r - 162*b*d^2*n*r*x^3 + 351*a*d^2*r^2*x^3 + 243*b*e^2*x^3*x^(2*r)*log(
c) + 486*b*d*e*x^3*x^r*log(c) + 486*b*d^2*r*x^3*log(c) + 243*b*d^2*n*x^3*lo
g(x) + 243*a*e^2*x^3*x^(2*r) + 486*a*d*e*x^3*x^r - 81*b*d^2*n*x^3 + 486*a*d
^2*r*x^3 + 243*b*d^2*x^3*log(c) + 243*a*d^2*x^3)/(4*r^4 + 36*r^3 + 117*r^2
+ 162*r + 81)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^r)^2\left(a+b\log\left(cx^n\right)\right)\,dx=\int x^2\,\left(d+e\,x^r\right)^2\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

```
[In] int(x^2*(d + e*x^r)^2*(a + b*log(c*x^n)),x)
[Out] int(x^2*(d + e*x^r)^2*(a + b*log(c*x^n)), x)
```

3.387 $\int (d+ex^r)^2 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int (d+ex^r)^2 (a+b\log(cx^n))\,dx = -bd^2nx - \frac{2bdenx^{1+r}}{(1+r)^2} - \frac{be^2nx^{1+2r}}{(1+2r)^2} + d^2x(a+b\log(cx^n)) + \frac{2dex^{1+r}(a+b\log(cx^n))}{1+r} + \frac{e^2x^{1+2r}(a+b\log(cx^n))}{1+2r}$$

```
[Out] -b*d^2*n*x-2*b*d*e*n*x^(1+r)/(1+r)^2-b*e^2*n*x^(1+2*r)/(1+2*r)^2+d^2*x*(a+b
*ln(c*x^n))+2*d*e*x^(1+r)*(a+b*ln(c*x^n))/(1+r)+e^2*x^(1+2*r)*(a+b*ln(c*x^n
))/(1+2*r)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {250, 2350}

$$\int (d+ex^r)^2 (a+b\log(cx^n))\,dx = d^2x(a+b\log(cx^n)) + \frac{2dex^{r+1}(a+b\log(cx^n))}{r+1} + \frac{e^2x^{2r+1}(a+b\log(cx^n))}{2r+1} - bd^2nx - \frac{2bdenx^{r+1}}{(r+1)^2} - \frac{be^2nx^{2r+1}}{(2r+1)^2}$$

```
[In] Int[(d + e*x^r)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -(b*d^2*n*x) - (2*b*d*e*n*x^(1 + r))/(1 + r)^2 - (b*e^2*n*x^(1 + 2*r))/(1 +
 2*r)^2 + d^2*x*(a + b*Log[c*x^n]) + (2*d*e*x^(1 + r)*(a + b*Log[c*x^n]))/(
1 + r) + (e^2*x^(1 + 2*r)*(a + b*Log[c*x^n]))/(1 + 2*r)
```

Rule 250

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

Rule 2350

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Dist[a + b*Log[c*x^n], u
, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= d^2x(a+b\log(cx^n))+\frac{2dex^{1+r}(a+b\log(cx^n))}{1+r}\\&\quad+\frac{e^2x^{1+2r}(a+b\log(cx^n))}{1+2r}-(bn)\int\left(d^2+\frac{2dex^r}{1+r}+\frac{e^2x^{2r}}{1+2r}\right)dx\\&=-bd^2nx-\frac{2bdenx^{1+r}}{(1+r)^2}-\frac{be^2nx^{1+2r}}{(1+2r)^2}+d^2x(a+b\log(cx^n))\\&\quad+\frac{2dex^{1+r}(a+b\log(cx^n))}{1+r}+\frac{e^2x^{1+2r}(a+b\log(cx^n))}{1+2r}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int(d+ex^r)^2(a+b\log(cx^n))\,dx=x\left(ad^2-bd^2n-\frac{2bdenx^r}{(1+r)^2}-\frac{be^2nx^{2r}}{(1+2r)^2}+bd^2\log(cx^n)+\frac{2dex^r(a+b\log(cx^n))}{1+r}+\frac{e^2x^{2r}(a+b\log(cx^n))}{1+2r}\right)$$

```
[In] Integrate[(d + e*x^r)^2*(a + b*Log[c*x^n]),x]
[Out] x*(a*d^2 - b*d^2*n - (2*b*d*e*n*x^r)/(1 + r)^2 - (b*e^2*n*x^(2*r))/(1 + 2*r
)^2 + b*d^2*Log[c*x^n] + (2*d*e*x^r*(a + b*Log[c*x^n]))/(1 + r) + (e^2*x^(2
*r)*(a + b*Log[c*x^n]))/(1 + 2*r))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(113) = 226.

Time = 1.01 (sec) , antiderivative size = 509, normalized size of antiderivative = 4.50

method	result
parallelrisch	$-\frac{-8x\,x^r\ln(c\,x^n)bde\,r^3-16x\,x^r\ln(c\,x^n)bde\,r^2-8x\,x^r r^3 ade-4x\ln(c\,x^n)b\,d^2r^4+2x\,x^r bden-12x\ln(c\,x^n)b\,d^2r^3-2xde\,x^r b\ln(c}{}$
risch	Expression too large to display

```
[In] int((d+e*x^r)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -(-8*x*x^r*ln(c*x^n)*b*d*e*r^3-16*x*x^r*ln(c*x^n)*b*d*e*r^2-4*x*(x^r)^2*ln(
c*x^n)*b*e^2*r-8*x*x^r*r^3*a*d*e-4*x*(x^r)^2*a*e^2*r-4*x*ln(c*x^n)*b*d^2*r^
4+x*(x^r)^2*b*e^2*n*r^2+2*x*(x^r)^2*b*e^2*n*r+2*x*x^r*b*d*e*n-12*x*ln(c*x^n
)*b*d^2*r^3-2*x*d*e*x^r*b*ln(c*x^n)-x*b*ln(c*x^n)*d^2-x*e^2*(x^r)^2*a-10*x*
x^r*ln(c*x^n)*b*d*e*r-2*x*(x^r)^2*ln(c*x^n)*b*e^2*r^3-5*x*(x^r)^2*ln(c*x^n)
*b*e^2*r^2-x*e^2*(x^r)^2*b*ln(c*x^n)-6*x*ln(c*x^n)*b*d^2*r+12*x*b*d^2*n*r^3
+x*(x^r)^2*b*e^2*n+13*x*b*d^2*n*r^2+6*x*b*d^2*n*r+4*x*b*d^2*n*r^4+8*x*x^r*b
*d*e*n*r^2+8*x*x^r*b*d*e*n*r-a*d^2*x-2*x*d*e*x^r*a-2*x*(x^r)^2*a*e^2*r^3-13
*x*ln(c*x^n)*b*d^2*r^2-4*x*a*d^2*r^4-12*x*a*d^2*r^3-13*x*a*d^2*r^2-6*x*a*d^
2*r-5*x*(x^r)^2*a*e^2*r^2-10*x*x^r*r*a*d*e-16*x*x^r*a*d*e*r^2+b*d^2*n*x)/(1
+2*r)^2/(r^2+2*r+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(113) = 226.

Time = 0.32 (sec) , antiderivative size = 466, normalized size of antiderivative = 4.12

$$\int (d+ex^r)^2\,(a+b\log(cx^n))\,dx$$
$$=\frac{(4\,bd^2r^4+12\,bd^2r^3+13\,bd^2r^2+6\,bd^2r+bd^2)x\log(c)+(4\,bd^2nr^4+12\,bd^2nr^3+13\,bd^2nr^2+6\,bd^2nr+bc}{}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] ((4*b*d^2*r^4 + 12*b*d^2*r^3 + 13*b*d^2*r^2 + 6*b*d^2*r + b*d^2)*x*log(c) +
 (4*b*d^2*n*r^4 + 12*b*d^2*n*r^3 + 13*b*d^2*n*r^2 + 6*b*d^2*n*r + b*d^2*n)*
x*log(x) - (4*(b*d^2*n - a*d^2)*r^4 + b*d^2*n + 12*(b*d^2*n - a*d^2)*r^3 -
a*d^2 + 13*(b*d^2*n - a*d^2)*r^2 + 6*(b*d^2*n - a*d^2)*r)*x + ((2*b*e^2*r^3
 + 5*b*e^2*r^2 + 4*b*e^2*r + b*e^2)*x*log(c) + (2*b*e^2*n*r^3 + 5*b*e^2*n*r
^2 + 4*b*e^2*n*r + b*e^2*n)*x*log(x) + (2*a*e^2*r^3 - b*e^2*n + a*e^2 - (b*
e^2*n - 5*a*e^2)*r^2 - 2*(b*e^2*n - 2*a*e^2)*r)*x)*x^(2*r) + 2*((4*b*d*e*r^
3 + 8*b*d*e*r^2 + 5*b*d*e*r + b*d*e)*x*log(c) + (4*b*d*e*n*r^3 + 8*b*d*e*n*
r^2 + 5*b*d*e*n*r + b*d*e*n)*x*log(x) + (4*a*d*e*r^3 - b*d*e*n + a*d*e - 4*
```

```
(b*d*e*n - 2*a*d*e)*r^2 - (4*b*d*e*n - 5*a*d*e)*r)*x)*x^r)/(4*r^4 + 12*r^3
+ 13*r^2 + 6*r + 1)
```

Sympy [A] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.87

$$\int (d+ex^r)^2\,(a+b\log(cx^n))\;dx$$
$$= ad^2x + 2ade\left(\begin{cases}\frac{x^{r+1}}{r+1} & \text{for}\ r\neq -1\\ \log(x) & \text{otherwise}\end{cases}\right) + ae^2\left(\begin{cases}\frac{x^{2r+1}}{2r+1} & \text{for}\ r\neq -\frac{1}{2}\\ \log(x) & \text{otherwise}\end{cases}\right) - bd^2nx$$
$$+ bd^2x\log(cx^n) - 2bden\left(\begin{cases}\frac{\begin{cases}\frac{x^{r+1}}{r+1} & \text{for}\ r\neq -1\\ \log(x) & \text{otherwise}\end{cases}}{r+1} & \text{for}\ r>-\infty\wedge r<\infty\wedge r\neq -1\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ 2bde\left(\begin{cases}\frac{x^{r+1}}{r+1} & \text{for}\ r\neq -1\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$
$$- be^2n\left(\begin{cases}\frac{\begin{cases}\frac{x^{2r+1}}{2r+1} & \text{for}\ r\neq -\frac{1}{2}\\ \log(x) & \text{otherwise}\end{cases}}{2r+1} & \text{for}\ r>-\infty\wedge r<\infty\wedge r\neq -\frac{1}{2}\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ be^2\left(\begin{cases}\frac{x^{2r+1}}{2r+1} & \text{for}\ r\neq -\frac{1}{2}\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate((d+e*x**r)**2*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**2*x + 2*a*d*e*Piecewise((x**(r + 1)/(r + 1), Ne(r, -1)), (log(x), True
)) + a*e**2*Piecewise((x**(2*r + 1)/(2*r + 1), Ne(r, -1/2)), (log(x), True)
) - b*d**2*n*x + b*d**2*x*log(c*x**n) - 2*b*d*e*n*Piecewise((Piecewise((x**
(r + 1)/(r + 1), Ne(r, -1)), (log(x), True))/(r + 1), (r > -oo) & (r < oo)
& Ne(r, -1)), (log(x)**2/2, True)) + 2*b*d*e*Piecewise((x**(r + 1)/(r + 1),
 Ne(r, -1)), (log(x), True))*log(c*x**n) - b*e**2*n*Piecewise((Piecewise((x
**(2*r + 1)/(2*r + 1), Ne(r, -1/2)), (log(x), True))/(2*r + 1), (r > -oo) &
 (r < oo) & Ne(r, -1/2)), (log(x)**2/2, True)) + b*e**2*Piecewise((x**(2*r
+ 1)/(2*r + 1), Ne(r, -1/2)), (log(x), True))*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.27

$$\int (d+ex^r)^2 (a+b\log(cx^n))\,dx = -bd^2nx + bd^2x\log(cx^n) + ad^2x + \frac{be^2x^{2r+1}\log(cx^n)}{2r+1} + \frac{2\,bdex^{r+1}\log(cx^n)}{r+1} - \frac{be^2nx^{2r+1}}{(2r+1)^2} + \frac{ae^2x^{2r+1}}{2r+1} - \frac{2\,bdenx^{r+1}}{(r+1)^2} + \frac{2\,adex^{r+1}}{r+1}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -b*d^2*n*x + b*d^2*x*log(c*x^n) + a*d^2*x + b*e^2*x^(2*r + 1)*log(c*x^n)/(2
*r + 1) + 2*b*d*e*x^(r + 1)*log(c*x^n)/(r + 1) - b*e^2*n*x^(2*r + 1)/(2*r +
 1)^2 + a*e^2*x^(2*r + 1)/(2*r + 1) - 2*b*d*e*n*x^(r + 1)/(r + 1)^2 + 2*a*d
*e*x^(r + 1)/(r + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(113) = 226.

Time = 0.34 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.16

$$\int (d+ex^r)^2 (a+b\log(cx^n))\,dx = \frac{2\,be^2nrxx^{2r}\log(x)}{4r^2+4r+1} + \frac{2\,bdenrxx^r\log(x)}{r^2+2r+1} + bd^2nx\log(x) + \frac{be^2nxx^{2r}\log(x)}{4r^2+4r+1} + \frac{2\,bdenxx^r\log(x)}{r^2+2r+1} - bd^2nx - \frac{be^2nxx^{2r}}{4r^2+4r+1} - \frac{2\,bdenxx^r}{r^2+2r+1} + bd^2x\log(c) + \frac{be^2xx^{2r}\log(c)}{2r+1} + \frac{2\,bdexx^r\log(c)}{r+1} + ad^2x + \frac{ae^2xx^{2r}}{2r+1} + \frac{2\,adexx^r}{r+1}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 2*b*e^2*n*r*x*x^(2*r)*log(x)/(4*r^2 + 4*r + 1) + 2*b*d*e*n*r*x*x^r*log(x)/(
r^2 + 2*r + 1) + b*d^2*n*x*log(x) + b*e^2*n*x*x^(2*r)*log(x)/(4*r^2 + 4*r +
 1) + 2*b*d*e*n*x*x^r*log(x)/(r^2 + 2*r + 1) - b*d^2*n*x - b*e^2*n*x*x^(2*r
)/(4*r^2 + 4*r + 1) - 2*b*d*e*n*x*x^r/(r^2 + 2*r + 1) + b*d^2*x*log(c) + b*
e^2*x*x^(2*r)*log(c)/(2*r + 1) + 2*b*d*e*x*x^r*log(c)/(r + 1) + a*d^2*x + a
*e^2*x*x^(2*r)/(2*r + 1) + 2*a*d*e*x*x^r/(r + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^r)^2 (a+b\log(cx^n))\, dx = \int (d+e\,x^r)^2 \left(a+b\,\ln\left(c\,x^n\right)\right) dx$$

```
[In] int((d + e*x^r)^2*(a + b*log(c*x^n)),x)
[Out] int((d + e*x^r)^2*(a + b*log(c*x^n)), x)
```

3.388 $\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{(d+ex^r)^2\,(a+b\log(cx^n))}{x^2}\,dx = -\frac{bd^2n}{x}-\frac{2bdenx^{-1+r}}{(1-r)^2}-\frac{be^2nx^{-1+2r}}{(1-2r)^2}-\frac{d^2(a+b\log(cx^n))}{x}-\frac{2dex^{-1+r}(a+b\log(cx^n))}{1-r}-\frac{e^2x^{-1+2r}(a+b\log(cx^n))}{1-2r}$$

```
[Out] -b*d^2*n/x-2*b*d*e*n*x^(-1+r)/(1-r)^2-b*e^2*n*x^(-1+2*r)/(1-2*r)^2-d^2*(a+b
*ln(c*x^n))/x-2*d*e*x^(-1+r)*(a+b*ln(c*x^n))/(1-r)-e^2*x^(-1+2*r)*(a+b*ln(c
*x^n))/(1-2*r)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {276, 2372, 14}

$$\int \frac{(d+ex^r)^2\,(a+b\log(cx^n))}{x^2}\,dx = -\frac{d^2(a+b\log(cx^n))}{x}-\frac{2dex^{r-1}(a+b\log(cx^n))}{1-r}-\frac{e^2x^{2r-1}(a+b\log(cx^n))}{1-2r}-\frac{bd^2n}{x}-\frac{2bdenx^{r-1}}{(1-r)^2}-\frac{be^2nx^{2r-1}}{(1-2r)^2}$$

```
[In] Int[((d + e*x^r)^2*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -((b*d^2*n)/x) - (2*b*d*e*n*x^(-1 + r))/(1 - r)^2 - (b*e^2*n*x^(-1 + 2*r))/
(1 - 2*r)^2 - (d^2*(a + b*Log[c*x^n]))/x - (2*d*e*x^(-1 + r)*(a + b*Log[c*x
^n]))/(1 - r) - (e^2*x^(-1 + 2*r)*(a + b*Log[c*x^n]))/(1 - 2*r)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{x} - \frac{2dex^{-1+r}(a+b\log(cx^n))}{1-r} \\
&\quad - \frac{e^2x^{-1+2r}(a+b\log(cx^n))}{1-2r} - (bn)\int \frac{-d^2+\frac{2dex^r}{-1+r}+\frac{e^2x^{2r}}{-1+2r}}{x^2}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{x} - \frac{2dex^{-1+r}(a+b\log(cx^n))}{1-r} \\
&\quad - \frac{e^2x^{-1+2r}(a+b\log(cx^n))}{1-2r} - (bn)\int \left(-\frac{d^2}{x^2}+\frac{2dex^{-2+r}}{-1+r}+\frac{e^2x^{2(-1+r)}}{-1+2r}\right)dx \\
&= -\frac{bd^2n}{x} - \frac{2bdenx^{-1+r}}{(1-r)^2} - \frac{be^2nx^{-1+2r}}{(1-2r)^2} - \frac{d^2(a+b\log(cx^n))}{x} \\
&\quad - \frac{2dex^{-1+r}(a+b\log(cx^n))}{1-r} - \frac{e^2x^{-1+2r}(a+b\log(cx^n))}{1-2r}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^2}\,dx$$
$$=\frac{bn\left(-d^2-\frac{2dex^r}{(-1+r)^2}-\frac{e^2x^{2r}}{(1-2r)^2}\right)+a\left(-d^2+\frac{2dex^r}{-1+r}+\frac{e^2x^{2r}}{-1+2r}\right)+b\left(-d^2+\frac{2dex^r}{-1+r}+\frac{e^2x^{2r}}{-1+2r}\right)\log(cx^n)}{x}$$

```
[In] Integrate[((d + e*x^r)^2*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] (b*n*(-d^2 - (2*d*e*x^r)/(-1 + r)^2 - (e^2*x^(2*r))/(1 - 2*r)^2) + a*(-d^2
+ (2*d*e*x^r)/(-1 + r) + (e^2*x^(2*r))/(-1 + 2*r)) + b*(-d^2 + (2*d*e*x^r)/
(-1 + r) + (e^2*x^(2*r))/(-1 + 2*r))*Log[c*x^n])/x
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(123) = 246.

Time = 1.11 (sec) , antiderivative size = 471, normalized size of antiderivative = 3.83

method	result
parallelrisch	$-\frac{b\ln(cx^n)d^2+2bden\,x^r+13b\,d^2n\,r^2+2de\,x^r a-6b\,d^2nr+2de\,x^r b\ln(cx^n)+4a\,d^2r^4-12a\,d^2r^3-8ade\,r^3x^r+b\,d^2n+a\,d^2+4\ln(c}{}$
risch	Expression too large to display

```
[In] int((d+e*x^r)^2*(a+b*ln(c*x^n))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x*(b*ln(c*x^n)*d^2-2*b*e^2*n*r*(x^r)^2+2*b*d*e*n*x^r+5*a*e^2*r^2*(x^r)^2
-4*a*e^2*r*(x^r)^2+b*e^2*n*(x^r)^2-2*a*e^2*r^3*(x^r)^2+e^2*(x^r)^2*a+13*b*d
^2*n*r^2+2*d*e*x^r*a+e^2*(x^r)^2*b*ln(c*x^n)-6*b*d^2*n*r+2*d*e*x^r*b*ln(c*x
^n)+4*a*d^2*r^4-12*a*d^2*r^3-8*a*d*e*r^3*x^r+b*d^2*n+a*d^2+4*ln(c*x^n)*b*d^
2*r^4-12*ln(c*x^n)*b*d^2*r^3+13*ln(c*x^n)*b*d^2*r^2-6*ln(c*x^n)*b*d^2*r+4*b
*d^2*n*r^4-12*b*d^2*n*r^3-8*b*d*e*n*r*x^r+13*a*d^2*r^2-6*a*d^2*r-2*(x^r)^2*
ln(c*x^n)*b*e^2*r^3+5*(x^r)^2*ln(c*x^n)*b*e^2*r^2-4*(x^r)^2*ln(c*x^n)*b*e^2
*r+8*b*d*e*n*r^2*x^r+16*a*d*e*r^2*x^r-10*a*d*e*r*x^r+b*e^2*n*r^2*(x^r)^2-8*
x^r*ln(c*x^n)*b*d*e*r^3+16*x^r*ln(c*x^n)*b*d*e*r^2-10*x^r*ln(c*x^n)*b*d*e*r
)/(-1+2*r)^2/(r^2-2*r+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(118) = 236$.

Time = 0.32 (sec) , antiderivative size = 455, normalized size of antiderivative = 3.70

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^2}\,dx = -\frac{4\,(bd^2n+ad^2)r^4+bd^2n-12\,(bd^2n+ad^2)r^3+ad^2+13\,(bd^2n+ad^2)r^2-6\,(bd^2n+ad^2)r-(2\,ae^2r^3-b}{}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
```

```
[Out] -(4*(b*d^2*n + a*d^2)*r^4 + b*d^2*n - 12*(b*d^2*n + a*d^2)*r^3 + a*d^2 + 13
*(b*d^2*n + a*d^2)*r^2 - 6*(b*d^2*n + a*d^2)*r - (2*a*e^2*r^3 - b*e^2*n - a
*e^2 - (b*e^2*n + 5*a*e^2)*r^2 + 2*(b*e^2*n + 2*a*e^2)*r + (2*b*e^2*r^3 - 5
*b*e^2*r^2 + 4*b*e^2*r - b*e^2)*log(c) + (2*b*e^2*n*r^3 - 5*b*e^2*n*r^2 + 4
*b*e^2*n*r - b*e^2*n)*log(x))*x^(2*r) - 2*(4*a*d*e*r^3 - b*d*e*n - a*d*e -
4*(b*d*e*n + 2*a*d*e)*r^2 + (4*b*d*e*n + 5*a*d*e)*r + (4*b*d*e*r^3 - 8*b*d*
e*r^2 + 5*b*d*e*r - b*d*e)*log(c) + (4*b*d*e*n*r^3 - 8*b*d*e*n*r^2 + 5*b*d*
e*n*r - b*d*e*n)*log(x))*x^r + (4*b*d^2*r^4 - 12*b*d^2*r^3 + 13*b*d^2*r^2 -
 6*b*d^2*r + b*d^2)*log(c) + (4*b*d^2*n*r^4 - 12*b*d^2*n*r^3 + 13*b*d^2*n*r
^2 - 6*b*d^2*n*r + b*d^2*n)*log(x))/((4*r^4 - 12*r^3 + 13*r^2 - 6*r + 1)*x)
```

Sympy [A] (verification not implemented)

Time = 3.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.70

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^2}\,dx$$
$$= -\frac{ad^2}{x} + 2ade\left(\begin{cases}\frac{x^r}{rx-x} & \text{for } r\neq 1\\ \frac{x^r\log(x)}{x} & \text{otherwise}\end{cases}\right) + ae^2\left(\begin{cases}\frac{x^{2r}}{2rx-x} & \text{for } r\neq \frac{1}{2}\\ \frac{x^{2r}\log(x)}{x} & \text{otherwise}\end{cases}\right) - \frac{bd^2n}{x}$$
$$- \frac{bd^2\log(cx^n)}{x} - 2bden\left(\begin{cases}\frac{\begin{cases}\frac{x^{r-1}}{r-1} & \text{for } r\neq 1\\ \log(x) & \text{otherwise}\end{cases}}{r-1} & \text{for } r>-\infty\wedge r<\infty\wedge r\neq 1\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ 2bde\left(\begin{cases}\frac{x^{r-1}}{r-1} & \text{for } r\neq 1\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$
$$- be^2n\left(\begin{cases}\frac{\begin{cases}\frac{x^{2r-1}}{2r-1} & \text{for } r\neq \frac{1}{2}\\ \log(x) & \text{otherwise}\end{cases}}{2r-1} & \text{for } r>-\infty\wedge r<\infty\wedge r\neq \frac{1}{2}\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ be^2\left(\begin{cases}\frac{x^{2r-1}}{2r-1} & \text{for } r\neq \frac{1}{2}\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate((d+e*x**r)**2*(a+b*ln(c*x**n))/x**2,x)
```

```
[Out] -a*d**2/x + 2*a*d*e*Piecewise((x**r/(r*x - x), Ne(r, 1)), (x**r*log(x)/x, T
rue)) + a*e**2*Piecewise((x**(2*r)/(2*r*x - x), Ne(r, 1/2)), (x**(2*r)*log(
x)/x, True)) - b*d**2*n/x - b*d**2*log(c*x**n)/x - 2*b*d*e*n*Piecewise((Pie
cewise((x**(r - 1)/(r - 1), Ne(r, 1)), (log(x), True))/(r - 1), (r > -oo) &
 (r < oo) & Ne(r, 1)), (log(x)**2/2, True)) + 2*b*d*e*Piecewise((x**(r - 1)
/(r - 1), Ne(r, 1)), (log(x), True))*log(c*x**n) - b*e**2*n*Piecewise((Piec
ewise((x**(2*r - 1)/(2*r - 1), Ne(r, 1/2)), (log(x), True))/(2*r - 1), (r >
 -oo) & (r < oo) & Ne(r, 1/2)), (log(x)**2/2, True)) + b*e**2*Piecewise((x*
*(2*r - 1)/(2*r - 1), Ne(r, 1/2)), (log(x), True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d + ex^r)^2 (a + b\log(cx^n))}{x^2} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-2>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{(d + ex^r)^2 (a + b\log(cx^n))}{x^2} \, dx = \int \frac{(ex^r + d)^2 (b\log(cx^n) + a)}{x^2} \, dx$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^r + d)^2*(b*log(c*x^n) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex^r)^2 (a + b\log(cx^n))}{x^2} \, dx = \int \frac{(d + e\,x^r)^2 (a + b\,\ln(c\,x^n))}{x^2} \, dx$$

```
[In] int(((d + e*x^r)^2*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int(((d + e*x^r)^2*(a + b*log(c*x^n)))/x^2, x)
```

3.389 $\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{(d+ex^r)^2\,(a+b\log(cx^n))}{x^4}\,dx = -\frac{bd^2n}{9x^3}-\frac{2bdenx^{-3+r}}{(3-r)^2}-\frac{be^2nx^{-3+2r}}{(3-2r)^2}-\frac{d^2(a+b\log(cx^n))}{3x^3}-\frac{2dex^{-3+r}(a+b\log(cx^n))}{3-r}-\frac{e^2x^{-3+2r}(a+b\log(cx^n))}{3-2r}$$

```
[Out] -1/9*b*d^2*n/x^3-2*b*d*e*n*x^(-3+r)/(3-r)^2-b*e^2*n*x^(-3+2*r)/(3-2*r)^2-1/
3*d^2*(a+b*ln(c*x^n))/x^3-2*d*e*x^(-3+r)*(a+b*ln(c*x^n))/(3-r)-e^2*x^(-3+2*
r)*(a+b*ln(c*x^n))/(3-2*r)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2372, 12, 14}

$$\int \frac{(d+ex^r)^2\,(a+b\log(cx^n))}{x^4}\,dx = -\frac{d^2(a+b\log(cx^n))}{3x^3}-\frac{2dex^{r-3}(a+b\log(cx^n))}{3-r}-\frac{e^2x^{2r-3}(a+b\log(cx^n))}{3-2r}-\frac{bd^2n}{9x^3}-\frac{2bdenx^{r-3}}{(3-r)^2}-\frac{be^2nx^{2r-3}}{(3-2r)^2}$$

```
[In] Int[((d + e*x^r)^2*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] -1/9*(b*d^2*n)/x^3 - (2*b*d*e*n*x^(-3 + r))/(3 - r)^2 - (b*e^2*n*x^(-3 + 2*
r))/(3 - 2*r)^2 - (d^2*(a + b*Log[c*x^n]))/(3*x^3) - (2*d*e*x^(-3 + r)*(a +
 b*Log[c*x^n]))/(3 - r) - (e^2*x^(-3 + 2*r)*(a + b*Log[c*x^n]))/(3 - 2*r)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{3x^3}-\frac{2dex^{-3+r}(a+b\log(cx^n))}{3-r}\\
&\quad-\frac{e^2x^{-3+2r}(a+b\log(cx^n))}{3-2r}-(bn)\int\frac{-d^2+\frac{6dex^r}{-3+r}+\frac{3e^2x^{2r}}{-3+2r}}{3x^4}\,dx\\
&=-\frac{d^2(a+b\log(cx^n))}{3x^3}-\frac{2dex^{-3+r}(a+b\log(cx^n))}{3-r}\\
&\quad-\frac{e^2x^{-3+2r}(a+b\log(cx^n))}{3-2r}-\frac{1}{3}(bn)\int\frac{-d^2+\frac{6dex^r}{-3+r}+\frac{3e^2x^{2r}}{-3+2r}}{x^4}\,dx\\
&=-\frac{d^2(a+b\log(cx^n))}{3x^3}-\frac{2dex^{-3+r}(a+b\log(cx^n))}{3-r}-\frac{e^2x^{-3+2r}(a+b\log(cx^n))}{3-2r}\\
&\quad-\frac{1}{3}(bn)\int\left(-\frac{d^2}{x^4}+\frac{6dex^{-4+r}}{-3+r}+\frac{3e^2x^{2(-2+r)}}{-3+2r}\right)dx\\
&=-\frac{bd^2n}{9x^3}-\frac{2bdenx^{-3+r}}{(3-r)^2}-\frac{be^2nx^{-3+2r}}{(3-2r)^2}-\frac{d^2(a+b\log(cx^n))}{3x^3}\\
&\quad-\frac{2dex^{-3+r}(a+b\log(cx^n))}{3-r}-\frac{e^2x^{-3+2r}(a+b\log(cx^n))}{3-2r}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^4}\,dx$$
$$=\frac{bn\left(-d^2-\frac{18dex^r}{(-3+r)^2}-\frac{9e^2x^{2r}}{(3-2r)^2}\right)+a\left(-3d^2+\frac{18dex^r}{-3+r}+\frac{9e^2x^{2r}}{-3+2r}\right)+3b\left(-d^2+\frac{6dex^r}{-3+r}+\frac{3e^2x^{2r}}{-3+2r}\right)\log(cx^n)}{9x^3}$$

```
[In] Integrate[((d + e*x^r)^2*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] (b*n*(-d^2 - (18*d*e*x^r)/(-3 + r)^2 - (9*e^2*x^(2*r))/(3 - 2*r)^2) + a*(-3
*d^2 + (18*d*e*x^r)/(-3 + r) + (9*e^2*x^(2*r))/(-3 + 2*r)) + 3*b*(-d^2 + (6
*d*e*x^r)/(-3 + r) + (3*e^2*x^(2*r))/(-3 + 2*r))*Log[c*x^n])/(9*x^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. 2(123) = 246.

Time = 1.12 (sec) , antiderivative size = 478, normalized size of antiderivative = 3.76

method	result
parallelrisch	$-\frac{243b\ln(c\,x^n)d^2+162bden\,x^r+117b\,d^2n\,r^2+486de\,x^r a-162b\,d^2nr+486de\,x^r b\ln(c\,x^n)+12a\,d^2r^4-108a\,d^2r^3-72ade\,r^3x^r+8}{}$
risch	Expression too large to display

```
[In] int((d+e*x^r)^2*(a+b*ln(c*x^n))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/x^3*(243*b*ln(c*x^n)*d^2-54*b*e^2*n*r*(x^r)^2+162*b*d*e*n*x^r+135*a*e^
2*r^2*(x^r)^2-324*a*e^2*r*(x^r)^2+81*b*e^2*n*(x^r)^2-18*a*e^2*r^3*(x^r)^2+2
43*e^2*(x^r)^2*a+117*b*d^2*n*r^2+486*d*e*x^r*a+243*e^2*(x^r)^2*b*ln(c*x^n)-
162*b*d^2*n*r+486*d*e*x^r*b*ln(c*x^n)+12*a*d^2*r^4-108*a*d^2*r^3-72*a*d*e*r
^3*x^r+81*b*d^2*n+243*a*d^2+12*ln(c*x^n)*b*d^2*r^4-108*ln(c*x^n)*b*d^2*r^3+
351*ln(c*x^n)*b*d^2*r^2-486*ln(c*x^n)*b*d^2*r+4*b*d^2*n*r^4-36*b*d^2*n*r^3-
216*b*d*e*n*r*x^r+351*a*d^2*r^2-486*a*d^2*r-18*(x^r)^2*ln(c*x^n)*b*e^2*r^3+
135*(x^r)^2*ln(c*x^n)*b*e^2*r^2-324*(x^r)^2*ln(c*x^n)*b*e^2*r+72*b*d*e*n*r^
2*x^r+432*a*d*e*r^2*x^r-810*a*d*e*r*x^r+9*b*e^2*n*r^2*(x^r)^2-72*x^r*ln(c*x
^n)*b*d*e*r^3+432*x^r*ln(c*x^n)*b*d*e*r^2-810*x^r*ln(c*x^n)*b*d*e*r)/(-3+2*
r)^2/(r^2-6*r+9)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(118) = 236$.

Time = 0.30 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.67

$$\int \frac{(d + ex^r)^2 (a + b\log(cx^n))}{x^4} dx = -\frac{4(bd^2n + 3ad^2)r^4 + 81bd^2n - 36(bd^2n + 3ad^2)r^3 + 243ad^2 + 117(bd^2n + 3ad^2)r^2 - 162(bd^2n + 3ad^2}{}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(4*(b*d^2*n + 3*a*d^2)*r^4 + 81*b*d^2*n - 36*(b*d^2*n + 3*a*d^2)*r^3 +
 243*a*d^2 + 117*(b*d^2*n + 3*a*d^2)*r^2 - 162*(b*d^2*n + 3*a*d^2)*r - 9*(2
*a*e^2*r^3 - 9*b*e^2*n - 27*a*e^2 - (b*e^2*n + 15*a*e^2)*r^2 + 6*(b*e^2*n +
 6*a*e^2)*r + (2*b*e^2*r^3 - 15*b*e^2*r^2 + 36*b*e^2*r - 27*b*e^2)*log(c) +
 (2*b*e^2*n*r^3 - 15*b*e^2*n*r^2 + 36*b*e^2*n*r - 27*b*e^2*n)*log(x))*x^(2*
r) - 18*(4*a*d*e*r^3 - 9*b*d*e*n - 27*a*d*e - 4*(b*d*e*n + 6*a*d*e)*r^2 + 3
*(4*b*d*e*n + 15*a*d*e)*r + (4*b*d*e*r^3 - 24*b*d*e*r^2 + 45*b*d*e*r - 27*b
*d*e)*log(c) + (4*b*d*e*n*r^3 - 24*b*d*e*n*r^2 + 45*b*d*e*n*r - 27*b*d*e*n)
*log(x))*x^r + 3*(4*b*d^2*r^4 - 36*b*d^2*r^3 + 117*b*d^2*r^2 - 162*b*d^2*r
+ 81*b*d^2)*log(c) + 3*(4*b*d^2*n*r^4 - 36*b*d^2*n*r^3 + 117*b*d^2*n*r^2 -
162*b*d^2*n*r + 81*b*d^2*n)*log(x))/((4*r^4 - 36*r^3 + 117*r^2 - 162*r + 81
)*x^3)
```

Sympy [A] (verification not implemented)

Time = 18.04 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.83

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^4}\,dx$$
$$= -\frac{ad^2}{3x^3} + 2ade\left(\begin{cases} \frac{x^r}{rx^3-3x^3} & \text{for } r\neq 3 \\ \frac{x^r\log(x)}{x^3} & \text{otherwise}\end{cases}\right) + ae^2\left(\begin{cases} \frac{x^{2r}}{2rx^3-3x^3} & \text{for } r\neq \frac{3}{2} \\ \frac{x^{2r}\log(x)}{x^3} & \text{otherwise}\end{cases}\right) - \frac{bd^2n}{9x^3}$$
$$- \frac{bd^2\log(cx^n)}{3x^3} - 2bden\left(\begin{cases} \frac{\begin{cases} \frac{x^{r-3}}{r-3} & \text{for } r\neq 3 \\ \log(x) & \text{otherwise}\end{cases}}{r-3} & \text{for } r>-\infty \wedge r<\infty \wedge r\neq 3 \\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ 2bde\left(\begin{cases} \frac{x^{r-3}}{r-3} & \text{for } r\neq 3 \\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$
$$- be^2n\left(\begin{cases} \frac{\begin{cases} \frac{x^{2r-3}}{2r-3} & \text{for } r\neq \frac{3}{2} \\ \log(x) & \text{otherwise}\end{cases}}{2r-3} & \text{for } r>-\infty \wedge r<\infty \wedge r\neq \frac{3}{2} \\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ be^2\left(\begin{cases} \frac{x^{2r-3}}{2r-3} & \text{for } r\neq \frac{3}{2} \\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate((d+e*x**r)**2*(a+b*ln(c*x**n))/x**4,x)
```

```
[Out] -a*d**2/(3*x**3) + 2*a*d*e*Piecewise((x**r/(r*x**3 - 3*x**3), Ne(r, 3)), (x
**r*log(x)/x**3, True)) + a*e**2*Piecewise((x**(2*r)/(2*r*x**3 - 3*x**3), N
e(r, 3/2)), (x**(2*r)*log(x)/x**3, True)) - b*d**2*n/(9*x**3) - b*d**2*log(
c*x**n)/(3*x**3) - 2*b*d*e*n*Piecewise((Piecewise((x**(r - 3)/(r - 3), Ne(r
, 3)), (log(x), True))/(r - 3), (r > -oo) & (r < oo) & Ne(r, 3)), (log(x)**
2/2, True)) + 2*b*d*e*Piecewise((x**(r - 3)/(r - 3), Ne(r, 3)), (log(x), Tr
ue))*log(c*x**n) - b*e**2*n*Piecewise((Piecewise((x**(2*r - 3)/(2*r - 3), N
e(r, 3/2)), (log(x), True))/(2*r - 3), (r > -oo) & (r < oo) & Ne(r, 3/2)),
(log(x)**2/2, True)) + b*e**2*Piecewise((x**(2*r - 3)/(2*r - 3), Ne(r, 3/2)
), (log(x), True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^4} dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-4>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^4} dx = \int \frac{(ex^r+d)^2(b\log(cx^n)+a)}{x^4} dx$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^r + d)^2*(b*log(c*x^n) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^4} dx = \int \frac{(d+e\,x^r)^2\,(a+b\,\ln(c\,x^n))}{x^4} dx$$

```
[In] int(((d + e*x^r)^2*(a + b*log(c*x^n)))/x^4,x)
```

```
[Out] int(((d + e*x^r)^2*(a + b*log(c*x^n)))/x^4, x)
```

3.390 $\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x^6}\,dx = -\frac{bd^2n}{25x^5}-\frac{2bdenx^{-5+r}}{(5-r)^2}-\frac{be^2nx^{-5+2r}}{(5-2r)^2}-\frac{d^2(a+b\log(cx^n))}{5x^5}-\frac{2dex^{-5+r}(a+b\log(cx^n))}{5-r}-\frac{e^2x^{-5+2r}(a+b\log(cx^n))}{5-2r}$$

```
[Out] -1/25*b*d^2*n/x^5-2*b*d*e*n*x^(-5+r)/(5-r)^2-b*e^2*n*x^(-5+2*r)/(5-2*r)^2-1
/5*d^2*(a+b*ln(c*x^n))/x^5-2*d*e*x^(-5+r)*(a+b*ln(c*x^n))/(5-r)-e^2*x^(-5+2
*r)*(a+b*ln(c*x^n))/(5-2*r)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2372, 12, 14}

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x^6}\,dx = -\frac{d^2(a+b\log(cx^n))}{5x^5}-\frac{2dex^{r-5}(a+b\log(cx^n))}{5-r}-\frac{e^2x^{2r-5}(a+b\log(cx^n))}{5-2r}-\frac{bd^2n}{25x^5}-\frac{2bdenx^{r-5}}{(5-r)^2}-\frac{be^2nx^{2r-5}}{(5-2r)^2}$$

```
[In] Int[((d + e*x^r)^2*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] -1/25*(b*d^2*n)/x^5 - (2*b*d*e*n*x^(-5 + r))/(5 - r)^2 - (b*e^2*n*x^(-5 + 2
*r))/(5 - 2*r)^2 - (d^2*(a + b*Log[c*x^n]))/(5*x^5) - (2*d*e*x^(-5 + r)*(a
+ b*Log[c*x^n]))/(5 - r) - (e^2*x^(-5 + 2*r)*(a + b*Log[c*x^n]))/(5 - 2*r)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{5x^5} - \frac{2dex^{-5+r}(a+b\log(cx^n))}{5-r} \\
&\quad - \frac{e^2x^{-5+2r}(a+b\log(cx^n))}{5-2r} - (bn)\int \frac{-d^2+\frac{10dex^r}{-5+r}+\frac{5e^2x^{2r}}{-5+2r}}{5x^6}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{5x^5} - \frac{2dex^{-5+r}(a+b\log(cx^n))}{5-r} \\
&\quad - \frac{e^2x^{-5+2r}(a+b\log(cx^n))}{5-2r} - \frac{1}{5}(bn)\int \frac{-d^2+\frac{10dex^r}{-5+r}+\frac{5e^2x^{2r}}{-5+2r}}{x^6}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{5x^5} - \frac{2dex^{-5+r}(a+b\log(cx^n))}{5-r} - \frac{e^2x^{-5+2r}(a+b\log(cx^n))}{5-2r} \\
&\quad - \frac{1}{5}(bn)\int \left(-\frac{d^2}{x^6}+\frac{10dex^{-6+r}}{-5+r}+\frac{5e^2x^{2(-3+r)}}{-5+2r}\right)dx \\
&= -\frac{bd^2n}{25x^5} - \frac{2bdenx^{-5+r}}{(5-r)^2} - \frac{be^2nx^{-5+2r}}{(5-2r)^2} - \frac{d^2(a+b\log(cx^n))}{5x^5} \\
&\quad - \frac{2dex^{-5+r}(a+b\log(cx^n))}{5-r} - \frac{e^2x^{-5+2r}(a+b\log(cx^n))}{5-2r}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^6}\,dx = \frac{bn\left(-d^2-\frac{50dex^r}{(-5+r)^2}-\frac{25e^2x^{2r}}{(5-2r)^2}\right)+a\left(-5d^2+\frac{50dex^r}{-5+r}+\frac{25e^2x^{2r}}{-5+2r}\right)+5b\left(-d^2+\frac{10dex^r}{-5+r}+\frac{5e^2x^{2r}}{-5+2r}\right)\log(cx^n)}{25x^5}$$

```
[In] Integrate[((d + e*x^r)^2*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] (b*n*(-d^2 - (50*d*e*x^r)/(-5 + r)^2 - (25*e^2*x^(2*r))/(5 - 2*r)^2) + a*(-
5*d^2 + (50*d*e*x^r)/(-5 + r) + (25*e^2*x^(2*r))/(-5 + 2*r)) + 5*b*(-d^2 +
(10*d*e*x^r)/(-5 + r) + (5*e^2*x^(2*r))/(-5 + 2*r))*Log[c*x^n])/(25*x^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. 2(123) = 246.

Time = 1.10 (sec) , antiderivative size = 478, normalized size of antiderivative = 3.76

method	result
parallelrisch	$-\frac{3125b\ln(cx^n)d^2+1250bden\,x^r+325b\,d^2n\,r^2+6250de\,x^r a-750b\,d^2nr+6250de\,x^r b\ln(cx^n)+20a\,d^2r^4-300a\,d^2r^3-200ade\,r^3}{}$
risch	Expression too large to display

```
[In] int((d+e*x^r)^2*(a+b*ln(c*x^n))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/25/x^5*(3125*b*ln(c*x^n)*d^2-250*b*e^2*n*r*(x^r)^2+1250*b*d*e*n*x^r+625*
a*e^2*r^2*(x^r)^2-2500*a*e^2*r*(x^r)^2+625*b*e^2*n*(x^r)^2-50*a*e^2*r^3*(x^
r)^2+3125*e^2*(x^r)^2*a+325*b*d^2*n*r^2+6250*d*e*x^r*a+3125*e^2*(x^r)^2*b*l
n(c*x^n)-750*b*d^2*n*r+6250*d*e*x^r*b*ln(c*x^n)+20*a*d^2*r^4-300*a*d^2*r^3-
200*a*d*e*r^3*x^r+625*b*d^2*n+3125*a*d^2+20*ln(c*x^n)*b*d^2*r^4-300*ln(c*x^
n)*b*d^2*r^3+1625*ln(c*x^n)*b*d^2*r^2-3750*ln(c*x^n)*b*d^2*r+4*b*d^2*n*r^4-
60*b*d^2*n*r^3-1000*b*d*e*n*r*x^r+1625*a*d^2*r^2-3750*a*d^2*r-50*(x^r)^2*ln
(c*x^n)*b*e^2*r^3+625*(x^r)^2*ln(c*x^n)*b*e^2*r^2-2500*(x^r)^2*ln(c*x^n)*b*
e^2*r+200*b*d*e*n*r^2*x^r+2000*a*d*e*r^2*x^r-6250*a*d*e*r*x^r+25*b*e^2*n*r^
2*(x^r)^2-200*x^r*ln(c*x^n)*b*d*e*r^3+2000*x^r*ln(c*x^n)*b*d*e*r^2-6250*x^r
*ln(c*x^n)*b*d*e*r)/(-5+2*r)^2/(r^2-10*r+25)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(118) = 236$.

Time = 0.33 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.67

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^6}\,dx = -\frac{4\,(bd^2n+5\,ad^2)r^4+625\,bd^2n-60\,(bd^2n+5\,ad^2)r^3+3125\,ad^2+325\,(bd^2n+5\,ad^2)r^2-750\,(bd^2n+5\,a}{}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^6,x, algorithm="fricas")
```

```
[Out] -1/25*(4*(b*d^2*n + 5*a*d^2)*r^4 + 625*b*d^2*n - 60*(b*d^2*n + 5*a*d^2)*r^3
 + 3125*a*d^2 + 325*(b*d^2*n + 5*a*d^2)*r^2 - 750*(b*d^2*n + 5*a*d^2)*r - 2
5*(2*a*e^2*r^3 - 25*b*e^2*n - 125*a*e^2 - (b*e^2*n + 25*a*e^2)*r^2 + 10*(b*
e^2*n + 10*a*e^2)*r + (2*b*e^2*r^3 - 25*b*e^2*r^2 + 100*b*e^2*r - 125*b*e^2
)*log(c) + (2*b*e^2*n*r^3 - 25*b*e^2*n*r^2 + 100*b*e^2*n*r - 125*b*e^2*n)*l
og(x))*x^(2*r) - 50*(4*a*d*e*r^3 - 25*b*d*e*n - 125*a*d*e - 4*(b*d*e*n + 10
*a*d*e)*r^2 + 5*(4*b*d*e*n + 25*a*d*e)*r + (4*b*d*e*r^3 - 40*b*d*e*r^2 + 12
5*b*d*e*r - 125*b*d*e)*log(c) + (4*b*d*e*n*r^3 - 40*b*d*e*n*r^2 + 125*b*d*e
*n*r - 125*b*d*e*n)*log(x))*x^r + 5*(4*b*d^2*r^4 - 60*b*d^2*r^3 + 325*b*d^2
*r^2 - 750*b*d^2*r + 625*b*d^2)*log(c) + 5*(4*b*d^2*n*r^4 - 60*b*d^2*n*r^3
+ 325*b*d^2*n*r^2 - 750*b*d^2*n*r + 625*b*d^2*n)*log(x))/((4*r^4 - 60*r^3 +
 325*r^2 - 750*r + 625)*x^5)
```

Sympy [A] (verification not implemented)

Time = 139.22 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.83

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^6}\,dx$$
$$= -\frac{ad^2}{5x^5} + 2ade\left(\begin{cases}\frac{x^r}{rx^5-5x^5} & \text{for } r\neq 5\\ \frac{x^r\log(x)}{x^5} & \text{otherwise}\end{cases}\right) + ae^2\left(\begin{cases}\frac{x^{2r}}{2rx^5-5x^5} & \text{for } r\neq \frac{5}{2}\\ \frac{x^{2r}\log(x)}{x^5} & \text{otherwise}\end{cases}\right) - \frac{bd^2n}{25x^5}$$
$$- \frac{bd^2\log(cx^n)}{5x^5} - 2bden\left(\begin{cases}\frac{\begin{cases}\frac{x^{r-5}}{r-5} & \text{for } r\neq 5\\ \log(x) & \text{otherwise}\end{cases}}{r-5} & \text{for } r>-\infty\wedge r<\infty\wedge r\neq 5\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ 2bde\left(\begin{cases}\frac{x^{r-5}}{r-5} & \text{for } r\neq 5\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$
$$- be^2n\left(\begin{cases}\frac{\begin{cases}\frac{x^{2r-5}}{2r-5} & \text{for } r\neq \frac{5}{2}\\ \log(x) & \text{otherwise}\end{cases}}{2r-5} & \text{for } r>-\infty\wedge r<\infty\wedge r\neq \frac{5}{2}\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ be^2\left(\begin{cases}\frac{x^{2r-5}}{2r-5} & \text{for } r\neq \frac{5}{2}\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate((d+e*x**r)**2*(a+b*ln(c*x**n))/x**6,x)
```

```
[Out] -a*d**2/(5*x**5) + 2*a*d*e*Piecewise((x**r/(r*x**5 - 5*x**5), Ne(r, 5)), (x
**r*log(x)/x**5, True)) + a*e**2*Piecewise((x**(2*r)/(2*r*x**5 - 5*x**5), N
e(r, 5/2)), (x**(2*r)*log(x)/x**5, True)) - b*d**2*n/(25*x**5) - b*d**2*log
(c*x**n)/(5*x**5) - 2*b*d*e*n*Piecewise((Piecewise((x**(r - 5)/(r - 5), Ne(
r, 5)), (log(x), True))/(r - 5), (r > -oo) & (r < oo) & Ne(r, 5)), (log(x)*
*2/2, True)) + 2*b*d*e*Piecewise((x**(r - 5)/(r - 5), Ne(r, 5)), (log(x), T
rue))*log(c*x**n) - b*e**2*n*Piecewise((Piecewise((x**(2*r - 5)/(2*r - 5),
Ne(r, 5/2)), (log(x), True))/(2*r - 5), (r > -oo) & (r < oo) & Ne(r, 5/2)),
 (log(x)**2/2, True)) + b*e**2*Piecewise((x**(2*r - 5)/(2*r - 5), Ne(r, 5/2
)), (log(x), True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)^2\,(a+b\log{(cx^n)})}{x^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-6>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{(d+ex^r)^2\,(a+b\log{(cx^n)})}{x^6}\,dx = \int \frac{(ex^r+d)^2(b\log{(cx^n)}+a)}{x^6}\,dx$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^r + d)^2*(b*log(c*x^n) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^2\,(a+b\log{(cx^n)})}{x^6}\,dx = \int \frac{(d+e\,x^r)^2\,(a+b\,\ln{(c\,x^n)})}{x^6}\,dx$$

```
[In] int(((d + e*x^r)^2*(a + b*log(c*x^n)))/x^6,x)
```

```
[Out] int(((d + e*x^r)^2*(a + b*log(c*x^n)))/x^6, x)
```

3.391 $\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x^8}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{(d+ex^r)^2\,(a+b\log(cx^n))}{x^8}\,dx = -\frac{bd^2n}{49x^7}-\frac{2bdenx^{-7+r}}{(7-r)^2}-\frac{be^2nx^{-7+2r}}{(7-2r)^2}-\frac{d^2(a+b\log(cx^n))}{7x^7}-\frac{2dex^{-7+r}(a+b\log(cx^n))}{7-r}-\frac{e^2x^{-7+2r}(a+b\log(cx^n))}{7-2r}$$

```
[Out] -1/49*b*d^2*n/x^7-2*b*d*e*n*x^(-7+r)/(7-r)^2-b*e^2*n*x^(-7+2*r)/(7-2*r)^2-1
/7*d^2*(a+b*ln(c*x^n))/x^7-2*d*e*x^(-7+r)*(a+b*ln(c*x^n))/(7-r)-e^2*x^(-7+2
*r)*(a+b*ln(c*x^n))/(7-2*r)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2372, 12, 14}

$$\int \frac{(d+ex^r)^2\,(a+b\log(cx^n))}{x^8}\,dx = -\frac{d^2(a+b\log(cx^n))}{7x^7}-\frac{2dex^{r-7}(a+b\log(cx^n))}{7-r}-\frac{e^2x^{2r-7}(a+b\log(cx^n))}{7-2r}-\frac{bd^2n}{49x^7}-\frac{2bdenx^{r-7}}{(7-r)^2}-\frac{be^2nx^{2r-7}}{(7-2r)^2}$$

```
[In] Int[((d + e*x^r)^2*(a + b*Log[c*x^n]))/x^8,x]
```

```
[Out] -1/49*(b*d^2*n)/x^7 - (2*b*d*e*n*x^(-7 + r))/(7 - r)^2 - (b*e^2*n*x^(-7 + 2
*r))/(7 - 2*r)^2 - (d^2*(a + b*Log[c*x^n]))/(7*x^7) - (2*d*e*x^(-7 + r)*(a
+ b*Log[c*x^n]))/(7 - r) - (e^2*x^(-7 + 2*r)*(a + b*Log[c*x^n]))/(7 - 2*r)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\log(cx^n))}{7x^7}-\frac{2dex^{-7+r}(a+b\log(cx^n))}{7-r} \\
&\quad -\frac{e^2x^{-7+2r}(a+b\log(cx^n))}{7-2r}-(bn)\int\frac{-d^2+\frac{14dex^r}{-7+r}+\frac{7e^2x^{2r}}{-7+2r}}{7x^8}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{7x^7}-\frac{2dex^{-7+r}(a+b\log(cx^n))}{7-r} \\
&\quad -\frac{e^2x^{-7+2r}(a+b\log(cx^n))}{7-2r}-\frac{1}{7}(bn)\int\frac{-d^2+\frac{14dex^r}{-7+r}+\frac{7e^2x^{2r}}{-7+2r}}{x^8}\,dx \\
&= -\frac{d^2(a+b\log(cx^n))}{7x^7}-\frac{2dex^{-7+r}(a+b\log(cx^n))}{7-r}-\frac{e^2x^{-7+2r}(a+b\log(cx^n))}{7-2r} \\
&\quad -\frac{1}{7}(bn)\int\left(-\frac{d^2}{x^8}+\frac{14dex^{-8+r}}{-7+r}+\frac{7e^2x^{2(-4+r)}}{-7+2r}\right)dx \\
&= -\frac{bd^2n}{49x^7}-\frac{2bdenx^{-7+r}}{(7-r)^2}-\frac{be^2nx^{-7+2r}}{(7-2r)^2}-\frac{d^2(a+b\log(cx^n))}{7x^7} \\
&\quad -\frac{2dex^{-7+r}(a+b\log(cx^n))}{7-r}-\frac{e^2x^{-7+2r}(a+b\log(cx^n))}{7-2r}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x^8}\,dx$$
$$=\frac{bn\left(-d^2-\frac{98dex^r}{(-7+r)^2}-\frac{49e^2x^{2r}}{(7-2r)^2}\right)+a\left(-7d^2+\frac{98dex^r}{-7+r}+\frac{49e^2x^{2r}}{-7+2r}\right)+7b\left(-d^2+\frac{14dex^r}{-7+r}+\frac{7e^2x^{2r}}{-7+2r}\right)\log(cx^n)}{49x^7}$$

```
[In] Integrate[((d + e*x^r)^2*(a + b*Log[c*x^n]))/x^8,x]
```

```
[Out] (b*n*(-d^2 - (98*d*e*x^r)/(-7 + r)^2 - (49*e^2*x^(2*r))/(7 - 2*r)^2) + a*(-
7*d^2 + (98*d*e*x^r)/(-7 + r) + (49*e^2*x^(2*r))/(-7 + 2*r)) + 7*b*(-d^2 +
(14*d*e*x^r)/(-7 + r) + (7*e^2*x^(2*r))/(-7 + 2*r))*Log[c*x^n])/(49*x^7)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(123) = 246.

Time = 3.26 (sec) , antiderivative size = 473, normalized size of antiderivative = 3.72

method	result
parallelrisch	$-\frac{16807b\ln(cx^n)d^2+4802bdenx^r+637bd^2nr^2+33614dex^ra-2058bd^2nr+33614dex^rb\ln(cx^n)+28ad^2r^4-588ad^2r^3-392a}{}$
risch	Expression too large to display

```
[In] int((d+e*x^r)^2*(a+b*ln(c*x^n))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/49/x^7*(16807*b*ln(c*x^n)*d^2-686*b*e^2*n*r*(x^r)^2+4802*b*d*e*n*x^r+171
5*a*e^2*r^2*(x^r)^2-9604*a*e^2*r*(x^r)^2+2401*b*e^2*n*(x^r)^2-98*a*e^2*r^3*
(x^r)^2+16807*e^2*(x^r)^2*a+637*b*d^2*n*r^2+33614*d*e*x^r*a+16807*e^2*(x^r)
^2*b*ln(c*x^n)-2058*b*d^2*n*r+33614*d*e*x^r*b*ln(c*x^n)+28*a*d^2*r^4-588*a*
d^2*r^3-392*a*d*e*r^3*x^r+2401*b*d^2*n+16807*a*d^2+28*ln(c*x^n)*b*d^2*r^4-5
88*ln(c*x^n)*b*d^2*r^3+4459*ln(c*x^n)*b*d^2*r^2-14406*ln(c*x^n)*b*d^2*r+4*b
*d^2*n*r^4-84*b*d^2*n*r^3-2744*b*d*e*n*r*x^r+4459*a*d^2*r^2-14406*a*d^2*r-9
8*(x^r)^2*ln(c*x^n)*b*e^2*r^3+1715*(x^r)^2*ln(c*x^n)*b*e^2*r^2-9604*(x^r)^2
*ln(c*x^n)*b*e^2*r+392*b*d*e*n*r^2*x^r+5488*a*d*e*r^2*x^r-24010*a*d*e*r*x^r
+49*b*e^2*n*r^2*(x^r)^2-392*x^r*ln(c*x^n)*b*d*e*r^3+5488*x^r*ln(c*x^n)*b*d*
e*r^2-24010*x^r*ln(c*x^n)*b*d*e*r)/(-7+2*r)^2/(-7+r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(118) = 236.

Time = 0.32 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.67

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x^8}\,dx = -\frac{4\,(bd^2n+7\,ad^2)r^4+2401\,bd^2n-84\,(bd^2n+7\,ad^2)r^3+16807\,ad^2+637\,(bd^2n+7\,ad^2)r^2-2058\,(bd^2n+$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^8,x, algorithm="fricas")
```

```
[Out] -1/49*(4*(b*d^2*n + 7*a*d^2)*r^4 + 2401*b*d^2*n - 84*(b*d^2*n + 7*a*d^2)*r^
3 + 16807*a*d^2 + 637*(b*d^2*n + 7*a*d^2)*r^2 - 2058*(b*d^2*n + 7*a*d^2)*r
- 49*(2*a*e^2*r^3 - 49*b*e^2*n - 343*a*e^2 - (b*e^2*n + 35*a*e^2)*r^2 + 14*
(b*e^2*n + 14*a*e^2)*r + (2*b*e^2*r^3 - 35*b*e^2*r^2 + 196*b*e^2*r - 343*b*
e^2)*log(c) + (2*b*e^2*n*r^3 - 35*b*e^2*n*r^2 + 196*b*e^2*n*r - 343*b*e^2*n
)*log(x))*x^(2*r) - 98*(4*a*d*e*r^3 - 49*b*d*e*n - 343*a*d*e - 4*(b*d*e*n +
 14*a*d*e)*r^2 + 7*(4*b*d*e*n + 35*a*d*e)*r + (4*b*d*e*r^3 - 56*b*d*e*r^2 +
 245*b*d*e*r - 343*b*d*e)*log(c) + (4*b*d*e*n*r^3 - 56*b*d*e*n*r^2 + 245*b*
d*e*n*r - 343*b*d*e*n)*log(x))*x^r + 7*(4*b*d^2*r^4 - 84*b*d^2*r^3 + 637*b*
d^2*r^2 - 2058*b*d^2*r + 2401*b*d^2)*log(c) + 7*(4*b*d^2*n*r^4 - 84*b*d^2*n
*r^3 + 637*b*d^2*n*r^2 - 2058*b*d^2*n*r + 2401*b*d^2*n)*log(x))/((4*r^4 - 8
4*r^3 + 637*r^2 - 2058*r + 2401)*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x^8}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x**r)**2*(a+b*ln(c*x**n))/x**8,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x^8}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-8>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{(d+ex^r)^2\,(a+b\log{(cx^n)})}{x^8}\,dx = \int \frac{(ex^r+d)^2(b\log{(cx^n)}+a)}{x^8}\,dx$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x^8,x, algorithm="giac")
[Out] integrate((e*x^r + d)^2*(b*log(c*x^n) + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^2\,(a+b\log{(cx^n)})}{x^8}\,dx = \int \frac{(d+e\,x^r)^2\,(a+b\,\ln{(c\,x^n)})}{x^8}\,dx$$

```
[In] int(((d + e*x^r)^2*(a + b*log(c*x^n)))/x^8,x)
[Out] int(((d + e*x^r)^2*(a + b*log(c*x^n)))/x^8, x)
```

3.392 $\int x^5(d+ex^r)^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int x^5(d+ex^r)^3(a+b\log(cx^n))\,dx = -\frac{1}{36}bd^3nx^6-\frac{be^3nx^{3(2+r)}}{9(2+r)^2}-\frac{3bde^2nx^{2(3+r)}}{4(3+r)^2}-\frac{3bd^2enx^{6+r}}{(6+r)^2}+\frac{1}{6}\left(d^3x^6+\frac{2e^3x^{3(2+r)}}{2+r}+\frac{9de^2x^{2(3+r)}}{3+r}+\frac{18d^2ex^{6+r}}{6+r}\right)(a+b\log(cx^n))$$

```
[Out] -1/36*b*d^3*n*x^6-1/9*b*e^3*n*x^(6+3*r)/(2+r)^2-3/4*b*d*e^2*n*x^(6+2*r)/(3+
r)^2-3*b*d^2*e*n*x^(6+r)/(6+r)^2+1/6*(d^3*x^6+2*e^3*x^(6+3*r)/(2+r)+9*d*e^2
*x^(6+2*r)/(3+r)+18*d^2*e*x^(6+r)/(6+r))*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2371, 12, 14}

$$\int x^5(d+ex^r)^3(a+b\log(cx^n))\,dx = \frac{1}{6}\left(d^3x^6+\frac{18d^2ex^{r+6}}{r+6}+\frac{9de^2x^{2(r+3)}}{r+3}+\frac{2e^3x^{3(r+2)}}{r+2}\right)(a+b\log(cx^n))-\frac{1}{36}bd^3nx^6-\frac{3bd^2enx^{r+6}}{(r+6)^2}-\frac{3bde^2nx^{2(r+3)}}{4(r+3)^2}-\frac{be^3nx^{3(r+2)}}{9(r+2)^2}$$

```
[In] Int[x^5*(d + e*x^r)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/36*(b*d^3*n*x^6) - (b*e^3*n*x^(3*(2 + r)))/(9*(2 + r)^2) - (3*b*d*e^2*n*
x^(2*(3 + r)))/(4*(3 + r)^2) - (3*b*d^2*e*n*x^(6 + r))/(6 + r)^2 + ((d^3*x^
6 + (2*e^3*x^(3*(2 + r)))/(2 + r) + (9*d*e^2*x^(2*(3 + r)))/(3 + r) + (18*d
^2*e*x^(6 + r))/(6 + r))*(a + b*Log[c*x^n]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{6}\left(d^3x^6+\frac{2e^3x^{3(2+r)}}{2+r}+\frac{9de^2x^{2(3+r)}}{3+r}+\frac{18d^2ex^{6+r}}{6+r}\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int \frac{1}{6}x^5\left(d^3+\frac{18d^2ex^r}{6+r}+\frac{9de^2x^{2r}}{3+r}+\frac{2e^3x^{3r}}{2+r}\right)dx \\
&= \frac{1}{6}\left(d^3x^6+\frac{2e^3x^{3(2+r)}}{2+r}+\frac{9de^2x^{2(3+r)}}{3+r}+\frac{18d^2ex^{6+r}}{6+r}\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{6}(bn)\int x^5\left(d^3+\frac{18d^2ex^r}{6+r}+\frac{9de^2x^{2r}}{3+r}+\frac{2e^3x^{3r}}{2+r}\right)dx \\
&= \frac{1}{6}\left(d^3x^6+\frac{2e^3x^{3(2+r)}}{2+r}+\frac{9de^2x^{2(3+r)}}{3+r}+\frac{18d^2ex^{6+r}}{6+r}\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{6}(bn)\int \left(d^3x^5+\frac{18d^2ex^{5+r}}{6+r}+\frac{9de^2x^{5+2r}}{3+r}+\frac{2e^3x^{5+3r}}{2+r}\right)dx
\end{aligned}
$$

$$= -\frac{1}{36}bd^3nx^6 - \frac{be^3nx^{3(2+r)}}{9(2+r)^2} - \frac{3bde^2nx^{2(3+r)}}{4(3+r)^2} - \frac{3bd^2enx^{6+r}}{(6+r)^2} + \frac{1}{6}\left(d^3x^6 + \frac{2e^3x^{3(2+r)}}{2+r} + \frac{9de^2x^{2(3+r)}}{3+r} + \frac{18d^2ex^{6+r}}{6+r}\right)(a + b\log(cx^n))$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.17

$$\int x^5(d+ex^r)^3(a+b\log(cx^n))\,dx = \frac{1}{36}x^6\left(bn\left(-d^3 - \frac{108d^2ex^r}{(6+r)^2} - \frac{27de^2x^{2r}}{(3+r)^2} - \frac{4e^3x^{3r}}{(2+r)^2}\right) + 6a\left(d^3 + \frac{18d^2ex^r}{6+r} + \frac{9de^2x^{2r}}{3+r} + \frac{2e^3x^{3r}}{2+r}\right) + 6b\left(d^3 + \frac{18d^2ex^r}{6+r} + \frac{9de^2x^{2r}}{3+r} + \frac{2e^3x^{3r}}{2+r}\right)\log(cx^n)\right)$$

```
[In] Integrate[x^5*(d + e*x^r)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^6*(b*n*(-d^3 - (108*d^2*e*x^r)/(6 + r)^2 - (27*d*e^2*x^(2*r))/(3 + r)^2
- (4*e^3*x^(3*r))/(2 + r)^2) + 6*a*(d^3 + (18*d^2*e*x^r)/(6 + r) + (9*d*e^2
*x^(2*r))/(3 + r) + (2*e^3*x^(3*r))/(2 + r)) + 6*b*(d^3 + (18*d^2*e*x^r)/(6
 + r) + (9*d*e^2*x^(2*r))/(3 + r) + (2*e^3*x^(3*r))/(2 + r))*Log[c*x^n]))/3
6
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. 2(139) = 278.

Time = 40.92 (sec) , antiderivative size = 1249, normalized size of antiderivative = 8.50

method	result	size
parallelrisch	Expression too large to display	1249
risch	Expression too large to display	4021

```
[In] int(x^5*(d+e*x^r)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/36*(-1026*x^6*(x^r)^2*a*d*e^2*r^4-7344*x^6*(x^r)^2*a*d*e^2*r^3-11664*x^6
*(x^r)^3*ln(c*x^n)*b*e^3*r-23328*e^2*d*b*ln(c*x^n)*(x^r)^2*x^6+4*x^6*(x^r)^
3*b*e^3*n*r^4+72*x^6*(x^r)^3*b*e^3*n*r^3+468*x^6*(x^r)^3*b*e^3*n*r^2+1296*x
^6*(x^r)^3*b*e^3*n*r-7776*a*d^3*x^6-30456*x^6*x^r*a*d^2*e*r^2-42768*x^6*x^r
*a*d^2*e*r+3888*x^6*x^r*b*d^2*e*n-23328*e*d^2*b*ln(c*x^n)*x^r*x^6-12*x^6*(x
^r)^3*ln(c*x^n)*b*e^3*r^5-108*x^6*x^r*a*d^2*e*r^5-1728*x^6*x^r*a*d^2*e*r^4-
10476*x^6*x^r*a*d^2*e*r^3-7776*e^3*b*ln(c*x^n)*(x^r)^3*x^6-12528*x^6*ln(c*x
^n)*b*d^3*r^2-15552*x^6*ln(c*x^n)*b*d^3*r-12*x^6*(x^r)^3*a*e^3*r^5-240*x^6*
(x^r)^3*a*e^3*r^4-1836*x^6*(x^r)^3*a*e^3*r^3-6696*x^6*(x^r)^3*a*e^3*r^2-116
```

```
64*x^6*(x^r)^3*a*e^3*r+1296*x^6*(x^r)^3*b*e^3*n-23328*x^6*x^r*a*d^2*e-23328
*x^6*(x^r)^2*a*d*e^2+x^6*b*d^3*n*r^6+22*x^6*b*d^3*n*r^5+193*x^6*b*d^3*n*r^4
+864*x^6*b*d^3*n*r^3+2088*x^6*b*d^3*n*r^2+2592*x^6*b*d^3*n*r-6*x^6*ln(c*x^n
)*b*d^3*r^6-132*x^6*ln(c*x^n)*b*d^3*r^5-240*x^6*(x^r)^3*ln(c*x^n)*b*e^3*r^4
-1836*x^6*(x^r)^3*ln(c*x^n)*b*e^3*r^3-6696*x^6*(x^r)^3*ln(c*x^n)*b*e^3*r^2-
38880*x^6*(x^r)^2*a*d*e^2*r+3888*x^6*(x^r)^2*b*d*e^2*n-24624*x^6*(x^r)^2*a*
d*e^2*r^2-54*x^6*(x^r)^2*a*d*e^2*r^5-7776*x^6*(x^r)^3*a*e^3-6*x^6*a*d^3*r^6
-132*x^6*a*d^3*r^5-1158*x^6*a*d^3*r^4-5184*x^6*a*d^3*r^3-12528*x^6*a*d^3*r^
2-15552*x^6*a*d^3*r-1158*x^6*ln(c*x^n)*b*d^3*r^4-5184*x^6*ln(c*x^n)*b*d^3*r
^3-108*x^6*x^r*ln(c*x^n)*b*d^2*e*r^5-1728*x^6*x^r*ln(c*x^n)*b*d^2*e*r^4-104
76*x^6*x^r*ln(c*x^n)*b*d^2*e*r^3-30456*x^6*x^r*ln(c*x^n)*b*d^2*e*r^2-42768*
x^6*x^r*ln(c*x^n)*b*d^2*e*r+108*x^6*x^r*b*d^2*e*n*r^4+1080*x^6*x^r*b*d^2*e*
n*r^3+3996*x^6*x^r*b*d^2*e*n*r^2+6480*x^6*x^r*b*d^2*e*n*r+27*x^6*(x^r)^2*b*
d*e^2*n*r^4+432*x^6*(x^r)^2*b*d*e^2*n*r^3+2376*x^6*(x^r)^2*b*d*e^2*n*r^2+51
84*x^6*(x^r)^2*b*d*e^2*n*r-54*x^6*(x^r)^2*ln(c*x^n)*b*d*e^2*r^5-1026*x^6*(x
^r)^2*ln(c*x^n)*b*d*e^2*r^4-7344*x^6*(x^r)^2*ln(c*x^n)*b*d*e^2*r^3-24624*x^
6*(x^r)^2*ln(c*x^n)*b*d*e^2*r^2-38880*x^6*(x^r)^2*ln(c*x^n)*b*d*e^2*r+1296*
b*d^3*n*x^6-7776*b*d^3*ln(c*x^n)*x^6)/(2+r)^2/(6+r)^2/(3+r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1011 vs. $2(139) = 278$.

Time = 0.32 (sec) , antiderivative size = 1011, normalized size of antiderivative = 6.88

$$\int x^5(d+ex^r)^3\,(a+b\log{(cx^n)})\;dx$$
$$= \frac{6\,(bd^3r^6+22\,bd^3r^5+193\,bd^3r^4+864\,bd^3r^3+2088\,bd^3r^2+2592\,bd^3r+1296\,bd^3)x^6\log{(c)}+6\,(bd^3nr^6+2}{}$$

```
[In] integrate(x^5*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 1/36*(6*(b*d^3*r^6 + 22*b*d^3*r^5 + 193*b*d^3*r^4 + 864*b*d^3*r^3 + 2088*b*
d^3*r^2 + 2592*b*d^3*r + 1296*b*d^3)*x^6*log(c) + 6*(b*d^3*n*r^6 + 22*b*d^3
*n*r^5 + 193*b*d^3*n*r^4 + 864*b*d^3*n*r^3 + 2088*b*d^3*n*r^2 + 2592*b*d^3*
n*r + 1296*b*d^3*n)*x^6*log(x) - ((b*d^3*n - 6*a*d^3)*r^6 + 22*(b*d^3*n - 6
*a*d^3)*r^5 + 1296*b*d^3*n + 193*(b*d^3*n - 6*a*d^3)*r^4 - 7776*a*d^3 + 864
*(b*d^3*n - 6*a*d^3)*r^3 + 2088*(b*d^3*n - 6*a*d^3)*r^2 + 2592*(b*d^3*n - 6
*a*d^3)*r)*x^6 + 4*(3*(b*e^3*r^5 + 20*b*e^3*r^4 + 153*b*e^3*r^3 + 558*b*e^3
*r^2 + 972*b*e^3*r + 648*b*e^3)*x^6*log(c) + 3*(b*e^3*n*r^5 + 20*b*e^3*n*r^
4 + 153*b*e^3*n*r^3 + 558*b*e^3*n*r^2 + 972*b*e^3*n*r + 648*b*e^3*n)*x^6*lo
g(x) + (3*a*e^3*r^5 - 324*b*e^3*n - (b*e^3*n - 60*a*e^3)*r^4 + 1944*a*e^3 -
 9*(2*b*e^3*n - 51*a*e^3)*r^3 - 9*(13*b*e^3*n - 186*a*e^3)*r^2 - 324*(b*e^3
*n - 9*a*e^3)*r)*x^6)*x^(3*r) + 27*(2*(b*d*e^2*r^5 + 19*b*d*e^2*r^4 + 136*b
*d*e^2*r^3 + 456*b*d*e^2*r^2 + 720*b*d*e^2*r + 432*b*d*e^2)*x^6*log(c) + 2*
```

```
(b*d*e^2*n*r^5 + 19*b*d*e^2*n*r^4 + 136*b*d*e^2*n*r^3 + 456*b*d*e^2*n*r^2 +
 720*b*d*e^2*n*r + 432*b*d*e^2*n)*x^6*log(x) + (2*a*d*e^2*r^5 - 144*b*d*e^2
*n - (b*d*e^2*n - 38*a*d*e^2)*r^4 + 864*a*d*e^2 - 16*(b*d*e^2*n - 17*a*d*e^
2)*r^3 - 8*(11*b*d*e^2*n - 114*a*d*e^2)*r^2 - 96*(2*b*d*e^2*n - 15*a*d*e^2)
*r)*x^6)*x^(2*r) + 108*((b*d^2*e*r^5 + 16*b*d^2*e*r^4 + 97*b*d^2*e*r^3 + 28
2*b*d^2*e*r^2 + 396*b*d^2*e*r + 216*b*d^2*e)*x^6*log(c) + (b*d^2*e*n*r^5 +
16*b*d^2*e*n*r^4 + 97*b*d^2*e*n*r^3 + 282*b*d^2*e*n*r^2 + 396*b*d^2*e*n*r +
 216*b*d^2*e*n)*x^6*log(x) + (a*d^2*e*r^5 - 36*b*d^2*e*n - (b*d^2*e*n - 16*
a*d^2*e)*r^4 + 216*a*d^2*e - (10*b*d^2*e*n - 97*a*d^2*e)*r^3 - (37*b*d^2*e*
n - 282*a*d^2*e)*r^2 - 12*(5*b*d^2*e*n - 33*a*d^2*e)*r)*x^6)*x^r)/(r^6 + 22
*r^5 + 193*r^4 + 864*r^3 + 2088*r^2 + 2592*r + 1296)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4100 vs. 2(143) = 286.

Time = 92.42 (sec) , antiderivative size = 4100, normalized size of antiderivative = 27.89

$$\int x^5(d+ex^r)^3\,(a+b\log{(cx^n)})\;dx = \text{Too large to display}$$

```
[In] integrate(x**5*(d+e*x**r)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((a*d**3*x**6/6 + 3*a*d**2*e*log(c*x**n)/n - a*d*e**2/(2*x**6) - a
*e**3/(12*x**12) - b*d**3*n*x**6/36 + b*d**3*x**6*log(c*x**n)/6 + 3*b*d**2*
e*log(c*x**n)**2/(2*n) - b*d*e**2*n/(12*x**6) - b*d*e**2*log(c*x**n)/(2*x**
6) - b*e**3*n/(144*x**12) - b*e**3*log(c*x**n)/(12*x**12), Eq(r, -6)), (a*d
**3*x**6/6 + a*d**2*e*x**3 + 3*a*d*e**2*log(c*x**n)/n - a*e**3/(3*x**3) - b
*d**3*n*x**6/36 + b*d**3*x**6*log(c*x**n)/6 - b*d**2*e*n*x**3/3 + b*d**2*e*
x**3*log(c*x**n) + 3*b*d*e**2*log(c*x**n)**2/(2*n) - b*e**3*n/(9*x**3) - b*
e**3*log(c*x**n)/(3*x**3), Eq(r, -3)), (a*d**3*x**6/6 + 3*a*d**2*e*x**4/4 +
 3*a*d*e**2*x**2/2 + a*e**3*log(c*x**n)/n - b*d**3*n*x**6/36 + b*d**3*x**6*
log(c*x**n)/6 - 3*b*d**2*e*n*x**4/16 + 3*b*d**2*e*x**4*log(c*x**n)/4 - 3*b*
d*e**2*n*x**2/4 + 3*b*d*e**2*x**2*log(c*x**n)/2 + b*e**3*log(c*x**n)**2/(2*
n), Eq(r, -2)), (6*a*d**3*r**6*x**6/(36*r**6 + 792*r**5 + 6948*r**4 + 31104
*r**3 + 75168*r**2 + 93312*r + 46656) + 132*a*d**3*r**5*x**6/(36*r**6 + 792
*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 1158*a*d**
3*r**4*x**6/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 933
12*r + 46656) + 5184*a*d**3*r**3*x**6/(36*r**6 + 792*r**5 + 6948*r**4 + 311
04*r**3 + 75168*r**2 + 93312*r + 46656) + 12528*a*d**3*r**2*x**6/(36*r**6 +
 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 15552*
a*d**3*r*x**6/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 9
3312*r + 46656) + 7776*a*d**3*x**6/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*
r**3 + 75168*r**2 + 93312*r + 46656) + 108*a*d**2*e*r**5*x**6*x**r/(36*r**6
 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 1728
*a*d**2*e*r**4*x**6*x**r/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 751
```

68*r**2 + 93312*r + 46656) + 10476*a*d**2*e*r**3*x**6*x**r/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 30456*a*d**2*e*r**2*x**6*x**r/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 42768*a*d**2*e*r*x**6*x**r/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 23328*a*d**2*e*x**6*x**r/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 54*a*d*e**2*r**5*x**6*x**(2*r)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 1026*a*d*e**2*r**4*x**6*x**(2*r)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 7344*a*d*e**2*r**3*x**6*x**(2*r)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 24624*a*d*e**2*r**2*x**6*x**(2*r)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 38880*a*d*e**2*r*x**6*x**(2*r)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 23328*a*d*e**2*x**6*x**(2*r)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 12*a*e**3*r**5*x**6*x**(3*r)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 240*a*e**3*r**4*x**6*x**(3*r)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 1836*a*e**3*r**3*x**6*x**(3*r)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 6696*a*e**3*r**2*x**6*x**(3*r)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 11664*a*e**3*r*x**6*x**(3*r)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 7776*a*e**3*x**6*x**(3*r)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) - b*d**3*n*r**6*x**6/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) - 22*b*d**3*n*r**5*x**6/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) - 193*b*d**3*n*r**4*x**6/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) - 864*b*d**3*n*r**3*x**6/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) - 2088*b*d**3*n*r**2*x**6/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) - 2592*b*d**3*n*r*x**6/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) - 1296*b*d**3*n*x**6/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 6*b*d**3*r**6*x**6*log(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 132*b*d**3*r**5*x**6*log(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 1158*b*d**3*r**4*x**6*log(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 5184*b*d**3*r**3*x**6*log(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 12528*b*d**3*r**2*x**6*log(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 15552*b*d**3*r*x**6*log(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 7776*b*d**3*x**6*log(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) - 108*b*d**2*e*n*r**4*x**6*x**r/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) - 1080*b*d*

*2*e*n*r**3*x**6*x**r/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*
r**2 + 93312*r + 46656) - 3996*b*d**2*e*n*r**2*x**6*x**r/(36*r**6 + 792*r**
5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) - 6480*b*d**2*e*
n*r*x**6*x**r/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 9
3312*r + 46656) - 3888*b*d**2*e*n*x**6*x**r/(36*r**6 + 792*r**5 + 6948*r**4
 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 108*b*d**2*e*r**5*x**6*x**r
*log(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93
312*r + 46656) + 1728*b*d**2*e*r**4*x**6*x**r*log(c*x**n)/(36*r**6 + 792*r*
*5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 10476*b*d**2*
e*r**3*x**6*x**r*log(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 +
 75168*r**2 + 93312*r + 46656) + 30456*b*d**2*e*r**2*x**6*x**r*log(c*x**n)/
(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656
) + 42768*b*d**2*e*r*x**6*x**r*log(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4
+ 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 23328*b*d**2*e*x**6*x**r*log
(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*
r + 46656) - 27*b*d*e**2*n*r**4*x**6*x**(2*r)/(36*r**6 + 792*r**5 + 6948*r*
*4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) - 432*b*d*e**2*n*r**3*x**6*
x**(2*r)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*
r + 46656) - 2376*b*d*e**2*n*r**2*x**6*x**(2*r)/(36*r**6 + 792*r**5 + 6948*
r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) - 5184*b*d*e**2*n*r*x**6*
x**(2*r)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*
r + 46656) - 3888*b*d*e**2*n*x**6*x**(2*r)/(36*r**6 + 792*r**5 + 6948*r**4
+ 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 54*b*d*e**2*r**5*x**6*x**(2*
r)*log(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 +
93312*r + 46656) + 1026*b*d*e**2*r**4*x**6*x**(2*r)*log(c*x**n)/(36*r**6 +
792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 7344*b*
d*e**2*r**3*x**6*x**(2*r)*log(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 311
04*r**3 + 75168*r**2 + 93312*r + 46656) + 24624*b*d*e**2*r**2*x**6*x**(2*r)
*log(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93
312*r + 46656) + 38880*b*d*e**2*r*x**6*x**(2*r)*log(c*x**n)/(36*r**6 + 792*
r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 23328*b*d*e
2*x6*x**(2*r)*log(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3
+ 75168*r**2 + 93312*r + 46656) - 4*b*e**3*n*r**4*x**6*x**(3*r)/(36*r**6 +
792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) - 72*b*e*
*3*n*r**3*x**6*x**(3*r)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 7516
8*r**2 + 93312*r + 46656) - 468*b*e**3*n*r**2*x**6*x**(3*r)/(36*r**6 + 792*
r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) - 1296*b*e**3
*n*r*x**6*x**(3*r)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**
2 + 93312*r + 46656) - 1296*b*e**3*n*x**6*x**(3*r)/(36*r**6 + 792*r**5 + 69
48*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 12*b*e**3*r**5*x**6*
x**(3*r)*log(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r
2 + 93312*r + 46656) + 240*b*e3*r**4*x**6*x**(3*r)*log(c*x**n)/(36*r**6
 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 1836
*b*e**3*r**3*x**6*x**(3*r)*log(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 31
104*r**3 + 75168*r**2 + 93312*r + 46656) + 6696*b*e**3*r**2*x**6*x**(3*r)*l

```
og(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 9331
2*r + 46656) + 11664*b*e**3*r*x**6*x**(3*r)*log(c*x**n)/(36*r**6 + 792*r**5
 + 6948*r**4 + 31104*r**3 + 75168*r**2 + 93312*r + 46656) + 7776*b*e**3*x**
6*x**(3*r)*log(c*x**n)/(36*r**6 + 792*r**5 + 6948*r**4 + 31104*r**3 + 75168
*r**2 + 93312*r + 46656), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.48

$$\int x^5(d+ex^r)^3\,(a+b\log{(cx^n)})\;dx = -\frac{1}{36}\,bd^3nx^6+\frac{1}{6}\,bd^3x^6\log{(cx^n)}+\frac{1}{6}\,ad^3x^6+\frac{be^3x^{3\,r+6}\log{(cx^n)}}{3\,(r+2)}+\frac{3\,bde^2x^{2\,r+6}\log{(cx^n)}}{2\,(r+3)}+\frac{3\,bd^2ex^{r+6}\log{(cx^n)}}{r+6}-\frac{be^3nx^{3\,r+6}}{9\,(r+2)^2}+\frac{ae^3x^{3\,r+6}}{3\,(r+2)}-\frac{3\,bde^2nx^{2\,r+6}}{4\,(r+3)^2}+\frac{3\,ade^2x^{2\,r+6}}{2\,(r+3)}-\frac{3\,bd^2enx^{r+6}}{(r+6)^2}+\frac{3\,ad^2ex^{r+6}}{r+6}$$

```
[In] integrate(x^5*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/36*b*d^3*n*x^6 + 1/6*b*d^3*x^6*log(c*x^n) + 1/6*a*d^3*x^6 + 1/3*b*e^3*x^
(3*r + 6)*log(c*x^n)/(r + 2) + 3/2*b*d*e^2*x^(2*r + 6)*log(c*x^n)/(r + 3) +
 3*b*d^2*e*x^(r + 6)*log(c*x^n)/(r + 6) - 1/9*b*e^3*n*x^(3*r + 6)/(r + 2)^2
 + 1/3*a*e^3*x^(3*r + 6)/(r + 2) - 3/4*b*d*e^2*n*x^(2*r + 6)/(r + 3)^2 + 3/
2*a*d*e^2*x^(2*r + 6)/(r + 3) - 3*b*d^2*e*n*x^(r + 6)/(r + 6)^2 + 3*a*d^2*e
*x^(r + 6)/(r + 6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1609 vs. $2(139) = 278$.

Time = 0.37 (sec) , antiderivative size = 1609, normalized size of antiderivative = 10.95

$$\int x^5(d+ex^r)^3\,(a+b\log{(cx^n)})\;dx=\text{Too large to display}$$

```
[In] integrate(x^5*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/36*(12*b*e^3*n*r^5*x^6*x^(3*r)*log(x) + 54*b*d*e^2*n*r^5*x^6*x^(2*r)*log(
x) + 108*b*d^2*e*n*r^5*x^6*x^r*log(x) + 6*b*d^3*n*r^6*x^6*log(x) - b*d^3*n*
```

r^6*x^6 + 12*b*e^3*r^5*x^6*x^(3*r)*log(c) + 54*b*d*e^2*r^5*x^6*x^(2*r)*log(
c) + 108*b*d^2*e*r^5*x^6*x^r*log(c) + 6*b*d^3*r^6*x^6*log(c) + 240*b*e^3*n*
r^4*x^6*x^(3*r)*log(x) + 1026*b*d*e^2*n*r^4*x^6*x^(2*r)*log(x) + 1728*b*d^2
*e*n*r^4*x^6*x^r*log(x) + 132*b*d^3*n*r^5*x^6*log(x) - 4*b*e^3*n*r^4*x^6*x^
(3*r) + 12*a*e^3*r^5*x^6*x^(3*r) - 27*b*d*e^2*n*r^4*x^6*x^(2*r) + 54*a*d*e^
2*r^5*x^6*x^(2*r) - 108*b*d^2*e*n*r^4*x^6*x^r + 108*a*d^2*e*r^5*x^6*x^r - 2
2*b*d^3*n*r^5*x^6 + 6*a*d^3*r^6*x^6 + 240*b*e^3*r^4*x^6*x^(3*r)*log(c) + 10
26*b*d*e^2*r^4*x^6*x^(2*r)*log(c) + 1728*b*d^2*e*r^4*x^6*x^r*log(c) + 132*b
*d^3*r^5*x^6*log(c) + 1836*b*e^3*n*r^3*x^6*x^(3*r)*log(x) + 7344*b*d*e^2*n*
r^3*x^6*x^(2*r)*log(x) + 10476*b*d^2*e*n*r^3*x^6*x^r*log(x) + 1158*b*d^3*n*
r^4*x^6*log(x) - 72*b*e^3*n*r^3*x^6*x^(3*r) + 240*a*e^3*r^4*x^6*x^(3*r) - 4
32*b*d*e^2*n*r^3*x^6*x^(2*r) + 1026*a*d*e^2*r^4*x^6*x^(2*r) - 1080*b*d^2*e*
n*r^3*x^6*x^r + 1728*a*d^2*e*r^4*x^6*x^r - 193*b*d^3*n*r^4*x^6 + 132*a*d^3*
r^5*x^6 + 1836*b*e^3*r^3*x^6*x^(3*r)*log(c) + 7344*b*d*e^2*r^3*x^6*x^(2*r)*
log(c) + 10476*b*d^2*e*r^3*x^6*x^r*log(c) + 1158*b*d^3*r^4*x^6*log(c) + 669
6*b*e^3*n*r^2*x^6*x^(3*r)*log(x) + 24624*b*d*e^2*n*r^2*x^6*x^(2*r)*log(x) +
 30456*b*d^2*e*n*r^2*x^6*x^r*log(x) + 5184*b*d^3*n*r^3*x^6*log(x) - 468*b*e
^3*n*r^2*x^6*x^(3*r) + 1836*a*e^3*r^3*x^6*x^(3*r) - 2376*b*d*e^2*n*r^2*x^6*
x^(2*r) + 7344*a*d*e^2*r^3*x^6*x^(2*r) - 3996*b*d^2*e*n*r^2*x^6*x^r + 10476
*a*d^2*e*r^3*x^6*x^r - 864*b*d^3*n*r^3*x^6 + 1158*a*d^3*r^4*x^6 + 6696*b*e^
3*r^2*x^6*x^(3*r)*log(c) + 24624*b*d*e^2*r^2*x^6*x^(2*r)*log(c) + 30456*b*d
^2*e*r^2*x^6*x^r*log(c) + 5184*b*d^3*r^3*x^6*log(c) + 11664*b*e^3*n*r*x^6*x
^(3*r)*log(x) + 38880*b*d*e^2*n*r*x^6*x^(2*r)*log(x) + 42768*b*d^2*e*n*r*x^
6*x^r*log(x) + 12528*b*d^3*n*r^2*x^6*log(x) - 1296*b*e^3*n*r*x^6*x^(3*r) +
6696*a*e^3*r^2*x^6*x^(3*r) - 5184*b*d*e^2*n*r*x^6*x^(2*r) + 24624*a*d*e^2*r
^2*x^6*x^(2*r) - 6480*b*d^2*e*n*r*x^6*x^r + 30456*a*d^2*e*r^2*x^6*x^r - 208
8*b*d^3*n*r^2*x^6 + 5184*a*d^3*r^3*x^6 + 11664*b*e^3*r*x^6*x^(3*r)*log(c) +
 38880*b*d*e^2*r*x^6*x^(2*r)*log(c) + 42768*b*d^2*e*r*x^6*x^r*log(c) + 1252
8*b*d^3*r^2*x^6*log(c) + 7776*b*e^3*n*x^6*x^(3*r)*log(x) + 23328*b*d*e^2*n*
x^6*x^(2*r)*log(x) + 23328*b*d^2*e*n*x^6*x^r*log(x) + 15552*b*d^3*n*r*x^6*l
og(x) - 1296*b*e^3*n*x^6*x^(3*r) + 11664*a*e^3*r*x^6*x^(3*r) - 3888*b*d*e^2
*n*x^6*x^(2*r) + 38880*a*d*e^2*r*x^6*x^(2*r) - 3888*b*d^2*e*n*x^6*x^r + 427
68*a*d^2*e*r*x^6*x^r - 2592*b*d^3*n*r*x^6 + 12528*a*d^3*r^2*x^6 + 7776*b*e^
3*x^6*x^(3*r)*log(c) + 23328*b*d*e^2*x^6*x^(2*r)*log(c) + 23328*b*d^2*e*x^6
*x^r*log(c) + 15552*b*d^3*r*x^6*log(c) + 7776*b*d^3*n*x^6*log(x) + 7776*a*e
^3*x^6*x^(3*r) + 23328*a*d*e^2*x^6*x^(2*r) + 23328*a*d^2*e*x^6*x^r - 1296*b
*d^3*n*x^6 + 15552*a*d^3*r*x^6 + 7776*b*d^3*x^6*log(c) + 7776*a*d^3*x^6)/(r
^6 + 22*r^5 + 193*r^4 + 864*r^3 + 2088*r^2 + 2592*r + 1296)

Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex^r)^3\,(a+b\log{(cx^n)})\;dx=\int x^5\,{\left(d+e\,x^r\right)}^3\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

```
[In] int(x^5*(d + e*x^r)^3*(a + b*log(c*x^n)),x)
[Out] int(x^5*(d + e*x^r)^3*(a + b*log(c*x^n)), x)
```

3.393 $\int x^3(d+ex^r)^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 149

$$\int x^3(d+ex^r)^3(a+b\log(cx^n))\,dx = -\frac{1}{16}bd^3nx^4-\frac{3bde^2nx^{2(2+r)}}{4(2+r)^2}-\frac{3bd^2enx^{4+r}}{(4+r)^2}-\frac{be^3nx^{4+3r}}{(4+3r)^2}+\frac{1}{4}\left(d^3x^4+\frac{6de^2x^{2(2+r)}}{2+r}+\frac{12d^2ex^{4+r}}{4+r}+\frac{4e^3x^{4+3r}}{4+3r}\right)(a+b\log(cx^n))$$

```
[Out] -1/16*b*d^3*n*x^4-3/4*b*d*e^2*n*x^(4+2*r)/(2+r)^2-3*b*d^2*e*n*x^(4+r)/(4+r)
^2-b*e^3*n*x^(4+3*r)/(4+3*r)^2+1/4*(d^3*x^4+6*d*e^2*x^(4+2*r)/(2+r)+12*d^2*
e*x^(4+r)/(4+r)+4*e^3*x^(4+3*r)/(4+3*r))*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2371, 12, 14}

$$\int x^3(d+ex^r)^3(a+b\log(cx^n))\,dx = \frac{1}{4}\left(d^3x^4+\frac{12d^2ex^{r+4}}{r+4}+\frac{6de^2x^{2(r+2)}}{r+2}+\frac{4e^3x^{3r+4}}{3r+4}\right)(a+b\log(cx^n))-\frac{1}{16}bd^3nx^4-\frac{3bd^2enx^{r+4}}{(r+4)^2}-\frac{3bde^2nx^{2(r+2)}}{4(r+2)^2}-\frac{be^3nx^{3r+4}}{(3r+4)^2}$$

```
[In] Int[x^3*(d + e*x^r)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/16*(b*d^3*n*x^4) - (3*b*d*e^2*n*x^(2*(2 + r)))/(4*(2 + r)^2) - (3*b*d^2*
e*n*x^(4 + r))/(4 + r)^2 - (b*e^3*n*x^(4 + 3*r))/(4 + 3*r)^2 + ((d^3*x^4 +
(6*d*e^2*x^(2*(2 + r)))/(2 + r) + (12*d^2*e*x^(4 + r))/(4 + r) + (4*e^3*x^(
4 + 3*r))/(4 + 3*r))*(a + b*Log[c*x^n]))/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\left(d^3x^4+\frac{6de^2x^{2(2+r)}}{2+r}+\frac{12d^2ex^{4+r}}{4+r}+\frac{4e^3x^{4+3r}}{4+3r}\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int \frac{1}{4}x^3\left(d^3+\frac{12d^2ex^r}{4+r}+\frac{6de^2x^{2r}}{2+r}+\frac{4e^3x^{3r}}{4+3r}\right)\,dx \\
&= \frac{1}{4}\left(d^3x^4+\frac{6de^2x^{2(2+r)}}{2+r}+\frac{12d^2ex^{4+r}}{4+r}+\frac{4e^3x^{4+3r}}{4+3r}\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{4}(bn)\int x^3\left(d^3+\frac{12d^2ex^r}{4+r}+\frac{6de^2x^{2r}}{2+r}+\frac{4e^3x^{3r}}{4+3r}\right)\,dx \\
&= \frac{1}{4}\left(d^3x^4+\frac{6de^2x^{2(2+r)}}{2+r}+\frac{12d^2ex^{4+r}}{4+r}+\frac{4e^3x^{4+3r}}{4+3r}\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{4}(bn)\int \left(d^3x^3+\frac{4e^3x^{3(1+r)}}{4+3r}+\frac{12d^2ex^{3+r}}{4+r}+\frac{6de^2x^{3+2r}}{2+r}\right)\,dx
\end{aligned}$$

$$= -\frac{1}{16}bd^3nx^4 - \frac{3bde^2nx^{2(2+r)}}{4(2+r)^2} - \frac{3bd^2enx^{4+r}}{(4+r)^2} - \frac{be^3nx^{4+3r}}{(4+3r)^2} + \frac{1}{4}\left(d^3x^4 + \frac{6de^2x^{2(2+r)}}{2+r} + \frac{12d^2ex^{4+r}}{4+r} + \frac{4e^3x^{4+3r}}{4+3r}\right)(a + b\log(cx^n))$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.19

$$\int x^3(d+ex^r)^3(a+b\log(cx^n))\,dx = \frac{1}{16}x^4\left(bn\left(-d^3 - \frac{48d^2ex^r}{(4+r)^2} - \frac{12de^2x^{2r}}{(2+r)^2} - \frac{16e^3x^{3r}}{(4+3r)^2}\right) + 4a\left(d^3 + \frac{12d^2ex^r}{4+r} + \frac{6de^2x^{2r}}{2+r} + \frac{4e^3x^{3r}}{4+3r}\right) + 4b\left(d^3 + \frac{12d^2ex^r}{4+r} + \frac{6de^2x^{2r}}{2+r} + \frac{4e^3x^{3r}}{4+3r}\right)\log(cx^n)\right)$$

```
[In] Integrate[x^3*(d + e*x^r)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^4*(b*n*(-d^3 - (48*d^2*e*x^r)/(4 + r)^2 - (12*d*e^2*x^(2*r))/(2 + r)^2 -
 (16*e^3*x^(3*r))/(4 + 3*r)^2) + 4*a*(d^3 + (12*d^2*e*x^r)/(4 + r) + (6*d*e
^2*x^(2*r))/(2 + r) + (4*e^3*x^(3*r))/(4 + 3*r)) + 4*b*(d^3 + (12*d^2*e*x^r
)/(4 + r) + (6*d*e^2*x^(2*r))/(2 + r) + (4*e^3*x^(3*r))/(4 + 3*r))*Log[c*x^
n]))/16
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1261 vs. 2(143) = 286.

Time = 15.46 (sec) , antiderivative size = 1262, normalized size of antiderivative = 8.47

method	result	size
parallelrisch	Expression too large to display	1262
risch	Expression too large to display	4027

```
[In] int(x^3*(d+e*x^r)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(-4096*a*d^3*x^4-12288*e^2*d*b*ln(c*x^n)*(x^r)^2*x^4-12288*e*d^2*b*ln
(c*x^n)*x^r*x^4-432*x^4*x^r*a*d^2*e*r^5-4608*x^4*x^r*a*d^2*e*r^4-18624*x^4*
x^r*a*d^2*e*r^3-36096*x^4*x^r*a*d^2*e*r^2-33792*x^4*x^r*a*d^2*e*r+3072*x^4*
x^r*b*d^2*e*n-48*x^4*(x^r)^3*ln(c*x^n)*b*e^3*r^5-640*x^4*(x^r)^3*ln(c*x^n)*
b*e^3*r^4-3264*x^4*(x^r)^3*ln(c*x^n)*b*e^3*r^3-7936*x^4*(x^r)^3*ln(c*x^n)*b
*e^3*r^2-9216*x^4*(x^r)^3*ln(c*x^n)*b*e^3*r-216*x^4*(x^r)^2*a*d*e^2*r^5-273
6*x^4*(x^r)^2*a*d*e^2*r^4-13056*x^4*(x^r)^2*a*d*e^2*r^3-29184*x^4*(x^r)^2*a
*d*e^2*r^2-30720*x^4*(x^r)^2*a*d*e^2*r+3072*x^4*(x^r)^2*b*d*e^2*n-4096*x^4*
ln(c*x^n)*b*d^3-12288*x^4*x^r*a*d^2*e+9*x^4*b*d^3*n*r^6+132*x^4*b*d^3*n*r^5
```

```
+772*x^4*b*d^3*n*r^4+2304*x^4*b*d^3*n*r^3+3712*x^4*b*d^3*n*r^2+3072*x^4*b*d
^3*n*r-12288*x^4*(x^r)^2*a*d*e^2-48*x^4*(x^r)^3*a*e^3*r^5-640*x^4*(x^r)^3*a
*e^3*r^4-3264*x^4*(x^r)^3*a*e^3*r^3-7936*x^4*(x^r)^3*a*e^3*r^2+432*x^4*x^r*
b*d^2*e*n*r^4+2880*x^4*x^r*b*d^2*e*n*r^3+7104*x^4*x^r*b*d^2*e*n*r^2+7680*x^
4*x^r*b*d^2*e*n*r-216*x^4*(x^r)^2*ln(c*x^n)*b*d*e^2*r^5-2736*x^4*(x^r)^2*ln
(c*x^n)*b*d*e^2*r^4-13056*x^4*(x^r)^2*ln(c*x^n)*b*d*e^2*r^3-29184*x^4*(x^r)
^2*ln(c*x^n)*b*d*e^2*r^2-30720*x^4*(x^r)^2*ln(c*x^n)*b*d*e^2*r-432*x^4*x^r*
ln(c*x^n)*b*d^2*e*r^5-4608*x^4*x^r*ln(c*x^n)*b*d^2*e*r^4-18624*x^4*x^r*ln(c
*x^n)*b*d^2*e*r^3-36096*x^4*x^r*ln(c*x^n)*b*d^2*e*r^2-33792*x^4*x^r*ln(c*x^
n)*b*d^2*e*r+108*x^4*(x^r)^2*b*d*e^2*n*r^4+1152*x^4*(x^r)^2*b*d*e^2*n*r^3+4
224*x^4*(x^r)^2*b*d*e^2*n*r^2+6144*x^4*(x^r)^2*b*d*e^2*n*r-36*x^4*a*d^3*r^6
-528*x^4*a*d^3*r^5-3088*x^4*a*d^3*r^4-9216*x^4*a*d^3*r^3-14848*x^4*a*d^3*r^
2-12288*x^4*a*d^3*r-4096*x^4*(x^r)^3*a*e^3-9216*x^4*(x^r)^3*a*e^3*r+1024*x^
4*(x^r)^3*b*e^3*n-4096*e^3*b*ln(c*x^n)*(x^r)^3*x^4-36*x^4*ln(c*x^n)*b*d^3*r
^6-528*x^4*ln(c*x^n)*b*d^3*r^5-3088*x^4*ln(c*x^n)*b*d^3*r^4-9216*x^4*ln(c*x
^n)*b*d^3*r^3-14848*x^4*ln(c*x^n)*b*d^3*r^2-12288*x^4*ln(c*x^n)*b*d^3*r+16*
x^4*(x^r)^3*b*e^3*n*r^4+192*x^4*(x^r)^3*b*e^3*n*r^3+832*x^4*(x^r)^3*b*e^3*n
*r^2+1536*x^4*(x^r)^3*b*e^3*n*r+1024*b*d^3*n*x^4)/(9*r^2+24*r+16)/(r^2+4*r+
4)/(4+r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1022 vs. $2(143) = 286$.

Time = 0.34 (sec) , antiderivative size = 1022, normalized size of antiderivative = 6.86

$$\int x^3(d+ex^r)^3\left(a+b\log\left(cx^n\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*(9*b*d^3*r^6 + 132*b*d^3*r^5 + 772*b*d^3*r^4 + 2304*b*d^3*r^3 + 371
2*b*d^3*r^2 + 3072*b*d^3*r + 1024*b*d^3)*x^4*log(c) + 4*(9*b*d^3*n*r^6 + 13
2*b*d^3*n*r^5 + 772*b*d^3*n*r^4 + 2304*b*d^3*n*r^3 + 3712*b*d^3*n*r^2 + 307
2*b*d^3*n*r + 1024*b*d^3*n)*x^4*log(x) - (9*(b*d^3*n - 4*a*d^3)*r^6 + 132*(
b*d^3*n - 4*a*d^3)*r^5 + 1024*b*d^3*n + 772*(b*d^3*n - 4*a*d^3)*r^4 - 4096*
a*d^3 + 2304*(b*d^3*n - 4*a*d^3)*r^3 + 3712*(b*d^3*n - 4*a*d^3)*r^2 + 3072*
(b*d^3*n - 4*a*d^3)*r)*x^4 + 16*((3*b*e^3*r^5 + 40*b*e^3*r^4 + 204*b*e^3*r^
3 + 496*b*e^3*r^2 + 576*b*e^3*r + 256*b*e^3)*x^4*log(c) + (3*b*e^3*n*r^5 +
40*b*e^3*n*r^4 + 204*b*e^3*n*r^3 + 496*b*e^3*n*r^2 + 576*b*e^3*n*r + 256*b*
e^3*n)*x^4*log(x) + (3*a*e^3*r^5 - 64*b*e^3*n - (b*e^3*n - 40*a*e^3)*r^4 +
256*a*e^3 - 12*(b*e^3*n - 17*a*e^3)*r^3 - 4*(13*b*e^3*n - 124*a*e^3)*r^2 -
96*(b*e^3*n - 6*a*e^3)*r)*x^4)*x^(3*r) + 12*(2*(9*b*d*e^2*r^5 + 114*b*d*e^2
*r^4 + 544*b*d*e^2*r^3 + 1216*b*d*e^2*r^2 + 1280*b*d*e^2*r + 512*b*d*e^2)*x
^4*log(c) + 2*(9*b*d*e^2*n*r^5 + 114*b*d*e^2*n*r^4 + 544*b*d*e^2*n*r^3 + 12
16*b*d*e^2*n*r^2 + 1280*b*d*e^2*n*r + 512*b*d*e^2*n)*x^4*log(x) + (18*a*d*e
```

```
^2*r^5 - 256*b*d*e^2*n - 3*(3*b*d*e^2*n - 76*a*d*e^2)*r^4 + 1024*a*d*e^2 -
32*(3*b*d*e^2*n - 34*a*d*e^2)*r^3 - 32*(11*b*d*e^2*n - 76*a*d*e^2)*r^2 - 51
2*(b*d*e^2*n - 5*a*d*e^2)*r)*x^4)*x^(2*r) + 48*((9*b*d^2*e*r^5 + 96*b*d^2*e
*r^4 + 388*b*d^2*e*r^3 + 752*b*d^2*e*r^2 + 704*b*d^2*e*r + 256*b*d^2*e)*x^4
*log(c) + (9*b*d^2*e*n*r^5 + 96*b*d^2*e*n*r^4 + 388*b*d^2*e*n*r^3 + 752*b*d
^2*e*n*r^2 + 704*b*d^2*e*n*r + 256*b*d^2*e*n)*x^4*log(x) + (9*a*d^2*e*r^5 -
 64*b*d^2*e*n - 3*(3*b*d^2*e*n - 32*a*d^2*e)*r^4 + 256*a*d^2*e - 4*(15*b*d^
2*e*n - 97*a*d^2*e)*r^3 - 4*(37*b*d^2*e*n - 188*a*d^2*e)*r^2 - 32*(5*b*d^2*
e*n - 22*a*d^2*e)*r)*x^4)*x^r)/(9*r^6 + 132*r^5 + 772*r^4 + 2304*r^3 + 3712
*r^2 + 3072*r + 1024)
```

Sympy [F(-1)]

Timed out.

$$\int x^3(d+ex^r)^3\left(a+b\log\left(cx^n\right)\right)\,dx=\text{Timed out}$$

```
[In] integrate(x**3*(d+e*x**r)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.49

$$\begin{aligned}\int x^3(d+ex^r)^3\left(a+b\log\left(cx^n\right)\right)\,dx &= -\frac{1}{16}\,bd^3nx^4+\frac{1}{4}\,bd^3x^4\log\left(cx^n\right)\\&\quad+\frac{1}{4}\,ad^3x^4+\frac{be^3x^{3\,r+4}\log\left(cx^n\right)}{3\,r+4}\\&\quad+\frac{3\,bde^2x^{2\,r+4}\log\left(cx^n\right)}{2\,(r+2)}+\frac{3\,bd^2ex^{r+4}\log\left(cx^n\right)}{r+4}\\&\quad-\frac{be^3nx^{3\,r+4}}{(3\,r+4)^2}+\frac{ae^3x^{3\,r+4}}{3\,r+4}-\frac{3\,bde^2nx^{2\,r+4}}{4\,(r+2)^2}\\&\quad+\frac{3\,ade^2x^{2\,r+4}}{2\,(r+2)}-\frac{3\,bd^2enx^{r+4}}{(r+4)^2}+\frac{3\,ad^2ex^{r+4}}{r+4}\end{aligned}$$

```
[In] integrate(x^3*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/16*b*d^3*n*x^4 + 1/4*b*d^3*x^4*log(c*x^n) + 1/4*a*d^3*x^4 + b*e^3*x^(3*r
 + 4)*log(c*x^n)/(3*r + 4) + 3/2*b*d*e^2*x^(2*r + 4)*log(c*x^n)/(r + 2) + 3
*b*d^2*e*x^(r + 4)*log(c*x^n)/(r + 4) - b*e^3*n*x^(3*r + 4)/(3*r + 4)^2 + a
*e^3*x^(3*r + 4)/(3*r + 4) - 3/4*b*d*e^2*n*x^(2*r + 4)/(r + 2)^2 + 3/2*a*d*
e^2*x^(2*r + 4)/(r + 2) - 3*b*d^2*e*n*x^(r + 4)/(r + 4)^2 + 3*a*d^2*e*x^(r
+ 4)/(r + 4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1611 vs. $2(143) = 286$.

Time = 0.48 (sec) , antiderivative size = 1611, normalized size of antiderivative = 10.81

$$\int x^3(d+ex^r)^3\left(a+b\log\left(cx^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate(x^3*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/16*(48*b*e^3*n*r^5*x^4*x^(3*r)*log(x) + 216*b*d*e^2*n*r^5*x^4*x^(2*r)*log
(x) + 432*b*d^2*e*n*r^5*x^4*x^r*log(x) + 36*b*d^3*n*r^6*x^4*log(x) - 9*b*d^
3*n*r^6*x^4 + 48*b*e^3*r^5*x^4*x^(3*r)*log(c) + 216*b*d*e^2*r^5*x^4*x^(2*r)
*log(c) + 432*b*d^2*e*r^5*x^4*x^r*log(c) + 36*b*d^3*r^6*x^4*log(c) + 640*b*
e^3*n*r^4*x^4*x^(3*r)*log(x) + 2736*b*d*e^2*n*r^4*x^4*x^(2*r)*log(x) + 4608
*b*d^2*e*n*r^4*x^4*x^r*log(x) + 528*b*d^3*n*r^5*x^4*log(x) - 16*b*e^3*n*r^4
*x^4*x^(3*r) + 48*a*e^3*r^5*x^4*x^(3*r) - 108*b*d*e^2*n*r^4*x^4*x^(2*r) + 2
16*a*d*e^2*r^5*x^4*x^(2*r) - 432*b*d^2*e*n*r^4*x^4*x^r + 432*a*d^2*e*r^5*x^
4*x^r - 132*b*d^3*n*r^5*x^4 + 36*a*d^3*r^6*x^4 + 640*b*e^3*r^4*x^4*x^(3*r)*
log(c) + 2736*b*d*e^2*r^4*x^4*x^(2*r)*log(c) + 4608*b*d^2*e*r^4*x^4*x^r*log
(c) + 528*b*d^3*r^5*x^4*log(c) + 3264*b*e^3*n*r^3*x^4*x^(3*r)*log(x) + 1305
6*b*d*e^2*n*r^3*x^4*x^(2*r)*log(x) + 18624*b*d^2*e*n*r^3*x^4*x^r*log(x) + 3
088*b*d^3*n*r^4*x^4*log(x) - 192*b*e^3*n*r^3*x^4*x^(3*r) + 640*a*e^3*r^4*x^
4*x^(3*r) - 1152*b*d*e^2*n*r^3*x^4*x^(2*r) + 2736*a*d*e^2*r^4*x^4*x^(2*r) -
 2880*b*d^2*e*n*r^3*x^4*x^r + 4608*a*d^2*e*r^4*x^4*x^r - 772*b*d^3*n*r^4*x^
4 + 528*a*d^3*r^5*x^4 + 3264*b*e^3*r^3*x^4*x^(3*r)*log(c) + 13056*b*d*e^2*r
^3*x^4*x^(2*r)*log(c) + 18624*b*d^2*e*r^3*x^4*x^r*log(c) + 3088*b*d^3*r^4*x
^4*log(c) + 7936*b*e^3*n*r^2*x^4*x^(3*r)*log(x) + 29184*b*d*e^2*n*r^2*x^4*x
^(2*r)*log(x) + 36096*b*d^2*e*n*r^2*x^4*x^r*log(x) + 9216*b*d^3*n*r^3*x^4*l
og(x) - 832*b*e^3*n*r^2*x^4*x^(3*r) + 3264*a*e^3*r^3*x^4*x^(3*r) - 4224*b*d
*e^2*n*r^2*x^4*x^(2*r) + 13056*a*d*e^2*r^3*x^4*x^(2*r) - 7104*b*d^2*e*n*r^2
*x^4*x^r + 18624*a*d^2*e*r^3*x^4*x^r - 2304*b*d^3*n*r^3*x^4 + 3088*a*d^3*r^
4*x^4 + 7936*b*e^3*r^2*x^4*x^(3*r)*log(c) + 29184*b*d*e^2*r^2*x^4*x^(2*r)*l
og(c) + 36096*b*d^2*e*r^2*x^4*x^r*log(c) + 9216*b*d^3*r^3*x^4*log(c) + 9216
*b*e^3*n*r*x^4*x^(3*r)*log(x) + 30720*b*d*e^2*n*r*x^4*x^(2*r)*log(x) + 3379
2*b*d^2*e*n*r*x^4*x^r*log(x) + 14848*b*d^3*n*r^2*x^4*log(x) - 1536*b*e^3*n*
r*x^4*x^(3*r) + 7936*a*e^3*r^2*x^4*x^(3*r) - 6144*b*d*e^2*n*r*x^4*x^(2*r) +
 29184*a*d*e^2*r^2*x^4*x^(2*r) - 7680*b*d^2*e*n*r*x^4*x^r + 36096*a*d^2*e*r
^2*x^4*x^r - 3712*b*d^3*n*r^2*x^4 + 9216*a*d^3*r^3*x^4 + 9216*b*e^3*r*x^4*x
^(3*r)*log(c) + 30720*b*d*e^2*r*x^4*x^(2*r)*log(c) + 33792*b*d^2*e*r*x^4*x^
r*log(c) + 14848*b*d^3*r^2*x^4*log(c) + 4096*b*e^3*n*x^4*x^(3*r)*log(x) + 1
2288*b*d*e^2*n*x^4*x^(2*r)*log(x) + 12288*b*d^2*e*n*x^4*x^r*log(x) + 12288*
b*d^3*n*r*x^4*log(x) - 1024*b*e^3*n*x^4*x^(3*r) + 9216*a*e^3*r*x^4*x^(3*r)
- 3072*b*d*e^2*n*x^4*x^(2*r) + 30720*a*d*e^2*r*x^4*x^(2*r) - 3072*b*d^2*e*n
*x^4*x^r + 33792*a*d^2*e*r*x^4*x^r - 3072*b*d^3*n*r*x^4 + 14848*a*d^3*r^2*x
```

```
^4 + 4096*b*e^3*x^4*x^(3*r)*log(c) + 12288*b*d*e^2*x^4*x^(2*r)*log(c) + 122
88*b*d^2*e*x^4*x^r*log(c) + 12288*b*d^3*r*x^4*log(c) + 4096*b*d^3*n*x^4*log
(x) + 4096*a*e^3*x^4*x^(3*r) + 12288*a*d*e^2*x^4*x^(2*r) + 12288*a*d^2*e*x^
4*x^r - 1024*b*d^3*n*x^4 + 12288*a*d^3*r*x^4 + 4096*b*d^3*x^4*log(c) + 4096
*a*d^3*x^4)/(9*r^6 + 132*r^5 + 772*r^4 + 2304*r^3 + 3712*r^2 + 3072*r + 102
4)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^r)^3\left(a+b\log\left(cx^n\right)\right)\,dx=\int x^3\left(d+e\,x^r\right)^3\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

```
[In] int(x^3*(d + e*x^r)^3*(a + b*log(c*x^n)),x)
[Out] int(x^3*(d + e*x^r)^3*(a + b*log(c*x^n)), x)
```

3.394 $\int x(d+ex^r)^3 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 149

$$\int x(d+ex^r)^3 (a+b\log(cx^n))\,dx = -\frac{1}{4}bd^3nx^2 - \frac{3bde^2nx^{2(1+r)}}{4(1+r)^2} - \frac{3bd^2enx^{2+r}}{(2+r)^2} - \frac{be^3nx^{2+3r}}{(2+3r)^2} + \frac{1}{2}\left(d^3x^2 + \frac{3de^2x^{2(1+r)}}{1+r} + \frac{6d^2ex^{2+r}}{2+r} + \frac{2e^3x^{2+3r}}{2+3r}\right)(a+b\log(cx^n))$$

```
[Out] -1/4*b*d^3*n*x^2-3/4*b*d*e^2*n*x^(2+2*r)/(1+r)^2-3*b*d^2*e*n*x^(2+r)/(2+r)^
2-b*e^3*n*x^(2+3*r)/(2+3*r)^2+1/2*(d^3*x^2+3*d*e^2*x^(2+2*r)/(1+r)+6*d^2*e*
x^(2+r)/(2+r)+2*e^3*x^(2+3*r)/(2+3*r))*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {276, 2371, 12, 14}

$$\int x(d+ex^r)^3 (a+b\log(cx^n))\,dx = \frac{1}{2}\left(d^3x^2 + \frac{6d^2ex^{r+2}}{r+2} + \frac{3de^2x^{2(r+1)}}{r+1} + \frac{2e^3x^{3r+2}}{3r+2}\right)(a+b\log(cx^n)) - \frac{1}{4}bd^3nx^2 - \frac{3bd^2enx^{r+2}}{(r+2)^2} - \frac{3bde^2nx^{2(r+1)}}{4(r+1)^2} - \frac{be^3nx^{3r+2}}{(3r+2)^2}$$

```
[In] Int[x*(d + e*x^r)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/4*(b*d^3*n*x^2) - (3*b*d*e^2*n*x^(2*(1 + r)))/(4*(1 + r)^2) - (3*b*d^2*e
*n*x^(2 + r))/(2 + r)^2 - (b*e^3*n*x^(2 + 3*r))/(2 + 3*r)^2 + ((d^3*x^2 + (
```

```
3*d*e^2*x^(2*(1 + r)))/(1 + r) + (6*d^2*e*x^(2 + r))/(2 + r) + (2*e^3*x^(2
+ 3*r))/(2 + 3*r))*(a + b*Log[c*x^n]))/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\left(d^3x^2+\frac{3de^2x^{2(1+r)}}{1+r}+\frac{6d^2ex^{2+r}}{2+r}+\frac{2e^3x^{2+3r}}{2+3r}\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int \frac{1}{2}x\left(d^3+\frac{6d^2ex^r}{2+r}+\frac{3de^2x^{2r}}{1+r}+\frac{2e^3x^{3r}}{2+3r}\right)dx \\
&= \frac{1}{2}\left(d^3x^2+\frac{3de^2x^{2(1+r)}}{1+r}+\frac{6d^2ex^{2+r}}{2+r}+\frac{2e^3x^{2+3r}}{2+3r}\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{2}(bn)\int x\left(d^3+\frac{6d^2ex^r}{2+r}+\frac{3de^2x^{2r}}{1+r}+\frac{2e^3x^{3r}}{2+3r}\right)dx \\
&= \frac{1}{2}\left(d^3x^2+\frac{3de^2x^{2(1+r)}}{1+r}+\frac{6d^2ex^{2+r}}{2+r}+\frac{2e^3x^{2+3r}}{2+3r}\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{2}(bn)\int \left(d^3x+\frac{6d^2ex^{1+r}}{2+r}+\frac{3de^2x^{1+2r}}{1+r}+\frac{2e^3x^{1+3r}}{2+3r}\right)dx \\
&= -\frac{1}{4}bd^3nx^2-\frac{3bde^2nx^{2(1+r)}}{4(1+r)^2}-\frac{3bd^2enx^{2+r}}{(2+r)^2}-\frac{be^3nx^{2+3r}}{(2+3r)^2} \\
&\quad +\frac{1}{2}\left(d^3x^2+\frac{3de^2x^{2(1+r)}}{1+r}+\frac{6d^2ex^{2+r}}{2+r}+\frac{2e^3x^{2+3r}}{2+3r}\right)(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.19

$$\int x(d+ex^r)^3(a+b\log(cx^n))\,dx = \frac{1}{4}x^2\left(bn\left(-d^3-\frac{12d^2ex^r}{(2+r)^2}-\frac{3de^2x^{2r}}{(1+r)^2}-\frac{4e^3x^{3r}}{(2+3r)^2}\right)+2a\left(d^3+\frac{6d^2ex^r}{2+r}+\frac{3de^2x^{2r}}{1+r}+\frac{2e^3x^{3r}}{2+3r}\right)+2b\left(d^3+\frac{6d^2ex^r}{2+r}+\frac{3de^2x^{2r}}{1+r}+\frac{2e^3x^{3r}}{2+3r}\right)\log(cx^n)\right)$$

```
[In] Integrate[x*(d + e*x^r)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^2*(b*n*(-d^3 - (12*d^2*e*x^r)/(2 + r)^2 - (3*d*e^2*x^(2*r))/(1 + r)^2 -
(4*e^3*x^(3*r))/(2 + 3*r)^2) + 2*a*(d^3 + (6*d^2*e*x^r)/(2 + r) + (3*d*e^2*
x^(2*r))/(1 + r) + (2*e^3*x^(3*r))/(2 + 3*r)) + 2*b*(d^3 + (6*d^2*e*x^r)/(2
 + r) + (3*d*e^2*x^(2*r))/(1 + r) + (2*e^3*x^(3*r))/(2 + 3*r))*Log[c*x^n]))
/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. 2(143) = 286.

Time = 5.65 (sec) , antiderivative size = 1267, normalized size of antiderivative = 8.50

method	result	size
parallelrisch	Expression too large to display	1267
risch	Expression too large to display	4027

```
[In] int(x*(d+e*x^r)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-18*x^2*a*d^3*r^6-132*x^2*a*d^3*r^5-386*x^2*a*d^3*r^4-576*x^2*a*d^3*r
^3-464*x^2*a*d^3*r^2-192*x^2*a*d^3*r-32*x^2*(x^r)^3*a*e^3-32*x^2*b*ln(c*x^n
)*d^3-32*a*d^3*x^2-80*x^2*(x^r)^3*a*e^3*r^4-204*x^2*(x^r)^3*a*e^3*r^3-248*x
^2*(x^r)^3*a*e^3*r^2-144*x^2*(x^r)^3*a*e^3*r+16*x^2*(x^r)^3*b*e^3*n+66*x^2*
b*d^3*n*r^5+193*x^2*b*d^3*n*r^4+288*x^2*b*d^3*n*r^3+232*x^2*b*d^3*n*r^2+96*
x^2*b*d^3*n*r-18*x^2*ln(c*x^n)*b*d^3*r^6-132*x^2*ln(c*x^n)*b*d^3*r^5-386*x^
2*ln(c*x^n)*b*d^3*r^4-576*x^2*ln(c*x^n)*b*d^3*r^3-464*x^2*ln(c*x^n)*b*d^3*r
^2-192*x^2*ln(c*x^n)*b*d^3*r-96*x^2*x^r*a*d^2*e-96*x^2*(x^r)^2*a*d*e^2-12*x
^2*(x^r)^3*a*e^3*r^5+9*x^2*b*d^3*n*r^6-32*e^3*b*ln(c*x^n)*(x^r)^3*x^2+16*b*
d^3*n*x^2-108*x^2*x^r*ln(c*x^n)*b*d^2*e*r^5-576*x^2*x^r*ln(c*x^n)*b*d^2*e*r
^4-1164*x^2*x^r*ln(c*x^n)*b*d^2*e*r^3-1128*x^2*x^r*ln(c*x^n)*b*d^2*e*r^2-52
8*x^2*x^r*ln(c*x^n)*b*d^2*e*r-54*x^2*(x^r)^2*ln(c*x^n)*b*d*e^2*r^5-342*x^2*
(x^r)^2*ln(c*x^n)*b*d*e^2*r^4-816*x^2*(x^r)^2*ln(c*x^n)*b*d*e^2*r^3-912*x^2
*(x^r)^2*ln(c*x^n)*b*d*e^2*r^2-480*x^2*(x^r)^2*ln(c*x^n)*b*d*e^2*r+108*x^2*
```

```
x^r*b*d^2*e*n*r^4+360*x^2*x^r*b*d^2*e*n*r^3+444*x^2*x^r*b*d^2*e*n*r^2+240*x
^2*x^r*b*d^2*e*n*r+27*x^2*(x^r)^2*b*d*e^2*n*r^4+144*x^2*(x^r)^2*b*d*e^2*n*r
^3+264*x^2*(x^r)^2*b*d*e^2*n*r^2-96*e*d^2*b*ln(c*x^n)*x^r*x^2-96*e^2*d*b*ln
(c*x^n)*(x^r)^2*x^2-12*x^2*(x^r)^3*ln(c*x^n)*b*e^3*r^5-80*x^2*(x^r)^3*ln(c*
x^n)*b*e^3*r^4-204*x^2*(x^r)^3*ln(c*x^n)*b*e^3*r^3-248*x^2*(x^r)^3*ln(c*x^n
)*b*e^3*r^2-144*x^2*(x^r)^3*ln(c*x^n)*b*e^3*r-108*x^2*x^r*a*d^2*e*r^5-576*x
^2*x^r*a*d^2*e*r^4-1164*x^2*x^r*a*d^2*e*r^3-1128*x^2*x^r*a*d^2*e*r^2-528*x^
2*x^r*a*d^2*e*r+48*x^2*x^r*b*d^2*e*n-54*x^2*(x^r)^2*a*d*e^2*r^5-342*x^2*(x^
r)^2*a*d*e^2*r^4-816*x^2*(x^r)^2*a*d*e^2*r^3-912*x^2*(x^r)^2*a*d*e^2*r^2-48
0*x^2*(x^r)^2*a*d*e^2*r+48*x^2*(x^r)^2*b*d*e^2*n+4*x^2*(x^r)^3*b*e^3*n*r^4+
24*x^2*(x^r)^3*b*e^3*n*r^3+52*x^2*(x^r)^3*b*e^3*n*r^2+48*x^2*(x^r)^3*b*e^3*
n*r+192*x^2*(x^r)^2*b*d*e^2*n*r)/(9*r^2+12*r+4)/(r^2+2*r+1)/(r^2+4*r+4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1024 vs. $2(143) = 286$.

Time = 0.34 (sec) , antiderivative size = 1024, normalized size of antiderivative = 6.87

$$\int x(d+ex^r)^3 (a+b\log(cx^n))\,dx = \frac{2\,(9\,bd^3r^6+66\,bd^3r^5+193\,bd^3r^4+288\,bd^3r^3+232\,bd^3r^2+96\,bd^3r+16\,bd^3)x^2\log(c)+2\,(9\,bd^3nr^6+66\,bc}{}$$

```
[In] integrate(x*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(9*b*d^3*r^6 + 66*b*d^3*r^5 + 193*b*d^3*r^4 + 288*b*d^3*r^3 + 232*b*
d^3*r^2 + 96*b*d^3*r + 16*b*d^3)*x^2*log(c) + 2*(9*b*d^3*n*r^6 + 66*b*d^3*n
*r^5 + 193*b*d^3*n*r^4 + 288*b*d^3*n*r^3 + 232*b*d^3*n*r^2 + 96*b*d^3*n*r +
 16*b*d^3*n)*x^2*log(x) - (9*(b*d^3*n - 2*a*d^3)*r^6 + 66*(b*d^3*n - 2*a*d^
3)*r^5 + 16*b*d^3*n + 193*(b*d^3*n - 2*a*d^3)*r^4 - 32*a*d^3 + 288*(b*d^3*n
 - 2*a*d^3)*r^3 + 232*(b*d^3*n - 2*a*d^3)*r^2 + 96*(b*d^3*n - 2*a*d^3)*r)*x
^2 + 4*((3*b*e^3*r^5 + 20*b*e^3*r^4 + 51*b*e^3*r^3 + 62*b*e^3*r^2 + 36*b*e^
3*r + 8*b*e^3)*x^2*log(c) + (3*b*e^3*n*r^5 + 20*b*e^3*n*r^4 + 51*b*e^3*n*r^
3 + 62*b*e^3*n*r^2 + 36*b*e^3*n*r + 8*b*e^3*n)*x^2*log(x) + (3*a*e^3*r^5 -
4*b*e^3*n - (b*e^3*n - 20*a*e^3)*r^4 + 8*a*e^3 - 3*(2*b*e^3*n - 17*a*e^3)*r
^3 - (13*b*e^3*n - 62*a*e^3)*r^2 - 12*(b*e^3*n - 3*a*e^3)*r)*x^2)*x^(3*r) +
 3*(2*(9*b*d*e^2*r^5 + 57*b*d*e^2*r^4 + 136*b*d*e^2*r^3 + 152*b*d*e^2*r^2 +
 80*b*d*e^2*r + 16*b*d*e^2)*x^2*log(c) + 2*(9*b*d*e^2*n*r^5 + 57*b*d*e^2*n*
r^4 + 136*b*d*e^2*n*r^3 + 152*b*d*e^2*n*r^2 + 80*b*d*e^2*n*r + 16*b*d*e^2*n
)*x^2*log(x) + (18*a*d*e^2*r^5 - 16*b*d*e^2*n - 3*(3*b*d*e^2*n - 38*a*d*e^2
)*r^4 + 32*a*d*e^2 - 16*(3*b*d*e^2*n - 17*a*d*e^2)*r^3 - 8*(11*b*d*e^2*n -
38*a*d*e^2)*r^2 - 32*(2*b*d*e^2*n - 5*a*d*e^2)*r)*x^2)*x^(2*r) + 12*((9*b*d
^2*e*r^5 + 48*b*d^2*e*r^4 + 97*b*d^2*e*r^3 + 94*b*d^2*e*r^2 + 44*b*d^2*e*r
+ 8*b*d^2*e)*x^2*log(c) + (9*b*d^2*e*n*r^5 + 48*b*d^2*e*n*r^4 + 97*b*d^2*e*
```

```
n*r^3 + 94*b*d^2*e*n*r^2 + 44*b*d^2*e*n*r + 8*b*d^2*e*n)*x^2*log(x) + (9*a*
d^2*e*r^5 - 4*b*d^2*e*n - 3*(3*b*d^2*e*n - 16*a*d^2*e)*r^4 + 8*a*d^2*e - (3
0*b*d^2*e*n - 97*a*d^2*e)*r^3 - (37*b*d^2*e*n - 94*a*d^2*e)*r^2 - 4*(5*b*d^
2*e*n - 11*a*d^2*e)*r)*x^2)*x^r)/(9*r^6 + 66*r^5 + 193*r^4 + 288*r^3 + 232*
r^2 + 96*r + 16)
```

Sympy [A] (verification not implemented)

Time = 86.83 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.40

$$\int x(d+ex^r)^3\,(a+b\log(cx^n))\,dx$$

$$= \frac{ad^3x^2}{2} + 3ad^2e\left(\begin{cases}\frac{x^2x^r}{r+2} & \text{for } r\neq -2\\ x^2x^r\log(x) & \text{otherwise}\end{cases}\right) + 3ade^2\left(\begin{cases}\frac{x^2x^{2r}}{2r+2} & \text{for } r\neq -1\\ x^2x^{2r}\log(x) & \text{otherwise}\end{cases}\right)$$
$$+ ae^3\left(\begin{cases}\frac{x^2x^{3r}}{3r+2} & \text{for } r\neq -\frac{2}{3}\\ x^2x^{3r}\log(x) & \text{otherwise}\end{cases}\right) - \frac{bd^3nx^2}{4} + \frac{bd^3x^2\log(cx^n)}{2}$$
$$- 3bd^2en\left(\begin{cases}\frac{\begin{cases}\frac{x^{r+2}}{r+2} & \text{for } r\neq -2\\ \log(x) & \text{otherwise}\end{cases}}{r+2} & \text{for } r>-\infty\wedge r<\infty\wedge r\neq -2\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ 3bd^2e\left(\begin{cases}\frac{x^{r+2}}{r+2} & \text{for } r\neq -2\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$
$$- 3bde^2n\left(\begin{cases}\frac{\begin{cases}\frac{x^{2r+2}}{2r+2} & \text{for } r\neq -1\\ \log(x) & \text{otherwise}\end{cases}}{2r+2} & \text{for } r>-\infty\wedge r<\infty\wedge r\neq -1\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ 3bde^2\left(\begin{cases}\frac{x^{2r+2}}{2r+2} & \text{for } r\neq -1\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$
$$- be^3n\left(\begin{cases}\frac{\begin{cases}\frac{x^{3r+2}}{3r+2} & \text{for } r\neq -\frac{2}{3}\\ \log(x) & \text{otherwise}\end{cases}}{3r+2} & \text{for } r>-\infty\wedge r<\infty\wedge r\neq -\frac{2}{3}\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ be^3\left(\begin{cases}\frac{x^{3r+2}}{3r+2} & \text{for } r\neq -\frac{2}{3}\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate(x*(d+e*x**r)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**3*x**2/2 + 3*a*d**2*e*Piecewise((x**2*x**r/(r + 2), Ne(r, -2)), (x**2*
x**r*log(x), True)) + 3*a*d*e**2*Piecewise((x**2*x**(2*r)/(2*r + 2), Ne(r,
-1)), (x**2*x**(2*r)*log(x), True)) + a*e**3*Piecewise((x**2*x**(3*r)/(3*r
+ 2), Ne(r, -2/3)), (x**2*x**(3*r)*log(x), True)) - b*d**3*n*x**2/4 + b*d**
3*x**2*log(c*x**n)/2 - 3*b*d**2*e*n*Piecewise((Piecewise((x**(r + 2)/(r + 2
), Ne(r, -2)), (log(x), True))/(r + 2), (r > -oo) & (r < oo) & Ne(r, -2)),
(log(x)**2/2, True)) + 3*b*d**2*e*Piecewise((x**(r + 2)/(r + 2), Ne(r, -2))
, (log(x), True))*log(c*x**n) - 3*b*d*e**2*n*Piecewise((Piecewise((x**(2*r
+ 2)/(2*r + 2), Ne(r, -1)), (log(x), True))/(2*r + 2), (r > -oo) & (r < oo)
 & Ne(r, -1)), (log(x)**2/2, True)) + 3*b*d*e**2*Piecewise((x**(2*r + 2)/(2
*r + 2), Ne(r, -1)), (log(x), True))*log(c*x**n) - b*e**3*n*Piecewise((Piec
ewise((x**(3*r + 2)/(3*r + 2), Ne(r, -2/3)), (log(x), True))/(3*r + 2), (r
> -oo) & (r < oo) & Ne(r, -2/3)), (log(x)**2/2, True)) + b*e**3*Piecewise((
x**(3*r + 2)/(3*r + 2), Ne(r, -2/3)), (log(x), True))*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.49

$$\int x(d+ex^r)^3\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{1}{4}\,bd^3nx^2+\frac{1}{2}\,bd^3x^2\log\left(cx^n\right)+\frac{1}{2}\,ad^3x^2+\frac{be^3x^{3\,r+2}\log\left(cx^n\right)}{3\,r+2}+\frac{3\,bde^2x^{2\,r+2}\log\left(cx^n\right)}{2\,(r+1)}+\frac{3\,bd^2ex^{r+2}\log\left(cx^n\right)}{r+2}-\frac{be^3nx^{3\,r+2}}{(3\,r+2)^2}+\frac{ae^3x^{3\,r+2}}{3\,r+2}-\frac{3\,bde^2nx^{2\,r+2}}{4\,(r+1)^2}+\frac{3\,ade^2x^{2\,r+2}}{2\,(r+1)}-\frac{3\,bd^2enx^{r+2}}{(r+2)^2}+\frac{3\,ad^2ex^{r+2}}{r+2}$$

```
[In] integrate(x*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d^3*n*x^2 + 1/2*b*d^3*x^2*log(c*x^n) + 1/2*a*d^3*x^2 + b*e^3*x^(3*r
+ 2)*log(c*x^n)/(3*r + 2) + 3/2*b*d*e^2*x^(2*r + 2)*log(c*x^n)/(r + 1) + 3*
b*d^2*e*x^(r + 2)*log(c*x^n)/(r + 2) - b*e^3*n*x^(3*r + 2)/(3*r + 2)^2 + a*
e^3*x^(3*r + 2)/(3*r + 2) - 3/4*b*d*e^2*n*x^(2*r + 2)/(r + 1)^2 + 3/2*a*d*e
^2*x^(2*r + 2)/(r + 1) - 3*b*d^2*e*n*x^(r + 2)/(r + 2)^2 + 3*a*d^2*e*x^(r +
 2)/(r + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1611 vs. 2(143) = 286.

Time = 0.37 (sec) , antiderivative size = 1611, normalized size of antiderivative = 10.81

$$\int x(d+ex^r)^3\,(a+b\log{(cx^n)})\;dx=\text{Too large to display}$$

```
[In] integrate(x*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/4*(12*b*e^3*n*r^5*x^2*x^(3*r)*log(x) + 54*b*d*e^2*n*r^5*x^2*x^(2*r)*log(x
) + 108*b*d^2*e*n*r^5*x^2*x^r*log(x) + 18*b*d^3*n*r^6*x^2*log(x) - 9*b*d^3*
n*r^6*x^2 + 12*b*e^3*r^5*x^2*x^(3*r)*log(c) + 54*b*d*e^2*r^5*x^2*x^(2*r)*lo
g(c) + 108*b*d^2*e*r^5*x^2*x^r*log(c) + 18*b*d^3*r^6*x^2*log(c) + 80*b*e^3*
n*r^4*x^2*x^(3*r)*log(x) + 342*b*d*e^2*n*r^4*x^2*x^(2*r)*log(x) + 576*b*d^2
*e*n*r^4*x^2*x^r*log(x) + 132*b*d^3*n*r^5*x^2*log(x) - 4*b*e^3*n*r^4*x^2*x^
(3*r) + 12*a*e^3*r^5*x^2*x^(3*r) - 27*b*d*e^2*n*r^4*x^2*x^(2*r) + 54*a*d*e^
2*r^5*x^2*x^(2*r) - 108*b*d^2*e*n*r^4*x^2*x^r + 108*a*d^2*e*r^5*x^2*x^r - 6
6*b*d^3*n*r^5*x^2 + 18*a*d^3*r^6*x^2 + 80*b*e^3*r^4*x^2*x^(3*r)*log(c) + 34
2*b*d*e^2*r^4*x^2*x^(2*r)*log(c) + 576*b*d^2*e*r^4*x^2*x^r*log(c) + 132*b*d
^3*r^5*x^2*log(c) + 204*b*e^3*n*r^3*x^2*x^(3*r)*log(x) + 816*b*d*e^2*n*r^3*
x^2*x^(2*r)*log(x) + 1164*b*d^2*e*n*r^3*x^2*x^r*log(x) + 386*b*d^3*n*r^4*x^
2*log(x) - 24*b*e^3*n*r^3*x^2*x^(3*r) + 80*a*e^3*r^4*x^2*x^(3*r) - 144*b*d*
e^2*n*r^3*x^2*x^(2*r) + 342*a*d*e^2*r^4*x^2*x^(2*r) - 360*b*d^2*e*n*r^3*x^2
*x^r + 576*a*d^2*e*r^4*x^2*x^r - 193*b*d^3*n*r^4*x^2 + 132*a*d^3*r^5*x^2 +
204*b*e^3*r^3*x^2*x^(3*r)*log(c) + 816*b*d*e^2*r^3*x^2*x^(2*r)*log(c) + 116
4*b*d^2*e*r^3*x^2*x^r*log(c) + 386*b*d^3*r^4*x^2*log(c) + 248*b*e^3*n*r^2*x
^2*x^(3*r)*log(x) + 912*b*d*e^2*n*r^2*x^2*x^(2*r)*log(x) + 1128*b*d^2*e*n*r
^2*x^2*x^r*log(x) + 576*b*d^3*n*r^3*x^2*log(x) - 52*b*e^3*n*r^2*x^2*x^(3*r)
 + 204*a*e^3*r^3*x^2*x^(3*r) - 264*b*d*e^2*n*r^2*x^2*x^(2*r) + 816*a*d*e^2*
r^3*x^2*x^(2*r) - 444*b*d^2*e*n*r^2*x^2*x^r + 1164*a*d^2*e*r^3*x^2*x^r - 28
8*b*d^3*n*r^3*x^2 + 386*a*d^3*r^4*x^2 + 248*b*e^3*r^2*x^2*x^(3*r)*log(c) +
912*b*d*e^2*r^2*x^2*x^(2*r)*log(c) + 1128*b*d^2*e*r^2*x^2*x^r*log(c) + 576*
b*d^3*r^3*x^2*log(c) + 144*b*e^3*n*r*x^2*x^(3*r)*log(x) + 480*b*d*e^2*n*r*x
^2*x^(2*r)*log(x) + 528*b*d^2*e*n*r*x^2*x^r*log(x) + 464*b*d^3*n*r^2*x^2*lo
g(x) - 48*b*e^3*n*r*x^2*x^(3*r) + 248*a*e^3*r^2*x^2*x^(3*r) - 192*b*d*e^2*n
*r*x^2*x^(2*r) + 912*a*d*e^2*r^2*x^2*x^(2*r) - 240*b*d^2*e*n*r*x^2*x^r + 11
28*a*d^2*e*r^2*x^2*x^r - 232*b*d^3*n*r^2*x^2 + 576*a*d^3*r^3*x^2 + 144*b*e^
3*r*x^2*x^(3*r)*log(c) + 480*b*d*e^2*r*x^2*x^(2*r)*log(c) + 528*b*d^2*e*r*x
^2*x^r*log(c) + 464*b*d^3*r^2*x^2*log(c) + 32*b*e^3*n*x^2*x^(3*r)*log(x) +
96*b*d*e^2*n*x^2*x^(2*r)*log(x) + 96*b*d^2*e*n*x^2*x^r*log(x) + 192*b*d^3*n
*r*x^2*log(x) - 16*b*e^3*n*x^2*x^(3*r) + 144*a*e^3*r*x^2*x^(3*r) - 48*b*d*e
^2*n*x^2*x^(2*r) + 480*a*d*e^2*r*x^2*x^(2*r) - 48*b*d^2*e*n*x^2*x^r + 528*a
*d^2*e*r*x^2*x^r - 96*b*d^3*n*r*x^2 + 464*a*d^3*r^2*x^2 + 32*b*e^3*x^2*x^(3
*r)*log(c) + 96*b*d*e^2*x^2*x^(2*r)*log(c) + 96*b*d^2*e*x^2*x^r*log(c) + 19
```

```
2*b*d^3*r*x^2*log(c) + 32*b*d^3*n*x^2*log(x) + 32*a*e^3*x^2*x^(3*r) + 96*a*
d*e^2*x^2*x^(2*r) + 96*a*d^2*e*x^2*x^r - 16*b*d^3*n*x^2 + 192*a*d^3*r*x^2 +
 32*b*d^3*x^2*log(c) + 32*a*d^3*x^2)/(9*r^6 + 66*r^5 + 193*r^4 + 288*r^3 +
232*r^2 + 96*r + 16)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^r)^3\,(a+b\log{(cx^n)})\;dx=\int x\,(d+e\,x^r)^3\,(a+b\,\ln{(c\,x^n)})\;dx$$

```
[In] int(x*(d + e*x^r)^3*(a + b*log(c*x^n)),x)
```

```
[Out] int(x*(d + e*x^r)^3*(a + b*log(c*x^n)), x)
```

3.395 $\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 152

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x}\,dx = -\frac{3bd^2enx^r}{r^2} - \frac{3bde^2nx^{2r}}{4r^2} - \frac{be^3nx^{3r}}{9r^2} - \frac{1}{2}bd^3n\log^2(x) + \frac{3d^2ex^r(a+b\log(cx^n))}{r} + \frac{3de^2x^{2r}(a+b\log(cx^n))}{2r} + \frac{e^3x^{3r}(a+b\log(cx^n))}{3r} + d^3\log(x)(a+b\log(cx^n))$$

```
[Out] -3*b*d^2*e*n*x^r/r^2-3/4*b*d*e^2*n*x^(2*r)/r^2-1/9*b*e^3*n*x^(3*r)/r^2-1/2*
b*d^3*n*ln(x)^2+3*d^2*e*x^r*(a+b*ln(c*x^n))/r+3/2*d*e^2*x^(2*r)*(a+b*ln(c*x
^n))/r+1/3*e^3*x^(3*r)*(a+b*ln(c*x^n))/r+d^3*ln(x)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {272, 45, 2372, 12, 14, 2338}

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x}\,dx = d^3\log(x)(a+b\log(cx^n)) + \frac{3d^2ex^r(a+b\log(cx^n))}{r} + \frac{3de^2x^{2r}(a+b\log(cx^n))}{2r} + \frac{e^3x^{3r}(a+b\log(cx^n))}{3r} - \frac{1}{2}bd^3n\log^2(x) - \frac{3bd^2enx^r}{r^2} - \frac{3bde^2nx^{2r}}{4r^2} - \frac{be^3nx^{3r}}{9r^2}$$

```
[In] Int[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] (-3*b*d^2*e*n*x^r)/r^2 - (3*b*d*e^2*n*x^(2*r))/(4*r^2) - (b*e^3*n*x^(3*r))/
(9*r^2) - (b*d^3*n*Log[x]^2)/2 + (3*d^2*e*x^r*(a + b*Log[c*x^n]))/r + (3*d*
```

```
e^2*x^(2*r)*(a + b*Log[c*x^n]))/(2*r) + (e^3*x^(3*r)*(a + b*Log[c*x^n]))/(3
*r) + d^3*Log[x]*(a + b*Log[c*x^n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3d^2ex^r(a+b\log(cx^n))}{r}+\frac{3de^2x^{2r}(a+b\log(cx^n))}{2r}+\frac{e^3x^{3r}(a+b\log(cx^n))}{3r}\\&\quad+d^3\log(x)\,(a+b\log(cx^n))-(bn)\int\frac{ex^r(18d^2+9dex^r+2e^2x^{2r})+6d^3r\log(x)}{6rx}\,dx\end{aligned}$$

$$= \frac{3d^2ex^r(a+b\log(cx^n))}{r} + \frac{3de^2x^{2r}(a+b\log(cx^n))}{2r} + \frac{e^3x^{3r}(a+b\log(cx^n))}{3r} + d^3\log(x)(a+b\log(cx^n)) - \frac{(bn)\int \frac{ex^r(18d^2+9dex^r+2e^2x^{2r})+6d^3r\log(x)}{x}dx}{6r}$$

$$= \frac{3d^2ex^r(a+b\log(cx^n))}{r} + \frac{3de^2x^{2r}(a+b\log(cx^n))}{2r} + \frac{e^3x^{3r}(a+b\log(cx^n))}{3r} + d^3\log(x)(a+b\log(cx^n)) - \frac{(bn)\int\left(18d^2ex^{-1+r}+9de^2x^{-1+2r}+2e^3x^{-1+3r}+\frac{6d^3r\log(x)}{x}\right)dx}{6r}$$

$$= -\frac{3bd^2enx^r}{r^2} - \frac{3bde^2nx^{2r}}{4r^2} - \frac{be^3nx^{3r}}{9r^2} + \frac{3d^2ex^r(a+b\log(cx^n))}{r} + \frac{3de^2x^{2r}(a+b\log(cx^n))}{2r} + \frac{e^3x^{3r}(a+b\log(cx^n))}{3r} + d^3\log(x)(a+b\log(cx^n)) - (bd^3n)\int\frac{\log(x)}{x}dx$$

$$= -\frac{3bd^2enx^r}{r^2} - \frac{3bde^2nx^{2r}}{4r^2} - \frac{be^3nx^{3r}}{9r^2} - \frac{1}{2}bd^3n\log^2(x) + \frac{3d^2ex^r(a+b\log(cx^n))}{r} + \frac{3de^2x^{2r}(a+b\log(cx^n))}{2r} + \frac{e^3x^{3r}(a+b\log(cx^n))}{3r} + d^3\log(x)(a+b\log(cx^n))$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x}dx = ad^3\log(x) + \frac{1}{36}\left(\frac{ex^r(6ar(18d^2+9dex^r+2e^2x^{2r})-bn(108d^2+27dex^r+4e^2x^{2r}))}{r^2} + \frac{6bex^r(18d^2+9dex^r+2e^2x^{2r})\log(cx^n)}{r} + \frac{18bd^3\log^2(cx^n)}{n}\right)$$

```
[In] Integrate[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] a*d^3*Log[x] + ((e*x^r*(6*a*r*(18*d^2 + 9*d*e*x^r + 2*e^2*x^(2*r)) - b*n*(1
08*d^2 + 27*d*e*x^r + 4*e^2*x^(2*r))))/r^2 + (6*b*e*x^r*(18*d^2 + 9*d*e*x^r
 + 2*e^2*x^(2*r))*Log[c*x^n])/r + (18*b*d^3*Log[c*x^n]^2)/n)/36
```

Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.16

method	result
parallelrisch	$\frac{12e^3 b\ln(c\,x^n)x^{3r}nr+12x^{3r}a\,e^3nr-4x^{3r}b\,e^3n^2+54bd\,e^2\ln(c\,x^n)x^{2r}nr+36\ln(x)a\,d^3n\,r^2+54x^{2r}ad\,e^2nr-27x^{2r}bd\,e^2n^2+108b\,d^2}{36n\,r^2}$
risch	$-\frac{3b\,d^2en\,x^r}{r^2}-\frac{3bd\,e^2n\,x^{2r}}{4r^2}-\frac{b\,e^3n\,x^{3r}}{9r^2}-\frac{i\ln(x)\pi b\,d^3\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)}{2}-\frac{i\pi b\,e^3\,\mathrm{csgn}(ic\,x^n)^3x^{3r}}{6r}+\ln(x)$

```
[In] int((d+e*x^r)^3*(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*(12*e^3*b*ln(c*x^n)*(x^r)^3*n*r+12*(x^r)^3*a*e^3*n*r-4*(x^r)^3*b*e^3*n
^2+54*b*d*e^2*ln(c*x^n)*(x^r)^2*n*r+36*ln(x)*a*d^3*n*r^2+54*(x^r)^2*a*d*e^2
*n*r-27*(x^r)^2*b*d*e^2*n^2+108*b*d^2*e*ln(c*x^n)*x^r*n*r+18*b*d^3*ln(c*x^n
)^2*r^2+108*x^r*a*d^2*e*n*r-108*x^r*b*d^2*e*n^2)/n/r^2
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex^r)^3\,(a+b\log{(cx^n)})}{x}\,dx$$
$$=\frac{18\,bd^3nr^2\log{(x)}^2+4\,(3\,be^3nr\log{(x)}+3\,be^3r\log{(c)}-be^3n+3\,ae^3r)x^{3\,r}+27\,(2\,bde^2nr\log{(x)}+2\,bde^2r\,\mathrm{lo}}{}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] 1/36*(18*b*d^3*n*r^2*log(x)^2 + 4*(3*b*e^3*n*r*log(x) + 3*b*e^3*r*log(c) -
b*e^3*n + 3*a*e^3*r)*x^(3*r) + 27*(2*b*d*e^2*n*r*log(x) + 2*b*d*e^2*r*log(c
) - b*d*e^2*n + 2*a*d*e^2*r)*x^(2*r) + 108*(b*d^2*e*n*r*log(x) + b*d^2*e*r*
log(c) - b*d^2*e*n + a*d^2*e*r)*x^r + 36*(b*d^3*r^2*log(c) + a*d^3*r^2)*log
(x))/r^2
```

Sympy [A] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.97

$$\int \frac{(d+ex^r)^3\,(a+b\log(cx^n))}{x}\,dx = \begin{cases} (a+b\log(c))\,(d+e)^3\log(x) \\ (a+b\log(c))\left(d^3\log(x)+\frac{3d^2ex^r}{r}+\frac{3de^2x^{2r}}{2r}+\frac{e^3x^{3r}}{3r}\right) \\ (d+e)^3\left(\begin{cases} a\log(x) & \text{for } b=0 \\ -(-a-b\log(c))\log(x) & \text{for } n=0 \\ \frac{(-a-b\log(cx^n))^2}{2bn} & \text{otherwise} \end{cases}\right) \\ \frac{ad^3\log(cx^n)}{n}+\frac{3ad^2ex^r}{r}+\frac{3ade^2x^{2r}}{2r}+\frac{ae^3x^{3r}}{3r}+\frac{bd^3\log(cx^n)^2}{2n}-\frac{3bd^2enx^r}{r^2}+\frac{3bd^2ex^r\log(cx^n)}{r}-\frac{3bde^2nx^{2r}}{4r^2}+\frac{3bde^2x^{2r}\log(c}{2r} \end{cases}$$

```
[In] integrate((d+e*x**r)**3*(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise(((a + b*log(c))*(d + e)**3*log(x), Eq(n, 0) & Eq(r, 0)), ((a + b*
log(c))*(d**3*log(x) + 3*d**2*e*x**r/r + 3*d*e**2*x**(2*r)/(2*r) + e**3*x**
(3*r)/(3*r)), Eq(n, 0)), ((d + e)**3*Piecewise((a*log(x), Eq(b, 0)), (-(-a
- b*log(c))*log(x), Eq(n, 0)), ((-a - b*log(c*x**n))**2/(2*b*n), True)), Eq
(r, 0)), (a*d**3*log(c*x**n)/n + 3*a*d**2*e*x**r/r + 3*a*d*e**2*x**(2*r)/(2
*r) + a*e**3*x**(3*r)/(3*r) + b*d**3*log(c*x**n)**2/(2*n) - 3*b*d**2*e*n*x*
*r/r**2 + 3*b*d**2*e*x**r*log(c*x**n)/r - 3*b*d*e**2*n*x**(2*r)/(4*r**2) +
3*b*d*e**2*x**(2*r)*log(c*x**n)/(2*r) - b*e**3*n*x**(3*r)/(9*r**2) + b*e**3
*x**(3*r)*log(c*x**n)/(3*r), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex^r)^3\,(a+b\log(cx^n))}{x}\,dx = \frac{be^3x^{3r}\log(cx^n)}{3r}+\frac{3bde^2x^{2r}\log(cx^n)}{2r}+\frac{3bd^2ex^r\log(cx^n)}{r} + \frac{bd^3\log(cx^n)^2}{2n}+ad^3\log(x)-\frac{be^3nx^{3r}}{9r^2}+\frac{ae^3x^{3r}}{3r} - \frac{3bde^2nx^{2r}}{4r^2}+\frac{3ade^2x^{2r}}{2r}-\frac{3bd^2enx^r}{r^2}+\frac{3ad^2ex^r}{r}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] 1/3*b*e^3*x^(3*r)*log(c*x^n)/r + 3/2*b*d*e^2*x^(2*r)*log(c*x^n)/r + 3*b*d^2
*e*x^r*log(c*x^n)/r + 1/2*b*d^3*log(c*x^n)^2/n + a*d^3*log(x) - 1/9*b*e^3*n
*x^(3*r)/r^2 + 1/3*a*e^3*x^(3*r)/r - 3/4*b*d*e^2*n*x^(2*r)/r^2 + 3/2*a*d*e^
2*x^(2*r)/r - 3*b*d^2*e*n*x^r/r^2 + 3*a*d^2*e*x^r/r
```

Giac [F]

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x} dx = \int \frac{(ex^r+d)^3 (b\log(cx^n)+a)}{x} dx$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x,x, algorithm="giac")
[Out] integrate((e*x^r + d)^3*(b*log(c*x^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x} dx = \int \frac{(d+e\,x^r)^3 (a+b\,\ln(c\,x^n))}{x} dx$$

```
[In] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x,x)
[Out] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x, x)
```

3.396 $\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 191

$$\int \frac{(d+ex^r)^3\,(a+b\log{(cx^n)})}{x^3}\,dx = -\frac{bd^3n}{4x^2}-\frac{3bde^2nx^{-2(1-r)}}{4(1-r)^2}-\frac{3bd^2enx^{-2+r}}{(2-r)^2}-\frac{be^3nx^{-2+3r}}{(2-3r)^2}-\frac{d^3(a+b\log{(cx^n)})}{2x^2}-\frac{3de^2x^{-2(1-r)}(a+b\log{(cx^n)})}{2(1-r)}-\frac{3d^2ex^{-2+r}(a+b\log{(cx^n)})}{2-r}-\frac{e^3x^{-2+3r}(a+b\log{(cx^n)})}{2-3r}$$

```
[Out] -1/4*b*d^3*n/x^2-3/4*b*d*e^2*n/(1-r)^2/(x^(2-2*r))-3*b*d^2*e*n*x^(-2+r)/(2-
r)^2-b*e^3*n*x^(-2+3*r)/(2-3*r)^2-1/2*d^3*(a+b*ln(c*x^n))/x^2-3/2*d*e^2*(a+
b*ln(c*x^n))/(1-r)/(x^(2-2*r))-3*d^2*e*x^(-2+r)*(a+b*ln(c*x^n))/(2-r)-e^3*x
^(-2+3*r)*(a+b*ln(c*x^n))/(2-3*r)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2372, 12, 14}

$$\int \frac{(d+ex^r)^3\,(a+b\log{(cx^n)})}{x^3}\,dx = -\frac{d^3(a+b\log{(cx^n)})}{2x^2}-\frac{3d^2ex^{r-2}(a+b\log{(cx^n)})}{2-r}-\frac{3de^2x^{-2(1-r)}(a+b\log{(cx^n)})}{2(1-r)}-\frac{e^3x^{3r-2}(a+b\log{(cx^n)})}{2-3r}-\frac{bd^3n}{4x^2}-\frac{3bd^2enx^{r-2}}{(2-r)^2}-\frac{3bde^2nx^{-2(1-r)}}{4(1-r)^2}-\frac{be^3nx^{3r-2}}{(2-3r)^2}$$

```
[In] Int[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/4*(b*d^3*n)/x^2 - (3*b*d*e^2*n)/(4*(1 - r)^2*x^(2*(1 - r))) - (3*b*d^2*e
*n*x^(-2 + r))/(2 - r)^2 - (b*e^3*n*x^(-2 + 3*r))/(2 - 3*r)^2 - (d^3*(a + b
*Log[c*x^n]))/(2*x^2) - (3*d*e^2*(a + b*Log[c*x^n]))/(2*(1 - r)*x^(2*(1 - r
))) - (3*d^2*e*x^(-2 + r)*(a + b*Log[c*x^n]))/(2 - r) - (e^3*x^(-2 + 3*r)*(
a + b*Log[c*x^n]))/(2 - 3*r)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{2x^2} - \frac{3de^2x^{-2(1-r)}(a+b\log(cx^n))}{2(1-r)} - \frac{3d^2ex^{-2+r}(a+b\log(cx^n))}{2-r} \\
&\quad - \frac{e^3x^{-2+3r}(a+b\log(cx^n))}{2-3r} - (bn)\int \frac{-d^3+\frac{6d^2ex^r}{-2+r}+\frac{3de^2x^{2r}}{-1+r}+\frac{2e^3x^{3r}}{-2+3r}}{2x^3}\,dx \\
&= -\frac{d^3(a+b\log(cx^n))}{2x^2} - \frac{3de^2x^{-2(1-r)}(a+b\log(cx^n))}{2(1-r)} - \frac{3d^2ex^{-2+r}(a+b\log(cx^n))}{2-r} \\
&\quad - \frac{e^3x^{-2+3r}(a+b\log(cx^n))}{2-3r} - \frac{1}{2}(bn)\int \frac{-d^3+\frac{6d^2ex^r}{-2+r}+\frac{3de^2x^{2r}}{-1+r}+\frac{2e^3x^{3r}}{-2+3r}}{x^3}\,dx
\end{aligned}$$

$$= -\frac{d^3(a+b\log(cx^n))}{2x^2} - \frac{3de^2x^{-2(1-r)}(a+b\log(cx^n))}{2(1-r)} - \frac{3d^2ex^{-2+r}(a+b\log(cx^n))}{2-r} - \frac{e^3x^{-2+3r}(a+b\log(cx^n))}{2-3r} - \frac{1}{2}(bn)\int\left(-\frac{d^3}{x^3}+\frac{6d^2ex^{-3+r}}{-2+r}+\frac{2e^3x^{3(-1+r)}}{-2+3r}+\frac{3de^2x^{-3+2r}}{-1+r}\right)dx$$

$$= -\frac{bd^3n}{4x^2} - \frac{3bde^2nx^{-2(1-r)}}{4(1-r)^2} - \frac{3bd^2enx^{-2+r}}{(2-r)^2} - \frac{be^3nx^{-2+3r}}{(2-3r)^2} - \frac{d^3(a+b\log(cx^n))}{2x^2} - \frac{3de^2x^{-2(1-r)}(a+b\log(cx^n))}{2(1-r)} - \frac{3d^2ex^{-2+r}(a+b\log(cx^n))}{2-r} - \frac{e^3x^{-2+3r}(a+b\log(cx^n))}{2-3r}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^3}\,dx = \frac{bn\left(-d^3-\frac{12d^2ex^r}{(-2+r)^2}-\frac{3de^2x^{2r}}{(-1+r)^2}-\frac{4e^3x^{3r}}{(2-3r)^2}\right)+a\left(-2d^3+\frac{12d^2ex^r}{-2+r}+\frac{6de^2x^{2r}}{-1+r}+\frac{4e^3x^{3r}}{-2+3r}\right)+2b\left(-d^3+\frac{6d^2ex^r}{-2+r}+\frac{3de^2x^{2r}}{-1+r}\right.}{4x^2}$$

```
[In] Integrate[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] (b*n*(-d^3 - (12*d^2*e*x^r)/(-2 + r)^2 - (3*d*e^2*x^(2*r))/(-1 + r)^2 - (4*
e^3*x^(3*r))/(2 - 3*r)^2) + a*(-2*d^3 + (12*d^2*e*x^r)/(-2 + r) + (6*d*e^2*
x^(2*r))/(-1 + r) + (4*e^3*x^(3*r))/(-2 + 3*r)) + 2*b*(-d^3 + (6*d^2*e*x^r)
/(-2 + r) + (3*d*e^2*x^(2*r))/(-1 + r) + (2*e^3*x^(3*r))/(-2 + 3*r))*Log[c*
x^n])/(4*x^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. 2(183) = 366.

Time = 3.59 (sec) , antiderivative size = 1039, normalized size of antiderivative = 5.44

method	result	size
parallelrisch	Expression too large to display	1039
risch	Expression too large to display	4027

```
[In] int((d+e*x^r)^3*(a+b*ln(c*x^n))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(32*b*ln(c*x^n)*d^3+96*b*d*e^2*ln(c*x^n)*(x^r)^2+32*e^3*(x^r)^3*a-288*
b*d^3*n*r^3+232*b*d^3*n*r^2-96*b*d^3*n*r+912*a*d*e^2*r^2*(x^r)^2-816*a*d*e^
2*r^3*(x^r)^2+96*d*e^2*(x^r)^2*a+96*d^2*e*x^r*a+32*a*d^3-204*a*e^3*r^3*(x^r
```

```
)^3+248*a*e^3*r^2*(x^r)^3-144*a*e^3*r*(x^r)^3-12*a*e^3*r^5*(x^r)^3+80*a*e^3
*r^4*(x^r)^3-12*(x^r)^3*ln(c*x^n)*b*e^3*r^5+80*(x^r)^3*ln(c*x^n)*b*e^3*r^4-
204*(x^r)^3*ln(c*x^n)*b*e^3*r^3+248*(x^r)^3*ln(c*x^n)*b*e^3*r^2-144*(x^r)^3
*ln(c*x^n)*b*e^3*r+96*b*d^2*e*ln(c*x^n)*x^r+9*b*d^3*n*r^6-66*b*d^3*n*r^5+19
3*b*d^3*n*r^4-108*x^r*ln(c*x^n)*b*d^2*e*r^5+576*x^r*ln(c*x^n)*b*d^2*e*r^4-1
164*x^r*ln(c*x^n)*b*d^2*e*r^3+1128*x^r*ln(c*x^n)*b*d^2*e*r^2-528*x^r*ln(c*x
^n)*b*d^2*e*r-54*(x^r)^2*ln(c*x^n)*b*d*e^2*r^5+342*(x^r)^2*ln(c*x^n)*b*d*e^
2*r^4-816*(x^r)^2*ln(c*x^n)*b*d*e^2*r^3+912*(x^r)^2*ln(c*x^n)*b*d*e^2*r^2-4
80*(x^r)^2*ln(c*x^n)*b*d*e^2*r+16*b*d^3*n+18*ln(c*x^n)*b*d^3*r^6-132*ln(c*x
^n)*b*d^3*r^5+386*ln(c*x^n)*b*d^3*r^4-576*ln(c*x^n)*b*d^3*r^3+464*ln(c*x^n)
*b*d^3*r^2-192*ln(c*x^n)*b*d^3*r+32*e^3*b*ln(c*x^n)*(x^r)^3-576*a*d^3*r^3+4
64*a*d^3*r^2-192*a*d^3*r+18*a*d^3*r^6-132*a*d^3*r^5+386*a*d^3*r^4-1164*a*d^
2*e*r^3*x^r+48*b*d*e^2*n*(x^r)^2+48*b*d^2*e*n*x^r+16*b*e^3*n*(x^r)^3-24*b*e
^3*n*r^3*(x^r)^3+52*b*e^3*n*r^2*(x^r)^3-48*b*e^3*n*r*(x^r)^3+1128*a*d^2*e*r
^2*x^r+576*a*d^2*e*r^4*x^r-528*a*d^2*e*r*x^r+4*b*e^3*n*r^4*(x^r)^3-480*a*d*
e^2*r*(x^r)^2-54*a*d*e^2*r^5*(x^r)^2+342*a*d*e^2*r^4*(x^r)^2-108*a*d^2*e*r^
5*x^r+264*b*d*e^2*n*r^2*(x^r)^2+444*b*d^2*e*n*r^2*x^r-192*b*d*e^2*n*r*(x^r)
^2-240*b*d^2*e*n*r*x^r-360*b*d^2*e*n*r^3*x^r+27*b*d*e^2*n*r^4*(x^r)^2-144*b
*d*e^2*n*r^3*(x^r)^2+108*b*d^2*e*n*r^4*x^r)/x^2/(-2+3*r)^2/(-1+r)^2/(-2+r)^
2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 981 vs. $2(174) = 348$.

Time = 0.36 (sec) , antiderivative size = 981, normalized size of antiderivative = 5.14

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^3}\,dx = -\frac{9\,(bd^3n+2\,ad^3)r^6-66\,(bd^3n+2\,ad^3)r^5+16\,bd^3n+193\,(bd^3n+2\,ad^3)r^4+32\,ad^3-288\,(bd^3n+2\,ad^3)}{}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(9*(b*d^3*n + 2*a*d^3)*r^6 - 66*(b*d^3*n + 2*a*d^3)*r^5 + 16*b*d^3*n +
 193*(b*d^3*n + 2*a*d^3)*r^4 + 32*a*d^3 - 288*(b*d^3*n + 2*a*d^3)*r^3 + 232
*(b*d^3*n + 2*a*d^3)*r^2 - 96*(b*d^3*n + 2*a*d^3)*r - 4*(3*a*e^3*r^5 - 4*b*
e^3*n - (b*e^3*n + 20*a*e^3)*r^4 - 8*a*e^3 + 3*(2*b*e^3*n + 17*a*e^3)*r^3 -
 (13*b*e^3*n + 62*a*e^3)*r^2 + 12*(b*e^3*n + 3*a*e^3)*r + (3*b*e^3*r^5 - 20
*b*e^3*r^4 + 51*b*e^3*r^3 - 62*b*e^3*r^2 + 36*b*e^3*r - 8*b*e^3)*log(c) + (
3*b*e^3*n*r^5 - 20*b*e^3*n*r^4 + 51*b*e^3*n*r^3 - 62*b*e^3*n*r^2 + 36*b*e^3
*n*r - 8*b*e^3*n)*log(x))*x^(3*r) - 3*(18*a*d*e^2*r^5 - 16*b*d*e^2*n - 3*(3
*b*d*e^2*n + 38*a*d*e^2)*r^4 - 32*a*d*e^2 + 16*(3*b*d*e^2*n + 17*a*d*e^2)*r
^3 - 8*(11*b*d*e^2*n + 38*a*d*e^2)*r^2 + 32*(2*b*d*e^2*n + 5*a*d*e^2)*r + 2
*(9*b*d*e^2*r^5 - 57*b*d*e^2*r^4 + 136*b*d*e^2*r^3 - 152*b*d*e^2*r^2 + 80*b
```

```
*d*e^2*r - 16*b*d*e^2)*log(c) + 2*(9*b*d*e^2*n*r^5 - 57*b*d*e^2*n*r^4 + 136
*b*d*e^2*n*r^3 - 152*b*d*e^2*n*r^2 + 80*b*d*e^2*n*r - 16*b*d*e^2*n)*log(x))
*x^(2*r) - 12*(9*a*d^2*e*r^5 - 4*b*d^2*e*n - 3*(3*b*d^2*e*n + 16*a*d^2*e)*r
^4 - 8*a*d^2*e + (30*b*d^2*e*n + 97*a*d^2*e)*r^3 - (37*b*d^2*e*n + 94*a*d^2
*e)*r^2 + 4*(5*b*d^2*e*n + 11*a*d^2*e)*r + (9*b*d^2*e*r^5 - 48*b*d^2*e*r^4
+ 97*b*d^2*e*r^3 - 94*b*d^2*e*r^2 + 44*b*d^2*e*r - 8*b*d^2*e)*log(c) + (9*b
*d^2*e*n*r^5 - 48*b*d^2*e*n*r^4 + 97*b*d^2*e*n*r^3 - 94*b*d^2*e*n*r^2 + 44*
b*d^2*e*n*r - 8*b*d^2*e*n)*log(x))*x^r + 2*(9*b*d^3*r^6 - 66*b*d^3*r^5 + 19
3*b*d^3*r^4 - 288*b*d^3*r^3 + 232*b*d^3*r^2 - 96*b*d^3*r + 16*b*d^3)*log(c)
 + 2*(9*b*d^3*n*r^6 - 66*b*d^3*n*r^5 + 193*b*d^3*n*r^4 - 288*b*d^3*n*r^3 +
232*b*d^3*n*r^2 - 96*b*d^3*n*r + 16*b*d^3*n)*log(x))/((9*r^6 - 66*r^5 + 193
*r^4 - 288*r^3 + 232*r^2 - 96*r + 16)*x^2)
```

Sympy [A] (verification not implemented)

Time = 46.14 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.82

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^3}\,dx$$
$$= -\frac{ad^3}{2x^2} + 3ad^2e\left(\begin{cases} \frac{x^r}{rx^2-2x^2} & \text{for } r\neq 2 \\ \frac{x^r\log(x)}{x^2} & \text{otherwise} \end{cases}\right) + 3ade^2\left(\begin{cases} \frac{x^{2r}}{2rx^2-2x^2} & \text{for } r\neq 1 \\ \frac{x^{2r}\log(x)}{x^2} & \text{otherwise} \end{cases}\right)$$
$$+ ae^3\left(\begin{cases} \frac{x^{3r}}{3rx^2-2x^2} & \text{for } r\neq \frac{2}{3} \\ \frac{x^{3r}\log(x)}{x^2} & \text{otherwise} \end{cases}\right) - \frac{bd^3n}{4x^2} - \frac{bd^3\log(cx^n)}{2x^2}$$
$$- 3bd^2en\left(\begin{cases} \frac{\begin{cases} \frac{x^{r-2}}{r-2} & \text{for } r\neq 2 \\ \log(x) & \text{otherwise} \end{cases}}{r-2} & \text{for } r>-\infty \wedge r<\infty \wedge r\neq 2 \\ \frac{\log(x)^2}{2} & \text{otherwise} \end{cases}\right)$$
$$+ 3bd^2e\left(\begin{cases} \frac{x^{r-2}}{r-2} & \text{for } r\neq 2 \\ \log(x) & \text{otherwise} \end{cases}\right)\log(cx^n)$$
$$- 3bde^2n\left(\begin{cases} \frac{\begin{cases} \frac{x^{2r-2}}{2r-2} & \text{for } r\neq 1 \\ \log(x) & \text{otherwise} \end{cases}}{2r-2} & \text{for } r>-\infty \wedge r<\infty \wedge r\neq 1 \\ \frac{\log(x)^2}{2} & \text{otherwise} \end{cases}\right)$$
$$+ 3bde^2\left(\begin{cases} \frac{x^{2r-2}}{2r-2} & \text{for } r\neq 1 \\ \log(x) & \text{otherwise} \end{cases}\right)\log(cx^n)$$
$$- be^3n\left(\begin{cases} \frac{\begin{cases} \frac{x^{3r-2}}{3r-2} & \text{for } r\neq \frac{2}{3} \\ \log(x) & \text{otherwise} \end{cases}}{3r-2} & \text{for } r>-\infty \wedge r<\infty \wedge r\neq \frac{2}{3} \\ \frac{\log(x)^2}{2} & \text{otherwise} \end{cases}\right)$$
$$+ be^3\left(\begin{cases} \frac{x^{3r-2}}{3r-2} & \text{for } r\neq \frac{2}{3} \\ \log(x) & \text{otherwise} \end{cases}\right)\log(cx^n)$$

```
[In] integrate((d+e*x**r)**3*(a+b*ln(c*x**n))/x**3,x)
```

```
[Out] -a*d**3/(2*x**2) + 3*a*d**2*e*Piecewise((x**r/(r*x**2 - 2*x**2), Ne(r, 2)),
 (x**r*log(x)/x**2, True)) + 3*a*d*e**2*Piecewise((x**(2*r)/(2*r*x**2 - 2*x
**2), Ne(r, 1)), (x**(2*r)*log(x)/x**2, True)) + a*e**3*Piecewise((x**(3*r)
```

```
/(3*r*x**2 - 2*x**2), Ne(r, 2/3)), (x**(3*r)*log(x)/x**2, True)) - b*d**3*n
/(4*x**2) - b*d**3*log(c*x**n)/(2*x**2) - 3*b*d**2*e*n*Piecewise((Piecewise
((x**(r - 2)/(r - 2), Ne(r, 2)), (log(x), True))/(r - 2), (r > -oo) & (r <
oo) & Ne(r, 2)), (log(x)**2/2, True)) + 3*b*d**2*e*Piecewise((x**(r - 2)/(r
 - 2), Ne(r, 2)), (log(x), True))*log(c*x**n) - 3*b*d*e**2*n*Piecewise((Pie
cewise((x**(2*r - 2)/(2*r - 2), Ne(r, 1)), (log(x), True))/(2*r - 2), (r >
-oo) & (r < oo) & Ne(r, 1)), (log(x)**2/2, True)) + 3*b*d*e**2*Piecewise((x
**(2*r - 2)/(2*r - 2), Ne(r, 1)), (log(x), True))*log(c*x**n) - b*e**3*n*Pi
ecewise((Piecewise((x**(3*r - 2)/(3*r - 2), Ne(r, 2/3)), (log(x), True))/(3
*r - 2), (r > -oo) & (r < oo) & Ne(r, 2/3)), (log(x)**2/2, True)) + b*e**3*
Piecewise((x**(3*r - 2)/(3*r - 2), Ne(r, 2/3)), (log(x), True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^3} dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-3>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^3} dx = \int \frac{(ex^r+d)^3(b\log(cx^n)+a)}{x^3} dx$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^r + d)^3*(b*log(c*x^n) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^3}\,dx = \int \frac{(d+e\,x^r)^3 (a+b\,\ln(c\,x^n))}{x^3}\,dx$$

```
[In] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x^3,x)
[Out] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x^3, x)
```

3.397 $\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 191

$$\int \frac{(d+ex^r)^3\,(a+b\log(cx^n))}{x^5}\,dx = -\frac{bd^3n}{16x^4} - \frac{3bde^2nx^{-2(2-r)}}{4(2-r)^2} - \frac{3bd^2enx^{-4+r}}{(4-r)^2} - \frac{be^3nx^{-4+3r}}{(4-3r)^2} - \frac{d^3(a+b\log(cx^n))}{4x^4} - \frac{3de^2x^{-2(2-r)}(a+b\log(cx^n))}{2(2-r)} - \frac{3d^2ex^{-4+r}(a+b\log(cx^n))}{4-r} - \frac{e^3x^{-4+3r}(a+b\log(cx^n))}{4-3r}$$

```
[Out] -1/16*b*d^3*n/x^4-3/4*b*d*e^2*n/(2-r)^2/(x^(4-2*r))-3*b*d^2*e*n*x^(-4+r)/(4
-r)^2-b*e^3*n*x^(-4+3*r)/(4-3*r)^2-1/4*d^3*(a+b*ln(c*x^n))/x^4-3/2*d*e^2*(a
+b*ln(c*x^n))/(2-r)/(x^(4-2*r))-3*d^2*e*x^(-4+r)*(a+b*ln(c*x^n))/(4-r)-e^3*
x^(-4+3*r)*(a+b*ln(c*x^n))/(4-3*r)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2372, 12, 14}

$$\int \frac{(d+ex^r)^3\,(a+b\log(cx^n))}{x^5}\,dx = -\frac{d^3(a+b\log(cx^n))}{4x^4} - \frac{3d^2ex^{r-4}(a+b\log(cx^n))}{4-r} - \frac{3de^2x^{-2(2-r)}(a+b\log(cx^n))}{2(2-r)} - \frac{e^3x^{3r-4}(a+b\log(cx^n))}{4-3r} - \frac{bd^3n}{16x^4} - \frac{3bd^2enx^{r-4}}{(4-r)^2} - \frac{3bde^2nx^{-2(2-r)}}{4(2-r)^2} - \frac{be^3nx^{3r-4}}{(4-3r)^2}$$

```
[In] Int[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x^5,x]
```

```
[Out] -1/16*(b*d^3*n)/x^4 - (3*b*d*e^2*n)/(4*(2 - r)^2*x^(2*(2 - r))) - (3*b*d^2*
e*n*x^(-4 + r))/(4 - r)^2 - (b*e^3*n*x^(-4 + 3*r))/(4 - 3*r)^2 - (d^3*(a +
b*Log[c*x^n]))/(4*x^4) - (3*d*e^2*(a + b*Log[c*x^n]))/(2*(2 - r)*x^(2*(2 -
r))) - (3*d^2*e*x^(-4 + r)*(a + b*Log[c*x^n]))/(4 - r) - (e^3*x^(-4 + 3*r)*
(a + b*Log[c*x^n]))/(4 - 3*r)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{4x^4}-\frac{3de^2x^{-2(2-r)}(a+b\log(cx^n))}{2(2-r)}-\frac{3d^2ex^{-4+r}(a+b\log(cx^n))}{4-r}\\&\quad-\frac{e^3x^{-4+3r}(a+b\log(cx^n))}{4-3r}-(bn)\int\frac{-d^3+\frac{12d^2ex^r}{-4+r}+\frac{6de^2x^{2r}}{-2+r}+\frac{4e^3x^{3r}}{-4+3r}}{4x^5}\,dx\\&=-\frac{d^3(a+b\log(cx^n))}{4x^4}-\frac{3de^2x^{-2(2-r)}(a+b\log(cx^n))}{2(2-r)}-\frac{3d^2ex^{-4+r}(a+b\log(cx^n))}{4-r}\\&\quad-\frac{e^3x^{-4+3r}(a+b\log(cx^n))}{4-3r}-\frac{1}{4}(bn)\int\frac{-d^3+\frac{12d^2ex^r}{-4+r}+\frac{6de^2x^{2r}}{-2+r}+\frac{4e^3x^{3r}}{-4+3r}}{x^5}\,dx\end{aligned}$$

$$= -\frac{d^3(a+b\log(cx^n))}{4x^4} - \frac{3de^2x^{-2(2-r)}(a+b\log(cx^n))}{2(2-r)} - \frac{3d^2ex^{-4+r}(a+b\log(cx^n))}{4-r} - \frac{e^3x^{-4+3r}(a+b\log(cx^n))}{4-3r} - \frac{1}{4}(bn)\int\left(-\frac{d^3}{x^5}+\frac{12d^2ex^{-5+r}}{-4+r}+\frac{6de^2x^{-5+2r}}{-2+r}+\frac{4e^3x^{-5+3r}}{-4+3r}\right)dx$$

$$= -\frac{bd^3n}{16x^4} - \frac{3bde^2nx^{-2(2-r)}}{4(2-r)^2} - \frac{3bd^2enx^{-4+r}}{(4-r)^2} - \frac{be^3nx^{-4+3r}}{(4-3r)^2} - \frac{d^3(a+b\log(cx^n))}{4x^4} - \frac{3de^2x^{-2(2-r)}(a+b\log(cx^n))}{2(2-r)} - \frac{3d^2ex^{-4+r}(a+b\log(cx^n))}{4-r} - \frac{e^3x^{-4+3r}(a+b\log(cx^n))}{4-3r}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.95

$$\int\frac{(d+ex^r)^3(a+b\log(cx^n))}{x^5}\,dx$$

$$= \frac{bn\left(-d^3-\frac{48d^2ex^r}{(-4+r)^2}-\frac{12de^2x^{2r}}{(-2+r)^2}-\frac{16e^3x^{3r}}{(4-3r)^2}\right)+a\left(-4d^3+\frac{48d^2ex^r}{-4+r}+\frac{24de^2x^{2r}}{-2+r}+\frac{16e^3x^{3r}}{-4+3r}\right)+4b\left(-d^3+\frac{12d^2ex^r}{-4+r}+\frac{6d}{-}\right.}{16x^4}$$

```
[In] Integrate[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x^5,x]
```

```
[Out] (b*n*(-d^3 - (48*d^2*e*x^r)/(-4 + r)^2 - (12*d*e^2*x^(2*r))/(-2 + r)^2 - (1
6*e^3*x^(3*r))/(4 - 3*r)^2) + a*(-4*d^3 + (48*d^2*e*x^r)/(-4 + r) + (24*d*e
^2*x^(2*r))/(-2 + r) + (16*e^3*x^(3*r))/(-4 + 3*r)) + 4*b*(-d^3 + (12*d^2*e
*x^r)/(-4 + r) + (6*d*e^2*x^(2*r))/(-2 + r) + (4*e^3*x^(3*r))/(-4 + 3*r))*L
og[c*x^n])/(16*x^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. 2(183) = 366.

Time = 3.44 (sec) , antiderivative size = 1039, normalized size of antiderivative = 5.44

method	result	size
parallelrisch	Expression too large to display	1039
risch	Expression too large to display	4027

```
[In] int((d+e*x^r)^3*(a+b*ln(c*x^n))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(4096*b*ln(c*x^n)*d^3+12288*b*d*e^2*ln(c*x^n)*(x^r)^2+4096*e^3*(x^r)^
3*a-2304*b*d^3*n*r^3+3712*b*d^3*n*r^2-3072*b*d^3*n*r+29184*a*d*e^2*r^2*(x^r
)^2-13056*a*d*e^2*r^3*(x^r)^2+12288*d*e^2*(x^r)^2*a+12288*d^2*e*x^r*a+4096*
```

```
a*d^3-3264*a*e^3*r^3*(x^r)^3+7936*a*e^3*r^2*(x^r)^3-9216*a*e^3*r*(x^r)^3-48
*a*e^3*r^5*(x^r)^3+640*a*e^3*r^4*(x^r)^3-48*(x^r)^3*ln(c*x^n)*b*e^3*r^5+640
*(x^r)^3*ln(c*x^n)*b*e^3*r^4-3264*(x^r)^3*ln(c*x^n)*b*e^3*r^3+7936*(x^r)^3*
ln(c*x^n)*b*e^3*r^2-9216*(x^r)^3*ln(c*x^n)*b*e^3*r+12288*b*d^2*e*ln(c*x^n)*
x^r+9*b*d^3*n*r^6-132*b*d^3*n*r^5+772*b*d^3*n*r^4-432*x^r*ln(c*x^n)*b*d^2*e
*r^5+4608*x^r*ln(c*x^n)*b*d^2*e*r^4-18624*x^r*ln(c*x^n)*b*d^2*e*r^3+36096*x
^r*ln(c*x^n)*b*d^2*e*r^2-33792*x^r*ln(c*x^n)*b*d^2*e*r-216*(x^r)^2*ln(c*x^n
)*b*d*e^2*r^5+2736*(x^r)^2*ln(c*x^n)*b*d*e^2*r^4-13056*(x^r)^2*ln(c*x^n)*b*
d*e^2*r^3+29184*(x^r)^2*ln(c*x^n)*b*d*e^2*r^2-30720*(x^r)^2*ln(c*x^n)*b*d*e
^2*r+1024*b*d^3*n+36*ln(c*x^n)*b*d^3*r^6-528*ln(c*x^n)*b*d^3*r^5+3088*ln(c*
x^n)*b*d^3*r^4-9216*ln(c*x^n)*b*d^3*r^3+14848*ln(c*x^n)*b*d^3*r^2-12288*ln(
c*x^n)*b*d^3*r+4096*e^3*b*ln(c*x^n)*(x^r)^3-9216*a*d^3*r^3+14848*a*d^3*r^2-
12288*a*d^3*r+36*a*d^3*r^6-528*a*d^3*r^5+3088*a*d^3*r^4-18624*a*d^2*e*r^3*x
^r+3072*b*d*e^2*n*(x^r)^2+3072*b*d^2*e*n*x^r+1024*b*e^3*n*(x^r)^3-192*b*e^3
*n*r^3*(x^r)^3+832*b*e^3*n*r^2*(x^r)^3-1536*b*e^3*n*r*(x^r)^3+36096*a*d^2*e
*r^2*x^r+4608*a*d^2*e*r^4*x^r-33792*a*d^2*e*r*x^r+16*b*e^3*n*r^4*(x^r)^3-30
720*a*d*e^2*r*(x^r)^2-216*a*d*e^2*r^5*(x^r)^2+2736*a*d*e^2*r^4*(x^r)^2-432*
a*d^2*e*r^5*x^r+4224*b*d*e^2*n*r^2*(x^r)^2+7104*b*d^2*e*n*r^2*x^r-6144*b*d*
e^2*n*r*(x^r)^2-7680*b*d^2*e*n*r*x^r-2880*b*d^2*e*n*r^3*x^r+108*b*d*e^2*n*r
^4*(x^r)^2-1152*b*d*e^2*n*r^3*(x^r)^2+432*b*d^2*e*n*r^4*x^r)/x^4/(-4+3*r)^2
/(-2+r)^2/(-4+r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(174) = 348$.

Time = 0.32 (sec) , antiderivative size = 980, normalized size of antiderivative = 5.13

$$\int \frac{(d+ex^r)^3\,(a+b\log(cx^n))}{x^5}\,dx = -\frac{9\,(bd^3n+4\,ad^3)r^6-132\,(bd^3n+4\,ad^3)r^5+1024\,bd^3n+772\,(bd^3n+4\,ad^3)r^4+4096\,ad^3-2304\,(bd^3n+\cdots}{}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(9*(b*d^3*n + 4*a*d^3)*r^6 - 132*(b*d^3*n + 4*a*d^3)*r^5 + 1024*b*d^3
*n + 772*(b*d^3*n + 4*a*d^3)*r^4 + 4096*a*d^3 - 2304*(b*d^3*n + 4*a*d^3)*r^
3 + 3712*(b*d^3*n + 4*a*d^3)*r^2 - 3072*(b*d^3*n + 4*a*d^3)*r - 16*(3*a*e^3
*r^5 - 64*b*e^3*n - (b*e^3*n + 40*a*e^3)*r^4 - 256*a*e^3 + 12*(b*e^3*n + 17
*a*e^3)*r^3 - 4*(13*b*e^3*n + 124*a*e^3)*r^2 + 96*(b*e^3*n + 6*a*e^3)*r + (
3*b*e^3*r^5 - 40*b*e^3*r^4 + 204*b*e^3*r^3 - 496*b*e^3*r^2 + 576*b*e^3*r -
256*b*e^3)*log(c) + (3*b*e^3*n*r^5 - 40*b*e^3*n*r^4 + 204*b*e^3*n*r^3 - 496
*b*e^3*n*r^2 + 576*b*e^3*n*r - 256*b*e^3*n)*log(x))*x^(3*r) - 12*(18*a*d*e^
2*r^5 - 256*b*d*e^2*n - 3*(3*b*d*e^2*n + 76*a*d*e^2)*r^4 - 1024*a*d*e^2 + 3
2*(3*b*d*e^2*n + 34*a*d*e^2)*r^3 - 32*(11*b*d*e^2*n + 76*a*d*e^2)*r^2 + 512
```

```
*(b*d*e^2*n + 5*a*d*e^2)*r + 2*(9*b*d*e^2*r^5 - 114*b*d*e^2*r^4 + 544*b*d*e
^2*r^3 - 1216*b*d*e^2*r^2 + 1280*b*d*e^2*r - 512*b*d*e^2)*log(c) + 2*(9*b*d
*e^2*n*r^5 - 114*b*d*e^2*n*r^4 + 544*b*d*e^2*n*r^3 - 1216*b*d*e^2*n*r^2 + 1
280*b*d*e^2*n*r - 512*b*d*e^2*n)*log(x))*x^(2*r) - 48*(9*a*d^2*e*r^5 - 64*b
*d^2*e*n - 3*(3*b*d^2*e*n + 32*a*d^2*e)*r^4 - 256*a*d^2*e + 4*(15*b*d^2*e*n
 + 97*a*d^2*e)*r^3 - 4*(37*b*d^2*e*n + 188*a*d^2*e)*r^2 + 32*(5*b*d^2*e*n +
 22*a*d^2*e)*r + (9*b*d^2*e*r^5 - 96*b*d^2*e*r^4 + 388*b*d^2*e*r^3 - 752*b*
d^2*e*r^2 + 704*b*d^2*e*r - 256*b*d^2*e)*log(c) + (9*b*d^2*e*n*r^5 - 96*b*d
^2*e*n*r^4 + 388*b*d^2*e*n*r^3 - 752*b*d^2*e*n*r^2 + 704*b*d^2*e*n*r - 256*
b*d^2*e*n)*log(x))*x^r + 4*(9*b*d^3*r^6 - 132*b*d^3*r^5 + 772*b*d^3*r^4 - 2
304*b*d^3*r^3 + 3712*b*d^3*r^2 - 3072*b*d^3*r + 1024*b*d^3)*log(c) + 4*(9*b
*d^3*n*r^6 - 132*b*d^3*n*r^5 + 772*b*d^3*n*r^4 - 2304*b*d^3*n*r^3 + 3712*b*
d^3*n*r^2 - 3072*b*d^3*n*r + 1024*b*d^3*n)*log(x))/((9*r^6 - 132*r^5 + 772*
r^4 - 2304*r^3 + 3712*r^2 - 3072*r + 1024)*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x**r)**3*(a+b*ln(c*x**n))/x**5,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^5}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^5,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-5>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{(d + ex^r)^3 (a + b \log(cx^n))}{x^5} \, dx = \int \frac{(ex^r + d)^3 (b \log(cx^n) + a)}{x^5} \, dx$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^5,x, algorithm="giac")
[Out] integrate((e*x^r + d)^3*(b*log(c*x^n) + a)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex^r)^3 (a + b \log(cx^n))}{x^5} \, dx = \int \frac{(d + e\,x^r)^3 \,(a + b\,\ln(c\,x^n))}{x^5} \, dx$$

```
[In] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x^5,x)
[Out] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x^5, x)
```

3.398 $\int x^4(d+ex^r)^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 151

$$\int x^4(d+ex^r)^3(a+b\log(cx^n))\,dx=-\frac{1}{25}bd^3nx^5-\frac{3bd^2enx^{5+r}}{(5+r)^2}-\frac{3bde^2nx^{5+2r}}{(5+2r)^2}-\frac{be^3nx^{5+3r}}{(5+3r)^2}+\frac{1}{5}\left(d^3x^5+\frac{15d^2ex^{5+r}}{5+r}+\frac{15de^2x^{5+2r}}{5+2r}+\frac{5e^3x^{5+3r}}{5+3r}\right)(a+b\log(cx^n))$$

```
[Out] -1/25*b*d^3*n*x^5-3*b*d^2*e*n*x^(5+r)/(5+r)^2-3*b*d*e^2*n*x^(5+2*r)/(5+2*r)
^2-b*e^3*n*x^(5+3*r)/(5+3*r)^2+1/5*(d^3*x^5+15*d^2*e*x^(5+r)/(5+r)+15*d*e^2
*x^(5+2*r)/(5+2*r)+5*e^3*x^(5+3*r)/(5+3*r))*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2371, 12, 14}

$$\int x^4(d+ex^r)^3(a+b\log(cx^n))\,dx=\frac{1}{5}\left(d^3x^5+\frac{15d^2ex^{r+5}}{r+5}+\frac{15de^2x^{2r+5}}{2r+5}+\frac{5e^3x^{3r+5}}{3r+5}\right)(a+b\log(cx^n))-\frac{1}{25}bd^3nx^5-\frac{3bd^2enx^{r+5}}{(r+5)^2}-\frac{3bde^2nx^{2r+5}}{(2r+5)^2}-\frac{be^3nx^{3r+5}}{(3r+5)^2}$$

```
[In] Int[x^4*(d + e*x^r)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/25*(b*d^3*n*x^5) - (3*b*d^2*e*n*x^(5 + r))/(5 + r)^2 - (3*b*d*e^2*n*x^(5
 + 2*r))/(5 + 2*r)^2 - (b*e^3*n*x^(5 + 3*r))/(5 + 3*r)^2 + ((d^3*x^5 + (15*
```

```
d^2*e*x^(5 + r))/(5 + r) + (15*d*e^2*x^(5 + 2*r))/(5 + 2*r) + (5*e^3*x^(5 +
 3*r))/(5 + 3*r))*(a + b*Log[c*x^n]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}\left(d^3x^5+\frac{15d^2ex^{5+r}}{5+r}+\frac{15de^2x^{5+2r}}{5+2r}+\frac{5e^3x^{5+3r}}{5+3r}\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int\frac{1}{5}x^4\left(d^3+\frac{15d^2ex^r}{5+r}+\frac{15de^2x^{2r}}{5+2r}+\frac{5e^3x^{3r}}{5+3r}\right)dx \\
&= \frac{1}{5}\left(d^3x^5+\frac{15d^2ex^{5+r}}{5+r}+\frac{15de^2x^{5+2r}}{5+2r}+\frac{5e^3x^{5+3r}}{5+3r}\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{5}(bn)\int x^4\left(d^3+\frac{15d^2ex^r}{5+r}+\frac{15de^2x^{2r}}{5+2r}+\frac{5e^3x^{3r}}{5+3r}\right)dx \\
&= \frac{1}{5}\left(d^3x^5+\frac{15d^2ex^{5+r}}{5+r}+\frac{15de^2x^{5+2r}}{5+2r}+\frac{5e^3x^{5+3r}}{5+3r}\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{5}(bn)\int\left(d^3x^4+\frac{15de^2x^{2(2+r)}}{5+2r}+\frac{15d^2ex^{4+r}}{5+r}+\frac{5e^3x^{4+3r}}{5+3r}\right)dx \\
&= -\frac{1}{25}bd^3nx^5-\frac{3bd^2enx^{5+r}}{(5+r)^2}-\frac{3bde^2nx^{5+2r}}{(5+2r)^2}-\frac{be^3nx^{5+3r}}{(5+3r)^2} \\
&\quad +\frac{1}{5}\left(d^3x^5+\frac{15d^2ex^{5+r}}{5+r}+\frac{15de^2x^{5+2r}}{5+2r}+\frac{5e^3x^{5+3r}}{5+3r}\right)(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.22

$$\int x^4(d+ex^r)^3(a+b\log(cx^n))\,dx = \frac{1}{25}x^5\left(bn\left(-d^3-\frac{75d^2ex^r}{(5+r)^2}-\frac{75de^2x^{2r}}{(5+2r)^2}-\frac{25e^3x^{3r}}{(5+3r)^2}\right)+5a\left(d^3+\frac{15d^2ex^r}{5+r}+\frac{15de^2x^{2r}}{5+2r}+\frac{5e^3x^{3r}}{5+3r}\right)+5b\left(d^3+\frac{15d^2ex^r}{5+r}+\frac{15de^2x^{2r}}{5+2r}+\frac{5e^3x^{3r}}{5+3r}\right)\log(cx^n)\right)$$

```
[In] Integrate[x^4*(d + e*x^r)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^5*(b*n*(-d^3 - (75*d^2*e*x^r)/(5 + r)^2 - (75*d*e^2*x^(2*r))/(5 + 2*r)^2
 - (25*e^3*x^(3*r))/(5 + 3*r)^2) + 5*a*(d^3 + (15*d^2*e*x^r)/(5 + r) + (15*
d*e^2*x^(2*r))/(5 + 2*r) + (5*e^3*x^(3*r))/(5 + 3*r)) + 5*b*(d^3 + (15*d^2*
e*x^r)/(5 + r) + (15*d*e^2*x^(2*r))/(5 + 2*r) + (5*e^3*x^(3*r))/(5 + 3*r))*
Log[c*x^n]))/25
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. 2(147) = 294.

Time = 26.56 (sec) , antiderivative size = 1269, normalized size of antiderivative = 8.40

method	result	size
parallelrisch	Expression too large to display	1269
risch	Expression too large to display	4031

```
[In] int(x^4*(d+e*x^r)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/25*(-78125*x^5*a*d^3-31875*x^5*(x^r)^3*ln(c*x^n)*b*e^3*r^3-187500*x^5*ln
(c*x^n)*b*d^3*r-300*x^5*(x^r)^3*a*e^3*r^5-96875*x^5*(x^r)^3*a*e^3*r^2-14062
5*x^5*(x^r)^3*a*e^3*r-181875*x^5*x^r*r^3*a*d^2*e-515625*e*d^2*b*ln(c*x^n)*x
^r*r*x^5-234375*x^5*d^2*e*x^r*b*ln(c*x^n)-234375*x^5*d*e^2*(x^r)^2*b*ln(c*x
^n)-78125*x^5*e^3*(x^r)^3*a-78125*x^5*b*ln(c*x^n)*d^3-78125*x^5*e^3*(x^r)^3
*b*ln(c*x^n)-234375*x^5*d^2*e*x^r*a-234375*x^5*d*e^2*(x^r)^2*a-181250*x^5*l
n(c*x^n)*b*d^3*r^2-24125*x^5*ln(c*x^n)*b*d^3*r^4-90000*x^5*ln(c*x^n)*b*d^3*
r^3-5000*x^5*(x^r)^3*a*e^3*r^4-468750*x^5*(x^r)^2*a*d*e^2*r-2700*x^5*x^r*ln
(c*x^n)*b*d^2*e*r^5-36000*x^5*x^r*ln(c*x^n)*b*d^2*e*r^4-181875*x^5*x^r*ln(c
*x^n)*b*d^2*e*r^3-440625*x^5*x^r*ln(c*x^n)*b*d^2*e*r^2+36*x^5*b*d^3*n*r^6+6
60*x^5*b*d^3*n*r^5+4825*x^5*b*d^3*n*r^4+18000*x^5*b*d^3*n*r^3+36250*x^5*b*d
^3*n*r^2+37500*x^5*b*d^3*n*r+15625*x^5*(x^r)^3*b*e^3*n-31875*x^5*(x^r)^3*a*
e^3*r^3-180*x^5*a*d^3*r^6-3300*x^5*a*d^3*r^5-24125*x^5*a*d^3*r^4-90000*x^5*
a*d^3*r^3-181250*x^5*a*d^3*r^2-187500*x^5*a*d^3*r-180*x^5*ln(c*x^n)*b*d^3*r
```

```
^6-3300*x^5*ln(c*x^n)*b*d^3*r^5+15625*b*d^3*n*x^5-1350*x^5*(x^r)^2*ln(c*x^n
)*b*d*e^2*r^5-21375*x^5*(x^r)^2*ln(c*x^n)*b*d*e^2*r^4-127500*x^5*(x^r)^2*ln
(c*x^n)*b*d*e^2*r^3-356250*x^5*(x^r)^2*ln(c*x^n)*b*d*e^2*r^2-1350*x^5*(x^r)
^2*a*d*e^2*r^5-96875*x^5*(x^r)^3*ln(c*x^n)*b*e^3*r^2-468750*e^2*d*b*ln(c*x^
n)*(x^r)^2*x^5*r-356250*x^5*(x^r)^2*a*d*e^2*r^2-440625*x^5*x^r*a*d^2*e*r^2-
515625*x^5*x^r*a*d^2*e*r-300*x^5*(x^r)^3*ln(c*x^n)*b*e^3*r^5-2700*x^5*x^r*a
*d^2*e*r^5-36000*x^5*x^r*a*d^2*e*r^4-5000*x^5*(x^r)^3*ln(c*x^n)*b*e^3*r^4+1
00*x^5*(x^r)^3*b*e^3*n*r^4+1500*x^5*(x^r)^3*b*e^3*n*r^3+8125*x^5*(x^r)^3*b*
e^3*n*r^2+18750*x^5*(x^r)^3*b*e^3*n*r+46875*x^5*x^r*b*d^2*e*n+46875*x^5*(x^
r)^2*b*d*e^2*n-21375*x^5*(x^r)^2*a*d*e^2*r^4-127500*x^5*(x^r)^2*a*d*e^2*r^3
-140625*x^5*(x^r)^3*ln(c*x^n)*b*e^3*r+2700*x^5*x^r*b*d^2*e*n*r^4+22500*x^5*
x^r*b*d^2*e*n*r^3+69375*x^5*x^r*b*d^2*e*n*r^2+93750*x^5*x^r*b*d^2*e*n*r+675
*x^5*(x^r)^2*b*d*e^2*n*r^4+9000*x^5*(x^r)^2*b*d*e^2*n*r^3+41250*x^5*(x^r)^2
*b*d*e^2*n*r^2+75000*x^5*(x^r)^2*b*d*e^2*n*r)/(4*r^2+20*r+25)/(r^2+10*r+25)
/(9*r^2+30*r+25)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1023 vs. $2(147) = 294$.

Time = 0.36 (sec) , antiderivative size = 1023, normalized size of antiderivative = 6.77

$$\int x^4(d+ex^r)^3\left(a+b\log\left(cx^n\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 1/25*(5*(36*b*d^3*r^6 + 660*b*d^3*r^5 + 4825*b*d^3*r^4 + 18000*b*d^3*r^3 +
36250*b*d^3*r^2 + 37500*b*d^3*r + 15625*b*d^3)*x^5*log(c) + 5*(36*b*d^3*n*r
^6 + 660*b*d^3*n*r^5 + 4825*b*d^3*n*r^4 + 18000*b*d^3*n*r^3 + 36250*b*d^3*n
*r^2 + 37500*b*d^3*n*r + 15625*b*d^3*n)*x^5*log(x) - (36*(b*d^3*n - 5*a*d^3
)*r^6 + 660*(b*d^3*n - 5*a*d^3)*r^5 + 15625*b*d^3*n + 4825*(b*d^3*n - 5*a*d
^3)*r^4 - 78125*a*d^3 + 18000*(b*d^3*n - 5*a*d^3)*r^3 + 36250*(b*d^3*n - 5*
a*d^3)*r^2 + 37500*(b*d^3*n - 5*a*d^3)*r)*x^5 + 25*((12*b*e^3*r^5 + 200*b*e
^3*r^4 + 1275*b*e^3*r^3 + 3875*b*e^3*r^2 + 5625*b*e^3*r + 3125*b*e^3)*x^5*l
og(c) + (12*b*e^3*n*r^5 + 200*b*e^3*n*r^4 + 1275*b*e^3*n*r^3 + 3875*b*e^3*n
*r^2 + 5625*b*e^3*n*r + 3125*b*e^3*n)*x^5*log(x) + (12*a*e^3*r^5 - 625*b*e^
3*n - 4*(b*e^3*n - 50*a*e^3)*r^4 + 3125*a*e^3 - 15*(4*b*e^3*n - 85*a*e^3)*r
^3 - 25*(13*b*e^3*n - 155*a*e^3)*r^2 - 375*(2*b*e^3*n - 15*a*e^3)*r)*x^5)*x
^(3*r) + 75*((18*b*d*e^2*r^5 + 285*b*d*e^2*r^4 + 1700*b*d*e^2*r^3 + 4750*b*
d*e^2*r^2 + 6250*b*d*e^2*r + 3125*b*d*e^2)*x^5*log(c) + (18*b*d*e^2*n*r^5 +
 285*b*d*e^2*n*r^4 + 1700*b*d*e^2*n*r^3 + 4750*b*d*e^2*n*r^2 + 6250*b*d*e^2
*n*r + 3125*b*d*e^2*n)*x^5*log(x) + (18*a*d*e^2*r^5 - 625*b*d*e^2*n - 3*(3*
b*d*e^2*n - 95*a*d*e^2)*r^4 + 3125*a*d*e^2 - 20*(6*b*d*e^2*n - 85*a*d*e^2)*
r^3 - 50*(11*b*d*e^2*n - 95*a*d*e^2)*r^2 - 250*(4*b*d*e^2*n - 25*a*d*e^2)*r
)*x^5)*x^(2*r) + 75*((36*b*d^2*e*r^5 + 480*b*d^2*e*r^4 + 2425*b*d^2*e*r^3 +
```

```
5875*b*d^2*e*r^2 + 6875*b*d^2*e*r + 3125*b*d^2*e)*x^5*log(c) + (36*b*d^2*e
*n*r^5 + 480*b*d^2*e*n*r^4 + 2425*b*d^2*e*n*r^3 + 5875*b*d^2*e*n*r^2 + 6875
*b*d^2*e*n*r + 3125*b*d^2*e*n)*x^5*log(x) + (36*a*d^2*e*r^5 - 625*b*d^2*e*n
 - 12*(3*b*d^2*e*n - 40*a*d^2*e)*r^4 + 3125*a*d^2*e - 25*(12*b*d^2*e*n - 97
*a*d^2*e)*r^3 - 25*(37*b*d^2*e*n - 235*a*d^2*e)*r^2 - 625*(2*b*d^2*e*n - 11
*a*d^2*e)*r)*x^5)*x^r)/(36*r^6 + 660*r^5 + 4825*r^4 + 18000*r^3 + 36250*r^2
 + 37500*r + 15625)
```

Sympy [F(-1)]

Timed out.

$$\int x^4(d+ex^r)^3\left(a+b\log\left(cx^n\right)\right)\,dx=\text{Timed out}$$

```
[In] integrate(x**4*(d+e*x**r)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.51

$$\begin{aligned}\int x^4(d+ex^r)^3\left(a+b\log\left(cx^n\right)\right)\,dx=&-\frac{1}{25}\,bd^3nx^5+\frac{1}{5}\,bd^3x^5\log\left(cx^n\right)\\&+\frac{1}{5}\,ad^3x^5+\frac{be^3x^{3\,r+5}\log\left(cx^n\right)}{3\,r+5}\\&+\frac{3\,bde^2x^{2\,r+5}\log\left(cx^n\right)}{2\,r+5}+\frac{3\,bd^2ex^{r+5}\log\left(cx^n\right)}{r+5}\\&-\frac{be^3nx^{3\,r+5}}{(3\,r+5)^2}+\frac{ae^3x^{3\,r+5}}{3\,r+5}-\frac{3\,bde^2nx^{2\,r+5}}{(2\,r+5)^2}\\&+\frac{3\,ade^2x^{2\,r+5}}{2\,r+5}-\frac{3\,bd^2enx^{r+5}}{(r+5)^2}+\frac{3\,ad^2ex^{r+5}}{r+5}\end{aligned}$$

```
[In] integrate(x^4*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/25*b*d^3*n*x^5 + 1/5*b*d^3*x^5*log(c*x^n) + 1/5*a*d^3*x^5 + b*e^3*x^(3*r
 + 5)*log(c*x^n)/(3*r + 5) + 3*b*d*e^2*x^(2*r + 5)*log(c*x^n)/(2*r + 5) + 3
*b*d^2*e*x^(r + 5)*log(c*x^n)/(r + 5) - b*e^3*n*x^(3*r + 5)/(3*r + 5)^2 + a
*e^3*x^(3*r + 5)/(3*r + 5) - 3*b*d*e^2*n*x^(2*r + 5)/(2*r + 5)^2 + 3*a*d*e^
2*x^(2*r + 5)/(2*r + 5) - 3*b*d^2*e*n*x^(r + 5)/(r + 5)^2 + 3*a*d^2*e*x^(r
+ 5)/(r + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1611 vs. $2(147) = 294$.

Time = 0.35 (sec) , antiderivative size = 1611, normalized size of antiderivative = 10.67

$$\int x^4(d+ex^r)^3\,(a+b\log{(cx^n)})\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/25*(300*b*e^3*n*r^5*x^5*x^(3*r)*log(x) + 1350*b*d*e^2*n*r^5*x^5*x^(2*r)*l
og(x) + 2700*b*d^2*e*n*r^5*x^5*x^r*log(x) + 180*b*d^3*n*r^6*x^5*log(x) - 36
*b*d^3*n*r^6*x^5 + 300*b*e^3*r^5*x^5*x^(3*r)*log(c) + 1350*b*d*e^2*r^5*x^5*
x^(2*r)*log(c) + 2700*b*d^2*e*r^5*x^5*x^r*log(c) + 180*b*d^3*r^6*x^5*log(c)
 + 5000*b*e^3*n*r^4*x^5*x^(3*r)*log(x) + 21375*b*d*e^2*n*r^4*x^5*x^(2*r)*lo
g(x) + 36000*b*d^2*e*n*r^4*x^5*x^r*log(x) + 3300*b*d^3*n*r^5*x^5*log(x) - 1
00*b*e^3*n*r^4*x^5*x^(3*r) + 300*a*e^3*r^5*x^5*x^(3*r) - 675*b*d*e^2*n*r^4*
x^5*x^(2*r) + 1350*a*d*e^2*r^5*x^5*x^(2*r) - 2700*b*d^2*e*n*r^4*x^5*x^r + 2
700*a*d^2*e*r^5*x^5*x^r - 660*b*d^3*n*r^5*x^5 + 180*a*d^3*r^6*x^5 + 5000*b*
e^3*r^4*x^5*x^(3*r)*log(c) + 21375*b*d*e^2*r^4*x^5*x^(2*r)*log(c) + 36000*b
*d^2*e*r^4*x^5*x^r*log(c) + 3300*b*d^3*r^5*x^5*log(c) + 31875*b*e^3*n*r^3*x
^5*x^(3*r)*log(x) + 127500*b*d*e^2*n*r^3*x^5*x^(2*r)*log(x) + 181875*b*d^2*
e*n*r^3*x^5*x^r*log(x) + 24125*b*d^3*n*r^4*x^5*log(x) - 1500*b*e^3*n*r^3*x^
5*x^(3*r) + 5000*a*e^3*r^4*x^5*x^(3*r) - 9000*b*d*e^2*n*r^3*x^5*x^(2*r) + 2
1375*a*d*e^2*r^4*x^5*x^(2*r) - 22500*b*d^2*e*n*r^3*x^5*x^r + 36000*a*d^2*e*
r^4*x^5*x^r - 4825*b*d^3*n*r^4*x^5 + 3300*a*d^3*r^5*x^5 + 31875*b*e^3*r^3*x
^5*x^(3*r)*log(c) + 127500*b*d*e^2*r^3*x^5*x^(2*r)*log(c) + 181875*b*d^2*e*
r^3*x^5*x^r*log(c) + 24125*b*d^3*r^4*x^5*log(c) + 96875*b*e^3*n*r^2*x^5*x^(
3*r)*log(x) + 356250*b*d*e^2*n*r^2*x^5*x^(2*r)*log(x) + 440625*b*d^2*e*n*r^
2*x^5*x^r*log(x) + 90000*b*d^3*n*r^3*x^5*log(x) - 8125*b*e^3*n*r^2*x^5*x^(3
*r) + 31875*a*e^3*r^3*x^5*x^(3*r) - 41250*b*d*e^2*n*r^2*x^5*x^(2*r) + 12750
0*a*d*e^2*r^3*x^5*x^(2*r) - 69375*b*d^2*e*n*r^2*x^5*x^r + 181875*a*d^2*e*r^
3*x^5*x^r - 18000*b*d^3*n*r^3*x^5 + 24125*a*d^3*r^4*x^5 + 96875*b*e^3*r^2*x
^5*x^(3*r)*log(c) + 356250*b*d*e^2*r^2*x^5*x^(2*r)*log(c) + 440625*b*d^2*e*
r^2*x^5*x^r*log(c) + 90000*b*d^3*r^3*x^5*log(c) + 140625*b*e^3*n*r*x^5*x^(3
*r)*log(x) + 468750*b*d*e^2*n*r*x^5*x^(2*r)*log(x) + 515625*b*d^2*e*n*r*x^5
*x^r*log(x) + 181250*b*d^3*n*r^2*x^5*log(x) - 18750*b*e^3*n*r*x^5*x^(3*r) +
 96875*a*e^3*r^2*x^5*x^(3*r) - 75000*b*d*e^2*n*r*x^5*x^(2*r) + 356250*a*d*e
^2*r^2*x^5*x^(2*r) - 93750*b*d^2*e*n*r*x^5*x^r + 440625*a*d^2*e*r^2*x^5*x^r
 - 36250*b*d^3*n*r^2*x^5 + 90000*a*d^3*r^3*x^5 + 140625*b*e^3*r*x^5*x^(3*r)
*log(c) + 468750*b*d*e^2*r*x^5*x^(2*r)*log(c) + 515625*b*d^2*e*r*x^5*x^r*lo
g(c) + 181250*b*d^3*r^2*x^5*log(c) + 78125*b*e^3*n*x^5*x^(3*r)*log(x) + 234
375*b*d*e^2*n*x^5*x^(2*r)*log(x) + 234375*b*d^2*e*n*x^5*x^r*log(x) + 187500
*b*d^3*n*r*x^5*log(x) - 15625*b*e^3*n*x^5*x^(3*r) + 140625*a*e^3*r*x^5*x^(3
*r) - 46875*b*d*e^2*n*x^5*x^(2*r) + 468750*a*d*e^2*r*x^5*x^(2*r) - 46875*b*
```

```
d^2*e*n*x^5*x^r + 515625*a*d^2*e*r*x^5*x^r - 37500*b*d^3*n*r*x^5 + 181250*a
*d^3*r^2*x^5 + 78125*b*e^3*x^5*x^(3*r)*log(c) + 234375*b*d*e^2*x^5*x^(2*r)*
log(c) + 234375*b*d^2*e*x^5*x^r*log(c) + 187500*b*d^3*r*x^5*log(c) + 78125*
b*d^3*n*x^5*log(x) + 78125*a*e^3*x^5*x^(3*r) + 234375*a*d*e^2*x^5*x^(2*r) +
 234375*a*d^2*e*x^5*x^r - 15625*b*d^3*n*x^5 + 187500*a*d^3*r*x^5 + 78125*b*
d^3*x^5*log(c) + 78125*a*d^3*x^5)/(36*r^6 + 660*r^5 + 4825*r^4 + 18000*r^3
+ 36250*r^2 + 37500*r + 15625)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex^r)^3\left(a+b\log\left(cx^n\right)\right)\,dx=\int x^4\left(d+e\,x^r\right)^3\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

```
[In] int(x^4*(d + e*x^r)^3*(a + b*log(c*x^n)),x)
[Out] int(x^4*(d + e*x^r)^3*(a + b*log(c*x^n)), x)
```

3.399 $\int x^2(d+ex^r)^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int x^2(d+ex^r)^3(a+b\log(cx^n))\,dx=-\frac{1}{9}bd^3nx^3-\frac{be^3nx^{3(1+r)}}{9(1+r)^2}-\frac{3bd^2enx^{3+r}}{(3+r)^2}-\frac{3bde^2nx^{3+2r}}{(3+2r)^2}+\frac{1}{3}\left(d^3x^3+\frac{e^3x^{3(1+r)}}{1+r}+\frac{9d^2ex^{3+r}}{3+r}+\frac{9de^2x^{3+2r}}{3+2r}\right)(a+b\log(cx^n))$$

```
[Out] -1/9*b*d^3*n*x^3-1/9*b*e^3*n*x^(3+3*r)/(1+r)^2-3*b*d^2*e*n*x^(3+r)/(3+r)^2-
3*b*d*e^2*n*x^(3+2*r)/(3+2*r)^2+1/3*(d^3*x^3+e^3*x^(3+3*r)/(1+r)+9*d^2*e*x^
(3+r)/(3+r)+9*d*e^2*x^(3+2*r)/(3+2*r))*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2371, 12, 14}

$$\int x^2(d+ex^r)^3(a+b\log(cx^n))\,dx=\frac{1}{3}\left(d^3x^3+\frac{9d^2ex^{r+3}}{r+3}+\frac{9de^2x^{2r+3}}{2r+3}+\frac{e^3x^{3(r+1)}}{r+1}\right)(a+b\log(cx^n))-\frac{1}{9}bd^3nx^3-\frac{3bd^2enx^{r+3}}{(r+3)^2}-\frac{3bde^2nx^{2r+3}}{(2r+3)^2}-\frac{be^3nx^{3(r+1)}}{9(r+1)^2}$$

```
[In] Int[x^2*(d + e*x^r)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/9*(b*d^3*n*x^3) - (b*e^3*n*x^(3*(1 + r)))/(9*(1 + r)^2) - (3*b*d^2*e*n*x
^(3 + r))/(3 + r)^2 - (3*b*d*e^2*n*x^(3 + 2*r))/(3 + 2*r)^2 + ((d^3*x^3 + (
```

```
e^3*x^(3*(1 + r)))/(1 + r) + (9*d^2*e*x^(3 + r))/(3 + r) + (9*d*e^2*x^(3 +
2*r))/(3 + 2*r))*(a + b*Log[c*x^n]))/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2371

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\left(d^3x^3+\frac{e^3x^{3(1+r)}}{1+r}+\frac{9d^2ex^{3+r}}{3+r}+\frac{9de^2x^{3+2r}}{3+2r}\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int \frac{1}{3}x^2\left(d^3+\frac{9d^2ex^r}{3+r}+\frac{9de^2x^{2r}}{3+2r}+\frac{e^3x^{3r}}{1+r}\right)dx \\
&= \frac{1}{3}\left(d^3x^3+\frac{e^3x^{3(1+r)}}{1+r}+\frac{9d^2ex^{3+r}}{3+r}+\frac{9de^2x^{3+2r}}{3+2r}\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{3}(bn)\int x^2\left(d^3+\frac{9d^2ex^r}{3+r}+\frac{9de^2x^{2r}}{3+2r}+\frac{e^3x^{3r}}{1+r}\right)dx \\
&= \frac{1}{3}\left(d^3x^3+\frac{e^3x^{3(1+r)}}{1+r}+\frac{9d^2ex^{3+r}}{3+r}+\frac{9de^2x^{3+2r}}{3+2r}\right)(a+b\log(cx^n)) \\
&\quad -\frac{1}{3}(bn)\int\left(d^3x^2+\frac{9de^2x^{2(1+r)}}{3+2r}+\frac{9d^2ex^{2+r}}{3+r}+\frac{e^3x^{2+3r}}{1+r}\right)dx \\
&= -\frac{1}{9}bd^3nx^3-\frac{be^3nx^{3(1+r)}}{9(1+r)^2}-\frac{3bd^2enx^{3+r}}{(3+r)^2}-\frac{3bde^2nx^{3+2r}}{(3+2r)^2} \\
&\quad +\frac{1}{3}\left(d^3x^3+\frac{e^3x^{3(1+r)}}{1+r}+\frac{9d^2ex^{3+r}}{3+r}+\frac{9de^2x^{3+2r}}{3+2r}\right)(a+b\log(cx^n))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.19

$$\int x^2(d+ex^r)^3\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{1}{9}x^3\left(bn\left(-d^3-\frac{27d^2ex^r}{(3+r)^2}-\frac{27de^2x^{2r}}{(3+2r)^2}-\frac{e^3x^{3r}}{(1+r)^2}\right)+3a\left(d^3+\frac{9d^2ex^r}{3+r}+\frac{9de^2x^{2r}}{3+2r}+\frac{e^3x^{3r}}{1+r}\right)+3b\left(d^3+\frac{9d^2ex^r}{3+r}+\frac{9de^2x^{2r}}{3+2r}+\frac{e^3x^{3r}}{1+r}\right)\log\left(cx^n\right)\right)$$

```
[In] Integrate[x^2*(d + e*x^r)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] (x^3*(b*n*(-d^3 - (27*d^2*e*x^r)/(3 + r)^2 - (27*d*e^2*x^(2*r))/(3 + 2*r)^2
 - (e^3*x^(3*r))/(1 + r)^2) + 3*a*(d^3 + (9*d^2*e*x^r)/(3 + r) + (9*d*e^2*x
^(2*r))/(3 + 2*r) + (e^3*x^(3*r))/(1 + r)) + 3*b*(d^3 + (9*d^2*e*x^r)/(3 +
r) + (9*d*e^2*x^(2*r))/(3 + 2*r) + (e^3*x^(3*r))/(1 + r))*Log[c*x^n]))/9
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1256 vs. 2(142) = 284.

Time = 9.67 (sec) , antiderivative size = 1257, normalized size of antiderivative = 8.49

method	result	size
parallelrisch	Expression too large to display	1257
risch	Expression too large to display	4027

```
[In] int(x^2*(d+e*x^r)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*(-243*a*d^3*x^3-108*x^3*x^r*a*d^2*e*r^5-864*x^3*x^r*a*d^2*e*r^4+324*x^
3*b*d^3*n*r-579*x^3*a*d^3*r^4-1296*x^3*a*d^3*r^3-1566*x^3*a*d^3*r^2-972*x^3
*a*d^3*r-729*x^3*d^2*e*x^r*b*ln(c*x^n)-243*x^3*e^3*(x^r)^3*a-243*x^3*b*ln(c
*x^n)*d^3+81*x^3*(x^r)^3*b*e^3*n+4*x^3*b*d^3*n*r^6+44*x^3*b*d^3*n*r^5+193*x
^3*b*d^3*n*r^4+432*x^3*b*d^3*n*r^3+522*x^3*b*d^3*n*r^2-729*x^3*d^2*e*x^r*a-
243*x^3*e^3*(x^r)^3*b*ln(c*x^n)-729*x^3*d*e^2*(x^r)^2*a+540*x^3*x^r*b*d^2*e
*n*r^3+999*x^3*x^r*b*d^2*e*n*r^2+810*x^3*x^r*b*d^2*e*n*r-120*x^3*(x^r)^3*a*
e^3*r^4-459*x^3*(x^r)^3*a*e^3*r^3-837*x^3*(x^r)^3*a*e^3*r^2-12*x^3*ln(c*x^n
)*b*d^3*r^6-132*x^3*ln(c*x^n)*b*d^3*r^5-579*x^3*ln(c*x^n)*b*d^3*r^4-1296*x^
3*ln(c*x^n)*b*d^3*r^3-12*x^3*a*d^3*r^6-132*x^3*a*d^3*r^5-729*x^3*d*e^2*(x^r
)^2*b*ln(c*x^n)+27*x^3*(x^r)^2*b*d*e^2*n*r^4+216*x^3*(x^r)^2*b*d*e^2*n*r^3+
594*x^3*(x^r)^2*b*d*e^2*n*r^2+648*x^3*(x^r)^2*b*d*e^2*n*r-108*x^3*x^r*ln(c*
x^n)*b*d^2*e*r^5-864*x^3*x^r*ln(c*x^n)*b*d^2*e*r^4-2619*x^3*x^r*ln(c*x^n)*b
*d^2*e*r^3-3807*x^3*x^r*ln(c*x^n)*b*d^2*e*r^2-54*x^3*(x^r)^2*ln(c*x^n)*b*d*
e^2*r^5+108*x^3*x^r*b*d^2*e*n*r^4-513*x^3*(x^r)^2*ln(c*x^n)*b*d*e^2*r^4-183
```

```
6*x^3*(x^r)^2*ln(c*x^n)*b*d*e^2*r^3-3078*x^3*(x^r)^2*ln(c*x^n)*b*d*e^2*r^2-
2673*e*d^2*b*ln(c*x^n)*x^r*r*x^3-729*x^3*(x^r)^3*a*e^3*r+117*x^3*(x^r)^3*b*
e^3*n*r^2+162*x^3*(x^r)^3*b*e^3*n*r+243*x^3*(x^r)^2*b*d*e^2*n+243*x^3*x^r*b
*d^2*e*n+4*x^3*(x^r)^3*b*e^3*n*r^4+81*b*d^3*n*x^3-12*x^3*(x^r)^3*a*e^3*r^5-
1566*x^3*ln(c*x^n)*b*d^3*r^2-972*x^3*ln(c*x^n)*b*d^3*r-2619*x^3*x^r*a*d^2*e
*r^3-3807*x^3*x^r*a*d^2*e*r^2-2673*x^3*x^r*a*d^2*e*r-513*x^3*(x^r)^2*a*d*e^
2*r^4-1836*x^3*(x^r)^2*a*d*e^2*r^3-3078*x^3*(x^r)^2*a*d*e^2*r^2-2430*x^3*(x
^r)^2*a*d*e^2*r+36*x^3*(x^r)^3*b*e^3*n*r^3-12*x^3*(x^r)^3*ln(c*x^n)*b*e^3*r
^5-120*x^3*(x^r)^3*ln(c*x^n)*b*e^3*r^4-459*x^3*(x^r)^3*ln(c*x^n)*b*e^3*r^3-
837*x^3*(x^r)^3*ln(c*x^n)*b*e^3*r^2-729*x^3*(x^r)^3*ln(c*x^n)*b*e^3*r-54*x^
3*(x^r)^2*a*d*e^2*r^5-2430*e^2*d*b*ln(c*x^n)*(x^r)^2*x^3*r)/(1+r)^2/(r^2+6*
r+9)/(3+2*r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1022 vs. $2(142) = 284$.

Time = 0.32 (sec) , antiderivative size = 1022, normalized size of antiderivative = 6.91

$$\int x^2(d+ex^r)^3\,(a+b\log{(cx^n)})\,dx$$
$$= \frac{3\,(4\,bd^3r^6+44\,bd^3r^5+193\,bd^3r^4+432\,bd^3r^3+522\,bd^3r^2+324\,bd^3r+81\,bd^3)x^3\log{(c)}+3\,(4\,bd^3nr^6+4}{}$$

```
[In] integrate(x^2*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(4*b*d^3*r^6 + 44*b*d^3*r^5 + 193*b*d^3*r^4 + 432*b*d^3*r^3 + 522*b*
d^3*r^2 + 324*b*d^3*r + 81*b*d^3)*x^3*log(c) + 3*(4*b*d^3*n*r^6 + 44*b*d^3*
n*r^5 + 193*b*d^3*n*r^4 + 432*b*d^3*n*r^3 + 522*b*d^3*n*r^2 + 324*b*d^3*n*r
 + 81*b*d^3*n)*x^3*log(x) - (4*(b*d^3*n - 3*a*d^3)*r^6 + 44*(b*d^3*n - 3*a*
d^3)*r^5 + 81*b*d^3*n + 193*(b*d^3*n - 3*a*d^3)*r^4 - 243*a*d^3 + 432*(b*d^
3*n - 3*a*d^3)*r^3 + 522*(b*d^3*n - 3*a*d^3)*r^2 + 324*(b*d^3*n - 3*a*d^3)*
r)*x^3 + (3*(4*b*e^3*r^5 + 40*b*e^3*r^4 + 153*b*e^3*r^3 + 279*b*e^3*r^2 + 2
43*b*e^3*r + 81*b*e^3)*x^3*log(c) + 3*(4*b*e^3*n*r^5 + 40*b*e^3*n*r^4 + 153
*b*e^3*n*r^3 + 279*b*e^3*n*r^2 + 243*b*e^3*n*r + 81*b*e^3*n)*x^3*log(x) + (
12*a*e^3*r^5 - 81*b*e^3*n - 4*(b*e^3*n - 30*a*e^3)*r^4 + 243*a*e^3 - 9*(4*b
*e^3*n - 51*a*e^3)*r^3 - 9*(13*b*e^3*n - 93*a*e^3)*r^2 - 81*(2*b*e^3*n - 9*
a*e^3)*r)*x^3)*x^(3*r) + 27*((2*b*d*e^2*r^5 + 19*b*d*e^2*r^4 + 68*b*d*e^2*r
^3 + 114*b*d*e^2*r^2 + 90*b*d*e^2*r + 27*b*d*e^2)*x^3*log(c) + (2*b*d*e^2*n
*r^5 + 19*b*d*e^2*n*r^4 + 68*b*d*e^2*n*r^3 + 114*b*d*e^2*n*r^2 + 90*b*d*e^2
*n*r + 27*b*d*e^2*n)*x^3*log(x) + (2*a*d*e^2*r^5 - 9*b*d*e^2*n - (b*d*e^2*n
 - 19*a*d*e^2)*r^4 + 27*a*d*e^2 - 4*(2*b*d*e^2*n - 17*a*d*e^2)*r^3 - 2*(11*
b*d*e^2*n - 57*a*d*e^2)*r^2 - 6*(4*b*d*e^2*n - 15*a*d*e^2)*r)*x^3)*x^(2*r)
+ 27*((4*b*d^2*e*r^5 + 32*b*d^2*e*r^4 + 97*b*d^2*e*r^3 + 141*b*d^2*e*r^2 +
99*b*d^2*e*r + 27*b*d^2*e)*x^3*log(c) + (4*b*d^2*e*n*r^5 + 32*b*d^2*e*n*r^4
```

```
 + 97*b*d^2*e*n*r^3 + 141*b*d^2*e*n*r^2 + 99*b*d^2*e*n*r + 27*b*d^2*e*n)*x^
3*log(x) + (4*a*d^2*e*r^5 - 9*b*d^2*e*n - 4*(b*d^2*e*n - 8*a*d^2*e)*r^4 + 2
7*a*d^2*e - (20*b*d^2*e*n - 97*a*d^2*e)*r^3 - (37*b*d^2*e*n - 141*a*d^2*e)*
r^2 - 3*(10*b*d^2*e*n - 33*a*d^2*e)*r)*x^3)*x^r)/(4*r^6 + 44*r^5 + 193*r^4
+ 432*r^3 + 522*r^2 + 324*r + 81)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(d+ex^r)^3(a+b\log(cx^n))\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(d+e*x**r)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.51

$$\begin{aligned}\int x^2(d+ex^r)^3(a+b\log(cx^n))\,dx = & -\frac{1}{9}bd^3nx^3+\frac{1}{3}bd^3x^3\log(cx^n)+\frac{1}{3}ad^3x^3 \\ & +\frac{be^3x^{3r+3}\log(cx^n)}{3(r+1)}+\frac{3bde^2x^{2r+3}\log(cx^n)}{2r+3} \\ & +\frac{3bd^2ex^{r+3}\log(cx^n)}{r+3}-\frac{be^3nx^{3r+3}}{9(r+1)^2} \\ & +\frac{ae^3x^{3r+3}}{3(r+1)}-\frac{3bde^2nx^{2r+3}}{(2r+3)^2}+\frac{3ade^2x^{2r+3}}{2r+3} \\ & -\frac{3bd^2enx^{r+3}}{(r+3)^2}+\frac{3ad^2ex^{r+3}}{r+3}\end{aligned}$$

```
[In] integrate(x^2*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/9*b*d^3*n*x^3 + 1/3*b*d^3*x^3*log(c*x^n) + 1/3*a*d^3*x^3 + 1/3*b*e^3*x^(
3*r + 3)*log(c*x^n)/(r + 1) + 3*b*d*e^2*x^(2*r + 3)*log(c*x^n)/(2*r + 3) +
3*b*d^2*e*x^(r + 3)*log(c*x^n)/(r + 3) - 1/9*b*e^3*n*x^(3*r + 3)/(r + 1)^2
+ 1/3*a*e^3*x^(3*r + 3)/(r + 1) - 3*b*d*e^2*n*x^(2*r + 3)/(2*r + 3)^2 + 3*a
*d*e^2*x^(2*r + 3)/(2*r + 3) - 3*b*d^2*e*n*x^(r + 3)/(r + 3)^2 + 3*a*d^2*e*
x^(r + 3)/(r + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1611 vs. $2(142) = 284$.

Time = 0.33 (sec) , antiderivative size = 1611, normalized size of antiderivative = 10.89

$$\int x^2(d+ex^r)^3 (a+b\log(cx^n)) \, dx = \text{Too large to display}$$

```
[In] integrate(x^2*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/9*(12*b*e^3*n*r^5*x^3*x^(3*r)*log(x) + 54*b*d*e^2*n*r^5*x^3*x^(2*r)*log(x
) + 108*b*d^2*e*n*r^5*x^3*x^r*log(x) + 12*b*d^3*n*r^6*x^3*log(x) - 4*b*d^3*
n*r^6*x^3 + 12*b*e^3*r^5*x^3*x^(3*r)*log(c) + 54*b*d*e^2*r^5*x^3*x^(2*r)*lo
g(c) + 108*b*d^2*e*r^5*x^3*x^r*log(c) + 12*b*d^3*r^6*x^3*log(c) + 120*b*e^3
*n*r^4*x^3*x^(3*r)*log(x) + 513*b*d*e^2*n*r^4*x^3*x^(2*r)*log(x) + 864*b*d^
2*e*n*r^4*x^3*x^r*log(x) + 132*b*d^3*n*r^5*x^3*log(x) - 4*b*e^3*n*r^4*x^3*x
^(3*r) + 12*a*e^3*r^5*x^3*x^(3*r) - 27*b*d*e^2*n*r^4*x^3*x^(2*r) + 54*a*d*e
^2*r^5*x^3*x^(2*r) - 108*b*d^2*e*n*r^4*x^3*x^r + 108*a*d^2*e*r^5*x^3*x^r -
44*b*d^3*n*r^5*x^3 + 12*a*d^3*r^6*x^3 + 120*b*e^3*r^4*x^3*x^(3*r)*log(c) +
513*b*d*e^2*r^4*x^3*x^(2*r)*log(c) + 864*b*d^2*e*r^4*x^3*x^r*log(c) + 132*b
*d^3*r^5*x^3*log(c) + 459*b*e^3*n*r^3*x^3*x^(3*r)*log(x) + 1836*b*d*e^2*n*r
^3*x^3*x^(2*r)*log(x) + 2619*b*d^2*e*n*r^3*x^3*x^r*log(x) + 579*b*d^3*n*r^4
*x^3*log(x) - 36*b*e^3*n*r^3*x^3*x^(3*r) + 120*a*e^3*r^4*x^3*x^(3*r) - 216*
b*d*e^2*n*r^3*x^3*x^(2*r) + 513*a*d*e^2*r^4*x^3*x^(2*r) - 540*b*d^2*e*n*r^3
*x^3*x^r + 864*a*d^2*e*r^4*x^3*x^r - 193*b*d^3*n*r^4*x^3 + 132*a*d^3*r^5*x^
3 + 459*b*e^3*r^3*x^3*x^(3*r)*log(c) + 1836*b*d*e^2*r^3*x^3*x^(2*r)*log(c)
+ 2619*b*d^2*e*r^3*x^3*x^r*log(c) + 579*b*d^3*r^4*x^3*log(c) + 837*b*e^3*n*
r^2*x^3*x^(3*r)*log(x) + 3078*b*d*e^2*n*r^2*x^3*x^(2*r)*log(x) + 3807*b*d^2
*e*n*r^2*x^3*x^r*log(x) + 1296*b*d^3*n*r^3*x^3*log(x) - 117*b*e^3*n*r^2*x^3
*x^(3*r) + 459*a*e^3*r^3*x^3*x^(3*r) - 594*b*d*e^2*n*r^2*x^3*x^(2*r) + 1836
*a*d*e^2*r^3*x^3*x^(2*r) - 999*b*d^2*e*n*r^2*x^3*x^r + 2619*a*d^2*e*r^3*x^3
*x^r - 432*b*d^3*n*r^3*x^3 + 579*a*d^3*r^4*x^3 + 837*b*e^3*r^2*x^3*x^(3*r)*
log(c) + 3078*b*d*e^2*r^2*x^3*x^(2*r)*log(c) + 3807*b*d^2*e*r^2*x^3*x^r*log
(c) + 1296*b*d^3*r^3*x^3*log(c) + 729*b*e^3*n*r*x^3*x^(3*r)*log(x) + 2430*b
*d*e^2*n*r*x^3*x^(2*r)*log(x) + 2673*b*d^2*e*n*r*x^3*x^r*log(x) + 1566*b*d^
3*n*r^2*x^3*log(x) - 162*b*e^3*n*r*x^3*x^(3*r) + 837*a*e^3*r^2*x^3*x^(3*r)
- 648*b*d*e^2*n*r*x^3*x^(2*r) + 3078*a*d*e^2*r^2*x^3*x^(2*r) - 810*b*d^2*e*
n*r*x^3*x^r + 3807*a*d^2*e*r^2*x^3*x^r - 522*b*d^3*n*r^2*x^3 + 1296*a*d^3*r
^3*x^3 + 729*b*e^3*r*x^3*x^(3*r)*log(c) + 2430*b*d*e^2*r*x^3*x^(2*r)*log(c)
 + 2673*b*d^2*e*r*x^3*x^r*log(c) + 1566*b*d^3*r^2*x^3*log(c) + 243*b*e^3*n*
x^3*x^(3*r)*log(x) + 729*b*d*e^2*n*x^3*x^(2*r)*log(x) + 729*b*d^2*e*n*x^3*x
^r*log(x) + 972*b*d^3*n*r*x^3*log(x) - 81*b*e^3*n*x^3*x^(3*r) + 729*a*e^3*r
*x^3*x^(3*r) - 243*b*d*e^2*n*x^3*x^(2*r) + 2430*a*d*e^2*r*x^3*x^(2*r) - 243
*b*d^2*e*n*x^3*x^r + 2673*a*d^2*e*r*x^3*x^r - 324*b*d^3*n*r*x^3 + 1566*a*d^
3*r^2*x^3 + 243*b*e^3*x^3*x^(3*r)*log(c) + 729*b*d*e^2*x^3*x^(2*r)*log(c) +
```

```
 729*b*d^2*e*x^3*x^r*log(c) + 972*b*d^3*r*x^3*log(c) + 243*b*d^3*n*x^3*log(
x) + 243*a*e^3*x^3*x^(3*r) + 729*a*d*e^2*x^3*x^(2*r) + 729*a*d^2*e*x^3*x^r
- 81*b*d^3*n*x^3 + 972*a*d^3*r*x^3 + 243*b*d^3*x^3*log(c) + 243*a*d^3*x^3)/
(4*r^6 + 44*r^5 + 193*r^4 + 432*r^3 + 522*r^2 + 324*r + 81)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^r)^3(a+b\log(cx^n))\,dx=\int x^2\,(d+e\,x^r)^3\,(a+b\,\ln(c\,x^n))\,dx$$

```
[In] int(x^2*(d + e*x^r)^3*(a + b*log(c*x^n)),x)
[Out] int(x^2*(d + e*x^r)^3*(a + b*log(c*x^n)), x)
```

3.400 $\int (d+ex^r)^3 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 169

$$\int (d+ex^r)^3 (a+b\log(cx^n))\,dx = -bd^3nx - \frac{3bd^2enx^{1+r}}{(1+r)^2} - \frac{3bde^2nx^{1+2r}}{(1+2r)^2} - \frac{be^3nx^{1+3r}}{(1+3r)^2} + d^3x(a+b\log(cx^n)) + \frac{3d^2ex^{1+r}(a+b\log(cx^n))}{1+r} + \frac{3de^2x^{1+2r}(a+b\log(cx^n))}{1+2r} + \frac{e^3x^{1+3r}(a+b\log(cx^n))}{1+3r}$$

```
[Out] -b*d^3*n*x-3*b*d^2*e*n*x^(1+r)/(1+r)^2-3*b*d*e^2*n*x^(1+2*r)/(1+2*r)^2-b*e^
3*n*x^(1+3*r)/(1+3*r)^2+d^3*x*(a+b*ln(c*x^n))+3*d^2*e*x^(1+r)*(a+b*ln(c*x^n
))/(1+r)+3*d*e^2*x^(1+2*r)*(a+b*ln(c*x^n))/(1+2*r)+e^3*x^(1+3*r)*(a+b*ln(c*
x^n))/(1+3*r)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {250, 2350}

$$\int (d+ex^r)^3 (a+b\log(cx^n))\,dx = d^3x(a+b\log(cx^n)) + \frac{3d^2ex^{r+1}(a+b\log(cx^n))}{r+1} + \frac{3de^2x^{2r+1}(a+b\log(cx^n))}{2r+1} + \frac{e^3x^{3r+1}(a+b\log(cx^n))}{3r+1} - bd^3nx - \frac{3bd^2enx^{r+1}}{(r+1)^2} - \frac{3bde^2nx^{2r+1}}{(2r+1)^2} - \frac{be^3nx^{3r+1}}{(3r+1)^2}$$

```
[In] Int[(d + e*x^r)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] -(b*d^3*n*x) - (3*b*d^2*e*n*x^(1 + r))/(1 + r)^2 - (3*b*d*e^2*n*x^(1 + 2*r)
)/(1 + 2*r)^2 - (b*e^3*n*x^(1 + 3*r))/(1 + 3*r)^2 + d^3*x*(a + b*Log[c*x^n]
) + (3*d^2*e*x^(1 + r)*(a + b*Log[c*x^n]))/(1 + r) + (3*d*e^2*x^(1 + 2*r)*(
a + b*Log[c*x^n]))/(1 + 2*r) + (e^3*x^(1 + 3*r)*(a + b*Log[c*x^n]))/(1 + 3*
r)
```

Rule 250

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

Rule 2350

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Dist[a + b*Log[c*x^n], u
, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= d^3x(a+b\log(cx^n))+\frac{3d^2ex^{1+r}(a+b\log(cx^n))}{1+r}+\frac{3de^2x^{1+2r}(a+b\log(cx^n))}{1+2r}\\&\quad+\frac{e^3x^{1+3r}(a+b\log(cx^n))}{1+3r}-(bn)\int\left(d^3+\frac{3d^2ex^r}{1+r}+\frac{3de^2x^{2r}}{1+2r}+\frac{e^3x^{3r}}{1+3r}\right)dx\\&=-bd^3nx-\frac{3bd^2enx^{1+r}}{(1+r)^2}-\frac{3bde^2nx^{1+2r}}{(1+2r)^2}-\frac{be^3nx^{1+3r}}{(1+3r)^2}+d^3x(a+b\log(cx^n))\\&\quad+\frac{3d^2ex^{1+r}(a+b\log(cx^n))}{1+r}+\frac{3de^2x^{1+2r}(a+b\log(cx^n))}{1+2r}+\frac{e^3x^{1+3r}(a+b\log(cx^n))}{1+3r}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.94

$$\int(d+ex^r)^3(a+b\log(cx^n))\,dx=x\left(ad^3-bd^3n-\frac{3bd^2enx^r}{(1+r)^2}-\frac{3bde^2nx^{2r}}{(1+2r)^2}-\frac{be^3nx^{3r}}{(1+3r)^2}+bd^3\log(cx^n)+\frac{3d^2ex^r(a+b\log(cx^n))}{1+r}+\frac{3de^2x^{2r}(a+b\log(cx^n))}{1+2r}+\frac{e^3x^{3r}(a+b\log(cx^n))}{1+3r}\right)$$

```
[In] Integrate[(d + e*x^r)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] x*(a*d^3 - b*d^3*n - (3*b*d^2*e*n*x^r)/(1 + r)^2 - (3*b*d*e^2*n*x^(2*r))/(1
 + 2*r)^2 - (b*e^3*n*x^(3*r))/(1 + 3*r)^2 + b*d^3*Log[c*x^n] + (3*d^2*e*x^r
*(a + b*Log[c*x^n]))/(1 + r) + (3*d*e^2*x^(2*r)*(a + b*Log[c*x^n]))/(1 + 2*
r) + (e^3*x^(3*r)*(a + b*Log[c*x^n]))/(1 + 3*r))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. 2(169) = 338.

Time = 3.41 (sec) , antiderivative size = 1108, normalized size of antiderivative = 6.56

method	result	size
parallelrisch	Expression too large to display	1108
risch	Expression too large to display	4023

```
[In] int((d+e*x^r)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -(-36*x*a*d^3*r^6-132*x*a*d^3*r^5-193*x*a*d^3*r^4-144*x*a*d^3*r^3-58*x*a*d^
3*r^2-12*x*a*d^3*r-40*x*(x^r)^3*a*e^3*r^4-51*x*(x^r)^3*a*e^3*r^3-31*x*(x^r)
^3*a*e^3*r^2+36*x*b*d^3*n*r^6+132*x*b*d^3*n*r^5+193*x*b*d^3*n*r^4-3*x*d^2*e
*x^r*b*ln(c*x^n)-3*x*d*e^2*(x^r)^2*b*ln(c*x^n)-x*e^3*(x^r)^3*a-x*b*ln(c*x^n
)*d^3-a*d^3*x-3*x*d*e^2*(x^r)^2*a-x*e^3*(x^r)^3*b*ln(c*x^n)-3*x*d^2*e*x^r*a
+4*x*(x^r)^3*b*e^3*n*r^4+x*(x^r)^3*b*e^3*n+144*x*b*d^3*n*r^3+58*x*b*d^3*n*r
^2+12*x*b*d^3*n*r-9*x*(x^r)^3*a*e^3*r-33*x*x^r*r*a*d^2*e+30*x*x^r*b*d^2*e*n
*r+27*x*(x^r)^2*b*d*e^2*n*r^4+72*x*(x^r)^2*b*d*e^2*n*r^3+108*x*x^r*b*d^2*e*
n*r^4+66*x*(x^r)^2*b*d*e^2*n*r^2+180*x*x^r*b*d^2*e*n*r^3+24*x*(x^r)^2*b*d*e
^2*n*r+111*x*x^r*b*d^2*e*n*r^2+b*d^3*n*x-114*x*(x^r)^2*ln(c*x^n)*b*d*e^2*r^
2-30*x*(x^r)^2*ln(c*x^n)*b*d*e^2*r-12*x*ln(c*x^n)*b*d^3*r-36*x*ln(c*x^n)*b*
d^3*r^6-132*x*ln(c*x^n)*b*d^3*r^5-193*x*ln(c*x^n)*b*d^3*r^4-144*x*ln(c*x^n)
*b*d^3*r^3-58*x*ln(c*x^n)*b*d^3*r^2-291*x*x^r*ln(c*x^n)*b*d^2*e*r^3-141*x*x
^r*ln(c*x^n)*b*d^2*e*r^2-33*x*x^r*ln(c*x^n)*b*d^2*e*r-54*x*(x^r)^2*ln(c*x^n
)*b*d*e^2*r^5-171*x*(x^r)^2*ln(c*x^n)*b*d*e^2*r^4-204*x*(x^r)^2*ln(c*x^n)*b
*d*e^2*r^3-12*x*(x^r)^3*a*e^3*r^5-30*x*(x^r)^2*a*d*e^2*r-9*e^3*b*ln(c*x^n)*
(x^r)^3*x*r-108*x*x^r*ln(c*x^n)*b*d^2*e*r^5-288*x*x^r*ln(c*x^n)*b*d^2*e*r^4
+12*x*(x^r)^3*b*e^3*n*r^3+13*x*(x^r)^3*b*e^3*n*r^2+6*x*(x^r)^3*b*e^3*n*r+3*
x*(x^r)^2*b*d*e^2*n-141*x*x^r*a*d^2*e*r^2-54*x*(x^r)^2*a*d*e^2*r^5-171*x*(x
^r)^2*a*d*e^2*r^4-204*x*(x^r)^2*a*d*e^2*r^3-114*x*(x^r)^2*a*d*e^2*r^2-12*x*
(x^r)^3*ln(c*x^n)*b*e^3*r^5-40*x*(x^r)^3*ln(c*x^n)*b*e^3*r^4-51*x*(x^r)^3*l
n(c*x^n)*b*e^3*r^3-31*x*(x^r)^3*ln(c*x^n)*b*e^3*r^2-108*x*x^r*a*d^2*e*r^5-2
88*x*x^r*a*d^2*e*r^4-291*x*x^r*a*d^2*e*r^3+3*x*x^r*b*d^2*e*n)/(1+3*r)^2/(1+
2*r)^2/(1+r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(169) = 338$.

Time = 0.31 (sec) , antiderivative size = 983, normalized size of antiderivative = 5.82

$$\int (d + ex^r)^3 (a + b\log(cx^n)) \, dx = \frac{(36\,bd^3r^6 + 132\,bd^3r^5 + 193\,bd^3r^4 + 144\,bd^3r^3 + 58\,bd^3r^2 + 12\,bd^3r + bd^3)x\log(c) + (36\,bd^3nr^6 + 132\,bd^3nr}{}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] ((36*b*d^3*r^6 + 132*b*d^3*r^5 + 193*b*d^3*r^4 + 144*b*d^3*r^3 + 58*b*d^3*r
^2 + 12*b*d^3*r + b*d^3)*x*log(c) + (36*b*d^3*n*r^6 + 132*b*d^3*n*r^5 + 193
*b*d^3*n*r^4 + 144*b*d^3*n*r^3 + 58*b*d^3*n*r^2 + 12*b*d^3*n*r + b*d^3*n)*x
*log(x) - (36*(b*d^3*n - a*d^3)*r^6 + 132*(b*d^3*n - a*d^3)*r^5 + b*d^3*n +
 193*(b*d^3*n - a*d^3)*r^4 - a*d^3 + 144*(b*d^3*n - a*d^3)*r^3 + 58*(b*d^3*
n - a*d^3)*r^2 + 12*(b*d^3*n - a*d^3)*r)*x + ((12*b*e^3*r^5 + 40*b*e^3*r^4
+ 51*b*e^3*r^3 + 31*b*e^3*r^2 + 9*b*e^3*r + b*e^3)*x*log(c) + (12*b*e^3*n*r
^5 + 40*b*e^3*n*r^4 + 51*b*e^3*n*r^3 + 31*b*e^3*n*r^2 + 9*b*e^3*n*r + b*e^3
*n)*x*log(x) + (12*a*e^3*r^5 - b*e^3*n - 4*(b*e^3*n - 10*a*e^3)*r^4 + a*e^3
 - 3*(4*b*e^3*n - 17*a*e^3)*r^3 - (13*b*e^3*n - 31*a*e^3)*r^2 - 3*(2*b*e^3*
n - 3*a*e^3)*r)*x)*x^(3*r) + 3*((18*b*d*e^2*r^5 + 57*b*d*e^2*r^4 + 68*b*d*e
^2*r^3 + 38*b*d*e^2*r^2 + 10*b*d*e^2*r + b*d*e^2)*x*log(c) + (18*b*d*e^2*n*
r^5 + 57*b*d*e^2*n*r^4 + 68*b*d*e^2*n*r^3 + 38*b*d*e^2*n*r^2 + 10*b*d*e^2*n
*r + b*d*e^2*n)*x*log(x) + (18*a*d*e^2*r^5 - b*d*e^2*n - 3*(3*b*d*e^2*n - 1
9*a*d*e^2)*r^4 + a*d*e^2 - 4*(6*b*d*e^2*n - 17*a*d*e^2)*r^3 - 2*(11*b*d*e^2
*n - 19*a*d*e^2)*r^2 - 2*(4*b*d*e^2*n - 5*a*d*e^2)*r)*x)*x^(2*r) + 3*((36*b
*d^2*e*r^5 + 96*b*d^2*e*r^4 + 97*b*d^2*e*r^3 + 47*b*d^2*e*r^2 + 11*b*d^2*e*
r + b*d^2*e)*x*log(c) + (36*b*d^2*e*n*r^5 + 96*b*d^2*e*n*r^4 + 97*b*d^2*e*n
*r^3 + 47*b*d^2*e*n*r^2 + 11*b*d^2*e*n*r + b*d^2*e*n)*x*log(x) + (36*a*d^2*
e*r^5 - b*d^2*e*n - 12*(3*b*d^2*e*n - 8*a*d^2*e)*r^4 + a*d^2*e - (60*b*d^2*
e*n - 97*a*d^2*e)*r^3 - (37*b*d^2*e*n - 47*a*d^2*e)*r^2 - (10*b*d^2*e*n - 1
1*a*d^2*e)*r)*x)*x^r)/(36*r^6 + 132*r^5 + 193*r^4 + 144*r^3 + 58*r^2 + 12*r
 + 1)
```

Sympy [A] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.92

$$\int (d+ex^r)^3\,(a+b\log(cx^n))\,dx$$
$$= ad^3x + 3ad^2e\left(\begin{cases}\frac{x^{r+1}}{r+1} & \text{for}\ r\neq -1\\ \log(x) & \text{otherwise}\end{cases}\right) + 3ade^2\left(\begin{cases}\frac{x^{2r+1}}{2r+1} & \text{for}\ r\neq -\frac{1}{2}\\ \log(x) & \text{otherwise}\end{cases}\right)$$
$$+ ae^3\left(\begin{cases}\frac{x^{3r+1}}{3r+1} & \text{for}\ r\neq -\frac{1}{3}\\ \log(x) & \text{otherwise}\end{cases}\right) - bd^3nx + bd^3x\log(cx^n)$$
$$- 3bd^2en\left(\begin{cases}\frac{\begin{cases}\frac{x^{r+1}}{r+1} & \text{for}\ r\neq -1\\ \log(x) & \text{otherwise}\end{cases}}{r+1} & \text{for}\ r>-\infty\wedge r<\infty\wedge r\neq -1\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ 3bd^2e\left(\begin{cases}\frac{x^{r+1}}{r+1} & \text{for}\ r\neq -1\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$
$$- 3bde^2n\left(\begin{cases}\frac{\begin{cases}\frac{x^{2r+1}}{2r+1} & \text{for}\ r\neq -\frac{1}{2}\\ \log(x) & \text{otherwise}\end{cases}}{2r+1} & \text{for}\ r>-\infty\wedge r<\infty\wedge r\neq -\frac{1}{2}\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ 3bde^2\left(\begin{cases}\frac{x^{2r+1}}{2r+1} & \text{for}\ r\neq -\frac{1}{2}\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$
$$- be^3n\left(\begin{cases}\frac{\begin{cases}\frac{x^{3r+1}}{3r+1} & \text{for}\ r\neq -\frac{1}{3}\\ \log(x) & \text{otherwise}\end{cases}}{3r+1} & \text{for}\ r>-\infty\wedge r<\infty\wedge r\neq -\frac{1}{3}\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ be^3\left(\begin{cases}\frac{x^{3r+1}}{3r+1} & \text{for}\ r\neq -\frac{1}{3}\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate((d+e*x**r)**3*(a+b*ln(c*x**n)),x)
```

```
[Out] a*d**3*x + 3*a*d**2*e*Piecewise((x**(r + 1)/(r + 1), Ne(r, -1)), (log(x), T
rue)) + 3*a*d*e**2*Piecewise((x**(2*r + 1)/(2*r + 1), Ne(r, -1/2)), (log(x)
, True)) + a*e**3*Piecewise((x**(3*r + 1)/(3*r + 1), Ne(r, -1/3)), (log(x),
```

```
True)) - b*d**3*n*x + b*d**3*x*log(c*x**n) - 3*b*d**2*e*n*Piecewise((Piece
wise((x**(r + 1)/(r + 1), Ne(r, -1)), (log(x), True))/(r + 1), (r > -oo) &
(r < oo) & Ne(r, -1)), (log(x)**2/2, True)) + 3*b*d**2*e*Piecewise((x**(r +
 1)/(r + 1), Ne(r, -1)), (log(x), True))*log(c*x**n) - 3*b*d*e**2*n*Piecewi
se((Piecewise((x**(2*r + 1)/(2*r + 1), Ne(r, -1/2)), (log(x), True))/(2*r +
 1), (r > -oo) & (r < oo) & Ne(r, -1/2)), (log(x)**2/2, True)) + 3*b*d*e**2
*Piecewise((x**(2*r + 1)/(2*r + 1), Ne(r, -1/2)), (log(x), True))*log(c*x**
n) - b*e**3*n*Piecewise((Piecewise((x**(3*r + 1)/(3*r + 1), Ne(r, -1/3)), (
log(x), True))/(3*r + 1), (r > -oo) & (r < oo) & Ne(r, -1/3)), (log(x)**2/2
, True)) + b*e**3*Piecewise((x**(3*r + 1)/(3*r + 1), Ne(r, -1/3)), (log(x),
 True))*log(c*x**n)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.30

$$\int (d+ex^r)^3 (a+b\log(cx^n))\, dx = -bd^3nx + bd^3x\log(cx^n) + ad^3x + \frac{be^3x^{3r+1}\log(cx^n)}{3r+1} + \frac{3bde^2x^{2r+1}\log(cx^n)}{2r+1} + \frac{3bd^2ex^{r+1}\log(cx^n)}{r+1} - \frac{be^3nx^{3r+1}}{(3r+1)^2} + \frac{ae^3x^{3r+1}}{3r+1} - \frac{3bde^2nx^{2r+1}}{(2r+1)^2} + \frac{3ade^2x^{2r+1}}{2r+1} - \frac{3bd^2enx^{r+1}}{(r+1)^2} + \frac{3ad^2ex^{r+1}}{r+1}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -b*d^3*n*x + b*d^3*x*log(c*x^n) + a*d^3*x + b*e^3*x^(3*r + 1)*log(c*x^n)/(3
*r + 1) + 3*b*d*e^2*x^(2*r + 1)*log(c*x^n)/(2*r + 1) + 3*b*d^2*e*x^(r + 1)*
log(c*x^n)/(r + 1) - b*e^3*n*x^(3*r + 1)/(3*r + 1)^2 + a*e^3*x^(3*r + 1)/(3
*r + 1) - 3*b*d*e^2*n*x^(2*r + 1)/(2*r + 1)^2 + 3*a*d*e^2*x^(2*r + 1)/(2*r
+ 1) - 3*b*d^2*e*n*x^(r + 1)/(r + 1)^2 + 3*a*d^2*e*x^(r + 1)/(r + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(169) = 338.

Time = 0.31 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.24

$$\int (d+ex^r)^3 (a+b\log(cx^n))\,dx = \frac{3\,be^3nrxx^{3\,r}\log(x)}{9\,r^2+6\,r+1} + \frac{6\,bde^2nrxx^{2\,r}\log(x)}{4\,r^2+4\,r+1} + \frac{3\,bd^2enrxx^r\log(x)}{r^2+2\,r+1} + bd^3nx\log(x) + \frac{be^3nxx^{3\,r}\log(x)}{9\,r^2+6\,r+1} + \frac{3\,bde^2nxx^{2\,r}\log(x)}{4\,r^2+4\,r+1} + \frac{3\,bd^2enxx^r\log(x)}{r^2+2\,r+1} - bd^3nx - \frac{be^3nxx^{3\,r}}{9\,r^2+6\,r+1} - \frac{3\,bde^2nxx^{2\,r}}{4\,r^2+4\,r+1} - \frac{3\,bd^2enxx^r}{r^2+2\,r+1} + bd^3x\log(c) + \frac{be^3xx^{3\,r}\log(c)}{3\,r+1} + \frac{3\,bde^2xx^{2\,r}\log(c)}{2\,r+1} + \frac{3\,bd^2exx^r\log(c)}{r+1} + ad^3x + \frac{ae^3xx^{3\,r}}{3\,r+1} + \frac{3\,ade^2xx^{2\,r}}{2\,r+1} + \frac{3\,ad^2exx^r}{r+1}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 3*b*e^3*n*r*x*x^(3*r)*log(x)/(9*r^2 + 6*r + 1) + 6*b*d*e^2*n*r*x*x^(2*r)*lo
g(x)/(4*r^2 + 4*r + 1) + 3*b*d^2*e*n*r*x*x^r*log(x)/(r^2 + 2*r + 1) + b*d^3
*n*x*log(x) + b*e^3*n*x*x^(3*r)*log(x)/(9*r^2 + 6*r + 1) + 3*b*d*e^2*n*x*x^
(2*r)*log(x)/(4*r^2 + 4*r + 1) + 3*b*d^2*e*n*x*x^r*log(x)/(r^2 + 2*r + 1) -
 b*d^3*n*x - b*e^3*n*x*x^(3*r)/(9*r^2 + 6*r + 1) - 3*b*d*e^2*n*x*x^(2*r)/(4
*r^2 + 4*r + 1) - 3*b*d^2*e*n*x*x^r/(r^2 + 2*r + 1) + b*d^3*x*log(c) + b*e^
3*x*x^(3*r)*log(c)/(3*r + 1) + 3*b*d*e^2*x*x^(2*r)*log(c)/(2*r + 1) + 3*b*d
^2*e*x*x^r*log(c)/(r + 1) + a*d^3*x + a*e^3*x*x^(3*r)/(3*r + 1) + 3*a*d*e^2
*x*x^(2*r)/(2*r + 1) + 3*a*d^2*e*x*x^r/(r + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^r)^3 (a+b\log(cx^n))\,dx = \int (d+e\,x^r)^3\,(a+b\,\ln(c\,x^n))\,dx$$

```
[In] int((d + e*x^r)^3*(a + b*log(c*x^n)),x)
```

```
[Out] int((d + e*x^r)^3*(a + b*log(c*x^n)), x)
```

3.401 $\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 179

$$\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^2}\,dx = -\frac{bd^3n}{x}-\frac{3bd^2enx^{-1+r}}{(1-r)^2}-\frac{3bde^2nx^{-1+2r}}{(1-2r)^2}-\frac{be^3nx^{-1+3r}}{(1-3r)^2}-\frac{d^3(a+b\log(cx^n))}{x}-\frac{3d^2ex^{-1+r}(a+b\log(cx^n))}{1-r}-\frac{3de^2x^{-1+2r}(a+b\log(cx^n))}{1-2r}-\frac{e^3x^{-1+3r}(a+b\log(cx^n))}{1-3r}$$

```
[Out] -b*d^3*n/x-3*b*d^2*e*n*x^(-1+r)/(1-r)^2-3*b*d*e^2*n*x^(-1+2*r)/(1-2*r)^2-b*
e^3*n*x^(-1+3*r)/(1-3*r)^2-d^3*(a+b*ln(c*x^n))/x-3*d^2*e*x^(-1+r)*(a+b*ln(c
*x^n))/(1-r)-3*d*e^2*x^(-1+2*r)*(a+b*ln(c*x^n))/(1-2*r)-e^3*x^(-1+3*r)*(a+b
*ln(c*x^n))/(1-3*r)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {276, 2372, 14}

$$\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^2}\,dx = -\frac{d^3(a+b\log(cx^n))}{x}-\frac{3d^2ex^{r-1}(a+b\log(cx^n))}{1-r}-\frac{3de^2x^{2r-1}(a+b\log(cx^n))}{1-2r}-\frac{e^3x^{3r-1}(a+b\log(cx^n))}{1-3r}-\frac{bd^3n}{x}-\frac{3bd^2enx^{r-1}}{(1-r)^2}-\frac{3bde^2nx^{2r-1}}{(1-2r)^2}-\frac{be^3nx^{3r-1}}{(1-3r)^2}$$

```
[In] Int[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -((b*d^3*n)/x) - (3*b*d^2*e*n*x^(-1 + r))/(1 - r)^2 - (3*b*d*e^2*n*x^(-1 +
2*r))/(1 - 2*r)^2 - (b*e^3*n*x^(-1 + 3*r))/(1 - 3*r)^2 - (d^3*(a + b*Log[c*
x^n]))/x - (3*d^2*e*x^(-1 + r)*(a + b*Log[c*x^n]))/(1 - r) - (3*d*e^2*x^(-1
 + 2*r)*(a + b*Log[c*x^n]))/(1 - 2*r) - (e^3*x^(-1 + 3*r)*(a + b*Log[c*x^n]
))/(1 - 3*r)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{x}-\frac{3d^2ex^{-1+r}(a+b\log(cx^n))}{1-r}-\frac{3de^2x^{-1+2r}(a+b\log(cx^n))}{1-2r}\\
&\quad-\frac{e^3x^{-1+3r}(a+b\log(cx^n))}{1-3r}-(bn)\int\frac{-d^3+\frac{3d^2ex^r}{-1+r}+\frac{3de^2x^{2r}}{-1+2r}+\frac{e^3x^{3r}}{-1+3r}}{x^2}\,dx\\
&= -\frac{d^3(a+b\log(cx^n))}{x}-\frac{3d^2ex^{-1+r}(a+b\log(cx^n))}{1-r}-\frac{3de^2x^{-1+2r}(a+b\log(cx^n))}{1-2r}\\
&\quad-\frac{e^3x^{-1+3r}(a+b\log(cx^n))}{1-3r}-(bn)\int\left(-\frac{d^3}{x^2}+\frac{3d^2ex^{-2+r}}{-1+r}+\frac{3de^2x^{2(-1+r)}}{-1+2r}+\frac{e^3x^{-2+3r}}{-1+3r}\right)dx\\
&= -\frac{bd^3n}{x}-\frac{3bd^2enx^{-1+r}}{(1-r)^2}-\frac{3bde^2nx^{-1+2r}}{(1-2r)^2}-\frac{be^3nx^{-1+3r}}{(1-3r)^2}-\frac{d^3(a+b\log(cx^n))}{x}\\
&\quad-\frac{3d^2ex^{-1+r}(a+b\log(cx^n))}{1-r}-\frac{3de^2x^{-1+2r}(a+b\log(cx^n))}{1-2r}\\
&\quad-\frac{e^3x^{-1+3r}(a+b\log(cx^n))}{1-3r}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^2} dx$$
$$= \frac{bn\left(-d^3-\frac{3d^2ex^r}{(-1+r)^2}-\frac{3de^2x^{2r}}{(1-2r)^2}-\frac{e^3x^{3r}}{(1-3r)^2}\right)+a\left(-d^3+\frac{3d^2ex^r}{-1+r}+\frac{3de^2x^{2r}}{-1+2r}+\frac{e^3x^{3r}}{-1+3r}\right)+b\left(-d^3+\frac{3d^2ex^r}{-1+r}+\frac{3de^2x^{2r}}{-1+2r}+\cdots\right.}{x}$$

```
[In] Integrate[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] (b*n*(-d^3 - (3*d^2*e*x^r)/(-1 + r)^2 - (3*d*e^2*x^(2*r))/(1 - 2*r)^2 - (e^
3*x^(3*r))/(1 - 3*r)^2) + a*(-d^3 + (3*d^2*e*x^r)/(-1 + r) + (3*d*e^2*x^(2*
r))/(-1 + 2*r) + (e^3*x^(3*r))/(-1 + 3*r)) + b*(-d^3 + (3*d^2*e*x^r)/(-1 +
r) + (3*d*e^2*x^(2*r))/(-1 + 2*r) + (e^3*x^(3*r))/(-1 + 3*r))*Log[c*x^n])/x
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. 2(179) = 358.

Time = 3.77 (sec) , antiderivative size = 1035, normalized size of antiderivative = 5.78

method	result	size
parallelrisch	Expression too large to display	1035
risch	Expression too large to display	4031

```
[In] int((d+e*x^r)^3*(a+b*ln(c*x^n))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*ln(c*x^n)*d^3+3*b*d*e^2*ln(c*x^n)*(x^r)^2+e^3*(x^r)^3*a-144*b*d^3*n*r^3
+58*b*d^3*n*r^2-12*b*d^3*n*r+114*a*d*e^2*r^2*(x^r)^2-204*a*d*e^2*r^3*(x^r)^
2+3*d*e^2*(x^r)^2*a+3*d^2*e*x^r*a+a*d^3-51*a*e^3*r^3*(x^r)^3+31*a*e^3*r^2*(
x^r)^3-9*a*e^3*r*(x^r)^3-12*a*e^3*r^5*(x^r)^3+40*a*e^3*r^4*(x^r)^3-12*(x^r)
^3*ln(c*x^n)*b*e^3*r^5+40*(x^r)^3*ln(c*x^n)*b*e^3*r^4-51*(x^r)^3*ln(c*x^n)*
b*e^3*r^3+31*(x^r)^3*ln(c*x^n)*b*e^3*r^2-9*(x^r)^3*ln(c*x^n)*b*e^3*r+3*b*d^
2*e*ln(c*x^n)*x^r+36*b*d^3*n*r^6-132*b*d^3*n*r^5+193*b*d^3*n*r^4-108*x^r*ln
(c*x^n)*b*d^2*e*r^5+288*x^r*ln(c*x^n)*b*d^2*e*r^4-291*x^r*ln(c*x^n)*b*d^2*e
*r^3+141*x^r*ln(c*x^n)*b*d^2*e*r^2-33*x^r*ln(c*x^n)*b*d^2*e*r-54*(x^r)^2*ln
(c*x^n)*b*d*e^2*r^5+171*(x^r)^2*ln(c*x^n)*b*d*e^2*r^4-204*(x^r)^2*ln(c*x^n)
*b*d*e^2*r^3+114*(x^r)^2*ln(c*x^n)*b*d*e^2*r^2-30*(x^r)^2*ln(c*x^n)*b*d*e^2
*r+b*d^3*n+36*ln(c*x^n)*b*d^3*r^6-132*ln(c*x^n)*b*d^3*r^5+193*ln(c*x^n)*b*d
^3*r^4-144*ln(c*x^n)*b*d^3*r^3+58*ln(c*x^n)*b*d^3*r^2-12*ln(c*x^n)*b*d^3*r+
e^3*b*ln(c*x^n)*(x^r)^3-144*a*d^3*r^3+58*a*d^3*r^2-12*a*d^3*r+36*a*d^3*r^6-
132*a*d^3*r^5+193*a*d^3*r^4-291*a*d^2*e*r^3*x^r+3*b*d*e^2*n*(x^r)^2+3*b*d^2
*e*n*x^r+b*e^3*n*(x^r)^3-12*b*e^3*n*r^3*(x^r)^3+13*b*e^3*n*r^2*(x^r)^3-6*b*
e^3*n*r*(x^r)^3+141*a*d^2*e*r^2*x^r+288*a*d^2*e*r^4*x^r-33*a*d^2*e*r*x^r+4*
```

```
b*e^3*n*r^4*(x^r)^3-30*a*d*e^2*r*(x^r)^2-54*a*d*e^2*r^5*(x^r)^2+171*a*d*e^2
*r^4*(x^r)^2-108*a*d^2*e*r^5*x^r+66*b*d*e^2*n*r^2*(x^r)^2+111*b*d^2*e*n*r^2
*x^r-24*b*d*e^2*n*r*(x^r)^2-30*b*d^2*e*n*r*x^r-180*b*d^2*e*n*r^3*x^r+27*b*d
*e^2*n*r^4*(x^r)^2-72*b*d*e^2*n*r^3*(x^r)^2+108*b*d^2*e*n*r^4*x^r)/x/(-1+3*
r)^2/(-1+2*r)^2/(-1+r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(174) = 348$.

Time = 0.31 (sec) , antiderivative size = 967, normalized size of antiderivative = 5.40

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^2} \, dx = -\frac{36\,(bd^3n+ad^3)r^6 - 132\,(bd^3n+ad^3)r^5 + bd^3n + 193\,(bd^3n+ad^3)r^4 + ad^3 - 144\,(bd^3n+ad^3)r^3 + 58\,(\cdots}{\cdots}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
```

```
[Out] -(36*(b*d^3*n + a*d^3)*r^6 - 132*(b*d^3*n + a*d^3)*r^5 + b*d^3*n + 193*(b*d
^3*n + a*d^3)*r^4 + a*d^3 - 144*(b*d^3*n + a*d^3)*r^3 + 58*(b*d^3*n + a*d^3
)*r^2 - 12*(b*d^3*n + a*d^3)*r - (12*a*e^3*r^5 - b*e^3*n - 4*(b*e^3*n + 10*
a*e^3)*r^4 - a*e^3 + 3*(4*b*e^3*n + 17*a*e^3)*r^3 - (13*b*e^3*n + 31*a*e^3)
*r^2 + 3*(2*b*e^3*n + 3*a*e^3)*r + (12*b*e^3*r^5 - 40*b*e^3*r^4 + 51*b*e^3*
r^3 - 31*b*e^3*r^2 + 9*b*e^3*r - b*e^3)*log(c) + (12*b*e^3*n*r^5 - 40*b*e^3
*n*r^4 + 51*b*e^3*n*r^3 - 31*b*e^3*n*r^2 + 9*b*e^3*n*r - b*e^3*n)*log(x))*x
^(3*r) - 3*(18*a*d*e^2*r^5 - b*d*e^2*n - 3*(3*b*d*e^2*n + 19*a*d*e^2)*r^4 -
 a*d*e^2 + 4*(6*b*d*e^2*n + 17*a*d*e^2)*r^3 - 2*(11*b*d*e^2*n + 19*a*d*e^2)
*r^2 + 2*(4*b*d*e^2*n + 5*a*d*e^2)*r + (18*b*d*e^2*r^5 - 57*b*d*e^2*r^4 + 6
8*b*d*e^2*r^3 - 38*b*d*e^2*r^2 + 10*b*d*e^2*r - b*d*e^2)*log(c) + (18*b*d*e
^2*n*r^5 - 57*b*d*e^2*n*r^4 + 68*b*d*e^2*n*r^3 - 38*b*d*e^2*n*r^2 + 10*b*d*
e^2*n*r - b*d*e^2*n)*log(x))*x^(2*r) - 3*(36*a*d^2*e*r^5 - b*d^2*e*n - 12*(
3*b*d^2*e*n + 8*a*d^2*e)*r^4 - a*d^2*e + (60*b*d^2*e*n + 97*a*d^2*e)*r^3 -
(37*b*d^2*e*n + 47*a*d^2*e)*r^2 + (10*b*d^2*e*n + 11*a*d^2*e)*r + (36*b*d^2
*e*r^5 - 96*b*d^2*e*r^4 + 97*b*d^2*e*r^3 - 47*b*d^2*e*r^2 + 11*b*d^2*e*r -
b*d^2*e)*log(c) + (36*b*d^2*e*n*r^5 - 96*b*d^2*e*n*r^4 + 97*b*d^2*e*n*r^3 -
 47*b*d^2*e*n*r^2 + 11*b*d^2*e*n*r - b*d^2*e*n)*log(x))*x^r + (36*b*d^3*r^6
 - 132*b*d^3*r^5 + 193*b*d^3*r^4 - 144*b*d^3*r^3 + 58*b*d^3*r^2 - 12*b*d^3*
r + b*d^3)*log(c) + (36*b*d^3*n*r^6 - 132*b*d^3*n*r^5 + 193*b*d^3*n*r^4 - 1
44*b*d^3*n*r^3 + 58*b*d^3*n*r^2 - 12*b*d^3*n*r + b*d^3*n)*log(x))/((36*r^6
- 132*r^5 + 193*r^4 - 144*r^3 + 58*r^2 - 12*r + 1)*x)
```

Sympy [A] (verification not implemented)

Time = 10.76 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.80

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^2}\,dx$$
$$= -\frac{ad^3}{x} + 3ad^2e\left(\begin{cases}\frac{x^r}{rx-x} & \text{for } r\neq 1\\ \frac{x^r\log(x)}{x} & \text{otherwise}\end{cases}\right) + 3ade^2\left(\begin{cases}\frac{x^{2r}}{2rx-x} & \text{for } r\neq \frac{1}{2}\\ \frac{x^{2r}\log(x)}{x} & \text{otherwise}\end{cases}\right)$$
$$+ ae^3\left(\begin{cases}\frac{x^{3r}}{3rx-x} & \text{for } r\neq \frac{1}{3}\\ \frac{x^{3r}\log(x)}{x} & \text{otherwise}\end{cases}\right) - \frac{bd^3n}{x} - \frac{bd^3\log(cx^n)}{x}$$
$$- 3bd^2en\left(\begin{cases}\frac{\begin{cases}\frac{x^{r-1}}{r-1} & \text{for } r\neq 1\\ \log(x) & \text{otherwise}\end{cases}}{r-1} & \text{for } r>-\infty \wedge r<\infty \wedge r\neq 1\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ 3bd^2e\left(\begin{cases}\frac{x^{r-1}}{r-1} & \text{for } r\neq 1\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$
$$- 3bde^2n\left(\begin{cases}\frac{\begin{cases}\frac{x^{2r-1}}{2r-1} & \text{for } r\neq \frac{1}{2}\\ \log(x) & \text{otherwise}\end{cases}}{2r-1} & \text{for } r>-\infty \wedge r<\infty \wedge r\neq \frac{1}{2}\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ 3bde^2\left(\begin{cases}\frac{x^{2r-1}}{2r-1} & \text{for } r\neq \frac{1}{2}\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$
$$- be^3n\left(\begin{cases}\frac{\begin{cases}\frac{x^{3r-1}}{3r-1} & \text{for } r\neq \frac{1}{3}\\ \log(x) & \text{otherwise}\end{cases}}{3r-1} & \text{for } r>-\infty \wedge r<\infty \wedge r\neq \frac{1}{3}\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ be^3\left(\begin{cases}\frac{x^{3r-1}}{3r-1} & \text{for } r\neq \frac{1}{3}\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate((d+e*x**r)**3*(a+b*ln(c*x**n))/x**2,x)
```

```
[Out] -a*d**3/x + 3*a*d**2*e*Piecewise((x**r/(r*x - x), Ne(r, 1)), (x**r*log(x)/x
, True)) + 3*a*d*e**2*Piecewise((x**(2*r)/(2*r*x - x), Ne(r, 1/2)), (x**(2*
r)*log(x)/x, True)) + a*e**3*Piecewise((x**(3*r)/(3*r*x - x), Ne(r, 1/3)),
```

```
(x**(3*r)*log(x)/x, True)) - b*d**3*n/x - b*d**3*log(c*x**n)/x - 3*b*d**2*e
*n*Piecewise((Piecewise((x**(r - 1)/(r - 1), Ne(r, 1)), (log(x), True))/(r
- 1), (r > -oo) & (r < oo) & Ne(r, 1)), (log(x)**2/2, True)) + 3*b*d**2*e*P
iecewise((x**(r - 1)/(r - 1), Ne(r, 1)), (log(x), True))*log(c*x**n) - 3*b*
d*e**2*n*Piecewise((Piecewise((x**(2*r - 1)/(2*r - 1), Ne(r, 1/2)), (log(x)
, True))/(2*r - 1), (r > -oo) & (r < oo) & Ne(r, 1/2)), (log(x)**2/2, True)
) + 3*b*d*e**2*Piecewise((x**(2*r - 1)/(2*r - 1), Ne(r, 1/2)), (log(x), Tru
e))*log(c*x**n) - b*e**3*n*Piecewise((Piecewise((x**(3*r - 1)/(3*r - 1), Ne
(r, 1/3)), (log(x), True))/(3*r - 1), (r > -oo) & (r < oo) & Ne(r, 1/3)), (
log(x)**2/2, True)) + b*e**3*Piecewise((x**(3*r - 1)/(3*r - 1), Ne(r, 1/3))
, (log(x), True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-2>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^2}\,dx = \int \frac{(ex^r+d)^3(b\log(cx^n)+a)}{x^2}\,dx$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^r + d)^3*(b*log(c*x^n) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^3\,(a+b\log{(cx^n)})}{x^2}\,dx = \int \frac{(d+e\,x^r)^3\,(a+b\,\ln{(c\,x^n)})}{x^2}\,dx$$

```
[In] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x^2,x)
[Out] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x^2, x)
```

3.402 $\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 191

$$\int \frac{(d+ex^r)^3\,(a+b\log(cx^n))}{x^4}\,dx = -\frac{bd^3n}{9x^3}-\frac{be^3nx^{-3(1-r)}}{9(1-r)^2}-\frac{3bd^2enx^{-3+r}}{(3-r)^2}-\frac{3bde^2nx^{-3+2r}}{(3-2r)^2}-\frac{d^3(a+b\log(cx^n))}{3x^3}-\frac{e^3x^{-3(1-r)}(a+b\log(cx^n))}{3(1-r)}-\frac{3d^2ex^{-3+r}(a+b\log(cx^n))}{3-r}-\frac{3de^2x^{-3+2r}(a+b\log(cx^n))}{3-2r}$$

```
[Out] -1/9*b*d^3*n/x^3-1/9*b*e^3*n/(1-r)^2/(x^(3-3*r))-3*b*d^2*e*n*x^(-3+r)/(3-r)
^2-3*b*d*e^2*n*x^(-3+2*r)/(3-2*r)^2-1/3*d^3*(a+b*ln(c*x^n))/x^3-1/3*e^3*(a+
b*ln(c*x^n))/(1-r)/(x^(3-3*r))-3*d^2*e*x^(-3+r)*(a+b*ln(c*x^n))/(3-r)-3*d*e
^2*x^(-3+2*r)*(a+b*ln(c*x^n))/(3-2*r)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used

= {276, 2372, 12, 14}

$$\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^4}\,dx = -\frac{d^3(a+b\log(cx^n))}{3x^3} - \frac{3d^2ex^{r-3}(a+b\log(cx^n))}{3-r} - \frac{3de^2x^{2r-3}(a+b\log(cx^n))}{3-2r} - \frac{e^3x^{-3(1-r)}(a+b\log(cx^n))}{3(1-r)} - \frac{bd^3n}{9x^3} - \frac{3bd^2enx^{r-3}}{(3-r)^2} - \frac{3bde^2nx^{2r-3}}{(3-2r)^2} - \frac{be^3nx^{-3(1-r)}}{9(1-r)^2}$$

```
[In] Int[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] -1/9*(b*d^3*n)/x^3 - (b*e^3*n)/(9*(1 - r)^2*x^(3*(1 - r))) - (3*b*d^2*e*n*x
^(-3 + r))/(3 - r)^2 - (3*b*d*e^2*n*x^(-3 + 2*r))/(3 - 2*r)^2 - (d^3*(a + b
*Log[c*x^n]))/(3*x^3) - (e^3*(a + b*Log[c*x^n]))/(3*(1 - r)*x^(3*(1 - r)))
- (3*d^2*e*x^(-3 + r)*(a + b*Log[c*x^n]))/(3 - r) - (3*d*e^2*x^(-3 + 2*r)*(
a + b*Log[c*x^n]))/(3 - 2*r)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{3x^3} - \frac{e^3x^{-3(1-r)}(a+b\log(cx^n))}{3(1-r)} - \frac{3d^2ex^{-3+r}(a+b\log(cx^n))}{3-r} \\
&\quad - \frac{3de^2x^{-3+2r}(a+b\log(cx^n))}{3-2r} - (bn)\int \frac{-d^3+\frac{9d^2ex^r}{-3+r}+\frac{9de^2x^{2r}}{-3+2r}+\frac{e^3x^{3r}}{-1+r}}{3x^4}\,dx \\
&= -\frac{d^3(a+b\log(cx^n))}{3x^3} - \frac{e^3x^{-3(1-r)}(a+b\log(cx^n))}{3(1-r)} - \frac{3d^2ex^{-3+r}(a+b\log(cx^n))}{3-r} \\
&\quad - \frac{3de^2x^{-3+2r}(a+b\log(cx^n))}{3-2r} - \frac{1}{3}(bn)\int \frac{-d^3+\frac{9d^2ex^r}{-3+r}+\frac{9de^2x^{2r}}{-3+2r}+\frac{e^3x^{3r}}{-1+r}}{x^4}\,dx \\
&= -\frac{d^3(a+b\log(cx^n))}{3x^3} - \frac{e^3x^{-3(1-r)}(a+b\log(cx^n))}{3(1-r)} \\
&\quad - \frac{3d^2ex^{-3+r}(a+b\log(cx^n))}{3-r} - \frac{3de^2x^{-3+2r}(a+b\log(cx^n))}{3-2r} \\
&\quad - \frac{1}{3}(bn)\int \left(-\frac{d^3}{x^4}+\frac{9d^2ex^{-4+r}}{-3+r}+\frac{9de^2x^{2(-2+r)}}{-3+2r}+\frac{e^3x^{-4+3r}}{-1+r}\right)dx \\
&= -\frac{bd^3n}{9x^3} - \frac{be^3nx^{-3(1-r)}}{9(1-r)^2} - \frac{3bd^2enx^{-3+r}}{(3-r)^2} - \frac{3bde^2nx^{-3+2r}}{(3-2r)^2} - \frac{d^3(a+b\log(cx^n))}{3x^3} \\
&\quad - \frac{e^3x^{-3(1-r)}(a+b\log(cx^n))}{3(1-r)} - \frac{3d^2ex^{-3+r}(a+b\log(cx^n))}{3-r} - \frac{3de^2x^{-3+2r}(a+b\log(cx^n))}{3-2r}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^4}\,dx = \frac{bn\left(-d^3-\frac{27d^2ex^r}{(-3+r)^2}-\frac{27de^2x^{2r}}{(3-2r)^2}-\frac{e^3x^{3r}}{(-1+r)^2}\right)+3a\left(-d^3+\frac{9d^2ex^r}{-3+r}+\frac{9de^2x^{2r}}{-3+2r}+\frac{e^3x^{3r}}{-1+r}\right)+3b\left(-d^3+\frac{9d^2ex^r}{-3+r}+\frac{9de^2x^{2r}}{-3+2r}\right.}{9x^3}$$

```
[In] Integrate[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] (b*n*(-d^3 - (27*d^2*e*x^r)/(-3 + r)^2 - (27*d*e^2*x^(2*r))/(3 - 2*r)^2 - (
e^3*x^(3*r))/(-1 + r)^2) + 3*a*(-d^3 + (9*d^2*e*x^r)/(-3 + r) + (9*d*e^2*x^
(2*r))/(-3 + 2*r) + (e^3*x^(3*r))/(-1 + r)) + 3*b*(-d^3 + (9*d^2*e*x^r)/(-3
 + r) + (9*d*e^2*x^(2*r))/(-3 + 2*r) + (e^3*x^(3*r))/(-1 + r))*Log[c*x^n])/
(9*x^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. $2(183) = 366$.

Time = 3.47 (sec) , antiderivative size = 1039, normalized size of antiderivative = 5.44

method	result	size
parallelrisch	Expression too large to display	1039
risch	Expression too large to display	4027

```
[In] int((d+e*x^r)^3*(a+b*ln(c*x^n))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*(243*b*ln(c*x^n)*d^3+729*b*d*e^2*ln(c*x^n)*(x^r)^2+243*e^3*(x^r)^3*a-4
32*b*d^3*n*r^3+522*b*d^3*n*r^2-324*b*d^3*n*r+3078*a*d*e^2*r^2*(x^r)^2-1836*
a*d*e^2*r^3*(x^r)^2+729*d*e^2*(x^r)^2*a+729*d^2*e*x^r*a+243*a*d^3-459*a*e^3
*r^3*(x^r)^3+837*a*e^3*r^2*(x^r)^3-729*a*e^3*r*(x^r)^3-12*a*e^3*r^5*(x^r)^3
+120*a*e^3*r^4*(x^r)^3-12*(x^r)^3*ln(c*x^n)*b*e^3*r^5+120*(x^r)^3*ln(c*x^n)
*b*e^3*r^4-459*(x^r)^3*ln(c*x^n)*b*e^3*r^3+837*(x^r)^3*ln(c*x^n)*b*e^3*r^2-
729*(x^r)^3*ln(c*x^n)*b*e^3*r+729*b*d^2*e*ln(c*x^n)*x^r+4*b*d^3*n*r^6-44*b*
d^3*n*r^5+193*b*d^3*n*r^4-108*x^r*ln(c*x^n)*b*d^2*e*r^5+864*x^r*ln(c*x^n)*b
*d^2*e*r^4-2619*x^r*ln(c*x^n)*b*d^2*e*r^3+3807*x^r*ln(c*x^n)*b*d^2*e*r^2-26
73*x^r*ln(c*x^n)*b*d^2*e*r-54*(x^r)^2*ln(c*x^n)*b*d*e^2*r^5+513*(x^r)^2*ln(
c*x^n)*b*d*e^2*r^4-1836*(x^r)^2*ln(c*x^n)*b*d*e^2*r^3+3078*(x^r)^2*ln(c*x^n
)*b*d*e^2*r^2-2430*(x^r)^2*ln(c*x^n)*b*d*e^2*r+81*b*d^3*n+12*ln(c*x^n)*b*d^
3*r^6-132*ln(c*x^n)*b*d^3*r^5+579*ln(c*x^n)*b*d^3*r^4-1296*ln(c*x^n)*b*d^3*
r^3+1566*ln(c*x^n)*b*d^3*r^2-972*ln(c*x^n)*b*d^3*r+243*e^3*b*ln(c*x^n)*(x^r
)^3-1296*a*d^3*r^3+1566*a*d^3*r^2-972*a*d^3*r+12*a*d^3*r^6-132*a*d^3*r^5+57
9*a*d^3*r^4-2619*a*d^2*e*r^3*x^r+243*b*d*e^2*n*(x^r)^2+243*b*d^2*e*n*x^r+81
*b*e^3*n*(x^r)^3-36*b*e^3*n*r^3*(x^r)^3+117*b*e^3*n*r^2*(x^r)^3-162*b*e^3*n
*r*(x^r)^3+3807*a*d^2*e*r^2*x^r+864*a*d^2*e*r^4*x^r-2673*a*d^2*e*r*x^r+4*b*
e^3*n*r^4*(x^r)^3-2430*a*d*e^2*r*(x^r)^2-54*a*d*e^2*r^5*(x^r)^2+513*a*d*e^2
*r^4*(x^r)^2-108*a*d^2*e*r^5*x^r+594*b*d*e^2*n*r^2*(x^r)^2+999*b*d^2*e*n*r^
2*x^r-648*b*d*e^2*n*r*(x^r)^2-810*b*d^2*e*n*r*x^r-540*b*d^2*e*n*r^3*x^r+27*
b*d*e^2*n*r^4*(x^r)^2-216*b*d*e^2*n*r^3*(x^r)^2+108*b*d^2*e*n*r^4*x^r)/x^3/
(-1+r)^2/(-3+2*r)^2/(-3+r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(175) = 350$.

Time = 0.34 (sec) , antiderivative size = 980, normalized size of antiderivative = 5.13

$$\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^4}\,dx =$$

$$-\frac{4\,(bd^3n+3\,ad^3)r^6-44\,(bd^3n+3\,ad^3)r^5+81\,bd^3n+193\,(bd^3n+3\,ad^3)r^4+243\,ad^3-432\,(bd^3n+3\,ad^3}{}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(4*(b*d^3*n + 3*a*d^3)*r^6 - 44*(b*d^3*n + 3*a*d^3)*r^5 + 81*b*d^3*n +
 193*(b*d^3*n + 3*a*d^3)*r^4 + 243*a*d^3 - 432*(b*d^3*n + 3*a*d^3)*r^3 + 52
2*(b*d^3*n + 3*a*d^3)*r^2 - 324*(b*d^3*n + 3*a*d^3)*r - (12*a*e^3*r^5 - 81*
b*e^3*n - 4*(b*e^3*n + 30*a*e^3)*r^4 - 243*a*e^3 + 9*(4*b*e^3*n + 51*a*e^3)
*r^3 - 9*(13*b*e^3*n + 93*a*e^3)*r^2 + 81*(2*b*e^3*n + 9*a*e^3)*r + 3*(4*b*
e^3*r^5 - 40*b*e^3*r^4 + 153*b*e^3*r^3 - 279*b*e^3*r^2 + 243*b*e^3*r - 81*b
*e^3)*log(c) + 3*(4*b*e^3*n*r^5 - 40*b*e^3*n*r^4 + 153*b*e^3*n*r^3 - 279*b*
e^3*n*r^2 + 243*b*e^3*n*r - 81*b*e^3*n)*log(x))*x^(3*r) - 27*(2*a*d*e^2*r^5
 - 9*b*d*e^2*n - (b*d*e^2*n + 19*a*d*e^2)*r^4 - 27*a*d*e^2 + 4*(2*b*d*e^2*n
 + 17*a*d*e^2)*r^3 - 2*(11*b*d*e^2*n + 57*a*d*e^2)*r^2 + 6*(4*b*d*e^2*n + 1
5*a*d*e^2)*r + (2*b*d*e^2*r^5 - 19*b*d*e^2*r^4 + 68*b*d*e^2*r^3 - 114*b*d*e
^2*r^2 + 90*b*d*e^2*r - 27*b*d*e^2)*log(c) + (2*b*d*e^2*n*r^5 - 19*b*d*e^2*
n*r^4 + 68*b*d*e^2*n*r^3 - 114*b*d*e^2*n*r^2 + 90*b*d*e^2*n*r - 27*b*d*e^2*
n)*log(x))*x^(2*r) - 27*(4*a*d^2*e*r^5 - 9*b*d^2*e*n - 4*(b*d^2*e*n + 8*a*d
^2*e)*r^4 - 27*a*d^2*e + (20*b*d^2*e*n + 97*a*d^2*e)*r^3 - (37*b*d^2*e*n +
141*a*d^2*e)*r^2 + 3*(10*b*d^2*e*n + 33*a*d^2*e)*r + (4*b*d^2*e*r^5 - 32*b*
d^2*e*r^4 + 97*b*d^2*e*r^3 - 141*b*d^2*e*r^2 + 99*b*d^2*e*r - 27*b*d^2*e)*l
og(c) + (4*b*d^2*e*n*r^5 - 32*b*d^2*e*n*r^4 + 97*b*d^2*e*n*r^3 - 141*b*d^2*
e*n*r^2 + 99*b*d^2*e*n*r - 27*b*d^2*e*n)*log(x))*x^r + 3*(4*b*d^3*r^6 - 44*
b*d^3*r^5 + 193*b*d^3*r^4 - 432*b*d^3*r^3 + 522*b*d^3*r^2 - 324*b*d^3*r + 8
1*b*d^3)*log(c) + 3*(4*b*d^3*n*r^6 - 44*b*d^3*n*r^5 + 193*b*d^3*n*r^4 - 432
*b*d^3*n*r^3 + 522*b*d^3*n*r^2 - 324*b*d^3*n*r + 81*b*d^3*n)*log(x))/((4*r^
6 - 44*r^5 + 193*r^4 - 432*r^3 + 522*r^2 - 324*r + 81)*x^3)
```

Sympy [A] (verification not implemented)

Time = 49.68 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.82

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^4}\,dx$$
$$= -\frac{ad^3}{3x^3} + 3ad^2e\left(\begin{cases}\frac{x^r}{rx^3-3x^3} & \text{for } r\neq 3\\ \frac{x^r\log(x)}{x^3} & \text{otherwise}\end{cases}\right) + 3ade^2\left(\begin{cases}\frac{x^{2r}}{2rx^3-3x^3} & \text{for } r\neq \frac{3}{2}\\ \frac{x^{2r}\log(x)}{x^3} & \text{otherwise}\end{cases}\right)$$
$$+ ae^3\left(\begin{cases}\frac{x^{3r}}{3rx^3-3x^3} & \text{for } r\neq 1\\ \frac{x^{3r}\log(x)}{x^3} & \text{otherwise}\end{cases}\right) - \frac{bd^3n}{9x^3} - \frac{bd^3\log(cx^n)}{3x^3}$$
$$- 3bd^2en\left(\begin{cases}\frac{\begin{cases}\frac{x^{r-3}}{r-3} & \text{for } r\neq 3\\ \log(x) & \text{otherwise}\end{cases}}{r-3} & \text{for } r>-\infty \wedge r<\infty \wedge r\neq 3\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ 3bd^2e\left(\begin{cases}\frac{x^{r-3}}{r-3} & \text{for } r\neq 3\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$
$$- 3bde^2n\left(\begin{cases}\frac{\begin{cases}\frac{x^{2r-3}}{2r-3} & \text{for } r\neq \frac{3}{2}\\ \log(x) & \text{otherwise}\end{cases}}{2r-3} & \text{for } r>-\infty \wedge r<\infty \wedge r\neq \frac{3}{2}\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ 3bde^2\left(\begin{cases}\frac{x^{2r-3}}{2r-3} & \text{for } r\neq \frac{3}{2}\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$
$$- be^3n\left(\begin{cases}\frac{\begin{cases}\frac{x^{3r-3}}{3r-3} & \text{for } r\neq 1\\ \log(x) & \text{otherwise}\end{cases}}{3r-3} & \text{for } r>-\infty \wedge r<\infty \wedge r\neq 1\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ be^3\left(\begin{cases}\frac{x^{3r-3}}{3r-3} & \text{for } r\neq 1\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$

```
[In] integrate((d+e*x**r)**3*(a+b*ln(c*x**n))/x**4,x)
```

```
[Out] -a*d**3/(3*x**3) + 3*a*d**2*e*Piecewise((x**r/(r*x**3 - 3*x**3), Ne(r, 3)),
 (x**r*log(x)/x**3, True)) + 3*a*d*e**2*Piecewise((x**(2*r)/(2*r*x**3 - 3*x
**3), Ne(r, 3/2)), (x**(2*r)*log(x)/x**3, True)) + a*e**3*Piecewise((x**(3*
```

```
r)/(3*r*x**3 - 3*x**3), Ne(r, 1)), (x**(3*r)*log(x)/x**3, True)) - b*d**3*n
/(9*x**3) - b*d**3*log(c*x**n)/(3*x**3) - 3*b*d**2*e*n*Piecewise((Piecewise
((x**(r - 3)/(r - 3), Ne(r, 3)), (log(x), True))/(r - 3), (r > -oo) & (r <
oo) & Ne(r, 3)), (log(x)**2/2, True)) + 3*b*d**2*e*Piecewise((x**(r - 3)/(r
 - 3), Ne(r, 3)), (log(x), True))*log(c*x**n) - 3*b*d*e**2*n*Piecewise((Pie
cewise((x**(2*r - 3)/(2*r - 3), Ne(r, 3/2)), (log(x), True))/(2*r - 3), (r
> -oo) & (r < oo) & Ne(r, 3/2)), (log(x)**2/2, True)) + 3*b*d*e**2*Piecewis
e((x**(2*r - 3)/(2*r - 3), Ne(r, 3/2)), (log(x), True))*log(c*x**n) - b*e**
3*n*Piecewise((Piecewise((x**(3*r - 3)/(3*r - 3), Ne(r, 1)), (log(x), True)
)/(3*r - 3), (r > -oo) & (r < oo) & Ne(r, 1)), (log(x)**2/2, True)) + b*e**
3*Piecewise((x**(3*r - 3)/(3*r - 3), Ne(r, 1)), (log(x), True))*log(c*x**n)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^4} dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-4>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^4} dx = \int \frac{(ex^r+d)^3(b\log(cx^n)+a)}{x^4} dx$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^r + d)^3*(b*log(c*x^n) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^4}\,dx = \int \frac{(d+e\,x^r)^3 (a+b\,\ln(c\,x^n))}{x^4}\,dx$$

```
[In] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x^4,x)
[Out] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x^4, x)
```

3.403 $\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 183

$$\int \frac{(d+ex^r)^3\,(a+b\log(cx^n))}{x^6}\,dx = -\frac{bd^3n}{25x^5}-\frac{3bd^2enx^{-5+r}}{(5-r)^2}-\frac{3bde^2nx^{-5+2r}}{(5-2r)^2}-\frac{be^3nx^{-5+3r}}{(5-3r)^2}-\frac{d^3(a+b\log(cx^n))}{5x^5}-\frac{3d^2ex^{-5+r}(a+b\log(cx^n))}{5-r}-\frac{3de^2x^{-5+2r}(a+b\log(cx^n))}{5-2r}-\frac{e^3x^{-5+3r}(a+b\log(cx^n))}{5-3r}$$

```
[Out] -1/25*b*d^3*n/x^5-3*b*d^2*e*n*x^(-5+r)/(5-r)^2-3*b*d*e^2*n*x^(-5+2*r)/(5-2*
r)^2-b*e^3*n*x^(-5+3*r)/(5-3*r)^2-1/5*d^3*(a+b*ln(c*x^n))/x^5-3*d^2*e*x^(-5
+r)*(a+b*ln(c*x^n))/(5-r)-3*d*e^2*x^(-5+2*r)*(a+b*ln(c*x^n))/(5-2*r)-e^3*x^
(-5+3*r)*(a+b*ln(c*x^n))/(5-3*r)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2372, 12, 14}

$$\int \frac{(d+ex^r)^3\,(a+b\log(cx^n))}{x^6}\,dx = -\frac{d^3(a+b\log(cx^n))}{5x^5}-\frac{3d^2ex^{r-5}(a+b\log(cx^n))}{5-r}-\frac{3de^2x^{2r-5}(a+b\log(cx^n))}{5-2r}-\frac{e^3x^{3r-5}(a+b\log(cx^n))}{5-3r}-\frac{bd^3n}{25x^5}-\frac{3bd^2enx^{r-5}}{(5-r)^2}-\frac{3bde^2nx^{2r-5}}{(5-2r)^2}-\frac{be^3nx^{3r-5}}{(5-3r)^2}$$

```
[In] Int[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] -1/25*(b*d^3*n)/x^5 - (3*b*d^2*e*n*x^(-5 + r))/(5 - r)^2 - (3*b*d*e^2*n*x^(
-5 + 2*r))/(5 - 2*r)^2 - (b*e^3*n*x^(-5 + 3*r))/(5 - 3*r)^2 - (d^3*(a + b*L
og[c*x^n]))/(5*x^5) - (3*d^2*e*x^(-5 + r)*(a + b*Log[c*x^n]))/(5 - r) - (3*
d*e^2*x^(-5 + 2*r)*(a + b*Log[c*x^n]))/(5 - 2*r) - (e^3*x^(-5 + 3*r)*(a + b
*Log[c*x^n]))/(5 - 3*r)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{5x^5}-\frac{3d^2ex^{-5+r}(a+b\log(cx^n))}{5-r}-\frac{3de^2x^{-5+2r}(a+b\log(cx^n))}{5-2r}\\&\quad-\frac{e^3x^{-5+3r}(a+b\log(cx^n))}{5-3r}-(bn)\int\frac{-d^3+\frac{15d^2ex^r}{-5+r}+\frac{15de^2x^{2r}}{-5+2r}+\frac{5e^3x^{3r}}{-5+3r}}{5x^6}\,dx\\&=-\frac{d^3(a+b\log(cx^n))}{5x^5}-\frac{3d^2ex^{-5+r}(a+b\log(cx^n))}{5-r}-\frac{3de^2x^{-5+2r}(a+b\log(cx^n))}{5-2r}\\&\quad-\frac{e^3x^{-5+3r}(a+b\log(cx^n))}{5-3r}-\frac{1}{5}(bn)\int\frac{-d^3+\frac{15d^2ex^r}{-5+r}+\frac{15de^2x^{2r}}{-5+2r}+\frac{5e^3x^{3r}}{-5+3r}}{x^6}\,dx\end{aligned}$$

$$= -\frac{d^3(a+b\log(cx^n))}{5x^5} - \frac{3d^2ex^{-5+r}(a+b\log(cx^n))}{5-r} - \frac{3de^2x^{-5+2r}(a+b\log(cx^n))}{5-2r} - \frac{e^3x^{-5+3r}(a+b\log(cx^n))}{5-3r} - \frac{1}{5}(bn)\int\left(-\frac{d^3}{x^6}+\frac{15d^2ex^{-6+r}}{-5+r}+\frac{15de^2x^{2(-3+r)}}{-5+2r}+\frac{5e^3x^{3(-2+r)}}{-5+3r}\right)dx$$

$$= -\frac{bd^3n}{25x^5} - \frac{3bd^2enx^{-5+r}}{(5-r)^2} - \frac{3bde^2nx^{-5+2r}}{(5-2r)^2} - \frac{be^3nx^{-5+3r}}{(5-3r)^2} - \frac{d^3(a+b\log(cx^n))}{5x^5} - \frac{3d^2ex^{-5+r}(a+b\log(cx^n))}{5-r} - \frac{3de^2x^{-5+2r}(a+b\log(cx^n))}{5-2r} - \frac{e^3x^{-5+3r}(a+b\log(cx^n))}{5-3r}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^6}\,dx = \frac{bn\left(-d^3-\frac{75d^2ex^r}{(-5+r)^2}-\frac{75de^2x^{2r}}{(5-2r)^2}-\frac{25e^3x^{3r}}{(5-3r)^2}\right)+a\left(-5d^3+\frac{75d^2ex^r}{-5+r}+\frac{75de^2x^{2r}}{-5+2r}+\frac{25e^3x^{3r}}{-5+3r}\right)+5b\left(-d^3+\frac{15d^2ex^r}{-5+r}+15\cdots\right.}{25x^5}$$

```
[In] Integrate[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x^6,x]
```

```
[Out] (b*n*(-d^3 - (75*d^2*e*x^r)/(-5 + r)^2 - (75*d*e^2*x^(2*r))/(5 - 2*r)^2 - (
25*e^3*x^(3*r))/(5 - 3*r)^2) + a*(-5*d^3 + (75*d^2*e*x^r)/(-5 + r) + (75*d*
e^2*x^(2*r))/(-5 + 2*r) + (25*e^3*x^(3*r))/(-5 + 3*r)) + 5*b*(-d^3 + (15*d^
2*e*x^r)/(-5 + r) + (15*d*e^2*x^(2*r))/(-5 + 2*r) + (5*e^3*x^(3*r))/(-5 + 3
*r))*Log[c*x^n])/(25*x^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1040 vs. 2(179) = 358.

Time = 3.65 (sec) , antiderivative size = 1041, normalized size of antiderivative = 5.69

method	result	size
parallelrisch	Expression too large to display	1041
risch	Expression too large to display	4031

```
[In] int((d+e*x^r)^3*(a+b*ln(c*x^n))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/25*(78125*b*ln(c*x^n)*d^3+234375*b*d*e^2*ln(c*x^n)*(x^r)^2+78125*e^3*(x^
r)^3*a-18000*b*d^3*n*r^3+36250*b*d^3*n*r^2-37500*b*d^3*n*r+356250*a*d*e^2*r
```

```
^2*(x^r)^2-127500*a*d*e^2*r^3*(x^r)^2+234375*d*e^2*(x^r)^2*a+234375*d^2*e*x
^r*a+78125*a*d^3-31875*a*e^3*r^3*(x^r)^3+96875*a*e^3*r^2*(x^r)^3-140625*a*e
^3*r*(x^r)^3-300*a*e^3*r^5*(x^r)^3+5000*a*e^3*r^4*(x^r)^3-300*(x^r)^3*ln(c*
x^n)*b*e^3*r^5+5000*(x^r)^3*ln(c*x^n)*b*e^3*r^4-31875*(x^r)^3*ln(c*x^n)*b*e
^3*r^3+96875*(x^r)^3*ln(c*x^n)*b*e^3*r^2-140625*(x^r)^3*ln(c*x^n)*b*e^3*r+2
34375*b*d^2*e*ln(c*x^n)*x^r+36*b*d^3*n*r^6-660*b*d^3*n*r^5+4825*b*d^3*n*r^4
-2700*x^r*ln(c*x^n)*b*d^2*e*r^5+36000*x^r*ln(c*x^n)*b*d^2*e*r^4-181875*x^r*
ln(c*x^n)*b*d^2*e*r^3+440625*x^r*ln(c*x^n)*b*d^2*e*r^2-515625*x^r*ln(c*x^n)
*b*d^2*e*r-1350*(x^r)^2*ln(c*x^n)*b*d*e^2*r^5+21375*(x^r)^2*ln(c*x^n)*b*d*e
^2*r^4-127500*(x^r)^2*ln(c*x^n)*b*d*e^2*r^3+356250*(x^r)^2*ln(c*x^n)*b*d*e^
2*r^2-468750*(x^r)^2*ln(c*x^n)*b*d*e^2*r+15625*b*d^3*n+180*ln(c*x^n)*b*d^3*
r^6-3300*ln(c*x^n)*b*d^3*r^5+24125*ln(c*x^n)*b*d^3*r^4-90000*ln(c*x^n)*b*d^
3*r^3+181250*ln(c*x^n)*b*d^3*r^2-187500*ln(c*x^n)*b*d^3*r+78125*e^3*b*ln(c*
x^n)*(x^r)^3-90000*a*d^3*r^3+181250*a*d^3*r^2-187500*a*d^3*r+180*a*d^3*r^6-
3300*a*d^3*r^5+24125*a*d^3*r^4-181875*a*d^2*e*r^3*x^r+46875*b*d*e^2*n*(x^r)
^2+46875*b*d^2*e*n*x^r+15625*b*e^3*n*(x^r)^3-1500*b*e^3*n*r^3*(x^r)^3+8125*
b*e^3*n*r^2*(x^r)^3-18750*b*e^3*n*r*(x^r)^3+440625*a*d^2*e*r^2*x^r+36000*a*
d^2*e*r^4*x^r-515625*a*d^2*e*r*x^r+100*b*e^3*n*r^4*(x^r)^3-468750*a*d*e^2*r
*(x^r)^2-1350*a*d*e^2*r^5*(x^r)^2+21375*a*d*e^2*r^4*(x^r)^2-2700*a*d^2*e*r^
5*x^r+41250*b*d*e^2*n*r^2*(x^r)^2+69375*b*d^2*e*n*r^2*x^r-75000*b*d*e^2*n*r
*(x^r)^2-93750*b*d^2*e*n*r*x^r-22500*b*d^2*e*n*r^3*x^r+675*b*d*e^2*n*r^4*(x
^r)^2-9000*b*d*e^2*n*r^3*(x^r)^2+2700*b*d^2*e*n*r^4*x^r)/x^5/(-5+3*r)^2/(-5
+2*r)^2/(-5+r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 981 vs. $2(174) = 348$.

Time = 0.36 (sec) , antiderivative size = 981, normalized size of antiderivative = 5.36

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^6} dx = \text{Too large to display}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^6,x, algorithm="fricas")
```

```
[Out] -1/25*(36*(b*d^3*n + 5*a*d^3)*r^6 - 660*(b*d^3*n + 5*a*d^3)*r^5 + 15625*b*d
^3*n + 4825*(b*d^3*n + 5*a*d^3)*r^4 + 78125*a*d^3 - 18000*(b*d^3*n + 5*a*d^
3)*r^3 + 36250*(b*d^3*n + 5*a*d^3)*r^2 - 37500*(b*d^3*n + 5*a*d^3)*r - 25*(
12*a*e^3*r^5 - 625*b*e^3*n - 4*(b*e^3*n + 50*a*e^3)*r^4 - 3125*a*e^3 + 15*(
4*b*e^3*n + 85*a*e^3)*r^3 - 25*(13*b*e^3*n + 155*a*e^3)*r^2 + 375*(2*b*e^3*
n + 15*a*e^3)*r + (12*b*e^3*r^5 - 200*b*e^3*r^4 + 1275*b*e^3*r^3 - 3875*b*e
^3*r^2 + 5625*b*e^3*r - 3125*b*e^3)*log(c) + (12*b*e^3*n*r^5 - 200*b*e^3*n*
r^4 + 1275*b*e^3*n*r^3 - 3875*b*e^3*n*r^2 + 5625*b*e^3*n*r - 3125*b*e^3*n)*
log(x))*x^(3*r) - 75*(18*a*d*e^2*r^5 - 625*b*d*e^2*n - 3*(3*b*d*e^2*n + 95*
a*d*e^2)*r^4 - 3125*a*d*e^2 + 20*(6*b*d*e^2*n + 85*a*d*e^2)*r^3 - 50*(11*b*
```

```
d*e^2*n + 95*a*d*e^2)*r^2 + 250*(4*b*d*e^2*n + 25*a*d*e^2)*r + (18*b*d*e^2*
r^5 - 285*b*d*e^2*r^4 + 1700*b*d*e^2*r^3 - 4750*b*d*e^2*r^2 + 6250*b*d*e^2*
r - 3125*b*d*e^2)*log(c) + (18*b*d*e^2*n*r^5 - 285*b*d*e^2*n*r^4 + 1700*b*d
*e^2*n*r^3 - 4750*b*d*e^2*n*r^2 + 6250*b*d*e^2*n*r - 3125*b*d*e^2*n)*log(x)
)*x^(2*r) - 75*(36*a*d^2*e*r^5 - 625*b*d^2*e*n - 12*(3*b*d^2*e*n + 40*a*d^2
*e)*r^4 - 3125*a*d^2*e + 25*(12*b*d^2*e*n + 97*a*d^2*e)*r^3 - 25*(37*b*d^2*
e*n + 235*a*d^2*e)*r^2 + 625*(2*b*d^2*e*n + 11*a*d^2*e)*r + (36*b*d^2*e*r^5
 - 480*b*d^2*e*r^4 + 2425*b*d^2*e*r^3 - 5875*b*d^2*e*r^2 + 6875*b*d^2*e*r -
 3125*b*d^2*e)*log(c) + (36*b*d^2*e*n*r^5 - 480*b*d^2*e*n*r^4 + 2425*b*d^2*
e*n*r^3 - 5875*b*d^2*e*n*r^2 + 6875*b*d^2*e*n*r - 3125*b*d^2*e*n)*log(x))*x
^r + 5*(36*b*d^3*r^6 - 660*b*d^3*r^5 + 4825*b*d^3*r^4 - 18000*b*d^3*r^3 + 3
6250*b*d^3*r^2 - 37500*b*d^3*r + 15625*b*d^3)*log(c) + 5*(36*b*d^3*n*r^6 -
660*b*d^3*n*r^5 + 4825*b*d^3*n*r^4 - 18000*b*d^3*n*r^3 + 36250*b*d^3*n*r^2
- 37500*b*d^3*n*r + 15625*b*d^3*n)*log(x))/((36*r^6 - 660*r^5 + 4825*r^4 -
18000*r^3 + 36250*r^2 - 37500*r + 15625)*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^6}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x**r)**3*(a+b*ln(c*x**n))/x**6,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-6>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^6} dx = \int \frac{(ex^r+d)^3(b\log(cx^n)+a)}{x^6} dx$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^6,x, algorithm="giac")
[Out] integrate((e*x^r + d)^3*(b*log(c*x^n) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^6} dx = \int \frac{(d+e\,x^r)^3 (a+b\,\ln(c\,x^n))}{x^6} dx$$

```
[In] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x^6,x)
[Out] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x^6, x)
```

3.404 $\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^8}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 183

$$\int \frac{(d+ex^r)^3\,(a+b\log(cx^n))}{x^8}\,dx = -\frac{bd^3n}{49x^7}-\frac{3bd^2enx^{-7+r}}{(7-r)^2}-\frac{3bde^2nx^{-7+2r}}{(7-2r)^2}-\frac{be^3nx^{-7+3r}}{(7-3r)^2}-\frac{d^3(a+b\log(cx^n))}{7x^7}-\frac{3d^2ex^{-7+r}(a+b\log(cx^n))}{7-r}-\frac{3de^2x^{-7+2r}(a+b\log(cx^n))}{7-2r}-\frac{e^3x^{-7+3r}(a+b\log(cx^n))}{7-3r}$$

```
[Out] -1/49*b*d^3*n/x^7-3*b*d^2*e*n*x^(-7+r)/(7-r)^2-3*b*d*e^2*n*x^(-7+2*r)/(7-2*
r)^2-b*e^3*n*x^(-7+3*r)/(7-3*r)^2-1/7*d^3*(a+b*ln(c*x^n))/x^7-3*d^2*e*x^(-7
+r)*(a+b*ln(c*x^n))/(7-r)-3*d*e^2*x^(-7+2*r)*(a+b*ln(c*x^n))/(7-2*r)-e^3*x^
(-7+3*r)*(a+b*ln(c*x^n))/(7-3*r)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {276, 2372, 12, 14}

$$\int \frac{(d+ex^r)^3\,(a+b\log(cx^n))}{x^8}\,dx = -\frac{d^3(a+b\log(cx^n))}{7x^7}-\frac{3d^2ex^{r-7}(a+b\log(cx^n))}{7-r}-\frac{3de^2x^{2r-7}(a+b\log(cx^n))}{7-2r}-\frac{e^3x^{3r-7}(a+b\log(cx^n))}{7-3r}-\frac{bd^3n}{49x^7}-\frac{3bd^2enx^{r-7}}{(7-r)^2}-\frac{3bde^2nx^{2r-7}}{(7-2r)^2}-\frac{be^3nx^{3r-7}}{(7-3r)^2}$$

```
[In] Int[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x^8,x]
```

```
[Out] -1/49*(b*d^3*n)/x^7 - (3*b*d^2*e*n*x^(-7 + r))/(7 - r)^2 - (3*b*d*e^2*n*x^(
-7 + 2*r))/(7 - 2*r)^2 - (b*e^3*n*x^(-7 + 3*r))/(7 - 3*r)^2 - (d^3*(a + b*L
og[c*x^n]))/(7*x^7) - (3*d^2*e*x^(-7 + r)*(a + b*Log[c*x^n]))/(7 - r) - (3*
d*e^2*x^(-7 + 2*r)*(a + b*Log[c*x^n]))/(7 - 2*r) - (e^3*x^(-7 + 3*r)*(a + b
*Log[c*x^n]))/(7 - 3*r)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{7x^7} - \frac{3d^2ex^{-7+r}(a+b\log(cx^n))}{7-r} - \frac{3de^2x^{-7+2r}(a+b\log(cx^n))}{7-2r} \\
&\quad - \frac{e^3x^{-7+3r}(a+b\log(cx^n))}{7-3r} - (bn)\int \frac{-d^3+\frac{21d^2ex^r}{-7+r}+\frac{21de^2x^{2r}}{-7+2r}+\frac{7e^3x^{3r}}{-7+3r}}{7x^8}\,dx \\
&= -\frac{d^3(a+b\log(cx^n))}{7x^7} - \frac{3d^2ex^{-7+r}(a+b\log(cx^n))}{7-r} - \frac{3de^2x^{-7+2r}(a+b\log(cx^n))}{7-2r} \\
&\quad - \frac{e^3x^{-7+3r}(a+b\log(cx^n))}{7-3r} - \frac{1}{7}(bn)\int \frac{-d^3+\frac{21d^2ex^r}{-7+r}+\frac{21de^2x^{2r}}{-7+2r}+\frac{7e^3x^{3r}}{-7+3r}}{x^8}\,dx
\end{aligned}
$$

$$= -\frac{d^3(a+b\log(cx^n))}{7x^7} - \frac{3d^2ex^{-7+r}(a+b\log(cx^n))}{7-r} - \frac{3de^2x^{-7+2r}(a+b\log(cx^n))}{7-2r} - \frac{e^3x^{-7+3r}(a+b\log(cx^n))}{7-3r} - \frac{1}{7}(bn)\int\left(-\frac{d^3}{x^8} + \frac{21d^2ex^{-8+r}}{-7+r} + \frac{21de^2x^{2(-4+r)}}{-7+2r} + \frac{7e^3x^{-8+3r}}{-7+3r}\right)dx$$

$$= -\frac{bd^3n}{49x^7} - \frac{3bd^2enx^{-7+r}}{(7-r)^2} - \frac{3bde^2nx^{-7+2r}}{(7-2r)^2} - \frac{be^3nx^{-7+3r}}{(7-3r)^2} - \frac{d^3(a+b\log(cx^n))}{7x^7} - \frac{3d^2ex^{-7+r}(a+b\log(cx^n))}{7-r} - \frac{3de^2x^{-7+2r}(a+b\log(cx^n))}{7-2r} - \frac{e^3x^{-7+3r}(a+b\log(cx^n))}{7-3r}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^8}\,dx = \frac{bn\left(-d^3 - \frac{147d^2ex^r}{(-7+r)^2} - \frac{147de^2x^{2r}}{(7-2r)^2} - \frac{49e^3x^{3r}}{(7-3r)^2}\right) + 7a\left(-d^3 + \frac{21d^2ex^r}{-7+r} + \frac{21de^2x^{2r}}{-7+2r} + \frac{7e^3x^{3r}}{-7+3r}\right) + 7b\left(-d^3 + \frac{21d^2ex^r}{-7+r} + \frac{21}{}\right.}{49x^7}$$

```
[In] Integrate[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x^8,x]
```

```
[Out] (b*n*(-d^3 - (147*d^2*e*x^r)/(-7 + r)^2 - (147*d*e^2*x^(2*r))/(7 - 2*r)^2 -
 (49*e^3*x^(3*r))/(7 - 3*r)^2) + 7*a*(-d^3 + (21*d^2*e*x^r)/(-7 + r) + (21*
d*e^2*x^(2*r))/(-7 + 2*r) + (7*e^3*x^(3*r))/(-7 + 3*r)) + 7*b*(-d^3 + (21*d
^2*e*x^r)/(-7 + r) + (21*d*e^2*x^(2*r))/(-7 + 2*r) + (7*e^3*x^(3*r))/(-7 +
3*r))*Log[c*x^n])/(49*x^7)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1040 vs. 2(179) = 358.

Time = 3.29 (sec) , antiderivative size = 1041, normalized size of antiderivative = 5.69

method	result	size
parallelrisch	Expression too large to display	1041
risch	Expression too large to display	4031

```
[In] int((d+e*x^r)^3*(a+b*ln(c*x^n))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/49*(823543*b*ln(c*x^n)*d^3+2470629*b*d*e^2*ln(c*x^n)*(x^r)^2+823543*e^3*
(x^r)^3*a-49392*b*d^3*n*r^3+139258*b*d^3*n*r^2-201684*b*d^3*n*r+1915998*a*d
```

```
*e^2*r^2*(x^r)^2-489804*a*d*e^2*r^3*(x^r)^2+2470629*d*e^2*(x^r)^2*a+2470629
*d^2*e*x^r*a+823543*a*d^3-122451*a*e^3*r^3*(x^r)^3+521017*a*e^3*r^2*(x^r)^3
-1058841*a*e^3*r*(x^r)^3-588*a*e^3*r^5*(x^r)^3+13720*a*e^3*r^4*(x^r)^3-588*
(x^r)^3*ln(c*x^n)*b*e^3*r^5+13720*(x^r)^3*ln(c*x^n)*b*e^3*r^4-122451*(x^r)^
3*ln(c*x^n)*b*e^3*r^3+521017*(x^r)^3*ln(c*x^n)*b*e^3*r^2-1058841*(x^r)^3*ln
(c*x^n)*b*e^3*r+2470629*b*d^2*e*ln(c*x^n)*x^r+36*b*d^3*n*r^6-924*b*d^3*n*r^
5+9457*b*d^3*n*r^4-5292*x^r*ln(c*x^n)*b*d^2*e*r^5+98784*x^r*ln(c*x^n)*b*d^2
*e*r^4-698691*x^r*ln(c*x^n)*b*d^2*e*r^3+2369787*x^r*ln(c*x^n)*b*d^2*e*r^2-3
882417*x^r*ln(c*x^n)*b*d^2*e*r-2646*(x^r)^2*ln(c*x^n)*b*d*e^2*r^5+58653*(x^
r)^2*ln(c*x^n)*b*d*e^2*r^4-489804*(x^r)^2*ln(c*x^n)*b*d*e^2*r^3+1915998*(x^
r)^2*ln(c*x^n)*b*d*e^2*r^2-3529470*(x^r)^2*ln(c*x^n)*b*d*e^2*r+117649*b*d^3
*n+252*ln(c*x^n)*b*d^3*r^6-6468*ln(c*x^n)*b*d^3*r^5+66199*ln(c*x^n)*b*d^3*r
^4-345744*ln(c*x^n)*b*d^3*r^3+974806*ln(c*x^n)*b*d^3*r^2-1411788*ln(c*x^n)*
b*d^3*r+823543*e^3*b*ln(c*x^n)*(x^r)^3-345744*a*d^3*r^3+974806*a*d^3*r^2-14
11788*a*d^3*r+252*a*d^3*r^6-6468*a*d^3*r^5+66199*a*d^3*r^4-698691*a*d^2*e*r
^3*x^r+352947*b*d*e^2*n*(x^r)^2+352947*b*d^2*e*n*x^r+117649*b*e^3*n*(x^r)^3
-4116*b*e^3*n*r^3*(x^r)^3+31213*b*e^3*n*r^2*(x^r)^3-100842*b*e^3*n*r*(x^r)^
3+2369787*a*d^2*e*r^2*x^r+98784*a*d^2*e*r^4*x^r-3882417*a*d^2*e*r*x^r+196*b
*e^3*n*r^4*(x^r)^3-3529470*a*d*e^2*r*(x^r)^2-2646*a*d*e^2*r^5*(x^r)^2+58653
*a*d*e^2*r^4*(x^r)^2-5292*a*d^2*e*r^5*x^r+158466*b*d*e^2*n*r^2*(x^r)^2+2665
11*b*d^2*e*n*r^2*x^r-403368*b*d*e^2*n*r*(x^r)^2-504210*b*d^2*e*n*r*x^r-6174
0*b*d^2*e*n*r^3*x^r+1323*b*d*e^2*n*r^4*(x^r)^2-24696*b*d*e^2*n*r^3*(x^r)^2+
5292*b*d^2*e*n*r^4*x^r)/x^7/(-7+3*r)^2/(-7+2*r)^2/(-7+r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 981 vs. $2(174) = 348$.

Time = 0.30 (sec) , antiderivative size = 981, normalized size of antiderivative = 5.36

$$\int \frac{(d+ex^r)^3\,(a+b\log{(cx^n)})}{x^8}\,dx = \text{Too large to display}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^8,x, algorithm="fricas")
```

```
[Out] -1/49*(36*(b*d^3*n + 7*a*d^3)*r^6 - 924*(b*d^3*n + 7*a*d^3)*r^5 + 117649*b*
d^3*n + 9457*(b*d^3*n + 7*a*d^3)*r^4 + 823543*a*d^3 - 49392*(b*d^3*n + 7*a*
d^3)*r^3 + 139258*(b*d^3*n + 7*a*d^3)*r^2 - 201684*(b*d^3*n + 7*a*d^3)*r -
49*(12*a*e^3*r^5 - 2401*b*e^3*n - 4*(b*e^3*n + 70*a*e^3)*r^4 - 16807*a*e^3
+ 21*(4*b*e^3*n + 119*a*e^3)*r^3 - 49*(13*b*e^3*n + 217*a*e^3)*r^2 + 1029*(
2*b*e^3*n + 21*a*e^3)*r + (12*b*e^3*r^5 - 280*b*e^3*r^4 + 2499*b*e^3*r^3 -
10633*b*e^3*r^2 + 21609*b*e^3*r - 16807*b*e^3)*log(c) + (12*b*e^3*n*r^5 - 2
80*b*e^3*n*r^4 + 2499*b*e^3*n*r^3 - 10633*b*e^3*n*r^2 + 21609*b*e^3*n*r - 1
6807*b*e^3*n)*log(x))*x^(3*r) - 147*(18*a*d*e^2*r^5 - 2401*b*d*e^2*n - 3*(3
*b*d*e^2*n + 133*a*d*e^2)*r^4 - 16807*a*d*e^2 + 28*(6*b*d*e^2*n + 119*a*d*e
```

```
^2)*r^3 - 98*(11*b*d*e^2*n + 133*a*d*e^2)*r^2 + 686*(4*b*d*e^2*n + 35*a*d*e
^2)*r + (18*b*d*e^2*r^5 - 399*b*d*e^2*r^4 + 3332*b*d*e^2*r^3 - 13034*b*d*e^
2*r^2 + 24010*b*d*e^2*r - 16807*b*d*e^2)*log(c) + (18*b*d*e^2*n*r^5 - 399*b
*d*e^2*n*r^4 + 3332*b*d*e^2*n*r^3 - 13034*b*d*e^2*n*r^2 + 24010*b*d*e^2*n*r
 - 16807*b*d*e^2*n)*log(x))*x^(2*r) - 147*(36*a*d^2*e*r^5 - 2401*b*d^2*e*n
- 12*(3*b*d^2*e*n + 56*a*d^2*e)*r^4 - 16807*a*d^2*e + 7*(60*b*d^2*e*n + 679
*a*d^2*e)*r^3 - 49*(37*b*d^2*e*n + 329*a*d^2*e)*r^2 + 343*(10*b*d^2*e*n + 7
7*a*d^2*e)*r + (36*b*d^2*e*r^5 - 672*b*d^2*e*r^4 + 4753*b*d^2*e*r^3 - 16121
*b*d^2*e*r^2 + 26411*b*d^2*e*r - 16807*b*d^2*e)*log(c) + (36*b*d^2*e*n*r^5
- 672*b*d^2*e*n*r^4 + 4753*b*d^2*e*n*r^3 - 16121*b*d^2*e*n*r^2 + 26411*b*d^
2*e*n*r - 16807*b*d^2*e*n)*log(x))*x^r + 7*(36*b*d^3*r^6 - 924*b*d^3*r^5 +
9457*b*d^3*r^4 - 49392*b*d^3*r^3 + 139258*b*d^3*r^2 - 201684*b*d^3*r + 1176
49*b*d^3)*log(c) + 7*(36*b*d^3*n*r^6 - 924*b*d^3*n*r^5 + 9457*b*d^3*n*r^4 -
 49392*b*d^3*n*r^3 + 139258*b*d^3*n*r^2 - 201684*b*d^3*n*r + 117649*b*d^3*n
)*log(x))/((36*r^6 - 924*r^5 + 9457*r^4 - 49392*r^3 + 139258*r^2 - 201684*r
 + 117649)*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^8}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x**r)**3*(a+b*ln(c*x**n))/x**8,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^8}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^8,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-8>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^8}\,dx = \int \frac{(ex^r+d)^3(b\log(cx^n)+a)}{x^8}\,dx$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^r + d)^3*(b*log(c*x^n) + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^8}\,dx = \int \frac{(d+e\,x^r)^3\,(a+b\,\ln(c\,x^n))}{x^8}\,dx$$

```
[In] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x^8,x)
```

```
[Out] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x^8, x)
```

3.405 $\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^{10}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 191

$$\int \frac{(d+ex^r)^3\,(a+b\log(cx^n))}{x^{10}}\,dx = -\frac{bd^3n}{81x^9}-\frac{be^3nx^{-3(3-r)}}{9(3-r)^2}-\frac{3bd^2enx^{-9+r}}{(9-r)^2}-\frac{3bde^2nx^{-9+2r}}{(9-2r)^2} -\frac{d^3(a+b\log(cx^n))}{9x^9}-\frac{e^3x^{-3(3-r)}(a+b\log(cx^n))}{3(3-r)} -\frac{3d^2ex^{-9+r}(a+b\log(cx^n))}{9-r} -\frac{3de^2x^{-9+2r}(a+b\log(cx^n))}{9-2r}$$

```
[Out] -1/81*b*d^3*n/x^9-1/9*b*e^3*n/(3-r)^2/(x^(9-3*r))-3*b*d^2*e*n*x^(-9+r)/(9-r
)^2-3*b*d*e^2*n*x^(-9+2*r)/(9-2*r)^2-1/9*d^3*(a+b*ln(c*x^n))/x^9-1/3*e^3*(a
+b*ln(c*x^n))/(3-r)/(x^(9-3*r))-3*d^2*e*x^(-9+r)*(a+b*ln(c*x^n))/(9-r)-3*d*
e^2*x^(-9+2*r)*(a+b*ln(c*x^n))/(9-2*r)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used

= {276, 2372, 12, 14}

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x^{10}}\,dx = -\frac{d^3(a+b\log(cx^n))}{9x^9} - \frac{3d^2ex^{r-9}(a+b\log(cx^n))}{9-r} - \frac{3de^2x^{2r-9}(a+b\log(cx^n))}{9-2r} - \frac{e^3x^{-3(3-r)}(a+b\log(cx^n))}{3(3-r)} - \frac{bd^3n}{81x^9} - \frac{3bd^2enx^{r-9}}{(9-r)^2} - \frac{3bde^2nx^{2r-9}}{(9-2r)^2} - \frac{be^3nx^{-3(3-r)}}{9(3-r)^2}$$

```
[In] Int[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x^10,x]
```

```
[Out] -1/81*(b*d^3*n)/x^9 - (b*e^3*n)/(9*(3 - r)^2*x^(3*(3 - r))) - (3*b*d^2*e*n*
x^(-9 + r))/(9 - r)^2 - (3*b*d*e^2*n*x^(-9 + 2*r))/(9 - 2*r)^2 - (d^3*(a +
b*Log[c*x^n]))/(9*x^9) - (e^3*(a + b*Log[c*x^n]))/(3*(3 - r)*x^(3*(3 - r)))
 - (3*d^2*e*x^(-9 + r)*(a + b*Log[c*x^n]))/(9 - r) - (3*d*e^2*x^(-9 + 2*r)*
(a + b*Log[c*x^n]))/(9 - 2*r)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^3(a+b\log(cx^n))}{9x^9} - \frac{e^3x^{-3(3-r)}(a+b\log(cx^n))}{3(3-r)} - \frac{3d^2ex^{-9+r}(a+b\log(cx^n))}{9-r} \\
&\quad - \frac{3de^2x^{-9+2r}(a+b\log(cx^n))}{9-2r} - (bn)\int \frac{-d^3+\frac{27d^2ex^r}{-9+r}+\frac{27de^2x^{2r}}{-9+2r}+\frac{3e^3x^{3r}}{-3+r}}{9x^{10}}\,dx \\
&= -\frac{d^3(a+b\log(cx^n))}{9x^9} - \frac{e^3x^{-3(3-r)}(a+b\log(cx^n))}{3(3-r)} - \frac{3d^2ex^{-9+r}(a+b\log(cx^n))}{9-r} \\
&\quad - \frac{3de^2x^{-9+2r}(a+b\log(cx^n))}{9-2r} - \frac{1}{9}(bn)\int \frac{-d^3+\frac{27d^2ex^r}{-9+r}+\frac{27de^2x^{2r}}{-9+2r}+\frac{3e^3x^{3r}}{-3+r}}{x^{10}}\,dx \\
&= -\frac{d^3(a+b\log(cx^n))}{9x^9} - \frac{e^3x^{-3(3-r)}(a+b\log(cx^n))}{3(3-r)} \\
&\quad - \frac{3d^2ex^{-9+r}(a+b\log(cx^n))}{9-r} - \frac{3de^2x^{-9+2r}(a+b\log(cx^n))}{9-2r} \\
&\quad - \frac{1}{9}(bn)\int \left(-\frac{d^3}{x^{10}}+\frac{27d^2ex^{-10+r}}{-9+r}+\frac{27de^2x^{2(-5+r)}}{-9+2r}+\frac{3e^3x^{-10+3r}}{-3+r}\right)dx \\
&= -\frac{bd^3n}{81x^9} - \frac{be^3nx^{-3(3-r)}}{9(3-r)^2} - \frac{3bd^2enx^{-9+r}}{(9-r)^2} - \frac{3bde^2nx^{-9+2r}}{(9-2r)^2} - \frac{d^3(a+b\log(cx^n))}{9x^9} \\
&\quad - \frac{e^3x^{-3(3-r)}(a+b\log(cx^n))}{3(3-r)} - \frac{3d^2ex^{-9+r}(a+b\log(cx^n))}{9-r} - \frac{3de^2x^{-9+2r}(a+b\log(cx^n))}{9-2r}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.95

$$
\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^{10}}\,dx
= \frac{bn\left(-d^3-\frac{243d^2ex^r}{(-9+r)^2}-\frac{243de^2x^{2r}}{(9-2r)^2}-\frac{9e^3x^{3r}}{(-3+r)^2}\right)+9a\left(-d^3+\frac{27d^2ex^r}{-9+r}+\frac{27de^2x^{2r}}{-9+2r}+\frac{3e^3x^{3r}}{-3+r}\right)+9b\left(-d^3+\frac{27d^2ex^r}{-9+r}+ \cdots\right.}{81x^9}
$$

```
[In] Integrate[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x^10,x]
```

```
[Out] (b*n*(-d^3 - (243*d^2*e*x^r)/(-9 + r)^2 - (243*d*e^2*x^(2*r))/(9 - 2*r)^2 -
 (9*e^3*x^(3*r))/(-3 + r)^2) + 9*a*(-d^3 + (27*d^2*e*x^r)/(-9 + r) + (27*d*
e^2*x^(2*r))/(-9 + 2*r) + (3*e^3*x^(3*r))/(-3 + r)) + 9*b*(-d^3 + (27*d^2*e
*x^r)/(-9 + r) + (27*d*e^2*x^(2*r))/(-9 + 2*r) + (3*e^3*x^(3*r))/(-3 + r))*
Log[c*x^n])/(81*x^9)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. 2(183) = 366.

Time = 10.00 (sec) , antiderivative size = 1044, normalized size of antiderivative = 5.47

method	result	size
parallelrisch	Expression too large to display	1044
risch	Expression too large to display	4027

```
[In] int((d+e*x^r)^3*(a+b*ln(c*x^n))/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/81*(531441*b*ln(c*x^n)*d^3+1594323*b*d*e^2*ln(c*x^n)*(x^r)^2+531441*e^3*
(x^r)^3*a-11664*b*d^3*n*r^3+42282*b*d^3*n*r^2-78732*b*d^3*n*r+747954*a*d*e^
2*r^2*(x^r)^2-148716*a*d*e^2*r^3*(x^r)^2+1594323*d*e^2*(x^r)^2*a+1594323*d^
2*e*x^r*a+531441*a*d^3-37179*a*e^3*r^3*(x^r)^3+203391*a*e^3*r^2*(x^r)^3-531
441*a*e^3*r*(x^r)^3-108*a*e^3*r^5*(x^r)^3+3240*a*e^3*r^4*(x^r)^3-108*(x^r)^
3*ln(c*x^n)*b*e^3*r^5+3240*(x^r)^3*ln(c*x^n)*b*e^3*r^4-37179*(x^r)^3*ln(c*x
^n)*b*e^3*r^3+203391*(x^r)^3*ln(c*x^n)*b*e^3*r^2-531441*(x^r)^3*ln(c*x^n)*b
*e^3*r+1594323*b*d^2*e*ln(c*x^n)*x^r+4*b*d^3*n*r^6-132*b*d^3*n*r^5+1737*b*d
^3*n*r^4-972*x^r*ln(c*x^n)*b*d^2*e*r^5+23328*x^r*ln(c*x^n)*b*d^2*e*r^4-2121
39*x^r*ln(c*x^n)*b*d^2*e*r^3+925101*x^r*ln(c*x^n)*b*d^2*e*r^2-1948617*x^r*l
n(c*x^n)*b*d^2*e*r-486*(x^r)^2*ln(c*x^n)*b*d*e^2*r^5+13851*(x^r)^2*ln(c*x^n
)*b*d*e^2*r^4-148716*(x^r)^2*ln(c*x^n)*b*d*e^2*r^3+747954*(x^r)^2*ln(c*x^n)
*b*d*e^2*r^2-1771470*(x^r)^2*ln(c*x^n)*b*d*e^2*r+59049*b*d^3*n+36*ln(c*x^n)
*b*d^3*r^6-1188*ln(c*x^n)*b*d^3*r^5+15633*ln(c*x^n)*b*d^3*r^4-104976*ln(c*x
^n)*b*d^3*r^3+380538*ln(c*x^n)*b*d^3*r^2-708588*ln(c*x^n)*b*d^3*r+531441*e^
3*b*ln(c*x^n)*(x^r)^3-104976*a*d^3*r^3+380538*a*d^3*r^2-708588*a*d^3*r+36*a
*d^3*r^6-1188*a*d^3*r^5+15633*a*d^3*r^4-212139*a*d^2*e*r^3*x^r+177147*b*d*e
^2*n*(x^r)^2+177147*b*d^2*e*n*x^r+59049*b*e^3*n*(x^r)^3-972*b*e^3*n*r^3*(x^
r)^3+9477*b*e^3*n*r^2*(x^r)^3-39366*b*e^3*n*r*(x^r)^3+925101*a*d^2*e*r^2*x^
r+23328*a*d^2*e*r^4*x^r-1948617*a*d^2*e*r*x^r+36*b*e^3*n*r^4*(x^r)^3-177147
0*a*d*e^2*r*(x^r)^2-486*a*d*e^2*r^5*(x^r)^2+13851*a*d*e^2*r^4*(x^r)^2-972*a
*d^2*e*r^5*x^r+48114*b*d*e^2*n*r^2*(x^r)^2+80919*b*d^2*e*n*r^2*x^r-157464*b
*d*e^2*n*r*(x^r)^2-196830*b*d^2*e*n*r*x^r-14580*b*d^2*e*n*r^3*x^r+243*b*d*e
^2*n*r^4*(x^r)^2-5832*b*d*e^2*n*r^3*(x^r)^2+972*b*d^2*e*n*r^4*x^r)/x^9/(-3+
r)^2/(r^2-18*r+81)/(-9+2*r)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 981 vs. $2(175) = 350$.

Time = 0.29 (sec) , antiderivative size = 981, normalized size of antiderivative = 5.14

$$\int \frac{(d+ex^r)^3 (a+b\log (cx^n))}{x^{10}}\,dx = -\frac{4\,(bd^3n+9\,ad^3)r^6-132\,(bd^3n+9\,ad^3)r^5+59049\,bd^3n+1737\,(bd^3n+9\,ad^3)r^4+531441\,ad^3-11664}{}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^10,x, algorithm="fricas")
```

```
[Out] -1/81*(4*(b*d^3*n + 9*a*d^3)*r^6 - 132*(b*d^3*n + 9*a*d^3)*r^5 + 59049*b*d^
3*n + 1737*(b*d^3*n + 9*a*d^3)*r^4 + 531441*a*d^3 - 11664*(b*d^3*n + 9*a*d^
3)*r^3 + 42282*(b*d^3*n + 9*a*d^3)*r^2 - 78732*(b*d^3*n + 9*a*d^3)*r - 9*(1
2*a*e^3*r^5 - 6561*b*e^3*n - 4*(b*e^3*n + 90*a*e^3)*r^4 - 59049*a*e^3 + 27*
(4*b*e^3*n + 153*a*e^3)*r^3 - 81*(13*b*e^3*n + 279*a*e^3)*r^2 + 2187*(2*b*e
^3*n + 27*a*e^3)*r + 3*(4*b*e^3*r^5 - 120*b*e^3*r^4 + 1377*b*e^3*r^3 - 7533
*b*e^3*r^2 + 19683*b*e^3*r - 19683*b*e^3)*log(c) + 3*(4*b*e^3*n*r^5 - 120*b
*e^3*n*r^4 + 1377*b*e^3*n*r^3 - 7533*b*e^3*n*r^2 + 19683*b*e^3*n*r - 19683*
b*e^3*n)*log(x))*x^(3*r) - 243*(2*a*d*e^2*r^5 - 729*b*d*e^2*n - (b*d*e^2*n
+ 57*a*d*e^2)*r^4 - 6561*a*d*e^2 + 12*(2*b*d*e^2*n + 51*a*d*e^2)*r^3 - 18*(
11*b*d*e^2*n + 171*a*d*e^2)*r^2 + 162*(4*b*d*e^2*n + 45*a*d*e^2)*r + (2*b*d
*e^2*r^5 - 57*b*d*e^2*r^4 + 612*b*d*e^2*r^3 - 3078*b*d*e^2*r^2 + 7290*b*d*e
^2*r - 6561*b*d*e^2)*log(c) + (2*b*d*e^2*n*r^5 - 57*b*d*e^2*n*r^4 + 612*b*d
*e^2*n*r^3 - 3078*b*d*e^2*n*r^2 + 7290*b*d*e^2*n*r - 6561*b*d*e^2*n)*log(x)
)*x^(2*r) - 243*(4*a*d^2*e*r^5 - 729*b*d^2*e*n - 4*(b*d^2*e*n + 24*a*d^2*e)
*r^4 - 6561*a*d^2*e + 3*(20*b*d^2*e*n + 291*a*d^2*e)*r^3 - 9*(37*b*d^2*e*n
+ 423*a*d^2*e)*r^2 + 81*(10*b*d^2*e*n + 99*a*d^2*e)*r + (4*b*d^2*e*r^5 - 96
*b*d^2*e*r^4 + 873*b*d^2*e*r^3 - 3807*b*d^2*e*r^2 + 8019*b*d^2*e*r - 6561*b
*d^2*e)*log(c) + (4*b*d^2*e*n*r^5 - 96*b*d^2*e*n*r^4 + 873*b*d^2*e*n*r^3 -
3807*b*d^2*e*n*r^2 + 8019*b*d^2*e*n*r - 6561*b*d^2*e*n)*log(x))*x^r + 9*(4*
b*d^3*r^6 - 132*b*d^3*r^5 + 1737*b*d^3*r^4 - 11664*b*d^3*r^3 + 42282*b*d^3*
r^2 - 78732*b*d^3*r + 59049*b*d^3)*log(c) + 9*(4*b*d^3*n*r^6 - 132*b*d^3*n*
r^5 + 1737*b*d^3*n*r^4 - 11664*b*d^3*n*r^3 + 42282*b*d^3*n*r^2 - 78732*b*d^
3*n*r + 59049*b*d^3*n)*log(x))/((4*r^6 - 132*r^5 + 1737*r^4 - 11664*r^3 + 4
2282*r^2 - 78732*r + 59049)*x^9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^{10}}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x**r)**3*(a+b*ln(c*x**n))/x**10,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^{10}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(r-10>0)', see `assume?` for more de
tails)I
```

Giac [F]

$$\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^{10}}\,dx = \int \frac{(ex^r+d)^3(b\log(cx^n)+a)}{x^{10}}\,dx$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x^10,x, algorithm="giac")
```

```
[Out] integrate((e*x^r + d)^3*(b*log(c*x^n) + a)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x^{10}}\,dx = \int \frac{(d+e\,x^r)^3\,(a+b\,\ln(c\,x^n))}{x^{10}}\,dx$$

```
[In] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x^10,x)
```

```
[Out] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x^10, x)
```

3.406 $\int \frac{x^3(a+b\log(cx^n))}{d+ex^r}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex^r}\,dx = \text{Int}\left(\frac{x^3(a+b\log(cx^n))}{d+ex^r},x\right)$$

```
[Out] Unintegrable(x^3*(a+b*ln(c*x^n))/(d+e*x^r),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex^r}\,dx = \int \frac{x^3(a+b\log(cx^n))}{d+ex^r}\,dx$$

```
[In] Int[(x^3*(a + b*Log[c*x^n]))/(d + e*x^r),x]
[Out] Defer[Int][(x^3*(a + b*Log[c*x^n]))/(d + e*x^r), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^3(a+b\log(cx^n))}{d+ex^r}\,dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. $2(26) = 52$.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.78

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex^r}\,dx$$
$$= \frac{x^4\left(-bn\,{}_3F_2\left(1,\frac{4}{r},\frac{4}{r};1+\frac{4}{r},1+\frac{4}{r};-\frac{ex^r}{d}\right)+4\,\text{Hypergeometric2F1}\left(1,\frac{4}{r},\frac{4+r}{r},-\frac{ex^r}{d}\right)(a+b\log(cx^n))\right)}{16d}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/(d + e*x^r),x]
```

```
[Out] (x^4*(-(b*n*HypergeometricPFQ[{1, 4/r, 4/r}, {1 + 4/r, 1 + 4/r}, -((e*x^r)/
d)]) + 4*Hypergeometric2F1[1, 4/r, (4 + r)/r, -((e*x^r)/d)]*(a + b*Log[c*x^
n])))/(16*d)
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^3(a+b\ln(c\,x^n))}{d+e\,x^r}dx$$

```
[In] int(x^3*(a+b*ln(c*x^n))/(d+e*x^r),x)
```

```
[Out] int(x^3*(a+b*ln(c*x^n))/(d+e*x^r),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex^r}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{ex^r+d}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(d+e*x^r),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*log(c*x^n) + a*x^3)/(e*x^r + d), x)
```

Sympy [N/A]

Not integrable

Time = 6.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex^r}\,dx = \int \frac{x^3(a+b\log(cx^n))}{d+ex^r}\,dx$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))/(d+e*x**r),x)
[Out] Integral(x**3*(a + b*log(c*x**n))/(d + e*x**r), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex^r}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{ex^r+d}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(d+e*x^r),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*x^3/(e*x^r + d), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex^r}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{ex^r+d}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(d+e*x^r),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^3/(e*x^r + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^3(a+b\log(cx^n))}{d+ex^r}\,dx=\int \frac{x^3\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{d+e\,x^r}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n)))/(d + e*x^r),x)
[Out] int((x^3*(a + b*log(c*x^n)))/(d + e*x^r), x)
```

3.407 $\int \frac{x(a+b\log(cx^n))}{d+ex^r}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{x(a+b\log(cx^n))}{d+ex^r}\,dx = \operatorname{Int}\left(\frac{x(a+b\log(cx^n))}{d+ex^r},x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*x^n))/(d+e*x^r),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(a+b\log(cx^n))}{d+ex^r}\,dx = \int \frac{x(a+b\log(cx^n))}{d+ex^r}\,dx$$

```
[In] Int[(x*(a + b*Log[c*x^n]))/(d + e*x^r),x]
[Out] Defer[Int][(x*(a + b*Log[c*x^n]))/(d + e*x^r), x]
```

Rubi steps

$$\text{integral} = \int \frac{x(a+b\log(cx^n))}{d+ex^r}\,dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. $2(24) = 48$.

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.14

$$\int \frac{x(a+b\log(cx^n))}{d+ex^r}\,dx = \frac{x^2\left(-bn\,{}_3F_2\left(1,\frac{2}{r},\frac{2}{r};1+\frac{2}{r},1+\frac{2}{r};-\frac{ex^r}{d}\right)+2\,\text{Hypergeometric2F1}\left(1,\frac{2}{r},\frac{2+r}{r},-\frac{ex^r}{d}\right)(a+b\log(cx^n))\right)}{4d}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e*x^r),x]
```

```
[Out] (x^2*(-(b*n*HypergeometricPFQ[{1, 2/r, 2/r}, {1 + 2/r, 1 + 2/r}, -((e*x^r)/
d)]) + 2*Hypergeometric2F1[1, 2/r, (2 + r)/r, -((e*x^r)/d)]*(a + b*Log[c*x^
n])))/(4*d)
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x(a+b\ln(c\,x^n))}{d+e\,x^r}dx$$

```
[In] int(x*(a+b*ln(c*x^n))/(d+e*x^r),x)
```

```
[Out] int(x*(a+b*ln(c*x^n))/(d+e*x^r),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{x(a+b\log(cx^n))}{d+ex^r}\,dx = \int \frac{(b\log(cx^n)+a)x}{ex^r+d}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(d+e*x^r),x, algorithm="fricas")
```

```
[Out] integral((b*x*log(c*x^n) + a*x)/(e*x^r + d), x)
```

Sympy [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x(a+b\log(cx^n))}{d+ex^r}\,dx = \int \frac{x(a+b\log(cx^n))}{d+ex^r}\,dx$$

```
[In] integrate(x*(a+b*ln(c*x**n))/(d+e*x**r),x)
[Out] Integral(x*(a + b*log(c*x**n))/(d + e*x**r), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x(a+b\log(cx^n))}{d+ex^r}\,dx = \int \frac{(b\log(cx^n)+a)x}{ex^r+d}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(d+e*x^r),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*x/(e*x^r + d), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x(a+b\log(cx^n))}{d+ex^r}\,dx = \int \frac{(b\log(cx^n)+a)x}{ex^r+d}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(d+e*x^r),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x/(e*x^r + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x(a+b\log(cx^n))}{d+ex^r}\,dx = \int \frac{x\,(a+b\,\ln(c\,x^n))}{d+e\,x^r}\,dx$$

```
[In] int((x*(a + b*log(c*x^n)))/(d + e*x^r),x)
[Out] int((x*(a + b*log(c*x^n)))/(d + e*x^r), x)
```

3.408 $\int \frac{a+b\log(cx^n)}{x(d+ex^r)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)}\,dx = -\frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{dr}+\frac{bn\operatorname{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)}{dr^2}$$

```
[Out] -(a+b*ln(c*x^n))*ln(1+d/e/(x^r))/d/r+b*n*polylog(2,-d/e/(x^r))/d/r^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)}\,dx = \frac{bn\operatorname{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)}{dr^2}-\frac{\log\left(\frac{dx^{-r}}{e}+1\right)(a+b\log(cx^n))}{dr}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x^r)),x]
```

```
[Out] -(((a + b*Log[c*x^n])*Log[1 + d/(e*x^r)])/(d*r)) + (b*n*PolyLog[2, -(d/(e*x
^r))])/(d*r^2)
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{dr}+\frac{(bn)\int\frac{\log\left(1+\frac{dx^{-r}}{e}\right)}{x}\,dx}{dr}\\ &= -\frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{dr}+\frac{bn\text{Li}_2\left(-\frac{dx^{-r}}{e}\right)}{dr^2}\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.00

$$\int\frac{a+b\log(cx^n)}{x(d+ex^r)}\,dx$$
$$=\frac{bnr^2\log^2(x)-2r(a+b\log(cx^n))\log(d-dx^r)+2bnr\log(x)\left(\log(d-dx^r)-\log(d+ex^r)\right)+2bn\log\left(-\frac{e}{}\right.}{2dr^2}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x^r)),x]
```

```
[Out] (b*n*r^2*Log[x]^2 - 2*r*(a + b*Log[c*x^n])*Log[d - d*x^r] + 2*b*n*r*Log[x]*
(Log[d - d*x^r] - Log[d + e*x^r]) + 2*b*n*Log[-((e*x^r)/d)]*Log[d + e*x^r]
+ 2*b*n*PolyLog[2, 1 + (e*x^r)/d])/(2*d*r^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 246, normalized size of antiderivative = 4.56

method	result
risch	$\frac{b\ln(d+e\,x^r)n\ln(x)}{rd}-\frac{b\ln(d+e\,x^r)\ln(x^n)}{rd}-\frac{b\ln(x^r)n\ln(x)}{rd}+\frac{b\ln(x^r)\ln(x^n)}{rd}+\frac{bn\ln(x)^2}{2d}-\frac{bn\ln(x)\ln\left(1+\frac{e\,x^r}{d}\right)}{rd}-\frac{bn\,\text{Li}_2\left(-\right.}{r^2}$

```
[In] int((a+b*ln(c*x^n))/x/(d+e*x^r),x,method=_RETURNVERBOSE)
```

```
[Out] b/r/d*ln(d+e*x^r)*n*ln(x)-b/r/d*ln(d+e*x^r)*ln(x^n)-b/r/d*ln(x^r)*n*ln(x)+b
/r/d*ln(x^r)*ln(x^n)+1/2*b*n/d*ln(x)^2-b/r*n/d*ln(x)*ln(1+e*x^r/d)-b/r^2*n/
d*polylog(2,-e*x^r/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*
I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2
*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-1/r/d*ln(d+e*x^r)+1/r/d*ln(x^r))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.72

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)}\,dx = \frac{bnr^2\log(x)^2 - 2\,bnr\log(x)\log\left(\frac{ex^r+d}{d}\right) - 2\,bn\mathrm{Li}_2\left(-\frac{ex^r+d}{d}+1\right) - 2\,(br\log(c)+ar)\log(ex^r+d) + 2\,(br^2}{2\,dr^2}$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r),x, algorithm="fricas")
```

```
[Out] 1/2*(b*n*r^2*log(x)^2 - 2*b*n*r*log(x)*log((e*x^r + d)/d) - 2*b*n*dilog(-(e
*x^r + d)/d + 1) - 2*(b*r*log(c) + a*r)*log(e*x^r + d) + 2*(b*r^2*log(c) +
a*r^2)*log(x))/(d*r^2)
```

Sympy [A] (verification not implemented)

Time = 149.77 (sec) , antiderivative size = 452, normalized size of antiderivative = 8.37

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*ln(c*x**n))/x/(d+e*x**r),x)
```

```
[Out] -2*a*e*Piecewise((1/(2*e) + x**r/d, Eq(e, 0)), (-log(-2*e*x**r)/(2*e), True
))/(d*r) - 2*a*e*Piecewise((1/(2*e) + x**r/d, Eq(e, 0)), (log(2*d + 2*e*x**
r)/(2*e), True))/(d*r) + 2*b*e*n*Piecewise((Piecewise((log(x)/(2*e) + x**r/
(d*r), Ne(r, 0)), (log(x)/(2*e) + log(x)/d, True)), Eq(e, 0)), (-Piecewise(
(0, (Abs(e*x**r) < 1) & (1/Abs(e*x**r) < 1)), (log(e*x**r)**2/(2*r) + I*pi*
log(e*x**r)/r, Abs(e*x**r) < 1), (log(1/(e*x**r))**2/(2*r) - I*pi*log(1/(e*
x**r))/r, 1/Abs(e*x**r) < 1), (meijerg(((), (1, 1, 1)), ((0, 0, 0), ()), e*
x**r*exp_polar(I*pi))/r + meijerg(((1, 1, 1), ()), ((), (0, 0, 0)), e*x**r*
exp_polar(I*pi))/r, True))/(2*e) - log(2)*log(x)/(2*e), True))/(d*r) + 2*b*
e*n*Piecewise((Piecewise((log(x)/(2*e) + x**r/(d*r), Ne(r, 0)), (log(x)/(2*
e) + log(x)/d, True)), Eq(e, 0)), (Piecewise((-polylog(2, e*x**r*exp_polar(
I*pi)/d)/r, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x
**r*exp_polar(I*pi)/d)/r, Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x**
r*exp_polar(I*pi)/d)/r, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ())
, x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*
x**r*exp_polar(I*pi)/d)/r, True))/(2*e) + log(2)*log(x)/(2*e), True))/(d*r)
 - 2*b*e*Piecewise((1/(2*e) + x**r/d, Eq(e, 0)), (-log(-2*e*x**r)/(2*e), Tr
ue))*log(c*x**n)/(d*r) - 2*b*e*Piecewise((1/(2*e) + x**r/d, Eq(e, 0)), (log
(2*d + 2*e*x**r)/(2*e), True))*log(c*x**n)/(d*r)
```

Maxima [F]

$$\int \frac{a + b\log(cx^n)}{x(d + ex^r)}\,dx = \int \frac{b\log(cx^n) + a}{(ex^r + d)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r),x, algorithm="maxima")
```

```
[Out] a*(log(x)/d - log((e*x^r + d)/e)/(d*r)) + b*integrate((log(c) + log(x^n))/(
e*x*x^r + d*x), x)
```

Giac [F]

$$\int \frac{a + b\log(cx^n)}{x(d + ex^r)}\,dx = \int \frac{b\log(cx^n) + a}{(ex^r + d)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x^r + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{x(d + ex^r)}\,dx = \int \frac{a + b\,\ln(c\,x^n)}{x\,(d + e\,x^r)}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x^r)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x*(d + e*x^r)), x)
```

3.409 $\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)}\,dx = \text{Int}\left(\frac{a+b\log(cx^n)}{x^3(d+ex^r)},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*x^n))/x^3/(d+e*x^r),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)}\,dx = \int \frac{a+b\log(cx^n)}{x^3(d+ex^r)}\,dx$$

```
[In] Int[(a + b*Log[c*x^n])/(x^3*(d + e*x^r)),x]
[Out] Defer[Int][(a + b*Log[c*x^n])/(x^3*(d + e*x^r)), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\log(cx^n)}{x^3(d+ex^r)}\,dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 86 vs. 2(26) = 52.

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.74

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)}\,dx = -\frac{bn\,{}_3F_2\left(1,-\frac{2}{r},-\frac{2}{r};1-\frac{2}{r},1-\frac{2}{r};-\frac{ex^r}{d}\right)+2\,\text{Hypergeometric2F1}\left(1,-\frac{2}{r},\frac{-2+r}{r},-\frac{ex^r}{d}\right)(a+b\log(cx^n))}{4dx^2}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*(d + e*x^r)),x]
```

```
[Out] -1/4*(b*n*HypergeometricPFQ[{1, -2/r, -2/r}, {1 - 2/r, 1 - 2/r}, -((e*x^r)/
d)] + 2*Hypergeometric2F1[1, -2/r, (-2 + r)/r, -((e*x^r)/d)]*(a + b*Log[c*x
^n]))/(d*x^2)
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\ln(c\,x^n)}{x^3(d+e\,x^r)}dx$$

```
[In] int((a+b*ln(c*x^n))/x^3/(d+e*x^r),x)
```

```
[Out] int((a+b*ln(c*x^n))/x^3/(d+e*x^r),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)}\,dx = \int \frac{b\log(cx^n)+a}{(ex^r+d)x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(d+e*x^r),x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e*x^3*x^r + d*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 5.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^r\right)}\,dx=\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^r\right)}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**3/(d+e*x**r),x)
[Out] Integral((a + b*log(c*x**n))/(x**3*(d + e*x**r)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^r\right)}\,dx=\int \frac{b\log\left(cx^n\right)+a}{(ex^r+d)x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(d+e*x^r),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)/((e*x^r + d)*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^r\right)}\,dx=\int \frac{b\log\left(cx^n\right)+a}{(ex^r+d)x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(d+e*x^r),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x^r + d)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)}\,dx=\int \frac{a+b\,\ln\left(c\,x^n\right)}{x^3\,\left(d+e\,x^r\right)}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e*x^r)),x)
[Out] int((a + b*log(c*x^n))/(x^3*(d + e*x^r)), x)
```

3.410 $\int \frac{x^2(a+b\log(cx^n))}{d+ex^r}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex^r}\,dx = \text{Int}\left(\frac{x^2(a+b\log(cx^n))}{d+ex^r},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*ln(c*x^n))/(d+e*x^r),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex^r}\,dx = \int \frac{x^2(a+b\log(cx^n))}{d+ex^r}\,dx$$

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/(d + e*x^r),x]
[Out] Defer[Int][(x^2*(a + b*Log[c*x^n]))/(d + e*x^r), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2(a+b\log(cx^n))}{d+ex^r}\,dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. $2(26) = 52$.

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.78

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex^r}\,dx = \frac{x^3\left(-bn\,{}_3F_2\left(1,\frac{3}{r},\frac{3}{r};1+\frac{3}{r},1+\frac{3}{r};-\frac{ex^r}{d}\right)+3\,\text{Hypergeometric2F1}\left(1,\frac{3}{r},\frac{3+r}{r},-\frac{ex^r}{d}\right)(a+b\log(cx^n))\right)}{9d}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*x^r),x]
[Out] (x^3*(-(b*n*HypergeometricPFQ[{1, 3/r, 3/r}, {1 + 3/r, 1 + 3/r}, -((e*x^r)/
d)]) + 3*Hypergeometric2F1[1, 3/r, (3 + r)/r, -((e*x^r)/d)]*(a + b*Log[c*x^
n])))/(9*d)
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\ln(c\,x^n))}{d+e\,x^r}dx$$

```
[In] int(x^2*(a+b*ln(c*x^n))/(d+e*x^r),x)
[Out] int(x^2*(a+b*ln(c*x^n))/(d+e*x^r),x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex^r}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{ex^r+d}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(d+e*x^r),x, algorithm="fricas")
[Out] integral((b*x^2*log(c*x^n) + a*x^2)/(e*x^r + d), x)
```

Sympy [N/A]

Not integrable

Time = 4.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex^r}\,dx=\int \frac{x^2(a+b\log(cx^n))}{d+ex^r}\,dx$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(d+e*x**r),x)
[Out] Integral(x**2*(a + b*log(c*x**n))/(d + e*x**r), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex^r}\,dx=\int \frac{(b\log(cx^n)+a)x^2}{ex^r+d}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(d+e*x^r),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*x^2/(e*x^r + d), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex^r}\,dx=\int \frac{(b\log(cx^n)+a)x^2}{ex^r+d}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(d+e*x^r),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^2/(e*x^r + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\log(cx^n))}{d+ex^r}\,dx=\int \frac{x^2\,(a+b\,\ln(c\,x^n))}{d+e\,x^r}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e*x^r),x)
[Out] int((x^2*(a + b*log(c*x^n)))/(d + e*x^r), x)
```

3.411 $\int \frac{a+b\log(cx^n)}{d+ex^r}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\log(cx^n)}{d+ex^r}\,dx=\operatorname{Int}\left(\frac{a+b\log(cx^n)}{d+ex^r},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*x^n))/(d+e*x^r),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\log(cx^n)}{d+ex^r}\,dx=\int \frac{a+b\log(cx^n)}{d+ex^r}\,dx$$

```
[In] Int[(a + b*Log[c*x^n])/(d + e*x^r),x]
```

```
[Out] Defer[Int][(a + b*Log[c*x^n])/(d + e*x^r), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\log(cx^n)}{d+ex^r}\,dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 69 vs. $2(23) = 46$.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.45

$$\int \frac{a+b\log(cx^n)}{d+ex^r}\,dx = \frac{x\left(-bn\,{}_3F_2\left(1,\frac{1}{r},\frac{1}{r};1+\frac{1}{r},1+\frac{1}{r};-\frac{ex^r}{d}\right)+\text{Hypergeometric2F1}\left(1,\frac{1}{r},1+\frac{1}{r},-\frac{ex^r}{d}\right)(a+b\log(cx^n))\right)}{d}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*x^r),x]
[Out] (x*(-(b*n*HypergeometricPFQ[{1, r^(-1), r^(-1)}, {1 + r^(-1), 1 + r^(-1)},
-((e*x^r)/d)]) + Hypergeometric2F1[1, r^(-1), 1 + r^(-1), -((e*x^r)/d)]*(a
+ b*Log[c*x^n])))/d
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\ln(c\,x^n)}{d+e\,x^r}dx$$

```
[In] int((a+b*ln(c*x^n))/(d+e*x^r),x)
[Out] int((a+b*ln(c*x^n))/(d+e*x^r),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\log(cx^n)}{d+ex^r}\,dx = \int \frac{b\log(cx^n)+a}{ex^r+d}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(d+e*x^r),x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(e*x^r + d), x)
```

Sympy [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{a+b\log\left(cx^{n}\right)}{d+ex^{r}}\,dx=\int \frac{a+b\log\left(cx^{n}\right)}{d+ex^{r}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/(d+e*x**r),x)
[Out] Integral((a + b*log(c*x**n))/(d + e*x**r), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\log\left(cx^{n}\right)}{d+ex^{r}}\,dx=\int \frac{b\log\left(cx^{n}\right)+a}{ex^{r}+d}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(d+e*x^r),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)/(e*x^r + d), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\log\left(cx^{n}\right)}{d+ex^{r}}\,dx=\int \frac{b\log\left(cx^{n}\right)+a}{ex^{r}+d}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(d+e*x^r),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/(e*x^r + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\log(cx^n)}{d+ex^r}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{d+e\,x^r}\,dx$$

```
[In] int((a + b*log(c*x^n))/(d + e*x^r),x)
[Out] int((a + b*log(c*x^n))/(d + e*x^r), x)
```

3.412 $\int \frac{a+b\log(cx^n)}{x^2(d+ex^r)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^r)}\,dx = \text{Int}\left(\frac{a+b\log(cx^n)}{x^2(d+ex^r)},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*x^n))/x^2/(d+e*x^r),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^r)}\,dx = \int \frac{a+b\log(cx^n)}{x^2(d+ex^r)}\,dx$$

```
[In] Int[(a + b*Log[c*x^n])/(x^2*(d + e*x^r)),x]
[Out] Defer[Int][(a + b*Log[c*x^n])/(x^2*(d + e*x^r)), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\log(cx^n)}{x^2(d+ex^r)}\,dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 83 vs. 2(26) = 52.

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.61

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^r)}\,dx = -\frac{bn\,{}_3F_2\left(1,-\frac{1}{r},-\frac{1}{r};1-\frac{1}{r},1-\frac{1}{r};-\frac{ex^r}{d}\right)+\text{Hypergeometric2F1}\left(1,-\frac{1}{r},\frac{-1+r}{r},-\frac{ex^r}{d}\right)(a+b\log(cx^n))}{dx}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*x^r)),x]
[Out] -((b*n*HypergeometricPFQ[{1, -r^(-1), -r^(-1)}, {1 - r^(-1), 1 - r^(-1)}, -
((e*x^r)/d)] + Hypergeometric2F1[1, -r^(-1), (-1 + r)/r, -((e*x^r)/d)]*(a +
 b*Log[c*x^n]))/(d*x))
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\ln(c\,x^n)}{x^2(d+e\,x^r)}dx$$

```
[In] int((a+b*ln(c*x^n))/x^2/(d+e*x^r),x)
[Out] int((a+b*ln(c*x^n))/x^2/(d+e*x^r),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^r)}\,dx = \int \frac{b\log(cx^n)+a}{(ex^r+d)x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(d+e*x^r),x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)/(e*x^2*x^r + d*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 3.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^r)}\,dx=\int \frac{a+b\log(cx^n)}{x^2(d+ex^r)}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**2/(d+e*x**r),x)
[Out] Integral((a + b*log(c*x**n))/(x**2*(d + e*x**r)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^r)}\,dx=\int \frac{b\log(cx^n)+a}{(ex^r+d)x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(d+e*x^r),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)/((e*x^r + d)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^r)}\,dx=\int \frac{b\log(cx^n)+a}{(ex^r+d)x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(d+e*x^r),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x^r + d)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^r)}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x^2\,(d+e\,x^r)}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*x^r)),x)
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*x^r)), x)
```

3.413 $\int \frac{x^3(a+b\log(cx^n))}{(d+ex^r)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^r)^2}\,dx = \text{Int}\left(\frac{x^3(a+b\log(cx^n))}{(d+ex^r)^2},x\right)$$

```
[Out] Unintegrable(x^3*(a+b*ln(c*x^n))/(d+e*x^r)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^r)^2}\,dx = \int \frac{x^3(a+b\log(cx^n))}{(d+ex^r)^2}\,dx$$

```
[In] Int[(x^3*(a + b*Log[c*x^n]))/(d + e*x^r)^2,x]
```

```
[Out] Defer[Int][(x^3*(a + b*Log[c*x^n]))/(d + e*x^r)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^3(a+b\log(cx^n))}{(d+ex^r)^2}\,dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(26) = 52.

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 6.09

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^r)^2}\,dx$$
$$= \frac{x^4\left(-bn(-4+r)\left(d+ex^r\right){}_3F_2\left(1,\frac{4}{r},\frac{4}{r};1+\frac{4}{r},1+\frac{4}{r};-\frac{ex^r}{d}\right)+16d(a+b\log(cx^n))+4(d+ex^r)\,\text{Hypergeome}\right.}{16d^2r\left(d+ex^r\right)}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/(d + e*x^r)^2,x]
```

```
[Out] (x^4*(-(b*n*(-4 + r)*(d + e*x^r)*HypergeometricPFQ[{1, 4/r, 4/r}, {1 + 4/r,
 1 + 4/r}, -((e*x^r)/d)]) + 16*d*(a + b*Log[c*x^n]) + 4*(d + e*x^r)*Hyperge
ometric2F1[1, 4/r, (4 + r)/r, -((e*x^r)/d)]*(-(b*n) + a*(-4 + r) + b*(-4 +
r)*Log[c*x^n])))/(16*d^2*r*(d + e*x^r))
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^3(a+b\ln(c\,x^n))}{(d+e\,x^r)^2}dx$$

```
[In] int(x^3*(a+b*ln(c*x^n))/(d+e*x^r)^2,x)
```

```
[Out] int(x^3*(a+b*ln(c*x^n))/(d+e*x^r)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^r)^2}\,dx = \int \frac{(b\log(cx^n)+a)x^3}{(ex^r+d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(d+e*x^r)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*log(c*x^n) + a*x^3)/(e^2*x^(2*r) + 2*d*e*x^r + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 56.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^r)^2}\,dx=\int \frac{x^3(a+b\log(cx^n))}{(d+ex^r)^2}\,dx$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))/(d+e*x**r)**2,x)
[Out] Integral(x**3*(a + b*log(c*x**n))/(d + e*x**r)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^r)^2}\,dx=\int \frac{(b\log(cx^n)+a)x^3}{(ex^r+d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(d+e*x^r)^2,x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*x^3/(e*x^r + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^r)^2}\,dx=\int \frac{(b\log(cx^n)+a)x^3}{(ex^r+d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*x^n))/(d+e*x^r)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^3/(e*x^r + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^3(a+b\log(cx^n))}{(d+ex^r)^2}\,dx=\int \frac{x^3\,(a+b\,\ln(c\,x^n))}{(d+e\,x^r)^2}\,dx$$

```
[In] int((x^3*(a + b*log(c*x^n)))/(d + e*x^r)^2,x)
[Out] int((x^3*(a + b*log(c*x^n)))/(d + e*x^r)^2, x)
```

3.414 $\int \frac{x(a+b\log(cx^n))}{(d+ex^r)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^r)^2}\,dx=\text{Int}\left(\frac{x(a+b\log(cx^n))}{(d+ex^r)^2},x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*x^n))/(d+e*x^r)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^r)^2}\,dx=\int \frac{x(a+b\log(cx^n))}{(d+ex^r)^2}\,dx$$

```
[In] Int[(x*(a + b*Log[c*x^n]))/(d + e*x^r)^2,x]
```

```
[Out] Defer[Int][(x*(a + b*Log[c*x^n]))/(d + e*x^r)^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{x(a+b\log(cx^n))}{(d+ex^r)^2}\,dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 140 vs. $2(24) = 48$.

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 6.67

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^r)^2}\,dx = \frac{x^2\left(-bn(-2+r)\left(d+ex^r\right)\,{}_3F_2\left(1,\frac{2}{r},\frac{2}{r};1+\frac{2}{r},1+\frac{2}{r};-\frac{ex^r}{d}\right)+4d(a+b\log(cx^n))+2(d+ex^r)\,\text{Hypergeomet}\right.}{4d^2r\left(d+ex^r\right)}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e*x^r)^2,x]
```

```
[Out] (x^2*(-(b*n*(-2 + r)*(d + e*x^r)*HypergeometricPFQ[{1, 2/r, 2/r}, {1 + 2/r,
 1 + 2/r}, -((e*x^r)/d)]) + 4*d*(a + b*Log[c*x^n]) + 2*(d + e*x^r)*Hypergeo
metric2F1[1, 2/r, (2 + r)/r, -((e*x^r)/d)]*(-(b*n) + a*(-2 + r) + b*(-2 + r
)*Log[c*x^n])))/(4*d^2*r*(d + e*x^r))
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x(a+b\ln(c\,x^n))}{(d+e\,x^r)^2}dx$$

```
[In] int(x*(a+b*ln(c*x^n))/(d+e*x^r)^2,x)
```

```
[Out] int(x*(a+b*ln(c*x^n))/(d+e*x^r)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^r)^2}\,dx = \int \frac{(b\log(cx^n)+a)x}{(ex^r+d)^2}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(d+e*x^r)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x*log(c*x^n) + a*x)/(e^2*x^(2*r) + 2*d*e*x^r + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 10.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^r)^2}\,dx=\int \frac{x(a+b\log(cx^n))}{(d+ex^r)^2}\,dx$$

```
[In] integrate(x*(a+b*ln(c*x**n))/(d+e*x**r)**2,x)
[Out] Integral(x*(a + b*log(c*x**n))/(d + e*x**r)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^r)^2}\,dx=\int \frac{(b\log(cx^n)+a)x}{(ex^r+d)^2}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(d+e*x^r)^2,x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*x/(e*x^r + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^r)^2}\,dx=\int \frac{(b\log(cx^n)+a)x}{(ex^r+d)^2}\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))/(d+e*x^r)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x/(e*x^r + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x(a+b\log(cx^n))}{(d+ex^r)^2}\,dx = \int \frac{x\,(a+b\,\ln(c\,x^n))}{(d+e\,x^r)^2}\,dx$$

```
[In] int((x*(a + b*log(c*x^n)))/(d + e*x^r)^2,x)
[Out] int((x*(a + b*log(c*x^n)))/(d + e*x^r)^2, x)
```

3.415 $\int \frac{a+b\log(cx^n)}{x(d+ex^r)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^2}\,dx = -\frac{ex^r(a+b\log(cx^n))}{d^2r(d+ex^r)} - \frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{d^2r} + \frac{bn\log(d+ex^r)}{d^2r^2} + \frac{bn\,\text{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)}{d^2r^2}$$

```
[Out] -e*x^r*(a+b*ln(c*x^n))/d^2/r/(d+e*x^r)-(a+b*ln(c*x^n))*ln(1+d/e/(x^r))/d^2/
r+b*n*ln(d+e*x^r)/d^2/r^2+b*n*polylog(2,-d/e/(x^r))/d^2/r^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2391, 2379, 2438, 2373, 266}

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^2}\,dx = -\frac{\log\left(\frac{dx^{-r}}{e}+1\right)(a+b\log(cx^n))}{d^2r} - \frac{ex^r(a+b\log(cx^n))}{d^2r(d+ex^r)} + \frac{bn\,\text{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)}{d^2r^2} + \frac{bn\log(d+ex^r)}{d^2r^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x^r)^2),x]
```

```
[Out] -((e*x^r*(a + b*Log[c*x^n]))/(d^2*r*(d + e*x^r))) - ((a + b*Log[c*x^n])*Log
[1 + d/(e*x^r)])/(d^2*r) + (b*n*Log[d + e*x^r])/(d^2*r^2) + (b*n*PolyLog[2,
 -(d/(e*x^r))])/(d^2*r^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^
(q_))/(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x^r)^(q + 1)*((a + b*Log[c*x^
n])^p/x), x], x] - Dist[e/d, Int[x^(r - 1)*(d + e*x^r)^q*(a + b*Log[c*x^n])
^p, x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0] && ILtQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{a+b\log(cx^n)}{x(d+ex^r)}\,dx}{d} - \frac{e\int \frac{x^{-1+r}(a+b\log(cx^n))}{(d+ex^r)^2}\,dx}{d} \\
&= -\frac{ex^r(a+b\log(cx^n))}{d^2r(d+ex^r)} - \frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{d^2r} \\
&\quad + \frac{(bn)\int \frac{\log\left(1+\frac{dx^{-r}}{e}\right)}{x}\,dx}{d^2r} + \frac{(ben)\int \frac{x^{-1+r}}{d+ex^r}\,dx}{d^2r} \\
&= -\frac{ex^r(a+b\log(cx^n))}{d^2r(d+ex^r)} - \frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{d^2r} + \frac{bn\log(d+ex^r)}{d^2r^2} + \frac{bn\operatorname{Li}_2\left(-\frac{dx^{-r}}{e}\right)}{d^2r^2}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.29

$$\int \frac{a+b\log\left(cx^n\right)}{x\left(d+ex^r\right)^2}\,dx$$
$$=\frac{\frac{dr(a+b\log(cx^n))}{d+ex^r}+bn\log\left(d-dx^r\right)-ar\log\left(d-dx^r\right)+br(n\log(x)-\log\left(cx^n\right))\log\left(d-dx^r\right)+bn\left(\frac{1}{2}r^2\log^2(\right.}{d^2r^2}$$

[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x^r)^2),x]

[Out] ((d*r*(a + b*Log[c*x^n]))/(d + e*x^r) + b*n*Log[d - d*x^r] - a*r*Log[d - d*x^r] + b*r*(n*Log[x] - Log[c*x^n])*Log[d - d*x^r] + b*n*((r^2*Log[x]^2)/2 + (-(r*Log[x]) + Log[-((e*x^r)/d)])*Log[d + e*x^r] + PolyLog[2, 1 + (e*x^r)/d]))/(d^2*r^2)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.20 (sec) , antiderivative size = 342, normalized size of antiderivative = 3.35

method	result
risch	$\frac{b\ln(d+e\,x^r)n\ln(x)}{r\,d^2}-\frac{b\ln(d+e\,x^r)\ln(x^n)}{r\,d^2}-\frac{bn\ln(x)}{rd(d+e\,x^r)}+\frac{b\ln(x^n)}{rd(d+e\,x^r)}-\frac{b\ln(x^r)n\ln(x)}{r\,d^2}+\frac{b\ln(x^r)\ln(x^n)}{r\,d^2}+\frac{bn\ln(d+e\,x^r)}{d^2r^2}$

[In] int((a+b*ln(c*x^n))/x/(d+e*x^r)^2,x,method=_RETURNVERBOSE)

[Out] b/r/d^2*ln(d+e*x^r)*n*ln(x)-b/r/d^2*ln(d+e*x^r)*ln(x^n)-b/r/d/(d+e*x^r)*n*ln(x)+b/r/d/(d+e*x^r)*ln(x^n)-b/r/d^2*ln(x^r)*n*ln(x)+b/r/d^2*ln(x^r)*ln(x^n)+b*n*ln(d+e*x^r)/d^2/r^2-b/r*n*e/d^2*ln(x)*x^r/(d+e*x^r)-b/r^2*n/d^2*dilog((d+e*x^r)/d)-b/r*n/d^2*ln(x)*ln((d+e*x^r)/d)+1/2*b*n/d^2*ln(x)^2+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)/r*(-1/d^2*ln(d+e*x^r)+1/d/(d+e*x^r)+1/d^2*ln(x^r))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(101) = 202.

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.10

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^2}\,dx$$
$$= \frac{bdnr^2\log(x)^2 + 2\,bdr\log(c) + 2\,adr + \left(benr^2\log(x)^2 + 2\,(ber^2\log(c) - benr + aer^2)\log(x)\right)x^r - 2\,(benx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*n*r^2*log(x)^2 + 2*b*d*r*log(c) + 2*a*d*r + (b*e*n*r^2*log(x)^2 +
2*(b*e*r^2*log(c) - b*e*n*r + a*e*r^2)*log(x))*x^r - 2*(b*e*n*x^r + b*d*n)*
dilog(-(e*x^r + d)/d + 1) - 2*(b*d*r*log(c) - b*d*n + a*d*r + (b*e*r*log(c)
 - b*e*n + a*e*r)*x^r)*log(e*x^r + d) + 2*(b*d*r^2*log(c) + a*d*r^2)*log(x)
 - 2*(b*e*n*r*x^r*log(x) + b*d*n*r*log(x))*log((e*x^r + d)/d))/(d^2*e*r^2*x
^r + d^3*r^2)
```

Sympy [A] (verification not implemented)

Time = 144.22 (sec) , antiderivative size = 360, normalized size of antiderivative = 3.53

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^2}\,dx = -\frac{ae\left(\begin{cases}\frac{x^r}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x^r} & \text{otherwise}\end{cases}\right)}{dr} - \frac{ae\left(\begin{cases}\frac{x^r}{d} & \text{for } e=0\\ \frac{\log(d+ex^r)}{e} & \text{otherwise}\end{cases}\right)}{d^2r}$$
$$+\frac{a\log(x^r)}{d^2r} + \frac{ben\left(\begin{cases}\frac{\begin{cases}\frac{x^r}{r} & \text{for } r\neq 0\\ \log(x) & \text{otherwise}\end{cases}}{d^2} & \text{for } e=0\\ -\begin{cases}\frac{\log(x)}{e^2} & \text{for } d=0\wedge r=0\\ -\frac{x^{-r}}{e^2r} & \text{for } d=0\\ \frac{\log(x)}{de+e^2} & \text{for } r=0\\ \frac{\log(x)}{de}-\frac{\log\left(\frac{d}{e}+x^r\right)}{der} & \text{otherwise}\end{cases} & \text{otherwise}\end{cases}\right)}{dr}$$
$$-\frac{be\left(\begin{cases}\frac{x^r}{d^2} & \text{for } e=0\\ -\frac{1}{de+e^2x^r} & \text{otherwise}\end{cases}\right)\log(cx^n)}{dr}$$
$$+\frac{ben\left(\begin{cases}\frac{\begin{cases}\frac{x^r}{r} & \text{for } r\neq 0\\ \log(x) & \text{otherwise}\end{cases}}{d} & \text{for } e=0\\ \frac{\begin{cases}-\frac{\mathrm{Li}_2\left(\frac{ex^re^{i\pi}}{d}\right)}{r} & \text{for } \frac{1}{|x|}<1\wedge|x|<1\\ \log(d)\log(x)-\frac{\mathrm{Li}_2\left(\frac{ex^re^{i\pi}}{d}\right)}{r} & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right)-\frac{\mathrm{Li}_2\left(\frac{ex^re^{i\pi}}{d}\right)}{r} & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(d)-\frac{\mathrm{Li}_2\left(\frac{ex^re^{i\pi}}{d}\right)}{r} & \text{otherwise}\end{cases}}{e} & \text{otherwise}\end{cases}\right)}{d^2r}$$
$$-\frac{be\left(\begin{cases}\frac{x^r}{d} & \text{for } e=0\\ \frac{\log(d+ex^r)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^2r}$$
$$+\frac{bn\left(\begin{cases}0 & \text{for } r=0\\ -\frac{\log(x^r)^2}{2r} & \text{otherwise}\end{cases}\right)}{d^2r} + \frac{b\log(x^r)\log(cx^n)}{d^2r}$$

```
[In] integrate((a+b*ln(c*x**n))/x/(d+e*x**r)**2,x)
```

```
[Out] -a*e*Piecewise((x**r/d**2, Eq(e, 0)), (-1/(d*e + e**2*x**r), True))/(d*r) -
 a*e*Piecewise((x**r/d, Eq(e, 0)), (log(d + e*x**r)/e, True))/(d**2*r) + a*
log(x**r)/(d**2*r) + b*e*n*Piecewise((Piecewise((x**r/r, Ne(r, 0)), (log(x)
, True))/d**2, Eq(e, 0)), (-Piecewise((log(x)/e**2, Eq(d, 0) & Eq(r, 0)), (
-1/(e**2*r*x**r), Eq(d, 0)), (log(x)/(d*e + e**2), Eq(r, 0)), (log(x)/(d*e)
 - log(d/e + x**r)/(d*e*r), True)), True))/(d*r) - b*e*Piecewise((x**r/d**2
, Eq(e, 0)), (-1/(d*e + e**2*x**r), True))*log(c*x**n)/(d*r) + b*e*n*Piecew
ise((Piecewise((x**r/r, Ne(r, 0)), (log(x), True))/d, Eq(e, 0)), (Piecewise
((-polylog(2, e*x**r*exp_polar(I*pi)/d)/r, (Abs(x) < 1) & (1/Abs(x) < 1)),
(log(d)*log(x) - polylog(2, e*x**r*exp_polar(I*pi)/d)/r, Abs(x) < 1), (-log
(d)*log(1/x) - polylog(2, e*x**r*exp_polar(I*pi)/d)/r, 1/Abs(x) < 1), (-mei
jerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0,
 0)), x)*log(d) - polylog(2, e*x**r*exp_polar(I*pi)/d)/r, True))/e, True))/
(d**2*r) - b*e*Piecewise((x**r/d, Eq(e, 0)), (log(d + e*x**r)/e, True))*log
(c*x**n)/(d**2*r) + b*n*Piecewise((0, Eq(r, 0)), (-log(x**r)**2/(2*r), True
))/(d**2*r) + b*log(x**r)*log(c*x**n)/(d**2*r)
```

Maxima [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x\left(d+ex^r\right)^2}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex^r+d\right)^2x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^2,x, algorithm="maxima")
```

```
[Out] a*(1/(d*e*r*x^r + d^2*r) + log(x)/d^2 - log((e*x^r + d)/e)/(d^2*r)) + b*int
egrate((log(c) + log(x^n))/(e^2*x*x^(2*r) + 2*d*e*x*x^r + d^2*x), x)
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x\left(d+ex^r\right)^2}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex^r+d\right)^2x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x^r + d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^2}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x\,(d+e\,x^r)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x^r)^2),x)
[Out] int((a + b*log(c*x^n))/(x*(d + e*x^r)^2), x)
```

3.416 $\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)^2}\,dx=\operatorname{Int}\left(\frac{a+b\log(cx^n)}{x^3(d+ex^r)^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*x^n))/x^3/(d+e*x^r)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)^2}\,dx=\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)^2}\,dx$$

```
[In] Int[(a + b*Log[c*x^n])/(x^3*(d + e*x^r)^2),x]
```

```
[Out] Defer[Int][(a + b*Log[c*x^n])/(x^3*(d + e*x^r)^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)^2}\,dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 139 vs. 2(26) = 52.

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 6.04

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)^2}\,dx = -\frac{bn(2+r)(d+ex^r)\,{}_3F_2\left(1,-\frac{2}{r},-\frac{2}{r};1-\frac{2}{r},1-\frac{2}{r};-\frac{ex^r}{d}\right)-4d(a+b\log(cx^n))+2(d+ex^r)\,\text{Hypergeome}}{4d^2rx^2(d+ex^r)}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*(d + e*x^r)^2),x]
```

```
[Out] -1/4*(b*n*(2 + r)*(d + e*x^r)*HypergeometricPFQ[{1, -2/r, -2/r}, {1 - 2/r,
1 - 2/r}, -((e*x^r)/d)] - 4*d*(a + b*Log[c*x^n]) + 2*(d + e*x^r)*Hypergeome
tric2F1[1, -2/r, (-2 + r)/r, -((e*x^r)/d)]*(-(b*n) + a*(2 + r) + b*(2 + r)*
Log[c*x^n]))/(d^2*r*x^2*(d + e*x^r))
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\ln(c\,x^n)}{x^3\,(d+e\,x^r)^2}dx$$

```
[In] int((a+b*ln(c*x^n))/x^3/(d+e*x^r)^2,x)
```

```
[Out] int((a+b*ln(c*x^n))/x^3/(d+e*x^r)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.96

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)^2}\,dx = \int \frac{b\log(cx^n)+a}{(ex^r+d)^2x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(d+e*x^r)^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e^2*x^3*x^(2*r) + 2*d*e*x^3*x^r + d^2*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 117.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)^2}\,dx=\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**3/(d+e*x**r)**2,x)
[Out] Integral((a + b*log(c*x**n))/(x**3*(d + e*x**r)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)^2}\,dx=\int \frac{b\log(cx^n)+a}{(ex^r+d)^2x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(d+e*x^r)^2,x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)/((e*x^r + d)^2*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)^2}\,dx=\int \frac{b\log(cx^n)+a}{(ex^r+d)^2x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^3/(d+e*x^r)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x^r + d)^2*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)^2}\,dx=\int \frac{a+b\,\ln\left(c\,x^n\right)}{x^3\,{\left(d+e\,x^r\right)}^2}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e*x^r)^2),x)
[Out] int((a + b*log(c*x^n))/(x^3*(d + e*x^r)^2), x)
```

3.417 $\int \frac{x^2(a+b\log(cx^n))}{(d+ex^r)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^r)^2} dx = \text{Int}\left(\frac{x^2(a+b\log(cx^n))}{(d+ex^r)^2}, x\right)$$

```
[Out] Unintegrable(x^2*(a+b*ln(c*x^n))/(d+e*x^r)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^r)^2} dx = \int \frac{x^2(a+b\log(cx^n))}{(d+ex^r)^2} dx$$

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/(d + e*x^r)^2,x]
```

```
[Out] Defer[Int][(x^2*(a + b*Log[c*x^n]))/(d + e*x^r)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2(a+b\log(cx^n))}{(d+ex^r)^2} dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(26) = 52.

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 6.09

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^r)^2}\,dx$$
$$= \frac{x^3\left(-bn(-3+r)\left(d+ex^r\right)\,{}_3F_2\left(1,\frac{3}{r},\frac{3}{r};1+\frac{3}{r},1+\frac{3}{r};-\frac{ex^r}{d}\right)+9d(a+b\log(cx^n))+3(d+ex^r)\,\text{Hypergeom}\right.}{9d^2r\left(d+ex^r\right)}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*x^r)^2,x]
```

```
[Out] (x^3*(-(b*n*(-3 + r)*(d + e*x^r)*HypergeometricPFQ[{1, 3/r, 3/r}, {1 + 3/r,
 1 + 3/r}, -((e*x^r)/d)]) + 9*d*(a + b*Log[c*x^n]) + 3*(d + e*x^r)*Hypergeo
metric2F1[1, 3/r, (3 + r)/r, -((e*x^r)/d)]*(-(b*n) + a*(-3 + r) + b*(-3 + r
)*Log[c*x^n])))/(9*d^2*r*(d + e*x^r))
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\ln(c\,x^n))}{(d+e\,x^r)^2}dx$$

```
[In] int(x^2*(a+b*ln(c*x^n))/(d+e*x^r)^2,x)
```

```
[Out] int(x^2*(a+b*ln(c*x^n))/(d+e*x^r)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^r)^2}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{(ex^r+d)^2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(d+e*x^r)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*log(c*x^n) + a*x^2)/(e^2*x^(2*r) + 2*d*e*x^r + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 23.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^r)^2}\,dx = \int \frac{x^2(a+b\log(cx^n))}{(d+ex^r)^2}\,dx$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(d+e*x**r)**2,x)
[Out] Integral(x**2*(a + b*log(c*x**n))/(d + e*x**r)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^r)^2}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{(ex^r+d)^2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(d+e*x^r)^2,x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*x^2/(e*x^r + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^r)^2}\,dx = \int \frac{(b\log(cx^n)+a)x^2}{(ex^r+d)^2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))/(d+e*x^r)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^2/(e*x^r + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\log(cx^n))}{(d+ex^r)^2}\,dx=\int \frac{x^2\,(a+b\,\ln(c\,x^n))}{(d+e\,x^r)^2}\,dx$$

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e*x^r)^2,x)
[Out] int((x^2*(a + b*log(c*x^n)))/(d + e*x^r)^2, x)
```

3.418 $\int \frac{a+b\log(cx^n)}{(d+ex^r)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\log(cx^n)}{(d+ex^r)^2}\,dx=\operatorname{Int}\left(\frac{a+b\log(cx^n)}{(d+ex^r)^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*x^n))/(d+e*x^r)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\log(cx^n)}{(d+ex^r)^2}\,dx=\int \frac{a+b\log(cx^n)}{(d+ex^r)^2}\,dx$$

```
[In] Int[(a + b*Log[c*x^n])/(d + e*x^r)^2,x]
```

```
[Out] Defer[Int][(a + b*Log[c*x^n])/(d + e*x^r)^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\log(cx^n)}{(d+ex^r)^2}\,dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 161 vs. $2(23) = 46$.

Time = 1.71 (sec) , antiderivative size = 161, normalized size of antiderivative = 8.05

$$\int \frac{a+b\log(cx^n)}{(d+ex^r)^2}\,dx$$
$$= \frac{x\left(adr\,\text{Hypergeometric2F1}\left(2,\frac{1}{r},1+\frac{1}{r},-\frac{ex^r}{d}\right)+aerx^r\,\text{Hypergeometric2F1}\left(2,\frac{1}{r},1+\frac{1}{r},-\frac{ex^r}{d}\right)-bn(-1+\right.}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*x^r)^2,x]
```

```
[Out] (x*(a*d*r*Hypergeometric2F1[2, r^(-1), 1 + r^(-1), -((e*x^r)/d)] + a*e*r*x^
r*Hypergeometric2F1[2, r^(-1), 1 + r^(-1), -((e*x^r)/d)] - b*n*(-1 + r)*(d
+ e*x^r)*HypergeometricPFQ[{1, r^(-1), r^(-1)}, {1 + r^(-1), 1 + r^(-1)}, -
((e*x^r)/d)] + b*d*Log[c*x^n] - b*(d + e*x^r)*Hypergeometric2F1[1, r^(-1),
1 + r^(-1), -((e*x^r)/d)]*(n - (-1 + r)*Log[c*x^n])))/(d^2*r*(d + e*x^r))
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\ln(c\,x^n)}{(d+e\,x^r)^2}dx$$

```
[In] int((a+b*ln(c*x^n))/(d+e*x^r)^2,x)
```

```
[Out] int((a+b*ln(c*x^n))/(d+e*x^r)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{a+b\log(cx^n)}{(d+ex^r)^2}\,dx = \int \frac{b\log(cx^n)+a}{(ex^r+d)^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(d+e*x^r)^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e^2*x^(2*r) + 2*d*e*x^r + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 9.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\log(cx^n)}{(d+ex^r)^2}\,dx = \int \frac{a+b\log(cx^n)}{(d+ex^r)^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/(d+e*x**r)**2,x)
[Out] Integral((a + b*log(c*x**n))/(d + e*x**r)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\log(cx^n)}{(d+ex^r)^2}\,dx = \int \frac{b\log(cx^n)+a}{(ex^r+d)^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(d+e*x^r)^2,x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)/(e*x^r + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\log(cx^n)}{(d+ex^r)^2}\,dx = \int \frac{b\log(cx^n)+a}{(ex^r+d)^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/(d+e*x^r)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/(e*x^r + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\log(cx^n)}{(d+ex^r)^2}\,dx=\int \frac{a+b\,\ln(c\,x^n)}{(d+e\,x^r)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))/(d + e*x^r)^2,x)
[Out] int((a + b*log(c*x^n))/(d + e*x^r)^2, x)
```

3.419 $\int \frac{a+b\log(cx^n)}{x^2(d+ex^r)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^r\right)^2}\,dx=\operatorname{Int}\left(\frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^r\right)^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*x^n))/x^2/(d+e*x^r)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^r\right)^2}\,dx=\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^r\right)^2}\,dx$$

```
[In] Int[(a + b*Log[c*x^n])/(x^2*(d + e*x^r)^2),x]
```

```
[Out] Defer[Int][(a + b*Log[c*x^n])/(x^2*(d + e*x^r)^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^r\right)^2}\,dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 135 vs. 2(26) = 52.

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 5.87

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^r\right)^2}\,dx = \frac{-bn(1+r)\left(d+ex^r\right)\,{}_3F_2\left(1,-\frac{1}{r},-\frac{1}{r};1-\frac{1}{r},1-\frac{1}{r};-\frac{ex^r}{d}\right)+d(a+b\log\left(cx^n\right))-\left(d+ex^r\right)\text{Hypergeometric}}{d^2rx\left(d+ex^r\right)}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*x^r)^2),x]
```

```
[Out] (-(b*n*(1 + r)*(d + e*x^r)*HypergeometricPFQ[{1, -r^(-1), -r^(-1)}, {1 - r^
(-1), 1 - r^(-1)}, -((e*x^r)/d)]) + d*(a + b*Log[c*x^n]) - (d + e*x^r)*Hype
rgeometric2F1[1, -r^(-1), (-1 + r)/r, -((e*x^r)/d)]*(a - b*n + a*r + b*(1 +
 r)*Log[c*x^n]))/(d^2*r*x*(d + e*x^r))
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\ln\left(c\,x^n\right)}{x^2\left(d+e\,x^r\right)^2}dx$$

```
[In] int((a+b*ln(c*x^n))/x^2/(d+e*x^r)^2,x)
```

```
[Out] int((a+b*ln(c*x^n))/x^2/(d+e*x^r)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.96

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^r\right)^2}\,dx = \int \frac{b\log\left(cx^n\right)+a}{\left(ex^r+d\right)^2x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(d+e*x^r)^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(e^2*x^2*x^(2*r) + 2*d*e*x^2*x^r + d^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 54.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^r)^2}\,dx=\int \frac{a+b\log(cx^n)}{x^2(d+ex^r)^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x**2/(d+e*x**r)**2,x)
[Out] Integral((a + b*log(c*x**n))/(x**2*(d + e*x**r)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^r)^2}\,dx=\int \frac{b\log(cx^n)+a}{(ex^r+d)^2x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(d+e*x^r)^2,x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)/((e*x^r + d)^2*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^r)^2}\,dx=\int \frac{b\log(cx^n)+a}{(ex^r+d)^2x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x^2/(d+e*x^r)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x^r + d)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^r)^2}\,dx=\int \frac{a+b\,\ln(c\,x^n)}{x^2\,(d+e\,x^r)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*x^r)^2),x)
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*x^r)^2), x)
```

3.420 $\int \frac{a+b\log(cx^n)}{x(c-x^{-n})}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \frac{a+b\log(cx^n)}{x(c-x^{-n})}\,dx=\frac{a\log(1-cx^n)}{cn}-\frac{b\,\text{PolyLog}(2,1-cx^n)}{cn}$$

```
[Out] a*ln(1-c*x^n)/c/n-b*polylog(2,1-c*x^n)/c/n
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2378, 2370, 2353, 2352}

$$\int \frac{a+b\log(cx^n)}{x(c-x^{-n})}\,dx=\frac{a\log(1-cx^n)}{cn}-\frac{b\,\text{PolyLog}(2,1-cx^n)}{cn}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(c - x^(-n))),x]
[Out] (a*Log[1 - c*x^n])/(c*n) - (b*PolyLog[2, 1 - c*x^n])/(c*n)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
(a + b*Log[(-c)*(d/e)])*(Log[d + e*x]/e), x] + Dist[b, Int[Log[(-e)*(x/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[(-c)*(d/e), 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b\log(cx)}{\left(c-\frac{1}{x}\right)x}\,dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \frac{a+b\log(cx)}{-1+cx}\,dx, x, x^n\right)}{n} \\
&= \frac{a\log(1-cx^n)}{cn} + \frac{b\text{Subst}\left(\int \frac{\log(cx)}{-1+cx}\,dx, x, x^n\right)}{n} \\
&= \frac{a\log(1-cx^n)}{cn} - \frac{b\text{Li}_2(1-cx^n)}{cn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log(cx^n)}{x(c-x^{-n})}\,dx = \frac{(a+b\log(cx^n))\log(1-cx^n)+b\,\text{PolyLog}(2,cx^n)}{cn}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(c - x^(-n))),x]
[Out] ((a + b*Log[c*x^n])*Log[1 - c*x^n] + b*PolyLog[2, c*x^n])/(c*n)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{a\ln(c\,x^n-1)}{c}-\frac{b\,\mathrm{dilog}(c\,x^n)}{c}}{n}$
default	$\frac{\frac{a\ln(c\,x^n-1)}{c}-\frac{b\,\mathrm{dilog}(c\,x^n)}{c}}{n}$
parts	$\frac{a\ln(c\,x^n-1)}{nc}-\frac{b\,\mathrm{dilog}(c\,x^n)}{nc}$
risch	$\frac{b\ln(1-c\,x^n)\ln(x^n)}{nc}-\frac{b\ln(1-c\,x^n)\ln(c\,x^n)}{nc}-\frac{b\,\mathrm{dilog}(c\,x^n)}{nc}+\frac{\left(-\frac{ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)}{2}+\frac{ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\right.}{2}}{}$

```
[In] int((a+b*ln(c*x^n))/x/(c-1/(x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(1/c*a*ln(c*x^n-1)-1/c*b*dilog(c*x^n))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{a+b\log(cx^n)}{x(c-x^{-n})}\,dx = \frac{bn\log(-cx^n+1)\log(x)+b\mathrm{Li}_2(cx^n)+(b\log(c)+a)\log(cx^n-1)}{cn}$$

```
[In] integrate((a+b*log(c*x^n))/x/(c-1/(x^n)),x, algorithm="fricas")
```

```
[Out] (b*n*log(-c*x^n + 1)*log(x) + b*dilog(c*x^n) + (b*log(c) + a)*log(c*x^n - 1
))/(c*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{a+b\log(cx^n)}{x(c-x^{-n})}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*ln(c*x**n))/x/(c-1/(x**n)),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{a + b\log(cx^n)}{x(c - x^{-n})}\,dx = \int \frac{b\log(cx^n) + a}{\left(c - \frac{1}{x^n}\right)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(c-1/(x^n)),x, algorithm="maxima")
```

```
[Out] b*integrate((x^n*log(c) + x^n*log(x^n))/(c*x*x^n - x), x) + a*log((c*x^n -
1)/c)/(c*n)
```

Giac [F]

$$\int \frac{a + b\log(cx^n)}{x(c - x^{-n})}\,dx = \int \frac{b\log(cx^n) + a}{\left(c - \frac{1}{x^n}\right)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(c-1/(x^n)),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((c - 1/x^n)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{x(c - x^{-n})}\,dx = \int \frac{a + b\,\ln(c\,x^n)}{x\,\left(c - \frac{1}{x^n}\right)}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(c - 1/x^n)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x*(c - 1/x^n)), x)
```

3.421 $\int \frac{(d+ex^r)^3(a+b\log(cx^n))}{x} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 152

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x} \, dx = -\frac{3bd^2enx^r}{r^2} - \frac{3bde^2nx^{2r}}{4r^2} - \frac{be^3nx^{3r}}{9r^2} - \frac{1}{2}bd^3n\log^2(x) + \frac{3d^2ex^r(a+b\log(cx^n))}{r} + \frac{3de^2x^{2r}(a+b\log(cx^n))}{2r} + \frac{e^3x^{3r}(a+b\log(cx^n))}{3r} + d^3\log(x)\,(a+b\log(cx^n))$$

```
[Out] -3*b*d^2*e*n*x^r/r^2-3/4*b*d*e^2*n*x^(2*r)/r^2-1/9*b*e^3*n*x^(3*r)/r^2-1/2*
b*d^3*n*ln(x)^2+3*d^2*e*x^r*(a+b*ln(c*x^n))/r+3/2*d*e^2*x^(2*r)*(a+b*ln(c*x
^n))/r+1/3*e^3*x^(3*r)*(a+b*ln(c*x^n))/r+d^3*ln(x)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {272, 45, 2372, 12, 14, 2338}

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x} \, dx = d^3\log(x)\,(a+b\log(cx^n)) + \frac{3d^2ex^r(a+b\log(cx^n))}{r} + \frac{3de^2x^{2r}(a+b\log(cx^n))}{2r} + \frac{e^3x^{3r}(a+b\log(cx^n))}{3r} - \frac{1}{2}bd^3n\log^2(x) - \frac{3bd^2enx^r}{r^2} - \frac{3bde^2nx^{2r}}{4r^2} - \frac{be^3nx^{3r}}{9r^2}$$

```
[In] Int[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] (-3*b*d^2*e*n*x^r)/r^2 - (3*b*d*e^2*n*x^(2*r))/(4*r^2) - (b*e^3*n*x^(3*r))/
(9*r^2) - (b*d^3*n*Log[x]^2)/2 + (3*d^2*e*x^r*(a + b*Log[c*x^n]))/r + (3*d*
```

```
e^2*x^(2*r)*(a + b*Log[c*x^n]))/(2*r) + (e^3*x^(3*r)*(a + b*Log[c*x^n]))/(3
*r) + d^3*Log[x]*(a + b*Log[c*x^n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3d^2ex^r(a+b\log(cx^n))}{r}+\frac{3de^2x^{2r}(a+b\log(cx^n))}{2r}+\frac{e^3x^{3r}(a+b\log(cx^n))}{3r}\\&\quad+d^3\log(x)\,(a+b\log(cx^n))-(bn)\int\frac{ex^r(18d^2+9dex^r+2e^2x^{2r})+6d^3r\log(x)}{6rx}\,dx\end{aligned}$$

$$= \frac{3d^2ex^r(a+b\log(cx^n))}{r} + \frac{3de^2x^{2r}(a+b\log(cx^n))}{2r} + \frac{e^3x^{3r}(a+b\log(cx^n))}{3r} + d^3\log(x)(a+b\log(cx^n)) - \frac{(bn)\int \frac{ex^r(18d^2+9dex^r+2e^2x^{2r})+6d^3r\log(x)}{x}dx}{6r}$$

$$= \frac{3d^2ex^r(a+b\log(cx^n))}{r} + \frac{3de^2x^{2r}(a+b\log(cx^n))}{2r} + \frac{e^3x^{3r}(a+b\log(cx^n))}{3r} + d^3\log(x)(a+b\log(cx^n)) - \frac{(bn)\int\left(18d^2ex^{-1+r}+9de^2x^{-1+2r}+2e^3x^{-1+3r}+\frac{6d^3r\log(x)}{x}\right)dx}{6r}$$

$$= -\frac{3bd^2enx^r}{r^2} - \frac{3bde^2nx^{2r}}{4r^2} - \frac{be^3nx^{3r}}{9r^2} + \frac{3d^2ex^r(a+b\log(cx^n))}{r} + \frac{3de^2x^{2r}(a+b\log(cx^n))}{2r} + \frac{e^3x^{3r}(a+b\log(cx^n))}{3r} + d^3\log(x)(a+b\log(cx^n)) - (bd^3n)\int\frac{\log(x)}{x}dx$$

$$= -\frac{3bd^2enx^r}{r^2} - \frac{3bde^2nx^{2r}}{4r^2} - \frac{be^3nx^{3r}}{9r^2} - \frac{1}{2}bd^3n\log^2(x) + \frac{3d^2ex^r(a+b\log(cx^n))}{r} + \frac{3de^2x^{2r}(a+b\log(cx^n))}{2r} + \frac{e^3x^{3r}(a+b\log(cx^n))}{3r} + d^3\log(x)(a+b\log(cx^n))$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.87

$$\int\frac{(d+ex^r)^3(a+b\log(cx^n))}{x}dx = ad^3\log(x) + \frac{1}{36}\left(\frac{ex^r(6ar(18d^2+9dex^r+2e^2x^{2r})-bn(108d^2+27dex^r+4e^2x^{2r}))}{r^2} + \frac{6bex^r(18d^2+9dex^r+2e^2x^{2r})\log(cx^n)}{r} + \frac{18bd^3\log^2(cx^n)}{n}\right)$$

```
[In] Integrate[((d + e*x^r)^3*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] a*d^3*Log[x] + ((e*x^r*(6*a*r*(18*d^2 + 9*d*e*x^r + 2*e^2*x^(2*r)) - b*n*(1
08*d^2 + 27*d*e*x^r + 4*e^2*x^(2*r))))/r^2 + (6*b*e*x^r*(18*d^2 + 9*d*e*x^r
 + 2*e^2*x^(2*r))*Log[c*x^n])/r + (18*b*d^3*Log[c*x^n]^2)/n)/36
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.16

method	result
parallelrisch	$\frac{12e^3b\ln(c\,x^n)x^{3r}nr+12x^{3r}a\,e^3nr-4x^{3r}b\,e^3n^2+54bd\,e^2\ln(c\,x^n)x^{2r}nr+36\ln(x)a\,d^3n\,r^2+54x^{2r}ad\,e^2nr-27x^{2r}bd\,e^2n^2+108b}{36n\,r^2}$
risch	$-\frac{3b\,d^2en\,x^r}{r^2}-\frac{3bd\,e^2n\,x^{2r}}{4r^2}-\frac{b\,e^3n\,x^{3r}}{9r^2}-\frac{i\ln(x)\pi b\,d^3\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)}{2}-\frac{i\pi b\,e^3\,\mathrm{csgn}(ic\,x^n)^3x^{3r}}{6r}+\ln($

[In] int((d+e*x^r)^3*(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)

[Out] 1/36*(12*e^3*b*ln(c*x^n)*(x^r)^3*n*r+12*(x^r)^3*a*e^3*n*r-4*(x^r)^3*b*e^3*n^2+54*b*d*e^2*ln(c*x^n)*(x^r)^2*n*r+36*ln(x)*a*d^3*n*r^2+54*(x^r)^2*a*d*e^2*n*r-27*(x^r)^2*b*d*e^2*n^2+108*b*d^2*e*ln(c*x^n)*x^r*n*r+18*b*d^3*ln(c*x^n)^2*r^2+108*x^r*a*d^2*e*n*r-108*x^r*b*d^2*e*n^2)/n/r^2

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex^r)^3\,(a+b\log{(cx^n)})}{x}\,dx$$
$$=\frac{18\,bd^3nr^2\log{(x)}^2+4\,(3\,be^3nr\log{(x)}+3\,be^3r\log{(c)}-be^3n+3\,ae^3r)x^{3\,r}+27\,(2\,bde^2nr\log{(x)}+2\,bde^2r}{}$$

[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x,x, algorithm="fricas")

[Out] 1/36*(18*b*d^3*n*r^2*log(x)^2 + 4*(3*b*e^3*n*r*log(x) + 3*b*e^3*r*log(c) - b*e^3*n + 3*a*e^3*r)*x^(3*r) + 27*(2*b*d*e^2*n*r*log(x) + 2*b*d*e^2*r*log(c) - b*d*e^2*n + 2*a*d*e^2*r)*x^(2*r) + 108*(b*d^2*e*n*r*log(x) + b*d^2*e*r*log(c) - b*d^2*e*n + a*d^2*e*r)*x^r + 36*(b*d^3*r^2*log(c) + a*d^3*r^2)*log(x))/r^2

Sympy [A] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.97

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x}\,dx = \begin{cases} (a+b\log(c))(d+e)^3\log(x) \\ (a+b\log(c))\left(d^3\log(x)+\frac{3d^2ex^r}{r}+\frac{3de^2x^{2r}}{2r}+\frac{e^3x^{3r}}{3r}\right) \\ (d+e)^3\left(\begin{cases} a\log(x) & \text{for } b=0 \\ -(-a-b\log(c))\log(x) & \text{for } n=0 \\ \frac{(-a-b\log(cx^n))^2}{2bn} & \text{otherwise} \end{cases}\right) \\ \frac{ad^3\log(cx^n)}{n}+\frac{3ad^2ex^r}{r}+\frac{3ade^2x^{2r}}{2r}+\frac{ae^3x^{3r}}{3r}+\frac{bd^3\log(cx^n)^2}{2n}-\frac{3bd^2enx^r}{r^2}+\frac{3bd^2ex^r\log(cx^n)}{r}-\frac{3bde^2nx^{2r}}{4r^2}+\frac{3bde^2x^{2r}\log(cx^{r}}{2r} \end{cases}$$

```
[In] integrate((d+e*x**r)**3*(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise(((a + b*log(c))*(d + e)**3*log(x), Eq(n, 0) & Eq(r, 0)), ((a + b*
log(c))*(d**3*log(x) + 3*d**2*e*x**r/r + 3*d*e**2*x**(2*r)/(2*r) + e**3*x**
(3*r)/(3*r)), Eq(n, 0)), ((d + e)**3*Piecewise((a*log(x), Eq(b, 0)), (-(-a
- b*log(c))*log(x), Eq(n, 0)), ((-a - b*log(c*x**n))**2/(2*b*n), True)), Eq
(r, 0)), (a*d**3*log(c*x**n)/n + 3*a*d**2*e*x**r/r + 3*a*d*e**2*x**(2*r)/(2
*r) + a*e**3*x**(3*r)/(3*r) + b*d**3*log(c*x**n)**2/(2*n) - 3*b*d**2*e*n*x*
*r/r**2 + 3*b*d**2*e*x**r*log(c*x**n)/r - 3*b*d*e**2*n*x**(2*r)/(4*r**2) +
3*b*d*e**2*x**(2*r)*log(c*x**n)/(2*r) - b*e**3*n*x**(3*r)/(9*r**2) + b*e**3
*x**(3*r)*log(c*x**n)/(3*r), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x}\,dx = \frac{be^3x^{3r}\log(cx^n)}{3r}+\frac{3bde^2x^{2r}\log(cx^n)}{2r}+\frac{3bd^2ex^r\log(cx^n)}{r}+\frac{bd^3\log(cx^n)^2}{2n}+ad^3\log(x)-\frac{be^3nx^{3r}}{9r^2}+\frac{ae^3x^{3r}}{3r}-\frac{3bde^2nx^{2r}}{4r^2}+\frac{3ade^2x^{2r}}{2r}-\frac{3bd^2enx^r}{r^2}+\frac{3ad^2ex^r}{r}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] 1/3*b*e^3*x^(3*r)*log(c*x^n)/r + 3/2*b*d*e^2*x^(2*r)*log(c*x^n)/r + 3*b*d^2
*e*x^r*log(c*x^n)/r + 1/2*b*d^3*log(c*x^n)^2/n + a*d^3*log(x) - 1/9*b*e^3*n
*x^(3*r)/r^2 + 1/3*a*e^3*x^(3*r)/r - 3/4*b*d*e^2*n*x^(2*r)/r^2 + 3/2*a*d*e^
2*x^(2*r)/r - 3*b*d^2*e*n*x^r/r^2 + 3*a*d^2*e*x^r/r
```

Giac [F]

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x} \, dx = \int \frac{(ex^r+d)^3(b\log(cx^n)+a)}{x} \, dx$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))/x,x, algorithm="giac")
[Out] integrate((e*x^r + d)^3*(b*log(c*x^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))}{x} \, dx = \int \frac{(d+e\,x^r)^3 \,(a+b\,\ln(c\,x^n))}{x} \, dx$$

```
[In] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x,x)
[Out] int(((d + e*x^r)^3*(a + b*log(c*x^n)))/x, x)
```

3.422 $\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x}\,dx = -\frac{2bdenx^r}{r^2}-\frac{be^2nx^{2r}}{4r^2}-\frac{1}{2}bd^2n\log^2(x)+\frac{2dex^r(a+b\log(cx^n))}{r}+\frac{e^2x^{2r}(a+b\log(cx^n))}{2r}+d^2\log(x)(a+b\log(cx^n))$$

```
[Out] -2*b*d*e*n*x^r/r^2-1/4*b*e^2*n*x^(2*r)/r^2-1/2*b*d^2*n*ln(x)^2+2*d*e*x^r*(a
+b*ln(c*x^n))/r+1/2*e^2*x^(2*r)*(a+b*ln(c*x^n))/r+d^2*ln(x)*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {272, 45, 2372, 12, 14, 2338}

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x}\,dx = d^2\log(x)(a+b\log(cx^n))+\frac{2dex^r(a+b\log(cx^n))}{r}+\frac{e^2x^{2r}(a+b\log(cx^n))}{2r}-\frac{1}{2}bd^2n\log^2(x)-\frac{2bdenx^r}{r^2}-\frac{be^2nx^{2r}}{4r^2}$$

```
[In] Int[((d + e*x^r)^2*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] (-2*b*d*e*n*x^r)/r^2 - (b*e^2*n*x^(2*r))/(4*r^2) - (b*d^2*n*Log[x]^2)/2 + (
2*d*e*x^r*(a + b*Log[c*x^n]))/r + (e^2*x^(2*r)*(a + b*Log[c*x^n]))/(2*r) +
d^2*Log[x]*(a + b*Log[c*x^n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2dex^r(a+b\log(cx^n))}{r}+\frac{e^2x^{2r}(a+b\log(cx^n))}{2r}\\&\quad+d^2\log(x)\,(a+b\log(cx^n))-(bn)\int\frac{ex^r(4d+ex^r)+2d^2r\log(x)}{2rx}\,dx\\&=\frac{2dex^r(a+b\log(cx^n))}{r}+\frac{e^2x^{2r}(a+b\log(cx^n))}{2r}\\&\quad+d^2\log(x)\,(a+b\log(cx^n))-\frac{(bn)\int\frac{ex^r(4d+ex^r)+2d^2r\log(x)}{x}\,dx}{2r}\end{aligned}$$

$$= \frac{2dex^r(a+b\log(cx^n))}{r} + \frac{e^2x^{2r}(a+b\log(cx^n))}{2r} + d^2\log(x)\,(a+b\log(cx^n)) - \frac{(bn)\int\left(4dex^{-1+r}+e^2x^{-1+2r}+\frac{2d^2r\log(x)}{x}\right)dx}{2r}$$

$$= -\frac{2bdenx^r}{r^2} - \frac{be^2nx^{2r}}{4r^2} + \frac{2dex^r(a+b\log(cx^n))}{r} + \frac{e^2x^{2r}(a+b\log(cx^n))}{2r} + d^2\log(x)\,(a+b\log(cx^n)) - (bd^2n)\int\frac{\log(x)}{x}\,dx$$

$$= -\frac{2bdenx^r}{r^2} - \frac{be^2nx^{2r}}{4r^2} - \frac{1}{2}bd^2n\log^2(x) + \frac{2dex^r(a+b\log(cx^n))}{r} + \frac{e^2x^{2r}(a+b\log(cx^n))}{2r} + d^2\log(x)\,(a+b\log(cx^n))$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int\frac{(d+ex^r)^2\,(a+b\log(cx^n))}{x}\,dx = \frac{1}{4}\left(\frac{ex^r(2ar(4d+ex^r)-bn(8d+ex^r))}{r^2} + 4ad^2\log(x) + \frac{2bex^r(4d+ex^r)\log(cx^n)}{r} + \frac{2bd^2\log^2(cx^n)}{n}\right)$$

```
[In] Integrate[((d + e*x^r)^2*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] ((e*x^r*(2*a*r*(4*d + e*x^r) - b*n*(8*d + e*x^r)))/r^2 + 4*a*d^2*Log[x] + (
2*b*e*x^r*(4*d + e*x^r)*Log[c*x^n])/r + (2*b*d^2*Log[c*x^n]^2)/n)/4
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17

method	result
parallelrisch	$\frac{2x^{2r}\ln(c\,x^n)b\,e^2rn+4\ln(x)a\,d^2n\,r^2+2x^{2r}a\,e^2nr-x^{2r}b\,e^2n^2+8x^r\ln(c\,x^n)bdern+2b\,d^2\ln(c\,x^n)^2r^2+8x^radenr-8x^rbde\,n^2}{4r^2n}$
risch	$\frac{b(2d^2\ln(x)r+e^2x^{2r}+4de\,x^r)\ln(x^n)}{2r}+\frac{i\pi b\,e^2\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2x^{2r}}{4r}-\frac{i\pi bde\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)x^r}{r}-\frac{i\pi b\,e^2}{}$

```
[In] int((d+e*x^r)^2*(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*(x^r)^2*ln(c*x^n)*b*e^2*r*n+4*ln(x)*a*d^2*n*r^2+2*(x^r)^2*a*e^2*n*r-
(x^r)^2*b*e^2*n^2+8*x^r*ln(c*x^n)*b*d*e*r*n+2*b*d^2*ln(c*x^n)^2*r^2+8*x^r*a
*d*e*n*r-8*x^r*b*d*e*n^2)/r^2/n
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x}\,dx = \frac{2\,bd^2nr^2\log(x)^2 + (2\,be^2nr\log(x) + 2\,be^2r\log(c) - be^2n + 2\,ae^2r)x^{2\,r} + 8\,(bdenr\log(x) + bder\log(c) -}{4\,r^2}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*d^2*n*r^2*log(x)^2 + (2*b*e^2*n*r*log(x) + 2*b*e^2*r*log(c) - b*e^
2*n + 2*a*e^2*r)*x^(2*r) + 8*(b*d*e*n*r*log(x) + b*d*e*r*log(c) - b*d*e*n +
 a*d*e*r)*x^r + 4*(b*d^2*r^2*log(c) + a*d^2*r^2)*log(x))/r^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(104) = 208$.

Time = 2.35 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.08

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))}{x}\,dx = \begin{cases} (a+b\log(c))\,(d+e)^2\log(x) & \text{for } n=0 \wedge \\ (a+b\log(c))\left(d^2\log(x) + \frac{2dex^r}{r} + \frac{e^2x^{2r}}{2r}\right) & \text{for } n=0 \\ (d+e)^2 \left(\begin{cases} a\log(x) & \text{for } b=0 \\ -(-a-b\log(c))\log(x) & \text{for } n=0 \\ \frac{(-a-b\log(cx^n))^2}{2bn} & \text{otherwise} \end{cases}\right) & \text{for } r=0 \\ \frac{ad^2\log(cx^n)}{n} + \frac{2adex^r}{r} + \frac{ae^2x^{2r}}{2r} + \frac{bd^2\log(cx^n)^2}{2n} - \frac{2bdenx^r}{r^2} + \frac{2bdex^r\log(cx^n)}{r} - \frac{be^2nx^{2r}}{4r^2} + \frac{be^2x^{2r}\log(cx^n)}{2r} & \text{otherwise} \end{cases}$$

```
[In] integrate((d+e*x**r)**2*(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise(((a + b*log(c))*(d + e)**2*log(x), Eq(n, 0) & Eq(r, 0)), ((a + b*
log(c))*(d**2*log(x) + 2*d*e*x**r/r + e**2*x**(2*r)/(2*r)), Eq(n, 0)), ((d
+ e)**2*Piecewise((a*log(x), Eq(b, 0)), (-(-a - b*log(c))*log(x), Eq(n, 0))
, ((-a - b*log(c*x**n))**2/(2*b*n), True)), Eq(r, 0)), (a*d**2*log(c*x**n)/
n + 2*a*d*e*x**r/r + a*e**2*x**(2*r)/(2*r) + b*d**2*log(c*x**n)**2/(2*n) -
2*b*d*e*n*x**r/r**2 + 2*b*d*e*x**r*log(c*x**n)/r - b*e**2*n*x**(2*r)/(4*r**
2) + b*e**2*x**(2*r)*log(c*x**n)/(2*r), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x}\,dx = \frac{be^2x^{2r}\log(cx^n)}{2r} + \frac{2\,bdex^r\log(cx^n)}{r} + \frac{bd^2\log(cx^n)^2}{2n} + ad^2\log(x) - \frac{be^2nx^{2r}}{4r^2} + \frac{ae^2x^{2r}}{2r} - \frac{2\,bdenx^r}{r^2} + \frac{2\,adex^r}{r}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*e^2*x^(2*r)*log(c*x^n)/r + 2*b*d*e*x^r*log(c*x^n)/r + 1/2*b*d^2*log(c
*x^n)^2/n + a*d^2*log(x) - 1/4*b*e^2*n*x^(2*r)/r^2 + 1/2*a*e^2*x^(2*r)/r -
2*b*d*e*n*x^r/r^2 + 2*a*d*e*x^r/r
```

Giac [F]

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x}\,dx = \int \frac{(ex^r+d)^2(b\log(cx^n)+a)}{x}\,dx$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^r + d)^2*(b*log(c*x^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))}{x}\,dx = \int \frac{(d+e\,x^r)^2\,(a+b\,\ln(c\,x^n))}{x}\,dx$$

```
[In] int(((d + e*x^r)^2*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int(((d + e*x^r)^2*(a + b*log(c*x^n)))/x, x)
```

3.423 $\int \frac{(d+ex^r)(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{(d+ex^r)\,(a+b\log(cx^n))}{x}\,dx=-\frac{benx^r}{r^2}+\frac{ex^r(a+b\log(cx^n))}{r}+\frac{d(a+b\log(cx^n))^2}{2bn}$$

```
[Out] -b*e*n*x^r/r^2+e*x^r*(a+b*ln(c*x^n))/r+1/2*d*(a+b*ln(c*x^n))^2/b/n
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {14, 2393, 2338, 2341}

$$\int \frac{(d+ex^r)\,(a+b\log(cx^n))}{x}\,dx=\frac{d(a+b\log(cx^n))^2}{2bn}+\frac{ex^r(a+b\log(cx^n))}{r}-\frac{benx^r}{r^2}$$

```
[In] Int[((d + e*x^r)*(a + b*Log[c*x^n]))/x,x]
[Out] -((b*e*n*x^r)/r^2) + (e*x^r*(a + b*Log[c*x^n]))/r + (d*(a + b*Log[c*x^n])^2
)/(2*b*n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d(a+b\log(cx^n))}{x}+ex^{-1+r}(a+b\log(cx^n))\right)dx\\
&= d\int \frac{a+b\log(cx^n)}{x}\,dx+e\int x^{-1+r}(a+b\log(cx^n))\,dx\\
&= -\frac{benx^r}{r^2}+\frac{ex^r(a+b\log(cx^n))}{r}+\frac{d(a+b\log(cx^n))^2}{2bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x}\,dx = \frac{e(-bn+ar)x^r}{r^2}+ad\log(x)+\frac{bex^r\log(cx^n)}{r}+\frac{bd\log^2(cx^n)}{2n}$$

```
[In] Integrate[((d + e*x^r)*(a + b*Log[c*x^n]))/x,x]
[Out] (e*(-(b*n) + a*r)*x^r)/r^2 + a*d*Log[x] + (b*e*x^r*Log[c*x^n])/r + (b*d*Log
[c*x^n]^2)/(2*n)
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

method	result
parallelrisch	$\frac{2\ln(x)adn\,r^2+2x^r\ln(c\,x^n)bern+bd\ln(c\,x^n)^2r^2+2x^r aenr-2x^r be\,n^2}{2r^2n}$
risch	$\frac{b(dr\ln(x)+e\,x^r)\ln(x^n)}{r}-\frac{i\ln(x)\pi bd\,\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)}{2}+\frac{i\ln(x)\pi bd\,\operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^n)^2}{2}+\frac{i\ln(x)\pi bd\,\operatorname{csgn}(ix}{2}$

```
[In] int((d+e*x^r)*(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*ln(x)*a*d*n*r^2+2*x^r*ln(c*x^n)*b*e*r*n+b*d*ln(c*x^n)^2*r^2+2*x^r*a*
e*n*r-2*x^r*b*e*n^2)/r^2/n
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x}\,dx = \frac{bdnr^2\log(x)^2+2\,(benr\log(x)+ber\log(c)-ben+aer)x^r+2\,(bdr^2\log(c)+adr^2)\log(x)}{2\,r^2}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*n*r^2*log(x)^2 + 2*(b*e*n*r*log(x) + b*e*r*log(c) - b*e*n + a*e*r)
*x^r + 2*(b*d*r^2*log(c) + a*d*r^2)*log(x))/r^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(46) = 92.

Time = 2.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.47

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x}\,dx = \begin{cases} (a+b\log(c))\,(d+e)\log(x) & \text{for } n=0 \wedge r=0 \\ (a+b\log(c))\left(d\log(x)+\frac{ex^r}{r}\right) & \text{for } n=0 \\ (d+e)\left(\begin{cases} a\log(x) & \text{for } b=0 \\ -(-a-b\log(c))\log(x) & \text{for } n=0 \\ \frac{(-a-b\log(cx^n))^2}{2bn} & \text{otherwise} \end{cases}\right) & \text{for } r=0 \\ \frac{ad\log(cx^n)}{n}+\frac{aex^r}{r}+\frac{bd\log(cx^n)^2}{2n}-\frac{benx^r}{r^2}+\frac{bex^r\log(cx^n)}{r} & \text{otherwise} \end{cases}$$

```
[In] integrate((d+e*x**r)*(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise(((a + b*log(c))*(d + e)*log(x), Eq(n, 0) & Eq(r, 0)), ((a + b*log
(c))*(d*log(x) + e*x**r/r), Eq(n, 0)), ((d + e)*Piecewise((a*log(x), Eq(b,
0)), (-(-a - b*log(c))*log(x), Eq(n, 0)), ((-a - b*log(c*x**n))**2/(2*b*n),
 True)), Eq(r, 0)), (a*d*log(c*x**n)/n + a*e*x**r/r + b*d*log(c*x**n)**2/(2
*n) - b*e*n*x**r/r**2 + b*e*x**r*log(c*x**n)/r, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x}\,dx = \frac{bex^r\log(cx^n)}{r} + \frac{bd\log(cx^n)^2}{2n} + ad\log(x) - \frac{benx^r}{r^2} + \frac{aex^r}{r}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x,x, algorithm="maxima")
[Out] b*e*x^r*log(c*x^n)/r + 1/2*b*d*log(c*x^n)^2/n + a*d*log(x) - b*e*n*x^r/r^2
+ a*e*x^r/r
```

Giac [F]

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x}\,dx = \int \frac{(ex^r+d)(b\log(cx^n)+a)}{x}\,dx$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))/x,x, algorithm="giac")
[Out] integrate((e*x^r + d)*(b*log(c*x^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)(a+b\log(cx^n))}{x}\,dx = \int \frac{(d+e\,x^r)\,(a+b\,\ln(c\,x^n))}{x}\,dx$$

```
[In] int(((d + e*x^r)*(a + b*log(c*x^n)))/x,x)
[Out] int(((d + e*x^r)*(a + b*log(c*x^n)))/x, x)
```

3.424 $\int \frac{a+b\log(cx^n)}{x(d+ex^r)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)}\,dx=-\frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{dr}+\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)}{dr^2}$$

```
[Out] -(a+b*ln(c*x^n))*ln(1+d/e/(x^r))/d/r+b*n*polylog(2,-d/e/(x^r))/d/r^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2379, 2438}

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)}\,dx=\frac{bn\,\mathrm{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)}{dr^2}-\frac{\log\left(\frac{dx^{-r}}{e}+1\right)(a+b\log(cx^n))}{dr}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x^r)),x]
```

```
[Out] -(((a + b*Log[c*x^n])*Log[1 + d/(e*x^r)])/(d*r)) + (b*n*PolyLog[2, -(d/(e*x
^r))])/(d*r^2)
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{dr}+\frac{(bn)\int\frac{\log\left(1+\frac{dx^{-r}}{e}\right)}{x}\,dx}{dr}\\ &= -\frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{dr}+\frac{bn\text{Li}_2\left(-\frac{dx^{-r}}{e}\right)}{dr^2}\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.00

$$\int\frac{a+b\log(cx^n)}{x(d+ex^r)}\,dx$$
$$=\frac{bnr^2\log^2(x)-2r(a+b\log(cx^n))\log(d-dx^r)+2bnr\log(x)(\log(d-dx^r)-\log(d+ex^r))+2bn\log\left(-\frac{e}{}\right.}{2dr^2}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x^r)),x]
```

```
[Out] (b*n*r^2*Log[x]^2 - 2*r*(a + b*Log[c*x^n])*Log[d - d*x^r] + 2*b*n*r*Log[x]*
(Log[d - d*x^r] - Log[d + e*x^r]) + 2*b*n*Log[-((e*x^r)/d)]*Log[d + e*x^r]
+ 2*b*n*PolyLog[2, 1 + (e*x^r)/d])/(2*d*r^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.00 (sec) , antiderivative size = 246, normalized size of antiderivative = 4.56

method	result
risch	$\frac{b\ln(d+e\,x^r)n\ln(x)}{rd}-\frac{b\ln(d+e\,x^r)\ln(x^n)}{rd}-\frac{b\ln(x^r)n\ln(x)}{rd}+\frac{b\ln(x^r)\ln(x^n)}{rd}+\frac{bn\ln(x)^2}{2d}-\frac{bn\ln(x)\ln\left(1+\frac{e\,x^r}{d}\right)}{rd}-\frac{bn\,\text{Li}_2\left(-\right.}{r^2}$

```
[In] int((a+b*ln(c*x^n))/x/(d+e*x^r),x,method=_RETURNVERBOSE)
```

```
[Out] b/r/d*ln(d+e*x^r)*n*ln(x)-b/r/d*ln(d+e*x^r)*ln(x^n)-b/r/d*ln(x^r)*n*ln(x)+b
/r/d*ln(x^r)*ln(x^n)+1/2*b*n/d*ln(x)^2-b/r*n/d*ln(x)*ln(1+e*x^r/d)-b/r^2*n/
d*polylog(2,-e*x^r/d)+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*
I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2
*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*(-1/r/d*ln(d+e*x^r)+1/r/d*ln(x^r))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.72

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)}\,dx = \frac{bnr^2\log(x)^2 - 2\,bnr\log(x)\log\left(\frac{ex^r+d}{d}\right) - 2\,bn\mathrm{Li}_2\left(-\frac{ex^r+d}{d}+1\right) - 2\,(br\log(c)+ar)\log(ex^r+d) + 2\,(br^2}{2\,dr^2}$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r),x, algorithm="fricas")
```

```
[Out] 1/2*(b*n*r^2*log(x)^2 - 2*b*n*r*log(x)*log((e*x^r + d)/d) - 2*b*n*dilog(-(e
*x^r + d)/d + 1) - 2*(b*r*log(c) + a*r)*log(e*x^r + d) + 2*(b*r^2*log(c) +
a*r^2)*log(x))/(d*r^2)
```

Sympy [A] (verification not implemented)

Time = 148.70 (sec) , antiderivative size = 452, normalized size of antiderivative = 8.37

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*ln(c*x**n))/x/(d+e*x**r),x)
```

```
[Out] -2*a*e*Piecewise((1/(2*e) + x**r/d, Eq(e, 0)), (-log(-2*e*x**r)/(2*e), True
))/(d*r) - 2*a*e*Piecewise((1/(2*e) + x**r/d, Eq(e, 0)), (log(2*d + 2*e*x**
r)/(2*e), True))/(d*r) + 2*b*e*n*Piecewise((Piecewise((log(x)/(2*e) + x**r/
(d*r), Ne(r, 0)), (log(x)/(2*e) + log(x)/d, True)), Eq(e, 0)), (-Piecewise(
(0, (Abs(e*x**r) < 1) & (1/Abs(e*x**r) < 1)), (log(e*x**r)**2/(2*r) + I*pi*
log(e*x**r)/r, Abs(e*x**r) < 1), (log(1/(e*x**r))**2/(2*r) - I*pi*log(1/(e*
x**r))/r, 1/Abs(e*x**r) < 1), (meijerg(((), (1, 1, 1)), ((0, 0, 0), ()), e*
x**r*exp_polar(I*pi))/r + meijerg(((1, 1, 1), ()), ((), (0, 0, 0)), e*x**r*
exp_polar(I*pi))/r, True))/(2*e) - log(2)*log(x)/(2*e), True))/(d*r) + 2*b*
e*n*Piecewise((Piecewise((log(x)/(2*e) + x**r/(d*r), Ne(r, 0)), (log(x)/(2*
e) + log(x)/d, True)), Eq(e, 0)), (Piecewise((-polylog(2, e*x**r*exp_polar(
I*pi)/d)/r, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x
**r*exp_polar(I*pi)/d)/r, Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x**
r*exp_polar(I*pi)/d)/r, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ())
, x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*
x**r*exp_polar(I*pi)/d)/r, True))/(2*e) + log(2)*log(x)/(2*e), True))/(d*r)
 - 2*b*e*Piecewise((1/(2*e) + x**r/d, Eq(e, 0)), (-log(-2*e*x**r)/(2*e), Tr
ue))*log(c*x**n)/(d*r) - 2*b*e*Piecewise((1/(2*e) + x**r/d, Eq(e, 0)), (log
(2*d + 2*e*x**r)/(2*e), True))*log(c*x**n)/(d*r)
```

Maxima [F]

$$\int \frac{a + b \log (cx^n)}{x (d + ex^r)} \, dx = \int \frac{b \log (cx^n) + a}{(ex^r + d)x} \, dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r),x, algorithm="maxima")
```

```
[Out] a*(log(x)/d - log((e*x^r + d)/e)/(d*r)) + b*integrate((log(c) + log(x^n))/(
e*x*x^r + d*x), x)
```

Giac [F]

$$\int \frac{a + b \log (cx^n)}{x (d + ex^r)} \, dx = \int \frac{b \log (cx^n) + a}{(ex^r + d)x} \, dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x^r + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \log (cx^n)}{x (d + ex^r)} \, dx = \int \frac{a + b \ln (c\, x^n)}{x \, (d + e\, x^r)} \, dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x^r)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x*(d + e*x^r)), x)
```

3.425 $\int \frac{a+b\log(cx^n)}{x(d+ex^r)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^2}\,dx = -\frac{ex^r(a+b\log(cx^n))}{d^2r(d+ex^r)} - \frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{d^2r} + \frac{bn\log(d+ex^r)}{d^2r^2} + \frac{bn\,\text{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)}{d^2r^2}$$

```
[Out] -e*x^r*(a+b*ln(c*x^n))/d^2/r/(d+e*x^r)-(a+b*ln(c*x^n))*ln(1+d/e/(x^r))/d^2/
r+b*n*ln(d+e*x^r)/d^2/r^2+b*n*polylog(2,-d/e/(x^r))/d^2/r^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2391, 2379, 2438, 2373, 266}

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^2}\,dx = -\frac{\log\left(\frac{dx^{-r}}{e}+1\right)(a+b\log(cx^n))}{d^2r} - \frac{ex^r(a+b\log(cx^n))}{d^2r(d+ex^r)} + \frac{bn\,\text{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)}{d^2r^2} + \frac{bn\log(d+ex^r)}{d^2r^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x^r)^2),x]
```

```
[Out] -((e*x^r*(a + b*Log[c*x^n]))/(d^2*r*(d + e*x^r))) - ((a + b*Log[c*x^n])*Log
[1 + d/(e*x^r)])/(d^2*r) + (b*n*Log[d + e*x^r])/(d^2*r^2) + (b*n*PolyLog[2,
 -(d/(e*x^r))])/(d^2*r^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^
(q_))/(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x^r)^(q + 1)*((a + b*Log[c*x^
n])^p/x), x], x] - Dist[e/d, Int[x^(r - 1)*(d + e*x^r)^q*(a + b*Log[c*x^n])
^p, x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0] && ILtQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{a+b\log(cx^n)}{x(d+ex^r)}\,dx}{d} - \frac{e\int \frac{x^{-1+r}(a+b\log(cx^n))}{(d+ex^r)^2}\,dx}{d}\\
&= -\frac{ex^r(a+b\log(cx^n))}{d^2r(d+ex^r)} - \frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{d^2r}\\
&\quad + \frac{(bn)\int \frac{\log\left(1+\frac{dx^{-r}}{e}\right)}{x}\,dx}{d^2r} + \frac{(ben)\int \frac{x^{-1+r}}{d+ex^r}\,dx}{d^2r}\\
&= -\frac{ex^r(a+b\log(cx^n))}{d^2r(d+ex^r)} - \frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{d^2r} + \frac{bn\log(d+ex^r)}{d^2r^2} + \frac{bn\text{Li}_2\left(-\frac{dx^{-r}}{e}\right)}{d^2r^2}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.29

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^2}\,dx$$
$$= \frac{\frac{dr(a+b\log(cx^n))}{d+ex^r} + bn\log(d-dx^r) - ar\log(d-dx^r) + br(n\log(x)-\log(cx^n))\log(d-dx^r) + bn\left(\frac{1}{2}r^2\log^2(\right.}{d^2r^2}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x^r)^2),x]
```

```
[Out] ((d*r*(a + b*Log[c*x^n]))/(d + e*x^r) + b*n*Log[d - d*x^r] - a*r*Log[d - d*
x^r] + b*r*(n*Log[x] - Log[c*x^n])*Log[d - d*x^r] + b*n*((r^2*Log[x]^2)/2 +
 (-(r*Log[x]) + Log[-((e*x^r)/d)])*Log[d + e*x^r] + PolyLog[2, 1 + (e*x^r)/
d]))/(d^2*r^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 342, normalized size of antiderivative = 3.35

method	result
risch	$\frac{b\ln(d+e\,x^r)n\ln(x)}{r\,d^2} - \frac{b\ln(d+e\,x^r)\ln(x^n)}{r\,d^2} - \frac{bn\ln(x)}{rd(d+e\,x^r)} + \frac{b\ln(x^n)}{rd(d+e\,x^r)} - \frac{b\ln(x^r)n\ln(x)}{r\,d^2} + \frac{b\ln(x^r)\ln(x^n)}{r\,d^2} + \frac{bn\ln(d+e\,x^r)}{d^2r^2}$

```
[In] int((a+b*ln(c*x^n))/x/(d+e*x^r)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b/r/d^2*ln(d+e*x^r)*n*ln(x)-b/r/d^2*ln(d+e*x^r)*ln(x^n)-b/r/d/(d+e*x^r)*n*l
n(x)+b/r/d/(d+e*x^r)*ln(x^n)-b/r/d^2*ln(x^r)*n*ln(x)+b/r/d^2*ln(x^r)*ln(x^n
)+b*n*ln(d+e*x^r)/d^2/r^2-b/r*n*e/d^2*ln(x)*x^r/(d+e*x^r)-b/r^2*n/d^2*dilog
((d+e*x^r)/d)-b/r*n/d^2*ln(x)*ln((d+e*x^r)/d)+1/2*b*n/d^2*ln(x)^2+(-1/2*I*b
*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^
2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*csgn(I*c*x^n)^3+b*ln(c)
+a)/r*(-1/d^2*ln(d+e*x^r)+1/d/(d+e*x^r)+1/d^2*ln(x^r))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(101) = 202.

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.10

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^2}\,dx$$
$$= \frac{bdnr^2\log(x)^2 + 2\,bdr\log(c) + 2\,adr + \left(benr^2\log(x)^2 + 2\,(ber^2\log(c) - benr + aer^2)\log(x)\right)x^r - 2\,(benx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*n*r^2*log(x)^2 + 2*b*d*r*log(c) + 2*a*d*r + (b*e*n*r^2*log(x)^2 +
2*(b*e*r^2*log(c) - b*e*n*r + a*e*r^2)*log(x))*x^r - 2*(b*e*n*x^r + b*d*n)*
dilog(-(e*x^r + d)/d + 1) - 2*(b*d*r*log(c) - b*d*n + a*d*r + (b*e*r*log(c)
 - b*e*n + a*e*r)*x^r)*log(e*x^r + d) + 2*(b*d*r^2*log(c) + a*d*r^2)*log(x)
 - 2*(b*e*n*r*x^r*log(x) + b*d*n*r*log(x))*log((e*x^r + d)/d))/(d^2*e*r^2*x
^r + d^3*r^2)
```

Sympy [A] (verification not implemented)

Time = 145.56 (sec) , antiderivative size = 360, normalized size of antiderivative = 3.53

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^2}\,dx = -\frac{ae\left(\begin{cases}\frac{x^r}{d^2} & \text{for}\ e=0\\ -\frac{1}{de+e^2x^r} & \text{otherwise}\end{cases}\right)}{dr} - \frac{ae\left(\begin{cases}\frac{x^r}{d} & \text{for}\ e=0\\ \frac{\log(d+ex^r)}{e} & \text{otherwise}\end{cases}\right)}{d^2r} + \frac{a\log(x^r)}{d^2r} + \frac{ben\left(\begin{cases}\frac{\begin{cases}\frac{x^r}{r} & \text{for}\ r\neq 0\\ \log(x) & \text{otherwise}\end{cases}}{d^2} & \text{for}\ e=0\\ -\begin{cases}\frac{\log(x)}{e^2} & \text{for}\ d=0\wedge r=0\\ -\frac{x^{-r}}{e^2r} & \text{for}\ d=0\\ \frac{\log(x)}{de+e^2} & \text{for}\ r=0\\ \frac{\log(x)}{de}-\frac{\log\left(\frac{d}{e}+x^r\right)}{der} & \text{otherwise}\end{cases} & \text{otherwise}\end{cases}\right)}{dr} - \frac{be\left(\begin{cases}\frac{x^r}{d^2} & \text{for}\ e=0\\ -\frac{1}{de+e^2x^r} & \text{otherwise}\end{cases}\right)\log(cx^n)}{dr} + \frac{ben\left(\begin{cases}\frac{\begin{cases}\frac{x^r}{r} & \text{for}\ r\neq 0\\ \log(x) & \text{otherwise}\end{cases}}{d} & \text{for}\ e=0\\ \frac{\begin{cases}-\frac{\operatorname{Li}_2\left(\frac{ex^re^{i\pi}}{d}\right)}{r} & \text{for}\ \frac{1}{|x|}<1\wedge|x|<1\\ \log(d)\log(x)-\frac{\operatorname{Li}_2\left(\frac{ex^re^{i\pi}}{d}\right)}{r} & \text{for}\ |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right)-\frac{\operatorname{Li}_2\left(\frac{ex^re^{i\pi}}{d}\right)}{r} & \text{for}\ \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log(d)-\frac{\operatorname{Li}_2\left(\frac{ex^re^{i\pi}}{d}\right)}{r} & \text{otherwise}\end{cases}}{e} & \text{otherwise}\end{cases}\right)}{d^2r} - \frac{be\left(\begin{cases}\frac{x^r}{d} & \text{for}\ e=0\\ \frac{\log(d+ex^r)}{e} & \text{otherwise}\end{cases}\right)\log(cx^n)}{d^2r} + \frac{bn\left(\begin{cases}0 & \text{for}\ r=0\\ -\frac{\log(x^r)^2}{2r} & \text{otherwise}\end{cases}\right)}{d^2r} + \frac{b\log(x^r)\log(cx^n)}{d^2r}$$

```
[In] integrate((a+b*ln(c*x**n))/x/(d+e*x**r)**2,x)
```

```
[Out] -a*e*Piecewise((x**r/d**2, Eq(e, 0)), (-1/(d*e + e**2*x**r), True))/(d*r) -
 a*e*Piecewise((x**r/d, Eq(e, 0)), (log(d + e*x**r)/e, True))/(d**2*r) + a*
log(x**r)/(d**2*r) + b*e*n*Piecewise((Piecewise((x**r/r, Ne(r, 0)), (log(x)
, True))/d**2, Eq(e, 0)), (-Piecewise((log(x)/e**2, Eq(d, 0) & Eq(r, 0)), (
-1/(e**2*r*x**r), Eq(d, 0)), (log(x)/(d*e + e**2), Eq(r, 0)), (log(x)/(d*e)
 - log(d/e + x**r)/(d*e*r), True)), True))/(d*r) - b*e*Piecewise((x**r/d**2
, Eq(e, 0)), (-1/(d*e + e**2*x**r), True))*log(c*x**n)/(d*r) + b*e*n*Piecew
ise((Piecewise((x**r/r, Ne(r, 0)), (log(x), True))/d, Eq(e, 0)), (Piecewise
((-polylog(2, e*x**r*exp_polar(I*pi)/d)/r, (Abs(x) < 1) & (1/Abs(x) < 1)),
(log(d)*log(x) - polylog(2, e*x**r*exp_polar(I*pi)/d)/r, Abs(x) < 1), (-log
(d)*log(1/x) - polylog(2, e*x**r*exp_polar(I*pi)/d)/r, 1/Abs(x) < 1), (-mei
jerg(((), (1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0,
 0)), x)*log(d) - polylog(2, e*x**r*exp_polar(I*pi)/d)/r, True))/e, True))/
(d**2*r) - b*e*Piecewise((x**r/d, Eq(e, 0)), (log(d + e*x**r)/e, True))*log
(c*x**n)/(d**2*r) + b*n*Piecewise((0, Eq(r, 0)), (-log(x**r)**2/(2*r), True
))/(d**2*r) + b*log(x**r)*log(c*x**n)/(d**2*r)
```

Maxima [F]

$$\int \frac{a + b\log\left(cx^n\right)}{x\left(d + ex^r\right)^2}\, dx = \int \frac{b\log\left(cx^n\right) + a}{\left(ex^r + d\right)^2 x}\, dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^2,x, algorithm="maxima")
```

```
[Out] a*(1/(d*e*r*x^r + d^2*r) + log(x)/d^2 - log((e*x^r + d)/e)/(d^2*r)) + b*int
egrate((log(c) + log(x^n))/(e^2*x*x^(2*r) + 2*d*e*x*x^r + d^2*x), x)
```

Giac [F]

$$\int \frac{a + b\log\left(cx^n\right)}{x\left(d + ex^r\right)^2}\, dx = \int \frac{b\log\left(cx^n\right) + a}{\left(ex^r + d\right)^2 x}\, dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x^r + d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^2}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{x\,(d+e\,x^r)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x^r)^2),x)
[Out] int((a + b*log(c*x^n))/(x*(d + e*x^r)^2), x)
```

3.426 $\int \frac{a+b\log(cx^n)}{x(d+ex^r)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 169

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^3}\,dx = -\frac{bn}{2d^2r^2(d+ex^r)} - \frac{bn\log(x)}{2d^3r} + \frac{a+b\log(cx^n)}{2dr(d+ex^r)^2} - \frac{ex^r(a+b\log(cx^n))}{d^3r(d+ex^r)} - \frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{d^3r} + \frac{3bn\log(d+ex^r)}{2d^3r^2} + \frac{bn\,\mathrm{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)}{d^3r^2}$$

```
[Out] -1/2*b*n/d^2/r^2/(d+e*x^r)-1/2*b*n*ln(x)/d^3/r+1/2*(a+b*ln(c*x^n))/d/r/(d+e
*x^r)^2-e*x^r*(a+b*ln(c*x^n))/d^3/r/(d+e*x^r)-(a+b*ln(c*x^n))*ln(1+d/e/(x^r
))/d^3/r+3/2*b*n*ln(d+e*x^r)/d^3/r^2+b*n*polylog(2,-d/e/(x^r))/d^3/r^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2391, 2379, 2438, 2373, 266, 2376, 272, 46}

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^3}\,dx = -\frac{\log\left(\frac{dx^{-r}}{e}+1\right)(a+b\log(cx^n))}{d^3r} - \frac{ex^r(a+b\log(cx^n))}{d^3r(d+ex^r)} + \frac{a+b\log(cx^n)}{2dr(d+ex^r)^2} + \frac{bn\,\mathrm{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)}{d^3r^2} + \frac{3bn\log(d+ex^r)}{2d^3r^2} - \frac{bn\log(x)}{2d^3r} - \frac{bn}{2d^2r^2(d+ex^r)}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x^r)^3),x]
```

```
[Out] -1/2*(b*n)/(d^2*r^2*(d + e*x^r)) - (b*n*Log[x])/(2*d^3*r) + (a + b*Log[c*x^
n])/(2*d*r*(d + e*x^r)^2) - (e*x^r*(a + b*Log[c*x^n]))/(d^3*r*(d + e*x^r))
- ((a + b*Log[c*x^n])*Log[1 + d/(e*x^r)])/(d^3*r) + (3*b*n*Log[d + e*x^r])/
(2*d^3*r^2) + (b*n*PolyLog[2, -(d/(e*x^r))])/(d^3*r^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
```

```
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^
(q_))/(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x^r)^(q + 1)*((a + b*Log[c*x^
n])^p/x), x], x] - Dist[e/d, Int[x^(r - 1)*(d + e*x^r)^q*(a + b*Log[c*x^n])
^p, x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0] && ILtQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{a+b\log(cx^n)}{x(d+ex^r)^2}\,dx}{d} - \frac{e\int \frac{x^{-1+r}(a+b\log(cx^n))}{(d+ex^r)^3}\,dx}{d}\\
&= \frac{a+b\log(cx^n)}{2dr(d+ex^r)^2} + \frac{\int \frac{a+b\log(cx^n)}{x(d+ex^r)}\,dx}{d^2} - \frac{e\int \frac{x^{-1+r}(a+b\log(cx^n))}{(d+ex^r)^2}\,dx}{d^2} - \frac{(bn)\int \frac{1}{x(d+ex^r)^2}\,dx}{2dr}\\
&= \frac{a+b\log(cx^n)}{2dr(d+ex^r)^2} - \frac{ex^r(a+b\log(cx^n))}{d^3r(d+ex^r)} - \frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{d^3r}\\
&\quad - \frac{(bn)\text{Subst}\left(\int \frac{1}{x(d+ex)^2}\,dx, x, x^r\right)}{2dr^2} + \frac{(bn)\int \frac{\log\left(1+\frac{dx^{-r}}{e}\right)}{x}\,dx}{d^3r} + \frac{(ben)\int \frac{x^{-1+r}}{d+ex^r}\,dx}{d^3r}\\
&= \frac{a+b\log(cx^n)}{2dr(d+ex^r)^2} - \frac{ex^r(a+b\log(cx^n))}{d^3r(d+ex^r)} - \frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{d^3r}\\
&\quad + \frac{bn\log(d+ex^r)}{d^3r^2} + \frac{bn\text{Li}_2\left(-\frac{dx^{-r}}{e}\right)}{d^3r^2} - \frac{(bn)\text{Subst}\left(\int \left(\frac{1}{d^2x} - \frac{e}{d(d+ex)^2} - \frac{e}{d^2(d+ex)}\right)dx, x, x^r\right)}{2dr^2}\\
&= -\frac{bn}{2d^2r^2(d+ex^r)} - \frac{bn\log(x)}{2d^3r} + \frac{a+b\log(cx^n)}{2dr(d+ex^r)^2} - \frac{ex^r(a+b\log(cx^n))}{d^3r(d+ex^r)}\\
&\quad - \frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{d^3r} + \frac{3bn\log(d+ex^r)}{2d^3r^2} + \frac{bn\text{Li}_2\left(-\frac{dx^{-r}}{e}\right)}{d^3r^2}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 0.16 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.01

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^3}\,dx = \frac{\frac{d^2r(a+b\log(cx^n))}{(d+ex^r)^2} + \frac{d(-bn+2ar+2br\log(cx^n))}{d+ex^r} + 3bn\log(d-dx^r) - 2ar\log(d-dx^r) + 2br(n\log(x)-\log(cx^n)) \cdots}{2d^3r^2}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x^r)^3),x]
```

```
[Out] ((d^2*r*(a + b*Log[c*x^n]))/(d + e*x^r)^2 + (d*(-(b*n) + 2*a*r + 2*b*r*Log[
c*x^n]))/(d + e*x^r) + 3*b*n*Log[d - d*x^r] - 2*a*r*Log[d - d*x^r] + 2*b*r*
(n*Log[x] - Log[c*x^n])*Log[d - d*x^r] + 2*b*n*((r^2*Log[x]^2)/2 + (-(r*Log
[x]) + Log[-((e*x^r)/d)])*Log[d + e*x^r] + PolyLog[2, 1 + (e*x^r)/d]))/(2*d
^3*r^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.00 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.80

method	result
risch	$\frac{b\ln(d+e\,x^r)n\ln(x)}{r\,d^3} - \frac{b\ln(d+e\,x^r)\ln(x^n)}{r\,d^3} - \frac{bn\ln(x)}{r\,d^2(d+e\,x^r)} + \frac{b\ln(x^n)}{r\,d^2(d+e\,x^r)} - \frac{bn\ln(x)}{2rd(d+e\,x^r)^2} + \frac{b\ln(x^n)}{2rd(d+e\,x^r)^2} - \frac{b\ln(x^r)n\ln(\cdots}{r\,d^3}$ …

```
[In] int((a+b*ln(c*x^n))/x/(d+e*x^r)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b/r/d^3*ln(d+e*x^r)*n*ln(x)-b/r/d^3*ln(d+e*x^r)*ln(x^n)-b/r/d^2/(d+e*x^r)*n
*ln(x)+b/r/d^2/(d+e*x^r)*ln(x^n)-1/2*b/r/d/(d+e*x^r)^2*n*ln(x)+1/2*b/r/d/(d
+e*x^r)^2*ln(x^n)-b/r/d^3*ln(x^r)*n*ln(x)+b/r/d^3*ln(x^r)*ln(x^n)+3/2*b*n*l
n(d+e*x^r)/d^3/r^2-b/r*n*e/d^3*ln(x)*x^r/(d+e*x^r)-b/r^2*n/d^3*dilog((d+e*x
^r)/d)-b/r*n/d^3*ln(x)*ln((d+e*x^r)/d)-1/2*b*n/d^2/r^2/(d+e*x^r)-1/2*b/r*n*
e^2/d^3*ln(x)*(x^r)^2/(d+e*x^r)^2-b/r*n*e/d^2*ln(x)*x^r/(d+e*x^r)^2+1/2*b*n
/d^3*ln(x)^2+(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*Pi*cs
gn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b*Pi*c
sgn(I*c*x^n)^3+b*ln(c)+a)/r*(-1/d^3*ln(d+e*x^r)+1/d^2/(d+e*x^r)+1/2/d/(d+e*
x^r)^2+1/d^3*ln(x^r))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(160) = 320$.

Time = 0.28 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.37

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^3}\,dx = \frac{bd^2nr^2\log(x)^2+3\,bd^2r\log(c)-bd^2n+3\,ad^2r+\left(be^2nr^2\log(x)^2+(2\,be^2r^2\log(c)-3\,be^2nr+2\,ae^2r^2)\log\right.}{}$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d^2*n*r^2*log(x)^2 + 3*b*d^2*r*log(c) - b*d^2*n + 3*a*d^2*r + (b*e^2
*n*r^2*log(x)^2 + (2*b*e^2*r^2*log(c) - 3*b*e^2*n*r + 2*a*e^2*r^2)*log(x))*
x^(2*r) + (2*b*d*e*n*r^2*log(x)^2 + 2*b*d*e*r*log(c) - b*d*e*n + 2*a*d*e*r
+ 4*(b*d*e*r^2*log(c) - b*d*e*n*r + a*d*e*r^2)*log(x))*x^r - 2*(b*e^2*n*x^(
2*r) + 2*b*d*e*n*x^r + b*d^2*n)*dilog(-(e*x^r + d)/d + 1) - (2*b*d^2*r*log(
c) - 3*b*d^2*n + 2*a*d^2*r + (2*b*e^2*r*log(c) - 3*b*e^2*n + 2*a*e^2*r)*x^(
2*r) + 2*(2*b*d*e*r*log(c) - 3*b*d*e*n + 2*a*d*e*r)*x^r)*log(e*x^r + d) + 2
*(b*d^2*r^2*log(c) + a*d^2*r^2)*log(x) - 2*(b*e^2*n*r*x^(2*r)*log(x) + 2*b*
d*e*n*r*x^r*log(x) + b*d^2*n*r*log(x))*log((e*x^r + d)/d))/(d^3*e^2*r^2*x^(
2*r) + 2*d^4*e*r^2*x^r + d^5*r^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*x**n))/x/(d+e*x**r)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^3}\,dx = \int \frac{b\log(cx^n)+a}{(ex^r+d)^3x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*((2*e*x^r + 3*d)/(d^2*e^2*r*x^(2*r) + 2*d^3*e*r*x^r + d^4*r) + 2*log(
x)/d^3 - 2*log((e*x^r + d)/e)/(d^3*r)) + b*integrate((log(c) + log(x^n))/(e
^3*x*x^(3*r) + 3*d*e^2*x*x^(2*r) + 3*d^2*e*x*x^r + d^3*x), x)
```

Giac [F]

$$\int \frac{a + b\log(cx^n)}{x(d + ex^r)^3}\,dx = \int \frac{b\log(cx^n) + a}{(ex^r + d)^3 x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*x^r + d)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{x(d + ex^r)^3}\,dx = \int \frac{a + b\,\ln(c\,x^n)}{x\,(d + e\,x^r)^3}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x^r)^3),x)
[Out] int((a + b*log(c*x^n))/(x*(d + e*x^r)^3), x)
```

3.427 $\int \frac{(d+ex^r)^3(a+b\log(cx^n))^2}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 245

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))^2}{x}\,dx = \frac{6b^2d^2en^2x^r}{r^3}+\frac{3b^2de^2n^2x^{2r}}{4r^3}+\frac{2b^2e^3n^2x^{3r}}{27r^3} - \frac{6bd^2enx^r(a+b\log(cx^n))}{r^2} - \frac{3bde^2nx^{2r}(a+b\log(cx^n))}{2r^2} - \frac{2be^3nx^{3r}(a+b\log(cx^n))}{9r^2} + \frac{3d^2ex^r(a+b\log(cx^n))^2}{r}+\frac{3de^2x^{2r}(a+b\log(cx^n))^2}{2r} + \frac{e^3x^{3r}(a+b\log(cx^n))^2}{3r}+\frac{d^3(a+b\log(cx^n))^3}{3bn}$$

```
[Out] 6*b^2*d^2*e*n^2*x^r/r^3+3/4*b^2*d*e^2*n^2*x^(2*r)/r^3+2/27*b^2*e^3*n^2*x^(3
*r)/r^3-6*b*d^2*e*n*x^r*(a+b*ln(c*x^n))/r^2-3/2*b*d*e^2*n*x^(2*r)*(a+b*ln(c
*x^n))/r^2-2/9*b*e^3*n*x^(3*r)*(a+b*ln(c*x^n))/r^2+3*d^2*e*x^r*(a+b*ln(c*x^
n))^2/r+3/2*d*e^2*x^(2*r)*(a+b*ln(c*x^n))^2/r+1/3*e^3*x^(3*r)*(a+b*ln(c*x^n
))^2/r+1/3*d^3*(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2395, 2339, 30, 2342, 2341}

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))^2}{x}\,dx = \frac{d^3(a+b\log(cx^n))^3}{3bn} - \frac{6bd^2enx^r(a+b\log(cx^n))}{r^2} + \frac{3d^2ex^r(a+b\log(cx^n))^2}{r} - \frac{3bde^2nx^{2r}(a+b\log(cx^n))}{2r^2} + \frac{3de^2x^{2r}(a+b\log(cx^n))^2}{2r} - \frac{2be^3nx^{3r}(a+b\log(cx^n))}{9r^2} + \frac{e^3x^{3r}(a+b\log(cx^n))^2}{3r} + \frac{6b^2d^2en^2x^r}{r^3} + \frac{3b^2de^2n^2x^{2r}}{4r^3} + \frac{2b^2e^3n^2x^{3r}}{27r^3}$$

```
[In] Int[((d + e*x^r)^3*(a + b*Log[c*x^n])^2)/x,x]
```

```
[Out] (6*b^2*d^2*e*n^2*x^r)/r^3 + (3*b^2*d*e^2*n^2*x^(2*r))/(4*r^3) + (2*b^2*e^3*
n^2*x^(3*r))/(27*r^3) - (6*b*d^2*e*n*x^r*(a + b*Log[c*x^n]))/r^2 - (3*b*d*e
^2*n*x^(2*r)*(a + b*Log[c*x^n]))/(2*r^2) - (2*b*e^3*n*x^(3*r)*(a + b*Log[c*
x^n]))/(9*r^2) + (3*d^2*e*x^r*(a + b*Log[c*x^n])^2)/r + (3*d*e^2*x^(2*r)*(a
 + b*Log[c*x^n])^2)/(2*r) + (e^3*x^(3*r)*(a + b*Log[c*x^n])^2)/(3*r) + (d^3
*(a + b*Log[c*x^n])^3)/(3*b*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^3(a+b\log(cx^n))^2}{x} + 3d^2ex^{-1+r}(a+b\log(cx^n))^2 + 3de^2x^{-1+2r}(a+b\log(cx^n))^2 + e^3x^{-1+3r}(a+b\log(cx^n))^2\right) dx$$

$$= d^3\int \frac{(a+b\log(cx^n))^2}{x}\,dx + (3d^2e)\int x^{-1+r}(a+b\log(cx^n))^2\,dx + (3de^2)\int x^{-1+2r}(a+b\log(cx^n))^2\,dx + e^3\int x^{-1+3r}(a+b\log(cx^n))^2\,dx$$

$$= \frac{3d^2ex^r(a+b\log(cx^n))^2}{r} + \frac{3de^2x^{2r}(a+b\log(cx^n))^2}{2r} + \frac{e^3x^{3r}(a+b\log(cx^n))^2}{3r} + \frac{d^3\text{Subst}\left(\int x^2\,dx, x, a+b\log(cx^n)\right)}{bn} - \frac{(6bd^2en)\int x^{-1+r}(a+b\log(cx^n))\,dx}{r} - \frac{(3bde^2n)\int x^{-1+2r}(a+b\log(cx^n))\,dx}{r} - \frac{(2be^3n)\int x^{-1+3r}(a+b\log(cx^n))\,dx}{3r}$$

$$= \frac{6b^2d^2en^2x^r}{r^3} + \frac{3b^2de^2n^2x^{2r}}{4r^3} + \frac{2b^2e^3n^2x^{3r}}{27r^3} - \frac{6bd^2enx^r(a+b\log(cx^n))}{r^2} - \frac{3bde^2nx^{2r}(a+b\log(cx^n))}{2r^2} - \frac{2be^3nx^{3r}(a+b\log(cx^n))}{9r^2} + \frac{3d^2ex^r(a+b\log(cx^n))^2}{r} + \frac{3de^2x^{2r}(a+b\log(cx^n))^2}{2r} + \frac{e^3x^{3r}(a+b\log(cx^n))^2}{3r} + \frac{d^3(a+b\log(cx^n))^3}{3bn}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))^2}{x}\,dx$$
$$= \frac{enx^r(18a^2r^2(18d^2+9dex^r+2e^2x^{2r})-6abnr(108d^2+27dex^r+4e^2x^{2r})+b^2n^2(648d^2+81dex^r+8e^2x^{2r}}{}$$

```
[In] Integrate[((d + e*x^r)^3*(a + b*Log[c*x^n])^2)/x,x]
```

```
[Out] (e*n*x^r*(18*a^2*r^2*(18*d^2 + 9*d*e*x^r + 2*e^2*x^(2*r)) - 6*a*b*n*r*(108*
d^2 + 27*d*e*x^r + 4*e^2*x^(2*r)) + b^2*n^2*(648*d^2 + 81*d*e*x^r + 8*e^2*x
^(2*r))) + 108*a^2*d^3*n*r^3*Log[x] - 6*b*e*n*r*x^r*(-6*a*r*(18*d^2 + 9*d*e
*x^r + 2*e^2*x^(2*r)) + b*n*(108*d^2 + 27*d*e*x^r + 4*e^2*x^(2*r)))*Log[c*x
^n] + 18*b*r^2*(6*a*d^3*r + b*e*n*x^r*(18*d^2 + 9*d*e*x^r + 2*e^2*x^(2*r)))
*Log[c*x^n]^2 + 36*b^2*d^3*r^3*Log[c*x^n]^3)/(108*n*r^3)
```

Maple [A] (verified)

Time = 9.88 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.71

method	result
parallelrisch	$\frac{36b^2e^3\ln(cx^n)^2x^{3r}r^2n+72x^{3r}\ln(cx^n)ab\,e^3nr^2-24x^{3r}\ln(cx^n)b^2e^3n^2r+162b^2d\,e^2\ln(cx^n)^2x^{2r}r^2n+36x^{3r}a^2e^3nr^2-24x^{3r}a}{}$
risch	Expression too large to display

```
[In] int((d+e*x^r)^3*(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/108*(36*b^2*e^3*ln(c*x^n)^2*(x^r)^3*r^2*n+72*(x^r)^3*ln(c*x^n)*a*b*e^3*n*
r^2-24*(x^r)^3*ln(c*x^n)*b^2*e^3*n^2*r+162*b^2*d*e^2*ln(c*x^n)^2*(x^r)^2*r^
2*n+36*(x^r)^3*a^2*e^3*n*r^2-24*(x^r)^3*a*b*e^3*n^2*r+8*(x^r)^3*b^2*e^3*n^3
+324*(x^r)^2*ln(c*x^n)*a*b*d*e^2*n*r^2-162*(x^r)^2*ln(c*x^n)*b^2*d*e^2*n^2*
r+324*b^2*d^2*e*ln(c*x^n)^2*x^r*r^2*n+36*b^2*d^3*ln(c*x^n)^3*r^3+108*ln(x)*
a^2*d^3*n*r^3+162*(x^r)^2*a^2*d*e^2*n*r^2-162*(x^r)^2*a*b*d*e^2*n^2*r+81*(x
^r)^2*b^2*d*e^2*n^3+648*x^r*ln(c*x^n)*a*b*d^2*e*n*r^2-648*x^r*ln(c*x^n)*b^2
*d^2*e*n^2*r+108*d^3*a*b*ln(c*x^n)^2*r^3+324*x^r*a^2*d^2*e*n*r^2-648*x^r*a*
b*d^2*e*n^2*r+648*x^r*b^2*d^2*e*n^3)/r^3/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(231) = 462.

Time = 0.27 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.13

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))^2}{x}\,dx$$
$$= \frac{36\,b^2d^3n^2r^3\log(x)^3 + 108\,(b^2d^3nr^3\log(c) + abd^3nr^3)\log(x)^2 + 4\,(9\,b^2e^3n^2r^2\log(x)^2 + 9\,b^2e^3r^2\log(c)^2 + 2}{}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] 1/108*(36*b^2*d^3*n^2*r^3*log(x)^3 + 108*(b^2*d^3*n*r^3*log(c) + a*b*d^3*n*
r^3)*log(x)^2 + 4*(9*b^2*e^3*n^2*r^2*log(x)^2 + 9*b^2*e^3*r^2*log(c)^2 + 2*
b^2*e^3*n^2 - 6*a*b*e^3*n*r + 9*a^2*e^3*r^2 - 6*(b^2*e^3*n*r - 3*a*b*e^3*r^
2)*log(c) + 6*(3*b^2*e^3*n*r^2*log(c) - b^2*e^3*n^2*r + 3*a*b*e^3*n*r^2)*lo
g(x))*x^(3*r) + 81*(2*b^2*d*e^2*n^2*r^2*log(x)^2 + 2*b^2*d*e^2*r^2*log(c)^2
 + b^2*d*e^2*n^2 - 2*a*b*d*e^2*n*r + 2*a^2*d*e^2*r^2 - 2*(b^2*d*e^2*n*r - 2
*a*b*d*e^2*r^2)*log(c) + 2*(2*b^2*d*e^2*n*r^2*log(c) - b^2*d*e^2*n^2*r + 2*
a*b*d*e^2*n*r^2)*log(x))*x^(2*r) + 324*(b^2*d^2*e*n^2*r^2*log(x)^2 + b^2*d^
2*e*r^2*log(c)^2 + 2*b^2*d^2*e*n^2 - 2*a*b*d^2*e*n*r + a^2*d^2*e*r^2 - 2*(b
^2*d^2*e*n*r - a*b*d^2*e*r^2)*log(c) + 2*(b^2*d^2*e*n*r^2*log(c) - b^2*d^2*
e*n^2*r + a*b*d^2*e*n*r^2)*log(x))*x^r + 108*(b^2*d^3*r^3*log(c)^2 + 2*a*b*
d^3*r^3*log(c) + a^2*d^3*r^3)*log(x))/r^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(246) = 492.

Time = 8.03 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.40

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))^2}{x}\,dx$$
$$= \begin{cases} (a+b\log(c))^2 (d+e)^3 \log(x) \\ (a+b\log(c))^2 \left(d^3\log(x) + \frac{3d^2ex^r}{r} + \frac{3de^2x^{2r}}{2r} + \frac{e^3x^{3r}}{3r}\right) \\ (d+e)^3 \left(\begin{cases} \frac{a^2\log(cx^n)+ab\log(cx^n)^2+\frac{b^2\log(cx^n)^3}{3}}{n} & \text{for}\ n\neq 0 \\ (a^2+2ab\log(c)+b^2\log(c)^2)\log(x) & \text{otherwise} \end{cases}\right) \\ \frac{a^2d^3\log(cx^n)}{n} + \frac{3a^2d^2ex^r}{r} + \frac{3a^2de^2x^{2r}}{2r} + \frac{a^2e^3x^{3r}}{3r} + \frac{abd^3\log(cx^n)^2}{n} - \frac{6abd^2enx^r}{r^2} + \frac{6abd^2ex^r\log(cx^n)}{r} - \frac{3abde^2nx^{2r}}{2r^2} + \frac{3abde^2}{} \end{cases}$$

```
[In] integrate((d+e*x**r)**3*(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Piecewise(((a + b*log(c))**2*(d + e)**3*log(x), Eq(n, 0) & Eq(r, 0)), ((a +
 b*log(c))**2*(d**3*log(x) + 3*d**2*e*x**r/r + 3*d*e**2*x**(2*r)/(2*r) + e*
*3*x**(3*r)/(3*r)), Eq(n, 0)), ((d + e)**3*Piecewise(((a**2*log(c*x**n) + a
*b*log(c*x**n)**2 + b**2*log(c*x**n)**3/3)/n, Ne(n, 0)), ((a**2 + 2*a*b*log
(c) + b**2*log(c)**2)*log(x), True)), Eq(r, 0)), (a**2*d**3*log(c*x**n)/n +
 3*a**2*d**2*e*x**r/r + 3*a**2*d*e**2*x**(2*r)/(2*r) + a**2*e**3*x**(3*r)/(
3*r) + a*b*d**3*log(c*x**n)**2/n - 6*a*b*d**2*e*n*x**r/r**2 + 6*a*b*d**2*e*
x**r*log(c*x**n)/r - 3*a*b*d*e**2*n*x**(2*r)/(2*r**2) + 3*a*b*d*e**2*x**(2*
r)*log(c*x**n)/r - 2*a*b*e**3*n*x**(3*r)/(9*r**2) + 2*a*b*e**3*x**(3*r)*log
(c*x**n)/(3*r) + b**2*d**3*log(c*x**n)**3/(3*n) + 6*b**2*d**2*e*n**2*x**r/r
**3 - 6*b**2*d**2*e*n*x**r*log(c*x**n)/r**2 + 3*b**2*d**2*e*x**r*log(c*x**n
)**2/r + 3*b**2*d*e**2*n**2*x**(2*r)/(4*r**3) - 3*b**2*d*e**2*n*x**(2*r)*lo
g(c*x**n)/(2*r**2) + 3*b**2*d*e**2*x**(2*r)*log(c*x**n)**2/(2*r) + 2*b**2*e
**3*n**2*x**(3*r)/(27*r**3) - 2*b**2*e**3*n*x**(3*r)*log(c*x**n)/(9*r**2) +
 b**2*e**3*x**(3*r)*log(c*x**n)**2/(3*r), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))^2}{x}\,dx = \frac{b^2e^3x^{3r}\log(cx^n)^2}{3r} + \frac{3b^2de^2x^{2r}\log(cx^n)^2}{2r} + \frac{3b^2d^2ex^r\log(cx^n)^2}{r} + \frac{b^2d^3\log(cx^n)^3}{3n} - \frac{2}{27}b^2e^3\left(\frac{3nx^{3r}\log(cx^n)}{r^2} - \frac{n^2x^{3r}}{r^3}\right) - \frac{3}{4}b^2de^2\left(\frac{2nx^{2r}\log(cx^n)}{r^2} - \frac{n^2x^{2r}}{r^3}\right) - 6b^2d^2e\left(\frac{nx^r\log(cx^n)}{r^2} - \frac{n^2x^r}{r^3}\right) + \frac{2abe^3x^{3r}\log(cx^n)}{3r} + \frac{3abde^2x^{2r}\log(cx^n)}{r} + \frac{6abd^2ex^r\log(cx^n)}{r} + \frac{abd^3\log(cx^n)^2}{n} + a^2d^3\log(x) - \frac{2abe^3nx^{3r}}{9r^2} + \frac{a^2e^3x^{3r}}{3r} - \frac{3abde^2nx^{2r}}{2r^2} + \frac{3a^2de^2x^{2r}}{2r} - \frac{6abd^2enx^r}{r^2} + \frac{3a^2d^2ex^r}{r}$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*e^3*x^(3*r)*log(c*x^n)^2/r + 3/2*b^2*d*e^2*x^(2*r)*log(c*x^n)^2/r +
 3*b^2*d^2*e*x^r*log(c*x^n)^2/r + 1/3*b^2*d^3*log(c*x^n)^3/n - 2/27*b^2*e^3
*(3*n*x^(3*r)*log(c*x^n)/r^2 - n^2*x^(3*r)/r^3) - 3/4*b^2*d*e^2*(2*n*x^(2*r
)*log(c*x^n)/r^2 - n^2*x^(2*r)/r^3) - 6*b^2*d^2*e*(n*x^r*log(c*x^n)/r^2 - n
```

```
^2*x^r/r^3) + 2/3*a*b*e^3*x^(3*r)*log(c*x^n)/r + 3*a*b*d*e^2*x^(2*r)*log(c*
x^n)/r + 6*a*b*d^2*e*x^r*log(c*x^n)/r + a*b*d^3*log(c*x^n)^2/n + a^2*d^3*lo
g(x) - 2/9*a*b*e^3*n*x^(3*r)/r^2 + 1/3*a^2*e^3*x^(3*r)/r - 3/2*a*b*d*e^2*n*
x^(2*r)/r^2 + 3/2*a^2*d*e^2*x^(2*r)/r - 6*a*b*d^2*e*n*x^r/r^2 + 3*a^2*d^2*e
*x^r/r
```

Giac [F]

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))^2}{x}\,dx = \int \frac{(ex^r+d)^3(b\log(cx^n)+a)^2}{x}\,dx$$

```
[In] integrate((d+e*x^r)^3*(a+b*log(c*x^n))^2/x,x, algorithm="giac")
[Out] integrate((e*x^r + d)^3*(b*log(c*x^n) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^3 (a+b\log(cx^n))^2}{x}\,dx = \int \frac{(d+e\,x^r)^3\,(a+b\,\ln(c\,x^n))^2}{x}\,dx$$

```
[In] int(((d + e*x^r)^3*(a + b*log(c*x^n))^2)/x,x)
[Out] int(((d + e*x^r)^3*(a + b*log(c*x^n))^2)/x, x)
```

3.428 $\int \frac{(d+ex^r)^2(a+b\log(cx^n))^2}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))^2}{x}\,dx = \frac{4b^2den^2x^r}{r^3}+\frac{b^2e^2n^2x^{2r}}{4r^3}-\frac{4bdenx^r(a+b\log(cx^n))}{r^2}-\frac{be^2nx^{2r}(a+b\log(cx^n))}{2r^2}+\frac{2dex^r(a+b\log(cx^n))^2}{r}+\frac{e^2x^{2r}(a+b\log(cx^n))^2}{2r}+\frac{d^2(a+b\log(cx^n))^3}{3bn}$$

```
[Out] 4*b^2*d*e*n^2*x^r/r^3+1/4*b^2*e^2*n^2*x^(2*r)/r^3-4*b*d*e*n*x^r*(a+b*ln(c*x
^n))/r^2-1/2*b*e^2*n*x^(2*r)*(a+b*ln(c*x^n))/r^2+2*d*e*x^r*(a+b*ln(c*x^n))^
2/r+1/2*e^2*x^(2*r)*(a+b*ln(c*x^n))^2/r+1/3*d^2*(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2395, 2339, 30, 2342, 2341}

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))^2}{x}\,dx = \frac{d^2(a+b\log(cx^n))^3}{3bn}-\frac{4bdenx^r(a+b\log(cx^n))}{r^2}+\frac{2dex^r(a+b\log(cx^n))^2}{r}-\frac{be^2nx^{2r}(a+b\log(cx^n))}{2r^2}+\frac{e^2x^{2r}(a+b\log(cx^n))^2}{2r}+\frac{4b^2den^2x^r}{r^3}+\frac{b^2e^2n^2x^{2r}}{4r^3}$$

```
[In] Int[((d + e*x^r)^2*(a + b*Log[c*x^n])^2)/x,x]
```

```
[Out] (4*b^2*d*e*n^2*x^r)/r^3 + (b^2*e^2*n^2*x^(2*r))/(4*r^3) - (4*b*d*e*n*x^r*(a
 + b*Log[c*x^n]))/r^2 - (b*e^2*n*x^(2*r)*(a + b*Log[c*x^n]))/(2*r^2) + (2*d
*e*x^r*(a + b*Log[c*x^n])^2)/r + (e^2*x^(2*r)*(a + b*Log[c*x^n])^2)/(2*r) +
 (d^2*(a + b*Log[c*x^n])^3)/(3*b*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2(a+b\log(cx^n))^2}{x} + 2dex^{-1+r}(a+b\log(cx^n))^2 + e^2x^{-1+2r}(a+b\log(cx^n))^2 \right) dx$$

$$
\begin{aligned}
&= d^2 \int \frac{(a+b\log(cx^n))^2}{x}\,dx + (2de)\int x^{-1+r}(a+b\log(cx^n))^2\,dx \\
&\quad + e^2\int x^{-1+2r}(a+b\log(cx^n))^2\,dx \\
&= \frac{2dex^r(a+b\log(cx^n))^2}{r} + \frac{e^2x^{2r}(a+b\log(cx^n))^2}{2r} + \frac{d^2\text{Subst}\left(\int x^2\,dx, x, a+b\log(cx^n)\right)}{bn} \\
&\quad - \frac{(4bden)\int x^{-1+r}(a+b\log(cx^n))\,dx}{r} - \frac{(be^2n)\int x^{-1+2r}(a+b\log(cx^n))\,dx}{r} \\
&= \frac{4b^2den^2x^r}{r^3} + \frac{b^2e^2n^2x^{2r}}{4r^3} - \frac{4bdenx^r(a+b\log(cx^n))}{r^2} - \frac{be^2nx^{2r}(a+b\log(cx^n))}{2r^2} \\
&\quad + \frac{2dex^r(a+b\log(cx^n))^2}{r} + \frac{e^2x^{2r}(a+b\log(cx^n))^2}{2r} + \frac{d^2(a+b\log(cx^n))^3}{3bn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.11

$$
\int \frac{(d+ex^r)^2(a+b\log(cx^n))^2}{x}\,dx
= \frac{3enx^r(2a^2r^2(4d+ex^r) - 2abnr(8d+ex^r) + b^2n^2(16d+ex^r)) + 12a^2d^2nr^3\log(x) - 6benrx^r(-2ar(4d+}{12nr^3}
$$

```
[In] Integrate[((d + e*x^r)^2*(a + b*Log[c*x^n])^2)/x,x]
```

```
[Out] (3*e*n*x^r*(2*a^2*r^2*(4*d + e*x^r) - 2*a*b*n*r*(8*d + e*x^r) + b^2*n^2*(16
*d + e*x^r)) + 12*a^2*d^2*n*r^3*Log[x] - 6*b*e*n*r*x^r*(-2*a*r*(4*d + e*x^r
) + b*n*(8*d + e*x^r))*Log[c*x^n] + 6*b*r^2*(2*a*d^2*r + b*e*n*x^r*(4*d + e
*x^r))*Log[c*x^n]^2 + 4*b^2*d^2*r^3*Log[c*x^n]^3)/(12*n*r^3)
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.75

method	result
parallelrisch	$\frac{6e^2b^2\ln(c\,x^n)^2x^{2r}r^2n+12x^{2r}\ln(c\,x^n)ab\,e^2n\,r^2-6x^{2r}\ln(c\,x^n)b^2e^2n^2r+24b^2de\ln(c\,x^n)^2x^rr^2n+4b^2d^2\ln(c\,x^n)^3r^3+12\ln(x)a^2}{}$
risch	Expression too large to display

```
[In] int((d+e*x^r)^2*(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(6*e^2*b^2*ln(c*x^n)^2*(x^r)^2*r^2*n+12*(x^r)^2*ln(c*x^n)*a*b*e^2*n*r^
2-6*(x^r)^2*ln(c*x^n)*b^2*e^2*n^2*r+24*b^2*d*e*ln(c*x^n)^2*x^r*r^2*n+4*b^2*
d^2*ln(c*x^n)^3*r^3+12*ln(x)*a^2*d^2*n*r^3+6*(x^r)^2*a^2*e^2*n*r^2-6*(x^r)^
2*a*b*e^2*n^2*r+3*(x^r)^2*b^2*e^2*n^3+48*x^r*ln(c*x^n)*a*b*d*e*n*r^2-48*x^r
```

```
*ln(c*x^n)*b^2*d*e*n^2*r+12*a*b*d^2*ln(c*x^n)^2*r^3+24*x^r*a^2*d*e*n*r^2-48
*x^r*a*b*d*e*n^2*r+48*x^r*b^2*d*e*n^3)/r^3/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(153) = 306.

Time = 0.27 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.19

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))^2}{x}\,dx = \frac{4\,b^2d^2n^2r^3\log(x)^3 + 12\,(b^2d^2nr^3\log(c) + abd^2nr^3)\log(x)^2 + 3\,(2\,b^2e^2n^2r^2\log(x)^2 + 2\,b^2e^2r^2\log(c)^2 + b^2e}{}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^2*d^2*n^2*r^3*log(x)^3 + 12*(b^2*d^2*n*r^3*log(c) + a*b*d^2*n*r^3
)*log(x)^2 + 3*(2*b^2*e^2*n^2*r^2*log(x)^2 + 2*b^2*e^2*r^2*log(c)^2 + b^2*e
^2*n^2 - 2*a*b*e^2*n*r + 2*a^2*e^2*r^2 - 2*(b^2*e^2*n*r - 2*a*b*e^2*r^2)*lo
g(c) + 2*(2*b^2*e^2*n*r^2*log(c) - b^2*e^2*n^2*r + 2*a*b*e^2*n*r^2)*log(x))
*x^(2*r) + 24*(b^2*d*e*n^2*r^2*log(x)^2 + b^2*d*e*r^2*log(c)^2 + 2*b^2*d*e*
n^2 - 2*a*b*d*e*n*r + a^2*d*e*r^2 - 2*(b^2*d*e*n*r - a*b*d*e*r^2)*log(c) +
2*(b^2*d*e*n*r^2*log(c) - b^2*d*e*n^2*r + a*b*d*e*n*r^2)*log(x))*x^r + 12*(
b^2*d^2*r^3*log(c)^2 + 2*a*b*d^2*r^3*log(c) + a^2*d^2*r^3)*log(x))/r^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(156) = 312.

Time = 7.70 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.53

$$\int \frac{(d+ex^r)^2(a+b\log(cx^n))^2}{x}\,dx = \begin{cases} (a+b\log(c))^2(d+e)^2\log(x) \\ (a+b\log(c))^2\left(d^2\log(x) + \frac{2dex^r}{r} + \frac{e^2x^{2r}}{2r}\right) \\ (d+e)^2\left(\begin{cases} \frac{a^2\log(cx^n)+ab\log(cx^n)^2+\frac{b^2\log(cx^n)^3}{3}}{n} & \text{for } n\neq 0 \\ (a^2+2ab\log(c)+b^2\log(c)^2)\log(x) & \text{otherwise} \end{cases}\right) \\ \frac{a^2d^2\log(cx^n)}{n} + \frac{2a^2dex^r}{r} + \frac{a^2e^2x^{2r}}{2r} + \frac{abd^2\log(cx^n)^2}{n} - \frac{4abdenx^r}{r^2} + \frac{4abdex^r\log(cx^n)}{r} - \frac{abe^2nx^{2r}}{2r^2} + \frac{abe^2x^{2r}\log(cx^n)}{r} + \frac{b^2d^2\,\mathrm{lo}}{} \end{cases}$$

```
[In] integrate((d+e*x**r)**2*(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Piecewise(((a + b*log(c))**2*(d + e)**2*log(x), Eq(n, 0) & Eq(r, 0)), ((a +
 b*log(c))**2*(d**2*log(x) + 2*d*e*x**r/r + e**2*x**(2*r)/(2*r)), Eq(n, 0))
, ((d + e)**2*Piecewise(((a**2*log(c*x**n) + a*b*log(c*x**n)**2 + b**2*log(
c*x**n)**3/3)/n, Ne(n, 0)), ((a**2 + 2*a*b*log(c) + b**2*log(c)**2)*log(x),
 True)), Eq(r, 0)), (a**2*d**2*log(c*x**n)/n + 2*a**2*d*e*x**r/r + a**2*e**
2*x**(2*r)/(2*r) + a*b*d**2*log(c*x**n)**2/n - 4*a*b*d*e*n*x**r/r**2 + 4*a*
b*d*e*x**r*log(c*x**n)/r - a*b*e**2*n*x**(2*r)/(2*r**2) + a*b*e**2*x**(2*r)
*log(c*x**n)/r + b**2*d**2*log(c*x**n)**3/(3*n) + 4*b**2*d*e*n**2*x**r/r**3
 - 4*b**2*d*e*n*x**r*log(c*x**n)/r**2 + 2*b**2*d*e*x**r*log(c*x**n)**2/r +
b**2*e**2*n**2*x**(2*r)/(4*r**3) - b**2*e**2*n*x**(2*r)*log(c*x**n)/(2*r**2
) + b**2*e**2*x**(2*r)*log(c*x**n)**2/(2*r), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))^2}{x}\,dx = \frac{b^2e^2x^{2r}\log(cx^n)^2}{2r} + \frac{2b^2dex^r\log(cx^n)^2}{r} + \frac{b^2d^2\log(cx^n)^3}{3n} - \frac{1}{4}b^2e^2\left(\frac{2nx^{2r}\log(cx^n)}{r^2} - \frac{n^2x^{2r}}{r^3}\right) - 4b^2de\left(\frac{nx^r\log(cx^n)}{r^2} - \frac{n^2x^r}{r^3}\right) + \frac{abe^2x^{2r}\log(cx^n)}{r} + \frac{4abdex^r\log(cx^n)}{r} + \frac{abd^2\log(cx^n)^2}{n} + a^2d^2\log(x) - \frac{abe^2nx^{2r}}{2r^2} + \frac{a^2e^2x^{2r}}{2r} - \frac{4abdenx^r}{r^2} + \frac{2a^2dex^r}{r}$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*e^2*x^(2*r)*log(c*x^n)^2/r + 2*b^2*d*e*x^r*log(c*x^n)^2/r + 1/3*b^2
*d^2*log(c*x^n)^3/n - 1/4*b^2*e^2*(2*n*x^(2*r)*log(c*x^n)/r^2 - n^2*x^(2*r)
/r^3) - 4*b^2*d*e*(n*x^r*log(c*x^n)/r^2 - n^2*x^r/r^3) + a*b*e^2*x^(2*r)*lo
g(c*x^n)/r + 4*a*b*d*e*x^r*log(c*x^n)/r + a*b*d^2*log(c*x^n)^2/n + a^2*d^2*
log(x) - 1/2*a*b*e^2*n*x^(2*r)/r^2 + 1/2*a^2*e^2*x^(2*r)/r - 4*a*b*d*e*n*x^
r/r^2 + 2*a^2*d*e*x^r/r
```

Giac [F]

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))^2}{x}\,dx = \int \frac{(ex^r+d)^2(b\log(cx^n)+a)^2}{x}\,dx$$

```
[In] integrate((d+e*x^r)^2*(a+b*log(c*x^n))^2/x,x, algorithm="giac")
[Out] integrate((e*x^r + d)^2*(b*log(c*x^n) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^2 (a+b\log(cx^n))^2}{x}\,dx = \int \frac{(d+e\,x^r)^2\,(a+b\,\ln(c\,x^n))^2}{x}\,dx$$

```
[In] int(((d + e*x^r)^2*(a + b*log(c*x^n))^2)/x,x)
[Out] int(((d + e*x^r)^2*(a + b*log(c*x^n))^2)/x, x)
```

3.429 $\int \frac{(d+ex^r)(a+b\log(cx^n))^2}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \frac{(d+ex^r)\,(a+b\log(cx^n))^2}{x}\,dx = \frac{2b^2en^2x^r}{r^3} - \frac{2benx^r(a+b\log(cx^n))}{r^2} + \frac{ex^r(a+b\log(cx^n))^2}{r} + \frac{d(a+b\log(cx^n))^3}{3bn}$$

```
[Out] 2*b^2*e*n^2*x^r/r^3-2*b*e*n*x^r*(a+b*ln(c*x^n))/r^2+e*x^r*(a+b*ln(c*x^n))^2
/r+1/3*d*(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2395, 2339, 30, 2342, 2341}

$$\int \frac{(d+ex^r)\,(a+b\log(cx^n))^2}{x}\,dx = \frac{d(a+b\log(cx^n))^3}{3bn} - \frac{2benx^r(a+b\log(cx^n))}{r^2} + \frac{ex^r(a+b\log(cx^n))^2}{r} + \frac{2b^2en^2x^r}{r^3}$$

```
[In] Int[((d + e*x^r)*(a + b*Log[c*x^n])^2)/x,x]
```

```
[Out] (2*b^2*e*n^2*x^r)/r^3 - (2*b*e*n*x^r*(a + b*Log[c*x^n]))/r^2 + (e*x^r*(a +
b*Log[c*x^n])^2)/r + (d*(a + b*Log[c*x^n])^3)/(3*b*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d(a+b\log(cx^n))^2}{x}+ex^{-1+r}(a+b\log(cx^n))^2\right)\,dx\\
&= d\int \frac{(a+b\log(cx^n))^2}{x}\,dx+e\int x^{-1+r}(a+b\log(cx^n))^2\,dx\\
&= \frac{ex^r(a+b\log(cx^n))^2}{r}+\frac{d\text{Subst}\left(\int x^2\,dx,x,a+b\log(cx^n)\right)}{bn}\\
&\quad-\frac{(2ben)\int x^{-1+r}(a+b\log(cx^n))\,dx}{r}\\
&= \frac{2b^2en^2x^r}{r^3}-\frac{2benx^r(a+b\log(cx^n))}{r^2}+\frac{ex^r(a+b\log(cx^n))^2}{r}+\frac{d(a+b\log(cx^n))^3}{3bn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex^r)(a+b\log(cx^n))^2}{x}\,dx = \frac{e(2b^2n^2-2abnr+a^2r^2)x^r}{r^3} + a^2d\log(x) - \frac{2be(bn-ar)x^r\log(cx^n)}{r^2} + \frac{b(adr+benx^r)\log^2(cx^n)}{nr} + \frac{b^2d\log^3(cx^n)}{3n}$$

```
[In] Integrate[((d + e*x^r)*(a + b*Log[c*x^n])^2)/x,x]
```

```
[Out] (e*(2*b^2*n^2 - 2*a*b*n*r + a^2*r^2)*x^r)/r^3 + a^2*d*Log[x] - (2*b*e*(b*n
- a*r)*x^r*Log[c*x^n])/r^2 + (b*(a*d*r + b*e*n*x^r)*Log[c*x^n]^2)/(n*r) + (
b^2*d*Log[c*x^n]^3)/(3*n)
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.86

method	result
parallelrisch	$\frac{3x^r\ln(cx^n)^2b^2er^2n+b^2d\ln(cx^n)^3r^3+3\ln(x)a^2dnr^3+6x^r\ln(cx^n)abenr^2-6x^r\ln(cx^n)b^2en^2r+3abd\ln(cx^n)^2r^3+3x^ra^2en}{3r^3n}$
risch	Expression too large to display

```
[In] int((d+e*x^r)*(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(3*x^r*ln(c*x^n)^2*b^2*e*r^2*n+b^2*d*ln(c*x^n)^3*r^3+3*ln(x)*a^2*d*n*r^
3+6*x^r*ln(c*x^n)*a*b*e*n*r^2-6*x^r*ln(c*x^n)*b^2*e*n^2*r+3*a*b*d*ln(c*x^n)
^2*r^3+3*x^r*a^2*e*n*r^2-6*x^r*a*b*e*n^2*r+6*x^r*b^2*e*n^3)/r^3/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(78) = 156.

Time = 0.31 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.41

$$\int \frac{(d+ex^r)(a+b\log(cx^n))^2}{x}\,dx = \frac{b^2dn^2r^3\log(x)^3 + 3(b^2dnr^3\log(c)+abdnr^3)\log(x)^2 + 3(b^2en^2r^2\log(x)^2 + b^2er^2\log(c)^2 + 2b^2en^2 - 2}{}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*d*n^2*r^3*log(x)^3 + 3*(b^2*d*n*r^3*log(c) + a*b*d*n*r^3)*log(x)^2
 + 3*(b^2*e*n^2*r^2*log(x)^2 + b^2*e*r^2*log(c)^2 + 2*b^2*e*n^2 - 2*a*b*e*n
*r + a^2*e*r^2 - 2*(b^2*e*n*r - a*b*e*r^2)*log(c) + 2*(b^2*e*n*r^2*log(c) -
 b^2*e*n^2*r + a*b*e*n*r^2)*log(x))*x^r + 3*(b^2*d*r^3*log(c)^2 + 2*a*b*d*r
^3*log(c) + a^2*d*r^3)*log(x))/r^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(76) = 152$.

Time = 6.87 (sec) , antiderivative size = 245, normalized size of antiderivative = 3.06

$$\int \frac{(d+ex^r)(a+b\log(cx^n))^2}{x}\,dx = \begin{cases} (a+b\log(c))^2(d+e)\log(x) \\ (a+b\log(c))^2\left(d\log(x)+\frac{ex^r}{r}\right) \\ (d+e)\left(\begin{cases} \frac{a^2\log(cx^n)+ab\log(cx^n)^2+\frac{b^2\log(cx^n)^3}{3}}{n} & \text{for } n\neq 0 \\ \left(a^2+2ab\log(c)+b^2\log(c)^2\right)\log(x) & \text{otherwise} \end{cases}\right) \\ \frac{a^2d\log(cx^n)}{n}+\frac{a^2ex^r}{r}+\frac{abd\log(cx^n)^2}{n}-\frac{2abenx^r}{r^2}+\frac{2abex^r\log(cx^n)}{r}+\frac{b^2d\log(cx^n)^3}{3n}+\frac{2b^2en^2x^r}{r^3}-\frac{2b^2enx^r\log(cx^n)}{r^2}+\frac{b^2ex^r\log}{} \end{cases}$$

```
[In] integrate((d+e*x**r)*(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Piecewise(((a + b*log(c))**2*(d + e)*log(x), Eq(n, 0) & Eq(r, 0)), ((a + b*
log(c))**2*(d*log(x) + e*x**r/r), Eq(n, 0)), ((d + e)*Piecewise(((a**2*log(
c*x**n) + a*b*log(c*x**n)**2 + b**2*log(c*x**n)**3/3)/n, Ne(n, 0)), ((a**2
+ 2*a*b*log(c) + b**2*log(c)**2)*log(x), True)), Eq(r, 0)), (a**2*d*log(c*x
**n)/n + a**2*e*x**r/r + a*b*d*log(c*x**n)**2/n - 2*a*b*e*n*x**r/r**2 + 2*a
*b*e*x**r*log(c*x**n)/r + b**2*d*log(c*x**n)**3/(3*n) + 2*b**2*e*n**2*x**r/
r**3 - 2*b**2*e*n*x**r*log(c*x**n)/r**2 + b**2*e*x**r*log(c*x**n)**2/r, Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex^r)(a+b\log(cx^n))^2}{x}\,dx = \frac{b^2ex^r\log(cx^n)^2}{r}+\frac{b^2d\log(cx^n)^3}{3\,n} - 2\,b^2e\left(\frac{nx^r\log(cx^n)}{r^2}-\frac{n^2x^r}{r^3}\right)+\frac{2\,abex^r\log(cx^n)}{r} + \frac{abd\log(cx^n)^2}{n}+a^2d\log(x)-\frac{2\,abenx^r}{r^2}+\frac{a^2ex^r}{r}$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*e*x^r*log(c*x^n)^2/r + 1/3*b^2*d*log(c*x^n)^3/n - 2*b^2*e*(n*x^r*log(c*
x^n)/r^2 - n^2*x^r/r^3) + 2*a*b*e*x^r*log(c*x^n)/r + a*b*d*log(c*x^n)^2/n +
 a^2*d*log(x) - 2*a*b*e*n*x^r/r^2 + a^2*e*x^r/r
```

Giac [F]

$$\int \frac{(d + ex^r)(a + b\log(cx^n))^2}{x}\, dx = \int \frac{(ex^r + d)(b\log(cx^n) + a)^2}{x}\, dx$$

```
[In] integrate((d+e*x^r)*(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^r + d)*(b*log(c*x^n) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex^r)(a + b\log(cx^n))^2}{x}\, dx = \int \frac{(d + e\,x^r)\,(a + b\,\ln(c\,x^n))^2}{x}\, dx$$

```
[In] int(((d + e*x^r)*(a + b*log(c*x^n))^2)/x,x)
```

```
[Out] int(((d + e*x^r)*(a + b*log(c*x^n))^2)/x, x)
```

3.430 $\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)}\,dx = -\frac{(a+b\log(cx^n))^2\log\left(1+\frac{dx^{-r}}{e}\right)}{dr} + \frac{2bn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)}{dr^2} + \frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{dx^{-r}}{e}\right)}{dr^3}$$

```
[Out] -(a+b*ln(c*x^n))^2*ln(1+d/e/(x^r))/d/r+2*b*n*(a+b*ln(c*x^n))*polylog(2,-d/e
/(x^r))/d/r^2+2*b^2*n^2*polylog(3,-d/e/(x^r))/d/r^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2379, 2421, 6724}

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)}\,dx = \frac{2bn\operatorname{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)(a+b\log(cx^n))}{dr^2} - \frac{\log\left(\frac{dx^{-r}}{e}+1\right)(a+b\log(cx^n))^2}{dr} + \frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{dx^{-r}}{e}\right)}{dr^3}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(x*(d + e*x^r)),x]
```

```
[Out] -(((a + b*Log[c*x^n])^2*Log[1 + d/(e*x^r)])/(d*r)) + (2*b*n*(a + b*Log[c*x^
n])*PolyLog[2, -(d/(e*x^r))])/(d*r^2) + (2*b^2*n^2*PolyLog[3, -(d/(e*x^r))]
)/(d*r^3)
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\log(cx^n))^2\log\left(1+\frac{dx^{-r}}{e}\right)}{dr}+\frac{(2bn)\int\frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{x}\,dx}{dr}\\
&= -\frac{(a+b\log(cx^n))^2\log\left(1+\frac{dx^{-r}}{e}\right)}{dr}\\
&\quad+\frac{2bn(a+b\log(cx^n))\,\text{Li}_2\left(-\frac{dx^{-r}}{e}\right)}{dr^2}-\frac{(2b^2n^2)\int\frac{\text{Li}_2\left(-\frac{dx^{-r}}{e}\right)}{x}\,dx}{dr^2}\\
&= -\frac{(a+b\log(cx^n))^2\log\left(1+\frac{dx^{-r}}{e}\right)}{dr}+\frac{2bn(a+b\log(cx^n))\,\text{Li}_2\left(-\frac{dx^{-r}}{e}\right)}{dr^2}+\frac{2b^2n^2\text{Li}_3\left(-\frac{dx^{-r}}{e}\right)}{dr^3}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 270 vs. 2(94) = 188.

Time = 0.19 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.87

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)}\,dx = -\frac{a^2r^2\log(d-dx^r)-2abr^2(n\log(x)-\log(cx^n))\log(d-dx^r)+b^2r^2(-n\log(x)+\log(cx^n))^2\log(d-dx^r)}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(x*(d + e*x^r)),x]
```

```
[Out] -((a^2*r^2*Log[d - d*x^r] - 2*a*b*r^2*(n*Log[x] - Log[c*x^n])*Log[d - d*x^r
] + b^2*r^2*(-(n*Log[x]) + Log[c*x^n])^2*Log[d - d*x^r] - 2*a*b*n*r*((r^2*L
og[x]^2)/2 + (-(r*Log[x]) + Log[-((e*x^r)/d)])*Log[d + e*x^r] + PolyLog[2,
1 + (e*x^r)/d]) + 2*b^2*n*r*(n*Log[x] - Log[c*x^n])*((r^2*Log[x]^2)/2 + (-(
r*Log[x]) + Log[-((e*x^r)/d)])*Log[d + e*x^r] + PolyLog[2, 1 + (e*x^r)/d])
+ b^2*n^2*(r^2*Log[x]^2*Log[1 + d/(e*x^r)] - 2*r*Log[x]*PolyLog[2, -(d/(e*x
^r))] - 2*PolyLog[3, -(d/(e*x^r))]))/(d*r^3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.80 (sec) , antiderivative size = 580, normalized size of antiderivative = 6.17

method	result
risch	$-\frac{b^2\ln(d+e\,x^r)\ln(x)^2n^2}{rd}+\frac{2b^2\ln(d+e\,x^r)\ln(x)\ln(x^n)n}{rd}-\frac{b^2\ln(d+e\,x^r)\ln(x^n)^2}{rd}+\frac{b^2\ln(x^r)\ln(x)^2n^2}{rd}-\frac{2b^2\ln(x^r)\ln(x)\ln(x^n)}{rd}$

```
[In] int((a+b*ln(c*x^n))^2/x/(d+e*x^r),x,method=_RETURNVERBOSE)
```

```
[Out] -b^2/r/d*ln(d+e*x^r)*ln(x)^2*n^2+2*b^2/r/d*ln(d+e*x^r)*ln(x)*ln(x^n)*n-b^2/
r/d*ln(d+e*x^r)*ln(x^n)^2+b^2/r/d*ln(x^r)*ln(x)^2*n^2-2*b^2/r/d*ln(x^r)*ln(
x)*ln(x^n)*n+b^2/r/d*ln(x^r)*ln(x^n)^2-2/3*b^2/d*ln(x)^3*n^2+b^2/r*n^2/d*ln
(x)^2*ln(1+e*x^r/d)+2*b^2/r^3*n^2/d*polylog(3,-e*x^r/d)+b^2*n/d*ln(x^n)*ln(
x)^2-2*b^2/r*n/d*ln(x)*ln(1+e*x^r/d)*ln(x^n)-2*b^2/r^2*n/d*polylog(2,-e*x^r
/d)*ln(x^n)+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*c
sgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*
b*ln(c)+2*a)*b/r*((ln(x^n)-n*ln(x))*(-1/d*ln(d+e*x^r)+1/d*ln(x^r))-n/r/d*(-
1/2*r^2*ln(x)^2+r*ln(x)*ln(1+e*x^r/d)+polylog(2,-e*x^r/d)))+1/4*(-I*b*Pi*cs
gn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*c
sgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*(-1/r/d*
ln(d+e*x^r)+1/r/d*ln(x^r))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(93) = 186$.

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.43

$$\int \frac{(a + b\log(cx^n))^2}{x(d + ex^r)}\,dx$$
$$= \frac{b^2n^2r^3\log(x)^3 + 6\,b^2n^2\mathrm{polylog}\left(3, -\frac{ex^r}{d}\right) + 3\,(b^2nr^3\log(c) + abnr^3)\log(x)^2 - 6\,(b^2n^2r\log(x) + b^2nr\log(}{}$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(d+e*x^r),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*n^2*r^3*log(x)^3 + 6*b^2*n^2*polylog(3, -e*x^r/d) + 3*(b^2*n*r^3*l
og(c) + a*b*n*r^3)*log(x)^2 - 6*(b^2*n^2*r*log(x) + b^2*n*r*log(c) + a*b*n*
r)*dilog(-(e*x^r + d)/d + 1) - 3*(b^2*r^2*log(c)^2 + 2*a*b*r^2*log(c) + a^2
*r^2)*log(e*x^r + d) + 3*(b^2*r^3*log(c)^2 + 2*a*b*r^3*log(c) + a^2*r^3)*lo
g(x) - 3*(b^2*n^2*r^2*log(x)^2 + 2*(b^2*n*r^2*log(c) + a*b*n*r^2)*log(x))*l
og((e*x^r + d)/d))/(d*r^3)
```

Sympy [F]

$$\int \frac{(a + b\log(cx^n))^2}{x(d + ex^r)}\,dx = \int \frac{(a + b\log(cx^n))^2}{x(d + ex^r)}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**2/x/(d+e*x**r),x)
```

```
[Out] Integral((a + b*log(c*x**n))**2/(x*(d + e*x**r)), x)
```

Maxima [F]

$$\int \frac{(a + b\log(cx^n))^2}{x(d + ex^r)}\,dx = \int \frac{(b\log(cx^n) + a)^2}{(ex^r + d)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(d+e*x^r),x, algorithm="maxima")
```

```
[Out] a^2*(log(x)/d - log((e*x^r + d)/e)/(d*r)) + integrate((b^2*log(c)^2 + b^2*l
og(x^n)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*log(x^n))/(e*x*x^r + d*x),
x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex^r+d)x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(d+e*x^r),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2/((e*x^r + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{x\,(d+e\,x^r)}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(x*(d + e*x^r)),x)
[Out] int((a + b*log(c*x^n))^2/(x*(d + e*x^r)), x)
```

3.431 $\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 182

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^2}\,dx = \frac{(a+b\log(cx^n))^2}{dr(d+ex^r)} + \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{d^2r^2} - \frac{(a+b\log(cx^n))^2\log\left(1+\frac{dx^{-r}}{e}\right)}{d^2r} - \frac{2b^2n^2\operatorname{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)}{d^2r^3} + \frac{2bn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)}{d^2r^2} + \frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{dx^{-r}}{e}\right)}{d^2r^3}$$

```
[Out] (a+b*ln(c*x^n))^2/d/r/(d+e*x^r)+2*b*n*(a+b*ln(c*x^n))*ln(1+d/e/(x^r))/d^2/r
^2-(a+b*ln(c*x^n))^2*ln(1+d/e/(x^r))/d^2/r-2*b^2*n^2*polylog(2,-d/e/(x^r))/
d^2/r^3+2*b*n*(a+b*ln(c*x^n))*polylog(2,-d/e/(x^r))/d^2/r^2+2*b^2*n^2*polyl
og(3,-d/e/(x^r))/d^2/r^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used

$= \{2391, 2379, 2421, 6724, 2376, 2438\}$

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^2}\,dx = \frac{2bn\,\mathrm{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)(a+b\log(cx^n))}{d^2r^2} + \frac{2bn\log\left(\frac{dx^{-r}}{e}+1\right)(a+b\log(cx^n))}{d^2r^2} - \frac{\log\left(\frac{dx^{-r}}{e}+1\right)(a+b\log(cx^n))^2}{d^2r} + \frac{(a+b\log(cx^n))^2}{dr(d+ex^r)} - \frac{2b^2n^2\,\mathrm{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)}{d^2r^3} + \frac{2b^2n^2\,\mathrm{PolyLog}\left(3,-\frac{dx^{-r}}{e}\right)}{d^2r^3}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(x*(d + e*x^r)^2),x]
```

```
[Out] (a + b*Log[c*x^n])^2/(d*r*(d + e*x^r)) + (2*b*n*(a + b*Log[c*x^n])*Log[1 +
d/(e*x^r)])/(d^2*r^2) - ((a + b*Log[c*x^n])^2*Log[1 + d/(e*x^r)])/(d^2*r) -
 (2*b^2*n^2*PolyLog[2, -(d/(e*x^r))])/(d^2*r^3) + (2*b*n*(a + b*Log[c*x^n])
*PolyLog[2, -(d/(e*x^r))])/(d^2*r^2) + (2*b^2*n^2*PolyLog[3, -(d/(e*x^r))])
/(d^2*r^3)
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^
(q_))/(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x^r)^(q + 1)*((a + b*Log[c*x^
n])^p/x), x], x] - Dist[e/d, Int[x^(r - 1)*(d + e*x^r)^q*(a + b*Log[c*x^n])
^p, x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0] && ILtQ[q, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)}\,dx}{d} - \frac{e\int \frac{x^{-1+r}(a+b\log(cx^n))^2}{(d+ex^r)^2}\,dx}{d}\\
&= \frac{(a+b\log(cx^n))^2}{dr(d+ex^r)} - \frac{(a+b\log(cx^n))^2\log\left(1+\frac{dx^{-r}}{e}\right)}{d^2r}\\
&\quad + \frac{(2bn)\int \frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{x}\,dx}{d^2r} - \frac{(2bn)\int \frac{a+b\log(cx^n)}{x(d+ex^r)}\,dx}{dr}\\
&= \frac{(a+b\log(cx^n))^2}{dr(d+ex^r)} + \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{d^2r^2} - \frac{(a+b\log(cx^n))^2\log\left(1+\frac{dx^{-r}}{e}\right)}{d^2r}\\
&\quad + \frac{2bn(a+b\log(cx^n))\,\mathrm{Li}_2\left(-\frac{dx^{-r}}{e}\right)}{d^2r^2} - \frac{(2b^2n^2)\int \frac{\log\left(1+\frac{dx^{-r}}{e}\right)}{x}\,dx}{d^2r^2} - \frac{(2b^2n^2)\int \frac{\mathrm{Li}_2\left(-\frac{dx^{-r}}{e}\right)}{x}\,dx}{d^2r^2}\\
&= \frac{(a+b\log(cx^n))^2}{dr(d+ex^r)} + \frac{2bn(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{d^2r^2} - \frac{(a+b\log(cx^n))^2\log\left(1+\frac{dx^{-r}}{e}\right)}{d^2r}\\
&\quad - \frac{2b^2n^2\mathrm{Li}_2\left(-\frac{dx^{-r}}{e}\right)}{d^2r^3} + \frac{2bn(a+b\log(cx^n))\,\mathrm{Li}_2\left(-\frac{dx^{-r}}{e}\right)}{d^2r^2} + \frac{2b^2n^2\mathrm{Li}_3\left(-\frac{dx^{-r}}{e}\right)}{d^2r^3}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 397 vs. $2(182) = 364$.

Time = 0.25 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.18

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^2}\,dx = \frac{\frac{dr^2(a+b\log(cx^n))^2}{d+ex^r} + 2abnr\log(d-dx^r) - a^2r^2\log(d-dx^r) + 2abr^2(n\log(x) - \log(cx^n))\log(d-dx^r) + 2b^2}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(x*(d + e*x^r)^2),x]
```

```
[Out] ((d*r^2*(a + b*Log[c*x^n])^2)/(d + e*x^r) + 2*a*b*n*r*Log[d - d*x^r] - a^2*
r^2*Log[d - d*x^r] + 2*a*b*r^2*(n*Log[x] - Log[c*x^n])*Log[d - d*x^r] + 2*b
^2*n*r*(-(n*Log[x]) + Log[c*x^n])*Log[d - d*x^r] - b^2*r^2*(-(n*Log[x]) + L
og[c*x^n])^2*Log[d - d*x^r] - 2*b^2*n^2*((r^2*Log[x]^2)/2 + (-(r*Log[x]) +
Log[-((e*x^r)/d)])*Log[d + e*x^r] + PolyLog[2, 1 + (e*x^r)/d]) + 2*a*b*n*r*
((r^2*Log[x]^2)/2 + (-(r*Log[x]) + Log[-((e*x^r)/d)])*Log[d + e*x^r] + Poly
Log[2, 1 + (e*x^r)/d]) + 2*b^2*n*r*(-(n*Log[x]) + Log[c*x^n])*((r^2*Log[x]^
2)/2 + (-(r*Log[x]) + Log[-((e*x^r)/d)])*Log[d + e*x^r] + PolyLog[2, 1 + (e
*x^r)/d]) - b^2*n^2*(r^2*Log[x]^2*Log[1 + d/(e*x^r)] - 2*r*Log[x]*PolyLog[2
, -(d/(e*x^r))] - 2*PolyLog[3, -(d/(e*x^r))]))/(d^2*r^3)
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))^2}{x\,(d+e\,x^r)^2}dx$$

```
[In] int((a+b*ln(c*x^n))^2/x/(d+e*x^r)^2,x)
```

```
[Out] int((a+b*ln(c*x^n))^2/x/(d+e*x^r)^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(180) = 360$.

Time = 0.27 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.30

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^2}\,dx = \frac{b^2dn^2r^3\log(x)^3 + 3\,b^2dr^2\log(c)^2 + 6\,abdr^2\log(c) + 3\,a^2dr^2 + 3\,(b^2dnr^3\log(c) + abdnr^3)\log(x)^2 + (b^2en^2}{}$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(d+e*x^r)^2,x, algorithm="fricas")
```

[Out] 1/3*(b^2*d*n^2*r^3*log(x)^3 + 3*b^2*d*r^2*log(c)^2 + 6*a*b*d*r^2*log(c) + 3*a^2*d*r^2 + 3*(b^2*d*n*r^3*log(c) + a*b*d*n*r^3)*log(x)^2 + (b^2*e*n^2*r^3*log(x)^3 + 3*(b^2*e*n*r^3*log(c) - b^2*e*n^2*r^2 + a*b*e*n*r^3)*log(x)^2 + 3*(b^2*e*r^3*log(c)^2 - 2*a*b*e*n*r^2 + a^2*e*r^3 - 2*(b^2*e*n*r^2 - a*b*e*r^3)*log(c))*log(x))*x^r - 6*(b^2*d*n^2*r*log(x) + b^2*d*n*r*log(c) - b^2*d*n^2 + a*b*d*n*r + (b^2*e*n^2*r*log(x) + b^2*e*n*r*log(c) - b^2*e*n^2 + a*b*e*n*r)*x^r)*dilog(-(e*x^r + d)/d + 1) - 3*(b^2*d*r^2*log(c)^2 - 2*a*b*d*n*r + a^2*d*r^2 + (b^2*e*r^2*log(c)^2 - 2*a*b*e*n*r + a^2*e*r^2 - 2*(b^2*e*n*r - a*b*e*r^2)*log(c))*x^r - 2*(b^2*d*n*r - a*b*d*r^2)*log(c))*log(e*x^r + d) + 3*(b^2*d*r^3*log(c)^2 + 2*a*b*d*r^3*log(c) + a^2*d*r^3)*log(x) - 3*(b^2*d*n^2*r^2*log(x)^2 + (b^2*e*n^2*r^2*log(x)^2 + 2*(b^2*e*n*r^2*log(c) - b^2*e*n^2*r + a*b*e*n*r^2)*log(x))*x^r + 2*(b^2*d*n*r^2*log(c) - b^2*d*n^2*r + a*b*d*n*r^2)*log(x))*log((e*x^r + d)/d) + 6*(b^2*e*n^2*x^r + b^2*d*n^2)*polylog(3, -e*x^r/d))/(d^2*e*r^3*x^r + d^3*r^3)

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^2}\,dx = \int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^2}\,dx$$

[In] integrate((a+b*ln(c*x**n))**2/x/(d+e*x**r)**2,x)

[Out] Integral((a + b*log(c*x**n))**2/(x*(d + e*x**r)**2), x)

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex^r+d)^2x}\,dx$$

[In] integrate((a+b*log(c*x^n))^2/x/(d+e*x^r)^2,x, algorithm="maxima")

[Out] a^2*(1/(d*e*r*x^r + d^2*r) + log(x)/d^2 - log((e*x^r + d)/e)/(d^2*r)) + integrate((b^2*log(c)^2 + b^2*log(x^n)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*log(x^n))/(e^2*x*x^(2*r) + 2*d*e*x*x^r + d^2*x), x)

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^2}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex^r+d)^2x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(d+e*x^r)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2/((e*x^r + d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^2}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{x\,(d+e\,x^r)^2}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(x*(d + e*x^r)^2),x)
[Out] int((a + b*log(c*x^n))^2/(x*(d + e*x^r)^2), x)
```

3.432 $\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 267

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^3}\,dx = \frac{benx^r(a+b\log(cx^n))}{d^3r^2(d+ex^r)} + \frac{(a+b\log(cx^n))^2}{2dr(d+ex^r)^2} + \frac{(a+b\log(cx^n))^2}{d^2r(d+ex^r)} + \frac{3bn(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{d^3r^2} - \frac{(a+b\log(cx^n))^2\log\left(1+\frac{dx^{-r}}{e}\right)}{d^3r} - \frac{b^2n^2\log(d+ex^r)}{d^3r^3} - \frac{3b^2n^2\operatorname{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)}{d^3r^3} + \frac{2bn(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)}{d^3r^2} + \frac{2b^2n^2\operatorname{PolyLog}\left(3,-\frac{dx^{-r}}{e}\right)}{d^3r^3}$$

```
[Out] b*e*n*x^r*(a+b*ln(c*x^n))/d^3/r^2/(d+e*x^r)+1/2*(a+b*ln(c*x^n))^2/d/r/(d+e*
x^r)^2+(a+b*ln(c*x^n))^2/d^2/r/(d+e*x^r)+3*b*n*(a+b*ln(c*x^n))*ln(1+d/e/(x^
r))/d^3/r^2-(a+b*ln(c*x^n))^2*ln(1+d/e/(x^r))/d^3/r-b^2*n^2*ln(d+e*x^r)/d^3
/r^3-3*b^2*n^2*polylog(2,-d/e/(x^r))/d^3/r^3+2*b*n*(a+b*ln(c*x^n))*polylog(
2,-d/e/(x^r))/d^3/r^2+2*b^2*n^2*polylog(3,-d/e/(x^r))/d^3/r^3
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2391, 2379, 2421, 6724, 2376, 2438, 2373, 266}

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^3}\,dx = \frac{2bn\,\text{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)(a+b\log(cx^n))}{d^3r^2} + \frac{3bn\log\left(\frac{dx^{-r}}{e}+1\right)(a+b\log(cx^n))}{d^3r^2} + \frac{benx^r(a+b\log(cx^n))}{d^3r^2(d+ex^r)} - \frac{\log\left(\frac{dx^{-r}}{e}+1\right)(a+b\log(cx^n))^2}{d^3r} + \frac{(a+b\log(cx^n))^2}{d^2r(d+ex^r)} + \frac{(a+b\log(cx^n))^2}{2dr(d+ex^r)^2} - \frac{3b^2n^2\,\text{PolyLog}\left(2,-\frac{dx^{-r}}{e}\right)}{d^3r^3} + \frac{2b^2n^2\,\text{PolyLog}\left(3,-\frac{dx^{-r}}{e}\right)}{d^3r^3} - \frac{b^2n^2\log(d+ex^r)}{d^3r^3}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(x*(d + e*x^r)^3),x]
```

```
[Out] (b*e*n*x^r*(a + b*Log[c*x^n]))/(d^3*r^2*(d + e*x^r)) + (a + b*Log[c*x^n])^2
/(2*d*r*(d + e*x^r)^2) + (a + b*Log[c*x^n])^2/(d^2*r*(d + e*x^r)) + (3*b*n*
(a + b*Log[c*x^n])*Log[1 + d/(e*x^r)])/(d^3*r^2) - ((a + b*Log[c*x^n])^2*Lo
g[1 + d/(e*x^r)])/(d^3*r) - (b^2*n^2*Log[d + e*x^r])/(d^3*r^3) - (3*b^2*n^2
*PolyLog[2, -(d/(e*x^r))])/(d^3*r^3) + (2*b*n*(a + b*Log[c*x^n])*PolyLog[2,
 -(d/(e*x^r))])/(d^3*r^2) + (2*b^2*n^2*PolyLog[3, -(d/(e*x^r))])/(d^3*r^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
```

```
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^
(q_))/(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x^r)^(q + 1)*((a + b*Log[c*x^
n])^p/x), x], x] - Dist[e/d, Int[x^(r - 1)*(d + e*x^r)^q*(a + b*Log[c*x^n])
^p, x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0] && ILtQ[q, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^2}\,dx}{d} - \frac{e\int \frac{x^{-1+r}(a+b\log(cx^n))^2}{(d+ex^r)^3}\,dx}{d}$$

$$= \frac{(a+b\log(cx^n))^2}{2dr(d+ex^r)^2} + \frac{\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)}\,dx}{d^2} - \frac{e\int \frac{x^{-1+r}(a+b\log(cx^n))^2}{(d+ex^r)^2}\,dx}{d^2} - \frac{(bn)\int \frac{a+b\log(cx^n)}{x(d+ex^r)^2}\,dx}{dr}$$

$$= \frac{(a+b\log(cx^n))^2}{2dr(d+ex^r)^2} + \frac{(a+b\log(cx^n))^2}{d^2r(d+ex^r)} - \frac{(a+b\log(cx^n))^2\log\left(1+\frac{dx^{-r}}{e}\right)}{d^3r} + \frac{(2bn)\int\frac{(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{x}dx}{d^3r} - \frac{(bn)\int\frac{a+b\log(cx^n)}{x(d+ex^r)}dx}{d^2r} - \frac{(2bn)\int\frac{a+b\log(cx^n)}{x(d+ex^r)}dx}{d^2r} + \frac{(ben)\int\frac{x^{-1+r}(a+b\log(cx^n))}{(d+ex^r)^2}dx}{d^2r}$$

$$= \frac{benx^r(a+b\log(cx^n))}{d^3r^2(d+ex^r)} + \frac{(a+b\log(cx^n))^2}{2dr(d+ex^r)^2} + \frac{(a+b\log(cx^n))^2}{d^2r(d+ex^r)} + \frac{3bn(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{d^3r^2} - \frac{(a+b\log(cx^n))^2\log\left(1+\frac{dx^{-r}}{e}\right)}{d^3r} + \frac{2bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{dx^{-r}}{e}\right)}{d^3r^2} - \frac{(b^2n^2)\int\frac{\log\left(1+\frac{dx^{-r}}{e}\right)}{x}dx}{d^3r^2} - \frac{(2b^2n^2)\int\frac{\log\left(1+\frac{dx^{-r}}{e}\right)}{x}dx}{d^3r^2} - \frac{(2b^2n^2)\int\frac{\operatorname{Li}_2\left(-\frac{dx^{-r}}{e}\right)}{x}dx}{d^3r^2} - \frac{(b^2en^2)\int\frac{x^{-1+r}}{d+ex^r}dx}{d^3r^2}$$

$$= \frac{benx^r(a+b\log(cx^n))}{d^3r^2(d+ex^r)} + \frac{(a+b\log(cx^n))^2}{2dr(d+ex^r)^2} + \frac{(a+b\log(cx^n))^2}{d^2r(d+ex^r)} + \frac{3bn(a+b\log(cx^n))\log\left(1+\frac{dx^{-r}}{e}\right)}{d^3r^2} - \frac{(a+b\log(cx^n))^2\log\left(1+\frac{dx^{-r}}{e}\right)}{d^3r} - \frac{b^2n^2\log(d+ex^r)}{d^3r^3} - \frac{3b^2n^2\operatorname{Li}_2\left(-\frac{dx^{-r}}{e}\right)}{d^3r^3} + \frac{2bn(a+b\log(cx^n))\operatorname{Li}_2\left(-\frac{dx^{-r}}{e}\right)}{d^3r^2} + \frac{2b^2n^2\operatorname{Li}_3\left(-\frac{dx^{-r}}{e}\right)}{d^3r^3}$$

Mathematica [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.72

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^3}\,dx$$
$$= \frac{\frac{d^2r^2(a+b\log(cx^n))^2}{(d+ex^r)^2} + \frac{2dr(a+b\log(cx^n))(-bn+ar+br\log(cx^n))}{d+ex^r} - 2b^2n^2\log(d-dx^r) + 6abnr\log(d-dx^r) - 2a^2r^2\log(}{}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(x*(d + e*x^r)^3),x]
```

```
[Out] ((d^2*r^2*(a + b*Log[c*x^n])^2)/(d + e*x^r)^2 + (2*d*r*(a + b*Log[c*x^n])*(
-(b*n) + a*r + b*r*Log[c*x^n]))/(d + e*x^r) - 2*b^2*n^2*Log[d - d*x^r] + 6*
a*b*n*r*Log[d - d*x^r] - 2*a^2*r^2*Log[d - d*x^r] + 4*a*b*r^2*(n*Log[x] - L
```

```
og[c*x^n])*Log[d - d*x^r] + 6*b^2*n*r*(-(n*Log[x]) + Log[c*x^n])*Log[d - d*
x^r] - 2*b^2*r^2*(-(n*Log[x]) + Log[c*x^n])^2*Log[d - d*x^r] - 6*b^2*n^2*((
r^2*Log[x]^2)/2 + (-(r*Log[x]) + Log[-((e*x^r)/d)])*Log[d + e*x^r] + PolyLo
g[2, 1 + (e*x^r)/d]) + 4*a*b*n*r*((r^2*Log[x]^2)/2 + (-(r*Log[x]) + Log[-((
e*x^r)/d)])*Log[d + e*x^r] + PolyLog[2, 1 + (e*x^r)/d]) + 4*b^2*n*r*(-(n*Lo
g[x]) + Log[c*x^n])*((r^2*Log[x]^2)/2 + (-(r*Log[x]) + Log[-((e*x^r)/d)])*L
og[d + e*x^r] + PolyLog[2, 1 + (e*x^r)/d]) - 2*b^2*n^2*(r^2*Log[x]^2*Log[1
+ d/(e*x^r)] - 2*r*Log[x]*PolyLog[2, -(d/(e*x^r))] - 2*PolyLog[3, -(d/(e*x^
r))]))/(2*d^3*r^3)
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))^2}{x\,(d+e\,x^r)^3}dx$$

[In] int((a+b*ln(c*x^n))^2/x/(d+e*x^r)^3,x)

[Out] int((a+b*ln(c*x^n))^2/x/(d+e*x^r)^3,x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. $2(263) = 526$.

Time = 0.28 (sec) , antiderivative size = 1165, normalized size of antiderivative = 4.36

$$\int \frac{(a+b\log(cx^n))^2}{x\,(d+ex^r)^3}\,dx = \text{Too large to display}$$

[In] integrate((a+b*log(c*x^n))^2/x/(d+e*x^r)^3,x, algorithm="fricas")

```
[Out] 1/6*(2*b^2*d^2*n^2*r^3*log(x)^3 + 9*b^2*d^2*r^2*log(c)^2 - 6*a*b*d^2*n*r +
9*a^2*d^2*r^2 + 6*(b^2*d^2*n*r^3*log(c) + a*b*d^2*n*r^3)*log(x)^2 + (2*b^2*
e^2*n^2*r^3*log(x)^3 + 3*(2*b^2*e^2*n*r^3*log(c) - 3*b^2*e^2*n^2*r^2 + 2*a*
b*e^2*n*r^3)*log(x)^2 + 6*(b^2*e^2*r^3*log(c)^2 + b^2*e^2*n^2*r - 3*a*b*e^2
*n*r^2 + a^2*e^2*r^3 - (3*b^2*e^2*n*r^2 - 2*a*b*e^2*r^3)*log(c))*log(x))*x^
(2*r) + 2*(2*b^2*d*e*n^2*r^3*log(x)^3 + 3*b^2*d*e*r^2*log(c)^2 - 3*a*b*d*e*
n*r + 3*a^2*d*e*r^2 + 6*(b^2*d*e*n*r^3*log(c) - b^2*d*e*n^2*r^2 + a*b*d*e*n
*r^3)*log(x)^2 - 3*(b^2*d*e*n*r - 2*a*b*d*e*r^2)*log(c) + 3*(2*b^2*d*e*r^3*
log(c)^2 + b^2*d*e*n^2*r - 4*a*b*d*e*n*r^2 + 2*a^2*d*e*r^3 - 4*(b^2*d*e*n*r
^2 - a*b*d*e*r^3)*log(c))*log(x))*x^r - 6*(2*b^2*d^2*n^2*r*log(x) + 2*b^2*d
^2*n*r*log(c) - 3*b^2*d^2*n^2 + 2*a*b*d^2*n*r + (2*b^2*e^2*n^2*r*log(x) + 2
*b^2*e^2*n*r*log(c) - 3*b^2*e^2*n^2 + 2*a*b*e^2*n*r)*x^(2*r) + 2*(2*b^2*d*e
*n^2*r*log(x) + 2*b^2*d*e*n*r*log(c) - 3*b^2*d*e*n^2 + 2*a*b*d*e*n*r)*x^r)*
dilog(-(e*x^r + d)/d + 1) - 6*(b^2*d^2*r^2*log(c)^2 + b^2*d^2*n^2 - 3*a*b*d
^2*n*r + a^2*d^2*r^2 + (b^2*e^2*r^2*log(c)^2 + b^2*e^2*n^2 - 3*a*b*e^2*n*r
+ a^2*e^2*r^2 - (3*b^2*e^2*n*r - 2*a*b*e^2*r^2)*log(c))*x^(2*r) + 2*(b^2*d*
```

```
e*r^2*log(c)^2 + b^2*d*e*n^2 - 3*a*b*d*e*n*r + a^2*d*e*r^2 - (3*b^2*d*e*n*r
 - 2*a*b*d*e*r^2)*log(c))*x^r - (3*b^2*d^2*n*r - 2*a*b*d^2*r^2)*log(c))*log
(e*x^r + d) - 6*(b^2*d^2*n*r - 3*a*b*d^2*r^2)*log(c) + 6*(b^2*d^2*r^3*log(c
)^2 + 2*a*b*d^2*r^3*log(c) + a^2*d^2*r^3)*log(x) - 6*(b^2*d^2*n^2*r^2*log(x
)^2 + (b^2*e^2*n^2*r^2*log(x)^2 + (2*b^2*e^2*n*r^2*log(c) - 3*b^2*e^2*n^2*r
 + 2*a*b*e^2*n*r^2)*log(x))*x^(2*r) + 2*(b^2*d*e*n^2*r^2*log(x)^2 + (2*b^2*
d*e*n*r^2*log(c) - 3*b^2*d*e*n^2*r + 2*a*b*d*e*n*r^2)*log(x))*x^r + (2*b^2*
d^2*n*r^2*log(c) - 3*b^2*d^2*n^2*r + 2*a*b*d^2*n*r^2)*log(x))*log((e*x^r +
d)/d) + 12*(b^2*e^2*n^2*x^(2*r) + 2*b^2*d*e*n^2*x^r + b^2*d^2*n^2)*polylog(
3, -e*x^r/d))/(d^3*e^2*r^3*x^(2*r) + 2*d^4*e*r^3*x^r + d^5*r^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*x**n))**2/x/(d+e*x**r)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^3}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex^r+d)^3x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(d+e*x^r)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*((2*e*x^r + 3*d)/(d^2*e^2*r*x^(2*r) + 2*d^3*e*r*x^r + d^4*r) + 2*lo
g(x)/d^3 - 2*log((e*x^r + d)/e)/(d^3*r)) + integrate((b^2*log(c)^2 + b^2*lo
g(x^n)^2 + 2*a*b*log(c) + 2*(b^2*log(c) + a*b)*log(x^n))/(e^3*x*x^(3*r) + 3
*d*e^2*x*x^(2*r) + 3*d^2*e*x*x^r + d^3*x), x)
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^2}{x(d+ex^r)^3}\,dx = \int \frac{(b\log(cx^n)+a)^2}{(ex^r+d)^3x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^2/x/(d+e*x^r)^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2/((e*x^r + d)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(cx^n))^2}{x(d + ex^r)^3} \, dx = \int \frac{(a + b\ln(c\,x^n))^2}{x(d + e\,x^r)^3} \, dx$$

```
[In] int((a + b*log(c*x^n))^2/(x*(d + e*x^r)^3),x)
[Out] int((a + b*log(c*x^n))^2/(x*(d + e*x^r)^3), x)
```

3.433 $\int \frac{(d+ex^r)^{5/2}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 327

$$\int \frac{(d+ex^r)^{5/2}(a+b\log(cx^n))}{x}\,dx = -\frac{92bd^2n\sqrt{d+ex^r}}{15r^2} - \frac{32bdn(d+ex^r)^{3/2}}{45r^2} - \frac{4bn(d+ex^r)^{5/2}}{25r^2} + \frac{92bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{15r^2} + \frac{2bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2} + \frac{2}{15}\left(\frac{15d^2\sqrt{d+ex^r}}{r} + \frac{5d(d+ex^r)^{3/2}}{r} + \frac{3(d+ex^r)^{5/2}}{r} - \frac{15d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)) - \frac{4bd^{5/2}na}{}$$

```
[Out] -32/45*b*d*n*(d+e*x^r)^(3/2)/r^2-4/25*b*n*(d+e*x^r)^(5/2)/r^2+92/15*b*d^(5/
2)*n*arctanh((d+e*x^r)^(1/2)/d^(1/2))/r^2+2*b*d^(5/2)*n*arctanh((d+e*x^r)^(
1/2)/d^(1/2))^2/r^2-4*b*d^(5/2)*n*arctanh((d+e*x^r)^(1/2)/d^(1/2))*ln(2*d^(
1/2)/(d^(1/2)-(d+e*x^r)^(1/2)))/r^2-2*b*d^(5/2)*n*polylog(2,1-2*d^(1/2)/(d^
(1/2)-(d+e*x^r)^(1/2)))/r^2-92/15*b*d^2*n*(d+e*x^r)^(1/2)/r^2+2/15*(a+b*ln(
c*x^n))*(5*d*(d+e*x^r)^(3/2)/r+3*(d+e*x^r)^(5/2)/r-15*d^(5/2)*arctanh((d+e*
x^r)^(1/2)/d^(1/2))/r+15*d^2*(d+e*x^r)^(1/2)/r)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used

= {272, 52, 65, 214, 2390, 6131, 6055, 2449, 2352}

$$\int \frac{(d+ex^r)^{5/2}(a+b\log(cx^n))}{x}\,dx = \frac{2}{15}\left(-\frac{15d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r} + \frac{15d^2\sqrt{d+ex^r}}{r} + \frac{5d(d+ex^r)^{3/2}}{r} + \frac{3(d+ex^r)^{5/2}}{r}\right)(a+b\log(cx^n)) + \frac{2bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2} + \frac{92bd^{5/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{15r^2} - \frac{4bd^{5/2}n\operatorname{arctanh}}{}$$

```
[In] Int[((d + e*x^r)^(5/2)*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] (-92*b*d^2*n*Sqrt[d + e*x^r])/(15*r^2) - (32*b*d*n*(d + e*x^r)^(3/2))/(45*r
^2) - (4*b*n*(d + e*x^r)^(5/2))/(25*r^2) + (92*b*d^(5/2)*n*ArcTanh[Sqrt[d +
 e*x^r]/Sqrt[d]])/(15*r^2) + (2*b*d^(5/2)*n*ArcTanh[Sqrt[d + e*x^r]/Sqrt[d]
]^2)/r^2 + (2*((15*d^2*Sqrt[d + e*x^r])/r + (5*d*(d + e*x^r)^(3/2))/r + (3*
(d + e*x^r)^(5/2))/r - (15*d^(5/2)*ArcTanh[Sqrt[d + e*x^r]/Sqrt[d]])/r)*(a
+ b*Log[c*x^n]))/15 - (4*b*d^(5/2)*n*ArcTanh[Sqrt[d + e*x^r]/Sqrt[d]]*Log[(
2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^r])])/r^2 - (2*b*d^(5/2)*n*PolyLog[2, 1
- (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^r])])/r^2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{2}{15}\left(\frac{15d^2\sqrt{d+ex^r}}{r}+\frac{5d(d+ex^r)^{3/2}}{r}+\frac{3(d+ex^r)^{5/2}}{r}-\frac{15d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n))-(bn)\int\left(\frac{2d^2\sqrt{d+ex^r}}{rx}+\frac{2d(d+ex^r)^{3/2}}{3rx}+\frac{2(d+ex^r)^{5/2}}{5rx}-\cdots\right.$$

$$= \frac{2}{15}\left(\frac{15d^2\sqrt{d+ex^r}}{r}+\frac{5d(d+ex^r)^{3/2}}{r}+\frac{3(d+ex^r)^{5/2}}{r}\right.$$

$$\left.-\frac{15d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n))-\frac{(2bn)\int\frac{(d+ex^r)^{5/2}}{x}\,dx}{5r}-\frac{(2bdn)\int\frac{(d+ex^r)^{3/2}}{x}\,dx}{3r}-\frac{(2bd$$

$$= \frac{2}{15}\left(\frac{15d^2\sqrt{d+ex^r}}{r}+\frac{5d(d+ex^r)^{3/2}}{r}+\frac{3(d+ex^r)^{5/2}}{r}\right.$$

$$\left.-\frac{15d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n))-\frac{(2bn)\text{Subst}\left(\int\frac{(d+ex)^{5/2}}{x}\,dx,x,x^r\right)}{5r^2}-\frac{(2bdn)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{4bd^2n\sqrt{d+ex^r}}{r^2}-\frac{4bdn(d+ex^r)^{3/2}}{9r^2}-\frac{4bn(d+ex^r)^{5/2}}{25r^2}$$

$$+\frac{2}{15}\left(\frac{15d^2\sqrt{d+ex^r}}{r}+\frac{5d(d+ex^r)^{3/2}}{r}+\frac{3(d+ex^r)^{5/2}}{r}-\frac{15d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)$$

$$= -\frac{16bd^2n\sqrt{d+ex^r}}{3r^2}-\frac{32bdn(d+ex^r)^{3/2}}{45r^2}-\frac{4bn(d+ex^r)^{5/2}}{25r^2}+\frac{2bd^{5/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2}$$

$$+\frac{2}{15}\left(\frac{15d^2\sqrt{d+ex^r}}{r}+\frac{5d(d+ex^r)^{3/2}}{r}+\frac{3(d+ex^r)^{5/2}}{r}-\frac{15d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)$$

$$= -\frac{92bd^2n\sqrt{d+ex^r}}{15r^2}-\frac{32bdn(d+ex^r)^{3/2}}{45r^2}-\frac{4bn(d+ex^r)^{5/2}}{25r^2}$$

$$+\frac{4bd^{5/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r^2}+\frac{2bd^{5/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2}$$

$$+\frac{2}{15}\left(\frac{15d^2\sqrt{d+ex^r}}{r}+\frac{5d(d+ex^r)^{3/2}}{r}+\frac{3(d+ex^r)^{5/2}}{r}-\frac{15d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)$$

$$= -\frac{92bd^2n\sqrt{d+ex^r}}{15r^2} - \frac{32bdn(d+ex^r)^{3/2}}{45r^2} - \frac{4bn(d+ex^r)^{5/2}}{25r^2} + \frac{16bd^{5/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{3r^2} + \frac{2bd^{5/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2} + \frac{2}{15}\left(\frac{15d^2\sqrt{d+ex^r}}{r} + \frac{5d(d+ex^r)^{3/2}}{r} + \frac{3(d+ex^r)^{5/2}}{r} - \frac{15d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n))$$

$$= -\frac{92bd^2n\sqrt{d+ex^r}}{15r^2} - \frac{32bdn(d+ex^r)^{3/2}}{45r^2} - \frac{4bn(d+ex^r)^{5/2}}{25r^2} + \frac{92bd^{5/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{15r^2} + \frac{2bd^{5/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2} + \frac{2}{15}\left(\frac{15d^2\sqrt{d+ex^r}}{r} + \frac{5d(d+ex^r)^{3/2}}{r} + \frac{3(d+ex^r)^{5/2}}{r} - \frac{15d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n))$$

Mathematica [F]

$$\int \frac{(d+ex^r)^{5/2}(a+b\log(cx^n))}{x}\,dx = \int \frac{(d+ex^r)^{5/2}(a+b\log(cx^n))}{x}\,dx$$

```
[In] Integrate[((d + e*x^r)^(5/2)*(a + b*Log[c*x^n]))/x,x]
[Out] Integrate[((d + e*x^r)^(5/2)*(a + b*Log[c*x^n]))/x, x]
```

Maple [F]

$$\int \frac{(d+e\,x^r)^{\frac{5}{2}}(a+b\ln(c\,x^n))}{x}dx$$

```
[In] int((d+e*x^r)^(5/2)*(a+b*ln(c*x^n))/x,x)
[Out] int((d+e*x^r)^(5/2)*(a+b*ln(c*x^n))/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)^{5/2}(a+b\log(cx^n))}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+e*x^r)^(5/2)*(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^{5/2}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x**r)**(5/2)*(a+b*ln(c*x**n))/x,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^r)^{5/2}(a+b\log(cx^n))}{x}\,dx = \int \frac{(ex^r+d)^{\frac{5}{2}}(b\log(cx^n)+a)}{x}\,dx$$

```
[In] integrate((d+e*x^r)^(5/2)*(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] 1/15*(15*d^(5/2)*log((sqrt(e*x^r + d) - sqrt(d))/(sqrt(e*x^r + d) + sqrt(d)
))/r + 2*(3*(e*x^r + d)^(5/2) + 5*(e*x^r + d)^(3/2)*d + 15*sqrt(e*x^r + d)*
d^2)/r)*a + b*integrate((e^2*x^(2*r)*log(c) + 2*d*e*x^r*log(c) + d^2*log(c)
 + (e^2*x^(2*r) + 2*d*e*x^r + d^2)*log(x^n))*sqrt(e*x^r + d)/x, x)
```

Giac [F]

$$\int \frac{(d+ex^r)^{5/2}(a+b\log(cx^n))}{x}\,dx = \int \frac{(ex^r+d)^{\frac{5}{2}}(b\log(cx^n)+a)}{x}\,dx$$

```
[In] integrate((d+e*x^r)^(5/2)*(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^r + d)^(5/2)*(b*log(c*x^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^{5/2}\,(a+b\log(cx^n))}{x}\,dx=\int \frac{(d+e\,x^r)^{5/2}\,(a+b\,\ln(c\,x^n))}{x}\,dx$$

```
[In] int(((d + e*x^r)^(5/2)*(a + b*log(c*x^n)))/x,x)
[Out] int(((d + e*x^r)^(5/2)*(a + b*log(c*x^n)))/x, x)
```

3.434 $\int \frac{(d+ex^r)^{3/2}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 284

$$\int \frac{(d+ex^r)^{3/2}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = -\frac{16bdn\sqrt{d+ex^r}}{3r^2}-\frac{4bn(d+ex^r)^{3/2}}{9r^2} + \frac{16bd^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{3r^2}+\frac{2bd^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2} + \frac{2}{3}\left(\frac{3d\sqrt{d+ex^r}}{r}+\frac{(d+ex^r)^{3/2}}{r}-\frac{3d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{4bd^{3/2}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\left(\right.}{r^2}$$

```
[Out] -4/9*b*n*(d+e*x^r)^(3/2)/r^2+16/3*b*d^(3/2)*n*arctanh((d+e*x^r)^(1/2)/d^(1/
2))/r^2+2*b*d^(3/2)*n*arctanh((d+e*x^r)^(1/2)/d^(1/2))^2/r^2-4*b*d^(3/2)*n*
arctanh((d+e*x^r)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)-(d+e*x^r)^(1/2)))/r^
2-2*b*d^(3/2)*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-(d+e*x^r)^(1/2)))/r^2-16/3*b
*d*n*(d+e*x^r)^(1/2)/r^2+2/3*(a+b*ln(c*x^n))*((d+e*x^r)^(3/2)/r-3*d^(3/2)*a
rctanh((d+e*x^r)^(1/2)/d^(1/2))/r+3*d*(d+e*x^r)^(1/2)/r)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used

$= \{272, 52, 65, 214, 2390, 6131, 6055, 2449, 2352\}$

$$\int \frac{(d+ex^r)^{3/2}(a+b\log(cx^n))}{x}\,dx = \frac{2}{3}\left(-\frac{3d^{3/2}\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}+\frac{3d\sqrt{d+ex^r}}{r}+\frac{(d+ex^r)^{3/2}}{r}\right)(a+b\log(cx^n))+\frac{2bd^{3/2}n\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2}+\frac{16bd^{3/2}n\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{3r^2}-\frac{4bd^{3/2}n\text{arctanh}\left(\cdots\right.}{}$$

```
[In] Int[((d + e*x^r)^(3/2)*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] (-16*b*d*n*Sqrt[d + e*x^r])/(3*r^2) - (4*b*n*(d + e*x^r)^(3/2))/(9*r^2) + (
16*b*d^(3/2)*n*ArcTanh[Sqrt[d + e*x^r]/Sqrt[d]])/(3*r^2) + (2*b*d^(3/2)*n*A
rcTanh[Sqrt[d + e*x^r]/Sqrt[d]]^2)/r^2 + (2*((3*d*Sqrt[d + e*x^r])/r + (d +
 e*x^r)^(3/2)/r - (3*d^(3/2)*ArcTanh[Sqrt[d + e*x^r]/Sqrt[d]])/r)*(a + b*Lo
g[c*x^n]))/3 - (4*b*d^(3/2)*n*ArcTanh[Sqrt[d + e*x^r]/Sqrt[d]]*Log[(2*Sqrt[
d])/(Sqrt[d] - Sqrt[d + e*x^r])])/r^2 - (2*b*d^(3/2)*n*PolyLog[2, 1 - (2*Sq
rt[d])/(Sqrt[d] - Sqrt[d + e*x^r])])/r^2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{3}\left(\frac{3d\sqrt{d+ex^r}}{r}+\frac{(d+ex^r)^{3/2}}{r}-\frac{3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)) \\
&\quad-(bn)\int\left(\frac{2d\sqrt{d+ex^r}}{rx}+\frac{2(d+ex^r)^{3/2}}{3rx}-\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{rx}\right)dx \\
&= \frac{2}{3}\left(\frac{3d\sqrt{d+ex^r}}{r}+\frac{(d+ex^r)^{3/2}}{r}-\frac{3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)) \\
&\quad-\frac{(2bn)\int\frac{(d+ex^r)^{3/2}}{x}\,dx}{3r}-\frac{(2bdn)\int\frac{\sqrt{d+ex^r}}{x}\,dx}{r}+\frac{\left(2bd^{3/2}n\right)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{x}\,dx}{r}
\end{aligned}$$

$$= \frac{2}{3}\left(\frac{3d\sqrt{d+ex^r}}{r} + \frac{(d+ex^r)^{3/2}}{r} - \frac{3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)) - \frac{(2bn)\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x}\,dx, x, x^r\right)}{3r^2} - \frac{(2bdn)\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x}\,dx, x, x^r\right)}{r^2} + \frac{(2bd^{3/2}n)\,\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx, x, x^r\right)}{r^2}$$

$$= -\frac{4bdn\sqrt{d+ex^r}}{r^2} - \frac{4bn(d+ex^r)^{3/2}}{9r^2} + \frac{2}{3}\left(\frac{3d\sqrt{d+ex^r}}{r} + \frac{(d+ex^r)^{3/2}}{r} - \frac{3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)) - \frac{(2bdn)\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x}\,dx, x, x^r\right)}{3r^2} + \frac{(4bd^{3/2}n)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{16bdn\sqrt{d+ex^r}}{3r^2} - \frac{4bn(d+ex^r)^{3/2}}{9r^2} + \frac{2bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2} + \frac{2}{3}\left(\frac{3d\sqrt{d+ex^r}}{r} + \frac{(d+ex^r)^{3/2}}{r} - \frac{3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)) - \frac{(4bdn)\text{Subst}\left(\int \frac{\tanh^{-1}}{1-}\right.}{}$$

$$= -\frac{16bdn\sqrt{d+ex^r}}{3r^2} - \frac{4bn(d+ex^r)^{3/2}}{9r^2} + \frac{4bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r^2} + \frac{2bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2}$$

$$+ \frac{2}{3}\left(\frac{3d\sqrt{d+ex^r}}{r} + \frac{(d+ex^r)^{3/2}}{r} - \frac{3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)) - \frac{4bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+}}{\sqrt{}}\right.}{r}$$

$$= -\frac{16bdn\sqrt{d+ex^r}}{3r^2} - \frac{4bn(d+ex^r)^{3/2}}{9r^2} + \frac{16bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{3r^2} + \frac{2bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2} + \frac{2}{3}\left(\frac{3d\sqrt{d+ex^r}}{r} + \frac{(d+ex^r)^{3/2}}{r} - \frac{3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)) - \frac{4bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+}}{\sqrt{}}\right.}{r}$$

$$= -\frac{16bdn\sqrt{d+ex^r}}{3r^2} - \frac{4bn(d+ex^r)^{3/2}}{9r^2} + \frac{16bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{3r^2} + \frac{2bd^{3/2}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2} + \frac{2}{3}\left(\frac{3d\sqrt{d+ex^r}}{r} + \frac{(d+ex^r)^{3/2}}{r} - \frac{3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)) - \frac{4bd^{3/2}n\tanh^{-1}\left(\sqrt{d}\right.}{}$$

Mathematica [F]

$$\int \frac{(d+ex^r)^{3/2}(a+b\log(cx^n))}{x}\,dx = \int \frac{(d+ex^r)^{3/2}(a+b\log(cx^n))}{x}\,dx$$

```
[In] Integrate[((d + e*x^r)^(3/2)*(a + b*Log[c*x^n]))/x,x]
[Out] Integrate[((d + e*x^r)^(3/2)*(a + b*Log[c*x^n]))/x, x]
```

Maple [F]

$$\int \frac{(d+e\,x^r)^{\frac{3}{2}}\,(a+b\ln(c\,x^n))}{x}dx$$

```
[In] int((d+e*x^r)^(3/2)*(a+b*ln(c*x^n))/x,x)
[Out] int((d+e*x^r)^(3/2)*(a+b*ln(c*x^n))/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d+ex^r)^{3/2}(a+b\log(cx^n))}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d+e*x^r)^(3/2)*(a+b*log(c*x^n))/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(d+ex^r)^{3/2}\,(a+b\log{(cx^n)})}{x}\,dx = \int \frac{(a+b\log{(cx^n)})\,(d+ex^r)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((d+e*x**r)**(3/2)*(a+b*ln(c*x**n))/x,x)
[Out] Integral((a + b*log(c*x**n))*(d + e*x**r)**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{(d+ex^r)^{3/2}\,(a+b\log{(cx^n)})}{x}\,dx = \int \frac{(ex^r+d)^{\frac{3}{2}}(b\log{(cx^n)}+a)}{x}\,dx$$

```
[In] integrate((d+e*x^r)^(3/2)*(a+b*log(c*x^n))/x,x, algorithm="maxima")
[Out] 1/3*(3*d^(3/2)*log((sqrt(e*x^r + d) - sqrt(d))/(sqrt(e*x^r + d) + sqrt(d)))
/r + 2*((e*x^r + d)^(3/2) + 3*sqrt(e*x^r + d)*d)/r)*a + b*integrate((e*x^r*
log(c) + d*log(c) + (e*x^r + d)*log(x^n))*sqrt(e*x^r + d)/x, x)
```

Giac [F]

$$\int \frac{(d+ex^r)^{3/2}\,(a+b\log{(cx^n)})}{x}\,dx = \int \frac{(ex^r+d)^{\frac{3}{2}}(b\log{(cx^n)}+a)}{x}\,dx$$

```
[In] integrate((d+e*x^r)^(3/2)*(a+b*log(c*x^n))/x,x, algorithm="giac")
[Out] integrate((e*x^r + d)^(3/2)*(b*log(c*x^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^r)^{3/2}\,(a+b\log{(cx^n)})}{x}\,dx = \int \frac{(d+e\,x^r)^{3/2}\,(a+b\,\ln{(c\,x^n)})}{x}\,dx$$

```
[In] int(((d + e*x^r)^(3/2)*(a + b*log(c*x^n)))/x,x)
[Out] int(((d + e*x^r)^(3/2)*(a + b*log(c*x^n)))/x, x)
```

3.435 $\int \frac{\sqrt{d+ex^r}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 240

$$\int \frac{\sqrt{d+ex^r}(a+b\log(cx^n))}{x}\,dx = -\frac{4bn\sqrt{d+ex^r}}{r^2}+\frac{4b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r^2}+\frac{2b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2}+2\left(\frac{\sqrt{d+ex^r}}{r}-\frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n))-\frac{4b\sqrt{d}n\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^r}}\right)}{r^2}-\frac{2b\sqrt{d}n\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^r}}\right)}{r^2}$$

```
[Out] 4*b*n*arctanh((d+e*x^r)^(1/2)/d^(1/2))*d^(1/2)/r^2+2*b*n*arctanh((d+e*x^r)^
(1/2)/d^(1/2))^2*d^(1/2)/r^2-4*b*n*arctanh((d+e*x^r)^(1/2)/d^(1/2))*ln(2*d^
(1/2)/(d^(1/2)-(d+e*x^r)^(1/2)))*d^(1/2)/r^2-2*b*n*polylog(2,1-2*d^(1/2)/(d
^(1/2)-(d+e*x^r)^(1/2)))*d^(1/2)/r^2-4*b*n*(d+e*x^r)^(1/2)/r^2+2*(a+b*ln(c*
x^n))*(-arctanh((d+e*x^r)^(1/2)/d^(1/2))*d^(1/2)/r+(d+e*x^r)^(1/2)/r)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {272, 52, 65, 214, 2390, 6131, 6055, 2449, 2352}

$$\int \frac{\sqrt{d+ex^r}(a+b\log(cx^n))}{x}\,dx = 2\left(\frac{\sqrt{d+ex^r}}{r}-\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)) +\frac{2b\sqrt{d}n\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2}+\frac{4b\sqrt{d}n\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r^2} -\frac{4b\sqrt{d}n\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^r}}\right)}{r^2} -\frac{2b\sqrt{d}n\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{ex^r+d}}\right)}{r^2}-\frac{4bn\sqrt{d+ex^r}}{r^2}$$

```
[In] Int[(Sqrt[d + e*x^r]*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] (-4*b*n*Sqrt[d + e*x^r])/r^2 + (4*b*Sqrt[d]*n*ArcTanh[Sqrt[d + e*x^r]/Sqrt[
d]])/r^2 + (2*b*Sqrt[d]*n*ArcTanh[Sqrt[d + e*x^r]/Sqrt[d]]^2)/r^2 + 2*(Sqrt
[d + e*x^r]/r - (Sqrt[d]*ArcTanh[Sqrt[d + e*x^r]/Sqrt[d]])/r)*(a + b*Log[c*
x^n]) - (4*b*Sqrt[d]*n*ArcTanh[Sqrt[d + e*x^r]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sq
rt[d] - Sqrt[d + e*x^r])])/r^2 - (2*b*Sqrt[d]*n*PolyLog[2, 1 - (2*Sqrt[d])/
(Sqrt[d] - Sqrt[d + e*x^r])])/r^2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = 2\left(\frac{\sqrt{d+ex^r}}{r} - \frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)) - (bn)\int\left(\frac{2\sqrt{d+ex^r}}{rx} - \frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{rx}\right)dx$$

$$= 2\left(\frac{\sqrt{d+ex^r}}{r} - \frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)) - \frac{(2bn)\int\frac{\sqrt{d+ex^r}}{x}\,dx}{r} + \frac{\left(2b\sqrt{d}n\right)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{x}\,dx}{r}$$

$$= 2\left(\frac{\sqrt{d+ex^r}}{r} - \frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)) - \frac{(2bn)\text{Subst}\left(\int\frac{\sqrt{d+ex}}{x}\,dx, x, x^r\right)}{r^2} + \frac{\left(2b\sqrt{d}n\right)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx, x, x^r\right)}{r^2}$$

$$= -\frac{4bn\sqrt{d+ex^r}}{r^2} + 2\left(\frac{\sqrt{d+ex^r}}{r} - \frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)) + \frac{\left(4b\sqrt{d}n\right)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx, x, \sqrt{d+ex^r}\right)}{r^2} - \frac{(2bdn)\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx, x, x^r\right)}{r^2}$$

$$= -\frac{4bn\sqrt{d+ex^r}}{r^2} + \frac{2b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2} + 2\left(\frac{\sqrt{d+ex^r}}{r} - \frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n)) - \frac{(4bn)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx, x, \sqrt{d+ex^r}\right)}{r^2} - \frac{(4bdn)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^r}\right)}{er^2}$$

$$= -\frac{4bn\sqrt{d+ex^r}}{r^2} + \frac{4b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r^2} + \frac{2b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2}$$
$$+ 2\left(\frac{\sqrt{d+ex^r}}{r} - \frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n))$$
$$- \frac{4b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^r}}\right)}{r^2}$$
$$+ \frac{(4bn)\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}\,dx, x, \sqrt{d+ex^r}\right)}{r^2}$$

$$= -\frac{4bn\sqrt{d+ex^r}}{r^2} + \frac{4b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r^2} + \frac{2b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2}$$
$$+ 2\left(\frac{\sqrt{d+ex^r}}{r} - \frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n))$$
$$- \frac{4b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^r}}\right)}{r^2}$$
$$- \frac{\left(4b\sqrt{d}n\right)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{\sqrt{d+ex^r}}{\sqrt{d}}}\right)}{r^2}$$

$$= -\frac{4bn\sqrt{d+ex^r}}{r^2} + \frac{4b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r^2} + \frac{2b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{r^2}$$
$$+ 2\left(\frac{\sqrt{d+ex^r}}{r} - \frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{r}\right)(a+b\log(cx^n))$$
$$- \frac{4b\sqrt{d}n\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^r}}\right)}{r^2} - \frac{2b\sqrt{d}n\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex^r}}{\sqrt{d}}}\right)}{r^2}$$

Mathematica [F]

$$\int \frac{\sqrt{d+ex^r}(a+b\log(cx^n))}{x}\,dx=\int \frac{\sqrt{d+ex^r}(a+b\log(cx^n))}{x}\,dx$$

```
[In] Integrate[(Sqrt[d + e*x^r]*(a + b*Log[c*x^n]))/x,x]
[Out] Integrate[(Sqrt[d + e*x^r]*(a + b*Log[c*x^n]))/x, x]
```

Maple [F]

$$\int \frac{\sqrt{d+e\,x^r}\,(a+b\ln(c\,x^n))}{x}dx$$

```
[In] int((d+e*x^r)^(1/2)*(a+b*ln(c*x^n))/x,x)
[Out] int((d+e*x^r)^(1/2)*(a+b*ln(c*x^n))/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^r}(a+b\log(cx^n))}{x}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((d+e*x^r)^(1/2)*(a+b*log(c*x^n))/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^r}(a+b\log(cx^n))}{x}\,dx=\int \frac{(a+b\log(cx^n))\sqrt{d+ex^r}}{x}\,dx$$

```
[In] integrate((d+e*x**r)**(1/2)*(a+b*ln(c*x**n))/x,x)
[Out] Integral((a + b*log(c*x**n))*sqrt(d + e*x**r)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^r}(a+b\log{(cx^n)})}{x}\,dx = \int \frac{\sqrt{ex^r+d}(b\log{(cx^n)}+a)}{x}\,dx$$

```
[In] integrate((d+e*x^r)^(1/2)*(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] a*(sqrt(d)*log((sqrt(e*x^r + d) - sqrt(d))/(sqrt(e*x^r + d) + sqrt(d)))/r +
 2*sqrt(e*x^r + d)/r) + b*integrate(sqrt(e*x^r + d)*(log(c) + log(x^n))/x,
x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex^r}(a+b\log{(cx^n)})}{x}\,dx = \int \frac{\sqrt{ex^r+d}(b\log{(cx^n)}+a)}{x}\,dx$$

```
[In] integrate((d+e*x^r)^(1/2)*(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^r + d)*(b*log(c*x^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^r}(a+b\log{(cx^n)})}{x}\,dx = \int \frac{\sqrt{d+e\,x^r}\,\left(a+b\,\ln{\left(c\,x^n\right)}\right)}{x}\,dx$$

```
[In] int(((d + e*x^r)^(1/2)*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int(((d + e*x^r)^(1/2)*(a + b*log(c*x^n)))/x, x)
```

3.436 $\int \frac{a+b\log(cx^n)}{x\sqrt{d+ex^r}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{a+b\log(cx^n)}{x\sqrt{d+ex^r}}\,dx = \frac{2bn\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{\sqrt{d}r^2} - \frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}r} - \frac{4bn\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^r}}\right)}{\sqrt{d}r^2} - \frac{2bn\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^r}}\right)}{\sqrt{d}r^2}$$

```
[Out] 2*b*n*arctanh((d+e*x^r)^(1/2)/d^(1/2))^2/r^2/d^(1/2)-2*arctanh((d+e*x^r)^(1
/2)/d^(1/2))*(a+b*ln(c*x^n))/r/d^(1/2)-4*b*n*arctanh((d+e*x^r)^(1/2)/d^(1/2
))*ln(2*d^(1/2)/(d^(1/2)-(d+e*x^r)^(1/2)))/r^2/d^(1/2)-2*b*n*polylog(2,1-2*
d^(1/2)/(d^(1/2)-(d+e*x^r)^(1/2)))/r^2/d^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {272, 65, 214, 2390, 12, 6131, 6055, 2449, 2352}

$$\int \frac{a+b\log(cx^n)}{x\sqrt{d+ex^r}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}r} + \frac{2bn\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{\sqrt{d}r^2} - \frac{4bn\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^r}}\right)}{\sqrt{d}r^2} - \frac{2bn\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{ex^r+d}}\right)}{\sqrt{d}r^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*Sqrt[d + e*x^r]),x]
```

```
[Out] (2*b*n*ArcTanh[Sqrt[d + e*x^r]/Sqrt[d]]^2)/(Sqrt[d]*r^2) - (2*ArcTanh[Sqrt[
d + e*x^r]/Sqrt[d]]*(a + b*Log[c*x^n]))/(Sqrt[d]*r) - (4*b*n*ArcTanh[Sqrt[d
 + e*x^r]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^r])])/(Sqrt[d]*r
^2) - (2*b*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^r])])/(Sqrt
[d]*r^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}r} - (bn)\int -\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{\sqrt{d}rx}\,dx \\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}r} + \frac{(2bn)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{x}\,dx}{\sqrt{d}r} \\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}r} + \frac{(2bn)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx, x, x^r\right)}{\sqrt{d}r^2}
\end{aligned}$$

$$=-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}r}+\frac{(4bn)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}dx,x,\sqrt{d+ex^r}\right)}{\sqrt{d}r^2}$$

$$=\frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{\sqrt{d}r^2}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}r}-\frac{(4bn)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}dx,x,\sqrt{d+ex^r}\right)}{dr^2}$$

$$=\frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{\sqrt{d}r^2}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}r}-\frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^r}}\right)}{\sqrt{d}r^2}+\frac{(4bn)\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-\frac{x}{\sqrt{d}}}\right)}{1-\frac{x^2}{d}}dx,x,\sqrt{d+ex^r}\right)}{dr^2}$$

$$=\frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{\sqrt{d}r^2}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}r}-\frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^r}}\right)}{\sqrt{d}r^2}-\frac{(4bn)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}dx,x,\frac{1}{1-\frac{\sqrt{d+ex^r}}{\sqrt{d}}}\right)}{\sqrt{d}r^2}$$

$$=\frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{\sqrt{d}r^2}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)(a+b\log(cx^n))}{\sqrt{d}r}-\frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^r}}\right)}{\sqrt{d}r^2}-\frac{2bn\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{d+ex^r}}{\sqrt{d}}}\right)}{\sqrt{d}r^2}$$

Mathematica [F]

$$\int\frac{a+b\log(cx^n)}{x\sqrt{d+ex^r}}dx=\int\frac{a+b\log(cx^n)}{x\sqrt{d+ex^r}}dx$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*Sqrt[d + e*x^r]),x]
[Out] Integrate[(a + b*Log[c*x^n])/(x*Sqrt[d + e*x^r]), x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c\,x^{n}\right)}{x\sqrt{d+e\,x^{r}}}dx$$

[In] int((a+b*ln(c*x^n))/x/(d+e*x^r)^(1/2),x)

[Out] int((a+b*ln(c*x^n))/x/(d+e*x^r)^(1/2),x)

Fricas [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(cx^{n}\right)}{x\sqrt{d+ex^{r}}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

Sympy [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x\sqrt{d+ex^{r}}}\,dx = \int \frac{a+b\log\left(cx^{n}\right)}{x\sqrt{d+ex^{r}}}\,dx$$

[In] integrate((a+b*ln(c*x**n))/x/(d+e*x**r)**(1/2),x)

[Out] Integral((a + b*log(c*x**n))/(x*sqrt(d + e*x**r)), x)

Maxima [F]

$$\int \frac{a+b\log\left(cx^{n}\right)}{x\sqrt{d+ex^{r}}}\,dx = \int \frac{b\log\left(cx^{n}\right)+a}{\sqrt{ex^{r}+d}x}\,dx$$

[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^(1/2),x, algorithm="maxima")

[Out] b*integrate((log(c) + log(x^n))/(sqrt(e*x^r + d)*x), x) + a*log((sqrt(e*x^r + d) - sqrt(d))/(sqrt(e*x^r + d) + sqrt(d)))/(sqrt(d)*r)

Giac [F]

$$\int \frac{a + b\log(cx^n)}{x\sqrt{d + ex^r}}\,dx = \int \frac{b\log(cx^n) + a}{\sqrt{ex^r + d}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^(1/2),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/(sqrt(e*x^r + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{x\sqrt{d + ex^r}}\,dx = \int \frac{a + b\,\ln(c\,x^n)}{x\,\sqrt{d + e\,x^r}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x^r)^(1/2)),x)
[Out] int((a + b*log(c*x^n))/(x*(d + e*x^r)^(1/2)), x)
```

3.437 $\int \frac{a+b\log(cx^n)}{x(d+ex^r)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 225

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^{3/2}}\,dx = \frac{4bn\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{3/2}r^2}+\frac{2bn\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{3/2}r^2} +2\left(\frac{1}{dr\sqrt{d+ex^r}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{3/2}r}\right)(a+b\log(cx^n))-\frac{4bn\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^r}}\right)}{d^{3/2}r^2}-\frac{2bn\operatorname{PolyLo}}{}$$

```
[Out] 4*b*n*arctanh((d+e*x^r)^(1/2)/d^(1/2))/d^(3/2)/r^2+2*b*n*arctanh((d+e*x^r)^
(1/2)/d^(1/2))^2/d^(3/2)/r^2-4*b*n*arctanh((d+e*x^r)^(1/2)/d^(1/2))*ln(2*d^
(1/2)/(d^(1/2)-(d+e*x^r)^(1/2)))/d^(3/2)/r^2-2*b*n*polylog(2,1-2*d^(1/2)/(d
^(1/2)-(d+e*x^r)^(1/2)))/d^(3/2)/r^2+2*(a+b*ln(c*x^n))*(-arctanh((d+e*x^r)^
(1/2)/d^(1/2))/d^(3/2)/r+1/d/r/(d+e*x^r)^(1/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {272, 53, 65, 214, 2390, 6131, 6055, 2449, 2352}

$$\begin{aligned}\int \frac{a+b\log(cx^n)}{x(d+ex^r)^{3/2}}\,dx &= 2\left(\frac{1}{dr\sqrt{d+ex^r}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{3/2}r}\right)(a+b\log(cx^n)) \\ &+\frac{2bn\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{3/2}r^2}+\frac{4bn\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{3/2}r^2} \\ &-\frac{4bn\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^r}}\right)}{d^{3/2}r^2}-\frac{2bn\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{ex^r+d}}\right)}{d^{3/2}r^2}\end{aligned}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x^r)^(3/2)),x]
```

```
[Out] (4*b*n*ArcTanh[Sqrt[d + e*x^r]/Sqrt[d]])/(d^(3/2)*r^2) + (2*b*n*ArcTanh[Sqr
t[d + e*x^r]/Sqrt[d]]^2)/(d^(3/2)*r^2) + 2*(1/(d*r*Sqrt[d + e*x^r]) - ArcTa
nh[Sqrt[d + e*x^r]/Sqrt[d]]/(d^(3/2)*r))*(a + b*Log[c*x^n]) - (4*b*n*ArcTan
h[Sqrt[d + e*x^r]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^r])])/(d
^(3/2)*r^2) - (2*b*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^r])
])/(d^(3/2)*r^2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
```

```
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\left(\frac{1}{dr\sqrt{d+ex^r}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{3/2}r}\right)(a+b\log(cx^n)) \\
&\quad -(bn)\int\left(\frac{2}{drx\sqrt{d+ex^r}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{3/2}rx}\right)dx \\
&= 2\left(\frac{1}{dr\sqrt{d+ex^r}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{3/2}r}\right)(a+b\log(cx^n)) \\
&\quad +\frac{(2bn)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{x}dx}{d^{3/2}r}-\frac{(2bn)\int\frac{1}{x\sqrt{d+ex^r}}dx}{dr} \\
&= 2\left(\frac{1}{dr\sqrt{d+ex^r}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{3/2}r}\right)(a+b\log(cx^n)) \\
&\quad +\frac{(2bn)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}dx,x,x^r\right)}{d^{3/2}r^2}-\frac{(2bn)\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}dx,x,x^r\right)}{dr^2}
\end{aligned}
$$

$$= 2\left(\frac{1}{dr\sqrt{d+ex^r}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{3/2}r}\right)(a+b\log(cx^n))$$
$$+ \frac{(4bn)\text{Subst}\left(\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx, x, \sqrt{d+ex^r}\right)}{d^{3/2}r^2}$$
$$- \frac{(4bn)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^r}\right)}{der^2}$$

$$= \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{3/2}r^2} + \frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{3/2}r^2}$$
$$+2\left(\frac{1}{dr\sqrt{d+ex^r}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{3/2}r}\right)(a+b\log(cx^n)) - \frac{(4bn)\text{Subst}\left(\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{1-\frac{x}{\sqrt{d}}}\,dx, x, \sqrt{d+ex}\right.}{d^2r^2}$$

$$= \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{3/2}r^2} + \frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{3/2}r^2}$$

$$+2\left(\frac{1}{dr\sqrt{d+ex^r}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{3/2}r}\right)(a+b\log(cx^n)) - \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^r}}\right)}{d^{3/2}r^2} +$$

$$= \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{3/2}r^2} + \frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{3/2}r^2}$$
$$+2\left(\frac{1}{dr\sqrt{d+ex^r}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{3/2}r}\right)(a+b\log(cx^n)) - \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^r}}\right)}{d^{3/2}r^2} -$$

$$= \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{3/2}r^2} + \frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{3/2}r^2}$$
$$+2\left(\frac{1}{dr\sqrt{d+ex^r}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{3/2}r}\right)(a+b\log(cx^n)) - \frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex^r}}\right)}{d^{3/2}r^2} -$$

Mathematica [F]

$$\int \frac{a+b\log \left(cx^n\right)}{x\left(d+ex^r\right)^{3/2}}\,dx=\int \frac{a+b\log \left(cx^n\right)}{x\left(d+ex^r\right)^{3/2}}\,dx$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x^r)^(3/2)),x]
[Out] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x^r)^(3/2)), x]
```

Maple [F]

$$\int \frac{a+b\ln \left(c\,x^{n}\right)}{x\left(d+e\,x^{r}\right)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*ln(c*x^n))/x/(d+e*x^r)^(3/2),x)
[Out] int((a+b*ln(c*x^n))/x/(d+e*x^r)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{a+b\log \left(cx^n\right)}{x\left(d+ex^r\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{a+b\log \left(cx^n\right)}{x\left(d+ex^r\right)^{3/2}}\,dx=\int \frac{a+b\log \left(cx^n\right)}{x\left(d+ex^r\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))/x/(d+e*x**r)**(3/2),x)
[Out] Integral((a + b*log(c*x**n))/(x*(d + e*x**r)**(3/2)), x)
```

Maxima [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^{3/2}}\,dx = \int \frac{b\log(cx^n)+a}{(ex^r+d)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^(3/2),x, algorithm="maxima")
```

```
[Out] a*(log((sqrt(e*x^r + d) - sqrt(d))/(sqrt(e*x^r + d) + sqrt(d)))/(d^(3/2)*r)
 + 2/(sqrt(e*x^r + d)*d*r)) + b*integrate((log(c) + log(x^n))/((e*x*x^r + d
*x)*sqrt(e*x^r + d)), x)
```

Giac [F]

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^{3/2}}\,dx = \int \frac{b\log(cx^n)+a}{(ex^r+d)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x^r + d)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^{3/2}}\,dx = \int \frac{a+b\ln(c\,x^n)}{x\,(d+e\,x^r)^{3/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x^r)^(3/2)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x*(d + e*x^r)^(3/2)), x)
```

3.438 $\int \frac{a+b\log(cx^n)}{x(d+ex^r)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 271

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^{5/2}}\,dx = -\frac{4bn}{3d^2r^2\sqrt{d+ex^r}}+\frac{16bn\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{3d^{5/2}r^2}+\frac{2bn\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{5/2}r^2}$$
$$+\frac{2}{3}\left(\frac{1}{dr(d+ex^r)^{3/2}}+\frac{3}{d^2r\sqrt{d+ex^r}}-\frac{3\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{5/2}r}\right)(a+b\log(cx^n))-\frac{4bn\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{\ldots}}{\sqrt{d}-\sqrt{\ldots}}\right.}{d^{5/2}r^2}$$

```
[Out] 16/3*b*n*arctanh((d+e*x^r)^(1/2)/d^(1/2))/d^(5/2)/r^2+2*b*n*arctanh((d+e*x^
r)^(1/2)/d^(1/2))^2/d^(5/2)/r^2-4*b*n*arctanh((d+e*x^r)^(1/2)/d^(1/2))*ln(2
*d^(1/2)/(d^(1/2)-(d+e*x^r)^(1/2)))/d^(5/2)/r^2-2*b*n*polylog(2,1-2*d^(1/2)
/(d^(1/2)-(d+e*x^r)^(1/2)))/d^(5/2)/r^2+2/3*(a+b*ln(c*x^n))*(1/d/r/(d+e*x^r
)^(3/2)-3*arctanh((d+e*x^r)^(1/2)/d^(1/2))/d^(5/2)/r+3/d^2/r/(d+e*x^r)^(1/2
))-4/3*b*n/d^2/r^2/(d+e*x^r)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {272, 53, 65, 214, 2390, 6131, 6055, 2449, 2352}

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^{5/2}}\,dx = \frac{2}{3}\left(-\frac{3\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{5/2}r}+\frac{3}{d^2r\sqrt{d+ex^r}}\right.$$
$$\left.+\frac{1}{dr(d+ex^r)^{3/2}}\right)(a+b\log(cx^n))+\frac{2bn\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{5/2}r^2}+\frac{16bn\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{3d^{5/2}r^2}-\frac{4bn\text{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\log\ldots}{d^{5/2}r^2}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x^r)^(5/2)),x]
```

```
[Out] (-4*b*n)/(3*d^2*r^2*Sqrt[d + e*x^r]) + (16*b*n*ArcTanh[Sqrt[d + e*x^r]/Sqrt
[d]])/(3*d^(5/2)*r^2) + (2*b*n*ArcTanh[Sqrt[d + e*x^r]/Sqrt[d]]^2)/(d^(5/2)
*r^2) + (2*(1/(d*r*(d + e*x^r)^(3/2)) + 3/(d^2*r*Sqrt[d + e*x^r]) - (3*ArcT
anh[Sqrt[d + e*x^r]/Sqrt[d]])/(d^(5/2)*r))*(a + b*Log[c*x^n]))/3 - (4*b*n*A
rcTanh[Sqrt[d + e*x^r]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^r])
])/(d^(5/2)*r^2) - (2*b*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*
x^r])])/(d^(5/2)*r^2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
```

```
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{2}{3}\left(\frac{1}{dr\left(d+ex^r\right)^{3/2}}+\frac{3}{d^2r\sqrt{d+ex^r}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{5/2}r}\right)\left(a+b\log\left(cx^n\right)\right) - (bn)\int\left(\frac{2}{3drx\left(d+ex^r\right)^{3/2}}+\frac{2}{d^2rx\sqrt{d+ex^r}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{5/2}rx}\right)dx$$

$$= \frac{2}{3}\left(\frac{1}{dr\left(d+ex^r\right)^{3/2}}+\frac{3}{d^2r\sqrt{d+ex^r}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{5/2}r}\right)\left(a+b\log\left(cx^n\right)\right) + \frac{(2bn)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{x}dx}{d^{5/2}r} - \frac{(2bn)\int\frac{1}{x\sqrt{d+ex^r}}dx}{d^2r} - \frac{(2bn)\int\frac{1}{x\left(d+ex^r\right)^{3/2}}dx}{3dr}$$

$$=\frac{2}{3}\left(\frac{1}{dr\left(d+ex^r\right)^{3/2}}+\frac{3}{d^2r\sqrt{d+ex^r}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{5/2}r}\right)\left(a+b\log\left(cx^n\right)\right)+\frac{(2bn)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx,x,x^r\right)}{d^{5/2}r^2}-\frac{(2bn)\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx,x,x^r\right)}{d^2r^2}-\frac{(2bn)\text{Subst}\left(\int\frac{1}{x(d+ex)^{3/2}}\,dx,x,x^r\right)}{3dr^2}$$

$$=-\frac{4bn}{3d^2r^2\sqrt{d+ex^r}}+\frac{2}{3}\left(\frac{1}{dr\left(d+ex^r\right)^{3/2}}+\frac{3}{d^2r\sqrt{d+ex^r}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{5/2}r}\right)\left(a+b\log\left(cx^n\right)\right)+\frac{(4bn)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx,x,\sqrt{d+ex^r}\right)}{d^{5/2}r^2}-\frac{(2bn)\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx,x,x^r\right)}{3d^2r^2}-\frac{(4bn)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^r}\right)}{d^2er^2}$$

$$=-\frac{4bn}{3d^2r^2\sqrt{d+ex^r}}+\frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{5/2}r^2}+\frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{5/2}r^2}+\frac{2}{3}\left(\frac{1}{dr\left(d+ex^r\right)^{3/2}}+\frac{3}{d^2r\sqrt{d+ex^r}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{5/2}r}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{(4bn)\text{Subst}\left(\int\frac{\tanh^{-1}\ldots}{1-\ldots}\ldots\right)}{\ldots}$$

$$=-\frac{4bn}{3d^2r^2\sqrt{d+ex^r}}+\frac{16bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{3d^{5/2}r^2}+\frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{5/2}r^2}+\frac{2}{3}\left(\frac{1}{dr\left(d+ex^r\right)^{3/2}}+\frac{3}{d^2r\sqrt{d+ex^r}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{5/2}r}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)\ldots}{d^{5/2}\ldots}$$

$$=-\frac{4bn}{3d^2r^2\sqrt{d+ex^r}}+\frac{16bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{3d^{5/2}r^2}+\frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{5/2}r^2}+\frac{2}{3}\left(\frac{1}{dr\left(d+ex^r\right)^{3/2}}+\frac{3}{d^2r\sqrt{d+ex^r}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{5/2}r}\right)\left(a+b\log\left(cx^n\right)\right)-\frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\ldots}\right)\ldots}{d^{5/2}\ldots}$$

$$= -\frac{4bn}{3d^2r^2\sqrt{d+ex^r}} + \frac{16bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{3d^{5/2}r^2} + \frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{5/2}r^2}$$

$$+\frac{2}{3}\left(\frac{1}{dr\,(d+ex^r)^{3/2}}+\frac{3}{d^2r\sqrt{d+ex^r}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{5/2}r}\right)(a+b\log(cx^n))-\frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{5/2}r}$$

Mathematica [F]

$$\int \frac{a+b\log(cx^n)}{x\,(d+ex^r)^{5/2}}\,dx = \int \frac{a+b\log(cx^n)}{x\,(d+ex^r)^{5/2}}\,dx$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x^r)^(5/2)),x]
[Out] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x^r)^(5/2)), x]
```

Maple [F]

$$\int \frac{a+b\ln(c\,x^n)}{x\,(d+e\,x^r)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*ln(c*x^n))/x/(d+e*x^r)^(5/2),x)
[Out] int((a+b*ln(c*x^n))/x/(d+e*x^r)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{a+b\log(cx^n)}{x\,(d+ex^r)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{x(d + ex^r)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*x**n))/x/(d+e*x**r)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a + b\log(cx^n)}{x(d + ex^r)^{5/2}}\,dx = \int \frac{b\log(cx^n) + a}{(ex^r + d)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(3*log((sqrt(e*x^r + d) - sqrt(d))/(sqrt(e*x^r + d) + sqrt(d)))/(d^(5
/2)*r) + 2*(3*e*x^r + 4*d)/((e*x^r + d)^(3/2)*d^2*r)) + b*integrate((log(c)
 + log(x^n))/((e^2*x*x^(2*r) + 2*d*e*x*x^r + d^2*x)*sqrt(e*x^r + d)), x)
```

Giac [F]

$$\int \frac{a + b\log(cx^n)}{x(d + ex^r)^{5/2}}\,dx = \int \frac{b\log(cx^n) + a}{(ex^r + d)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x^r + d)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log(cx^n)}{x(d + ex^r)^{5/2}}\,dx = \int \frac{a + b\,\ln(c\,x^n)}{x\,(d + e\,x^r)^{5/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x^r)^(5/2)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x*(d + e*x^r)^(5/2)), x)
```

3.439 $\int \frac{a+b\log(cx^n)}{x(d+ex^r)^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 314

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^{7/2}}\,dx = -\frac{4bn}{15d^2r^2(d+ex^r)^{3/2}} - \frac{32bn}{15d^3r^2\sqrt{d+ex^r}} + \frac{92bn\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{15d^{7/2}r^2} + \frac{2bn\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{7/2}r^2} + \frac{2}{15}\left(\frac{3}{dr(d+ex^r)^{5/2}} + \frac{5}{d^2r(d+ex^r)^{3/2}} + \frac{15}{d^3r\sqrt{d+ex^r}} - \frac{15\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{7/2}r}\right)(a+b\log(cx^n)) - \frac{4bn\operatorname{arctanh}}{}$$

```
[Out] -4/15*b*n/d^2/r^2/(d+e*x^r)^(3/2)+92/15*b*n*arctanh((d+e*x^r)^(1/2)/d^(1/2)
)/d^(7/2)/r^2+2*b*n*arctanh((d+e*x^r)^(1/2)/d^(1/2))^2/d^(7/2)/r^2-4*b*n*ar
ctanh((d+e*x^r)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)-(d+e*x^r)^(1/2)))/d^(7
/2)/r^2-2*b*n*polylog(2,1-2*d^(1/2)/(d^(1/2)-(d+e*x^r)^(1/2)))/d^(7/2)/r^2+
2/15*(a+b*ln(c*x^n))*(3/d/r/(d+e*x^r)^(5/2)+5/d^2/r/(d+e*x^r)^(3/2)-15*arct
anh((d+e*x^r)^(1/2)/d^(1/2))/d^(7/2)/r+15/d^3/r/(d+e*x^r)^(1/2))-32/15*b*n/
d^3/r^2/(d+e*x^r)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used

= {272, 53, 65, 214, 2390, 6131, 6055, 2449, 2352}

$$\int \frac{a+b\log(cx^n)}{x(d+ex^r)^{7/2}}\,dx = \frac{2}{15}\left(-\frac{15\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{7/2}r} + \frac{15}{d^3r\sqrt{d+ex^r}} + \frac{5}{d^2r(d+ex^r)^{3/2}} + \frac{3}{dr(d+ex^r)^{5/2}}\right)(a+b\log(cx^n)) + \frac{2bn\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{7/2}r^2} + \frac{92bn\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{15d^{7/2}r^2} - \frac{4bn\operatorname{arctanh}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{7/2}r}$$

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*x^r)^(7/2)),x]
```

```
[Out] (-4*b*n)/(15*d^2*r^2*(d + e*x^r)^(3/2)) - (32*b*n)/(15*d^3*r^2*Sqrt[d + e*x
^r]) + (92*b*n*ArcTanh[Sqrt[d + e*x^r]/Sqrt[d]])/(15*d^(7/2)*r^2) + (2*b*n*
ArcTanh[Sqrt[d + e*x^r]/Sqrt[d]]^2)/(d^(7/2)*r^2) + (2*(3/(d*r*(d + e*x^r)^
(5/2)) + 5/(d^2*r*(d + e*x^r)^(3/2)) + 15/(d^3*r*Sqrt[d + e*x^r]) - (15*Arc
Tanh[Sqrt[d + e*x^r]/Sqrt[d]])/(d^(7/2)*r))*(a + b*Log[c*x^n]))/15 - (4*b*n
*ArcTanh[Sqrt[d + e*x^r]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*x^r
])])/(d^(7/2)*r^2) - (2*b*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d +
e*x^r])])/(d^(7/2)*r^2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.))
/(x_), x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q/x, x]}, Simp[u*(a + b*L
og[c*x^n]), x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IntegerQ[q - 1/2]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{2}{15}\left(\frac{3}{dr\,(d+ex^r)^{5/2}}+\frac{5}{d^2r\,(d+ex^r)^{3/2}}+\frac{15}{d^3r\sqrt{d+ex^r}}\right.$$
$$\left.-\frac{15\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{7/2}r}\right)(a+b\log(cx^n))-(bn)\int\left(\frac{2}{5drx\,(d+ex^r)^{5/2}}+\frac{2}{3d^2rx\,(d+ex^r)^{3/2}}+\frac{2}{d^3rx\sqrt{d+ex^r}}\right.$$

$$=\frac{2}{15}\left(\frac{3}{dr\left(d+ex^r\right)^{5/2}}+\frac{5}{d^2r\left(d+ex^r\right)^{3/2}}+\frac{15}{d^3r\sqrt{d+ex^r}}\right.$$
$$\left.-\frac{15\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{7/2}r}\right)(a+b\log\left(cx^n\right))+\frac{(2bn)\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{x}\,dx}{d^{7/2}r}-\frac{(2bn)\int\frac{1}{x\sqrt{d+ex^r}}\,dx}{d^3r}-\frac{(2bn)}{}$$

$$=\frac{2}{15}\left(\frac{3}{dr\left(d+ex^r\right)^{5/2}}+\frac{5}{d^2r\left(d+ex^r\right)^{3/2}}+\frac{15}{d^3r\sqrt{d+ex^r}}\right.$$
$$\left.-\frac{15\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{7/2}r}\right)(a+b\log\left(cx^n\right))+\frac{(2bn)\text{Subst}\left(\int\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{x}\,dx,x,x^r\right)}{d^{7/2}r^2}-\frac{(2bn)\text{Subst}\left(\right.}{}$$

$$=-\frac{4bn}{15d^2r^2\left(d+ex^r\right)^{3/2}}-\frac{4bn}{3d^3r^2\sqrt{d+ex^r}}$$
$$+\frac{2}{15}\left(\frac{3}{dr\left(d+ex^r\right)^{5/2}}+\frac{5}{d^2r\left(d+ex^r\right)^{3/2}}+\frac{15}{d^3r\sqrt{d+ex^r}}\right.$$
$$\left.-\frac{15\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{7/2}r}\right)(a+b\log\left(cx^n\right))+\frac{(4bn)\text{Subst}\left(\int\frac{x\tanh^{-1}\left(\frac{x}{\sqrt{d}}\right)}{-d+x^2}\,dx,x,\sqrt{d+ex^r}\right)}{d^{7/2}r^2}-\frac{(2bn)\text{Su}}{}$$

$$=-\frac{4bn}{15d^2r^2\left(d+ex^r\right)^{3/2}}-\frac{32bn}{15d^3r^2\sqrt{d+ex^r}}+\frac{4bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{7/2}r^2}+\frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{7/2}r^2}$$
$$+\frac{2}{15}\left(\frac{3}{dr\left(d+ex^r\right)^{5/2}}+\frac{5}{d^2r\left(d+ex^r\right)^{3/2}}+\frac{15}{d^3r\sqrt{d+ex^r}}-\frac{15\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{7/2}r}\right)(a+b\log\left(cx^n\right))-$$

$$=-\frac{4bn}{15d^2r^2\left(d+ex^r\right)^{3/2}}-\frac{32bn}{15d^3r^2\sqrt{d+ex^r}}+\frac{16bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{3d^{7/2}r^2}+\frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{7/2}r^2}$$

$$+\frac{2}{15}\left(\frac{3}{dr\left(d+ex^r\right)^{5/2}}+\frac{5}{d^2r\left(d+ex^r\right)^{3/2}}+\frac{15}{d^3r\sqrt{d+ex^r}}-\frac{15\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{7/2}r}\right)(a+b\log\left(cx^n\right))-$$

$$=-\frac{4bn}{15d^2r^2(d+ex^r)^{3/2}}-\frac{32bn}{15d^3r^2\sqrt{d+ex^r}}+\frac{92bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{15d^{7/2}r^2}+\frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{7/2}r^2}$$

$$+\frac{2}{15}\left(\frac{3}{dr(d+ex^r)^{5/2}}+\frac{5}{d^2r(d+ex^r)^{3/2}}+\frac{15}{d^3r\sqrt{d+ex^r}}-\frac{15\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{7/2}r}\right)(a+b\log(cx^n))-$$

$$=-\frac{4bn}{15d^2r^2(d+ex^r)^{3/2}}-\frac{32bn}{15d^3r^2\sqrt{d+ex^r}}+\frac{92bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{15d^{7/2}r^2}+\frac{2bn\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)^2}{d^{7/2}r^2}$$

$$+\frac{2}{15}\left(\frac{3}{dr(d+ex^r)^{5/2}}+\frac{5}{d^2r(d+ex^r)^{3/2}}+\frac{15}{d^3r\sqrt{d+ex^r}}-\frac{15\tanh^{-1}\left(\frac{\sqrt{d+ex^r}}{\sqrt{d}}\right)}{d^{7/2}r}\right)(a+b\log(cx^n))-$$

Mathematica [F]

$$\int\frac{a+b\log(cx^n)}{x(d+ex^r)^{7/2}}\,dx=\int\frac{a+b\log(cx^n)}{x(d+ex^r)^{7/2}}\,dx$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x^r)^(7/2)),x]
[Out] Integrate[(a + b*Log[c*x^n])/(x*(d + e*x^r)^(7/2)), x]
```

Maple [F]

$$\int\frac{a+b\ln(c\,x^n)}{x\,(d+e\,x^r)^{\frac{7}{2}}}dx$$

```
[In] int((a+b*ln(c*x^n))/x/(d+e*x^r)^(7/2),x)
[Out] int((a+b*ln(c*x^n))/x/(d+e*x^r)^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int\frac{a+b\log(cx^n)}{x(d+ex^r)^{7/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x\left(d+ex^r\right)^{7/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*x**n))/x/(d+e*x**r)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x\left(d+ex^r\right)^{7/2}}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex^r+d\right)^{\frac{7}{2}}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*a*(15*log((sqrt(e*x^r + d) - sqrt(d))/(sqrt(e*x^r + d) + sqrt(d)))/(d^
(7/2)*r) + 2*(15*(e*x^r + d)^2 + 5*(e*x^r + d)*d + 3*d^2)/((e*x^r + d)^(5/2
)*d^3*r)) + b*integrate((log(c) + log(x^n))/((e^3*x*x^(3*r) + 3*d*e^2*x*x^(
2*r) + 3*d^2*e*x*x^r + d^3*x)*sqrt(e*x^r + d)), x)
```

Giac [F]

$$\int \frac{a+b\log\left(cx^n\right)}{x\left(d+ex^r\right)^{7/2}}\,dx=\int \frac{b\log\left(cx^n\right)+a}{\left(ex^r+d\right)^{\frac{7}{2}}x}\,dx$$

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*x^r)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*x^r + d)^(7/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{x\left(d+ex^r\right)^{7/2}}\,dx=\int \frac{a+b\,\ln\left(c\,x^n\right)}{x\,{\left(d+e\,x^r\right)}^{7/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(x*(d + e*x^r)^(7/2)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x*(d + e*x^r)^(7/2)), x)
```

3.440 $\int (fx)^m (d+ex^r)^3 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 233

$$\int (fx)^m (d+ex^r)^3 (a+b\log(cx^n))\,dx = -\frac{3bd^2enx^{1+r}(fx)^m}{(1+m+r)^2} - \frac{3bde^2nx^{1+2r}(fx)^m}{(1+m+2r)^2} - \frac{be^3nx^{1+3r}(fx)^m}{(1+m+3r)^2} - \frac{bd^3n(fx)^{1+m}}{f(1+m)^2} + \frac{3d^2ex^{1+r}(fx)^m(a+b\log(cx^n))}{1+m+r} + \frac{3de^2x^{1+2r}(fx)^m(a+b\log(cx^n))}{1+m+2r} + \frac{e^3x^{1+3r}(fx)^m(a+b\log(cx^n))}{1+m+3r} + \frac{d^3(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)}$$

```
[Out] -3*b*d^2*e*n*x^(1+r)*(f*x)^m/(1+m+r)^2-3*b*d*e^2*n*x^(1+2*r)*(f*x)^m/(1+m+2
*r)^2-b*e^3*n*x^(1+3*r)*(f*x)^m/(1+m+3*r)^2-b*d^3*n*(f*x)^(1+m)/f/(1+m)^2+3
*d^2*e*x^(1+r)*(f*x)^m*(a+b*ln(c*x^n))/(1+m+r)+3*d*e^2*x^(1+2*r)*(f*x)^m*(a
+b*ln(c*x^n))/(1+m+2*r)+e^3*x^(1+3*r)*(f*x)^m*(a+b*ln(c*x^n))/(1+m+3*r)+d^3
*(f*x)^(1+m)*(a+b*ln(c*x^n))/f/(1+m)
```

Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {276, 20, 30, 2392, 14}

$$\int (fx)^m \left(d+ex^r\right)^3 \left(a+b\log\left(cx^n\right)\right) dx = \frac{d^3(fx)^{m+1}\left(a+b\log\left(cx^n\right)\right)}{f(m+1)} + \frac{3d^2ex^{r+1}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{m+r+1} + \frac{3de^2x^{2r+1}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{m+2r+1} + \frac{e^3x^{3r+1}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{m+3r+1} - \frac{bd^3n(fx)^{m+1}}{f(m+1)^2} - \frac{3bd^2enx^{r+1}(fx)^m}{(m+r+1)^2} - \frac{3bde^2nx^{2r+1}(fx)^m}{(m+2r+1)^2} - \frac{be^3nx^{3r+1}(fx)^m}{(m+3r+1)^2}$$

```
[In] Int[(f*x)^m*(d + e*x^r)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] (-3*b*d^2*e*n*x^(1 + r)*(f*x)^m)/(1 + m + r)^2 - (3*b*d*e^2*n*x^(1 + 2*r)*(
f*x)^m)/(1 + m + 2*r)^2 - (b*e^3*n*x^(1 + 3*r)*(f*x)^m)/(1 + m + 3*r)^2 - (
b*d^3*n*(f*x)^(1 + m))/(f*(1 + m)^2) + (3*d^2*e*x^(1 + r)*(f*x)^m*(a + b*Lo
g[c*x^n]))/(1 + m + r) + (3*d*e^2*x^(1 + 2*r)*(f*x)^m*(a + b*Log[c*x^n]))/(
1 + m + 2*r) + (e^3*x^(1 + 3*r)*(f*x)^m*(a + b*Log[c*x^n]))/(1 + m + 3*r) +
 (d^3*(f*x)^(1 + m)*(a + b*Log[c*x^n]))/(f*(1 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3d^2ex^{1+r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+r}+\frac{3de^2x^{1+2r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+2r} \\
&\quad+\frac{e^3x^{1+3r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+3r}+\frac{d^3(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)} \\
&\quad-(bn)\int(fx)^m\left(\frac{d^3}{1+m}+\frac{3d^2ex^r}{1+m+r}+\frac{3de^2x^{2r}}{1+m+2r}+\frac{e^3x^{3r}}{1+m+3r}\right)dx \\
&= \frac{3d^2ex^{1+r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+r}+\frac{3de^2x^{1+2r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+2r} \\
&\quad+\frac{e^3x^{1+3r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+3r}+\frac{d^3(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)} \\
&\quad-(bn)\int\left(\frac{d^3(fx)^m}{1+m}+\frac{3d^2ex^r(fx)^m}{1+m+r}+\frac{3de^2x^{2r}(fx)^m}{1+m+2r}+\frac{e^3x^{3r}(fx)^m}{1+m+3r}\right)dx \\
&= -\frac{bd^3n(fx)^{1+m}}{f(1+m)^2}+\frac{3d^2ex^{1+r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+r} \\
&\quad+\frac{3de^2x^{1+2r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+2r}+\frac{e^3x^{1+3r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+3r} \\
&\quad+\frac{d^3(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)}-\frac{\left(3bd^2en\right)\int x^r(fx)^m\,dx}{1+m+r} \\
&\quad-\frac{\left(3bde^2n\right)\int x^{2r}(fx)^m\,dx}{1+m+2r}-\frac{\left(be^3n\right)\int x^{3r}(fx)^m\,dx}{1+m+3r}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{bd^3n(fx)^{1+m}}{f(1+m)^2} + \frac{3d^2ex^{1+r}(fx)^m(a+b\log(cx^n))}{1+m+r} \\
&+ \frac{3de^2x^{1+2r}(fx)^m(a+b\log(cx^n))}{1+m+2r} + \frac{e^3x^{1+3r}(fx)^m(a+b\log(cx^n))}{1+m+3r} \\
&+ \frac{d^3(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)} - \frac{(3bd^2enx^{-m}(fx)^m)\int x^{m+r}\,dx}{1+m+r} \\
&- \frac{(3bde^2nx^{-m}(fx)^m)\int x^{m+2r}\,dx}{1+m+2r} - \frac{(be^3nx^{-m}(fx)^m)\int x^{m+3r}\,dx}{1+m+3r} \\
&= -\frac{3bd^2enx^{1+r}(fx)^m}{(1+m+r)^2} - \frac{3bde^2nx^{1+2r}(fx)^m}{(1+m+2r)^2} - \frac{be^3nx^{1+3r}(fx)^m}{(1+m+3r)^2} - \frac{bd^3n(fx)^{1+m}}{f(1+m)^2} \\
&+ \frac{3d^2ex^{1+r}(fx)^m(a+b\log(cx^n))}{1+m+r} + \frac{3de^2x^{1+2r}(fx)^m(a+b\log(cx^n))}{1+m+2r} \\
&+ \frac{e^3x^{1+3r}(fx)^m(a+b\log(cx^n))}{1+m+3r} + \frac{d^3(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.76

$$
\begin{aligned}
\int (fx)^m(d+ex^r)^3(a+b\log(cx^n))\,dx = x(fx)^m\Bigg(&-\frac{bd^3n}{(1+m)^2} - \frac{3bd^2enx^r}{(1+m+r)^2} \\
&- \frac{3bde^2nx^{2r}}{(1+m+2r)^2} - \frac{be^3nx^{3r}}{(1+m+3r)^2} \\
&+ \frac{d^3(a+b\log(cx^n))}{1+m} + \frac{3d^2ex^r(a+b\log(cx^n))}{1+m+r} \\
&+ \frac{3de^2x^{2r}(a+b\log(cx^n))}{1+m+2r} \\
&+ \frac{e^3x^{3r}(a+b\log(cx^n))}{1+m+3r}\Bigg)
\end{aligned}
$$

```
[In] Integrate[(f*x)^m*(d + e*x^r)^3*(a + b*Log[c*x^n]),x]
```

```
[Out] x*(f*x)^m*(-((b*d^3*n)/(1 + m)^2) - (3*b*d^2*e*n*x^r)/(1 + m + r)^2 - (3*b*
d*e^2*n*x^(2*r))/(1 + m + 2*r)^2 - (b*e^3*n*x^(3*r))/(1 + m + 3*r)^2 + (d^3
*(a + b*Log[c*x^n]))/(1 + m) + (3*d^2*e*x^r*(a + b*Log[c*x^n]))/(1 + m + r)
 + (3*d*e^2*x^(2*r)*(a + b*Log[c*x^n]))/(1 + m + 2*r) + (e^3*x^(3*r)*(a + b
*Log[c*x^n]))/(1 + m + 3*r))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8182 vs. 2(233) = 466.

Time = 57.15 (sec) , antiderivative size = 8183, normalized size of antiderivative = 35.12

method	result	size
parallelrisch	Expression too large to display	8183
risch	Expression too large to display	22640

```
[In] int((f*x)^m*(d+e*x^r)^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4918 vs. 2(233) = 466.

Time = 0.48 (sec) , antiderivative size = 4918, normalized size of antiderivative = 21.11

$$\int (fx)^m (d+ex^r)^3 (a+b\log(cx^n))\,dx = \text{Too large to display}$$

```
[In] integrate((f*x)^m*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (((b*e^3*m^7 + 7*b*e^3*m^6 + 21*b*e^3*m^5 + 35*b*e^3*m^4 + 35*b*e^3*m^3 + 2
1*b*e^3*m^2 + 12*(b*e^3*m^2 + 2*b*e^3*m + b*e^3)*r^5 + 7*b*e^3*m + 40*(b*e^
3*m^3 + 3*b*e^3*m^2 + 3*b*e^3*m + b*e^3)*r^4 + b*e^3 + 51*(b*e^3*m^4 + 4*b*
e^3*m^3 + 6*b*e^3*m^2 + 4*b*e^3*m + b*e^3)*r^3 + 31*(b*e^3*m^5 + 5*b*e^3*m^
4 + 10*b*e^3*m^3 + 10*b*e^3*m^2 + 5*b*e^3*m + b*e^3)*r^2 + 9*(b*e^3*m^6 + 6
*b*e^3*m^5 + 15*b*e^3*m^4 + 20*b*e^3*m^3 + 15*b*e^3*m^2 + 6*b*e^3*m + b*e^3
)*r)*x*log(c) + (12*(b*e^3*m^2 + 2*b*e^3*m + b*e^3)*n*r^5 + 40*(b*e^3*m^3 +
 3*b*e^3*m^2 + 3*b*e^3*m + b*e^3)*n*r^4 + 51*(b*e^3*m^4 + 4*b*e^3*m^3 + 6*b
*e^3*m^2 + 4*b*e^3*m + b*e^3)*n*r^3 + 31*(b*e^3*m^5 + 5*b*e^3*m^4 + 10*b*e^
3*m^3 + 10*b*e^3*m^2 + 5*b*e^3*m + b*e^3)*n*r^2 + 9*(b*e^3*m^6 + 6*b*e^3*m^
5 + 15*b*e^3*m^4 + 20*b*e^3*m^3 + 15*b*e^3*m^2 + 6*b*e^3*m + b*e^3)*n*r + (
b*e^3*m^7 + 7*b*e^3*m^6 + 21*b*e^3*m^5 + 35*b*e^3*m^4 + 35*b*e^3*m^3 + 21*b
*e^3*m^2 + 7*b*e^3*m + b*e^3)*n)*x*log(x) + (a*e^3*m^7 + 7*a*e^3*m^6 + 21*a
*e^3*m^5 + 35*a*e^3*m^4 + 35*a*e^3*m^3 + 21*a*e^3*m^2 + 12*(a*e^3*m^2 + 2*a
*e^3*m + a*e^3)*r^5 + 7*a*e^3*m + 4*(10*a*e^3*m^3 + 30*a*e^3*m^2 + 30*a*e^3
*m + 10*a*e^3 - (b*e^3*m^2 + 2*b*e^3*m + b*e^3)*n)*r^4 + a*e^3 + 3*(17*a*e^
3*m^4 + 68*a*e^3*m^3 + 102*a*e^3*m^2 + 68*a*e^3*m + 17*a*e^3 - 4*(b*e^3*m^3
 + 3*b*e^3*m^2 + 3*b*e^3*m + b*e^3)*n)*r^3 + (31*a*e^3*m^5 + 155*a*e^3*m^4
+ 310*a*e^3*m^3 + 310*a*e^3*m^2 + 155*a*e^3*m + 31*a*e^3 - 13*(b*e^3*m^4 +
4*b*e^3*m^3 + 6*b*e^3*m^2 + 4*b*e^3*m + b*e^3)*n)*r^2 - (b*e^3*m^6 + 6*b*e^
3*m^5 + 15*b*e^3*m^4 + 20*b*e^3*m^3 + 15*b*e^3*m^2 + 6*b*e^3*m + b*e^3)*n +
 3*(3*a*e^3*m^6 + 18*a*e^3*m^5 + 45*a*e^3*m^4 + 60*a*e^3*m^3 + 45*a*e^3*m^2
```

+ 18*a*e^3*m + 3*a*e^3 - 2*(b*e^3*m^5 + 5*b*e^3*m^4 + 10*b*e^3*m^3 + 10*b*
e^3*m^2 + 5*b*e^3*m + b*e^3)*n)*r)*x)*x^(3*r)*e^(m*log(f) + m*log(x)) + 3*(
(b*d*e^2*m^7 + 7*b*d*e^2*m^6 + 21*b*d*e^2*m^5 + 35*b*d*e^2*m^4 + 35*b*d*e^2
*m^3 + 21*b*d*e^2*m^2 + 18*(b*d*e^2*m^2 + 2*b*d*e^2*m + b*d*e^2)*r^5 + 7*b*
d*e^2*m + 57*(b*d*e^2*m^3 + 3*b*d*e^2*m^2 + 3*b*d*e^2*m + b*d*e^2)*r^4 + b*
d*e^2 + 68*(b*d*e^2*m^4 + 4*b*d*e^2*m^3 + 6*b*d*e^2*m^2 + 4*b*d*e^2*m + b*d
*e^2)*r^3 + 38*(b*d*e^2*m^5 + 5*b*d*e^2*m^4 + 10*b*d*e^2*m^3 + 10*b*d*e^2*m
^2 + 5*b*d*e^2*m + b*d*e^2)*r^2 + 10*(b*d*e^2*m^6 + 6*b*d*e^2*m^5 + 15*b*d*
e^2*m^4 + 20*b*d*e^2*m^3 + 15*b*d*e^2*m^2 + 6*b*d*e^2*m + b*d*e^2)*r)*x*log
(c) + (18*(b*d*e^2*m^2 + 2*b*d*e^2*m + b*d*e^2)*n*r^5 + 57*(b*d*e^2*m^3 + 3
*b*d*e^2*m^2 + 3*b*d*e^2*m + b*d*e^2)*n*r^4 + 68*(b*d*e^2*m^4 + 4*b*d*e^2*m
^3 + 6*b*d*e^2*m^2 + 4*b*d*e^2*m + b*d*e^2)*n*r^3 + 38*(b*d*e^2*m^5 + 5*b*d
*e^2*m^4 + 10*b*d*e^2*m^3 + 10*b*d*e^2*m^2 + 5*b*d*e^2*m + b*d*e^2)*n*r^2 +
10*(b*d*e^2*m^6 + 6*b*d*e^2*m^5 + 15*b*d*e^2*m^4 + 20*b*d*e^2*m^3 + 15*b*d
*e^2*m^2 + 6*b*d*e^2*m + b*d*e^2)*n*r + (b*d*e^2*m^7 + 7*b*d*e^2*m^6 + 21*b
*d*e^2*m^5 + 35*b*d*e^2*m^4 + 35*b*d*e^2*m^3 + 21*b*d*e^2*m^2 + 7*b*d*e^2*m
+ b*d*e^2)*n)*x*log(x) + (a*d*e^2*m^7 + 7*a*d*e^2*m^6 + 21*a*d*e^2*m^5 + 3
5*a*d*e^2*m^4 + 35*a*d*e^2*m^3 + 21*a*d*e^2*m^2 + 18*(a*d*e^2*m^2 + 2*a*d*e
^2*m + a*d*e^2)*r^5 + 7*a*d*e^2*m + 3*(19*a*d*e^2*m^3 + 57*a*d*e^2*m^2 + 57
*a*d*e^2*m + 19*a*d*e^2 - 3*(b*d*e^2*m^2 + 2*b*d*e^2*m + b*d*e^2)*n)*r^4 +
a*d*e^2 + 4*(17*a*d*e^2*m^4 + 68*a*d*e^2*m^3 + 102*a*d*e^2*m^2 + 68*a*d*e^2
*m + 17*a*d*e^2 - 6*(b*d*e^2*m^3 + 3*b*d*e^2*m^2 + 3*b*d*e^2*m + b*d*e^2)*n
)*r^3 + 2*(19*a*d*e^2*m^5 + 95*a*d*e^2*m^4 + 190*a*d*e^2*m^3 + 190*a*d*e^2*
m^2 + 95*a*d*e^2*m + 19*a*d*e^2 - 11*(b*d*e^2*m^4 + 4*b*d*e^2*m^3 + 6*b*d*e
^2*m^2 + 4*b*d*e^2*m + b*d*e^2)*n)*r^2 - (b*d*e^2*m^6 + 6*b*d*e^2*m^5 + 15*
b*d*e^2*m^4 + 20*b*d*e^2*m^3 + 15*b*d*e^2*m^2 + 6*b*d*e^2*m + b*d*e^2)*n +
2*(5*a*d*e^2*m^6 + 30*a*d*e^2*m^5 + 75*a*d*e^2*m^4 + 100*a*d*e^2*m^3 + 75*a
*d*e^2*m^2 + 30*a*d*e^2*m + 5*a*d*e^2 - 4*(b*d*e^2*m^5 + 5*b*d*e^2*m^4 + 10
*b*d*e^2*m^3 + 10*b*d*e^2*m^2 + 5*b*d*e^2*m + b*d*e^2)*n)*r)*x)*x^(2*r)*e^(
m*log(f) + m*log(x)) + 3*((b*d^2*e*m^7 + 7*b*d^2*e*m^6 + 21*b*d^2*e*m^5 + 3
5*b*d^2*e*m^4 + 35*b*d^2*e*m^3 + 21*b*d^2*e*m^2 + 36*(b*d^2*e*m^2 + 2*b*d^2
*e*m + b*d^2*e)*r^5 + 7*b*d^2*e*m + 96*(b*d^2*e*m^3 + 3*b*d^2*e*m^2 + 3*b*d
^2*e*m + b*d^2*e)*r^4 + b*d^2*e + 97*(b*d^2*e*m^4 + 4*b*d^2*e*m^3 + 6*b*d^2
*e*m^2 + 4*b*d^2*e*m + b*d^2*e)*r^3 + 47*(b*d^2*e*m^5 + 5*b*d^2*e*m^4 + 10*
b*d^2*e*m^3 + 10*b*d^2*e*m^2 + 5*b*d^2*e*m + b*d^2*e)*r^2 + 11*(b*d^2*e*m^6
+ 6*b*d^2*e*m^5 + 15*b*d^2*e*m^4 + 20*b*d^2*e*m^3 + 15*b*d^2*e*m^2 + 6*b*d
^2*e*m + b*d^2*e)*r)*x*log(c) + (36*(b*d^2*e*m^2 + 2*b*d^2*e*m + b*d^2*e)*n
r^5 + 96(b*d^2*e*m^3 + 3*b*d^2*e*m^2 + 3*b*d^2*e*m + b*d^2*e)*n*r^4 + 97*
(b*d^2*e*m^4 + 4*b*d^2*e*m^3 + 6*b*d^2*e*m^2 + 4*b*d^2*e*m + b*d^2*e)*n*r^3
+ 47*(b*d^2*e*m^5 + 5*b*d^2*e*m^4 + 10*b*d^2*e*m^3 + 10*b*d^2*e*m^2 + 5*b*
d^2*e*m + b*d^2*e)*n*r^2 + 11*(b*d^2*e*m^6 + 6*b*d^2*e*m^5 + 15*b*d^2*e*m^4
+ 20*b*d^2*e*m^3 + 15*b*d^2*e*m^2 + 6*b*d^2*e*m + b*d^2*e)*n*r + (b*d^2*e*
m^7 + 7*b*d^2*e*m^6 + 21*b*d^2*e*m^5 + 35*b*d^2*e*m^4 + 35*b*d^2*e*m^3 + 21
*b*d^2*e*m^2 + 7*b*d^2*e*m + b*d^2*e)*n)*x*log(x) + (a*d^2*e*m^7 + 7*a*d^2*
e*m^6 + 21*a*d^2*e*m^5 + 35*a*d^2*e*m^4 + 35*a*d^2*e*m^3 + 21*a*d^2*e*m^2 +

```
 36*(a*d^2*e*m^2 + 2*a*d^2*e*m + a*d^2*e)*r^5 + 7*a*d^2*e*m + 12*(8*a*d^2*e
*m^3 + 24*a*d^2*e*m^2 + 24*a*d^2*e*m + 8*a*d^2*e - 3*(b*d^2*e*m^2 + 2*b*d^2
*e*m + b*d^2*e)*n)*r^4 + a*d^2*e + (97*a*d^2*e*m^4 + 388*a*d^2*e*m^3 + 582*
a*d^2*e*m^2 + 388*a*d^2*e*m + 97*a*d^2*e - 60*(b*d^2*e*m^3 + 3*b*d^2*e*m^2
+ 3*b*d^2*e*m + b*d^2*e)*n)*r^3 + (47*a*d^2*e*m^5 + 235*a*d^2*e*m^4 + 470*a
*d^2*e*m^3 + 470*a*d^2*e*m^2 + 235*a*d^2*e*m + 47*a*d^2*e - 37*(b*d^2*e*m^4
 + 4*b*d^2*e*m^3 + 6*b*d^2*e*m^2 + 4*b*d^2*e*m + b*d^2*e)*n)*r^2 - (b*d^2*e
*m^6 + 6*b*d^2*e*m^5 + 15*b*d^2*e*m^4 + 20*b*d^2*e*m^3 + 15*b*d^2*e*m^2 + 6
*b*d^2*e*m + b*d^2*e)*n + (11*a*d^2*e*m^6 + 66*a*d^2*e*m^5 + 165*a*d^2*e*m^
4 + 220*a*d^2*e*m^3 + 165*a*d^2*e*m^2 + 66*a*d^2*e*m + 11*a*d^2*e - 10*(b*d
^2*e*m^5 + 5*b*d^2*e*m^4 + 10*b*d^2*e*m^3 + 10*b*d^2*e*m^2 + 5*b*d^2*e*m +
b*d^2*e)*n)*r)*x)*x^r*e^(m*log(f) + m*log(x)) + ((b*d^3*m^7 + 7*b*d^3*m^6 +
 21*b*d^3*m^5 + 35*b*d^3*m^4 + 35*b*d^3*m^3 + 36*(b*d^3*m + b*d^3)*r^6 + 21
*b*d^3*m^2 + 132*(b*d^3*m^2 + 2*b*d^3*m + b*d^3)*r^5 + 7*b*d^3*m + 193*(b*d
^3*m^3 + 3*b*d^3*m^2 + 3*b*d^3*m + b*d^3)*r^4 + b*d^3 + 144*(b*d^3*m^4 + 4*
b*d^3*m^3 + 6*b*d^3*m^2 + 4*b*d^3*m + b*d^3)*r^3 + 58*(b*d^3*m^5 + 5*b*d^3*
m^4 + 10*b*d^3*m^3 + 10*b*d^3*m^2 + 5*b*d^3*m + b*d^3)*r^2 + 12*(b*d^3*m^6
+ 6*b*d^3*m^5 + 15*b*d^3*m^4 + 20*b*d^3*m^3 + 15*b*d^3*m^2 + 6*b*d^3*m + b*
d^3)*r)*x*log(c) + (36*(b*d^3*m + b*d^3)*n*r^6 + 132*(b*d^3*m^2 + 2*b*d^3*m
 + b*d^3)*n*r^5 + 193*(b*d^3*m^3 + 3*b*d^3*m^2 + 3*b*d^3*m + b*d^3)*n*r^4 +
 144*(b*d^3*m^4 + 4*b*d^3*m^3 + 6*b*d^3*m^2 + 4*b*d^3*m + b*d^3)*n*r^3 + 58
*(b*d^3*m^5 + 5*b*d^3*m^4 + 10*b*d^3*m^3 + 10*b*d^3*m^2 + 5*b*d^3*m + b*d^3
)*n*r^2 + 12*(b*d^3*m^6 + 6*b*d^3*m^5 + 15*b*d^3*m^4 + 20*b*d^3*m^3 + 15*b*
d^3*m^2 + 6*b*d^3*m + b*d^3)*n*r + (b*d^3*m^7 + 7*b*d^3*m^6 + 21*b*d^3*m^5
+ 35*b*d^3*m^4 + 35*b*d^3*m^3 + 21*b*d^3*m^2 + 7*b*d^3*m + b*d^3)*n)*x*log(
x) + (a*d^3*m^7 + 7*a*d^3*m^6 + 21*a*d^3*m^5 + 35*a*d^3*m^4 + 35*a*d^3*m^3
+ 36*(a*d^3*m - b*d^3*n + a*d^3)*r^6 + 21*a*d^3*m^2 + 132*(a*d^3*m^2 + 2*a*
d^3*m + a*d^3 - (b*d^3*m + b*d^3)*n)*r^5 + 7*a*d^3*m + 193*(a*d^3*m^3 + 3*a
*d^3*m^2 + 3*a*d^3*m + a*d^3 - (b*d^3*m^2 + 2*b*d^3*m + b*d^3)*n)*r^4 + a*d
^3 + 144*(a*d^3*m^4 + 4*a*d^3*m^3 + 6*a*d^3*m^2 + 4*a*d^3*m + a*d^3 - (b*d^
3*m^3 + 3*b*d^3*m^2 + 3*b*d^3*m + b*d^3)*n)*r^3 + 58*(a*d^3*m^5 + 5*a*d^3*m
^4 + 10*a*d^3*m^3 + 10*a*d^3*m^2 + 5*a*d^3*m + a*d^3 - (b*d^3*m^4 + 4*b*d^3
*m^3 + 6*b*d^3*m^2 + 4*b*d^3*m + b*d^3)*n)*r^2 - (b*d^3*m^6 + 6*b*d^3*m^5 +
 15*b*d^3*m^4 + 20*b*d^3*m^3 + 15*b*d^3*m^2 + 6*b*d^3*m + b*d^3)*n + 12*(a*
d^3*m^6 + 6*a*d^3*m^5 + 15*a*d^3*m^4 + 20*a*d^3*m^3 + 15*a*d^3*m^2 + 6*a*d^
3*m + a*d^3 - (b*d^3*m^5 + 5*b*d^3*m^4 + 10*b*d^3*m^3 + 10*b*d^3*m^2 + 5*b*
d^3*m + b*d^3)*n)*r)*x)*e^(m*log(f) + m*log(x)))/(m^8 + 8*m^7 + 36*(m^2 + 2
*m + 1)*r^6 + 28*m^6 + 132*(m^3 + 3*m^2 + 3*m + 1)*r^5 + 56*m^5 + 193*(m^4
+ 4*m^3 + 6*m^2 + 4*m + 1)*r^4 + 70*m^4 + 144*(m^5 + 5*m^4 + 10*m^3 + 10*m^
2 + 5*m + 1)*r^3 + 56*m^3 + 58*(m^6 + 6*m^5 + 15*m^4 + 20*m^3 + 15*m^2 + 6*
m + 1)*r^2 + 28*m^2 + 12*(m^7 + 7*m^6 + 21*m^5 + 35*m^4 + 35*m^3 + 21*m^2 +
 7*m + 1)*r + 8*m + 1)
```

Sympy [F(-2)]

Exception generated.

$$\int (fx)^m (d+ex^r)^3 (a+b\log(cx^n))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x)**m*(d+e*x**r)**3*(a+b*ln(c*x**n)),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.47

$$\int (fx)^m (d+ex^r)^3 (a+b\log(cx^n))\,dx = \frac{be^3f^mxe^{(m\log(x)+3\,r\log(x))}\log(cx^n)}{m+3\,r+1} + \frac{3\,bde^2f^mxe^{(m\log(x)+2\,r\log(x))}\log(cx^n)}{m+2\,r+1} + \frac{3\,bd^2ef^mxe^{(m\log(x)+r\log(x))}\log(cx^n)}{m+r+1} - \frac{bd^3f^mnxx^m}{(m+1)^2} + \frac{ae^3f^mxe^{(m\log(x)+3\,r\log(x))}}{m+3\,r+1} - \frac{be^3f^mnxe^{(m\log(x)+3\,r\log(x))}}{(m+3\,r+1)^2} + \frac{3\,ade^2f^mxe^{(m\log(x)+2\,r\log(x))}}{m+2\,r+1} - \frac{3\,bde^2f^mnxe^{(m\log(x)+2\,r\log(x))}}{(m+2\,r+1)^2} + \frac{3\,ad^2ef^mxe^{(m\log(x)+r\log(x))}}{m+r+1} - \frac{3\,bd^2ef^mnxe^{(m\log(x)+r\log(x))}}{(m+r+1)^2} + \frac{(fx)^{m+1}bd^3\log(cx^n)}{f(m+1)} + \frac{(fx)^{m+1}ad^3}{f(m+1)}$$

```
[In] integrate((f*x)^m*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] b*e^3*f^m*x*e^(m*log(x) + 3*r*log(x))*log(c*x^n)/(m + 3*r + 1) + 3*b*d*e^2*
f^m*x*e^(m*log(x) + 2*r*log(x))*log(c*x^n)/(m + 2*r + 1) + 3*b*d^2*e*f^m*x*
e^(m*log(x) + r*log(x))*log(c*x^n)/(m + r + 1) - b*d^3*f^m*n*x*x^m/(m + 1)^
2 + a*e^3*f^m*x*e^(m*log(x) + 3*r*log(x))/(m + 3*r + 1) - b*e^3*f^m*n*x*e^(
```

```
m*log(x) + 3*r*log(x))/(m + 3*r + 1)^2 + 3*a*d*e^2*f^m*x*e^(m*log(x) + 2*r*
log(x))/(m + 2*r + 1) - 3*b*d*e^2*f^m*n*x*e^(m*log(x) + 2*r*log(x))/(m + 2*
r + 1)^2 + 3*a*d^2*e*f^m*x*e^(m*log(x) + r*log(x))/(m + r + 1) - 3*b*d^2*e*
f^m*n*x*e^(m*log(x) + r*log(x))/(m + r + 1)^2 + (f*x)^(m + 1)*b*d^3*log(c*x
^n)/(f*(m + 1)) + (f*x)^(m + 1)*a*d^3/(f*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(233) = 466$.

Time = 0.37 (sec) , antiderivative size = 772, normalized size of antiderivative = 3.31

$$\int (fx)^m (d+ex^r)^3 (a+b\log(cx^n))\,dx = \frac{be^3f^m mnxx^m x^{3r}\log(x)}{m^2+6mr+9r^2+2m+6r+1} + \frac{3be^3f^m nrxx^m x^{3r}\log(x)}{m^2+6mr+9r^2+2m+6r+1} + \frac{3bde^2f^m mnxx^m x^{2r}\log(x)}{m^2+4mr+4r^2+2m+4r+1} + \frac{6bde^2f^m nrxx^m x^{2r}\log(x)}{m^2+4mr+4r^2+2m+4r+1} + \frac{3bd^2ef^m mnxx^m x^r\log(x)}{m^2+2mr+r^2+2m+2r+1} + \frac{3bd^2ef^m nrxx^m x^r\log(x)}{m^2+2mr+r^2+2m+2r+1} + \frac{bd^3f^m mnxx^m\log(x)}{m^2+2m+1} + \frac{be^3f^m nxx^m x^{3r}\log(x)}{m^2+6mr+9r^2+2m+6r+1} + \frac{3bde^2f^m nxx^m x^{2r}\log(x)}{m^2+4mr+4r^2+2m+4r+1} + \frac{3bd^2ef^m nxx^m x^r\log(x)}{m^2+2mr+r^2+2m+2r+1} - \frac{be^3f^m nxx^m x^{3r}}{m^2+6mr+9r^2+2m+6r+1} - \frac{3bde^2f^m nxx^m x^{2r}}{m^2+4mr+4r^2+2m+4r+1} - \frac{3bd^2ef^m nxx^m x^r}{m^2+2mr+r^2+2m+2r+1} + \frac{be^3f^m xx^m x^{3r}\log(c)}{m+3r+1} + \frac{3bde^2f^m xx^m x^{2r}\log(c)}{m+2r+1} + \frac{3bd^2ef^m xx^m x^r\log(c)}{m+r+1} + \frac{bd^3f^m nxx^m\log(x)}{m^2+2m+1} - \frac{bd^3f^m nxx^m}{m^2+2m+1} + \frac{ae^3f^m xx^m x^{3r}}{m+3r+1} + \frac{3ade^2f^m xx^m x^{2r}}{m+2r+1} + \frac{3ad^2ef^m xx^m x^r}{m+r+1} + \frac{(fx)^m bd^3x\log(c)}{m+1} + \frac{(fx)^m ad^3x}{m+1}$$

```
[In] integrate((f*x)^m*(d+e*x^r)^3*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*e^3*f^m*m*n*x*x^m*x^(3*r)*log(x)/(m^2 + 6*m*r + 9*r^2 + 2*m + 6*r + 1) +
3*b*e^3*f^m*n*r*x*x^m*x^(3*r)*log(x)/(m^2 + 6*m*r + 9*r^2 + 2*m + 6*r + 1)
+ 3*b*d*e^2*f^m*m*n*x*x^m*x^(2*r)*log(x)/(m^2 + 4*m*r + 4*r^2 + 2*m + 4*r +
```

```
 1) + 6*b*d*e^2*f^m*n*r*x*x^m*x^(2*r)*log(x)/(m^2 + 4*m*r + 4*r^2 + 2*m + 4
*r + 1) + 3*b*d^2*e*f^m*m*n*x*x^m*x^r*log(x)/(m^2 + 2*m*r + r^2 + 2*m + 2*r
 + 1) + 3*b*d^2*e*f^m*n*r*x*x^m*x^r*log(x)/(m^2 + 2*m*r + r^2 + 2*m + 2*r +
 1) + b*d^3*f^m*m*n*x*x^m*log(x)/(m^2 + 2*m + 1) + b*e^3*f^m*n*x*x^m*x^(3*r
)*log(x)/(m^2 + 6*m*r + 9*r^2 + 2*m + 6*r + 1) + 3*b*d*e^2*f^m*n*x*x^m*x^(2
*r)*log(x)/(m^2 + 4*m*r + 4*r^2 + 2*m + 4*r + 1) + 3*b*d^2*e*f^m*n*x*x^m*x^
r*log(x)/(m^2 + 2*m*r + r^2 + 2*m + 2*r + 1) - b*e^3*f^m*n*x*x^m*x^(3*r)/(m
^2 + 6*m*r + 9*r^2 + 2*m + 6*r + 1) - 3*b*d*e^2*f^m*n*x*x^m*x^(2*r)/(m^2 +
4*m*r + 4*r^2 + 2*m + 4*r + 1) - 3*b*d^2*e*f^m*n*x*x^m*x^r/(m^2 + 2*m*r + r
^2 + 2*m + 2*r + 1) + b*e^3*f^m*x*x^m*x^(3*r)*log(c)/(m + 3*r + 1) + 3*b*d*
e^2*f^m*x*x^m*x^(2*r)*log(c)/(m + 2*r + 1) + 3*b*d^2*e*f^m*x*x^m*x^r*log(c)
/(m + r + 1) + b*d^3*f^m*n*x*x^m*log(x)/(m^2 + 2*m + 1) - b*d^3*f^m*n*x*x^m
/(m^2 + 2*m + 1) + a*e^3*f^m*x*x^m*x^(3*r)/(m + 3*r + 1) + 3*a*d*e^2*f^m*x*
x^m*x^(2*r)/(m + 2*r + 1) + 3*a*d^2*e*f^m*x*x^m*x^r/(m + r + 1) + (f*x)^m*b
*d^3*x*log(c)/(m + 1) + (f*x)^m*a*d^3*x/(m + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^r)^3 (a+b\log(cx^n))\, dx = \int (f\,x)^m\, (d+e\,x^r)^3\, (a+b\,\ln(c\,x^n))\, dx$$

```
[In] int((f*x)^m*(d + e*x^r)^3*(a + b*log(c*x^n)),x)
[Out] int((f*x)^m*(d + e*x^r)^3*(a + b*log(c*x^n)), x)
```

3.441 $\int (fx)^m (d+ex^r)^2 (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 165

$$\int (fx)^m (d+ex^r)^2 (a+b\log(cx^n))\,dx = -\frac{2bdenx^{1+r}(fx)^m}{(1+m+r)^2} - \frac{be^2nx^{1+2r}(fx)^m}{(1+m+2r)^2} - \frac{bd^2n(fx)^{1+m}}{f(1+m)^2} + \frac{2dex^{1+r}(fx)^m (a+b\log(cx^n))}{1+m+r} + \frac{e^2x^{1+2r}(fx)^m (a+b\log(cx^n))}{1+m+2r} + \frac{d^2(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)}$$

```
[Out] -2*b*d*e*n*x^(1+r)*(f*x)^m/(1+m+r)^2-b*e^2*n*x^(1+2*r)*(f*x)^m/(1+m+2*r)^2-
b*d^2*n*(f*x)^(1+m)/f/(1+m)^2+2*d*e*x^(1+r)*(f*x)^m*(a+b*ln(c*x^n))/(1+m+r)
+e^2*x^(1+2*r)*(f*x)^m*(a+b*ln(c*x^n))/(1+m+2*r)+d^2*(f*x)^(1+m)*(a+b*ln(c*
x^n))/f/(1+m)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

$= \{276, 20, 30, 2392, 14\}$

$$\int (fx)^m (d+ex^r)^2 (a+b\log(cx^n))\, dx = \frac{d^2(fx)^{m+1}(a+b\log(cx^n))}{f(m+1)} + \frac{2dex^{r+1}(fx)^m(a+b\log(cx^n))}{m+r+1} + \frac{e^2x^{2r+1}(fx)^m(a+b\log(cx^n))}{m+2r+1} - \frac{bd^2n(fx)^{m+1}}{f(m+1)^2} - \frac{2bdenx^{r+1}(fx)^m}{(m+r+1)^2} - \frac{be^2nx^{2r+1}(fx)^m}{(m+2r+1)^2}$$

```
[In] Int[(f*x)^m*(d + e*x^r)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] (-2*b*d*e*n*x^(1 + r)*(f*x)^m)/(1 + m + r)^2 - (b*e^2*n*x^(1 + 2*r)*(f*x)^m
)/(1 + m + 2*r)^2 - (b*d^2*n*(f*x)^(1 + m))/(f*(1 + m)^2) + (2*d*e*x^(1 + r
)*(f*x)^m*(a + b*Log[c*x^n]))/(1 + m + r) + (e^2*x^(1 + 2*r)*(f*x)^m*(a + b
*Log[c*x^n]))/(1 + m + 2*r) + (d^2*(f*x)^(1 + m)*(a + b*Log[c*x^n]))/(f*(1
+ m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
```

```
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2dex^{1+r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+r} \\
&\quad+\frac{e^2x^{1+2r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+2r}+\frac{d^2(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)} \\
&\quad-(bn)\int(fx)^m\left(\frac{d^2}{1+m}+\frac{2dex^r}{1+m+r}+\frac{e^2x^{2r}}{1+m+2r}\right)dx \\
&= \frac{2dex^{1+r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+r} \\
&\quad+\frac{e^2x^{1+2r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+2r}+\frac{d^2(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)} \\
&\quad-(bn)\int\left(\frac{d^2(fx)^m}{1+m}+\frac{2dex^r(fx)^m}{1+m+r}+\frac{e^2x^{2r}(fx)^m}{1+m+2r}\right)dx \\
&= -\frac{bd^2n(fx)^{1+m}}{f(1+m)^2}+\frac{2dex^{1+r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+r}+\frac{e^2x^{1+2r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+2r} \\
&\quad+\frac{d^2(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)}-\frac{(2bden)\int x^r(fx)^m\,dx}{1+m+r}-\frac{\left(be^2n\right)\int x^{2r}(fx)^m\,dx}{1+m+2r} \\
&= -\frac{bd^2n(fx)^{1+m}}{f(1+m)^2}+\frac{2dex^{1+r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+r} \\
&\quad+\frac{e^2x^{1+2r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+2r}+\frac{d^2(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)} \\
&\quad-\frac{\left(2bdenx^{-m}(fx)^m\right)\int x^{m+r}\,dx}{1+m+r}-\frac{\left(be^2nx^{-m}(fx)^m\right)\int x^{m+2r}\,dx}{1+m+2r} \\
&= -\frac{2bdenx^{1+r}(fx)^m}{(1+m+r)^2}-\frac{be^2nx^{1+2r}(fx)^m}{(1+m+2r)^2} \\
&\quad-\frac{bd^2n(fx)^{1+m}}{f(1+m)^2}+\frac{2dex^{1+r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+r} \\
&\quad+\frac{e^2x^{1+2r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+2r}+\frac{d^2(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.75

$$\int (fx)^m (d+ex^r)^2 (a+b\log(cx^n))\,dx = x(fx)^m\left(-\frac{bd^2n}{(1+m)^2}-\frac{2bdenx^r}{(1+m+r)^2}-\frac{be^2nx^{2r}}{(1+m+2r)^2}+\frac{d^2(a+b\log(cx^n))}{1+m}+\frac{2dex^r(a+b\log(cx^n))}{1+m+r}+\frac{e^2x^{2r}(a+b\log(cx^n))}{1+m+2r}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^r)^2*(a + b*Log[c*x^n]),x]
```

```
[Out] x*(f*x)^m*(-((b*d^2*n)/(1 + m)^2) - (2*b*d*e*n*x^r)/(1 + m + r)^2 - (b*e^2*
n*x^(2*r))/(1 + m + 2*r)^2 + (d^2*(a + b*Log[c*x^n]))/(1 + m) + (2*d*e*x^r*
(a + b*Log[c*x^n]))/(1 + m + r) + (e^2*x^(2*r)*(a + b*Log[c*x^n]))/(1 + m +
 2*r))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3058 vs. 2(165) = 330.

Time = 13.00 (sec) , antiderivative size = 3059, normalized size of antiderivative = 18.54

method	result	size
parallelrisch	Expression too large to display	3059
risch	Expression too large to display	8671

```
[In] int((f*x)^m*(d+e*x^r)^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -(-39*x*(f*x)^m*a*d^2*m*r^2+13*x*(f*x)^m*b*d^2*n*r^2+4*x*(f*x)^m*b*d^2*n*r^
4-10*x*(x^r)^2*(f*x)^m*a*e^2*m^2-5*x*(x^r)^2*(f*x)^m*a*e^2*r^2-12*x*(f*x)^m
*ln(c*x^n)*b*d^2*r^3-24*x*(f*x)^m*a*d^2*m^3*r-39*x*(f*x)^m*a*d^2*m^2*r^2-e^
2*b*ln(c*x^n)*(f*x)^m*(x^r)^2*x-10*x*(x^r)^2*(f*x)^m*a*e^2*m^3-5*x*(x^r)^2*
(f*x)^m*a*e^2*m-6*x*(f*x)^m*ln(c*x^n)*b*d^2*r-24*x*(f*x)^m*a*d^2*m*r-x*(f*x
)^m*a*d^2-15*x*(x^r)^2*(f*x)^m*a*e^2*m^2*r^2-4*x*(x^r)^2*(f*x)^m*a*e^2*m*r^
3-2*b*d*e*ln(c*x^n)*(f*x)^m*x^r*x-x*(x^r)^2*(f*x)^m*ln(c*x^n)*b*e^2*m^5-10*
x*(x^r)^2*(f*x)^m*ln(c*x^n)*b*e^2*m^3-2*x*(x^r)^2*(f*x)^m*ln(c*x^n)*b*e^2*r
^3-16*x*(x^r)^2*(f*x)^m*a*e^2*m^3*r-5*x*(x^r)^2*(f*x)^m*a*e^2*m^3*r^2-24*x*
(f*x)^m*a*d^2*m*r^3-36*x*(f*x)^m*ln(c*x^n)*b*d^2*m^2*r-39*x*(f*x)^m*ln(c*x^
n)*b*d^2*m*r^2+18*x*(f*x)^m*b*d^2*m^2*n*r+26*x*(f*x)^m*b*d^2*m*n*r^2-20*x*x
^r*(f*x)^m*a*d*e*m^2-16*x*x^r*(f*x)^m*a*d*e*r^2-24*x*(f*x)^m*ln(c*x^n)*b*d^
2*m*r+18*x*(f*x)^m*b*d^2*m*n*r-10*x*x^r*(f*x)^m*a*d*e*m-10*x*x^r*(f*x)^m*a*
d*e*r+2*x*x^r*(f*x)^m*b*d*e*n-2*x*(x^r)^2*(f*x)^m*a*e^2*r^3-4*x*(f*x)^m*ln(
c*x^n)*b*d^2*r^4-6*x*(f*x)^m*a*d^2*m^4*r-13*x*(f*x)^m*a*d^2*m^3*r^2-12*x*(f
```

*x)^m*a*d^2*m^2*r^3-4*x*(f*x)^m*a*d^2*m*r^4+12*x*(f*x)^m*b*d^2*m*n*r^3-5*x*
(x^r)^2*(f*x)^m*ln(c*x^n)*b*e^2*m-4*x*(x^r)^2*(f*x)^m*ln(c*x^n)*b*e^2*r+12*
x*(f*x)^m*b*d^2*n*r^3-16*x*(x^r)^2*(f*x)^m*a*e^2*m*r+4*x*(x^r)^2*(f*x)^m*b*
e^2*m*n+2*x*(x^r)^2*(f*x)^m*b*e^2*n*r-20*x*x^r*(f*x)^m*a*d*e*m^3-8*x*x^r*(f
*x)^m*a*d*e*r^3-2*x*(x^r)^2*(f*x)^m*a*e^2*m^2*r^3+x*(x^r)^2*(f*x)^m*b*e^2*m
^4*n-5*x*(x^r)^2*(f*x)^m*ln(c*x^n)*b*e^2*m^4-4*x*(x^r)^2*(f*x)^m*a*e^2*m^4*
r-x*(f*x)^m*ln(c*x^n)*b*d^2-x*(f*x)^m*a*d^2*m^5-5*x*(f*x)^m*a*d^2*m^4-10*x*
(f*x)^m*a*d^2*m^3-10*x*(f*x)^m*a*d^2*m^2-5*x*(f*x)^m*a*d^2*m+x*(f*x)^m*b*d^
2*n-4*x*(f*x)^m*a*d^2*r^4+13*x*(f*x)^m*b*d^2*m^2*n*r^2-24*x*(x^r)^2*(f*x)^m
*a*e^2*m^2*r-15*x*(x^r)^2*(f*x)^m*a*e^2*m*r^2+6*x*(x^r)^2*(f*x)^m*b*e^2*m^2
n+x(x^r)^2*(f*x)^m*b*e^2*n*r^2-10*x*x^r*(f*x)^m*a*d*e*m^4-24*x*(f*x)^m*ln
(c*x^n)*b*d^2*m^3*r-39*x*(f*x)^m*ln(c*x^n)*b*d^2*m^2*r^2-24*x*(f*x)^m*ln(c*
x^n)*b*d^2*m*r^3+6*x*(f*x)^m*b*d^2*m^3*n*r+4*x*(x^r)^2*(f*x)^m*b*e^2*m^3*n-
2*x*x^r*(f*x)^m*a*d*e*m^5-6*x*(f*x)^m*ln(c*x^n)*b*d^2*m^4*r-13*x*(f*x)^m*ln
(c*x^n)*b*d^2*m^3*r^2-12*x*(f*x)^m*ln(c*x^n)*b*d^2*m^2*r^3-4*x*(f*x)^m*ln(c
*x^n)*b*d^2*m*r^4-10*x*(x^r)^2*(f*x)^m*ln(c*x^n)*b*e^2*m^2-5*x*(x^r)^2*(f*x
)^m*ln(c*x^n)*b*e^2*r^2-4*x*(x^r)^2*(f*x)^m*a*e^2*r+x*(x^r)^2*(f*x)^m*b*e^2
*n-13*x*(f*x)^m*ln(c*x^n)*b*d^2*r^2-5*x*(x^r)^2*(f*x)^m*a*e^2*m^4-x*(x^r)^2
*(f*x)^m*a*e^2*m^5+6*x*(f*x)^m*b*d^2*n*r-2*x*x^r*(f*x)^m*a*d*e-36*x*(f*x)^m
*a*d^2*m^2*r-12*x*(f*x)^m*a*d^2*r^3-x*(x^r)^2*(f*x)^m*a*e^2-13*x*(f*x)^m*a*
d^2*r^2-6*x*(f*x)^m*a*d^2*r-4*x*(x^r)^2*(f*x)^m*ln(c*x^n)*b*e^2*m^4*r-x*(f*
x)^m*ln(c*x^n)*b*d^2*m^5-5*x*(f*x)^m*ln(c*x^n)*b*d^2*m^4+x*(f*x)^m*b*d^2*m^
4*n-10*x*(f*x)^m*ln(c*x^n)*b*d^2*m^3+4*x*(f*x)^m*b*d^2*m^3*n-10*x*(f*x)^m*l
n(c*x^n)*b*d^2*m^2+6*x*(f*x)^m*b*d^2*m^2*n-5*x*(f*x)^m*ln(c*x^n)*b*d^2*m+4*
x*(f*x)^m*b*d^2*m*n-40*x*x^r*(f*x)^m*ln(c*x^n)*b*d*e*m*r+24*x*x^r*(f*x)^m*b
*d*e*m*n*r-10*x*x^r*(f*x)^m*ln(c*x^n)*b*d*e*m^4*r-16*x*x^r*(f*x)^m*ln(c*x^n
)*b*d*e*m^3*r^2-8*x*x^r*(f*x)^m*ln(c*x^n)*b*d*e*m^2*r^3-40*x*x^r*(f*x)^m*ln
(c*x^n)*b*d*e*m^3*r-48*x*x^r*(f*x)^m*ln(c*x^n)*b*d*e*m^2*r^2-16*x*x^r*(f*x)
^m*ln(c*x^n)*b*d*e*m*r^3+8*x*x^r*(f*x)^m*b*d*e*m^3*n*r+8*x*x^r*(f*x)^m*b*d*
e*m^2*n*r^2-60*x*x^r*(f*x)^m*ln(c*x^n)*b*d*e*m^2*r-48*x*x^r*(f*x)^m*ln(c*x^
n)*b*d*e*m*r^2+24*x*x^r*(f*x)^m*b*d*e*m^2*n*r+16*x*x^r*(f*x)^m*b*d*e*m*n*r^
2-16*x*x^r*(f*x)^m*a*d*e*m^3*r^2-8*x*x^r*(f*x)^m*a*d*e*m^2*r^3+2*x*x^r*(f*x
)^m*b*d*e*m^4*n-16*x*(x^r)^2*(f*x)^m*ln(c*x^n)*b*e^2*m*r+6*x*(x^r)^2*(f*x)^
m*b*e^2*m*n*r-20*x*x^r*(f*x)^m*ln(c*x^n)*b*d*e*m^3-8*x*x^r*(f*x)^m*ln(c*x^n
)*b*d*e*r^3-40*x*x^r*(f*x)^m*a*d*e*m^3*r-48*x*x^r*(f*x)^m*a*d*e*m^2*r^2-16*
x*x^r*(f*x)^m*a*d*e*m*r^3+8*x*x^r*(f*x)^m*b*d*e*m^3*n-20*x*x^r*(f*x)^m*ln(c
*x^n)*b*d*e*m^2-16*x*x^r*(f*x)^m*ln(c*x^n)*b*d*e*r^2-60*x*x^r*(f*x)^m*a*d*e
*m^2*r-48*x*x^r*(f*x)^m*a*d*e*m*r^2+12*x*x^r*(f*x)^m*b*d*e*m^2*n+8*x*x^r*(f
*x)^m*b*d*e*n*r^2-10*x*x^r*(f*x)^m*ln(c*x^n)*b*d*e*m-10*x*x^r*(f*x)^m*ln(c*
x^n)*b*d*e*r-40*x*x^r*(f*x)^m*a*d*e*m*r+8*x*x^r*(f*x)^m*b*d*e*m*n+8*x*x^r*(
f*x)^m*b*d*e*n*r-5*x*(x^r)^2*(f*x)^m*ln(c*x^n)*b*e^2*m^3*r^2-2*x*(x^r)^2*(f
*x)^m*ln(c*x^n)*b*e^2*m^2*r^3-16*x*(x^r)^2*(f*x)^m*ln(c*x^n)*b*e^2*m^3*r-15
x(x^r)^2*(f*x)^m*ln(c*x^n)*b*e^2*m^2*r^2-4*x*(x^r)^2*(f*x)^m*ln(c*x^n)*b*
e^2*m*r^3+2*x*(x^r)^2*(f*x)^m*b*e^2*m^3*n*r+x*(x^r)^2*(f*x)^m*b*e^2*m^2*n*r
^2-2*x*x^r*(f*x)^m*ln(c*x^n)*b*d*e*m^5-24*x*(x^r)^2*(f*x)^m*ln(c*x^n)*b*e^2

```
*m^2*r-15*x*(x^r)^2*(f*x)^m*ln(c*x^n)*b*e^2*m*r^2+6*x*(x^r)^2*(f*x)^m*b*e^2
*m^2*n*r+2*x*(x^r)^2*(f*x)^m*b*e^2*m*n*r^2-10*x*x^r*(f*x)^m*ln(c*x^n)*b*d*e
*m^4-10*x*x^r*(f*x)^m*a*d*e*m^4*r)/(m^2+4*m*r+4*r^2+2*m+4*r+1)/(m^2+2*m*r+r
^2+2*m+2*r+1)/(m^2+2*m+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1875 vs. $2(165) = 330$.

Time = 0.34 (sec) , antiderivative size = 1875, normalized size of antiderivative = 11.36

$$\int (fx)^m (d+ex^r)^2 (a+b\log(cx^n))\, dx = \text{Too large to display}$$

```
[In] integrate((f*x)^m*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (((b*e^2*m^5 + 5*b*e^2*m^4 + 10*b*e^2*m^3 + 10*b*e^2*m^2 + 5*b*e^2*m + 2*(b
*e^2*m^2 + 2*b*e^2*m + b*e^2)*r^3 + b*e^2 + 5*(b*e^2*m^3 + 3*b*e^2*m^2 + 3*
b*e^2*m + b*e^2)*r^2 + 4*(b*e^2*m^4 + 4*b*e^2*m^3 + 6*b*e^2*m^2 + 4*b*e^2*m
 + b*e^2)*r)*x*log(c) + (2*(b*e^2*m^2 + 2*b*e^2*m + b*e^2)*n*r^3 + 5*(b*e^2
*m^3 + 3*b*e^2*m^2 + 3*b*e^2*m + b*e^2)*n*r^2 + 4*(b*e^2*m^4 + 4*b*e^2*m^3
+ 6*b*e^2*m^2 + 4*b*e^2*m + b*e^2)*n*r + (b*e^2*m^5 + 5*b*e^2*m^4 + 10*b*e^
2*m^3 + 10*b*e^2*m^2 + 5*b*e^2*m + b*e^2)*n)*x*log(x) + (a*e^2*m^5 + 5*a*e^
2*m^4 + 10*a*e^2*m^3 + 10*a*e^2*m^2 + 5*a*e^2*m + 2*(a*e^2*m^2 + 2*a*e^2*m
+ a*e^2)*r^3 + a*e^2 + (5*a*e^2*m^3 + 15*a*e^2*m^2 + 15*a*e^2*m + 5*a*e^2 -
 (b*e^2*m^2 + 2*b*e^2*m + b*e^2)*n)*r^2 - (b*e^2*m^4 + 4*b*e^2*m^3 + 6*b*e^
2*m^2 + 4*b*e^2*m + b*e^2)*n + 2*(2*a*e^2*m^4 + 8*a*e^2*m^3 + 12*a*e^2*m^2
+ 8*a*e^2*m + 2*a*e^2 - (b*e^2*m^3 + 3*b*e^2*m^2 + 3*b*e^2*m + b*e^2)*n)*r)
*x)*x^(2*r)*e^(m*log(f) + m*log(x)) + 2*((b*d*e*m^5 + 5*b*d*e*m^4 + 10*b*d*
e*m^3 + 10*b*d*e*m^2 + 5*b*d*e*m + 4*(b*d*e*m^2 + 2*b*d*e*m + b*d*e)*r^3 +
b*d*e + 8*(b*d*e*m^3 + 3*b*d*e*m^2 + 3*b*d*e*m + b*d*e)*r^2 + 5*(b*d*e*m^4
+ 4*b*d*e*m^3 + 6*b*d*e*m^2 + 4*b*d*e*m + b*d*e)*r)*x*log(c) + (4*(b*d*e*m^
2 + 2*b*d*e*m + b*d*e)*n*r^3 + 8*(b*d*e*m^3 + 3*b*d*e*m^2 + 3*b*d*e*m + b*d
*e)*n*r^2 + 5*(b*d*e*m^4 + 4*b*d*e*m^3 + 6*b*d*e*m^2 + 4*b*d*e*m + b*d*e)*n
*r + (b*d*e*m^5 + 5*b*d*e*m^4 + 10*b*d*e*m^3 + 10*b*d*e*m^2 + 5*b*d*e*m + b
*d*e)*n)*x*log(x) + (a*d*e*m^5 + 5*a*d*e*m^4 + 10*a*d*e*m^3 + 10*a*d*e*m^2
+ 5*a*d*e*m + 4*(a*d*e*m^2 + 2*a*d*e*m + a*d*e)*r^3 + a*d*e + 4*(2*a*d*e*m^
3 + 6*a*d*e*m^2 + 6*a*d*e*m + 2*a*d*e - (b*d*e*m^2 + 2*b*d*e*m + b*d*e)*n)*
r^2 - (b*d*e*m^4 + 4*b*d*e*m^3 + 6*b*d*e*m^2 + 4*b*d*e*m + b*d*e)*n + (5*a*
d*e*m^4 + 20*a*d*e*m^3 + 30*a*d*e*m^2 + 20*a*d*e*m + 5*a*d*e - 4*(b*d*e*m^3
 + 3*b*d*e*m^2 + 3*b*d*e*m + b*d*e)*n)*r)*x)*x^r*e^(m*log(f) + m*log(x)) +
((b*d^2*m^5 + 5*b*d^2*m^4 + 10*b*d^2*m^3 + 10*b*d^2*m^2 + 4*(b*d^2*m + b*d^
2)*r^4 + 5*b*d^2*m + 12*(b*d^2*m^2 + 2*b*d^2*m + b*d^2)*r^3 + b*d^2 + 13*(b
*d^2*m^3 + 3*b*d^2*m^2 + 3*b*d^2*m + b*d^2)*r^2 + 6*(b*d^2*m^4 + 4*b*d^2*m^
3 + 6*b*d^2*m^2 + 4*b*d^2*m + b*d^2)*r)*x*log(c) + (4*(b*d^2*m + b*d^2)*n*r
^4 + 12*(b*d^2*m^2 + 2*b*d^2*m + b*d^2)*n*r^3 + 13*(b*d^2*m^3 + 3*b*d^2*m^2
```

```
 + 3*b*d^2*m + b*d^2)*n*r^2 + 6*(b*d^2*m^4 + 4*b*d^2*m^3 + 6*b*d^2*m^2 + 4*
b*d^2*m + b*d^2)*n*r + (b*d^2*m^5 + 5*b*d^2*m^4 + 10*b*d^2*m^3 + 10*b*d^2*m
^2 + 5*b*d^2*m + b*d^2)*n)*x*log(x) + (a*d^2*m^5 + 5*a*d^2*m^4 + 10*a*d^2*m
^3 + 10*a*d^2*m^2 + 4*(a*d^2*m - b*d^2*n + a*d^2)*r^4 + 5*a*d^2*m + 12*(a*d
^2*m^2 + 2*a*d^2*m + a*d^2 - (b*d^2*m + b*d^2)*n)*r^3 + a*d^2 + 13*(a*d^2*m
^3 + 3*a*d^2*m^2 + 3*a*d^2*m + a*d^2 - (b*d^2*m^2 + 2*b*d^2*m + b*d^2)*n)*r
^2 - (b*d^2*m^4 + 4*b*d^2*m^3 + 6*b*d^2*m^2 + 4*b*d^2*m + b*d^2)*n + 6*(a*d
^2*m^4 + 4*a*d^2*m^3 + 6*a*d^2*m^2 + 4*a*d^2*m + a*d^2 - (b*d^2*m^3 + 3*b*d
^2*m^2 + 3*b*d^2*m + b*d^2)*n)*r)*x)*e^(m*log(f) + m*log(x)))/(m^6 + 6*m^5
+ 4*(m^2 + 2*m + 1)*r^4 + 15*m^4 + 12*(m^3 + 3*m^2 + 3*m + 1)*r^3 + 20*m^3
+ 13*(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)*r^2 + 15*m^2 + 6*(m^5 + 5*m^4 + 10*m^3
 + 10*m^2 + 5*m + 1)*r + 6*m + 1)
```

Sympy [F(-2)]

Exception generated.

$$\int (fx)^m \left(d+ex^r\right)^2 \left(a+b\log\left(cx^n\right)\right)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x)**m*(d+e*x**r)**2*(a+b*ln(c*x**n)),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.45

$$\begin{aligned}\int (fx)^m \left(d+ex^r\right)^2 \left(a+b\log\left(cx^n\right)\right)\,dx &= \frac{be^2f^mxe^{(m\log(x)+2\,r\log(x))}\log\left(cx^n\right)}{m+2\,r+1} \\ &+ \frac{2\,bdef^mxe^{(m\log(x)+r\log(x))}\log\left(cx^n\right)}{m+r+1} \\ &- \frac{bd^2f^mnxx^m}{(m+1)^2} + \frac{ae^2f^mxe^{(m\log(x)+2\,r\log(x))}}{m+2\,r+1} \\ &- \frac{be^2f^mnxe^{(m\log(x)+2\,r\log(x))}}{(m+2\,r+1)^2} \\ &+ \frac{2\,adef^mxe^{(m\log(x)+r\log(x))}}{m+r+1} \\ &- \frac{2\,bdef^mnxe^{(m\log(x)+r\log(x))}}{(m+r+1)^2} \\ &+ \frac{(fx)^{m+1}bd^2\log\left(cx^n\right)}{f(m+1)} + \frac{(fx)^{m+1}ad^2}{f(m+1)}\end{aligned}$$

```
[In] integrate((f*x)^m*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] b*e^2*f^m*x*e^(m*log(x) + 2*r*log(x))*log(c*x^n)/(m + 2*r + 1) + 2*b*d*e*f^
m*x*e^(m*log(x) + r*log(x))*log(c*x^n)/(m + r + 1) - b*d^2*f^m*n*x*x^m/(m +
 1)^2 + a*e^2*f^m*x*e^(m*log(x) + 2*r*log(x))/(m + 2*r + 1) - b*e^2*f^m*n*x
*e^(m*log(x) + 2*r*log(x))/(m + 2*r + 1)^2 + 2*a*d*e*f^m*x*e^(m*log(x) + r*
log(x))/(m + r + 1) - 2*b*d*e*f^m*n*x*e^(m*log(x) + r*log(x))/(m + r + 1)^2
 + (f*x)^(m + 1)*b*d^2*log(c*x^n)/(f*(m + 1)) + (f*x)^(m + 1)*a*d^2/(f*(m +
 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(165) = 330$.

Time = 0.37 (sec) , antiderivative size = 528, normalized size of antiderivative = 3.20

$$\int (fx)^m (d+ex^r)^2 (a+b\log(cx^n))\,dx = \frac{be^2f^mmnxx^mx^{2r}\log(x)}{m^2+4mr+4r^2+2m+4r+1} + \frac{2be^2f^mnrxx^mx^{2r}\log(x)}{m^2+4mr+4r^2+2m+4r+1} + \frac{2bdef^mmnxx^mx^r\log(x)}{m^2+2mr+r^2+2m+2r+1} + \frac{2bdef^mnrxx^mx^r\log(x)}{m^2+2mr+r^2+2m+2r+1} + \frac{bd^2f^mmnxx^m\log(x)}{m^2+2m+1} + \frac{be^2f^mnxx^mx^{2r}\log(x)}{m^2+4mr+4r^2+2m+4r+1} + \frac{2bdef^mnxx^mx^r\log(x)}{m^2+2mr+r^2+2m+2r+1} - \frac{be^2f^mnxx^mx^{2r}}{m^2+4mr+4r^2+2m+4r+1} - \frac{2bdef^mnxx^mx^r}{m^2+2mr+r^2+2m+2r+1} + \frac{be^2f^mxx^mx^{2r}\log(c)}{m+2r+1} + \frac{2bdef^mxx^mx^r\log(c)}{m+r+1} + \frac{bd^2f^mnxx^m\log(x)}{m^2+2m+1} - \frac{bd^2f^mnxx^m}{m^2+2m+1} + \frac{ae^2f^mxx^mx^{2r}}{m+2r+1} + \frac{2adef^mxx^mx^r}{m+r+1} + \frac{(fx)^m bd^2x\log(c)}{m+1} + \frac{(fx)^m ad^2x}{m+1}$$

```
[In] integrate((f*x)^m*(d+e*x^r)^2*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*e^2*f^m*m*n*x*x^m*x^(2*r)*log(x)/(m^2 + 4*m*r + 4*r^2 + 2*m + 4*r + 1) +
2*b*e^2*f^m*n*r*x*x^m*x^(2*r)*log(x)/(m^2 + 4*m*r + 4*r^2 + 2*m + 4*r + 1)
+ 2*b*d*e*f^m*m*n*x*x^m*x^r*log(x)/(m^2 + 2*m*r + r^2 + 2*m + 2*r + 1) + 2*
b*d*e*f^m*n*r*x*x^m*x^r*log(x)/(m^2 + 2*m*r + r^2 + 2*m + 2*r + 1) + b*d^2*
f^m*m*n*x*x^m*log(x)/(m^2 + 2*m + 1) + b*e^2*f^m*n*x*x^m*x^(2*r)*log(x)/(m^
2 + 4*m*r + 4*r^2 + 2*m + 4*r + 1) + 2*b*d*e*f^m*n*x*x^m*x^r*log(x)/(m^2 +
2*m*r + r^2 + 2*m + 2*r + 1) - b*e^2*f^m*n*x*x^m*x^(2*r)/(m^2 + 4*m*r + 4*r
^2 + 2*m + 4*r + 1) - 2*b*d*e*f^m*n*x*x^m*x^r/(m^2 + 2*m*r + r^2 + 2*m + 2*
r + 1) + b*e^2*f^m*x*x^m*x^(2*r)*log(c)/(m + 2*r + 1) + 2*b*d*e*f^m*x*x^m*x
^r*log(c)/(m + r + 1) + b*d^2*f^m*n*x*x^m*log(x)/(m^2 + 2*m + 1) - b*d^2*f^
m*n*x*x^m/(m^2 + 2*m + 1) + a*e^2*f^m*x*x^m*x^(2*r)/(m + 2*r + 1) + 2*a*d*e
```

```
*f^m*x*x^m*x^r/(m + r + 1) + (f*x)^m*b*d^2*x*log(c)/(m + 1) + (f*x)^m*a*d^2
*x/(m + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^r)^2 (a+b\log(cx^n))\,dx = \int (f\,x)^m\,(d+e\,x^r)^2\,(a+b\,\ln(c\,x^n))\,dx$$

```
[In] int((f*x)^m*(d + e*x^r)^2*(a + b*log(c*x^n)),x)
[Out] int((f*x)^m*(d + e*x^r)^2*(a + b*log(c*x^n)), x)
```

3.442 $\int (fx)^m (d+ex^r) (a+b\log(cx^n)) \, dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int (fx)^m (d+ex^r) (a+b\log(cx^n)) \, dx = -\frac{benx^{1+r}(fx)^m}{(1+m+r)^2} - \frac{bdn(fx)^{1+m}}{f(1+m)^2} + \frac{ex^{1+r}(fx)^m (a+b\log(cx^n))}{1+m+r} + \frac{d(fx)^{1+m} (a+b\log(cx^n))}{f(1+m)}$$

```
[Out] -b*e*n*x^(1+r)*(f*x)^m/(1+m+r)^2-b*d*n*(f*x)^(1+m)/f/(1+m)^2+e*x^(1+r)*(f*x
)^m*(a+b*ln(c*x^n))/(1+m+r)+d*(f*x)^(1+m)*(a+b*ln(c*x^n))/f/(1+m)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {14, 20, 30, 2392}

$$\int (fx)^m (d+ex^r) (a+b\log(cx^n)) \, dx = \frac{d(fx)^{m+1} (a+b\log(cx^n))}{f(m+1)} + \frac{ex^{r+1}(fx)^m (a+b\log(cx^n))}{m+r+1} - \frac{bdn(fx)^{m+1}}{f(m+1)^2} - \frac{benx^{r+1}(fx)^m}{(m+r+1)^2}$$

```
[In] Int[(f*x)^m*(d + e*x^r)*(a + b*Log[c*x^n]),x]
```

```
[Out] -((b*e*n*x^(1 + r)*(f*x)^m)/(1 + m + r)^2) - (b*d*n*(f*x)^(1 + m))/(f*(1 +
m)^2) + (e*x^(1 + r)*(f*x)^m*(a + b*Log[c*x^n]))/(1 + m + r) + (d*(f*x)^(1
+ m)*(a + b*Log[c*x^n]))/(f*(1 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^r)^q, x]
}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x],
x], x] /; ((EqQ[r, 1] || EqQ[r, 2]) && IntegerQ[m] && IntegerQ[q - 1/2]) ||
 InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] &&
 IntegerQ[2*q] && ((IntegerQ[m] && IntegerQ[r]) || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ex^{1+r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+r}+\frac{d(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)} \\
&\quad-(bn)\int(fx)^m\left(\frac{d}{1+m}+\frac{ex^r}{1+m+r}\right)dx \\
&= \frac{ex^{1+r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+r}+\frac{d(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)} \\
&\quad-(bn)\int\left(\frac{d(fx)^m}{1+m}+\frac{ex^r(fx)^m}{1+m+r}\right)dx \\
&= -\frac{bdn(fx)^{1+m}}{f(1+m)^2}+\frac{ex^{1+r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+r} \\
&\quad+\frac{d(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)}-\frac{(ben)\int x^r(fx)^m\,dx}{1+m+r} \\
&= -\frac{bdn(fx)^{1+m}}{f(1+m)^2}+\frac{ex^{1+r}(fx)^m\left(a+b\log\left(cx^n\right)\right)}{1+m+r} \\
&\quad+\frac{d(fx)^{1+m}\left(a+b\log\left(cx^n\right)\right)}{f(1+m)}-\frac{\left(benx^{-m}(fx)^m\right)\int x^{m+r}\,dx}{1+m+r}
\end{aligned}$$

$$
= -\frac{benx^{1+r}(fx)^m}{(1+m+r)^2} - \frac{bdn(fx)^{1+m}}{f(1+m)^2} + \frac{ex^{1+r}(fx)^m(a+b\log(cx^n))}{1+m+r} + \frac{d(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72

$$
\int (fx)^m(d+ex^r)(a+b\log(cx^n))\,dx = x(fx)^m\left(-\frac{bdn}{(1+m)^2} - \frac{benx^r}{(1+m+r)^2} + \frac{d(a+b\log(cx^n))}{1+m} + \frac{ex^r(a+b\log(cx^n))}{1+m+r}\right)
$$

[In] Integrate[(f*x)^m*(d + e*x^r)*(a + b*Log[c*x^n]),x]

[Out] x*(f*x)^m*(-((b*d*n)/(1 + m)^2) - (b*e*n*x^r)/(1 + m + r)^2 + (d*(a + b*Log[c*x^n]))/(1 + m) + (e*x^r*(a + b*Log[c*x^n]))/(1 + m + r))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. 2(97) = 194.

Time = 1.92 (sec) , antiderivative size = 660, normalized size of antiderivative = 6.80

method	result
parallelrisch	$-\frac{-x(fx)^m ad - x(fx)^m ad\,m^3 - 3x(fx)^m ad\,m^2 - 3x(fx)^m adm + x(fx)^m bdn - x(fx)^m \ln(c\,x^n)bd - x(fx)^m \ln(c\,x^n)bd\,m^3 - 3x(fx}{}$
risch	Expression too large to display

[In] int((f*x)^m*(d+e*x^r)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)

[Out] -(-x*(f*x)^m*a*d-x*(f*x)^m*a*d*m^3-3*x*(f*x)^m*a*d*m^2-3*x*(f*x)^m*a*d*m+x*(f*x)^m*b*d*n-x*(f*x)^m*ln(c*x^n)*b*d-x*(f*x)^m*ln(c*x^n)*b*d*m^3-3*x*(f*x)^m*ln(c*x^n)*b*d*m^2+x*(f*x)^m*b*d*m^2*n-3*x*(f*x)^m*ln(c*x^n)*b*d*m+2*x*(f*x)^m*b*d*m*n-2*x*x^r*(f*x)^m*ln(c*x^n)*b*e*m*r-x*x^r*(f*x)^m*ln(c*x^n)*b*e*m^2*r-x*x^r*(f*x)^m*ln(c*x^n)*b*e*m^3-3*x*x^r*(f*x)^m*ln(c*x^n)*b*e*m^2-x*x^r*(f*x)^m*a*e*m^2*r+x*x^r*(f*x)^m*b*e*m^2*n-2*x*(f*x)^m*ln(c*x^n)*b*d*m^2*r-x*(f*x)^m*ln(c*x^n)*b*d*m*r^2-3*x*x^r*(f*x)^m*ln(c*x^n)*b*e*m-x*x^r*(f*x)^m*ln(c*x^n)*b*e*r-2*x*x^r*(f*x)^m*a*e*m*r+2*x*x^r*(f*x)^m*b*e*m*n-4*x*(f*x)^m*ln(c*x^n)*b*d*m*r+2*x*(f*x)^m*b*d*m*n*r-x*(f*x)^m*a*d*r^2-x*x^r*(f*x)^m*a*e-2*x*(f*x)^m*a*d*r-x*x^r*(f*x)^m*ln(c*x^n)*b*e-x*x^r*(f*x)^m*a*e*m^3-3*x*x^r*(f*x)^m*a*e*m^2-x*(f*x)^m*ln(c*x^n)*b*d*r^2-2*x*(f*x)^m*a*d*m^2*r-x*(f*x)^m*a*d*m*r^2+x*(f*x)^m*b*d*n*r^2-3*x*x^r*(f*x)^m*a*e*m-x*x^r*(f*x)^m*a*e*r+x*x^r*(f*x)^m*b*e*n-2*x*(f*x)^m*ln(c*x^n)*b*d*r-4*x*(f*x)^m*a*d*m*r+2*x*(f*x)^m*b*d*n*r)/(1+m)^2/(1+m+r)^2

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(97) = 194.

Time = 0.28 (sec) , antiderivative size = 431, normalized size of antiderivative = 4.44

$$\int (fx)^m (d + ex^r) (a + b\log(cx^n)) \, dx$$
$$= \frac{((bem^3 + 3\,bem^2 + 3\,bem + be + (bem^2 + 2\,bem + be)r)x\log(c) + ((bem^2 + 2\,bem + be)nr + (bem^3 + 3\,b}{}$$

```
[In] integrate((f*x)^m*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (((b*e*m^3 + 3*b*e*m^2 + 3*b*e*m + b*e + (b*e*m^2 + 2*b*e*m + b*e)*r)*x*log
(c) + ((b*e*m^2 + 2*b*e*m + b*e)*n*r + (b*e*m^3 + 3*b*e*m^2 + 3*b*e*m + b*e
)*n)*x*log(x) + (a*e*m^3 + 3*a*e*m^2 + 3*a*e*m + a*e - (b*e*m^2 + 2*b*e*m +
 b*e)*n + (a*e*m^2 + 2*a*e*m + a*e)*r)*x)*x^r*e^(m*log(f) + m*log(x)) + ((b
*d*m^3 + 3*b*d*m^2 + 3*b*d*m + (b*d*m + b*d)*r^2 + b*d + 2*(b*d*m^2 + 2*b*d
*m + b*d)*r)*x*log(c) + ((b*d*m + b*d)*n*r^2 + 2*(b*d*m^2 + 2*b*d*m + b*d)*
n*r + (b*d*m^3 + 3*b*d*m^2 + 3*b*d*m + b*d)*n)*x*log(x) + (a*d*m^3 + 3*a*d*
m^2 + 3*a*d*m + (a*d*m - b*d*n + a*d)*r^2 + a*d - (b*d*m^2 + 2*b*d*m + b*d)
*n + 2*(a*d*m^2 + 2*a*d*m + a*d - (b*d*m + b*d)*n)*r)*x)*e^(m*log(f) + m*lo
g(x)))/(m^4 + 4*m^3 + (m^2 + 2*m + 1)*r^2 + 6*m^2 + 2*(m^3 + 3*m^2 + 3*m +
1)*r + 4*m + 1)
```

Sympy [F(-2)]

Exception generated.

$$\int (fx)^m (d + ex^r) (a + b\log(cx^n)) \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x)**m*(d+e*x**r)*(a+b*ln(c*x**n)),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.41

$$\int (fx)^m (d + ex^r) (a + b\log(cx^n)) \, dx = \frac{bef^m xe^{(m\log(x)+r\log(x))}\log(cx^n)}{m+r+1} - \frac{bdf^m nxx^m}{(m+1)^2} + \frac{aef^m xe^{(m\log(x)+r\log(x))}}{m+r+1} - \frac{bef^m nxe^{(m\log(x)+r\log(x))}}{(m+r+1)^2} + \frac{(fx)^{m+1}\,bd\log(cx^n)}{f(m+1)} + \frac{(fx)^{m+1}\,ad}{f(m+1)}$$

```
[In] integrate((f*x)^m*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] b*e*f^m*x*e^(m*log(x) + r*log(x))*log(c*x^n)/(m + r + 1) - b*d*f^m*n*x*x^m/
(m + 1)^2 + a*e*f^m*x*e^(m*log(x) + r*log(x))/(m + r + 1) - b*e*f^m*n*x*e^(
m*log(x) + r*log(x))/(m + r + 1)^2 + (f*x)^(m + 1)*b*d*log(c*x^n)/(f*(m + 1
)) + (f*x)^(m + 1)*a*d/(f*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(97) = 194$.

Time = 0.31 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.94

$$\int (fx)^m (d + ex^r) (a + b\log(cx^n)) \, dx = \frac{bef^m mnxx^m x^r \log(x)}{m^2 + 2\,mr + r^2 + 2\,m + 2\,r + 1} + \frac{bef^m nrxx^m x^r \log(x)}{m^2 + 2\,mr + r^2 + 2\,m + 2\,r + 1} + \frac{bdf^m mnxx^m \log(x)}{m^2 + 2\,m + 1} + \frac{bef^m nxx^m x^r \log(x)}{m^2 + 2\,mr + r^2 + 2\,m + 2\,r + 1} - \frac{bef^m nxx^m x^r}{m^2 + 2\,mr + r^2 + 2\,m + 2\,r + 1} + \frac{bef^m xx^m x^r \log(c)}{m + r + 1} + \frac{bdf^m nxx^m \log(x)}{m^2 + 2\,m + 1} - \frac{bdf^m nxx^m}{m^2 + 2\,m + 1} + \frac{aef^m xx^m x^r}{m + r + 1} + \frac{(fx)^m bdx \log(c)}{m + 1} + \frac{(fx)^m adx}{m + 1}$$

```
[In] integrate((f*x)^m*(d+e*x^r)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*e*f^m*m*n*x*x^m*x^r*log(x)/(m^2 + 2*m*r + r^2 + 2*m + 2*r + 1) + b*e*f^m*
n*r*x*x^m*x^r*log(x)/(m^2 + 2*m*r + r^2 + 2*m + 2*r + 1) + b*d*f^m*m*n*x*x^
m*log(x)/(m^2 + 2*m + 1) + b*e*f^m*n*x*x^m*x^r*log(x)/(m^2 + 2*m*r + r^2 +
2*m + 2*r + 1) - b*e*f^m*n*x*x^m*x^r/(m^2 + 2*m*r + r^2 + 2*m + 2*r + 1) +
b*e*f^m*x*x^m*x^r*log(c)/(m + r + 1) + b*d*f^m*n*x*x^m*log(x)/(m^2 + 2*m +
1) - b*d*f^m*n*x*x^m/(m^2 + 2*m + 1) + a*e*f^m*x*x^m*x^r/(m + r + 1) + (f*x
)^m*b*d*x*log(c)/(m + 1) + (f*x)^m*a*d*x/(m + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^r) (a+b\log(cx^n)) \, dx = \int (f\,x)^m \left(d+e\,x^r\right) \left(a+b\,\ln\left(c\,x^n\right)\right) \, dx$$

[In] int((f*x)^m*(d + e*x^r)*(a + b*log(c*x^n)),x)

[Out] int((f*x)^m*(d + e*x^r)*(a + b*log(c*x^n)), x)

3.443 $\int (fx)^m (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int (fx)^m (a+b\log(cx^n))\,dx = -\frac{bn(fx)^{1+m}}{f(1+m)^2}+\frac{(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)}$$

```
[Out] -b*n*(f*x)^(1+m)/f/(1+m)^2+(f*x)^(1+m)*(a+b*ln(c*x^n))/f/(1+m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2341}

$$\int (fx)^m (a+b\log(cx^n))\,dx = \frac{(fx)^{m+1}(a+b\log(cx^n))}{f(m+1)}-\frac{bn(fx)^{m+1}}{f(m+1)^2}$$

```
[In] Int[(f*x)^m*(a + b*Log[c*x^n]),x]
```

```
[Out] -((b*n*(f*x)^(1 + m))/(f*(1 + m)^2)) + ((f*x)^(1 + m)*(a + b*Log[c*x^n]))/(
f*(1 + m))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{bn(fx)^{1+m}}{f(1+m)^2}+\frac{(fx)^{1+m}(a+b\log(cx^n))}{f(1+m)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int (fx)^m (a+b\log(cx^n))\,dx = \frac{x(fx)^m (a+am-bn+b(1+m)\log(cx^n))}{(1+m)^2}$$

```
[In] Integrate[(f*x)^m*(a + b*Log[c*x^n]),x]
[Out] (x*(f*x)^m*(a + a*m - b*n + b*(1 + m)*Log[c*x^n]))/(1 + m)^2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

method	result
parallelrisch	$-\frac{-x(fx)^m \ln(c\,x^n)bm-x(fx)^m \ln(c\,x^n)b-x(fx)^m am+x(fx)^m bn-x(fx)^m a}{(1+m)^2}$
risch	$\frac{bx\,x^m f^m e^{\frac{i\,\mathrm{csgn}(ifx)\pi m(\mathrm{csgn}(ifx)-\mathrm{csgn}(ix))(-\mathrm{csgn}(ifx)+\mathrm{csgn}(if))}{2}} \ln(x^n)}{1+m} - \frac{\left(i\pi b\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)m-i\pi b\ \mathrm{csgn}(i\right.}{}$

```
[In] int((f*x)^m*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] -(-x*(f*x)^m*ln(c*x^n)*b*m-x*(f*x)^m*ln(c*x^n)*b-x*(f*x)^m*a*m+x*(f*x)^m*b*
n-x*(f*x)^m*a)/(1+m)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int (fx)^m (a+b\log(cx^n))\,dx = \frac{((bm+b)nx\log(x)+(bm+b)x\log(c)+(am-bn+a)x)e^{(m\log(f)+m\log(x))}}{m^2+2m+1}$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] ((b*m + b)*n*x*log(x) + (b*m + b)*x*log(c) + (a*m - b*n + a)*x)*e^(m*log(f)
 + m*log(x))/(m^2 + 2*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(37) = 74$.

Time = 2.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.07

$$\int (fx)^m (a + b\log(cx^n))\,dx = \begin{cases} \frac{amx(fx)^m}{m^2+2m+1} + \frac{ax(fx)^m}{m^2+2m+1} + \frac{bmx(fx)^m\log(cx^n)}{m^2+2m+1} - \frac{bnx(fx)^m}{m^2+2m+1} + \frac{bx(fx)^m\log(cx^n)}{m^2+2m+1} & \text{for } m \neq -1 \\ \frac{\begin{cases} a\log(x) & \text{for } b = 0 \\ -(-a - b\log(c))\log(x) & \text{for } n = 0 \\ \frac{(-a-b\log(cx^n))^2}{2bn} & \text{otherwise} \end{cases}}{f} & \text{otherwise} \end{cases}$$

```
[In] integrate((f*x)**m*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((a*m*x*(f*x)**m/(m**2 + 2*m + 1) + a*x*(f*x)**m/(m**2 + 2*m + 1)
+ b*m*x*(f*x)**m*log(c*x**n)/(m**2 + 2*m + 1) - b*n*x*(f*x)**m/(m**2 + 2*m
+ 1) + b*x*(f*x)**m*log(c*x**n)/(m**2 + 2*m + 1), Ne(m, -1)), (Piecewise((a
*log(x), Eq(b, 0)), (-(-a - b*log(c))*log(x), Eq(n, 0)), ((-a - b*log(c*x**
n))**2/(2*b*n), True))/f, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int (fx)^m (a + b\log(cx^n))\,dx = -\frac{bf^m nxx^m}{(m+1)^2} + \frac{(fx)^{m+1}b\log(cx^n)}{f(m+1)} + \frac{(fx)^{m+1}a}{f(m+1)}$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -b*f^m*n*x*x^m/(m + 1)^2 + (f*x)^(m + 1)*b*log(c*x^n)/(f*(m + 1)) + (f*x)^(
m + 1)*a/(f*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(46) = 92$.

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int (fx)^m (a + b\log(cx^n)) \, dx = \frac{bf^m mnxx^m \log(x)}{m^2 + 2m + 1} + \frac{bf^m nxx^m \log(x)}{m^2 + 2m + 1} - \frac{bf^m nxx^m}{m^2 + 2m + 1} + \frac{(fx)^m bx\log(c)}{m + 1} + \frac{(fx)^m ax}{m + 1}$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*f^m*m*n*x*x^m*log(x)/(m^2 + 2*m + 1) + b*f^m*n*x*x^m*log(x)/(m^2 + 2*m +
1) - b*f^m*n*x*x^m/(m^2 + 2*m + 1) + (f*x)^m*b*x*log(c)/(m + 1) + (f*x)^m*a
*x/(m + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (a + b\log(cx^n)) \, dx = \int (f\,x)^m \, (a + b\,\ln(c\,x^n)) \, dx$$

```
[In] int((f*x)^m*(a + b*log(c*x^n)),x)
```

```
[Out] int((f*x)^m*(a + b*log(c*x^n)), x)
```

3.444 $\int \frac{(fx)^m(a+b\log(cx^n))}{d+ex^r} dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m (a+b\log(cx^n))}{d+ex^r} dx = \text{Int}\left(\frac{(fx)^m (a+b\log(cx^n))}{d+ex^r}, x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*ln(c*x^n))/(d+e*x^r),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m (a+b\log(cx^n))}{d+ex^r} dx = \int \frac{(fx)^m (a+b\log(cx^n))}{d+ex^r} dx$$

```
[In] Int[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x^r),x]
[Out] Defer[Int][((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x^r), x]
```

Rubi steps

$$\text{integral} = \int \frac{(fx)^m (a+b\log(cx^n))}{d+ex^r} dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 111 vs. 2(28) = 56.

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.44

$$\int \frac{(fx)^m (a+b\log(cx^n))}{d+ex^r}\,dx$$
$$=\frac{x(fx)^m \left(-bn\,{}_3F_2\left(1,\frac{1}{r}+\frac{m}{r},\frac{1}{r}+\frac{m}{r};1+\frac{1}{r}+\frac{m}{r},1+\frac{1}{r}+\frac{m}{r};-\frac{ex^r}{d}\right)+(1+m)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{r},\right.\right.}{d(1+m)^2}$$

```
[In] Integrate[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x^r),x]
```

```
[Out] (x*(f*x)^m*(-(b*n*HypergeometricPFQ[{1, r^(-1) + m/r, r^(-1) + m/r}, {1 + r
^(-1) + m/r, 1 + r^(-1) + m/r}, -((e*x^r)/d)]) + (1 + m)*Hypergeometric2F1[
1, (1 + m)/r, (1 + m + r)/r, -((e*x^r)/d)]*(a + b*Log[c*x^n])))/(d*(1 + m)^
2)
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a+b\ln(c\,x^n))}{d+e\,x^r}dx$$

```
[In] int((f*x)^m*(a+b*ln(c*x^n))/(d+e*x^r),x)
```

```
[Out] int((f*x)^m*(a+b*ln(c*x^n))/(d+e*x^r),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{(fx)^m (a+b\log(cx^n))}{d+ex^r}\,dx=\int \frac{(b\log(cx^n)+a)(fx)^m}{ex^r+d}\,dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(d+e*x^r),x, algorithm="fricas")
```

```
[Out] integral(((f*x)^m*b*log(c*x^n) + (f*x)^m*a)/(e*x^r + d), x)
```

Sympy [N/A]

Not integrable

Time = 6.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(fx)^m (a + b\log(cx^n))}{d + ex^r}\, dx = \int \frac{(fx)^m (a + b\log(cx^n))}{d + ex^r}\, dx$$

```
[In] integrate((f*x)**m*(a+b*ln(c*x**n))/(d+e*x**r),x)
[Out] Integral((f*x)**m*(a + b*log(c*x**n))/(d + e*x**r), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m (a + b\log(cx^n))}{d + ex^r}\, dx = \int \frac{(b\log(cx^n) + a)(fx)^m}{ex^r + d}\, dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(d+e*x^r),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*(f*x)^m/(e*x^r + d), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m (a + b\log(cx^n))}{d + ex^r}\, dx = \int \frac{(b\log(cx^n) + a)(fx)^m}{ex^r + d}\, dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(d+e*x^r),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*(f*x)^m/(e*x^r + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m (a + b\log(cx^n))}{d + ex^r}\, dx = \int \frac{(f\,x)^m\,(a + b\,\ln(c\,x^n))}{d + e\,x^r}\, dx$$

```
[In] int(((f*x)^m*(a + b*log(c*x^n)))/(d + e*x^r),x)
[Out] int(((f*x)^m*(a + b*log(c*x^n)))/(d + e*x^r), x)
```

3.445 $\int \frac{(fx)^m(a+b\log(cx^n))}{(d+ex^r)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m (a+b\log(cx^n))}{(d+ex^r)^2}\,dx = \text{Int}\left(\frac{(fx)^m (a+b\log(cx^n))}{(d+ex^r)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*ln(c*x^n))/(d+e*x^r)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m (a+b\log(cx^n))}{(d+ex^r)^2}\,dx = \int \frac{(fx)^m (a+b\log(cx^n))}{(d+ex^r)^2}\,dx$$

```
[In] Int[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x^r)^2,x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x^r)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(fx)^m (a+b\log(cx^n))}{(d+ex^r)^2}\,dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(28) = 56.

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 7.08

$$\int \frac{(fx)^m (a + b\log(cx^n))}{(d + ex^r)^2}\,dx$$
$$= \frac{x(fx)^m \left(bn(1+m-r)(d+ex^r)\ {}_3F_2\left(1,\frac{1}{r}+\frac{m}{r},\frac{1}{r}+\frac{m}{r};1+\frac{1}{r}+\frac{m}{r},1+\frac{1}{r}+\frac{m}{r};-\frac{ex^r}{d}\right)-(1+m)\left(-d(1+\right.\right.}{d^2}$$

```
[In] Integrate[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x^r)^2,x]
```

```
[Out] (x*(f*x)^m*(b*n*(1 + m - r)*(d + e*x^r)*HypergeometricPFQ[{1, r^(-1) + m/r,
 r^(-1) + m/r}, {1 + r^(-1) + m/r, 1 + r^(-1) + m/r}, -((e*x^r)/d)] - (1 +
m)*(-(d*(1 + m)*(a + b*Log[c*x^n])) + (d + e*x^r)*Hypergeometric2F1[1, (1 +
 m)/r, (1 + m + r)/r, -((e*x^r)/d)]*(b*n + a*(1 + m - r) + b*(1 + m - r)*Lo
g[c*x^n]))))/(d^2*(1 + m)^2*r*(d + e*x^r))
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b\ln(c\,x^n))}{(d + e\,x^r)^2}dx$$

```
[In] int((f*x)^m*(a+b*ln(c*x^n))/(d+e*x^r)^2,x)
```

```
[Out] int((f*x)^m*(a+b*ln(c*x^n))/(d+e*x^r)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{(fx)^m (a + b\log(cx^n))}{(d + ex^r)^2}\,dx = \int \frac{(b\log(cx^n) + a)(fx)^m}{(ex^r + d)^2}\,dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(d+e*x^r)^2,x, algorithm="fricas")
```

```
[Out] integral(((f*x)^m*b*log(c*x^n) + (f*x)^m*a)/(e^2*x^(2*r) + 2*d*e*x^r + d^2)
, x)
```

Sympy [N/A]

Not integrable

Time = 23.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m (a + b\log (cx^n))}{(d + ex^r)^2} \, dx = \int \frac{(fx)^m (a + b\log (cx^n))}{(d + ex^r)^2} \, dx$$

```
[In] integrate((f*x)**m*(a+b*ln(c*x**n))/(d+e*x**r)**2,x)
[Out] Integral((f*x)**m*(a + b*log(c*x**n))/(d + e*x**r)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m (a + b\log (cx^n))}{(d + ex^r)^2} \, dx = \int \frac{(b\log (cx^n) + a)(fx)^m}{(ex^r + d)^2} \, dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(d+e*x^r)^2,x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*(f*x)^m/(e*x^r + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m (a + b\log (cx^n))}{(d + ex^r)^2} \, dx = \int \frac{(b\log (cx^n) + a)(fx)^m}{(ex^r + d)^2} \, dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))/(d+e*x^r)^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*(f*x)^m/(e*x^r + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m (a+b\log (cx^n))}{(d+ex^r)^2}\,dx = \int \frac{(f\,x)^m\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{{\left(d+e\,x^r\right)}^2}\,dx$$

```
[In] int(((f*x)^m*(a + b*log(c*x^n)))/(d + e*x^r)^2,x)
[Out] int(((f*x)^m*(a + b*log(c*x^n)))/(d + e*x^r)^2, x)
```

3.446 $\int \left(d+ex^{-\frac{1}{1+q}}\right)^q (a+b\log(cx^n))\, dx$

Optimal result

Integrand size = 26, antiderivative size = 102

$$\int \left(d+ex^{-\frac{1}{1+q}}\right)^q (a+b\log(cx^n))\, dx = -bnx\left(d+ex^{-\frac{1}{1+q}}\right)^q \left(1+\frac{ex^{-\frac{1}{1+q}}}{d}\right)^{-q} \text{Hypergeometric2F1}\left(-1-q,-1-q,-q,-\frac{ex^{-\frac{1}{1+q}}}{d}\right) + \frac{x\left(d+ex^{-\frac{1}{1+q}}\right)^{1+q}(a+b\log(cx^n))}{d}$$

```
[Out] -b*n*x*(d+e/(x^(1/(1+q))))^q*hypergeom([-1-q, -1-q],[-q],-e/d/(x^(1/(1+q)))
)/((1+e/d/(x^(1/(1+q))))^q)+x*(d+e/(x^(1/(1+q))))^(1+q)*(a+b*ln(c*x^n))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used

$= \{2351, 252, 251\}$

$$\int \left(d+ex^{-\frac{1}{1+q}}\right)^q (a+b\log(cx^n))\,dx = \frac{x\left(d+ex^{-\frac{1}{q+1}}\right)^{q+1}(a+b\log(cx^n))}{d} - bnx\left(d+ex^{-\frac{1}{q+1}}\right)^q \left(\frac{ex^{-\frac{1}{q+1}}}{d}+1\right)^{-q} \text{Hypergeometric2F1}\left(-q-1,-q-1,-q,-\frac{ex^{-\frac{1}{q+1}}}{d}\right)$$

```
[In] Int[(d + e/x^(1 + q)^(-1))^q*(a + b*Log[c*x^n]),x]
```

```
[Out] -((b*n*x*(d + e/x^(1 + q)^(-1))^q*Hypergeometric2F1[-1 - q, -1 - q, -q, -(e
/(d*x^(1 + q)^(-1)))])/(1 + e/(d*x^(1 + q)^(-1)))^q) + (x*(d + e/x^(1 + q)^
(-1))^(1 + q)*(a + b*Log[c*x^n]))/d
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{x\left(d+ex^{-\frac{1}{1+q}}\right)^{1+q}(a+b\log(cx^n))}{d} - \frac{(bn)\int \left(d+ex^{-\frac{1}{1+q}}\right)^{1+q} dx}{d}$$

$$= \frac{x\left(d+ex^{-\frac{1}{1+q}}\right)^{1+q}(a+b\log(cx^n))}{d} - \left(bn\left(d+ex^{-\frac{1}{1+q}}\right)^q\left(1+\frac{ex^{-\frac{1}{1+q}}}{d}\right)^{-q}\right)\int\left(1+\frac{ex^{-\frac{1}{1+q}}}{d}\right)^{1+q}dx$$

$$= -bnx\left(d+ex^{-\frac{1}{1+q}}\right)^q\left(1+\frac{ex^{-\frac{1}{1+q}}}{d}\right)^{-q} {}_2F_1\left(-1-q,-1-q;-q;-\frac{ex^{-\frac{1}{1+q}}}{d}\right) + \frac{x\left(d+ex^{-\frac{1}{1+q}}\right)^{1+q}(a+b\log(cx^n))}{d}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.40

$$\int\left(d+ex^{-\frac{1}{1+q}}\right)^q(a+b\log(cx^n))\,dx = \frac{x^{-\frac{1}{1+q}}\left(d+ex^{-\frac{1}{1+q}}\right)^q\left(1+\frac{dx^{\frac{1}{1+q}}}{e}\right)^{-q}\left(-bdn(1+q)^2x^{\frac{2+q}{1+q}}\,{}_3F_2\left(1,1,-q;2,2;-\frac{dx^{\frac{1}{1+q}}}{e}\right)-benx\log(x)+\left(1+\right.\right.}{d}$$

```
[In] Integrate[(d + e/x^(1 + q)^(-1))^q*(a + b*Log[c*x^n]),x]
```

```
[Out] ((d + e/x^(1 + q)^(-1))^q*(-(b*d*n*(1 + q)^2*x^((2 + q)/(1 + q))*Hypergeome
tricPFQ[{1, 1, -q}, {2, 2}, -((d*x^(1 + q)^(-1))/e)]) - b*e*n*x*Log[x] + (1
 + (d*x^(1 + q)^(-1))/e)^q*(e*x + d*x^((2 + q)/(1 + q)))*(a + b*Log[c*x^n])
))/(d*x^(1 + q)^(-1)*(1 + (d*x^(1 + q)^(-1))/e)^q)
```

Maple [F]

$$\int\left(d+e\,x^{-\frac{1}{1+q}}\right)^q(a+b\ln(c\,x^n))\,dx$$

```
[In] int((d+e/(x^(1/(1+q))))^q*(a+b*ln(c*x^n)),x)
```

```
[Out] int((d+e/(x^(1/(1+q))))^q*(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int \left(d+ex^{-\frac{1}{1+q}}\right)^q (a+b\log(cx^n))\,dx = \int (b\log(cx^n)+a)\left(d+\frac{e}{x^{\left(\frac{1}{q+1}\right)}}\right)^q dx$$

```
[In] integrate((d+e/(x^(1/(1+q))))^q*(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)*((d*x^(1/(q + 1)) + e)/x^(1/(q + 1)))^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d+ex^{-\frac{1}{1+q}}\right)^q (a+b\log(cx^n))\,dx = \text{Timed out}$$

```
[In] integrate((d+e/(x**(1/(1+q))))**q*(a+b*ln(c*x**n)),x)
[Out] Timed out
```

Maxima [F]

$$\int \left(d+ex^{-\frac{1}{1+q}}\right)^q (a+b\log(cx^n))\,dx = \int (b\log(cx^n)+a)\left(d+\frac{e}{x^{\left(\frac{1}{q+1}\right)}}\right)^q dx$$

```
[In] integrate((d+e/(x^(1/(1+q))))^q*(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*(d + e/x^(1/(q + 1)))^q, x)
```

Giac [F]

$$\int \left(d+ex^{-\frac{1}{1+q}}\right)^q (a+b\log(cx^n))\,dx = \int (b\log(cx^n)+a)\left(d+\frac{e}{x^{\left(\frac{1}{q+1}\right)}}\right)^q dx$$

```
[In] integrate((d+e/(x^(1/(1+q))))^q*(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*(d + e/x^(1/(q + 1)))^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^{-\frac{1}{1+q}}\right)^q (a+b\log(cx^n))\,dx=\int \left(d+\frac{e}{x^{\frac{1}{q+1}}}\right)^q (a+b\,\ln(c\,x^n))\,dx$$

```
[In] int((d + e/x^(1/(q + 1)))^q*(a + b*log(c*x^n)),x)
[Out] int((d + e/x^(1/(q + 1)))^q*(a + b*log(c*x^n)), x)
```

3.447 $\int (fx)^{-1-(1+q)r}\left(d+ex^r\right)^q\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 119

$$\begin{aligned}&\int (fx)^{-1-(1+q)r}\left(d+ex^r\right)^q\left(a+b\log\left(cx^n\right)\right)\,dx\\&=-\frac{bn(fx)^{-((1+q)r)}\left(d+ex^r\right)^q\left(1+\frac{ex^r}{d}\right)^{-q}\text{Hypergeometric2F1}\left(-1-q,-1-q,-q,-\frac{ex^r}{d}\right)}{f(1+q)^2r^2}\\&\quad-\frac{(fx)^{-((1+q)r)}\left(d+ex^r\right)^{1+q}\left(a+b\log\left(cx^n\right)\right)}{df(1+q)r}\end{aligned}$$

```
[Out] -b*n*(d+e*x^r)^q*hypergeom([-1-q, -1-q],[-q],-e*x^r/d)/f/(1+q)^2/r^2/((f*x)
^((1+q)*r))/((1+e*x^r/d)^q)-(d+e*x^r)^(1+q)*(a+b*ln(c*x^n))/d/f/(1+q)/r/((f
*x)^((1+q)*r))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2373, 372, 371}

$$\begin{aligned}&\int (fx)^{-1-(1+q)r}\left(d+ex^r\right)^q\left(a+b\log\left(cx^n\right)\right)\,dx\\&=-\frac{(fx)^{-((q+1)r)}\left(d+ex^r\right)^{q+1}\left(a+b\log\left(cx^n\right)\right)}{df(q+1)r}\\&\quad-\frac{bn(fx)^{-((q+1)r)}\left(d+ex^r\right)^q\left(\frac{ex^r}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(-q-1,-q-1,-q,-\frac{ex^r}{d}\right)}{f(q+1)^2r^2}\end{aligned}$$

```
[In] Int[(f*x)^(-1 - (1 + q)*r)*(d + e*x^r)^q*(a + b*Log[c*x^n]),x]
```

```
[Out] -((b*n*(d + e*x^r)^q*Hypergeometric2F1[-1 - q, -1 - q, -q, -((e*x^r)/d)])/(
f*(1 + q)^2*r^2*(f*x)^((1 + q)*r)*(1 + (e*x^r)/d)^q)) - ((d + e*x^r)^(1 + q
)*(a + b*Log[c*x^n]))/(d*f*(1 + q)*r*(f*x)^((1 + q)*r))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(fx)^{-((1+q)r)}\left(d+ex^r\right)^{1+q}\left(a+b\log\left(cx^n\right)\right)}{df(1+q)r}+\frac{(bn)\int (fx)^{-1-(1+q)r}\left(d+ex^r\right)^{1+q}\,dx}{d(1+q)r}\\
&= -\frac{(fx)^{-((1+q)r)}\left(d+ex^r\right)^{1+q}\left(a+b\log\left(cx^n\right)\right)}{df(1+q)r}\\
&\quad+\frac{\left(bn(d+ex^r)^q\left(1+\frac{ex^r}{d}\right)^{-q}\right)\int (fx)^{-1-(1+q)r}\left(1+\frac{ex^r}{d}\right)^{1+q}\,dx}{(1+q)r}\\
&= -\frac{bn(fx)^{-((1+q)r)}\left(d+ex^r\right)^{q}\left(1+\frac{ex^r}{d}\right)^{-q}\,{}_2F_1\left(-1-q,-1-q;-q;-\frac{ex^r}{d}\right)}{f(1+q)^2r^2}\\
&\quad-\frac{(fx)^{-((1+q)r)}\left(d+ex^r\right)^{1+q}\left(a+b\log\left(cx^n\right)\right)}{df(1+q)r}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int (fx)^{-1-(1+q)r}\,(d+ex^r)^q\,(a+b\log(cx^n))\,dx =$$
$$-\frac{(fx)^{-((1+q)r)}\,(d+ex^r)^q\left(bn\left(1+\frac{ex^r}{d}\right)^{-q}\text{Hypergeometric2F1}\left(-1-q,-1-q,-q,-\frac{ex^r}{d}\right)+\frac{(1+q)r(d+ex^r)(}{d}\right.}{f(1+q)^2r^2}$$

```
[In] Integrate[(f*x)^(-1 - (1 + q)*r)*(d + e*x^r)^q*(a + b*Log[c*x^n]),x]
```

```
[Out] -(((d + e*x^r)^q*((b*n*Hypergeometric2F1[-1 - q, -1 - q, -q, -((e*x^r)/d)])
/(1 + (e*x^r)/d)^q + ((1 + q)*r*(d + e*x^r)*(a + b*Log[c*x^n]))/d))/(f*(1 +
 q)^2*r^2*(f*x)^((1 + q)*r)))
```

Maple [F]

$$\int (fx)^{-1-(1+q)r}\,(d+e\,x^r)^q\,(a+b\ln(c\,x^n))\,dx$$

```
[In] int((f*x)^(-1-(1+q)*r)*(d+e*x^r)^q*(a+b*ln(c*x^n)),x)
```

```
[Out] int((f*x)^(-1-(1+q)*r)*(d+e*x^r)^q*(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int (fx)^{-1-(1+q)r}\,(d+ex^r)^q\,(a+b\log(cx^n))\,dx = \int (b\log(cx^n)+a)(ex^r+d)^q(fx)^{-(q+1)r-1}\,dx$$

```
[In] integrate((f*x)^(-1-(1+q)*r)*(d+e*x^r)^q*(a+b*log(c*x^n)),x, algorithm="fri
cas")
```

```
[Out] integral(((f*x)^(-(q + 1)*r - 1)*b*log(c*x^n) + (f*x)^(-(q + 1)*r - 1)*a)*(
e*x^r + d)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^{-1-(1+q)r}\,(d+ex^r)^q\,(a+b\log(cx^n))\,dx = \text{Timed out}$$

```
[In] integrate((f*x)**(-1-(1+q)*r)*(d+e*x**r)**q*(a+b*ln(c*x**n)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (fx)^{-1-(1+q)r} (d+ex^r)^q (a+b\log(cx^n)) \, dx = \int (b\log(cx^n) + a)(ex^r + d)^q (fx)^{-(q+1)r-1} \, dx$$

```
[In] integrate((f*x)^(-1-(1+q)*r)*(d+e*x^r)^q*(a+b*log(c*x^n)),x, algorithm="max
ima")
```

```
[Out] integrate((b*log(c*x^n) + a)*(e*x^r + d)^q*(f*x)^(-(q + 1)*r - 1), x)
```

Giac [F]

$$\int (fx)^{-1-(1+q)r} (d+ex^r)^q (a+b\log(cx^n)) \, dx = \int (b\log(cx^n) + a)(ex^r + d)^q (fx)^{-(q+1)r-1} \, dx$$

```
[In] integrate((f*x)^(-1-(1+q)*r)*(d+e*x^r)^q*(a+b*log(c*x^n)),x, algorithm="gia
c")
```

```
[Out] integrate((b*log(c*x^n) + a)*(e*x^r + d)^q*(f*x)^(-(q + 1)*r - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1-(1+q)r} (d+ex^r)^q (a+b\log(cx^n)) \, dx = \int \frac{(d+e\,x^r)^q\,(a+b\,\ln(c\,x^n))}{(f\,x)^{r\,(q+1)+1}} \, dx$$

```
[In] int(((d + e*x^r)^q*(a + b*log(c*x^n)))/(f*x)^(r*(q + 1) + 1),x)
```

```
[Out] int(((d + e*x^r)^q*(a + b*log(c*x^n)))/(f*x)^(r*(q + 1) + 1), x)
```

3.448 $\int (fx)^m (d+ex^r)^3 (a+b\log(cx^n))^p \, dx$

Optimal result

Integrand size = 27, antiderivative size = 480

$$\int (fx)^m (d+ex^r)^3 (a+b\log(cx^n))^p \, dx$$

$$= \frac{d^3 e^{-\frac{a(1+m)}{bn}} (fx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \Gamma\left(1+p, -\frac{(1+m)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{-p}}{f(1+m)}$$

$$+ \frac{3d^2 e e^{-\frac{a(1+m+r)}{bn}} x^{1+r} (fx)^m (cx^n)^{-\frac{1+m+r}{n}} \Gamma\left(1+p, -\frac{(1+m+r)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+m+r)(a+b\log \cdots}{bn}\right.}{1+m+r}$$

$$+ \frac{3de^2 e^{-\frac{a(1+m+2r)}{bn}} x^{1+2r} (fx)^m (cx^n)^{-\frac{1+m+2r}{n}} \Gamma\left(1+p, -\frac{(1+m+2r)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+m+2r)(\cdots}{\cdots}\right.}{1+m+2r}$$

$$+ \frac{e^3 e^{-\frac{a(1+m+3r)}{bn}} x^{1+3r} (fx)^m (cx^n)^{-\frac{1+m+3r}{n}} \Gamma\left(1+p, -\frac{(1+m+3r)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+m+3r)(a+\cdots}{bn}\right.}{1+m+3r}$$

```
[Out] d^3*(f*x)^(1+m)*GAMMA(p+1,-(1+m)*(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p/exp
(a*(1+m)/b/n)/f/(1+m)/((c*x^n)^((1+m)/n))/((-(1+m)*(a+b*ln(c*x^n))/b/n)^p)+
3*d^2*e*x^(1+r)*(f*x)^m*GAMMA(p+1,-(1+m+r)*(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x
^n))^p/exp(a*(1+m+r)/b/n)/(1+m+r)/((c*x^n)^((1+m+r)/n))/((-(1+m+r)*(a+b*ln(
c*x^n))/b/n)^p)+3*d*e^2*x^(1+2*r)*(f*x)^m*GAMMA(p+1,-(1+m+2*r)*(a+b*ln(c*x^
n))/b/n)*(a+b*ln(c*x^n))^p/exp(a*(1+m+2*r)/b/n)/(1+m+2*r)/((c*x^n)^((1+m+2*
r)/n))/((-(1+m+2*r)*(a+b*ln(c*x^n))/b/n)^p)+e^3*x^(1+3*r)*(f*x)^m*GAMMA(p+1
,-(1+m+3*r)*(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p/exp(a*(1+m+3*r)/b/n)/(1+
m+3*r)/((c*x^n)^((1+m+3*r)/n))/((-(1+m+3*r)*(a+b*ln(c*x^n))/b/n)^p)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2395, 2347, 2212, 20}

$$\int (fx)^m (d+ex^r)^3 (a+b\log(cx^n))^p \, dx$$

$$= \frac{d^3 (fx)^{m+1} e^{-\frac{a(m+1)}{bn}} (cx^n)^{-\frac{m+1}{n}} (a+b\log(cx^n))^p \left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p} \Gamma\left(p+1, -\frac{(m+1)(a+b\log(cx^n))}{bn}\right)}{f(m+1)}$$
$$+ \frac{3d^2 e x^{r+1} (fx)^m e^{-\frac{a(m+r+1)}{bn}} (cx^n)^{-\frac{m+r+1}{n}} (a+b\log(cx^n))^p \left(-\frac{(m+r+1)(a+b\log(cx^n))}{bn}\right)^{-p} \Gamma\left(p+1, -\frac{(m+r+1)(a+b\log}{bn}\right.}{m+r+1}$$
$$+ \frac{3de^2 x^{2r+1} (fx)^m e^{-\frac{a(m+2r+1)}{bn}} (cx^n)^{-\frac{m+2r+1}{n}} (a+b\log(cx^n))^p \left(-\frac{(m+2r+1)(a+b\log(cx^n))}{bn}\right)^{-p} \Gamma\left(p+1, -\frac{(m+2r+1)(}{}\right.}{m+2r+1}$$
$$+ \frac{e^3 x^{3r+1} (fx)^m e^{-\frac{a(m+3r+1)}{bn}} (cx^n)^{-\frac{m+3r+1}{n}} (a+b\log(cx^n))^p \left(-\frac{(m+3r+1)(a+b\log(cx^n))}{bn}\right)^{-p} \Gamma\left(p+1, -\frac{(m+3r+1)(a+}{bn}\right.}{m+3r+1}$$

```
[In] Int[(f*x)^m*(d + e*x^r)^3*(a + b*Log[c*x^n])^p,x]
```

```
[Out] (d^3*(f*x)^(1 + m)*Gamma[1 + p, -(((1 + m)*(a + b*Log[c*x^n]))/(b*n))]*(a +
 b*Log[c*x^n])^p)/(E^((a*(1 + m))/(b*n))*f*(1 + m)*(c*x^n)^((1 + m)/n)*(-((
(1 + m)*(a + b*Log[c*x^n]))/(b*n)))^p) + (3*d^2*e*x^(1 + r)*(f*x)^m*Gamma[1
 + p, -(((1 + m + r)*(a + b*Log[c*x^n]))/(b*n))]*(a + b*Log[c*x^n])^p)/(E^(
(a*(1 + m + r))/(b*n))*(1 + m + r)*(c*x^n)^((1 + m + r)/n)*(-(((1 + m + r)*
(a + b*Log[c*x^n]))/(b*n)))^p) + (3*d*e^2*x^(1 + 2*r)*(f*x)^m*Gamma[1 + p,
-(((1 + m + 2*r)*(a + b*Log[c*x^n]))/(b*n))]*(a + b*Log[c*x^n])^p)/(E^((a*(
1 + m + 2*r))/(b*n))*(1 + m + 2*r)*(c*x^n)^((1 + m + 2*r)/n)*(-(((1 + m + 2
*r)*(a + b*Log[c*x^n]))/(b*n)))^p) + (e^3*x^(1 + 3*r)*(f*x)^m*Gamma[1 + p,
-(((1 + m + 3*r)*(a + b*Log[c*x^n]))/(b*n))]*(a + b*Log[c*x^n])^p)/(E^((a*(
1 + m + 3*r))/(b*n))*(1 + m + 3*r)*(c*x^n)^((1 + m + 3*r)/n)*(-(((1 + m + 3
*r)*(a + b*Log[c*x^n]))/(b*n)))^p)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
```

```
((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(d^3(fx)^m \left(a+b\log\left(cx^n\right)\right)^p+3d^2ex^r(fx)^m \left(a+b\log\left(cx^n\right)\right)^p \right. \\
&\qquad \left. +3de^2x^{2r}(fx)^m \left(a+b\log\left(cx^n\right)\right)^p+e^3x^{3r}(fx)^m \left(a+b\log\left(cx^n\right)\right)^p\right)\,dx \\
&= d^3\int (fx)^m \left(a+b\log\left(cx^n\right)\right)^p\,dx+\left(3d^2e\right)\int x^r(fx)^m \left(a+b\log\left(cx^n\right)\right)^p\,dx \\
&\quad +\left(3de^2\right)\int x^{2r}(fx)^m \left(a+b\log\left(cx^n\right)\right)^p\,dx+e^3\int x^{3r}(fx)^m \left(a+b\log\left(cx^n\right)\right)^p\,dx \\
&= \left(3d^2ex^{-m}(fx)^m\right)\int x^{m+r}\left(a+b\log\left(cx^n\right)\right)^p\,dx \\
&\quad +\left(3de^2x^{-m}(fx)^m\right)\int x^{m+2r}\left(a+b\log\left(cx^n\right)\right)^p\,dx \\
&\quad +\left(e^3x^{-m}(fx)^m\right)\int x^{m+3r}\left(a+b\log\left(cx^n\right)\right)^p\,dx \\
&\quad +\frac{\left(d^3(fx)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int e^{\frac{(1+m)x}{n}}(a+bx)^p\,dx,x,\log\left(cx^n\right)\right)}{fn}
\end{aligned}$$

$$= \frac{d^3 e^{-\frac{a(1+m)}{bn}} (fx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \Gamma\left(1+p, -\frac{(1+m)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{-}}{f(1+m)}$$
$$+ \frac{\left(3d^2 e x^{1+r} (fx)^m (cx^n)^{-\frac{1+m+r}{n}}\right) \text{Subst}\left(\int e^{\frac{(1+m+r)x}{n}} (a+bx)^p \, dx, x, \log(cx^n)\right)}{n}$$
$$+ \frac{\left(3d e^2 x^{1+2r} (fx)^m (cx^n)^{-\frac{1+m+2r}{n}}\right) \text{Subst}\left(\int e^{\frac{(1+m+2r)x}{n}} (a+bx)^p \, dx, x, \log(cx^n)\right)}{n}$$
$$+ \frac{\left(e^3 x^{1+3r} (fx)^m (cx^n)^{-\frac{1+m+3r}{n}}\right) \text{Subst}\left(\int e^{\frac{(1+m+3r)x}{n}} (a+bx)^p \, dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{d^3 e^{-\frac{a(1+m)}{bn}} (fx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \Gamma\left(1+p, -\frac{(1+m)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{-}}{f(1+m)}$$
$$+ \frac{3d^2 e e^{-\frac{a(1+m+r)}{bn}} x^{1+r} (fx)^m (cx^n)^{-\frac{1+m+r}{n}} \Gamma\left(1+p, -\frac{(1+m+r)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+m+}{}\right.}{1+m+r}$$
$$+ \frac{3d e^2 e^{-\frac{a(1+m+2r)}{bn}} x^{1+2r} (fx)^m (cx^n)^{-\frac{1+m+2r}{n}} \Gamma\left(1+p, -\frac{(1+m+2r)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+}{}\right.}{1+m+2r}$$
$$+ \frac{e^3 e^{-\frac{a(1+m+3r)}{bn}} x^{1+3r} (fx)^m (cx^n)^{-\frac{1+m+3r}{n}} \Gamma\left(1+p, -\frac{(1+m+3r)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+m}{}\right.}{1+m+3r}$$

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.85

$$\int (fx)^m (d+ex^r)^3 (a+b\log(cx^n))^p \, dx = x^{-m}(fx)^m (a$$
$$+b\log(cx^n))^p \left(\frac{d^3 e^{-\frac{(1+m)(a-bn\log(x)+b\log(cx^n))}{bn}} \Gamma\left(1+p, -\frac{(1+m)(a+b\log(cx^n))}{bn}\right) \left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{-p}}{1+m} \right.$$
$$+e\left(\frac{3d^2 e^{-\frac{(1+m+r)(a-bn\log(x)+b\log(cx^n))}{bn}} \Gamma\left(1+p, -\frac{(1+m+r)(a+b\log(cx^n))}{bn}\right) \left(-\frac{(1+m+r)(a+b\log(cx^n))}{bn}\right)^{-p}}{1+m+r} \right.$$
$$+e\left(\frac{3d e^{-\frac{(1+m+2r)(a-bn\log(x)+b\log(cx^n))}{bn}} \Gamma\left(1+p, -\frac{(1+m+2r)(a+b\log(cx^n))}{bn}\right) \left(-\frac{(1+m+2r)(a+b\log(cx^n))}{bn}\right)^{-p}}{1+m+2r} + \frac{e e^{-\frac{(1+m+3r}{}}}{} \right.$$

```
[In] Integrate[(f*x)^m*(d + e*x^r)^3*(a + b*Log[c*x^n])^p,x]
```

```
[Out] ((f*x)^m*(a + b*Log[c*x^n])^p*((d^3*Gamma[1 + p, -(((1 + m)*(a + b*Log[c*x^
n]))/(b*n))])/(E^(((1 + m)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n))*(1 + m)*
(-(((1 + m)*(a + b*Log[c*x^n]))/(b*n)))^p) + e*((3*d^2*Gamma[1 + p, -(((1 +
 m + r)*(a + b*Log[c*x^n]))/(b*n))])/(E^(((1 + m + r)*(a - b*n*Log[x] + b*L
og[c*x^n]))/(b*n))*(1 + m + r)*(-(((1 + m + r)*(a + b*Log[c*x^n]))/(b*n)))^
p) + e*((3*d*Gamma[1 + p, -(((1 + m + 2*r)*(a + b*Log[c*x^n]))/(b*n))])/(E^
(((1 + m + 2*r)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n))*(1 + m + 2*r)*(-(((
1 + m + 2*r)*(a + b*Log[c*x^n]))/(b*n)))^p) + (e*Gamma[1 + p, -(((1 + m + 3
*r)*(a + b*Log[c*x^n]))/(b*n))])/(E^(((1 + m + 3*r)*(a - b*n*Log[x] + b*Log
[c*x^n]))/(b*n))*(1 + m + 3*r)*(-(((1 + m + 3*r)*(a + b*Log[c*x^n]))/(b*n))
)^p)))))/x^m
```

Maple [F]

$$\int (fx)^m \left(d+e\,x^r\right)^3 \left(a+b\ln \left(c\,x^n\right)\right)^p dx$$

```
[In] int((f*x)^m*(d+e*x^r)^3*(a+b*ln(c*x^n))^p,x)
```

```
[Out] int((f*x)^m*(d+e*x^r)^3*(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^r\right)^3 \left(a+b\log \left(cx^n\right)\right)^p \, dx = \int \left(ex^r+d\right)^3 (fx)^m \left(b\log \left(cx^n\right)+a\right)^p \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^r)^3*(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^(3*r) + 3*d*e^2*x^(2*r) + 3*d^2*e*x^r + d^3)*(f*x)^m*(b*log
(c*x^n) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^r\right)^3 \left(a+b\log \left(cx^n\right)\right)^p \, dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(d+e*x**r)**3*(a+b*ln(c*x**n))**p,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (fx)^m (d+ex^r)^3 (a+b\log(cx^n))^p \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((f*x)^m*(d+e*x^r)^3*(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int (fx)^m (d+ex^r)^3 (a+b\log(cx^n))^p \, dx = \int (ex^r+d)^3 (fx)^m (b\log(cx^n)+a)^p \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^r)^3*(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^r + d)^3*(f*x)^m*(b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^r)^3 (a+b\log(cx^n))^p \, dx = \int (f\,x)^m \left(d+e\,x^r\right)^3 \left(a+b\,\ln\left(c\,x^n\right)\right)^p \, dx$$

```
[In] int((f*x)^m*(d + e*x^r)^3*(a + b*log(c*x^n))^p,x)
```

```
[Out] int((f*x)^m*(d + e*x^r)^3*(a + b*log(c*x^n))^p, x)
```

3.449 $\int (fx)^m (d+ex^r)^2 (a+b\log(cx^n))^p \, dx$

Optimal result

Integrand size = 27, antiderivative size = 350

$$\int (fx)^m (d+ex^r)^2 (a+b\log(cx^n))^p \, dx = \frac{d^2 e^{-\frac{a(1+m)}{bn}} (fx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{-p}}{f(1+m)} + \frac{2de e^{-\frac{a(1+m+r)}{bn}} x^{1+r} (fx)^m (cx^n)^{-\frac{1+m+r}{n}} \Gamma\left(1+p,-\frac{(1+m+r)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+m+r)(a+b\log(cx^n))}{bn}\right)^{-p}}{1+m+r} + \frac{e^2 e^{-\frac{a(1+m+2r)}{bn}} x^{1+2r} (fx)^m (cx^n)^{-\frac{1+m+2r}{n}} \Gamma\left(1+p,-\frac{(1+m+2r)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+m+2r)(a+b\log(cx^n))}{bn}\right)^{-p}}{1+m+2r}$$

```
[Out] d^2*(f*x)^(1+m)*GAMMA(p+1,-(1+m)*(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p/exp
(a*(1+m)/b/n)/f/(1+m)/((c*x^n)^((1+m)/n))/((-(1+m)*(a+b*ln(c*x^n))/b/n)^p)+
2*d*e*x^(1+r)*(f*x)^m*GAMMA(p+1,-(1+m+r)*(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n
))^p/exp(a*(1+m+r)/b/n)/(1+m+r)/((c*x^n)^((1+m+r)/n))/((-(1+m+r)*(a+b*ln(c*
x^n))/b/n)^p)+e^2*x^(1+2*r)*(f*x)^m*GAMMA(p+1,-(1+m+2*r)*(a+b*ln(c*x^n))/b/
n)*(a+b*ln(c*x^n))^p/exp(a*(1+m+2*r)/b/n)/(1+m+2*r)/((c*x^n)^((1+m+2*r)/n))
/((-(1+m+2*r)*(a+b*ln(c*x^n))/b/n)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2395, 2347, 2212, 20}

$$\int (fx)^m (d+ex^r)^2 (a+b\log(cx^n))^p\,dx$$
$$=\frac{d^2(fx)^{m+1}e^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}(a+b\log(cx^n))^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)}{f(m+1)}$$
$$+\frac{2dex^{r+1}(fx)^me^{-\frac{a(m+r+1)}{bn}}(cx^n)^{-\frac{m+r+1}{n}}(a+b\log(cx^n))^p\left(-\frac{(m+r+1)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\Big(p+1,-\frac{(m+r+1)(a+b\,\text{lo}}{bn}}{m+r+1}$$
$$+\frac{e^2x^{2r+1}(fx)^me^{-\frac{a(m+2r+1)}{bn}}(cx^n)^{-\frac{m+2r+1}{n}}(a+b\log(cx^n))^p\left(-\frac{(m+2r+1)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\Big(p+1,-\frac{(m+2r+1)(a+}{bn}}{m+2r+1}$$

```
[In] Int[(f*x)^m*(d + e*x^r)^2*(a + b*Log[c*x^n])^p,x]
```

```
[Out] (d^2*(f*x)^(1 + m)*Gamma[1 + p, -(((1 + m)*(a + b*Log[c*x^n]))/(b*n))]*(a +
 b*Log[c*x^n])^p)/(E^((a*(1 + m))/(b*n))*f*(1 + m)*(c*x^n)^((1 + m)/n)*(-((
(1 + m)*(a + b*Log[c*x^n]))/(b*n)))^p) + (2*d*e*x^(1 + r)*(f*x)^m*Gamma[1 +
 p, -(((1 + m + r)*(a + b*Log[c*x^n]))/(b*n))]*(a + b*Log[c*x^n])^p)/(E^((a
*(1 + m + r))/(b*n))*(1 + m + r)*(c*x^n)^((1 + m + r)/n)*(-(((1 + m + r)*(a
 + b*Log[c*x^n]))/(b*n)))^p) + (e^2*x^(1 + 2*r)*(f*x)^m*Gamma[1 + p, -(((1
+ m + 2*r)*(a + b*Log[c*x^n]))/(b*n))]*(a + b*Log[c*x^n])^p)/(E^((a*(1 + m
+ 2*r))/(b*n))*(1 + m + 2*r)*(c*x^n)^((1 + m + 2*r)/n)*(-(((1 + m + 2*r)*(a
 + b*Log[c*x^n]))/(b*n)))^p)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$\text{integral} = \int \left(d^2(fx)^m\left(a+b\log\left(cx^n\right)\right)^p + 2dex^r(fx)^m\left(a+b\log\left(cx^n\right)\right)^p + e^2x^{2r}(fx)^m\left(a+b\log\left(cx^n\right)\right)^p\right)\,dx$$

$$= d^2\int (fx)^m\left(a+b\log\left(cx^n\right)\right)^p\,dx + (2de)\int x^r(fx)^m\left(a+b\log\left(cx^n\right)\right)^p\,dx + e^2\int x^{2r}(fx)^m\left(a+b\log\left(cx^n\right)\right)^p\,dx$$

$$= \left(2dex^{-m}(fx)^m\right)\int x^{m+r}\left(a+b\log\left(cx^n\right)\right)^p\,dx + \left(e^2x^{-m}(fx)^m\right)\int x^{m+2r}\left(a+b\log\left(cx^n\right)\right)^p\,dx + \frac{\left(d^2(fx)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int e^{\frac{(1+m)x}{n}}(a+bx)^p\,dx,x,\log\left(cx^n\right)\right)}{fn}$$

$$= \frac{d^2e^{-\frac{a(1+m)}{bn}}(fx)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right.}{f(1+m)} + \frac{\left(2dex^{1+r}(fx)^m\left(cx^n\right)^{-\frac{1+m+r}{n}}\right)\text{Subst}\left(\int e^{\frac{(1+m+r)x}{n}}(a+bx)^p\,dx,x,\log\left(cx^n\right)\right)}{n} + \frac{\left(e^2x^{1+2r}(fx)^m\left(cx^n\right)^{-\frac{1+m+2r}{n}}\right)\text{Subst}\left(\int e^{\frac{(1+m+2r)x}{n}}(a+bx)^p\,dx,x,\log\left(cx^n\right)\right)}{n}$$

$$= \frac{d^2 e^{-\frac{a(1+m)}{bn}} (fx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \Gamma\left(1+p, -\frac{(1+m)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{-}}{f(1+m)}$$
$$+ \frac{2dee^{-\frac{a(1+m+r)}{bn}} x^{1+r} (fx)^m (cx^n)^{-\frac{1+m+r}{n}} \Gamma\left(1+p, -\frac{(1+m+r)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+m+r}{}\right.}{1+m+r}$$
$$+ \frac{e^2 e^{-\frac{a(1+m+2r)}{bn}} x^{1+2r} (fx)^m (cx^n)^{-\frac{1+m+2r}{n}} \Gamma\left(1+p, -\frac{(1+m+2r)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+m-}{}\right.}{1+m+2r}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.87

$$\int (fx)^m (d+ex^r)^2 (a+b\log(cx^n))^p \, dx = x^{-m}(fx)^m (a+b\log(cx^n))^p \left(\frac{d^2 e^{-\frac{(1+m)(a-bn\log(x)+b\log(cx^n))}{bn}} \Gamma\left(1+p, -\frac{(1+m)(a+b\log(cx^n))}{bn}\right) \left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{-p}}{1+m} + e\left(\frac{2de^{-\frac{(1+m+r)(a-bn\log(x)+b\log(cx^n))}{bn}} \Gamma\left(1+p, -\frac{(1+m+r)(a+b\log(cx^n))}{bn}\right) \left(-\frac{(1+m+r)(a+b\log(cx^n))}{bn}\right)^{-p}}{1+m+r} + \frac{ee^{-\frac{(1+m+2r)(a-bn\log(x)+b\log(cx^n))}{bn}} \Gamma\left(1+p, -\frac{(1+m+2r)(a+b\log(cx^n))}{bn}\right) \left(-\frac{(1+m+2r)(a+b\log(cx^n))}{bn}\right)^{-p}}{1+m+2r} \right) \right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^r)^2*(a + b*Log[c*x^n])^p,x]
```

```
[Out] ((f*x)^m*(a + b*Log[c*x^n])^p*((d^2*Gamma[1 + p, -(((1 + m)*(a + b*Log[c*x^
n]))/(b*n))])/(E^(((1 + m)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n))*(1 + m)*
(-(((1 + m)*(a + b*Log[c*x^n]))/(b*n)))^p) + e*((2*d*Gamma[1 + p, -(((1 + m
 + r)*(a + b*Log[c*x^n]))/(b*n))])/(E^(((1 + m + r)*(a - b*n*Log[x] + b*Log
[c*x^n]))/(b*n))*(1 + m + r)*(-(((1 + m + r)*(a + b*Log[c*x^n]))/(b*n)))^p)
 + (e*Gamma[1 + p, -(((1 + m + 2*r)*(a + b*Log[c*x^n]))/(b*n))])/(E^(((1 +
m + 2*r)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n))*(1 + m + 2*r)*(-(((1 + m +
 2*r)*(a + b*Log[c*x^n]))/(b*n)))^p))))/x^m
```

Maple [F]

$$\int (fx)^m (d+e\,x^r)^2 (a+b\ln(c\,x^n))^p dx$$

```
[In] int((f*x)^m*(d+e*x^r)^2*(a+b*ln(c*x^n))^p,x)
[Out] int((f*x)^m*(d+e*x^r)^2*(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int (fx)^m (d+ex^r)^2 (a+b\log(cx^n))^p \, dx = \int (ex^r+d)^2 (fx)^m (b\log(cx^n)+a)^p \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^r)^2*(a+b*log(c*x^n))^p,x, algorithm="fricas")
[Out] integral((e^2*x^(2*r) + 2*d*e*x^r + d^2)*(f*x)^m*(b*log(c*x^n) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^r)^2 (a+b\log(cx^n))^p \, dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(d+e*x**r)**2*(a+b*ln(c*x**n))**p,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (fx)^m (d+ex^r)^2 (a+b\log(cx^n))^p \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((f*x)^m*(d+e*x^r)^2*(a+b*log(c*x^n))^p,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int (fx)^m (d+ex^r)^2 (a+b\log(cx^n))^p \, dx = \int (ex^r+d)^2 (fx)^m (b\log(cx^n)+a)^p \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^r)^2*(a+b*log(c*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x^r + d)^2*(f*x)^m*(b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^r)^2 (a+b\log(cx^n))^p \, dx = \int (f\,x)^m \left(d+e\,x^r\right)^2 \left(a+b\,\ln\left(c\,x^n\right)\right)^p \, dx$$

```
[In] int((f*x)^m*(d + e*x^r)^2*(a + b*log(c*x^n))^p,x)
[Out] int((f*x)^m*(d + e*x^r)^2*(a + b*log(c*x^n))^p, x)
```

3.450 $\int (fx)^m (d+ex^r) (a+b\log(cx^n))^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 220

$$\int (fx)^m (d+ex^r) (a+b\log(cx^n))^p \, dx = \frac{de^{-\frac{a(1+m)}{bn}}(fx)^{1+m}(cx^n)^{-\frac{1+m}{n}}\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{-p}}{f(1+m)} + \frac{ee^{-\frac{a(1+m+r)}{bn}}x^{1+r}(fx)^m(cx^n)^{-\frac{1+m+r}{n}}\Gamma\left(1+p,-\frac{(1+m+r)(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{(1+m+r)(a+b\log(cx^n))}{bn}\right.}{1+m+r}$$

```
[Out] d*(f*x)^(1+m)*GAMMA(p+1,-(1+m)*(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p/exp(a
*(1+m)/b/n)/f/(1+m)/((c*x^n)^((1+m)/n))/((-(1+m)*(a+b*ln(c*x^n))/b/n)^p)+e*
x^(1+r)*(f*x)^m*GAMMA(p+1,-(1+m+r)*(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p/e
xp(a*(1+m+r)/b/n)/(1+m+r)/((c*x^n)^((1+m+r)/n))/((-(1+m+r)*(a+b*ln(c*x^n))/
b/n)^p)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2395, 2347, 2212, 20}

$$\int (fx)^m (d+ex^r) (a+b\log(cx^n))^p \, dx = \frac{d(fx)^{m+1}e^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}(a+b\log(cx^n))^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)}{f(m+1)} + \frac{ex^{r+1}(fx)^m e^{-\frac{a(m+r+1)}{bn}}(cx^n)^{-\frac{m+r+1}{n}}(a+b\log(cx^n))^p\left(-\frac{(m+r+1)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{(m+r+1)(a+b\log}{bn}\right.}{m+r+1}$$

```
[In] Int[(f*x)^m*(d + e*x^r)*(a + b*Log[c*x^n])^p,x]
```

```
[Out] (d*(f*x)^(1 + m)*Gamma[1 + p, -(((1 + m)*(a + b*Log[c*x^n]))/(b*n))]*(a + b
*Log[c*x^n])^p)/(E^((a*(1 + m))/(b*n))*f*(1 + m)*(c*x^n)^((1 + m)/n)*(-(((1
 + m)*(a + b*Log[c*x^n]))/(b*n)))^p) + (e*x^(1 + r)*(f*x)^m*Gamma[1 + p, -(
((1 + m + r)*(a + b*Log[c*x^n]))/(b*n))]*(a + b*Log[c*x^n])^p)/(E^((a*(1 +
m + r))/(b*n))*(1 + m + r)*(c*x^n)^((1 + m + r)/n)*(-(((1 + m + r)*(a + b*L
og[c*x^n]))/(b*n)))^p)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(d(fx)^m\left(a+b\log\left(cx^n\right)\right)^p+ex^r(fx)^m\left(a+b\log\left(cx^n\right)\right)^p\right)\,dx\\ &= d\int (fx)^m\left(a+b\log\left(cx^n\right)\right)^p\,dx+e\int x^r(fx)^m\left(a+b\log\left(cx^n\right)\right)^p\,dx\end{aligned}$$

$$= \left(e x^{-m} (f x)^m\right) \int x^{m+r} (a+b\log(cx^n))^p\,dx + \frac{\left(d (fx)^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int e^{\frac{(1+m)x}{n}} (a+bx)^p\,dx, x, \log(cx^n)\right)}{fn}$$

$$= \frac{d e^{-\frac{a(1+m)}{bn}} (fx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \Gamma\left(1+p, -\frac{(1+m)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right) \cdots}{f(1+m)} + \frac{\left(e x^{1+r} (fx)^m (cx^n)^{-\frac{1+m+r}{n}}\right) \text{Subst}\left(\int e^{\frac{(1+m+r)x}{n}} (a+bx)^p\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{d e^{-\frac{a(1+m)}{bn}} (fx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \Gamma\left(1+p, -\frac{(1+m)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right) \cdots}{f(1+m)} + \frac{e e^{-\frac{a(1+m+r)}{bn}} x^{1+r} (fx)^m (cx^n)^{-\frac{1+m+r}{n}} \Gamma\left(1+p, -\frac{(1+m+r)(a+b\log(cx^n))}{bn}\right) (a+b\log(cx^n))^p \left(-\frac{(1+m+r)(\cdots}{\cdots}\right) \cdots}{1+m+r}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.91

$$\int (fx)^m (d+ex^r) (a+b\log(cx^n))^p\,dx = x^{-m} (fx)^m (a+b\log(cx^n))^p \left(\frac{d e^{-\frac{(1+m)(a-bn\log(x)+b\log(cx^n))}{bn}} \Gamma\left(1+p, -\frac{(1+m)(a+b\log(cx^n))}{bn}\right) \left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{-p}}{1+m} + \frac{e e^{-\frac{(1+m+r)(a-bn\log(x)+b\log(cx^n))}{bn}} \Gamma\left(1+p, -\frac{(1+m+r)(a+b\log(cx^n))}{bn}\right) \left(-\frac{(1+m+r)(a+b\log(cx^n))}{bn}\right)^{-p}}{1+m+r}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^r)*(a + b*Log[c*x^n])^p,x]
```

```
[Out] ((f*x)^m*(a + b*Log[c*x^n])^p*((d*Gamma[1 + p, -(((1 + m)*(a + b*Log[c*x^n]
))/(b*n))])/(E^(((1 + m)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n))*(1 + m)*(-
(((1 + m)*(a + b*Log[c*x^n]))/(b*n)))^p) + (e*Gamma[1 + p, -(((1 + m + r)*(
a + b*Log[c*x^n]))/(b*n))])/(E^(((1 + m + r)*(a - b*n*Log[x] + b*Log[c*x^n]
))/(b*n))*(1 + m + r)*(-(((1 + m + r)*(a + b*Log[c*x^n]))/(b*n)))^p)))/x^m
```

Maple [F]

$$\int (fx)^m (d + e\,x^r) (a + b\ln (c\,x^n))^p dx$$

```
[In] int((f*x)^m*(d+e*x^r)*(a+b*ln(c*x^n))^p,x)
[Out] int((f*x)^m*(d+e*x^r)*(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int (fx)^m (d + ex^r) (a + b\log (cx^n))^p \, dx = \int (ex^r + d)(fx)^m (b\log (cx^n) + a)^p \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^r)*(a+b*log(c*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x^r + d)*(f*x)^m*(b*log(c*x^n) + a)^p, x)
```

Sympy [F]

$$\int (fx)^m (d + ex^r) (a + b\log (cx^n))^p \, dx = \int (fx)^m (a + b\log (cx^n))^p (d + ex^r) \, dx$$

```
[In] integrate((f*x)**m*(d+e*x**r)*(a+b*ln(c*x**n))**p,x)
[Out] Integral((f*x)**m*(a + b*log(c*x**n))**p*(d + e*x**r), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (fx)^m (d + ex^r) (a + b\log (cx^n))^p \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((f*x)^m*(d+e*x^r)*(a+b*log(c*x^n))^p,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int (fx)^m (d+ex^r) (a+b\log(cx^n))^p \, dx = \int (ex^r+d)(fx)^m (b\log(cx^n)+a)^p \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^r)*(a+b*log(c*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x^r + d)*(f*x)^m*(b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^r) (a+b\log(cx^n))^p \, dx = \int (f\,x)^m \left(d+e\,x^r\right) \left(a+b\,\ln(c\,x^n)\right)^p dx$$

```
[In] int((f*x)^m*(d + e*x^r)*(a + b*log(c*x^n))^p,x)
[Out] int((f*x)^m*(d + e*x^r)*(a + b*log(c*x^n))^p, x)
```

3.451 $\int (fx)^m (a+b\log(cx^n))^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 106

$$\int (fx)^m (a+b\log(cx^n))^p\,dx = \frac{e^{-\frac{a(1+m)}{bn}}(fx)^{1+m}(cx^n)^{-\frac{1+m}{n}}\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{-p}}{f(1+m)}$$

```
[Out] (f*x)^(1+m)*GAMMA(p+1,-(1+m)*(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p/exp(a*(
1+m)/b/n)/f/(1+m)/((c*x^n)^((1+m)/n))/((-(1+m)*(a+b*ln(c*x^n))/b/n)^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2347, 2212}

$$\int (fx)^m (a+b\log(cx^n))^p\,dx = \frac{(fx)^{m+1}e^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}(a+b\log(cx^n))^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)}{f(m+1)}$$

```
[In] Int[(f*x)^m*(a + b*Log[c*x^n])^p,x]
```

```
[Out] ((f*x)^(1 + m)*Gamma[1 + p, -(((1 + m)*(a + b*Log[c*x^n]))/(b*n))]*(a + b*L
og[c*x^n])^p)/(E^((a*(1 + m))/(b*n))*f*(1 + m)*(c*x^n)^((1 + m)/n)*(-(((1 +
 m)*(a + b*Log[c*x^n]))/(b*n)))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\left((fx)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int e^{\frac{(1+m)x}{n}}(a+bx)^p\,dx, x, \log\left(cx^n\right)\right)}{fn}$$

$$= \frac{e^{-\frac{a(1+m)}{bn}}(fx)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{-}}{f(1+m)}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01

$$\int (fx)^m\left(a+b\log\left(cx^n\right)\right)^p\,dx$$

$$= \frac{e^{-\frac{(1+m)(a+b(-n\log(x)+\log(cx^n)))}{bn}}x^{-m}(fx)^m\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{-}}{1+m}$$

```
[In] Integrate[(f*x)^m*(a + b*Log[c*x^n])^p,x]
```

```
[Out] ((f*x)^m*Gamma[1 + p, -(((1 + m)*(a + b*Log[c*x^n]))/(b*n))]*(a + b*Log[c*x
^n])^p)/(E^(((1 + m)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m)*x^m
*(-(((1 + m)*(a + b*Log[c*x^n]))/(b*n)))^p)
```

Maple [F]

$$\int (fx)^m (a+b\ln(c\,x^n))^p dx$$

```
[In] int((f*x)^m*(a+b*ln(c*x^n))^p,x)
[Out] int((f*x)^m*(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int (fx)^m (a+b\log(cx^n))^p \, dx = \int (fx)^m (b\log(cx^n)+a)^p \, dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))^p,x, algorithm="fricas")
[Out] integral((f*x)^m*(b*log(c*x^n) + a)^p, x)
```

Sympy [F]

$$\int (fx)^m (a+b\log(cx^n))^p \, dx = \int (fx)^m (a+b\log(cx^n))^p \, dx$$

```
[In] integrate((f*x)**m*(a+b*ln(c*x**n))**p,x)
[Out] Integral((f*x)**m*(a + b*log(c*x**n))**p, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (fx)^m (a+b\log(cx^n))^p \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))^p,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int (fx)^m (a + b\log(cx^n))^p \, dx = \int (fx)^m (b\log(cx^n) + a)^p \, dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))^p,x, algorithm="giac")
[Out] integrate((f*x)^m*(b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (a + b\log(cx^n))^p \, dx = \int (f\,x)^m \,(a + b\,\ln(c\,x^n))^p \, dx$$

```
[In] int((f*x)^m*(a + b*log(c*x^n))^p,x)
[Out] int((f*x)^m*(a + b*log(c*x^n))^p, x)
```

3.452 $\int \frac{(fx)^m(a+b\log(cx^n))^p}{d+ex^r}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(fx)^m\left(a+b\log\left(cx^n\right)\right)^p}{d+ex^r}\,dx=\text{Int}\left(\frac{(fx)^m\left(a+b\log\left(cx^n\right)\right)^p}{d+ex^r},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*ln(c*x^n))^p/(d+e*x^r),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\log\left(cx^n\right)\right)^p}{d+ex^r}\,dx=\int \frac{(fx)^m\left(a+b\log\left(cx^n\right)\right)^p}{d+ex^r}\,dx$$

```
[In] Int[((f*x)^m*(a + b*Log[c*x^n])^p)/(d + e*x^r),x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*Log[c*x^n])^p)/(d + e*x^r), x]
```

Rubi steps

$$\text{integral}=\int \frac{(fx)^m\left(a+b\log\left(cx^n\right)\right)^p}{d+ex^r}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m (a+b\log(cx^n))^p}{d+ex^r}\,dx = \int \frac{(fx)^m (a+b\log(cx^n))^p}{d+ex^r}\,dx$$

```
[In] Integrate[((f*x)^m*(a + b*Log[c*x^n])^p)/(d + e*x^r),x]
[Out] Integrate[((f*x)^m*(a + b*Log[c*x^n])^p)/(d + e*x^r), x]
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a+b\ln(c\,x^n))^p}{d+e\,x^r}dx$$

```
[In] int((f*x)^m*(a+b*ln(c*x^n))^p/(d+e*x^r),x)
[Out] int((f*x)^m*(a+b*ln(c*x^n))^p/(d+e*x^r),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m (a+b\log(cx^n))^p}{d+ex^r}\,dx = \int \frac{(fx)^m (b\log(cx^n)+a)^p}{ex^r+d}\,dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))^p/(d+e*x^r),x, algorithm="fricas")
[Out] integral((f*x)^m*(b*log(c*x^n) + a)^p/(e*x^r + d), x)
```

Sympy [N/A]

Not integrable

Time = 92.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(fx)^m (a+b\log(cx^n))^p}{d+ex^r}\,dx = \int \frac{(fx)^m (a+b\log(cx^n))^p}{d+ex^r}\,dx$$

```
[In] integrate((f*x)**m*(a+b*ln(c*x**n))**p/(d+e*x**r),x)
[Out] Integral((f*x)**m*(a + b*log(c*x**n))**p/(d + e*x**r), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(fx)^m (a + b\log(cx^n))^p}{d + ex^r}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))^p/(d+e*x^r),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m (a + b\log(cx^n))^p}{d + ex^r}\,dx = \int \frac{(fx)^m (b\log(cx^n) + a)^p}{ex^r + d}\,dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))^p/(d+e*x^r),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*(b*log(c*x^n) + a)^p/(e*x^r + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m (a + b\log(cx^n))^p}{d + ex^r}\,dx = \int \frac{(f\,x)^m\,(a + b\,\ln(c\,x^n))^p}{d + e\,x^r}\,dx$$

```
[In] int(((f*x)^m*(a + b*log(c*x^n))^p)/(d + e*x^r),x)
```

```
[Out] int(((f*x)^m*(a + b*log(c*x^n))^p)/(d + e*x^r), x)
```

3.453 $\int \frac{(fx)^m(a+b\log(cx^n))^p}{(d+ex^r)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(fx)^m (a+b\log(cx^n))^p}{(d+ex^r)^2}\,dx = \text{Int}\left(\frac{(fx)^m (a+b\log(cx^n))^p}{(d+ex^r)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*ln(c*x^n))^p/(d+e*x^r)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m (a+b\log(cx^n))^p}{(d+ex^r)^2}\,dx = \int \frac{(fx)^m (a+b\log(cx^n))^p}{(d+ex^r)^2}\,dx$$

```
[In] Int[((f*x)^m*(a + b*Log[c*x^n])^p)/(d + e*x^r)^2,x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*Log[c*x^n])^p)/(d + e*x^r)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(fx)^m (a+b\log(cx^n))^p}{(d+ex^r)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m (a+b\log(cx^n))^p}{(d+ex^r)^2}\,dx = \int \frac{(fx)^m (a+b\log(cx^n))^p}{(d+ex^r)^2}\,dx$$

```
[In] Integrate[((f*x)^m*(a + b*Log[c*x^n])^p)/(d + e*x^r)^2,x]
[Out] Integrate[((f*x)^m*(a + b*Log[c*x^n])^p)/(d + e*x^r)^2, x]
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a+b\ln(c\,x^n))^p}{(d+e\,x^r)^2}dx$$

```
[In] int((f*x)^m*(a+b*ln(c*x^n))^p/(d+e*x^r)^2,x)
[Out] int((f*x)^m*(a+b*ln(c*x^n))^p/(d+e*x^r)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{(fx)^m (a+b\log(cx^n))^p}{(d+ex^r)^2}\,dx = \int \frac{(fx)^m (b\log(cx^n)+a)^p}{(ex^r+d)^2}\,dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))^p/(d+e*x^r)^2,x, algorithm="fricas")
[Out] integral((f*x)^m*(b*log(c*x^n) + a)^p/(e^2*x^(2*r) + 2*d*e*x^r + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.04

$$\int \frac{(fx)^m (a+b\log(cx^n))^p}{(d+ex^r)^2}\,dx = \int \frac{(fx)^m (a+b\log(cx^n))^p}{(d+ex^r)^2}\,dx$$

```
[In] integrate((f*x)**m*(a+b*ln(c*x**n))**p/(d+e*x**r)**2,x)
[Out] integrate((f*x)**m*(a+b*ln(c*x**n))**p/(d+e*x**r)**2,x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(fx)^m (a+b\log(cx^n))^p}{(d+ex^r)^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))^p/(d+e*x^r)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m (a+b\log(cx^n))^p}{(d+ex^r)^2}\,dx = \int \frac{(fx)^m (b\log(cx^n)+a)^p}{(ex^r+d)^2}\,dx$$

```
[In] integrate((f*x)^m*(a+b*log(c*x^n))^p/(d+e*x^r)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*(b*log(c*x^n) + a)^p/(e*x^r + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(fx)^m (a+b\log(cx^n))^p}{(d+ex^r)^2}\,dx = \int \frac{(f\,x)^m\,(a+b\,\ln(c\,x^n))^p}{(d+e\,x^r)^2}\,dx$$

```
[In] int(((f*x)^m*(a + b*log(c*x^n))^p)/(d + e*x^r)^2,x)
```

```
[Out] int(((f*x)^m*(a + b*log(c*x^n))^p)/(d + e*x^r)^2, x)
```

3.454 $\int \frac{(f+gx)(a+b\log(cx^n))}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \frac{(f+gx)\left(a+b\log\left(cx^n\right)\right)}{(d+ex)^3}\,dx=\frac{b(ef-dg)n}{2de^2(d+ex)}+\frac{bf^2n\log(x)}{2d^2(ef-dg)}-\frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)}{2(ef-dg)(d+ex)^2}-\frac{b(ef+dg)n\log(d+ex)}{2d^2e^2}$$

```
[Out] 1/2*b*(-d*g+e*f)*n/d/e^2/(e*x+d)+1/2*b*f^2*n*ln(x)/d^2/(-d*g+e*f)-1/2*(g*x+
f)^2*(a+b*ln(c*x^n))/(-d*g+e*f)/(e*x+d)^2-1/2*b*(d*g+e*f)*n*ln(e*x+d)/d^2/e
^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2398, 90}

$$\int \frac{(f+gx)\left(a+b\log\left(cx^n\right)\right)}{(d+ex)^3}\,dx=-\frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)}{2(d+ex)^2(ef-dg)}-\frac{bn(dg+ef)\log(d+ex)}{2d^2e^2}+\frac{bf^2n\log(x)}{2d^2(ef-dg)}+\frac{bn(ef-dg)}{2de^2(d+ex)}$$

```
[In] Int[((f + g*x)*(a + b*Log[c*x^n]))/(d + e*x)^3,x]
```

```
[Out] (b*(e*f - d*g)*n)/(2*d*e^2*(d + e*x)) + (b*f^2*n*Log[x])/(2*d^2*(e*f - d*g)
) - ((f + g*x)^2*(a + b*Log[c*x^n]))/(2*(e*f - d*g)*(d + e*x)^2) - (b*(e*f
+ d*g)*n*Log[d + e*x])/(2*d^2*e^2)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)}{2(ef-dg)(d+ex)^2}+\frac{(bn)\int\frac{(f+gx)^2}{x(d+ex)^2}\,dx}{2(ef-dg)}\\
&= -\frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)}{2(ef-dg)(d+ex)^2}+\frac{(bn)\int\left(\frac{f^2}{d^2x}-\frac{(-ef+dg)^2}{de(d+ex)^2}+\frac{-e^2f^2+d^2g^2}{d^2e(d+ex)}\right)dx}{2(ef-dg)}\\
&= \frac{b(ef-dg)n}{2de^2(d+ex)}+\frac{bf^2n\log(x)}{2d^2(ef-dg)}-\frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)}{2(ef-dg)(d+ex)^2}-\frac{b(ef+dg)n\log(d+ex)}{2d^2e^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

$$
\int\frac{(f+gx)\left(a+b\log\left(cx^n\right)\right)}{(d+ex)^3}\,dx = \frac{-\frac{(ef-dg)(a+b\log(cx^n))}{(d+ex)^2}-\frac{2g(a+b\log(cx^n))}{d+ex}+\frac{2bgn(\log(x)-\log(d+ex))}{d}+\frac{b(ef-dg)n\left(\frac{d}{d+ex}+\log(x)-\log(d+ex)\right)}{d^2}}{2e^2}
$$

```
[In] Integrate[((f + g*x)*(a + b*Log[c*x^n]))/(d + e*x)^3,x]
[Out] (-(((e*f - d*g)*(a + b*Log[c*x^n]))/(d + e*x)^2) - (2*g*(a + b*Log[c*x^n]))
/(d + e*x) + (2*b*g*n*(Log[x] - Log[d + e*x]))/d + (b*(e*f - d*g)*n*(d/(d +
 e*x) + Log[x] - Log[d + e*x]))/d^2)/(2*e^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(107) = 214.

Time = 0.68 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.47

method	result
parallelrisch	$\frac{-b\,d^{3} g n - 2 a\,d^{2} e f + 2\ln(x) x^{2} b d\,e^{2} g n + b d\,e^{2} g n\,x^{2} - 4 x\ln(c\,x^{n}) b\,d^{2} e g + 2\ln(x) x^{2} b\,e^{3} f n - 2\ln(e x+d) b d\,e^{2} g n\,x^{2} - 4\ln(e x+d) b\,d^{2} e g n}{}$
risch	$-\frac{b(2 e g x+d g+e f)\ln(x^{n})}{2(e x+d)^{2} e^{2}} + \frac{-2 b\,d^{2} e g n x + 2 b d\,e^{2} f n x - i\pi b\,d^{3} g\ \mathrm{csgn}(i c)\,\mathrm{csgn}(i c\,x^{n})^{2} + i\pi b\,d^{3} g\ \mathrm{csgn}(i c)\,\mathrm{csgn}(i x^{n})\,\mathrm{csgn}(i c\,x^{n}) - i}{}$

```
[In] int((g*x+f)*(a+b*ln(c*x^n))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-b*d^3*g*n-2*a*d^2*e*f+2*ln(x)*x^2*b*d*e^2*g*n+b*d*e^2*g*n*x^2-4*x*ln(
c*x^n)*b*d^2*e*g+2*ln(x)*x^2*b*e^3*f*n-2*ln(e*x+d)*b*d*e^2*g*n*x^2-4*ln(e*x
+d)*b*d^2*e*g*n*x-4*ln(e*x+d)*b*d*e^2*f*n*x+b*d^2*e*f*n-2*ln(e*x+d)*b*d^3*g
*n+2*ln(x)*b*d^2*e*f*n-2*ln(c*x^n)*b*d^3*g-2*ln(e*x+d)*b*e^3*f*n*x^2-2*ln(e
*x+d)*b*d^2*e*f*n+4*ln(x)*x*b*d^2*e*g*n+4*ln(x)*x*b*d*e^2*f*n+2*x^2*a*d*e^2
*g-2*ln(c*x^n)*b*d^2*e*f+2*ln(x)*b*d^3*g*n-b*e^3*f*n*x^2)/d^2/e^2/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(107) = 214.

Time = 0.28 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.87

$$\int \frac{(f+gx)\,(a+b\log{(cx^{n})})}{(d+ex)^{3}}\,dx = -\frac{ad^{2}ef + ad^{3}g - (bd^{2}ef - bd^{3}g)n + (2\,ad^{2}eg - (bde^{2}f - bd^{2}eg)n)x + ((be^{3}f + bde^{2}g)nx^{2} + 2\,(bde^{2}f + b}{2\,(d^{2}e}$$

```
[In] integrate((g*x+f)*(a+b*log(c*x^n))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*d^2*e*f + a*d^3*g - (b*d^2*e*f - b*d^3*g)*n + (2*a*d^2*e*g - (b*d*e
^2*f - b*d^2*e*g)*n)*x + ((b*e^3*f + b*d*e^2*g)*n*x^2 + 2*(b*d*e^2*f + b*d^
2*e*g)*n*x + (b*d^2*e*f + b*d^3*g)*n)*log(e*x + d) + (2*b*d^2*e*g*x + b*d^2
*e*f + b*d^3*g)*log(c) - (2*b*d*e^2*f*n*x + (b*e^3*f + b*d*e^2*g)*n*x^2)*lo
g(x))/(d^2*e^4*x^2 + 2*d^3*e^3*x + d^4*e^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 908 vs. $2(100) = 200$.

Time = 2.37 (sec) , antiderivative size = 908, normalized size of antiderivative = 7.90

$$\int \frac{(f+gx)(a+b\log(cx^n))}{(d+ex)^3}\,dx$$

$$= \begin{cases} \tilde{\infty}\left(-\frac{af}{2x^2}-\frac{ag}{x}-\frac{bfn}{4x^2}-\frac{bf\log(cx^n)}{2x^2}-\frac{bgn}{x}-\frac{bg\log(cx^n)}{x}\right) \\ \frac{afx+\frac{agx^2}{2}-bfnx+bfx\log(cx^n)-\frac{bgnx^2}{4}+\frac{bgx^2\log(cx^n)}{2}}{d^3} \\ \frac{-\frac{af}{2x^2}-\frac{ag}{x}-\frac{bfn}{4x^2}-\frac{bf\log(cx^n)}{2x^2}-\frac{bgn}{x}-\frac{bg\log(cx^n)}{x}}{e^3} \\ -\frac{ad^3g}{2d^4e^2+4d^3e^3x+2d^2e^4x^2}-\frac{ad^2ef}{2d^4e^2+4d^3e^3x+2d^2e^4x^2}-\frac{2ad^2egx}{2d^4e^2+4d^3e^3x+2d^2e^4x^2}-\frac{bd^3gn\log\left(\frac{d}{e}+x\right)}{2d^4e^2+4d^3e^3x+2d^2e^4x^2}-\frac{bd^3gn}{2d^4e^2+4d^3e^3x+2d^2e^4x^2} \end{cases}$$

```
[In] integrate((g*x+f)*(a+b*ln(c*x**n))/(e*x+d)**3,x)
```

```
[Out] Piecewise((zoo*(-a*f/(2*x**2) - a*g/x - b*f*n/(4*x**2) - b*f*log(c*x**n)/(2
*x**2) - b*g*n/x - b*g*log(c*x**n)/x), Eq(d, 0) & Eq(e, 0)), ((a*f*x + a*g*
x**2/2 - b*f*n*x + b*f*x*log(c*x**n) - b*g*n*x**2/4 + b*g*x**2*log(c*x**n)/
2)/d**3, Eq(e, 0)), ((-a*f/(2*x**2) - a*g/x - b*f*n/(4*x**2) - b*f*log(c*x*
*n)/(2*x**2) - b*g*n/x - b*g*log(c*x**n)/x)/e**3, Eq(d, 0)), (-a*d**3*g/(2*
d**4*e**2 + 4*d**3*e**3*x + 2*d**2*e**4*x**2) - a*d**2*e*f/(2*d**4*e**2 + 4
*d**3*e**3*x + 2*d**2*e**4*x**2) - 2*a*d**2*e*g*x/(2*d**4*e**2 + 4*d**3*e**
3*x + 2*d**2*e**4*x**2) - b*d**3*g*n*log(d/e + x)/(2*d**4*e**2 + 4*d**3*e**
3*x + 2*d**2*e**4*x**2) - b*d**3*g*n/(2*d**4*e**2 + 4*d**3*e**3*x + 2*d**2*
e**4*x**2) - b*d**2*e*f*n*log(d/e + x)/(2*d**4*e**2 + 4*d**3*e**3*x + 2*d**
2*e**4*x**2) + b*d**2*e*f*n/(2*d**4*e**2 + 4*d**3*e**3*x + 2*d**2*e**4*x**2
) - 2*b*d**2*e*g*n*x*log(d/e + x)/(2*d**4*e**2 + 4*d**3*e**3*x + 2*d**2*e**
4*x**2) - b*d**2*e*g*n*x/(2*d**4*e**2 + 4*d**3*e**3*x + 2*d**2*e**4*x**2) -
 2*b*d*e**2*f*n*x*log(d/e + x)/(2*d**4*e**2 + 4*d**3*e**3*x + 2*d**2*e**4*x
**2) + b*d*e**2*f*n*x/(2*d**4*e**2 + 4*d**3*e**3*x + 2*d**2*e**4*x**2) + 2*
b*d*e**2*f*x*log(c*x**n)/(2*d**4*e**2 + 4*d**3*e**3*x + 2*d**2*e**4*x**2) -
 b*d*e**2*g*n*x**2*log(d/e + x)/(2*d**4*e**2 + 4*d**3*e**3*x + 2*d**2*e**4*
x**2) + b*d*e**2*g*x**2*log(c*x**n)/(2*d**4*e**2 + 4*d**3*e**3*x + 2*d**2*e
**4*x**2) - b*e**3*f*n*x**2*log(d/e + x)/(2*d**4*e**2 + 4*d**3*e**3*x + 2*d
**2*e**4*x**2) + b*e**3*f*x**2*log(c*x**n)/(2*d**4*e**2 + 4*d**3*e**3*x + 2
*d**2*e**4*x**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(107) = 214$.

Time = 0.20 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.90

$$\int \frac{(f+gx)\left(a+b\log\left(cx^{n}\right)\right)}{(d+ex)^{3}}\,dx = \frac{1}{2}\,bfn\left(\frac{1}{de^{2}x+d^{2}e} - \frac{\log\left(ex+d\right)}{d^{2}e} + \frac{\log\left(x\right)}{d^{2}e}\right) - \frac{1}{2}\,bgn\left(\frac{1}{e^{3}x+de^{2}} + \frac{\log\left(ex+d\right)}{de^{2}} - \frac{\log\left(x\right)}{de^{2}}\right) - \frac{(2\,ex+d)bg\log\left(cx^{n}\right)}{2\left(e^{4}x^{2}+2\,de^{3}x+d^{2}e^{2}\right)} - \frac{(2\,ex+d)ag}{2\left(e^{4}x^{2}+2\,de^{3}x+d^{2}e^{2}\right)} - \frac{bf\log\left(cx^{n}\right)}{2\left(e^{3}x^{2}+2\,de^{2}x+d^{2}e\right)} - \frac{af}{2\left(e^{3}x^{2}+2\,de^{2}x+d^{2}e\right)}$$

```
[In] integrate((g*x+f)*(a+b*log(c*x^n))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*f*n*(1/(d*e^2*x + d^2*e) - log(e*x + d)/(d^2*e) + log(x)/(d^2*e)) - 1
/2*b*g*n*(1/(e^3*x + d*e^2) + log(e*x + d)/(d*e^2) - log(x)/(d*e^2)) - 1/2*
(2*e*x + d)*b*g*log(c*x^n)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) - 1/2*(2*e*x + d
)*a*g/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) - 1/2*b*f*log(c*x^n)/(e^3*x^2 + 2*d*e
^2*x + d^2*e) - 1/2*a*f/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.73

$$\int \frac{(f+gx)\left(a+b\log\left(cx^{n}\right)\right)}{(d+ex)^{3}}\,dx = -\frac{(2\,begnx+befn+bdgn)\log\left(x\right)}{2\left(e^{4}x^{2}+2\,de^{3}x+d^{2}e^{2}\right)} + \frac{be^{2}fnx-bdegnx-2\,bdegx\log\left(c\right)+bdefn-bd^{2}gn-2\,adegx-bdef\log\left(c\right)-bd^{2}g\log\left(c\right)-adef-ac}{2\left(de^{4}x^{2}+2\,d^{2}e^{3}x+d^{3}e^{2}\right)} - \frac{(befn+bdgn)\log\left(ex+d\right)}{2\,d^{2}e^{2}} + \frac{(befn+bdgn)\log\left(x\right)}{2\,d^{2}e^{2}}$$

```
[In] integrate((g*x+f)*(a+b*log(c*x^n))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*e*g*n*x + b*e*f*n + b*d*g*n)*log(x)/(e^4*x^2 + 2*d*e^3*x + d^2*e^
2) + 1/2*(b*e^2*f*n*x - b*d*e*g*n*x - 2*b*d*e*g*x*log(c) + b*d*e*f*n - b*d^
2*g*n - 2*a*d*e*g*x - b*d*e*f*log(c) - b*d^2*g*log(c) - a*d*e*f - a*d^2*g)/
(d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2) - 1/2*(b*e*f*n + b*d*g*n)*log(e*x + d)/
(d^2*e^2) + 1/2*(b*e*f*n + b*d*g*n)*log(x)/(d^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.51

$$\int \frac{(f+gx)\left(a+b\log\left(cx^n\right)\right)}{(d+ex)^3}\,dx = -\frac{a\,d\,g+a\,e\,f+\frac{x\,\left(2\,a\,d\,e\,g-b\,e^2\,f\,n+b\,d\,e\,g\,n\right)}{d}+b\,d\,g\,n-b\,e\,f\,n}{2\,d^2\,e^2+4\,d\,e^3\,x+2\,e^4\,x^2} - \frac{\ln\left(c\,x^n\right)\,\left(\frac{b\,f}{2\,e}+\frac{b\,d\,g}{2\,e^2}+\frac{b\,g\,x}{e}\right)}{d^2+2\,d\,e\,x+e^2\,x^2} - \frac{b\,n\,\mathrm{atanh}\left(\frac{b\,n\,\left(d\,g+e\,f\right)\,\left(d+2\,e\,x\right)}{d\,\left(b\,d\,g\,n+b\,e\,f\,n\right)}\right)\,\left(d\,g+e\,f\right)}{d^2\,e^2}$$

```
[In] int(((f + g*x)*(a + b*log(c*x^n)))/(d + e*x)^3,x)
[Out] - (a*d*g + a*e*f + (x*(2*a*d*e*g - b*e^2*f*n + b*d*e*g*n))/d + b*d*g*n - b*
e*f*n)/(2*d^2*e^2 + 2*e^4*x^2 + 4*d*e^3*x) - (log(c*x^n)*((b*f)/(2*e) + (b*
d*g)/(2*e^2) + (b*g*x)/e))/(d^2 + e^2*x^2 + 2*d*e*x) - (b*n*atanh((b*n*(d*g
 + e*f)*(d + 2*e*x))/(d*(b*d*g*n + b*e*f*n)))*(d*g + e*f))/(d^2*e^2)
```

3.455 $\int \frac{(f+gx)(a+b\log(cx^n))^2}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 202

$$\begin{aligned}\int \frac{(f+gx)\left(a+b\log\left(cx^n\right)\right)^2}{(d+ex)^3}\,dx = &-\frac{b(ef-dg)nx(a+b\log\left(cx^n\right))}{d^2e(d+ex)} \\ &+\frac{f^2\left(a+b\log\left(cx^n\right)\right)^2}{2d^2(ef-dg)}-\frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)^2}{2(ef-dg)(d+ex)^2} \\ &+\frac{b^2(ef-dg)n^2\log(d+ex)}{d^2e^2} \\ &-\frac{b(ef+dg)n(a+b\log\left(cx^n\right))\log\left(1+\frac{ex}{d}\right)}{d^2e^2} \\ &-\frac{b^2(ef+dg)n^2\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^2e^2}\end{aligned}$$

```
[Out] -b*(-d*g+e*f)*n*x*(a+b*ln(c*x^n))/d^2/e/(e*x+d)+1/2*f^2*(a+b*ln(c*x^n))^2/d
^2/(-d*g+e*f)-1/2*(g*x+f)^2*(a+b*ln(c*x^n))^2/(-d*g+e*f)/(e*x+d)^2+b^2*(-d*
g+e*f)*n^2*ln(e*x+d)/d^2/e^2-b*(d*g+e*f)*n*(a+b*ln(c*x^n))*ln(1+e*x/d)/d^2/
e^2-b^2*(d*g+e*f)*n^2*polylog(2,-e*x/d)/d^2/e^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

= {2398, 2404, 2338, 2351, 31, 2354, 2438}

$$\int \frac{(f+gx)\left(a+b\log\left(cx^n\right)\right)^2}{(d+ex)^3}\,dx = -\frac{bn(dg+ef)\log\left(\frac{ex}{d}+1\right)\left(a+b\log\left(cx^n\right)\right)}{d^2e^2} + \frac{f^2\left(a+b\log\left(cx^n\right)\right)^2}{2d^2(ef-dg)} - \frac{bnx(ef-dg)\left(a+b\log\left(cx^n\right)\right)}{d^2e(d+ex)} - \frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)^2}{2(d+ex)^2(ef-dg)} - \frac{b^2n^2(dg+ef)\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^2e^2} + \frac{b^2n^2(ef-dg)\log(d+ex)}{d^2e^2}$$

```
[In] Int[((f + g*x)*(a + b*Log[c*x^n])^2)/(d + e*x)^3,x]
```

```
[Out] -((b*(e*f - d*g)*n*x*(a + b*Log[c*x^n]))/(d^2*e*(d + e*x))) + (f^2*(a + b*L
og[c*x^n])^2)/(2*d^2*(e*f - d*g)) - ((f + g*x)^2*(a + b*Log[c*x^n])^2)/(2*(
e*f - d*g)*(d + e*x)^2) + (b^2*(e*f - d*g)*n^2*Log[d + e*x])/(d^2*e^2) - (b
*(e*f + d*g)*n*(a + b*Log[c*x^n])*Log[1 + (e*x)/d])/(d^2*e^2) - (b^2*(e*f +
 d*g)*n^2*PolyLog[2, -((e*x)/d)])/(d^2*e^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)^2}{2(ef-dg)(d+ex)^2}+\frac{(bn)\int\frac{(f+gx)^2(a+b\log(cx^n))}{x(d+ex)^2}\,dx}{ef-dg}\\
&= -\frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)^2}{2(ef-dg)(d+ex)^2}\\
&\quad+\frac{(bn)\int\left(\frac{f^2(a+b\log(cx^n))}{d^2x}-\frac{(-ef+dg)^2(a+b\log(cx^n))}{de(d+ex)^2}+\frac{(-e^2f^2+d^2g^2)(a+b\log(cx^n))}{d^2e(d+ex)}\right)dx}{ef-dg}\\
&= -\frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)^2}{2(ef-dg)(d+ex)^2}+\frac{(bf^2n)\int\frac{a+b\log(cx^n)}{x}\,dx}{d^2(ef-dg)}\\
&\quad-\frac{(b(ef-dg)n)\int\frac{a+b\log(cx^n)}{(d+ex)^2}\,dx}{de}-\frac{(b(ef+dg)n)\int\frac{a+b\log(cx^n)}{d+ex}\,dx}{d^2e}\\
&= -\frac{b(ef-dg)nx(a+b\log\left(cx^n\right))}{d^2e(d+ex)}+\frac{f^2\left(a+b\log\left(cx^n\right)\right)^2}{2d^2(ef-dg)}\\
&\quad-\frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)^2}{2(ef-dg)(d+ex)^2}-\frac{b(ef+dg)n(a+b\log\left(cx^n\right))\log\left(1+\frac{ex}{d}\right)}{d^2e^2}\\
&\quad+\frac{(b^2(ef-dg)n^2)\int\frac{1}{d+ex}\,dx}{d^2e}+\frac{(b^2(ef+dg)n^2)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{d^2e^2}
\end{aligned}
$$

$$= -\frac{b(ef-dg)nx(a+b\log(cx^n))}{d^2e(d+ex)} + \frac{f^2(a+b\log(cx^n))^2}{2d^2(ef-dg)} - \frac{(f+gx)^2(a+b\log(cx^n))^2}{2(ef-dg)(d+ex)^2} + \frac{b^2(ef-dg)n^2\log(d+ex)}{d^2e^2} - \frac{b(ef+dg)n(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^2e^2} - \frac{b^2(ef+dg)n^2\text{Li}_2\left(-\frac{ex}{d}\right)}{d^2e^2}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.21

$$\int \frac{(f+gx)(a+b\log(cx^n))^2}{(d+ex)^3}\,dx = \frac{-\frac{(ef-dg)(a+b\log(cx^n))^2}{(d+ex)^2} - \frac{2g(a+b\log(cx^n))^2}{d+ex} + \frac{2g\left((a+b\log(cx^n))\left(a+b\log(cx^n)-2bn\log\left(1+\frac{ex}{d}\right)\right)-2b^2n^2\,\text{PolyLog}\left(2,-\frac{ex}{d}\right)\right)}{d} + \frac{(ef-dg)}{}}{}$$

```
[In] Integrate[((f + g*x)*(a + b*Log[c*x^n])^2)/(d + e*x)^3,x]
```

```
[Out] (-(((e*f - d*g)*(a + b*Log[c*x^n])^2)/(d + e*x)^2) - (2*g*(a + b*Log[c*x^n]
)^2)/(d + e*x) + (2*g*((a + b*Log[c*x^n])*(a + b*Log[c*x^n] - 2*b*n*Log[1 +
 (e*x)/d]) - 2*b^2*n^2*PolyLog[2, -((e*x)/d)]))/d + ((e*f - d*g)*(2*b*d*n*(
a + b*Log[c*x^n]) + (d + e*x)*(a + b*Log[c*x^n])^2 - 2*b^2*n^2*(d + e*x)*(L
og[x] - Log[d + e*x]) - 2*b*n*(d + e*x)*(a + b*Log[c*x^n])*Log[1 + (e*x)/d]
 - 2*b^2*n^2*(d + e*x)*PolyLog[2, -((e*x)/d)]))/(d^2*(d + e*x)))/(2*e^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.77 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.66

method	result
risch	$-\frac{b^2\ln(x^n)^2 g}{e^2(ex+d)} + \frac{b^2\ln(x^n)^2 dg}{2e^2(ex+d)^2} - \frac{b^2\ln(x^n)^2 f}{2e(ex+d)^2} - \frac{b^2 n\ln(x^n)g}{e^2(ex+d)} + \frac{b^2 n\ln(x^n)f}{ed(ex+d)} - \frac{b^2 n\ln(x^n)\ln(ex+d)g}{e^2 d} - \frac{b^2 n\ln(x^n)\ln(ex+d)}{e\,d^2}$

```
[In] int((g*x+f)*(a+b*ln(c*x^n))^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b^2*ln(x^n)^2*g/e^2/(e*x+d)+1/2*b^2*ln(x^n)^2/e^2/(e*x+d)^2*d*g-1/2*b^2*ln
(x^n)^2/e/(e*x+d)^2*f-b^2*n*ln(x^n)/e^2/(e*x+d)*g+b^2*n*ln(x^n)/e/d/(e*x+d)
*f-b^2*n*ln(x^n)/e^2/d*ln(e*x+d)*g-b^2*n*ln(x^n)/e/d^2*ln(e*x+d)*f+b^2*n*ln
(x^n)/e^2/d*ln(x)*g+b^2*n*ln(x^n)/e/d^2*ln(x)*f-1/2*b^2*n^2/e^2/d*ln(x)^2*g
-1/2*b^2*n^2/e/d^2*ln(x)^2*f-b^2*n^2/e^2/d*ln(e*x+d)*g+b^2*n^2/e/d^2*ln(e*x
+d)*f+b^2*n^2/e^2/d*ln(x)*g-b^2*n^2/e/d^2*ln(x)*f+b^2*n^2/e^2/d*ln(e*x+d)*l
n(-e*x/d)*g+b^2*n^2/e/d^2*ln(e*x+d)*ln(-e*x/d)*f+b^2*n^2/e^2/d*dilog(-e*x/d
```

```
)*g+b^2*n^2/e/d^2*dilog(-e*x/d)*f+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x
^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b
*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b*(-ln(x^n)/e^2/(e*x+d)*g+1/2*ln(x^n)/e^
2/(e*x+d)^2*d*g-1/2*ln(x^n)/e/(e*x+d)^2*f-1/2*n/e^2*((d*g+e*f)/d^2*ln(e*x+d
)+(d*g-e*f)/d/(e*x+d)+1/d^2*(-d*g-e*f)*ln(x)))+1/4*(-I*b*Pi*csgn(I*c)*csgn(
I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*cs
gn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*(-g/e^2/(e*x+d)-1/2*(
-d*g+e*f)/e^2/(e*x+d)^2)
```

Fricas [F]

$$\int \frac{(f+gx)\,(a+b\log(cx^n))^2}{(d+ex)^3}\,dx=\int \frac{(gx+f)(b\log(cx^n)+a)^2}{(ex+d)^3}\,dx$$

```
[In] integrate((g*x+f)*(a+b*log(c*x^n))^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*g*x + a^2*f + (b^2*g*x + b^2*f)*log(c*x^n)^2 + 2*(a*b*g*x + a
*b*f)*log(c*x^n))/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F]

$$\int \frac{(f+gx)\,(a+b\log(cx^n))^2}{(d+ex)^3}\,dx=\int \frac{(a+b\log(cx^n))^2\,(f+gx)}{(d+ex)^3}\,dx$$

```
[In] integrate((g*x+f)*(a+b*ln(c*x**n))**2/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*log(c*x**n))**2*(f + g*x)/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{(f+gx)\,(a+b\log(cx^n))^2}{(d+ex)^3}\,dx=\int \frac{(gx+f)(b\log(cx^n)+a)^2}{(ex+d)^3}\,dx$$

```
[In] integrate((g*x+f)*(a+b*log(c*x^n))^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] a*b*f*n*(1/(d*e^2*x + d^2*e) - log(e*x + d)/(d^2*e) + log(x)/(d^2*e)) - a*b
*g*n*(1/(e^3*x + d*e^2) + log(e*x + d)/(d*e^2) - log(x)/(d*e^2)) - (2*e*x +
 d)*a*b*g*log(c*x^n)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) - 1/2*(2*e*x + d)*a^2*
g/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) - a*b*f*log(c*x^n)/(e^3*x^2 + 2*d*e^2*x +
 d^2*e) - 1/2*a^2*f/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 1/2*(2*b^2*e*g*x + (e*f
 + d*g)*b^2)*log(x^n)^2/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) + integrate((b^2*e^
2*g*x^2*log(c)^2 + b^2*e^2*f*x*log(c)^2 + (2*(e^2*g*n + e^2*g*log(c))*b^2*x
^2 + (e^2*f*n + 3*d*e*g*n + 2*e^2*f*log(c))*b^2*x + (d*e*f*n + d^2*g*n)*b^2
)*log(x^n))/(e^5*x^4 + 3*d*e^4*x^3 + 3*d^2*e^3*x^2 + d^3*e^2*x), x)
```

Giac [F]

$$\int \frac{(f + gx)(a + b\log(cx^n))^2}{(d + ex)^3} dx = \int \frac{(gx + f)(b\log(cx^n) + a)^2}{(ex + d)^3} dx$$

```
[In] integrate((g*x+f)*(a+b*log(c*x^n))^2/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((g*x + f)*(b*log(c*x^n) + a)^2/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f + gx)(a + b\log(cx^n))^2}{(d + ex)^3} dx = \int \frac{(f + g\,x)\,(a + b\,\ln(c\,x^n))^2}{(d + e\,x)^3} dx$$

```
[In] int(((f + g*x)*(a + b*log(c*x^n))^2)/(d + e*x)^3,x)
[Out] int(((f + g*x)*(a + b*log(c*x^n))^2)/(d + e*x)^3, x)
```

3.456 $\int \frac{(f+gx)(a+b\log(cx^n))^3}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 295

$$\int \frac{(f+gx)(a+b\log(cx^n))^3}{(d+ex)^3}\,dx = -\frac{3b(ef-dg)nx(a+b\log(cx^n))^2}{2d^2e(d+ex)} + \frac{f^2(a+b\log(cx^n))^3}{2d^2(ef-dg)} - \frac{(f+gx)^2(a+b\log(cx^n))^3}{2(ef-dg)(d+ex)^2} + \frac{3b^2(ef-dg)n^2(a+b\log(cx^n))\log\left(1+\frac{ex}{d}\right)}{d^2e^2} - \frac{3b(ef+dg)n(a+b\log(cx^n))^2\log\left(1+\frac{ex}{d}\right)}{2d^2e^2} + \frac{3b^3(ef-dg)n^3\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^2e^2} - \frac{3b^2(ef+dg)n^2(a+b\log(cx^n))\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^2e^2} + \frac{3b^3(ef+dg)n^3\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{d^2e^2}$$

```
[Out] -3/2*b*(-d*g+e*f)*n*x*(a+b*ln(c*x^n))^2/d^2/e/(e*x+d)+1/2*f^2*(a+b*ln(c*x^n
))^3/d^2/(-d*g+e*f)-1/2*(g*x+f)^2*(a+b*ln(c*x^n))^3/(-d*g+e*f)/(e*x+d)^2+3*
b^2*(-d*g+e*f)*n^2*(a+b*ln(c*x^n))*ln(1+e*x/d)/d^2/e^2-3/2*b*(d*g+e*f)*n*(a
+b*ln(c*x^n))^2*ln(1+e*x/d)/d^2/e^2+3*b^3*(-d*g+e*f)*n^3*polylog(2,-e*x/d)/
d^2/e^2-3*b^2*(d*g+e*f)*n^2*(a+b*ln(c*x^n))*polylog(2,-e*x/d)/d^2/e^2+3*b^3
*(d*g+e*f)*n^3*polylog(3,-e*x/d)/d^2/e^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2398, 2404, 2339, 30, 2355, 2354, 2438, 2421, 6724}

$$\int \frac{(f+gx)\left(a+b\log\left(cx^n\right)\right)^3}{(d+ex)^3}\,dx = -\frac{3b^2n^2(dg+ef)\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)\left(a+b\log\left(cx^n\right)\right)}{d^2e^2} + \frac{3b^2n^2(ef-dg)\log\left(\frac{ex}{d}+1\right)\left(a+b\log\left(cx^n\right)\right)}{d^2e^2} - \frac{3bn(dg+ef)\log\left(\frac{ex}{d}+1\right)\left(a+b\log\left(cx^n\right)\right)^2}{2d^2e^2} + \frac{f^2\left(a+b\log\left(cx^n\right)\right)^3}{2d^2(ef-dg)} - \frac{3bnx(ef-dg)\left(a+b\log\left(cx^n\right)\right)^2}{2d^2e(d+ex)} - \frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)^3}{2(d+ex)^2(ef-dg)} + \frac{3b^3n^3(ef-dg)\operatorname{PolyLog}\left(2,-\frac{ex}{d}\right)}{d^2e^2} + \frac{3b^3n^3(dg+ef)\operatorname{PolyLog}\left(3,-\frac{ex}{d}\right)}{d^2e^2}$$

```
[In] Int[((f + g*x)*(a + b*Log[c*x^n])^3)/(d + e*x)^3,x]
```

```
[Out] (-3*b*(e*f - d*g)*n*x*(a + b*Log[c*x^n])^2)/(2*d^2*e*(d + e*x)) + (f^2*(a +
 b*Log[c*x^n])^3)/(2*d^2*(e*f - d*g)) - ((f + g*x)^2*(a + b*Log[c*x^n])^3)/
(2*(e*f - d*g)*(d + e*x)^2) + (3*b^2*(e*f - d*g)*n^2*(a + b*Log[c*x^n])*Log
[1 + (e*x)/d])/(d^2*e^2) - (3*b*(e*f + d*g)*n*(a + b*Log[c*x^n])^2*Log[1 +
(e*x)/d])/(2*d^2*e^2) + (3*b^3*(e*f - d*g)*n^3*PolyLog[2, -((e*x)/d)])/(d^2
*e^2) - (3*b^2*(e*f + d*g)*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)])/(
d^2*e^2) + (3*b^3*(e*f + d*g)*n^3*PolyLog[3, -((e*x)/d)])/(d^2*e^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
```

```
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)^3}{2(ef-dg)(d+ex)^2}+\frac{(3bn)\int\frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)^2}{x(d+ex)^2}\,dx}{2(ef-dg)}\\
&= -\frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)^3}{2(ef-dg)(d+ex)^2}\\
&\quad+\frac{(3bn)\int\left(\frac{f^2\left(a+b\log\left(cx^n\right)\right)^2}{d^2x}-\frac{(-ef+dg)^2\left(a+b\log\left(cx^n\right)\right)^2}{de(d+ex)^2}+\frac{\left(-e^2f^2+d^2g^2\right)\left(a+b\log\left(cx^n\right)\right)^2}{d^2e(d+ex)}\right)dx}{2(ef-dg)}\\
&= -\frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)^3}{2(ef-dg)(d+ex)^2}+\frac{\left(3bf^2n\right)\int\frac{\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx}{2d^2(ef-dg)}\\
&\quad-\frac{(3b(ef-dg)n)\int\frac{\left(a+b\log\left(cx^n\right)\right)^2}{(d+ex)^2}\,dx}{2de}-\frac{(3b(ef+dg)n)\int\frac{\left(a+b\log\left(cx^n\right)\right)^2}{d+ex}\,dx}{2d^2e}\\
&= -\frac{3b(ef-dg)nx\left(a+b\log\left(cx^n\right)\right)^2}{2d^2e(d+ex)}-\frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)^3}{2(ef-dg)(d+ex)^2}\\
&\quad-\frac{3b(ef+dg)n\left(a+b\log\left(cx^n\right)\right)^2\log\left(1+\frac{ex}{d}\right)}{2d^2e^2}\\
&\quad+\frac{\left(3f^2\right)\text{Subst}\left(\int x^2\,dx,x,a+b\log\left(cx^n\right)\right)}{2d^2(ef-dg)}+\frac{\left(3b^2(ef-dg)n^2\right)\int\frac{a+b\log\left(cx^n\right)}{d+ex}\,dx}{d^2e}\\
&\quad+\frac{\left(3b^2(ef+dg)n^2\right)\int\frac{\left(a+b\log\left(cx^n\right)\right)\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{d^2e^2}\\
&= -\frac{3b(ef-dg)nx\left(a+b\log\left(cx^n\right)\right)^2}{2d^2e(d+ex)}+\frac{f^2\left(a+b\log\left(cx^n\right)\right)^3}{2d^2(ef-dg)}\\
&\quad-\frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)^3}{2(ef-dg)(d+ex)^2}+\frac{3b^2(ef-dg)n^2\left(a+b\log\left(cx^n\right)\right)\log\left(1+\frac{ex}{d}\right)}{d^2e^2}\\
&\quad-\frac{3b(ef+dg)n\left(a+b\log\left(cx^n\right)\right)^2\log\left(1+\frac{ex}{d}\right)}{2d^2e^2}\\
&\quad-\frac{3b^2(ef+dg)n^2\left(a+b\log\left(cx^n\right)\right)\text{Li}_2\left(-\frac{ex}{d}\right)}{d^2e^2}\\
&\quad-\frac{\left(3b^3(ef-dg)n^3\right)\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx}{d^2e^2}+\frac{\left(3b^3(ef+dg)n^3\right)\int\frac{\text{Li}_2\left(-\frac{ex}{d}\right)}{x}\,dx}{d^2e^2}\\
&= -\frac{3b(ef-dg)nx\left(a+b\log\left(cx^n\right)\right)^2}{2d^2e(d+ex)}+\frac{f^2\left(a+b\log\left(cx^n\right)\right)^3}{2d^2(ef-dg)}\\
&\quad-\frac{(f+gx)^2\left(a+b\log\left(cx^n\right)\right)^3}{2(ef-dg)(d+ex)^2}+\frac{3b^2(ef-dg)n^2\left(a+b\log\left(cx^n\right)\right)\log\left(1+\frac{ex}{d}\right)}{d^2e^2}\\
&\quad-\frac{3b(ef+dg)n\left(a+b\log\left(cx^n\right)\right)^2\log\left(1+\frac{ex}{d}\right)}{2d^2e^2}+\frac{3b^3(ef-dg)n^3\text{Li}_2\left(-\frac{ex}{d}\right)}{d^2e^2}\\
&\quad-\frac{3b^2(ef+dg)n^2\left(a+b\log\left(cx^n\right)\right)\text{Li}_2\left(-\frac{ex}{d}\right)}{d^2e^2}+\frac{3b^3(ef+dg)n^3\text{Li}_3\left(-\frac{ex}{d}\right)}{d^2e^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx)\left(a+b\log\left(cx^n\right)\right)^3}{(d+ex)^3}\,dx$$
$$=\frac{-\frac{(ef-dg)(a+b\log(cx^n))^3}{(d+ex)^2}-\frac{2g(a+b\log(cx^n))^3}{d+ex}+\frac{2g\left((a+b\log(cx^n))^2\left(a+b\log(cx^n)-3bn\log\left(1+\frac{ex}{d}\right)\right)-6b^2n^2(a+b\log(cx^n))\,\mathrm{PolyLog}\left(2,-\frac{ex}{d}\right.}{d}}{}$$

```
[In] Integrate[((f + g*x)*(a + b*Log[c*x^n])^3)/(d + e*x)^3,x]
```

```
[Out] (-(((e*f - d*g)*(a + b*Log[c*x^n])^3)/(d + e*x)^2) - (2*g*(a + b*Log[c*x^n]
)^3)/(d + e*x) + (2*g*((a + b*Log[c*x^n])^2*(a + b*Log[c*x^n] - 3*b*n*Log[1
 + (e*x)/d]) - 6*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)] + 6*b^3*
n^3*PolyLog[3, -((e*x)/d)]))/d + ((e*f - d*g)*(3*b*d*n*(a + b*Log[c*x^n])^2
 + (d + e*x)*(a + b*Log[c*x^n])^3 - 3*b*n*(d + e*x)*(a + b*Log[c*x^n])^2*Lo
g[1 + (e*x)/d] - 3*b*n*(d + e*x)*((a + b*Log[c*x^n])*(a + b*Log[c*x^n] - 2*
b*n*Log[1 + (e*x)/d]) - 2*b^2*n^2*PolyLog[2, -((e*x)/d)]) - 6*b^2*n^2*(d +
e*x)*((a + b*Log[c*x^n])*PolyLog[2, -((e*x)/d)] - b*n*PolyLog[3, -((e*x)/d)
])))/(d^2*(d + e*x)))/(2*e^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.20 (sec) , antiderivative size = 1652, normalized size of antiderivative = 5.60

method	result	size
risch	Expression too large to display	1652

```
[In] int((g*x+f)*(a+b*ln(c*x^n))^3/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b^3*ln(x^n)^3*g/e^2/(e*x+d)-1/2*b^3*ln(x^n)^3/e/(e*x+d)^2*f+3*b^3*n^2/e/d^
2*ln(x^n)*ln(e*x+d)*f-3*b^3*n^2/e/d^2*ln(x^n)*ln(x)*f+3*b^3*n^3/e^2/d*ln(e*
x+d)*ln(-e*x/d)*g-3*b^3*n^3/e^2/d*ln(x)*dilog(-e*x/d)*g+3*b^3*n^2/e/d^2*dil
og(-e*x/d)*ln(x^n)*f+3/2*b^3*n^3/e^2/d*ln(x)^2*ln(e*x+d)*g-3/2*b^3*n^2/e^2/
d*ln(x)^2*ln(x^n)*g-3*b^3*n^2/e^2/d*ln(x^n)*ln(e*x+d)*g+3*b^3*n^2/e^2/d*ln(
x^n)*ln(x)*g+1/2*b^3*n^3/e^2/d*ln(x)^3*g-3/2*b^3*n^3/e^2/d*ln(x)^2*g+3*b^3*
n^3/e^2/d*dilog(-e*x/d)*g+3*b^3*n^3/e^2/d*polylog(3,-e*x/d)*g+3*b^3*n^2/e^2
/d*dilog(-e*x/d)*ln(x^n)*g-3*b^3*n^3/e/d^2*ln(e*x+d)*ln(-e*x/d)*f-3/2*b^3*n
^3/e^2/d*ln(x)^2*ln(1+e*x/d)*g-3*b^3*n^3/e^2/d*ln(x)*polylog(2,-e*x/d)*g-3*
b^3*n^3/e/d^2*ln(x)*dilog(-e*x/d)*f+3/2*b^3*n^3/e/d^2*ln(x)^2*ln(e*x+d)*f-3
/2*b^3*n^3/e/d^2*ln(x)^2*ln(1+e*x/d)*f-3*b^3*n^3/e/d^2*ln(x)*polylog(2,-e*x
/d)*f-3/2*b^3*n^2/e/d^2*ln(x)^2*ln(x^n)*f+3/2*b^3*n*ln(x^n)^2/e/d/(e*x+d)*f
-3/2*b^3*n*ln(x^n)^2/e^2/d*ln(e*x+d)*g-3/2*b^3*n*ln(x^n)^2/e/d^2*ln(e*x+d)*
```

```
f+3/2*b^3*n*ln(x^n)^2/e^2/d*ln(x)*g+3/2*b^3*n*ln(x^n)^2/e/d^2*ln(x)*f+1/8*(
-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^
2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^
3*(-g/e^2/(e*x+d)-1/2*(-d*g+e*f)/e^2/(e*x+d)^2)+3/4*(-I*b*Pi*csgn(I*c)*csgn
(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*c
sgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)^2*b*(-ln(x^n)/e^2/(e*x
+d)*g+1/2*ln(x^n)/e^2/(e*x+d)^2*d*g-1/2*ln(x^n)/e/(e*x+d)^2*f-1/2*n/e^2*((d
*g+e*f)/d^2*ln(e*x+d)+(d*g-e*f)/d/(e*x+d)+1/d^2*(-d*g-e*f)*ln(x)))+3/2*(-I*
b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I
*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*a)*b^2
*(-ln(x^n)^2*g/e^2/(e*x+d)+1/2*ln(x^n)^2/e^2/(e*x+d)^2*d*g-1/2*ln(x^n)^2/e/
(e*x+d)^2*f-n*(ln(x^n)/e^2/(e*x+d)*g-ln(x^n)/e/d/(e*x+d)*f+ln(x^n)/e^2/d*ln
(e*x+d)*g+ln(x^n)/e/d^2*ln(e*x+d)*f-ln(x^n)/e^2/d*ln(x)*g-ln(x^n)/e/d^2*ln(
x)*f+n/e^2*(1/2*(d*g+e*f)/d^2*ln(x)^2-(d*g-e*f)/d*(-1/d*ln(e*x+d)+1/d*ln(x)
)-(d*g+e*f)/d^2*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d)))))+3*b^3*n^3/e/d^2*pol
ylog(3,-e*x/d)*f+1/2*b^3*ln(x^n)^3/e^2/(e*x+d)^2*d*g-3/2*b^3*n*ln(x^n)^2*g/
e^2/(e*x+d)+1/2*b^3*n^3/e/d^2*ln(x)^3*f+3*b^3*n^2/e^2/d*ln(e*x+d)*ln(-e*x/d
)*ln(x^n)*g-3*b^3*n^3/e/d^2*ln(x)*ln(e*x+d)*ln(-e*x/d)*f-3*b^3*n^3/e^2/d*ln
(x)*ln(e*x+d)*ln(-e*x/d)*g+3*b^3*n^2/e/d^2*ln(e*x+d)*ln(-e*x/d)*ln(x^n)*f+3
/2*b^3*n^3/e/d^2*ln(x)^2*f-3*b^3*n^3/e/d^2*dilog(-e*x/d)*f
```

Fricas [F]

$$\int \frac{(f+gx)\left(a+b\log\left(cx^n\right)\right)^3}{(d+ex)^3}\,dx=\int \frac{(gx+f)(b\log\left(cx^n\right)+a)^3}{(ex+d)^3}\,dx$$

```
[In] integrate((g*x+f)*(a+b*log(c*x^n))^3/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*g*x + a^3*f + (b^3*g*x + b^3*f)*log(c*x^n)^3 + 3*(a*b^2*g*x +
 a*b^2*f)*log(c*x^n)^2 + 3*(a^2*b*g*x + a^2*b*f)*log(c*x^n))/(e^3*x^3 + 3*d
*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F]

$$\int \frac{(f+gx)\left(a+b\log\left(cx^n\right)\right)^3}{(d+ex)^3}\,dx=\int \frac{\left(a+b\log\left(cx^n\right)\right)^3(f+gx)}{(d+ex)^3}\,dx$$

```
[In] integrate((g*x+f)*(a+b*ln(c*x**n))**3/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*log(c*x**n))**3*(f + g*x)/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{(f+gx)(a+b\log(cx^n))^3}{(d+ex)^3}\,dx = \int \frac{(gx+f)(b\log(cx^n)+a)^3}{(ex+d)^3}\,dx$$

```
[In] integrate((g*x+f)*(a+b*log(c*x^n))^3/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 3/2*a^2*b*f*n*(1/(d*e^2*x + d^2*e) - log(e*x + d)/(d^2*e) + log(x)/(d^2*e))
 - 3/2*a^2*b*g*n*(1/(e^3*x + d*e^2) + log(e*x + d)/(d*e^2) - log(x)/(d*e^2)
) - 3/2*(2*e*x + d)*a^2*b*g*log(c*x^n)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) - 1/
2*(2*e*x + d)*a^3*g/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) - 3/2*a^2*b*f*log(c*x^n
)/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 1/2*a^3*f/(e^3*x^2 + 2*d*e^2*x + d^2*e) -
 1/2*(2*b^3*e*g*x + (e*f + d*g)*b^3)*log(x^n)^3/(e^4*x^2 + 2*d*e^3*x + d^2*
e^2) + integrate(1/2*(2*(b^3*e^2*g*log(c)^3 + 3*a*b^2*e^2*g*log(c)^2)*x^2 +
 3*((d*e*f*n + d^2*g*n)*b^3 + 2*(a*b^2*e^2*g + (e^2*g*n + e^2*g*log(c))*b^3
)*x^2 + (2*a*b^2*e^2*f + (e^2*f*n + 3*d*e*g*n + 2*e^2*f*log(c))*b^3)*x)*log
(x^n)^2 + 2*(b^3*e^2*f*log(c)^3 + 3*a*b^2*e^2*f*log(c)^2)*x + 6*((b^3*e^2*g
*log(c)^2 + 2*a*b^2*e^2*g*log(c))*x^2 + (b^3*e^2*f*log(c)^2 + 2*a*b^2*e^2*f
*log(c))*x)*log(x^n))/(e^5*x^4 + 3*d*e^4*x^3 + 3*d^2*e^3*x^2 + d^3*e^2*x),
x)
```

Giac [F]

$$\int \frac{(f+gx)(a+b\log(cx^n))^3}{(d+ex)^3}\,dx = \int \frac{(gx+f)(b\log(cx^n)+a)^3}{(ex+d)^3}\,dx$$

```
[In] integrate((g*x+f)*(a+b*log(c*x^n))^3/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)*(b*log(c*x^n) + a)^3/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)(a+b\log(cx^n))^3}{(d+ex)^3}\,dx = \int \frac{(f+g\,x)\,\left(a+b\ln\left(c\,x^n\right)\right)^3}{(d+e\,x)^3}\,dx$$

```
[In] int(((f + g*x)*(a + b*log(c*x^n))^3)/(d + e*x)^3,x)
```

```
[Out] int(((f + g*x)*(a + b*log(c*x^n))^3)/(d + e*x)^3, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                      Small rewrite of logic in main function to make it*)
(*                      match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                  end if
              else #result contains complex but optimal is not
                  if debug then
                          print("result contains complex but optimal is not");
                  fi;
                  return "C","Result contains complex when optimal does not.";
              fi;
          else # result do not contain complex
               # this assumes optimal do not as well. No check is needed here.
              if debug then
                     print("result do not contain complex, this assumes optimal do not as well")
              fi;
              if leaf_count_result<=2*leaf_count_optimal then
                  if debug then
                      print("leaf_count_result<=2*leaf_count_optimal");
                  fi;
                  return "A"," ";
              else
                  if debug then
                      print("leaf_count_result>2*leaf_count_optimal");
                  fi;
                  return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
              fi;
          fi;
      else #ExpnType(result) > ExpnType(optimal)
          if debug then
              print("ExpnType(result) > ExpnType(optimal)");
          fi;
          return "C",cat("Result contains higher order function than in optimal. Order ",
                         convert(ExpnType_result,string)," vs. order ",
                         convert(ExpnType_optimal,string),".");
      fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```